

A fuzzy multi-objective programming for optimization of fire station locations through genetic algorithms

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Abstract: The location of fire stations at a fire & rescue service is an important factor in its fire protection capability. This paper aims to determine the optimal location of fire station facilities. The method proposed here is the combination of a fuzzy multi-objective programming and a genetic algorithm. The reason of using this method is that the decision of where to locate fire stations depends upon a set of criteria, such as travel times or travel distances, setting up costs and operating costs, and the ordinary optimization methods may not be able to handle them effectively due to the relatively large scale of real problems. The original fuzzy multiple objectives are appropriately converted to a single unified ‘min-max’ goal, which makes it easy to apply a genetic algorithm for the resulting problem solving. Compared with the existing methods of fire station location our approach has three distinguish features: (1) considering the fuzzy nature of a decision maker (DM) in the location optimization model; (2) fully considering the demands to the facilities from the areas with various fire risk categories; (3) being more understandable and practical to DM. The case study was based on the data collected from the Derbyshire fire & rescue service and used to illustrate the application of the method for fire station locations.

Keywords: Multi-objective programming; genetic algorithm; fuzzy programming; location; fire stations.

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1 Introduction

Determination of where to locate fire stations and how many fire stations to have in a given area is perhaps the most important decision faced by any Chief Fire Officer. It is appreciated that the optimum solution is the one which minimizes the sum of losses from fire and the cost of providing the service (Hogg, 1968). In detail, appropriate fire station locations can bring the following benefits (Tzeng and Chen, 1999): (a) it can shorten the distance between fire stations and accident sites so as to improve reaction time efficiency; (b) fire stations can be loaded to minimize overlap of fire station services, so as to utilize efficiently fire station resources; and (c) it can help determine the reasonable number of fire stations at a given area by considering an economical trade-off between the accident-loss cost and the total setup and operating costs of fire stations.

Fire station facility location problems have multiple objectives and are complex and NP-hard. Multiple objectives often conflict with each other and require multi-objective approaches rather than a single objective approach. Quite often multiple objectives are converted into a single unified goal as the weighted sum of the objectives or the total deviations from the goals or into a series of single objective optimization problem by picking one of the objectives to minimize while each of the others is turned into a constraint. Diwekar (2003) introduced the general ways of converting multi-objective optimisation problems into a single objective optimisation problem. Chen (2001) presented an application of this conversion in a multi-objective optimisation problem. Ordinary optimisation methods may not be able to handle these NP-hard problems effectively. As a consequence, several meta-heuristic methods such as Genetic Algorithms, Tabu search and Simulated Annealing have been introduced to solve these problems. Hales and Moberg (2003) summarized these meta-heuristic optimisation methods in their review of location science research.

The main objectives of this paper are to fully consider the various fire risk categories of the given area in the location optimisation and to establish a fire station location model which is understandable and practical to fire service authorities. The original fuzzy multiple objectives need be appropriately converted to a single unified 'min-max' goal. This makes it easy to apply a genetic algorithm for the resulting problem solving. Our approach is similar in spirit to the works of Sakawa et al. (1997) and Tzeng and Chen (1999), but is revised to address the various fire risk categories of a given area and to convert into a genetic algorithm based search problem.

The main advantage of our approach over the existing approaches including the works of Sakawa et al. (1997) and Tzeng and Chen (1999) is that different risk categories and obstacles within a given region have been considered in both the multiple objectives and the constraints. The way of ensuring a reasonable distance between any two adjacent fire stations through the characteristic function is more simple and efficient. In summary, our approach proposed in this paper has three distinguish features: a) distinguishing the areas with different risk categories in the optimal location problem is more reasonable and understandable; b) introducing the fuzzy nature of the recommendations of the Home Office in the UK on the speed of fire engine attack to accidents in the optimal location model has

greatly improved the precision of the optimal model and possesses a potential to reduce the amount of facilities; c) choosing a suitable chromosome format and embedding constraints into the fitness function of a genetic algorithm has dramatically reduced the complexity of the optimal fire station location problem.

This paper is organized as follows: Section 2 reviews location science research, particularly in optimal fire station location. The proposed multi-objective fuzzy model is presented in Section 3. GA based resolution is described in Section 4. Application to the Derbyshire fire and rescue service is given in Section 5. Finally, some discussions and future work are presented in Section 6.

2 Location science research

2.1 Location science research

Location science research investigates where to physically locate a set of facilities (resources) so as to minimize the cost of satisfying a set of demands subject to certain constraints (Hale and Moberg, 2003). It includes locating fire stations to minimize the maximum response time to fire accidents. A variety of different exact and heuristic solution approaches have been developed to solve location problems (Brotcorne et al., 2003; Jayaraman et al., 2003; Salhi and Gamal, 2003; Cheung et al, 2001). The exact solution approaches have been used to obtain the global optimal solution in location problems and the heuristic solution approaches have been taken to yield reasonable solutions with moderately computational time. However, these two approaches bring two further issues for carrying out location problem solving, complex optimization nature and problem sizes (Kim, 2000). These two issues have led to the development of different heuristic solution techniques and algorithms, including the evolutionary computing technologies. Kim (2000) gave a comprehensive literature review in these areas. Hale and Moberg (2003) gave a review on location science research, which provides readers with a more general review of the location science research landscape rather than an exhaustive list of location science topics.

There are very rich literatures on the use of GA in location science research. The research of Hosage and Goodchild (1986) appeared as a pioneering attempt that applied GA to location problems. They developed a binary genetic algorithm for solving a location problem. In practice, it is not always appropriate to convert a solution to a binary representation. An integer representation (Bianchi and Church, 1992) and a real number representation (Houck et al., 1996; Brimberg et al., 2000; and Salhi and Gamal, 2003) have been developed to represent the location of facilities.

2.2 Optimal fire stations location

The determination of optimal base locations for fixed emergency facilities has a long history in location science research. It is to determine the “best” base location for fire fighting engines so that some service level objective is optimized. It is assumed that each fire engine waits at its base until called into

service. After completing service, the fire engine returns to the base to await another call. The main task of fire stations is to operate in case of fire and in case of some other emergencies. The speed of arrival of the assistance is the dominant criterion.

In recent years, multiple criteria modelling approaches have been used in fire station location problems. Tzeng and Chen (1999) consider three objectives in the optimal location of airport fire stations:

- Minimizing the total setup cost of fire stations and total loss cost of an accident;
- Minimizing the longest distance from the fire stations to any point at the airport;
- Minimizing the longest distance from any fire station to the high-risk area.

Badri et al. (1998) considers 11 objectives in a general multi-objective model for locating fire stations. These multiple objectives incorporate both travel times and travel distances from stations to accident sites, and consider other cost-related objectives and technical or political criteria such as water availability and service overlaps of fire stations. These 11 objectives are as follows:

- Minimize fixed cost;
- Minimize total annual operating cost;
- Maximize service of those areas that require in most based on number of forecasted accidents;
- Minimize average distance travelled from station to accident sites;
- Minimize maximum distance travelled from station to accident sites;
- Minimize average time travelled from station to accident sites;
- Minimize maximum time travelled from station to accident sites;
- Attain targeted number of fire stations required;
- Minimize service overlaps of fire stations;
- Attain favoured area status;
- Minimize locating where water availability could be a problem.

The methods of solving the multi-objective location problems vary from traditional optimization programming such as branch and bound, MINLP (mixed integer nonlinear programming) (Badri et al, 1998; Diwekar, 2003) to evolutionary computation techniques such as genetic algorithms (GA) (Tzeng and Chen, 1999; Cheung et al., 2001; Salhi and Gamal, 2003). The hybrid evolutionary methods are also employed in optimal location problems (Gong et al. 1997; Sakawa et al., 1997) that are described in a hierarchy structure; problems at lower levels are solved by traditional optimization techniques; while problems at higher levels become more complex and solved by an evolutionary technique.

3 Fuzzy Multi-Objective Optimization Model

Fuzzy multi-criteria models have been used in several studies in location optimization problems (Bhattacharya et al., 1992; Sakawa et al., 1997; Tzeng and Chen, 1999). People usually think that the main reason of using a fuzzy multi-objective approach in fire station location optimization problems is its simplicity when compared with traditional weighting methods (Tzeng and Chen, 1999). More than

the simplicity, if investigating the recommendations of the Home Office in the UK on the speed of fire engine attack to accidents which are summarized in Table 1 (Home Office, 1985), the fuzzy nature is exactly there. For example, the risk category B areas require the time limits for the fire engine attendance are 5 to 8 minutes, which mean that 5 minutes of the travel time is good enough and 8 minutes is still acceptable. This requirement is fuzzy in nature. Table 1 also shows that different risk categories have different travel time requirements. Taking the average travel speed as 60 mile per hour the travel time requirements are converted to the distance limits shown in Table 1 as well. Tzeng and Chen (1999) used the risk rank for a location to multiply the distance between the fire station and the location. The risk rank is computed based on accident statistics for different areas. Our approach will follow the recommendation made by the Home Office in the UK rather than based on the accident statistics which might not be available and/or reliable. Our approach assumes that once positioned, fire engines would almost always be available when a call arrived. Therefore the number of pumps in Table 1 is not used in this approach. The numbers of fire engines and fire fighters in any fire stations are usually allocated and regularly renewed in terms of the latest accident statistic reports in the area and the financial budget of the fire stations. In a major accident such as terrorist attacks the assumption made here may be not true. In these situations local and national collaborations between nearby fire stations and even fire & rescue services become extremely important. The requirements of the availability of fire engines in individual fire stations will be met through the local and/or national cooperation of fire stations. The UK government funded project ‘Firelink’ (Office of the Deputy Prime Minister, 2003) intends to achieve an efficient and reliable collaboration between fire & rescue services through advanced information sharing, mobile communication technologies and central control in order to respond any major accidents.

Table 1: Recommendation of the Home Office in the UK on the speed of attack to fire accidents

Risk Category	Number of Pumps	Time Limits for Attendance (in Minutes)	Distance Limits to accident sites (in Miles)
A	3	4-5	4-5
B	2	5-8	5-8
C	1	8-10	8-10
D	1	10-20	10-20

3.1 Multi-objective binary programming

The decision variable, denoted as S_{ij} , takes a binary value in fire station location optimization problems. If a fire station is set up on a x-y coordinate (i, j), then $S_{ij}=1$, otherwise, $S_{ij}=0$. The following is the generalized representation of a multi-objective binary programming problem (Diwekar, 2003):

$$\begin{aligned}
 & \text{Min} \quad (f_1(S_{ij}), f_2(S_{ij}), \dots, f_M(S_{ij}))^T & (1) \\
 & \text{subject to} \quad g_l(S_{ij}) \leq 0 \quad l=1, 2, \dots, P \\
 & \quad \quad \quad S_{ij} \in \{0, 1\},
 \end{aligned}$$

where $f_k(S_{ij})$, $k=1, 2, 3, \dots, M$, are M conflicting objective functions; $g_l(S_{ij})$, $l=1, 2, 3, \dots, P$, are P constraints; i and j are a set of integer coordinates within a given area. The optimal variable in equation (1) and the rest of the paper is the whole set of S_{ij} , i.e. a matrix $\mathbf{S} = (S_{ij})_{i_{\max} \times j_{\max}}$, rather than any single decision variable S_{ij} , unless specified.

In a minimization problem such as minimizing the travel time from a fire station to an accident site, the objective stated by a Decision Maker (DM) such as the Home Office in the UK is to be best less than an optimistic value f_k^+ , and be surely less than a pessimistic value f_k^- . This requirement possesses the fuzzy nature and can be treated by introducing the following linear membership function (Sakawa et al., 1997):

$$\mu_k(f_k(S_{ij})) = \begin{cases} 1 & f_k(S_{ij}) \leq f_k^+ \\ 1 - \frac{f_k(S_{ij}) - f_k^+}{f_k^- - f_k^+} & f_k^+ \leq f_k(S_{ij}) \leq f_k^- \\ 0 & f_k(S_{ij}) \geq f_k^- \end{cases} \quad (2)$$

It is assumed that the k^{th} membership function should be 1 if the objective achieves the optimistic value, 0 if the pessimistic value is not achieved, and linear from 0 to 1. Such a linear membership function is illustrated in Figure 1.

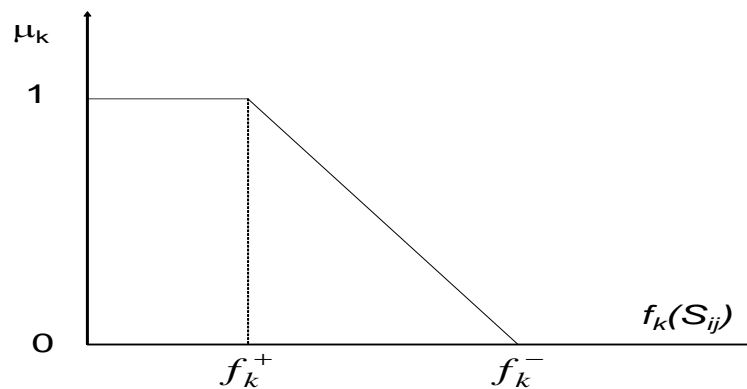


Figure 1: The achievement level for fuzzy objectives ($k=1, 2, 3, \dots, M$)

The basic concept of fuzzy multi-objective optimization is to find the maximal achievement level among constraints of conflicting objectives. The multi-objective binary programming problems shown in equation (1) can be transformed as:

$$\begin{aligned} \text{Max} \quad & \min_{k=1, \dots, M} \{ \mu_k(f_k(S_{ij})) \} \\ \text{subject to} \quad & g_l(S_{ij}) \leq 0 \quad l=1, 2, \dots, P \\ & S_{ij} \in \{0, 1\}, \end{aligned} \quad (3)$$

3.2 Fuzzy multi-objective model for fire station location optimization

In the fire station location problem multiple as well as conflicting goals are present. Two common goals used in many emergency-location related studies are (a) to minimize the fixed cost and the total loss cost of accidents, (b) to minimize the distance from the fire station to any accident site. In our approach we adopt the model built by Tzeng and Chen (1999) for the first objective to obtain the optimal number of fire stations, and use the above fuzzy multi-objective model for the second objective to address the recommendation made by the Home Office on the time limits for attendance at accident sites.

3.2.1 Minimizing the total setup and operating cost of fire stations and total loss cost of accidents in a given area

The setup and operating cost for each fire station is denoted as SC, the total loss cost of accidents in a given area as TLC. SC and TLC are estimated by fire brigade authorities. The estimate of TLC is based on the total number of accidents and statistic property loss in these accidents. Obviously an insufficient number of fire stations will lead to inefficient reaction times and cause more loss cost. If no fire station was setup in a given area, the total loss cost will be equal to TLC; on the contrary, the total loss cost can be reduced when more fire stations are setup. On the other hand, too many fire stations will result in the increase of the setup and operating cost. Therefore, there must be an optimal number of fire stations after the trade-off between the total setup and operating cost of fire stations and the total loss cost. We modify the model established by Tzeng and Chen (1999) by introducing an adjustable parameter α in the TLC term for optimizing the number of general fire stations.

$$\text{Min} \{ f_1(S_{ij}) = \sum_i \sum_j S_{ij} \times SC + \alpha \times TLC \times e^{-\sum_i \sum_j S_{ij}} \} \quad (4)$$

where S_{ij} is the decision variable, α is the adjustable parameter. The adjustable parameter α is used to tune the value of the effect of the total number of fire stations on the actual loss cost of accidents in order to match the actual situations. The value of the adjustable parameter α is obtained from the linear regression in terms of the history record.

If the total number of the fire stations $\sum_i \sum_j S_{ij}$ is denoted as N, equation (4) can be re-written as the

following ‘continuous’ function with the only optimal variable N:

$$\begin{aligned} \text{Min} \{ f_1(N) = N \times SC + \alpha \times TLC \times e^{-N} \} \\ N = \sum_i \sum_j S_{ij} \end{aligned} \quad (5)$$

The optimal value N in equation (5) can be easily obtained through setting the derivative of $f_1(N)$ being zero and rounding the result into an integer value once SC and TLC have been estimated by fire brigade authorities and α has been calculated from the history record.

3.2.2 Minimizing the longest distance from a fire station to any accident site

The recommendation made by the Home Office in the UK in Table 1 shows the time limits for fire engine attendance at accident sites. The time taken to get to an accident site is necessarily dependent upon the distance to be traveled and the road conditions experienced during the journey. Many researchers used maximum distance to reflect the worst scenario case associated with bad conditions experienced during the journey. Some researchers consider the elements of time and distance simultaneously. The new feature here is that in order to fully consider the demands to fire facilities from the areas with various fire risk categories four individual objectives are built since these areas have different attendance time limits.

The objective for the areas with the fire risk category ‘A’ is shown in equation (6), the ones for the areas with the fire risk category ‘B’, ‘C’, and ‘D’ are shown in equations (7), (8), and (9) respectively.

$$\text{Min } \{f_2(S_{ij}) = \underset{\{(x,y)|r_{xy}=A, \forall x,y\}}{\text{Max}} \left\{ \underset{\{(i,j)|S_{ij}=1, \forall i,j\}}{\text{Min}} ((x-i)^2 + (y-j)^2)^{\frac{1}{2}} \right\} \} \quad (6)$$

$$\text{Min } \{f_3(S_{ij}) = \underset{\{(x,y)|r_{xy}=B, \forall x,y\}}{\text{Max}} \left\{ \underset{\{(i,j)|S_{ij}=1, \forall i,j\}}{\text{Min}} ((x-i)^2 + (y-j)^2)^{\frac{1}{2}} \right\} \} \quad (7)$$

$$\text{Min } \{f_4(S_{ij}) = \underset{\{(x,y)|r_{xy}=C, \forall x,y\}}{\text{Max}} \left\{ \underset{\{(i,j)|S_{ij}=1, \forall i,j\}}{\text{Min}} ((x-i)^2 + (y-j)^2)^{\frac{1}{2}} \right\} \} \quad (8)$$

$$\text{Min } \{f_5(S_{ij}) = \underset{\{(x,y)|r_{xy}=D, \forall x,y\}}{\text{Max}} \left\{ \underset{\{(i,j)|S_{ij}=1, \forall i,j\}}{\text{Min}} ((x-i)^2 + (y-j)^2)^{\frac{1}{2}} \right\} \} \quad (9)$$

where S_{ij} is the decision variable; x, y are the coordinate of a fire site location; i, j are the coordinate of a location where a fire station might be located; r_{xy} is the fire risk category for the location with the coordinate (x, y) , it may take a value from ‘A’, ‘B’, ‘C’, and ‘D’.

Equation (6) is an expression that calculates the distance from a location with the coordinate (x, y) to the closest fire station with the coordinate (i, j) , i.e. $\underset{\{(i,j)|S_{ij}=1, \forall i,j\}}{\text{Min}} ((x-i)^2 + (y-j)^2)^{\frac{1}{2}}$, and then calculates the maximal distance for all the locations (x, y) with the fire risk category ‘A’, i.e. $\underset{\{(x,y)|r_{xy}=A, \forall x,y\}}{\text{Max}}$. The optimal objective is to minimize the maximal distance, i.e. $\text{Min}\{f_2(S_{ij})\}$.

Equations (7), (8) and (9) have a similar formula but deal with the areas with the fire risk category ‘B’, ‘C’, and ‘D’. Euclidean distance is applied in equations (6) to (9). The actual value of the distance is the result of the value $((x-i)^2 + (y-j)^2)^{\frac{1}{2}}$ multiplied by the size of each cell in the coordinate system. Other kinds of distance such as Manhattan distance $|x-i| + |y-j|$ can be equivalently applied for this purpose.

3.2.3 Constraints

The first constraint for the optimal objectives shown in equations (6) to (9) applies to the total number of fire stations being N obtained from the first optimal objective shown in equation (5).

$$\sum_i \sum_j S_{ij} = N \quad (10)$$

The second constraint considers obstacles in a given area. A fire station should not be built up within any obstacles such as waterways and reserved areas, which are described in equation (11).

$$S_{ij} = 0 \quad \forall (i, j) \in \Omega \quad (11)$$

where the symbol Ω represents a set of obstacle coordinates. The principle of satisfying the constraint shown in equation (11) is to check the feasibility of any possible fire station locations. If a possible location is infeasible, i.e. the possible location is within the set of obstacles, we repair it in two ways: (a) randomly generate another possible location and check its feasibility, repeat this process until a feasible location is found; (b) move the infeasible possible location of the fire station to its nearest feasible locations. The way (b) is illustrated in Figure 2, in which the only obstacle is at the location with the coordinate $(i+3, j+1)$ if choosing the bottom left corner of the cell to represent the location. The possible nearest feasible locations are obtained by increasing or decreasing the horizontal or vertical coordinates by 1, which are the cell with the coordinates $(i+3, j)$, $(i+3, j+2)$, $(i+2, j+1)$, and $(i+4, j+1)$.

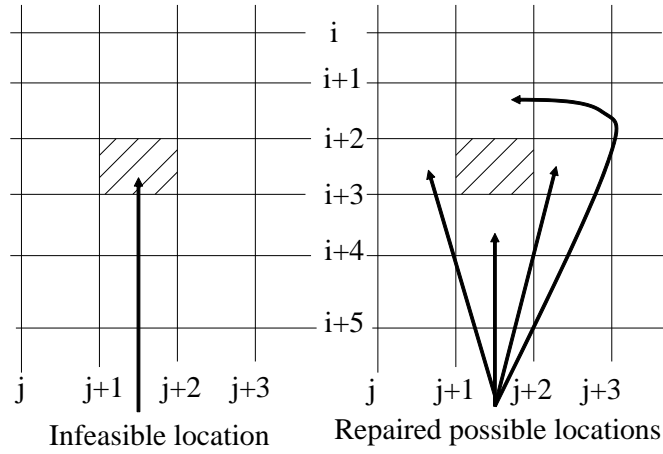


Figure 2: Avoiding obstacles

If the distance between two fire stations a and b is shorter than the distance between fire station a and any other fire station we call fire station b is adjacent to fire station a . The definition of fire station b being adjacent to fire station a is described in equation (12).

$$\begin{aligned} ((i_a - i_b)^2 + (j_a - j_b)^2)^{1/2} < ((i_a - i)^2 + (j_a - j)^2)^{1/2} \\ \forall (i, j), \quad (i, j) \neq (i_a, j_a), \quad (i, j) \neq (i_b, j_b), \quad (i_a, j_a) \neq (i_b, j_b) \end{aligned} \quad (12)$$

The third constraint is that there should be a reasonable distance, denoted as d_{ab}^r , between any two adjacent fire stations a and b . This distance should not be too long, for example not be greater than a longest distance, d_{ab}^l , for fire stations to support each other, and should not be so short, for example

not be less than a shortest distance, d_{ab}^s , as to cause overlapping of fire station services. Tzeng and Chen (1999) used the expressions d_{ab}^r , d_{ab}^l , d_{ab}^s to construct fuzzy constraints. For the sake of the simplicity, we use two inequalities to express the constraint as shown in equation (13).

$$d_{ab}^s \leq ((i_a - i_b)^2 + (j_a - j_b)^2)^{1/2} \leq d_{ab}^l \quad (13)$$

where (i_a, j_a) and (i_b, j_b) are the coordinates of any two adjacently located fire stations a and b respectively.

3.2.4 Fuzzy multi-objective model in a single unified goal

The optimistic and pessimistic values of the objectives f_2 , to f_5 are listed in Table 2, which are directly derived from the Home Office recommendations shown in Table 1. By applying the fuzzy membership function shown in equation (2) into the objectives f_2 to f_5 shown in equations (6) to (9), the multi-objective binary programming problem shown in equation (3) can be presented as equation (14), where d_{ab}^l and d_{ab}^s are the longest and the shortest distances between two adjacent fire stations; N is the optimal total number of fire stations; Ω represents the set of obstacle coordinates. Equation (14) includes the three constraints described in section 3.2.3.

$$\begin{aligned} \text{Max} \quad & \min\{\mu_2(f_2(S_{ij})), \mu_3(f_3(S_{ij})), \mu_4(f_4(S_{ij})), \mu_5(f_5(S_{ij}))\} \\ \text{subject to} \quad & \sum_i \sum_j S_{ij} = N \\ & S_{ij} \in \{0, 1\} \\ & S_{ij} = 0, \forall (i, j) \in \Omega \\ & \forall \text{adjacent } a, b, \quad d_{ab}^s \leq ((i_a - i_b)^2 + (j_a - j_b)^2)^{1/2} \leq d_{ab}^l \end{aligned} \quad (14)$$

Table 2: Optimistic and pessimistic objectives

Fire risk category	Objective	Optimistic value (f ⁺)	Pessimistic value (f ⁻)
A	f ₂	4	5
B	f ₃	5	8
C	f ₄	8	10
D	f ₅	10	20

4 Formulating as a Genetic Algorithm Based Search Problem

The above fuzzy multi-objective problem shown in a single unified goal can be formulised as a GA based search problem. In order to do so, it is necessary to define:

- a representation of the problem, i.e. a chromosome; and
- a fitness function defined in terms of this representation.

4.1 Representation

A chromosome in a genetic algorithm can be represented as a binary string, an integer string, or a real number string. In this study, the decision variable S_{ij} is a binary, and the location is a pair of integer coordinate (x, y) taken from the National Grid. If a double string (Sakawa et al., 1997) is chosen as the representation for a chromosome, a chromosome can be represented as

$$\begin{pmatrix} (x_1, y_1) & (x_1, y_2) & \cdots & (x_K, y_L) \\ S_{11} & S_{12} & \cdots & S_{KL} \end{pmatrix} \quad (15)$$

where K and L are the numbers of row and column of the National Grid in a given area. In the evolution of the chromosome in a double string shown in equation (15), the first row must be maintained unchanged, and only the decision variable row evolves. The difficulty of the chromosome evolution is that the sum of the decision variables at the second row must be equal to N , the optimal number of fire stations, as shown in equation (10). In order to avoid this complexity we choose the coordinates of the N locations where the fire stations are initially located to build the chromosome and only let the coordinate of fire station locations evolves. This is presented as follows:

$$[(i_1, j_1), (i_2, j_2), \cdots (i_N, j_N)] \quad (16)$$

where (i_a, j_a) is the coordinate of the a^{th} fire station and

$$\begin{aligned} S_{i_a j_a} &= 1, (a = 1, 2, \cdots, N) \\ i_a &\in \{x_1, x_2, \cdots, x_K\} \\ j_a &\in \{y_1, y_2, \cdots, y_L\} \end{aligned}$$

Three standard GA evolutionary operators including mutation, crossover and reproduction have a straightforward application to the chromosome shown in equation (16). The optimal variables S_{ij} in the single unified goal of the fuzzy multi-objective model shown in equation (14) is replaced by the coordinates of the initially selected N fire station locations shown in equation (16) in the evolution of the chromosome. If denoting (i_1, i_2, \cdots, i_N) as I , and (j_1, j_2, \cdots, j_N) as J , equation (14) is re-written as equation (17). The constraint shown in equation (10) has been taken away from equation (17) because it will be automatically satisfied in the chromosome representation shown in equation (16).

$$\begin{aligned} \text{Max} \quad & \min\{\mu_2(f_2(I, J)), \mu_3(f_3(I, J)), \mu_4(f_4(I, J)), \mu_5(f_5(I, J))\} \\ \text{subject to} \quad & (i_a, j_a) \notin \Omega, \quad a = 1, \cdots, N \\ & \forall \text{ adjacent } a, b, d_{ab}^s \leq ((i_a - i_b)^2 + (j_a - j_b)^2)^{1/2} \leq d_{ab}^l \end{aligned} \quad (17)$$

4.2 Fitness function

Using a characteristic function (Sakawa et al., 1997) $\chi_F(I, J)$, the constraints shown in equation (13) can be re-written as

$$\begin{aligned} \chi_F(I, J) &= \chi_F(i_1, j_1, i_2, j_2, \cdots, i_N, j_N) \\ &= \begin{cases} 1 & \text{if } d_{ab}^s \leq ((i_a - i_b)^2 + (j_a - j_b)^2)^{1/2} \leq d_{ab}^l \quad \forall \text{ adjacent } a, b \\ 0 & \text{otherwise} \end{cases} \end{aligned} \quad (18)$$

Equation (18) means that if there is a reasonable distance between any two adjacent fire stations, i.e. the constraint shown in equation (13) has been satisfied, the characteristic function equals 1, and otherwise, the characteristic function equals 0.

By introducing the characteristic function $\chi_F(I,J)$ shown in equation (18) into the objective function shown in equation (17) and denoting the fitness function as $f_{fitness}(I,J)$, the fitness function $f_{fitness}$ is defined as follows:

$$f_{fitness}(I,J) = \min \{ \mu_2(f_2(I,J)), \mu_3(f_3(I,J)), \mu_4(f_4(I,J)), \mu_5(f_5(I,J)), \chi_F(I,J) \} \quad (19)$$

The above fitness function is equal to 0 if the characteristic function is 0; otherwise it will take the minimal value of the fuzzy memberships of the objectives f_2 to f_5 .

The fitness functions of the optimal fire station locations are expected to achieve a maximum fuzzy membership, i.e. the minimal value among the longest distances from a fire station to any accident sites for any fire risk category areas, therefore achieves a value maximally close to the optimistic value recommended by the Home Office in the UK. That is:

$$Max \quad f_{fitness}(I,J) \quad (20)$$

It can be seen that once the constraint shown in equation (13) is not satisfied, i.e. two adjacent fire stations are too close to or too far away from each other, the characteristic function χ_F is 0, the fitness value is then equal to zero as well so that the chromosome is discarded in the next generation. In order to achieve the maximal fitness value, the constraint shown in equation (13) must be satisfied, i.e. $\chi_F(I,J)$ must be equal to 1. Therefore, by choosing the representation of the chromosome shown in equation (16) and introducing the characteristic function $\chi_F(I,J)$ shown in equation (18) into the fitness function, both constraints shown in equations (10) and (13) have been embedded in the fitness function. The only constraint that needs to be considered separately is the one of avoiding obstacles re-written as equation (21).

$$(i_a, j_a) \notin \Omega, \quad a = 1, \dots, N \quad (21)$$

5 Applications to the Derbyshire Fire Station Locations

5.1 Data collection

The National Grid System in the UK divides the Derbyshire region into 186×116 cells, each of them is a 0.5×0.5 km² square. Set the cell outside of the Derbyshire region with the value of zero, the ones within the region with the value of 1 for the fire risk category 'A', the value of 2 for the fire risk category 'B', 3 for 'C', and 4 for 'D'. The obstacle cells within the region have the value of zero. A part of the region is illustrated in Figure 3.

5.3 Implementation and result analysis

Table 2 is applied into equation (2) to compute the fuzzy membership for the areas with the fire risk categories 'A', 'B', 'C', and 'D' as follows:

$$\mu_2 = \begin{cases} 1 & \text{if } f_2(I, J) \leq 4 \\ 5 - f_2(I, J) & \text{if } 4 < f_2(I, J) < 5 \\ 0 & \text{if } f_2(I, J) \geq 5 \end{cases} \quad (23)$$

$$\mu_3 = \begin{cases} 1 & \text{if } f_3(I, J) \leq 5 \\ \frac{8 - f_3(I, J)}{3} & \text{if } 5 < f_3(I, J) < 8 \\ 0 & \text{if } f_3(I, J) \geq 8 \end{cases} \quad (24)$$

$$\mu_4 = \begin{cases} 1 & \text{if } f_4(I, J) \leq 8 \\ 5 - \frac{f_4(I, J)}{2} & \text{if } 8 < f_4(I, J) < 10 \\ 0 & \text{if } f_4(I, J) \geq 10 \end{cases} \quad (25)$$

$$\mu_5 = \begin{cases} 1 & \text{if } f_5(I, J) \leq 10 \\ 2 - \frac{f_5(I, J)}{10} & \text{if } 10 < f_5(I, J) < 20 \\ 0 & \text{if } f_5(I, J) \geq 20 \end{cases} \quad (26)$$

The value of the characteristic function $\chi_F(I, J)$ shown in equation (18) is determined by the minimal distance and the maximum distance between any two adjacent fire stations which are denoted as d_{ab}^{\min} and d_{ab}^{\max} respectively. After consulting fire service authorities the shortest distance d_{ab}^s is set to 0.5 miles, and the longest distance d_{ab}^l is set to 10 miles. After obtaining d_{ab}^{\min} and d_{ab}^{\max} , the characteristic function is computed as follows:

$$\chi_F(I, J) = \begin{cases} 1 & \text{if } 0.5 \leq d_{ab}^{\min}, \quad \text{and } d_{ab}^{\max} \leq 10 \\ 0 & \text{otherwise} \end{cases} \quad (27)$$

The ordinary three GA operators: reproduction, mutation, and crossover, have been implemented in the genetic algorithm. Based on our experience (Yang, 2004) the probability of mutation is set as 0.08, the probability of crossover as 0.6, and the size of the population as 50, and the number of generation as 100. Tables 3 and 4 give the numerical coordinates of the 30 fire stations. Figure 4 illustrates the distribution of the final 30 fire station sites identified by the circle symbols. The best fitness value is 0.44. The fuzzy memberships for the areas with the fire risk categories 'A', 'C', and 'D' equal 1.0, the one for the area with the fire risk category 'B' equals 0.44. The minimal distance and the maximum distance between any two adjacent fire stations equal 1.32 miles and 9.01 miles respectively, which are

calculated in the format $((i_a - i_b)^2 + (j_a - j_b)^2)^{1/2} \times$ the cell size (0.5 km here) and satisfy the constraint shown in equation (13), and therefore make the value of the characteristic function be equal to 1.

The upper left corner in Figure 4 is the national forest district. The population and the fire risk in this area are much lower than ones in the Derby city centre and the Chesterfield city centre located in the middle of the Derbyshire region. Only three recommended fire stations are located in the national forest district. Most of the recommended fire stations are located in the Derby city centre and the Chesterfield city centre. This is similar to the actual fire station locations in Derbyshire. The actual locations of the fire stations in Derbyshire are not illustrated in Figure 4 because the actual number of the fire stations is far below the optimal number and the locations were determined purely based on the population distribution rather than the risk categories in the area. The major difference between the recommended and actual locations in this case study is that all the actual locations of fire stations are either in a city centre or in a village centre, but part of the recommended fire stations are located between two villages or out of the centre centres. The reason of causing this difference is that the recommended locations are only focused on the various risk categories in a given area, and does not involve any social factors in the objectives. This difference has been acknowledged by the DFRS authority.

Table 3: Fuzzy memberships and fitness values for 30 fire stations

Gen- eration	f_2	f_3	f_4	f_5	μ_2	μ_3	μ_4	μ_5	d_{ab}^{\min} (mile)	d_{ab}^{\max} (mile)	χ_F	$f_{fitness}$
1	4.36	7.69	8.16	8.16	0.64	0.10	0.92	1.00	0.93	8.04	1	0.1
2	4.23	7.57	7.25	7.33	0.77	0.14	1.00	1.00	0.69	8.89	1	0.14
3	3.43	7.25	6.98	7.47	1.00	0.25	1.00	1.00	1.39	9.24	1	0.25
5	1.81	7.09	7.56	7.68	1.00	0.30	1.00	1.00	1.55	9.33	1	0.30
35	3.36	6.90	7.84	7.69	1.00	0.37	1.00	1.00	0.98	8.56	1	0.37
43	3.24	6.79	6.90	7.17	1.00	0.40	1.00	1.00	1.12	8.23	1	0.40
76	2.93	6.67	7.28	7.52	1.00	0.44	1.00	1.00	1.32	9.01	1	0.44

Table 4: 30 Fire station locations proposed by GA

1 (87, 112)	2 (60, 38)	3 (48, 122)	4 (91, 63)	5 (78, 56)	6 (56, 97)
7 (82, 88)	8 (66, 28)	9 (45, 58)	10 (52, 98)	11 (95, 54)	12 (94, 118)
13 (20, 167)	14 (60, 70)	15 (70, 80)	16 (55, 112)	17 (42, 48)	18 (89, 76)
19 (60, 8)	20 (96, 47)	21 (110, 129)	22 (49, 141)	23 (49, 63)	24 (17, 118)
25 (47, 111)	26 (65, 123)	27 (76, 120)	28 (36, 90)	29 (20, 138)	30 (82, 93)

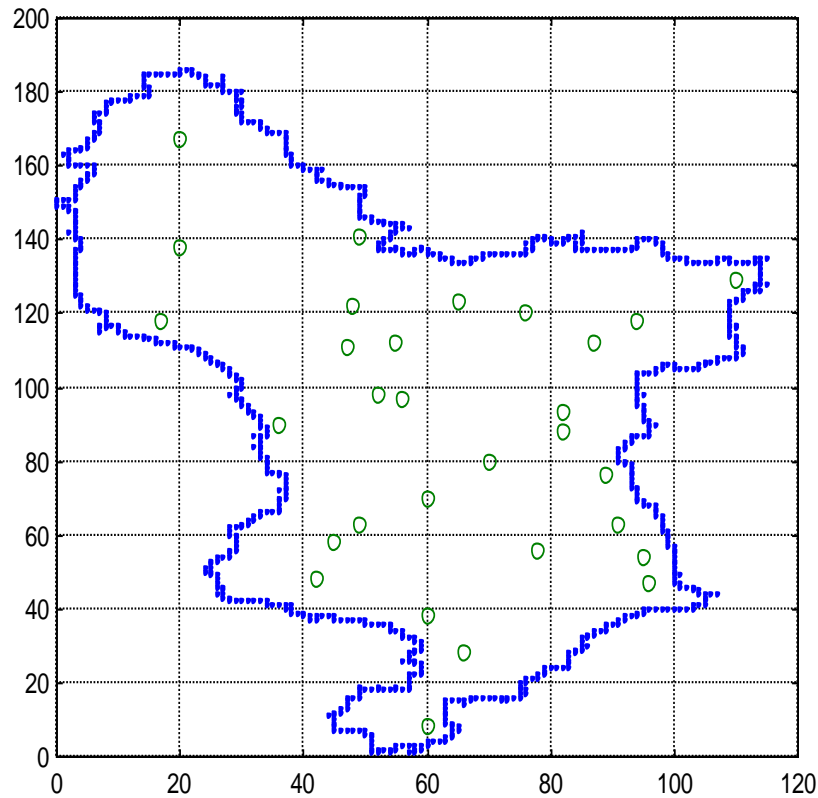


Figure 4: Distribution of the final 30 fire station sites

6 Discussions and future work

Any optimal location problems are usually constrained by a number of physical limitations and have multiple conflicting objectives. In this paper we converted a fuzzy multi-objective optimization model into a single unified goal and combined the goal with a genetic algorithm for the fire station location problem. Five objectives are created, the first one is to minimize the total setup and operating costs of fire stations and total loss cost of accidents in a given area, and the rest four are to minimize the longest distance from a fire station to any accident site. The optimal number of fire stations is obtained by

solving the first objective, which is transferred into a constraint for the rest of four objectives. There are other two extra constraints: one considers obstacles in a given area, and the other concerns the distance between any two adjacent fire stations. By choosing the coordinates of N possible fire station locations to represent a chromosome in the genetic algorithm we make the constraint about the optimal number of fire stations being automatically satisfied in the genetic algorithm. Also by using a characteristic function the constraint about the distance between any two adjacent fire stations is embedded in the fitness function of the genetic algorithm. Therefore the genetic algorithm only needs to consider a single constraint, which is to avoid the obstacles in a given area. Based on the recommendations of the Home Office in the UK on the speed of fire engine attack to accidents the areas with different fire risk categories have different travel time requirements. In order to fully consider these various demands to fire facilities four individual objectives are created. This approach is more reasonable and closer to practice. Our case study illustrates that the model established and the method proposed in this paper to deal with the constraints can be applied successfully at the DFRS.

Further work in optimal fire station locations mainly considers how to model and deal with the capacities of different types of fire stations. For example, the whole time stations will function 24 hours a day and 7 days a week, the day staffing stations will only be available in day time, and the retained stations will be available only once a request has been received. Also the facilities allocated in these different types of fire stations are various. The capacity limitations should be included in the optimal fire location problem. Furthermore, information about road conditions is helpful in the decision making of the fire station location. Bad weather and peak hours will make the road conditions much worse than good weather and non-peak hours. How to consider the effect of the road conditions on the travel time of fire engines is still a big challenge.

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