

Continuous nonsingular terminal sliding mode control for systems with mismatched disturbances [★]

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Abstract

A continuous nonsingular terminal sliding mode control approach is proposed for mismatched disturbance attenuation. A novel nonlinear dynamic sliding mode surface is designed based on a finite-time disturbance observer. The time taken to reach the desired setpoint from any initial states under mismatched disturbance is guaranteed to be finite time. In addition, the proposed method exhibits the fine properties of nominal performance recovery as well as chattering alleviation.

Key words: Nonsingular terminal sliding mode; Finite-time disturbance observer; Mismatched disturbance

1 Introduction

Sliding mode control (SMC) is well known for its fine robustness against disturbances and parameter uncertainties (Utkin, 1992). Among the SMC community, nonsingular terminal SMC (NTSMC) has been widely studied since it can achieve finite-time fast convergence property without causing any singularity problem encountered in the traditional terminal SMC (TSMC) (Feng, Yu, & Man, 2002; Yu, Yu, Shirinzadeh, & Man, 2005; Zong, Zhao, & Zhang, 2008). However, the existing NTSMC is only insensitive to matched disturbances and can not attenuate mismatched disturbances effectively. In addition, the existing NTSMC law is essentially discontinuous and the control chattering problem is unavoidable.

Mismatched disturbances widely exist in various control engineering systems (Yang, Li, & Yu, 2012) and bring big challenges for the SMC design. Due to the significance of attenuating mismatched uncertainties in

practical applications, combination of robust techniques with SMC has been studied in literature, for example see Choi (2002); Cao & Xu (2004); Cheng (2009); Estrada & Fridman (2010); Kim, Park, & Oh (2000) and the references therein. Roughly speaking, the aforementioned SMC methods can be divided into the following two categories. The first category mainly focuses on the stability (or robust stability) of different systems under mismatched structure uncertainties via some classical control design tools, such as Riccati approach (Kim, Park, & Oh, 2000), LMI-based approach (Choi, 2002), adaptive approach (Cheng, 2009) and backstepping approach (Estrada & Fridman, 2010). The second category is referred to as integral sliding-model control (I-SMC) (Cao & Xu, 2004). The idea behind the I-SMC is that a high frequency switching gain is designed to force the states to achieve the integral sliding surface, and then the integral action in the sliding surface drives the states to the desired equilibrium in the presence of mismatched uncertainties.

Note that all the above two categories of SMC methods handle the mismatched uncertainties in a robust way, which implies that the uncertainty attenuation ability is achieved at the price of sacrificing its nominal control performance. Yang, Li, & Yu (2012) proposed a linear dynamic sliding surface design method based on a disturbance observer to attenuate the mismatched disturbance without sacrificing the nominal performance. However, the sliding motion from the sliding surface to the desired setpoint therein is asymptotical convergence rather than finite-time convergence due to the existence of lin-

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ear switching manifold. In Estrada & Fridman (2010), a quasi-continuous high-order SMC method via backstepping design is proposed to achieve finite-time tracking control regardless of mismatched disturbance, where the controller is still discontinuous in the sliding manifolds.

In this paper, a novel continuous NTSMC method is proposed for the finite-time control of system subject to mismatched disturbances via a finite-time disturbance observer (FTDO). A new nonlinear dynamic sliding surface which introduces the disturbance estimation is designed to guarantee the finite-time sliding motion in the presence of mismatched disturbance. The basic principle of the proposed method is concluded by the following three steps. Firstly, the error dynamics of the FTDO converge to zero in finite time under appropriate designed parameters. Secondly, a non-smooth but continuous control law is designed to force the initial states outside the sliding surface to reach the designed sliding surface in finite time. Finally, with the designed control law, the system states are driven to the desired setpoint in finite time by sliding motion along the sliding surface even in the presence of mismatched disturbance.

2 Motivations

Consider the following dynamic system under mismatched disturbances, depicted by

$$\begin{aligned}\dot{x}_i &= x_{i+1} + d_i(\bar{x}_i, t), \quad i = 1, \dots, n-1, \\ \dot{x}_n &= a(x) + b(x)u + d_n(x, t), \\ y &= x_1,\end{aligned}\quad (1)$$

where $x = [x_1 \cdots x_n]^T$ is the state vector, $\bar{x}_i = [x_1 \cdots x_i]^T$, u is the control input, y is the controlled output, $d_n(x, t)$ and $d_i(\bar{x}_i, t)$ denote the matched and mismatched disturbances, respectively. $a(x)$ and $b(x) \neq 0$ are smooth nonlinear functions in terms of x .

Taking a second-order system as an illustration (i.e., $n = 2$ for system (1)), the nonlinear sliding surface for existing NTSMC method (Feng, Yu, & Man, 2002) is usually defined as follows

$$s = x_1 + \frac{1}{\beta} x_2^{p/q}, \quad (2)$$

where $\beta > 0$ is a design constant, p and q are positive odd integers which satisfy the condition that $1 < p/q < 2$.

The NTSMC law is usually designed as follows

$$u = -b^{-1}(x) \left[a(x) + \beta \frac{q}{p} x_2^{2-p/q} + k \operatorname{sgn}(s) \right], \quad (3)$$

where k is the switching gain to be designed. Combining (2) and (3) gives

$$\dot{s} = -\frac{1}{\beta} \frac{p}{q} x_2^{p/q-1} [k \operatorname{sgn}(s) - d_n] + d_1. \quad (4)$$

In the absence of mismatched disturbances, i.e., $d_1 = 0$, it can be concluded from Feng, Yu, & Man (2002) that the conventional NTSMC law (3) with appropriately chosen parameters (actually $k > \sup |d_n|$) can drive arbitrary initial states of system (1) to the equilibrium point $x = 0$ in finite time, which implies that the conventional NTSMC is insensitive to matched disturbances.

However, in the presence of mismatched disturbances, i.e., $d_1 \neq 0$, two problems appear for the conventional NTSMC method. Firstly, it is not easy to determine the switching gain k such that the states of system (1) initially outside the sliding surface will reach the sliding surface $s = 0$ in finite time. Secondly, even if the sliding surface $s = 0$ is reached, the system dynamics is determined by the following nonlinear differential equation $\dot{x}_1 = -x_2 + d_1 = -\beta^{q/p} x_1^{q/p} + d_1$, which implies that the output $y = x_1$ of system (1) is affected by the mismatched disturbance d_1 , and does not converge to zero in finite time. To this end, it is imperative to address the disturbance rejection problem of the NTSMC design method in the case of mismatching condition.

3 Main results

Suppose that the disturbance d_i in (1) is $(n-i+1)$ th order differentiable and $d_i^{[n-i+1]}$ has a Lipschitz constant L_i . A finite-time disturbance observer (FTDO) (Shtessel, Shkolnikov, & Levant, 2007) is firstly used to estimate the disturbance in system (1), given by

$$\begin{aligned}\dot{z}_0^i &= v_0^i + f_i(x, u), \quad \dot{z}_1^i = v_1^i, \dots, \dot{z}_{n-i+1}^i = v_{n-i+1}^i \\ v_0^i &= -\lambda_0^i L_i^{\frac{1}{n-i+2}} |z_0^i - x_i|^{\frac{n-i+1}{n-i+2}} \operatorname{sgn}(z_0^i - x_i) + z_1^i, \\ v_j^i &= -\lambda_j^i L_i^{\frac{1}{n-i+2-j}} |z_j^i - v_{j-1}^i|^{\frac{n-i+1-j}{n-i+2-j}} \operatorname{sgn}(z_j^i - v_{j-1}^i) + z_{j+1}^i, \\ v_{n-i+1}^i &= -\lambda_{n-i+1}^i L_i \operatorname{sgn}(z_{n-i+1}^i - v_{n-i}^i), \\ \hat{x}_i &= z_0^i, \quad \hat{d}_i = z_1^i, \quad \hat{d}_i = z_2^i, \dots, \quad \hat{d}_i^{[n-i]} = z_{n-i+1}^i,\end{aligned}\quad (5)$$

for $i = 1, \dots, n$ and $j = 0, 1, \dots, n-i+1$, where $f_i(x, u) = x_{i+1}$ for $i = 1, \dots, n-1$, $f_n(x, u) = a(x) + b(x)u$, $\lambda_j^i > 0$ is the observer coefficients to be designed, $\hat{x}_i, \hat{d}_i, \hat{d}_i, \hat{d}_i^{[n-i]}$ are the estimates of $x_i, d_i, \dot{d}_i, d_i^{[n-i]}$, respectively. Combining (1) with (5), the observer estimation error is governed by

$$\begin{aligned}\dot{e}_0^i &= -\lambda_0^i L_i^{\frac{1}{n-i+2}} |e_0^i|^{\frac{n-i+1}{n-i+2}} \operatorname{sgn}(e_0^i) + e_1^i, \\ \dot{e}_j^i &= -\lambda_j^i L_i^{\frac{1}{n-i+2-j}} |e_j^i - \dot{e}_{j-1}^i|^{\frac{n-i+1-j}{n-i+2-j}} \operatorname{sgn}(e_j^i - \dot{e}_{j-1}^i) + e_{j+1}^i, \\ \dot{e}_{n-i+1}^i &\in -\lambda_{n-i+1}^i L_i \operatorname{sgn}(e_{n-i+1}^i - \dot{e}_{n-i}^i) + [-L_i, L_i],\end{aligned}\quad (6)$$

where the estimation errors are defined as $e_0^i = z_0^i - x_i$, $e_j^i = z_j^i - \dot{d}_i^{[j-1]}$. It follows from Shtessel, Shkolnikov, & Levant (2007) that the observer error system (6) is finite-time stable, that is, there is a finite time such that $e_j^i(t) = 0$.

3.1 A second-order system case

A novel nonlinear dynamic sliding mode surface for system (1) in the case $n = 2$ is defined by

$$s = x_1 + \frac{1}{\beta}(x_2 + \hat{d}_1)^{p/q}, \quad (7)$$

where β , p , and q have been defined in (2), \hat{d}_1 is the disturbance estimation given by FTDO (5).

Theorem 1. For system (1) in the case $n = 2$ with the proposed novel nonlinear sliding mode surface (7), if the new NTSMC law is designed as

$$u = -b^{-1}(x) \times \left[a(x) + \beta \frac{q}{p}(x_2 + \hat{d}_1)^{2-p/q} + \hat{d}_2 + v_1^1 + K_1 s + K_2 \text{sgn}(s)|s|^\alpha \right], \quad (8)$$

where $K_1, K_2 > 0$, $0 < \alpha < 1$ is the parameters to be designed, and v_1^1 has been given in (5), then the system output $y = x_1$ will converge to zero in finite time.

Proof. For the proposed sliding surface (7), its derivative along the system dynamics (1) is

$$\begin{aligned} \dot{s} &= \dot{x}_1 + \frac{1}{\beta} \frac{p}{q}(x_2 + \hat{d}_1)^{p/q-1}(\dot{x}_2 + \dot{\hat{d}}_1) \\ &= -\frac{1}{\beta} \frac{p}{q} \tilde{x}_2^{p/q-1} [K_1 s + K_2 \text{sgn}(s)|s|^\alpha + e_1^2] - e_1^1, \end{aligned} \quad (9)$$

where $\tilde{x}_2 = x_2 + \hat{d}_1$. Substituting the control law (8) into the system (1), yields

$$\dot{\tilde{x}}_2 = -\beta \frac{q}{p} \tilde{x}_2^{2-p/q} - K_1 s - K_2 \text{sgn}(s)|s|^\alpha - e_1^2. \quad (10)$$

Define a finite time bounded (FTB) function (Li, & Tian, 2007) $V_1(s, x_1, \tilde{x}_2) = \frac{1}{2}(s^2 + x_1^2 + \tilde{x}_2^2)$ for the sliding mode dynamics (9) and the state dynamics (10). Note that $|s|^\alpha < 1 + |s|$. Taking the derivative of $V_1(s)$ along system (9) yields

$$\begin{aligned} \dot{V}_1 &= -\frac{1}{\beta} \frac{p}{q} \tilde{x}_2^{p/q-1} (K_1 s^2 + K_2 |s|^{\alpha+1} + e_1^2 s) + x_1(\tilde{x}_2 - e_1^1) \\ &\quad - e_1^1 s + \tilde{x}_2(-\beta \frac{q}{p} \tilde{x}_2^{2-p/q} - K_1 s - K_2 \text{sgn}(s)|s|^\alpha - e_1^2) \\ &\leq \frac{1}{\beta} \frac{p}{q} (1 + |\tilde{x}_2|) |e_1^2 s| + |x_1 \tilde{x}_2| + |x_1 e_1^1| + |e_1^1 s| \\ &\quad + |\tilde{x}_2| [K_1 |s| + K_2 (1 + |s|)] + |\tilde{x}_2 e_1^2| \\ &\leq \frac{1}{\beta} \frac{p}{q} \left[\frac{(e_1^2)^2 + s^2}{2} + |e_1^2| \frac{\tilde{x}_2^2 + (e_1^2)^2}{2} \right] + \frac{x_1^2 + \tilde{x}_2^2}{2} + \frac{x_1^2 + (e_1^1)^2}{2} \\ &\quad + \frac{(e_1^1)^2 + s^2}{2} + K_2 \frac{1 + \tilde{x}_2^2}{2} + (K_1 + K_2) \frac{s^2 + \tilde{x}_2^2}{2} + \frac{(e_1^2)^2 + \tilde{x}_2^2}{2} \\ &\leq K_{v1} V_1 + L_{v1}, \end{aligned} \quad (11)$$

where $K_{v1} = \max\{1 + K_1 + K_2 + \frac{p}{\beta q}, 2 + K_1 + 2K_2 + \frac{p}{\beta q}|e_1^2|, 2\}$, and $L_{v1} = \max\{(e_1^1)^2 + \frac{K_2}{2} + \frac{1}{2} \left(1 + \frac{p}{\beta q}\right) (e_1^2)^2 + \frac{p}{2\beta q} (e_1^2)^3\}$ are bounded constants due to the boundness of e_1^1 and e_1^2 . It can be concluded from

(11) that $V_1(s, x_1, \tilde{x}_2)$ and so s, x_1, \tilde{x}_2 will not escape in finite time (Li, & Tian, 2007).

Since the disturbance estimation errors e_1^1 and e_1^2 in (6) will converge to zero in a finite time, after then, system (9) then reduces to

$$\dot{s} = -\rho(\tilde{x}_2) [K_1 s + K_2 \text{sgn}(s)|s|^\alpha], \quad (12)$$

where $\rho(\tilde{x}_2) = \frac{1}{\beta} \frac{p}{q} \tilde{x}_2^{p/q-1}$. Next we will show that (12) is finite-time stable. The idea of the proof procedure is inspired from Feng, Yu, & Man (2002).

For the case of $\tilde{x}_2 \neq 0$, it can be followed from $\rho(\tilde{x}_2) > 0$ that (12) is finite-time stable. For $\tilde{x}_2 = 0$, it is obtained from (10) that $\dot{\tilde{x}}_2 = -K_1 s - K_2 \text{sgn}(s)|s|^\alpha$. Similar with the proof of Feng, Yu, & Man (2002), it can be shown that $\tilde{x}_2 = 0$ is not an attractor. Therefore, it can be concluded that system (12) is finite-time stable.

Once the sliding surface $s = 0$ is reached, it is derived from the sliding surface (7) and the system dynamics (1) that

$$s = x_1 + \frac{1}{\beta}(x_2 + d_1)^{p/q} = x_1 + \frac{1}{\beta} \dot{x}_1^{p/q} = 0. \quad (13)$$

With the chosen control parameters, system (13) is finite-time stable, which completes the proof. \square

3.2 A general high-order system case

A novel sliding surface for system (1) in the case $n > 2$ is designed as

$$\begin{aligned} s &= \tilde{x}_n + \int_0^\tau \sum_{i=1}^n [k_i \text{sgn}(\tilde{x}_i) |\tilde{x}_i|^{\alpha_i}] d\tau, \\ \tilde{x}_1 &= x_1, \tilde{x}_i = x_i + \sum_{j=1}^{i-1} \tilde{d}_j^{[i-j-1]}, \quad i = 2, \dots, n, \end{aligned} \quad (14)$$

where $\alpha_{i-1} = \alpha_i \alpha_{i+1} / (2\alpha_{i+1} - \alpha_i)$ ($i = 2, \dots, n$), $\alpha_{n+1} = 1$, $\alpha_n = \alpha_0 \in (1 - \epsilon, 1)$, $\epsilon \in (0, 1)$, and $k_i > 0$ should be selected such that polynomial $\lambda^n + k_n \lambda^{n-1} + \dots + k_2 \lambda + k_1$ is Hurwitz.

Theorem 2. For high-order system (1) with the proposed nonlinear sliding mode surface (14), if the generalized continuous NTSMC law is designed as

$$\begin{aligned} u &= -b^{-1}(x) \times \left\{ a(x) + \hat{d}_n + \sum_{i=1}^{n-1} v_{n-i}^i \right. \\ &\quad \left. + \sum_{i=1}^n [k_i \text{sgn}(\tilde{x}_i) |\tilde{x}_i|^{\alpha_i}] + K_1 s + K_2 \text{sgn}(s)|s|^\alpha \right\}, \end{aligned} \quad (15)$$

where v_{n-i}^i has been given in (5), then the system output $y = x_1$ will converge to zero in finite time.

Proof. Taking derivative of the sliding surface (14) along system dynamics (1) under control law (15) gives

$$\dot{s} = -K_1 s - K_2 \text{sgn}(s)|s|^\alpha - e_1^n. \quad (16)$$

The dynamics of states \tilde{x}_i are obtained from (14), governed by

$$\begin{aligned} \dot{\tilde{x}}_i &= \tilde{x}_{i+1} + \tilde{e}_i, \quad i = 1, \dots, n-1, \\ \dot{\tilde{x}}_n &= -\sum_{i=1}^n [k_i \text{sgn}(\tilde{x}_i) |\tilde{x}_i|^{\alpha_i}] + \dot{s}, \end{aligned} \quad (17)$$

where $\tilde{e}_1 = -e_1^1$, $\tilde{e}_i = \sum_{j=1}^{i-1} (\dot{e}_{i-j}^j - e_{i-j+1}^j) - e_1^i$ for $i = 2, \dots, n-1$. The dynamics (17) implies that the states suffer from the sliding surface dynamics (16) and the observer error dynamics (6). Next we will show the observer error dynamics (6) will not drive the sliding surface dynamics (16) and the state dynamics (17) to infinity in finite time.

Define a FTB function $V_2(s, \tilde{x}) = \frac{1}{2}s^2 + \sum_{i=1}^n \frac{1}{2}\tilde{x}_i^2$ for system (17). Note that the parameter α_i ($i = 1, \dots, n$) satisfy the condition $\alpha_i \in (0, 1)$, which implies that $|\tilde{x}_i|^{\alpha_i} < 1 + |\tilde{x}_i|$. Taking derivative of $V_2(\tilde{x})$ along dynamics (17), one obtains

$$\begin{aligned} \dot{V}_2 &= s\dot{s} + \sum_{i=1}^{n-1} \tilde{x}_i(\tilde{x}_{i+1} + \tilde{e}_i) \\ &\quad - \tilde{x}_n \sum_{i=1}^n k_i \text{sgn}(\tilde{x}_i) |\tilde{x}_i|^{\alpha_i} + \tilde{x}_n \dot{s} \\ &\leq |se_1^1| + \sum_{i=1}^{n-1} |\tilde{x}_i|(|\tilde{x}_{i+1}| + |\tilde{e}_i|) \\ &\quad + |\tilde{x}_n| \sum_{i=1}^n k_i (1 + |\tilde{x}_i|) \\ &\quad + |\tilde{x}_n| (K_1|s| + K_2(1 + |s|) + |e_1^n|) \\ &\leq \frac{s^2 + (e_1^n)^2}{2} + \sum_{i=1}^{n-1} \frac{\tilde{x}_i^2 + \tilde{x}_{i+1}^2}{2} \\ &\quad + \sum_{i=1}^{n-1} \frac{\tilde{x}_i^2 + \tilde{e}_i^2}{2} + \frac{\tilde{x}_n^2 + (\sum_{i=1}^n k_i)^2}{2} + \sum_{i=1}^n k_i \frac{\tilde{x}_i^2 + \tilde{x}_n^2}{2} \\ &\quad + \frac{K_1 + K_2}{2} (\tilde{x}_n^2 + s^2) + \frac{\tilde{x}_n^2 + (K_2 + |e_1^n|)^2}{2} \\ &\leq K_{v2} V_2 + L_{v2} \end{aligned} \quad (18)$$

where $K_{v2} = 3 + k_n + \sum_{i=1}^n k_i + K_1 + K_2$, and $L_{v2} = \frac{1}{2} \max \left[(e_1^n)^2 + \sum_{i=1}^{n-1} \tilde{e}_i^2 + (\sum_{i=1}^n k_i)^2 + (K_2 + |e_1^n|)^2 \right]$. Eq. (6) shows that the estimation error e_j^i will converge to zero in finite time regardless of the states x_i , which implies that \tilde{e}_i , e_1^n and so L_{v2} are bounded. Therefore, it can be concluded from (18) that $V_2(s, \tilde{x})$ and so the state \tilde{x}_i will not escape to infinity in finite time before the convergence of observer error dynamics (Li, & Tian, 2007). Since the disturbance estimation errors e_1^1 and e_1^2 in (6) will converge to zero in a finite time, after then, system (16) then reduces to

$$\dot{s} = -K_1 s - K_2 \text{sgn}(s) |s|^\alpha, \quad (19)$$

which is finite time stable. Once the sliding surface $s = 0$ and disturbance estimation error $e_j^i = 0$ are achieved in a finite time, the system dynamics (17) will reduce to the following system

$$\dot{\tilde{x}}_i = \tilde{x}_{i+1}, \quad \dot{\tilde{x}}_n = - \sum_{i=1}^n [k_i \text{sgn}(\tilde{x}_i) |\tilde{x}_i|^{\alpha_i}], \quad (20)$$

for $i = 1, \dots, n-1$. It is derived from Bhat & Bernstein (2005) that (20) is finite-time stable, which completes the proof. \square

Remark 1 (Nominal Performance Recovery). In the absence of disturbance, it is derived from the observer

error dynamics (6) that $e_j^i(t) = 0$ and $v_{n-i}^i(t) = 0$ if the initial values of the observer states are selected as $z_0^i(t_0) = x_i(t_0)$ and $z_1^i(t_0) = \dots = z_{n-i+1}^i(t_0) = 0$. In this case, the proposed sliding surface (7) and the control law (8) reduce to those of the traditional NTSMC, also the proposed sliding surface (14) and the control law (15) reduce to the high-order NTSMC in Zong, Zhao, & Zhang (2008). This implies that the nominal control performance of the proposed method is retained.

4 Simulation example

Consider the simulation example of a permanent magnet synchronous motor (PMSM) in Liu, & Li (2012)

$$\begin{aligned} \frac{d\omega}{dt} &= \frac{K_t}{J} i_q - \frac{B}{J} \omega - \frac{1}{J} T_L, \\ \frac{di_d}{dt} &= -\frac{R_s}{L_d} i_d + n_p \omega i_q + \frac{1}{L_d} u_d, \\ \frac{di_q}{dt} &= -\frac{R_s}{L_q} i_q - n_p \omega i_d - \frac{n_p \phi_v}{L_q} \omega + \frac{1}{L_q} u_q, \end{aligned} \quad (21)$$

where ω the rotor speed, i_d and i_q the d -axis and q -axis stator currents, u_d and u_q the d -axis and q -axis stator voltages, T_L the load torque, respectively. The significance and values of the PMSM parameters under consideration are referred to Liu, & Li (2012).

By appropriate coordinate transformation (Yang, Liu, Li, & Chen, 2012), the PMSM model (21) can be transferred to two subsystems, including a two-order speed one and a one-order current one, where the speed one satisfies the formation of (1). The proposed method is employed for the speed subsystem of the PMSM system. In addition, to show the effectiveness of the proposed method, the baseline NTSM controller, I-SMC and DO-LSMC (Yang, Li, & Yu, 2012) are employed for performance comparison. The same control design of the one-order current subsystem (i_d loop) is employed for the four methods, which is omitted here for space due to its simplicity.

The parameters of the proposed method are designed as $\beta = 4 \times 10^4$, $p = 5$, $q = 3$, $K_1 = K_2 = 4.2 \times 10^5$, $\lambda_0 = 2$, $\lambda_1 = 1.5$, $\lambda_2 = 0.01$, $L = 100$. The parameters of baseline NTSM controller are designed the same as the proposed method. The parameters of DO-LSMC are designed as $c = 55$, $k = 1.5 \times 10^7$, $l = 500$. The parameters of I-SMC are designed as $c_1 = 180$, $k = 1.5 \times 10^7$.

The disturbance (unknown load torque variation) $T_L(t) = 2$ for $0.2 \leq t < 0.4$ and $T_L(t) = 2 + \sin 20\pi t$ for $0.4 \leq t \leq 0.8$ is supposed to impose on the system. The simulations are carried out in a Gaussian measurement noise environment that with mean of 0 and covariance of $\text{diag}\{5, 0.001, 0.005\}$ for practicality. Response curves of the state variables under the four methods are shown by Figs. 1. The corresponding control signals are shown in Fig. 2.

It is observed from Figs. 1 that the state responses of the proposed method are the same as those of the baseline NTSMC method during the first 0.2 sec, which shows nominal performance recovery property as stated in Remark 1. As shown by Fig. 1, the DO-LSMC and

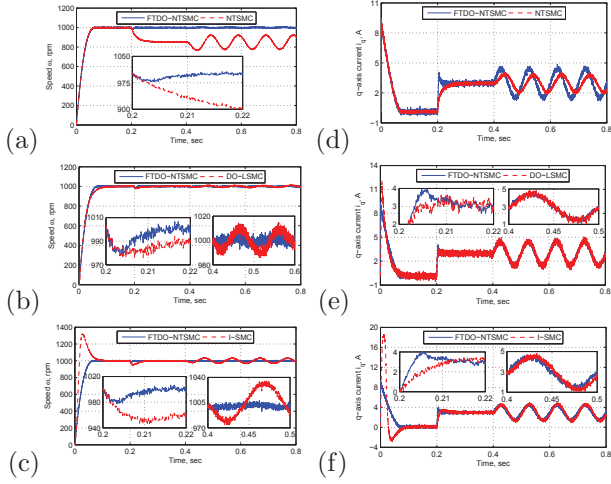


Fig. 1. State response curves of the PMSM under FTDO-NTSMC (solid line) and the other three control methods (dashed line) in measurement noise environment.

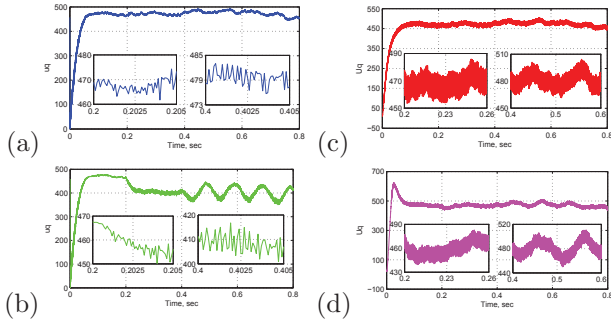


Fig. 2. Control signals of the PMSM under the four control methods in measurement noise environment: (a) FTDO-NTSMC; (b) NTSMC; (c) DO-LSMC; and (d) I-SMC.

the I-SMC methods only attenuate the harmonic disturbance to a specified small region, while the proposed method has removed such disturbance completely.

5 Conclusion

The continuous finite-time control problem of the system with mismatched disturbance has been addressed in this paper by using nonsingular terminal sliding mode technique. A novel nonlinear dynamic sliding mode surface design has been proposed for the mismatched disturbance attenuation via a finite-time disturbance observer. The proposed method has the following two remarkable properties. Firstly, the proposed method retains the nominal control performance since the FTDO serves like a patch to the baseline NTSMC and does not cause any adverse effects on the system in the absence of disturbance. Secondly, the proposed method largely alleviates the chattering problem of NTSMC since the mismatched disturbance has been compensated by FTDO-based compensation and no discontinuous control action is required to reject the disturbance.

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