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Doctoral Thesis

Distributed Optimisation Techniques for Wireless Networks

Author:
Bokamoso Basutli

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Signal Processing and Networks Research Group
Wolfson School of Mechanical, Manufacturing and Electrical Engineering

Certificate of Originality

This is to certify that I am responsible for the work submitted in this thesis, that the original work is my own work except as specified in acknowledgements or in footnotes, and that neither the thesis nor the original work contained therein has been submitted to this or any other institution for a degree.

Signed: ______________________________

Candidate: ______________________________

To my wife and children.

Abstract

Alongside the ever increasing traffic demand, the fifth generation (5G) cellular network architecture is being proposed to provide better quality of service, increased data rate, decreased latency, and increased capacity. Without any doubt, the 5G cellular network will comprise of ultra-dense networks and multiple input multiple output technologies. This will make the current centralised solutions impractical due to increased complexity. Moreover, the amount of coordination information that needs to be transported over the backhaul links will be increased. Distributed or decentralised solutions are promising to provide better alternatives.

This thesis proposes new distributed algorithms for wireless networks which aim to reduce the amount of system overheads in the backhaul links and the system complexity. The analysis of conflicts amongst transmitters, and resource allocation are conducted via the use of game theory, convex optimisation, and auction theory.

Firstly, game-theoretic model is used to analyse a mixed quality of service (QoS) strategic non-cooperative game (SNG), for a two-user multiple-input single-output (MISO) interference channel. The players are considered to have different objectives. Following this, the mixed QoS SNG is extended to a multicell multiuser network in terms of signal-to-interference-and-noise ratio (SINR) requirement. In the multicell multiuser setting, each transmitter is assumed to be serving real time users (RTUs) and non-real time users (NRTUs), simultaneously. A novel mixed QoS SNG algorithm is proposed, with its operating point identified as the Nash equilibrium-mixed QoS (NE-mixed QoS). Nash, Kalai-Smorodinsky, and Egalitarian bargain solutions are then proposed to improve the performance of the NE-mixed QoS. The performance of the bargain solutions are observed to be comparable to the centralised solutions.

Secondly, user offloading and user association problems are addressed for small cells using auction theory. The main base station wishes to offload some of its users to privately owned small cell access points. A novel bid-wait-auction (BWA) algorithm, which allows single-item bidding at each auction round, is designed to decompose the combinatorial mathematical nature of the problem. An analysis on the existence and uniqueness of the dominant strategy equilibrium is conducted. The BWA is then used to form the forward BWA (FBWA) and the backward BWA (BBWA). It is observed that the BBWA allows more users to be admitted as compared to the FBWA.

Finally, simultaneous multiple-round ascending auction (SMRA), altered SMRA (ASMRA), sequential combinatorial auction with item bidding (SCAIB), and repetitive combinatorial auction with item bidding (RCAIB) algorithms are proposed to perform user offloading and user association for small cells. These algorithms are able to allow bundle bidding. It is then proven that, truthful bidding is individually rational and leads to Walrasian equilibrium. The performance of the proposed auction based algorithms is evaluated. It is observed that the proposed algorithms match the performance of the centralised solutions when the guest users have low target rates. The SCAIB algorithm is shown to be the most preferred as it provides high admission rate and competitive revenue to the bidders.

Contents

Statement of Originality

The contributions of this thesis are mainly on the development of various distributed resource allocation algorithms for wireless networks. The novelty of the contributions is summarised below and it is supported by various international journal and conference publications.

In Chapter 3, various algorithms under mixed quality of service (QoS); namely, strategic non-cooperative game (SNG) and bargain game are proposed. The mixed QoS SNG is capable of achieving higher average sum rate, with less power consumption, as compared to maximum ratio transmission scheme. In addition, a mixed QoS distributed algorithm that utilises alternating direction method of multipliers (ADMM) is proposed and shown to converge to an optimal centralised solution using simulation results. The results have been presented in:

1 B. Basutli and S. Lambotharan, "Distributed beamformer design under mixed SINR balancing and SINR-Target-Constraints," in IEEE International Conference on Digital Signal Processing, Jul. 2015, pp. 530-534.

2 B. Basutli and S. Lambotharan, "Extending the bargain region using beamforming design under mixed QoS constraints," in IEEE International Conference on Digital Signal Processing, Jul. 2015, pp. 511-515.

In Chapter 4, an extended mixed QoS SNG is proposed for multicell multiuser wireless networks. In addition, Egalitarian and Kalai-Smorodinsky bargain games are proposed to improve the solution attained by the mixed QoS SNG. Numerical simulations revealed that, on average, the mixed QoS sum rate achieved by the bargaining games is comparable to that of the centralised Egalitarian and Kalai-Smorodinsky solutions. This work has been presented in:

3 B. Basutli and S. Lambotharan, "Game-Theoretic beamforming techniques for multiuser multi-cell networks under mixed QoS constraints," submitted to IET Signal Processing (provisionally accepted subject to revision).

In Chapter 5, both backward bid-wait-auction (BBWA) and forward bid-wait-auction (FBWA) are proposed for user offloading and user association mechanism in heterogeneous networks (HetNets). Both the proposed algorithms significantly increase the network capacity and recover the revenue that could be lost in the absence of small cells. The BBWA with fixed preference profile is the most preferred algorithm by the macro base station (MBS) and the small cell access points (SCAs). The proposed algorithm is able to provide closer to optimal solution with significant saving in complexity. This work has been presented in:

4 B. Basutli and S. Lambotharan, "Auction based competition of hybrid small cells for dropped macrocell users," submitted to IET Signal Processing (provisionally accepted subject to revision)

In Chapter 6, various auctioned based algorithms, namely; simultaneous multiple-round ascending auction (SMRA), altered SMRA (ASMRA), sequential combinatorial auction with item bidding (SCAIB), and repetitive combinatorial auction with item bidding (RCAIB) are proposed to increase the network capacity in HetNets. The SCAIB algorithm is the most preferred algorithm since it provides higher admission rate and competitive revenues for the SCAs. The proposed algorithms match the performance of the centralised solution at low target data rates. This work will be presented in:

5 B. Basutli and S. Lambotharan, "Offloading macrocell users to hybrid small cells via auctioning in HetNets," to be submitted to IEEE Access.

Acknowledgements

In prayer, I humbly thank the Almighty God who protects and usher me with challenges that shape my life. I would like to express my heartfelt gratitudes to Professor Sangarapillai Lambotharan and Professor Jonathan Chambers who warmly welcomed me to Loughborough and groomed me into the person I am now. Professor Sangarapillai Lambotharan's patience and encouragement towards me remain remarkable, and this has helped to overcome the challenges I faced during my studies. I am grateful to my wife who decided to quit her job to accompany me to Loughborough, UK. Her support gave me the hope to success and kept me striving in Botswana. I also like to thank my children who kept me smiling even during bad days. I would like to thank my family, especially my mother, who kept on encouraging me even though she went through some difficult times in my absence.

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Abbreviations

1G	First-Generation
2G	Second-Generation
3G	Third-Generation
3GPP	Third-Generation Partnership Project
4G	Fourth-Generation
5G	Fifth-Generation
ADMM	Alternating Direction Method of Multipliers
AMPS	Advanced Mobile Phone Systems
AP	Access Point
APP	Adaptive Preference Profile
ASMRA	Altered Simultaneous Multiple-Round Ascending Auction
ASR(s)	Average Sum Rate(s)
BBWA	Backward Bid-Wait-Auction
BER	Bit Error Rate
bpcu	Bits Per Channel Use
BnB	Branch and Bound
BR	Best Response
BS(s)	Base Station(s)
BSUM	Block Successive Upper Minimisation
BWA	Bid-Wait-Auction
CA	Combinatorial Auction
CAIB	Combinatorial Auction with Item Bidding
CAPEX	Capital Expenditure
CB	Coordinated Beamforming
CDMA	Code Division Multiple Access

CoMP	Coordinated Multipoint
CRE	Cell Range Expansion
CS	Coordinated Scheduling
CSI	Channel State Information
CQI	Channel Quality Indicator
D2D	Device-to-Device
DLPR	Dual Linear Programming Relaxation
DoF	Degrees of Freedom
DSE	Dominant Strategy Equilibrium
DSIC	Dominant-Strategy Incentive Compatible
DVSINR	Distributed Virtual SINR
DZF	Distributed Zero Forcing
EBG	Egalitarian Bargain Game
EDGE	Enhanced Data Rate for GSM Evolution
EM	Electromagnetic
EPIC	Ex-Post Incentive Compatible
EVDO	Evolution-Data Optimized
FCC	Federal Communications Commission
FBWA	Forward Bid-Wait-Auction
FCC	Federal Communications Commission
FDMA	Frequency Division Multiple Access
FPP	Fixed Preference Profile
GPRS	General Packet Radio Service
GPS	Global Positioning System
GSM	Global System for Mobile communications
GU(s)	Guest User(s)
HetNet(s)	Heterogeneous Network(s)
HSDPA	High Speed Downlink Packet Access
HSUPA	High Speed Uplink Packet Access
HU(s)	Home User(s)
ICI	Inter-cell Interference
IEEE	Institute of Electrical and Electronics Engineers
IFBC	Interfering Broadcast Channel

IFC	Interference Channel
IF-MAC	Interfering Multiple Access Channel
IMT	International Mobile Telecommunications
IP	Integer Program
ITU-R	International Telecommunication Union Radio Standards Sector
JT	Joint Transmission
JTACS	Japanese Total Access Communication System
KKT	Karush-Kuhn-Tucker
KS	Kalai-Smorodinsky
KSBG	Kalai-Smorodinsky Bargain Game
LOS	Line-of-Sight
LPR	Linear Programming Relaxation
LTE	Long Term Evolution
LTE-Hi	LTE Hotspot Improvement
MBS	Macrocell Base Station
MISO	Multiple-Input Single-Output
MIMO	Multiple-Input Multiple-Output
MIMO IFC	Multiple-Input Multiple-Output interference channel
MNO	Mobile Network Operator
MOP	Multi-objective optimisation
MRT	Maximum Ratio Transmission
MS(s)	Mobile Station(s)
MMSE	Minimum Mean Square Error
MSE	Mean Square Error
MU(s)	Macrocell User(s)
Multi-RAT	Multi-Radio Access Technology
NB	Nash Bargain
NE	Nash Equilibrium
NMT	Nordic Mobile Telephone
NRTU(s)	Non-Real Time User(s)
OFDM	Orthogonal Frequency Division Multiple
OFDMA	Orthogonal Frequency Division Multiple Access
PMP	Power Minimisation Problem

QoS	Quality of Service
RAIB	Repetitive Combinatorial Auction with Item Bidding
RTU(s)	Real Time User(s)
SAA	Simultaneous Ascending Auction
SCA(s)	Small Cell Access point(s)
SCAIB	Sequential Combinatorial Auction with Item Bidding
SC-FDMA	Single Carrier Frequency Division Multiple Access
SDP	Semi Definite Programming
sDSE	Sub-Auction Dominant Strategy Equilibrium
SDMA	Space Division Multiple Access
SINR(s)	Signal-to-Interference-and-Noise Ratio(s)
SISO	Single-Input Single-Output
SMRA	Simultaneous Multiple-Round Ascending Auction
SNG	Strategic Non-cooperative Game
SNR(s)	Signal-to-Noise Ratio(s)
SOC	Second Order Cone
SOFDMA	Scalable OFDMA
SOCP	Second Order Cone Programming
TACS	Total Access Cellular System
TDMA	Time Division Multiple Access
TPS	Transmission Point Selection
TV	Television
UA	User Admission
UM	User Maximization
UMTS	Universal Mobile Telecommunications Service
VCG	Vickery-Clarke-Groove
WCDMA	Wideband Code Division Multiple Access
WE	Walrasian Equilibrium
WiFi	Wireless Fidelity
WIMAX	Worldwide Interoperability for Microwave Access
WMMSE	Weighted Minimum Mean Square Error
WSP	Wireless Service Provider
ZF	Zero-Forcing

Mathematical Notations

$\mathbb{C}^{M\times N}$	A set of complex-valued $M \times N$ matrices.
$\mathbb{R}^{M\times N}$	The set of real-valued $M \times N$ matrices.
$\mathbb{R}_+^{M\times N}$	The set of non-negative real-valued $M \times N$ matrices.
$\mathbb{Z}_+^{M\times N}$	The set of non-negative integer $M \times N$ matrices.
$x_i = [\mathbf{x}]_i$	Two ways of writing the ith element of a vector $\mathbf{x}$.
$x_{ij} = [\mathbf{X}]_{ij}$	Two ways of writing the i,jth element of a matrix $\mathbf{X}$.
$\mathrm{diag}(x_1,\ldots,x_N)$	The diagonal matrix with $x_1,\ldots,x_N$ at the diagonal.
$\mathrm{diag}(\mathbf{X}_1,\ldots,\mathbf{X}_N)$	The block-diagonal matrix with $\mathbf{X}_1,\ldots,\mathbf{X}_N$ at the diagonal.
$\mathbf{X}^\mathsf{T}$	The transpose of $\mathbf{X}$.
$\mathbf{X}^*$	The conjugate of a each element of $\mathbf{X}$.
$\mathbf{X}^\mathsf{H}$	The conjugate transpose of $\mathbf{X}$.
$\mathbf{X}^{-1}$	The inverse of a square matrix $\mathbf{X}$.
$\mathbf{\Pi}_\mathbf{X}$	The orthogonal projection matrix onto the column space of $\mathbf{X}$ (i.e., $\mathbf{\Pi}_\mathbf{X} = \mathbf{X}(\mathbf{X}^H\mathbf{X})^{-1}\mathbf{X}^H$).
$\mathbf{\Pi}_\mathbf{X}^\perp$	Projection matrix onto the orthogonal complement of the column space of $\mathbf{X}$ (i.e., $\mathbf{\Pi}_\mathbf{X}^\perp = \mathbf{I} - \mathbf{\Pi}_\mathbf{X}$).
$\Re\{x\}$	Real part of a scalar x.
$\Im\{x\}$	Imaginary part of a scalar x.
$\mid x \mid$	Absolute value of a scalar x.
$\angle x$	Phase of a complex-valued scalar x.
$\lceil x \rceil$	The smallest integer not less than the scalar $x \in \mathbb{R}$.
$\log_a(x)$	Logarithm of x using the base $x \in \mathbb{R}_+$.
$\mathrm{tr}\{\mathbf{X}\}$	The trace of a square matrix $\mathbf{X}$.
$\mathrm{rank}\{\mathbf{X}\}$	The rank of a matrix $\mathbf{X}$ (i.e., non-zero singular values).
$\mathbf{relint}\{\mathbf{X}\}$	The relative interior of $\mathbf{X}$.

$\mathbf{card}\{\mathbf{X}\}$	The cardinality of set $\mathbf{X}$.
$\mathbf{dom}\, f$	The domain of function f.
$\mathcal{CN}(\mathbf{x}, \mathbf{R})$	The circular symmetric complex Gaussian counterpart.
$\mathbb{E}\{\mathbf{X}\}$	The mathematical expectation of a stochastic $\mathbf{X}$.
$\parallel \mathbf{x} \parallel_p$	The L_p-norm $\parallel \mathbf{x} \parallel_p = (\sum_i \mid x_i \mid^p)^{1/p}$ of $\mathbf{x}$.
$\parallel \mathbf{X} \parallel_F$	The Frobenius norm $\parallel \mathbf{X} \parallel_F = \sqrt{\sum_{i,j} \mid x_{i,j} \mid^2}$ of $\mathbf{X}$.
$\mid \mathcal{S} \mid$	The cardinality (i.e., number of members) of a set $\mathcal{S}$.
$\mathcal{S} \setminus \{k\}$	The remaining set when member k is removed.
$\mathcal{S}_1 \cup \mathcal{S}_2$	Union set with all members which are in $\mathcal{S}_1$ and/or $\mathcal{S}_2$.
$\mathcal{S}_1 \cap \mathcal{S}_2$	Intersection set with all members which are in both $\mathcal{S}_1$ and $\mathcal{S}_2$.
$\mathcal{S}(n)$	The nth member of a set $\mathcal{S}$.
$\forall x$	Means that a statement holds for all x (in the set that x belongs to).
$x \in \mathcal{S}$	If $\mathcal{S}$ is a set: x is a member.
	If $\mathcal{S}$ a stochastic distribution: x is a realization.
$\mathbf{X} \succ \mathbf{Y}$	Means that $\mathbf{X} - \mathbf{Y}$ is positive definite.
$\mathbf{X} \succeq \mathbf{Y}$	Means that $\mathbf{X} - \mathbf{Y}$ is positive semi-definite.
$\mathbf{x} \succeq \mathbf{y}$	Means that the vector $\mathbf{x}$ majorizes $\mathbf{y}$.
$\mathbf{x} > \mathbf{y} (\mathbf{x} \geq \mathbf{y})$	Means that $x_i > y_i (x_i \geq y_i)$ for all vector indices i.
$\mathbf{I}_M$	The $M \times M$ identity matrix.
$\mathbf{1}_M$	The $M \times 1$ matrix (i.e., vector) of only ones.
$\mathbf{0}_M$	The $M \times M$ matrix of only zeros.
$\mathbf{0}_{M \times N}$	The $M \times N$ matrix of only zeros.

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Chapter 1

Introduction to Wireless Communication

Modern wireless communications have a history that dates back to the 19th century. Prediction of the existence of electromagnetic (EM) waves in 1867 by Maxwell was verified by Hertz in 1887. In 1890, Branly developed a coherer for detecting radio waves. The first wireless radiotelegraph transmission by Guglielmo Marconi in 1895 [2], opened more doors to research in wireless communications. Successive research achievements in this area catapulted radio communications into our contemporary lives.

Paramount wireless services like global position system (GPS), commercial television (TV), and voice communication, have revolutionised our lives and they remain a necessity. In the past years, additional services like multimedia applications, online-gaming, mobile social networks, wireless internet access for real-time video and music and many more, immensely seized our attention. The demand of these services by end users is highly attached to other externalities like security, safety, health, and even personal feelings.

1.1 Motivation

Unfortunately, the service provision for the explosive demand of wireless communication services does not come without difficulties. Scarce and expensive resources such as spectrum, and inexorable constraints like interference leakage, quality of service

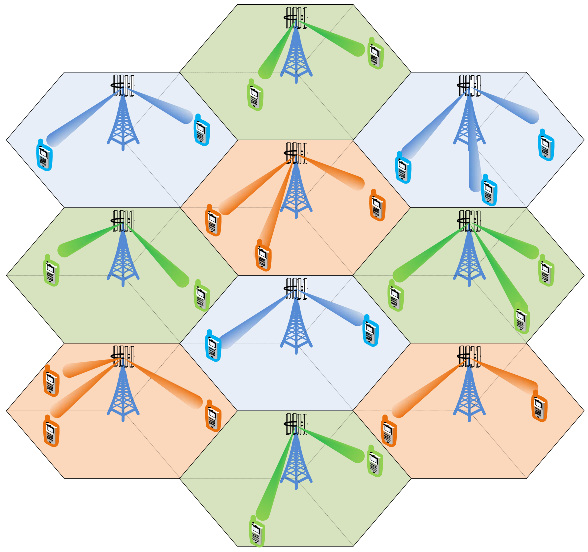


FIGURE 1.1: An example of a cellular network. Each colour represent a particular frequency band.

(QoS), channel capacity, transmission power, delay tolerance and many others, define the breadth and width of the wireless system limitations, as well as the economical implications. These challenges are foreseen to perpetuate into the future with increased magnitude. Therefore, resource management for wireless communication networks is a very critical area in network planning and optimisation. The management of radio resources is now in the forefront of research, and it is seen as a high investment by the service providers.

In the recent years, research advancements in the provision and management of resources for wireless communications have ushered us with a plethora of mathematical tools and methods. Choice of most appropriate tools and methods will be determined by underlying characteristics of the wireless communication systems including cellular network structure, interference model, etcetera. Hence a brief description is provided below.

1.2 Cellular Networks

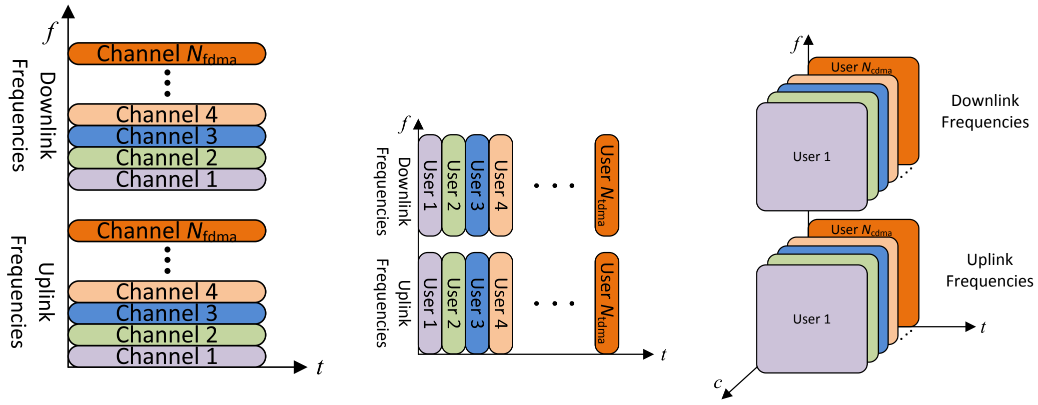
A wireless "cellular" system is related to a wireless communications system divided in small sections called "cells". Figure 1.1 shows an example of a cellular network. Each cell comprises of a base station (BS), mobile users and a set of frequencies. It is usually assumed that both the BS and mobile users have capabilities to transmit and receive information. The signal transmission from the BS to the mobile users is called the *downlink* transmission, and the inverse is called the *uplink* transmission.

1.2.1 First Generation Systems

The Nordic Mobile Telephone/Total Access Cellular System (NMT/TACS), Advanced Mobile Phone Systems (AMPS), and Japanese Total Access Communication System (JTACS) are some of the first-generation (1G) systems. These systems were mostly analogue. The 1G systems had separate downlink and uplink frequencies for each user as shown in Figure 1.2a. This access scheme of transmission is referred to as the frequency division multiple access (FDMA). Figure 1.2 shows different access schemes with frequency division duplexing (FDD). The 1G systems supported data rates up to 2.4 kbps. Some impediments in 1G systems included lack of security, low capacity, and reckless handoff [3].

1.2.2 Second Generation Systems

In the late 1990s, new second-generation (2G) systems were deployed with the intention to improve the capacity experienced in the 1G systems. The 2G systems such as the Global System for Mobile (GSM) communications utilised digital modulation formats and time division multiple access (TDMA) scheme (see Figure 1.2b). In TDMA, interference from other users is avoided by allocating the full bandwidth to a user for a particular time slot. These systems are capable of supporting data rates up to 64 kbps. The FDMA and the TDMA schemes, try to eliminate interference as much as possible. But these scheme can be inefficient in terms of capacity. Further improvements on the 2G systems, under the 2.5G systems, include the General Packet Radio Service (GPRS), Enhanced Data Rate for GSM Evolution (EDGE) [4], and utilization of code division multiple access (CDMA) scheme [3, 5] (see Figure 1.2c). In CDMA, different spreading codes are assigned to each user to allow simultaneous transmission on the same frequency. Since the orthogonality of the spreading codes in CDMA can be lost, due to system imperfections, systems that utilise CDMA suffer from significant level of interference.

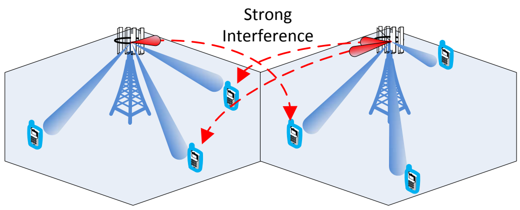


(A) FDMA with FDD. (B) TDMA with FDD. (C) CDMA with FDD.

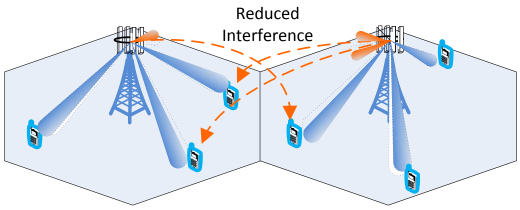
FIGURE 1.2: Multiple access schemes with Frequency Division Duplexing. In Figure 1.2a, each user is allocated separate uplink and downlink frequencies. In Figure 1.2b, different time slots are allocated for different users. In Figure 1.2c, all users utilize the whole time slots and frequency bands by using different spreading codes.

1.2.3 Third Generation Systems

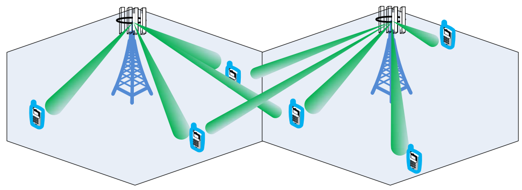
In order to address the challenges in the 1G and the 2G systems, the third-generation (3G) systems were introduced in the late 2000, with data rates up to 2 Mbps. Together with CDMA, Wideband CDMA (WCDMA), global roaming, and improved voice quality, elevated the 3G systems. An example of a 3G system is the Universal Mobile Telecommunications System (UMTS). Unfortunately, the services offered in 3G drain the mobile handset battery at a higher rate as compared to 2G handsets. The utilization of the Evolution-Data Optimized (EVDO), High Speed Uplink/Downlink Packet Access (HSUPA/HSDPA), instigated other intermediate wireless generations between 3G and 4G. This includes the 3.5G and the 3.75G, with data rate up to 5-30 Mbps. In 3.75G, Long Term Evolution (LTE) technology and Fixed Worldwide Interoperability for Microwave Access (WIMAX) technologies were introduced. These technologies utilise the Orthogonal/Single Carrier Frequency Division Multiple Access (OFDMA/SC-FDMA) and Scalable OFDMA (SOFDMA) schemes [3]. SOFDMA is the IEEE 802.16e OFDMA mode for fixed and mobile WIMAX. In SOFDMA, the sub-carrier frequency spacing remains constant, but the fast Fourier transform size of the channel bandwidth is scaled.



(A) Uncoordinated multicell network.



(B) Coordinated Beamforming/Coordinatede Scheduling.



(C) Joint Transmission.

FIGURE 1.3: Examples of CoMP techniques. In Figure 1.3a there is no coordination and cell-edge users experience strong interference. In Figure 1.3b BSs coordinate and there is reduced interference as BSs try to place nulls in the direction of users in different cells. In Figure 1.3c both BSs serve the cell edge users.

1.2.4 Fourth Generation Systems

In January 2012, the International Telecommunication Union Radio Standards Sector (ITU-R) approved the International Mobile Telecommunications (IMT)-Advanced Specifications of Fourth Generation (4G) wireless networks [6]. At present, the Third Generation Partnership Project (3GPP) is working on the LTE-Advanced technology. The initial phase of LTE-Advanced, the LTE-A increase the spectrum efficiency by using carrier aggregation, enhancement of multi-antenna techniques, and transmission bandwidth beyond 20 MHz [7]. With the argument that some of the technologies in LTE-A are reaching their theoretical limits, 3GPP is also working on LTE-B, to make further improvements. LTE-B is mainly developed for capacity boosting of at least 30 folds increase of LTE-A [8]. Increasing traffic demand due to social networking, online gaming, video streaming and sharing, cost and energy consumption, will hopefully be improved in LTE-B. Enhacements of multi-antenna techniques, Multi-Radio Access Technology (Multi-RAT), LTE Hotspot Improvement (LTE-Hi), and small cells are amongst the key features of LTE-B [8]. The last two chapters of this thesis investigate various resource allocation and beamforming methods for improving network capacity using existing small cells.

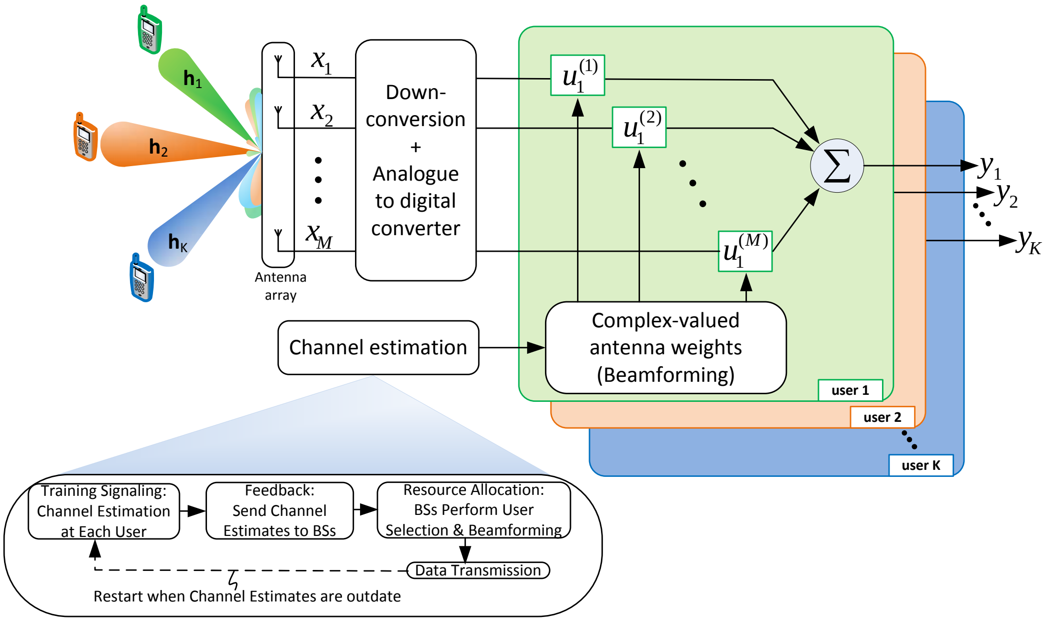
1.3 Interference Channels

One of the solutions to spectrum scarcity is frequency reuse. The interference channel (IFC) is a mathematical model wherein the base stations (BSs) share the same frequency band. Interference avoidance techniques in schemes such as TDMA and FDMA can be very inefficient. By allowing interference to occur via aggressive frequency reuse, and concurrently deploying interference management techniques, can improve the spectrum efficiency. When neighbouring cells use the same frequency band, it is highly probable for concurrent transmission to occur, thereby inducing interference across and within cells. To reduce the negative impacts brought about by this setup, interference management techniques such as the coordinated multipoint (CoMP) transmission and reception techniques are being used.

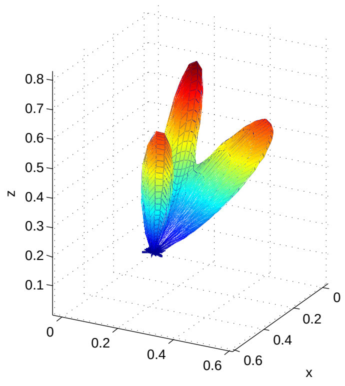
CoMP techniques can be classified as coordinated scheduling and coordinated beamforming (CS/CB), joint transmission (JT), and transmission point selection (TPS) [9], as depicted in Figure 1.3. Under CS/CB, as shown in Figure 1.3b, BSs share channel state information (CSI) for cell edge users, but the data for each user is only available at the serving BS. With the CSI knowledge at each BS, beams are designed such that nulls are placed in the direction of users in the other cell. In JT, BSs share both CSI and data so that cell edge users can be served by multiple BSs at once (see Figure 1.3c). These techniques were standardised the 3GPP for the LTE (3GPP LTE) technology. The 3GPP LTE techniques use multiple transmit and receive antennas to enhance the quality of the received signal and additionally suppress the interference.

1.3.1 Multi-antenna Communications

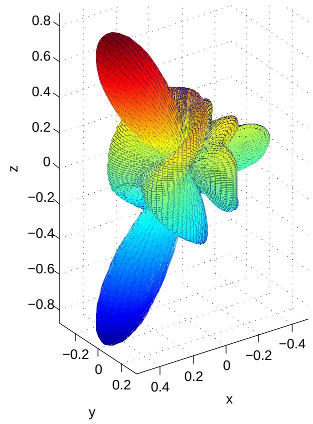
Multi-antenna techniques are used to provide spatial separation of users via beamforming technique. Beamforming is a signal processing technique for spatial filtering of signals in antenna arrays. It is a physical layer technique used for interference management in multi-antenna systems. This is achieved, for example, by controlling the radiation pattern of the antenna array, thereby concentrating transmission power to the intended user while placing nulls in the direction of other users. In essence, using different amplitudes and phases of the transmitted signal causes the signal to add in desired directions and cancel in undesired directions. Beam steering allows beams to form in



(A) An uplink smart antenna system.



(B) Beam patterns creatd by a 16 × 16 plannar array.



(C) Radiation pattern for a beamsteered circular array.

FIGURE 1.4: An example of a the antenna array system and array radiation patterns. Figure 1.4a depicts an uplink system with K mobile users and a BS with M-element antenna array for receiving. An example of radiation patterns created with planar array and circular are shown in Figure 1.4b and Figure 1.4c.

the directions of users with clear line-of-sight (LOS). In cases where there is no LOS, beam steering allows multipath components to add up coherently in areas around the affected users. Due to the required space between antenna elements in an array, it is more practical to implement arrays at the BS than at the mobile devices.

Beamforming techniques are used in switched beam and adaptive beamforming systems. Switched beam systems use a particular beam pattern, from a predefined beam patterns, at any given time. On the other hand, adaptive beamforming systems adjust the beam pattern according to the dynamic measurements made. Figure 1.4a shows an uplink system with K mobile users and a BS with M antennas. Each mobile users is assumed to be transmitting single data stream. The received signals are superimposed to each other. By using the beam weight at each antenna element, the signal can be intelligently decoded. The antenna array can either be linear, circular or planar. Radiation patterns produced by planar and circular arrays are illustrated in Figure 1.4b and Figure 1.4c, respectively. The fact that the beam resolution increases with the number of antenna elements at the transmitters, has triggered the instigation of research in massive MIMO. Massive MIMO system is when a BS uses arrays of antennas consisting of hundreds of antenna elements to communicate to single antenna users, over the same frequency band and time [10, 11]. Massive MIMO systems use a time division duplex (TDD) technique, wherein the uplink and downlink transmissions on the same frequency occur at separated times.

1.4 Problem Statement

With the continued efforts to optimise wireless resources, it is undeniable that new challenges in the wireless communications networks keep on emerging. For example, computational complexity grows as the wireless system become very large, indoor traffic dominates the total mobile traffic with 60 percent voice traffic and 70 percent data traffic originating from indoors [6], increased backhaul traffic and energy consumption. Though some of these challenges will remain inevitable, it is still vital to explore other means to reduce their impact on the overall system performance. Before stating the general problem in this thesis, first consider a generic optimisation problem [12]

$$\underset{\mathbf{r}}{\texttt{minimize}} f(\mathbf{r}), \quad \texttt{subject to} \quad \mathbf{r} \in \mathcal{Q}, \tag{1.1}$$

where $f(\mathbf{r})$ is the cost function, vector $\mathbf{r}$ is the optimisation variable, and $\mathcal{Q}$ is the feasible region. The optimisation variable resembles a list of user parameters and $\mathcal{Q}$ depends on factors like maximum transmission power, noise, transmission strategy, reception strategy, and many more.

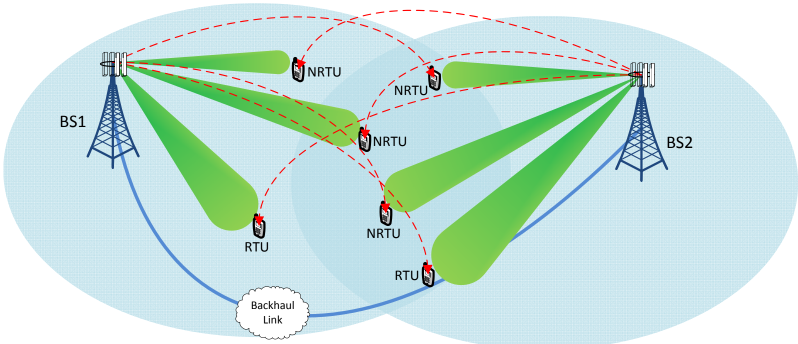


FIGURE 1.5: A multiuser multi-cell network topology with heterogenous users. There are to types of users: the non-real time users NRTUs and real time users RTUs.

1.4.1 Problem Description: Part 1

In the first part of this thesis, the optimisation variables are viewed as the worst case user signal-to-interference-plus-noise ratios (SINR). With the worst case SINR as the variable, then (1.1) becomes a `max-min` problem. A network with heterogeneous users, as shown in Figure 1.5, is considered. The network has two types of users; real-time users (RTUs) and non real-time users (NRTUs)[1]. The RTUs require delay intolerant services and have a specific QoS (SINR) target, whereas the NRTUs require delay tolerant services and accept any best-effort QoS they can get. Special cases of this problem occur when only a particular set of users is present at certain BS, for example, if a certain BS serves only RTUs or NRTUs. The aim is to develop novel and distributed algorithms when the BSs' objective is to maximise the worst case SINR of the NRTUs, subject to the SINR constraints of the RTUs, and transmission power constraints.

1.4.2 Problem Description: Part 2

With the aim of increasing the network capacity, the second part of this thesis treat the optimisation variables as the number of users. This type of variable turns (1.1) into a cardinality maximisation problem. As shown in Figure 1.6, the network consists of one macrocell owned by the mobile network operator (MNO), privately owned small cells access points (SCAs), macrocell users (MUs), small cell host/home users (HUs) and small cell guest users (GUs). The GUs are the MUs that the macrocell base station (MBS) is willing to offload to the small cells. The aim is to take advantage of the already

[1]It should be noted that latency requirements for the NRTUs in terms of propagation, transmission, and processing are not considered. Essentially, the RTUs are priority users and therefore there are given high preference over the NRTUs.

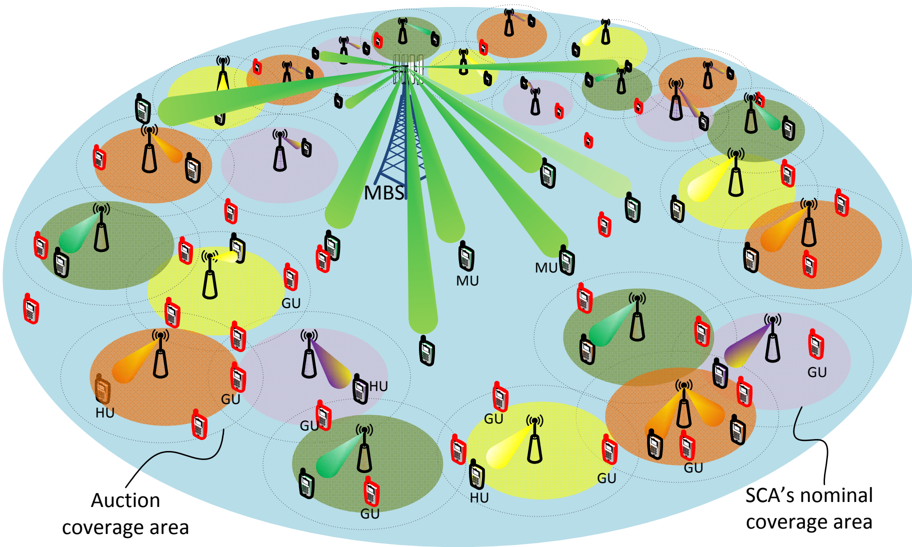


FIGURE 1.6: A heterogeneous network consisting of one macro base station (MBS) and hybrid small cells access points (SCAs). All transmitters operate on non-overlapping frequency bands. Each SCAs is serving its home users (HUs). The are guest users (GUs) that have been dropped by the MBS.

existing small cells to offload traffic from the MBS. The SCAs are willing to tender for service provision to the MBS as long as it is profitable to do so. Admission of GUs by the SCAs can only occur under the QoS constraints of the HUs, and the transmission power constraints. This problem is formulated as a surplus maximisation problem, via auction mechanism.

1.5 Research Objectives and Contributions

The main objective of this thesis is to use optimisation techniques for distributed beamforming and resource allocation in wireless communication networks. Specific research problems addressed in this thesis are summarised as follows:

1. The CoMP transmission and receive techniques in 3GPP LTE are a remarkable step towards the improvement of the wireless system performance and it has stimulated a profusion of research output found in the literature. Most of the literature proposes centralised solutions wherein the CSI for multiple users is shared amongst transmitters. The CSI needs to be transmitted via backhaul links. With an increasing traffic, the CSI can be very large. Sharing coordination messages can

be subjected to delays, thereby depriving the system from achieving a better performance. In addition, the backhaul links may have capacity constraints. The objective of this thesis is to find new algorithms that would relieve the backhaul links from the coordination information, while the system achieves a reasonable performance.

2. Coordination can be well managed when the interfering transmitters belong to the same operator. But, spectrum scarcity will result in neighbouring cells that belong to different operators. A similar scenario may arise when there is an outage on the backhaul links between neighbouring cells that operate in the same frequency band. Furthermore, hostile environments and financial constraints may discourage the implementation of backhaul links. This would create a competition amongst cells. The objective is to propose and analyse fully decentralized solutions. Performance analysis shall be considered to measure the performance loss under fully distributed solutions.

3. Most of the existing literature on radio resource management in wireless communication networks assumes that the users are of the same class or type. It is highly improbable to encounter this in real-life. Hence, the objective of this thesis is to develop distributed algorithms for wireless networks with RTUs and NRTUs. Since the RTUs require delay intolerant services, they have high priority and therefore become part of the constraints. Considering a network with RTUs and NRTUs allows BSs to have different objective functions.

4. Deployment of new access points (APs) for admitting more users requires a lot of capital investment and more spectrum acquisition. Taking advantage of low powered APs like SCAs, can help MNOs to realise increased network capacity, via network densification. SCAs allow aggressive frequency reuse within macrocells and thereby improve spectrum efficiency, power efficiency and network capacity. In order to reduce the operators' capital expenditure (CAPEX), usage of already deployed small cells will be ideal for offloading traffic from macrocells. Unfortunately, some of the SCAs belong to customers. A lot of contributions in the literature propose incentivised frameworks wherein the operator can offload their users to the already existing third party owned SCAs in exchange of payments. Since most of these private owners are business minded, incentives will therefore

attract private owners to participate in these offloading mechanisms. Hence, the objective is propose new algorithms that will address the competition amongst the private owners who are already eager to participate in incentivised frameworks. The aim is to study and analyse how the system performance is improved by engaging private owners via auctioning mechanisms.

5. In order to meet the expected high traffic volume increase in wireless communications, 5G networks are proposed to achieve gigabit-level throughput [13–15]. One major part of the 5G network is the hyper-dense deployment of small cells. But the main challenge of these multi-tier networks is the random deployment, dynamic on-off, flexible connection to cellular core networks and flat system architecture [14]. For example, small cells can be turned on/off by the customer according to their traffic demands. In networks with densely deployed small cells, in order to manage the quickly changing channel and traffic parameters, the need for a central control node becomes obsolete. Moreover, the diverse backhaul links in 5G networks (*id est*, S1 interface, X2 interface, internet IP, wireless, fiber) [16] are mostly capacity limited. Hence, when small cells are densely deployed, it becomes a problem to transport the massive backhaul traffic to and fro the core network [13]. It is therefore essential to develop distributed algorithms for carrier selection, synchronisation, power control, and etcetera [14]. To address these challenges, this thesis aims develop distributed algorithms that will encourage the small cells to always be turned on. Regardless of the presence of HUs, customers will generate revenue by turning their small cells on. Distributed algorithms will reduce the overheads in the backhaul links.

1.6 Thesis Outline

Chapter 2 provides a survey on resource allocation techniques and discusses the relevant mathematical models used in this work. Various resource allocation techniques using beamforming are introduced.

Chapter 3 addresses the problem described in Section 1.4.1 by applying game-theory and convex optimisation. In particular, downlink beamforming design under mixed QoS criterion is studied. First, a two-user model, wherein one BS is serving an RTU and the other BS is serving a NRTU, is considered. Maximum ratio transmission (MRT) and zero forcing (ZF) beamforming techniques are analysed to show the inefficiency of the performance of noncooperative solutions. A comparison to optimal beamforming techniques under sum-rate maximisation and `max-min` is conducted. Lastly, the problem is extend to a multicell multiuser scenario. Each BS is considered to be serving RTUs and NRTUs. A dual decomposition technique is used to develop a distributed algorithm. The performance of the proposed algorithm is compared to that of the centralised solution.

In Chapter 4, a game-theoretic approach for the downlink beamformer design in a multicell multiuser wireless network under a mixed QoS criterion is proposed. A novel mixed QoS strategic non-cooperative game (SNG) algorithm, wherein BSs determine their downlink beamformers in a fully distributed manner, is proposed. The mixed QoS SNG is supplemented with a fall back mechanism, which converts the problem to a pure `max-min` optimisation problem in cases of infeasibility of the mixed QoS problem. In order to improve the Nash equilibrium (NE) operating point obtained by the mixed QoS SNG, the mixed QoS bargain games (BGs) are studied. In particular, the Egalitarian and Kalai-Smorodinsky (KS) bargain solutions are proposed.

In Chapter 5, an auction based beamforming and user association algorithm for a wireless network with a macrocell deployed with multiple SCAs, is proposed. Accordingly, the MBS wishes to offload some of its users to a number of SCAs in exchange of payments based on auctioning. The SCAs compete for serving the MBS users. This user association and user offloading problem is solved by the proposed novel bid-wait auction (BWA) method. Two scenarios are considered. In the first scenario, the MBS initially admits the largest possible set of MUs, which it can serve simultaneously, and then auctions off the remaining (dropped) MUs to the SCAs. The SCAs are willing to admit GUs in addition to commitments of serving their HUs. Thus, the SCAs can admit GUs (i.e. dropped MUs) provided that the QoS of their HUs is not compromised. This problem is solved by the proposed forward BWA (FBWA) algorithm. In the second scenario, the MBS aims to auction off as many MUs as possible, and then admits the largest possible set of remaining MUs. This is solved by the proposed backward BWA (BBWA) algorithm. The proposed methods provide decentralized solutions resulting into exchange of only minimal signalling information between the SCAs and MBS, and the solution is shown to be close to optimum.

Chapter 6 further investigates the auction based algorithms for offloading MUs from the MBS. The same system model in Chapter 5 is considered. First, a simultaneous multiple-round ascending auction (SMRA), for allocating MUs to the SCAs, is proposed. Taking into account the overheads incurred by SCAs during valuation in the SMRA, further improvements are obtained through the proposed altered SMRA (ASMRA) and the combinatorial auction with item bidding (CAIB) algorithms. In particular, sequential CAIB (SCAIB) and repetitive CAIB (RCAIB) algorithms are proposed. The analysis shows that the valuation function used by the SCAs is gross substitute, hence the prove of existence of the Walrasian equilibrium (WE), is conducted. It is shown that truthful bidding is individually rational for all the proposed algorithms. Finally, validation of the proposed solutions with reference to the optimal solution, is conducted via numerical simulations.

Chapter 7 gives a summary of the contributions made by this thesis. Potential research problems are also stipulated.

Chapter 2

Radio Resource Management Techniques for Wireless Communications Network

This chapter introduces the techniques and theories which will be used in the rest of this thesis. The main focus is on multi-antenna techniques, convex optimisation, game theory and auction theory. Examples are used to show how these techniques can be applied within the context of resource allocation in wireless communication networks.

2.1 Resource Allocation

A generic system model for a MISO downlink network has the following characteristics: Consider a multicell multiuser wireless network consisting of $N = |\mathcal{N}|$ BSs, and $K = |\mathcal{U}|$ single antenna users. The users are partitioned amongst the BSs such that BS n serves a set $\mathcal{U}_n \subseteq \mathcal{U}$ of users. A BS that is serving user k is denoted by n_k. Assume that each BS is equipped with M antennas. The transmitted signal for user k is given by

$$\mathbf{x}_k(t) = \mathbf{w}_k s_k(t), \tag{2.1}$$

where $s_k(t) \in \mathbb{C}$ represents the information symbol at time t and $\mathbf{w}_k \in \mathbb{C}^M$ is the transmit beamforming vector for user k. Without loss of generality, assume that $s_k(t)$

is normalised, such that $\mathbb{E}\{|s_k(t)|^2\} = 1$, as the power of the signal can be incorporated into $\mathbf{w}_k$. All data streams are assumed to be independent, *id est*, $\mathbb{E}\{s_k(t)s_i(t)^*\} = 0$ if $k \neq i$. It is assumed that the BSs operate on the same frequency band. The received signal at the k-th user can be written as

$$y_k(t) = \mathbf{h}^{\mathsf{H}}_{n_k,k}\mathbf{x}_k(t) + \sum_{i\in\mathcal{U}\setminus k} \mathbf{h}^{\mathsf{H}}_{n_i,k}\mathbf{x}_i(t) + \eta_k(t), \tag{2.2}$$

where $\mathbf{h}_{n_k,k} \in \mathbb{C}^M$ is the random channel vector from BS n_k to user k, and $\eta_k(t) \in \mathcal{CN}(0, \sigma^2)$ is the circular symmetric zero mean complex Gaussian noise with variance σ^2. The notation $\mathcal{U} \setminus k$ denotes a set $\mathcal{U}$ excluding member k. The downlink SINR of the k-th user is given as

$$\mathsf{SINR}^{\mathsf{D}}_k = \frac{\overbrace{|\mathbf{h}^{\mathsf{H}}_{n_k,k}\mathbf{w}_k|^2}^{\text{Desired signal power}}}{\underbrace{\sum_{i\in\mathcal{U}_{n_k}\setminus k} |\mathbf{h}^{\mathsf{H}}_{n_k,k}\mathbf{w}_i|^2}_{\text{Intra-cell Interference power}} + \underbrace{\sum_{n\in\mathcal{N}\setminus n_k}\sum_{j\in\mathcal{U}_n} |\mathbf{h}^{\mathsf{H}}_{n,k}\mathbf{w}_j|^2}_{\text{Inter-cell interference power}} + \underbrace{\sigma^2}_{\text{Noise power}}}. \tag{2.3}$$

Note that the users are coupled by cross-talk [17]. This is a challenge when designing distributed algorithms. One of the most common resource allocation problem in wireless communications systems is based on the transmit beamforming. This problem can be formulated as minimisation of transmitted power at the BS subject to the SINR targets as

$$\mathscr{P}_{\text{PMP}} : \underset{\{\mathbf{w}_k\},\forall k}{\texttt{minimise}} \ \sum_{k\in\mathcal{U}} \|\mathbf{w}_k\|^2, \ \texttt{subject to}\, \mathsf{SINR} \geq \gamma_k, \quad \forall k \in \mathcal{U}, \forall n \in \mathcal{N}, \tag{2.4}$$

where γ_k is the minimum SINR threshold required by user k. The problem in (2.4) can be converted into its *convex* equivalent form and solved efficiently using *convex optimisation* tools [12, 18].

Sometimes the SINR targets in (2.4) may turn out to be infeasible. This may occur, for example, when some of the users are in deep fading and/or experience significant shadowing effects, the channels of some users are highly correlated, or there is lack of resources at the BSs. To overcome these problems, a `max-min` fairness approach is sometimes used. The approach, referred to as the SINR balancing technique, maximises the worst case SINR subject to the available total transmission power [17, 19] and is formulated as

$$\mathscr{P}_{\text{SB}} : \underset{\{\mathbf{w}\},\forall k}{\mathtt{maximise}}\,\underset{k}{\mathtt{min}}(\mathsf{SINR}_k),\ \mathtt{subject\ to}\ \sum_{k\in\mathcal{U}} \|\mathbf{w}_k\|^2 \leq \varphi_n^{\max},\quad n\in\mathcal{N}, \tag{2.5}$$

where $\varphi_n^{\max}$ is the maximum available transmit power at the BS n. This problem was analysed in [17] and it was solved using an iterative algorithm by exploiting the *uplink-downlink duality*. It should be noted that problems in (2.4) and (2.5) have the SINRs of the users either in the objective or as the constraints.

In the following sections, discussions on mathematical theories and tools used to solve the problems in (2.4) and (2.5) are provided. Even though there is a plethora of mathematical tools and techniques such as genetic algorithm, simulated annealing, and matching theory, the focus will be on convex optimisation, game-theory and auction theory. The reason is that game theory and auction theory are known to provide lightweight coordination information amongst agents, which is the main focus of this thesis. Furthermore, convex optimisation is a core subject on optimisation techniques and it offers solutions that are tractable.

2.2 Convex optimisation

Most of the resource allocation problems require mathematical optimisation, in particular *convex optimisation*. Hence, this section provides literature background in convex optimisation, as it is the core theory to the problems in this thesis. Wireless communication networks are often characterised by the wireless resources, system constraints and requirements that can be interpreted using mathematical functions. The strong relationship between these characteristics is ineluctable; hence, it can foster conflicting requirements and demands. *Optimisation* is a mathematical science that studies how to make good and informed decisions when confronted with such conflicting requirements and demands [20]. *Convex optimisation* is the minimisation of a convex objective function, subject to convex constraints [12, 21]. The optimal solution of a convex problem guarantees to be the best solution [12, 20] since the local optimum is also the global optimum[1].

[1] The reader is referred to [12] for detailed explanation, derivations and proofs.

Consider an optimisation problem as defined in the standard form [12],

$$\begin{aligned} \texttt{minimise} \quad & f_0(\mathbf{x}) \\ \texttt{subject to} \quad & f_i(\mathbf{x}) \leq 0 \quad i = 1, \ldots, m, \\ & h_i(\mathbf{x}) = 0 \quad\ \ i = 1, \ldots, p, \end{aligned} \tag{2.6}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the *optimisation variable*, function $f_0 : \mathbb{R}^n \mapsto \mathbb{R}$ is the *objective function* or *cost function*, functions $f_i : \mathbb{R}^n \mapsto \mathbb{R}$ are the *inequality constraints*, and functions $h_i : \mathbb{R}^n \mapsto \mathbb{R}$ are the *equality constraint functions*. The *domain* $\mathcal{D}$ or the *feasible set* of (2.6) contains all the feasible points that satisfy $f_i(\mathbf{x}) \leq 0, i = 1, \ldots, m$ and $h_i(\mathbf{x}) = 0, i = 1, \ldots, p$, and it is given by

$$\mathcal{D} = \bigcap_{i=0}^{m} \mathbf{dom}\, f_i \cap \bigcap_{i=0}^{p} \mathbf{dom}\, h_i. \tag{2.7}$$

The notation $\mathbf{dom}\, f$ means the domain of f. Problem (2.6) can be viewed as a maximisation problem by setting the objective function to $-f_0$, subject to the constraints therein. Now, a convex optimisation problem has the form

$$\begin{aligned} \texttt{minimise} \quad & f_0(\mathbf{x}) \\ \texttt{subject to} \quad & f_i(\mathbf{x}) \leq 0 \quad i = 1, \ldots, m, \\ & \mathbf{a}_i^{\mathsf{T}}\mathbf{x} = b_i \quad\ \ i = 1, \ldots, p, \end{aligned} \tag{2.8}$$

where $f_0, \ldots, f_m$ are convex functions[2]. The equality constraint functions $h_i(\mathbf{x}) = \mathbf{a}_i^{\mathsf{T}}\mathbf{x} - b_i$ must be an affine[3]. The domain of (2.8) is a *convex set* given by

$$\mathcal{D} = \bigcap_{i=0}^{m} \mathbf{dom}\, f_i. \tag{2.9}$$

2.2.1 Quasiconvex optimisation

The problem (2.8) is called quasiconvex optimisation problem if the objective function f_0 is quasiconvex[4]. In resource allocation, the SINR balancing problem $\mathscr{P}_{\text{SB}}$ defined in (2.5), is considered as a quasiconvex optimisation problem [22], and it is worthy studying

[2] $f : \mathbb{R}^n \mapsto \mathbb{R}$ is called convex if $\forall \mathbf{x}_1, \mathbf{x}_2 \in \mathbf{dom}\, f$, and $\theta \in [0, 1]$, $\ f(\theta \mathbf{x}_1 + (1-\theta)\mathbf{x}_2) \leq (1-\theta) f(\mathbf{x}_2)$.

[3] $f : \mathbb{R}^n \mapsto \mathbb{R}$ is called affine if $\forall \mathbf{x}_1, \mathbf{x}_2 \in \mathbf{dom}\, f$, and $\theta \in [0, 1]$, $\ f(\theta \mathbf{x}_1 + (1-\theta)\mathbf{x}_2) = (1-\theta) f(\mathbf{x}_2)$.

[4] $f : \mathbb{R}^n \mapsto \mathbb{R}$ is called quasiconvex if $\forall \mathbf{x}_1, \mathbf{x}_2 \in \mathbf{dom}\, f$, and $\theta \in [0, 1]$, $\ f(\theta \mathbf{x}_1 + (1-\theta)\mathbf{x}_2) \leq \texttt{max}\,(f(\mathbf{x}_1), f(\mathbf{x}_2))$.

its general solution structure through convex feasibility problems. Assume that f_0 in (2.8) is quasiconvex, then there is a family of functions $\phi_t : \mathbb{R}^n \mapsto \mathbb{R}$ such that, $\phi_t(\mathbf{x})$ is convex in $\mathbf{x}$ for fixed $t \in \mathbb{R}$, and t-sublevel set of f_0 is 0-sublevel of ϕ_t, *id est*,

$$f_0(\mathbf{x}) \leq 0 \Longleftrightarrow \phi_t(\mathbf{x}) \leq 0. \tag{2.10}$$

For every $\mathbf{x}$, $\phi_t(\mathbf{x})$ is a nonincreasing function of t. Therefore, for a variable $s \geq t$ then

$$\phi_s(\mathbf{x}) \leq \phi_t(\mathbf{x}). \tag{2.11}$$

For every fixed t, a convex *feasibility problem* for (2.8) has the form

$$\begin{aligned} \texttt{find} \quad & \mathbf{x} \\ \texttt{subject to} \quad & \phi_t(\mathbf{x}) \leq 0, \\ & f_i(\mathbf{x}) \leq 0, \quad i = 1, \ldots, m, \\ & \mathbf{A}\mathbf{x} = \mathbf{b}. \end{aligned} \tag{2.12}$$

If $p^\star$ is the optimal solution of (2.8), then a solution to (2.12) which gives $p^\star \leq t$ indicates feasibility, and a solution that result in $p^\star \geq t$ indicates infeasibility. Commonly a *bisection method* (also referred to as the *binary search method*) is used to solve the quasiconvex optimisation problem (2.8), by solving the convex feasibility problem (2.12) over an interval $[l, u]$ containing $p^\star$ [12, 20]. In Algorithm 1, (2.12) is solved at midpoint $t = (l+u)/2$ to check if $p^\star$ is in the lower or upper half of the interval, repeatedly until a set tolerance ϵ is reached. After every iteration the interval is halved; and hence, exactly $\lceil \log_2 ((u-l)/\epsilon) \rceil$ iterations are required.

Algorithm 1: Quasiconvex optimisation via Bisection method

Data: $\epsilon > 0$, $l \leq p^\star$, $u \geq p^\star$
Result: $t^\star \geq p^\star$
while $l - u > \epsilon$ **do**
  Set $t = \frac{l+u}{2}$
  Solve (2.12) at t
  if *feasible* **then**
    set $u = t$;
  else
    set $l = t$;

2.2.2 Duality Theory

Duality provides a powerful approach for dealing with resource allocation problems. A dual problem has three main advantages [12, 20]:

1. The dual problem is convex even if its primal is not.
2. The number of variables in the dual program is usually less than those in the primal program. The number of variables in the dual program is equal to the number of constraints in the primal program.
3. The maximum value achieved by the dual program is often equal to the minimum value achieved by the primal program.

In the *Lagrangian function*, the constraints are augmented to the objective function through Lagrangian multipliers. The Lagrangian function $\mathcal{L} : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^p \mapsto \mathbb{R}$ of (2.8) is defined as

$$\mathcal{L}(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\nu}) = f_0(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i f_i(\mathbf{x}) + \sum_{i=1}^{p} \nu_i h_i(\mathbf{x}), \tag{2.13}$$

where λ_i is the Lagrange multiplier associated with the inequality constraint, and ν_i is the Lagrange multiplier associated the equality constraint and $\mathbf{dom}\,\mathcal{L} = \mathcal{D} \times \mathbb{R}^m \times \mathbb{R}^n$. Now, the Lagrangian dual function $g : \mathbb{R}^m \times \mathbb{R}^p \mapsto \mathbb{R}$, is defined as the minimum value of the Lagrangian function over $\mathbf{x}$ and it is given by

$$g(\boldsymbol{\lambda}, \boldsymbol{\nu}) = \inf_{\mathbf{x} \in \mathcal{D}} \mathcal{L}(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\nu}). \tag{2.14}$$

The dual function is concave even if (2.8) is nonconvex. For any given feasible point $\tilde{\mathbf{x}}$ of (2.8), $g(\boldsymbol{\lambda}, \boldsymbol{\nu}) \leq f_0(\tilde{\mathbf{x}})$ should hold. The dual function yields lower bounds on the optimal value $p^\star$ of (2.8). For any $\boldsymbol{\lambda} \succeq 0$ and for any value $\boldsymbol{\nu}$

$$g(\boldsymbol{\lambda}, \boldsymbol{\nu}) \leq p^\star. \tag{2.15}$$

See [12, Section 5.1.3] for proof. If $\tilde{\mathbf{x}}$ is a feasible vector for (2.8), it can be concluded that

$$g(\boldsymbol{\lambda}, \boldsymbol{\nu}) = \inf_{\mathbf{x} \in \mathcal{D}} \mathcal{L}(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\nu}) \leq \mathcal{L}(\tilde{\mathbf{x}}, \boldsymbol{\lambda}, \boldsymbol{\nu}) \leq f_0(\tilde{x}), \tag{2.16}$$

and that the lower bound is obtained by maximising the lower bound. The *Lagrange dual problem* associated with the *primal problem* (2.8) becomes

$$\underset{\boldsymbol{\lambda},\boldsymbol{\nu}}{\texttt{maximise}}\, g(\boldsymbol{\lambda},\boldsymbol{\nu}),\ \texttt{subject to}\, \boldsymbol{\lambda} \succeq 0. \tag{2.17}$$

It follows that $(\boldsymbol{\lambda}^\star, \boldsymbol{\nu}^\star)$ are the *dual optimal* or *optimal Lagrange multipliers* if they are optimal for problem (2.14).

Definition 2.1 (*Weak duality*)

If $d^\star$ denotes the optimal value of the Lagrange dual problem (2.14), then it represents the best lower bound on the optimal value $p^\star$ of the primal problem (2.8). The weak duality is defined by the inequality

$$d^\star \leq p^\star. \tag{2.18}$$

The weak duality holds if the primal problem is nonconvex and $d^\star$ and $p^\star$ are infinite.

Definition 2.2 (*Strong duality*)

The optimal duality gap *is defined as the difference $p^\star - d^\star$ and it is always non-negative. If the duality gap is zero then strong duality holds, implying that*

$$d^\star = p^\star. \tag{2.19}$$

The strong duality does not always hold even when the primal problem is convex; and therefore, *constraint qualifications* are necessary to qualify this.

Example 2.1 (*Constraint qualification, Slater's condition*)

If the primal is convex, and there exists an $\mathbf{x} \in \textbf{relint}\,\mathcal{D}$ such that

$$f_i(\mathbf{x}) < 0,\ \ i = 1, \ldots, m, \quad \mathbf{A}\mathbf{x} = \mathbf{b}, \tag{2.20}$$

then, strong duality $p^\star = d^\star$ holds, and the dual problem is attained. The notation $\textbf{relint}\,\mathcal{D}$ is the relative interior of set $\mathcal{D}$.

Once the Slater's condition is satisfied, optimality conditions are assessed by the Karush-Kuhn-Tucker (KKT) conditions.

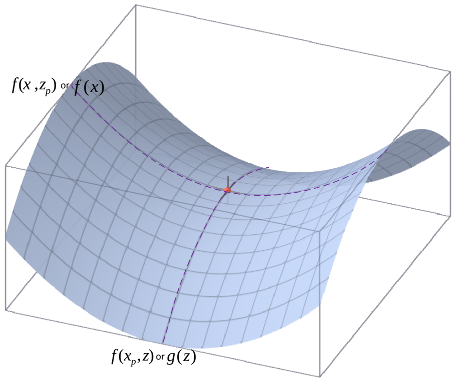


FIGURE 2.1: An illustration of strong duality between the primal and the dual problems. Figure by (Lucas V. Barbosa).

Definition 2.3 (*KKT Conditions*)

Assume $f_0, \ldots, f_m$ and $h_1, \ldots, h_p$ are differentiable. Let $\mathbf{x}^\star$ and $(\boldsymbol{\lambda}^\star, \boldsymbol{\nu}^\star)$ be the optimal solution to primal and dual problems, respectively, with zero duality gap. KKT conditions suggest that the gradient for $\mathcal{L}(\mathbf{x}, \boldsymbol{\lambda}^\star, \boldsymbol{\nu}^\star)$ must vanish at $\mathbf{x}^\star$, id est,

$$\nabla f_0(\mathbf{x}^\star) + \sum_{i=1}^{m} \lambda_i^\star \nabla f_i(\mathbf{x}^\star) + \sum_{i=1}^{p} \nu_i^\star \nabla h_i(\mathbf{x}^\star) = 0, \tag{2.21}$$

giving the following additional KKT conditions:

$$\begin{aligned} f_i(\mathbf{x}^\star) &\leq 0, i = 1, \ldots, m, \\ h_i(\mathbf{x}^\star) &= 0, i = 1, \ldots, p, \\ \lambda_i^\star &\geq 0, i = 1, \ldots, m, \\ \lambda_i^\star f_i(\mathbf{x}^\star) &= 0, i = 1, \ldots, m. \end{aligned} \tag{2.22}$$

Example 2.2 (*Duality*)

Consider a linear conic programming primal dual pair in (2.23)[20, 21]

$$\begin{aligned} \mathscr{P}_\text{p} : \underset{\mathbf{x}}{\texttt{minimise}} \quad & \boldsymbol{\alpha}^T \mathbf{x} \\ \texttt{subject to} \quad & \mathbf{x} \in \mathcal{K}, \\ & \mathbf{C}\mathbf{x} = \boldsymbol{\beta}. \end{aligned} \quad \Longleftrightarrow \quad \begin{aligned} \mathscr{P}_\text{d} : \underset{\mathbf{y}, \mathbf{z}}{\texttt{maximise}} \quad & \boldsymbol{\beta}^T \mathbf{z} \\ \texttt{subject to} \quad & \mathbf{y} \in \mathcal{K}', \\ & \mathbf{C}^T \mathbf{z} + \mathbf{y} = \boldsymbol{\alpha}. \end{aligned} \tag{2.23}$$

where $\mathscr{P}_\text{p}$ is a primal conic problem over cone $\mathcal{K}$, and $\mathscr{P}_\text{d}$ is a dual conic problem over cone $\mathcal{K}'$. Note that the objective function value of $\mathscr{P}_\text{d}$ never exceeds that of $\mathscr{P}_\text{p}$.

Therefore, the following conditions should hold

$$\left.\begin{aligned} &\boldsymbol{\alpha}^T\mathbf{x} \geq \boldsymbol{\beta}^T\mathbf{z}, \\ &\mathbf{x}^T(\mathbf{C}^T\mathbf{z}+\mathbf{y}) \geq (\mathbf{C}\mathbf{x})^T\mathbf{z}, \\ &\mathbf{x}^T\mathbf{y} \geq 0. \end{aligned}\right\} \tag{2.24}$$

Since $\mathscr{P}_{\text{p}}$ is convex, if the Slater's condition is satisfied, then ${\mathbf{x}^\star}^T\mathbf{y}^\star = 0$ is true. This yields strong duality. Combining $\mathscr{P}_{\text{p}}$ and $\mathscr{P}_{\text{d}}$ result in the following `min-max` *problem*

$$\mathscr{P}_{\text{p-d}}: \texttt{minimise} \qquad \boldsymbol{\alpha}^T\mathbf{x} - \boldsymbol{\beta}^T\mathbf{z} \tag{2.25}$$

$$\texttt{subject to} \qquad \mathbf{x} \in \mathcal{K}, \quad \mathbf{y} \in \mathcal{K}', \tag{2.26}$$

$$\mathbf{C}\mathbf{x} = \boldsymbol{\beta}, \quad \mathbf{C}^T\mathbf{z} + \mathbf{y} = \boldsymbol{\alpha}, \tag{2.27}$$

whose solution always satisfies (2.19)*. That is, the gap at the saddle point is 0.*

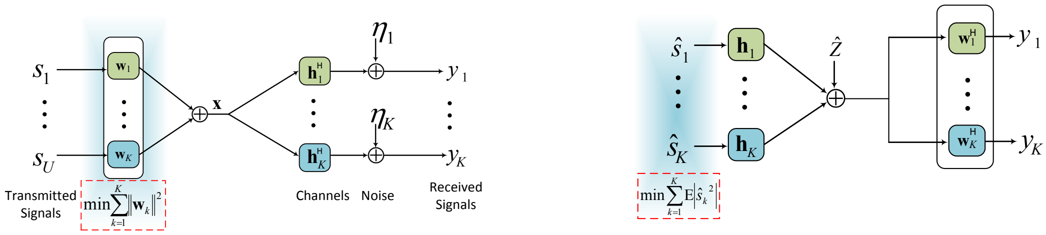
Figure 2.1 is a mnemonic that illustrates the relationship between the $\mathscr{P}_{\text{p}}$ and $\mathscr{P}_{\text{d}}$. Note that if strong duality exists, the duality gap is 0, meaning that $f(\mathbf{x})$ and $g(\mathbf{z})$ (dashed line) touch at the *saddle* value.

2.2.3 Downlink Beamformer Design Via Lagrangian Duality

Works in [17,21,23–30] have used Lagrangian duality for downlink beamformer design. These works derived several uplink-downlink duality properties that offer very insightful and tractable structures. Uplink-downlink duality suggests that, the minimum power required to satisfy a certain set of SINR targets in the downlink MIMO channel, is equal to the minimum power required to achieve the same set of SINR targets in the uplink channel. The uplink-downlink properties are demonstrated through the following example adopted from [21].

Example 2.3

Consider a downlink and uplink networks in Figures 2.2a *and* 2.2b *respectively. There is a set $\mathcal{U} := \{1, 2, \ldots, K\}$ of users, each denoted with k. Assume each user is equipped with a single antenna, while the BS is equipped with $M > 1$ antennas. The received*



(A) Single-cell downlink network.

(B) Single-cell uplink network.

FIGURE 2.2: Block diagram for single-cell network in the downlink and uplink.

signal at the k*-th user in the downlink is*

$$y_k = \mathbf{h}_k^H \mathbf{x} + \eta_k, \quad \forall k \in \mathcal{U}, \tag{2.28}$$

where $\mathbf{h}_k \in \mathbb{C}^M$ *is the channel vector between* k*-th user and the BS,* $\mathbf{x} \in \mathbb{C}^M$ *is the transmitted signal, and* η_k *is the additive white Gaussian noise. Let* $\mathbf{x} = \sum_{k\in\mathcal{U}} \mathbf{w}_k s_k$*, where* s_k *is the complex scalar denoting the information signal for the* k*-th user, and* $\mathbf{w}_k \in \mathbb{C}^M$ *is the beamforming vector for the* k*-user. Without loss of generality, let* $\mathbb{E}|s_k|^2 = 1$*, since the power can be embedded in the beamforming vector. Now (2.28) becomes*

$$y_k = \mathbf{h}_k^H \left(\sum_{k\in\mathcal{U}} \mathbf{w}_k s_k \right) + \eta_k, \quad \forall k \in \mathcal{U}. \tag{2.29}$$

The SINR for the k*-th user in the downlink is given by*

$$\mathit{SINR}_k^D = \frac{|\mathbf{h}_k^H \mathbf{w}_k|^2}{\sum\limits_{j\in\mathcal{U}\setminus k} |\mathbf{h}_k^H \mathbf{w}_j|^2 + \sigma_k}. \tag{2.30}$$

Let the primal problem of the network be of the form $\mathscr{P}_{\text{PMP}}$ *in (2.4). Note that* $\mathscr{P}_{\text{PMP}}$ *is not convex due to the SINRs constraints. The Lagrangian of* $\mathscr{P}_{\text{PMP}}$ *(without intercell interference term) is given by*

$$\begin{aligned} \mathcal{L}(\mathbf{w}_k, \lambda_k) &= \sum_{k\in\mathcal{U}} \|\mathbf{w}_k\|^2 + \frac{1}{\sigma_k^2} \sum_{k\in\mathcal{U}} \lambda_k \sum_{j\in\mathcal{U}\setminus k} |\mathbf{h}_k^H \mathbf{w}_j|^2 + \sum_{k\in\mathcal{U}} \lambda_k \left(1 - \frac{1}{\gamma_k \sigma_k^2} |\mathbf{h}_k^H \mathbf{w}_k|^2\right) \\ &= \sum_{k\in\mathcal{U}} \lambda_k \sigma_k^2 + \sum_{k\in\mathcal{U}} \mathbf{w}_k^H \left(\mathbf{I}_M + \sum_{j\in\mathcal{U}\setminus k} \frac{\lambda_i}{\sigma_k^2} \mathbf{h}_j \mathbf{h}_j^H - \frac{\lambda_k}{\gamma_k \sigma_k^2} \mathbf{h}_k \mathbf{h}_k^H \right) \mathbf{w}_k, \end{aligned} \tag{2.31}$$

where $\lambda_k \geq 0$ *is the Lagrange multiplier associated with the* k*-th SINR constraint. The dual objective is defined as* $g(\lambda_k) = \min_{\mathbf{w}_k} \mathcal{L}(\mathbf{w}_k, \lambda_k)$*. Assume that* $\sum_{k\in\mathcal{U}} \|\mathbf{w}_k\|^2$ *is differentiable, and let* $\mathbf{w}_k^\star$ *and* $\lambda_k^\star$ *be the optimal solutions to the primal and dual* *problems, respectively, with zero duality gap. Then KKT conditions suggest that the gradient for (2.31) should be zero at* $\mathbf{w}^\star$ *id est,* $\partial\mathcal{L}/\partial\mathbf{w}_k^\star = \mathbf{0}, \forall k \in \mathcal{U}$ *[12]. This results in*

$$
\begin{aligned}
\mathbf{w}_k + \sum_{j\in\mathcal{U}\setminus k} \frac{\lambda_i}{\sigma_k^2}\mathbf{h}_j\mathbf{h}_j^H\mathbf{w}_k^\star - \frac{\lambda_k}{\gamma_k\sigma_k^2}\mathbf{h}_k\mathbf{h}_k^H\mathbf{w}_k^\star = \mathbf{0},\\
\Rightarrow \left(\mathbf{I}_M + \sum_{j\in\mathcal{U}\setminus k}\frac{\lambda_j}{\sigma_k^2}\mathbf{h}_j\mathbf{h}_j^H\right)\mathbf{w}_k = \frac{\lambda_k}{\sigma_k^2}\left(1+\frac{1}{\gamma_k}\right)\mathbf{h}_k\mathbf{h}_k^H\mathbf{w}_k,\\
\Rightarrow \mathbf{w}_k = \left(\mathbf{I}_M + \sum_{j\in\mathcal{U}\setminus k}\frac{\lambda_j}{\sigma_k^2}\mathbf{h}_k\mathbf{h}_k^H\right)^{-1}\mathbf{h}_k\frac{\lambda_k}{\sigma_k^2}\left(1+\frac{1}{\gamma_k}\right)\mathbf{h}_k^H\mathbf{w}_k,
\end{aligned}
\tag{2.32}
$$

where $\mathbf{I}_M$ *is an* $M\times M$ *identity matrix. The term* $\frac{\lambda_k}{\sigma_k^2}(1+\frac{1}{\gamma_k})\mathbf{h}_k^H\mathbf{w}_k$ *is a scalar and it accounts for the allocated power [19]. Therefore, the optimal* $\mathbf{w}_k$ *should be colinear to* $(\mathbf{I}_M + \sum_{i\in\mathcal{U}_n\setminus k}\lambda_i/\sigma_k^2\mathbf{h}_j\mathbf{h}_j^H)^{-1}\mathbf{h}_k$*, hence the normalised optimal beamforming vectors* $\mathbf{w}_k^\star$*,* $k\in\mathcal{U}$ *are*

$$
\mathbf{w}_k^\star = \sqrt{\varphi_k}\underbrace{\frac{\left(\mathbf{I}_M + \sum\limits_{j\in\mathcal{U}\setminus k}\frac{\lambda_j}{\sigma_k^2}\mathbf{h}_j\mathbf{h}_j^H\right)^{-1}\mathbf{h}_k}{\left\|\left(\mathbf{I}_M + \sum\limits_{i\in\mathcal{U}\setminus k}\frac{\lambda_i}{\sigma_k^2}\mathbf{h}_j\mathbf{h}_j^H\right)^{-1}\mathbf{h}_k\right\|_2}}_{\tilde{\mathbf{w}}_k^\star},
\tag{2.33}
$$

where φ_k *is the beamforming power and* $\tilde{\mathbf{w}}_k^\star$ *is the unit-norm beamforming direction for the* k*-th user. The vector* $\tilde{\mathbf{w}}_k^\star$ *is referred to as the minimum mean square error (MMSE) filter [17,19,21,23–30]. The dual problem* $\mathscr{P}_{\text{d-PMP}}$ *is defined as*

$$
\begin{aligned}
\mathscr{P}_{\text{d-PMP}} : \underset{\{\mathbf{w}_k\}\forall k}{\texttt{maximise}} \quad & \sum_{k\in\mathcal{U}}\lambda_k\sigma_k^2,\\
\texttt{subject to} \quad & \left(\mathbf{I}_M + \sum_{j\in\mathcal{U}\setminus k}\lambda_j\mathbf{h}_j\mathbf{h}_j^H\right) \succeq \lambda_k\left(1+\frac{1}{\gamma_k}\right)\mathbf{h}_k\mathbf{h}_k^H.
\end{aligned}
\tag{2.34}
$$

With reference to Figure 2.2b, the total power minimization problem for the uplink $\mathscr{P}_{\text{PMP}}^{U}$ *is formulated as*

$$
\mathscr{P}_{\text{PMP}}^{U} : \underset{\{\varphi_k\}\forall k}{\texttt{minimise}} \sum_{k\in\mathcal{U}}\varphi_k, \ \texttt{subject to} \quad \mathsf{SINR}_k^U \geq \gamma_k.
\tag{2.35}
$$

where

$$
\mathsf{SINR}_k^U = \frac{\varphi_k|\tilde{\mathbf{w}}_k^{\mathsf{H}}\mathbf{h}_k|^2}{\sum\limits_{j\in\mathcal{U}\setminus k}\varphi_j|\tilde{\mathbf{w}}_j^{\mathsf{H}}\mathbf{h}_k|^2 + \sigma_k\tilde{\mathbf{w}}_k^{\mathsf{H}}\tilde{\mathbf{w}}_k}.
\tag{2.36}
$$

Now substituting MMSE filter into (2.35) yields

$$\begin{aligned}\mathscr{P}^{\mathcal{U}}_{\text{PMP}} : \underset{\{\mathbf{w}_k\}\forall k}{\texttt{maximise}} \quad & \sum_{k\in\mathcal{U}} \varphi_k, \\ \texttt{subject to} \quad & \left(\mathbf{I}_M + \sum_{j\in\mathcal{U}\setminus k} \lambda_j \mathbf{h}_j \mathbf{h}_j^H\right) \preceq \lambda_k \left(1 + \frac{1}{\gamma_k}\right) \mathbf{h}_k \mathbf{h}_k^H.\end{aligned} \tag{2.37}$$

By noting that $\varphi_k = \lambda_k \sigma_k^2$, then $\mathscr{P}^{\mathcal{U}}_{\text{PMP}}$(2.37) and $\mathscr{P}_{\text{d-PMP}}$(2.34) are identical, except that the `maximise` *is replaced with* `minimise`*, and the inequality of the constraints are reversed.*

2.3 Decomposition Techniques

As discussed above, the Lagrangian duality has proven to be a very useful tool in resource allocation. This section discusses how the Lagrangian function can be used in distributed algorithm design. Since the problems encountered in resource allocation have conflicting variables, it becomes a prerequisite to decompose these problems in order to derive distributed algorithms.

2.3.1 Primal Decomposition

Consider an unconstrained problem of the form

$$\texttt{minimise}\, f(\mathbf{x}) = f_1(\mathbf{x_1}, \mathbf{y}) + f_2(\mathbf{x_2}, \mathbf{y}), \tag{2.38}$$

where $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \mathbf{y})$ is a vector containing the optimisation variables. Note that the objective of (2.38) will become separable in $\mathbf{x}_1$ and $\mathbf{x}_2$ if $\mathbf{y}$ is fixed. This will yield two separate subproblems that can be solved independently. Since $\mathbf{y}$ couples the subproblems, it is called the *complicating variable*. The variables $\mathbf{x}_1$ and $\mathbf{x}_2$ are known as the *private* or *local variables*. Let $\phi_1(\mathbf{y})$ and $\phi_2(\mathbf{y})$ be the optimal values of $\mathscr{P}_1^{\text{pri}}$ and $\mathscr{P}_2^{\text{pri}}$, respectively, where

$$\mathscr{P}_1^{\text{pri}} : \underset{\mathbf{x}_1}{\texttt{minimise}}\, f_1(\mathbf{x_1}, \mathbf{y}), \qquad \text{and} \qquad \mathscr{P}_2^{\text{pri}} : \underset{\mathbf{x}_2}{\texttt{minimise}}\, f_2(\mathbf{x_2}, \mathbf{y}), \tag{2.39}$$

are known as *subproblems*. Now, (2.38) can equivalently be written as

$$\underset{\mathbf{y}}{\texttt{minimise}}\, \phi_1(\mathbf{y}) + \phi_2(\mathbf{y}). \tag{2.40}$$

The problem (2.40) is called the *master problem* [31, 32]. Note that if (2.38) is convex, so is (2.40). In order to solve (2.38) via the *decomposition method*, (2.40) is solved using a *subgradient method* [12]. In each iteration, $\phi_1(\mathbf{y})$ and $\phi_2(\mathbf{y})$ are evaluated (in parallel/sequentially) using $\mathscr{P}_1^{\text{pri}}$ and $\mathscr{P}_2^{\text{pri}}$.

2.3.2 Dual Decomposition

Another way to solve (2.38) is through dual decomposition. By introducing new variables $\mathbf{y}_1$ and $\mathbf{y}_2$, (2.38) can be expressed as

$$\texttt{minimise}\, f(\mathbf{x}) = f_1(\mathbf{x}_1, \mathbf{y}_1) + f_2(\mathbf{x}_2, \mathbf{y}_2), \texttt{ subject to}\, \mathbf{y}_1 = \mathbf{y}_2. \tag{2.41}$$

The new variables provide a *local copy* of the complicating variable and a *consistency constraint* $\mathbf{y}_1 = \mathbf{y}_2$ [31]. The objective of (2.41) is now separable in $(\mathbf{x}_1, \mathbf{y}_1)$ and $(\mathbf{x}_2, \mathbf{y}_2)$. The Lagrangian of (2.41) is

$$\mathcal{L}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{y}_1, \mathbf{y}_2, \boldsymbol{\lambda}) = f_1(\mathbf{x}_1, \mathbf{y}_1) + f_2(\mathbf{x}_2, \mathbf{y}_2) + \lambda^{\mathsf{T}}(\mathbf{y}_1 - \mathbf{y}_2), \tag{2.42}$$

which is separable. This yields a dual problem

$$\texttt{maximise}\, g(\boldsymbol{\lambda}) = g_1(\boldsymbol{\lambda}) + g_2(\boldsymbol{\lambda}), \tag{2.43}$$

where

$$g_1(\boldsymbol{\lambda}) = \underset{\mathbf{x}_1, \mathbf{y}_1}{\texttt{inf}}\, f_1(\mathbf{x}_1, \mathbf{y}_1) + \lambda^{\mathsf{T}}\mathbf{y}_1, \qquad \text{and} \qquad g_2(\boldsymbol{\lambda}) = \underset{\mathbf{x}_2, \mathbf{y}_2}{\texttt{inf}}\, f_2(\mathbf{x}_2, \mathbf{y}_2) + \lambda^{\mathsf{T}}\mathbf{y}_2. \tag{2.44}$$

Now, g_1 and g_2 can be solved independently. The master problem in (2.43) can be solved via subgradient, cutting-plane, any other method.

2.3.3 Alternating Direction Method of Multipliers

The alternating direction method of multipliers (ADMM) is an algorithm used in distributed convex optimisation [33], especially for large systems. It combines the benefits of dual decomposition and the augmented Lagrangian methods, when the problems are constrained. The ADMM considers an optimisation problem of the form

$$\underset{\mathbf{x},\,\mathbf{z}}{\texttt{minimise}} \quad f(\mathbf{x}) + g(\mathbf{z}), \; \texttt{subject to} \quad \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{z} = \mathbf{c}, \tag{2.45}$$

where $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{z} \in \mathbb{R}^m$ are variables, $\mathbf{A} \in \mathbb{R}^{p \times n}$, $\mathbf{B} \in \mathbb{R}^{p \times m}$, and $\mathbf{c} \in \mathbb{R}^p$. It is assumed that f and g are convex. The augmented Lagrangian of (2.55) is

$$\mathcal{L}_\rho(\mathbf{x}, \mathbf{z}, \boldsymbol{\lambda}) = f(\mathbf{x}) + g(\mathbf{z}) + \boldsymbol{\lambda}^\mathsf{T}(\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{z} - \mathbf{c}) + (\rho/2) \parallel \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{z} - \mathbf{c} \parallel_2^2, \tag{2.46}$$

where $\boldsymbol{\lambda}$ is the dual variable or the Lagrange multiplier, and $\rho > 0$ is the penalty parameter. The ADMM consists of the following successive iterations:

$$\mathbf{x}^{t+1} := \underset{\mathbf{x}}{\texttt{argmin}} \; \mathcal{L}_\rho\left(\mathbf{x}, \mathbf{z}^t, \boldsymbol{\lambda}^t\right), \tag{2.47}$$

$$\mathbf{z}^{t+1} := \underset{z}{\texttt{argmin}} \; \mathcal{L}_\rho\left(\mathbf{x}^{t+1}, \mathbf{z}^{t+1}, \boldsymbol{\kappa}^t\right), \tag{2.48}$$

$$\boldsymbol{\lambda}^{t+1} := \boldsymbol{\lambda}^t + \rho\left(\mathbf{A}\mathbf{x}^{t+1} + \mathbf{B}\mathbf{z}^{t+1} - \mathbf{c}\right), \tag{2.49}$$

where t is the time index. The variables $\mathbf{x}$ and $\mathbf{z}$ are updated in a sequential or *alternating* manner. By separating the minimisation over $\mathbf{x}$ and $\mathbf{z}$ into two steps, allows the decomposition of f and g. One of the setbacks in ADMM is that it can be very slow to converge to high accuracy [33, 34]. Another setback in ADMM is that, even though the convergence of ADMM for a convex optimisation problem with two blocks of variables and functions has been proved in [33, 35], convergence is not guaranteed when there are more than two block of variables and functions [36]. Nevertheless, ADMM often converges to moderate accuracy within a few tens of iterations. The performance of the ADMM is discussed in Example 2.4.

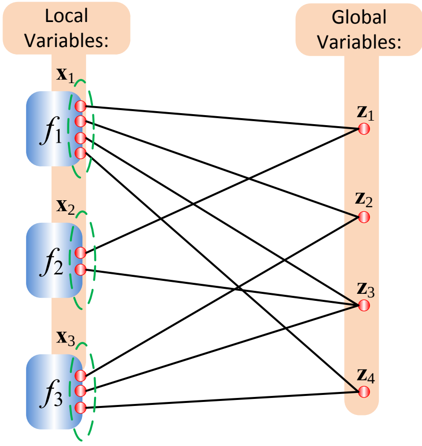


FIGURE 2.3: Network example for general form consensus optimisation. Each edge represent a consistency constraint.

2.3.4 Consensus Optimisation

The ADMM can be used to solve a consensus problem [33, 35], wherein agents have to agree on a single data value. Consider a general consensus problem of the form

$$\underset{\{\mathbf{x}_i\},\{\tilde{\mathbf{z}}_i\},\forall i}{\texttt{minimise}} \;\; f_i(\mathbf{x}_i), \; \texttt{subject to} \;\; \mathbf{x}_i - \tilde{\mathbf{z}} = \mathbf{0}, \;\; i = 1, \ldots, N, \tag{2.50}$$

where $\mathbf{x}_i \in \mathbb{R}^{n_i}$ and $\mathbf{z}_i \in \mathbb{R}^{n_i}$ are *local* and *global* variables, respectively. Figure 2.3, shows a network of $N = 3$ subsystems, global variable dimension of $n = 4$, and local variable dimensions of $n_1 = 4$, $n_2 = 2$, and $n_3 = 3$. As illustrated, the objective terms and the global variables can be represented as a bipartite graph [33]. The ADMM for (2.50) is

$$\mathbf{x}_i^{t+1} \;:=\; \underset{\mathbf{x}_i}{\texttt{argmin}} \;\; \left(f_i(\mathbf{x}_i) + \boldsymbol{\lambda}_i^{(t)\mathsf{T}}(\mathbf{x}_i - \mathbf{z}^t) + \frac{\rho}{2} \parallel \mathbf{x}_i - \mathbf{z}^t \parallel_2^2 \right), \tag{2.51}$$

$$\mathbf{z}^{t+1} \;:=\; \underset{\mathbf{z}}{\texttt{argmin}} \;\; \left(g(\mathbf{z}) + \sum_{i=1}^{N} \left(-\boldsymbol{\lambda}_i^{(t)\mathsf{T}}\mathbf{z} + \frac{\rho}{2} \parallel \mathbf{x}_i^{t+1} - \mathbf{z}^t \parallel_2^2 \right) \right), \tag{2.52}$$

$$\boldsymbol{\lambda}_i^{t+1} \;:=\; \boldsymbol{\lambda}_i^t + \rho \left(\mathbf{x}_i^{t+1} - \mathbf{z}^{t+1} \right), \tag{2.53}$$

where $\boldsymbol{\lambda}_i \in \mathbb{R}^{n_i}$ is the dual variable. Steps (2.51) and (2.53) can be evaluated independently.

In order to understand the application and performance of the ADMM in distributed algorithm design, a general network problem discussed in [34] is elaborated in the following example.

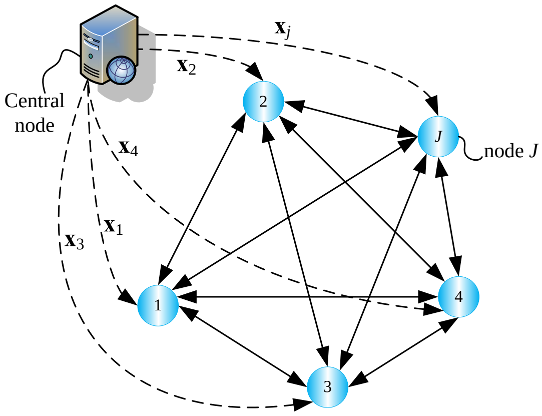


FIGURE 2.4: Network layout for a system model in Example 2.4

Example 2.4 (*Application of ADMM in Networks*)

Consider a network in Figure 2.4, with a set $\mathcal{J} := \{1, 2, \dots, J\}$ of fully interconnected nodes and a central node. The central node broadcasts message $\mathbf{s} \in \mathbb{R}^n$ to all nodes. Assume that the nodes are able to exchange information over inter-nodes, which are assumed to be ideal and time invariant. Each node will receive a modified $\mathbf{x}_j \in \mathbb{R}^{m\times 1}$ given by

$$\mathbf{x}_j = \mathbf{H}_j\mathbf{s} + \eta_j, \tag{2.54}$$

where $\mathbf{H}_j \in \mathbb{C}^{m\times n}$ is the disturbance matrix and $\eta_j \in \mathcal{CN}(0, \sigma_n^2)$ is additive Gaussian noise with zero mean and variance σ_n^2. Since estimating $\mathbf{s}$ locally at every node will result in different estimates, the authors in [34] formulated the problem of estimating $\mathbf{s}$ at each node as a consensus problem, with the objective as the sum of the estimation errors. Each node solves

$$\texttt{minimise} \quad \frac{1}{2}\sum_{j\in\mathcal{J}} \| \mathbf{x}_j - \mathbf{H}_j\mathbf{s}_j \|^2, \ \texttt{subject to} \quad \mathbf{s}_j - \mathbf{s}_i = \mathbf{0}, (i,j) \in \mathcal{J}, \tag{2.55}$$

where $\mathbf{s}_j \in \mathbb{R}^n$ are the variables, $\mathbf{H}_j \in \mathbb{C}^{m\times n}$, and $\mathbf{x}_j \in \mathbb{C}^m$ are the problem data. The consensus constraints $\mathbf{s}_j - \mathbf{s}_i$, ensure that the estimated $\mathbf{s}_j$ at any node is in agreement with all other nodes. In essence, $\mathbf{s}$ will be estimated at each node, but iteratively. Thus, at the end of the optimisation, each node will have the same $\mathbf{s}$. To decouple (2.55), a new variable $\mathbf{z}_j \in \mathbb{R}^n$, $\forall j$ is introduced. The problem (2.55) can now be cast as

$$\begin{aligned} \texttt{minimise} \quad & \frac{1}{2}\sum_{j\in\mathcal{J}} \| \mathbf{x}_j - \mathbf{H}_j\mathbf{s}_j \|^2 \\ \texttt{subject to} \ & \mathbf{s}_j - \mathbf{z}_i = \mathbf{0}, (i,j) \in \mathcal{J}, \\ & \mathbf{s}_i - \mathbf{z}_i = \mathbf{0}, (i,j) \in \mathcal{J}. \end{aligned} \tag{2.56}$$

Problem (2.56) is separable in $j \in \mathcal{J}$. Its augmented Lagrangian is written as

$$\mathcal{L}_\rho(\mathbf{s}, \mathbf{z}, \boldsymbol{\lambda}) = \sum_{j \in \mathcal{J}} \left(\frac{1}{2} \parallel \mathbf{x}_j - \mathbf{H}_j \mathbf{s}_j \parallel^2 - \sum_{i \in \mathcal{J}} \boldsymbol{\lambda}_{ij}^T (\mathbf{s}_j - \mathbf{z}_i) + \sum_{i \in \mathcal{J}} \frac{\rho_{ij}}{2} \parallel \mathbf{s}_j - \mathbf{s}_i \parallel^2 \right), \tag{2.57}$$

where $\boldsymbol{\lambda}_{ij} \in \mathbb{R}^{n \times 1}$ are the Lagrangian multipliers, and $\rho_{ij} \in \mathbb{R}$ are the penalty parameters. This yields an ADMM algorithm with the following successive iterations,

$$\mathbf{s}_j^{t+1} := \left(\mathbf{H}_j^H \mathbf{H}_j + \sum_{i \in \mathcal{J}} \rho_{ij} \mathbf{I} \right)^{-1} \left(\mathbf{H}_j^H \mathbf{x}_j + \sum_{i \in \mathcal{J}} (\boldsymbol{\lambda}_{ij}^t + \rho_{ij} \mathbf{z}_i^t) \right), \tag{2.58}$$

$$\mathbf{z}_j^{t+1} := \frac{1}{J} \sum_{i \in \mathcal{J}} \left(\mathbf{s}_i^t - \frac{1}{\rho_{ij}} \boldsymbol{\lambda}^t \right), \tag{2.59}$$

$$\boldsymbol{\lambda}^{t+1} := \boldsymbol{\lambda}^t + \rho_{ij} \left(\mathbf{s}^{t+1} - \mathbf{z}_j^{t+1} \right). \tag{2.60}$$

Let $\hat{\mathbf{s}}_j$ be the estimate of $\mathbf{s}$ at node j. The Least Squares (LS) and the Minimum Mean Square Error (MMSE) estimates of $\mathbf{s}$ are given by

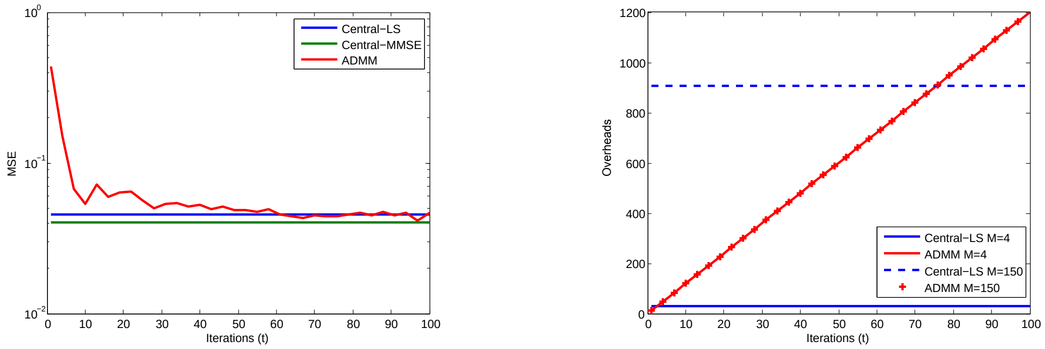
$$\hat{\mathbf{s}}_j^{LS} = (\mathbf{H}_j^H \mathbf{H}_j)^{-1} \mathbf{H}_j^H \mathbf{x}_j, \quad \text{and}, \tag{2.61}$$

$$\hat{\mathbf{s}}_j^{MMSE} = (\mathbf{H}_j^H \mathbf{H}_j + \sigma_j^2 \mathbf{I}_M)^{-1} \mathbf{H}_j^H \mathbf{x}_j. \tag{2.62}$$

Figure 2.5 shows the performance of the ADMM solution against the LS and MMSE estimations. Note that in Figure 2.5a, even though ADMM takes long to reach high accuracy, it reaches moderate accuracy within 20 iterations. Figure 2.5b shows that, when the size of the received message is small, the system overheads are worse than in the case of a centralised solution. But once the message m is increased, the ADMM experience a significant decrease in the overheads. This is why the ADMM is well suited for large systems such as in 5G. Involvement of hyper-dense small cell deployment, massive MIMO, and influx of mobile users in 5G, will require distributed algorithms [13, 14] such as the ADMM.

2.4 Game Theory

Another approach for developing distributed algorithm is game theory. Invented by John von Neumann and Oskar Morgenstern in 1944 [37] and advanced by John Nash [38, 39], game theory is a branch of mathematics that enables modelling and analysis



(A) Sample mean square error with $J = 6$ and $M = 4$.

(B) Communication overheads when $J = 6$ and $M = 4, 150$.

FIGURE 2.5: Comparison of the ADMM algorithm and the centralized solutions

of the interaction between self-interested agents (*id est* players) [1, 37]. This section introduces game theory models, particularly non-cooperative and cooperative models. The discussions of this section are based on the structure of general game theory shown in Figure 2.6.

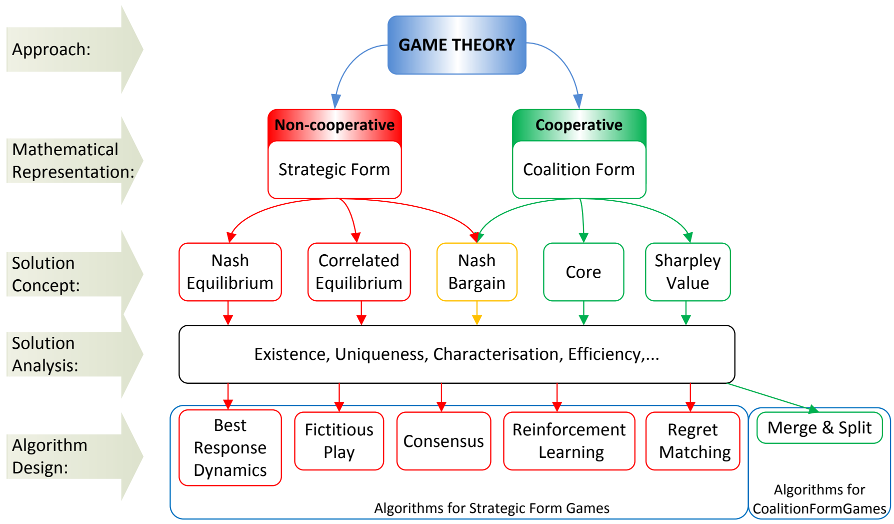


FIGURE 2.6: A partial structure of game theory. Adapted from [1].

2.4.1 Non-cooperative Games

Non-cooperative games can be represented in two ways, the *normal form* or the *extensive form* [40–42]. The normal form is sometimes referred to as the *matrix form* or the *strategic form*, and it is in this form, whereby a game is defined by listing payoffs against their corresponding strategies. It is usually assumed that the players move simultaneously. In the extensive form, players can be deemed to move in a sequential manner (*exempli gratia*, include timing), and as a result the game can easily be represented as a tree diagram. Moreover, in extensive form, players are capable of keeping track of moves other players made when they make decisions [40]. It can therefore be assumed that the normal form is a special case of the extensive form when no player is able to observe other players' moves.

Definition 2.4 (*Non-cooperative Game*)

An N-person normal form game is a modelled as

$$\mathcal{G} = \{\mathcal{N}, \{\mathcal{S}_i\}_{i\in\mathcal{N}}, \{u_i\}_{i\in\mathcal{N}}\}, \tag{2.63}$$

where:

1. $\mathcal{N}$ *is a finite set of* players *indexed by* $\mathcal{N} = \{1, \ldots, N\}$,
2. $\mathcal{S}_i$ *is a set of available* strategies *(actions) for player* i,
3. $u_i : \mathcal{S}_i \mapsto \mathbb{R}$ *is the* utility *(payoff) function of player* i.

In a non-cooperative game, the players compete with each other, and therefore, each player makes independent decision, given the possible actions taken by other players. This is a different case with a cooperative game, where the players coordinate in making decisions. Authors in [40] highlighted that the term *non-cooperative* does not necessary mean players do not cooperate, but rather it implies that any cooperation that suffices should be *self-enforcing*, without coordination between players. A distinguish between an action and a strategy is also very important in *dynamic games*, whereby choices of each player are dictated by the available information [40]. The games considered in this thesis are *static games*, (*id est* decisions are taken simultaneously), and therefore, strategy and action will mean the same thing, unless stated otherwise. A *mixed strategy* is the mapping from information available to player i to the action set $\mathcal{A}_i$ available to this player [40, 41, 43], and it can be defined as

$$\mathcal{S}_i = \left\{ s_i : \mathcal{A}_i \mapsto \mathbb{R}_+, \sum_{a_i \in \mathcal{A}_i} s_i(a_i) = 1 \right\}. \tag{2.64}$$

Players select strategies that will optimise[5] their utilities. It follows that if a player

[5] Depending on the system objective function, players select strategies to maximise their payoffs or minimise their utilities.

selects a strategy from the probability distribution in (2.64), with probability of 1, that strategy is called *pure strategy*. A useful approach to solve non-cooperative games is by utilising dominating strategies, which reduce or simplify the game, by eliminating some dominated strategies [40].

Definition 2.5 (*Dominant Strategy*)

A strategy s_i strictly dominates *s'_i for player i if*

$$u_i(s_i, \mathbf{s}_{-i}) > u_i(s'_i, \mathbf{s}_{-i}), \quad \forall s_i \in \mathcal{S}_i \quad \text{and} \quad \mathbf{s}_{-i} \in \mathcal{S}_{-i}. \tag{2.65}$$

A strategy s_i weakly dominates *s'_i for player i if*

$$u_i(s_i, \mathbf{s}_{-i}) \geq u_i(s'_i, \mathbf{s}_{-i}), \quad \forall s_i \in \mathcal{S}_i \quad \text{and} \quad \mathbf{s}_{-i} \in \mathcal{S}_{-i}, \tag{2.66}$$

where $\mathcal{S}_{-i} = \prod_{j \neq i} \mathcal{S}_j$ is a set of all strategy profiles for all players other than player i.

A game in Definition 2.4 is a *finite game*, as it is characterized by a finite set of strategies $\mathcal{S}_i$ for all $i \in \mathcal{N}$, finite set of players and finite set of utilities. According to [38], for any finite non-cooperative game, an equilibrium point will be attainable, in this case, it is the Nash equilibrium (NE). At NE, none of the players has any incentive to deviate.

Theorem 2.1 (*Nash Theorem*)

Every (finite) normal form game has at least one NE [38, 44].

Definition 2.6 (*Nash Equilibrium*)

An equilibrium is a state where no player can benefit by changing its current strategy if the other players maintain their NE strategies [40, 43].

$$u_i(s_i^{NE}, \mathbf{s}_{-i}^{NE}) \geq u_i(s_i, \mathbf{s}_i^{NE}) \quad \forall s_i \in \mathcal{S}_i, \forall i \in \mathcal{N}. \tag{2.67}$$

The mathematical description of the NE in (2.67), shows that the NE set contains all the *best response* (BR) strategies of all players, given that some players resort to NE strategy. It is therefore reasonable to say that, the NE is the solution to a non-cooperative game.

Definition 2.7 (*Best Response*)

The BR for player i to the strategies $\mathbf{s}_{-i}$ of other players is the set valued function

$$BR_i(\mathbf{s}_{-i}) = \underset{s_i \in \mathcal{S}_i}{\texttt{argmax}}\, u_i(s_i, \mathbf{s}_{-i}). \tag{2.68}$$

From Definition 2.7, the strategy profile $\mathbf{s}^{\text{NE}}$ is an NE if and only if $\mathbf{s}^{\text{NE}} \in \text{BR}(\mathbf{s}^{\text{NE}})$. Unfortunately, the NE solution has two major defects [45], and this makes it unattractive in certain problems.

1. More than one NE may exist in a game, which makes it difficult to judge the outcome of a game.
2. NE is often inefficient [45–47]. In [47], it was shown that the NE is bounded by a constant irrespective of the available transmission power. Hence, the NE inefficiently allocates the resources which results in low system performance.

2.4.2 Cooperative Games

In order to achieve more efficient payoffs, cooperation may be necessary between players. In this manner, players are able to share relevant information in order to facilitate the process of achieving better payoffs than the NE. As depicted in Figure 2.6, a cooperative game is represented in a *coalition form* or through the *bargain theory*. The interest of this thesis is mainly on bargain theory. In particular, Nash bargain (NB) process, which is an *axiomatic* [39] (*id est* characterized by the process of checking if the outcome satisfies a set of predefined axioms) bargain process, is considered. For brevity, the NB problem is defined by a tuple $\{\mathcal{S}, \mathbf{d}\}$, where the disagreement/threat point $\mathbf{d} \in \mathcal{S}$, is an outcome when the players cannot reach an agreement [48]. The Nash axiomatic bargain theory requires the utility set to be a convex set, in order to guarantee a unique solution that satisfies the four axioms discussed in [48]. The NB aims to maximise

$$\text{NB} = \prod_{i}^{N} (s_i - d_i)\,, \tag{2.69}$$

where d_i is the disagreement point. Maximising the NB function (2.69) maximises the volume of the box between the NE and the Pareto boundary. Solution to (2.69) can be found graphically, by choosing a constant c on the Pareto boundary where Nash curve and the Pareto boundary intersect. The Nash curve therefore satisfies

$$c = (s_1 - d_1), \ldots, (s_N - d_N). \tag{2.70}$$

From (2.69) and (2.70), it is observed that the NB solution will achieve higher utility than the NE for all players. These solutions give Pareto optimal utilities. The main interest in this work, is to investigate both the NE and the Pareto optimal solutions.

Definition 2.8 (*Pareto Optimal*)

A strategy $\mathbf{s}^{PO}$ *is deemed Pareto optimal [1,40,43,48] if for any strategy* $\mathbf{s}$, *it is such that*

$$u_i(\mathbf{s}^{PO}) \geq u_i(\mathbf{s}), \forall i \in \mathcal{N}. \tag{2.71}$$

When operating at Pareto optimal profile, it it not possible to increase the payoff of one player without decreasing utilities of other players.

2.5 Resource Allocation via Auction Theory

The resource management aspects discussed earlier, show how wireless communications need appraisal via economic considerations in order to deem them viable. Economic models have a root in the area of wireless communications network. The authors in [49] emphasise that wireless communications and networking systems are tightly coupled with economics such, that it becomes vital to consider the economic implication when making a technology choice. Ideally, service providers[6] should aim to maximise the *social welfare* of the network. Unfortunately, the wireless resources such as the spectrum license and equipments are procured at very high costs. In addition, the maintenance of the wireless systems and personnel recruitment dictate the objectives of the service providers. Due to these costs, service providers are propelled to maximise their profits. The tension between the supply and demand, the competition for users and spectrum acquisition by service providers, can be analysed by studying the economics of the network. Economic models can be used to improve the overall performance of the wireless networks and satisfaction levels for the users and the service providers [49].

[6]The term service providers is used to refer to mobile network operators (MNOs) and third party network operators, *id est* customers.

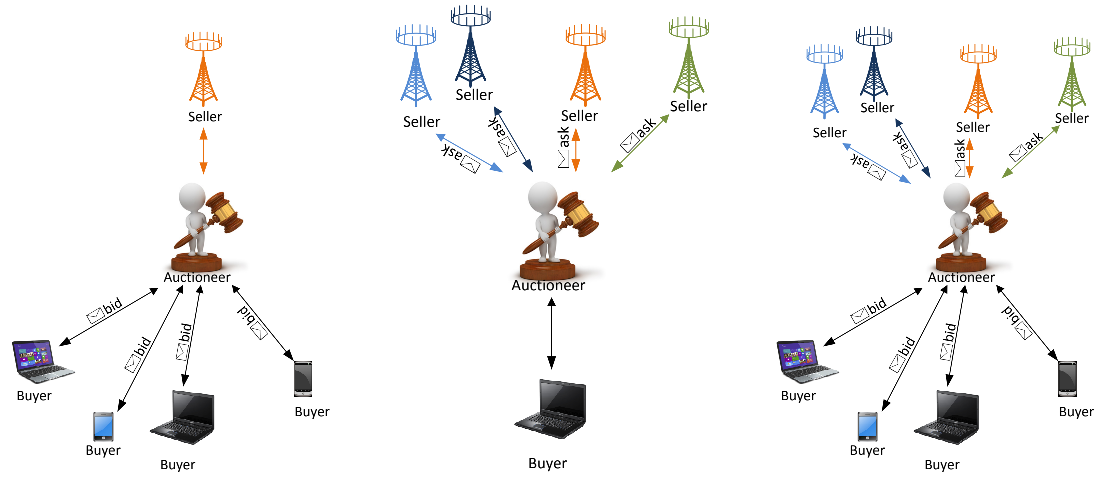
The scarcity of the spectrum has instigated new resource allocation techniques via auctioning. *Auction theory* is a subfield of economics and management, which deals with how agents behave in *auction* markets [50, 51]. *Auction* is a process of selling or buying goods or services. Spectrum auctioning, for example, has been successfully implemented, and it has shown improvement to the overall wireless system. Examples of spectrum actioning by the Federal Communications Commission (FCC) can be viewed at (`https://www.fcc.gov`). Unlike in the past, where spectrum was solely owned by government entities and commercial operators under static licensing, new technologies and services allow customers to own or rent part of the spectrum. Allowing diverse spectrum ownership can improve the network performance, but at the same time, it can elevate the scarcity of the spectrum, especially when static licensing is used. Through dynamic spectrum licensing, unlicensed users can opportunistically share the spectrum with licensed users. In order to encourage the license holders to release their spectrum for sharing, dynamic spectrum management should provide satisfactory economic incentives. This work proposes auction based mechanisms to reward the license holders, for allowing unlicensed users access to their spectrum.

2.5.1 Types of Auctions

Many kinds of auctions are found in the literature [50, 52]. To aid the explanations of auctions discussed here, some basic terminologies in auction theory are provided.

Definition 2.9 (*Terminologies in Auction Theory*)

- *Commodity: A commodity/good/item*[7] *is an object being offered by a seller.*
- *Seller: A seller*[8] *is the owner of the items, and he is interested in selling his items.*
- *Bidder: A buyer/bidder*[9] *is someone who wants to buy items in an auction.*
- *Auctioneer: An agent*[10]*, usually appointed by the seller, who is responsible for conducting the auction proceedings.*
- *Valuation: It is the monetary values as perceived by the bidders/sellers on the items on auctions, or a function that maps the values to the items. Both the bidder and the seller have a reserved valuation on the items. The valuations*



(A) Forward Auction. (B) Reverse Auction. (C) Double Auction

FIGURE 2.7: Classification of auctions. The arrows denote the transactions of commodities and money among participants.

can be private or public[11]. *A valuation for item g by bidder s is denoted by* v_{sg}.

- *Bid/Ask: It is the proposal made by the bidder/seller to the seller/bidder. A bid can be the valuation of the item, but it needs not to be true. A bid from bidder s, for item g is denoted by b_{sg}.*
- *Price: This is a value asked by the auctioneer/seller during an auction. It also refers to the payment*[12] *that has to made by a bidder for acquisition of an item. The price or payment for item g is denoted as q_g or p_g.*
- *Utility: The residual value after subtracting the price or payment from the valuation. The utility of bidder s from item g is of the form $u_{sg} = v_{sg} - p_{sg}$, while seller i utility is $u_{ig} = p_{ig} - v_{ig}$.*

Auctions can be classified in terms of who is initiating the proposal (*id est* bid/ask), number of items in the market, or mode of expressing bids. Figure 2.7 shows auction types, depending on who is exhibiting a bid/ask.

[7] The term item is used henceforth. In this thesis, guest users (GUs) are treated as items.
[8] In this work, the MNO assumes the position of a seller.
[9] The rest of the document use the term bidder. In this thesis, the SCA is assigned the role of a bidder.
[10] This work considers the MBS as an auctioneer. Since the auctioneer and the seller belong to the same entity, these terms are used as synonyms.
[11] In this thesis, the seller's valuation is public while the bidders' valuations are private.
[12] The terms price and payments are used as synonyms.

- Forward Auction: In this auction, bidders bid for items from sellers as shown in Figure 2.7a.
- Reverse Auction: Under this auction, sellers compete for bidders by taking ask actions as shown Figure 2.7b.
- Double-sided Auction: For this auction type, both the bidders and the sellers make bids/asks as shown in Figure 2.7c.

Additionally, auctions can be classified as open-cry or sealed-bid. In open-cry auctions, bidders bids are publicised, while in sealed-bid auctions, bids are confidentially submitted to the auctioneer(s). This thesis is concerned with sealed-bid auctions; and hence, examples of sealed-auctions are discussed below.

2.5.2 *k*-th Price Auctions

First-price and second-price sealed auctions are the most common auctions. In first-price auction, the winner is nominated as the bidder who proposed the highest bid. The payment is the winner's bid. In a second-price auction, *id est*, the Vickery auction [53], the winner is the highest bidder with the second highest bid as the payment. That is, if bidder s is the highest bidder on item g with bid b_{sg}, the payment is determined as $p_{sg} = \mathtt{max}_{j \neq s} p_{jg}$. The Vickery-Clarke-Groves (VCG) mechanism [43, 50, 53–57] generalises the Vickery auction. The VCG has the following setup: Assume there are $S = |\mathcal{S}|$ bidders, and a feasible allocation set $\mathcal{A}$. Player s has a valuation function $v_s \in \mathcal{V}_s : \mathcal{A} \mapsto \mathbb{R}$, where $\mathcal{V}_s \subseteq \mathbb{R}^{|\mathcal{A}|}$ is a commonly known set of the possible valuation functions for bidder s. With the assumption that all bidders have *quasilinear* utilities, the utility of bidder s of alternative $a \in \mathcal{A}$, with payment p_s is $u_s = v_s(a) - p_s$. Let $\mathcal{V} = \mathcal{V}_1 \times \cdots \times \mathcal{V}_S$, $\mathbf{p} = [p_1, \ldots, p_S]$, and $\mathbf{v} = [v_1, \ldots, v_S]$.

Definition 2.10 (*Direct Revelation Mechanism*)

A direct revelation mechanism is a social choice $(f, \mathbf{p})$, $f : \mathcal{V} \mapsto \mathcal{A}$ *that gets a vector* $\mathbf{v}$ *of valuation functions, selects some alternative* $a \in \mathcal{A}$*, and payments* $\mathbf{p}(\mathbf{v})$ *[43, 50, 56]. If the mechanism is a truthful mechanism it has the property that, for* $v_s, v'_s, \mathbf{v}_{-s}$ *it is the case*

$$v_s(f(\mathbf{v})) - p_s(\mathbf{v}) \geq v_s(f(v'_s, \mathbf{v}_{-s})) - p_s((v'_s, \mathbf{v}_{-s})), \tag{2.72}$$

when v_s is the true valuation. This latter property can be interpreted via an incentive compatible mechanism wherein bidders are incentivised for bidding truthfully and do not need to guess how other bidders are bidding.

Definition 2.11 (***Vickery-Clarke-Groves (VCG) Mechanism***)

A mechanism $(f, \mathbf{p})$ is a VCG mechanism if

- *f maximises the social welfare (id est surplus) such that*

$$f(\mathbf{v}) \in \underset{a \in \mathcal{A}}{\mathtt{argmax}} \sum_{s \in \mathcal{S}} v_s(a), \tag{2.73}$$

- *and for some functions $h_1, \ldots, h_S$, where $h_s : \mathcal{V}_{-s} \mapsto \mathbb{R}$ (id est h_s does not depend on v_s), it is a case that for all $v_1 \in \mathcal{V}_1, \ldots, v_S \in \mathcal{V}_S$:*

$$p_s(\mathbf{v}) = h_s(\mathbf{v}_{-s}) - \sum_{j \neq s} v_j(f(\mathbf{v})), \tag{2.74}$$

where $\mathcal{V}_{-s} = \mathcal{V}_1 \times \cdots \times \mathcal{V}_{s-1} \times \mathcal{V}_{s+1} \times \cdots \times \mathcal{V}_S$, and $\mathbf{v}_{-s} = [v_1, \ldots, v_{s-1}, v_{s+1}, \ldots, v_S]$ [53, 58].

Theorem 2.2 (***Vickery-Clarke-Grove***)

Every VCG mechanism is incentive-compatible [43, 50].

Proof 1 Fix bidder s, $\mathbf{v}_{-s}$, $\mathbf{v}_s$, v_s, and v'_s. Suppose bidder s has valuation v_s and is contemplating on bidding v'_s. Let $a = f(v_s, \mathbf{v}_{-s})$ and $a' = f(v'_s, \mathbf{v}_{-s})$. The utilities of bidder s for declaring v_s and v'_s are

$$u(a) = v_s(a) + \sum_{j \neq s} v_j(a) - h_s(\mathbf{v}_{-s}), \quad \text{and} \quad u(a') = v_s(a') + \sum_{j \neq s} v_j(a') - h_s(\mathbf{v}_{-s}), \tag{2.75}$$

respectively. Since $a = f(v_s, \mathbf{v}_{-s})$ maximises social welfare over all alternatives, then

$$u(a) \geq u(a'), \qquad \Longleftrightarrow \qquad v_s(a) + \sum_{j \neq s} v_j(a) \geq v_s(a') + \sum_{j \neq s} v_j(a'). \tag{2.76}$$

■

According to the Clarke pivot rule [59, 60], the winning bidder is charged its externalities by setting $h_s(\mathbf{v}_{-s}) = \mathtt{max}_{b \in \mathcal{A}} \sum_{j \neq s} v_j(b)$ in (2.74). This yields a payment of the form

$$p_s(\mathbf{v}) = \max_b \sum_{j \neq s} v_j(b) - \sum_{j \neq s} v_j(a), \tag{2.77}$$

where $a = f(\mathbf{v})$. Essentially, the Clarke pivot rule sets $h_s(\mathbf{v}_{-s})$ to the social welfare of every bidder if bidder s is not present. Hence, the payment $p_s(\mathbf{v})$ is the maximum social welfare in the absence of bidder s, which defines the externality of bidder s. Coupling the VCG mechanism with the Clarke pivot, yields a mechanism that is incentive compatible, maximises the social welfare, makes no payments to bidder, and is individually rational (if the valuation are positive).

2.5.3 Combinatorial Auctions

Often, auctions are classified as single-object or multi-object auctions. As it is the case in this thesis, bidders usually participate in multiple auctions which leads to a more sophisticated market. The bidders may need to buy a structured combination of heterogeneous items. Combinatorial auctions allows items to be auctioned concurrently and bidders can express preferences on bundles of items. These types of auctions are preferable when the items in auction are dependent [43,51,58,61,62]. Combinatorial auction mechanisms have impediments such as *communication overheads*, *market clearing complexity* and *exposure problem* [51].

To elaborate on some of these drawbacks, consider a combinatorial auction with $S = |\mathcal{S}|$ bidders, and $G = |\mathcal{G}|$ *non-identical* items. Let $\mathcal{A}$ denote the outcome set of S-vectors $(\mathcal{G}'_1, \ldots, \mathcal{G}'_S)$ with $\mathcal{G}'_s$ denoting the items allocated to bidder s. Each bidder will have a private valuation $v_s(\mathcal{G}'_s)$ for every possible bundle $\mathcal{G}'_s \subseteq \mathcal{G}$ it might get. This yields 2^G private values. For an auctioneer to gather such huge information from all bidders, and compute the optimal allocations will consequently lead to high communication overheads and computational complexity, especially when the number of items is large [43, 50, 62]. Further to this, a bidder will incur valuation overheads for computing all the possible values. These disadvantages lead to the development of indirect mechanisms, wherein information about bidders' preferences is gathered on the "need-to-know" basis. One classic auction using the indirect mechanism is the *simultaneous multiple-round ascending auction* (SMRA) (*id est simultaneous ascending auction* (SAA)) [43, 50].

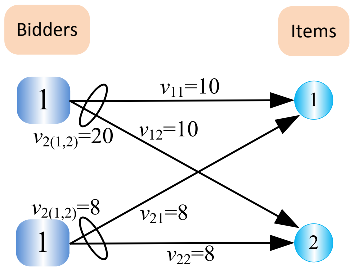


FIGURE 2.8: An bipartite for economy with two bidders and two items. Edges are labelled with bidders valuations.

2.5.4 Simultaneous Multiple-Round Ascending Auctions

SMRA is a collection of concurrent single-item English auctions. In an English auction, the auctioneer iteratively increments the price of an item by a known value. In each iteration the bidders who are interested will respond to the price by indicating their *demand*. The auction will continue until the supply matches the demand (*id est* market clears). While SMRA is widely used in spectrum auctioning [51, 62], it has drawbacks such as *collusion*, *demand reduction*, and *exposure problem* [43, 50].

Example 2.5 (*Demand Reduction*)

Consider an economy with two bidders and two identical items as shown in Figure 2.8. Bidder 1 has valuation 10 for each of the items and valuation of 20 for both items. Bidder 2 has valuation of 8 for only one of the item and is interested in only one item (id est its valuation is 8 for both items). The maximum surplus of this auction is 20, which is attained by allocating both items to bidder 1. Supposing the auction is an SMRA auction. Note that bidder 2 will be glad to acquire any of the items at any price less than 8. Therefore bidder 2 will drop out of the auction when both items have price at least 8. If bidder 1 insists in winning both items, its utility will be $u_1 = 20 - 16 = 4$. *Now suppose, bidder 1 reduces its demand by targeting only one item, then each bidder will win one of the items with prices close to zero. This will yields utilities of* $u_1 \approx 10$ *for bidder 1 and* $u_2 \approx 8$ *for bidder 2. Ultimately, this will lead to reduced welfare and revenue.*

Example 2.6 (*Exposure Problem*)

Consider the same model in Figure 2.8, but assume the items are non-identical. Further assume bidder 1 has valuation 100 for both items (the items are complementary)

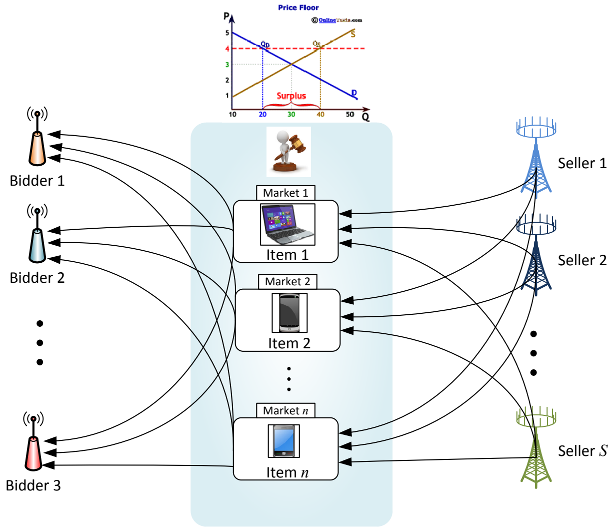


FIGURE 2.9: An example of an model with S bidders, G items and N sellers.

and valuation of 0 otherwise. Bidder 2 has valuation of 75 for only one of the item, and is interested in only one item (id est its valuation is 75 for both items). The maximum surplus is attained by allocating all the items to bidder 1, with surplus of 100 and revenue of 75. In SMRA, bidder 2 will not drop out of the auction until the price of each item reach 75. Bidder 1 will have to pay 150 for both items if he insists in winning both items. This will result in a negative utility for bidder 1.

2.5.5 General Equilibrium Theory

Usually markets comprise of sellers and bidders as depicted in Figure 2.9. This thesis considers only an *exchange model*, wherein there is no production. The items flow between the sellers and bidders as the prices are varied. The prices dictate how the items should be allocated. Price systems are useful for *coordinating* and *equilibrating* the markets. In most cases, bidders have monetary *budgets* that cannot be exceeded. It is therefore vital for bidders to derive their *demands* according to their budgets and *preferences*. The two main theories used in determining the equilibrium of an economy are the *partial equilibrium theory* and the *general equilibrium theory*. In partial equilibrium theory, the markets are independent, such that the changes in one market do not influence the price in another market. On the other hand, general equilibrium theory deals with markets that are coupled, such that the change of prices in one market lead to a change in the prices of other markets. Since the structure of problems considered in this thesis can be analysed via the general equilibrium theory, it is discussed below.

Consider a set of competitive markets with a set $\mathcal{S} := \{1, 2, \ldots, S\}$ of bidders, and a set $\mathcal{G} := \{1, 2, \ldots, G\}$ of items. Bidder s has $\mathbb{R}^G$ as the consumption set, and utility function $u_s : \mathbb{R}_+^G \mapsto \mathbb{R}_+$. Usually, bidder s is endowed with a budget b_s that constrains the number of items he can buy. By using the Arrow-Debreu market model, it can be assumed that bidder s is initially endowed with the amount of items instead of monetary budgets. Let the endowment for bidder s be $\mathbf{e}_s \in \mathbb{R}_+^G$ and $\mathbf{q} = [q_1, q_2, \ldots, q_G]$ be the price profile of the items, where q_g denotes the price for item g. This simulates an exchange economy $\mathcal{E}((u_s, \mathbf{e}_s)_{s \in \mathcal{S}})$ wherein each bidder has a budgets set

$$\mathcal{B}_s = \{\mathbf{x} \in \mathbb{R}_+^G : \mathbf{q}^\mathsf{T}\mathbf{x} \leq \mathbf{q}^\mathsf{T}\mathbf{e}_s\}. \tag{2.78}$$

Each bidder is faced with a problem of the form

$$\underset{\mathbf{x} \in \mathbb{R}_+^G}{\texttt{maximise}} \quad u_s(\mathbf{x}), \; \texttt{subject to} \quad \mathbf{q}^\mathsf{T}\mathbf{x} = \mathbf{q}^\mathsf{T}\mathbf{e}_s. \tag{2.79}$$

Note that each bidder makes the decision on the amount of goods to buy independent of other bidders actions. This distributed decision-making yields a distributed algorithm. The solution to the exchange economy $\mathcal{E}((u_s, \mathbf{e}_s)_{s \in \mathcal{S}})$ is usually characterised as the Walrasian equilibrium (WE) [43, 50].

Definition 2.12 (*Walrasian Equilibrium*)

A WE for economy $\mathcal{E}((u_s, \mathbf{e}_s)_{s \in \mathcal{S}})$ is a price vector and allocation $(\mathbf{q}, (\mathbf{x}_s)_{s \in \mathcal{S}})$ such that [43, 50]:

- *Bidders are maximising their utilities:*

$$x_s \in \texttt{argmax}\, u_s(\mathbf{x}), \forall s \in \mathcal{S}. \tag{2.80}$$

- *The market clears:*

$$\sum_{s \in \mathcal{S}} \mathbf{x}_{sg} = \sum_{s \in \mathcal{S}} \mathbf{e}_{sg}. \tag{2.81}$$

2.6 Conclusion

This chapter has laid the fundamentals required for the rest of the thesis. In particular, it covered beamforming techniques, convex optimisation, dual decomposition methods, game theory and auction theory. All these techniques will be applied to derive distributed algorithms for wireless networks. In Chapters 3 and 4, beamforming techniques, convex optimisation, dual decomposition methods, and game theory are used for the development and analysis of the mixed QoS strategic non-cooperative game (SNG) and mixed QoS cooperative game. In Chapters 5 and 6, beamforming techniques, convex optimisation, dual decomposition methods, and auction theory are employed to derive and analyse novel distributed algorithms for traffic offloading and beamformer design in HetNets. Related and parallel works for every chapter are discussed therein.

Chapter 3

Resource Allocation via Game-Theoretic and Convex Optimisation Techniques

This chapter demonstrates how mixed QoS optimisation can be solved via game-theoretic models and convex optimisation. Firstly, a two-user game is constructed to demonstrate how the *bargain region* can be extended by using mixed QoS criterion. Secondly, the two-user network is extended to a multicell multiuser network. A beamformer design problem under mixed QoS criterion is solved by formulating a mixed QoS SNG and mixed QoS cooperative game. The mixed QoS criterion is one way to present a multi-objective optimisation (MOP) problem, wherein agents have more than one objective. Later in Chapter 4, the two methods will be combined to form the Egalitarian and Kalai-Smorodinsky bargain games. The work discussed here has been published in [63, 64] (also see copyrights clearance in Appendix A).

3.1 Introduction

In multicell coordinated beamforming (MCBF), multiple BSs select transmit strategies jointly in order to mitigate intercell interference. In most cases, MCBF algorithms assume full and perfect knowledge of the CSI [65, 66] at the transmitters and the receivers. Global CSI is sent to a central processor where all the computation takes place.

Even though this approach provides an improved spectrum utilisation as compared to the traditional interference avoidance transmit strategies, it is evident that the heavy CSI and data sharing will be a burden to the backhaul links. Consequently, the complexity will significantly increase [67].

3.1.1 Related and Parallel Works

Game theoretic techniques offer a structure that readily allows for decentralised implementation. In some of the early works in [46, 68, 69], an SNG was applied to a power control problem for a single-input single-output (SISO) model. These works demonstrated the inefficiency of the NE point with respect to Pareto optimality; hence, pricing mechanism was proposed to obtain efficient solutions which required cooperation among the players.

Non-cooperative and cooperative games were used in [47, 70–79] to develop distributed algorithms for SISO and MISO-IFC systems. Works in [70, 71, 73] considered a SISO model. A MIMO-IFC setup was addressed in [72, 74] but [74] considered both non-cooperative and cooperative games.

In [47, 75–79], cooperative games were used to develop distributed optimisation algorithms. In [75, 76], a SISO model was considered and bargain theory was employed to derive decentralised solutions for spectrum sharing. Authors in [47, 77] used a MISO-IFC setup to investigate non-cooperative and cooperative games. Furthermore, works in [47, 77] proved that solutions of linear combination of selfishness and altruism lead to Pareto optimal solutions.

Earlier works on beamformer design for wireless communications can be found in [23, 28, 80, 81]. In [23], the problem was reformulated into virtual uplink problem, and the Perron Frobenius theorem was used to derive an iterative solution. Authors in [80] reformulated the problem into a convex semi-definite optimisation problem, while in [81], linear programming duality was used. The same problem with per-antenna power constraints was solved in [28] using Lagrangian duality. The SINR balancing problem was solved in [22, 82–84] by using various approaches including the Perron Frobenius, second order cone programming (SOCP), and the bisection methods [12]. Some of the cited works considered centralised solution, which have practical difficulties due to the reasons mentioned earlier. To overcome this, some distributed algorithms have been proposed in the literature. In distributed algorithms, the computational complexity is distributed among the access points (APs) and only relevant information is exchanged for coordination.

Authors in [30, 85–87] proposed distributed power minimisation algorithms using uplink-downlink duality, dual decomposition, and primal decomposition methods. The dual decomposition and primal decomposition methods are used in conjunction with the subgradient method, which is highly sensitive on the step size [33], and has slow convergence as studied in [88]. A distributed SINR balancing solution is proposed in [89] by combining uplink-downlink duality with bisection method [12]. In one of the recent works [88], the ADMM algorithm [33] was adopted to solve both the power minimisation and SINR balancing problems. To solve the quasi-convex SINR balancing problem [12], the work in [88] combined golden search ratio algorithm with the ADMM algorithm.

3.1.2 Contributions

This chapter proposes game theoretic and dual decomposition frameworks for a wireless network with users having different classes of QoS [90]. The mixed QoS criterion is attractive for energy efficient wireless communication as suggested in [91]. The work in [91] argued that; consideration of differentiated QoS by exploiting delay tolerant and delay intolerant applications is one of the key energy-efficient resource management methods in 5G. All the past works addressed a situation whereby all players or APs have similar intentions; that is, either to maximise their utilities or to minimise their costs. The contributions of this work addresses situations where APs can simultaneously be confronted by both power minimisation ($\mathscr{P}_{\text{PMP}}$ in (2.4)), and SINR balancing ($\mathscr{P}_{\text{SB}}$ in (2.5)) problems. This MOP is captured by the mixed QoS criterion. In the mixed QoS problem, it is required to achieve a specific SINR target for a certain group of users while the SINRs of the other group of users should be balanced and maximised [90]. The mixed QoS based beamformer design for delay tolerant and delay intolerant services was proposed in [90] using a centralised optimisation method. However, in this thesis, decentralised algorithms design is considered.

3.2 System Model for Two-user Game

Consider a MISO-IFC system shown in Figure 3.1 with $N = 2$ APs and $K = 2$ mobile stations (MSs), where each user is assigned to one AP at any given time. It is assumed that both APs share the same frequency band, and that each of them is equipped with $M = 2$ transmit antennas. This setup can be encountered when the two APs belong to different operators. Moreover, due to the scarcity of the spectrum, aggressive frequency reuse like in HetNets may result in the same setup. In the downlink, the transmitted signal for k-th user from AP n can be written as

$$\mathbf{x}_k(t) = \mathbf{w}_k s_k(t), \tag{3.1}$$

where $s_k(t) \in \mathbb{C}$ represents the information symbol at time t, and $\mathbf{w}_k \in \mathbb{C}^M$ is the unnormalised transmit beamforming vector for user k. Without loss of generality, assume that $s_k(t)$ is normalised such that $\mathbb{E}\{|s_k(t)|^2\} = 1$, and that all data streams are independent such that, $\mathbb{E}\{s_k(t)s_i(t)^*\} = 0$, if $k \neq i$. The subscripts $k = 1$ and $k = 2$ are used to refer to RTU (MS_1) and NRTU (MS_2), respectively. Similarly the indices $n = 1$ and $n = 2$ are used to refer to AP_1 and AP_2, respectively. Assume that AP_1 is serving a real-time user (RTU), whereas AP_2 is serving a non-real-time user (NRTU). In this setting, MS_1 will need to attain a specific SINR target while MS_2 will want to attain maximum possible rate. The received signals at MS_1 and MS_2 are respectively given by:

$$y_1(t) = \mathbf{h}_{11}^{\mathsf{H}}\mathbf{w}_1 s_1(t) + \mathbf{h}_{21}^{\mathsf{H}}\mathbf{w}_2 s_2(t) + \eta_1(t), \tag{3.2}$$

$$y_2(t) = \mathbf{h}_{22}^{\mathsf{H}}\mathbf{w}_2 s_2(t) + \mathbf{h}_{12}^{\mathsf{H}}\mathbf{w}_1 s_1(t) + \eta_2(t), \tag{3.3}$$

where $\mathbf{h}_{nk} \in \mathbb{C}^M$ is the channel vector from AP n to the k-th user, and $\eta_k(t) \in \mathcal{CN}(0, \sigma^2)$ is the circular symmetric zero mean complex Gaussian noise with variance σ^2. Assume the maximum transmit power at each AP is $\varphi_n^{\max} = 1$; hence, the power constraints need to satisfy $\|\mathbf{w}_k\|^2 \leq 1,\ k = 1, 2$.

3.2.1 Problem Formulation

The game environment has the following description: The APs are set to be the players and the transmit beamformers are set to be the strategies. Since the APs have

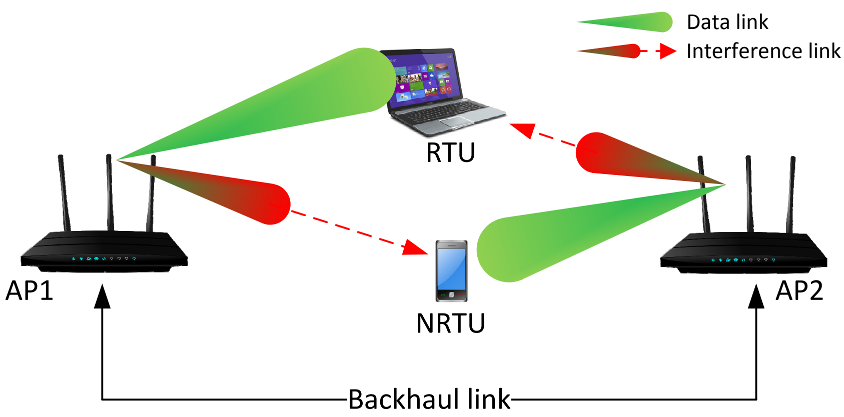


FIGURE 3.1: A scaled down model of Figure 1.5 to a two-user MISO IFC. AP_1 serves a RTU and AP_2 serves a NRTU.

different objectives, the utilities for AP_1 and AP_2 are transmit power and information rate, respectively. The target rate for the RTU is denoted as ψ_1. Now the mixed QoS SNG is formulated as

$$\mathcal{G} = \{\mathcal{N} := \{1,2\}, \{\mathbf{w}_k \ \mid \ \|\mathbf{w}_k\|^2 \leq 1\}_{k=1,2}, \{\|\mathbf{w}_1\|^2, \mathsf{R}_2\}\}, \tag{3.4}$$

where $\mathcal{N}$ is the set of all players, $\mathbf{w}_k$ denotes the strategy for the n-th player, and R_k is the utility function which denotes the rate achieved by the k-th user. The rate for the k-th user is defined by

$$\mathsf{R}_k = \log_2(1 + \mathsf{SINR}_k) = \log_2\left(1 + \frac{|\mathbf{w}_k^{\mathsf{H}}\mathbf{h}_{kk}|^2}{|\mathbf{w}_j^{\mathsf{H}}\mathbf{h}_{jk}|^2 + \sigma^2}\right), \quad j \neq k. \tag{3.5}$$

The optimisation problems at AP_1 and AP_2 are formulated as

$$\mathscr{P}_{\mathrm{PMP}} : \underset{\mathbf{w}_1}{\mathtt{minimise}} \ \ \|\mathbf{w}_1\|^2, \ \mathtt{subject\ to} \ \ \mathsf{R}_1 \geq \psi_1, \tag{3.6}$$

$$\mathscr{P}_{\mathrm{SB}} : \underset{\mathbf{w}_2}{\mathtt{maximise}} \ \ (\mathsf{R}_2), \ \mathtt{subject\ to} \ \ \|\mathbf{w}_2\|^2 \leq \varphi_2^{\max}. \tag{3.7}$$

3.3 Mixed QoS SNG

The unique Nash equilibrium-mixed QoS (NE-mixed QoS) operating point, that corresponds to the *scaled* maximum ratio transmission (MRT) [47], is given by the dominant strategy of the form

$$\mathbf{w}_k^{\text{NE-mixed QoS}} = \chi_k \frac{\mathbf{h}_{kk}^*}{\|\mathbf{h}_{kk}\|}, \tag{3.8}$$

where $(\cdot)^*$ is the complex conjugate, and χ_k is a scaling factor that accounts for transmission power. Since AP_1 only wants to achieve a specific target rate, it should be the case $0 \leq \chi_1 \leq 1$. On the other hand, AP_2 is interested in maximising its utility; therefore, it will use all its maximum allowable power, giving $\chi_2 = 1$. If both players consider altruism; then, they will choose zero forcing (ZF) strategies such that a null is always placed on the direction of the other player. The mixed QoS strategy corresponding to the ZF beamforming direction (ZF-mixed QoS) is given by

$$\mathbf{w}_k^{\text{ZF-mixed QoS}} = \chi_k \frac{\prod_{\mathbf{h}_{jk}^*}^{\perp} \mathbf{h}_{kk}^*}{\left\| \prod_{\mathbf{h}_{jk}^*}^{\perp} \mathbf{h}_{kk}^* \right\|}, \tag{3.9}$$

where $\mathbf{\Pi}_{\mathbf{X}}^{\perp} = \mathbf{I} - \mathbf{X}(\mathbf{X}^{\mathsf{H}}\mathbf{X})^{-1}\mathbf{X}^{\mathsf{H}}$ is the the orthogonal projection onto the orthogonal complement of the column space of $\mathbf{X}$. If mixed QoS criterion is not considered, the NE-mixed QoS will transform to conventional NE (*id est* NE), with $\chi_k = 1$, $k = 1, 2$. Both the NE and NE-mixed QoS are generally inefficient as discussed in [46, 47, 68, 69, 74–79]. Thus, there is a need to study cooperative games and other transmission strategies. Nevertheless, it will be shown via numerical simulations that operating at NE-mixed QoS is more power efficient than operating at NE.

3.3.1 Calculation of the NE-mixed QoS

There are two possible approaches to reach the NE-mixed QoS. Each approach has its own merits and demerits. First, the *bargain region* is defined as the sub-region that contains all points that satisfy $\mathsf{R}_k \geq \mathsf{R}_k^{\text{NE}}$, $k = 1, 2$. The two approaches for calculating the NE-mixed QoS are:

1. During the first iteration, AP_1 computes its beamformers by setting the RTU target rate, while AP_2 uses its maximum power to maximise the rate R_2. It is most probable that at the end of iteration 1, the RTU would not achieve its target rate due to interference from AP_2. On the second iteration, AP_1's BR will be to set the RTU target, while considering the interference caused by AP_2. Since AP_2 has used all its resources, it cannot improve its beamformers, and its final

rate depends entirely on the interference from AP_1. AP_1 can either achieve its target rate, if the target rate is lower than the NE rate, or achieve the NE rate, if otherwise. This means the NE-mixed QoS can be outside the bargain region or coincide with the usual NE point.

2. During the first iteration, both APs implement their *unscaled* MRT beamformers (*id est*, $\chi_k = 1, \forall k$), which will result in the NE point [47]. This is achieved by letting the utility of AP_1 to be R_1 in the first iteration, and by solving a problem similar to (3.7) with user subscript as 1. In the second iteration, AP_1 can then determine if the target rate required by MS_1 is lower or higher than its NE value. If the required target is less, AP_1 will reduce its transmission power by using (3.8), which will reduce interference imposed on MS_2. At the NE-mixed QoS, the MS_1 will achieve its target rate, while MS_2 will get NE-mixed QoS rate, where $\mathsf{R}_2^{\text{NE-mixed QoS}} \geq \mathsf{R}_2^{\text{NE}}$. This suggests that at the NE-mixed QoS, AP_1 may use less or all its power. If $\psi_1 > \mathsf{R}_1^{\text{NE}}$, the NE-mixed QoS will coincide with the NE point.

It is therefore reasonable to use the second approach since the NE-mixed QoS may be reached in one iteration (*id est*, when the target rate is higher than the NE rate). Once more, if the target rate is less than the NE rate, implementing the *unscaled* MRT in the first iteration by AP_1 will protect the RTU, as it will get a rate greater or equal to the target rate (*id est*, $\mathsf{R}_1 \geq \psi_1$) during both iterations.

3.4 Mixed QoS Cooperative Game

In this section, bargain theory is employed to devise a solution that will address the inefficiency of the NE-mixed QoS. Bargain theory requires the utility space to be convex, but interference in the denominator of (3.5) renders the convexity of the rate region. In [47], a convex hull [12] is used to convexify the original rate region by enlarging it. Assuming that the receiver treats interference as noise, the achievable rate region $\mathcal{R}$ is the union of the rate tuple R_1 and R_2. This feasible rate region has an upper-right boundary called the Pareto boundary $\mathcal{R}^\star$, where it is impossible to improve the performance of one user without affecting the performance of at least one other user

[12]. The convex hull is computed by performing time sharing over $\mathcal{R}$, giving

$$\tilde{\mathcal{R}} = \bigcup_{\substack{0\leq\beta\leq 1,\\ (\mathsf{R}_1^1,\mathsf{R}_2^1)\in\mathcal{R},\\ (\mathsf{R}_1^2,\mathsf{R}_2^2)\in\mathcal{R}.}} \left(\beta\mathsf{R}_1^1 + (1-\beta)\mathsf{R}_1^2, \beta\mathsf{R}_2^1 + (1-\beta)\mathsf{R}_2^2\right), \tag{3.10}$$

where β is the time sharing coefficient, $(\mathsf{R}_1^1, \mathsf{R}_2^1)$ and $(\mathsf{R}_1^2, \mathsf{R}_2^2)$ are two points in the rate region $\mathcal{R}$. The convex hull $\tilde{\mathcal{R}}$, gives a set of achievable outcomes when the players are allowed to partition degrees of freedom (DoF) (*exempli gratia*, time or bandwidth) into two parts [47]. For example, in the first portion of time, β, both players choose strategies $\mathbf{w}_k^1$, $\forall k$ with outcomes $\beta\mathsf{R}_1^1$, $\beta\mathsf{R}_2^1$, and for the remaining part, both players choose strategies $\mathbf{w}_k^2$, $\forall k$ with outcomes $(1-\beta)\mathsf{R}_1^2$ and $(1-\beta)\mathsf{R}_2^2$. The benefit of partitioning the degrees of freedom in this manner enlarges the original utility space by generating a convex hull [45].

3.4.1 Mixed QoS Solution via Bargaining

If the players cooperate, they are able to achieve an operating point on the Pareto boundary $\tilde{\mathcal{R}}$. A Nash bargain (NB) problem aims to maximise the Nash function as

$$\begin{aligned} \underset{\mathbf{w}_k,\beta}{\texttt{maximise}} \quad & \sum_k \log_2\left(\tilde{\mathsf{R}}_k - \mathsf{R}_k^{\mathrm{NE}}\right) \\ \texttt{subject to} \quad & \|\mathbf{w}_k\|^2 \leq \varphi_n^{\max}, \qquad \forall k, \forall n, \\ & \tilde{\mathsf{R}}_k > \mathsf{R}_k^{\mathrm{NE}}, \qquad \forall k, \\ & 0 \leq \beta \leq 1, \end{aligned} \tag{3.11}$$

where $\mathsf{R}_k^{\mathrm{NE}}$ is the NE rate of the k-th user, which is referred to as the disagreement point. Setting the disagreement point to the NE in Problem (3.11) reduces the search space to the sub region $\tilde{\mathcal{R}}^+$ [47] (*id est*, the *bargain region*). If any of the players deviates, the NE will become the default outcome. By using the weighted Nash function

$$\mathrm{NB} = \prod_k \left(\tilde{\mathsf{R}}_k - \mathsf{R}_k^{\mathrm{NE}}\right)^{\alpha_k}, \tag{3.12}$$

any point in the bargain region can be achieved. The exponent α_k should satisfy $\alpha_k \geq 0$, $\forall k$. The mixed QoS criterion is able to attain any point in the bargain region without explicitly defining α_k. Again, the mixed QoS BG will allow APs to reach the Pareto boundary outside the bargain region.

3.4.2 Exchange Model

In [77], beamforming parameterisation model for a two-user game was used to obtain the scalar parameters required to be exchanged between users during cooperation. The beamforming parameterisation is given by

$$\mathbf{w}_k(\lambda_k) = \frac{\lambda_k \mathbf{w}_k^{\mathrm{NE}} + (1-\lambda_k)\mathbf{w}_k^{\mathrm{ZF}}}{\left\|\lambda_k \mathbf{w}_k^{\mathrm{NE}} + (1-\lambda_k)\mathbf{w}_k^{\mathrm{ZF}}\right\|}, \tag{3.13}$$

which has been proven to attain any point on the Pareto boundary. The λ_k should satisfy $0 \leq \lambda_k \leq 1$. The exchange model in (3.13) means that, in order to achieve Pareto boundary, each player has to combine both its utilities and other players' utilities in the optimisation cost function. By using (3.13), a parameterisation of (3.5) can be achieved by expressing the achievable rate as a function of λ_k as

$$\mathsf{R}_k(\lambda_k, \lambda_j) = \log_2\left(1 + \frac{|\mathbf{w}_k(\lambda_k)^{\mathsf{H}}\mathbf{h}_{kk}|^2}{|\mathbf{w}_j(\lambda_j)^{\mathsf{H}}\mathbf{h}_{jk}|^2 + \sigma^2}\right), \quad j \neq k. \tag{3.14}$$

This real-valued parametrisation is used to design distributed algorithms. Problem in (3.11) can now be redefined as

$$\underset{0\leq\lambda_1,\lambda_2\geq 1}{\mathtt{maximise}} \;\; (\mathsf{R}_1(\lambda_1,\lambda_2) - \mathsf{R}_1^{\mathrm{NE}})(\mathsf{R}_2(\lambda_1,\lambda_2) - \mathsf{R}_2^{\mathrm{NE}}). \tag{3.15}$$

3.5 Conditions for Mixed QoS BG Operating Point

For the mixed QoS BG, the NE point is chosen as the disagreement point instead of the NE-mixed QoS. This is based on the point made in the second condition under Section 3.3.1, which suggests that the NE point is a *strong* disagreement to MS_2. Mixed QoS BG solution exists if and only if the rate constraint of MS_1 guarantees the following condition:

$$\mathsf{R}_2^{\star} \geq \mathsf{R}_2^{\mathrm{NE}}, \tag{3.16}$$

where $\mathsf{R}_2^{\star}$ is the optimal rate for MS_2. Therefore, the maximum acceptable target rate $\mathsf{R}_1^{\max}$ for MS_1 under mixed QoS, is when MS_2 is able to transmit at $\mathsf{R}_2 = \mathsf{R}_2^{\mathrm{NE}}$. Thus,

	Channels for Scenario 1
$\mathbf{h}_{11}$	1.051660+0.377669i 1.620710+0.343706i
$\mathbf{h}_{22}$	-0.239492+0.022606i 1.612110+0.005823i
$\mathbf{h}_{12}$	-0.121799+0.104716i 0.602630-0.828884i
$\mathbf{h}_{21}$	-0.331985-0.123566i -0.129720-0.258555i

TABLE 3.1: Channels used for producing results in Figure 3.2.

the mixed QoS BG solution is achievable if the target rate is $\psi_1 \leq \mathsf{R}_1^{\max}$. If the condition in (3.16) is not satisfied, MS_2 will defect.

Now, assume the aim is to always guarantee the maximum possible rate to MS_1. Condition (3.16) is invoked to recast the bargain problem as

$$\begin{aligned} \underset{0\leq\lambda_1,\lambda_2\geq 0}{\texttt{maximise}} \;\; & \mathsf{R}_1(\lambda_1,\lambda_2) - \mathsf{R}_1^{\text{NE}} \\ \texttt{subject to} \;\; & \mathsf{R}_2(\lambda_1,\lambda_2) \geq \mathsf{R}_2^{\text{NE}}. \end{aligned} \tag{3.17}$$

The benefit of using (3.17) is that there is no predefined target rate for the RTU, but this may not be power efficient. In the mixed QoS BG, the following cases are possible:

- *Case 1:* The target rate for MS_1 is less than its NE rate, $\psi_1 < \mathsf{R}_1^{\text{NE}}$. In this case, instead of using the NB problem in (3.11), the area of the triangle enclosed between the ψ_1, R_1^{NE}, and the Pareto boundary is maximised as shown in Figure 3.2. Under this condition, the *bargain region is extended*. The new bargain region is referred herein as the *mixed QoS bargain region*. Problem in (3.11) will now be defined as:

$$\begin{aligned} \underset{0\leq\lambda_1,\lambda_2\geq 1}{\texttt{maximise}} \;\; & \frac{1}{2}(\mathsf{R}_1^{\text{NE}} - \mathsf{R}_1(\lambda_1,\lambda_2))(\mathsf{R}_2(\lambda_1,\lambda_2) - \mathsf{R}_2^{\text{NE}}) \\ \texttt{subject to} \;\; & \mathsf{R}_1(\lambda_1,\lambda_2) \geq \gamma_1, \\ & \mathsf{R}_2(\lambda_1,\lambda_2) \geq \mathsf{R}_2^{\text{NE}}. \end{aligned} \tag{3.18}$$

- *Case 2:* The target rate for MS_1 is more than its NE rate, $\psi_1 > \mathsf{R}_1^{\text{NE}}$. In this case, NB problem in (3.18) is used to maximise the area of the rectangle between the ψ_1, R_1^{NE}, and the Pareto boundary by setting the objective to $(\mathsf{R}_1(\lambda_1,\lambda_2) - \mathsf{R}_1^{\text{NE}})(\mathsf{R}_2(\lambda_1,\lambda_2) - \mathsf{R}_2^{\text{NE}})$.

	Channels for Scenario 2	
$\mathbf{h}_{11}$	0.349731+0.220651i	-0.085718-1.634090i
$\mathbf{h}_{22}$	0.053835+0.267813i	2.287930+0.139450i
$\mathbf{h}_{12}$	-0.015381-0.671806i	-0.148975+0.422753i
$\mathbf{h}_{21}$	0.450793-0.327430i	-0.252294-0.118805i

TABLE 3.2: Channels used for producing results in Figure 3.3.

3.6 Numerical Examples and Discussions

In order to validate the performance of the proposed mixed QoS SNG and mixed QOS BG, the following numerical simulations were carried out. The rate region $\mathcal{R}$ was determined as in [47], for a given set of random channels. An alternative is to use the ergodic rate region. Tables 3.1 and 3.2 give the channels used for producing results in Figures 3.2 and 3.3, respectively. Figure 3.2 and Figure 3.3 show the rate regions for signal-to-noise ratio (SNR) at 0 dB and 10 dB respectively, where $\mathsf{SNR} = \varphi_n^{\max}/\sigma^2$.

3.6.1 Case 1

Results in Figure 3.2 show a case whereby the target rate of MS_1 is less than R_1^{NE}, *id est*, $\psi_1 = 2 < \mathsf{R}_1^{\text{NE}} = 2.2523$. For this case, the mixed QoS BG operating point was determined by solving (3.18). The mixed QoS operating point in the figure is at $\mathsf{R}_1 = 2.0288$ and $\mathsf{R}_2 = 1.7293$. The plot shows that even though there is bargaining, this operating point is outside the conventional bargain region. Thus, the bargain region has been extended.

3.6.2 Case 2

A case where the target rate of MS_1 is more than its NE rate, *id est*, $\psi_1 = 4.6 > R_1^{\text{NE}} = 3.8808$, was studied by solving (3.18) with $(\mathsf{R}_1(\lambda_1, \lambda_2) - \mathsf{R}_1^{\text{NE}})(\mathsf{R}_2(\lambda_1, \lambda_2) - \mathsf{R}_2^{\text{NE}})$ as the objective, and the results are shown in Figure 3.3. The mixed QoS BG operating point is at $\mathsf{R}_1 = 4.6164$ and $\mathsf{R}_2 = 4.4847$. This point is in the conventional bargain region and it corresponds to the weighted NB, which can be achieved by maximising (3.12). In both cases, it is vital to abide by the condition in (3.16) for a Pareto optimal solution.

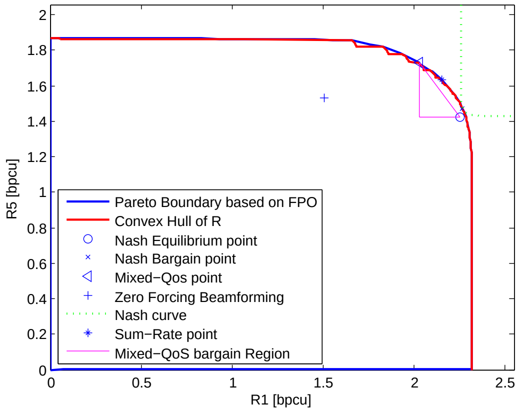


FIGURE 3.2: Rate region with SNR = 0 dB. Results for case 1.

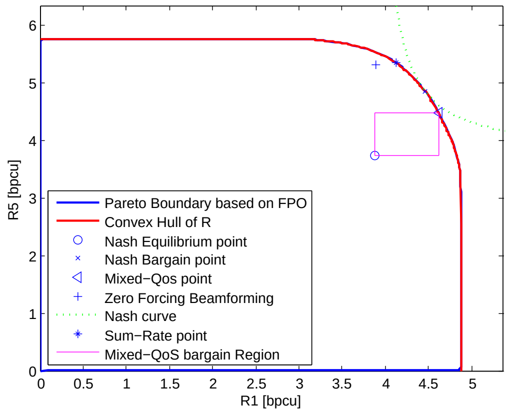


FIGURE 3.3: Rate region with SNR = 10 dB. Results for case 2.

3.6.3 Average Performance of Mixed QoS SNG

The performance of the conventional SNG, wherein both players would want to maximise their utilities, and the mixed QoS SNG, were studied in Figure 3.4 and Figure 3.5. The results in Figure 3.4 show average sum rates (ASRs) achieved over 10 random channel realisations, for a range of average SNRs. The target rate of the RTU was set to $\psi_1 = \{1, 2, 3, 4, 5\}$. If the target rate of the RTU is infeasible, the information rates of both users were maximised. Figure 3.4 shows that at low SNRs, the target rate of the RTU is infeasible at AP_1 and therefore the ASRs of both systems will be equal. Between an average SNR of -4 dB and 10 dB, all the target rates become feasible. For all target rates, the NE-mixed QoS ASRs start off below the NE ASRs, but from SNR of 12.5 dB onwards, the NE-mixed QoS ASRs begin to surpass NE ASRs. At 30 dB SNR, an average difference of about 2 bits/symbol use between the NE-mixed QoS ASR and NE

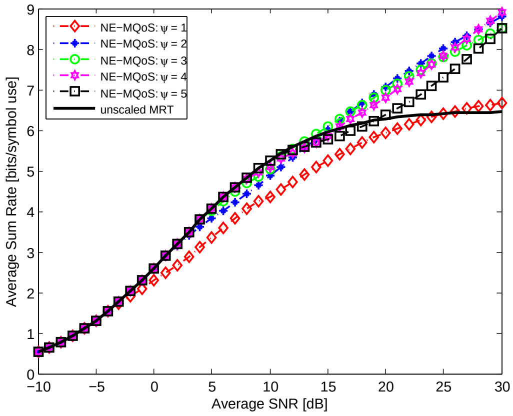


FIGURE 3.4: Average sum rate over 10 random channel realisations using *unscaled* MRT and *scaled* MRT, in a network topology described in Section 3.2.

ASR, for target rates greater than one, is observed. This achievement comes with a positive benefit on power saving as shown in Figure 3.5.

Results in Figure 3.5 illustrates the difference between the total power consumed between the two systems. From (3.8), it is suggested that by using the conventional MRT beamformers, both APs will always use their maximum available power to maximise their utilities. This is not always the case with NE-mixed QoS beamformers. The AP serving the RTU may not necessarily use its maximum power. Since at low SNRs both systems will use their maximum power (*id est*, NE-mixed QoS coincides with NE), the difference of the two system will be zero. It is observed that low targets rates will save more power at low SNRs and save the least power at high SNRs. Figure 3.5 shows that under mixed QoS SNG, there is potential of saving power between 1.7569 dB to 2.6949 dB at an average SNR of 30 dB, for target rates between 1 and 5 respectively. This highlights the advantage of mixed QoS beamforming criterion, and shows that mixed QoS criterion can achieve more NE-mixed QoS ASRs than the NE ASRs, while conserving some power.

3.7 System Model for Multi-cell Multiuser Network

In this section, decomposition methods are used for designing a distributed algorithm for the model in Figure 3.6. Consider a multicell MISO downlink system of N BSs. There are K MSs, where each user is assigned to only one BS at any given time. Denote the n-th serving BS to the k-th user by n_k, a set of BSs by $\mathcal{N}$, a set of all users by $\mathcal{U}$, and

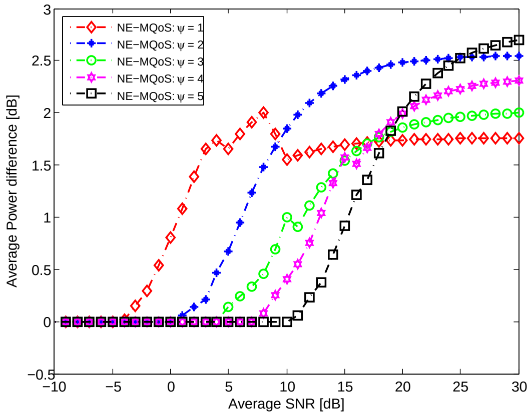


FIGURE 3.5: Average total power over 10 channel realisation using MRT and mixed QoS criteria, in a network topology described in Section 3.2.

a subset $\mathcal{U}_n \subseteq \mathcal{U}$, which to includes all users allocated to BS n. Sets $\mathcal{U}_n^{\mathsf{R}} \subset \mathcal{U}_n$ and $\mathcal{U}_n^{\mathsf{N}} \subset \mathcal{U}_n$ respectively denote all RTUs and NRTUs at the n-th BS. The cardinality of sets $\mathcal{U}_n$, $\mathcal{U}_n^{\mathsf{R}}$ and $\mathcal{U}_n^{\mathsf{N}}$ are K_n, $K1_n$, and $K_n - K1_n$, respectively. It is assumed that all BSs share the same frequency band. Each BS is equipped with M transmit antennas, and it has the maximum possible transmission power $\varphi_n^{\max}$. Each user is equipped with single antenna. In the downlink, the transmitted signal for user k from BS n_k can be written as

$$\mathbf{x}_k(t) = \mathbf{w}_k s_k(t), \tag{3.19}$$

where $s_k(t) \in \mathbb{C}$ represents the information symbol at time t, and $\mathbf{w}_k \in \mathbb{C}^M$ is the unnormalised transmit beamforming vector for user k. Without loss of generality, assume that $s_k(t)$ is normalised, such that $\mathbb{E}\{|s_k(t)|^2\} = 1$, and that all data streams are independent such that $\mathbb{E}\{s_k(t)s_i(t)^*\} = 0$ if $k \neq i$. The received signal at the k-th user can be written as

$$y_k(t) = \mathbf{h}_{n_k,k}^{\mathsf{H}} \mathbf{x}_k(t) + \sum_{i \in \mathcal{U} \setminus k} \mathbf{h}_{n_i,k}^{\mathsf{H}} \mathbf{x}_i(t) + \eta_k(t), \tag{3.20}$$

where $\mathbf{h}_{n_k,k} \in \mathbb{C}^M$ is the channel vector from the BS n_k to user k, and $\eta_k(t) \in \mathcal{CN}(0, \sigma^2)$ is the circular symmetric zero mean complex Gaussian noise with variance σ^2.

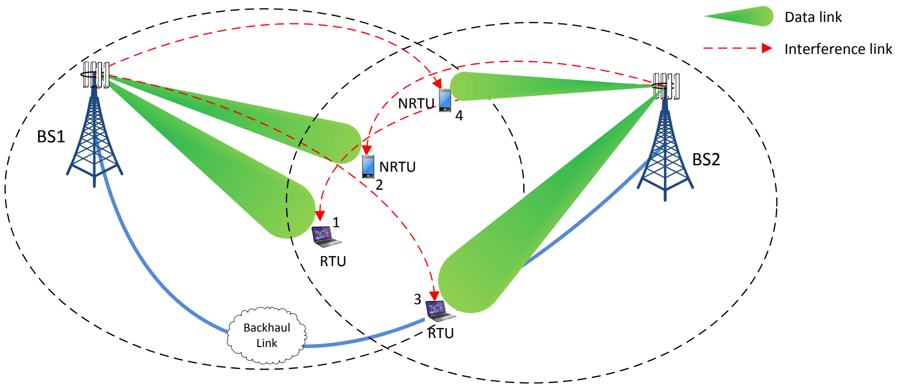


FIGURE 3.6: Network Topology consisting of two BSs, two RTUs and two NRTUs.

3.7.1 Problem Formulation

The instantaneous downlink SINR of the k-th is

$$\mathsf{SINR}_k = \frac{|\mathbf{h}_{n_k,k}^{\mathsf{H}}\mathbf{w}_k|^2}{\sum\limits_{i\in\mathcal{U}_n\backslash k}|\mathbf{h}_{n_k,k}^{\mathsf{H}}\mathbf{w}_i|^2 + \sum\limits_{\substack{n=1\\n\neq n_k}}^{N}\sum\limits_{i\in\mathcal{U}_n}|\mathbf{h}_{n,k}^{\mathsf{H}}\mathbf{w}_i|^2 + \sigma^2}. \tag{3.21}$$

The aim is to maximise the minimum SINR of the NRTUs, while ensuring a certain level of QoS for the RTUs, in a distributed manner. Define the SINR targets of the RTUs at BS n as $\mathbf{\Xi}_{n_{\mathsf{RTU}}} = [\xi_1, \ldots, \xi_{K1_n}]$, which are predefined values, and the preferred intermediary SINR targets of the NRTUs as $\mathbf{\Delta}_{n_{\mathsf{NRTU}}} = [\delta_{K1_n+1}, \ldots, \delta_{K_n}]$. By combining the power minimisation ($\mathscr{P}_{\text{PMP}}$ in (2.4)) and SINR balancing ($\mathscr{P}_{\text{SB}}$ in (2.5)) problems, the mixed QoS problem can be stated as

$$\begin{aligned}
&\underset{\{\mathbf{w}_k\}_{k\in\mathcal{U}_n},\forall n}{\texttt{maximise}} \ \underset{k}{\texttt{min}}\frac{\mathsf{SINR}_k}{\delta_k}, \quad k = K1_n+1,\ldots,K_n\\
&\texttt{subject to} \ \ \mathsf{SINR}_k \geq \xi_k, \quad k = 1,\ldots,K1_n,\\
&\qquad\qquad \sum_{k\in\mathcal{U}_n}\|\mathbf{w}_k\|^2 \leq \varphi_n^{\max}, \quad n\in\mathcal{N}.
\end{aligned} \tag{3.22}$$

Problem in (3.22) shows that the mixed QoS problem feasibility depends heavily on the feasibility of the SINRs targets of the RTUs than those of the NRTUs. By using the same argument in [22], it is concluded that the mixed QoS problem is a quasi-convex. Hence, solving (3.22) involves performing line search in a quasi-convex curve using a bisection method [12]. Thus, problem in (3.22) will be solved in two main steps. The first step involves running a feasibility check for given SINR targets, and the second step involves allocating excess resources to NRTUs, while ensuring the required user performance of RTUs. Problem in (3.22) is not convex due to the SINR terms in the objective and the constraints. Nevertheless, the SINRs can be written in their equivalent second order cone (SOC) as

$$\mathsf{SINR}_k \geq \xi_k \Longrightarrow \left\| \begin{matrix} \mathbf{h}_{n_k,k}^{\mathsf{H}} \mathbf{w}_1 \\ \vdots \\ \mathbf{h}_{n_k,k}^{\mathsf{H}} \mathbf{w}_{K_n} \\ \sigma_k \end{matrix} \right\| \leq \sqrt{\frac{1+\xi_k}{\xi_k}} \Re(\mathbf{h}_{n_k,k}^{\mathsf{H}} \mathbf{w}_k), \tag{3.23}$$

$$\Im(\mathbf{h}_{n_k,k}^{\mathsf{H}} \mathbf{w}_k) = 0, \quad \forall k, \tag{3.24}$$

For brevity, all the SINR targets of the all users at the n-th BS are denoted as $\mathbf{\Gamma}_n = \{\gamma_1, \ldots, \gamma_{K_n}\}$, and the beamforming matrix of BS n as $\mathbf{W}_n = [\mathbf{w}_k]_{k \in \mathcal{U}_n}$. The feasible set of beamformers at BS n is given as

$$\mathcal{W}_n = \left\{ \mathbf{W}_n : \sum_{k \in \mathcal{U}_n} \|\mathbf{w}_k\|_2^2 \leq \varphi_n^{\max}, \ \ \mathsf{SINR}_k \geq \gamma_k \ \ \forall k \in \mathcal{U}_n \right\}. \tag{3.25}$$

The feasibility problem is defined as

$$\underset{\mathbf{W}_n \in \mathcal{W}_n, \forall n}{\texttt{minimise}} \sum_{k \in \mathcal{U}_n} \|\mathbf{w}_k\|_2^2, \ \ \texttt{subject to} \ \ \mathsf{SINR}_k \geq \gamma_k \ \ \forall k \in \mathcal{U}_n. \tag{3.26}$$

If the SINR targets of the RTUs are not feasible, then the mixed QoS problem in (3.22) is not feasible. From (3.21), it can be understood that the user performance for all users are coupled by both the interference and the transmission powers. This interdependency requires more information to be shared between the BSs, while jointly optimising all links. To reduce the information shared between the BSs, the overall feasibility problem is presented as a *global consensus* problem [33].

3.8 Mixed QoS Distributed Algorithm

First, introduce auxiliary variables $\kappa_{n,k}$, and $\tilde{\kappa}_{n,k}$ to represent the actual inter-cell interference from the n-th BS to the k-th user, and its local copy, respectively. This adds a new set of constraints that enforce consistency between the global and local copies to ensure that they are in *consensus*. The overall feasibility problem which includes all

RTUs and NRTUs can be written as

$$\begin{aligned}
&\underset{\{\mathbf{w}_k\}_{k\in\mathcal{U}_n},\{\tilde{\boldsymbol{\kappa}}_n\}_{u\in\mathcal{N}}}{\texttt{minimise}} && \sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{U}_n}\|\mathbf{w}_k\|_2^2 \\
&\texttt{subject to} && \frac{|\mathbf{h}_{n_k,k}^{\mathsf{H}}\mathbf{w}_k|^2}{\sum\limits_{i\in\mathcal{U}_n\backslash k}|\mathbf{h}_{n_k,k}^{\mathsf{H}}\mathbf{w}_i|^2+\sum\limits_{\substack{n=1\\n\neq n_k}}^{N}\tilde{\kappa}_{n,k}^2+\sigma^2}\geq\gamma_k \quad \forall k\in\mathcal{U}, \\
& && \kappa_{n,k}^2\geq\sum_{i\in\mathcal{U}_n}|\mathbf{h}_{n,k}^{\mathsf{H}}\mathbf{w}_i|^2 \quad k\in\mathcal{U}_n, n\in\mathcal{N}, n\neq n_k, \\
& && \tilde{\kappa}_{n,k}\geq\kappa_{n,k} \quad \forall k,n,n\neq n_k.
\end{aligned} \tag{3.27}$$

The intercell interference is estimated as follows: Initially, the local and global copies are set to 0. In the subsequent iterations, the actual interference generated at the k-th user $\tilde{\kappa}_{n,k}^2$ is sent to the serving BS. The believed interference $\kappa_{n,k}^2$ is then updated while fixing the global copy $\tilde{\kappa}_{n,k}^2$.

Problem (3.27) can be transformed into a convex SOCP [12] problem as

$$\begin{aligned}
&\underset{\{\mathbf{w}_k\}_{k\in\mathcal{U}_n},\{\tilde{\boldsymbol{\kappa}}_n\}_{u\in\mathcal{N}}}{\texttt{minimise}} && \sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{U}_n}\|\mathbf{w}_k\|_2^2 \\
&\texttt{subject to} && \begin{bmatrix}\sqrt{1+\frac{1}{\gamma_k}}\mathbf{h}_{n_k,k}^{\mathsf{H}}\mathbf{w}_k\\ \mathbf{h}_{n_k,k}^{\mathsf{H}}\mathbf{W}_n\\ \tilde{\boldsymbol{\kappa}}_k\\ \sigma\end{bmatrix}\succeq_{\mathrm{SOC}}0, \quad \forall k\in\mathcal{U}_n,\forall n\in\mathcal{N}, \\
& && \begin{bmatrix}\kappa_{n,k}\\ \mathbf{h}_{n,k}^{\mathsf{H}}\mathbf{W}_n\end{bmatrix}\succeq_{\mathrm{SOC}}0, \quad \forall k\in\mathcal{U}_n,\forall n\in\mathcal{N},n\neq n_k, \\
& && \tilde{\kappa}_{n,k}\geq\kappa_{n,k}, \quad \forall k\in\mathcal{U}_n,\forall n\in\mathcal{N},n\neq n_k,
\end{aligned} \tag{3.28}$$

where $\tilde{\boldsymbol{\kappa}}_k=\{\tilde{\kappa}_{n,k}\}_{n\in\mathcal{N}\backslash n_k}$ and the notation $\succeq_{\mathrm{SOC}}$ refers to the generalised inequalities with respect to the second-order cone [12]. The problem in (3.28) is convex and separable in $n\in\mathcal{N}$. This allows (3.28) to be solved with a distributed algorithm. With reference to the third constraint in (3.28), define column vectors $\tilde{\boldsymbol{\kappa}}_n$ by concatenating all local variables associated with BS n, and $\boldsymbol{\kappa}_n$ by concatenating all global variables associated with elements of $\tilde{\boldsymbol{\kappa}}_n$. Thus, for fixed global variables $\boldsymbol{\kappa}_n$, the set of beamformers at BS n, given in (3.25), depends on the local variables $\tilde{\boldsymbol{\kappa}}_n$. Taking this into account, the feasibility indicator function $f_n(\mathbf{W}_n, \tilde{\boldsymbol{\kappa}}_n)$, at each BS n is formed as

$$f_n(\mathbf{W}_n, \tilde{\boldsymbol{\kappa}}_n) = \begin{cases} \sum\limits_{k \in \mathcal{U}_n} \|\mathbf{w}_k\|_2^2, & (\mathbf{W}_n, \tilde{\boldsymbol{\kappa}}_n) \in \mathcal{W}_n, \\ \infty, & \text{otherwise.} \end{cases} \tag{3.29}$$

This leads to a compact model of problem in (3.28) defined by

$$\begin{aligned} &\underset{\{\mathbf{w}_k\}_{k\in\mathcal{U}_n}, \tilde{\boldsymbol{\kappa}}_n}{\texttt{minimise}} \quad \sum_{n\in\mathcal{N}} f_n(\mathbf{W}_n, \tilde{\boldsymbol{\kappa}}_n) \\ &\texttt{subject to} \quad \tilde{\boldsymbol{\kappa}}_n \geq \boldsymbol{\kappa}_n, \forall n \in \mathcal{N}. \end{aligned} \tag{3.30}$$

The augmented Lagrangian of (3.30) is given by

$$\begin{aligned} &\mathcal{L}_\rho(\{\mathbf{W}_n, \tilde{\boldsymbol{\kappa}}_n\}_{n\in\mathcal{N}}, \{\boldsymbol{\kappa}_n\}_{n\in\mathcal{N}}, \{\mathbf{y}\}_{n\in\mathcal{N}}) = \\ &\sum_{n\in\mathcal{N}} \left(f_n(\mathbf{W}_n, \tilde{\boldsymbol{\kappa}}_n) + \mathbf{y}^{\mathsf{T}}(\tilde{\boldsymbol{\kappa}}_n - \boldsymbol{\kappa}_n) + \frac{\rho}{2}\|\boldsymbol{\kappa}_n - \tilde{\boldsymbol{\kappa}}_n\|_2^2 \right) \\ &= \sum_{n\in\mathcal{N}} \left(f_n(\mathbf{W}_n, \tilde{\boldsymbol{\kappa}}_n) + \frac{\rho}{2}\|\tilde{\boldsymbol{\kappa}}_n - \boldsymbol{\kappa}_n + \mathbf{u}_n\|_2^2 + c \right), \end{aligned} \tag{3.31}$$

where $\{\mathbf{y}\}_{n\in\mathcal{N}}$ are Lagrange multipliers for the interference constraints, $\rho > 0$ is a penalty parameter, $\mathbf{u}_n = (1/\rho)\mathbf{y}_n$ is the scaled dual variable, and $c = -\rho/2\|\mathbf{u}_n\|_2^2$ is a constant vector that can dropped during minimisation [33]. The ADMM algorithm for solving (3.31) involves a single *Gauss-Seidel* pass [33] over $\boldsymbol{\kappa}_n$ and $\tilde{\boldsymbol{\kappa}}_n$, and therefore consists of three successive iterations as

$$\mathbf{W}_n^{t+1}, \tilde{\boldsymbol{\kappa}}_n^{t+1} := \underset{\mathbf{W}_n, \tilde{\boldsymbol{\kappa}}_n}{\texttt{argmin}}\, \mathcal{L}_\rho\left(\mathbf{W}_n, \tilde{\boldsymbol{\kappa}}_n, \boldsymbol{\kappa}_n^t, \mathbf{u}_n^t\right), \quad n \in \mathcal{N}, \tag{3.32}$$

$$\{\boldsymbol{\kappa}_n^{t+1}\}_{n\in\mathcal{N}} := \underset{\boldsymbol{\kappa}_n}{\texttt{argmin}}\, \mathcal{L}_\rho\left(\mathbf{W}_n^{t+1}, \tilde{\boldsymbol{\kappa}}_n^{t+1}, \boldsymbol{\kappa}_n, \mathbf{u}_n^t\right), \tag{3.33}$$

$$\mathbf{u}_n^{t+1} := \mathbf{u}_n^t + \tilde{\boldsymbol{\kappa}}_n^{t+1} - \boldsymbol{\kappa}_n^{t+1} \quad n \in \mathcal{N}, \tag{3.34}$$

where t is the iteration index.

These steps for solving (3.32) to (3.34) are explained with the aid of Algorithm 2. For a fixed global variable $\boldsymbol{\kappa}$, iteration in (3.32) is solved at each BS by solving the following problem.

Algorithm 2: Mixed QoS distributed algorithm

Data: $\varphi_n^{\max}$, penalty $\rho > 0$, stopping criterion, $\delta > 0$, $\mathsf{SINR}_k \geq \gamma_k^\star, \forall k$, Γ^{lower}, Γ^{upper}

Data: Initialisation: set $\{\boldsymbol{\kappa}_n\}_{n\in\mathcal{N}} = 0$,$\{\tilde{\boldsymbol{\kappa}}_n\}_{n\in\mathcal{N}} = 0$,$\{\mathbf{y}_n\}_{n\in\mathcal{N}} = 0$

Result: Optimal beamforming vectors $\{\mathbf{W}_n\}_{n\in\mathcal{N}}$

while $\Gamma^{upper} - \Gamma^{lower} > \delta$ **do**
    Set $\Gamma^{\text{candidate}} = \frac{\Gamma^{\text{upper}}+\Gamma^{\text{lower}}}{2}$
    Set $\gamma_k^\star = a_k + \alpha\Gamma^{\text{candidate}}, \forall k$;
    while *stopping criterion not satisfied* **do**
        Set $t = t + 1$;
        Solve 3.35 at each BS to update $\mathbf{W}_n^{t+1}$,$\tilde{\boldsymbol{\kappa}}_n^{t+1}$
        Exchange relevant local copies $\boldsymbol{\kappa}_n^{t+1}$ between BSs coupled by consistency constraints
        Update global variable $\boldsymbol{\kappa}_n^{t+1}$, using ((3.36))
        Update dual variable $\mathbf{y}_n^{t+1}$, using ((3.34))
        if *stopping criterion is not satisfied* **then**
            set $\Gamma^{\text{upper}} = \Gamma^{\text{candidate}}$;
    if *stopping criterion is satisfied* **and** *current* $\mathbf{W}_n$ *is feasible* **then**
        Set $\{\mathbf{W}_n^{\text{lower}}\}$ as the solution;
        Set $\Gamma^{\text{lower}} = \Gamma^{\text{candidate}}$

Set $\Gamma^{\text{lower}}_{\text{final}} = \Gamma^{\text{lower}}$ and $\Gamma^{\text{upper}}_{\text{final}} = \Gamma^{\text{upper}}$

$$\begin{aligned}
&\underset{\{\mathbf{w}_k\}_{k\in\mathcal{U}_n},\tilde{\boldsymbol{\kappa}}_n}{\texttt{minimise}} \quad \sum_{k\in\mathcal{U}_n} \|\mathbf{w}_k\|_2^2 + \frac{\rho}{2}\|\tilde{\boldsymbol{\kappa}}_n - \boldsymbol{\kappa}_n^t + \mathbf{u}_n^t\|_2^2 \\
&\texttt{subject to} \quad \begin{bmatrix} \sqrt{1+\frac{1}{\gamma_k}}\mathbf{h}_{n_k,k}^{\mathsf{H}}\mathbf{w}_k \\ \mathbf{h}_{n_k,k}^{\mathsf{H}}\mathbf{W}_n \\ \tilde{\boldsymbol{\kappa}}_k \\ \sigma \end{bmatrix} \succeq_{\text{SOC}} 0, \quad \forall k \in \mathcal{U}_n, \forall n \in \mathcal{N}, \\
&\qquad\qquad \begin{bmatrix} \kappa_{n,k} \\ \mathbf{h}_{n,k}^{\mathsf{H}}\mathbf{W}_n \end{bmatrix} \succeq_{\text{SOC}} 0, \quad \forall k \in \mathcal{U}_n, \forall n \in \mathcal{N}, n \neq n_k, \\
&\qquad\qquad \tilde{\kappa}_{n,k} \geq \kappa_{n,k}, \quad \forall k \in \mathcal{U}_n, \forall n \in \mathcal{N}, n \neq n_k.
\end{aligned} \tag{3.35}$$

In (3.33), BSs gather $\tilde{\boldsymbol{\kappa}}^{t+1}$ from their neighbours to form the averages. In essence, since the each element of the global variable couples two local variables of the neighbouring BSs, its solution is simply the average of its local variables [33, 88] given by

$$\kappa_{n,k}^{t+1} = \left(\tilde{\kappa}_{n_k,k}^{t+1} + \tilde{\kappa}_{n,k}^{t+1}\right)/2. \tag{3.36}$$

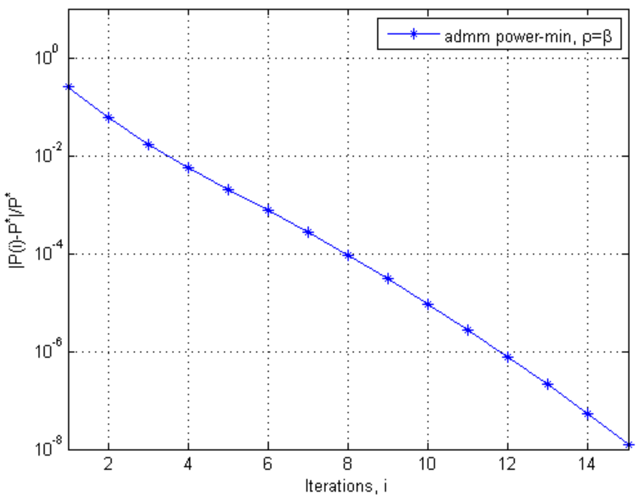


FIGURE 3.7: Suboptimality of the ADMM distributed algorithm versus iteration.

3.9 Numerical Example and Discussion

The performance of the proposed algorithm is evaluated using the network topology shown in Figure 3.6. The network consists of 2 BSs, 2 RTUs and 2 NRTUs. BS 1 serves MSs 1 and 2, and BS 2 serves MSs 3 and 4. Both MSs 1 and 3 are RTUs, while the rest are NRTUs. Each BS is equipped with $M = 5$ antennas and all MSs have single antennas. The SINR targets for RTU 1 and RTU 3 are 13 dB and 9 dB, respectively. The maximum transmit power was set to $\varphi_n^{\max} = 10$ dB at both BSs, $\Gamma^{\text{lower}} = 0$ and $\Gamma^{\text{upper}} = 40$ dB. The line search accuracy is $\delta = 10^{-6}$ and $\rho = \beta$ [88] where

$$\beta = \max_{n\in\mathcal{N}} \left\{ \sum_{n\in\mathcal{N}} (10^{0.1\times\gamma_k})/\|\mathbf{h}_{n_k,k}\|_2^2 \right\}. \tag{3.37}$$

All channel vectors $\mathbf{h}_{n_k,k}$ were modelled as independent and identically distributed (i.i.d.), Gaussian random variables. The noise variance was set to $\sigma^2 = 1$ for all users. It should be noted that, the **while** *loop* from step 4 of Algorithm 2 involves solving a feasibility problem, which require solving (3.32) to (3.34), until a required accuracy is achieved. Authors in [33] recommend 15 iterations. Power evolution of this part is illustrated in Figure 3.7, and it demonstrates that normalised power accuracy of 10^{-8} is achieved in 15 iterations. The rate of convergence can be altered by varying the penalty parameter ρ [33].

The convergence rate of the proposed algorithm is shown in Figure 3.8, which compares the proposed distributed algorithm with the centralised solution. In the centralised

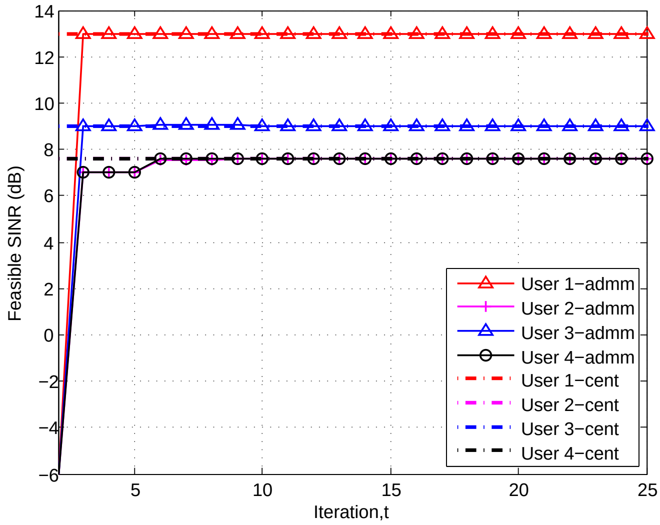


FIGURE 3.8: Achieved feasible SINR versus iteration.

system, it is assumed that the CSI at both BSs are sent to a central processing unit where the optimisation is performed. One benefit of the proposed algorithm is that, during the evolution of the NRTUs SINRs, the user performance of the RTUs remain fixed, as depicted in Figure 3.8. The normalised accuracy of the feasible SINRs is illustrated in Figure 3.9. This plot shows the normalised SINR accuracy, computed as $\mid \gamma_{\text{admm}}^{(i)} - \gamma_{\text{cent}}^{\star} \mid / \gamma_{\text{cent}}^{\star}$, against iteration i, where $\gamma_{\text{cent}}^{\star}$ is the optimal SINR achieved by using a centralised algorithm, and $\gamma_{\text{admm}}^{(i)}$ is the achievable SINR at the i-th iteration. Figure 3.9 shows that the proposed algorithm achieves 10^{-2} normalised accuracy on the NRTUs SINRs in less than 10 iterations. Moreover, the accuracy of the RTUs SINRs is always below 10^{-2}, and they achieve higher average accuracy as compared to the NRTUs. The abrupt increase of the normalised SINR accuracy of RTUs around iteration 10, is due to the significant interference introduced by NRTUs, as the BSs try to maximise their SINRs. The accuracy measure in Figure 3.9 highly depends on the accuracy of power evolution in Figure 3.7, and therefore, it can be improved by setting higher accuracy on the normalised power accuracy.

3.10 Conclusion

Game theory application under a mixed QoS BG in a two-user game has shown that, a Pareto optimal solution outside the bargain region can be reached by navigating the extended region of the bargain region. The mixed QoS criterion achieves a new equilibrium

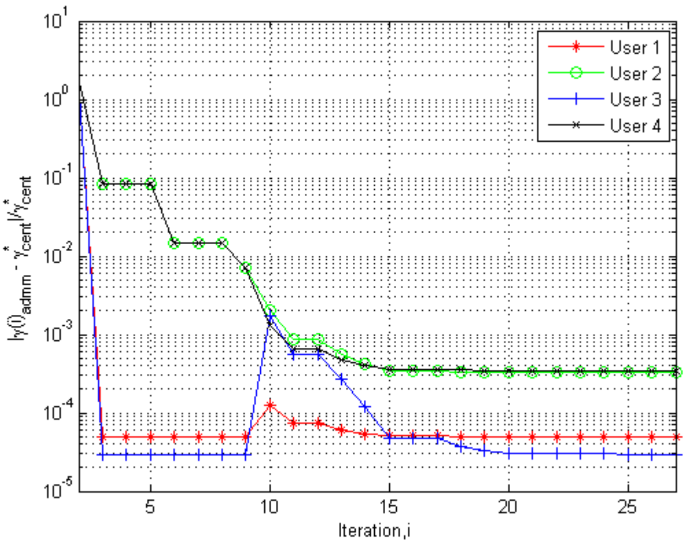


FIGURE 3.9: Suboptimality of the feasible SINR versus iteration.

point, NE-mixed QoS, which in general enables NRTUs to achieve a better information rate at high SNRs as compared to the unscaled MRT beamforming. The mixed QoS beamforming criterion is capable of achieving higher average sum rate, with less power consumption, as compared to unscaled MRT beamforming. A distributed algorithm for jointly solving SINR balancing and SINR target problem in multicell MISO has been proposed. The simulation results proved that the algorithm converges to optimal centralised solution. The algorithm uses alternating direction method of multipliers, which guarantees a fast solution. In Chapter 4, a mixed QoS problem will be solved using game-theoretic approach for a multi-cell mutliuser network.

Chapter 4

Game-Theoretic Beamforming Techniques for a Multicell Multiuser Network Under Mixed QoS Constraints

In the previous chapter, game theory and dual decomposition techniques were used to develop distributed algorithms for a two-user game, and a multicell multiuser wireless network, respectively. In this chapter, game-theoretic techniques for beamformer design are proposed for a multicell multiuser network. In particular, the solution structure for a mixed QoS SNG achieved using game theory is investigated, and a distributed algorithm is developed. Both non-cooperative and cooperative game-theoretic methods are studied. Bargain theory is used to derive a Pareto optimal solution. In particular, the Egalitarian and Kalai-Smorodinsky (KS) bargaining solutions are proposed.

4.1 Introduction

In recent years, a wide range of game-theoretic studies have been conducted, for multiuser systems. Game theory is a discipline in economics which is used to model and analyse situations where decision makers may have conflicting interests [48, 92]. The factors that usually inhibits distributed implementations are caused by the interdependencies between resources such as storage, computational, and spectral resources [1]. The conflicting nature of transmitters in a multiuser wireless network makes it relevant to represent the optimisation problem from a game-theoretic perspective.

4.1.1 Related Works

In addition to the works discussed in Section 3.1.1, distributed algorithms for downlink beamformer design in multicell multiuser networks have been proposed in [93–103]. Works in [93] and [102] considered optimisation problem that maximises the minimum SINR of the users. In order to eliminate partial interference, authors in [102] applied the Tomlinson-Harashima precoding technique before deriving a closed form solution to the `max-min` problem. A power minimisation problem was studied in [95, 100, 101]. An SINR pricing term that allows decentralised implementation is introduced in [95]. A second-order Taylor approximation method is used to approximate the pricing term with reduced information exchange between BSs.

Considered in [98,99] is an optimisation problem that maximises the weighted `max-min` fairness rate. The constraints considered therein include individual probability of rate outage and power budget. The authors in [98, 99] utilised the block successive upper minimisation (BSUM) method to propose a distributed BSUM algorithm. In addition, the distributed weighted minimum mean-square error (WMMSE) algorithm was proposed to optimise the weighted sum rate in parallel. The distributed zero forcing (DZF) and distributed virtual SINR (DVSINR) linear precoding schemes were deployed in [103] to select users that can potentially maximise the sum rate. With the utilisation of block-diagonalisation precoding scheme, an SNG was studied in [97] to maximise the network sum rate distributively. The authors used the game-theoretic framework to confirm the existence and uniqueness of the NE in the SNG.

In [104], dual decomposition technique was used to derive a distributed algorithm for energy-efficient resource allocation in device-to-device (D2D) networks. The optimisation problem aims to maximise the minimum weighted energy-efficiency of D2D links, while guaranteeing QoS requirements to the users. Han *et alii* [105] studied a non-cooperative game to perform sub-channel assignment. The objectives of the players is to minimise the transmission power under QoS constraints of the users. By observing that the non-cooperative game can potential result in non-convergence, or some undesirable NE, Han *et alii* introduced a virtual referee, to monitor and improve the outcome of the non-cooperative game. The improvement is acquired by either reducing the transmission rates or shedding off some of the users.

In order to achieve Pareto optimal solutions, works in [47, 64, 77–79] proposed cooperative game theory based distributed algorithms. In [47] and [77], bargain theory is used to develop a distributed algorithm using two-user game model, which was extended to an arbitrary number of users in [78]. Bargain theory was employed to derive Kalai-Smorodinsky (KS) solution to maximise users' rate, while ensuring allocation of same fraction of the rate that a user will get in the absence of interference. Authors in [79] compared Nash bargain (NB) solution to both the KS and the Egalitarian solutions. They concluded that the NB solution achieves a better trade-off between fairness and efficiency.

Competitive and coordinated beamformer design methods for a multicell downlink network were proposed in [87]. A pricing mechanism was devised to achieve the Pareto optimal solutions. Another related work in [106] considered SINR balancing for a downlink cognitive radio network, using an underlaying approach. This work proposed a beamforming technique that maximises the worst case secondary user SINR, while ensuring that the interference leakage to the primary user is kept below specific thresholds. Game theory was applied to solve a SINR balancing problem for multiuser multicell network in [107].

4.1.2 Contributions

Further to the works discussed above, this chapter proposes beamformer design based on mixed QoS beamformer criterion using both non-cooperative (competitive) game and the cooperative (bargain) game. The contributions of this chapter are as follows:

- A novel mixed QoS SNG is proposed and analysed to develop a fully distributed algorithm for a multicell multiuser network with RTUs and NRTUs.
- The existence of the NE for the proposed mixed QoS SNG is studied.

- In order to take care of the possibility of non-convergence of the mixed QoS SNG, or some undesirable NE, a fall back mechanism and some necessary assumptions are proposed.
- The Egalitarian and KS bargain solutions are proposed to improve the NE operating point.
- Extensive numerical analysis and comparative evaluations of the proposed algorithms, and the optimal solutions are conducted.

4.2 System Model and Problem Formulation

Consider a similar system model and problem formulation described in Section 3.7. The mixed QoS problem in (3.22) is restated as

$$\begin{aligned} &\underset{\{\mathbf{w}_k\}_{k\in\mathcal{U}_n},\forall n}{\texttt{maximise}} \ \min_{k} \frac{\mathsf{SINR}_k}{\delta_k}, \quad k \in \mathcal{U}_n^{\mathsf{N}}, n \in \mathcal{N}, \\ &\texttt{subject to} \ \ \mathsf{SINR}_k \geq \xi_k, \quad k \in \mathcal{U}_n^{\mathsf{R}}, n \in \mathcal{N}, \\ &\qquad \sum_{k\in\mathcal{U}_n} \|\mathbf{w}_k\|^2 \leq \varphi_n^{\max}, \quad n \in \mathcal{N}. \end{aligned} \tag{4.1}$$

The feasible set of beamformers of the n-th BS is given by

$$\mathcal{W}_n := \Big\{ \mathbf{W}_n : \sum_{k\in\mathcal{U}_n} \|\mathbf{w}_k\|_2^2 \leq \varphi_n^{\max} : \ \ \mathsf{SINR}_k \geq \xi_k, \quad k \in \mathcal{U}_n^{\mathsf{R}} \Big\}, \tag{4.2}$$

The QoS feasible region $\mathcal{Q} \subset \mathbb{R}_+^{NK}$ is

$$\mathcal{Q} := \{\mathsf{SINR}_1, \ldots, \mathsf{SINR}_K : \mathbf{W}_n \in \mathcal{W}_n, \quad \forall n\}. \tag{4.3}$$

Thus, the overall system performance is measured by the function $f : \mathcal{Q} \mapsto \mathbb{R}$ as it attains points in the feasible region. Now, each BS has a QoS feasible region $\mathcal{Q}_n \subset \mathbb{R}_+^{K_n}$, defined as a subset of $\mathcal{Q}$, given by

$$\mathcal{Q}_n := \{\mathsf{SINR}_1, \ldots, \mathsf{SINR}_{K_n} : \mathbf{W}_n \in \mathcal{W}_n\}. \tag{4.4}$$

The aim is to form a mixed QoS SNG where each BS is only aware of its QoS feasible region $\mathcal{Q}_n$. The utility function is defined as the worst case SINR of the NRTUs. For a given QoS requirement, the overall system indicator function or the system cost function is defined as

$$f(\mathsf{SINR}_k) = \begin{cases} 0, & \min\limits_{\{k:\xi_k^\star>0\}} \frac{\mathsf{SINR}_k}{\xi_k^\star} \geq 1, \quad \forall k \in \mathcal{U}_n^{\mathrm{R}}, \forall n, \\ 1, & \text{otherwise}, \end{cases} \tag{4.5}$$

where $\xi_k^\star$ is the feasible SINR target. The indicator function will attain a zero if the QoS requirements are feasible and returns an empty set if otherwise.

4.3 Strategic Non-Cooperative Game (SNG)

In an SNG, each player is aware of only its local CSI. The local CSI is assumed to be private. Denote the inter-cell interference (ICI) plus the noise power at the k-th MS as

$$r_{-n_k}(\mathbf{W}_{-n}) = \sum_{\substack{n=1 \\ n\neq n_k}}^{N} \sum_{j\in\mathcal{U}_n} |\mathbf{h}_{n,k}^{\mathsf{H}}\mathbf{w}_j|^2 + \sigma^2, \tag{4.6}$$

where $\mathbf{W}_{-n}$ is the beamforming vectors of all other BSs except that of the n-th BS. At each BS, the ICI plus the noise power vector is denoted as $\mathbf{r}_{-n} = [r_{-n_1}, \ldots, r_{-n_{K_n}}]^{\mathsf{T}}$. The main motivation of representing the ICI plus noise by $\mathbf{r}_{-n}$, is to enable BSs to perform distributed optimisation, since it decouples the strategy sets. Initially, each BS approximate r_{-n_k} to σ^2. Then, after implementing the optimal beamformers, each user will feedback r_{-n_k} to their serving BSs. The users can estimate their SINRs by using the downlink pilot signal. The BSs are players, the beamformers are the strategies, and the SINRs of the NRTUs are the utilities. The intention is to balance and maximise the SINRs of the NRTUs of each BS, while satisfying the SINR targets for the RTUs. It can be proven that all NRTUs will attain the same SINR [90]; hence, all the intermediary SINR targets of the NRTUs are denoted with an identical value $\Delta_{n_{\mathsf{NRTU}}}$. The overall mixed QoS SNG is described as

$$\mathcal{G} = \Big\{\mathcal{N}, \{\mathcal{W}_n : n \in \mathcal{N}\}, \{\Delta_{n_{\mathsf{NRTU}}}(\mathbf{W}_n, \mathbf{W}_{-n}) : n \in \mathcal{N}\}\Big\}, \tag{4.7}$$

Since (4.1) is quasi-convex [22], it is solved using a bisection search [12]. Each play round of the proposed game consists of two stages namely; the *qualification stage* and the *learning stage* (please see Figure 4.1a). In the qualification stage, each player separately performs a bisection search on the feasible region $\mathcal{Q}_n$, to determine the optimal beamformer $\mathbf{W}_n$ that solves (4.1) distributively, for a given $\mathbf{r}_{-n}$. The learning stage follows implementation of these beamformers, and learning $\mathbf{r}_{-n}$. In the qualification stage, each player performs a bisection search on $\Delta_{n_{\mathsf{NRTU}}}$ by solving

$$\begin{aligned} \underset{\mathbf{W}_n \in \mathcal{W}_n}{\texttt{minimise}} \quad & \sum_{k \in \mathcal{U}_n} \|\mathbf{w}_k\|_2^2 \\ \texttt{subject to} \quad & \mathsf{SINR}_k \geq \xi_k, \qquad \forall k \in \mathcal{U}_n^{\mathsf{R}}, \\ & \mathsf{SINR}_k \geq \Delta_{n_{\mathsf{NRTU}}}, \quad \forall k \in \mathcal{U}_n^{\mathsf{N}}. \end{aligned} \tag{4.8}$$

The optimal solution of (4.1) is obtained by performing bisection search over (4.8) for a given $\Delta_{n_{\mathsf{NRTU}}}$, until the maximum possible $\Delta_{n_{\mathsf{NRTU}}}$ is obtained. It should be noted that for a fixed $\mathbf{r}_{-n}$, this stage requires only the knowledge of the feasible region $\mathcal{Q}_n$. The steps for the mixed QoS SNG are summarised in Algorithm 3.

4.3.1 Existence of NE of the Sub-game

The background noise vector $\mathbf{r}_{-n}$ decouples the strategy set of the game. Thus, the following theorem is used to prove the existence of the NE:

Theorem 4.1 (*Debreu-Fan-Glicksberg*)

> *Consider an SNG given by $\mathcal{G}$, whose strategy spaces S_i are* non-empty, compact and convex sets. *If the utility function $u_i(\mathbf{s})$ is a continuous function in the profile of strategy $\mathbf{s}$ and* quasi-concave *in s_i, then the game $\mathcal{G}$ has at least one pure NE [44, 108].*

Proof 2 The problem in consideration admits at least one NE for the following reasons. The strategy profile of $\mathcal{W}_n$, is a *convex set* as shown in (4.2). Since constraint on SINR_k is a SOC, and is quasi-convex on the set $\mathcal{W}_n$, the Debreu-Fan-Glicksberg is satisfied. Therefore, game $\mathcal{G}$ is quasi-concave and it has at least one pure NE. ■

4.3.2 Determining the Pure NE of the Sub-game

For brevity, denote the SINR targets of all RTUs and the intermediary SINR targets of all the NRTUs at the n-th BS as $\Gamma_n = [\gamma_1, \dots, \gamma_{K_n}]$. By assuming that the maximum

Algorithm 3: Mixed QoS SNG algorithm

Data: $\varphi_n^{\max}$, noise power, stopping criterion (*exempli gratia* iterations), $\delta > 0$, $\mathsf{SINR}_k \geq \gamma_k^\star, \forall k$, Γ_n^{L}, Γ_n^{U}, $\alpha_k = 1, \forall k \in \mathcal{U}_n^{\mathsf{N}}, \forall n \in \mathcal{N}$, $\alpha_k = 0, \forall k \in \mathcal{U}_n^{\mathsf{R}}, \forall n \in \mathcal{N}$

Data: Initialisation: Interference $r_{n_k} = \sigma^2$,

Result: Optimal beamforming vectors $\{\mathbf{W}_n\}_{n \in \mathcal{N}}$

Set $i = 0$;, fall back indicator $f_n = 0, \forall n$

while *stopping criterion is not satisfied* **do**

- Set $i = i + 1$;
- BS $n = 1 \dots N$ update local variables ($\{\mathbf{W}_n^{t+1}\}$):
- **while** $\Gamma_n^U - \Gamma_n^L > \delta$ **do**
  - Set $\Gamma_n^{\text{candidate}} = \frac{\Gamma_n^{\mathsf{U}} + \Gamma_n^{\mathsf{L}}}{2}$
  - **if** $f_n = 1$ **then**
    - $\alpha_k = 1, \forall k \in \mathcal{U}_n^{\mathsf{R}}$
  - Set $\gamma_k^\star = \Gamma_k^{\text{NE}} + \alpha_k \Gamma_n^{\text{candidate}}(k), \quad \forall k$;
  - **if** *Problem* (4.8) *is feasible* **then**
    - Set $\{\mathbf{W}_n^{\mathsf{L}}\}$ as the solution;
    - Set $\Gamma_n^{\mathsf{L}} = \Gamma_n^{\text{candidate}}$
  - **else**
    - Set $\Gamma_n^{\mathsf{U}} = \Gamma_n^{\text{candidate}}$
- *Optional*: **if** *Problem* (4.8) *is infeasible* **and** $i = 1$ **then**
  - $f_n = 1$ at the affected BS_n;
- Compute SINR_k and update local variable r_{n_k}
- **if** *stopping criterion is satisfied* **and** *current* $\mathbf{W}_n$ *is feasible* **then**
  - Set $\{\mathbf{W}_n^{\mathsf{L}}\}$ as the solution;
  - Set $\Gamma_n^{\mathsf{L}} = \Gamma_n^{\text{candidate}}$

Set $\Gamma_{\text{n-final}}^{\mathsf{L}} = \Gamma_n^{\mathsf{L}}$ and $\Gamma_{\text{n-final}}^{\mathsf{U}} = \Gamma_n^{\mathsf{U}}$

SINR $\Delta_{n_{\mathsf{NRTU}}}^\star$ is known at every game round, the optimal beamformer vector $\mathbf{w}_k^\star$, $k \in \mathcal{U}_n$ [30, 87] will be given by

$$\mathbf{w}_k^\star = \sqrt{\varphi_k}\tilde{\mathbf{w}}_k^\star = \sqrt{\varphi_k}\frac{\left(\mathbf{I}_M + \sum\limits_{i \in \mathcal{U}_n \setminus k} \frac{\lambda_i}{r_{-n_k}} \mathbf{h}_{n_k,i}\mathbf{h}_{n_k,i}^{\mathsf{H}}\right)^{-1} \mathbf{h}_{n_k,k}}{\left\| \left(\mathbf{I}_M + \sum\limits_{i \in \mathcal{U}_n \setminus k} \frac{\lambda_i}{r_{-n_k}} \mathbf{h}_{n_k,i}\mathbf{h}_{n_k,i}^{\mathsf{H}}\right)^{-1} \mathbf{h}_{n_k,k} \right\|_2}, \tag{4.9}$$

where φ_k is the beamforming power, $\tilde{\mathbf{w}}_k^\star$ is the unit-norm beamforming direction for the k-th user, and $\lambda_k \geq 0$ is the Lagrange multiplier. With this assumption, problem in (4.8) can be cast as

$$\underset{\varphi_k}{\texttt{minimise}} \sum_{k \in \mathcal{U}_n} \varphi_k, \quad \texttt{subject to} \quad \frac{\varphi_k |\mathbf{h}_{n_k,k}^{\mathsf{H}} \tilde{\mathbf{w}}_k|^2}{\sum\limits_{i \in \mathcal{U}_n \setminus k} \varphi_i |\mathbf{h}_{n_k,k}^{\mathsf{H}} \tilde{\mathbf{w}}_i|^2 + r_{-n_k}} \geq \gamma_k \quad \forall k \in \mathcal{U}_n. \tag{4.10}$$

The transmission power to the users at the n-th BS are stacked in a power allocation vector $\boldsymbol{\varphi}_n = [\varphi_1, \ldots, \varphi_{K_n}]^\mathsf{T} \in \mathbb{R}^{K_n}$. An extended power allocation vector at each user is formed as $\underline{\boldsymbol{\varphi}}_n = [\boldsymbol{\varphi}_n, r_{-n_k}]^\mathsf{T} \in \mathbb{R}^{K_n+1}$. For the analysis, the SINR is defined in terms of $\underline{\boldsymbol{\varphi}}_n$ and the interference function $\mathcal{I}_k : \mathbb{R}_+^{K_n+1} \mapsto \mathbb{R}_+$ [109] as

$$\mathsf{SINR}_k(\underline{\boldsymbol{\varphi}}_n) = \frac{\varphi_k}{\mathcal{I}_k(\underline{\boldsymbol{\varphi}}_n)}, \tag{4.11}$$

where

$$\mathcal{I}_k(\underline{\boldsymbol{\varphi}}_n) = \min_{\|\tilde{\mathbf{w}}_k\|=1} [\boldsymbol{\Psi}_n(\tilde{\mathbf{w}}_k)\boldsymbol{\varphi}_n]_k + \frac{r_{-n_k}}{|\mathbf{h}_{n_k,k}^\mathsf{H}\tilde{\mathbf{w}}_k|^2}. \tag{4.12}$$

The constant link gain matrix (*id est*, a coupling matrix) $\boldsymbol{\Psi}_n$, for the n-th BS is defined as

$$[\boldsymbol{\Psi}_n]_{i,k}(\tilde{\mathbf{w}}_\mathbf{k}) = \begin{cases} \frac{|\mathbf{h}_{n_k,k}^\mathsf{H}\tilde{\mathbf{w}}_i|^2}{|\mathbf{h}_{n_k,k}^\mathsf{H}\tilde{\mathbf{w}}_k|^2}, & i \neq k, \\ 0, & i = k. \end{cases} \tag{4.13}$$

At every bisection level, the SINR targets of all users served by the n-th BS are denoted as $\mathbf{D}_n = \mathtt{diag}\{\gamma_1, \ldots, \gamma_{K_n}\}$. Case 1 of the indicator function in (4.5) can be represented as $\mathtt{maximise}_k\,(\gamma_k \mathcal{I}_k(\underline{\boldsymbol{\varphi}}_n))/\varphi_k < 1$. The target $\mathbf{D}_n$ is feasible if and only if the $\mathtt{min\text{-}max}$ optimum $O(\mathbf{D}_n) < 1$ [109]. The optimum $O(\mathbf{D}_n) < 1$ can be written as

$$O(\mathbf{D}_n) = \inf_{\underline{\boldsymbol{\varphi}}_n > 0} \left(\max_{k \in \mathcal{U}_n} \frac{\gamma_k \mathcal{I}_k(\underline{\boldsymbol{\varphi}}_n)}{\varphi_k} \right). \tag{4.14}$$

For fixed $\mathbf{r}_{-n}$, with $r_{-n_k}(\underline{\boldsymbol{\varphi}}) > 0$, $\forall k$, and a known maximum SINR $\Delta^\star_{n_{\mathsf{NRTU}}}$ of the NRTUs attached to BS n, $\mathcal{I}_k(\underline{\boldsymbol{\varphi}}_n)$ will entirely depend on $\boldsymbol{\varphi}_n$. In [109], it was shown that there exists a standard interference function $\mathcal{I}_k(\boldsymbol{\varphi}_n)$ [110] such that

$$\mathcal{I}_k(\boldsymbol{\varphi}_n) := \mathcal{I}_k(\underline{\boldsymbol{\varphi}}_n). \tag{4.15}$$

This identity allows $\mathcal{I}_k(\underline{\boldsymbol{\varphi}}_n)$ to be treated as a special case of the standard interference function $\mathcal{I}_k(\boldsymbol{\varphi}_n)$, and it can therefore be characterised by the following axioms [109] for any $\boldsymbol{\varphi}_n \geq \mathbf{0}$:

1. Positivity: $\mathcal{I}_k(\underline{\boldsymbol{\varphi}}_n)$ is nonnegative on $\mathbb{R}_+^{K_n+1}$.
2. Scalability: $\mathcal{I}_k(\beta\underline{\boldsymbol{\varphi}}_n)^{(1)} = \beta\mathcal{I}_k(\underline{\boldsymbol{\varphi}}_n)^{(2)}$ for $\beta \in \mathbb{R}_+$.
3. Monotonicity: $\mathcal{I}_k(\underline{\boldsymbol{\varphi}}_n)^{(1)} \geq \mathcal{I}_k(\underline{\boldsymbol{\varphi}}_n)^{(2)}$ if $\underline{\boldsymbol{\varphi}}_n^{(1)} \geq \underline{\boldsymbol{\varphi}}_n^{(2)}$.

4. Strict Monotonicity: $\mathcal{I}_k(\underline{\boldsymbol{\varphi}}_n)^{(1)} \geq \mathcal{I}_k(\underline{\boldsymbol{\varphi}}_n)^{(2)}$ if $\underline{\boldsymbol{\varphi}}_n^{(1)} \geq \underline{\boldsymbol{\varphi}}_n^{(2)}$ and $\underline{\boldsymbol{\varphi}}_{nK_n+1}^{(1)} \geq \underline{\boldsymbol{\varphi}}_{nK_n+1}^{(2)}$.

It is argued that if a function is standard, then it has a fixed point solution [108]. Define a set containing all the possible power allocations for which the SINR target $\mathbf{D}_n$ is satisfied, as

$$\mathcal{P}_n(\mathbf{D}_n) = \{\boldsymbol{\varphi}_n > 0 : \varphi_k \geq \gamma_k \mathcal{I}_k(\underline{\boldsymbol{\varphi}}_n), k \in \mathcal{U}_n\}. \tag{4.16}$$

Problem in (4.10) is redefined into its compact form as

$$\underset{\varphi_k}{\texttt{minimise}} \sum_{k \in \mathcal{U}_n} \varphi_k, \ \texttt{subject to} \ \boldsymbol{\varphi}_n \in \mathcal{P}_n(\mathbf{D}_n). \tag{4.17}$$

Yates [110] showed that if the set $\mathcal{P}_n(\mathbf{D}_n)$ is non-empty, (*id est*, $O(\mathbf{D}_n) < 1$), there exists an optimiser of problem in (4.17) given by a fixed-point iteration

$$\boldsymbol{\varphi}_n^{(t+1)} = \mathbf{D}_n \mathcal{I}(\boldsymbol{\varphi}_n^{(t)}), \tag{4.18}$$

where t is the time index. This provides an optimiser at each BS during qualification stage. Since during every play round, players will adaptively change their beamformers, a new $\mathbf{r}_{-n}$ will be learned; hence, the BR of each player is to solve (4.18) repeatedly for every new values of $\mathbf{r}_{-n}$. With the assumption that the maximum SINR $\Delta^{\star}_{n\mathsf{NRTU}}$ of the NRTUs attached to BS n is known, the BR of each BS at each game round is defined as

$$\boldsymbol{\varphi}_n^{\star} = \mathrm{BR}_n(\boldsymbol{\varphi}_{-n}) = \mathbf{D}_n \mathcal{I}(\underline{\boldsymbol{\varphi}}_n). \tag{4.19}$$

Hence, (4.1) will have an optimiser at the maximum possible SINR target $\mathbf{D}_n^{\star}$. The game will converge only when the newly learnt $\mathbf{r}_{-n}$ causes no change to the previous power vector $\boldsymbol{\varphi}_n$. It is possible that the mixed QoS will not converge. This occurs when the SINRs of the RTUs, at a particular qualification stage, become infeasible for a given $\mathbf{r}_{-n}$. In order to force the game to converge, it is assumed the affected players will adopt their previous feasible beamformers and power allocation.

4.3.3 Fall Back Mechanism

Unlike the conventional `max-min` problem, the mixed QoS SNG may become infeasible if the SINR targets of the RTUs are not satisfied. This could be due to inadequate transmission power, interference, and/or bad channels. Consequently, for the case of multiple BSs, if the mixed QoS problem turns out to be infeasible for any of the BSs; then, the game model in (4.7) will not hold. In order to retain a valid game model, a fall back mechanism is introduced for all BSs. If the mixed QoS feasibility problem in (4.8) is infeasible during the first play round/qualification stage; then, only `max-min` problem will be considered for both RTUs and NRTUs at the affected BS. Alternatively, the affected BS may choose to admit only a subset of its RTUs by solving the admission control problem [105]. Even though this may readily give a valid game model, and possibly give rise to an increased performance of the NRTUs, it totally excludes the dropped users from taking part in the bargaining process; hence, this is not considered in this work.

The mixed QoS SNG described in Algorithm 3 includes the fall back mechanism option at step 16. In Algorithm 3, the qualification stage (the first play round), involves each BS solving (4.8) with $r_{-n_k} = \sigma^2, \forall k$ (*id est*, ignoring interference from other BSs) iteratively using a bisection method. This is the stage that determines if the fall back mechanism should be activated or not.

4.4 Mixed QoS Bargain Games

The focus of this section is bargain games under mixed QoS criterion. Since the global optimisation problem in (4.1) is concerned with fairness, a mixed QoS Egalitarian bargain game (EBG) and mixed QoS Kalai-Smorodinsky bargain game (KSBG) are studied. In the mixed QoS EBG, the players will have equal share of the remaining feasible region, enclosed between the NE and the Pareto boundary, denoted hence-forth as $\mathcal{Q}_+$. On the other hand, the mixed QoS KSBG allows players to acquire a share that is proportional to what they will achieve in the absence of competition. ADMM [33,88] is adopted to decompose the mixed QoS problem.

4.4.1 Mixed QoS EBG

The ADMM is used to reduce the amount of information shared between BS. Unlike in [88], where the searching space of the algorithm is defined by $\mathcal{Q}$, it is demonstrated that during bargaining, the search space is reduced to $\mathcal{Q}_+$. The achieved SINRs at NE-mixed QoS at the n-th BS are denoted as $\Gamma_n^{\text{NE}} = [\gamma_1^{\text{NE}}, \ldots, \gamma_{K_n}^{\text{NE}}]$. The global system Egalitarian problem is formulated as

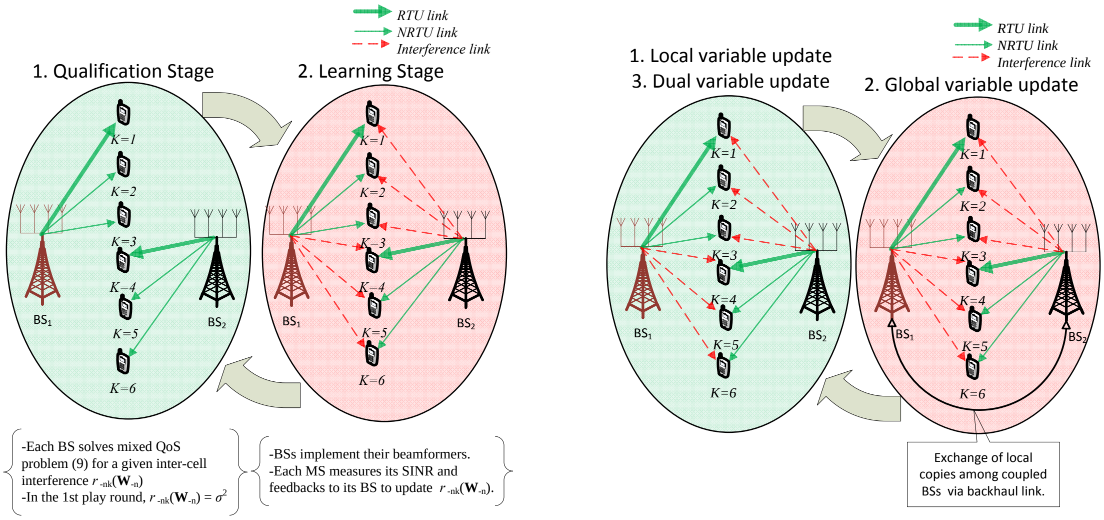
$$\begin{aligned} \underset{\mathbf{W}_n \in \mathcal{W}_n, \forall n}{\texttt{maximise}} \quad & \underset{k \in \mathcal{U}}{\texttt{min}} \frac{\mathsf{SINR}_k}{\alpha_k} \\ \texttt{subject to} \quad & \mathsf{SINR}_n \geq \Gamma_n^{\text{NE}}, \quad k \in \mathcal{U}, n \in \mathcal{N}, \\ & \sum_{k \in \mathcal{U}_n} \|\mathbf{w}_k\|_2^2 \leq \varphi_n^{\max}, \quad k \in \mathcal{U}, n \in \mathcal{N}. \end{aligned} \tag{4.20}$$

where α_k is either a 0 or 1. Ideally, if the SINR targets of the all the RTUs at each BS are attained at the NE-mixed QoS, then the values of α_k will be set to 0 for all RTUs and to 1 for all NRTUs. In this case, NRTUs will have equal share of $\mathcal{Q}_+$. When α_k is 0, then the term SINR_k/α_k will go to infinity, thereby removing it from the objective of (4.20). A case may arise whereby at NE-mixed QoS, one or more of the RTUs fail to achieve its SINR target. For this case, the players would give the affected RTU a high priority during bargaining by initially setting $\alpha_k = 1$ for the affected RTU, and $\alpha_k = 0$ for all NRTUs. Once the RTUs reach their SINR targets, their SINRs will be fixed, and the remaining resources will be shared equally amongst the NRTUs by setting $\alpha_k = 1$ for all NRTUs.

A mixed QoS optimisation is performed over $\mathcal{Q}_+$. Denote the optimal SINRs from Egalitarian bargain game solution at each BS as $\Gamma_n^{\text{EBG}} = [\gamma_k^{\text{EBG}}, \ldots, \gamma_{K_n}^{\text{EBG}}]$. The optimal mixed QoS Egalitarian bargain game solution will allocate equal share of resource to the NRTUs given as

$$c_n = \gamma_k^{\text{EBG}} - \gamma_k^{\text{NE}}, \quad k \in \mathcal{U}_n^{\mathsf{N}}. \tag{4.21}$$

This means that we expect the optimal SINRs of the NRTUs at each BS to give equal value c_n if the RTUs achieved their SINR targets at the Nash equilibrium. If this is not the case, the values of c_n may be different because the affected RTUs will have a share on $\mathcal{Q}_+$.



(A) One play round stages for the mixed QoS SNG.

(B) A particular play round for the mixed QoS BG.

FIGURE 4.1: Illustration of the steps involved during each play round in the mixed QoS and the bargaining games.

4.4.2 Mixed QoS KSBG

In the KSBG, players get pay-offs that are proportional to what they would achieve if there was no competition. Define an ideal operating point at which each player experiences no intercell interference as the utopia point, $\mathbf{u}_n = [u_1, \ldots, u_{K_n}]$. The overall mixed QoS KS optimisation problem for the whole system is defined as

$$\begin{aligned} \underset{\mathbf{W}_n \in \mathcal{W}_n, \forall n}{\texttt{maximise}} \ \underset{k \in \mathcal{U}}{\texttt{min}} & \ \frac{\mathsf{SINR}_k - \gamma_k^{\mathrm{NE}}}{\alpha_k \left(u_k - \gamma_k^{\mathrm{NE}}\right)} \\ \texttt{subject to} \ & \ \mathsf{SINR}_n \geq \Gamma_n^{\mathrm{NE}}, \quad k \in \mathcal{U}, \, n \in \mathcal{N}, \\ & \sum_{k \in \mathcal{U}_n} \|\mathbf{w}_k\|_2^2 \leq \varphi_n^{\max}, \quad k \in \mathcal{U}, \, n \in \mathcal{N}. \end{aligned} \tag{4.22}$$

We denote the optimal SINRs from Kalai-Smorodinsky solution at each BS as $\Gamma_n^{\mathrm{KS}} = [\gamma_k^{\mathrm{KS}}, \ldots, \gamma_{K_n}^{\mathrm{KS}}]$. The optimal mixed QoS Kalai-Smorodinsky solution will yield a value v_n defined as

$$v_n = \frac{\gamma_k^{\mathrm{KS}} - \gamma_k^{\mathrm{NE}}}{u_k - \gamma_k^{\mathrm{NE}}}. \tag{4.23}$$

The values c_n and v_n in (4.21) and (**??**) reveal the relationship between the Egalitarian bargain game and Kalai-Smorodinsky bargain solutions. Thus, an algorithm that will provide solution to both the mixed QoS EBG and the mixed QoS KSBG is proposed.

4.4.3 System Model for Bargaining Solutions

Consider the system in Section 4.2. The players strategies are decoupled by using the same approach discussed in Section 3.8. The same procedures therein, will yield the following ADMM:

$$\mathbf{W}_n^{t+1}, \tilde{\boldsymbol{\kappa}}_n^{t+1} := \underset{\mathbf{W}_n, \tilde{\boldsymbol{\kappa}}_n}{\texttt{argmin}} \ \mathcal{L}_\rho\left(\mathbf{W}_n, \tilde{\boldsymbol{\kappa}}_n, \boldsymbol{\kappa}_n^s, \mathbf{u}_n^t\right), \quad n \in \mathcal{N}, \tag{4.24}$$

$$\boldsymbol{\kappa}_n^{t+1} := \underset{\boldsymbol{\kappa}_n}{\texttt{argmin}} \ \mathcal{L}_\rho\left(\mathbf{W}_n^{t+1}, \tilde{\boldsymbol{\kappa}}_n^{t+1}, \boldsymbol{\kappa}_n, \mathbf{u}_n^t\right), \tag{4.25}$$

$$\mathbf{u}_n^{t+1} := \mathbf{u}_n^t + \tilde{\boldsymbol{\kappa}}_n^{t+1} - \boldsymbol{\kappa}_n^{t+1} \quad n \in \mathcal{N}, . \tag{4.26}$$

where t is the iteration index. The solution to the mixed QoS EBG and KSBG is summarised in Algorithm 4. The steps in (4.24) to (4.25) are combined with a bisection method to provide the optimal solution to the mixed QoS problems in (4.20) and (4.22).

4.4.4 Convergence of the Bargaining Games

As in [33], the critical assumptions required for the cooperative game to converge is that $f_n\left(\mathbf{W}_n, \tilde{\boldsymbol{\kappa}}_n\right)$ is closed, proper, convex and the unaugmented Lagrangian $\mathcal{L}_0$ has a saddle point. Under these assumptions, the iterations of ADMM satisfies the following:

1. The dual residual $(\tilde{\boldsymbol{\kappa}}_n^t - \boldsymbol{\kappa}_n^t) \to \mathbf{0}$ as $t \to \infty$.

2. The objective function in (3.35) converges *id est*, $f_n\left(\mathbf{W}_n, \tilde{\boldsymbol{\kappa}}_n\right) \to \mathbf{W}^\star$ as $t \to \infty$.

3. The dual variable converges, i.e, $\mathbf{y}_n^t \to \mathbf{y}_n^\star$ where $\mathbf{y}_n^\star$ is the dual optimal point.

4. The local variable $\tilde{\boldsymbol{\kappa}}_n^t$ and the global variables $\boldsymbol{\kappa}_n^t$ do not necessarily have to converge to the optimal values [33].

The indicator function in (3.29) has the characteristics defined above and satisfies all the three conditions. This means that the ADMM algorithm will converge to the globally optimal solution, and if there is a unique point, it will be reached as t approaches ∞. A summary of the mixed QoS BG algorithm is presented in Algorithm 4.

Algorithm 4: Mixed QoS BG algorithm

Data: $\varphi_n^{\max}$, penalty $\rho > 0$, stopping criterion, $\delta > 0$, $\mathsf{SINR}_k \geq \gamma_k^{\text{NE}}, \forall k$, Γ_n^{upper},*id est*,($\Gamma_n^{\text{upper}} = \mathbf{u}_n$ for mixed QoS KSBG), $i = 0$, $\alpha_k = 0, \forall k \in \mathcal{U}_n^{\mathsf{R}}$, $\alpha_k = 1, \forall k \in \mathcal{U}_n^{\mathsf{N}}$, $\forall k, \forall n$.
Data: Γ_n^{NE}, $f_n, \forall n$: determined via Algorithm 3.
Data: Initialisation: set $\{\boldsymbol{\kappa}_n\}_{n\in\mathcal{N}} = 0$,$\{\tilde{\boldsymbol{\kappa}}_n\}_{n\in\mathcal{N}} = 0$,$\{\mathbf{y}_n\}_{n\in\mathcal{N}} = 0$, $\Gamma_n^{\text{lower}} = 0$
Result: Optimal beamforming vectors $\{\mathbf{W}_n\}_{n\in\mathcal{N}}$

while $\Gamma_n^{upper} - \Gamma_n^{lower} > \delta$ **do**
- Set $i = i + 1$,
- Set $\Gamma_n^{\text{candidate}} = \frac{\Gamma_n^{\text{upper}} + \Gamma_n^{\text{lower}}}{2}$
- **if** $f_n = 0, \forall n \in \mathcal{N}$ ***and*** $i = 1$ **then**
  - $\alpha_k = 1, \forall k \in \mathcal{U}_n^{\mathsf{R}}$, $\Gamma_n^{\text{candidate}}(k) = \xi_k - \gamma_k^{\text{NE}}, \forall k \in \mathcal{U}_n^{\mathsf{R}}$
- Set $\gamma_k^* = \Gamma_n^{\text{NE-mixedQoS}}(k) + \alpha_k \Gamma_n^{\text{candidate}}(k), \quad \forall k;$
- Steps 4 - 11 in Algorithm 2.

$\alpha_k = 0, \forall k \in \mathcal{U}_n^{\mathsf{R}}$, $\alpha_k = 1, \forall k \in \mathcal{U}_n^{\mathsf{N}}$.
Steps 4 - 15 in Algorithm 2.

4.5 Numerical Examples

To investigate the performance of Algorithms 3 and 4, a network topology shown in Figure 4.2 (also refer to Figure 4.1) is used. The network has two BSs (*id est* two players), each equipped with $M = 4$ antennas. The maximum transmission power at each BS is $\varphi_n^{\max} = 20$ dB. Each BS serves three single antenna MSs that include two NRTUs and one RTU. The SINR targets for the RTUs at the first BS and the second BS were set to 10 dB and 12 dB, respectively. The line search accuracy at each BS is set to $\delta = 10^{-4}$. The maximum iteration is set to 15 for the mixed QoS SNG and all bargaining games. All channel vectors, $\mathbf{h}_{n_k,k}$, are chosen from the distribution $\mathcal{CN}(0, \mathbf{I})$, which resembles uncorrelated frequency flat Rayleigh fading. Only a single sub-carrier is considered, but the algorithm can be applied to multi-carrier transmission. In that case, the algorithm has to be applied to each resource block. The noise power was set to $\sigma^2 = 1$ W, for all users. The centralised solutions are used as a reference system for performance comparison. Under the mixed QoS framework, there are two main possible scenarios:

1. The SINR target of at least one RTU is not satisfied because:

   (a) of intercell interference,

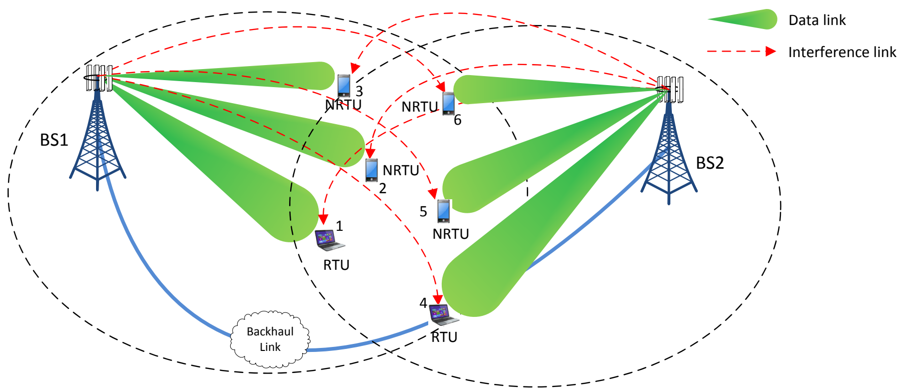
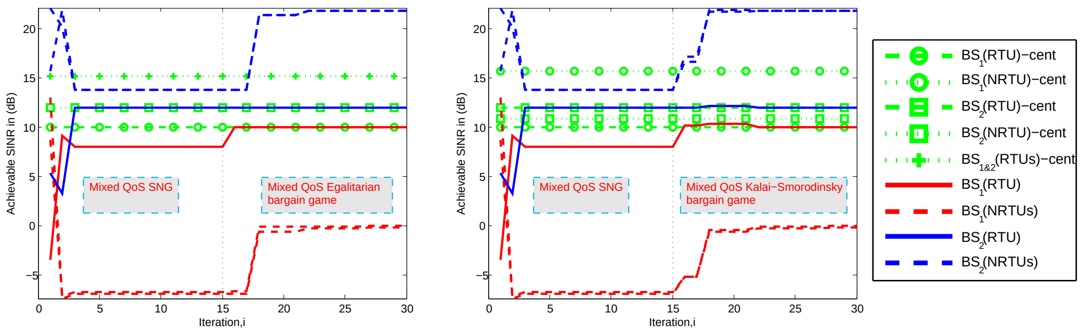


FIGURE 4.2: A multiuser multicell network topology. User 1 and user 4 are RTUs while the remaining users are the NRTUs.



(A) Comparison of the mixed QoS, mixed QoS EBG and the centralised Egalitarian solutions.

(B) Comparison of the mixed QoS, mixed QoS KSBG and the centralised Kalai-Smorodinsky solutions.

(C) Legends.

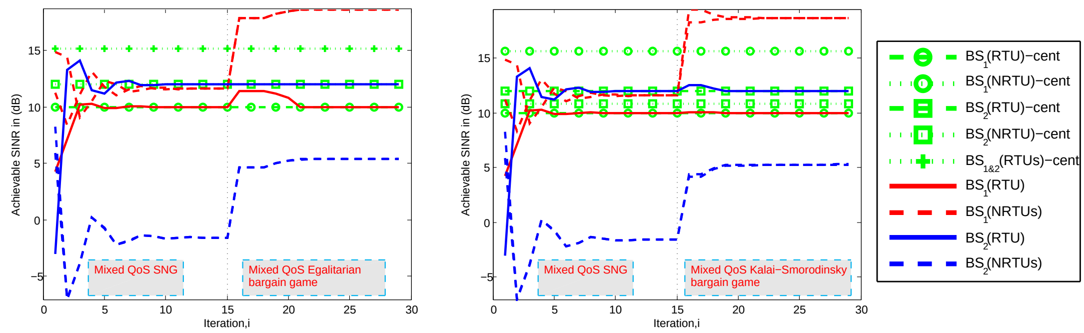
FIGURE 4.3: The SINR evolution using Algorithms 3 and 4 under scenario 1. All the sub-figures show the mixed QoS NE and a transition from the mixed QoS SNG to the mixed QoS EBG and the mixed QoS KSBG.

(b) or during the first play round, one or both of the BSs fail to pass the qualification stage. Using the fall back mechanism, the affected BS then considers a pure `max-min` problem. This will introduce interference to the other BSs and may encourage them to consider bargaining.

2. The SINR targets of both the RTUs are satisfied.

4.5.1 Results Under Scenario 1

The results in Figure 4.3, show the SINR evolution of the users, under Algorithms 3 and 4. In Figure 4.3a, it is noted that, during the mixed QoS SNG, only BS2 managed to balance the SINRs of its NRTUs while achieving the SINR targets of its RTU. BS1 failed



(A) Comparison of the mixed QoS, mixed QoS EBG and the centralised Egalitarian solutions.

(B) Comparison of the mixed QoS, mixed QoS KSBG and the centralised Kalai-Smorodinsky solutions.

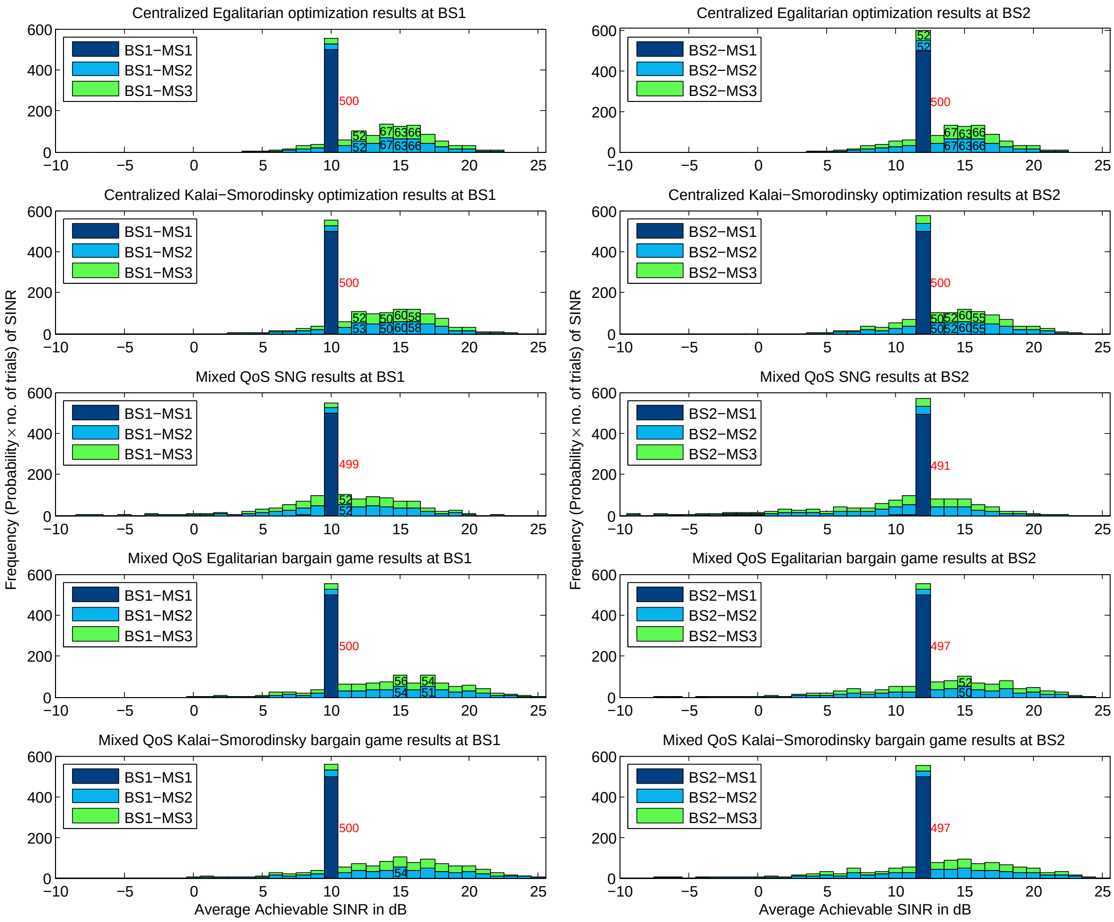
(C) Legends.

FIGURE 4.4: The SINR evolution using Algorithms 3 and 4 under scenario 2.

to achieve the SINR target of its RTU due to intercell interference. During mixed QoS EBG, BS1 managed to achieve the SINR target of its RTU during the first iteration into bargaining. The remaining resources were then shared equally amongst all the NRTUs. The SINRs achieved by the mixed QoS SNG and mixed QoS EBG were compared to the centralised Egalitarian solution. It is observed that, NRTUs at BS1 achieved SINRs that are lower than the balanced SINRs they will achieve under the centralised Egalitarian solution. During the mixed QoS SNG, the NRTUs at BS2 achieved balanced SINRs that are below the balanced SINRs provided by the centralised Egalitarian solution. But during the mixed QoS EBG, the NRTUs at BS2 managed to attain balanced SINRs that are more than the balanced SINRs offered by the centralised Egalitarian solution. Figure 4.3b shows the transition from the mixed QoS SNG to the mixed QoS KSBG, and comparison is made with the centralised Kalai-Smorodinsky solution. It is noted that in the centralised Kalai-Smorodinsky solution, the NRTUs are not necessarily balanced to the same SINR. The same trend in Figure 4.3a is observed.

4.5.2 Results Under Scenario 2

The outcome of scenario 2 is illustrated in Figure 4.4a and Figure 4.4b. Note that at the mixed QoS NE, both RTUs achieved their SINR targets and the SINRs of NRTUs were balanced. The convergence rate of the mixed QoS SNG is reduced under this scenario. This is because, since both BSs are able to achieve the SINR target of their RTUs at each qualification stage, every time a new value $\mathbf{r}_{-n}$ is observed, the BSs will have to adapt their beamformers. This is not the case in Figure 4.3a and Figure 4.3b,



(A) Frequency of achieved SINRs at BS1. (B) Frequency of achieved SINRs at BS2.

FIGURE 4.5: Comparison of proposed algorithms and centralised solutions in terms of frequency of achieved SINRs over 500 random channel realisations.

wherein during the second iteration, BS1 adapts its previous beamformers and power allocation.

4.5.3 General Performance of the Proposed Algorithms

A further analysis on the performance of mixed QoS SNG (Algorithm 3) and mixed QoS BG (Algorithm 4) was performed using 500 random channel realisations. Figures 4.5 depicts the achieved SINRs by all users under various solutions at BS1 and BS2. In Figure 4.5a, it is observed that both the centralised Egalitarian and Kalai-Smorodinsky solutions are able to achieve the SINR target of the RTUs for all channel realisations. Furthermore, the centralised solutions always balance the SINRs of the NRTUs. Note that the mixed QoS SNG failed to achieved the SINR target of the RTUs for one of the channel realisations. The mixed QoS SNG is inefficient as compared to the centralised

Player	BS-1		BS-2	
Parameter (bits/s/Hz)	Mean	Variance	Mean	Variance
Cent. Egalitarian	12.9310	4.2634	13.5462	4.2634
Cent. Kalai-Smorodinsky	12.9755	4.6055	13.6425	5.1448
Mixed QoS SNG	11.1905	7.0076	11.2477	9.1827
Mixed QoS EBG	13.3158	8.0057	13.2866	9.7089
Mixed QoS KSBG	13.3337	8.3967	13.2889	10.0967

Table 4.1: Comparison of mixed QoS sum rate attained for 500 random channel realisations using centralised, mixed QoS SNG and mixed QoS BG algorithms.

solutions. Similar trends are observed in Figure 4.5b. Notice that BS2 attained higher SINRs that are below 5 dB as compared to BS1. This is because, the SINR target of the RTU at BS2 is larger than the SINR target at BS1. Hence, this results in more transmission power being utilised for the RTU at BS2 as compared to the NRTUs. Nevertheless, all NRTUs at both BSs are able to achieve higher SINRs that are greater than 20 dB as compared to the centralised solutions.

The mean and the variance of the sum rate for the mixed QoS problem are summarised in Table 4.1. As anticipated, in terms of mean, the mixed QoS SNG is inefficient compared to all the centralised and bargaining solutions. Both the bargain solutions outperform the centralised solutions at BS1, but this is not the case at BS2. The latter observation comes as a consequence of the high SINR target of RTU2. As a remark, this does not mean that the bargain solutions outperform the centralised system, but in mixed QoS, a player may benefit more if there is no cooperation. The overall system performance under bargaining matches that of the centralised system. It is noted that the variance of the bargaining solutions are larger than those achieved by the mixed QoS SNG and the centralised solutions.

4.6 Conclusion

This chapter proposed a game-theoretic framework for the downlink beamformer design in a multicell network, under mixed QoS criterion. Both the mixed QoS SNG and the mixed QoS BGs were studied. A mixed QoS SNG was formulated, whereby each BS determines optimal downlink beamformers in a distributed manner by considering estimate of ICI plus noise power. Moreover, two bargaining solutions, namely, the mixed

QoS EBG and the mixed QoS KSBG, which use the NE-mixed QoS as a disagreement point, were proposed. On average, the mixed QoS sum rate achieved by the bargaining games is comparable to that of the centralised Egalitarian and Kalai-Smorodinsky solutions. In Chapter 5, a novel auction is proposed to improve the network capacity.

Chapter 5

Single Item bidding Auctions for User Offloading in HetNets

In this chapter, an auction based beamforming and user association algorithm for a heterogeneous network is proposed. In particular, a wireless network with a macrocell deployed with multiple small cells is considered. Accordingly, the macrocell base station (MBS) wishes to offload some of its users to a number of small cell access points (SCA), in exchange of payments based on auctioning. The proposed bid-wait auction (BWA) method, which offers a decentralised solution, is used to coordinate the competition of SCAs over macrocell users.

5.1 Introduction

The envisaged 5G is anticipated to address the continuously growing demand for high capacity in wireless mobile communications [15]. Apart from advanced interference mitigation and massive MIMO techniques; usage of small cells for cell densification is expected to provide high spectral efficiency. Small cells have the potential to offload MBS traffic and increase the network capacity. But, as many stakeholders have embraced this solution, small cells are either operator-deployed or user-deployed. The latter brings forth other challenges like inter-cell interference, mainly because the deployment process is uncoordinated. Small cells can operate in open-access mode, hybrid mode or closed-group mode [16,111]. In closed-access mode, only pre-registered users (*id est*, host users

(HUs)) can access the transmitter, while the open-access mode allows both pre-registered and unregistered users (*id est*, GUs) to access the network resource. The hybrid-access mode allows pre-registered users access, while unregistered users get access under some certain restrictions. Amongst these three modes, works in [16, 111] advocate for the hybrid mode as it allows shared resources between the GUs and the HUs. Most of the user-deployed small cells operate in closed-access mode, and this reduces the spectral efficiency of the network. Works in [112–115] show that incentives will motivate private owners to consider switching their small cells into hybrid mode. The operators will then reimburse the private owners for connecting their users.

The question now becomes, how should the MUs be assigned to the accessible small cells? Taking into consideration that reimbursement has to be made to SCAs owners, for admitting GUs, what criterion should be used to maximise the number of GUs admitted by the SCAs? These questions are addressed in this chapter by providing an auction based framework, that allows the SCAs to generate profit while admitting the maximum possible number of GUs, within the framework of multicell beamforming. Clearly, a combination of macrocells with small cells densification and massive MIMO techniques make the traditional network planning and optimisation techniques a complex task [14]. Furthermore, it is discussed in [16, 111] that the amount of data and information in different backhaul links (*exempli gratia*; S1, X2, and internet IP interfaces), and the latency, will be significantly increased. Consequently, there is a need for decentralised algorithms as addressed in this chapter.

5.1.1 Related and Parallel Works

MNOs can offload some of their mobile users to third party networks where under-utilised spectrum resources exist. The benefits of offloading traffic have been extensively studied in the literature. Some of the traffic offloading mechanisms and analysis can be found in [116–122]. A Wiffler system is proposed in [117] to augment mobile 3G capacity with WiFi. The Wiffler is used in vehicular networks for delay tolerant applications to address the availability and performance challenges. Erlang-like capacity, in a setting with multiple macrocells deployed with picocells and femtocells, is analysed in [120]. The findings in [120] show that small cells achieve higher network capacities with good energy efficiency. It was deduced that small cells are a good alternative to network densification. In [121], a small cell activation mechanism for offloading traffic from a macrocell to small cells, while avoiding user QoS degradation is proposed. The main idea in [121] is to offload traffic to small cells in energy saving mode only when there is a significant energy saving gains. This approach reduces the total energy consumption of the network. Work in [122] considered a centralised energy aware offloading scheme, based on cloud-radio access network.

The work in [123] considered a user load balancing problem for obtaining optimal `max-min` fair bandwidth allocation under bandwidth and backhaul constraints. An approach that rely on the Gibbs sampler, which does not require exact coordination information among the wireless devices, was proposed in [124]. A self-configuring algorithm was proposed to allow multiple interfering 802.11 access points to select their operating frequency. This was done in order to minimise interference, and to allow users to choose the AP in order to get a fair share on the whole network bandwidth. In [125], a network-wide utility maximisation problem was considered. A solution that jointly optimises partial frequency reuse and load balancing was proposed to achieve proportional fair association. In [126], an inverse problem was considered wherein the service providers compete for femtocell under a multi-leader follower game framework. A framework for user association in infrastructure-based wireless network considering rate-optimal, throughput-optimal, delay-optimal, and load-equalising association policies was studied in [127]. An iterative distributed user association policy that adapts to spatial traffic loads was proposed in [127].

In order to address the issue of incentivised offloading, auction based algorithms have been proposed in [128–134]. In [128] and [132], the authors formulated a combinatorial reverse auction problem, wherein a set of MNOs act as auctioneers and the wireless APs as bidders. The commodity in auction is the under-utilised bandwidth on the side of the APs. In the problem formulation, the APs submit bids to the MNOs who in turn select the AP of their interest. A reverse auction framework for fair and efficient access permission is proposed in [129]. In particular, the authors in [129] proposed a Vickrey–Clarke–Groves (VCG) based mechanism to maximise the social welfare in a network with one wireless service provider (WSP), and several femtocell owners. In their network model, the WSP is the buyer and the femtocells are selling their access permissions to allow the WSP users access. The authors in [129] dealt with the cell overlapping by partitioning the femtocell coverages into small granularity of same size (referred to as locations). This allows bids to be expressed as a function of access permissions in each location. In order to tackle the complexity of the VCG mechanism, the authors in [129] further proposed a suboptimal algorithm with low complexity.

In [130], a network with multiple MBSs, third party owned femtocells, and mobile users is considered. In order to allow the MBSs to offload some of their traffic to the femtocells, the femtocells bid to provide service to the MBSs. With emphasis that the sellers (femtocells) could incur significant overheads during valuation, the authors further propose a system which allows imprecise valuations. Therefore, the femtocells are allowed to estimate their valuations. Another VCG based mechanism is proposed in [131]. The work in [131] proposed a greedy algorithm in attempt to reduce the high time complexity problem in the VCG mechanism.

A well researched method to offload the users from macro cells to small cells is the biased cell association (also referred to as cell range expansion (CRE)) [135–138]. This method offer preferential load balancing by giving small cells high preference over macro cells. In [135], it was noted that the SINRs of the offloaded users, especially those in the CRE region, is dramatically decreased when interference management techniques are not in place. A spatial interference cancellation (SIC) scheme with biased cell association was then proposed in [135]. In this SIC scheme, strong interference from macro cells to users in the CRE region is mitigated in the spatial domain. Motivated by the fact that the impact of coupled downlink-uplink offloading is not well understood, authors in [136] proposed a tractable model which characterises the uplink SINR and rate distribution as a function of association rules and power control parameters.

Amongst the most recent works, authors in [139] considered a problem where the APs decide whether they need to be on open or closed mode in order to maximise their performance. The problem was solved using a game theoretic approach. Authors in [140] considered cell association and resource allocation problems to address jointly the problem of user association and load balancing. A network-wide utility maximisation problem was formulated, and dual decomposition method was used to derive a distributed solution. Further works in [114, 141–144] proposed distributed algorithms for assignment of users to the small cells using auctioning, heuristic beamforming designs, Stackelberg games, and evolutionary games. Amongst these works, consideration of joint auctioning and beamforming design techniques has not been addressed. Even though various auction based algorithms have been proposed in the literature, to solve spectrum auctions as in [62, 145–147], these algorithms cannot be directly applied to the problem under consideration.

5.1.2 Contributions

In this chapter, a framework for cell association optimisation from an auctioning perspective is developed. The aim is to develop a close to optimal, if not optimal, distributed algorithm that will associate MUs to the hybrid SCAs. In particular, the problem considered here is a multi-unit auction, wherein bidders are interested in multiple items. The contributions of this chapter are as follows:

- A novel auction called the BWA, that jointly performs downlink beamformer design and user association, is proposed and analysed. Previous works that propose auction based mechanisms do not consider beamformer design in their mechanism design.
- A novel valuation function for bidders, that automatically monitors the resource budgets of the bidder, is proposed.
- A novel payment rule, that allows the BWA to allocate items to bidders with sparse information, is proposed and analysed. Though the proposed payment rule is different from the VCG payment rule, it is shown that some of the principles of the VCG mechanism are preserved.
- It is shown that the BWA has dominant strategy equilibrium at every auction round, which decomposes the combinatorial nature of the problem, thereby allowing sequential and parallel auctions to manifest autonomously.
- Thorough numerical analysis and comparative evaluation of the proposed BWA, and the optimal solution for heterogeneous deployments is performed.

5.2 System Model and Assumptions

Consider a single-cell MISO downlink network consisting of a MBS deployed with a set $\mathcal{S} = [1, \ldots, S]$ of SCAs. It is assumed that the SCAs are privately owned and are operating in hybrid mode. It is further assumed that the MBS and the SCAs are operating on non-overlapping frequency bands, as shown in Figure 1.6. The assumption of having non-overlapping frequency bands allows the valuations of the bidders not to depend on each other[1]. The SCAs can admit GUs with the provision that performance of their HUs[2] is not degraded. The MBS is equipped with M_{MBS} antennas and each SCA is equipped with M_{SCA} antennas, where $M_{\text{MBS}} >> M_{\text{SCA}}$. It is assumed that the MBS has commitment to serve $M_0 = |\mathcal{M}_0|$ MUs, where $M_0 >> M_{\text{MBS}}$. The latter assumption of offloading the MBS is to illustrate the necessity for offloading users to the SCAs. The MBS and each SCA have maximum transmission powers of $\varphi_0^{\max}$ and $\varphi_s^{\max}$, respectively. Moreover, it is assumed that all SCAs are connected to the MBS via capacity limited wired backhaul links. These backhaul links are used for coordinating auctions (*id est* transporting bids, auction invitations and announcements). All users are assumed to have specific QoS requirements that have to be met, otherwise they will be dropped. In order to improve the readability of this chapter, a summary of acronyms and notations used herein, are provided in Table 5.1.

The following terms are used in the rest of the thesis:

- *Preference profile:* A set of all GUs that an SCA is willing to bid on, and sorted in the order of preference.
- *Valuation profile:* A set of all bids (*id est*, valuations) corresponding to the preference profile.
- *Auction coverage area:* A bidder is allowed to bid for users only if the users are within a prescribed area known as the auction coverage area.

5.2.1 Motivation

It is likely that some of the resources may not be fully utilised by SCA's pre-registered HUs. Therefore, a hybrid configuration is deemed suitable. This mode of operation allows other MUs in the vicinity of the SCAs to connect to the SCAs, if such a need

[1]A model wherein the valuation function are dependent is not covered in this work, but it is part of future work.

[2]The term home user (HU) is used interchangeable to refer to preregistered users and GUs that are already admitted by an SCA. This is because once a GU is admitted, the SCA has the mandate to serve that GU as its preregistered user.

Notation	Definition
HU(s)	Host users, *id est*, SCAs' pre-registered user(s).
GU(s)	Guest users, *id est*, MBS user(s) requesting connection to the SCAs.
MU(s)	Macrocell user(s) primarily served by the MBS. The dropped MUs that are being auctioned are called GUs.
$\mathcal{S}$	A set of all SCAs.
$\mathcal{M}_0$	A set of all MUs with cardinality $M_0 = \lvert\mathcal{M}_0\rvert$. $\mathcal{M}_0 = \{1, \ldots, M_0\}$.
$\mathcal{G}_s \subset \mathcal{G}$	A set of all GUs in the auction coverage area of the s-th SCA with cardinality $G_s = \lvert\mathcal{G}_s\rvert$.
$\mathcal{H}_s$	A set of all HU(s) and admitted GUs at the s-th SCA with cardinality $H_s = \lvert\mathcal{H}_s\rvert$.
$\mathcal{F}_s := \mathcal{H}_s \cup \mathcal{G}_s$	A set of HUs and GUs in the auction coverage area of the s-th SCA.
$\mathcal{A}_s$	The allocation/provisional set. A set of (provisionally) assigned GUs for SCA s.
$\mathcal{C}_g$	The competitors set. A set of SCAs competing for GU g.
$\mathcal{G}_s^i$	The conditional bidding set/conditional bid. A set of favourite GUs for SCA s.
$\mathcal{P}_s := \mathcal{G}_s$	The preference set. A set of GUs in the auction area of the SCA s arranged in the order of preference.
$\mathcal{R}_s \subset \mathcal{G}_s$	The remainder set. A set of GUs that are left over after determining the favourite set $\mathcal{G}_s^i$.
$\mathcal{L}_s \subset \mathcal{G}_s$	The loose set. A set of GUs that have been lost to other bidders.
$\mathcal{T}^r \subseteq \mathcal{S}$	Contact set during auction round r. A set of SCAs that have to answer queries from the MBS.

TABLE 5.1: Frequently used notations.

arises. In order to avoid degradation of its reputation, by having many dropped users, the MBS's objective is to admit as many MUs as possible. As it may be impossible at some point for the MBS to accommodate all of its users, it will offload some of its users to the SCAs. If the SCAs densely deployed, there is a high chance that a GU may be in the vicinity of more than one SCA. This chapter proposes a mechanism that resolves the competition of SCAs for GUs in the overlaps.

5.2.2 FBWA and BBWA Algorithms

The problem of offloading users to the SCAs can be in two different approaches[3]: Approach (1); the MBS can firstly admit the maximum possible MUs it can serve and then offloads the dropped MUs to the SCAs via auctioning. Approach (2); the MBS can firstly allow the SCAs to bid for the GUs they can serve, while guaranteeing HUs their QoS, and then later admits the remaining MUs. Note that the auction is mainly used to resolve the conflict that arises, when there is an overlap in the preference set of one or more SCAs. Generally, in the first approach, the MBS greedily wants to serve as many MUs as possible, while in the second approach, the MBS is interested in offloading as many MUs as possible. The proposed BWA algorithm is supplemented with the admission control algorithm proposed in [148–150] to form the forward BWA (FBWA) and backward BWA (BBWA) algorithms. The FBWA algorithm addresses the first approach, and the BBWA algorithm addresses the second approach.

User admission problem is a separate problem on its own, with a plethora of algorithms proposed in the literature. The recently proposed user admission and beamforming algorithm in [148–150] is adopted; hence, the focus of this work is entirely on surplus maximisation problem. Nevertheless, the admission problem is used by the bidders to complete their valuation functions. Each SCA has private valuation information, the complexity of which increases exponentially with the number of users. The question that needs to be addressed is, how to share enough information with the auctioneer so that satisfactory allocation can be made. The strategic behaviour of the SCAs also needs to be analysed. All these issues are incorporated within the BWA algorithm design. Figure 5.1 projects Figure 1.6 into an economy model that can be solved via the proposed BWA. In the BWA model, bidders are allowed to bid for one item at a time.

[3]There is another approach that falls in between which is excluded from this chapter, wherein the MBS acts as both the bidder and the auctioneer. Not all countries allows auctioneers or sellers to participate in the auction as bidders. But often, auctioneers can participate by using their bid as the reserve price. Therefore, this approach can be studied via auctions with reserve prices.

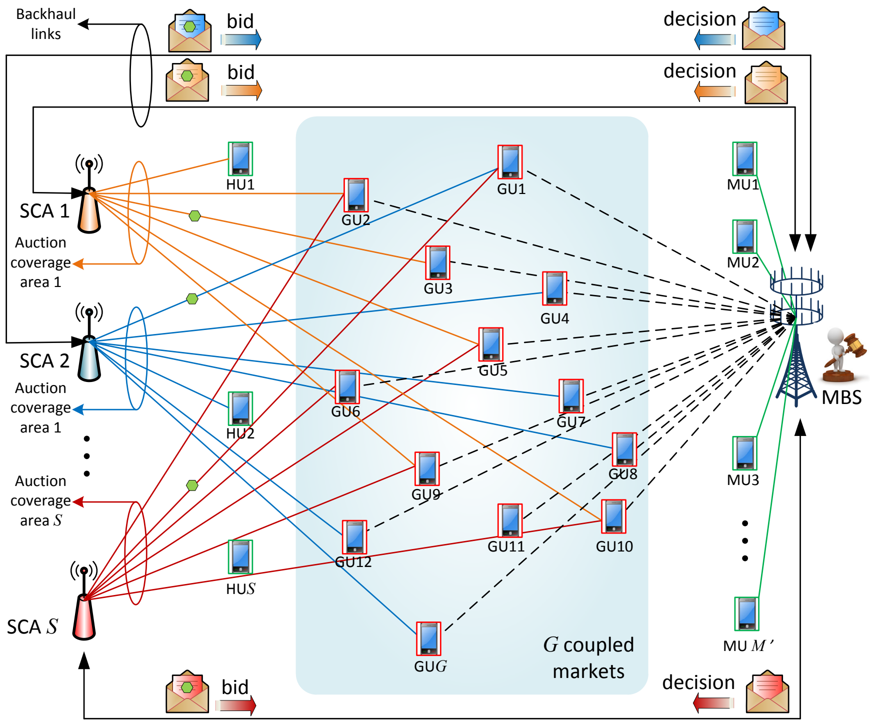


FIGURE 5.1: A competitive market comprising of one MBS, G guest users and S hybrid SCAs. Each SCA's most preferred GU is indicate with a green dot.

5.2.3 System Metric Design

Index the MBS by 0 and the s-th SCA by s. The instantaneous downlink SINR of the m-th MU is given by

$$\mathsf{SINR}_m^0 = \frac{|\mathbf{h}_{0m}^{\mathsf{H}} \mathbf{w}_m|^2}{\sum\limits_{i \in \mathcal{M}_0 \backslash m} |\mathbf{h}_{0m}^{\mathsf{H}} \mathbf{w}_i|^2 + \sigma_m^2}, \tag{5.1}$$

where $\mathbf{h}_{0m} \in \mathbb{C}^{M_{\text{MBS}}}$ is the channel vector from the MBS to the m-th MU, $\mathbf{w}_m \in \mathbb{C}^{M_{\text{MBS}} \times 1}$ is the transmit beamforming vector for the m-th MU, and σ_m^2 is the receiver noise power at the m-th MU. Note that as the MBS and the SCAs use non-overlapping frequency bands, there is no intercell interference. Let the set of HUs and GUs served by the s-th SCA be $\mathcal{H}_s$, each denoted by h. The instantaneous downlink SINR of the h-th HU, served by the s-th SCA is given by

$$\mathsf{SINR}_h^s = \frac{|\mathbf{h}_{sh}^{\mathsf{H}} \mathbf{w}_h|^2}{\sum\limits_{j \in \mathcal{H}_s \backslash h} |\mathbf{h}_{sh}^{\mathsf{H}} \mathbf{w}_j|^2 + \sigma_h^2}, \tag{5.2}$$

where $\mathbf{h}_{sh} \in \mathbb{C}^{M_{\text{SCA}}}$ is the channel vector from the s-th SCA to the h-th HU, $\mathbf{w}_h \in \mathbb{C}^{M_{\text{SCA}}}$ is the transmit beamforming vector for the h-th HU, and σ_h^2 is the receiver noise power at the h-th HU.

5.2.4 User Admission Problem

Now, let the predefined QoS targets of the MUs be defined as $\boldsymbol{\Xi}^0 = [\xi_1^0, \ldots, \xi_{M_0}^0]$. A set of admitted users is denoted by $\mathcal{M}_0' \subseteq \mathcal{M}_0$, which is a parameter to be maximised. The user admission problem $\mathscr{P}^{0-\text{UM}}$ at the MBS is formulated as

$$\begin{aligned} \mathscr{P}_1^{0-\text{UM}} : \texttt{maximise} \quad & \mathbf{card}(\mathcal{M}_0') \\ \texttt{subject to} \quad & \mathsf{SINR}_m^0 \geq \xi_m^0, \quad m \in \mathcal{M}_0, \\ & \sum_{m \in \mathcal{M}_0} \|\mathbf{w}_m\|_2^2 \leq \varphi_0^{\max}, \end{aligned} \tag{5.3}$$

where $\mathbf{card}(\mathcal{M}_0')$ denotes the cardinality of the set $\mathcal{M}_0' \subseteq \{1, \ldots, M_0\}$. It is assumed all MUs have identical QoS requirements. This latter assumption encourages the SCAs to admit as many GUs as possible as shown later. In this thesis, the capacity of the network is measured in terms of the number of admitted users. This motivates the objective in (5.3) and allows the commodity or the items to be users[4]. The problem in (5.3) is non-convex because the objective function and the QoS constraints are non-convex. The QoS constraints can be rewritten in their equivalent SOC as

$$\mathsf{SINR}_m^0 \geq \xi_m^0 \Longrightarrow \left\| \begin{matrix} \mathbf{h}_{0m}^{\mathsf{H}} \mathbf{w}_1 \\ \vdots \\ \mathbf{h}_{0m}^{\mathsf{H}} \mathbf{w}_{M_0} \\ \sigma_m \end{matrix} \right\| \leq \sqrt{\frac{1 + \xi_m^0}{\xi_m^0}} \Re(\mathbf{h}_{0m}^{\mathsf{H}} \mathbf{w}_m), \tag{5.4}$$

$$\Im(\mathbf{h}_{0m}^{\mathsf{H}} \mathbf{w}_m) = 0, \quad \forall m, \tag{5.5}$$

where the first line is the second order cone (SOC) constraints and the second line is the affine constraints. The operators $\Re(\cdot)$ and $\Im(\cdot)$, extracts the real part and the imaginary part of the argument, respectively. Let the matrix $\mathbf{W}_0 = [\mathbf{w}_m]_{m \in \mathcal{M}_0}$ be defined by concatenating the column vectors $\mathbf{w}_m$ at MBS.

[4]By choosing the objective as the maximum possible number of users admitted deprives the problem at hand from exploring multiuser diversity and channel conditions, bandwidth and frequency diversity. Nonetheless, these parameters will be considered in the future work.

By introducing the slack variables $\mathbf{a}^0 = [a_1^0, \ldots, a_{M_0}^0]$, problem in (5.3) can be rewritten as

$$\begin{aligned}
\mathscr{P}_2^{0-\mathrm{UM}} : \underset{\{\mathbf{w}_m\},\{\mathbf{a}^0\}}{\texttt{minimise}} \quad & \|\mathbf{a}^0\|_0 \\
\texttt{subject to} \quad & \begin{bmatrix} \sqrt{1+\frac{1}{\xi_m^0}}\mathbf{h}_{0m}^{\mathsf{H}}\mathbf{w}_m + a_m^0 \\ \mathbf{h}_{0m}^{\mathsf{H}}\mathbf{W}_0 \\ \sigma \end{bmatrix} \succeq_{\mathrm{SOC}} 0, \qquad m \in \mathcal{M}_0, \\
& \Im(\mathbf{h}_{0m}^{\mathsf{H}}\mathbf{w}_m) = 0, \quad \forall m, \\
& \mathbf{a}^0 \geq \mathbf{0}, \quad \forall m, \\
& \sum_{m\in\mathcal{M}_0} \|\mathbf{w}_m\|_2^2 \leq \varphi_0^{\max}, \quad \forall m,
\end{aligned} \tag{5.6}$$

where $\|\mathbf{a}^0\|_0$ denotes the ℓ_0-norm of $\mathbf{a}^0$, which is the number of nonzero elements in a^0. The value of each a_m^0 indicates the feasibility gap for the corresponding user, and therefore, it is the indication of how much a user is preferred by the transmitter. From the SINR constraints in (5.6), when the channel of the m-th user is good, then the value of a_m^0 will be as close to zero as possible. Contrary, when the m-th user has bad channel, then the value of a_m^0 will be large. The notation $\succeq_{\mathrm{SOC}}$ denotes the generalised inequalities with respect to the SOC [12]. The objective function in (5.6) is not a convex function. By replacing the ℓ_0-norm with its convex hull, *id est*, ℓ_1-norm, a good approximation can be attained [150]. Now the problem in (5.6) is reformulated as

$$\begin{aligned}
\mathscr{P}_3^{0-\mathrm{UM}} : \underset{\{\mathbf{w}_m\},\{\mathbf{a}^0\}}{\texttt{minimise}} \quad & \|\mathbf{a}^0\|_1 \\
\texttt{subject to} \quad & \begin{bmatrix} \sqrt{1+\frac{1}{\xi_m^0}}\mathbf{h}_{0m}^{\mathsf{H}}\mathbf{w}_m + a_m^0 \\ \mathbf{h}_{0m}^{\mathsf{H}}\mathbf{W}_0 \\ \sigma \end{bmatrix} \succeq_{\mathrm{SOC}} 0, \qquad m \in \mathcal{M}_0, \\
& \Im(\mathbf{h}_{0m}^{\mathsf{H}}\mathbf{w}_m) = 0, \quad \forall m, \\
& \mathbf{a}^0 \geq \mathbf{0}, \quad \forall m, \\
& \sum_{m\in\mathcal{M}_0} \|\mathbf{w}_m\|_2^2 \leq \varphi_0^{\max}, \quad \forall m.
\end{aligned} \tag{5.7}$$

Problem (5.7) is a convex problem and can be solved using the convex programming package CVX [151]. In order to obtain an optimal admission set $\mathcal{M}_0'$, as proved in [148, 150], the elements of $\mathbf{a}^0$ are rearranged in ascending order and the MUs are sequentially admitted beginning with the smallest a_m^0 to build up the optimal set $\mathcal{M}_0'$. This is done by checking for feasibility at every admission by solving

$$\begin{aligned}
\mathscr{P}^{0-\mathrm{UA}} : \underset{\{\mathbf{w}_m\}}{\texttt{minimise}} \quad & \sum_{\forall m \in \mathcal{M}_0' \cup m} \|\mathbf{w}_m\|_2^2 \\
\texttt{subject to} \quad & \mathsf{SINR}_m^0 \geq \xi_m^0, \quad \forall m \in \mathcal{M}_0' \cup m, \\
& \sum_{m \in \mathcal{M}_0'} \|\mathbf{w}_m\|_2^2 \leq \varphi_0^{\max}, \quad \forall m,
\end{aligned} \tag{5.8}$$

Once the newly admitted user makes the constraints in (5.8) infeasible, it is removed from the set $\mathcal{M}_0'$. The resulting admission set $\mathcal{M}_0'$ is now optimal. If the MBS is using FBWA, all the dropped MUs by the MBS will be considered as GUs for the SCA. The MBS will then send invitation to all SCAs to participate in a sealed-bid auction, for the GUs. The same process is used under BBWA algorithm, but the set of MUs available for admission at the MBS, are those that were unable to be admitted by the SCAs.

5.2.5 Source of Revenues

Each served user pays the MBS an amount of κ per unit of data rate, and therefore, the revenue generated from the MUs that are being served by the MBS is given by $\kappa \sum_{m \in \mathcal{M}_0'} r_m^0$, where r_m^0 is $\log_2(1 + \xi_m^0)$. It should be noted that using channel capacity to determine the customers' payments could result in overpayments. A more accurate approach is to transform the target SINRs to their corresponding channel quality indicator (CQI) [152, 153] and derive a payment using the CQI. But for simplicity, the former approximation is used in this work. Since the MBS auctions some of its users to the SCAs, for the purpose of valuation of serving users, it is assumed that the SCAs also charge the GUs κ per unit of data rate for the connection. The auctioneer will generate some revenue by collecting payments from the SCAs. Therefore, the total revenue generated by the MBS is given by $\kappa \sum_{m \in \mathcal{M}_0'} r_m^0 + \sum_{s \in S} p_s$, where p_s is the total payment made by s-th SCA to the MBS. However, in reality, the GUs admitted by an SCA will pay their bills to the MBS, which will in turn pay an SCA the difference between the GU's payment and the payment to be paid to the MBS by the SCAs. The main objective of the bidders is to maximise their revenue, while the auctioneer aims to maximise the surplus. The misalignment of the objectives between the auctioneer and the bidders may bring many difficulties in auctioning [50]. In the mechanism design for the BWA in Section 5.4, the objectives of the bidders and the auctioneer are aligned, such that both of them would aim to maximise the number of admitted GUs.

5.3 Surplus Maximisation via BWA

The proposed BWA aims to maximise the surplus of the auctioneer. The design of the BWA is aligned to the VCG mechanism proposed in [53].

Proposition 5.1

The surplus maximisation mechanism is dominant-strategy incentive compatible (DSIC) which optimises social surplus pointwise [53–55].

The BWA is a collection of concurrent sealed-bid single-item auctions, wherein the SCAs have a fixed set of items they can bid on. During the auction, every g-th GU on auction must be assigned to exactly one SCA; therefore, there will be multiple auction rounds in the BWA. Define a set $\mathcal{G}_s \subseteq \mathcal{G}$ to contain all GUs that can be assigned to s-th SCA, and a competitors' set $\mathcal{C}_g$, which contains all SCAs competing to connect the g-th GU. A feasible assignment $\mathcal{A}$, is the set of SCA-GU pairs (sg), with $g \in \mathcal{G}_s$. An SCA can be part of more than one pair $(sg) \in \mathcal{A}$. In the proposed BWA, the MBS plays the rule of the auctioneer, and its objective is to assign the GUs to those SCAs that value them the most in a sequential manner. This will indirectly associate GUs to SCAs and maximise the number of admitted GUs. Therefore, the number of admitted users is treated as the performance metric. For every admitted user, there is a cost incurred in terms of the transmission power. Denote the QoS target of the g-th GU at the s-th SCA as ξ_g^s. The minimum revenue a bidder would like to generate, by admitting a GU, is denoted by $\psi_{sg} \geq 0$. The connection cost incurred by the s-th SCA during r-th auction round, is denoted as c_{sg}^r. The marginal value of the g-th GU by the s-th SCA during the r-th auction round, is given by

$$v_{sg}^r = \kappa \log_2(1 + \xi_g^s) - c_{sg}^r - \psi_{sg}. \tag{5.9}$$

Note that the value v_{sg}^r is conditioned on the admitted users. Without loss of generality, it is assumed $\psi_{sg} = 0$. Let the cost per unit power be denoted as μ. The cost of connecting the g-th GU, during the r-th auction round, is given by

$$c_{sg}^{r} = \mu \left(\sum_{\forall i \in \mathcal{H}_s \cup g} \|\widehat{\mathbf{w}}_i\|^2 - \sum_{\forall k \in \mathcal{H}_s} \|\mathbf{w}_k\|^2 \right), \tag{5.10}$$

where $\widehat{\mathbf{w}}_i$ is the beamformer vector of the i-th HU given that GU g is admitted, and $\mathbf{w}_k$ is the beamformer vector of the k-th HU before the GU g is admitted. The first term in (5.10) is the total transmission power after the connection of the g-th GU, and the last term is the total transmission power before g-th GU is admitted.

The social surplus maximisation problem at the MBS is

$$\begin{aligned} \underset{x_{sg}}{\texttt{maximise}} \quad & \sum_{r=1}^{R} \sum_{s=1}^{S} v_{sg}^{r} x_{sg}^{r} \\ \texttt{subject to} \quad & \sum_{g \in \mathcal{G}'_s} x_{sg}^{r} \leq 1, \quad \forall s \in \mathcal{S}, \\ & \sum_{s \in \mathcal{C}_g} x_{sg}^{r} \leq 1, \quad \forall g \in \mathcal{G}, \forall r, \\ & x_{sg}^{r} \in \{0,1\}, \quad \forall (sg) \in \mathcal{A}', \end{aligned} \tag{5.11}$$

where R is the total number of auction rounds, $\mathcal{A}'$ is the set of all possible SCA-GU assignment pairs $(sg)(\mathcal{A}' \subseteq \mathcal{A})$ and $(x_{sg}^{r})_{g \in \mathcal{G}'_s}$ are binary decision variables, indicating association of SCAs. $x_{sg}^{r} = 1$ means SCA s is assigned to GU g and otherwise $x_{sg}^{r} = 0$. The second and third constraints ensure that each SCA can be assigned to one or more GUs, and each GU can be assigned to only one SCA.

The problem in (5.11) could be viewed as the multi-assignment problem. Multi-assignment problems are usually solved using the combination of the forward and reverse auctions as in [154]. Unlike the work in [154], the problem in discussion can take any asymmetry form, whereby the number of GUs could be more or equal to the number of SCAs, or vice versa. Another approach used in [20, 155, 156] is to relax the last constraint, by allowing it to take any real value in the interval $[0, 1]$. A rounding approach is then used to approximate the values of x_{sg}^{r}. In this work, (5.11) is solved by running simultaneous sealed-bid single-item auctions, wherein at each auction round, each bidder's action is a bid b_{sg}^{r} on the most preferred GU. If a bidder wins, he pays a price p_{sg}^{r} to the auctioneer (MBS), which is the second highest bid from the set $\mathcal{C}_g$. The bidders utility model at r-th auction round, on the bid/action profile $\mathbf{b}^{r} = [b_{1g}^{r}, \ldots, b_{S\overline{g}}^{r}]$ is a quasilinear utility model, define as

$$u^r_{sg}(\mathbf{b}^r) = v^r_{sg}(\mathbf{b}^r)x^r_{sg}(\mathbf{b}^r) - p^r_{sg}(\mathbf{b}^r), \tag{5.12}$$

where the subscripts g and $\overline{g}$ could refer to the same or different GUs.

Since BWA iteratively runs concurrent sealed-bid auctions, it is a requirement to define the *allocation rule* $\mathbf{x}^r$ and the *payment rule* $\mathbf{p}^r$ for every r-th auction round. The allocation rule declares the winner, and the payment rule determines the amount to be paid, as described in detail in Section 5.4. The overall objective of the SCA is to maximise

$$\sum_{r=1}^{R} u^r_{sg}(\mathbf{b}^r). \tag{5.13}$$

By assuming positive utility at each auction round, the utility in (5.13) is maximised by admitting as many GUs as possible. This is because all the MUs/GUs are assumed to have identical QoS targets[5]. The preference profile of each SCA tells the bidder the most preferred GUs at each auction round. In Section 5.4, it is shown that a bidder will also maximise his revenue by bidding according to the preference profile. For the purpose of comparison, two different preference profile criteria are investigated as follows:

5.3.1 Fixed Preference Profile (FPP) Criterion

In this criterion, it is assumed that bidders determine their preference profile once, at the beginning of the auction, and fix it for the entire auction. This approach could lead to an optimal solution if each GUs cause the same amount of interference to each other. Unfortunately, it is almost improbable to encounter that kind of environment in practical wireless communication systems. In the problem at hand, the locations and the CSI of the GUs, dictate how admitting GUs will affect the HUs. Therefore, it is anticipated that the preference profile will dynamically change as users are being admitted.

The fixed preference profile (FPP) is computed as follows: Each SCA identifies the GUs it can possibly accommodate, which is a set of all the GUs that fall within its auction coverage area. This is followed by determining the FPP, by solving the admission

[5] An offloading problem with user having differential QoS will be considered as future work. Considering differential QoS for the GUs will require vigorous analyses on the behaviour of bidders.

problem. Define the QoS targets of the HUs and GUs as $\boldsymbol{\Xi}^s = [\xi_1^s, \ldots, \xi_{F_s}^s]$. The auxiliary variables $\mathbf{a}^s = [a_1^s, \ldots, a_{F_s}^s]$ are introduced. The same procedures for deriving (5.3)-(5.7) are used to form an ℓ_1-norm admission problem for an SCA as

$$
\begin{aligned}
\mathscr{P}^{s-\mathrm{UM}} : \underset{\{\mathbf{w}_j\},\{\mathbf{a}^s\}}{\mathtt{minimise}} \quad & \|\mathbf{a}^s\|_1 \\
\mathtt{subject\ to} \quad & \begin{bmatrix} \sqrt{1+\frac{1}{\xi_j^s}}\mathbf{h}_{sj}^{\mathsf{H}}\mathbf{w}_j + a_j^s \\ \mathbf{h}_{sj}^{\mathsf{H}}\mathbf{W}_s \\ \sigma \end{bmatrix} \succeq_{\mathrm{SOC}} 0, \quad j \in \mathcal{F}_s, \\
& \Im(\mathbf{h}_{sj}^{\mathsf{H}}\mathbf{w}_j) = 0, \quad \forall j, \\
& \mathbf{a}^s = \mathbf{0}, \quad j = 1, \ldots, H_s, \\
& \mathbf{a}^s \geq \mathbf{0}, \quad j = H_s+1, \ldots, F_s, \\
& \sum_{j \in \mathcal{F}_s} \|\mathbf{w}_j\|_2^2 \leq \varphi_s^{\max}, \quad \forall j,
\end{aligned}
\tag{5.14}
$$

where the third constrained ensures that the HUs are given first priority. To build up a preference set of GUs $\mathcal{G}_s' \subseteq \mathcal{F}_s$, the vector $\mathbf{a}^s$ is sorted in ascending order. The corresponding indices of the sorted $\mathbf{a}^s$ with an exclusion of the index of HUs give the FPP $\mathbf{f}_s$. It should be noted that at this stage, no valuation profile that corresponds to $\mathbf{f}_s$ is determined. It is unnecessary to value the preferred users at this stage, as there is no guarantee that all GUs in the preference set will be won. The valuations are calculated on the "need-to-know" basis. At every auction round, a bidder will use (5.9) to place a value on the most preferred GU.

5.3.2 Adaptive Preference Profile (APP) Criterion

As highlighted earlier, it is anticipated that the level of preference over GUs will be reduced if an admission of a particular GU is already made due to the substitute nature of the GUs. Therefore, the preference profiles need to be revised every time a new GU is admitted. The valuations for every GU $g \in \mathcal{G}_s$ is performed separately, and then sorted in descending order to determine the current preference profile. A bid is then placed on the GUs with the highest valuation. Let the QoS targets of the HUs (this includes all the admitted GUs) and g-th GU be defined as $\boldsymbol{\Xi}^s = [\xi_1^s, \ldots, \xi_{H_s}^s, \xi_g^s]$. Also, let the set $\mathcal{H}_s$ be a set of HUs and admitted GUs. For every available GU $g \in \mathcal{G}_s$, each of the SCA determines the connection cost by solving the following feasibility problem:

$$\begin{aligned} \underset{\{\mathbf{w}_k\}}{\texttt{minimise}} \quad & \sum_{\forall k \in \mathcal{H}_s \cup g} \|\mathbf{w}_k\|_2^2 \\ \texttt{subject to} \quad & \mathsf{SINR}_k^s \geq \xi_k^s, \quad \forall k \in \mathcal{H}_s \cup g, \\ & \sum_{\forall k \in \mathcal{H}_s \cup g} \|\mathbf{w}_k\|_2^2 \leq \varphi_s^{\max}. \end{aligned} \tag{5.15}$$

Unlike the user maximisation problem (5.14) which does not necessarily give optimal beamformers, problem in (5.15) will always give an optimal beamformer. Therefore, the valuation can be determined straight away, by finding the difference in transmission power before and after admission, using (5.10). With the exception of the first auction round, note that for every auction round, losers (*id est*, bidders who lost the items they bid on) from the previous round do not need to revise their preference profiles. The bidders on WAIT (*id est*, bidders are on WAIT if the decision on their bid is withheld) do nothing, while the winners are required to revise their preference profiles and submit new bids. The losers from the previous round only need to submit the bid on the next most preferred, and available GUs, since the valuation has already been determined. Contrast should be made that when FPP is utilised, all contacted bidders need to calculate the new valuations for admitting the next most preferred and available GU.

In the case whereby the preference profile needs to be updated at the SCA, all the valuations of the available GUs need to be calculated. Though this could seem costly, it offers the SCAs with the capability to identify and prune away all the GUs that will never be feasible for admission. This is not applicable when FPP is used, wherein for every SCAs in the contact set $\mathcal{T}^r$ (*id est*, a set of SCAs that are eligible to submit new bids during auction round r), only the value of the next preferred, and available GU is determined. This will continue until admitting the next preferred and available GU becomes infeasible. In both the FPP and APP criteria, once admitting any GU becomes infeasible, the SCA will drop out from the auction.

Assuming all bidders bid according to their preference profiles, then the bids from each SCA are expected to be monotonically decreasing. Another common and crucial characteristic between the FPP and APP criteria is that, bidding on a subset of users could be allowed once there is no intersection between preference profiles. Since the MBS

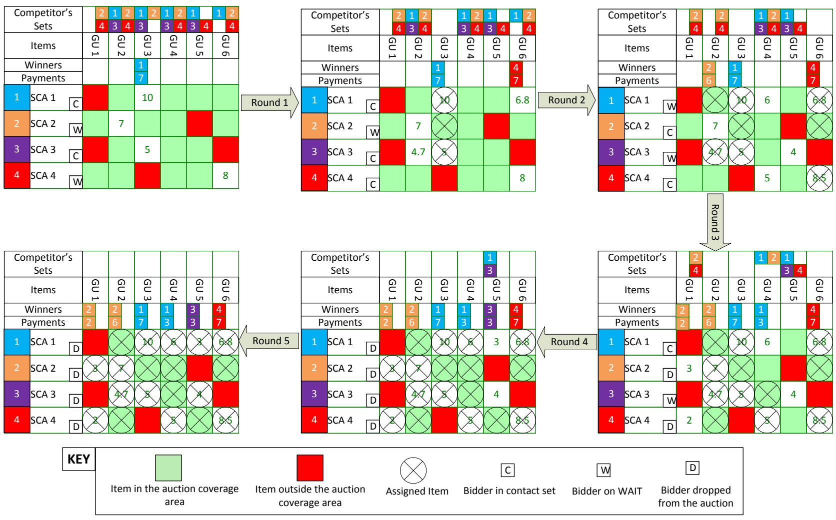


FIGURE 5.2: An example of the steps performed by the BWA.

has access to the preference sets of all SCAs, it can monitor the intersections between these sets. For any preference profile that does not conflict with others, the MBS will permit the corresponding SCA to submit bundle bids on the largest possible set of the GUs[6]. This functionality allows parallel sub-auctions, while the main auction progresses, and it will potentially increase the rate of convergence. If any SCA has knowledge that some of the GUs are not considered by other SCAs, there is a potential for unfaithful bidding. However, as it is difficult for any SCA to acquire preference profiles of other SCAs; therefore, this work excludes this possibility from the analysis.

5.4 Mechanism Design for the BWA Algorithm

The proposed BWA algorithm is utilised to solve the surplus maximisation problem in (5.11). In order to reduce the amount of information shared between the auctioneer and the bidder, the BWA uses iterative indirect mechanism, to gather useful information from bidders. As it is assumed that the MBS has knowledge of the locations of all the bidders and the GUs, it can formulate the preference sets of all the SCAs. With this knowledge at the MBS, and preference profiles at the SCAs, the auctioneer sets a rule such that, each bidder should submit one bid at a time. The bids should be monotonically decreasing in each auction round. Furthermore, it is required that only

[6]It is important to note that, once the preference set of a particular bidder is decoupled from others, it is unnecessary for that bidder to have fear on competition. Therefore, truthful bidding will be dominant strategy for that bidder.

one bid should be submitted per GU during the entire BWA. Even though the BWA uses some of the principles from the VCG mechanism, it is emphasised that the two are totally different. The following example, with the aid of Figure 5.2, is used to highlight the differences:

Example 5.1 (*A Sample of a BWA*)

Consider a bid-wait-auction (BWA) with six GUs. Assume there are four active bidders SCA1, SCA2, SCA3 and SCA4 with preference sets {GU2, GU3, GU4, GU5, GU6}, {GU1, GU2, GU3, GU4, GU6}, {GU2, GU3, GU4, GU5}, and {GU1, GU2, GU4, GU5}, respectively. The BWA will iterates as shown in Figure 5.2. Note that unlike the VCG, which charges the winner the second highest bid on the item won, the BWA charges the winner the second highest price from the competitor's set. The set $\mathcal{C}_{GU3} := \{SCA1, SCA2, SCA3\}$ is the competitors set for GU3. Therefore, in the first auction round, the BWA allocates GU1 to SCA1, and charges it 7 from bidder SCA2. SCA2 and SCA4 are then put on WAIT, while SCA1 and SCA3 are put on the contact set $\mathcal{T}^2$, making them the only two bidders who are allowed to submit new bids in the second round. This same process is repeated until the $\mathcal{T}^r$ is empty. The BBWA and the FBWA algorithms summarised in Algorithms 5 and 6, utilise this BWA in their main loops.

5.4.1 Existence of Equilibrium in the BWA

During the r-th auction round, the valuation of the s-th bidder on the g-th GU is denoted by v^r_{sg}, and a collection of all bidders' valuations are denoted as $\mathbf{v}^r = [v^r_{1g}, \ldots, v^r_{S\bar{g}}]$, where g and $\bar{g}$ are the identity of the GUs. The GUs g and $\bar{g}$ do not need to be different. The strategy of each bidder is denoted as s^r_{sg}. Define a collection of all bidders' strategies and actions at the r-th auction round as $\mathbf{s}^r = [s^r_{1g}, \ldots, s^r_{S\bar{g}}]$ and $\mathbf{b}^r = [b^r_{1g}, \ldots, b^r_{S\bar{g}}]$, respectively. Given R as the maximum auction rounds required for the market to clear, the s-th bidder valuations, strategies, and actions for the entire BWA are denoted as $\mathbf{v}_s = [v^1_{s1}, \ldots, v^R_{sG_s}]$, $\mathbf{s}_s = [s^1_{s1}, \ldots, s^R_{sG_s}]$, and $\mathbf{b}_s = [b^1_{s1}, \ldots, b^R_{sG_s}]$, respectively. The entire BWA has the valuation, strategy, and action spaces denoted $\mathbf{V} = [\mathbf{v}_1, \ldots, \mathbf{v}_S]$, $\mathbf{B} = [\mathbf{b}_1, \ldots, \mathbf{b}_S]$, and $\mathbf{S} = [\mathbf{s}_1, \ldots, \mathbf{s}_S]$, respectively.

Definition 5.1 (*Dominant Strategy Equilibrium*)

A sub-auction dominant strategy equilibrium (sDSE) at every auction round, is a *strategy profile* $\mathbf{s}^r$ *such that for all* s, v^r_{sg}, *and* $\mathbf{b}^r_{-s}$, *the utility of bidder* s *is maximised by following strategy* $s^r_{sg}(v^r_{sg})$ *[157]. Extending the definition to the entire BWA, a DSE is a strategy space* $\mathbf{S}$ *such that for all* s, $\mathbf{v}_s$, *and* $\mathbf{B}_{-s}$, *the utility of bidder* s *is maximised by following strategy* $\mathbf{s}_s(\mathbf{v}_s)$.

Now, the task is to develop a dominant-strategy incentive compatible (DSIC) mechanism for the BWA, and prove that the BWA has a unique sDSE at each auction round and a unique DSE for the entire BWA. Recall that the BWA is a collection of concurrent sealed-bid single-item auctions. The *single-parameter environment* [50], which treats single-item auction as a special case, can be used for BWA mechanism design. The outcome of such mechanism is the allocation and payment vectors $\mathbf{x}^r = [x^r_{1,g}, \ldots, x^r_{S,\overline{g}}]$ and $\mathbf{p}^r = [p^r_{1,g}, \ldots, p^r_{S,\overline{g}}]$. The mechanism and the profile of equilibrium strategies induce *ex post allocation* and *payment rules*, that map the valuation profile $\mathbf{v}^r$ to the allocation $\mathbf{x}^r(\mathbf{v}^r)$ and payments $\mathbf{p}^r(\mathbf{v}^r)$ [157].

5.4.2 Allocation Rule

If the bids from a particular bidder are not monotonically decreasing, its current bid will not be accepted and the bidder is dropped from the auction. This ensures that all SCAs will bid on the GUs, in the sequence defined in their preference profiles. Every bid submitted should be on the most currently preferred GU. At every auction round, the bidders aim to maximise their quasilinear utilities defined in (5.12), while at a higher level, the aim of bidders is to maximise the number of GUs admitted, as shown in (5.13). In every auction round, the BWA allocates the GU to the bidder with the highest bid, if the feasible assignment set $\mathcal{A}$ has the minimum required information. Hence forth, multiple assignments can be made in a single auction round if bidders bid for different GUs. The BWA takes advantage of the main objective of the auctioneer, who is not primarily interested in revenue maximisation, but surplus maximisation, by using the information from the sparse feasible assignment set $\mathcal{A}$, to allocate GUs.

Proposition 5.2

Assume the auctioneer has the preference sets of all bidders $\mathcal{G}_s, \forall s \in \mathcal{S}$. *Suppose bidders* j *and* k *are the only bidders who are eligible to bid on item* m. *If during the* r*-th auction round, item* m *is bidder* j*'s first preference with a bid of* b^r_{jm}, *and the current bid from* k*'s bidder is* b^r_{kp} *on item* p *(id est, item* p *has more preference than* *item m from bidder k's perspective), then the following conditions exist:*

1. *If $b^r_{jm} > b^r_{kp}$, it suggests that $b^r_{jm} > b^r_{km}$, concluding that bidder k stands no chance in winning item m. The item m is then assigned to bidder j. Under this condition, the auctioneer has complete bid information on item m. The bid b^r_{kp} is henceforth referred to as bidder j's critical bid. Critical bids can only come from bidders in the set $\mathcal{C}_m$.*

2. *If $b^r_{jm} < b^r_{kp}$, then bidder k still stands a chance to win item m. Therefore bidder j will have to WAIT (hence the term BID-WAIT), until the auctioneer has the right information to announce the winner between bidders j and k. Under this condition, the auctioneer has incomplete bid information on item m. The bids b^r_{jm} and b^r_{kp} are henceforth referred to as bidder j's wait bid and potential bid, respectively.*

Proof 3 Since the auctioneer has access to the preference sets and uses the one bid at a time rule, and by assuming truthful bidding, the preference profiles at the SCAs dictates that the bids submitted should be monotonically decreasing at each auction round. Therefore, the next bid on the next available preferred item is always less or equal to the current submitted bid. ■

Regardless of the sparseness of the collected bids at each auction round, the auctioneer will have complete bid information on at least one GU. This means at every auction round, there will be a winner. For every auction round, the BWA utilises an allocation rule $\mathbf{x}^r$, that allocates the GU to the bidder with the highest bid [50,53,157], defined as

$$\mathbf{x}^r(\mathbf{b}^r) = \underset{\mathcal{A}}{\texttt{argmax}} \sum_{s \in \mathcal{S}} b^r_{sg} x^r_{sg}. \tag{5.16}$$

5.4.3 Payment Rule

The second-price payment rule used in the VCG mechanism which charges the winner the second highest bid is invoked. The BWA extends the second-price rule, by charging the winner the second highest bid from the bidder in competitors' set $\mathcal{C}_g$, *id est*, the

Algorithm 5: Forward Bid-Wait Auction (FBWA) Algorithm

Data: Initialisation: Guest user set $\mathcal{G} := \emptyset$, assignment set $\mathcal{A} := \emptyset$, *auction round*: $i = 0$.
Result: Optimal Allocation set $\mathcal{A}' \subseteq \mathcal{A}$.
<u>MBS-MU admission</u>
1 Solve (5.3) and (5.7) to get $\mathcal{R}'_0$ and $\mathcal{G}$.
2 $\mathcal{T}^1 :=$ {all eligible SCAs}.
<u>SCA-GU admission: BWA</u>
3 **while** $\mathcal{T}^r \neq \emptyset$ **do**
4     $r = r + 1$
5   Auctioneer contacts bidders in $\mathcal{T}^r$
6   Active bidders submit their bids b^r_{sg} on most preferred GU(s).
7   **if** $b^r_{sg} = \emptyset$ *or* $b^r_{sg} > b^{r-1}_{sg}$ **then**
8   Bidder is dropped from auction.
9   Auctioneer declares winners on items with complete bid information.
10   **if** *item has incomplete bid information* **then**
11   Current best bidder WAITS.
12   Auctioneer determines $mathcalT^{r+1}$.

Algorithm 6: Backward Bid-Wait Auction (BBWA) Algorithm

Data: Initialisation: $\mathcal{T}^1 :=$ {all eligible SCAs}, $\mathcal{G} := \mathcal{M}_0$, $\mathcal{A} := \emptyset$, *auction round*: $r = 0$.
Result: Optimal Allocation set $\mathcal{A}' \subseteq \mathcal{A}$.
<u>SCA-GU admission: BWA</u>
1 Perform step 3-12 of Algorithm 5.
2 Set $\mathcal{M}_0 = \mathcal{M}_0 \setminus \mathcal{G}'$.
<u>MBS-MU admission</u>
3 Solve to (5.3) and (5.7) to get $\mathcal{M}'_0$.

critical bid. It is very important to note that the critical bid need not to be the second highest bid on a particular item, as elaborated in Example 5.1 and *Proposition* 5.2, condition 1. As explained in Section 2.5.2, using the Clarke pivot rule [59, 60] to charge the winning bidder its externalities is the only way to enforce truth-telling. In *Proposition* 5.2 condition 1, the payment to the auctioneer by bidder j would be b^k_{kp}.

Theorem 5.1

Truthful bidding is a weak dominant strategy in the bid-wait-auction.

Proof 4 Consider an arbitrary bidder s, its valuation at r-th auction round on the g-th GU is v^r_{sg} and $\mathbf{b}^r_{-s}$ are the bids of other bidders. The bids $\mathbf{b}^r_{-s}$ do not necessarily have to be placed on the g-th GU. The valuation v^r_{sg} is an immutable valuation for bidder s on the g-th GU. Let $B = \mathtt{max}_{t \neq s} v^r_{t\bar{g}}$ denote the highest bid by some other potential bidder of g-th GU (*id est*, g-th GU belongs to the preference set of bidder t). The bid B

$$u_s = \begin{cases} 0, & \text{if } b^r_{s\bar{g}} < B^{r+1}, & (5.17a) \\ (v^r_{s\bar{g}} - B^r) + 0, & \text{if } B^{r+1} \leq b^r_{s\bar{g}} < B^r, & (5.17b) \\ (v^r_{s\bar{g}} - B^r) + 0, & \text{if } b^r_{s\bar{g}} \geq B^r, b^{r+1}_{sg} < B^r, & (5.17c) \\ (v^r_{s\bar{g}} - B^r) + (v^r_{sg} - \epsilon^{r+1}_{sg|\bar{g}} - B^r), & \text{if } b^r_{s\bar{g}} \geq B^r, b^{r+1}_{sg} \geq B^r, & (5.17d) \end{cases}$$

$$\texttt{max}\{0, u_s\} = \begin{cases} 0, & \text{if } v^r_{sg} < B^{r+1}, \\ & \quad v^r_{s\bar{g}} < B^{r+1}, & (5.18a) \\ 0, & \text{if } B^{r+1} \leq v^r_{sg} < B^r, \\ & \quad v^r_{s\bar{g}} < B^{r+1}, & (5.18b) \\ (v^r_{s\bar{g}} - B^{r+1}), & \text{if } B^{r+1} \leq v^r_{sg} < B^r, & (5.18c) \\ & \quad B^{r+1} \leq v^r_{s\bar{g}} < B^r, \\ (v^r_{sg} - B^r) + (v^r_{s\bar{g}} - \epsilon^{r+1}_{s\bar{g}|g} - B^{r+1}), & \text{if } v^r_{sg} \geq B^r, \\ & \quad B^{r+1} \leq v^{r+1}_{s\bar{g}} < B^r, & (5.18d) \\ (v^r_{sg} - B^r) + (v^r_{s\bar{g}} - \epsilon^{r+1}_{s\bar{g}|g} - B^{r+1}), & \text{if } v^r_{sg} \geq B^r, v^{r+1}_{s\bar{g}} \geq B^r. & (5.18e) \end{cases}$$

---

is the *critical bid* of bidder s (*Proposition*, 5.2 condition 1). The GU $\bar{g}$ could be the g-th GU or any other GU. Now, given the bid B, there are only two distinct outcomes for bidder s. If bidder s bids $b^r_{sg} < B$, he loses and receives utility $u^r_{sg} = 0$. But if he bids $b^r_{sg} \geq B$, and by assuming that the ties are broken in favour of bidder s, then he wins and receives utility $u^r_{sg} = v^r_{sg} - B$. In the BWA, the ties are broken by random choice. Now the following cases exist. If $v^r_{sg} < B$, maximum utility that bidders s will obtain is $\texttt{max}\{0, v^r_{sg} - B\} = 0$. On the other hand, if $v^r_{sg} \geq B$, maximum utility that bidders s will obtain is $\texttt{max}\{0, v^r_{sg} - B\} = v^r_{sg} - B$, which occurs by bidding truthfully and winning. ■

Note that, by using the bidder's critical bid as the payment in the BWA, tends to have the same advantages over the second-price auction as the n-th random price auction proposed in [158]. One of the deficiencies of the second-price auction is that, bidders whose valuations are far below or above the market-clearing price, may bid untruthfully and remain unnoticed. This could be a setback, especially when the auctioneer tries to learn the bidding behaviour of the bidders in order to determine the entire demand curve of the auction. Contrary to this, both the proposed BWA and the n-th random price auction proposed in [158], are able to engage all bidders to bid truthfully. The n-th random price auction randomly picks a bid other than the highest bid and set it as a price for the goods. This filters all the users whose bids are below the set price, and will potentially punish those bidders who bid insincerely. In the proposed BWA, the assumption that the bidders will bid on the most preferred GUs, and the sparseness of the collected bids, greatly reduces the margin between the market-clearing price and the valuations of the bidders. The second-price auction is ultimately a special case of the BWA, which occurs when all the possible bids on a particular GU are available and both the winning bid and the critical bid are on that GU.

Theorem 5.2

Bidding on the most preferred GU is a dominant strategy in the bid-wait-auction.

Proof 5 Without loss of generality, consider two items with identities g and $\overline{g}$. Fix an arbitrary bidder s with the preference profile $\mathbf{f}_s = [g, \overline{g}]$ at the r-th auction round. Set its valuations profile as $\mathbf{v}_s = [v^r_{sg},\, v^r_{s\overline{g}}]$ where $v^r_{sg} > v^r_{s\overline{g}}$, and denote the bids from other bidders as $\mathbf{b}^r_{-s}$, $\mathbf{b}^{r+1}_{-s}$ during auction rounds r and $r+1$ respectively. Again without loss of generality, assume that all other bidders have the same preference profiles as bidder s at the r-th auction round. Let $B^r = \texttt{max}_{t\neq s} v^r_{tg}$ and $B^{r+1} = \texttt{max}_{z\neq s} v^{r+1}_{z\overline{g}}$ denote the critical bids for bidder s during auction rounds r and $r+1$, respectively. The critical bids B^r and B^{r+1}, should satisfy $B^r > B^{r+1}$. If during r auction round, bidder s bids $b^r_{s\overline{g}}$ on GU $\overline{g}$, its potential utility is $u_s = u^r_{s\overline{g}} + u^{r+1}_{sg}$. In this case, only three distinct outcomes as described in (5.17) exists. In (5.17d), $\epsilon^{r+1}_{sg|\overline{g}} > 0$ implies a decrease in valuation on GU g during auction round $r+1$, given that GU $\overline{g}$ is already admitted. In (5.17a), bidder s is put on WAIT during auction round r and he loses GU g. In the auction round $r+1$, he also loses GU $\overline{g}$. In (5.17b), bidder s is put on WAIT during auction round r and he loses GU g, but during auction round $r+1$, he wins GU $\overline{g}$. In (5.17c), bidder s wins GU $\overline{g}$ during auction round r and other bidders are put on WAIT. In the auction round $r+1$, only bidder s is allowed to submit a new bid $b^{r+1}_{sg} < b^r_{s\overline{g}}$. Still under (5.17c), if the new bid $b^{r+1}_{sg} < B^r$, then he loses GU g. In (5.17d), bidder s wins both the GUs.

On contrary, suppose bidder s places his order of preference truthfully by bidding on item g in the r-th auction round, and $\overline{g}$ in the $(r+1)$-th auction round. The potential utility the bidder s will obtain is given in (5.18), where $\epsilon^{r+1}_{s\overline{g}|g} > 0$ implies a decrease in valuation on GU $\overline{g}$ during auction round $r+1$, given that GU g is already admitted. By comparing the overall utilities in (5.17b)-(5.17c) with (5.18c), yields $(v^r_{s\overline{g}} - B^r) < (v^r_{sg} - B^{r+1})$. Similarly, by comparing the overall utilities in (5.17d) with (5.18d)-(5.18e), gives $(v^r_{s\overline{g}} - B^r) + (v^r_{sg} - \epsilon^{r+1}_{sg|\overline{g}} - B^r) < (v^r_{sg} - B^r) + (v^r_{s\overline{g}} - \epsilon^{r+1}_{s\overline{g}|g} - B^{r+1})$. It should be further noted that, not all the GUs in the preference profile of a bidder will always be feasible to admit. Therefore, bidding on a less preferred GU and winning could make it infeasible to admit the most preferred GU. This concludes that, bidder s can get the highest utility, only by being truthful in both the valuation and the order of preference. ■

5.4.4 Uniqueness of the sDSE and the DSE

Both the sDSE and DSE require that, for all s, $\mathbf{v}^r$, and $b^r_{s,g}$, bidder s has a high utility for playing strategy at $v^r_{s,g}$ than following any other strategy at $b^r_{s,g}$, *id est*,

$$v^r_{sg} \cdot x^r_{sg}(\mathbf{v}^r) - p^r_{sg}(\mathbf{v}^r) \geq v^r_{sg} \cdot x^r_{sg}(b^r_{sg}, \mathbf{v}^r_{-s}) - p^r_{sg}(b^r_{sg}, \mathbf{v}^r_{-s}). \tag{5.19}$$

In every auction round, the proposed BWA allocates at least one GU to a bidder with the highest bid, and charges the winner its *critical bid*. A unique payment rule that will guard against insincere bidding, such that the allocation rule $\mathbf{x}^r$ is implementable, is derived below.

Definition 5.2

An implementable allocation rule is a function $\mathbf{x}^r$ which when coupled with payment rule $\mathbf{p}^r$ is such that $(\mathbf{x}^r, \mathbf{p}^r)$ is DSIC. An allocation rule is monotone if for every bidder s, and for fixed bids $\mathbf{b}^r_{-s}$ of other bidders, the allocation $x^r_{sg}(b^r_{sg}, \mathbf{b}^r_{-s})$ to s is increasing in its bid b^r_{sg} [50].

The BWA payment rule should satisfy

$$p^r_{sg}(\mathbf{b}^r) \in [0, b^r_{sg} \cdot x_{sg}(\mathbf{b}^r)], \forall s \in \mathcal{S}, \tag{5.20}$$

where the lower bound ensures that no payment should be made by the auctioneer to the bidders. The upper bound guarantees a bidder that for as long as he bids truthfully, he will have non-negative utility. For completeness, Myerson's Lemma in [159] is invoked to derive a unique BWA payment rule.

Theorem 5.3 (*Myerson's Lemma [159]*)

In a single-parameter environment, where the agents have independent utility and quasilinear utility functions, a profile of allocation and payment rules $(\mathbf{x}^r, \mathbf{p}^r)$ are in sDSE and implementable only if, for all $s \in \mathcal{S}$;

$$\hat{b}^r_{sg} \cdot x_{sg}(\hat{b}^r_{sg}, \mathbf{b}^r_{-s}) - p^r_{sg}(\hat{b}^r_{sg}, \mathbf{b}^r_{-s}) \geq \hat{b}^r_{sg} \cdot x_{sg}(\check{b}^r_{sg}, \mathbf{b}^r_{-s}) - p^r_{sg}(\check{b}^r_{sg}, \mathbf{b}^r_{-s}). \tag{5.20}$$

$$\check{b}^r_{sg} \cdot x_{sg}(\check{b}^r_{sg}, \mathbf{b}^r_{-s}) - p^r_{sg}(\check{b}^r_{sg}, \mathbf{b}^r_{-s}) \geq \check{b}^r_{sg} \cdot x_{sg}(\hat{b}^r_{sg}, \mathbf{b}^r_{-s}) - p^r_{sg}(\hat{b}^r_{sg}, \mathbf{b}^r_{-s}). \tag{5.21}$$

$$\begin{aligned} \hat{b}^r_{sg}\left(x_{sg}(\check{b}^r_{sg}, \mathbf{b}^r_{-s}) - x_{sg}(\hat{b}^r_{sg}, \mathbf{b}^r_{-s})\right) &\leq \left(p^r_{sg}(\check{b}^r_{sg}, \mathbf{b}^r_{-s}) - p^r_{sg}(\hat{b}^r_{sg}, \mathbf{b}^r_{-s})\right) \\ &\leq \check{b}^r_{sg}\left(x_{sg}(\check{b}^r_{sg}, \mathbf{b}^r_{-s}) - x_{sg}(\hat{b}^r_{sg}, \mathbf{b}^r_{-s})\right) \end{aligned} \tag{5.22}$$

$$\begin{aligned} \lim_{\check{b}^r_{sg} \to \hat{b}^r_{sg}} \left[\hat{b}^r_{sg}\left(x_{sg}(\check{b}^r_{sg}, \mathbf{b}^r_{-s}) - x_{sg}(\hat{b}^r_{sg}, \mathbf{b}^r_{-s})\right)\right] &\leq \lim_{\check{b}^r_{sg} \to \hat{b}^r_{sg}} \left[\left(p^r_{sg}(\check{b}^r_{sg}, \mathbf{b}^r_{-s}) - p^r_{sg}(\hat{b}^r_{sg}, \mathbf{b}^r_{-s})\right)\right] \\ &\leq \lim_{\check{b}^r_{sg} \to \hat{b}^r_{sg}} \left[\check{b}^r_{sg}\left(x_{sg}(\check{b}^r_{sg}, \mathbf{b}^r_{-s}) - x_{sg}(\hat{b}^r_{sg}, \mathbf{b}^r_{-s})\right)\right] \\ &= \Delta_{|\hat{b}^r_{sg}} p^r_{sg} = \hat{b}^r_{sg} \cdot \Delta_{|\hat{b}^r_{sg}} x_{sg}(b^r_{sg}, \mathbf{b}^r_{-s}). \end{aligned} \tag{5.23}$$

> 1. $\mathbf{x}^r$ *is is monotone and non-decreasing, and*
>
> 2. *there is a unique payment rule given as* $p^r_s(b^r_{sg}, \mathbf{v}^r_{-s}) = \int_0^{b^r_{sg}} b^r_{sg} \cdot x'_s(b^r_{sg}, \mathbf{v}^r_{-s}) db^r_{sg}$, *where* $b^r_{sg} = 0$ *implies* $p^r_s(0, \mathbf{v}^r_{-s}) = 0$.

Proof 6 Assume that $(\mathbf{x}^r, \mathbf{p}^r)$ is DSIC. Consider two possible bids $(\check{b}^r_{sg}, \hat{b}^r_{sg})$ from bidder s on item g during r-th auction round such that $0 \leq \check{b}^r_{sg} < \hat{b}^r_{sg}$. Assume that the private valuation of bidder s on its most preferred GU g during the r-th auction round is $\hat{b}^r_{sg}$, but he underbids by submitting $\check{b}^r_{sg}$ instead. Using the DSIC principle in (5.19) yields (5.20). Similarly, if private valuation of the bidder s on GU g at the r-th auction round is $\check{b}^r_{sg}$, but he overbids by submitting $\hat{b}^r_{sg}$ instead, gives (5.21).

The payment difference $(p^r_{sg}(\check{b}^r_{sg}, \mathbf{b}^r_{-s}) - p^r_{sg}(\hat{b}^r_{sg}, \mathbf{b}^r_{-s}))$ from (5.20) and (5.21) is given by the sandwich theorem [160], as shown in (5.22). Noting that $x^r_{sg}(\cdot, \mathbf{b}^r_{-s})$ is a piecewise constant, applying the limit inequality theorem [160] in (5.22) yields the change in payment in (5.23), where $\Delta_{|\hat{b}^r_{sg}}$ is the magnitude of change at $\hat{b}^r_{sg}$. Now, the unique payment formula for each bidder at the r-th auction round is given by

$$p^r_{sg}(b^r_{sg}, \mathbf{b}^r_{-s}) = \sum_{c=1}^{C^r_g} b^r_{cg} \cdot \Delta_{|b^r_{sg}} x^r_{sg}(\cdot, \mathbf{b}^r_{-s}), \tag{5.24}$$

where b^r_{cg} is the c-th breakpoint of the allocation function $x^r_{sg}(\cdot, \mathbf{b}_{-s})$ in the range $[0, b^r_{sg}]$ during r-th auction round, and $C^r_g = |\mathcal{C}^r_g \subseteq \mathcal{C}_g| \leq C_g = |\mathcal{C}_g|$ is the maximum number of active bidders in the competitive set of the g-th GU. Note that the breakpoint occurs at the critical bid of bidder s. Since the critical bid can only come from a bidder in the competitive set $\mathcal{C}_g$, if a bidder is invited to submit a bundle bid, he is guaranteed that he will pay nothing for admitting new GUs. Now the overall payment formula for bidder s for the entire BWA is given as

$$p_s(\mathbf{b}_s, \mathbf{B}_{-s}) = \sum_{r=1}^{R} \sum_{c=1}^{C'_g} b^r_{cg} \cdot \Delta_{|b^r_{sg}} x^r_{sg}(\cdot, \mathbf{b}_{-s}), \tag{5.25}$$

The total revenue generated from the BWA is given by $\sum_{s \in \mathcal{S}} p_s(\mathbf{b}_s, \mathbf{B}_{-s})$. Since the allocation function $x^r_{sg}(\cdot, \mathbf{b}_{-s})$ is a bounded monotone function, it is continuous and differentiable. Assume that $\check{b}^r_{sg} = \hat{b}^r_{sg} + d\hat{b}^r_{sg}$. Now, dividing (5.22) through by $d\hat{b}^r_{sg}$ and following the same procedure as in (5.23), yields

$$\frac{d}{d\hat{b}^r_{sg}} p(\hat{b}^r_{sg}, \mathbf{b}^r_{-s}) = \hat{b}^r_{sg} \cdot \frac{d}{d\hat{b}^r_{sg}} x(\hat{b}^r_{sg}, \mathbf{b}^r_{-s}). \tag{5.26}$$

The unique payment formula of every bidder during the r-th auction in (5.24) can be re-written as

$$p^r_s(b^r_{sg}, \mathbf{b}^r_{-s}) = \int_0^{b^r_{sg}} b^r_{sg} \cdot \frac{d}{db^r_{sg}} x^r_{sg}(b^r_{sg}, \mathbf{b}^r_{-s}) db^r_{sg}. \tag{5.27}$$

This is in agreement to the second condition of *Theorem* 5.3. Hence the proof. ■

Equations (5.24) and (5.25) show that the BWA has the allocation and payment rules that lead to a unique sDSE and ultimately a unique DSE. Note that a bidder only pays when he is assigned a GU(s).

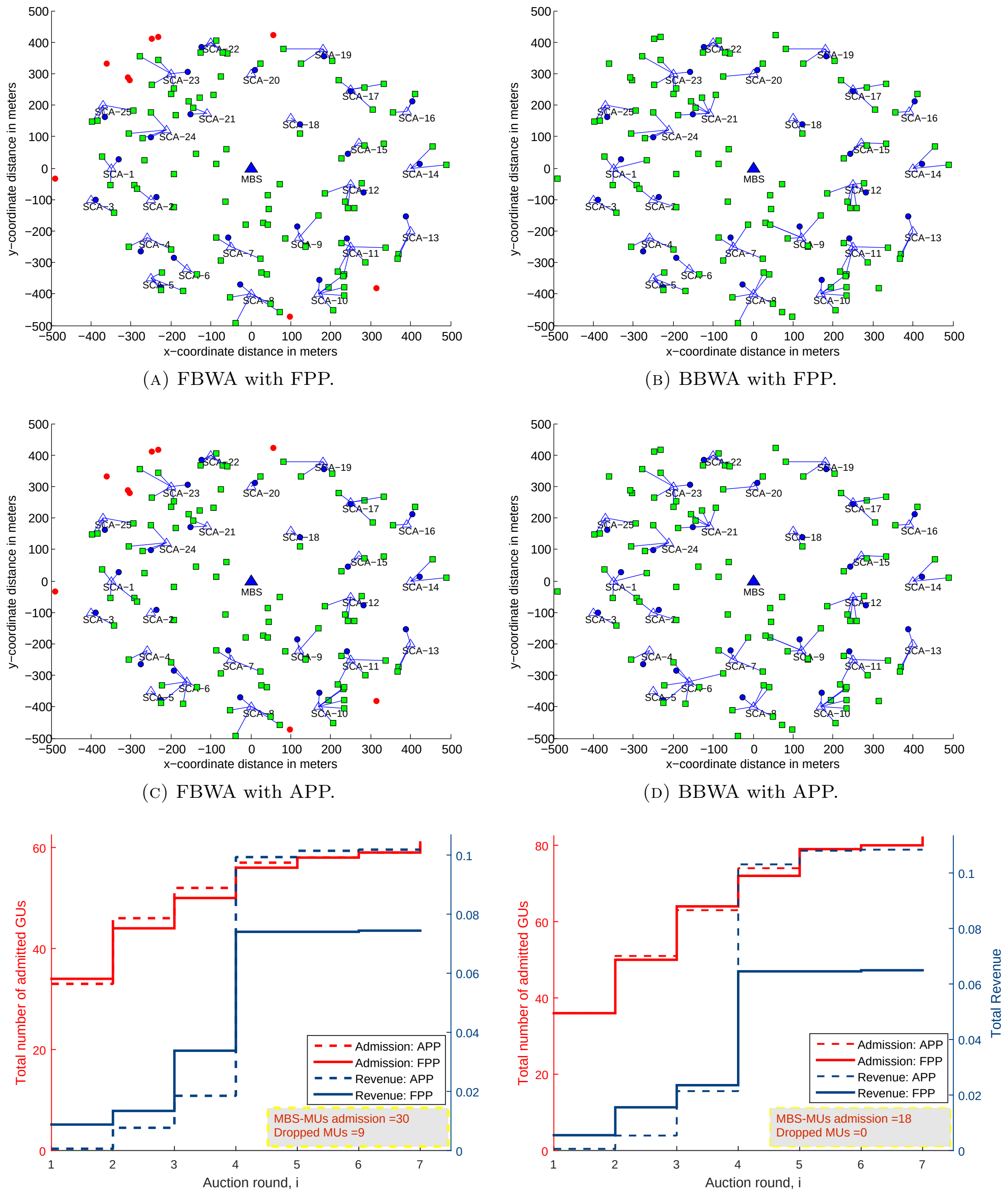
5.4.5 Optimality and Efficiency of the BWA

Usually the optimality of an auction is measured in terms of the revenue generated. Contrary to this, this chapter defines optimality as the ability to admit the maximum possible number of GUs. An auction is *allocative efficient* if the highest bidders always wins [40]. With these definitions in place, a remark that the BWA is suboptimal and inefficient is made. This is because valuations of the GUs on auction are stochastically dependent, but the BWA ignores this fact. In Section 5.5, numerical simulations are used compare the performance of the BWA with centralised algorithm proposed in [149].

5.5 Numerical Example

To demonstrate the performance of the proposed algorithms, consider a network with one MBS equipped with $M_{\text{MBS}} = 50$ antennae and 25 SCAs, each equipped with $M_{\text{SCA}} = 8$ antennae. There are 100 MUs that were randomly placed within the coverage area of the MBS. Each SCA is committed to serve one HU, with a data rate target of 2 bits/s/Hz. Low data rate are chosen for the HUs to ensure that it is always feasible for the SCA to serve the HU. The nominal coverage radius of the MBS and each SCA are 500 m and 30 m, respectively. The maximum transmission powers at the MBS and each SCA are $\varphi_0^{\max} = 46$ dBm and $\varphi_s^{\max} = 30$ dBm, respectively. The SCAs are only allowed to bid on users that fall within twice their nominal coverage radius. Choosing a large auction coverage area ensures that competition exists amongst bidders, and it also ensures that untruthful bidders will always be punished. It is assumed the MBS knows the locations of the SCAs and its 100 MUs. Based on this knowledge, the MBS will be able to determine those SCAs that can bid on any of its MUs. The noise power of all users was set to $\sigma^2 = 1$. The cost per unit of data rate and the cost per unit power were set to $\kappa = 0.1$ and $\mu = 0.00001$, respectively. All other model parameters are summarised in Table 5.2. The SCA-MUs is chosen such that there will be overlaps on the preference sets. This introduces competition amongst SCAs.

Figure 5.3 shows the results of the FBWA and BBWA when the SCAs utilise the FPP and APP criteria. The green squares indicate the locations of the admitted MUs served by either the MBS or the SCAs. Those users served by the SCAs are explicitly shown by connecting the users with the corresponding SCAs using blue lines. Those users that are not served by any of the transmitters (*id est* dropped users) are shown by red dots. The blue dots show the locations of the HUs. Figure 5.3a and Figure 5.3c show the results of FBWA algorithm for the cases of both the FPP and the APP criteria. The observation in both cases is that; as for the FBWA algorithm, the MBS performs admission control first, most of the users closer to the MBS have been admitted by MBSs and the remaining users are the contenders for bidding by SCAs. As opposed to this, the BBWA algorithm aims to auction off the MUs to the SCAs first. As seen from Figure 5.3a and Figure 5.3b, for the case of the FPP, with the BBWA algorithm, the SCAs have taken even those users that were served by MBS when the FBWA algorithm was used. This is because the choice of serving users was given to SCAs first. However,



(A) FBWA with FPP.

(B) BBWA with FPP.

(C) FBWA with APP.

(D) BBWA with APP.

(E) Cumulative admitted GUs/MUs and generated revenue using FBWA.

(F) Cumulative admitted GUs/MUs and generated revenue using BBWA.

FIGURE 5.3: Comparison of BBWA and FBWA allocation results when bidders use FPP and APP criteria to determine values of the GUs.

Description/Parameter	**Value**
Macrocell radius	500 m.
Smallcell radius	30 m.
MBS downlink transmit power $\varphi_0^{\max}$	46 dBm.
SCA downlink transmit power $\varphi_s^{\max}$	20 dBm.
MBS path and penetration loss at d (km)	$128.1 + 37.6 \log_{10}(d)$ dB.
SCA path and penetration loss at d (km)	$127 + 30 \log_{10}(d)$ dB.
Lognormal shadowing standard deviation	7 dB.
MBS-MUs minimum distance constraint	35 m.
SCA-MUs minimum distance constraint	3 m.
Noise variance σ^2	-127 dBm.
Wall attenuation	20 dB.
Number of MUs	100.
Number of HUs per SCA	1.
Number of MBS antennas M_{mbs}	50.
Number of SCA antennas M_{sca}	8.
Small scale fading distribution	$\mathbf{h}_{jk} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_{jk})$.
Physical channel model	[161, Eq.(34)]

TABLE 5.2: Numerical parameters for numerical evaluation.

this has resulted into some users even very far from the MBS to be served by the MBS. The same can be observed for the APP schemes as seen in Figure 5.3c and Figure 5.3d.

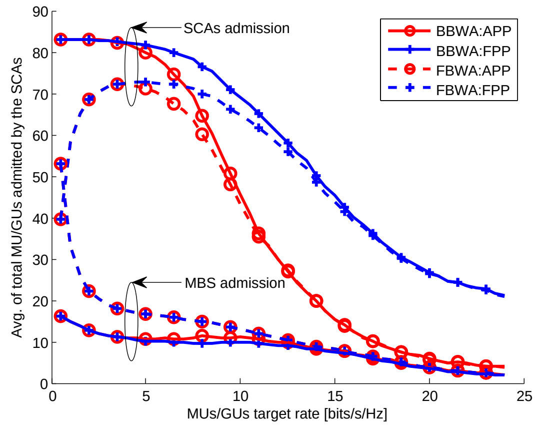
The revenue generated by the MBS due to those users served by the SCAs is now studied. The Figure 5.3e depicts the number of MUs admitted by the SCAs and the revenue generated by the MBS using the FBWA algorithm. The red lines show the number of admitted users and the blue lines show the revenue. The result of the APP criterion is shown by dashed lines and that of the FPP algorithm is shown by solid lines. As seen, more revenue is generated for the MBS if the SCAs use the APP criterion. This is because the margin between the winning bid and the critical bid is reduced, hence the payment made by the winner is increased. Similar observation is seen for the BBWA algorithm as well in Figure 5.3f. Also, notice from Figure 5.3e and Figure 5.3f that the number of dropped users is nine with the FBWA while none of the users is dropped with the BBWA. This is explained as follows: when the FBWA is used, as the MBS chooses the users closer to it first, there is a possibility that users that are far away from the SCAs may not be chosen by the SCAs because serving these users is not profitable to SCAs. However, with the BBWA, all users that are not served by the SCAs will be taken over by the MBS.

5.5.1 General Performance of the BBWA and FBWA

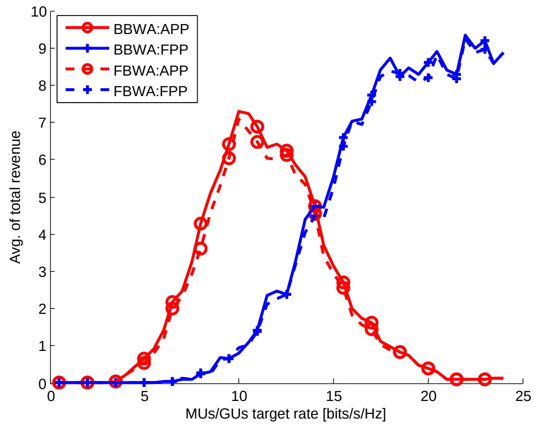
Now, the performance of the proposed methods by varying the target data rate for MUs while fixing the data rate for HUs at 2 bits/s/Hz is investigated. Figure 5.4a shows the average number of MUs admitted by the SCAs and the MBS (shown separately). The solid line depicts the performance of the BBWA while the dashed lines depict the performance of FBWA. The red lines indicate the APP criterion while the blue line indicate the FPP criterion. As similar to Figure 5.3, for a given preference criterion (FPP/APP), BBWA admits more users than the FBWA. Also, the FPP admits more users than APP for a given algorithm. Hence in terms of surplus maximisation, BBWA with FPP criterion is most preferred. As seen in Figure 5.4b, in terms of revenue generation for the SCAs, SCAs would prefer the BBWA with the APP criterion at lower target rates, and the BBWA with the FPP criterion at higher target rates. However, as the primary intention of the MBS is to minimise the dropped users, it will also prefer the BBWA algorithm. As the MBS cannot impose the preference criterion to SCAs, the SCAs will choose APP criterion at lower target rates and FPP criterion at higher target rates. A CVX optimisation toolbox [151] was used to solve the problems in (5.7), (5.14), and (5.15).

A comparison of the average system overhead measured in terms of the number of invitations for bidding, number of bids submitted and the number of announcements made is conducted. Each components of the different factors in the overhead carries different weights[7]. An invitation carries a weight of 1, while a bid and an announcement carries weight of 2. As seen in Figure 5.4c, the system overhead drops with increasing target data rate. This is because with increasing target data rate, the SCAs will reach

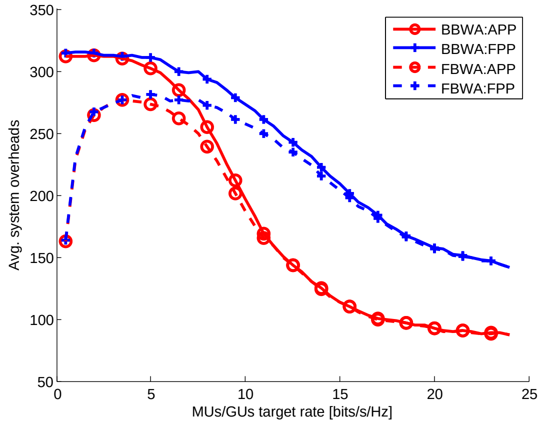
[7]An invitation is either 1 (*id est* inviting a bidder to submit a new bid) or 0 (*id est* dropping a bidder for not following the auction rules). A bid carries the identity of the GU of interest and its value. An announcement carries the identity of the GU and either 1 or 0 to indicate if the GU has been won or not.



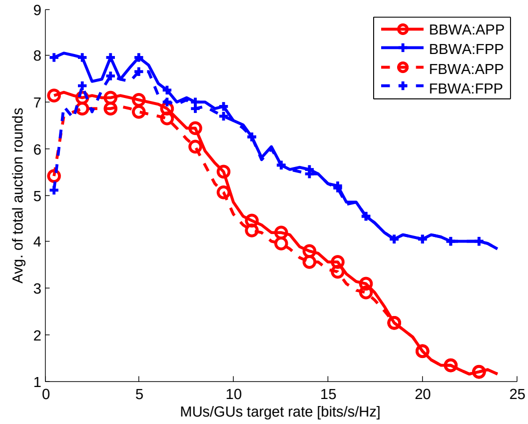
(A) Average MUs/GUs admitted by the SCAs.



(B) Average revenue generated by SCAs from the auctions.



(C) Average system overheads incurred.



(D) Average auction rounds.

FIGURE 5.4: Average performance of the proposed BBWA and FBWA for 20 channel realisations. There are 100 MUs and 25 SCAs.

its admission capacity quickly and there is no need for further auctioning. The average number of auction rounds is also compared in Figure 5.4d. For the same reason, the number of auction rounds drops with the increasing data rate.

In terms of computational complexity, it can be noted that in FBWA algorithms, the MBS will have a huge pool of users to choose from, and therefore, most of the computational burden will be centralised to the MBS. Once the MBS has admitted its users, the SCAs will have a reduce pool of users to choose from which will reduce the computational burden on the SCAs. In the BBWA algorithms, the SCAs will have a large pool of users to choose from, but due to the auction coverage area restrictions, the computational burden will be distributed across all SCAs. After the SCAs have admitted their preferred users, the MBS will be faced with a reduced pool of users; hence, it will incur less computational complexity as compared to FBWA.

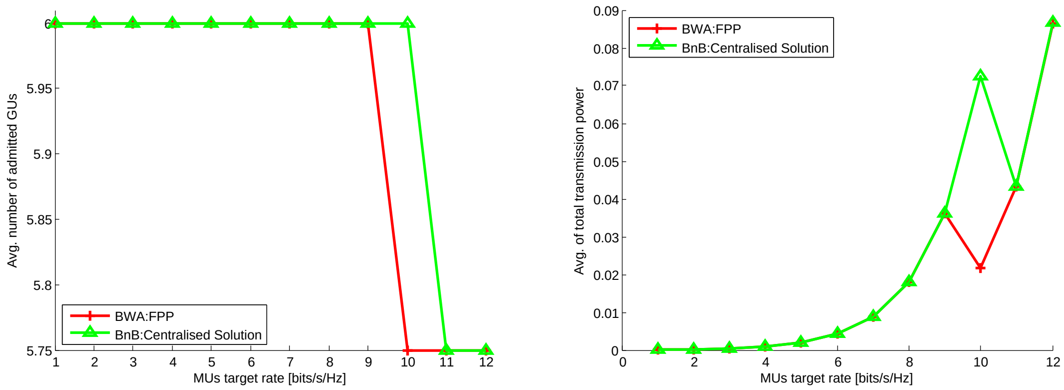
5.5.2 Evaluation of Optimality and Efficiency.

As the BWA is the main component of both the FBWA and the BBWA algorithms, its optimality and efficiency was analysed. The BWA was compared to a branch-and-bound (BnB) centralised solution proposed in [149], which uses YALMIP [18] to solve the admission and user association problems. All the simulations were carried out on a personal computer with 3.4 GHz Intel Core i5 processor. In order to reduce the computational burden for the centralised system, only 6 MUs and 2 SCAs were considered. The optimal solution was accelerated by searching for the feasible allocation space that is known to be feasible and gives high cardinality of admitted users.

Figure 5.5 shows the optimal average number of admitted MUs and the transmission power at each SCA as the target data rate of the MUs is varied. By comparing the BWA and the BnB in Figure 5.5a, it is be observed that the BWA matches the centralised solution at lower target rates. The only time when the BWA did not match the admission capacity offered by the optimal solution is between 9 bits/s/Hz and 11 bits/s/Hz. In Figure 5.5a, it is observed that as the target data rate of the MUs is increased, the total transmission power increases exponentially in both schemes. From target data rate of 10 bits/s/Hz, the average number of admitted users under BWA drops from 6 to 5.75. Consequently, the total transmission power is also dropped. A similar trend is observed in BnB from target data rate of 11 bits/s/Hz. It should be noted that when the system dimension is increased, the performance gap between the centralised solution and the proposed BWA may increase. The average total time for obtaining at solution at each target data rate is 162.2807 seconds for the accelerated optimal solution and 2.2461 seconds for the BWA.

5.6 Conclusion

A general framework that addresses user association problem in a wireless downlink heterogeneous network by utilising auctioning has been proposed. Two approaches were considered. In the first approach, the MBS admits as many users as it can serve and then auctions off the remaining users to SCAs. This is solved by the FBWA algorithm. In the second approach, the MBS auctions off as many users as possible to the SCAs and then admits the largest possible set of users from the remaining users. This is solved



(A) Average MUs admitted by the SCAs under BWA and centralised solution.

(B) Average of total power consumption under BWA and centralised solution.

FIGURE 5.5: Comparison of the BWA and the optimal solution.

by the BBWA algorithm. As the intention of the MBS is to admit as many users as possible, either directly serving them or by auctioning them off to the SCAs, it appears that the BBWA is the most preferred choice for MBS. In terms of revenue generation, SCAs would prefer the FPP criterion. Hence BBWA with the FPP criterion is the most preferred algorithm considering the preference of both the MBS and SCAs. The proposed algorithm is able to provide closer to optimal solution with significant saving in complexity and overheads. Unlike the proposed BWA discussed above, Chapter 6 proposes other auctions that always allow bidders to submit bundle bids.

Chapter 6

Multiple Item bidding Auctions for User Offloading in HetNets

Similar to Chapter 5, this chapter proposes auction based algorithms for offloading MUs from the MBS to various privately owned SCAs. In contrast to the BWA based algorithms, auctions proposed here always allow bundle bidding. A simultaneous multiple-round ascending auction (SMRA) algorithm for allocating MUs to the SCAs is proposed. Some of the fall-backs experienced in SMRA are addressed in the proposed altered SMRA (ASMRA), sequential combinatorial auction with item bidding (SCAIB), and RCAIB algorithms. The SMRA and the ASMRA use the first price payment rule while the SCAIB and the RCAIB use the second price payment rule.

6.1 Introduction and Related Works

Literature review on traffic offloading, and applications of auctioning in wireless networks is provided in Section 5.1.1. Since this chapter is focused on combinatorial auctions, related literature is provided. Many combinatorial auctions have received much attention in spectrum auctioning. Combinatorial auctions allow users to submit bids on a combinations of items [61]. Most of developed countries such as, USA, Australia, UK, and Germany, have adopted auctioning as a pricing scheme to dynamically allocate different spectrum bands [162–167]. Dynamic spectrum allocation extends to the field of cognitive radios. In cognitive radio, secondary users can opportunistically access the radio resource that belong to the primary users [2, 168–170].

In almost all the auction environments, the auctioneers wish to sell multiple objects to multiple bidders. The items can be *homogeneous* or *heterogeneous* [62]. On the other hand, the bidders can have *single-unit demand* or *multi-object demand*. Inclusion of all these characteristics and requirements makes combinatorial auction algorithm very complex. Most of the auction methods in spectrum auctioning are classified as simultaneous ascending auctions (SAAs). SAA is a large collection of auctions running in parallel [50, 163]. If implemented correctly, SAA generates market prices. Authors in [171] argued that, it is beneficial if the bidders can participate in different auctions at the same time. Therefore, combinatorial auction with item bidding (CAIB) was studied in [171]. This type of auction allows bidders to construct bundles with items from different auctions. A double auction framework, for spectrum auctioning and autonomous networks, was proposed in [62]. In order to develop distributed algorithms that are scalable to large systems, this chapter adopts the mechanism designs in SAAs and CAIB that are able to perform offloading and downlink beamforming.

6.1.1 Contributions

Most of the works in the literature assume that the customers are delighted to have their SCAs in closed-access mode, which constrain them to *reverse auctions*. Furthermore, most works apply the VCG mechanism which is deemed "the lovely but lonely Vickery auction" in [57]. Despite having good characteristics, the VCG is not widely applicable in practice [57]. This is because bidders are not always willing to reveal their true valuation. Also, the VCG is prune to collusions, wherein bidders may form illegal coalitions to reduce competition. Contrary to the works in the literature, this chapter explores other auctions that have been used in practice, especially in spectrum auctioning. The focus is on *forward auctions*. The incentivised offloading mechanisms have shown that the third party owners will benefit for participating in these auctions. This will attract a lot participants and increase competition amongst bidders. One of the motivations of this chapter is that, it is most probable that the SCAs owners are business oriented, and therefore they would use every available opportunity to minimise the cost of running their businesses. With high costs incurred in acquisition of the spectrum, under-utilization of this spectrum will be a liability to the business. In this regard, it is assumed that the privately owned SCAs are willing to buy users from the MNOs so as to fully occupy their under-utilized resource. On the other hand, the MNOs are highly interested in exploiting mechanisms that will increase their network capacity, without deploying extra BSs, so that their capital expenditure (CAPEX) can be reduced. These self interests of both parties create a marketplace environment, that will be used to develop auction based algorithms.

The focus of this chapter is on a multi-unit auction settings, wherein the bidders have budget constraints in terms of their ability on the number of users they can accommodate. These budget constraints are private i.e., they are not known to the auctioneer and other bidders. In particular, SMRA [172] and CAIB [171] are investigated. In the SMRA method, the items are simultaneously sold in an iterative fashion. In the CAIB, items are sold separately and independently in a one shot auction. Thus, every bidder submits a single bid for each item, and each item is sold independently as in a single-item auction.

The contributions of this chapter are as follows:

- New auction based algorithms that jointly perform downlink beamformer design and user association are proposed and analysed.
- Two SMRA based algorithms are proposed to facilitate the offloading process. The first algorithm directly applies the classical SMRA, which is used in spectrum auctioning. In order to reduce the valuation overheads incurred by the bidders, a second algorithm, referred herein as the altered SMRA (ASMRA), is proposed. These two algorithms preserve the privacy of bidders' valuations. In SMRA and ASMRA the item is given to the bidder with the highest bid and the payment is the winner's bid. In SMRA, Bidders are allowed to add and remove items on their bidding set as they wish. Contrary, in the ASMRA, bidders are allowed to add and remove items on their bidding set only when they have permanently lost an item on their current bidding set.
- In addition, two forward CAIB algorithms are proposed; the sequential CAIB (SCAIB) and the repetitive CAIB (RCAIB). These algorithms use the second-price rule (*id est*, VCG payment). In the RCAIB, standing highest bids are advertised

to competitors. Advertising the highest bids will provide more information to competitors, and thus encourage retaliations. The SCAIB tries to avoid this problem. In SCAIB and RCAIB, the item is allocated to the bidder with the highest bid and the payment is the second highest bid. The difference between SCAIB and RCAIB is that, in SCAIB a bidder can submit a bid on item once while in the RCAIB a bidder is allowed to bid on an item as many times as possible.

- It is shown that truthful bidding leads to individual rationality, and it is the best response for every bidder. Furthermore, it is demonstrated how truthful bidding leads to a Walrasian Equilibrium, where the supply equals the demand.
- Thorough numerical analysis is conducted, and validation of the proposed algorithms is carried out by comparing the proposed algorithms, with the optimal solution for heterogeneous deployments.

For readability, please refer to Table 5.1 in Chapter 5 for notations.

6.2 System Model and Problem Formulation

Consider the system model described in Section 5.2. In current HetNets, MUs can only be served by the MBS. This type of setting has shown to be very inefficient in terms of spectral and energy usage. With an ever increasing traffic, network operators can take advantage of the privately owned SCAs to serve some of their MUs, especially those at the boundary of the coverage area. To achieve this, incentives should be in place to encourage SCAs to operate in hybrid access modes, *id est* to serve own users and guest users. In this chapter, a compensation model to provide incentives is formulated using auction theory. In particular, a study on the utilisation of the forward ascending and the combinatorial auctions is conducted. If the SCAs are densely deployed, there is a high chance that a GU may be in the auction coverage area of multiple SCAs. This stimulates a competitive market as shown in Figure 6.1. To analyse this competitive market, an auction environment in which the MBS is the auctioneer, the SCAs are the bidders, and the GUs are the items, is formulated. In Chapter 5, it was recommended that, for high network capacity, the MBS should auction out the MUs to the SCAs before it performs admission control for the remaining users.

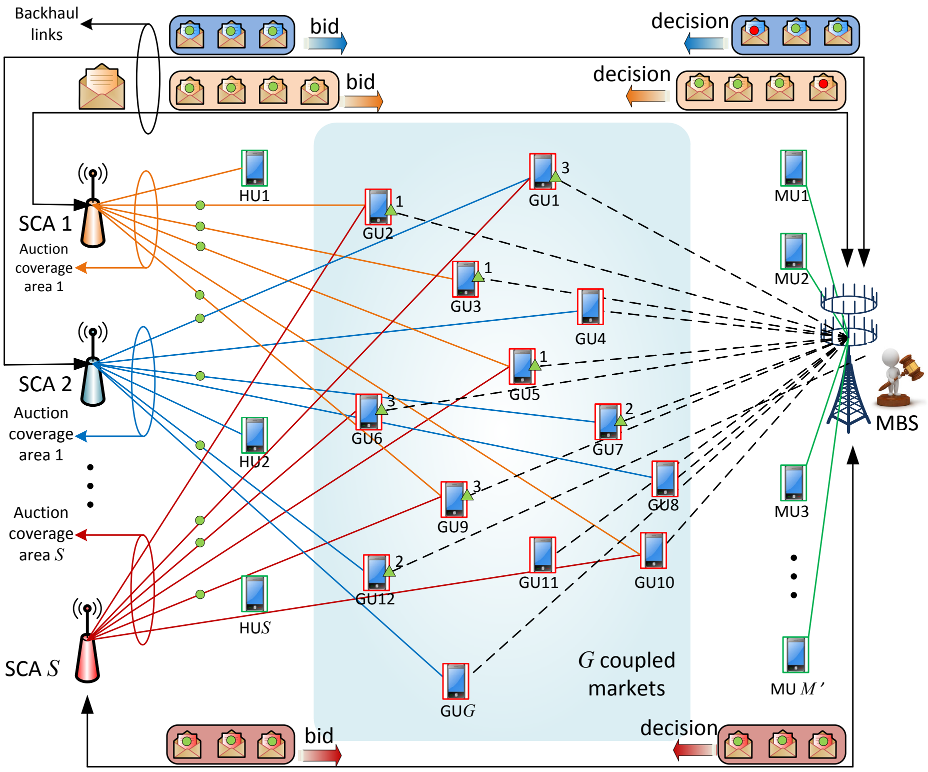


FIGURE 6.1: An auction market setting in a heterogeneous network. Guest users GU1, GU2, GU5, GU9, and GU10 are over-demanded items. GU11 is under-demanded.

6.2.1 General Auction Environment

The MBS intends to perform surplus-maximisation, for an economy with G heterogeneous items (*id est*, GUs), via auctioning. It is assumed that all SCAs have private *marginal values* (see (6.1)) on the items and private budget constraints. In order to maximise their utilities, all bidders wish to admit their favourite GUs subject to the transmission power constraints and QoS requirements of their own HUs. Note that the budget constraints of the bidders emanate from their transmission powers. Later, it will be shown that these budget constraints set the upper bound on the maximum number of GUs a bidder can accommodate.

Each bidder has private valuations $v_s(\mathcal{G}'_s)$ for every possible bundle of GUs $\mathcal{G}'_s \subseteq \mathcal{G}_s$ in its auction coverage area. This will result in immense private parameters. A valuation function of G_s number of items is a function $v_s(\mathcal{G}_s) : 2^{G_s} \to \mathbb{R}$, such that $v_s(\emptyset) = 0$. *Free disposal* is assumed, hence the monotonicity condition such that $v_s(\mathcal{G}'_s) \leq v_s(\mathcal{G}^\dagger_s)$, whenever $\mathcal{G}'_s \subseteq \mathcal{G}^\dagger_s$. Consider two disjoint sets $\mathcal{G}'_s$, and $\mathcal{G}^\dagger_s$ at the s-th SCA. The *marginal* *value* of $\mathcal{G}_s^{\dagger}$ with respect to the set $\mathcal{G}_s'$ is defined as

$$v_s(\mathcal{G}_s^{\dagger}|\mathcal{G}_s') = v_s(\mathcal{G}_s' \cup \mathcal{G}_s^{\dagger}) - v_s(\mathcal{G}_s'). \tag{6.1}$$

For a price profile $\mathbf{q} \in \mathbb{R}^G$, the utility of s-th bidder for acquiring G_s' GUs is a quasi-linear function defined as

$$u_s(\mathcal{G}_s') = v_s(\mathcal{G}_s') - \sum_{g \in \mathcal{G}_s'} \mathbf{q}(g). \tag{6.2}$$

It is assumed that there are no externalities on the valuation functions of the bidders. Thus, the valuation of each bidder depends only on the set of items it acquires.

6.2.2 System Metric Design

The system metrics are similar to those defined in Section 5.2.3.

6.2.3 Bidders' Valuation Functions

Prior to bidding, all SCAs have to determine their valuations on their favourite GUs. With the assumption that all the GUs require the same QoS in terms of SINR target and that the initial prices of all GUs are 0, the valuation function in the first auction round/iteration will be achieved by solving the user admission control problem. This involves solving the user maximisation ($\mathscr{P}^{\text{UM}}$) and user admission ($\mathscr{P}^{\text{UA}}$) problems. The QoS targets of the HUs and GUs at the s-th SCA are defined as $\mathbf{\Xi}^s = [\xi_1^s, \ldots, \xi_{H_s}^s, \xi_{H_s+1}^s, \ldots, \xi_{F_s}^s]$. Following the procedures for deriving (5.3)-(5.7) in Section 5.3.1, the optimisation will result into ℓ_1-norm user maximisation problem

$\mathscr{P}^{s-\mathrm{UM}}$ as in (5.14), which is restated here as

$$\begin{aligned}
\mathscr{P}^{s-\mathrm{UM}} : \underset{\{\mathbf{w}_h\},\{\mathbf{a}^s\}}{\mathtt{minimise}} \quad & \|\mathbf{a}^s\|_1 \\
\mathtt{subject\ to} \quad & \begin{bmatrix} \sqrt{1+\frac{1}{\xi_h^s}}\mathbf{h}_{sh}^{\mathsf{H}}\mathbf{w}_{sh} + a_h^s \\ \mathbf{h}_{sh}^{\mathsf{H}}\mathbf{W}_s \\ \sigma \end{bmatrix} \succeq_{\mathrm{SOC}} 0, \quad h \in \mathcal{F}_s, \\
& \Im(\mathbf{h}_{sh}^{\mathsf{H}}\mathbf{w}_{sh}) = 0, \quad \forall h, \\
& \mathbf{a}^s = \mathbf{0}, \quad h = 1, \ldots, H_s, \\
& \mathbf{a}^s \geq \mathbf{0}, \quad h = H_s + 1, \ldots, F_s, \\
& \sum_{h\in\mathcal{F}_s} \|\mathbf{w}_{sh}\|_2^2 \leq \varphi_s^{\max}, \quad \forall h.
\end{aligned} \tag{6.3}$$

To build up a preference set of GUs $\mathcal{P}_s \subseteq \mathcal{G}_s$, the vector $\mathbf{a}^s$ is sorted in ascending order. The corresponding indices of the sorted $\mathbf{a}^s$ with an exclusion of the index of HUs give the preference profile $\mathbf{f}_s$. To build up an optimal favourite set $\mathcal{G}_s' \subseteq \mathcal{P}_s$ and to determine the *marginal values* v_{sg} for each g-th user, the GUs are sequentially admitted beginning with the one corresponding to the smallest a_h^s. This is done by checking for feasibility at every admission by solving

$$\begin{aligned}
\mathscr{P}^{s-\mathrm{UA}} : \underset{\{\mathbf{w}_{sh}\}}{\mathtt{minimise}} \quad & \sum_{\forall h\in\mathcal{H}_s\cup g} \|\mathbf{w}_{sh}\|_2^2 \\
\mathtt{subject\ to} \quad & \mathsf{SINR}_h^s \geq \xi_h^s, \quad \forall h \in \mathcal{H}_s \cup g, \\
& \sum_{\forall h\in\mathcal{H}_s\cup g} \|\mathbf{w}_{sh}\|_2^2 \leq \varphi_s^{\max}.
\end{aligned} \tag{6.4}$$

When a newly admitted user makes the constraints in (6.4) infeasible, it is removed from the set $\mathcal{G}_s'$. Note that in the first auction round/iteration, solving $\mathscr{P}^{s-\mathrm{UM}}$ and $\mathscr{P}^{s-\mathrm{UA}}$ will give the favourite set with its corresponding marginal values. Essentially, the favourite set $\mathcal{G}_s'$ is determined using

$$\mathcal{G}_s' := \underset{\mathcal{P}_s\subseteq\mathcal{G}\setminus\mathcal{A}_s}{\mathtt{argmax}} \left\{ v_s(\mathcal{H}_s \cup \mathcal{P}_s) - \left(\sum_{g\in\mathcal{A}_s} \mathbf{q}(g) + \sum_{g\in\mathcal{P}_s} (\mathbf{q}(g) + \delta) \right) \right\}, \tag{6.5}$$

where $\mathcal{A}_s$ is the already admitted GUs, and δ is the price increment. In SMRA and ASMRA, (6.5) is used without alteration. In SCAIB, $\delta = 0$ and $\mathbf{q} = \mathbf{0}$, and in RCAIB, $\delta = 0$ and $\mathbf{q}(g) = \mathtt{argmax} \sum_{s\in\mathcal{C}_g} b_{sg}^r x_{sg}^r$, where r is the auction round index. Note that if the prices of all the GUs on the auction are identical, the utility of an SCA is maximised by admitting as many GUs as possible. Therefore, the very first favourite set is the upper bound on the maximum number of GUs that an SCA can admit. This first favourite set, denoted as $\hat{\mathcal{G}}_s$, forms a budget constraint on the bidder. In an iterative auction, the cardinality of the favourite set can only decrease as the auction progresses. During the auction, the prices are bound to be different, hence, the bidder is obliged to determine the favourite set by exhaustively trying all the possible combinations of the GUs available using (6.5). For a given favourite set $\mathcal{G}'_s$, the valuation process will provide values that are downward-sloping such that $v_{s1} \geq v_{s2} \geq \cdots \geq v_{sG'_s}$. The total valuation of the favourite set is given by $v_s(\mathcal{G}'_s) = \sum_{g\in\mathcal{G}'_s} v_{sg}$.

At every admission stage, the bidders determine the marginal value of the newly admitted GU. Let the charge per unit of the data rate paid by the every GU for connection, and the cost per unit power, be denoted as μ and κ, respectively. The marginal value of the admitted g-th GU is determined as

$$v_{sg} = \kappa \log_2(1 + \xi_g^s) - c_{sg}, \tag{6.6}$$

where the *marginal cost* c_{sg}, is given by

$$c_{sg} = \mu \left(\sum_{\forall k \in \mathcal{H}_s \cup g} \|\widehat{\mathbf{w}}_{sk}\|_2^2 - \sum_{\forall h \in \mathcal{H}_s} \|\mathbf{w}_{sh}\|_2^2 \right). \tag{6.7}$$

In (6.7), $\widehat{\mathbf{w}}_{sk}$ is the beamformer vector of the k-th HU given that GU g is admitted, and $\mathbf{w}_{sh}$ is the beamformer vector of the h-th HU before GU g is admitted. The first term in (6.7) is the total power consumed after the connection of the g-th GU and the last term is the total power consumption before g-th GU is admitted. Summing over all users before and after admission, the valuation in (6.6) is expressed as

$$v_s(g|\mathcal{H}_s) = v_s(\mathcal{H}_s \cup g) - v_s(g) \tag{6.8}$$

6.3 Surplus Maximisation Problem

The objective of the MBS is to maximise its surplus, which is measured in terms of the number of users that are offloaded to the SCAs. The general surplus maximisation problem at the MBS can be cast as the following integer program (IP):

$$\begin{aligned}
\mathscr{P}^{\text{IP}}: \underset{x_{s\mathcal{A}_s}}{\texttt{maximise}} \quad & \sum_{s\in\mathcal{S}}^{S} \sum_{\mathcal{A}_s\subseteq\mathcal{G}} v_{s\mathcal{A}_s} x_{s\mathcal{A}_s} \\
\texttt{subject to:} \quad & \sum_{j\in\mathcal{A}_s} \sum_{s} x_{s\mathcal{A}_s} \leq 1, \quad \forall j \in \mathcal{G}, \\
& \sum_{\mathcal{A}_s} x_{s\mathcal{A}_s} \leq 1, \quad \forall s \in \mathcal{S}, \\
& x_{s\mathcal{A}_s} \in \{0,1\}, \quad \forall s \in \mathcal{S}, \mathcal{A}_s \subseteq \mathcal{G}.
\end{aligned} \tag{6.9}$$

The first constraint ensures that every GU is matched with at most one SCA. The second constraint ensures that every SCA should get at most one bundle. In the SMRA, the objective will reduce to $\sum_{s\in\mathcal{S}}\sum_{g\in\mathcal{G}} v_{sg}x_{sg}$. In the CAIB, $\mathcal{A}_s(\mathbf{b})$ is used to denote the allocation for a bid profile $\mathbf{b}$. Let $\mathbf{b} = (\mathbf{b}_s, \mathbf{b}_{-s})$ denote the bid profile where SCA s bids $\mathbf{b}_s$ and all other SCAs bid $\mathbf{b}_{-s} = (\mathbf{b}_1, \ldots, \mathbf{b}_{s-1}, \mathbf{b}_{s+1}, \ldots, \mathbf{b}_S)$. In CAIB, the allocation and payment rules require the GU to be matched with the highest bidder at a price equal to the second highest bid. For a given allocation $\mathcal{A}_s(\mathbf{b}) \subseteq \mathcal{G}'_s$, the sum of the highest bids are denoted by

$$\begin{aligned}
B^{\text{high}}(\mathcal{A}_s(\mathbf{b}), \mathbf{b}) &= \sum_{g\in\mathcal{A}_s(\mathbf{b})} \max_{t}(\mathbf{b}_t(g)), \\
B^{\text{high}}_{-s}(\mathcal{A}_s(\mathbf{b}), \mathbf{b}_{-s}) &= \sum_{g\in\mathcal{A}_s(\mathbf{b})} \max_{t\neq s}(\mathbf{b}_t(g)).
\end{aligned} \tag{6.10}$$

Using (6.2) and the second price rule, the utility of the s-th SCA is given by

$$u_s(\mathbf{b}) = v_s(\mathcal{A}_s(\mathbf{b})) - B^{\text{high}}_{-s}(\mathcal{A}_s(\mathbf{b}), \mathbf{b}_{-s}). \tag{6.11}$$

6.3.1 Existence of the Walrasian Equilibrium

In [58, 173], it is argued that if a WE exists, any efficient allocation must solve the relaxed $\mathscr{P}^{\text{IP}}$. In order to address the existence of the WE in the SMRA and the CAIB, the following definitions are required.

Definition 6.1 (*Demand*)

Given a valuation function $v_s(\mathcal{G}_s) : 2^{G_s} \to \mathbb{R}$ and a vector of prices $\mathbf{q} \in \mathbb{R}$, the demand ($\mathcal{D}_s(v_s, \mathbf{q})$) of bidder s at the price of $\mathbf{q}$ is given by

$$\mathcal{D}_s(v_s, \mathbf{q}) := \{\mathcal{G}'_s \subseteq \mathcal{G}_s : u_s(\mathcal{G}'_s) \geq u_s(\mathcal{G}^{\dagger}_s), \forall \mathcal{G}^{\dagger}_s \subset \mathcal{G}_s\}. \tag{6.12}$$

Definition 6.2 (***Allocation***)

An allocation is a partition of $\mathcal{G}$ into pairwise disjoint sets of items $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_S$.

Definition 6.3 (***Submodular Valuation***)

Bidders utilities are deemed to be decreasing marginal utilities if the marginal value of an item decreases as the number of already accumulated items increases. This is equivalently defined via the submodular valuation definition. A valuation function v_s is submodular if for a pair $\mathcal{G}'_s \subseteq \mathcal{G}^{\dagger}_s$, and a GU g, $v_s(g|\mathcal{G}'_s) \leq v_s(g|\mathcal{G}^{\dagger}_s)$.

Definition 6.4 (***Complementary Free***)

A valuation function v_s is complementary free if for all sets of items $\mathcal{G}'_s$ and $\mathcal{G}^{\dagger}_s$, the following holds:

$$v_s(\mathcal{G}'_s) + v_s(\mathcal{G}^{\dagger}_s) \geq v_s(\mathcal{G}'_s \cup \mathcal{G}^{\dagger}_s). \tag{6.13}$$

6.3.2 Submodularity of the Valuation Function

Theorem 6.1

The valuation function v_s in (6.8) is a submodular valuation function.

Proof 7 In [174, 175], a valuation function v_s is submodular if and only if *any* of the following conditions hold.

1. Decreasing marginal utilities: For any $g, g^{\dagger} \in \mathcal{G}$ and $\mathcal{G}'_s \subseteq \mathcal{G}$, then $v_s(g|\mathcal{G}'_s) \geq v_s(g|\mathcal{G}'_s \cup \{g^{\dagger}\})$.
2. Monotonicity: For any $\mathcal{G}'_s, \mathcal{G}^{\dagger}_s, \mathcal{G}^{\ddagger}_s \subseteq \mathcal{G}$, such that $\mathcal{G}' \subseteq \mathcal{G}^{\dagger}$, then $v_s(\mathcal{G}^{\ddagger}_s|\mathcal{G}'_s) \geq v(\mathcal{G}^{\ddagger}_s|\mathcal{G}^{\dagger}_s)$.
3. Complementary free: For any $\mathcal{G}'_s, \mathcal{G}^{\dagger}_s \subseteq \mathcal{G}$, then $v_s(\mathcal{G}'_s) + v(\mathcal{G}^{\dagger}_s) \geq v_s(\mathcal{G}'_s \cup \mathcal{G}^{\dagger}_s) + v(\mathcal{G}'_s \cap \mathcal{G}^{\dagger}_s)$.

It is sufficient to qualify for one of the conditions above. The decreasing marginal utilities condition is considered. Using (4.9), define the $\mathbf{w}_{sh} = \sqrt{\varphi_h}\tilde{\mathbf{w}}_{sh}$, where φ_h is the power and $\tilde{\mathbf{w}}_{sh}$ is the unit-norm beamforming direction for the h-th HU. Further, denote power allocation vector $\boldsymbol{\varphi}_s = [\varphi_1, \ldots, \varphi_{H_s}]$. The SINR in (5.2) can be expressed as

$$\mathsf{SINR}^s_h(\boldsymbol{\varphi}_s) = \frac{\varphi_h}{\mathcal{I}_h(\boldsymbol{\varphi}_s)}, \tag{6.14}$$

where

$$\mathcal{I}_h(\boldsymbol{\varphi}_s) = \min_{\|\tilde{\mathbf{w}}_{sh}\|=1} [\boldsymbol{\Psi}_s(\tilde{\mathbf{w}}_{sh})\boldsymbol{\varphi}_s]_h + \frac{\sigma_h}{|\mathbf{h}_{sh}^{\mathsf{H}}\tilde{\mathbf{w}}_{sh}|^2}. \tag{6.15}$$

The constant link gain matrix (*id est*, a coupling matrix) $\boldsymbol{\Psi}_s$ for the s-th SCA is defined as

$$[\boldsymbol{\Psi}_s]_{sh}(\tilde{\mathbf{w}}_{sh}) = \begin{cases} \frac{|\mathbf{h}_{sh}^{\mathsf{H}}\tilde{\mathbf{w}}_{sj}|^2}{|\mathbf{h}_{sh}^{\mathsf{H}}\tilde{\mathbf{w}}_{sh}|^2}, & j \neq h, \\ 0, & j = h. \end{cases} \tag{6.16}$$

It was proven in [110] and [109] that $\mathcal{I}_k(\boldsymbol{\varphi}_s)$ is a standard function (also see Section 4.3.2). Now, consider that the preference profiles of bidder s as $\mathcal{P}_s := \{g_1, g_2, \ldots, g_{u-1}, g_u, g_{u+1}, \ldots, g_{G_s}\}$. Assume that during sequential admission of GUs, the set of HUs is $\mathcal{H}_s := \{g_1, g_2, \ldots, g_{u-1}\}$ with the corresponding power allocation vector of $\boldsymbol{\varphi}_s^{\mathcal{H}_s}$. Assume that in the next admission, SCA s considers GU g_u, with the resulting power allocation vector $\boldsymbol{\varphi}_s^{g_u|\mathcal{H}_s}$. Due to the monotonicity of $\mathcal{I}_k(\boldsymbol{\varphi}_s)$ on $\boldsymbol{\varphi}_s$ and using (6.14), it is argued that $\mathbf{1}^{\mathsf{T}}\boldsymbol{\varphi}_s^{g_u|\mathcal{H}_s} \geq \mathbf{1}^{\mathsf{T}}\boldsymbol{\varphi}_s^{\mathcal{H}_s}$. Now, suppose before admitting GU g_u, bidder s admits GU g_{u+1} first. Note that GU g_{u+1} has equal or lower preference to bidder s as compared to GU g_u. Denote the power allocation vector by $\boldsymbol{\varphi}_s^{g_{u+1}|\mathcal{H}_s}$, when GU g_{u+1} is admitted first. With the same argument given earlier, the new power allocation vector will satisfy $\mathbf{1}^{\mathsf{T}}\boldsymbol{\varphi}_s^{g_{u+1}|\mathcal{H}_s} \geq \mathbf{1}^{\mathsf{T}}\boldsymbol{\varphi}_s^{g_u|\mathcal{H}_s} \geq \mathbf{1}^{\mathsf{T}}\boldsymbol{\varphi}_s^{\mathcal{H}_s}$. If GU g_u is admitted after GU g_{u+1}, with the corresponding power allocation vector being $\boldsymbol{\varphi}_s^{g_u|\mathcal{H}_s\cup\{g_{u+1}\}}$, it should be the case that $\mathbf{1}^{\mathsf{T}}\boldsymbol{\varphi}_s^{g_u|\mathcal{H}_s\cup\{g_{u+1}\}} \geq \mathbf{1}^{\mathsf{T}}\boldsymbol{\varphi}_s^{g_{u+1}|\mathcal{H}_s} \geq \mathbf{1}^{\mathsf{T}}\boldsymbol{\varphi}_s^{g_u|\mathcal{H}_s} \geq \mathbf{1}^{\mathsf{T}}\boldsymbol{\varphi}_s^{\mathcal{H}_s}$.

Now by utilising (6.8) and (6.7), gives

$$\begin{aligned} v_s(g_u|\mathcal{H}_s) &= \kappa \log_2(1+\xi_{g_u}^s) - \left(\mathbf{1}^{\mathsf{T}}\boldsymbol{\varphi}_s^{g_u|\mathcal{H}_s} - \mathbf{1}^{\mathsf{T}}\boldsymbol{\varphi}_s^{\mathcal{H}_s}\right) \\ &\geq \kappa \log_2(1+\xi_{g_u}^s) - \left(\mathbf{1}^{\mathsf{T}}\boldsymbol{\varphi}_s^{g_u|\mathcal{H}_s\cup\{g_{u+1}\}} - \mathbf{1}^{\mathsf{T}}\boldsymbol{\varphi}_s^{g_{u+1}|\mathcal{H}_s}\right) \\ &= v_s(g_u|\mathcal{H}_s \cup g_{u+1}) \end{aligned} \tag{6.17}$$

■

Lemma 6.1

The valuation function v_s is submodular for every subset $\mathcal{Q}_s$, and the marginal valuation function $v_s(\cdot|\mathcal{Q}_s)$ is complementary free.

Proof 8 Using theorem 6.1, it is required to proof that for all $\mathcal{G}_s', \mathcal{G}_s^{\dagger} \in \mathcal{G}_s$, it is such that $v_s(\mathcal{G}_s') + v(\mathcal{G}_s^{\dagger}) \geq v_s(\mathcal{G}_s' \cup \mathcal{G}_s^{\dagger}) + v(\mathcal{G}_s' \cap \mathcal{G}_s^{\dagger})$. Let $\mathcal{Q}_s := \mathcal{G}_s' \cap \mathcal{G}_s^{\dagger}$, $\bar{\mathcal{G}}_s' := \mathcal{G}_s' \setminus \mathcal{Q}_s$, and $\bar{\mathcal{G}}_s^{\dagger} := \mathcal{G}_s^{\dagger} \setminus \mathcal{Q}_s$. By using (6.1), define the following marginal values: $v_s(\mathcal{G}_s') =$

$v_s(\bar{\mathcal{G}}'_s|\mathcal{Q}_s)+v_s(\mathcal{Q}_s)$, $v_s(\mathcal{G}^\dagger_s) = v_s(\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s)+v_s(\mathcal{Q}_s)$, $v_s(\mathcal{G}'_s\cup\mathcal{G}^\dagger_s) = v_s(\bar{\mathcal{G}}'_s\cup\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s)+v_s(\mathcal{Q}_s)$, and $v_s(\mathcal{G}'_s\cap\mathcal{G}^\dagger_s) = v_s(\mathcal{Q}_s)$. The third condition of theorem 6.1 is equivalently written as

$$\begin{aligned} v_s(\mathcal{G}'_s) + v(\mathcal{G}^\dagger_s) \geq v_s(\mathcal{G}'_s\cup\mathcal{G}^\dagger_s) + v(\mathcal{G}'_s\cap\mathcal{G}^\dagger_s) \\ \Rightarrow v_s(\bar{\mathcal{G}}'_s|\mathcal{Q}_s) + v_s(\mathcal{Q}_s) + v_s(\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s) + v_s(\mathcal{Q}_s) \geq v_s(\bar{\mathcal{G}}'_s\cup\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s) + v_s(\mathcal{Q}_s) + v_s(\mathcal{Q}_s) \\ \Rightarrow v_s(\bar{\mathcal{G}}'_s|\mathcal{Q}_s) + v_s(\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s) \geq v_s(\bar{\mathcal{G}}'_s\cup\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s). \end{aligned} \tag{6.18}$$

This suggests that $v(\cdot|\mathcal{Q}_s)$ is complement free as per (6.13). With the properties of the interference function given in (6.15), and the conclusion in (6.17), it is argued that $\mathbf{1}^\mathsf{T}\boldsymbol{\varphi}_s^{\bar{\mathcal{G}}'_s|\mathcal{Q}_s} \leq \mathbf{1}^\mathsf{T}\boldsymbol{\varphi}_s^{\bar{\mathcal{G}}'_s\cup\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s}$, $\mathbf{1}^\mathsf{T}\boldsymbol{\varphi}_s^{\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s} \leq \mathbf{1}^\mathsf{T}\boldsymbol{\varphi}_s^{\bar{\mathcal{G}}'_s\cup\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s}$, and $\mathbf{1}^\mathsf{T}\boldsymbol{\varphi}_s^{\bar{\mathcal{G}}'_s|\mathcal{Q}_s} + \mathbf{1}^\mathsf{T}\boldsymbol{\varphi}_s^{\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s} \leq \mathbf{1}^\mathsf{T}\boldsymbol{\varphi}_s^{\bar{\mathcal{G}}'_s\cup\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s}$. Thus, (6.18) is confirmed.

To conclude the proof, it is required to prove that for all $\mathcal{Q}_s$ and $\bar{\mathcal{G}}'_s, \bar{\mathcal{G}}^\dagger_s \subseteq \mathcal{Q}^\mathsf{c}_s$, it is such that $v_s(\bar{\mathcal{G}}'_s|\mathcal{Q}_s) + v_s(\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s) \geq v_s(\bar{\mathcal{G}}'_s\cup\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s)$. Let $\mathcal{G}'_s = \bar{\mathcal{G}}'_s\cup\mathcal{Q}_s$ and $\mathcal{G}^\dagger_s = \bar{\mathcal{G}}^\dagger_s\cup\mathcal{Q}_s$. With these definitions, the same marginal valuations as before are observed: $v_s(\mathcal{G}'_s) = v_s(\bar{\mathcal{G}}'_s|\mathcal{Q}_s)+v_s(\mathcal{Q}_s)$, $v_s(\mathcal{G}^\dagger_s) = v_s(\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s)+v_s(\mathcal{Q}_s)$, $v_s(\mathcal{G}'_s\cup\mathcal{G}^\dagger_s) = v_s(\bar{\mathcal{G}}'_s\cup\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s)+v_s(\mathcal{Q}_s)$, and $v_s(\mathcal{G}'_s\cap\mathcal{G}^\dagger_s) = v_s(\mathcal{Q}_s)$. Due to v_s being a submodular function, the condition $v_s(\bar{\mathcal{G}}'_s|\mathcal{Q}_s)+v_s(\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s) \geq v_s(\bar{\mathcal{G}}'_s\cup\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s)$ is equivalently written as $v_s(\bar{\mathcal{G}}'_s|\mathcal{Q}_s)+v_s(\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s) \geq v_s(\bar{\mathcal{G}}'_s\cup\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s)$. Use of the same arguments made above concludes the proof. ■

6.3.3 Gross-substitute of the Valuation Function

A much stronger property of the valuation function is the gross-substitute condition.

Definition 6.5 (*Walrasian Equilibrium*)

In a market with $G = |\mathcal{G}|$ items, $S = |\mathcal{S}|$ agents, and valuations v_s, a WE is a price $\mathbf{q}^\star \in \mathbb{R}_+$ and a partition of goods in disjoint sets $\mathcal{G} := \cup_{s=1}^{S}\mathcal{A}_s$ such that $\mathcal{A}_s \in \mathcal{D}_s(v_s, \mathbf{q})$. The WE corresponds to the market-clearing prices where every bidder receives a bundle in his demand set [50]. At WE the following conditions must hold:

- *Condition 1: Each bidder s is matched to its preferred item $g \in \texttt{argmax}\{v_{sg} - q_g\}_{g\in G\cup\{\emptyset\}}$.*
- *Condition 2: An item $g \in \mathcal{G}$ is unsold only if $q_g = 0$.*

Definition 6.6 (*Gross Substitute Condition, Kelso and Crawford [176]*)

A valuation v_s over the items $\mathcal{G}_s$ satisfies the gross substitution (GS) condition if and only if for any price profile $\mathbf{q} \in \mathbb{R}$ and $\mathcal{G}'_s \in \mathcal{D}_s(v_s, \mathbf{q})$, if $\mathbf{q}'$ is a price profile such that $\mathbf{q}' \leq \mathbf{q}$, then there is a set $\mathcal{G}^{\dagger}_s \in \mathcal{D}_s(v_s, \mathbf{q}')$ such that $\mathcal{G}'_s \cap \{g : \mathbf{q}(g) = \mathbf{q}'(g)\} \subseteq \mathcal{G}^{\dagger}$.

In brief, Definition 6.6 suggests that, if a bidder has GS valuation and demands a set $\mathcal{G}'_s$ of items at the price profile $\mathbf{q}$, if the price of some of the items subsequently increase, the bidder still demands some of the items in $\mathcal{G}'_s$ whose price remained unchanged.

Proposition 6.1

The valuation function in (6.8) is a gross valuation function.

Proof 9 Fix a bidder s, v_s and $\mathbf{v_{-s}}$. Let the corresponding marginal values for the favourite set $\mathcal{G}'_s$ be denoted as $v_{s1}, v_{s2}, \ldots, v_{s\mathcal{G}'_s}$. Suppose bidder s gets matched with all the GUs in its favourite set at price vector $\mathbf{q}$. Now, introduce a new bidder t who has a favourite set $\mathcal{G}'_t$ such that $\mathcal{G}'_t \cap \mathcal{G}'_s := \{g_2\}$. Assume that $v_{sg} \geq v_{tg}, g \in \mathcal{G}'_s \setminus g_2$[1]. Assuming truthful bidding, bidder s will loose GU g_2 to bidder t as the price of GU g_2 will increase. This change in allocation will result in a new power allocation vector $\boldsymbol{\varphi}^{\dagger}_s$ such that $\boldsymbol{\varphi}^{\dagger}_s \preceq \boldsymbol{\varphi}_s$, with $\boldsymbol{\varphi}^{\dagger}_s(g_2) = 0$. The monotonicity axiom for (6.15) from [109, 110] is invoked, and it is stated that $\mathcal{I}_h(\boldsymbol{\varphi}_s) \geq \mathcal{I}_h(\boldsymbol{\varphi}^{\dagger}_s)$. With this being true, loosing GU g_2 will increase the marginal values of all other GUs in the favourite set $\mathcal{G}'_s \setminus g_2$ and thus making them more attractive to SCA s. ■

6.3.4 Computation of the WE prices

The linear programming relaxation (LPR) of $\mathscr{P}^{\mathrm{IP}}$ is

$$\begin{aligned}
\mathscr{P}^{\mathrm{LPR}} : \underset{x_{pg}}{\texttt{maximise}} \quad & \sum_{s=1}^{S} \sum_{\mathcal{A}_s \subseteq \mathcal{G}} v_{s\mathcal{A}_s} x_{s\mathcal{A}_s} \\
\texttt{subject to:} \quad & \sum_{j \in \mathcal{A}_s} \sum_{s} x_{s\mathcal{A}_s} \leq 1, \quad \forall j \in \mathcal{G}, \\
& \sum_{\mathcal{A}_s} x_{s\mathcal{A}_s} \leq 1, \quad \forall s \in \mathcal{S}, \\
& 0 \leq x_{s\mathcal{A}_s} \leq 1, \quad \mathcal{A}_s \in \mathcal{G}, \forall s \in \mathcal{S}
\end{aligned} \tag{6.19}$$

[1]It is assumed that all tie breaks are in favour of bidders s. In this proof, g_2 is chosen to be the only GU that bidder s lost. This is to simplify the proof.

Even though $\mathscr{P}^{\text{LPR}}$ has $G+S$ variables, it has an exponential number of constraints. The works in [58, 173] propose solving the dual of $\mathscr{P}^{\text{LPR}}$ by utilising separation based linear programming algorithm. The dual linear programming relaxation (DLPR) is defined as

$$\begin{aligned} &\mathscr{P}^{\text{DLPR}}: \\ &\underset{x_{pg}}{\texttt{minimise}} \quad \sum_{s=1}^{S} u_s + \sum_{g\in G} \mathbf{p}(g) \\ &\texttt{subject to:} \quad u_s \geq v_s(\mathcal{A}_s) - \sum_{g\in\mathcal{A}_s} \mathbf{p}(g), \quad \forall s \in \mathcal{S}, \mathcal{A}_s \in \mathcal{G}, \\ &\qquad \mathbf{p}(g) \geq 0, u(s) \geq 0, \quad \forall s \in \mathcal{S}, g \in \mathcal{G}, \end{aligned} \tag{6.20}$$

where $\mathbf{p}$ and u_s are the Lagrange multipliers associated with the constraints in $\mathscr{P}^{\text{LPR}}$. For completeness, the following well known theorems are stated:

Theorem 6.2 (*First Welfare Theorem [50]*)

Suppose $(\mathbf{q}, \mathcal{A}_1, \ldots, \mathcal{A}_S)$ is a WE, then the allocation $(\mathcal{A}_1, \ldots, \mathcal{A}_S)$ maximises the social welfare, id est, maximises $\sum_{s\in\mathcal{S}} v_s(\mathcal{A}_s)$.

Proof 10 Let $Q = \sum_{g\in\mathcal{G}} \mathbf{q}(g)$ be the sum of prices of all GUs and let the allocation $(\mathcal{A}_1^\star, \ldots, \mathcal{A}_S^\star)$ be any welfare maximising allocation. Since $\mathcal{A}_s \in D(v_s, \mathbf{q})$, then by utilising condition 1 of Definition 6.5, the following holds

$$v_s(\mathcal{A}_s) - q(\mathcal{A}_s) \geq v_s(\mathcal{A}_s^\star) - q(\mathcal{A}_s^\star). \tag{6.21}$$

Summing over all s yields

$$\sum_{s\in\mathcal{S}} v_s(\mathcal{A}_s) - \sum_{s\in\mathcal{S}} q(\mathcal{A}_s) \geq \sum_{s\in\mathcal{S}} v_s(\mathcal{A}_s^\star) - \sum_{s\in\mathcal{S}} q(\mathcal{A}_s^\star). \tag{6.22}$$

When summing over all GUs that have non-zero price, it concludes that $\sum_{s\in\mathcal{S}} v_s(\mathcal{A}_s) \geq \sum_{s\in\mathcal{S}} v_s(\mathcal{A}_s^\star)$. ■

Theorem 6.2 is complemented by the Second Welfare Theorem via the duality theorem in linear programming.

Theorem 6.3 (*Second Welfare Theorem [50]*)

Suppose an optimal solution for $\mathscr{P}^{\text{LPR}}$ exists, then a WE whose allocation is the given solution also exists.

Proof 11 Let the optimal allocation to $\mathscr{P}^{\text{LPR}}$ be $(\mathcal{A}_1^\star, \ldots, \mathcal{A}_S^\star)$. Suppose the optimal solution to $\mathscr{P}^{\text{DLPR}}$ is given by $(\mathbf{p}^\star, u_1^\star, \ldots, u_S^\star)$. It is required to show that $(\mathbf{p}^\star, \mathcal{A}_1^\star, \ldots, \mathcal{A}_S^\star)$ is a WE. Since KKT conditions are necessary and sufficient for the optimality to $\mathscr{P}^{\text{LPR}}$ and $\mathscr{P}^{\text{DLPR}}$, then for each SCA for which $x_{s\mathcal{A}_s^\star} > 0$, it is such that $x_{s\mathcal{A}_s^\star} = 0$ in $\mathscr{P}^{\text{LPR}}$ and $u_s = v_s(\mathcal{A}_s^\star) - \sum_{g \in \mathcal{A}_s^\star} \mathbf{p}^\star(g)$ in $\mathscr{P}^{\text{DLPR}}$ being true. Therefore, for any other bundle A_s

$$u_s = v_s(\mathcal{A}_s^\star) - \sum_{g \in \mathcal{A}_s^\star} \mathbf{p}^\star(g) \geq v_s(\mathcal{A}_s) - \sum_{g \in \mathcal{A}_s} \mathbf{p}^\star(g). \tag{6.23}$$

■

Theorem 6.3 means that, if $(\mathbf{p}, \mathcal{A}_1, \ldots, \mathcal{A}_S)$ is a WE and $(\mathcal{A}_1^\star, \ldots, \mathcal{A}_S^\star)$ maximises the surplus $\sum_{s \in \mathcal{S}} v_s(\mathcal{A}_s^\star)$, then $(\mathbf{p}, \mathcal{A}_1^\star, \ldots, \mathcal{A}_S^\star)$ is also a WE. Both theorems 6.2 and 6.3 suggest that the WE exists if there is strong duality between $\mathscr{P}^{\text{LPR}}$ and $\mathscr{P}^{\text{DLPR}}$. In order to solve $\mathscr{P}^{\text{DLPR}}$, two ascending auction algorithms (SMRA) and two CAIB algorithms, based on the Walras' *tatônnement (id est, trial and error) procedure* [177], are proposed.

6.4 The SMRA and CAIB Algorithms

First the iterative SMRA and ASMRA algorithms are proposed. These two algorithms enable SCAs to preserve privacy of the valuations. Furthermore, two variations of the CAIB are proposed. Initially, a sequential CAIB (SCAIB), wherein the auctioneer runs different CAIB in a sequential manner, is proposed. In this setting, a bidder is allowed to submit a bid on a particular item only once for the entire auction. Finally, a repetitive CAIB (RCAIB) is proposed. In RCAIB, bidders are allowed to correct their bids by rebidding on items, for as long as they believe they constitute their favourite item set.

6.4.1 The Simultaneous Multiple-Round Ascending Auction Mechanism

Algorithm 7 describes how to compute WE using an SMRA. First, define a *conditional bidding set*[2] $\mathcal{G}_s^i$ at every iteration[3] i as the set that contains all the GUs that bidder s wishes to bid on given that it has admitted the *provisional set* $\mathcal{A}_s$. The assumption is that bidder s automatically bid on each GU $g \in \mathcal{A}_s$. In this regard, any bidder is not allowed to withdraw its bid on any GU that it has been matched with. For an SCA to relinquish a GU, one of its competitors has to outbid it on that GU. The MBS predefines the set on which each SCA can bid on by setting the auction coverage area for each bidder to $\alpha\varsigma_s$, where α is the scaling factor and ς_s is the SCA's nominal coverage area. The prices of all the GUs are initialised as $\mathbf{q}(g) = 0, \forall g \in \mathcal{G}$. The initial contact set $\mathcal{T}$ contains all SCAs with at least one auctioned GU in their auction coverage area. The set $\mathcal{G}$ of GUs that are on the auction is initialised as all MUs that fall within the auction coverage areas of all SCAs. For all SCAs, the provisional set $\mathcal{A}_s$, the conditional set $\mathcal{G}_s^i$, the loose set $\mathcal{L}_s$ are initialised as empty sets.

The Algorithm 7 iterates as follows: The MBS invites all bidders in the contact set $\mathcal{T}$ to indicate their conditional bidding sets. Each SCA submits their conditional bidding sets $\mathcal{G}_s^i \subseteq G_s \setminus \mathcal{A}_s$ to the MBS, with the assumption that the price of each GU $g \in G_s \setminus \mathcal{A}_s$ has price $\mathbf{q}^i(g) = \mathbf{q}^{i-1}(g) + \delta$. The prices for all GUs $g \in \mathcal{A}_s$ are assumed to be unchanged, *id est*, $\mathbf{q}^i(g) = \mathbf{q}^{i-1}(g)$. In step 7 to step 16, the MBS updates the provisional sets by randomly allocating a GU to any bidder that is interested in it. In cases when there is a tie, the winner is picked randomly. The MBS then updates the prices of all the GUs that are over demanded and updates the contact set.

Now suppose the current set of competitors for GU g is empty. This implies that GU g does not appear in any of the conditional sets, $g \notin \cup_{s=1}^{S} \mathcal{G}_s^i$. The price for GU g is set to $\mathbf{q}^{i+1}(g) = \mathbf{q}^i(g)$ and the *provisional winner* remains unchanged, *id est*, if $g \in \mathcal{A}_s$ during iteration i then $g \in \mathcal{A}_s$ in iteration $i + 1$. If $|\mathcal{C}_g| = 1$, supply equals demand, then GU g is matched with the SCA $s \in \mathcal{C}_g$. Otherwise if $|\mathcal{C}_g| > 1$, then GU g is over demanded. Under this condition the GU g changes hands by a random assignment to

[2] Conditional bidding set is the favourite set. The conditional bidding set is sometimes referred to as the conditional bid.

[3] The terms iteration and round are reserved to describe the state of an iterative auction and a sequential auction, respectively.

Algorithm 7: SMRA Algorithm

Data: Initialisation: $\delta > 0$, $\mathbf{q}(g) = 0, \forall g \in G$, $(S, G) \in \mathbb{Z}_+$, GUs' set $\mathcal{G} := \cup_{s=1}^{S} \mathcal{G}_s$, assignment set $\mathcal{A}_s = \emptyset, \forall s \in \mathcal{S}$, lost items set $\mathcal{L}_s = \emptyset, \forall s \in \mathcal{S}$, $\mathcal{G}_s^i = \emptyset, \forall s \in \mathcal{S}$, *auction round*: $i = 0$.

Result: Optimal Allocation set $\mathcal{A}_s^\star, \forall s \in \mathcal{S}$.

while $\mathcal{T} \neq \emptyset || \cup_{s=1}^{S} \mathcal{G}_s^i \neq \emptyset$ **do**
- $i \leftarrow i + 1$
- Auctioneer asks each bidder for its conditional bidding set $\mathcal{G}_s^i$.
- Bidders determines new preference profiles and marginal values for their favourite subset $\mathcal{G}_s^i$ (using (6.5)) of items not assigned to them, given the admitted GUs and the current prices $\mathbf{q}^i$.
- Bidders submit their conditional bidding set.
- Auctioneer set $\mathcal{T} = \emptyset$, $\mathcal{L}_s = \emptyset$
- **for** $g \in \cup_{s=1}^{S} \mathcal{G}_s^i$ **do**
  - **if** $|\mathcal{C}_g^i| > 1$ **then**
    - pick a random bidder s: $\mathcal{A}_s \leftarrow \mathcal{A}_s \cup g$
    - $\forall k \neq s$, $\mathcal{A}_k \leftarrow \mathcal{A}_k \setminus g$, $\mathcal{T} \leftarrow \mathcal{T} \cup \forall k \neq s$
    - $\mathcal{L}_k \leftarrow g, \forall k \in \mathcal{C}_g^i \setminus s$
    - $\mathbf{q}^{i+1}(g) \leftarrow \mathbf{q}^i(g) + \delta$
  - **else if** $|\mathcal{C}_g^i| = 1$ **then**
    - $\mathcal{A}_s \leftarrow \mathcal{A}_s \cup g$, $\mathcal{G} \leftarrow \mathcal{G} \setminus g$
  - **else**
    - pick a random winner s from subset of the bidders in $\mathcal{T}$: $\mathcal{A}_s \leftarrow \mathcal{A}_s \cup g$

a bidder in the set $\mathcal{C}_g$, with the exception of its previous provisional winner. The same process becomes repeated until the contact set is empty or when the conditional bidding sets of all bidders become empty.

6.4.2 Altered SMRA Algorithm

Note that in Algorithm 7, for the SCAs to maximise their utilities, they are forced to exhaustively check for every possible bundle in $\mathcal{G}_s$ in every iteration. This could be computationally costly on both the bidders and the auctioneer. To reduce the overheads incurred by the SCAs during valuation, the MBS use the activity rule as follows: once a bidder places a bid on a GU g, it must commit to bid on that GU g in every iteration. Otherwise if an SCA fails to submit a bid on GU g, then this SCA is erased from the competitors set $\mathcal{C}_g$ and it cannot join later. Therefore a bidder is forced to commit bidding to its current favourite set until it loses at least one of the GUs. The SCAs are

Algorithm 8: ASMRA Algorithm

Data: See Algorithm 7.

Result: See Algorithm 7.

while $\mathcal{T} \neq \emptyset || \cup_{s=1}^{S} \mathcal{G}_s^i \neq \emptyset$ **do**
  Algorithm 7 steps 2-3.
  if $\mathcal{L}_s \neq \emptyset || i = 1$ **then**
    Algorithm 7 step 4.
  else
    for $g \in \mathcal{G}_s^{i-1}$ **do**
      if $v_{sg} > \mathbf{q}(g)$ **then**
        $\mathcal{G}_s^i \leftarrow g$
      else
        $\mathcal{G}_s^i \leftarrow \mathcal{G}_s^{i-1} \setminus g$
  Algorithm 7 Steps 5 - 16.

only allowed access to the prices of the GUs they are currently bidding on. Once an SCA registers a lose, the MBS reveals all the prices of the GUs in its remainder set to that particular SCA. The favourite set can now be augmented with new favourite GUs in the remainder set $\mathcal{R}_s := \mathcal{P}_s \setminus \hat{\mathcal{G}}_s$. For as long as bidder does not exceed its budget, it is allowed to bid on the remainder set $\mathcal{R}_s$ whenever a lose occurs. The ASMRA is summarised in Algorithm 8.

6.4.3 The CAIB Algorithms

Two different CAIB under the second price mechanism are proposed. Even though the second price mechanism has dominant strategies under single-item auction, it is unlikely to expect the same property to hold under CA [171]. In [171], the authors analysed price of anarchy in a non-truthful combinatorial auction when bidders have subadditive (*id est*, submodular) valuations. In the proposed algorithms, it is assumed that the bidders are truthful. Firstly,a sequential CAIB (SCAIB), where the auctioneer runs separate CAIB in a sequential manner, is proposed. Secondly, a repeated CAIB (RCAIB), wherein the MBS runs a CAIB repetitively, is studied. Note that in SCAIB, once an SCA s has acquired GU g, it cannot be auctioned again unless it is a case such that $g \in \mathcal{R}_t, t \in \mathcal{C}_g \setminus s$. In contrast, in RCAIB environment, the MBS posit the winning bid to all potential bidders. Therefore, if any of the potential bidders is able to outbid the provisional winner, the GU is reassigned to the new provisional winner. This process is repeated until no new conditional bids are received from the SCAs.

Algorithm 9: SCAIB Algorithm

Data: Initialisation: $\mathbf{q}(g) = 0, \forall g \in G$, $(S, G) \in \mathbb{Z}_+$, GUs' set $\mathcal{G} := \cup_s^S \mathcal{G}_s$, assignment set $\mathcal{A}_s = \emptyset, \forall s \in \mathcal{S}$, lost items set $\mathcal{L}_s = \emptyset, \forall s \in \mathcal{S}$, $\mathcal{G}_s^i = \emptyset, \forall s \in \mathcal{S}$, *auction number*: $r = 0$.

Result: Optimal Allocation set $\mathcal{A}_s^\star, \forall s \in \mathcal{S}$.

1 **while** $\mathcal{T} \neq \emptyset || \cup_{s=1}^S \mathcal{G}_s^r \neq \emptyset$ **do**
2     $r \leftarrow r + 1$
3   MBS invites SCAs to submit bids, $\forall s \in \mathcal{T}$.
4   **if** $\mathcal{L}_s \neq \emptyset || r = 1$ **then**
5   Bidders determines new preference profiles and valuations for their favourite subset $\mathcal{G}_s^r$ (using (6.5)) of items in the remainder set $\mathcal{R}_s$ given the admitted GUs.
6   MBS collects bids from SCAs $\forall s \in \mathcal{T}$.
7   Auctioneer set $\mathcal{T} = \emptyset$, $\mathcal{L}_s = \emptyset$
8   **for** $g \in \cup_{s=1}^S \mathcal{G}_s^r$ **do**
9   **if** $|\mathcal{C}_g^r| > 1$ **then**
10   pick the current bidder s with the highest bid: $\mathcal{A}_s \leftarrow \mathcal{A}_s \cup g$
11             $\forall k \neq s$, $\mathcal{A}_k \leftarrow \mathcal{A}_k \setminus g$, contact only bidders who have submitted a bid $\mathcal{T} \leftarrow \mathcal{T} \cup \forall k \neq s$
12             $\mathcal{L}_k \leftarrow g, \forall k \in \mathcal{C}_g^r \setminus s$
          else
              $\mathcal{A}_s \leftarrow \mathcal{A}_s \cup g$, $\mathcal{G} \leftarrow \mathcal{G} \setminus g$

6.4.4 Sequential Combinatorial Auction With Item Bidding

Similar initialisations as in Algorithm 7 are carried out in Algorithm 9. Unlike in the SMRA, here the MBS does not post prices. Instead the MBS runs several single shot CAIB sequentially. In every CAIB, an SCA in the contact set sends bids on its conditional bidding set. An SCA can only bid on a GU at most once. In step 9, if a GU g has a competitors set such that $\mathcal{C}_g \neq \emptyset$, and it appears in at least one conditional bidding set $g \in \mathcal{G}_s, \forall s \in \mathcal{C}_g$, then it is provisionally assigned to the highest bidder at price equal the second highest bid. The provisional bidder will remain assigned to this GU if no competitor outbids it in the successive auction rounds. After losing some of its favourite GUs in the previous round, an SCA may advance some of the GUs from its reminder set to form an entirely new conditional bidding set. Due to the budget constraint, it is clear that the conditional bidding sets will diminish as the number of CAIB are being run. Once no new conditional bids are submitted the SCAIB halts.

Algorithm 10: RCAIB Algorithm

Data: See Algorithm 7.
Result: See Algorithm 7.
1 **while** $\mathcal{T} \neq \emptyset || \cup_{s=1}^{S} \mathcal{G}_s^i \neq \emptyset$ **do**
2 $\quad i \leftarrow i + 1$
3 $\quad$ MBS invites SCAs to submit bids, $\forall s \in \mathcal{T}$.
4 $\quad$ Algorithm 8 step 3 - 9.
5 $\quad$ Algorithm 9 steps 6 - 7
6 $\quad$ **for** $g \in \cup_{s=1}^{S} \mathcal{G}_s^i$ **do**
7 $\quad\quad$ Algorithm 9 steps 9 - 12
8 $\quad\quad \mathbf{q}^{i+1}(g) \leftarrow \texttt{argmax} \sum_{s \in \mathcal{C}_g^i} b_{sg}^r x_{sg}^r$

6.4.5 Repetitive Combinatorial Auction with Item Bidding

The RCAIB is summarised in Algorithm 10. The prices of all the GUs are initialised to zero. In the very first iteration, the MBS collects bids on bidders favourite sets. The MBS allocates a GU to the current highest bidder. In the successive iterations, the MBS use the current *standing highest bid* on a GU as the reserve price for that item. If an SCAs loses some of its favourites GUs, the marginal values of the accumulated GUs in its provisional set increases due to reduced interference. This creates capacity for a new conditional bidding set. In step 4 the SCAs determine their new bidding sets which may contain the GUs that were previously lost. Therefore it is possible for an SCA to recoup a GU after it was lost to a competitor. This process is repeated until no new conditional set is available.

6.5 Bidding Strategies

In order for an auction to accurately discover the market prices, truthful bidding should at least be guaranteed. In all auctions, unfaithfulness can manifest if any SCA has knowledge about the preference sets of its competitors. However, as it is difficult for any SCA to acquire preference profiles of other SCAs. This possibility is excluded in the mechanism design and analysis. Unfortunately, SMRA has two setbacks that may encourage bidders to deviate from their truthfully bidding strategies. In [172] it is claimed that if the valuation function of bidders satisfy gross-substitute condition, then truthful bidding becomes compatible with SMRA for any price trajectory. But due the *demand reduction* and *sniping* issues in the ascending auctions, truthful bidding is unlikely to occur. Demand reduction is a bidder request for fewer items in order to lower competition, hence maximises its utilities. Snipping is when a bidder observe the activities in the auction, without participation and then later makes an offer. In the SMRA, the bidder use the *local improvement method* wherein a bidder add, delete or replaces items. This strategy has proven to find the optimal demand set when the valuation functions are gross-substitutes [60]. In SMRA an SCA's strategy can be influenced by what it can infer from the auction history. In iterative auctions like the SMRA, the action sets of bidders can be history-dependent and as a result these sets get quite rich. The information learned by a bidder from his sequence of provisional sets of items, his sequence of conditional bids, and the price trajectory will have great influence on his bidding strategy function. The strategy of a bidder is defined as the function that maps the bidder's valuation to any of other bidders' possible actions.

Definition 6.7

Let $\mathcal{V}_1, \ldots, \mathcal{V}_S$ be the possible private valuations of the bidders. A strategy profile $\mathbf{s} = [s_1 \cdots s_S]$ is an ex-post Nash equilibrium (EPNE) if, for every SCA s and valuation $v_s \in \mathcal{V}_s$, the action $s(v_s)$ is the best-response to every action profile $\mathbf{s}_{-s}(\mathbf{v}_{-s})$ where $\mathbf{v}_{-s} \in \mathcal{V}_{-s}$ [50].

Now, the focus is to analyse if sincere bidding, as a strategy profile, can lead to an equilibrium. First, assume that all bidders are *ex-post individual-rational*. That is, bidders play risk-free strategies in order to avoid getting negative utilities. Authors in [178] argued that if bidders are ex-post individually-rational, and have submodular valuation functions, then every mixed Bayesian Nash equilibrium of a Bayesian auction (*id est*, auctions in which valuations are private) provides a 2-approximation[4] to the optimal social welfare.

6.5.1 Individual Rational Bidding

With the assumption of ex-post individual rationality, an SCA will never take action that will result in negative utility. Define a bidding strategy to be *supportive* if it is individual rational.

[4]An algorithm is referred to as an approximation algorithm if it produce a solution that is guarantee to be within a some approximation factor k. Therefore 2-approximation algorithm gives a solution within a factor of 2 of the optimum solution.

Definition 6.8

Given the s-th SCA with provisional set $\mathcal{A}_s^i$ at iteration i, a conditional bid $\mathcal{G}_s^i$ is secure if for any given $\mathcal{A}'_s \subseteq \mathcal{A}_s^i \cup \mathcal{G}_s^i$, it is a case such that $v_s(\mathcal{A}'_s) \geq q(\mathcal{A}'_s)$.

Proposition 6.2

Denote $\mathcal{A}_s^i$, $\mathcal{G}_s^i$, and $\mathbf{q}^i$ as the provisional set, the conditional bidding set, and the price profile at iteration $i \in \mathbb{Z}_+$ with $i \leq \hat{i}$. If an SCA makes a non-secure bid during iteration $\hat{i}$, then there exist secure SCAs who can bid consistently with the history such that SCA s gets a negative utility in the final allocation.

Proof 12 Assume all other SCAs other than SCA s bid sincerely. Suppose that the s-th SCA bid inconsistently on a particular GU g and finally acquires it[5]. Define the maximum possible marginal value of GU $g \in \mathcal{G}_s \setminus \mathcal{A}_s^i$ as $\hat{v}_{sg} = v_s(\emptyset \cup g)$. Suppose GU g has the highest preference amongst all other GUs in the set $\mathcal{G}_s^{\hat{i}}$, then the marginal value of GU g during iteration $\hat{i}$ is $v_{sg}^{\hat{i}} = v_s(\mathcal{A}_s^i \cup g) - v_s(g)$. Then the GU g will contribute maximum utility of $u_{sg} = v_{sg}^{\hat{i}} - \mathbf{q}(g)$ to the total utility $u_s = v_s(\mathcal{A}_s) - \sum_{g \in \mathcal{G}_s} \mathbf{q}(g)$ earned by SCA s. If bidder s bids inconsistently during iteration $\hat{i}$ with $\hat{v}_{sg} < \mathbf{q}(g)$, then there exist at least one SCA $t \in \mathcal{C}_g \setminus s$ who values GU g more. One of the following outcomes are feasible at the end of the auction:

$$u_s = \begin{cases} u_s^- = u_{sg}, & \text{if } \mathcal{A}_s^\star := g, \quad (6.24a) \\ u_s^-, & \text{if } \mathcal{A}_s^\star := \mathcal{G}_s^{\hat{i}} \cup g, |u_{sg}| > |u_{s\mathcal{G}_s^{\hat{i}}}|, \quad (6.24b) \\ u_s^+ \leq u_s^{\mathsf{c}}, & \text{if } \mathcal{A}_s^\star := \mathcal{G}_s^{\hat{i}} \cup g, |u_{sg}| \leq |u_{s\mathcal{G}_s^{\hat{i}}}|, \quad (6.24c) \end{cases}$$

where u_s^- means the utility is negative, u_s^+ means the utility is positive and u_s^{c} is the utility attained by bidding consistently and securely. The first case in (6.24a) follows immediately from the fact that $\hat{v}_{sg} < \mathbf{q}(g)$. The second case in (6.24b) suggests that if the absolute utility from winning GU g is greater than the absolute utility of other admitted GUs $\mathcal{G}_s$, then bidder s will get a negative utility. The third case in (6.24c) shows that bidder s might achieve a positive utility but the achievable utility cannot exceed that under consistent bidding. The proof shows that bidding securely is the best response for SCA s. ∎

[5]This proof can easily be extend to a case where the SCA bids inconsistently on a set of GUs. For simplicity, only one GU is chosen.

Proposition 6.2 carries forward to the ASMRA. Note that in ASMRA, if an SCA underbids on a particular GU so that it has access to the prices of the GUs in the remainder set, it is not allowed to rebid on that particular GU at a later stage. It will be a very risky move for an SCA to underbid on any of the GUs in the current conditional bidding set as the prices of the GUs in the remainder set may be very high.

Proposition 6.3

Truthful bidding is individual rationality in the SCAIB and the RCAIB.

Proof 13 The proof below is for the SCAIB but it can easily be extended to RCAIB. Consider an SCA s during auction round r with conditional bidding set $\mathcal{G}_s^r$. An SCA can have a truthful bidding function v_s^{c} that arranges the GUs in the set $\mathcal{G}_s^r$ according to their preference order and computes the truthful marginal values. On the other, the marginal values can be computed using another function v_s^{u}. For example, the function v_s^{u} could map the GUs to the values different from those that they will have when v_s^{c} is used, simply by changing their order of preference. Both valuation functions will have the following cases: SCA s

1. bids truthful and consistently according to the valuation function,
2. underbids on at least one of the GUs,
3. overbids on at least one of the GUs,
4. underbids and overbids on two different sets of GUs.

It is assumed that, there exists a set of competitors who have the conditional bidding set $\mathcal{G}_s^r$ as a subset of their conditional bidding sets. Fix a set $\mathcal{Y}_s^r \subseteq \mathcal{G}_s^r$ which contains the GUs that an SCA s underbids/overbids on. The following cases are possible:

- *Case 1:* If an SCA bids truthfully, and consistently using the valuation function v_s^{c}, it gets a utility of $u_s^{\mathsf{c}}(\mathbf{b}_s) = v_s^{\mathsf{c}}(\mathcal{A}_s(\mathbf{b}_s)) - B_{-s}^{\text{high}}(\mathcal{A}_s(\mathbf{b}_s), \mathbf{b}_{-s})$.
- *Case 2:* If an SCA underbids with $\underline{\mathbf{b}}_s$, then there is a possibility that another SCA will outbid it on some of the GUs in $\mathcal{Y}_s^r$. Since the allocation is monotone in $\mathbf{b}$, then $\mathcal{A}_s(\underline{\mathbf{b}}_s) \subseteq \mathcal{A}_s(\mathbf{b}_s)$. Note that if an SCA loses some of the GUs $\mathcal{Y}_s^{r\mathsf{L}} \subset \mathcal{Y}_s^r$, then the marginal values of the remaining GUs in $\mathcal{G}_s^r \setminus \mathcal{A}_s(\underline{\mathbf{b}}_s)$ will increase. Denote the total increase of the marginal values by ϵ_s^r, and the utility contributed by the set

$\mathcal{Y}_s^{r\mathsf{L}}$ as $u_s(\mathcal{Y}_s^{r\mathsf{L}})$. If $\epsilon_s^r > u_s(\mathcal{Y}_s^{r\mathsf{L}})$, the auction may suffer from *demand reduction*. Demand reduction is more prominent in iterative and sequential auctions wherein bidders use history reliant strategies for their next move. Fortunately the SCAIB uses one shot auctions in a sequential manner, therefore use of demand reduction could be very risky. Moreover, in SCAIB, if the SCAs use demand reduction to maximise their profit during auction r, they will simulate a reduction on their budget, which is learned by the MBS in the first auction round, thereby reducing the number GUs they can bid on in the forthcoming auction rounds. The attained utility for underbidding is $u_s^{\mathsf{c}}(\underline{\mathbf{b}}_s) = v_s^{\mathsf{c}}(\mathcal{A}_s(\underline{\mathbf{b}}_s)) - B_{-s}^{\text{high}}(\mathcal{A}_s(\underline{\mathbf{b}}_s), \mathbf{b}_{-s})$.

- *Case 3:* If an SCA overbids with $\bar{\mathbf{b}}_s$, then there is a possibility that it could outbid its competitors on some of the GUs in $\mathcal{Y}_s^r$, and attain a set $\mathcal{Y}_s^{r\mathsf{W}} \subseteq \mathcal{Y}_s^r$ in the allocation set $\mathcal{A}_s(\bar{\mathbf{b}}_s)$ such that $\mathcal{A}_s(\underline{\mathbf{b}}_s) \subseteq \mathcal{A}_s(\mathbf{b}_s) \subseteq \mathcal{A}_s(\bar{\mathbf{b}}_s)$. The utility contributed by the set $\mathcal{Y}_s^{r\mathsf{W}}$ is $u_s(\mathcal{Y}_s^{r\mathsf{W}})$. By overbidding, the utility of SCA s is $u_s^{\mathsf{c}}(\bar{\mathbf{b}}_s) = v_s^{\mathsf{c}}(\mathcal{A}_s(\bar{\mathbf{b}}_s)) - B_{-s}^{\text{high}}(\mathcal{A}_s(\bar{\mathbf{b}}_s), \mathbf{b}_{-s})$.

- *Case 4:* Suppose an SCA submits a bid profile $\bar{\underline{\mathbf{b}}}_s$ with underbids and overbids. First, denote the sets which an SCA s underbids and overbids on as $\underline{\mathcal{Y}}_s^r \subset \mathcal{Y}_s^r$ and $\bar{\mathcal{Y}}_s^r \subset \mathcal{Y}_s^r$, respectively. Further, denote the sets of GUs that an SCA s lose and win for underbidding and overbidding as $\underline{\mathcal{Y}}_s^{r\mathsf{L}}$ and $\bar{\mathcal{Y}}_s^{r\mathsf{W}}$, respectively in the allocation set $\mathcal{A}_s(\bar{\underline{\mathbf{b}}}_s)$. The resulting utility is given by $u_s^{\mathsf{c}}(\bar{\underline{\mathbf{b}}}_s) = v_s^{\mathsf{c}}(\mathcal{A}_s(\bar{\underline{\mathbf{b}}}_s)) - B_{-s}^{\text{high}}(\mathcal{A}_s(\bar{\underline{\mathbf{b}}}_s), \mathbf{b}_{-s})$.

At the end of the auction round, the pay-off received by SCA s is given in (6.32). Underbidding can lead to *demand reduction* as shown in (6.32a). Note that an SCA cannot improve its utility by overbidding during a particular auction round. In (6.32f) and (6.32h), if $|u_s(\mathcal{Y}_s^{r\mathsf{W}})| > |u_s(\mathcal{A}_s(\bar{\mathbf{b}}_s)|$ or $|u_s(\bar{\mathcal{Y}}_s^{r\mathsf{W}})| > |u_s(\mathcal{A}_s(\bar{\underline{\mathbf{b}}}_s)|$ then the utility of an SCA will be negative. Therefore the latter strategies are not *supportive*. Failure to maximise the utility during a particular auction round by unfaithful bidding will result in more utility loss in the forthcoming rounds because of the increased prices on the GUs and the decreasing valuations. When an SCA uses the bidding function v_s^{u}, then an SCA will have a combination of overbidding and underbidding. Following the same arguments stated above, there will be no improvement on the utility by unfaithful bidding. ■

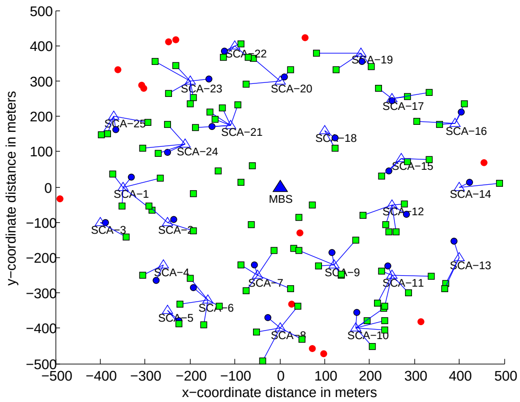
Since overbidding is not individual rational, *strong no-overbidding assumption* used in [178, 179] is assumed.

$$
u_s^r = \begin{cases}
u_s^{\mathsf{c}}(\underline{\mathbf{b}}_s) > u_s^{\mathsf{c}}(\mathbf{b}_s), & \text{if } \mathcal{A}_s(\underline{\mathbf{b}}_s) \subset \mathcal{A}_s(\mathbf{b}_s), \epsilon_s^r > u_s(\mathcal{Y}_s^{\mathsf{rL}}), \quad (6.32a) \\
u_s^{\mathsf{c}}(\underline{\mathbf{b}}_s) < u_s^{\mathsf{c}}(\mathbf{b}_s), & \text{if } \mathcal{A}_s(\underline{\mathbf{b}}_s) \subset \mathcal{A}_s(\mathbf{b}_s), \epsilon_s^r < u_s(\mathcal{Y}_s^{\mathsf{rL}}), \quad (6.32b) \\
u_s^{\mathsf{c}}(\underline{\mathbf{b}}_s) = u_s^{\mathsf{c}}(\mathbf{b}_s), & \text{if } \mathcal{A}_s(\underline{\mathbf{b}}_s) := \mathcal{A}_s(\mathbf{b}_s), \quad (6.32c) \\
 & \text{or if } \mathcal{A}_s(\underline{\mathbf{b}}_s) \subseteq \mathcal{A}_s(\mathbf{b}_s), \epsilon_s^r = u_s(\mathcal{Y}_s^{\mathsf{rL}}), \quad (6.32d) \\
u_s^{\mathsf{c}}(\bar{\mathbf{b}}_s) = u_s^{\mathsf{c}}(\mathbf{b}_s), & \text{if } \mathcal{A}_s(\bar{\mathbf{b}}_s) := \mathcal{A}_s(\mathbf{b}_s), \quad (6.32e) \\
u_s^{\mathsf{c}}(\bar{\mathbf{b}}_s) < u_s^{\mathsf{c}}(\mathbf{b}_s), & \text{if } \mathcal{A}_s(\mathbf{b}_s) \subset \mathcal{A}_s(\bar{\mathbf{b}}_s), u_s(\mathcal{Y}_s^{\mathsf{rW}}) < 0, \quad (6.32f) \\
u_s^{\mathsf{c}}(\underline{\bar{\mathbf{b}}}_s) < u_s^{\mathsf{c}}(\mathbf{b}_s), & \text{if } \mathcal{A}_s(\underline{\bar{\mathbf{b}}}_s) \subset \mathcal{A}_s(\mathbf{b}_s), \epsilon_s^r < u_s(\underline{\mathcal{Y}}_s^{\mathsf{rL}}), \bar{\mathcal{Y}}_s^{\mathsf{rW}} := \emptyset, \quad (6.32g) \\
 & \text{or if } \mathcal{A}_s(\underline{\bar{\mathbf{b}}}_s) \subset \mathcal{A}_s(\mathbf{b}_s), \epsilon_s^r \leq u_s(\underline{\mathcal{Y}}_s^{\mathsf{rL}}), \\
 & \quad u_s(\bar{\mathcal{Y}}_s^{\mathsf{rW}}) < 0, \quad (6.32h) \\
 & \text{or if } \mathcal{A}_s(\underline{\bar{\mathbf{b}}}_s) \subset \mathcal{A}_s(\mathbf{b}_s), u_s(\bar{\mathcal{Y}}_s^{\mathsf{rW}}) < 0, \underline{\mathcal{Y}}_s^{\mathsf{rL}}; = \emptyset, \quad (6.32i) \\
u_s^{\mathsf{c}}(\underline{\bar{\mathbf{b}}}_s) = u_s^{\mathsf{c}}(\mathbf{b}_s), & \text{if } \mathcal{A}_s(\underline{\bar{\mathbf{b}}}_s) := \mathcal{A}_s(\mathbf{b}_s), \quad (6.32j) \\
 & \text{or if } \epsilon_s^r = u_s(\underline{\mathcal{Y}}_s^{\mathsf{rL}}), \bar{\mathcal{Y}}_s^{\mathsf{rW}} := \emptyset, \quad (6.32k) \\
u_s^{\mathsf{c}}(\underline{\bar{\mathbf{b}}}_s) > u_s^{\mathsf{c}}(\mathbf{b}_s), & \text{if } \mathcal{A}_s(\underline{\bar{\mathbf{b}}}_s) \subset \mathcal{A}_s(\mathbf{b}_s), \epsilon_s^r > u_s(\underline{\mathcal{Y}}_s^{\mathsf{rL}}), \bar{\mathcal{Y}}_s^{\mathsf{rW}} := \emptyset, \quad (6.32l) \\
 & \text{or if } \mathcal{A}_s(\underline{\bar{\mathbf{b}}}_s) \subset \mathcal{A}_s(\mathbf{b}_s), \epsilon_s^r > u_s(\underline{\mathcal{Y}}_s^{\mathsf{rL}}), \\
 & \quad \epsilon_s^r > |u_s(\bar{\mathcal{Y}}_s^{\mathsf{rW}})|. \quad (6.32m)
\end{cases}
$$

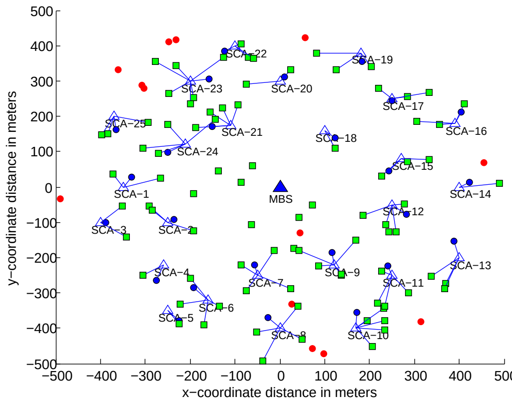
6.6 Numerical Example

The setup used in Section 5.5 is used here. In order to reduce the computational complexity and the system overheads, the price increment in the SMRA and ASMRA were adapted using $\delta = 0.001 \times (\text{target data rate})/(0.5)[\text{bits/s/Hz}]$. The model parameters are summarised in Table 5.2.

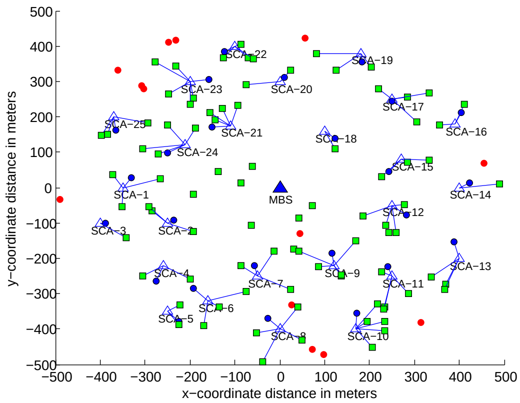
The results in Figure 6.2 gives the topographical overview of the network after running the SMRA, the ASMRA, the SCAIB and the RCAIB algorithms when the target QoS of all the MUs was set to 8 bits/s/Hz. The green squares and the red dots indicate the locations of the admitted MUs and the dropped MUs, respectively. The HUs locations are indicated by the blue dots. Those users served by the SCAs are explicitly indicated by connecting the users with the corresponding SCA using blue lines. All admitted users without connecting lines are served by the MBS. Figure 6.2a and Figure 6.2b show the results of the SMRA and the ASMRA, respectively. Since the ASMRA is the sub-optimal version of the SMRA, it is observed that the ASMRA sometimes to fail to associate users to the closest SCAs. This is observed between SCA-1 and SCA-3, between SCA-22 and SCA-23, and between SCA-21 and SCA-24. The reason is that in the ASMRA, since



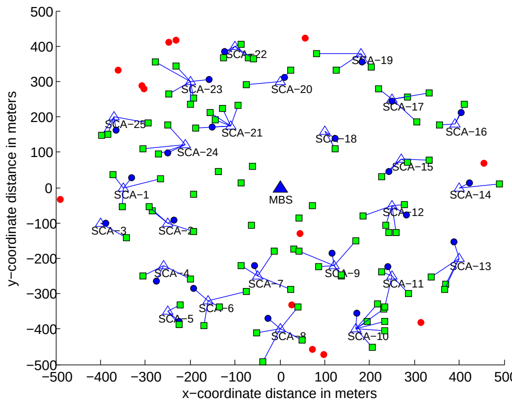
(A) User allocation under SMRA.



(B) User allocation under ASMRA.



(C) User allocation under SCAIB.



(D) User allocation under RCAIB.

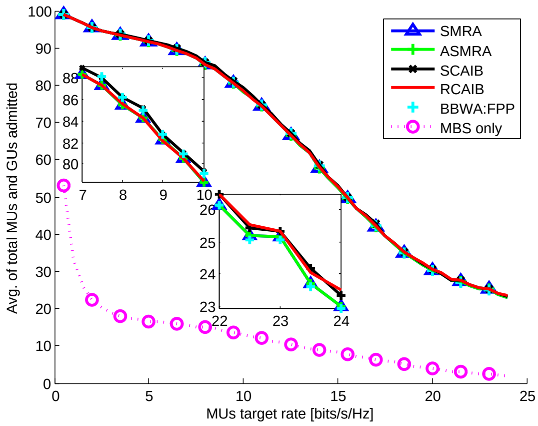
FIGURE 6.2: Comparison of user association under different auctions.

an SCA is confined to bid on a particular set of GUs until it experiences a lose, by the time it bids on the GUs in its remainder set, the cheapest GUs may be further away.

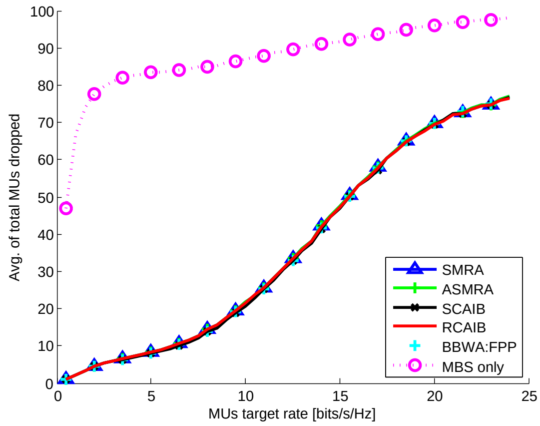
The user admission results for the SCAIB and the RCAIB are shown in Figure 6.2c and Figure 6.2d. In these figures, another pattern of user association is observed. Even though these two algorithms provide a very similar user association pattern, a difference is observed between SCA-21 and SCA-24. It is further observed that for this particular channel realisation, all four algorithms admit the same set of users, but the user association to various SCAs may differ. These close performances are explained by the sub-modularity and gross-substitute characteristics of the valuation functions.

6.6.1 General Performance of the proposed Algorithms

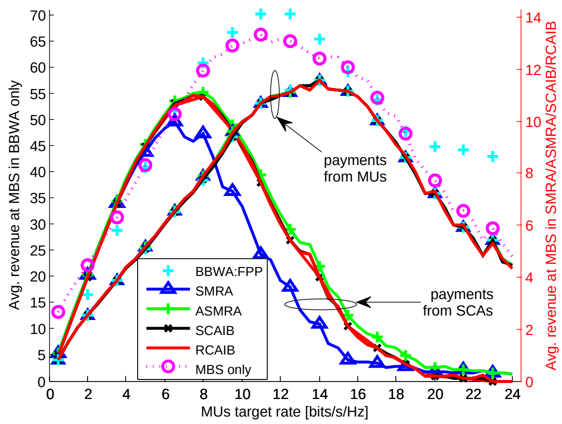
In Figure 6.3, the performances of the SMRA, the ASMRA, the SCAIB and the RCAIB are averaged over 20 random channel realisations. Since the BBWA with FPP



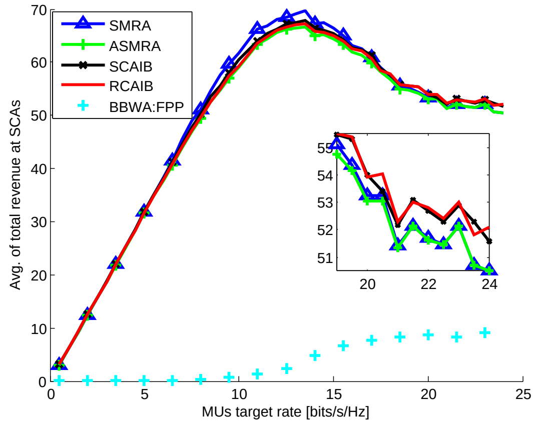
(A) Average of total MUs/GUs admitted by the SCAs and the MBS.



(B) Average number of dropped MUs.



(C) Average revenue generated by the MBS.



(D) Average revenue generated by the SCAs.

FIGURE 6.3: Average performance of the proposed BBWA and FBWA for 20 channel realisations. There are 100 MUs and 25 SCAs.

criterion is the recommended solution from Chapter 5, it is used here as a benchmark. Figure 6.3a shows the average of the total admitted MUs/GUs by the SCAs and the MBS. The dotted line shows the average admitted MUs in the absence of auctioning. The solid lines depicts the performance of the proposed algorithms. It is observed that there is a huge improvement on user admission when the SCAs are taking part in the auction. Even though the performances of SMRA, ASMRA, SCAIB and RCAIB are relatively close, it is observed that SCAIB always gives highest user admission performance at lower target rates. At higher target data rates, the two CAIB algorithms outperforms SMRA and ASMRA algorithms. This is because in CAIB algorithms, the MBS and the SCAs learn the market price of a particular GU earlier, hence providing the SCAs with a chance to explore other cheaper GUs earlier. The average performance of SMRA and ASMRA are equal.

In comparison with the BBWA with FPP criterion that was proposed in Chapter 5, it is observed that the BBWA with FPP criterion matches the performance of the SCAIB at lower targets rates. At higher target rate, the BBWA with FPP criterion matches the performance of the SMRA and the ASMRA. The Figure 6.3b depicts the total number of dropped MUs. As expected, and for the same reasons stated earlier, CAIB algorithms have the lowest dropped number of users as compared to SMRA and ASMRA.

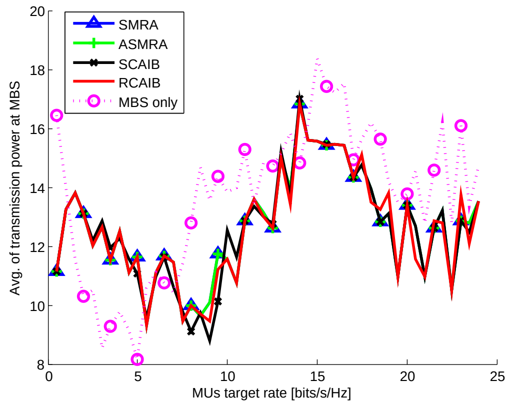
Figure 6.3c illustrates the revenue generated by the MBS from the payments made by the MUs and the SCAs. The left y-axis is for the BBWA with FPP criterion and the right y-axis is for the various algorithms proposed in this chapter. The MUs' payments are explicitly from the MUs that are admitted by the MBS. Though very minimal, the differences between the revenues earned from MUs, under the proposed algorithms, suggest that the sets of MUs left behind after auctioning are different from one auction to the other. From the graphs depicting the revenues generated from the SCAs' payments, the SCAs make the lowest payments to the MBS under SMRA and more payments under ASMRA. This is because by forcing the SCAs to commit to a bidding set until there is a lose, ramps up the competition and ultimately increases payments for bidder under ASMRA. Note that by conducting an auction, the MBS generates more revenue as compared to when it greedily serves the MUs alone. The BBWA with FPP criterion always generate more income to the MBS. This is because the set from which the critical bid (or payment) is being pulled from (*id est* the competitors' set) is very rich in the BBWA with FPP criterion.

The revenues generated by the SCAs under the SMRA, ASMRA, SCAIB and RCAIB algorithms, are illustrated in Figure 6.3d. At lower target data rates, the revenues earned by the SCAs under all the proposed algorithms are almost equal. It is noted that in SMRA, the SCAs are able to generate highest revenue in the data rate range from 6 to 16 bits/s/Hz. This is due to the local improvement method which is not present in SCAIB. The effect of the local improvement method in RCAIB is weaker, and much weaker in ASMRA, hence depriving the SCAs from maximising their profits especially at moderate target rates. Nonetheless, it is observed that the revenue generated under CAIB and SCAIB is more than that of SMRA and ASMRA at lower and higher target data rates when the competition is respectively higher or lower. This due to the quick price discovery in CAIB algorithms, which allows SCAs to quickly discover GUs with less competition, and lower prices. Ultimately the payments to the MBS are reduced.

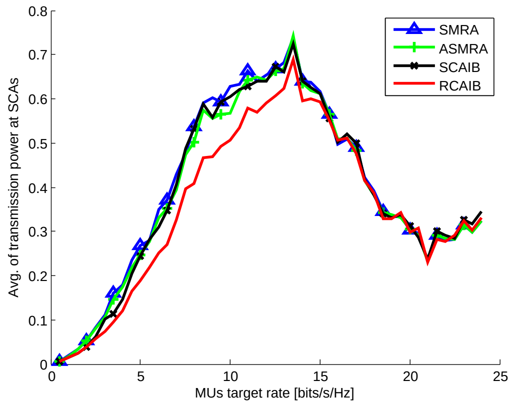
It is observed that ASMRA always generate the lowest revenue. This is because the SCAs are not allowed to explore other opportunities until they experience a lose on the set they are bidding on. Since in the BBWA with FPP criterion, bidders make more payments to the MBS, their profit is very low as compared to the SMRA, ASMRA, SCAIB and RCAIB algorithms.

Figure 6.4a and Figure 6.4b show the average transmission powers at the MBS and SCAs, respectively, under the utilisation of SMRA, ASMRA, SCAIB, and RCAIB. In Figure 6.4a, SMRA and the ASMRA lead to the same transmission power. This is because these two algorithms perform equally in terms of the admitted and the dropped GUs, respectively. The transmission power of the MBS under SMRA and the ASMRA are comparable to that of RCAIB at lower target rates. At high target data rates, the transmission power of SMRA and ASMRA are comparable to that of SCAIB. Note that when there is no auctioning, the MBS will utilise more power relative to the admitted MUs. In Figure 6.4b, RCAIB has the least transmission power, while SMRA has the most transmission power. This suggests that the under RCAIB, the SCAs chose the GUs that are closer to them while under SMRA, the SCAs chose the GUs that are further away from them. This reveals the effect of the local improvement method in SMRA, which allows the SCAs to identify cheaper GUs. Usually GUs that are further away are likely to have lower prices. The performance of ASMRA and SCAIB, in terms of the transmission power, lies between that of SMRA and RCAIB.

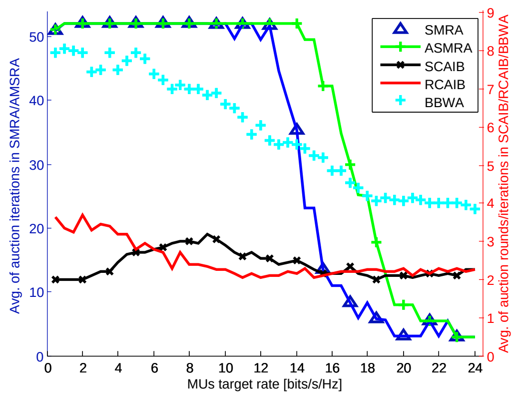
Figure 6.4c and Figure 6.4d show the number of auction rounds/iterations and the system overheads under each auction. The overheads are measured in terms of the number of invitations for bidding, number of bids submitted and the number of announcements made. SMRA and ASMRA use the left y-axes, while SCAIB and CAIB use the right y-axes. In Figure 6.4c, the number of iterations/rounds reduces as the target data rates are increased. This is because at high target data rates, the GUs (mainly those further away from the SCAs) get less attractive, which induce decoupled preference sets. Ultimately the SCAs will drop out of the auction quickly, thereby increasing the convergence rate. Note that in SMRA and ASMRA, the smaller the price increment δ, the higher the number of iterations. This will be even worse when the values of the GUs are increased. Earlier, it was specified that δ is adapted using $\delta = 0.001 \times (\text{target data rate})/(0.5)$[bits/s/Hz]. If the price increment is fixed to $\delta = 0.001$, the number of iterations required in SMRA and ASMRA can reach 10^4, at



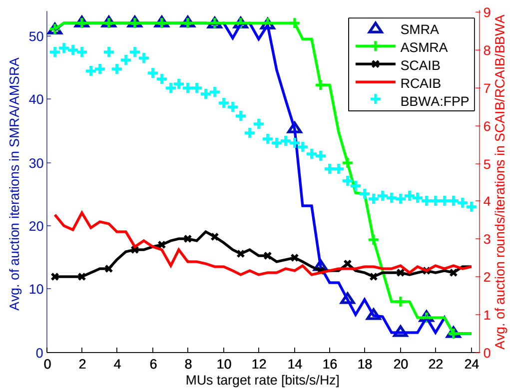
(A) Average of total tranmission power at the MBS.



(B) Average of total tranmission power at the SCAs.



(C) Average auction rounds.

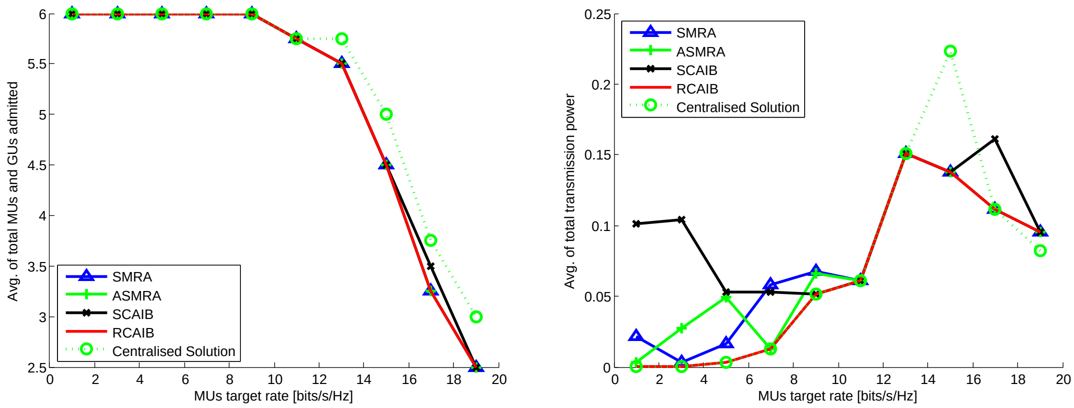


(D) Average system overheads incurred.

FIGURE 6.4: Comparison of SMRA, ASMRA, SCAIB, and RCAIB in terms of number of auction iterations/rounds and system overheads.

target rate between 5 bits/s/Hz and 10 bits/s/Hz. In Figure 6.4c the highest iterations required is 52. SCAIB and RCAIB registered maximum of 3.5 auction rounds/iterations. This is for the same reason explained earlier that in CAIB algorithms, the rate of price discovery is very high, therefore the auctions quickly reach the WE. The same course is observed in Figure 6.4d. SMRA and ASMRA have large system overheads as compared to SCAIB and RCAIB. This is because in SMRA and ASMRA, the price are increased with a very small value in every iteration.

Due to the one bid per round rule in the BBWA with FPP criterion, it is observed that the BBWA with FPP criterion requires more auction rounds as compared to SCAIB and RCAIB. But since the SCAIB and the RCAIB use combinatorial bids, they incur more system overheads as compared to the BBWA with FPP criterion. As a remark, it is observed that the number iterations/rounds are linked to the system overheads. But,



(A) Average GUs admitted by the SCAs.

(B) Average transmission power at the SCAs.

FIGURE 6.5: Comparison of SMRA, ASMRA, SCAIB, and RCAIB and the centralised solution.

since the SMRA and the ASMRA use the same payment rule, it is observed that the ASMRA incur less system overheads than the SMRA even when the number of iterations are identical. This is due to the altering rule in the ASMRA that prevents bidders from exploring other GUs until there is a loss in the provisional bidding set $\mathcal{G}_s$.

The performance given above clearly shows that by using auctioning mechanism to offload users from the macrocell to the SCAs, there is gain in user admission and revenue. From the results, CAIB algorithms outperform SMRA and ASMRA in almost all performance measurements. Though SMRA generates more profit for the SCAs, its benefits are overshadowed by the costs incurred in system overheads and computational complexity. In terms of surplus maximisation, SCAIB is the most preferred. Since SCAIB always generate the second highest revenues for the SCAs, the SCAs will prefer SCAIB.

6.6.2 Optimality of the Proposed Algorithms

The proposed algorithms were compared to a centralised solution proposed in [149], which uses the branch-and-bound (BnB) method to solve the admission and user association problems. In order to reduce the computational burden for the centralised system, a network with 6 GUs and 2 SCAs is considered. The system parameters of the SCAs and the GUs remain unaltered. Figure 6.5a shows the user admission performance of the auctions proposed here and the centralised solution. All the proposed algorithms match the performance of the centralised solution at lower target data rates. At higher data rates, all the proposed algorithms attain reduced user admission as compared to the centralised solution. This is because in auction based algorithms, the SCAs objective is to maximise their profits, whereas in the centralised solution, the objective of the SCAs is user maximisation. In Figure 6.5b, it is observed that at lower target rates, SCAIB transmission power is higher than that of the other auctions. This is for the same reasons explained earlier. Therefore, SCAIB has a shortfall in associative efficiency at lower target data rates. At higher data rates all the proposed algorithms utilise the same amount of power for the same number of admitted GUs. Moreover, as expected, it is noted that the centralised solution, is more power efficient as compared to the proposed algorithms[6][7].

6.7 Conclusion

This chapter has investigated a joint offloading and downlink beamformer problem in heterogeneous networks. The offloading problem was formulated as a combinatorial auction which readily provides decentralised algorithms. Various algorithms namely; SMRA, ASMRA, SCAIB, and RCAIB algorithms have been proposed to offer incentivised offloading mechanisms. In these algorithms, the SCAs design downlink beamformers during their valuations. The analysis proved the existence of the Walrasian equilibrium for the proposed valuation functions. SCAIB algorithm is the most preferred algorithm since it provides high admission rate and competitive revenues for the SCAs. The proposed algorithms match the performance of the centralised solution at low target data rates. The following chapter summarises this thesis and discusses future works.

[6]As mentioned in Section 5.5.2, increasing the system dimension may increase the performance gap between the centralised solution and the proposed algorithms.

[7]As highlighted in Chapter 5, using throughput as the performance metric would result in different performance for the proposed algorithms. This is part of the future works.

Chapter 7

Conclusions

7.1 Conclusion

The focus of this thesis has been on the development and analysis of distributed optimisation techniques for wireless networks. Distributed algorithms are very vital in large systems, as they relief the backhaul links from overhead informations. In addition, distributed algorithms decentralise the computation tasks amongst transmitters, hence reducing the computational complexity experienced in centralised systems. Game-theoretic models, decomposition techniques, and auction theory, have been used to decouple the optimisation problems under consideration. This thesis has addressed strategic non-cooperative games (SNGs), cooperative games, traffic offloading, admission control, user association, and downlink beamformer design in wireless networks. The main findings and contributions are as follows:

- In Chapter 3, novel game theoretic and dual decomposition frameworks for a wireless network with users having different classes of quality of service (QoS) were studied. The mixed QoS criterion is attractive for energy efficient wireless communication as suggested in [91]. This work addressed situations where access points (APs) can simultaneously be confronted by both power minimisation and SINR balancing problems. This multi-objective optimisation (MOP) is addressed by the mixed QoS criterion. A mixed QoS SNG and bargain game were proposed. It was shown that, in comparison to the conventional Nash equilibrium (NE), the NE-mixed QoS is capable of attaining higher sum rate with less power.

- Chapter 4 proposed and analysed a novel mixed QoS SNG for a multicell multiuser network, with RTUs and NRTUs. The mixed QoS SNG offers a fully distributed algorithm for downlink beamformer design. It was shown that the mixed QoS SNG reaches an inefficient NE-mixed QoS. Due to the possibility of non-convergence of the mixed QoS SNG, a fall back mechanism was proposed. In addition, players were assumed to apply their last known feasible beamformers whenever infeasibility arises during the course of the game. In order to improve the performance of the NE-mixed QoS, mixed QoS EBG and mixed QoS KSBG algorithms were proposed. Via numerical analysis and comparative evaluations, it was shown that the mixed QoS sum rate of the bargaining games is comparable to that of the centralised solutions.

- A novel auction, called the bid-wait-auction (BWA), that jointly performs downlink beamformer design and user association, was proposed and analysed in Chapter 5. Moreover, a novel payment rule, that allows the BWA to allocate items to bidders with sparse information, was proposed and analysed. It was shown that the proposed payment rule preserve some of the principles of the VCG mechanism. It was shown that the BWA has dominant strategy equilibrium at every auction round, which decomposes the combinatorial nature of the problem, thereby allowing sequential and parallel auctions to manifest autonomously. Numerical analysis revealed the proposed BWA is close to optimum. In terms of admission rate, it was shown that the backward (BBWA) is preferred over the forward (FBWA).

- In Chapter 6, two SMRA based algorithms were proposed, to address the offloading and user association problems. The first algorithm directly applies the classical SMRA, which is used in spectrum auctioning. A novel altered SMRA (ASMRA) was proposed to reduce the valuation overheads incurred by the bidders in SMRA. In addition, two novel CAIB algorithms were proposed; namely, the sequential CAIB (SCAIB) and the repetitive CAIB (RCAIB). SCAIB and RCAIB utilise the VCG payment rule. In RCAIB, the auctioneer posit the standing highest bids to competitors. Since the advertisement of the highest bids to competitors can encourage retaliations, the SCAIB avoids this problem. It was shown that truthful bidding is the best response for every bidder. An analysis on how truthful bidding leads to a Walrasian equilibrium (WE) was conducted. Through numerical analysis, it was demonstrated that the proposed algorithms match the centralised solution at low target rates of the GUs. Numerical results showed that, by auctioning its users to the SCAs, the MBS gain both in user admission and revenue. On the other hand, the SCA gain in terms of revenue and spectral efficiency.

- In contrast with the state of the art distributed optimisation techniques in the literature, this thesis has proposed distributed algorithms with heterogeneous users. In addition, auction based algorithms proposed here are able to reflect the economic implications on the choice of optimisation approaches. All the auction based algorithms proposed here can be directly used in networks with both private and operator owned small cells. Hence, the proposed solution are scalable to large/different systems.

7.2 Future Works

While working towards achieving the objectives of this thesis, other challenges and ideas that are worthwhile to be investigated further came up. These challenges and ideas are elaborated below.

- Game-theoretic approach was used in Chapter 3 and Chapter 4 to develop distributed algorithms. The mixed QoS criterion was used to demonstrate how game theory can be used to solve MOP problems. In this work, the MOP problem includes power minimisation and SINR balancing. It would be interesting to add more objectives to the mixed criterion. This may include: sum rate and proportional fairness utilities.

- It is worth considering other types of games to solve the mixed QoS problems in Chapter 3 and Chapter 4. Games that can be applied to solve the mixed QoS problem include; hierarchical games (*exempli gratia*, Stackelberg game), coalition games, and Bayesian games.

- Chapter 5 and Chapter 6 considered an economy with no production. It is worth investigating the same user offloading problem, while considering an economy with production. For example, the production rate can be captured as the rate of incoming traffic.

- Similar to the mixed QoS criterion used in Chapter 3 and Chapter 4, it is worth investigating differential objectives for the bidders and the auctioneer in Chapter 5 and Chapter 6. Furthermore, a multiuser and frequency diversity would make significant contribution. This would change the behaviour of the bidders and the equilibrium of the market.

- Extension of the auctions in Chapter 3 and Chapter 4 to a setting with multiple auctioneers and multiple bidders will bring substantial contributions. The auction can then be analysed for cases when the auctioneers are synchronised (cooperating) or unsynchronised (non-cooperative). Bidders can also be allowed to form coalitions to reduce competition. Further, an analysis wherein bidders collude is very crucial.

- The valuation functions used by the bidders are computationally intensive, as they use interior point methods. This resulted in prolonged simulation times. In practice, prolonged simulations would induce delays in decision making; hence, it may take longer to reach equilibrium. In addition, due to the mobility of users, delays in bidding can result in outdated valuations, and consequently inefficient equilibria. It will be worth considering estimation models for bidder valuations functions. Moreover, the assumption of independent valuation can be lifted, by allowing interference amongst bidders.

- The items considered in Chapter 5 and Chapter 6 are substitutes. It would be interesting to study and analyse a market with complementary items, or a combination of complementary and substitutes items. Complementary items can manifest when there are users who request the same data. If the transmitters are content aware, they can identify complementary items. Complementary effects can be investigated by considering items with positive synergies.

- The entire thesis concentrated on downlink transmission. Formulating uplink problems and applying the same methods may present different mathematical analysis. Also, it would be very crucial to consider offloading users from the macrocells to the WiFi systems.

Appendix A

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