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Auction-based competition of hybrid small cells for dropped macrocell users

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Abstract: We propose an auction-based beamforming and user association algorithm for a wireless network consisting of a macrocell and multiple small cell access points (SCAs). The SCAs compete for serving the macrocell base station (MBS) users (MUs). The corresponding user association problem is solved by the proposed bid-wait auction method. The authors considered two scenarios. In the first scenario, the MBS initially admits the largest possible set of MUs that it can serve simultaneously and then auctions off the remaining MUs to the SCAs, who are willing to admit guest users in addition to their commitments to serve their own host users. This problem is solved by the proposed forward bid-wait auction. In the second scenario, the MBS aims to offload as many MUs as possible to the SCAs and then admits the largest possible set of remaining MUs. This is solved by the proposed backward bid-wait auction. The proposed algorithms provide a solution that is very close to the optimum solution obtained by using a centralised global optimisation.

1 Introduction

The fifth generation wireless system is anticipated to address the growing demand for spectrum and wireless capacity [1]. Usage of small cell access points (SCAs) in terms of cell densification is expected to increase spectral efficiency as it allows aggressive reuse of frequencies within a macrocell. SCAs can be either operator deployed or user deployed. SCAs could operate in open-access mode, hybrid mode or closed-group mode [2]. Among these three modes, works in [2] advocate for hybrid mode as it allows shared resources between host users (HUs) and guest users (GUs). The macrocell operator can provide incentives to the SCAs for serving its users [3, 4]. Within this context and using the notion of game theory, the wireless system can be categorised into *buyers*, *sellers*, *goods* and *auctioneers* [5]. Auction is a process of selling or buying goods or services. In an auction, the goods are exchanged between the sellers and the buyers according to the variation of the prices. Hence, pricing is used for coordinating and equilibrating the markets.

1.1 Related works

The benefits of offloading traffic have been extensively studied in [6–9]. The findings in [7] show that small cells can achieve higher network capacity and energy efficiency. In [8], a small cell activation mechanism for offloading traffic from a macrocell to small cells, while avoiding user quality of service (QoS) degradation, was proposed. The work in [9] considered a centralised energy aware offloading mechanism for cloud-radio access network.

In [10], a problem wherein the service providers compete for femtocell under a multi-leader follower game framework was considered. A framework for user association in infrastructure-based wireless network that considered optimal throughput, delay and load equalisation was proposed in [11].

Auction-based algorithms have been proposed in [12–16]. A reverse auction framework based on Vickrey–Clarke–Groves (VCG) mechanism was proposed in [12] for a fair and efficient access permission that maximises the social welfare of the network consisting of one wireless service provider and several femtocell owners. Authors in [17] proposed a mechanism to switch between open and closed modes to maximise their performance. The problem was solved using a game theoretic approach.

The authors [4, 18, 19] proposed distributed algorithms for assigning users to SCAs using auctioning, heuristic beamforming designs, Stackelberg games and evolutionary games. Despite all these auction-based algorithms reported in [20, 21], algorithms that considered multiple user access through spatial beamforming and auctioning mechanism have not been reported in the literature, which is the focus of this paper.

1.2 Contributions

Our objective is to develop an auction framework for performing beamforming-based spatial multiplexing, user offloading and user association in a heterogeneous network. This framework enhances the network capacity by utilising transmitting infrastructure. The specific contributions of our work are as follows:

- We propose and analyse a novel auction mechanism called the bid-wait auction (BWA) that jointly performs downlink beamformer design and user association. To the best of our knowledge, auction mechanisms in the literature have not considered joint beamformer design and user allocation.
- We develop a novel valuation function for bidder that automatically monitors resource budgets for the bidder.
- We propose and analyse a novel payment rule that allows BWA to allocate items to bidders with sparse information. We proved the existence of the dominant-strategy equilibrium (DSE).

Notations: We use the following notations: We use the upper-case bold face and lower-case bold face letters for matrices and vectors, respectively. The notation $\| \cdot \|$ denotes the Euclidean norm. The operators $\mathfrak{R}(\cdot)$ and $\mathfrak{I}(\cdot)$ extract the real and the imaginary parts of their arguments, respectively. The regular and Hermitian transposes are denoted by $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$, respectively.

2 System model and assumptions

Consider a downlink network consisting of a macrocell base station (MBS) deployed with S number of SCAs as shown in Fig. 1. The SCAs are privately owned and are operated in a hybrid mode. The MBS and the SCAs employ non-overlapping frequency bands. The MBS is equipped with M_{MBS} antennas and each SCA is equipped with M_{SCA} antennas. The MBS has M_0 MBS users (MUs) wanting

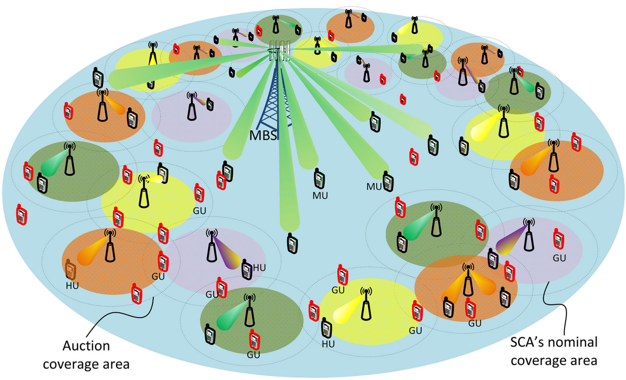

Fig. 1 *Heterogeneous network consisting of one MBS and several hybrid SCAs. All transmitters operate in non-overlapping frequency bands. Each of the SCAs is serving its HUs. There are GUs (coloured in red) that have been dropped by the MBS*

access to the network. The MBS and each SCA have maximum transmission powers of $p_0^{\max}$ and $p_s^{\max}$, respectively. All of the users have single antenna at the receiver and have specific QoS requirements.

2.1 Motivation

It is likely that the resources at the SCAs may be under utilised by the HUs. On the other hand, resources at the MBS may be over utilised. To avoid user dropouts, MBS will offload some of its users to SCAs. In the presence of dense deployment of SCAs, there is a high chance that a GU may be in the vicinity of more than one SCAs. This work proposes a mechanism that handles competition among SCAs to serve MUs in return for monetary benefits through user allocation and beamforming.

2.2 Forward bid-wait auction (FBWA) and backward bid-wait auction (BBWA) algorithms

We consider two scenarios: In the first scenario, the MBS admits the maximum possible MUs it can serve and then offloads the dropped MUs to SCAs via auctioning. In the second scenario, the MBS allows the SCAs to bid for serving GUs and then later aims to admit the remaining MUs. We propose the BWA and supplement it with an admission control to develop FBWA and BBWA algorithms. The FBWA and BBWA algorithms solve problems in the first and second scenarios, respectively.

2.3 System metric design

We index the MBS by 0 and the sth SCA by s. Let the set of MUs served by the MBS be $\mathcal{M}_0$. Each MU is denoted by index m. In the downlink, the transmitted signal for MU m from the MBS is written by

$$\boldsymbol{x}_m(t) = \boldsymbol{w}_m s_m(t), \tag{1}$$

where $s_m(t) \in \mathbb{C}$ represents the information symbol at time t, and $\boldsymbol{w}_m \in \mathbb{C}^{M_{\mathrm{MBS}} \times 1}$ is the transmit beamforming vector for user m, the squared norm of which provides the allocated power. Without loss of generality, assume that $s_k(t)$ is normalised such that $\mathbb{E}\{|s_k(t)|^2\} = 1$, and that all data streams are independent such that, $\mathbb{E}\{s_m(t)s_i(t)^*\} = 0$, if $m \neq i$. The received signal at the mth MU is given by

$$y_{0m} = \boldsymbol{h}_{0m}^{\mathrm{H}} \boldsymbol{x}_m(t) + \sum_{i \in \mathcal{M}_0 \setminus m} \boldsymbol{h}_{0m}^{\mathrm{H}} \boldsymbol{x}_i(t) + \eta_m(t), \tag{2}$$

where $\boldsymbol{h}_{0m} \in \mathbb{C}^{M_{\mathrm{MBS}}}$ is the random channel vector from the MBS to the mth MU, and $\eta_m(t) \in \mathcal{CN}(0, \sigma^2)$ is the circular symmetric zero mean complex Gaussian noise with variance σ_m^2. Let the set of HUs and GUs served by the sth SCA be $\mathcal{H}_s$, each denoted by h. The transmitted signal for HU h from the SCA s is $\boldsymbol{x}_h(t) = \boldsymbol{w}_h s_h(t)$. The received signal at the hth HU is given as

$$y_{sh} = \boldsymbol{h}_{sh}^{\mathrm{H}} \boldsymbol{x}_h(t) + \sum_{j \in \mathcal{H}_s \setminus h} \boldsymbol{h}_{sh}^{\mathrm{H}} \boldsymbol{x}_j(t) + \eta_h(t), \tag{3}$$

where $\boldsymbol{h}_{sh} \in \mathbb{C}^{M_{\mathrm{SCA}} \times 1}$ is the random channel vector from the SCA s to the hth HU, and $\eta_h(t) \in \mathcal{CN}(0, \sigma^2)$ is the circular symmetric zero mean complex Gaussian noise with variance σ_h^2.

The downlink signal-to-interference-plus-noise ratio (SINR) of the mth MU and the hth HU are given, respectively, by

$$\mathrm{SINR}_{0m} = \frac{|\boldsymbol{h}_{0m}^{\mathrm{H}} \boldsymbol{w}_m|^2}{\sum_{i \in \mathcal{M}_0 \setminus m} |\boldsymbol{h}_{0m}^{\mathrm{H}} \boldsymbol{w}_i|^2 + \sigma_m^2}, \tag{4}$$

$$\mathrm{SINR}_{sh} = \frac{|\boldsymbol{h}_{sh}^{\mathrm{H}} \boldsymbol{w}_h|^2}{\sum_{j \in \mathcal{H}_s \setminus h} |\boldsymbol{h}_{sh}^{\mathrm{H}} \boldsymbol{w}_j|^2 + \sigma_h^2}. \tag{5}$$

2.4 User admission by the MBS

Let us define the SINR targets of the MUs as $\Xi_0 = [\xi_1^0, \ldots, \xi_{M_0}^0]$ and $\mathcal{M}'_0 \subseteq \mathcal{M}_0$ as a set of admitted users, whose cardinality is a parameter to be maximised. The user admission problem at the MBS is formulated as

$$\begin{aligned} &\text{maximise} && |\mathcal{M}'_0| \\ &\text{subject to} && \mathrm{SINR}_{0m} \geq \xi_m^0, \quad m \in \mathcal{M}_0, \\ &&& \sum_{m \in \mathcal{M}_0} \| \boldsymbol{w}_m \|^2 \leq p_0^{\max}, \end{aligned} \tag{6}$$

where $|\mathcal{M}'_0|$ denotes the cardinality of the set $\mathcal{M}'_0$. We assume that all of the MUs have identical QoS requirements. This latter assumption encourages the SCAs to admit as many GUs as possible, as shown later. The problem in (6) is non-convex due to non-convex objective function. However, QoS constraints can be rewritten in their equivalent second-order cone (SOC) [22] as

$$\mathrm{SINR}_m^0 \geq \xi_m^0 \Rightarrow \left\| \begin{matrix} \boldsymbol{h}_{0m}^{\mathrm{H}} \boldsymbol{w}_1 \\ \vdots \\ \square \\ \boldsymbol{h}_{0m}^{\mathrm{H}} \boldsymbol{w}_{M_0} \end{matrix} \right\| \leq \sqrt{\frac{1 + \xi_m^0}{\xi_m^0}} \Re(\boldsymbol{h}_{0m}^{\mathrm{H}} \boldsymbol{w}_m), \tag{7}$$

$$\Im(\boldsymbol{h}_{0m}^{\mathrm{H}} \boldsymbol{w}_m) = 0, \quad \forall m. \tag{8}$$

Let the matrix $\boldsymbol{W}_0 = [\boldsymbol{w}_m]_{m \in \mathcal{M}_0}$ be defined by concatenating the column vectors $\boldsymbol{w}_m$ at MBS. We introduce slack variables $\boldsymbol{a}^0 = [a_1^0, \ldots, a_{M_0}^0]$ and rewrite the problem in (6) as

$$\begin{aligned} &\underset{\{\boldsymbol{w}_m\}, \{\boldsymbol{a}^0\}}{\text{minimise}} && \| \boldsymbol{a}^0 \|_0 \\ &\text{subject to} && \begin{bmatrix} \sqrt{1 + \frac{1}{\xi_m^0}} \boldsymbol{h}_{0m}^{\mathrm{H}} \boldsymbol{w}_m + a_m^0 \\ \boldsymbol{h}_{0m}^{\mathrm{H}} \boldsymbol{W}_0 \\ \sigma \end{bmatrix} \succeq_{\mathrm{SOC}} 0, \quad m \in \mathcal{M}_0, \\ &&& \Im(\boldsymbol{h}_{0m} H \boldsymbol{w}_m) = 0 \quad \forall m, \\ &&& \boldsymbol{a}^0 \geq \boldsymbol{0} \quad \forall m, \\ &&& \sum_{m \in \mathcal{M}_0} \| \boldsymbol{w}_m \|^2 \leq p_0^{\max} \quad \forall m, \end{aligned} \tag{9}$$

The objective in (9) is an ℓ_0-norm, which accounts for the number of non-zero elements in the vector $\boldsymbol{a}^0$. This ℓ_0-norm problem is a combinatorial problem, which is non-convex and non-deterministic polynomial-time (NP) hard. A widely adopted approach in the literature to deal with this form of non-convex problem is to approximate the ℓ_0-norm with an ℓ_1-norm [23, 24]. Hence, we have replaced the objective function with an ℓ_1-norm. This together with the SOC constraints makes the overall problem a convex problem, known as SOC programming [22] as follows:

$$\begin{aligned} &\underset{\{\boldsymbol{w}_m\},\{\boldsymbol{a}^0\}}{\text{minimise}} \quad \| \boldsymbol{a}^0 \|_1 \\ &\text{subject to constraints in (9)}. \end{aligned} \tag{10}$$

The above convex problem can be solved using the CVX tool [25], which is able to indicate if the problem is feasible or not. The value of each a_m^0 indicates the feasibility gap for the corresponding user and preference of any users by the transmitter. To obtain the optimal admission set $\mathscr{M}'_0$, as proved in [23], the elements of $\boldsymbol{a}^0$ are rearranged in ascending order and the MUs are sequentially admitted starting with those users that have smallest a_m^0. This is done by performing feasibility check at every admission stage by solving

$$\begin{aligned} &\underset{\{\boldsymbol{w}_m\}}{\text{minimise}} \quad \sum_{\forall m \in \mathscr{M}_{0'} \cup m} \| \boldsymbol{w}_m \|^2 \\ &\text{subject to} \quad \text{SINR}_m^0 \geq \xi_m^0 \quad \forall m \in \mathscr{M}_{0'} \cup m, \\ &\qquad \sum_{m \in \mathscr{M}'_0} \| \boldsymbol{w}_m \|_2^2 \leq p_0^{\max} \quad \forall m, \end{aligned} \tag{11}$$

If a newly admitted user makes the constraints in (11) infeasible (i.e. when feasibility test fails), then that user will be removed from the set $\mathscr{M}'_0$. The resulting admission set $\mathscr{M}'_0$ will be optimal in sense of maximising admitted users.

3 Bid-wait auction

The MBS wishes to offload as many users as possible to SCAs. This is usually formulated as a surplus maximisation in auctioning [26]. Therefore, we use the number of admitted GUs as our performance metric. We form a BWA considering the MBS as the auctioneer, the SCAs as the bidders, and the GUs as the items.

Let us denote the beamformer vector at the SCA for serving the ith HU given that GU g is admitted by the SCA as $\hat{\boldsymbol{w}}_i$. Also, we denote the beamformer vector at the SCA for serving the kth HU before the GU g is admitted as $\boldsymbol{w}_k$. The cost of connecting the gth GU during the rth auction round is given by

$$c_{sg}^r = \mu\left(\sum_{\forall i \in \mathscr{H}_s \cup g} \| \hat{\boldsymbol{w}}_i \|_2^2 - \sum_{\forall k \in \mathscr{H}_s} \| \boldsymbol{w}_k \|_2^2 \right), \tag{12}$$

where μ is the cost per unit power. The first term in (12) is the total transmission power after the admission of the gth GU. The last term is the total transmission power before gth GU is admitted.

Each served user pay the SCA an amount of κ per unit of data rate. Since the MBS auctions some of its users to the SCAs, the GUs will pay SCAs that will in turn pay MBS. The difference of the payment is the profit generated by the SCA for serving a GU. We denote the SINR target for the GU as ξ_g^s. Hence, each GU has a marginal value v_{sg}^r given by

$$v_{sg}^r = \kappa \log_2(1 + \xi_g^s) - c_{sg}^r, \tag{13}$$

which is a value contributed by that GU given the already admitted users. These values are private and they are unknown to other bidders and the auctioneer. The marginal value in (13) demonstrates that the GUs are substitutes, i.e. admitting a user by an SCA at a particular stage will change the required beamformer and power allocation of already admitted users as well as the remaining users that SCAs will be bidding. This will change the preference order of items for every SCAs. Therefore, it is critical that an SCA comes up with an effective preference profile. In Section 4, we propose two types of preference profiles.

3.1 Surplus maximisation in BWA

An intuitive approach in surplus maximisation is to allocate items to bidders that value them the most. This allocation rule indirectly allows maximisation of allocated items. The BWA is a collection of concurrent sealed-bid single-item auctions. In the proposed BWA, the objective of the MBS is to assign the GUs to those SCAs that value them the most.

Let us define a set $\mathscr{G}_s \subseteq \mathscr{G}$ to contain all GUs that can be assigned to sth SCA and a competitors' set $\mathscr{C}_g$ which contains all SCAs competing to connect the gth GU. A feasible assignment $\mathscr{A}$ is the set of SCA-GU pairs (sg), with $g \in \mathscr{G}_s$. An SCA can be part of more than one pair $(sg) \in \mathscr{A}$. The surplus maximisation problem at the MBS is formulated as the following integer program:

$$\begin{aligned} &\underset{x_{sg}^r}{\text{maximise}} \quad \sum_{r=1}^{R} \sum_{s=1}^{S} v_{sg}^r x_{sg}^r \\ &\text{subject to} \quad \sum_{g \in \mathscr{G}'_s} x_{sg}^r \leq 1 \quad \forall s \in \mathscr{S}, \\ &\qquad \sum_{s \in \mathscr{C}_g} x_{sg}^r \leq 1 \quad \forall g \in \mathscr{G}, \quad \forall r, \\ &\qquad x_{sg}^r \in \{0, 1\} \quad \forall (sg) \in \mathscr{A}', \end{aligned} \tag{14}$$

where R is the total number of auction rounds, $\mathscr{A}'$ is the set of all possible SCA-GU assignment pairs $(sg)(\mathscr{A}' \subseteq \mathscr{A})$ and $(x_{sg}^r)_{g \in \mathscr{G}'_s}$ are binary decision variables, indicating association of SCAs. $x_{sg}^r = 1$ means that SCA s is assigned to GU g and otherwise $x_{sg}^r = 0$. Hence, the term $\sum_{s \in \mathscr{S}} v_{sg}^r x_{sg}^r$ is the surplus at the rth auction round. We propose to solve (14) by running simultaneous sealed-bid single-item auctions wherein, at each auction round, each bidder's action is a bid b_{sg}^r (not necessarily the true value) on the most preferred GU. This accounts for the summation over the total number of auction rounds in the objective. Therefore, BWA mechanism decomposes the combinatorial nature of the problem and runs virtual single-item auctions repetitively. The second and the third constraints ensure that each SCA can be assigned to one or more GUs and each GU can be assigned to only one SCA.

4 Bidders valuation functions

If an SCA wins a GU during auction round r, it pays a price p_{sg}^r to the MBS. The bidders utility model at rth auction round, on the bid/action profile $\boldsymbol{b}^r = [b_{1g}^r, \ldots, b_{S\bar{g}}^r]$ is a quasilinear utility model define as

$$u_{sg}^r(\boldsymbol{b}^r) = v_{sg}^r(\boldsymbol{b}^r) x_{sg}^r(\boldsymbol{b}^r) - p_{sg}^r(\boldsymbol{b}^r), \tag{15}$$

where the subscript g and $\bar{g}$ could refer to the same or different GUs. The overall objective of the SCA is to maximise

$$\sum_{r=1}^{R} u_{sg}^r(\boldsymbol{b}^r). \tag{16}$$

By assuming positive utility at each auction round, and some payment $p_{sg}^r(\boldsymbol{b}^r)$ that is independent of $v_{sg}^r(\boldsymbol{b}^r) x_{sg}^r(\boldsymbol{b}^r)$ the utility in (16) is maximised by admitting as many GUs as possible. This is because we assumed that all of the MUs have identical QoS targets.

4.1 Fixed preference profile (FPP) criterion

In this case, we assume that bidders determine their preference profile once, at the beginning of the auction, and fix it for the entire BWA. Each SCA identifies the GUs that fall within its auction coverage area. This is followed by determining the FPP by solving the admission problem. Let us define the QoS targets of the HUs and GUs as $\boldsymbol{\Xi}^s = [\xi_1^s, \ldots, \xi_{F_s}^s]$. We introduce auxiliary variables $\boldsymbol{a}^s = [a_1^s, \ldots, a_{F_s}^s]$ and use the same procedures for deriving (6)–(10) to form an ℓ_1-norm admission problem for the SCA as

$$\begin{aligned}
\underset{\{\boldsymbol{w}_j\},\{\boldsymbol{a}^s\}}{\text{minimise}} \quad & \| \boldsymbol{a}^s \|_1 \\
\text{subject to} \quad & \begin{bmatrix} \sqrt{1+\frac{1}{\xi_j^s}}\boldsymbol{h}_{sj}^{\mathrm{H}}\boldsymbol{w}_j + a_j^s \\ \boldsymbol{h}_{sj}^{\mathrm{H}}\boldsymbol{W}_s \\ \sigma \end{bmatrix} \succeq_{\mathrm{SOC}} 0, \quad j \in \mathcal{F}_s, \\
& \Im(\boldsymbol{h}_{sj}^{\mathrm{H}}\boldsymbol{w}_j) = 0 \quad \forall j, \\
& \boldsymbol{a}^s = \boldsymbol{0}, \quad j = 1, \ldots, H_s, \\
& \boldsymbol{a}^s \geq \boldsymbol{0}, \quad j = H_s + 1, \ldots, F_s, \\
& \sum_{j \in \mathcal{F}_s} \| \boldsymbol{w}_j \|_2^2 \leq p_s^{\max} \quad \forall j,
\end{aligned} \tag{17}$$

where the third constraint ensures that the HUs are given first priority. To build up a preference set of GUs $\mathcal{G}'_s \subseteq \mathcal{F}_s$, we sort the vector $\boldsymbol{a}^s$ in ascending order of its elements. The corresponding indices of the sorted $\boldsymbol{a}^s$ with an exclusion of the HUs give the FPP $\boldsymbol{f}_s$. It should be noted that at this stage, no valuation profile that corresponds to $\boldsymbol{f}_s$ is determined. Since there is no guarantee that all GUs in the preference set will be won, the values are computed on a 'need-to-know' basis. At every auction round, an SCA will use (13) to place a value on the most preferred GU.

4.2 Adaptive preference profile (APP) criterion

It is anticipated that the level of preference over GUs will be reduced if an admission of a particular GU is already made due the substitute nature of the GUs. Therefore the preference profiles need to be revised every time a new GU is admitted. The values for every GU $g \in \mathcal{G}_s$ are computed separately and sorted in descending order to determine the current preference profile. A bid is then placed on a GU that is perceived to have the highest value. Let us define the QoS targets of the HUs and gth GU as $\boldsymbol{\Xi}^s = [\xi_1^s, \ldots, \xi_{H_s}^s, \xi_g^s]$. Also let the set $\mathcal{H}_s$ as a set of HUs and admitted GUs. For every available GU $g \in \mathcal{G}_s$, each of the SCA determines the connection cost by solving the following feasibility problem:

$$\begin{aligned}
\underset{\{\boldsymbol{w}_k\}}{\text{minimise}} \quad & \sum_{\forall k \in \mathcal{H}_s \cup g} \| \boldsymbol{w}_k \|_2^2 \\
\text{subject to} \quad & \mathrm{SINR}_k^s \geq \xi_k^s \quad \forall k \in \mathcal{H}_s \cup g, \\
& \sum_{\forall k \in \mathcal{H}_s \cup g} \| \boldsymbol{w}_k \|_2^2 \leq p_s^{\max}.
\end{aligned} \tag{18}$$

The connection cost can be determined using (12). With the exception of the first auction round, we note that for every auction round, losers from the previous round do not need to revise their preference profiles. The bidders on WAIT (i.e. bidders are on WAIT if the decision on their bid is withheld) do nothing while the winners are required to revise their preference profiles and submit new bids. The losers from the previous round only need to submit the bid on the next most preferred and available GUs since the values are already known. When the preference profile needs revision, all the values of the available GUs need to be calculated. Though it may appear, it offers the SCAs with the capability to identify and prune away all the GUs that will never be feasible for admission. This is not applicable when FPP criterion is used. In FPP criterion, only the value of the next preferred and available GU is determined at every SCA in the contact_list (i.e. a list of SCAs that are eligible to submit new bids).

For any non-conflicting preference profile, the MBS will permit the corresponding SCA to submit bundle bids on the largest set of the remaining GUs that it can admit simultaneously. If any SCA has knowledge that some GUs are not bid by all of the remaining SCAs, there is a possibility for unfaithful bidding. However, as it is difficult for any SCA to acquire preference profiles of other SCAs, we exclude this possibility in our work.

5 BWA mechanism design

We propose a BWA auction which inherits some properties of the second-price auction proposed in [27]. To reduce the amount of information shared between the MBS and the SCAs, the BWA uses iterative indirect mechanism to gather useful information from SCAs. It is assumed that the MBS has knowledge of the locations of all of the bidders and the GUs. Therefore it can formulate the preference sets of all of the SCAs. The MBS sets a rule that each bidder should submit one bid at a time. The bids should be monotonically decreasing in each auction round. Even though the BWA uses some of the principles from the VCG mechanism, we emphasise that the two methods are totally different. To highlight this difference, we present the following example with the aid of Fig. 2.

Example 1: Consider a bid-wait auction with six GUs and four bidders SCA1, SCA2, SCA3 and SCA4 with preference sets $\{\mathrm{GU2, GU3, GU4, GU5, GU6}\}$, $\{\mathrm{GU1, GU2, GU3, GU4, GU6}\}$, $\{\mathrm{GU2, GU3, GU4, GU5}\}$ and $\{\mathrm{GU2, GU3, GU4, GU5}\}$, respectively. The BWA will iterate as shown in Fig. 2. Note that unlike the VCG which charges the winner the second highest bid on the winning item, the BWA charges the winner the second highest price from the competitor's set. The set $\mathcal{G}_{\mathrm{GU1}} := \{\mathrm{SCA1, SCA2, SCA3, SCA4}\}$ is the competitors set for GU1. Therefore in the first auction round the BWA allocates GU1 to SCA1 and charges it 7 from bidder SCA2. SCA2 and SCA4 are then put on WAIT while SCA1 and SCA3 are put in the `contact_list` making them the only two bidders who are allowed to submit new bids on the second round. The same process is repeated until the `contact_list` becomes empty. The BBWA and the FBWA algorithms are summarised in Algorithms 1 (Fig. 3) and 2 utilise this BWA in their main loops.

5.1 Existence of equilibrium in the BWA

During the rth auction round, we denote the valuation of the sth bidder on the gth GU by v_{sg}^r and a collection of all bidders' valuations as $\boldsymbol{v}^r = [v_{1g}^r, \ldots, v_{S\bar{g}}^r]$, where g and $\bar{g}$ are the identities of the GUs. The GUs g and $\bar{g}$ do not need to be different. We define the strategy of each bidder $s_{s,g}^r$ as a function that maps a bidder's valuation to any of other bidder's possible *actions*. Let us define a collection of all bidders' strategies and actions in the rth auction round as $\boldsymbol{s}^r = [s_{1g}^r, \ldots, s_{S\bar{g}}^r]$ and $\boldsymbol{b}^r = [b_{1g}^r, \ldots, b_{S\bar{g}}^r]$ respectively. Given R as the maximum number of auction rounds required before the market closes, the sth bidder valuations, strategies and actions for the entire BWA are denoted by $\boldsymbol{v}_s = [v_{s1}^1, \ldots, v_{sG_s}^R]$, $\boldsymbol{s}_s = [s_{s1}^1, \ldots, s_{sG_s}^R]$ and $\boldsymbol{b}_s = [b_{s1}^1, \ldots, b_{sG_s}^R]$, respectively. The entire BWA has the valuation, strategy and action spaces denoted by $\boldsymbol{V} = [\boldsymbol{v}_1, \ldots, \boldsymbol{v}_S]$, $\boldsymbol{B} = [\boldsymbol{b}_1, \ldots, \boldsymbol{b}_S]$ and $\boldsymbol{S} = [\boldsymbol{s}_1, \ldots, \boldsymbol{s}_S]$, respectively.

Definition 1: A *sub-auction dominant-strategy equilibrium* (*sDSE*) at every auction round is a strategy profile $\boldsymbol{s}^r$ such that for all s, v_{sg}^r and $\boldsymbol{b}_{-s}^r$, the utility of bidder s is maximised by following the strategy $s_{sg}^r(v_{sg}^r)$ [26].

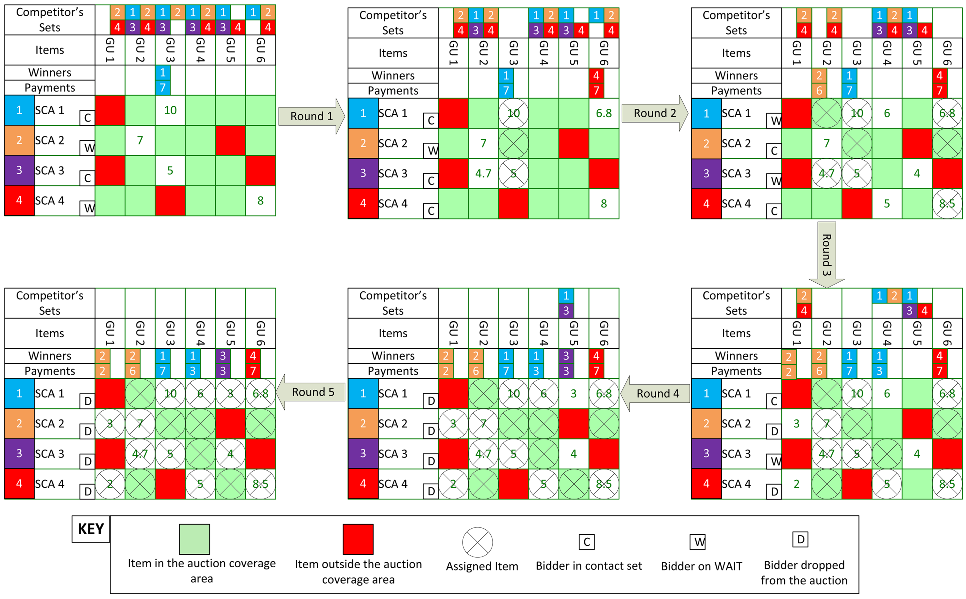

Fig. 2 *Example of the steps performed by the BWA*

Data: Initialization: Guest user set $\mathcal{G} = \emptyset$, assignment set $\mathcal{A} = \emptyset$, *auction round*: $i = 0$.
Result: Optimal allocation set $\mathcal{A}' \subseteq \mathcal{A}$.
MBS-MU admission
Solve (6) and (10) to get $\mathcal{R}'_0$ and $\mathcal{G}$.
`contact_list` = 1:*number eligible SCAs*.
SCA-GU admission: BWA
while $\texttt{contact\_list} \neq \emptyset$ **do**
  $i = i + 1$
  Auctioneer contacts bidders on `contact_list`
  Active bidders submit their bids b^r_{sg} on most preferred GU(s).
  if $b^r_{sg} = \emptyset$ *or* $b^r_{sg} > b^{r-1}_{sg}$ **then**
    Bidder is dropped.
  Auctioneer declares winners on items with complete bid information.
  if *item has incomplete bid information* **then**
    Current best bidder WAITS.
  Auctioneer determines new `contact_list`.

Fig. 3 *Algorithm 1: Forward bid-wait auction*

We now develop a dominant-strategy incentive compatible (DSIC) mechanism for the BWA and prove that the BWA has a unique sDSE at each auction round and a unique DSEfor the entire BWA. Since the BWA is a collection of concurrent sealed-bid single-item auctions, we can confine our problem into a *single-parameter environment* [26] for mechanism design. The outcome of such mechanism is the allocation and payment vectors $\boldsymbol{x}^r = [x^r_{1,g}, \dots, x^r_{S,\bar{g}}]$ and $\boldsymbol{p}^r = [p^r_{1,g}, \dots, p^r_{S,\bar{g}}]$.

5.2 Allocation rule

If the bids from a particular bidder are not monotonically decreasing, its current bid will not be accepted and the bidder is dropped from the auction. In every auction round, the BWA allocates the GU to the bidder with the highest bid if the feasible assignment set $\mathscr{A}$ has the minimum required information using the allocation rule

$$\boldsymbol{x}^r(\boldsymbol{b}^r) = \arg\max_{\mathscr{A}} \sum_{s \in \mathscr{S}} b^r_{sg} x^r_{sg} . \tag{19}$$

Proposition 1: Assume the auctioneer has the preference sets of all bidders $\mathscr{G}_s, \forall s \in \mathscr{S}$. Suppose bidders j and k are the only bidders who are eligible to bid on item m. If during the rth auction round, item m is bidder j's first preference with a bid of b^r_{jm} and the current bid from k's bidder is b^r_{kp} on item p (i.e. item p is more preferred than item m from bidder k's perspective), then the following conditions exist:

i. If $b^r_{jm} > b^r_{kp}$, it suggests that $b^r_{jm} > b^r_{km}$, concluding that bidder k stands no chance in winning item m. The item is then assigned to bidder j. Under this condition, the auctioneer has *complete bid information* on item m. We henceforth refer to bid b^r_{kp} as bidder j's *critical bid*.

ii. If $b^r_{jm} < b^r_{kp}$, then bidder k still stands a chance to win item m. Therefore bidder j will have to WAIT (hence the term BID-WAIT) until the auctioneer has the right information to announce the winner between bidders j and k. Under this condition, the auctioneer has *incomplete bid information* on item m.

Proof: Since the auctioneer has access to the preference sets and uses the one bid at a time rule, and by assuming truthful bidding, the preference profiles at the SCAs dictates that the bids submitted should be monotonically decreasing at each auction round. Therefore, the next bid on the next available preferred item is always less or equal to the current submitted bid. □

Algorithm 2: BBWA

Data: Initialisation: `contact_list`=1:number of eligible SCAs, $\mathscr{G} = \mathscr{M}_0$,

$\mathscr{A} = \emptyset$, *auction round*: $i = 0$.

Result: Optimal allocation set $\mathscr{A}' \subseteq \mathscr{A}$.

SCA-GU admission: BWA

1 Perform steps 3–12 of Algorithm 1 (Fig. 3).

2 Set $\mathscr{M}_0 = \mathscr{M}_0 \setminus \mathscr{G}'$.

MBS-MU admission

3 Solve to (6) and (10) to get $\mathscr{M}'_0$.

5.3 Payment rule

The BWA extends the second-price rule by charging the winner the second highest bid from the bidder in competitors' set $\mathscr{C}_g$, i.e. the critical bid. It is very important to note that the critical bid needs not to be the second highest bid on a particular item as elaborated in Proposition 1.

Theorem 1: Truthful bidding is a weak dominant strategy in the BWA.

Proof: Consider an arbitrary bidder s, its valuation at rth auction round on the gth GU is v^r_{sg} and $\boldsymbol{b}^r_{-s}$ is the bids of other bidders. The bids $\boldsymbol{b}^r_{-s}$ do not necessarily have to be placed on the gth GU. The valuation v^r_{sg} is an immutable valuation for bidder s on the gth GU. Let $B = \max_{t \neq s} v^r_{t\bar{g}}$ denote the highest bid by some other potential bidder of gth GU (i.e. gth GU belongs to the preference set of bidder t). The bid B is the *critical bid* of bidder s. The GU $\bar{g}$ could be the gth GU or any other GU. Now, given the bid B, there are only two distinct outcomes for bidder s. If bidder s bids $b^r_{sg} < B$, he loses and receives utility $u^r_{sg} = 0$. But if he bids $b^r_{sg} \geq B$, and by assuming that the ties are broken in favour of bidder s, then he wins and receives utility $u^r_{sg} = v^r_{sg} - B$. In the BWA, we break the ties by random choice. Now the following cases exist. If $v^r_{sg} < B$, maximum utility that bidders s will obtain is $\max\{0, v^r_{sg} - B\} = 0$. On the other hand if $v^r_{sg} \geq B$, maximum utility that bidders s will obtain is $\max\{0, v^r_{sg} - B\} = v^r_{sg} - B$, which occurs by bidding truthfully and winning. □

Theorem 2: Bidding on the most preferred GU is a dominant strategy in the bid-wait auction. (see (20) and (21)) (see (21))

Proof: Without loss of generality, consider two items with identities g and $\bar{g}$. Fix an arbitrary bidder s with the preference profile $\boldsymbol{f}_s = [g, \bar{g}]$ at the rth auction round. Set its valuations profile as $\boldsymbol{v}_s = [v^r_{sg}, v^r_{s\bar{g}}]$ where $v^r_{sg} > v^r_{s\bar{g}}$, and denote the bids from other bidders as $\boldsymbol{b}^r_{-s}$, $\boldsymbol{b}^{r+1}_{-s}$ during the auction rounds r and $r+1$, respectively. Again without loss of generality, let us assume that all other bidders have the same preference profiles as bidder s at the rth auction round. Let $B^r = \max_{t \neq s} v^r_{tg}$ and $B^{r+1} = \max_{z \neq s} v^{r+1}_{z\bar{g}}$ denote the critical bids for bidder s during auction the rounds r and $r+1$, respectively. The critical bids B^r and B^{r+1} should satisfy $B^r > B^{r+1}$. If during rth auction round bidder s bids $b^r_{s\bar{g}}$ on GU $\bar{g}$, its potential utility is $u_s = u^r_{s\bar{g}} + u^{r+1}_{sg}$. In this case, only three distinct outcomes as described in (20) exists. In (20d), $\epsilon^{r+1}_{sg|\bar{g}} > 0$ implies a decrease in valuation on GU g during auction round $r+1$ given that GU $\bar{g}$ is already admitted. In (20a), bidder s is put on WAIT during auction round r and he loses GU g. In the auction round $r+1$, he also loses GU $\bar{g}$. In (20b), bidder s is put on WAIT during auction round r and he loses GU g but during auction round $r+1$, he wins GU $\bar{g}$. In (20c), bidder s wins GU $\bar{g}$ during auction round r and other bidders are put on WAIT. In the auction round $r+1$, only bidder s is allowed to submit a new bid $b^{r+1}_{sg} < b^r_{s\bar{g}}$. Still under (20c), if the new bid $b^{r+1}_{sg} < B^r$, then he loses GU g. In (20d), bidder s wins both the GUs.

On the contrary, suppose bidder s places his order of preference truthfully by bidding on item g in the rth auction round, and $\bar{g}$ in the $(r+1)$th auction round. The potential utility the bidder s will obtain is given in (21), where $\epsilon^{r+1}_{s\bar{g}|g} > 0$ implies a decrease in valuation on GU $\bar{g}$ during auction round $r+1$ given that GU g is already admitted. By comparing the overall utilities in (20b)–(20c) with (21c), we get $(v^r_{s\bar{g}} - B^r) < (v^r_{sg} - B^{r+1})$. Similarly, by comparing the overall utilities in (20d) with (21d)–(21e), we get $(v^r_{s\bar{g}} - B^r) + (v^r_{sg} - \epsilon^{r+1}_{sg|\bar{g}} - B^r) < (v^r_{sg} - B^r) + (v^r_{s\bar{g}} - \epsilon^{r+1}_{s\bar{g}|g} - B^{r+1})$. This concludes that the bidder s can get highest utility only by being truthful in terms of both the valuation and preference order. □

5.4 Uniqueness of the sDSE and the DSE

Both the sDSE and DSE require that, for all s, $\boldsymbol{v}^r$, and b^r_{sg}, bidder s has a high utility for playing strategy at v^r_{sg} than following any other strategy at b^r_{sg}, i.e.

$$v^r_{sg} \cdot x^r_{sg}(\boldsymbol{v}^r) - p^r_{sg}(\boldsymbol{v}^r) \geq v^r_{sg} \cdot x^r_{sg}(b^r_{sg}, \boldsymbol{v}^r_{-s}) - p^r_{sg}(b^r_{sg}, \boldsymbol{v}^r_{-s})\,. \tag{22}$$

$$u_s = \begin{cases} 0, & \text{if } b^r_{s\bar{g}} < B^{r+1}, \quad \text{(a)} \\ (v^r_{s\bar{g}} - B^r) + 0, & \text{if } B^{r+1} \leq b^r_{s\bar{g}} < B^r, \quad \text{(b)} \\ (v^r_{s\bar{g}} - B^r) + 0, & \text{if } b^r_{s\bar{g}} \geq B^r, b^{r+1}_{sg} < B^r, \quad \text{(c)} \\ (v^r_{s\bar{g}} - B^r) + (v^r_{sg} - \epsilon^{r+1}_{sg|\bar{g}} - B^r), & \text{if } b^r_{s\bar{g}} \geq B^r, b^{r+1}_{sg} \geq B^r, \quad \text{(d)} \end{cases} \tag{20}$$

$$\max\{0, u_s\} = \begin{cases} 0, & \text{if } v^r_{sg} < B^{r+1}, v^r_{s\bar{g}} < B^{r+1}, \quad \text{(a)} \\ 0, & \text{if } B^{r+1} \leq v^r_{sg} < B^r, v^r_{s\bar{g}} < B^{r+1}, \quad \text{(b)} \\ (v^r_{s\bar{g}} - B^{r+1}), & \text{if } B^{r+1} \leq v^r_{sg} < B^r, \quad \text{(c)} \\ & B^{r+1} \leq v^r_{s\bar{g}} < B^r, \\ (v^r_{sg} - B^r) + (v^r_{s\bar{g}} - \epsilon^{r+1}_{s\bar{g}|g} - B^{r+1}), & \text{if } v^r_{sg} \geq B^r, B^{r+1} \leq v^{r+1}_{s\bar{g}} < B^r, \quad \text{(d)} \\ (v^r_{sg} - B^r) + (v^r_{s\bar{g}} - \epsilon^{r+1}_{s\bar{g}|g} - B^{r+1}), & \text{if } v^r_{sg} \geq B^r, v^{r+1}_{s\bar{g}} \geq B^r\,. \quad \text{(e)} \end{cases} \tag{21}$$

We now derive a unique payment rule that will guard against insincere bidding such that the allocation rule $\boldsymbol{x}^r$ is implementable.

Definition 2: An implementable allocation rule is a function $\boldsymbol{x}^r$ which when coupled with payment rule $\boldsymbol{p}^r$ is such that $(\boldsymbol{x}^r, \boldsymbol{p}^r)$ is DSIC. An allocation rule is monotone if for every bidder s and for fixed bids $\boldsymbol{b}^r_{-s}$ of other bidders, the allocation $x^r_{sg}(b^r_{sg}, \boldsymbol{b}^r_{-s})$ to s is increasing in its bid b^r_{sg} [26].

The BWA payment rule should satisfy

$$p^r_{sg}(\boldsymbol{b}^r) \in [0, b^r_{sg} \cdot x_{sg}(\boldsymbol{b}^r)], \quad \forall s \in \mathcal{S} \tag{23}$$

where the lower bound ensures that no payment should be made by the auctioneer to the bidders. The upper bound guarantees a bidder that for as long as he bids truthfully, he will have non-negative utility. For completeness, we invoke Myerson's Lemma in [28] to derive a unique BWA payment rule.

Theorem 3 (Myerson's lemma): In a single-parameter environment, where the bidders have independent utility and quasilinear utility functions, a profile of allocation and payment rules $(\boldsymbol{x}^r, \boldsymbol{p}^r)$ is in sDSE and implementable only if for all $s \in \mathcal{S}$;

i. $\boldsymbol{x}^r$ is monotone and non-decreasing
ii. there is a unique payment rule given as $p^r_s(b^r_{sg}, \boldsymbol{v}^r_{-s}) = \int_0^{b^r_{sg}} b^r_{sg} \cdot x'_s(b^r_{sg}, \boldsymbol{v}^r_{-s})\,\mathrm{d}z$, where $b^r_{sg} = 0$ implies $p^r_s(0, \boldsymbol{v}^r_{-s}) = 0$.

Proof: Let us assume that $(\boldsymbol{x}^r, \boldsymbol{p}^r)$ is DSIC. Consider two possible bids $(\check{b}^r_{sg}, \hat{b}^r_{sg})$ from bidder s on item g during rth auction round such that $0 \le \check{b}^r_{sg} < \hat{b}^r_{sg}$. Assume that the private valuation of bidder s on its most preferred GU g during the rth auction round is $\hat{b}^r_{sg}$ but he underbids by submitting $\check{b}^r_{sg}$ instead. Using the DSIC principle in (22) we get (24). Similarly, if private valuation of the bidder s on GU g at the rth auction round is $\check{b}^r_{sg}$ but he overbids by submitting $\hat{b}^r_{sg}$ instead, we get (25).

$$\hat{b}^r_{sg} \cdot x_{sg}(\hat{b}^r_{sg}, \boldsymbol{b}^r_{-s}) - p^r_{sg}(\hat{b}^r_{sg}, \boldsymbol{b}^r_{-s}) \ge \hat{b}^r_{sg} \cdot x_{sg}(\check{b}^r_{sg}, \boldsymbol{b}^r_{-s}) - p^r_{sg}(\check{b}^r_{sg}, \boldsymbol{b}^r_{-s}) \,. \tag{24}$$

$$\check{b}^r_{sg} \cdot x_{sg}(\check{b}^r_{sg}, \boldsymbol{b}^r_{-s}) - p^r_{sg}(\check{b}^r_{sg}, \boldsymbol{b}^r_{-s}) \ge \check{b}^r_{sg} \cdot x_{sg}(\hat{b}^r_{sg}, \boldsymbol{b}^r_{-s}) - p^r_{sg}(\hat{b}^r_{sg}, \boldsymbol{b}^r_{-s}) \,. \tag{25}$$

$$\begin{aligned}\hat{b}^r_{sg}\left(x_{sg}(\check{b}^r_{sg}, \boldsymbol{b}^r_{-s}) - x_{sg}(\hat{b}^r_{sg}, \boldsymbol{b}^r_{-s})\right) &\le \left(p^r_{sg}(\check{b}^r_{sg}, \boldsymbol{b}^r_{-s}) - p^r_{sg}(\hat{b}^r_{sg}, \boldsymbol{b}^r_{-s})\right) \\ &\le \check{b}^r_{sg}\left(x_{sg}(\check{b}^r_{sg}, \boldsymbol{b}^r_{-s}) - x_{sg}(\hat{b}^r_{sg}, \boldsymbol{b}^r_{-s})\right)\end{aligned} \tag{26}$$

(see (27)) The payment difference $(p^r_{sg}(\check{b}^r_{sg}, \boldsymbol{b}^r_{-s}) - p^r_{sg}(\hat{b}^r_{sg}, \boldsymbol{b}^r_{-s}))$ from (24) and (25) is given by the sandwich theorem as shown in (26). Noting that $x^r_{sg}(\,\cdot\,, \boldsymbol{b}^r_{-s})$ is a piecewise constant, now applying the limit inequality theorem on (26), the change in payment is given in (27), where $\Delta_{|\hat{b}^r_{sg}}$ is the magnitude of change at $\hat{b}^r_{sg}$. Now the unique payment formula for each bidder at the rth auction round is given by

$$p^r_{sg}(b^r_{sg}, \boldsymbol{b}^r_{-s}) = \sum_{c=1}^{C^r_g} b^r_{cg} \cdot \Delta_{|b^r_{sg}} x^r_{sg}(\,\cdot\,, \boldsymbol{b}^r_{-s}), \tag{28}$$

where b^r_{cg} is the cth breakpoint of the allocation function $x^r_{sg}(\,\cdot\,, \boldsymbol{b}_{-s})$ in the range $[0, b^r_{sg}]$ during rth auction round, and $C^r_g = |\mathscr{C}^r_g \subseteq \mathscr{C}_g| \le C_g = |\mathscr{C}_g|$ is maximum number of active bidders in the competitive set of the gth GU. Now the overall payment formula for bidder s for the entire BWA is given as

$$p_s(\boldsymbol{b}_s, \boldsymbol{B}_{-s}) = \sum_{r=1}^{R} \sum_{c=1}^{C'_g} b^r_{cg} \cdot \Delta_{|b^r_{sg}} x^r_{sg}(\,\cdot\,, \boldsymbol{b}_{-s}), \tag{29}$$

The total revenue generated from the BWA is given by $\sum_{s \in \mathcal{S}} p_s(\boldsymbol{b}_s, \boldsymbol{B}_{-s})$. Since the allocation function $x^r_{sg}(\,\cdot\,, \boldsymbol{b}_{-s})$ is a bounded monotone function, it is continuous and differentiable. Let us assume that $\check{b}^r_{sg} = \hat{b}^r_{sg} + \mathrm{d}\hat{b}^r_{sg}$. Now dividing (28) by $\mathrm{d}\hat{b}^r_{sg}$ and following the same procedure as in (29) we get

$$\frac{\mathrm{d}}{\mathrm{d}\hat{b}^r_{sg}} p(\hat{b}^r_{sg}, \boldsymbol{b}^r_{-s}) = \hat{b}^r_{sg} \cdot \frac{\mathrm{d}}{\mathrm{d}\hat{b}^r_{sg}} x(\hat{b}^r_{sg}, \boldsymbol{b}^r_{-s}) \,. \tag{30}$$

The unique payment formula of every bidder during the rth auction in (28) can be rewritten as

$$p^r_s(b^r_{sg}, \boldsymbol{b}^r_{-s}) = \int_0^{b^r_{sg}} b^r_{sg} \cdot \frac{\mathrm{d}}{\mathrm{d}b^r_{sg}} x^r_{sg}(b^r_{sg}, \boldsymbol{b}^r_{-s})\,\mathrm{d}b^r_{sg} \tag{31}$$

which is in agreement with the second condition of Theorem 3. Hence the proof. □

Equations (28) and (29) show that the allocation and payment rules of BWA lead to a unique sDSE and ultimately a unique DSE. Note that a bidder only pays when he is assigned a GU(s).

6 Numerical example

We consider a network with one MBS and 25 SCAs. Each SCA is committed to serve one HU with a data rate target of 2 bits/s/Hz. The SCAs are only allowed to bid for GUs that fall within twice their nominal coverage radius. The cost per unit data rate and the cost per unit power were set to $\kappa = 0.1$ and $\mu = 0.00001$, respectively. All other model parameters are summarised in Table 1. We used MATLAB and CVX to simulate the proposed algorithms. We investigated the performance of the proposed methods by varying the target data rate for MUs while fixing the data rate for HUs at 2 bits/s/Hz. The proposed algorithms are compared with the simultaneous ascending auction (SAA)-based algorithm proposed in [29, 30]. In the SAA, the auctioneer gradually increase the price or payment for serving users with a small value δ. The bidders then indicate which set of GUs they are interested on for a given price. This is repeated until there is only one bidder who expresses an interest to each GU. Fig. 4*a* shows the average number of MUs admitted by the SCAs and the MBS (shown separately). For a given preference criterion, the BBWA admits more users than the FBWA. Also, the FPP criterion admits more users than APP criterion for a given algorithm. In terms of surplus maximisation, the BBWA with FPP criterion is most

$$\begin{aligned}&\lim_{\check{b}^r_{sg} \to \hat{b}^r_{sg}} \left[\hat{b}^r_{sg}\left(x_{sg}(\check{b}^r_{sg}, \boldsymbol{b}^r_{-s}) - x_{sg}(\hat{b}^r_{sg}, \boldsymbol{b}^r_{-s})\right)\right] \le \lim_{\check{b}^r_{sg} \to \hat{b}^r_{sg}} \left[\left(p^r_{sg}(\check{b}^r_{sg}, \boldsymbol{b}^r_{-s}) - p^r_{sg}(\hat{b}^r_{sg}, \boldsymbol{b}^r_{-s})\right)\right] \\ &\le \lim_{\check{b}^r_{sg} \to \hat{b}^r_{sg}} \left[\check{b}^r_{sg}\left(x_{sg}(\check{b}^r_{sg}, \boldsymbol{b}^r_{-s}) - x_{sg}(\hat{b}^r_{sg}, \boldsymbol{b}^r_{-s})\right)\right] = \Delta_{|\hat{b}^r_{sg}} p^r_{sg} = \hat{b}^r_{sg} \cdot \Delta_{|\hat{b}^r_{sg}} x_{sg}(b^r_{sg}, \boldsymbol{b}^r_{-s}),\end{aligned} \tag{27}$$

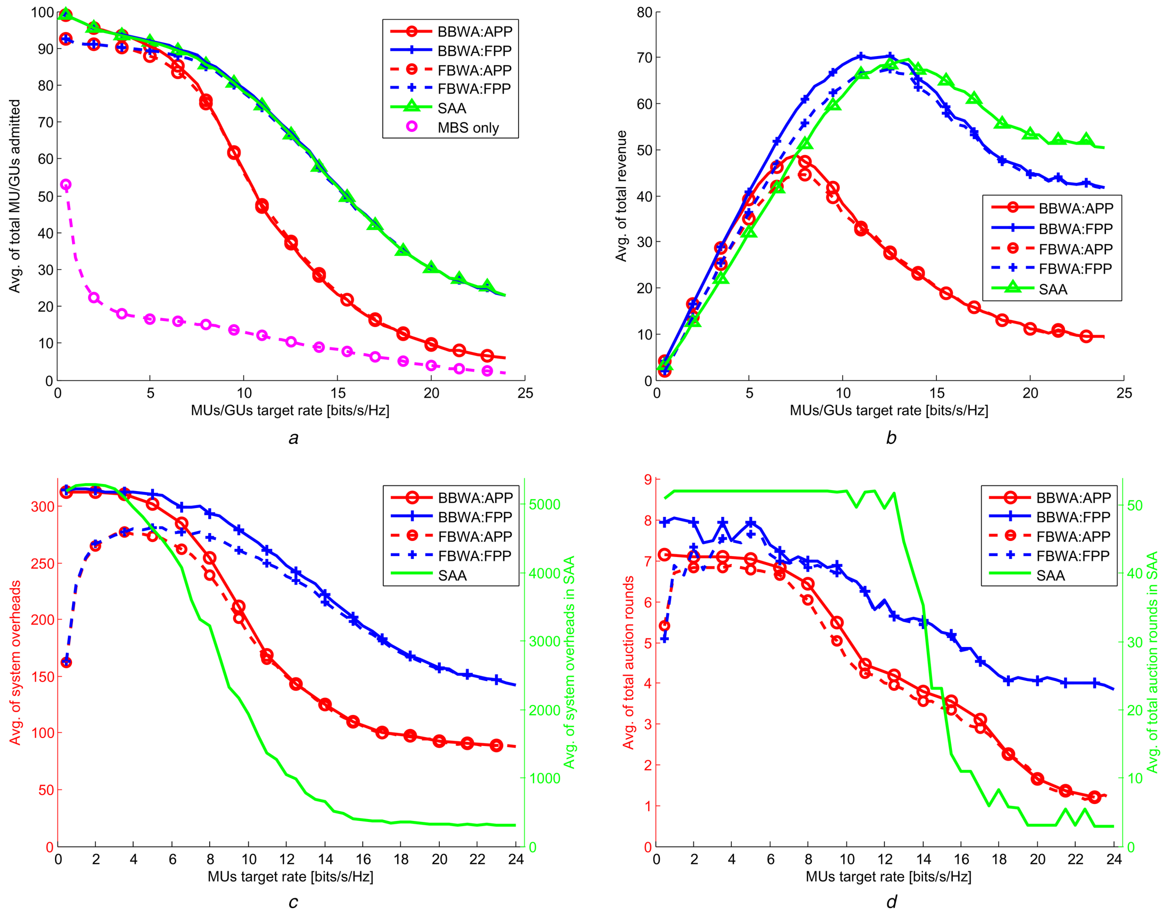

Fig. 4 *Average performance of the proposed BBWA and FBWA for 20 channel realisations. There are 100 MUs and 25 SCAs*
(a) Average MUs/GUs admitted by the SCAs, ***(b)*** Average revenue generated through auctioning, ***(c)*** Average system overheads incurred, ***(d)*** Average auction rounds

Table 1 The parameters for numerical evaluation

Description/parameter	Value
macrocell radius	500 m
small cell radius	30 m
MBS downlink transmit power $p_0^{\max}$	46 dBm
SCA downlink transmit power $p_s^{\max}$	20 dBm
MBS path and penetration loss at *d* (km)	$128.1 + 37.6\log_{10}(d)$ dB
SCA path and penetration loss at *d* (km)	$127 + 30\log_{10}(d)$ dB
log normal shadowing standard deviation	7 dB
MBS-MUs minimum distance constraint	35 m
SCA-MUs minimum distance constraint	3 m
noise variance σ^2	−127 dBm
wall attenuation	20 dB
number of MUs	100
number of HUs per SCA	1
number of MBS antennas M_{mbs}	50
number of SCA antennas M_{sca}	8
small-scale fading distribution	$\boldsymbol{h}_{jk} \sim \mathscr{CN}(\mathbf{0}, \boldsymbol{R}_{jk})$

preferred. The BBWA with FPP criterion admits more users when compared with the SAA algorithm.

As seen in Fig. 4*b*, in terms of revenue generation, the SCAs would prefer the BBWA with the FPP criterion. However, as the primary intention of the MBS is to minimise the dropped users, it will also prefer BBWA algorithm. In comparison to SAA algorithm, BBWA with FPP criterion generates more revenue at lower targets rates, while the SAA algorithm generates more revenue at higher targets rates. This is due to the following reason. The competition is very strong among the bidders at lower target rates than at higher target rates. In SAA algorithm, bidders (SCAs) pay the bid they submitted rather than the second highest bid from the set of competitors. Hence when there is high competition among SCAs, the price being paid for the GUs in the SAA is higher than that being paid in the BBWA with FPP criterion. In contrast, when the competition is low, the bidders will pay less under SAA than that under BBWA with FPP criterion.

We also compared the average system overheads measured in terms of the number of invitations for bidding, the number of bids submitted and the number of announcements made. As seen in Fig. 4*c*, the system overhead drops with increasing target data rate. This is because with increasing target data rate, the SCAs will reach its admission capacity quickly and there is no need for further auctioning. The average number of auction rounds is also compared in Fig. 4*d*. For the same reason, the number of auction rounds drops with the increasing data rate. To reduce the system overheads in SAA, the price increment step was set as $\delta = 0.001 \times (\text{target data rate})/(0.5)$ (bits/s/Hz). Regardless of this price increment adaptation, it is observed in Figs. 4*c* and *d* that the proposed algorithms outperform the SAA algorithm in terms of both the system overheads and the auction rounds.

In Fig. 5, we compared the performance of the BWA to a centralised solution proposed in [31]. We considered six MUs and two SCAs. As seen in Fig. 5*a*, as the target data rate of the MUs is increased, the total transmission power increases exponentially. Starting from target data rate of 10.5 bits/s/Hz, the average number of admitted users at SCA 1 drops from 3 to 2.75. Consequently, the total transmission power is also dropped. A similar trend is observed in Fig. 5*b* for the BWA. By comparing Figs. 5*a* and *b*, it is observed that the BWA is close to optimal.

7 Conclusion

We have proposed a framework that performs user association and beamforming in a wireless downlink heterogeneous network through auctioning. We considered two scenarios. In the first scenario, the MBS admits as many users as it can serve and then

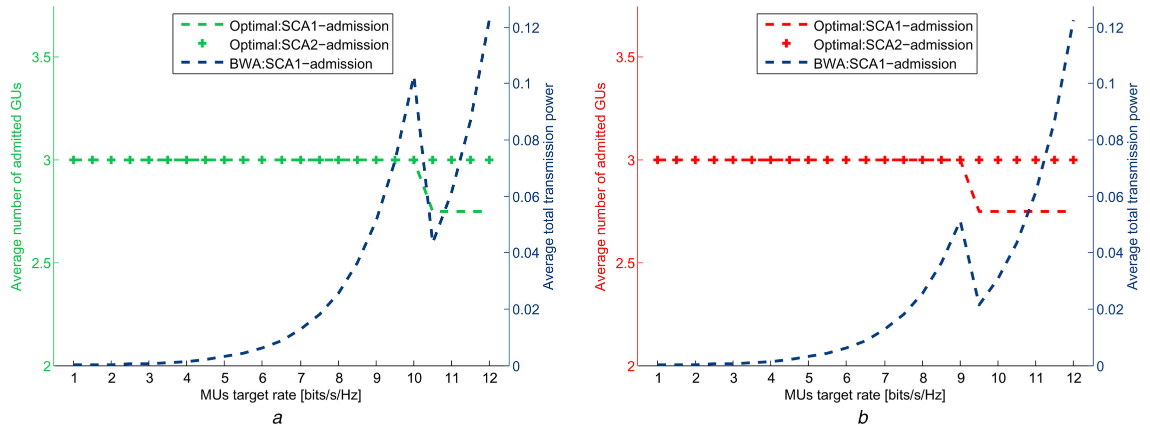

Fig. 5 *Comparison of the BWA and the optimal solution*
(a) Average MUs admitted by the SCAs and total power consumption under centralised solution, ***(b)*** Average MUs admitted by the SCAs and total power consumption under BWA

auctions off the remaining users to SCAs. This is solved using the FBWA algorithm. In the second scenario, the MBS auctions off as many users as possible to the SCAs and then admits a largest possible set of users from the remaining users. This is solved using the BBWA algorithm. The results show that the BBWA with FPP criterion is preferred by the MBS as well as the SCAs. The proposed algorithm is able to provide closer to optimal solution with significant saving in complexity.

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