FIXED POINT PERFORMANCE APPROXIMATIONS FOR SLOTTED RING NETWORKS

by

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A Doctoral Thesis

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CERTIFICATE OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this thesis, that the original work is my own except as specified in acknowledgements or in footnotes, and that neither the thesis nor the original work contained therein has been submitted to this or any other institution for a higher degree.

K.R.S. Rodrigo

To my parents, brother and sister.

ABSTRACT

The purpose of this research is twofold - the first objective being to develop Markovian models that can be used to analyze the performance of the various medium access control protocols of slotted ring type local area networks.

Although at the present time slotted rings do not feature in any international standard, the motivation for this work is that previous research has shown that such networks can perform exceptionally well in high speed, short packet length environments of the type used for transmitting integrated services such as mixes of voice, video and computer data. It is likely therefore that slotted rings, or networks based on these, will feature in the next generation of local area networks whose ability to efficiently transmit integrated services will be of prime importance.

Various medium access control protocols are modelled and analyzed, these being the source deletion protocol, the destination deletion protocol, and an extension of the latter for carrying integrated services, the Orwell protocol.

The second objective of the research is concerned with the modelling techniques used, which are based on discrete-time Markov chains. For the majority of cases, an exact solution to these models is intractable and the size of the state space is often such as to make a direct numerical solution prohibitively expensive. The approach taken is to use either station based or slot based models and to reduce the solution to finding the fixed point of some (in general) non-linear operator in a suitably chosen space. This fixed point approach is studied in some detail via its application to the slotted ring models, and the effects of various simplifying assumptions on the accuracy of the solutions is examined. This is done by comparing the results of the models with simulations.

Perhaps the major restriction associated with the models used is that the network's stations must be statistically identical. This restriction is removed in the latter part of the thesis when the method is extended to handle networks with stations that are statistically different. This is done by interpreting the models used as discrete-time queuing networks with state dependent routing and different customer classes. The generality of this approach is demonstrated by applying it to slotted Aloha type networks in addition to slotted rings.

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Last, but not least, I thank my mother, brother and sister for their love and support without which this research would never have been realised.

ABBREVIATIONS

ANSI	- American National Standards Institute
ATM	- Asynchronous Transfer Mode
CSMA/CD	- Carrier Sense Multiple Access / Collision Detection
DDSlotBMx $\{1 \leq x \leq 2\}$	- Destination Deletion, Slot Based Model 'x'
DDStnBMx $\{1 \leq x \leq 4\}$	- Destination Deletion, Station Based Model 'x'
EPA	- Equilibrium Point Analysis
FDDI	- Fibre Distributed Data Interface
IEE	- Institution of Electrical Engineers
IEEE	- Institution of Electrical and Electronic Engineers
ISDN	- Integrated Services Digital Network
LAN	- Local Area Network
MAC	- Medium Access Control
OSI	- Open Systems Interconnection
OSlotBM	- Orwell Slot Based Model
OStnBM	- Orwell Station Based Model
PRP	- Packet Rejection Probability
SAEM	- Slotted Aloha Extended Model
SARM	- Slotted Aloha Recursive Model
SAStnBM	- Slotted Aloha Station Based Model
SATM	- Slotted Aloha Tasaka's Model
SDSlotBM	- Source Deletion, Slot Based Model
SDStnBMx $\{1 \leq x \leq 4\}$	- Source Deletion, Station Based Model 'x'
SREM	- Slotted Ring Extended Model
SRRM	- Slotted Ring Recursive Model

NOTATIONS

$\bar{d}$	=	Average delay (channel access delay)
$\bar{d}_i$	=	Average delay (channel access delay) for station i
Di	=	The maximum number of calls a station in the Orwell ring is currently supporting
Dmax	=	Maximum number of calls a station is allowed to support in the Orwell ring
K	=	Total number of stations in the network less one
L	=	Total bits per slot
L_d	=	Class 1 data bits per slot
$m^i_{1,2..k}(K)$	=	Probability of station i being in the idle state (Slotted Aloha networks)
M	=	Total number of slots in the network
N	=	Total number of stations in the network
p	=	Packet arrival probability (Slotted Ring networks) or retransmission probability (Slotted Aloha networks), the units of which depends upon the model
p_i	=	Packet arrival probability (Slotted Ring networks) or retransmission probability (Slotted Aloha networks) of station i
PRP	=	Packet Rejection Probability
PRP_i	=	Packet Rejection Probability of station i
P(T,y,x)	=	Steady state number of stations occupying state (T,y,x)
P(W,y,x)	=	Steady state number of stations occupying state (W,y,x)
$P(x)_i$	=	Steady state probability of station i occupying state x
P(y,x)	=	Steady state probability of being in state (y,x)
$\bar{q}$	=	Average queue
$\bar{q}_i$	=	Average queue of station i
Q	=	Maximum buffer size per station (station based models) or average

maximum buffer size per slot (slot based models)

R = Network transmission rate

R_1 = Class 1 data arrival rate

S = Probability that a station in a non-transmitting state will find the next slot full

S'_i = Probability with which the next passing slot would appear full to station i when that station is not transmitting

$S_{1,2,..k}(K)$ = Throughput due to 1,2,...k stations in a K station network

$S^{(i)}_{1,2..k}(K)$ = Throughput contribution of station i in a K station network having the stations 1,2,...k

Throughput_i = Throughput of station i

$T_{1,2..k}(K)$ = Probability that at least on of the stations 1,2,..k of a K station network will attempt to transmit in the next slot (Slotted Aloha networks)

(T,y,x) = Transmitting state with a remaining transmission time of y time units (station times), and x packets in the buffer

(W,y,x) = Waiting state with y time units (station times) before the next slot header arrives and having x number of packets in the buffer

(y,x) = The state after x number of time units since the start of transmission and y number of packet in the buffer

σ = Packet arrival probability (Slotted Aloha networks)

σ_i = Packet arrival probability for station i (Slotted Aloha networks)

CONTENTS

CHAPTER 1

Introduction

1.0 Local Area Networks - A Brief Introduction

A Local Area Network (LAN), has been defined by Stallings (Stallings 87) to be "... a communications network that provides interconnection of a variety of data communication devices within a small area."

According to Edwards (Edwards 89), the development of LANs can be categorised into three generations based upon their transmission speeds and provision of facilities. These are -

- First Generation LANs: with speeds up to 10 Mbits/s, networking equipment such as fileservers, printers, etc., or distributed computer systems. e.g. Ethernet, Token Ring.

- Second Generation LANs: having transmission rates around 100 Mbits/s, capable of handling voice in addition to the services supported by the first generation LANs.

- Third Generation LANs: capable of speeds of 1 Gbits/s or higher, with enough capacity to support multimedia data transmission including real-time video. These networks are still under active development.

The topologies under which the stations may be interconnected can be broadly classified into four - bus, star, ring and tree networks, or a combination of thereof. Several access methods are available for each topology, the effectiveness of which depends on the packet length, transmission speed, delay tolerance of the data, etc.

1.1 Literature Survey

Since the advent of computer based communication networks (Abramson 73, Clark 78, Klessig 86), many simulation and mathematical modelling studies have been made for the various topologies under which they may be interconnected, as well as for the different access protocols that may be used under each such method. Such research include studies of the Aloha or Broadcast Bus type protocols (Ganz 89, Mukherjee 88, Ramana 82), ring networks such as the Token Ring (Bux 83), Slotted Ring (Arem 90a, King 87, Lee 91, Mitrani 87), as well as the Cambridge Ring (Harrus 85, Mitrani 84, Sorensen 85, Wilkes 79) - a practical network based on the former.

Kamal and Hamacher (Kamal 90), have suggested a more efficient extension to the basic Slotted Ring protocol where a station is allowed to use any number of slots repeatedly to transmit a given message. To prevent hogging, the station has to release those slots after completing the transmission of that particular message before attempting to transmit another message.

Studies have also been made to investigate the effect of having multiple rings on a token, slotted or register insertion type ring network (Bhuyan 89).

Due to the emergence of high speed networking hardware as well as the use of optical fibres as the transmission medium, several studies (Bux 81, Falconer 85, Friedman 89, Limb 84, Pattavina 88, Rodrigues 90, Shepherd 82) have been made on the relative merits of different network architectures and medium access protocols which enable the transmission of multimedia (EMmC 90, Feldman 91) information that include isochronous messages such as voice and video, in addition to the non-isochronous data. Littlewood et al (Littlewood 87) have listed and analysed the important characteristics such networks should possess. It has been found that ring type networks, especially Slotted Rings (Zafirovic 88), can perform exceptionally well under short packet length, high speed environments since they can provide bounded medium access delay for any data packet in a finite buffered station.

It is noted that the above studies do not consider the effects of signal compression, bandwidth saving due to the multiplexing of several channels (Chin 89, Johnson 85), and/or techniques such as layered video coding (Ghanbari 89). In this latter method, the video signals are coded into different classes (levels) and only the first layer need to be transmitted to create the basic picture, the other layer(s) only used to enhance the picture quality. Thus, the network need give guaranteed bandwidth only to the level 1 signals.

Even though such techniques will not, most probably, affect a comparison of two protocols or network topologies as long as all networks under study ignore (or take into consideration) such methods, when considering the suitability of a specific network in carrying integrated services, it is felt that attention should be given to such details.

In order to guarantee an upper bound to the delay suffered by isochronous data, a number of advanced medium access protocols for ring type networks have been suggested and analysed. For the token based rings, these include the FDDI and FDDI II protocols (Dykeman 88, Jayasumana 90, Johnson 87, Ross 89, Sevcik 87, Takagi 90), as well as other algorithms such as the ones suggested by Pattavina (Pattavina 88a) using multiple tokens, and by Ibe and Gibson (Ibe 86) where a "transmission window scheme" and a token with priority and reservation fields are used. In another paper (Ibe 86a) the latter two authors have also studied the possibility of using non-reservation and reservation schemes for voice and data transmission.

For the slot based rings, the Cambridge Fast Ring (Hopper 88), the MAGNET (Lazar 85), the LOCOST traffic scheduling scheme (Limb 87), the Orwell protocol (Adams 84, Arem 90, Falconer 85a, Gallagher 86, Lee 91, Mitrani 86) and its possible extensions, for example, using a movable boundary scheme (Woodward 91a), etc. have been suggested. A dynamic bandwidth allocation system for integrated services based upon a station, termed the "Head End station", responsible for generating slots has been suggested by Li (Li 88). However, such a mechanism does away with the distributed control that is present in the basic Slotted Rings. The possibility of transmitting voice and data on a Slotted Ring (source deletion) operating on a frame structure has been investigated by Abedin et al (Abedin 86), and, Takiyasu and Tanaka (Takiyasu 89) have studied the effects of slot concatenation and reuse of the same slot during low traffic periods, specifically with regard to the transmission of integrated services.

Investigations have also been made on the feasibility of transmitting voice and data through Broadcast Bus networks (Mukherjee 88a, Sharrock 89). Goel and Elhakeem (Goel 85) examined the possibility of achieving the same with the use of a "Frame Adaptable Reservation Aloha with Carrier Sensing" (FARA/CS) and CSMA/CD for voice and data transmission respectively.

In addition to the above mentioned publications on specific networks, of which some dealt in the aspect of mathematical modelling, several others, dealing with queueing networks in general, or in specific classes of such networks, are of particular interest with regard to this research.

These include the studies made by Bharath-Kumar (Bharath 80) and Walrand (Walrand 83) on discrete time queueing networks. In Ganz 90, slotted communication networks with finite buffer capacities have been analysed and Sykas et al (Sykas 86) have proposed a scheme that may be used to model several slotted multiple-access protocols. Similarly, the study carried out by Daduna and Schaßberger (Daduna 83) on discrete time queueing networks is worthy of reference. Sriram et al (Sriram 83) have modelled voice and data transmission using a movable boundary frame allocation system with and without speech activity detectors. In Kamal 89, a non-exhaustive multiserver polling system has been analysed and applied to slotted and partial-insertion rings.

In Lavenberg 80 and Sevcik 81, it was independently shown that "... the stationary state probabilities at instants at which customers of a particular type arrive at a particular service center and enter a particular class are equal to the stationary state probabilities at arbitrary times for the network with one less customer." (Lavenberg 80) This result, known as the Arrival Theorem, is of particular importance to some of the models in this thesis, especially those in chapter 7, where two algorithms will be developed to analyse statistically non-identical stations.

In addition to the above publications, several books were consulted, both for general background reading, or, in some cases, on specific areas of interest. These include books on communication network protocols and analysis (Edwards 89, Hammond 86, Held 91, Judge 88, Marsden 91, Mitrani 87, Schwartz 87, Stallings 87, Stallings 89, Tanenbaum 81, Tasaka 86), simulation (Mitrani 82, Schoemaker 82), as well as on basic queueing theory (Gnedenko 89, Hall 91, Kleinrock 75, Turin 90), probability and statistics (Daellenbach 83, Gray 67, Howard 60, Lipschutz 68), and, programming and numerical methods (Hume 83, NAG 87, Press 86, Reynolds 86, Williams 79).

1.2 Modelling Methods Used In The Thesis

A convenient way of analysing the performance of a time-slotted, multiple access protocol is to develop a model for the protocol based on a discrete-time Markov chain (Kleinrock 75a). In most practical cases, however, the state description is multidimensional, and an explicit solution of the balance equations is not feasible;

one must then proceed either numerically, or use some form of approximation to obtain a solution. All models in this thesis are concerned with methods that fall into the latter category, where the equilibrium distribution of the Markov chain is approximated by a Dirac delta function located at a point in the state space where the system is in equilibrium (an equilibrium point).

One such method is the equilibrium point analysis (EPA) (Tasaka 86). The method requires the solution of a set of coupled, non-linear equations, known as the equilibrium point equations, which can usually be reduced to a fixed-point equation for some parameter of interest. This, in turn, can be solved by the usual methods for such equations (iteration, bisection, etc.).

Another approximation method that has been widely used is the Fixed Point Approximation Method (Harrus 85, Lee 91, Mitrani 84). This scheme consists of solving the Markov chain for a single network station in isolation, the effects of the rest of the network being allowed for by the introduction of some global parameter(s), the value(s) of which is(are) then determined. To evaluate the network parameters, all the stations of the network are superimposed.

It is of interest to note that these two methods have been shown to be equivalent under the constraints that users in the latter method are independent and statistically identical (Woodward 91). This fact will illustrated in chapter 2 by showing how a model based on a single station can be easily converted into one of the type mentioned in Tasaka 86.

All models in this thesis will be based on discrete time techniques, most of which follow a discretised version of Mitrani's Fixed Point Approximation Method (Mitrani 84), which was originally specified in continuous time.

1.3 Objectives Of This Research

As suggested by the title of this thesis, one of the main aims of the research is to develop Markovian models to analyse the performance of various medium access protocols for the Slotted Ring.

As of to date, there has only been very little research done in modelling networks based on observing them from different subsystems. A second objective of this thesis to redress this deficiency by using Slotted Rings to illustrate how a given network may be modelled by viewing it from a user's (station's) or server's (slot's) point of view, or, globally.

1.4 Organisation Of The Thesis

The thesis will be organised in the following manner.

In chapter 2, a brief introduction to the Slotted Rings will be given and four source deletion models based on observing the network from a station (station based models) presented. Of these, the second and the third will be simplified versions of the first and the degradation of performance due to this will be analysed. A matrix method, suitable for solving models which have state dependent transition probabilities, will be suggested and used to solve the fourth model. It will also be shown how a station based model may be easily converted to a model based on observing the network globally. The results of all models will be validated with the use of simulations.

A slot based model for a Slotted Ring with a source deletion protocol will be developed in chapter 3 and its performance compared to simulation results obtained in the previous chapter.

Prior to modelling practical destination deletion protocols, chapters 4 and 5 will be used to investigate the basic destination deletion concept based on a station and a slot respectively. Two possible methods of calculating this destination deletion probability will be examined. As in chapter 2, the possibility of simplifying the main models and the cost of such an action on the performance will be analysed.

Two models for the Orwell protocol, based on a station and a slot respectively, will be described in chapter 6. These models will be the extended versions of the models developed for the destination deletion protocol in the previous two chapters. As usual, the analytical results will be compared to the simulation results.

In chapter 7, two algorithms which enable the modelling techniques used, to be extended to networks with statistically non-identical users, will be proposed and analysed. These algorithms will be based on interpreting the models used as discrete-time queueing networks. To show their generality, they will be applied to a Slotted Aloha network in addition to the Slotted Ring (source deletion) network, and the results examined in relation to simulations.

Finally, chapter 8 will summarise the work carried out in this thesis and suggest further areas for research.

CHAPTER 2

Slotted Ring Source Deletion Protocol : Station Based Models

2.0 Introduction

A Ring Network (Hammond 86, Stallings 87, Schwartz 87, Tanenbaum 81) consists of a number of stations interconnected in the manner shown in figure 2.0.1, the connecting media between the stations being twisted pair, coaxial or fibre optic cables.

In a Slotted Ring network (e.g. the Cambridge Ring (Wilkes 89)), the bits contained in the ring (due to the delays at the stations and possible repeaters as well as the bits that are in the cables), are grouped in fixed numbers to form the slots.

Each slot can be broadly divided into two main sections (figure 2.0.2) the control bits and the data bits, the former, containing bits for the source and destination addresses of the packet contained in the slot, a bit to indicate the Full/Empty state of the slot, bits for the response of the destination station, parity checking, monitoring, slot header and the trailer.

A station wishing to transmit will attempt to do so in the next empty slot that passes, by marking it as full and loading the appropriate information and data

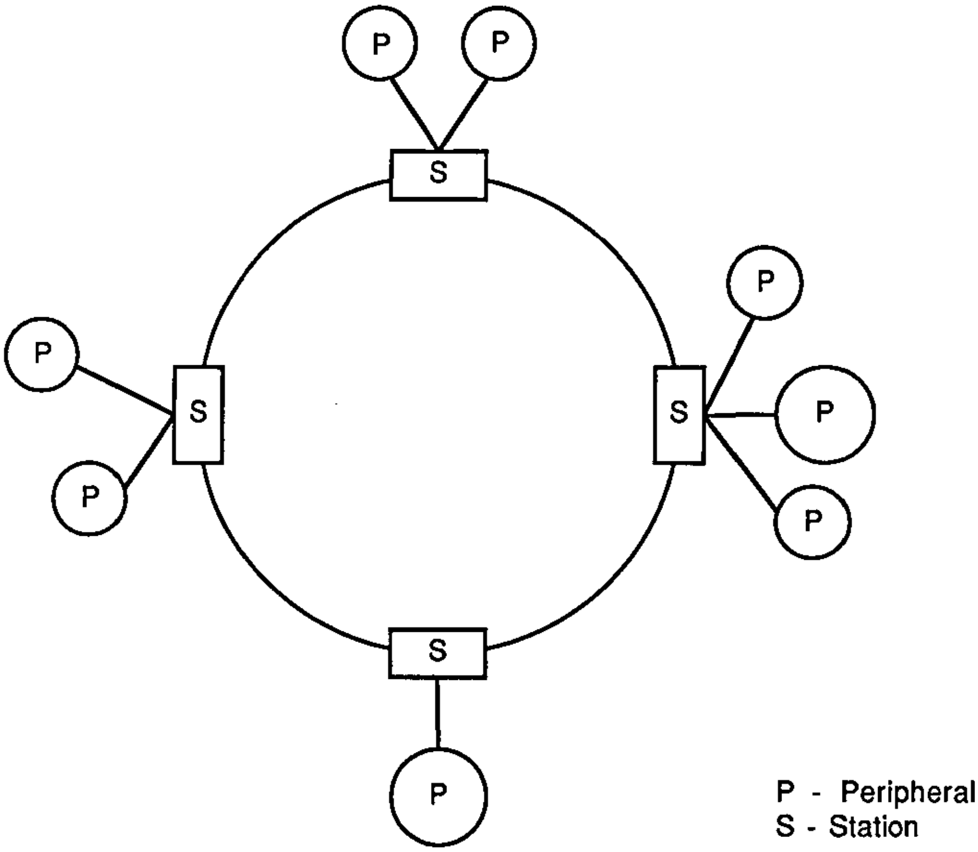


Fig 2.0.1 Ring Network Structure

Header	Destination Address	Source Address	Data	Other Control bits	Trailer

Fig 2.0.2 Slot Structure

bits. In the case of Source Deletion Protocols, the packets are deleted from the slots only by the stations that transmitted them.

The response bits from the destination station with regard to the packet in a specific slot will enable the transmitting station to decide upon the action it should take with the copy of that packet that has been retained.

To prevent hogging, a station is not allowed to reuse the slot it has just deleted.

In this chapter, four models will be presented for a Slotted Ring with a Source Deletion Protocol, all the models being based upon the network being observed from a station. The models in sections 2.1 and 2.2 will assume that the packet is removed from the main buffer as soon as it is transmitted (It may be stored elseware in case of a necessity to retransmit.), whereas the models of sections 2.3 and 2.5 will assume that the packet will not be removed from the main buffer until its transmitted copy has traversed around the ring. The model in section 2.2 is a simplified version of the one in section 2.1 and will greatly reduce the solving procedure involved. It will be shown that both these models give close results, thus justifying the use of the second simplified model in place of the first.

The model of section 2.5 is an extension to that in section 2.1 and will be presented by considering a segment of the network. This model will also be used to show how a station based model can be easily converted into a model based upon the network being observed globally.

To prevent repetition, the assumptions that are common to all the models in this chapter are as follows -

(2.0.1) Each data packet will fit exactly into a slot; as such, if there are no message receiving failures, only one slot will be needed to transmit a packet.

(2.0.2) No conditions will arise that would necessitate the retransmission of a packet, or, broadly speaking, the receiving station will never be too busy to reject a packet and no transmission errors would occur.

(2.0.3) All the stations are statistically identical and independent. They are distributed evenly around the ring.

(2.0.4) All slots are independent and identical. This also implies that any gap between any two consecutive slots is a constant.

(2.0.5) The packet buffer at a station is at least unity; if not, the model would change slightly to accommodate that fact although the basic concepts would remain the same.

(2.0.6) A packet in any given slot is deleted after it has circulated once around the ring by the station that transmitted it (Source Deletion).

(2.0.7) A station is not allowed to use the slot it has just emptied in the same cycle.
This is used in real systems to prevent hogging.

(2.0.8) A station is not allowed to have more than one packet under transmission at any given time.

(2.0.9) The number of stations is greater than, or equal to the number of slots, and is a large integer.

(2.0.10) State transitions take place just before the end of a time unit, and are completed just after beginning of the next time unit.
Here, the time unit used will be based in terms of slot times or station times.

(2.0.11) The state is observed immediately after state transitions.

(2.0.12) Packets arrive at the start of a time unit.

Note that a slot time is defined as the amount of time required for a given slot to occupy the position of the next slot downstream. In other words, this is equal to the time required by a slot to complete one cycle around the ring divided by the total number of slots in the network.

Similarly, the station time is the time needed by a slot adjacent to a given station to reach the next downstream station. Therefore, this is equal to the time a slot takes to complete a single cycle divided by the total number of stations in the network.

2.1 Source Deletion, Station Based Model 1 (SDStnBM1)

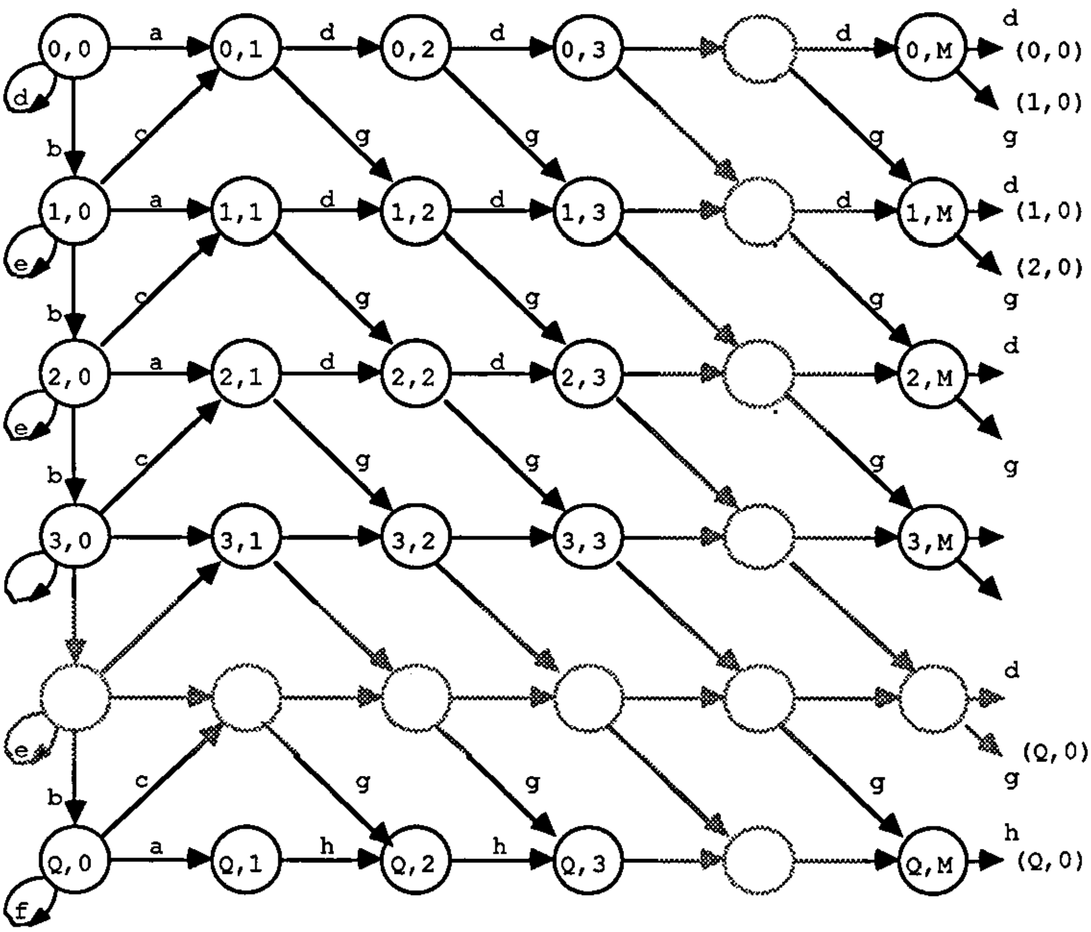
The following additional assumptions will be made -

(2.1.1) Only a single packet of data may arrive at any station per given slot time.

(2.1.2) Packet arrivals at a station have a Bernoulli distribution with parameter 'p', which has units of packets per station per slot time.

(2.1.3) A packet is deleted from the buffer at the start of its transmission.

The Markov state diagram for the model is shown in figure 2.1.1. The two values shown inside each state represent the queue size of the station and the number of time units (slot times in this model) that have elapsed since the start of the transmission respectively. The letters beside the arrows show the probability of



where - $a = p(1-S)$, $b = pS$, $c = (1-p)(1-S)$, $d = 1-p$, $e = (1-p)S$, $f = S$, $g = p$, $h = 1$

Fig 2.1.1 State transition diagram for a station based, discrete-time Markov model of a Slotted Ring with a Source Deletion Protocol. (SDStnBM1)

a transition in that direction within the next time unit.

Let,

N = Total number of stations in the network

M = Total number of slots in the network

Q = Maximum buffer size per station

S = Probability that a station in a non transmitting state will find the next slot full.

P(y,x) = Steady state probability of the station being in state (y,x) where y is the queue size of the station at that state and x is the number of slot times that have passed since the start of transmission.

A station in state (0,0) will start to transmit in the next slot time by moving to state (0,1) if there is a packet arrival and if the next slot is empty. Since these two events are independent, their intersection is the product of the two probabilities. i.e.,

$$a = p(1-S)$$

If there is a packet arrival, but the next slot is busy, then the station can be represented as transiting to state (1,0) with a probability 'b', where,

$$b = pS$$

With no packet arrival, it will remain in the same state (0,0) with a probability 'd', where,

$$d = 1-p$$

Similarly, if the next slot is empty, a station in a state (y,0) $\{0<y\leq Q\}$, will start transmitting in the next slot time by moving onto states (y-1,1) or (y,1) with probabilities 'c', where,

$$c = (1-p)(1-S)$$

or 'a' respectively. In the event of the next slot being busy, and the buffer of that station not being full, it will either remain in the same state with a probability 'e', where,

$$e = (1-p)S$$

or move vertically down in the state diagram to state (y+1,0) with probability 'b'. However, if the buffer is full and the next slot is busy, the station will remain in the same state with probability 'f', where,

$$f = S$$

A station in state (y,x) $\{0 \leq y < Q, 0 < x < M\}$ will, in the next slot time, move either to the state (y,x+1) or (y+1,x+1) with probabilities 'd' or 'g' respectively, where,

$$g = p$$

When the buffer is full and the station is transmitting, i.e., (Q,x) states with $\{0 < x < M\}$, no more new packets can be accepted for transmission. Therefore, the station will always move to the state (Q,x+1) in the next time unit.

i.e.,

$$h = 1$$

At the end of transmission (i.e. all (y,M) states), the station will go into an idle state (y+1,0) or (y,0) depending upon whether there is a packet arrival or not.

The few remaining unexplained transitions may be interpreted in a similar manner.

Assuming a steady state,

$$\textit{total probability of entering a given state} = \textit{total probability of exiting that state}$$

Using this principle and substituting for state transition probabilities, the following balance equations may be derived for the different states -

$$P(0,0)p = P(0,M)(1-p) \tag{2.1.1}$$

$$P(y,1) = P(y,0)p(1-s) + P(y+1,0)(1-p)(1-S) \quad \{0 \le y < Q\} \tag{2.1.2}$$

$$P(0,x) = P(0,x-1)(1-p) \quad \{2 \le x \le M\} \tag{2.1.3}$$

$$P(y,x) = P(y,x-1)(1-p) + P(y-1,x-1)p \quad \{0 < y < Q,\ 2 \le x \le M\} \tag{2.1.4}$$

$$P(y,0)[1-(1-p)S] = P(y,M)(1-p) + P(y-1,0)pS + P(y-1,M)p \quad \{0 < y < Q\} \tag{2.1.5}$$

$$P(Q,0)(1-S) = P(Q,M) + P(Q-1,0)pS + P(Q-1,M)p \tag{2.1.6}$$

$$P(Q,1) = P(Q,0)p(1-S) \tag{2.1.7}$$

$$P(Q,x) = P(Q,x-1) + P(Q-1,x-1)p \quad \{2 \le x \le M\} \tag{2.1.8}$$

From equations (2.1.1) and (2.1.3),

$$P(0,1) = P(0,0)\frac{p}{(1-p)^M} \tag{2.1.9}$$

then, from (2.1.1),

$$P(0,M) = P(0,0)\frac{p}{(1-P)} \tag{2.1.10}$$

from (2.1.2),

$$P(y,0) = \frac{P(y-1,1) - P(y-1,0)p(1-S)}{(1-p)(1-S)} \quad \{1 \le y \le Q\} \tag{2.1.11}$$

from (2.1.5),

$$P(y,M) = \frac{P(y,0)[1-(1-p)S] - P(y-1,0)pS - P(y-1,M)p}{(1-P)} \quad \{0 < y < Q\} \tag{2.1.12}$$

Long and tedious simplification of equation (2.1.4) shows that P(y,x) {0≤y<Q,

$2 \leq x \leq M\}$ has the following relationship -

$$P(y,x) = \sum_{j=0}^{y} P(j,1)\, p^{(y-j)} (1-p)^{(x-1-y+j)}\ {}^{(x-1)}C_{(y-j)} \qquad (2.1.13)$$

However, for the purpose of the thesis, this equation will be proved by induction.

Assume that the relationship (2.1.13) holds true for the states $(\alpha-1,\beta-1)$, $(\alpha,\beta-1)$. Then,

$$\begin{aligned}
P(\alpha,\beta) &= P(\alpha-1,\beta-1)p + P(\alpha,\beta-1)(1-p) \\
&= p\sum_{j=0}^{(\alpha-1)} P(j,1)\, p^{(\alpha-1-j)} (1-p)^{(\beta-1-\alpha+j)}\ {}^{(\beta-2)}C_{(\alpha-1-j)} + \\
&\qquad (1-p)\sum_{j=0}^{\alpha} P(j,1)\, p^{(\alpha-j)} (1-p)^{(\beta-2-\alpha+j)}\ {}^{(\beta-2)}C_{(\alpha-j)} \\
&= \sum_{j=0}^{\alpha-1} P(j,1)\, p^{(\alpha-j)} (1-p)^{(\beta-1-\alpha+j)} \left[{}^{(\beta-2)}C_{(\alpha-1-j)} + {}^{(\beta-2)}C_{(\alpha-j)}\right] \\
&\qquad - \left[P(j,1)\, p^{(\alpha-j)} (1-p)^{(\beta-1-\alpha+j)}\ {}^{(\beta-2)}C_{(\alpha-j)}\right]_{j=\alpha} \\
&= \sum_{j=0}^{\alpha} P(j,1)\, p^{(\alpha-j)} (1-p)^{(\beta-1-\alpha+j)}\ {}^{(\beta-1)}C_{(\alpha-j)}
\end{aligned}$$

Considering all (y,2) states,

$$\begin{aligned}
P(0,2) &= P(0,1)(1-p) \\
P(1,2) &= P(1,1)(1-p) \quad + P(0,1)p \\
&\cdots \\
P(Q-1,2) &= P(Q-1,1)(1-p) + P(Q-2,1)p
\end{aligned}$$

Thus, equation (2.1.13) can be said to hold true for all (y,x) states where $0 \leq y < Q$ and $x=2$. Therefore from induction, it may be said that (2.1.13) is true for all $0 \leq y < Q$ and $2 \leq x \leq M$.

From (2.1.13),

$$P(y,1) = \frac{P(y,M) - \sum_{j=0}^{y-1} P(j,1)\, p^{(y-j)} (1-p)^{(M-1-y+j)}\, {}^{(M-1)}C_{(y-j)}}{(1-p)^{(M-1)}} \qquad (2.1.14)$$

$$\{0<y<Q\}$$

Calculation of P(y,0) values

If the values of P(0,0) and S are known, the following sequence may be used to calculate all the other P(y,0) values-

(2.1.9) to obtain P(0,1)

(2.1.10) to obtain P(0,M)

the following loop to be executed repeatedly for all $1<y<Q$,

(2.1.11) to obtain P(y,0)

(2.1.12) to obtain P(y,M)

(2.1.14) to obtain P(y,1)

(2.1.11) to obtain P(Q,0)

Considering the sum of all the state probabilities,

$$\sum_{y=0}^{Q} \sum_{x=0}^{M} P(y,x) = 1 \qquad (2.1.15)$$

As it may be seen from figure 2.1.1, a station in a state (y,x) $\{0<x<M\}$ will always move to one of the states (y,x+1) or (y+1,x+1). Therefore, we can say that as long as the station is in steady state equilibrium,

$$\sum_{y=0}^{Q} P(y,x) = \sum_{y=0}^{Q} P(y,x+1) \qquad \{1 \le x < M\}$$

therefore, using equation (2.1.15),

$$\sum_{y=0}^{Q} P(y,0) + \sum_{y=0}^{Q} P(y,1)M = 1$$

substituting for P(y,1) from (2.1.2) and (2.1.7),

$$P(0,0) = \frac{1 - [1+M(1-S)]\sum_{y=1}^{Q} P(y,0)}{[1+Mp(1-S)]} \tag{2.1.16}$$

$$\begin{array}{l} \textit{Probability that a station} \\ \quad \textit{is transmitting} \end{array} = 1 - \sum_{y=0}^{Q} P(y,0)$$

Therefore, the probability of a given station in a non-transmitting state seeing the next slot full, is the probability that one of the other N-1 stations are transmitting in that slot (Harrus 85). Thus,

$$S = \frac{(N-1)}{M}\left(1 - \sum_{y=0}^{Q} P(y,0)\right) \tag{2.1.17}$$

Both equations (2.1.16) and (2.1.17) are functions of P(0,0) and S. (Note that P(y,0) is a function of P(0,0).) Therefore, an iterative method may be used to solve them.

A simple iterative method that may be used is as follows -

Step 1 - Guess an initial value for P(0,0) and S.

Step 2 - Apply the latest P(0,0) and S in equation (2.1.16) find the new value of P(0,0).

Step 3 - Substitute the latest P(0,0) and S in equation (2.1.17) find the new value of S.

Step 4 - Repeat Steps 2 and 3 until the values of P(0,0) and S converge onto an acceptable level of error.

It is noted that in order to carry out steps 2 and 3, the sequence of calculating the P(y,0) values mentioned above has to be used.

Once the solution for P(0,0) and S have been found, they may be used to calculate the rest of the steady state probabilities.

Since S is the probability of a slot being filled by one of N-1 stations,

$$Throughput = \frac{N}{(N-1)} S \qquad (2.1.18)$$

$$Average\ Queue\ (\overline{q}) = \sum_{y=0}^{Q} \left(y \sum_{x=0}^{M} P(y,x) \right) \quad (packets) \qquad (2.1.19)$$

From Little's result,

$$Average\ Queue = Mean\ Arrival\ Rate \times Waiting\ Time$$

therefore,

$$Delay\ (\overline{d}) = \frac{N\ \overline{q}}{Throughput} + 1 \quad (slot\ times) \qquad (2.1.20)$$

Since the packet is removed from the buffer as soon as it is transmitted, the delay calculated here is the channel access delay, the '+ 1' being due to the fact that any packet has to wait at least one slot-time before transmission (due to assumption 2.0.12).

$$\underset{(given\ packet\ arrival)}{Packet\ Rejection\ Probability} = \frac{Arrival\ Rate - Service\ Rate}{Arrival\ Rate}$$

$$PRP = \frac{NpM - Throughput \times M}{NpM}$$

therefore,

$$PRP = 1 - \frac{Throughput}{N\,p} \tag{2.1.21}$$

Another way to calculate the same measure is to obtain the probability of a packet arrival when the buffer of the station is full. Then,

$$\begin{aligned} PRP &= P(Q,0)\,pS + p\sum_{x=1}^{M} P(Q,x) \\ &= p\left(P(Q,0)\,S + \sum_{x=1}^{M} P(Q,x)\right) \end{aligned} \tag{2.1.22}$$

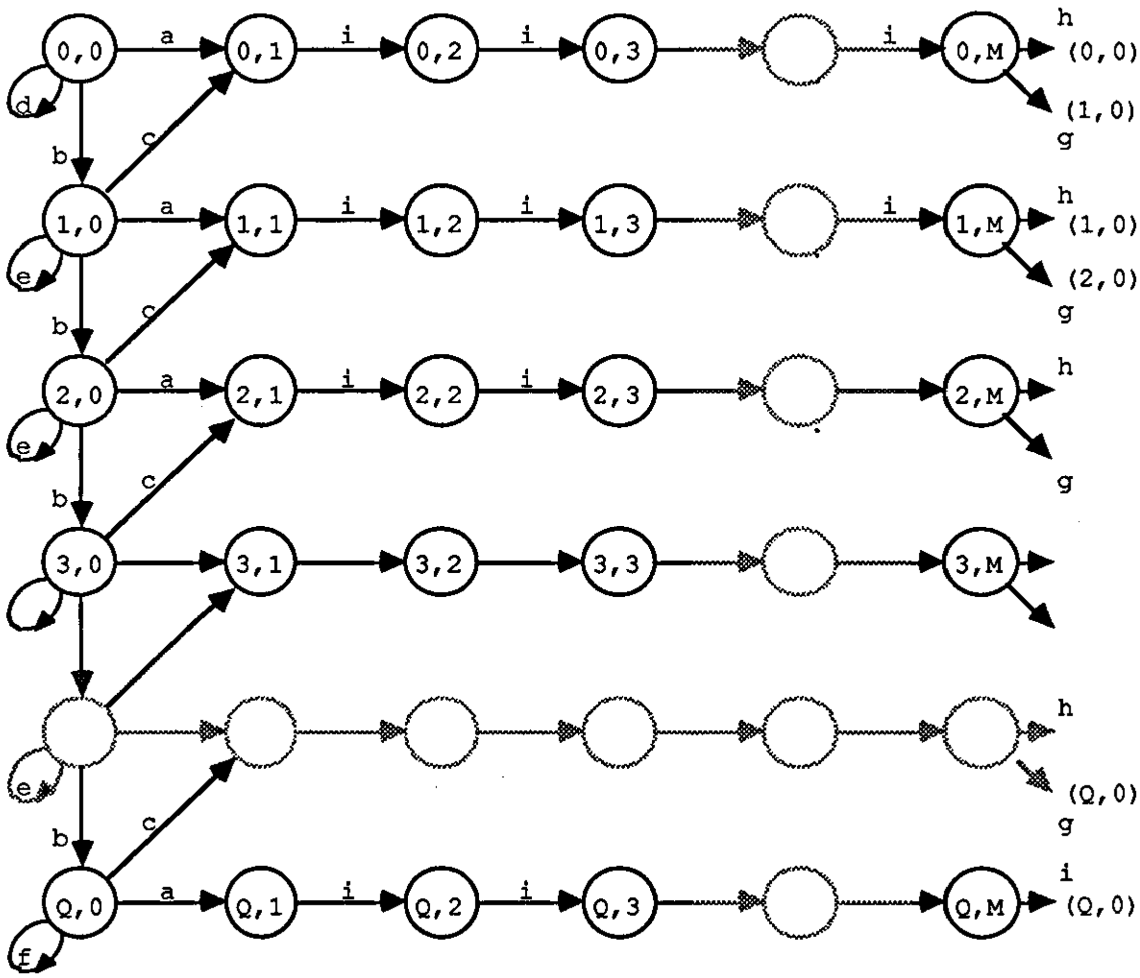
Both (2.1.21) and (2.1.22) give identical results.

One of the greatest drawbacks of this model is the large number of equations that have to be solved in order to calculate the system performance.

The fact that the iterative method suggested does not always converge, forces one to seek more complex iterative methods such as Mathematical Annealing, etc., which usually take a long time to reach the solution. It is not feasible to use the "Generalised Newton's Method" to solve for the above two unknowns, as the complexity of the equations involved prevent them from producing simple equations for their partial derivatives.

2.2 Source Deletion, Station Based Model 2 (SDStnBM2)

The model presented in this section is a simplified version of the previous one, the method of packet arrival being similar to that used by Tasaka (Tasaka 86) when modelling the CSMA-CD protocol. It provides a single variable for which one may iterate for a solution, thus, reducing the amount of computations involved considerably. As shown later, this provides a satisfactory match to the



where - a = p(1-S) d = 1-p g = Mp
b = pS e = (1-p)S h = 1- Mp
c = (1-p)(1-S) f = S i = 1

Fig 2.2.1 State transition diagram for a simplified, station based, discrete-time Markov model of a Slotted Ring with a Source Deletion Protocol. (SDStnBM2)

results obtained from the model in section 2.1.

In addition to the general assumptions mentioned in the introduction to this chapter, the following are assumed -

(2.2.1) When a station is idling, a single packet may arrive to that station every slot time with probability 'p'. This arrival process will follow a Bernoulli distribution.

(2.2.2) However, when the station is transmitting, no new packets will arrive at the station until the end of transmission. Since 'M' slot times are needed to complete a packet transmission, a single packet, with an arrival probability of 'Mp' packets per station, may be generated at the end of each transmission.

Assumption (2.1.3).

Note that assumption (2.2.2) effectively approximates a binomial distribution with parameters (M,p) with a Bernoulli distribution with parameter Mp.

Figure 2.2.1 shows the Markov state diagram for the model. The transition from one state to the other may be explained in a similar manner to that in section 2.1.

As before, let,

N = Total number of stations in the network

M = Total number of slots in the network

Q = Maximum buffer size per station

S = Probability that a station in a non transmitting state will find the next slot full

P(y,x) = Steady state probability of the station being in state (y,x) where y is the queue size of the station at that state and x is the number of slot times that have passed since the start of transmission.

Considering the equality of the probabilities of total arrivals and total exits to and from a given state at steady state equilibrium, the following balance equations may be derived.

$$P(0,0)p = P(0,M)(1-pM) \tag{2.2.1}$$

$$P(y,1) = P(y,0)p(1-S) + P(y+1,0)(1-p)(1-S) \quad \{0 \le y < Q\} \tag{2.2.2}$$

$$P(Q,1) = P(Q,0)p(1-S) \tag{2.2.3}$$

$$P(y,0)[1-(1-p)S] = P(y,M)(1-pM) + P(y-1,M)Mp + P(y-1,0)pS \quad \{1 \le y < Q\} \tag{2.2.4}$$

$$P(Q,0)(1-S) = P(Q,M) + P(Q-1,M)Mp + P(Q-1,0)pS \tag{2.2.5}$$

$$P(y,x) = P(y,x+1) \qquad \{0 \le y \le Q,\ 1 \le x < M\} \tag{2.2.6}$$

applying equation (2.2.6) to (2.2.4) and (2.2.5),

$$P(y,0)[1-(1-p)S] = P(y,1)(1-pM) + P(y-1,1)Mp + P(y-1,0)pS \quad \{1 \le y < Q\} \tag{2.2.7}$$

$$P(Q,0)(1-S) = P(Q,1) + P(Q-1,1)Mp + P(Q-1,0)pS \tag{2.2.8}$$

substituting in (2.2.8) for P(Q,1) and P(Q-1,1) from equations (2.2.3) and (2.2.2) respectively,

$$P(Q-1,0) = P(Q,0)\,A \tag{2.2.9}$$

where,

$$A = \frac{(1-p)(1-S)(Mp-1)}{p[S(Mp-1) - Mp]} \tag{2.2.10}$$

Re-substituting in (2.2.2),

$$P(Q-1,1) = P(Q-1,0)\frac{p}{(1-Mp)} \tag{2.2.11}$$

Induction will be used to prove that -

$$P(Q-r,0) = P(Q-r+1,0)\ A \qquad \{0<r\leq Q\} \tag{2.2.12}$$

$$P(Q-r,1) = P(Q-r,0)\frac{p}{(1-Mp)} \qquad \{0<r\leq Q\} \tag{2.2.13}$$

Assume that equation (2.2.13) holds true for a queue size of (Q-r) where $0<r<Q$. Then, substituting in equation (2.2.7) for P(Q-r-1,1) and P(Q-r,1) from the equations (2.2.2) and (2.2.13) respectively, it can be easily shown that,

$$P(Q-(r+1),0) = P(Q-r,0)\ A \qquad \{0\leq r<Q\}$$

re-substituting in (2.2.2),

$$P(Q-(r+1),1) = P(Q-(r+1),0)\frac{p}{(1-Mp)} \qquad \{0<r<Q\}$$

This implies that, if the above equation is true for a queue of (Q-r), then, it is true for a queue of (Q-(r+1)) as well. Since equations (2.2.9) and (2.2.11) show that it is valid for $r=1$, we may say that both equations (2.2.12) and (2.2.13) hold true for all $1\leq r\leq Q$.

Since S is the throughput contribution of N-1 stations,

$$\begin{aligned} S &= \frac{(N-1)}{M}\sum_{y=0}^{Q}\sum_{x=1}^{M}P(y,x) \\ &= \frac{(N-1)}{M}M\sum_{y=0}^{Q}P(y,1) \\ &= (N-1)\,[P(Q,1) + P(Q-1,1) + \ldots\ldots + P(1,1) + P(0,1) \end{aligned}$$

using equations (2.2.3), (2.2.12) and (2.2.13),

$$S = (N-1)\left[p(1-S) + \frac{p}{(1-Mp)}[A+A^2+A^3+\ .\ .\ .\ +A^Q]\right]P(Q,0)$$

$$= \begin{cases} (N-1)pP(Q,0)\left[(1-S) + \dfrac{1}{(1-Mp)}\dfrac{A(1-A^Q)}{(1-A)}\right] & (A\neq 1) \\ (N-1)pP(Q,0)\left[(1-S) + \dfrac{1}{(1-Mp)}Q\right] & (A=1) \end{cases} \qquad (2.2.14)$$

Considering the sum of all P(y,x) values and using equation (2.2.12),

$$1 = \sum_{y=0}^{Q}\sum_{x=0}^{M} P(y,x)$$

$$= \frac{SM}{(N-1)} + \sum_{y=0}^{Q} P(y,0)$$

$$= \frac{SM}{(N-1)} + P(Q,0)\ [A^Q+A^{Q-1}+\ .\ .\ .\ +A^2+A+1]$$

$$= \begin{cases} \dfrac{SM}{(N-1)} + P(Q,0)\dfrac{(1-A^{Q+1})}{(1-A)} & (A\neq 1) \\ \dfrac{SM}{(N-1)} + P(Q,0)\ (Q+1) & (A=1) \end{cases}$$

therefore,

$$P(Q,0) = \begin{cases} \left[1 - \dfrac{SM}{(N-1)}\right]\dfrac{(1-A)}{(1-A^{Q+1})} & (A\neq 1) \\ \left[1 - \dfrac{SM}{(N-1)}\right]\dfrac{1}{(Q+1)} & (A=1) \end{cases} \qquad (2.2.15)$$

substituting in (2.2.14),

$$S = \begin{cases} (N-1)p\left[(1-S) + \dfrac{1}{(1-Mp)}\dfrac{A(1-A^Q)}{(1-A)}\right]\left[1 - \dfrac{SM}{(N-1)}\right]\dfrac{(1-A)}{(1-A^{Q+1})} & (A\neq 1) \\ (N-1)p\left[(1-S) + \dfrac{1}{(1-Mp)}Q\right]\left[1 - \dfrac{SM}{(N-1)}\right]\dfrac{1}{(Q+1)} & (A=1) \end{cases} \qquad (2.2.16)$$

This fixed point equation can be solved for S using a simple iterative method. In the event of it not converging to the solution (as it happens at times when p is increased near to maximum possible), the following method may be used on the modified (2.2.16) equation.

$$f(S) = \begin{cases} S - (N-1)p\left[(1-S) + \frac{1}{(1-Mp)}\frac{A(1-A^{Q})}{(1-A)}\right]\left[1 - \frac{SM}{(N-1)}\right]\frac{(1-A)}{(1-A^{Q+1})} & (A\neq 1) \\ S - (N-1)p\left[(1-S) + \frac{1}{(1-Mp)}Q\right]\left[1 - \frac{SM}{(N-1)}\right]\frac{1}{(Q+1)} & (A=1) \end{cases} \qquad (2.2.17)$$

At the point of solution, f(S) will be zero, and the Bisection Method of iteration (Williams 79) may be used.

Once S is known, (2.2.15) may be used to compute the relevant P(Q,0) value. As before,

$$Throughput = S\frac{N}{(N-1)} \qquad (2.2.18)$$

$$\begin{aligned} AverageQueue(\overline{q}) &= \sum_{y=0}^{Q}\sum_{x=0}^{M} yP(y,x) \\ &= \sum_{y=0}^{Q} y[P(y,0) + MP(y,1)] \\ &= Q[P(Q,0) + MP(Q,1)] + \\ &\qquad \sum_{y=1}^{Q-1} y[P(y,0) + MP(y,1)] \end{aligned}$$

substituting for P(y,1) from equations (2.2.3) and (2.2.13),

$$\overline{q} = P(Q,0)\ Q\,[1 + Mp(1-S)] + \frac{1}{(1-Mp)}\sum_{y=1}^{Q-1} yP(y,0)$$

substituting from equation (2.2.12), and summing up the resulting series,

$$\overline{q} = \begin{cases} \left\{ Q\,[1+Mp(1-S)] + \frac{1}{(1-Mp)} \left[\frac{(A^{Q-1}-1) + (Q-1)\left(\frac{1}{A}-1\right)}{\left(1-\frac{1}{A}\right)^2} \right] \right\} P(Q,0) & (A \neq 1) \\ \left\{ Q\,[1+Mp(1-S)] + \frac{1}{(1-Mp)} \left[\frac{(Q-1)\,Q}{2} \right] \right\} P(Q,0) & (A = 1) \end{cases} \qquad (2.2.19)$$

Using Little's law,

$$\overline{d} = \frac{\overline{q}\,N}{Throughput} + 1 \qquad (slot\ times) \qquad (2.2.20)$$

As in the previous model,

$$\underset{(PRP)}{Packet\ Rejection\ Probability} = 1 - \frac{Throughput}{N\,p} \qquad (2.2.21)$$

The major disadvantage of this model is due to the fact that, since a probability cannot be greater than unity, the probability of a packet arrival at any given instance should also follow the same rule. i.e.,

$$Mp < 1$$
$$p < \frac{1}{M}$$

This means that this model is not valid for values of 'p' greater than or equal to 1/M, thus restricting its useful range. However, to prevent instability and buffer overflow losses, real life systems whose the number of stations are much higher than the number of slots do not operate near these throughput saturation regions, thus making this model usable for most practical purposes.

2.3 Source Deletion, Station Based Model 3 (SDStnBM3)

In this model, the only difference to the previous one is the assumption that -

(2.3.1) a packet is not deleted from its buffer until its transmission is complete.

In other words, the packet will be retained in the buffer until the transmitted copy of it has completely travelled once around the ring.

Assumptions (2.2.1) and (2.2.2) will apply in this model as well, assumption (2.1.3) being modified as above to suit the new condition.

The Markov state diagram for this model (Figure 2.3.1) may be explained in the usual manner. Let the variables N, M, Q, S and P(y,x) denote the usual variables.

Considering steady state equilibrium,

$$P(0,0)p = P(1,M)(1-Mp) \tag{2.3.1}$$

$$P(y,0)[1-(1-p)S] = P(y-1,0)pS + P(y,M)Mp + P(y+1,M)(1-Mp) \quad \{1\le y<Q\} \tag{2.3.2}$$

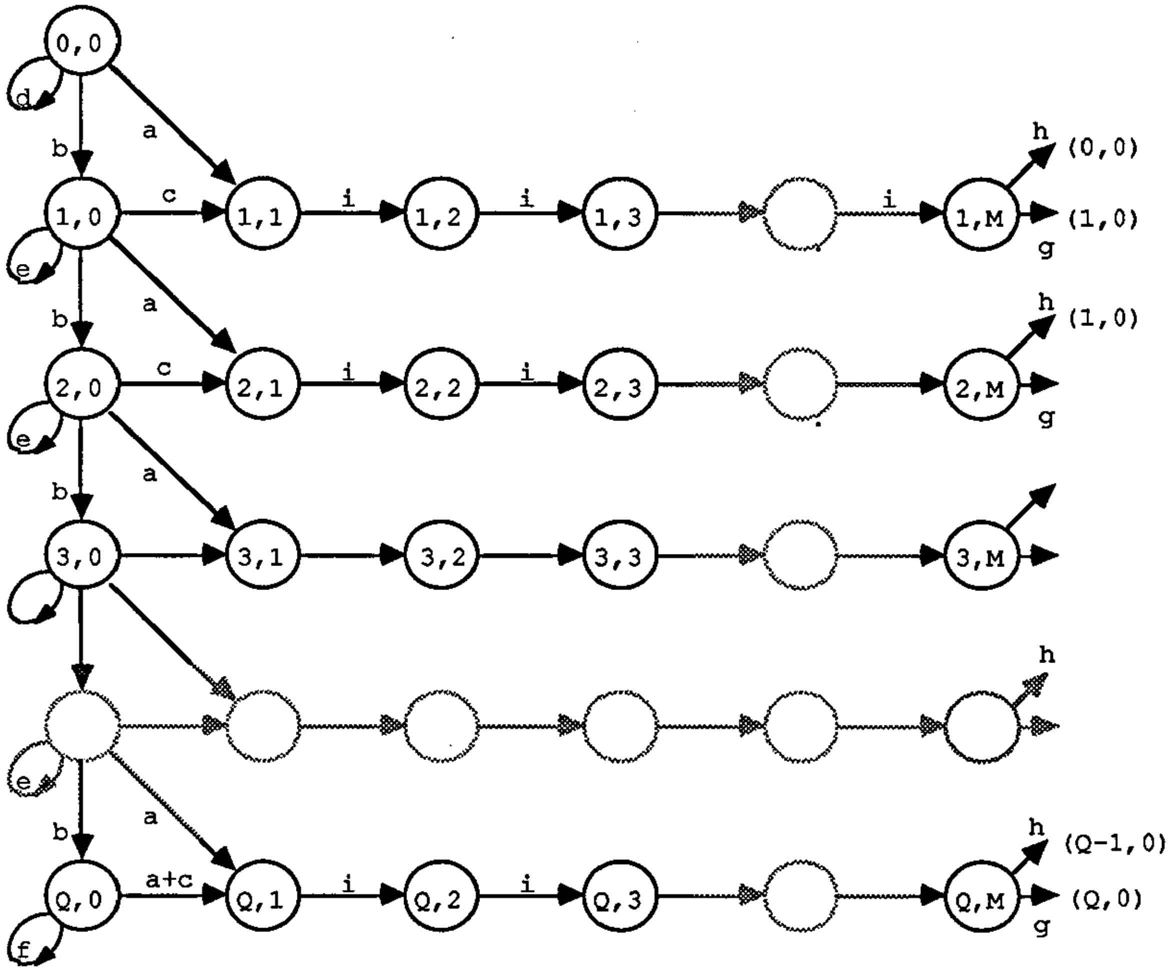
$$P(Q,0)(1-S) = P(Q-1,0)pS + P(Q,M)Mp \tag{2.3.3}$$

$$P(y,1) = P(y-1,0)p(1-S) + P(y,0)(1-p)(1-S) \quad \{1\le y<Q\} \tag{2.3.4}$$

$$P(Q,1) = P(Q-1,0)p(1-S) + P(Q,0)(1-S) \tag{2.3.5}$$

$$P(y,x) = P(y,x+1) \quad \{1\le y\le Q,\ 1\le x\le M-1\} \tag{2.3.6}$$

applying equation (2.3.6) in (2.3.1), (2.3.2) and (2.3.3) respectively,



where - a = p(1-S)   d = 1-p   g = Mp
b = pS   e = (1-p)S   h = 1- Mp
c = (1-p)(1-S)   f = S   i = 1

Fig 2.3.1 State transition diagram for a simplified, station based, discrete-time Markov model of a Slotted Ring with a Source Deletion Protocol. (SDStnBM3)

$$P(0,0)p = P(1,1)(1-Mp) \tag{2.3.7}$$

$$P(y,0)[1-(1-p)S] = P(y-1,0)pS + P(y,1)Mp + P(y+1,1)(1-Mp) \quad \{1 \le y < Q\} \tag{2.3.8}$$

$$P(Q,0)(1-S) = P(Q-1,0)pS + P(Q,1)Mp \tag{2.3.9}$$

Induction will be used to prove the following,

$$P(y,1) = P(y-1,0)\frac{p}{(1-Mp)} \qquad \{1 \le y \le Q\} \tag{2.3.10}$$

$$P(y,0) = P(y-1,0)A \qquad \{1 \le y < Q\} \tag{2.3.11}$$

where,

$$A = \frac{p[1-(1-S)(1-Mp)]}{(1-p)(1-S)(1-Mp)} \tag{2.3.12}$$

Equation (2.3.7) shows that (2.3.10) is true for y=1.
Substituting (2.3.7) in (2.3.4) for y=1, it can be easily shown that (2.3.11) hold true for y=1 as well.

Assume that equations (2.3.10) and (2.3.11) hold true for y=α. Then from equations (2.3.4) and (2.3.8),

$$P(\alpha+1,1) = \frac{P(\alpha,0)[1-(1-p)(S+(1-S)Mp)]-P(\alpha-1,0)p[S+Mp(1-S)]}{(1-Mp)} \qquad \{2 \le \alpha+1 \le Q\}$$

substituting for P(α-1,0) from equation (2.3.11),

$$P(\alpha+1,1) = P(\alpha,0)\frac{p}{(1-Mp)} \tag{2.3.13}$$

thus (2.3.10) is correct for 1≤y<Q.

From (2.3.5) and (2.3.9) it can be shown that,

$$P(Q,1) = P(Q-1,0)\frac{p}{(1-Mp)}$$

therefore, (2.3.10) hold true for all $1 \le y \le Q$.
Substituting (2.3.13) in (2.3.4),

$$P(\alpha+1,0) = P(\alpha,0)A \qquad \{2 \le \alpha+1 < Q\}$$

Since this relationship is true for y=1, it holds for the range $1 \le y < Q$.

Using the equation (2.3.11) repeatedly, it can be shown that,

$$P(y,0) = P(0,0)A^{y} \qquad \{1 \le y < Q\} \tag{2.3.14}$$

$$P(Q,0) = P(0,0)A^{Q}(1-p) \tag{2.3.15}$$

$$P(y,1) = P(0,0)A^{y-1}\frac{p}{(1-Mp)} \tag{2.3.16}$$

Considering the sum of all the state probabilities,

$$\begin{aligned} 1 &= \sum_{y=0}^{Q} P(y,0) + \sum_{y=1}^{Q}\sum_{x=1}^{M} P(y,x) \\ &= \sum_{y=0}^{Q} P(y,0) + M\sum_{y=1}^{Q} P(y,x) \end{aligned}$$

substituting from (2.3.14), (2.3.15) and (2.3.16),

$$P(0,0) = \begin{cases} \left[\dfrac{(1-A^{Q})}{(1-A)(1-Mp)} + A^{Q}(1-p)\right]^{-1} & (A \neq 1) \\ \left[\dfrac{Q}{(1-Mp)} + (1-p)\right]^{-1} & (A = 1) \end{cases} \tag{2.3.17}$$

Considering the throughput contribution of N-1 stations,

$$
\begin{aligned}
S &= \frac{N-1}{M}\sum_{y=1}^{Q}\sum_{x=1}^{M} P(y,x) \\
&= N-1 \sum_{y=1}^{Q} P(y,x)
\end{aligned}
$$

substituting from (2.3.14) and summing the resulting series,

$$
S = \begin{cases} \dfrac{p(N-1)\,(1-A^{Q})}{(1-Mp)\,(1-A)} P(0,0) & (A \neq 1) \\[2ex] \dfrac{p(N-1)\,Q}{(1-Mp)} P(0,0) & (A = 1) \end{cases} \tag{2.3.18}
$$

This last equation is a fixed point equation that can be solved for 'S' using an iterative method as in Section 2.2, the value of P(0,0) required being obtained from (2.3.15).

Then,

$$
Throughput = S\frac{N}{(N-1)} \tag{2.3.19}
$$

$$
\begin{aligned}
\overline{q} &= \sum_{y=1}^{Q} yP(y,0) + M\sum_{y=1}^{Q} yP(y,1) \\
&= \begin{cases} P(0,0)\left[\dfrac{A(1-A^{Q-1})}{(1-A)^2} - \dfrac{(Q-1)A^{Q}}{(1-A)} + QA^{Q}(1-p)\right] & \\ \qquad + P(0,0)\dfrac{Mp}{(1-Mp)}\left[\dfrac{(1-A^{Q})}{(1-A)^2} - \dfrac{QA^{Q}}{(1-A)}\right] & (A \neq 1) \\ P(0,0)\left[\dfrac{(Q-1)Q}{2} + Q(1-p)\right] + P(0,0)\dfrac{Mp}{(1-Mp)}\dfrac{Q(Q+1)}{2} & (A = 1) \end{cases}
\end{aligned} \tag{2.3.20}
$$

$$
\overline{d} = \frac{\overline{q}\,N}{Throughput} + 1 \qquad (slot\ times) \tag{2.3.21}
$$

$$\text{Packet Rejection Probability (given packet arrival)} = 1 - \frac{\text{Throughput}}{N\,p} \qquad (2.3.22)$$

It is noted that the value of 'p' should be less than 1/M in this model as well.

2.4 Matrix Solving Method

When a Markov model is large and complex, one practical method of obtaining a solution is to use iteration on the transition matrix. The first step in order to do this is to generate the State Transition Matrix [P]. (Daellenbach 83, chapter 13)

If $[t]_i$ is a row matrix of size 'n' containing the State Probabilities of this Markov Chain at the i^{th} time interval, then, from Markov theory,

$$[t]_i\,[P] = [t]_{i+1} \qquad (2.4.1)$$

At steady state equilibrium,

$$\begin{aligned} [t]_i &= [t]_{i+1} \\ &= [t] \end{aligned} \qquad (2.4.2)$$

if,

$$[t] = [t_1 \; t_2 \; t_3 \; . \; . \; . \; . \; t_n]$$

and

$$[P] = \begin{bmatrix} P_{11} & P_{12} & . & P_{1n} \\ P_{21} & P_{22} & . & P_{2n} \\ . & . & . & . \\ P_{n1} & P_{n2} & . & P_{nn} \end{bmatrix}$$

(Note that these t_i values are similar to the steady state equilibrium probabilities

P(y,x) of the Markov models in this thesis. They should not be confused with the above state transition probabilities P_{ij})

expanding the above relationship in equation (2.4.1),

$$\begin{aligned}
t_1P_{11} + t_2P_{21} + \ldots + t_nP_{n1} &= t_1 \\
t_1P_{12} + t_2P_{22} + \ldots + t_nP_{n2} &= t_2 \\
\ldots\ldots\ldots\ldots \\
t_1P_{1n} + t_2P_{2n} + \ldots + t_nP_{nn} &= t_n
\end{aligned}$$

which in turn provide the following set of equations -

$$\begin{aligned}
t_1(P_{11}-1) + t_2P_{21} + \ldots + t_nP_{n1} &= 0 \\
t_1P_{12} + t_2(P_{22}-1) + \ldots + t_nP_{n2} &= 0 \\
\ldots\ldots\ldots\ldots \\
t_1P_{1n} + t_2P_{2n} + \ldots + t_n(P_{nn}-1) &= 0
\end{aligned}$$

As it can be seen, this set of equations are not independent. Therefore, one of them has to be replaced by another independent equation. For convenience, the last equation will be replaced by the following equation indicating the sum of all state probabilities being unity -

$$t_1 + t_2 + t_3 + \ldots + t_n = 1$$

Then, by converting these equations back into their matrix form,

$$(t_1 \;\; t_2 \;\; t_3 \;\; . \;\; t_n)\begin{pmatrix} (P_{11}-1) & P_{12} & P_{13} & . & 1 \\ P_{21} & (P_{22}-1) & P_{23} & . & 1 \\ . & . & . & . & . \\ P_{n1} & P_{n2} & P_{n3} & . & 1 \end{pmatrix} = (0 \;\; 0 \;\; 0 \;\; . \;\; 1) \qquad (2.4.3)$$

When the State Transition Probabilities P(y,x) are known, this last equation may be solved for [t] using any of the large number of matrix solving procedures that are available for most computers. In this research, two such routines have been used - the NAG F04ATF procedure which have an exact solution (NAG 87), and, the iterative routine recommended for solving sparse matrices in reference (Press

86).

This latter procedure, which takes advantage of the sparseness of the [P] matrix, was used extensively when there were a large number of states (say over 200) involved, thus making it difficult to use the memory intensive NAG F04ATF routine.

It is noted that both these routines solved for an unknown column matrix [x] having the following relationship -

$$[A]\ [x] = [b]$$

where [b] is a row vector.

Therefore, in order to obtain a solution, the equation (2.4.3) was converted into the following form by transposing both sides of that equation -

$$\begin{pmatrix} (P_{11}-1) & P_{12} & P_{13} & . & 1 \\ P_{21} & (P_{22}-1) & P_{23} & . & 1 \\ . & . & . & . & . \\ P_{n1} & P_{n2} & P_{n3} & . & 1 \end{pmatrix}^T \begin{pmatrix} t_1 \\ t_2 \\ . \\ t_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ . \\ 1 \end{pmatrix} \qquad (2.4.4)$$

Matrix Solution for the Station Based Models

In these models, since the transition from a non transmitting state (i.e., x=0 state) to a transmitting state (x≠0) depend upon the probability of the next slot being empty, which, in turn, depend on the x≠0 states, we end up with a situation of having to iterate for a solution.

When the matrix method is used, the following steps may be followed to obtain a solution -

(1) Guess a value for 'S'; where, S = Probability that the next slot is full.

(2) Generate the State Transition Matrix (STM) using 'S'.

(3) Modify the STM as above to make it suitable for solving the system when it is at Steady State.

(4) Solve the matrix and find the new 'S' value using the given Steady State Probabilities.

(5) Compare the initial and the new 'S' values, and accept the new 'S' as the solution if the error is within the tolerance limits. Else go back to Step (1).

There are two main methods of choosing a value for 'S' in Step (1), viz, the Simple Iterative Method and the Bisection Method (Williams 79). The Newton-Raphson Method cannot be used due to the fact that it is extremely difficult, if not impossible, to even express the equation of 'S', let alone differentiate it as required by that method.

The Simple Iteration Method converges very rapidly toward the solution for all values of p in the region where the throughput is linearly increasing : this is a feature of all the station based models to follow in this thesis, and is due to the fact that a graph of S_{new} v S_{old} is quite flat within this range of p. (See later figures (6.4.42) to (6.4.46) of chapter 6.)

However, this method may not be used when the Throughput is saturated, as, at the point of intersection with the y=x line, the modulus of the gradient of the S_{new} v S_{old} curve is greater than 1, thus making the iteration diverge away from the solution. On such occasions, the relatively slower to converge Binary Iterative

Method should be used.

Due to the repetitive nature of iterative solutions, the above method is slower than the method used by Lee (Lee 91) in solving similar problems, but, is far more versatile and gives an accurate solution to the model as it is not limited to the assumptions made by Lee.

2.5 Source Deletion, Station Based Model Considering A Single Slot Segment Of The Network (SDStnBM4)

In this section, the waiting time for the stations that are not transmitting is considered in greater detail. This waiting time will be modelled considering a segment of the network corresponding to a single slot.

The logic behind this approach can be explained in the following manner -

Since all the stations are assumed statistically identical and are distributed evenly around the ring, and also the ring is divided uniformly into M identical and independent slots, the non transmitting stations in a Slotted Ring network can be divided and grouped according to the time they have to wait for the next slot header to arrive. Therefore, using this criteria in a network having eight stations and two slots as in figure (2.5.1), the stations can be considered as four groups of two stations (1 & 5, 2 & 6, 3 & 7, and 4 & 8) and represented as in figures (2.5.2) or (2.5.3).

Additional assumptions -

(2.5.1) Only a single packet of data may arrive per station at a given station time.

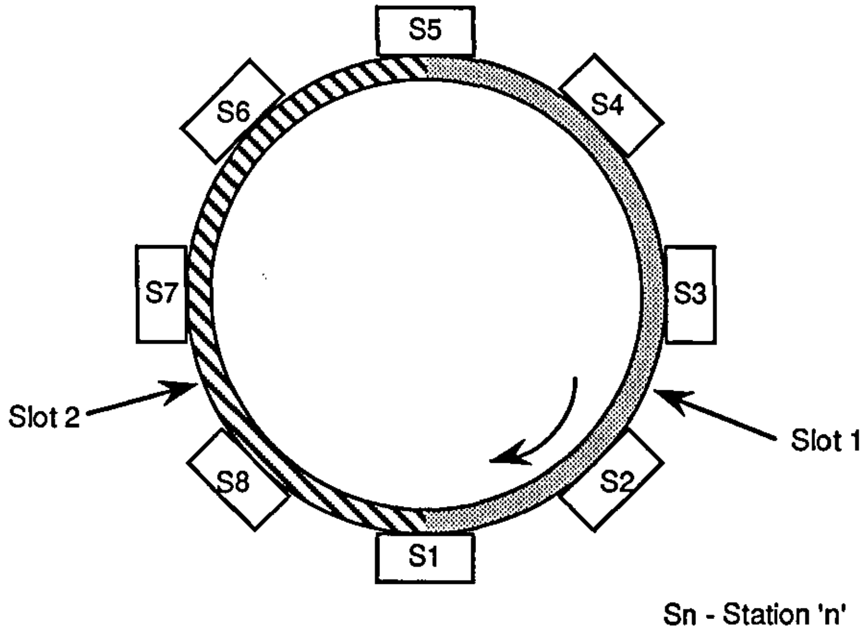


Fig 2.5.1

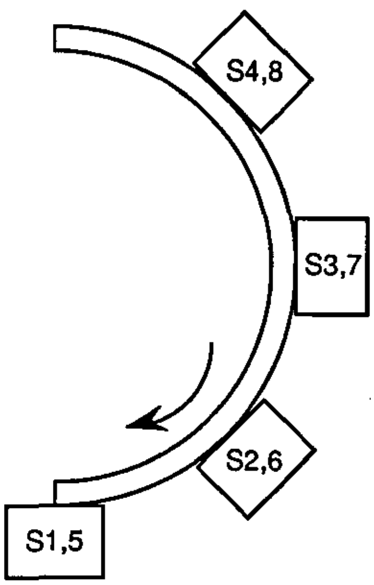


Fig 2.5.2

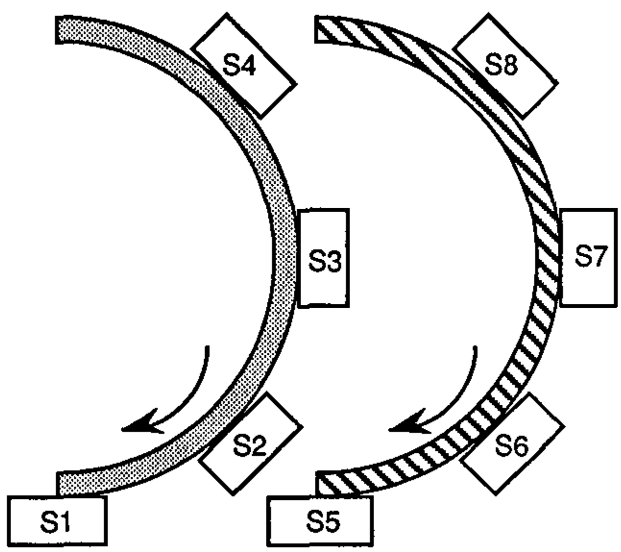
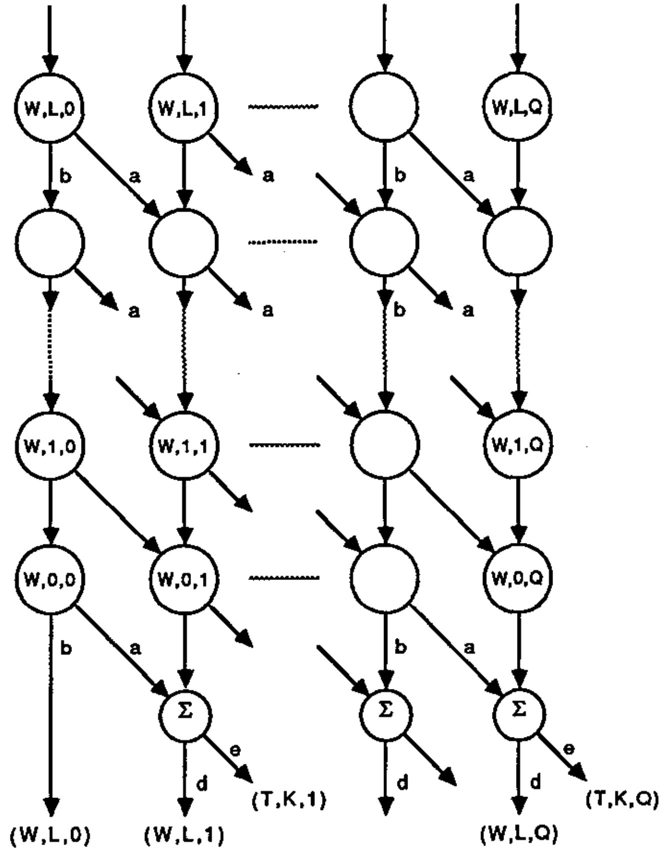
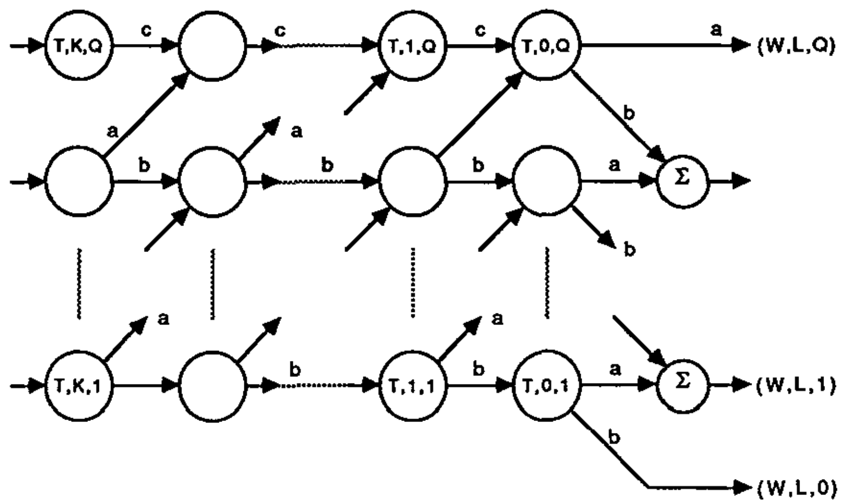


Fig 2.5.3





Where: a = p
b = 1-p
c = 1
d = S
e = 1-S
K = N-1
L = (N/M)-1

Fig 2.5.4 Markov model for Section 2.5

(2.5.2) This packet arrival rate will follow a Bernoulli distribution with parameter 'p' packets per station per station time.

(2.5.3) The number of stations will be an integer multiple of the number of slots.

Assumption (2.3.1).

Figure 2.5.4 shows the Markov state diagram for the model. The state (W,y,x) represents a waiting 'W' state, with 'y' station times before the next slot header arrives and having 'x' number of packets in the buffer. A (T,y,x) state is a transmitting 'T' state with 'x' packets in the buffer and having 'y' station times up to the end of transmission.

Let,

M = total number of slots

N = total number of stations

Q = maximum buffer size per station

S = probability that a station wishing to transmit finds the next slot busy

P(T,y,x) = number of stations occupying state (T,y,x)

P(W,y,x) = number of stations occupying state (W,y,x)

A station in (W,y,x) $\{y \neq 0, x \neq Q\}$ state may, in the next station time, transit to a state (W,y-1,x+1) or (W,y-1,x) depending upon whether there is a packet arrival or not. When it reaches the state (W,0,x) $\{x \neq Q\}$ that station will start transmitting and move to a state (T,N-1,x+1) or (T,N-1,x) if the next slot is empty, or else, move to (W,N/M,x+1) or (W,N/M,x) to wait 'N/M' station times for the next slot to arrive.

Similarly, a station in (T,y,x) state $\{y \neq 0, x \neq Q\}$, will transit to (T,y-1,x+1) or (T,y-1,x), again depending upon the probability of a packet arrival. At the end of transmission (y=0), the station will assume a state of (W,N/M -1,x+1) or (W,N/M -1,x) to wait for the next slot to arrive.

The transition probabilities indicated in figure (2.5.4) are as follows -

$$
\begin{aligned}
a &= p \\
b &= 1-p \\
c &= 1 \\
d &= S \\
e &= 1-S \\
K &= N-1 \\
L &= \frac{N}{M}-1
\end{aligned}
$$

Considering the sum of all P(T,y,x) and P(W,y,x) values,

$$\sum_{j=0}^{\frac{N}{M}-1} \sum_{i=0}^{Q} P(W,j,i) + \sum_{j=0}^{N-1} \sum_{i=0}^{Q} P(T,j,i) = \frac{N}{M} \qquad (2.5.1)$$

Since 'S' is the probability of the next slot being busy due to N-1 stations,

$$S = \frac{N-1}{N} \sum_{j=0}^{N-1} \sum_{i=1}^{Q} P(T,j,i) \qquad (2.5.2)$$

The matrix method of solving described in Section 2.4 may be used to solve this model. The only modification that has to be made to account for the change in the value of the sum of all state occupancies, is to replace the '1' in the Right Hand Side matrix of the equations (2.4.3) and (2.4.4) with the value 'N/M'.

Then,

$$Throughput = S \frac{N}{(N-1)} \qquad (2.5.3)$$

$$\bar{q} = \left\{ \sum_{j=0}^{\frac{N}{M}-1} \sum_{i=0}^{Q} iP(W,j,i) + \sum_{j=0}^{N-1} \sum_{i=0}^{Q} iP(T,j,i) \right\} \frac{M}{N} \quad (packets) \qquad (2.5.4)$$

$$\bar{d} = \frac{\bar{q}\,N}{Throughput} + \frac{M}{N} \quad (slot\ times) \tag{2.5.5}$$

The term M/N was added to account for the minimum packet delay of one station time that occurs due to the assumption (2.0.12).

$$\underset{(PRP)}{Packet\ Rejection\ Probability} = 1 - \frac{Throughput\ M}{p\ N^2} \tag{2.5.6}$$

Conversion into a basic Station Based Model

As mentioned in the introduction to this chapter, this model can be easily converted into an extended version of SDStnBM1 with the following minimal changes -

$$\sum_{j=0}^{\frac{N}{M}-1} \sum_{i=0}^{Q} P(W,j,i) + \sum_{j=0}^{N-1} \sum_{i=0}^{Q} P(T,j,i) = 1 \tag{2.5.7}$$

$$S = \frac{(N-1)}{M} \sum_{j=0}^{N-1} \sum_{i=1}^{Q} P(T,j,i) \tag{2.5.8}$$

$$Throughput = \frac{N}{(N-1)} S \tag{2.5.9}$$

$$\bar{q} = \sum_{j=0}^{\frac{N}{M}-1} \sum_{i=0}^{Q} iP(W,j,i) + \sum_{j=0}^{N-1} \sum_{i=0}^{Q} iP(T,j,i) \tag{2.5.10}$$

the Average Delay and the Packet Rejection Probability remaining the same as in equations (2.5.5) and (2.5.6) respectively. Note that in this case, both P(T,j,i) and P(W,j,i) are the probabilities of a station occupying those given states.

It is noted that the method formulated in Section 2.4 may be used without any modifications to solve this converted model.

Both the models in this section give identical results.

The major disadvantage of the model is the fact that it cannot correctly reflect the waiting states unless the number of stations is an integral multiple of the number of slots.

2.6 Results

The models are solved for the varying combinations of N, M and Q. For two different cases of N (16 and 64), three different values each of M (1, 8 and 16) and Q (1, 10 and 50) are considered. However, in order to limit the amount of graphs presented, the combinations containing M=16 or Q=50 are omitted from the thesis. The differences that occur in the performance measures when M is increased from 8 to 16, or when Q is increased from 10 to 50 are either negligible, or usually predictable. However, for the purpose of the analysis, all results obtained are considered. The results obtained for the Throughput, Average Queue, Average Delay and the Packet Rejection Probability will be compared to simulations.

Due to the fact that most models frequently show almost identical results, for clarity, their performance will be plotted as discrete points whereas the simulation results will be shown as a solid line.

It is noted that, in order to present the results of all the models having similar network parameters in a single graph, the packet arrival rate of SDStnBM4 (p) has been converted into the units of "packets per station per slot time" (say p') using the following transformation -

$$p' = \frac{pN}{M}$$

Such a step amounts to an assumption that the two binomial distributions of M trials with success probability P, and N trials with a probability of success MP/N are identical. Clearly such an assumption is incorrect. However, for convenience and also due to the fact that only the mean values are of greater importance here, the errors resulting from this transformation have been ignored.

Throughput

When the maximum buffer size is very small (say 1), the models and the simulation tend to show an increasingly poor match with the increase of the number of slots. Also, in the region where the throughput reaches saturation, the simulation varies in an irregular manner when $M>1$. This again is more persistent when the number of slots is large compared the number of stations.

Both these above effects are due to the quasi stable states that occur for high levels of packet arrival rates (Falconer 85). It is also due to this reason that the saturation throughputs of the simulations for the cases where $M>1$ is less than that for the cases where $M=1$. No quasi stable states can occur when there is only one slot in the network, thus under such cases, the simulation correctly shows the maximum possible throughput (Appendix A) which one may achieve with this type of network. That is -

$$Throughput = \frac{1}{1 + \frac{1}{N}}$$

When considering the effect of increasing the buffer size (Q) with the other factors (N and M) remaining constant, it can be seen that the throughput characteristics for the models tend to become parts of two intersecting straight lines. In the unsaturated region, this line has an equation that can be expressed as -

$$\begin{aligned} \mathit{Throughput} &= \mathit{Stations} * \mathit{Packet\ Arrival\ Rate} \\ &= N * p \end{aligned}$$

and in the saturated region,

$$\mathit{Throughput} = \mathit{constant}$$

The increase of the number of stations keeping the Q and M constant tend to give rise to more evenly changing throughput characteristics. This is due to the fact that the ratio of the number slots to the number of stations is becoming smaller, thus limiting the number of adverse quasi stable state combinations possible.

In general, all the models show an excellent match to the simulation in the unsaturated region, the only exception being when $Q=1$ and $M>1$. At saturation, except under the above conditions, the comparison can be considered as satisfactory.

It is noted that though the models do not consider the effects of quasi stable states, the saturation values shown fall close to those of the simulation. This is due to the fact that, even though we assume the stations to be totally independent of other stations in the network, the fact that a slot being full is a function of itself, $S=f(S)$, brings in, to some extent, the effects of the other stations on that particular station.

Average Queue

At low packet arrival rates models SDStnBM3 and SDStnBM4 show a higher average queue than the other two models and the simulation. This is due to the fact that in these models, a packet is not deleted from the buffer until the end of its transmission. Though this effect can be seen clearly only in the cases where $Q=1$, it does occur for the other values of Q as well, but is less visible due to the lack of resolution of the y axis.

Apart from this deviation, SDStnBM1 and SDStnBM4 tend to reflect fairly similar results, with SDStnBM4 at times giving out a slightly improved match to the simulations.

Therefore, other than in special circumstances, it is felt that SDStnBM4 does not justify its usage compared to the less complex SDStnBM1.

Except in the case of M=1, SDStnBM2 and SDStnBM3 show relatively poor results, the latter model being the worse.

All models tend to generally improve their ability to reflect the simulation results with the increase of N. The performance of SDStnBM2 and SDStnBM3 degrade with the increase of M.

With the increase of Q, it can be seen that, when the throughput is not in saturation, the average queue remains very low. However, this changes in a manner of a step function to near maximum as soon as the throughput saturates.

Average Delay

As in the case of average queue, SDStnBM3 and SDStnBM4 show a comparatively higher value at low packet arrival rates. In this case it is a minimum value equal to the summation of the number of slots in the network and the minimum delay encountered before transmission. This is due to the fact that in these two models, any packet, even if it is transmitted in the next slot after its arrival, has to remain in the buffer (M+1) or (M + M/N) slot times respectively before it is discarded.

All models tend to show improved match to the simulation with the increase of

N or Q. Conversely, all the models, especially SDStnBM2 and SDStnBM3, give out a deteriorating comparison to the simulation with the increase in the number of slots. This again is due to the quasi stable states that occur in greater frequencies with the increase of M.

The reason for the wider disparity between the models and the simulation near the saturation regions is again due to the quasi stable states that appear in the simulation thus reducing the throughput and increasing the delay.

Since the saturation throughput shown in the models are slightly higher than the simulation, the fact that the delays of all the models are calculated using this throughput is another reason as to the fact that the delay shown by them are slightly lower than that of the simulation.

Packet Rejection Probability

When Q=1, as can be expected, packets tend to get rejected due to buffer overflow even at low packet arrival rates. However, with the increase of Q, buffer overflow starts occurring only when the throughput reaches saturation.

All models in general give a good match to the simulation. However, for the case of Q=1, SDStnBM4 tends to produce relatively high results for low packet arrival rates. This effect worsens with the increase of the number of slots. In all other N, M and Q combinations, all the models tend to predict a higher packet arrival rate before buffer overflow starts to occur.

SDStnBM1 gives the best overall performance in predicting the packet rejection probability.

2.7 Summary

In this chapter, four models for the slotted ring with a source deletion protocol have been analysed. All the models were based upon observing the network from a station.

Models SDStnBM2 and SDStnBM3 were the simplified versions of the first model SDStnBM1 and resulted in much simpler solutions with only minimal degradation to the performance measures. Therefore, they may be used when the network constants permit their use.

The last model SDStnBM4 is an extended version of SDStnBM1, the former paying more attention to the changes in the queue size when the station is not transmitting. However, there was very little improvement in the performance metrics when compared to the first model.

A matrix method for solving the above type of Markov models was also introduced. This method will be widely used in the models in the following chapters.

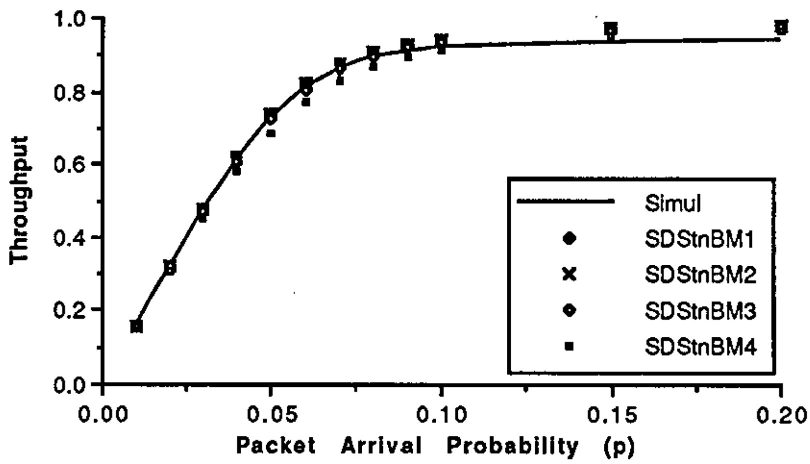


Fig. 2.6.1 Throughput Performance (N=16, M=1, Q=1)

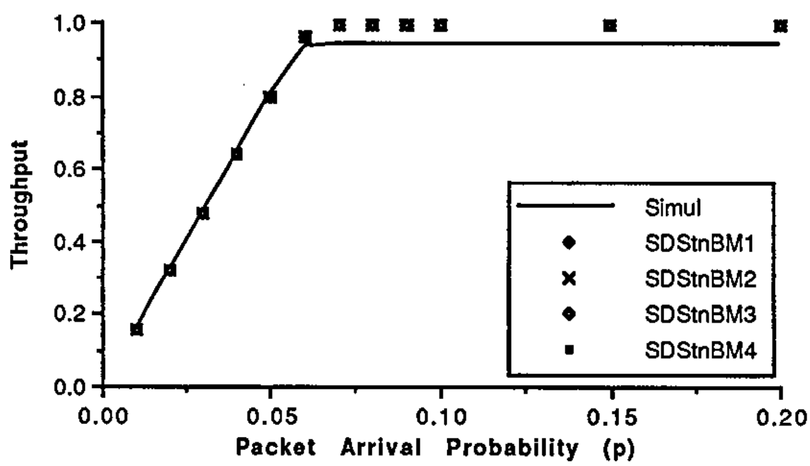


Fig. 2.6.2 Throughput Performance (N=16, M=1, Q=10)

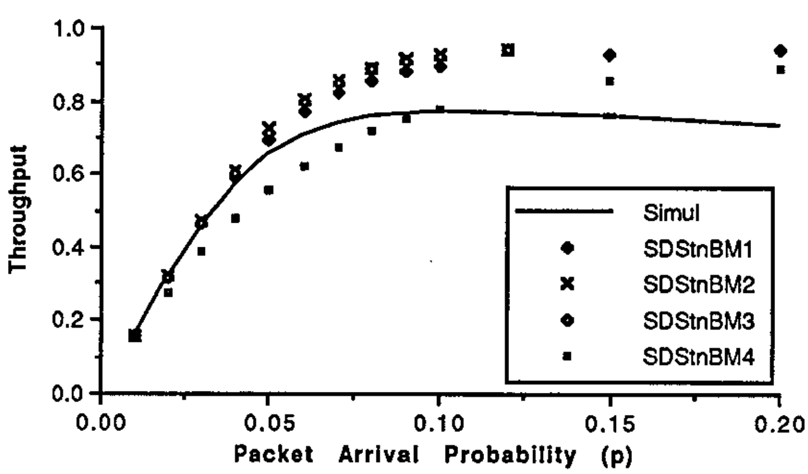


Fig. 2.6.3 Throughput Performance (N=16, M=8, Q=1)

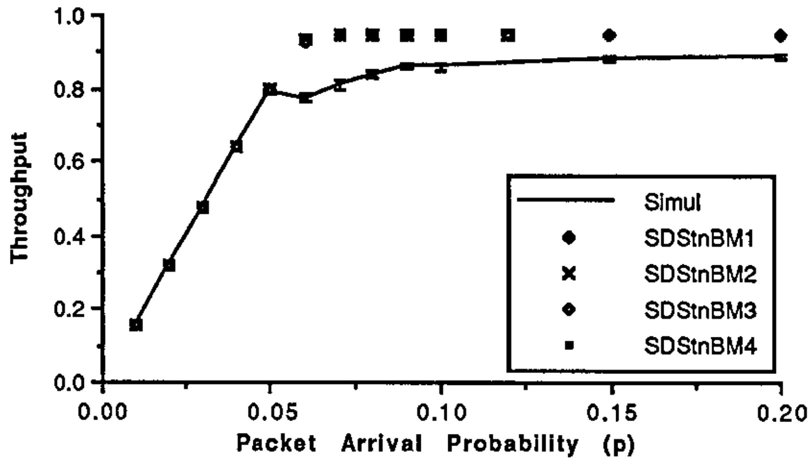


Fig. 2.6.4 Throughput Performance (N=16, M=8, Q=10)

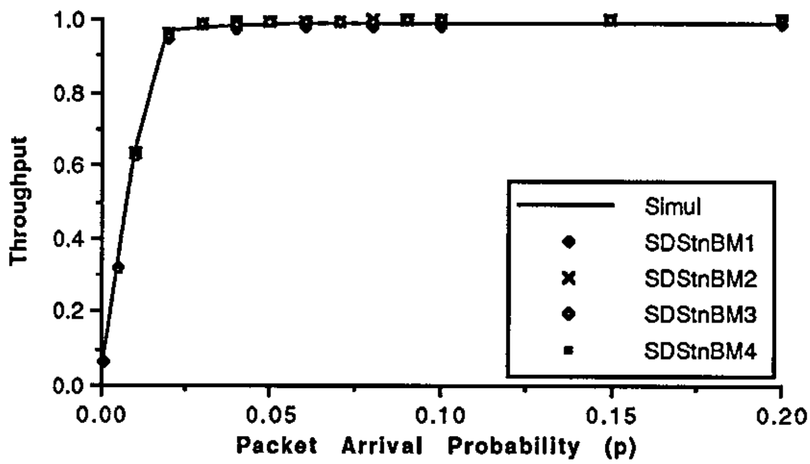


Fig. 2.6.5 Throughput Performance (N=64, M=1, Q=1)

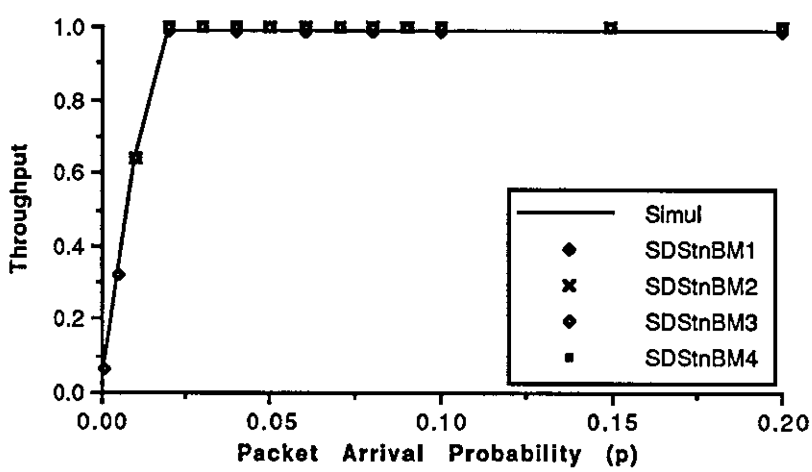


Fig. 2.6.6 Throughput Performance (N=64, M=1, Q=10)

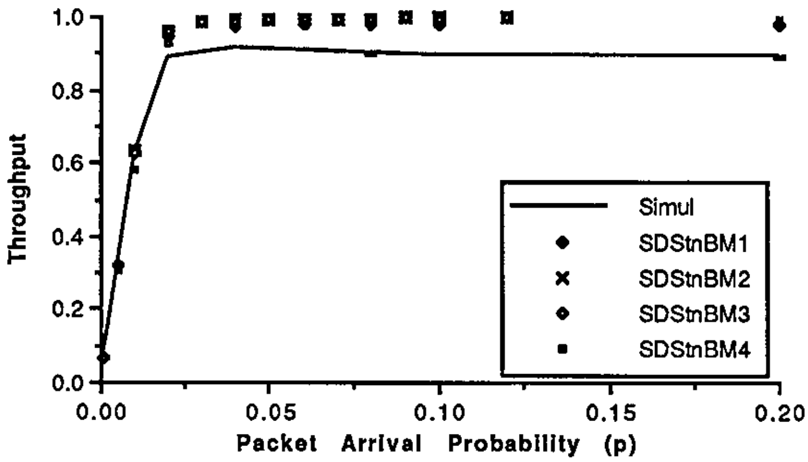


Fig. 2.6.7 Throughput Performance (N=64, M=8, Q=1)

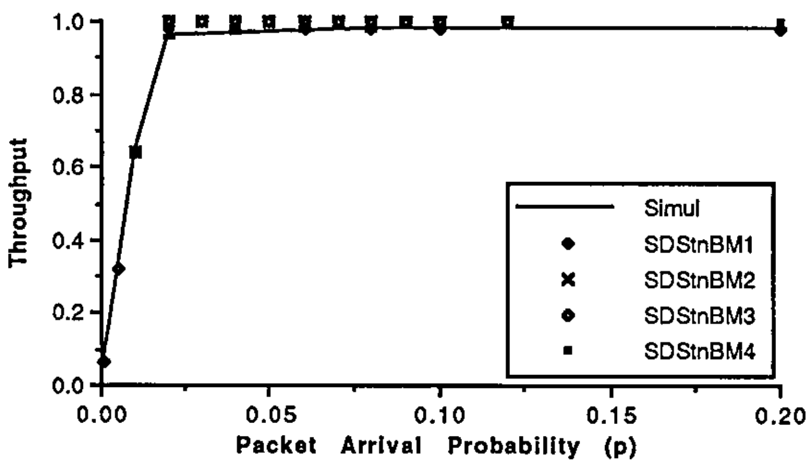


Fig. 2.6.8 Throughput Performance (N=64, M=8, Q=10)

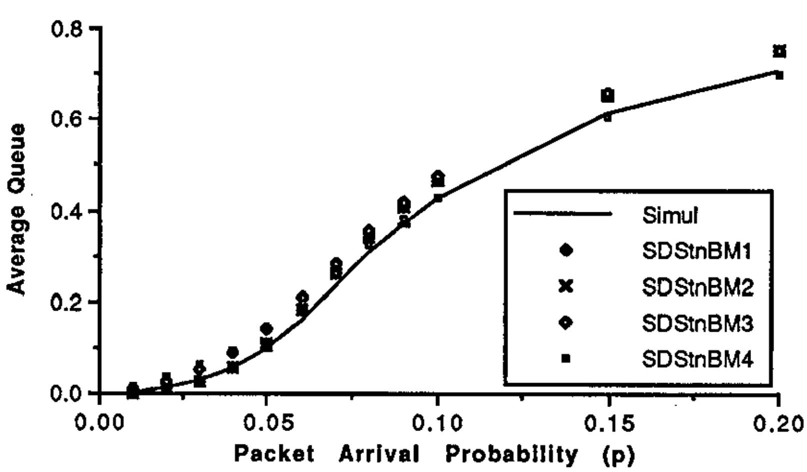


Fig. 2.6.9 Queueing Performance (N=16, M=1, Q=1)

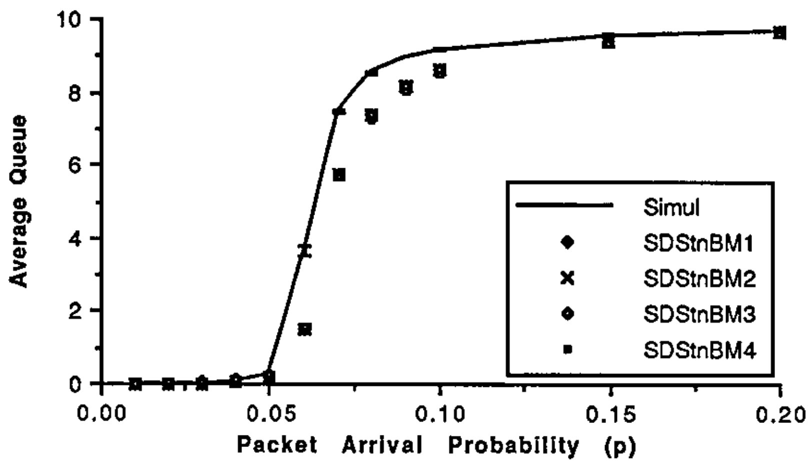


Fig. 2.6.10 Queueing Performance (N=16, M=1, Q=10)

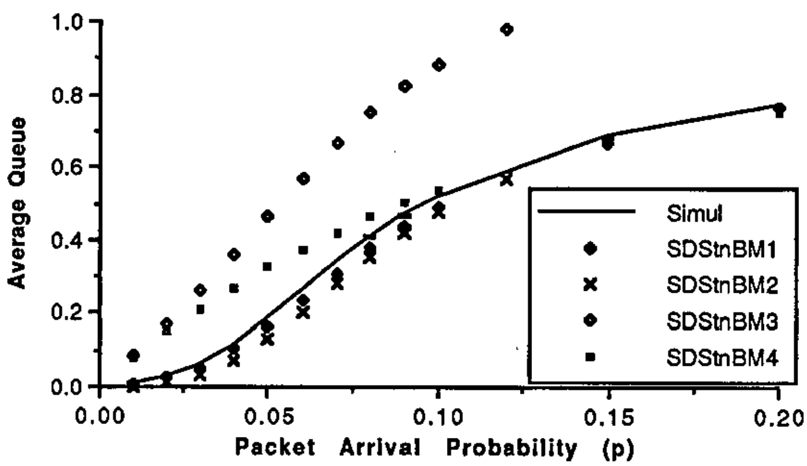


Fig. 2.6.11 Queueing Performance (N=16, M=8, Q=1)

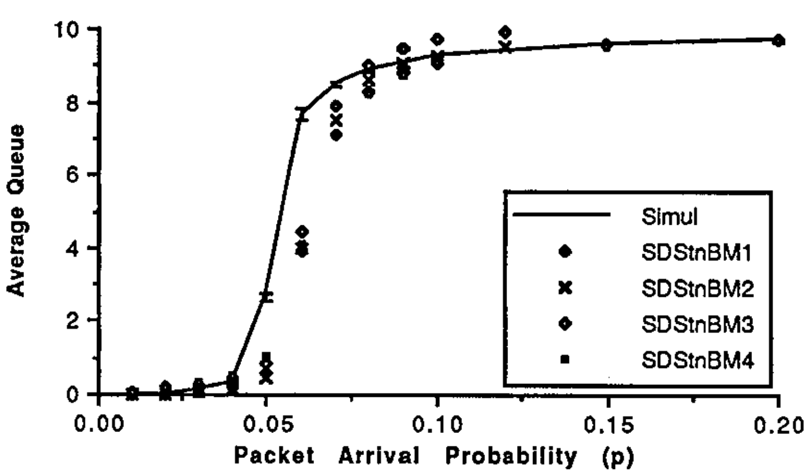


Fig. 2.6.12 Queueing Performance (N=16, M=8, Q=10)

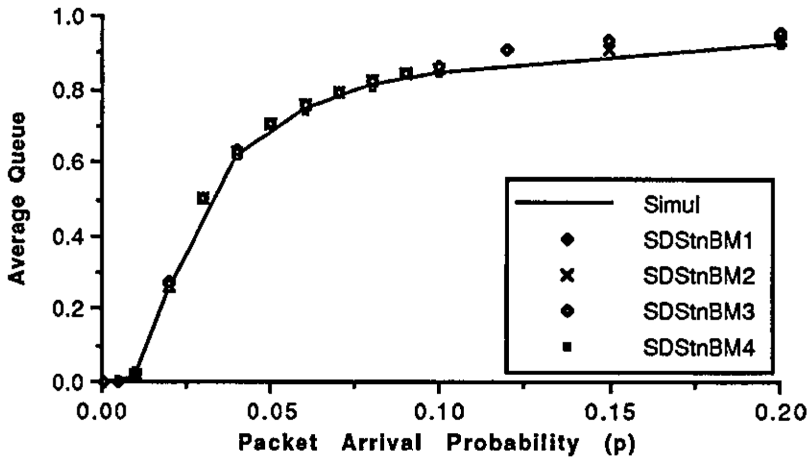


Fig. 2.6.13 Queueing Performance (N=64, M=1, Q=1)

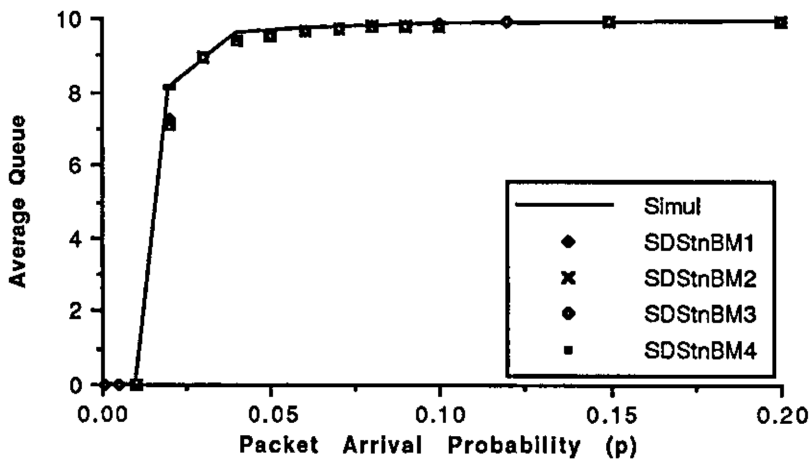


Fig. 2.6.14 Queueing Performance (N=64, M=1, Q=10)

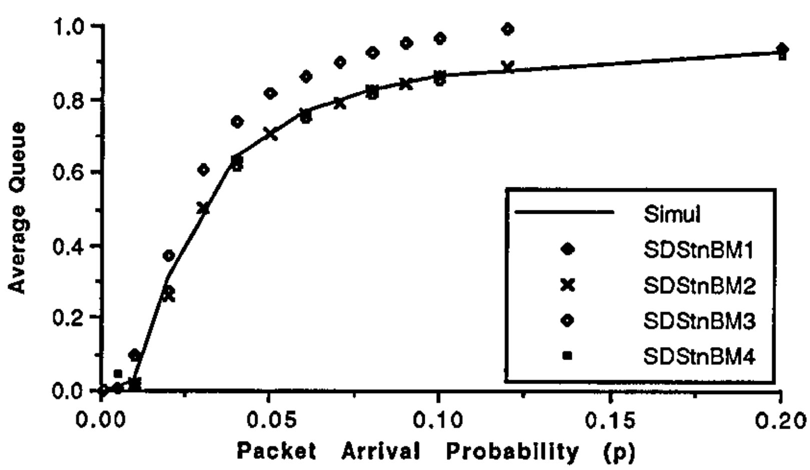


Fig. 2.6.15 Queueing Performance (N=64, M=8, Q=1)

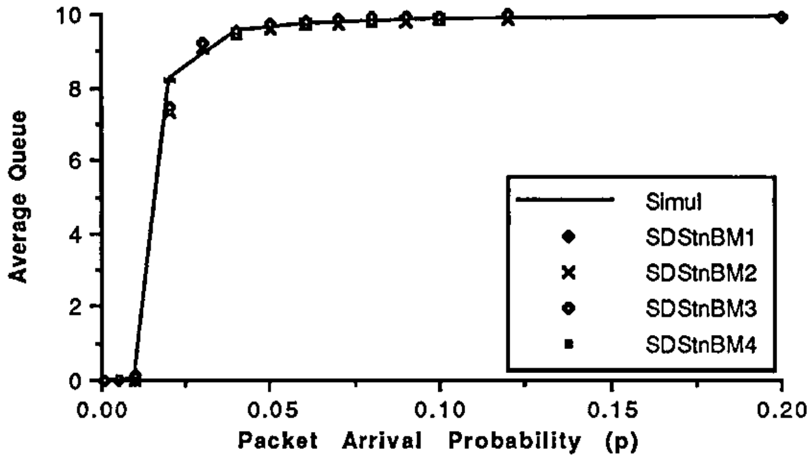


Fig. 2.6.16 Queueing Performance (N=64, M=8, Q=10)

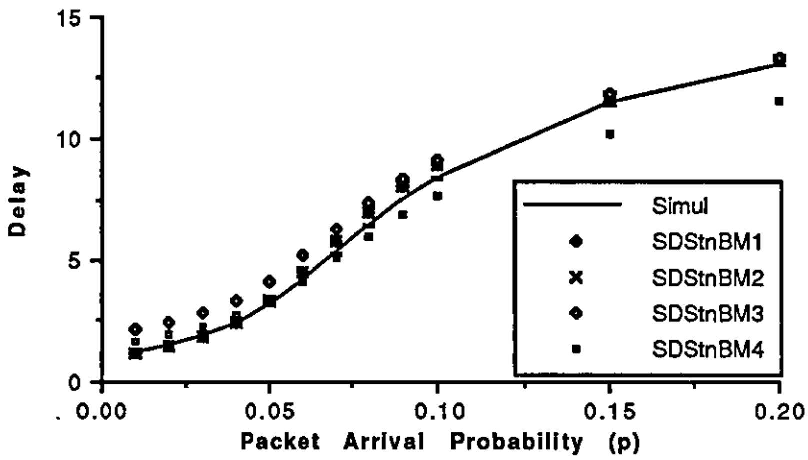


Fig. 2.6.17 Delay Performance (N=16, M=1, Q=1)

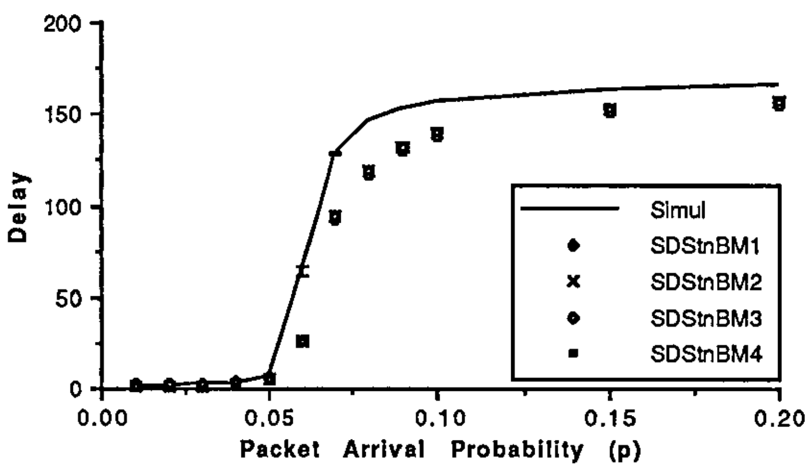


Fig. 2.6.18 Delay Performance (N=16, M=1, Q=10)

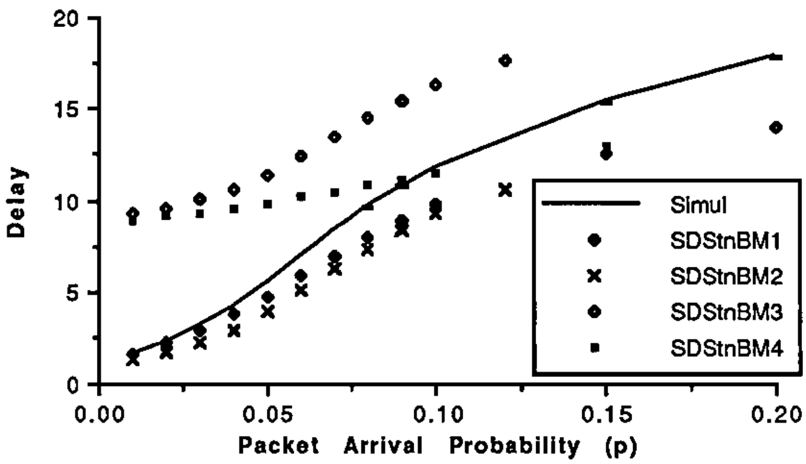


Fig. 2.6.19 Delay Performance (N=16, M=8, Q=1)

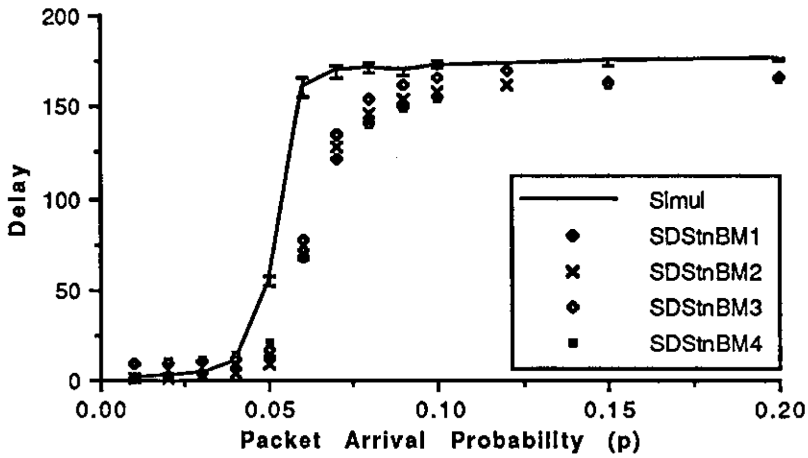


Fig. 2.6.20 Delay Performance (N=16, M=8, Q=10)

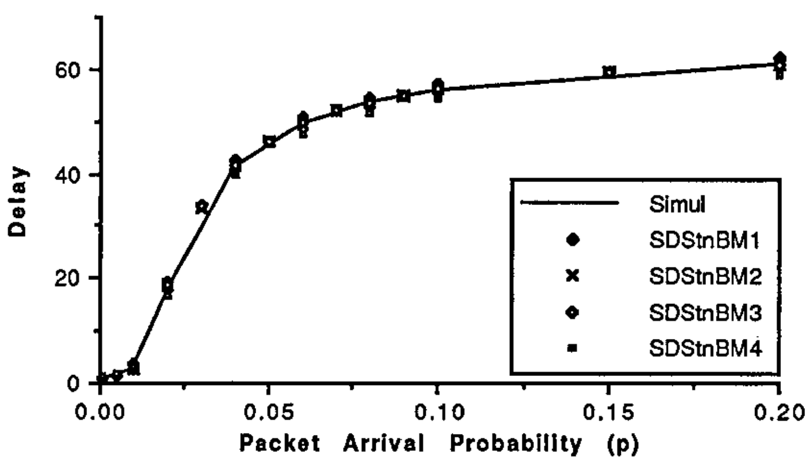


Fig. 2.6.21 Delay Performance (N=64, M=1, Q=1)

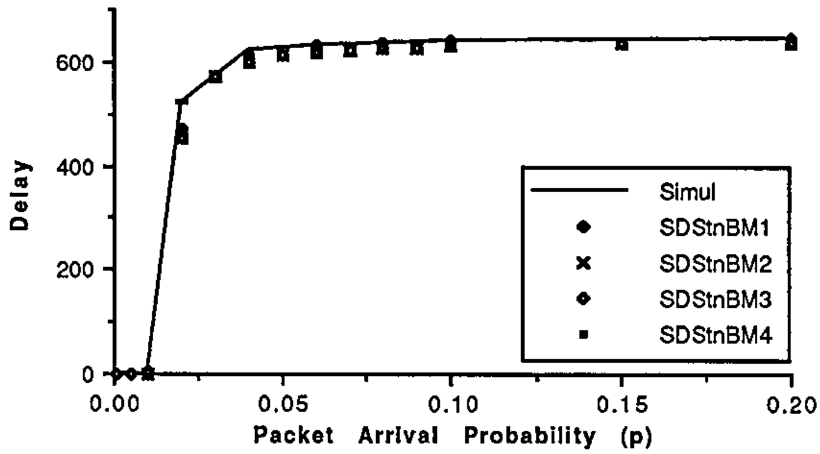


Fig. 2.6.22 Delay Performance (N=64, M=1, Q=10)

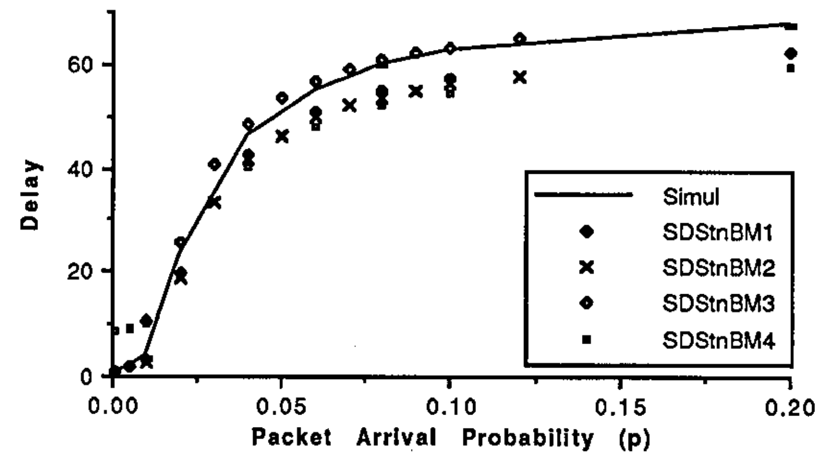


Fig. 2.6.23 Delay Performance (N=64, M=8, Q=1)

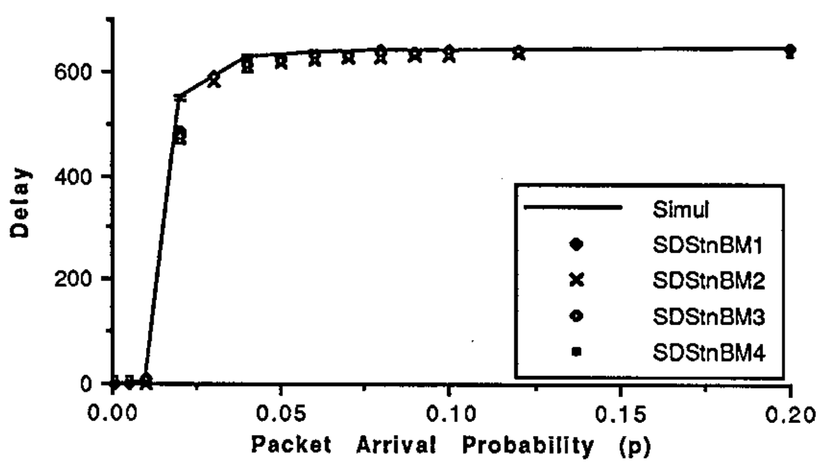


Fig. 2.6.24 Delay Performance (N=64, M=8, Q=10)

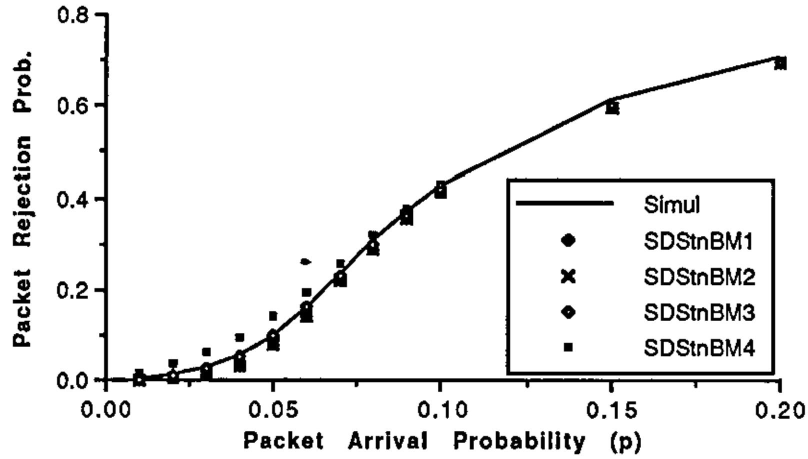


Fig. 2.6.25 Buffer Overflow Characteristics (N=16, M=1, Q=1)

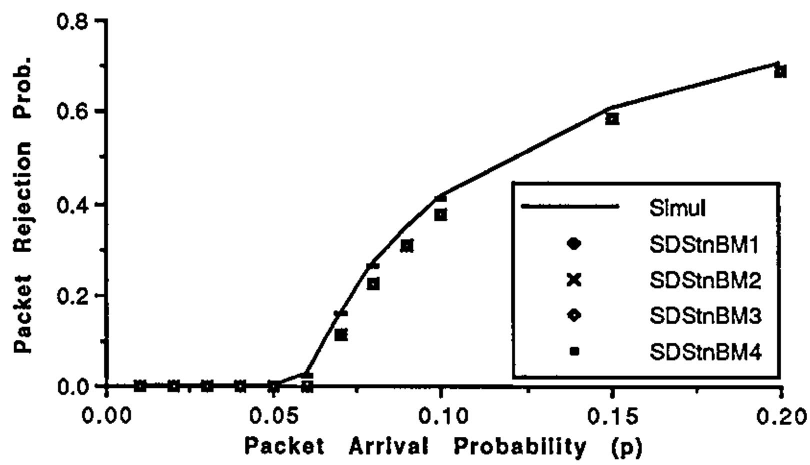


Fig. 2.6.26 Buffer Overflow Characteristics (N=16, M=1, Q=10)

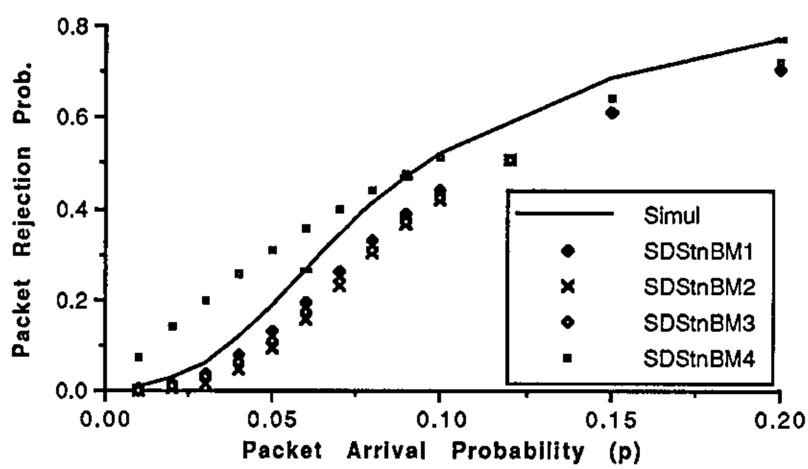


Fig. 2.6.27 Buffer Overflow Characteristics (N=16, M=8, Q=1)

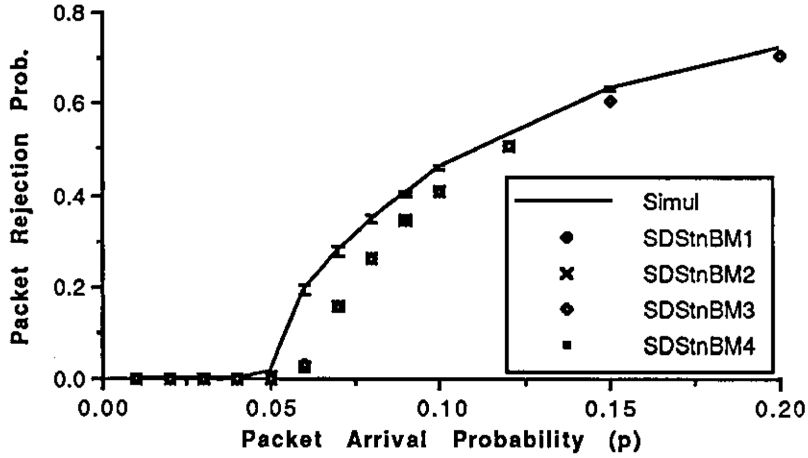


Fig. 2.6.28 Buffer Overflow Characteristics (N=16, M=8, Q=10)

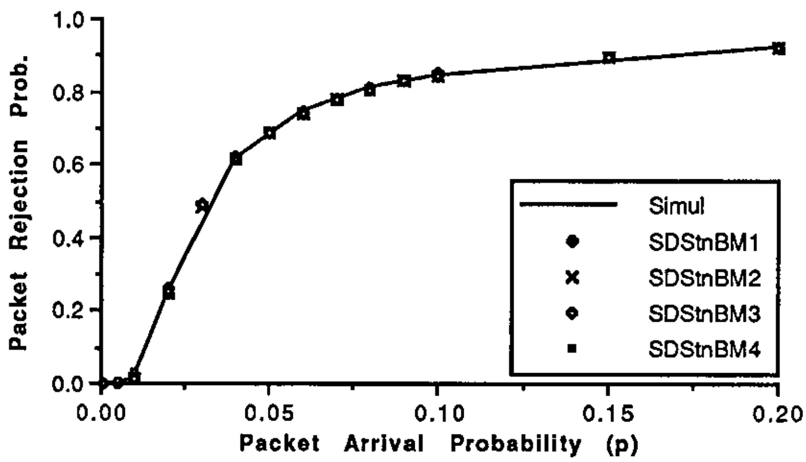


Fig. 2.6.29 Buffer Overflow Characteristics (N=64, M=1, Q=1)

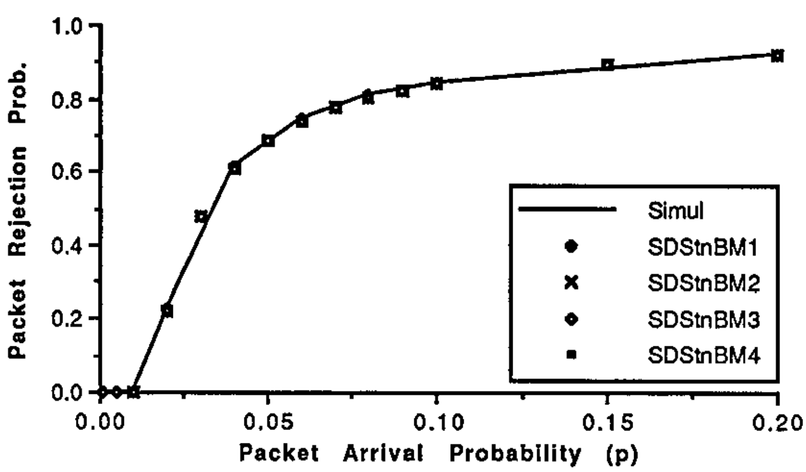


Fig. 2.6.30 Buffer Overflow Characteristics (N=64, M=1, Q=10)

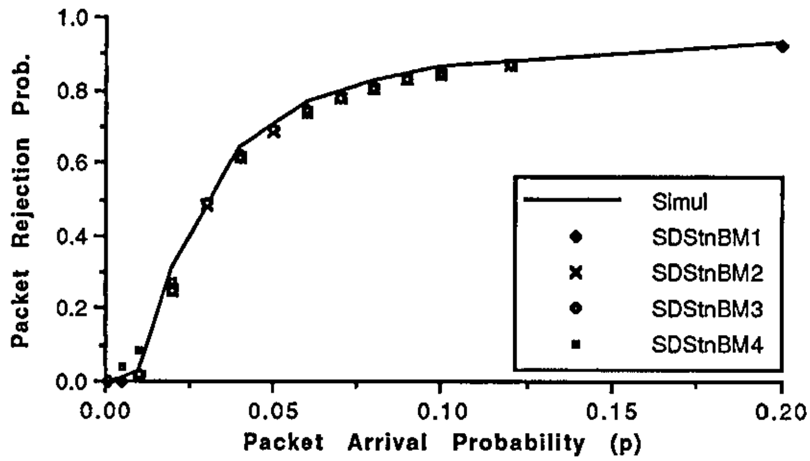


Fig. 2.6.31 Buffer Overflow Characteristics (N=64, M=8, Q=1)

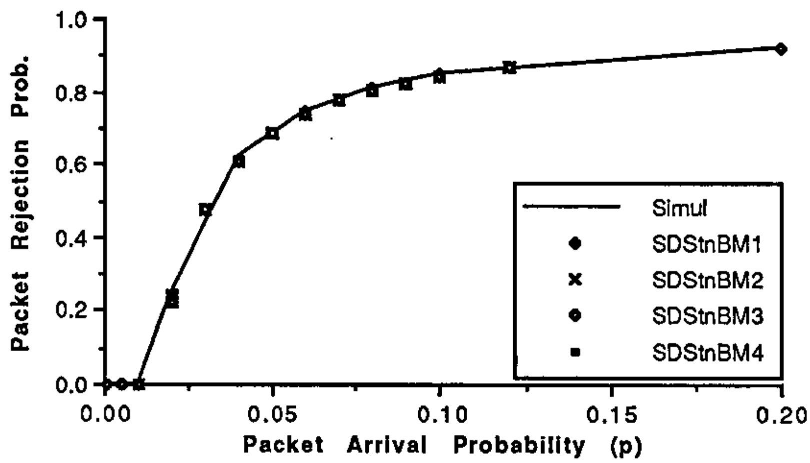


Fig. 2.6.32 Buffer Overflow Characteristics (N=64, M=8, Q=10)

CHAPTER 3

Slotted Ring, Source Deletion Protocol: Slot Based Model

3.0 Introduction

In this chapter, the Slotted Ring network with a source deletion protocol will be modelled observing the system from a single slot.

3.1 Source Deletion, Slot Based Model (SDSlotBM)

The following assumptions made in the previous chapter will apply in this chapter as well -

assumptions (2.0.1) to (2.0.12).
assumption (2.1.3).

In addition, the following assumption will also be made -

(3.1.1) The total buffer size of the entire network (i.e., N * maximum buffer size per station) is an integer multiple of the number of slots.

(3.1.2) Only a single packet of data may arrive per slot at any given station time.

(3.1.3) The packet arrival rate will follow a Bernoulli distribution and is denoted by 'p' packets per slot per station time.

Figure 3.1.1 shows the Markov state diagram applicable to this model. Of the two values shown inside each state, the first indicates the queue size per slot, and the second, the number of station times that have elapsed since the start of transmission.

For this model,

$P(y,x)$ = steady state probability of a slot being in state (y,x).

Q = Average maximum buffer size per slot

= (N * Maximum buffer size per station) / M

N and M have their usual meanings.

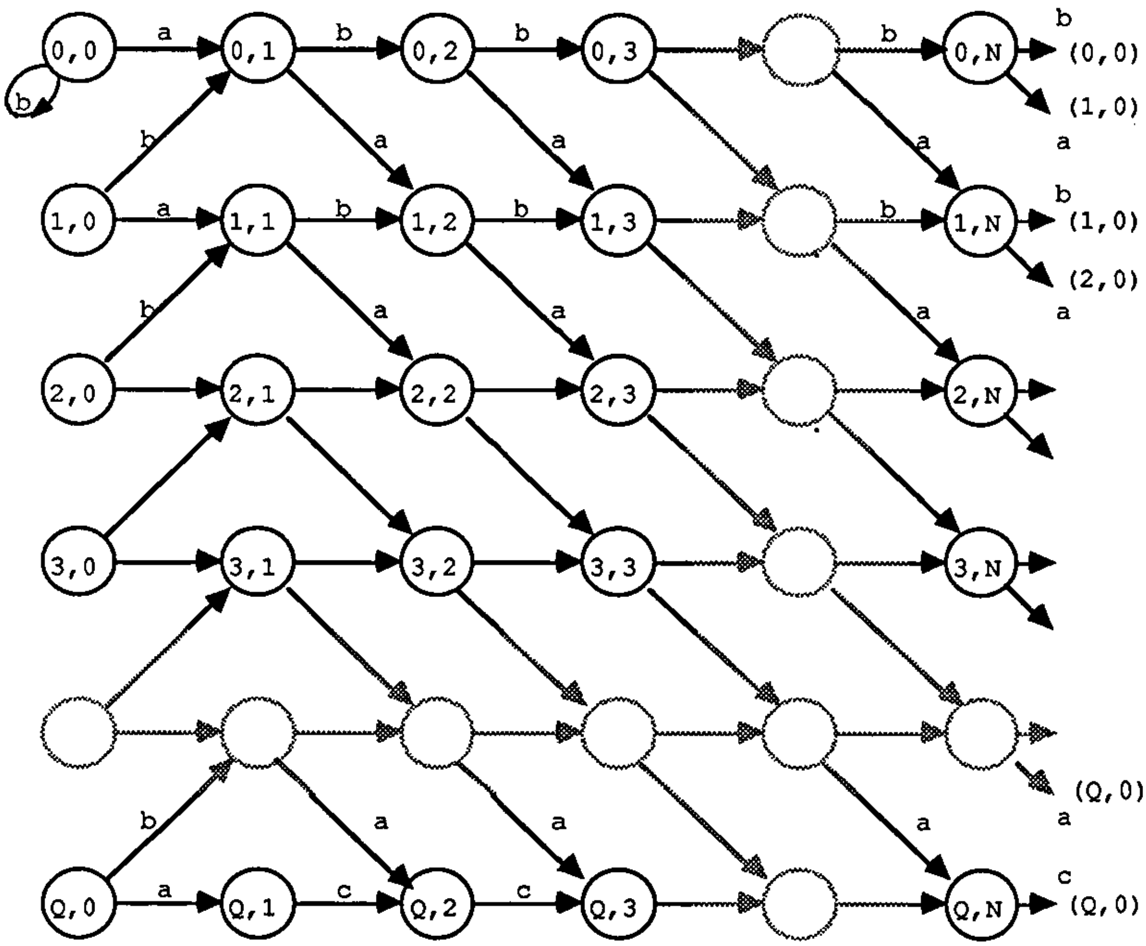
An empty slot (states $(y,0)$), will be filled with a packet for transmission in the next station time if there are packets awaiting to be transmitted ($y \geq 1$ states), or if there is a packet arrival when there is no queue ($y=0$ state).

To complete the transmission, the slot will need to travel once around the ring - i.e., N number of station times, after which it will be emptied by the transmitting station. Packets may arrive to the network at any given station time, thus making the slot change its state in the usual manner.

When solving the model for steady state, in order to avoid the lengthy simplification of the equations involved, the common properties of this model with that of SDStnBM1 will be used.

It may be easily observed that the two following changes to SDStnBM1's Markov state diagram will convert it into SDSlotBM -

(1) Let $S=0$.



where - a = p
b = 1-p
c = 1

Fig 3.1.1 Discrete-time slot based Markov model for a Slotted Ring with a Source Deletion Protocol (SDSlotBM)

(2) Change the number of horizontal 'x' states from M+1 to N+1. This will take into account the station times involved in this model over the slot times that were considered in SDStnBM1.

Then, the equations (2.1.9), (2.1.10), (2.1.11), (2.1.12) and (2.1.14) can be respectively converted to suit the current model as follows -

$$P(0,1) = P(0,0)\frac{p}{(1-p)^{N}} \tag{3.1.1}$$

$$P(0,N) = P(0,0)\frac{p}{(1-p)} \tag{3.1.2}$$

$$P(y,0) = \frac{P(y-1,1) - P(y-1,0)p}{(1-p)} \qquad \{1 \le y \le Q\} \tag{3.1.3}$$

$$P(y,N) = \frac{P(y,0) - P(y-1,N)p}{(1-p)} \qquad \{1 \le y < Q\} \tag{3.1.4}$$

$$P(y,1) = \frac{P(y,N) - \sum_{j=0}^{y-1} P(j,1)\, p^{(y-j)}\, (1-p)^{(N-1-y+j)}\; {}^{(N-1)}C_{(y-j)}}{(1-p)^{(N-1)}} \qquad \{1 \le y < Q\} \tag{3.1.5}$$

Considering the summation of all the state probabilities and converting equation (2.1.16) to suit,

$$\begin{aligned} 1 &= \sum_{x=0}^{N}\sum_{y=0}^{Q} P(y,x) \\ &= P(0,0)\,(1+Np) + (N+1)\sum_{y=1}^{Q} P(y,0) \end{aligned} \tag{3.1.6}$$

If P(0,0) is known, then the other P(y,0) states can be calculated thus -

(3.1.1) to calculate P(0,1)

(3.1.2) to calculate P(0,N)

The following loop to be executed repeatedly for all 1<y<Q,

(3.1.3) to calculate P(y,0)

(3.1.4) to calculate P(y,N)

(3.1.5) to calculate P(y,1)

Finally, (3.1.3) to obtain P(Q,0).

Using equation (3.1.6) and applying the values calculated for P(y,0) using the above sequence, a suitable numerical method (simple iteration, bisection method) may be used to solve for P(0,0), which in turn can be utilised to compute the rest of the steady state probabilities.

Then,

$$\begin{aligned} \textit{Throughput} &= \textit{Number of transmissions per cycle per slot} \\ &= N\left(P(0,0)p + \sum_{y=1}^{Q} P(y,0)\right) \end{aligned} \tag{3.1.7}$$

$$\textit{Probability of a slot being full} = 1 - \sum_{y=0}^{Q} P(y,0) \tag{3.1.8}$$

It can be shown that both the above equations give identical results thus confirming the fact that, for Source Deletion protocols, the throughput is the probability of a slot being full.

$$\underset{(\overline{q})}{\textit{Average Queue per Station}} = \frac{M}{N}\sum_{y=0}^{Q}\sum_{x=0}^{N} yP(y,x) \tag{3.1.9}$$

$$\textit{Average Delay } (\overline{d}) = \frac{\overline{q}N}{\textit{Throughput}} + \frac{M}{N} \tag{3.1.10}$$

$$\underset{(PRP)}{\textit{Packet Rejection Probability}} = 1 - \frac{\textit{Throughput}}{pN} \qquad (3.1.11)$$

One of the greatest shortcomings of solving for P(0,0) initially is the large errors that result when the value of P(0,0) becomes close to zero at network overload (i.e., when the throughput is at saturation and the average queue is at maximum possible levels). On such occasions, one has to either express the above formulae in a different manner to make them stable, or, resort to a matrix solution.

It is noted that this model can be solved by using the simplification used in the models SDStnBM2 and SDStnBM3. However, this reduces the upper limit of the packet arrival probability to 1/N so that the model is not useable in the saturated throughput region. Also, the fact that the complete model SDSlotBM can be reduced to a single variable solution, reduces the need for such simplification.

3.2 Matrix Solution

In the case of slot based models, solving for the steady state probabilities is theoretically quite straightforward once the problem has be transformed into a soluble form as in section 2.4. However, the greatest practical problem is the size of the matrix that has to be solved, which arises due to the fact that the maximum queue of the slot based model is at least N/M times greater than that for the similar station based models considered in chapter 2. Therefore, in order to get round this, use is made of sparse matrix solution techniques mentioned earlier.

Even though these sparse matrix methods can, to a large extent, reduce the computer memory usage, it does little to reduce the computation time, which can run into several hours of CPU time for just one solution. To alleviate this problem, a property of the network was used as follows.

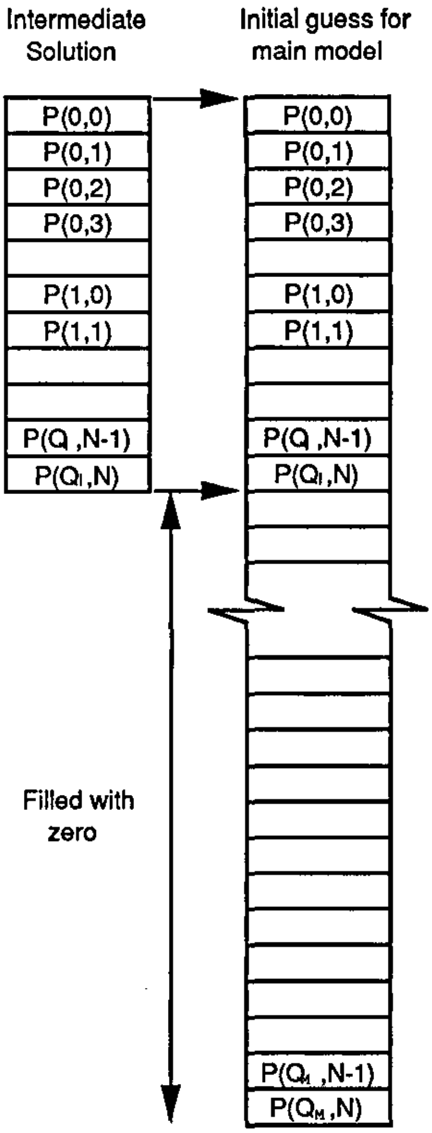
It was observed that, when the throughput is either in the linear or saturated regions, other variables remaining the same, the steady state probabilities of the model are independent of the maximum queue size, especially when it is high (say over 15). Due to this same effect, when the packet arrival probability is increased from 0 to 1, it was observed that the average queue tends to be either quite low (in the region where the throughput is linear), or, take a value close to its maximum possible under saturated conditions. As it may be seen from 'Average Queue v Packet Arrival Probability' graphs, the region between these two extremes, the transition region, is extremely small. This means that, except within the transition region, the higher values of the P(y,x) are always concentrated either in the region where the queue is low (states representing lower queue size), or, in the region where the queue is high (states representing a queue size close to the maximum queue), the rest of the P(y,x) values being zero, or very close to zero.

Therefore, except in this transition region, by observing the steady state probabilities for a model with a lower maximum queue with all other network parameters being identical, the above fact can be used to guess an almost 100% correct initial set of values for P(y,x) when solving for a relatively high maximum queue per station using the iterative sparse matrix method.

The following steps are used to bring about this solution -

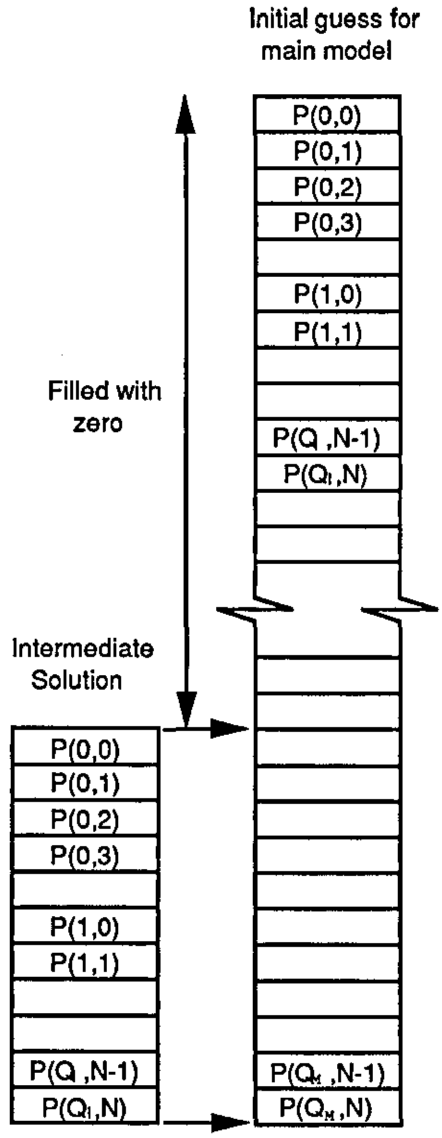
(1) Solve the network for a lower maximum queue size, the other variables of the system remaining the same, and record the steady state probabilities. The maximum queue size here should be low enough to bring about a speedy solution to this intermediate step, but not too low so as to prevent a natural state distribution taking place due to buffer overflow even when the throughput is not close to saturation.

(2) If the throughput for the above packet arrival rate is expected to be in the linear region, then the steady state probabilities recorded above may be



Initial guess when the throughput is expected to be non-saturated

Figure 3.2.1



Initial guess when the throughput is expected to be saturated

Figure 3.2.2

used as the initial guess for the similar states in the system under consideration. The states that are not given such an initial value, (amounting to the difference between the total number of states in the model under consideration and the intermediate model) will be given a value of zero. (see figure 3.2.1)

(3) Alternately, if the throughput for the given packet arrival rate is expected to lie in the saturated region, then since the steady state probability distribution shows more weight around the maximum queue region, the values obtained in step (1) are substituted as initial values to the corresponding maximum queue states as shown in figure 3.2.2. In other words, the state (y,x) of the intermediate model is assumed to be the state $(y+Q_M-Q_I,x)$ upon being substituted; where, Q_M is the maximum queue of the model under consideration and Q_I is the maximum queue of the Intermediate model.

As in step (2), the states that are not given an initial steady state probability value in this manner are given a value of zero.

It is noted that this method, depending upon the Q_M and Q_I involved, can easily increase the speed of execution over a thousandfold.

3.3 Results

As in the case of the previous chapter, the model was solved for all the possible combinations of N (16 and 64), M (1, 8 and 16) and Q (1, 10 and 50) and the usual parameters observed.

Throughput

The throughput characteristics for this model, as in the case of station based models, tend to become two intersecting straight lines with the increase of the maximum buffer size Q, the first line indicating the linear increase of the throughput with the packet arrival probability being identical. The line that represents the saturated region shows the ideal throughput that one may ever achieve -

$$Throughput = \frac{1}{1+\frac{1}{N}}$$

(See appendix A.) Due to this, the model performs poorly with increasing M when matched with the simulation, since the latter is prone to the earlier mentioned quasi stable states.

Average Queue

As mentioned before in section 3.2, the average queue remains independent of Q for lower packet arrival rates, but, changes nearly in the manner of a step function to the maximum buffer value when the system reaches saturation.

A comparison with the simulation indicates a general improvement of match with the increase of Q, a slight improvement with the increase of N and a slight degradation with the increase of M.

Average Delay

When comparing with the simulation, the average delay shows a marked improvement with the increase of Q, and a slight improvement with the increase of N. The effect of changing M on this comparison being varied - an improvement with the increase of M except when N=16 and Q≠1.

Packet Rejection Probability

The comparison with the simulation for this parameter shows an improvement in match with the increase of N. For the case of N=64, the increase of M or Q on its own does not make any significant effect on the results. However, for N=16, there is a general degradation of match with the increase of M except for Q=1 and low packet arrival rates where it actually improves. There is a similar improvement with the increase of Q for N=16 and M=1.

3.4 Summary

Slot or server based models are uncommon, especially in the area of Slotted Ring networks. One of the main purposes of this chapter was to lay the groundwork in viewing the system from this angle by modelling a basic Slotted Ring so that it may be later extended to predict the behaviour of more advanced protocols.

The model was first solved by the use of equations developed for the station based models that were modified to suit the new situation. An alternate matrix solution was then suggested with possible (network dependent) shortcuts which one may use to shorten the computation time when there are a large number of states due to a large packet buffer.

One of the main outcomes that can be concluded from such a method is the fact that it is quite sufficient to solve the model for a maximum buffer size (Q) of about 10, so as to enable the prediction of the effects a much higher maximum buffer size would have on the network performance.

Since the model assume independent stations and slots, the results tend to show ideal performance characteristics. Thus, when there is only a single slot, due to the absence of quasi stable states (Falconer 85), the simulation shows an excellent match to the model- especially when the network is more stable under a higher maximum buffer size.

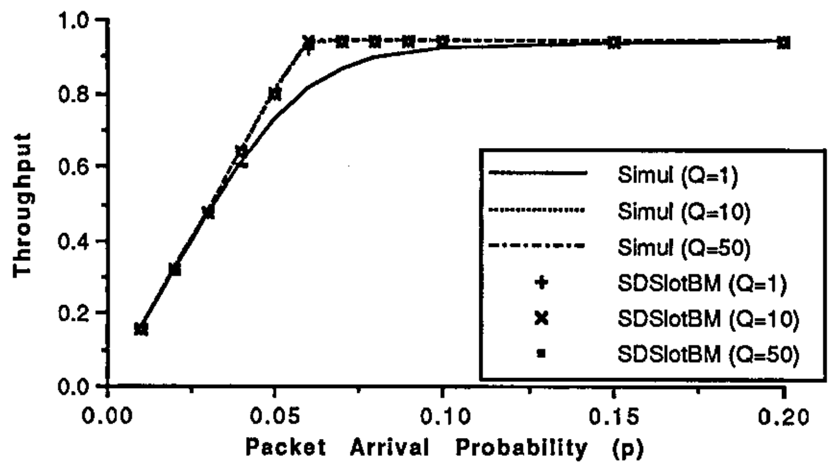


Fig. 3.3.1 Throughput Performance (N=16, M=1)

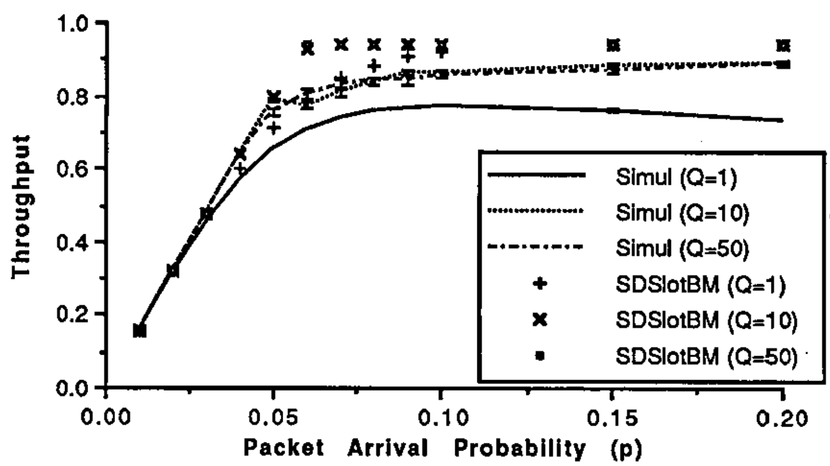


Fig. 3.3.2 Throughput Performance (N=16, M=8)

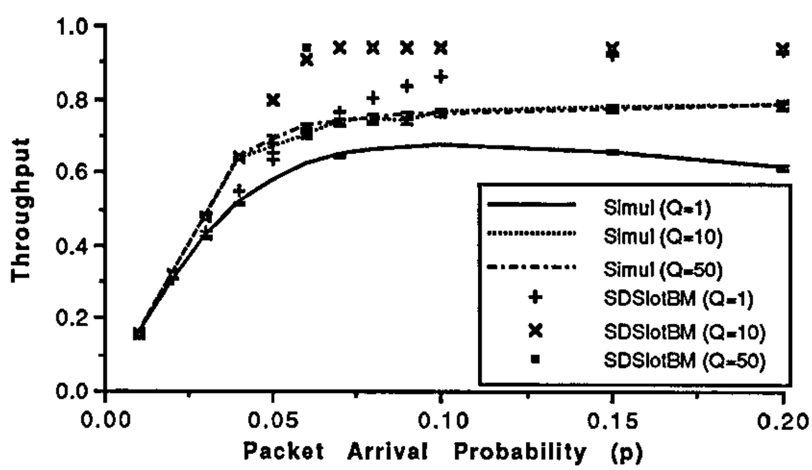


Fig. 3.3.3 Throughput Performance (N=16, M=16)

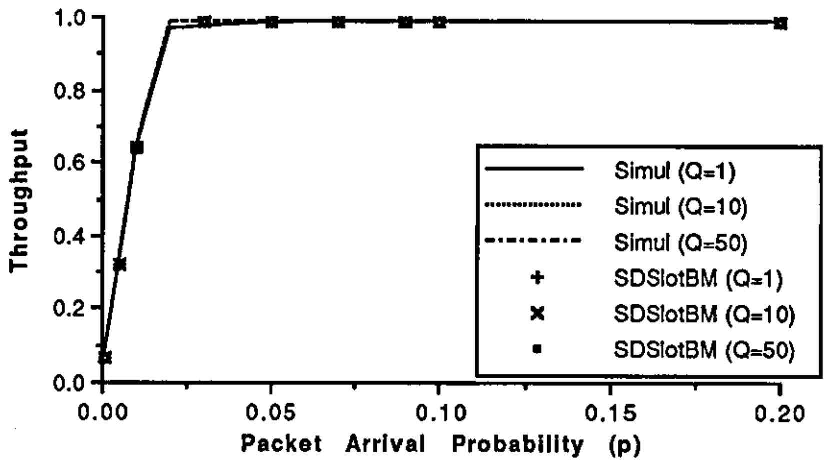


Fig. 3.3.4 Throughput Performance (N=64, M=1)

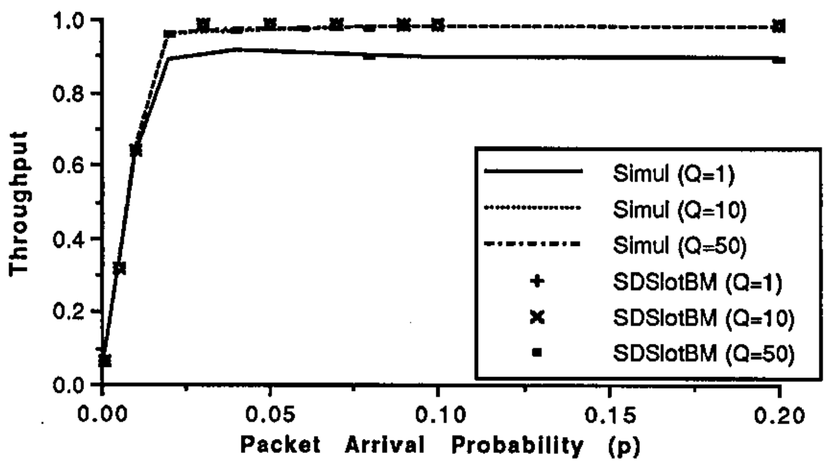


Fig. 3.3.5 Throughput Performance (N=64, M=8)

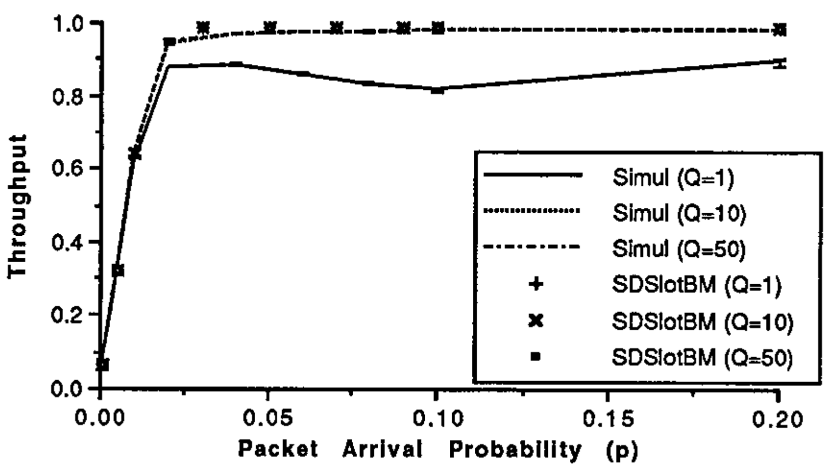


Fig. 3.3.6 Throughput Performance (N=64, M=16)

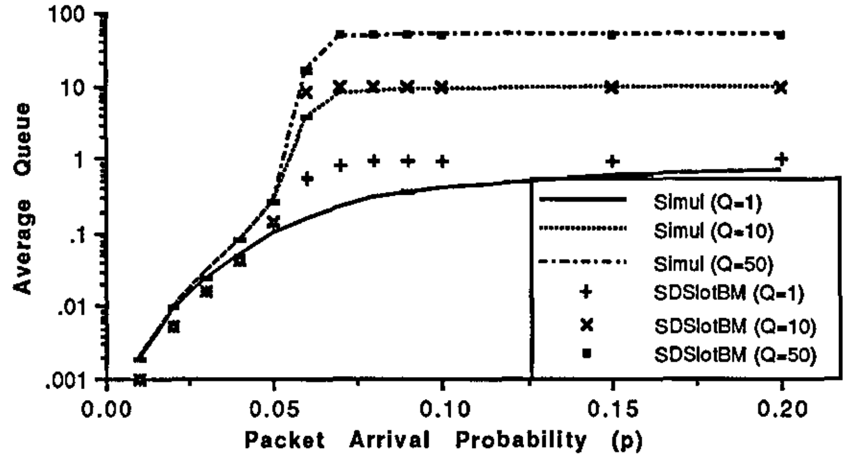


Fig. 3.3.7 Queueing Performance (N=16, M=1)

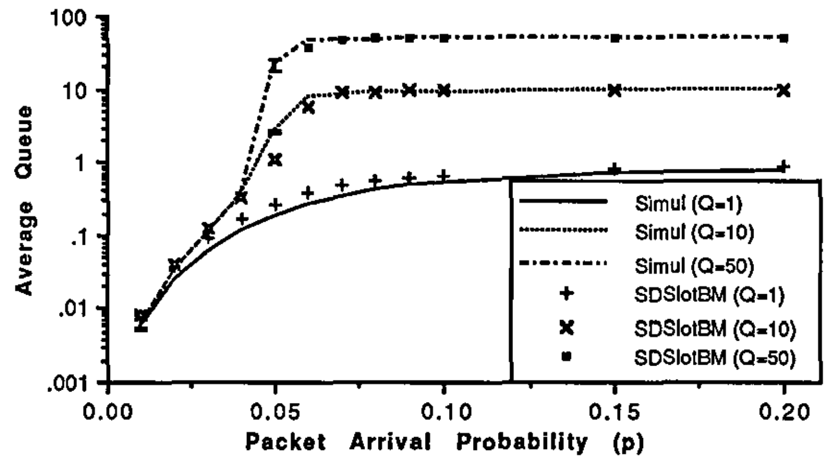


Fig. 3.3.8 Queueing Performance (N=16, M=8)

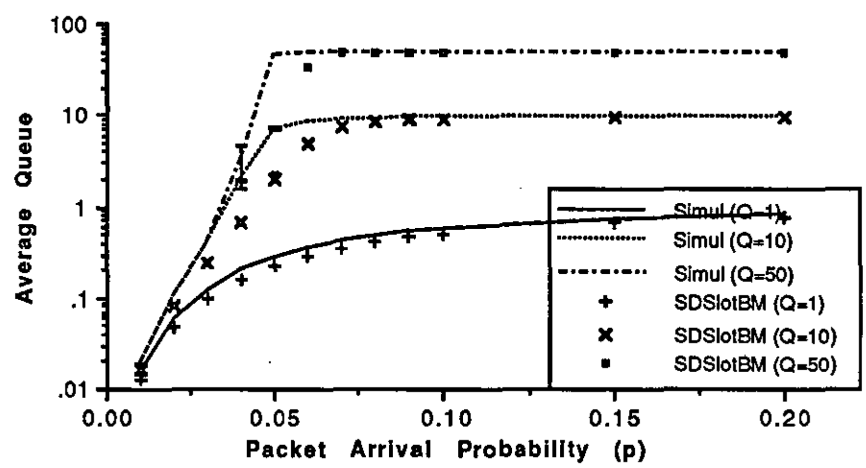


Fig. 3.3.9 Queueing Performance (N=16, M=16)

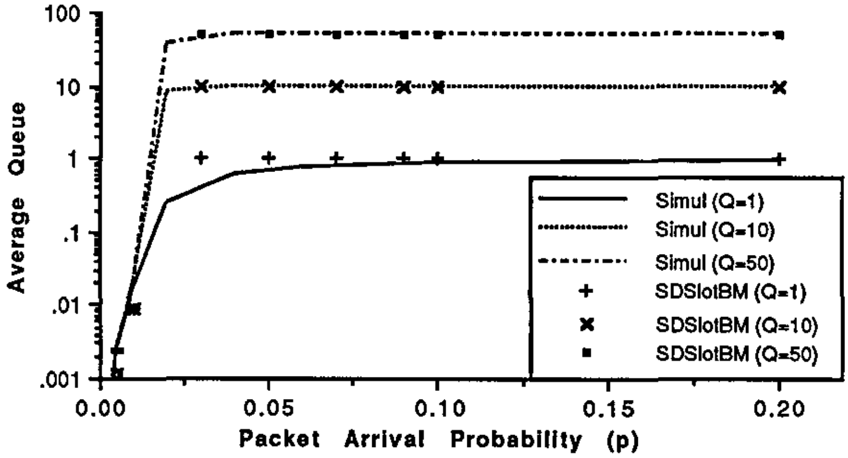


Fig. 3.3.10 Queueing Performance (N=64, M=1)

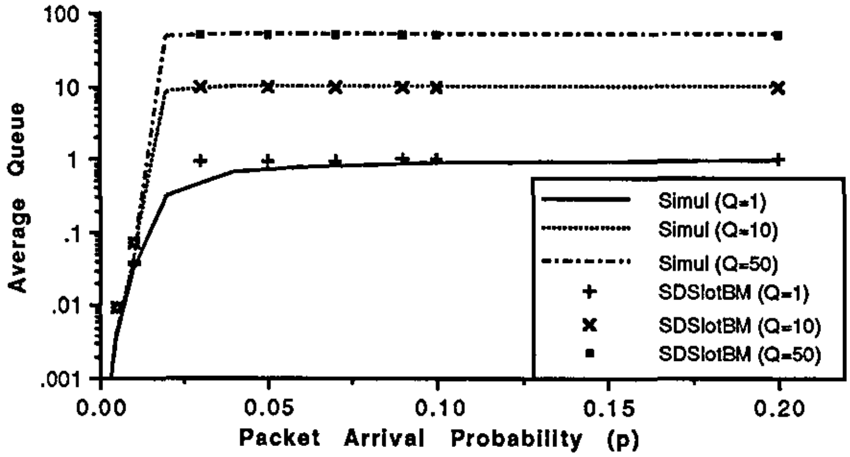


Fig. 3.3.11 Queueing Performance (N=64, M=8)

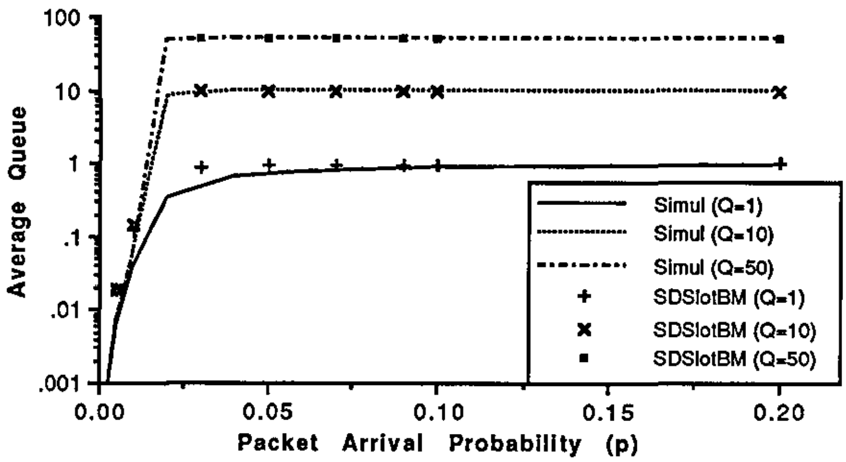


Fig. 3.3.12 Queueing Performance (N=64, M=16)

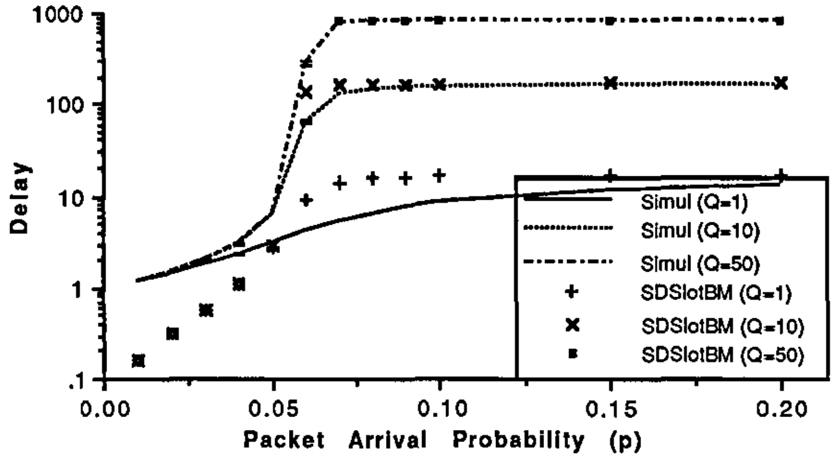


Fig. 3.3.13 Delay Performance (N=16, M=1)

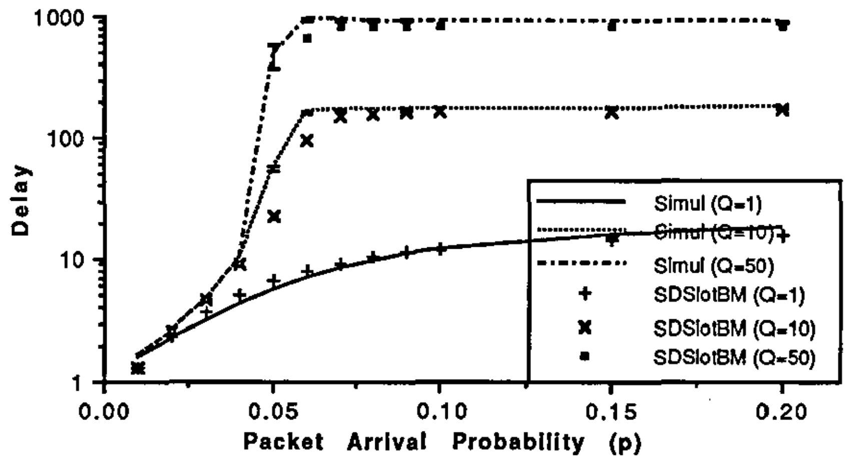


Fig. 3.3.14 Delay Performance (N=16, M=8)

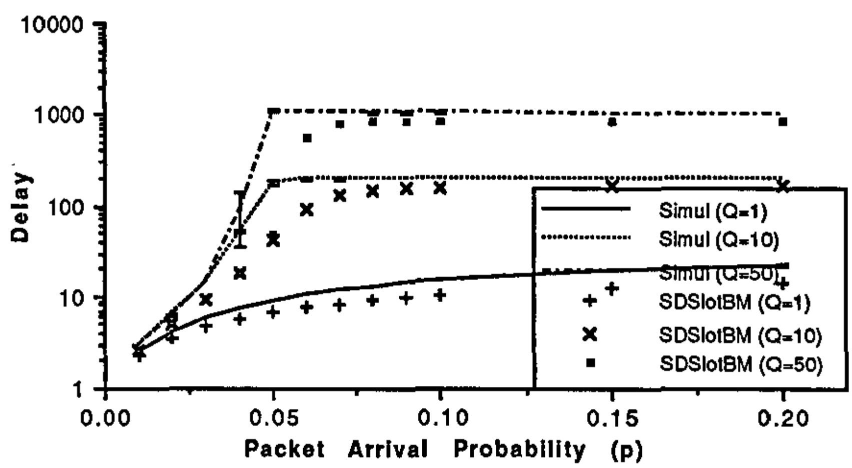


Fig. 3.3.15 Delay Performance (N=16, M=16)

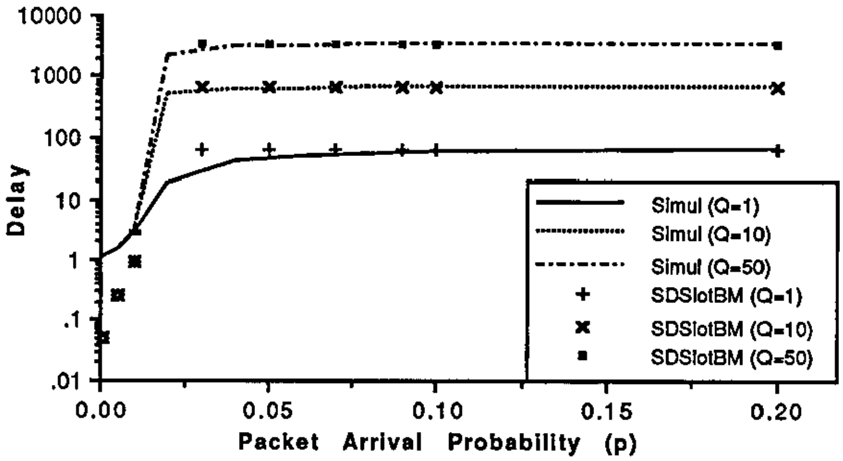


Fig. 3.3.16 Delay Performance (N=64, M=1)

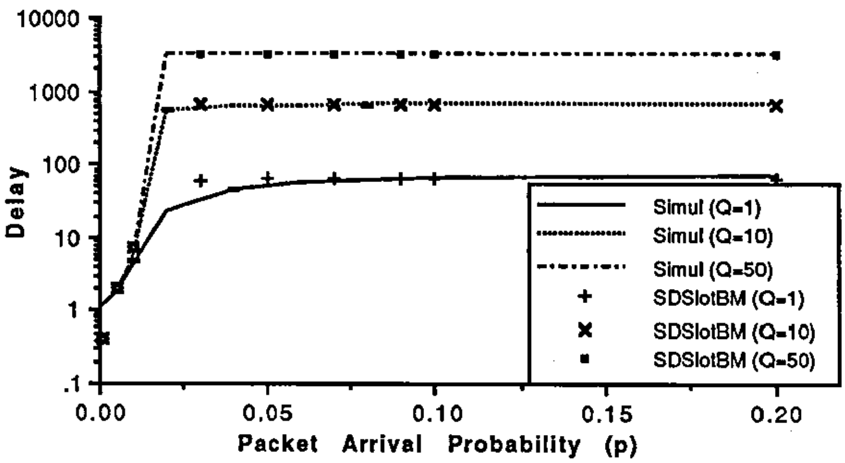


Fig. 3.3.17 Delay Performance (N=64, M=8)

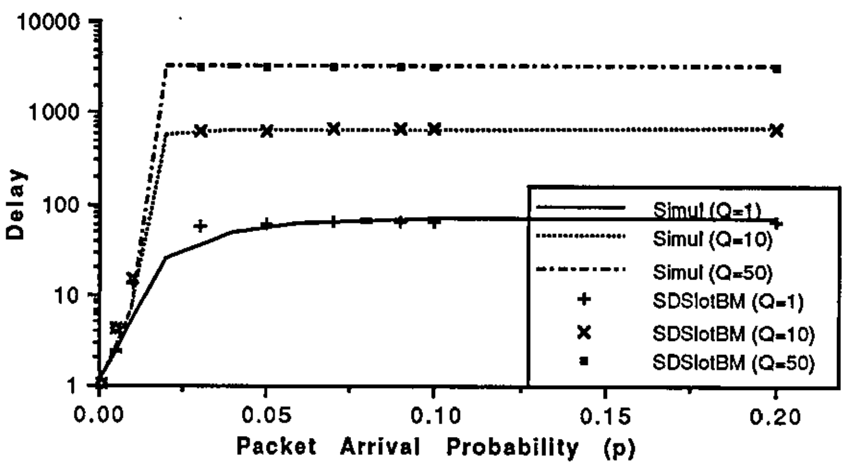


Fig. 3.3.18 Delay Performance (N=64, M=16)

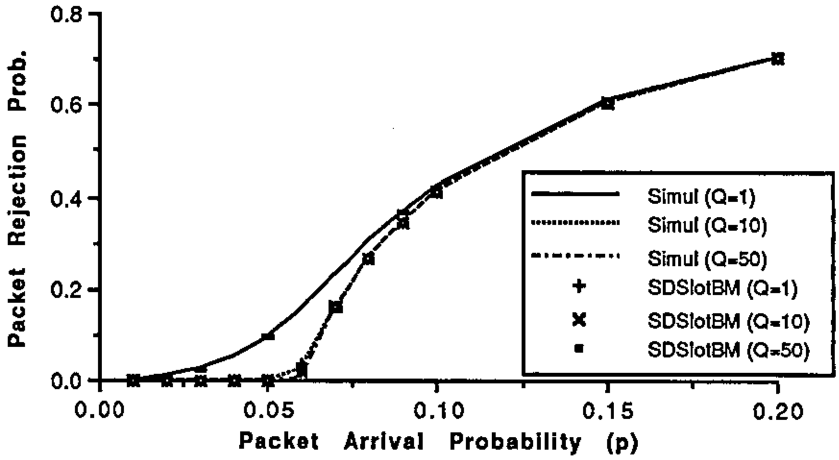


Fig. 3.3.19 Buffer Overflow Characteristics (N=16, M=1)

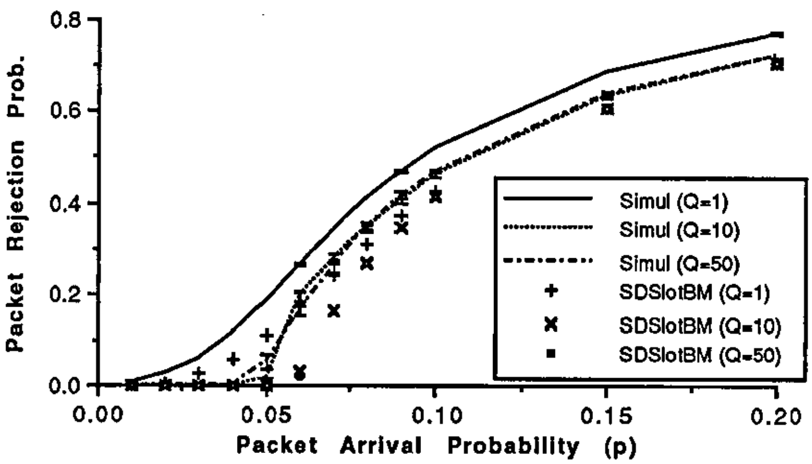


Fig. 3.3.20 Buffer Overflow Characteristics (N=16, M=8)

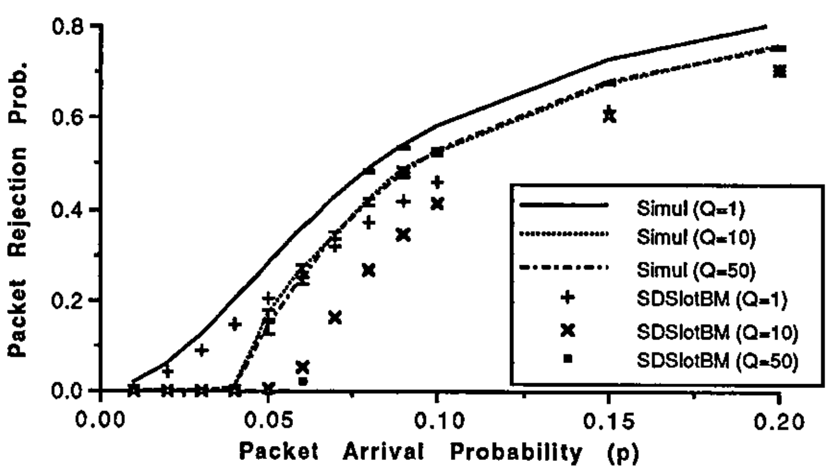


Fig. 3.3.21 Buffer Overflow Characteristics (N=16, M=16)

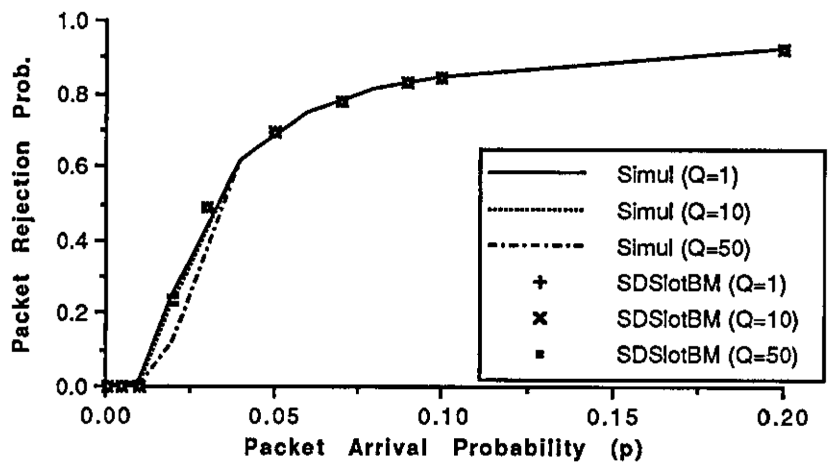


Fig. 3.3.22 Buffer Overflow Characteristics (N=64, M=1)

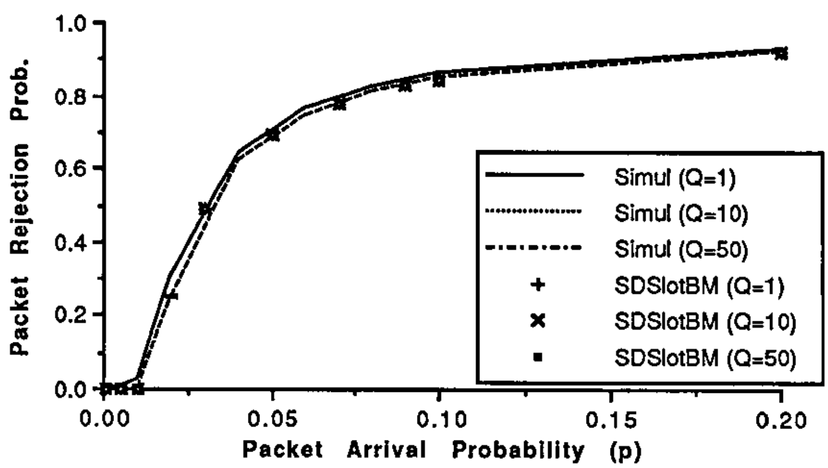


Fig. 3.3.23 Buffer Overflow Characteristics (N=64, M=8)

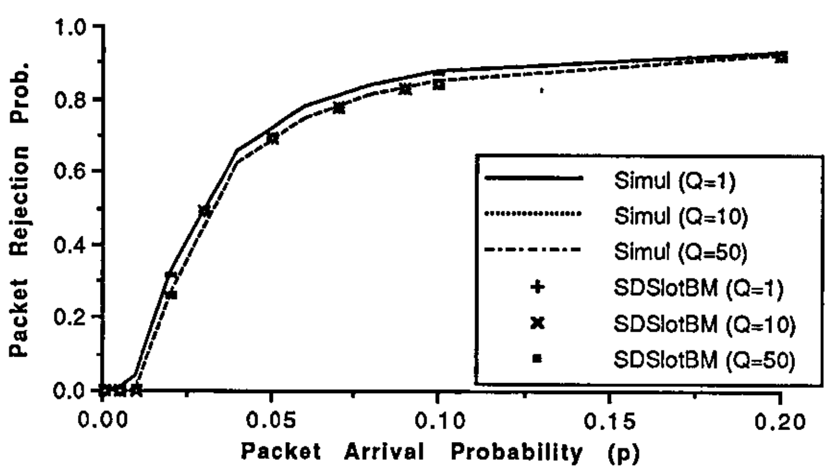


Fig. 3.3.24 Buffer Overflow Characteristics (N=64, M=16)

CHAPTER 4

Slotted Ring, Destination Deletion Protocol : Station Based Models

4.0 Introduction

In destination deletion slotted ring networks, the packet is removed from the network by the recipient station rather than by the station that transmitted it. Though this effectively prevents the error checking that can be done by the transmitting station by comparing the returned message with that of the original, the advances made in hardware have resulted in extremely low error rates, thus making such a check unnecessary in many situations. This is particularly the case for isochronous data, such as voice or video, when the effects of errors are likely to be no more catastrophic than a small degradation in reproduction quality.

In this chapter, four models for this type of network will be considered. The first two will be similar in all aspects except that they will assume that, throughout its traverse around the ring, the probability of a packet being addressed to a given station can be either constant or variable. Due to the complexity of the resulting Markov diagrams, the matrix method will be used to solve them.

Models 3 and 4 will be the simplified versions of the models 1 and 2 respectively, such that the solution can be expressed using basic equations, thus greatly reducing the amount of computation needed.

The assumptions (2.0.1) to (2.0.5), (2.0.7) to (2.0.12), (2.1.3) and (2.5.1) will be applicable to all the models in this chapter as well. In addition, the following are assumed -

(4.0.1) Packets are removed from the network by the receiving station.

(4.0.2) A station will not transmit to itself.

(4.0.3) No general broadcasts take place.

Although the use of the assumption (2.0.8) may defeat one of the main advantages of the destination deletion protocol, due to assumption (2.0.9) and (2.0.3), the average of having a maximum of 1 slot per station, conveniently alleviates any serious objections. Physically, this process may be interpreted as not transmitting another packet until the current packet has reached its destination.

All the models will consider time intervals measured in station times.

4.1 Destination Deletion, Station Based Model 1 (DDStnBM1)

Figure 4.1.1 shows the Markov chain applicable for this model. Considering assumption (4.0.2), the maximum distance a packet may travel is up to the last station before the transmitting station in the direction of data flow. Accordingly, the maximum time a packet can occupy a slot is (N-1) station times. Thus in figure 4.1.1,

$$K = N-1$$

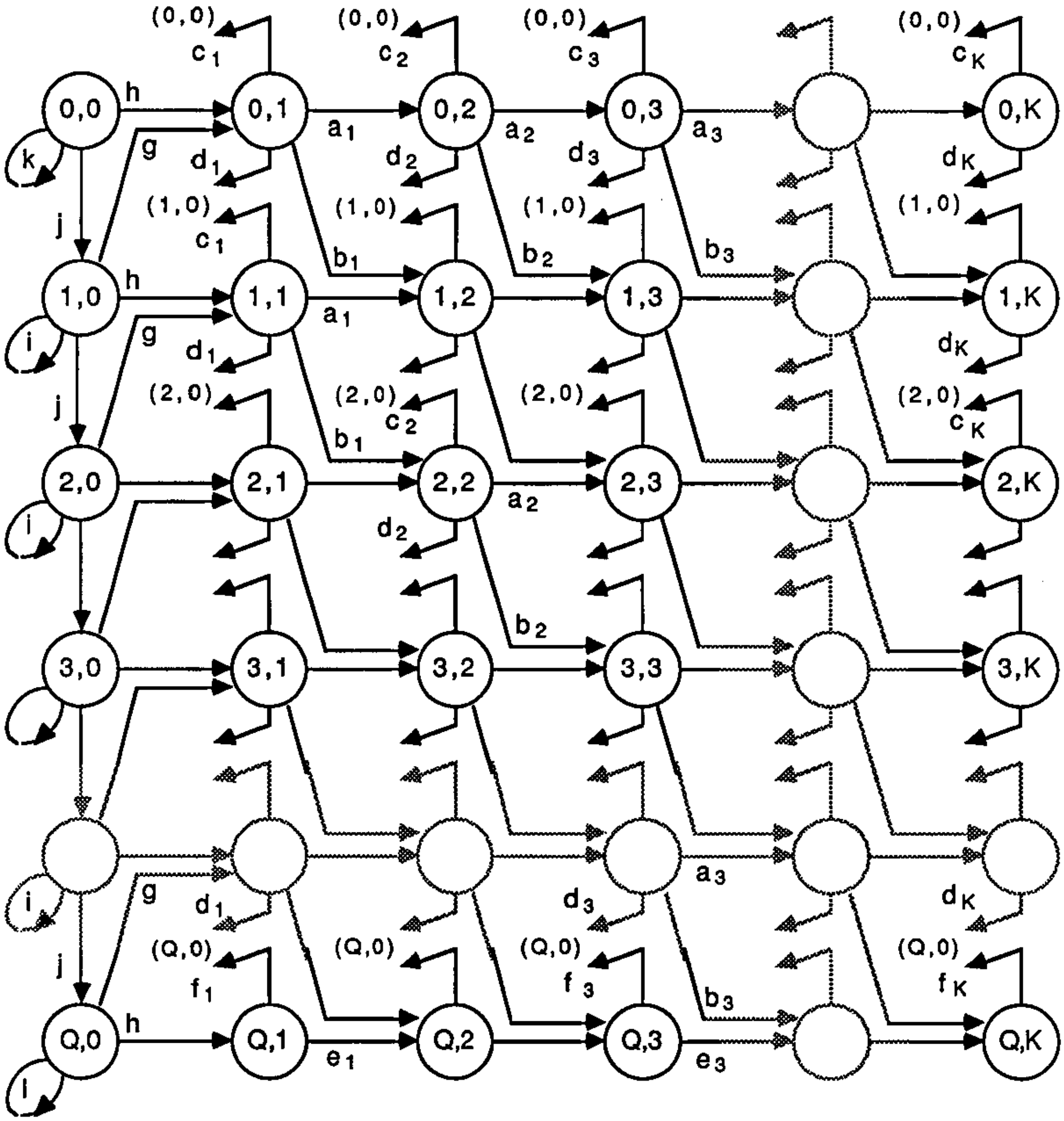


Fig 4.1.1 Discrete Time, Station Based Markov Model for a Slotted Ring with a Destination Deletion Protocol

The transitions c_x, d_x and f_x, $1 \leq x \leq K$, indicate the probability that the packet transmitted is addressed to the x^{th} downstream station from the current station. Due to destination deletion, these probabilities also represent the probability of an end of transmission of the packet and the station returning to its idle or waiting state.

In addition to the general assumptions mentioned in section 4.0, assumption (2.5.2) and the following are made -

(4.1.1) **Probability of destination deletion is 1/K at all stations except the last downstream station from which the packet will be deleted with unit probability.**

From assumptions (4.1.1) and (4.0.2),

$$\begin{matrix}\textit{Probability of a destination deletion} \\ \textit{(except in last downstream station)}\end{matrix} = \frac{1}{K}$$

Therefore,

$$c_x = \frac{1-p}{K} \qquad \{1 \leq x < K\}$$
$$c_K = 1-p \tag{4.1.1}$$

$$d_x = \frac{p}{K} \qquad \{1 \leq x < K\}$$
$$d_K = p \tag{4.1.2}$$

$$f_x = \frac{1}{K} \qquad \{1 \leq x < K\}$$
$$f_K = 1 \tag{4.1.3}$$

The transitions a_x, b_x, e_x, g, h, i, j, k and l can be explained as in the model SDStnBM1.

After accounting for the possible values of c_x, d_x and f_x above,

$$a_x = \left(1-\frac{1}{K}\right)(1-p) \qquad \{1 \le x < K\} \tag{4.1.4}$$

$$b_x = \left(1-\frac{1}{K}\right)p \qquad \{1 \le x < K\} \tag{4.1.5}$$

$$e_x = \left(1-\frac{1}{K}\right) \qquad \{1 \le x < K\} \tag{4.1.6}$$

As in the case of similar transitions in model SDStnBM1,

$$g = (1-p)(1-S) \tag{4.1.7}$$

$$h = p(1-S) \tag{4.1.8}$$

$$i = (1-p)S \tag{4.1.9}$$

$$j = pS \tag{4.1.10}$$

$$k = 1-p \tag{4.1.11}$$

$$l = S \tag{4.1.12}$$

where, as before,

$$S = \frac{N-1}{M}\left(1-\sum_{y=0}^{Q} P(y,0)\right) \tag{4.1.13}$$

Using these transition probabilities, one may solve the model using the matrix method described in section 2.4.

Then, considering the number of packets transmitted per slot per cycle,

$$Throughput = \left(\sum_{y=1}^{Q} P(y,0) + pP(0,0)\right)(1-S)\ \frac{N^2}{M} \tag{4.1.14}$$

As usual,

$$\textit{Average Queue }(\overline{q}) = \sum_{x=0}^{K}\sum_{y=0}^{Q} yP(y,x) \tag{4.1.15}$$

$$\textit{Average Delay }(\overline{d}) = \frac{\overline{q}\,N}{Throughput} + \frac{M}{N} \tag{4.1.16}$$

$$\begin{matrix}\textit{Packet Rejection}\\ \textit{Probability}(PRP)\end{matrix} = 1 - \frac{Throughput\ M}{N^2\ p} \tag{4.1.17}$$

4.2 Destination Deletion, Station Based Model 2 (DDStnBM2)

Even though a station may choose any one of the other stations as the destination for its packets with equal probability, it can be argued that further a packet travels around the ring, due to the decreasing number of possible destinations, higher becomes the probability that it will reach its destination. Therefore we may assume that -

(4.2.1) A station will transmit to all other stations with the same frequency.

In addition to all the assumptions of section 4.0, assumption (2.5.2) will also be applicable.

If a packet has travelled 'x' station times from the start of its transmission, then, including the current station, there are (K-x+1) possible stations left to which the packet may be addressed. Therefore,

$$\begin{matrix}\textit{probability that the packet is addressed} \\ \textit{to the current } x^{th} \textit{ downstream station}\end{matrix} = \frac{1}{K-x+1}$$

Since the Markov chain applicable for this model remains the same as that of Fig 4.1.1, using the above factor,

$$c_x = \frac{(1-p)}{(K-x+1)} \qquad \{1 \le x \le K\} \tag{4.2.1}$$

$$d_x = \frac{p}{(K-x+1)} \qquad \{1 \le x \le K\} \tag{4.2.2}$$

$$f_x = \frac{1}{(K-x+1)} \qquad \{1 \le x \le K\} \tag{4.2.3}$$

Thus,

$$a_x = \left(1 - \frac{1}{(K-x+1)}\right)(1-p) \qquad \{1 \le x \le K\} \tag{4.2.4}$$

$$b_x = \left(1 - \frac{1}{(K-x+1)}\right)p \qquad \{1 \le x \le K\} \tag{4.2.5}$$

$$e_x = 1 - \frac{1}{(K-x+1)} \qquad \{1 \le x \le K\} \tag{4.2.6}$$

The transition probabilities g, h, i, j, k and l remain the same as in DDStnBM1 (equations (4.1.7) to (4.1.12)).

A matrix solution is opted for this model, the equations for the network performance measures - throughput, average queue, average delay and the packet rejection probability being identical to those in the previous section (equations (4.1.14) to (4.1.17)).

4.3 Destination Deletion, Station Based Model 3 (DDStnBM3)

As mentioned in the introduction to this chapter, the model will be a simplified version of DDStnBM1, the simplification which follows, being fairly similar to that of SDStnBM2.

Assumptions -

(4.3.1) When a station is idling, a single packet may arrive to that station every station time with probability 'p', this arrival process following a Bernoulli distribution.

(4.3.2) However, when the station is transmitting, no new packets may arrive to that station until the end of transmission. Thus, at the point where the packet is deleted from the buffer, probability of a packet arrival will be considered to be 'px', where 'x' represents the number of station times that have elapsed since the start of transmission.

Assumption (4.1.1).

Figure 4.3.1 shows the Markov chain for this model. Since the probability of a slot being deleted from the network remains a constant, as in DDStnBM1,

$$
\begin{aligned}
f_x &= \frac{1}{K} \qquad \{1 \le x < K\} \\
&= f \qquad (say) \\
f_K &= 1
\end{aligned}
$$

However, due to the assumptions (4.3.1) and (4.3.2),

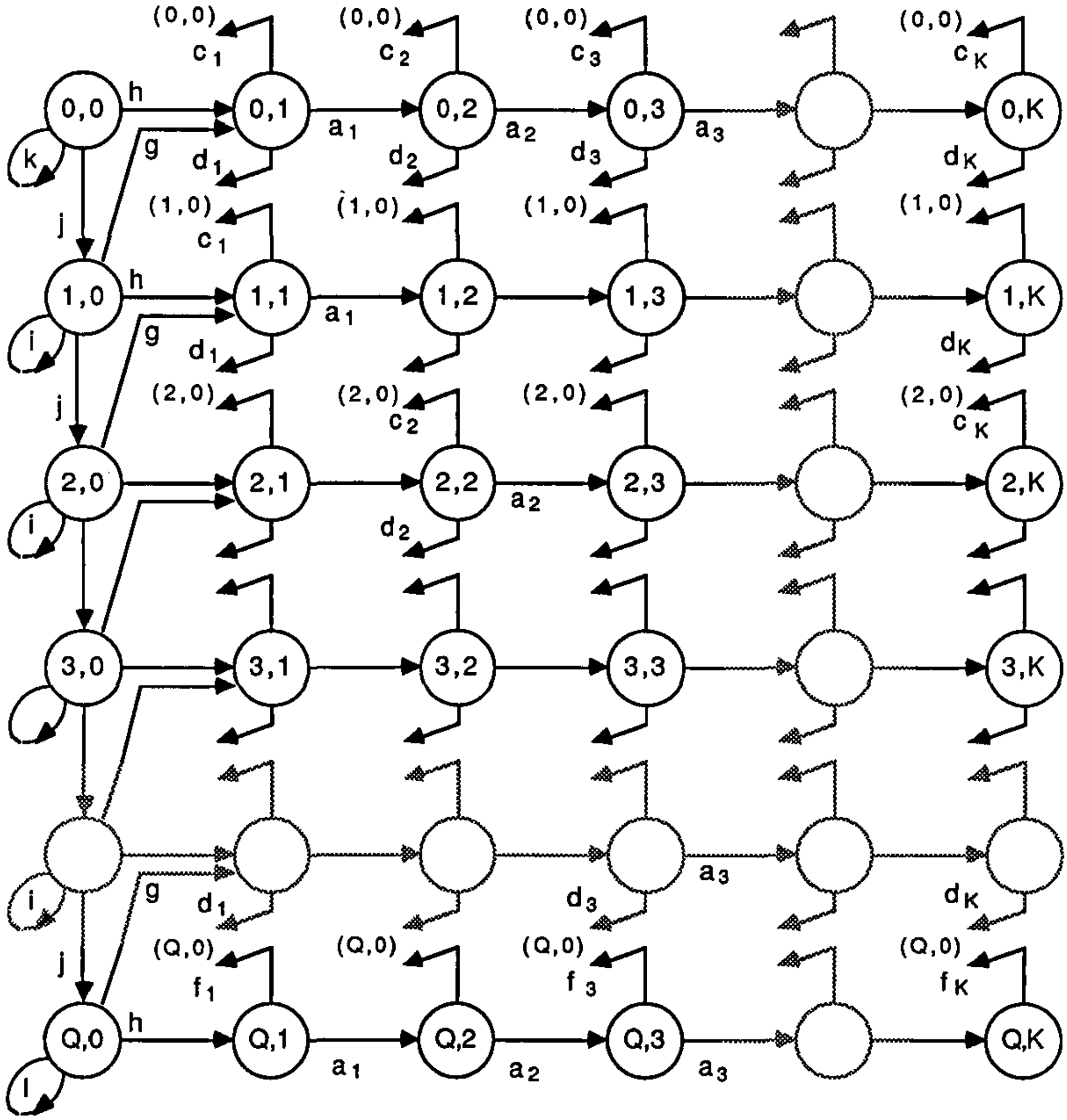


Fig 4.3.1 Simplified Discrete Time, Station Based Markov Model for a Slotted Ring with a Destination Deletion Protocol

$$c_x = \frac{(1-px)}{K} \qquad \{1 \le x < K\}$$

$$c_K = 1-pK$$

$$d_x = \frac{px}{K} \qquad \{1 \le x < K\}$$

$$d_K = pK$$

$$\begin{aligned} a_x &= 1 - (c_x+d_x) \\ &= 1 - \frac{1}{K} \qquad \{1 \le x < K\} \\ &= a \quad (say) \end{aligned}$$

The transition probabilities g, h, i, j, k and l remain identical to that of DDStnBM1 (equations (4.1.7) to (4.1.12)).

Considering the steady state,

$$P(0,0)p = \sum_{x=1}^{K} P(0,x)\,c_x \tag{4.3.1}$$

$$\begin{aligned} P(y,0)\,[1-(1-p)S] = P(y-1,0)pS &+ \sum_{x=1}^{K} P(y-1,x)\,d_x \\ &+ \sum_{x=1}^{K} P(y,x)\,c_x \qquad \{1 \le y < Q\} \end{aligned} \tag{4.3.2}$$

$$\begin{aligned} P(Q,0)\,(1-S) = P(Q-1,0)pS &+ \sum_{x=1}^{K} P(Q-1,x)\,d_x \\ &+ \sum_{x=1}^{K} P(Q,x)\,f_x \end{aligned} \tag{4.3.3}$$

$$P(y,1) = P(y,0)p(1-S) + P(y+1,0)(1-p)(1-S) \quad \{0 \le y < Q\} \tag{4.3.4}$$

$$P(Q,1) = P(Q,0)p(1-S) \tag{4.3.5}$$

$$P(y,x) = P(y,x-1)\,a_{x-1} \qquad \{0 \le y \le Q,\ 2 \le x \le K\} \tag{4.3.6}$$

By repeated use of (4.3.6),

$$P(y,x) = P(y,1)\,a^{x-1} \qquad \{0 \le y \le Q,\ 1 \le x \le K\} \tag{4.3.7}$$

Let,

$$\alpha = \frac{\sum_{x=1}^{K} P(y,x)\,c_x}{P(y,1)}$$

$$\beta = \frac{\sum_{x=1}^{K} P(y,x)\,d_x}{P(y,1)}$$

$$\gamma = \frac{\sum_{x=1}^{K} P(Q,x)\,f_x}{P(Q,1)}$$

then, by using equation (4.3.6) and summing the result, it can be easily shown that,

$$\alpha = 1-pK(1-a^{K}) \tag{4.3.8}$$

$$\beta = pK(1-a^{K}) \tag{4.3.9}$$

$$\gamma = 1 \tag{4.3.10}$$

From equation (4.3.4),

$$P(y+1,0) = \frac{P(y,1) - P(y,0)p(1-S)}{(1-p)(1-S)} \quad \{0 \le y < Q\} \qquad (4.3.11)$$

Induction will be used to prove that,

$$P(y+1,0) = P(y,0)\delta \qquad \{0 \le y < Q\} \qquad (4.3.12)$$

and,

$$P(y,1) = P(y,0)\epsilon \qquad \{0 \le y < Q\} \qquad (4.3.13)$$

where,

$$\delta = \frac{p(S+(1-S)\beta)}{(1-p)(1-S)(1-\beta)}$$

$$\epsilon = \frac{p}{(1-\beta)}$$

Assume that equation (4.3.13) is correct for y=r (0≤r<Q). Then,

$$P(r,1) = P(r,0)\epsilon \qquad (4.3.14)$$

Substituting this in (4.3.11), it can be easily shown that,

$$P(r+1,0) = \delta P(r,0)$$

Using y=0 in equation (4.3.11) and substituting from (4.3.1), (4.3.8) and (4.3.9),

$$P(1,0) = P(0,0)\delta$$

Thus by the rules of induction, equation (4.3.12) can be considered true within the given range.

Substituting y=r+1 in (4.3.2) and using the equations (4.3.8), (4.3.9), (4.3.12) and (4.3.14), it may be shown that,

$$P(r+1,1) = P(r+1,0)\epsilon \qquad \{0 \le r < Q\}$$

Since equations (4.3.1), (4.3.8) and (4.3.9) show that

$$P(0,1) = P(0,0)\epsilon$$

equation (4.3.13) too is valid in the specified region.

Using equation (4.3.12) repeatedly by increasing the value of y from 0, it can be shown that,

$$P(y,0) = P(0,0)\,\delta^{y} \qquad \{0 \le y \le Q\} \tag{4.3.15}$$

Then from (4.3.13),

$$P(y,1) = P(0,0)\,\epsilon\delta^{y} \qquad \{0 \le y < Q\} \tag{4.3.16}$$

Considering the sum of all state probabilities,

$$1 = \sum_{y=0}^{Q}\sum_{x=0}^{K} P(y,x) \tag{4.3.17}$$

But,

$$\begin{aligned}\sum_{x=1}^{K} P(y,x) &= P(y,1)\,(1+a+a^{2}+\dots+a^{K-1}) \\ &= P(y,1)\,(1-a^{K})\,K \\ &= P(y,1)\,\frac{\beta}{p}\end{aligned} \tag{4.3.18}$$

Using the equations (4.3.5), (4.3.15), (4.3.16) and (4.3.18),

$$\begin{aligned}\sum_{y=0}^{Q}\sum_{x=1}^{K} P(y,x) &= \frac{\beta}{p}\sum_{y=0}^{Q} P(y,1) \\ &= \frac{\beta}{p}P(0,0)\left[\frac{\epsilon(1-\delta^{Q})}{(1-\delta)} + \delta^{Q}p(1-S)\right]\end{aligned} \tag{4.3.19}$$

Similarly, using (4.3.15),

$$\sum_{y=0}^{Q} P(y,0) = P(0,0)\,\frac{(1-\delta^{Q+1})}{(1-\delta)} \tag{4.3.20}$$

Substituting (4.3.19) and (4.3.20) in (4.3.17),

$$1 = P(0,0)\left\{\frac{(1-\delta^{Q+1})}{(1-\delta)} + \frac{\beta}{p}\left[\frac{\epsilon(1-\delta^{Q})}{(1-\delta)} + \delta^{Q}p(1-S)\right]\right\} \tag{4.3.21}$$

As in previous station based models,

$$\begin{aligned} S &= \frac{N-1}{M}\left(1 - \sum_{y=0}^{Q} P(y,0)\right) \\ &= \frac{N-1}{M}\left(1 - P(0,0)\,\frac{(1-\delta^{Q+1})}{(1-\delta)}\right) \end{aligned} \tag{4.3.22}$$

Substituting for P(0,0) from (4.3.21), an iterative equation for S may be obtained from the above formula.

Considering the number of packets transmitted per cycle per slot,

$$\begin{aligned} Throughput &= \left[\sum_{y=1}^{Q} P(y,0) + pP(0,0)\right]\frac{N^2}{M}(1-S) \\ &= \left[\frac{\delta(1-\delta^{Q})}{(1-\delta)} + p\right]\frac{N^2}{M}(1-S)\,P(0,0) \end{aligned} \tag{4.3.23}$$

After the summation of several series, it can be shown that,

$$\begin{aligned} \overline{q} &= \sum_{y=0}^{Q}\sum_{x=0}^{K} yP(y,x) \\ &= \frac{P(0,0)}{(1-\beta)}\left[\frac{\delta(1-\delta^{Q-1})}{(1-\delta)} - (Q-1)\delta^{Q}\right]\frac{1}{(1-\delta)} \\ &\quad + P(0,0)\,Q\delta^{Q}[1+\beta(1-S)] \end{aligned} \tag{4.3.24}$$

Thus, by substituting for P(0,0) from the equation (4.3.21), the average queue may be obtained. The average delay and the packet rejection probability may be calculated using the equations in section 4.1.

As in some previous cases, since the value of P(0,0) can become quite small or even zero at higher packet arrival rates, when calculating the throughput and the average queue using the above equations it is not advisable to calculate P(0,0) separately to be substituted in (4.3.23) and (4.3.24). Instead, equation (4.3.21) should be incorporated in them, and, the resulting equations, (too long to be shown here) expressed so as to be stable for different values of $|\delta|$.

It is noted that in order to have the transition probability c_K positive under all circumstances, the packet arrival rate should not be allowed to exceed the value 1/K, thus limiting the range of p.

4.4 Destination Deletion, Station Based Model 4 (DDStnBM4)

In this model, a simplification similar to that in DDStnBM3 is used, this time however, the probability of a destination deletion considered to be varying (increasing) with the progression of a packet through the network.

In addition to all the assumptions in section 4.0, (4.2.1), (4.3.1) and (4.3.2) are applied.

The Markov state diagram for the model remains identical to that in figure 4.3.1, the new state transition probabilities being,

$$a_x = \frac{(K-x)}{(K-x+1)} \qquad \{1 \le x \le K\}$$

$$c_x = \frac{(1-px)}{(K-x+1)} \qquad \{1 \le x \le K\}$$

$$d_x = \frac{px}{(K-x+1)} \qquad \{1 \le x \le K\}$$

$$f_x = \frac{1}{(K-x+1)} \qquad \{1 \le x \le K\}$$

The values of g, h, i, j, k and l remain the same as in DDStnBM3.

Due to the considerable similarity between DDStnBM3 and DDStnBM4, it can be shown that, except for the equations (4.3.7) to (4.3.10) of the former, all other numbered equations and derivations remain the same for the current model. Even a summation similar to that in (4.3.18) under the new conditions, as shown below, results in an identical equation. Therefore, by calculating the new α, β and γ values and substituting them, the new network parameters can be obtained.

By the repeated use of (4.3.6),

$$P(y,x) = \frac{(K-x+1)}{K} P(y,1) \qquad \{1 \le x \le K\} \tag{4.4.7}$$

Summing up the resulting series,

$$\begin{aligned} \alpha &= \frac{\sum_{x=1}^{K} P(y,x)\, c_x}{P(y,1)} \\ &= 1 - \frac{p(K+1)}{2} \end{aligned} \tag{4.4.8}$$

Similarly,

$$\beta = \frac{p(K+1)}{2} \tag{4.4.9}$$

$$\gamma = 1 \tag{4.4.10}$$

$$\sum_{x=1}^{K} P(y,x) = \frac{P(y,1)}{K}[K+(K-1)+(K-2)+...+1]$$
$$= P(y,1)\frac{\beta}{p} \qquad (4.4.18)$$

Note that for convenience of comparison, the above equations have been numbered so that there is a one to one correspondence between equations in section 4.4 and those in section 4.3. For example, equation (4.4.18) and (4.3.18) represent the same quantity in the respective models.

4.5 Results

The four models presented above were solved for most combinations of the following parameters - N (16 and 64), M (1, 8 and 16) and Q (1, 10 and 50). As in the case of chapter 2, even though the graphs containing the combinations of M=16 or Q=50 have been excluded to limit the size of the thesis, the analysis has been made considering all the results obtained.

As in the case of model SDStnBM4, the packet arrival rates in the graphs presented (say p') are in units of packets per station per slot time in contrast to the units of packets per station per station time used in the models. The transformation has been done by -

$$p' = \frac{pN}{M}$$

and, as before, the errors resulting from this type of transformation have been ignored.

Although the 95% confidence limits were calculated for the simulation results, in most cases they were found to be too close to show any significant difference.

Throughput

As in the case of source deletion models, with the increase of packet arrival rate, the throughput increases steadily and becomes saturated. Again, as in the previous models, with the increase of Q, the characteristics tend to become two intersecting straight lines, one having an equation -

$$Throughput = Np$$

and the other,

$$Throughput = constant$$

Quasi stable states, first examined by Falconer et al (Falconer 85), cannot exist in destination deletion protocols since such an effect can only occur in source deletion systems. Therefore simulations reflect the theoretically highest mean throughput attainable,

$$Throughput = \frac{1}{\frac{1}{2} + \frac{1}{N}}$$

(For proof, see appendix A.)

With regard to the models, though all of these show similar results in the linear region, the ones that consider a variable destination deletion probability tend to give a higher saturation throughput. The same models also give a better match to the simulation with the increase of N.

The increase of M has no effect on the simulation (non existence of quasi stable states), but causes the saturation throughput of the models to decrease, this reduction becoming less with the increase of N.

Average Queue

With the increase of Q, the average queue shown by all the curves tend to become a step function with the limits of 0 and Q. This transition takes place when the network moves into saturation.

Models DDStnBM2 and DDStnBM4 show a better match to the simulation with the increase of N. The average queue of all the models increase with M, this change however being nominal for larger values of N. The simulation remains unaffected by changes in M.

Average Delay

As in the case for average queue, the delay tends to rise sharply from near zero to its maximum with the increase of Q. However, this transition takes place at a lower packet arrival rate than that for the average queue.

The comparison of N=16 and N=64 results show an improved match to the simulation for DDStnBM2 and DDStnBM4 for higher number of stations. The delay curves of the models increase with M in the unsaturated region, this being due the use of assumption (2.0.12). With respect to the simulation, all models show an increase in delay with p, although the difference is negligible for higher values of N.

Packet Rejection Probability (PRP)

As can be expected, for very low values of Q (1 in this case), PRP is considerable even at low packet arrival rates. The increase of Q results in no, or negligible packet loss when the system is not in saturation.

The models show an increase in PRP with M, this change, again being quite small when N=64. The simulation however tends to be unaffected by the changes to M.

An increase in N results in the models DDStnBM2 and DDStnBM4 giving a better match to the simulation, especially when Q is 10 or more.

4.6 Summary

In this chapter, four models based on a single station for a destination deletion slotted ring network have been presented.

The last two models were the simplified versions of the first two, thus enabling them to be solved using fixed point equations rather than a direct numerical approach. However, as in the case of previously similarly simplified models, this effectively reduced the range of the packet arrival rate applicable to these models. The first and the third models assumed a constant destination deletion probability of a packet at all stations except the last downstream station, whereas the other two models assumed this to change depending upon the distance the packet has travelled.

From the results obtained, it may be seen that the assumption of a variable destination deletion probability of a packet gives better results. At least one reason for this is the fact that a constant destination deletion probability does not give rise to an average packet being transmitted halfway around the ring (appendix B) - a condition that should exist since all stations are assumed identical.

The simplified models give a reasonably good match to their respective complete models, and thus may be used when the network constants permit this.

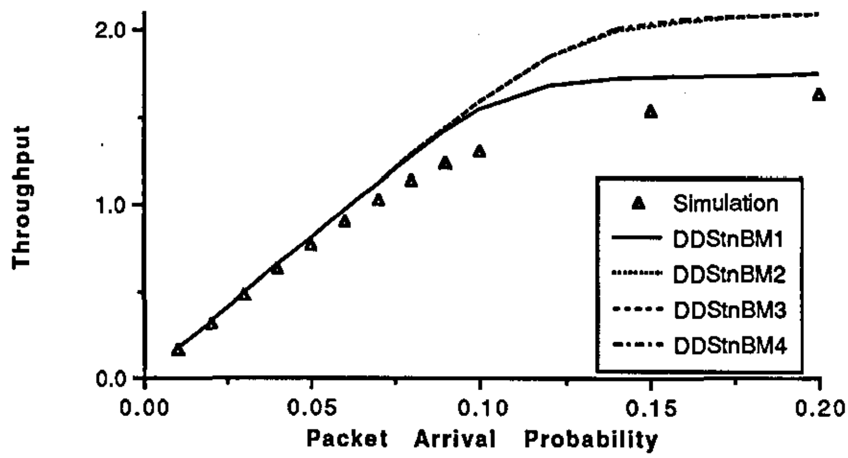


Fig. 4.5.1 Throughput Performance (N=16, M=1, Q=1)

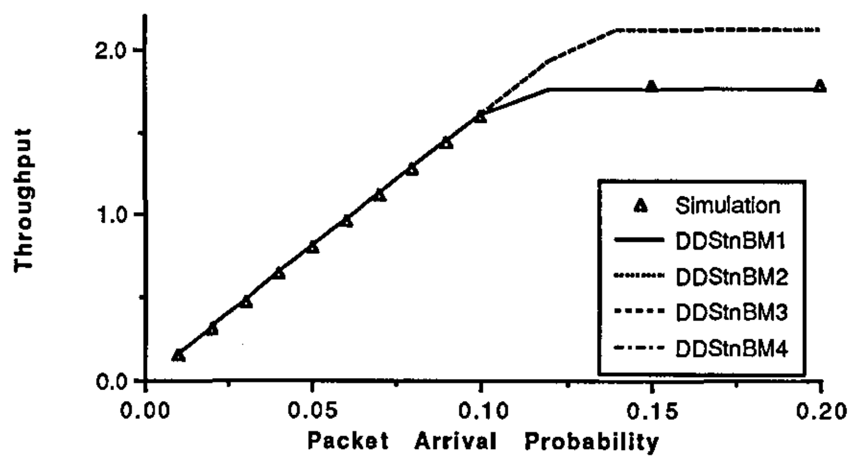


Fig. 4.5.2 Throughput Performance (N=16, M=1, Q=10)

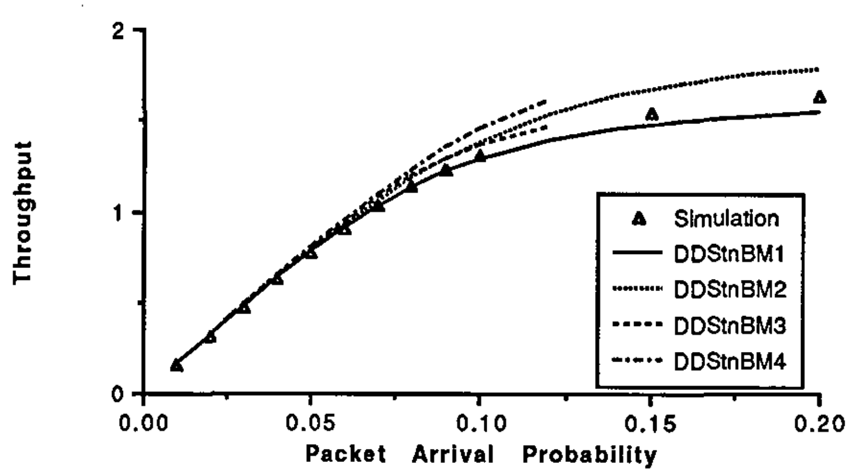


Fig. 4.5.3 Throughput Performance (N=16, M=8, Q=1)

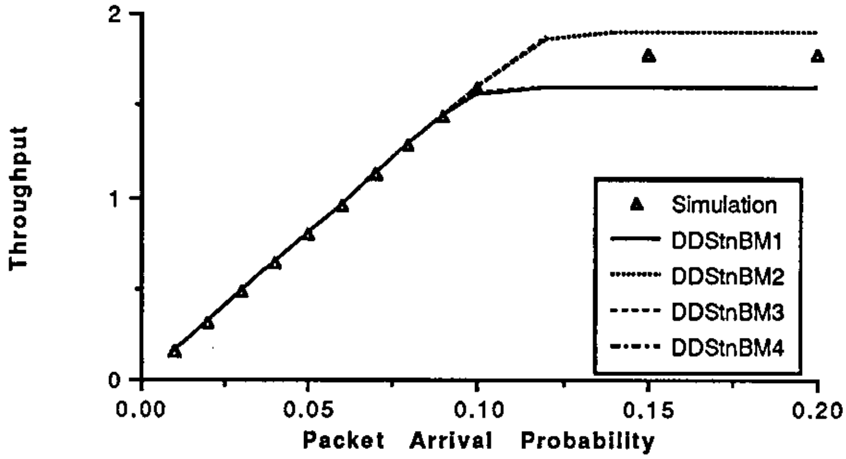


Fig. 4.5.4 Throughput Performance (N=16, M=8, Q=10)

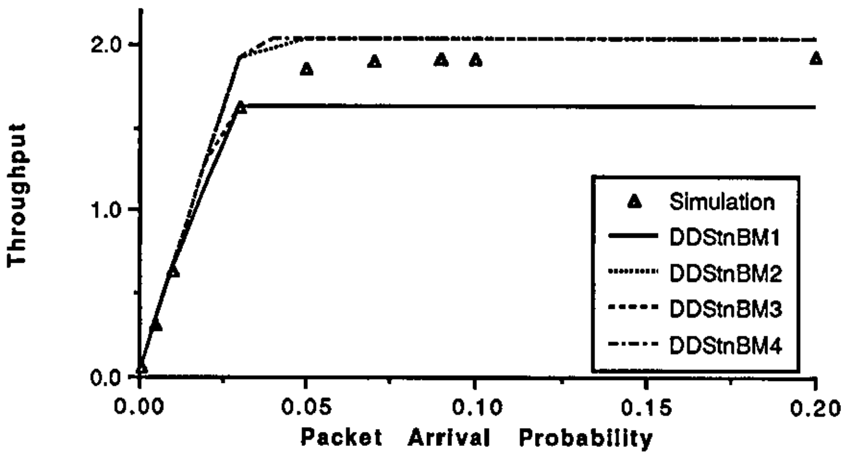


Fig. 4.5.5 Throughput Performance (N=64, M=1, Q=1)

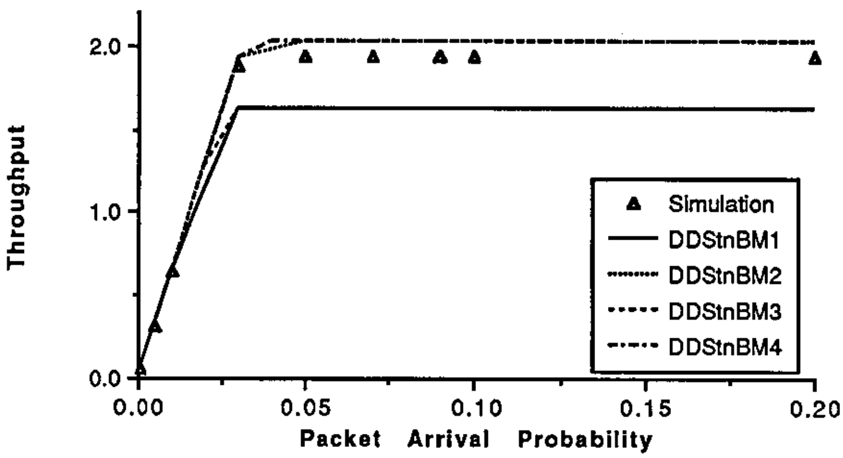


Fig. 4.5.6 Throughput Performance (N=64, M=1, Q=10)

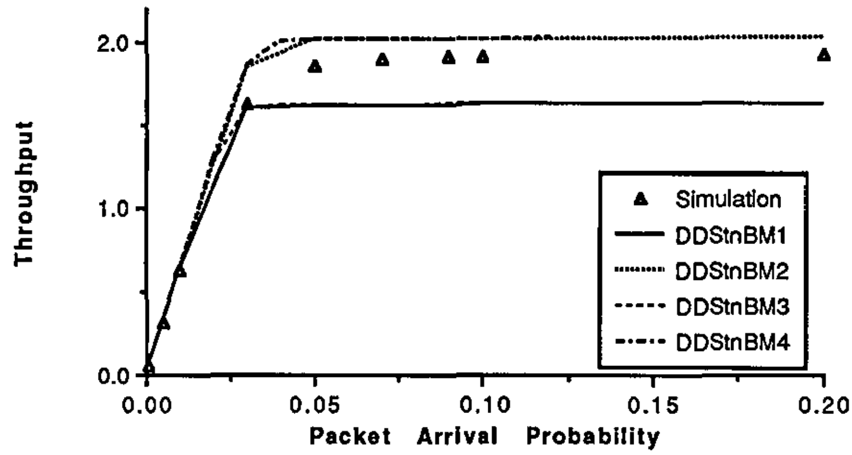


Fig. 4.5.7 Throughput Performance (N=64, M=8, Q=1)

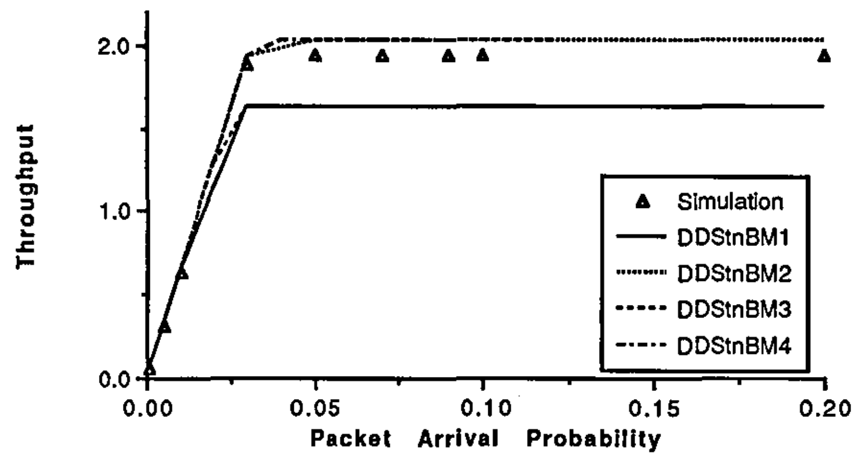


Fig. 4.5.8 Throughput Performance (N=64, M=8, Q=10)

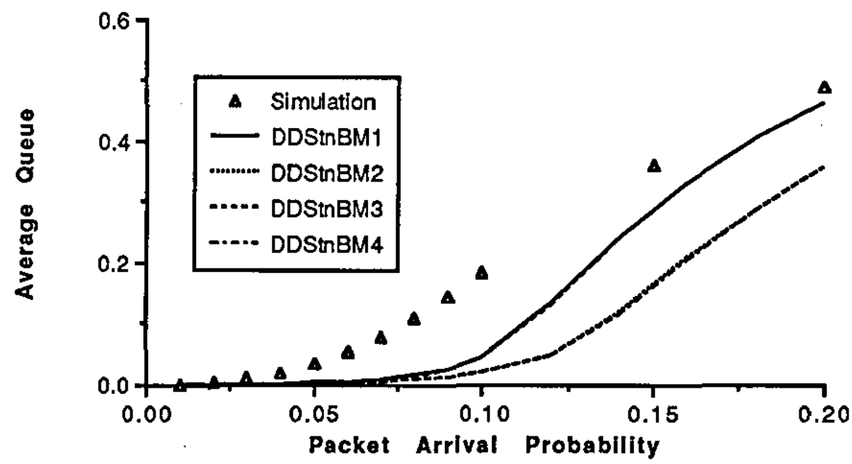


Fig. 4.5.9 Queueing Performance (N=16, M=1, Q=1)

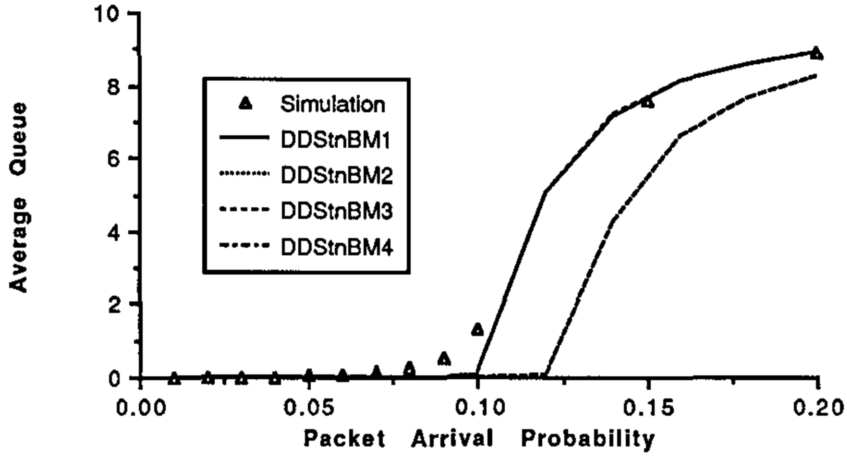


Fig. 4.5.10 Queueing Performance (N=16, M=1, Q=10)

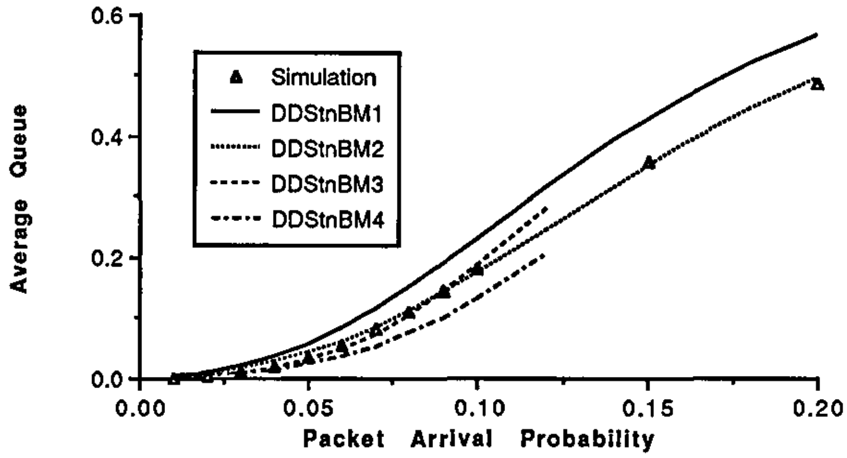


Fig. 4.5.11 Queueing Performance (N=16, M=8, Q=1)

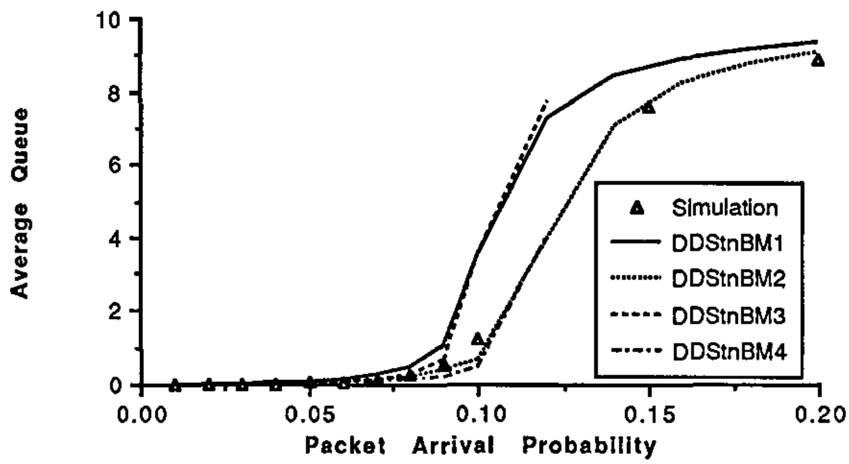


Fig. 4.5.12 Queueing Performance (N=16, M=8, Q=10)

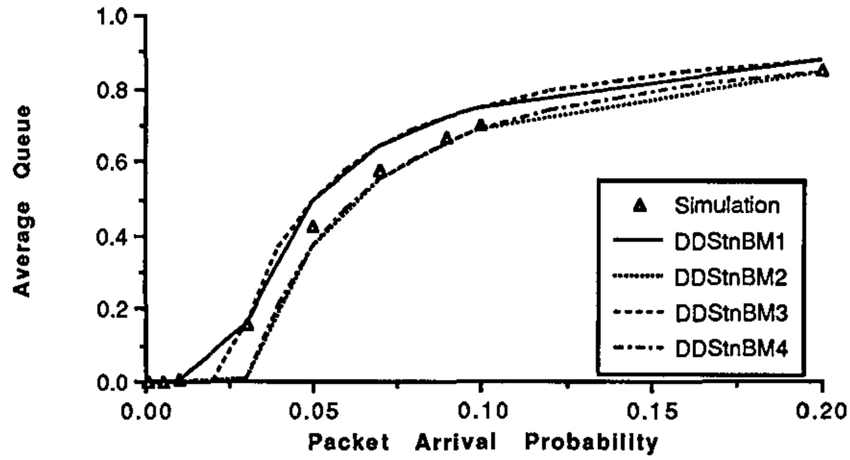


Fig. 4.5.13 Queueing Performance (N=64, M=1, Q=1)

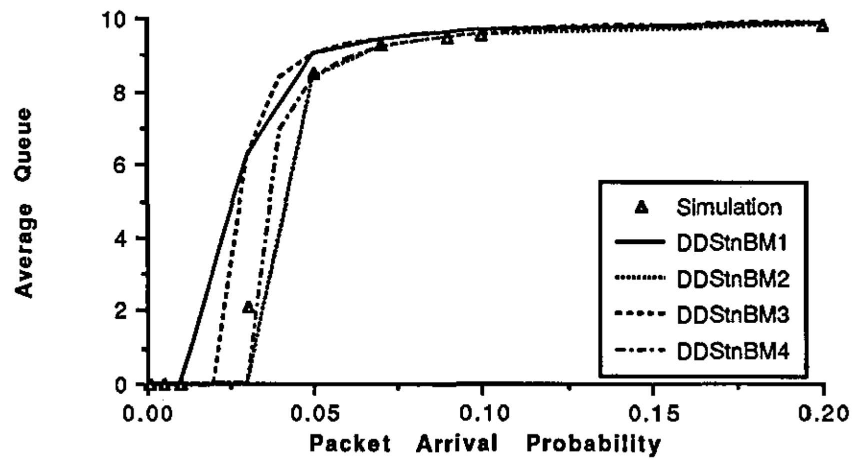


Fig. 4.5.14 Queueing Performance (N=64, M=1, Q=10)

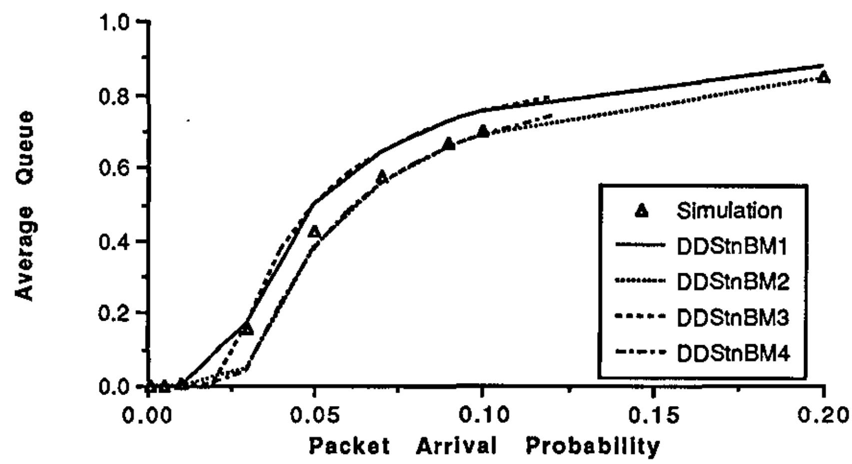


Fig. 4.5.15 Queueing Performance (N=64, M=8, Q=1)

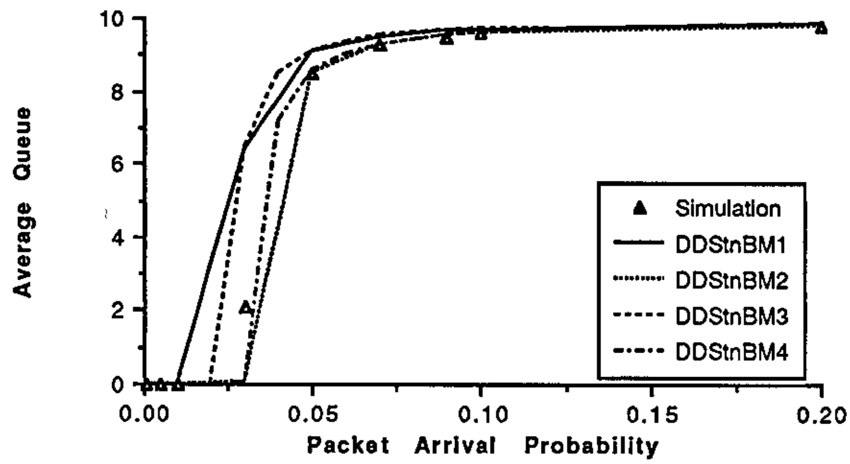


Fig. 4.5.16 Queueing Performance (N=64, M=8, Q=10)

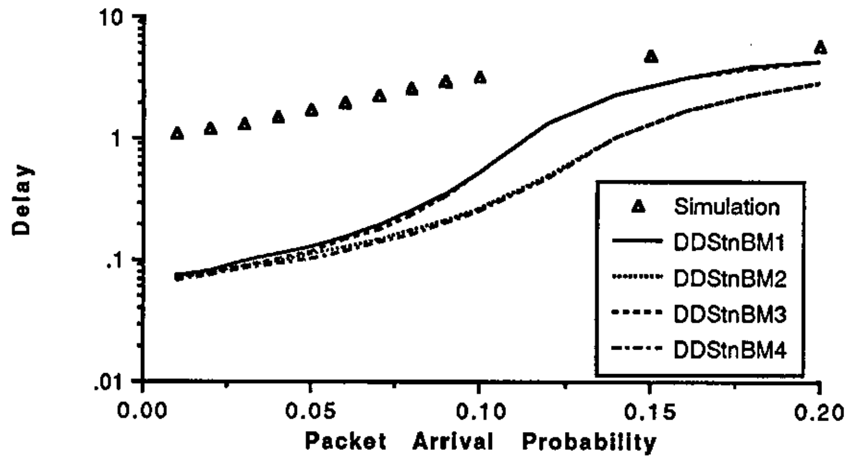


Fig. 4.5.17 Delay Performance (N=16, M=1, Q=1)

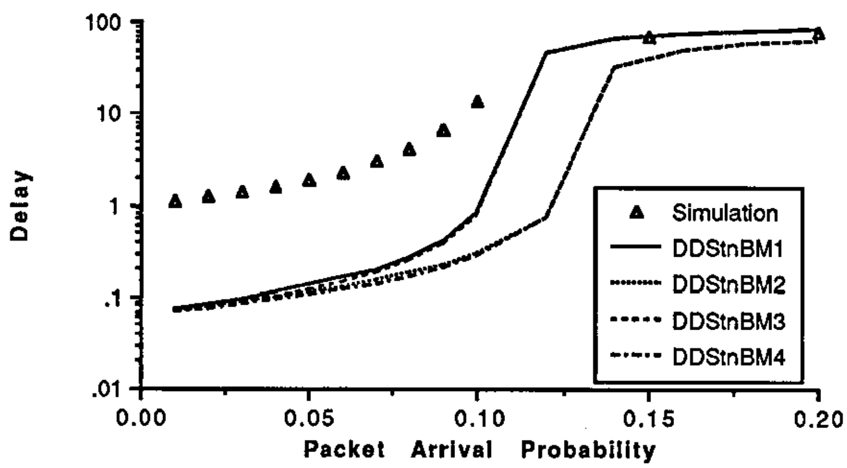


Fig. 4.5.18 Delay Performance (N=16, M=1, Q=10)

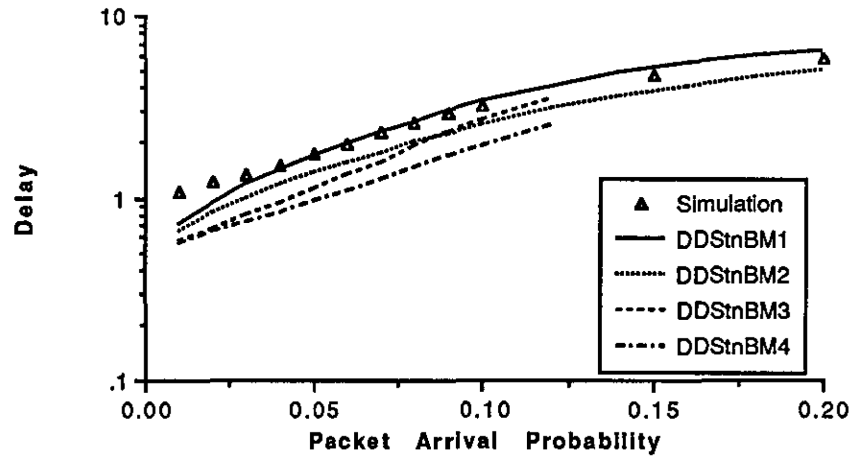


Fig. 4.5.19 Delay Performance (N=16, M=8, Q=1)

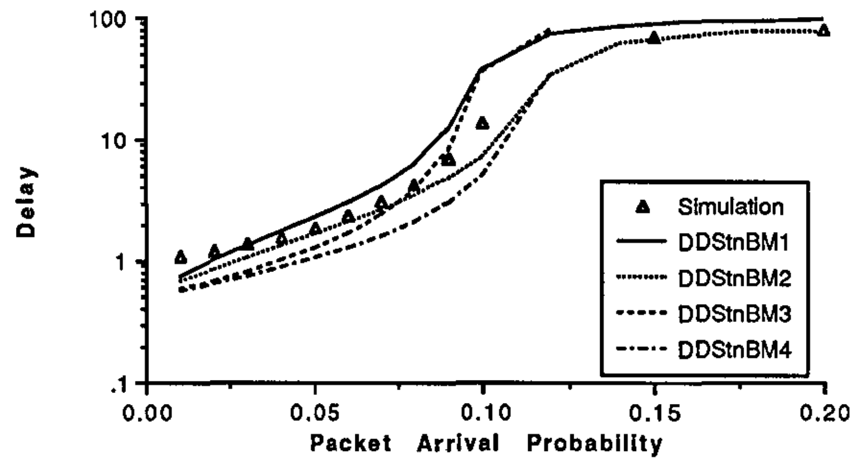


Fig. 4.5.20 Delay Performance (N=16, M=8, Q=10)

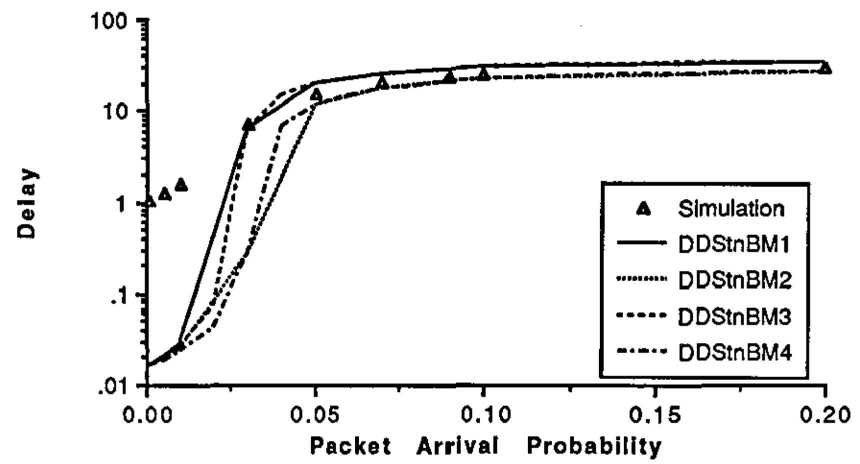


Fig. 4.5.21 Delay Performance (N=64, M=1, Q=1)

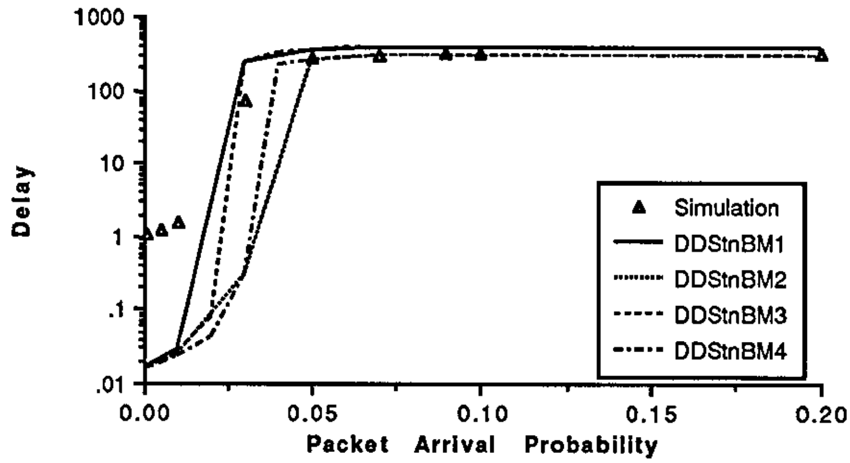


Fig. 4.5.22 Delay Performance (N=64, M=1, Q=10)

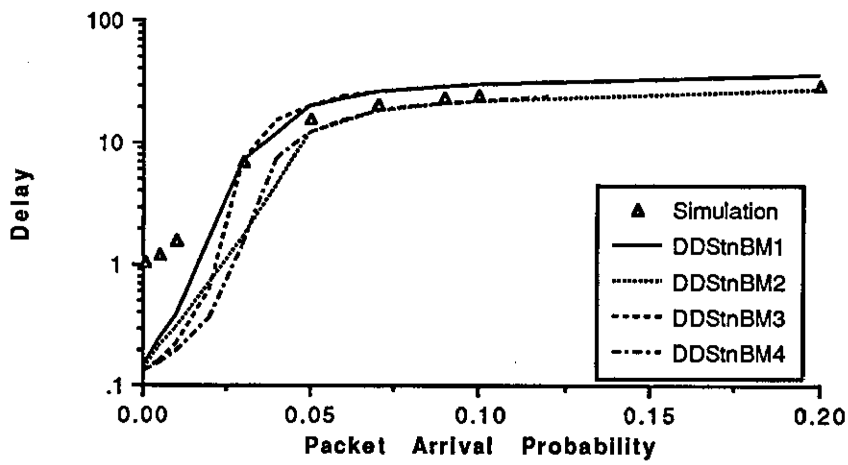


Fig. 4.5.23 Delay Performance (N=64, M=8, Q=1)

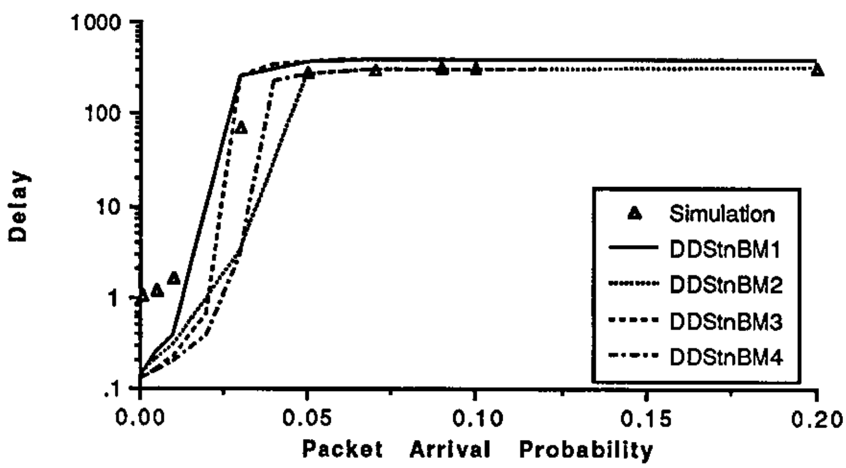


Fig. 4.5.24 Delay Performance (N=64, M=8, Q=10)

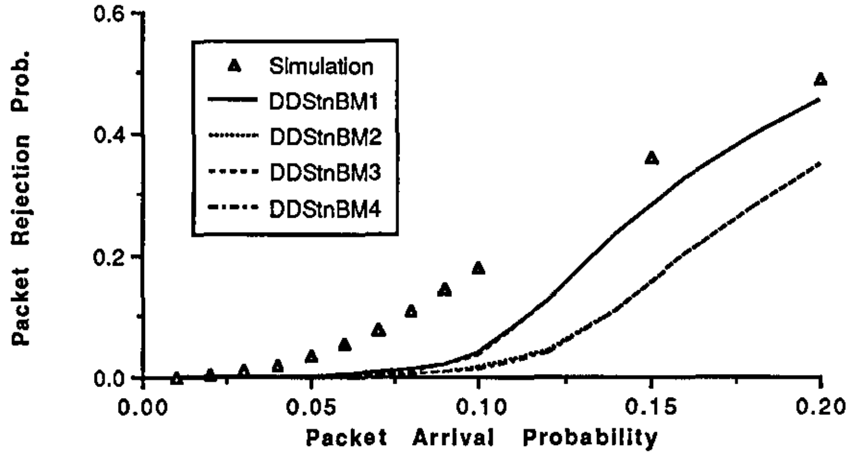


Fig. 4.5.25 Buffer Overflow Characteristics (N=16, M=1, Q=1)

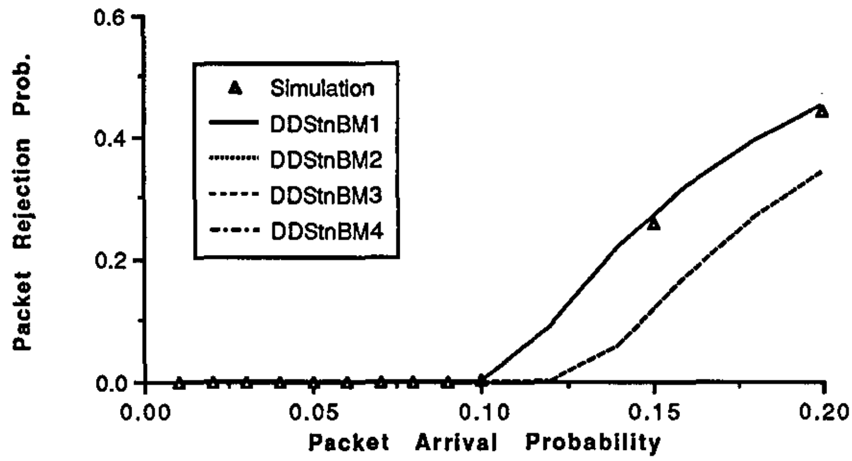


Fig. 4.5.26 Buffer Overflow Characteristics (N=16, M=1, Q=10)

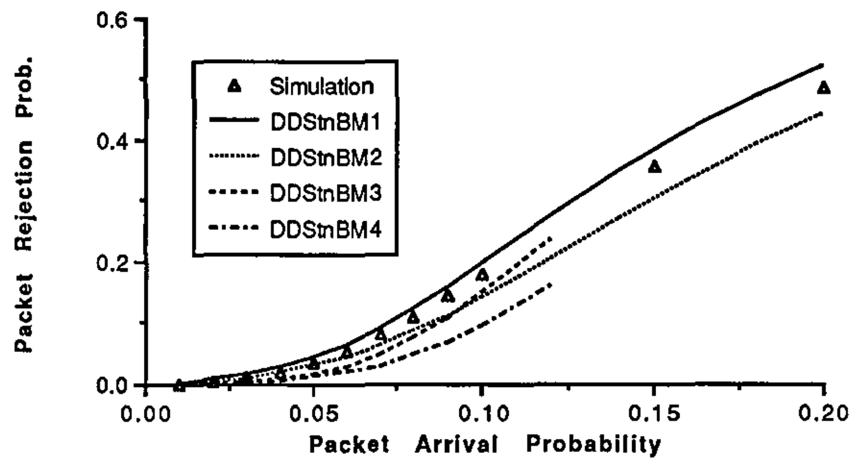


Fig. 4.5.27 Buffer Overflow Characteristics (N=16, M=8, Q=1)

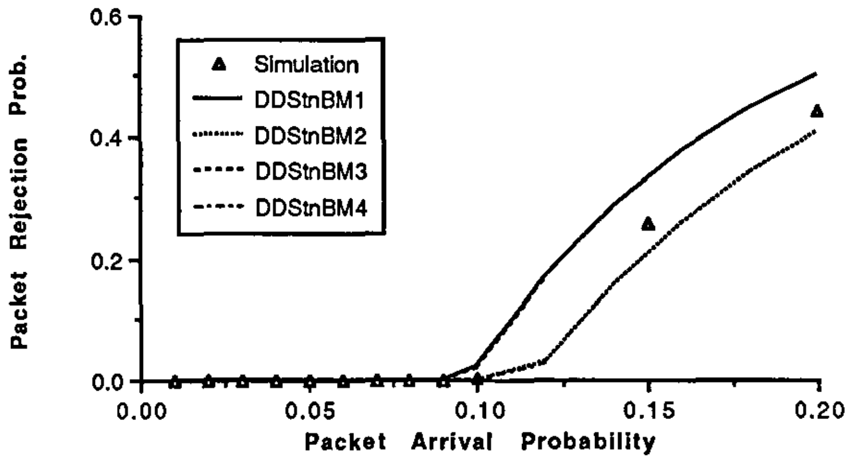


Fig. 4.5.28 Buffer Overflow Characteristics (N=16, M=8, Q=10)

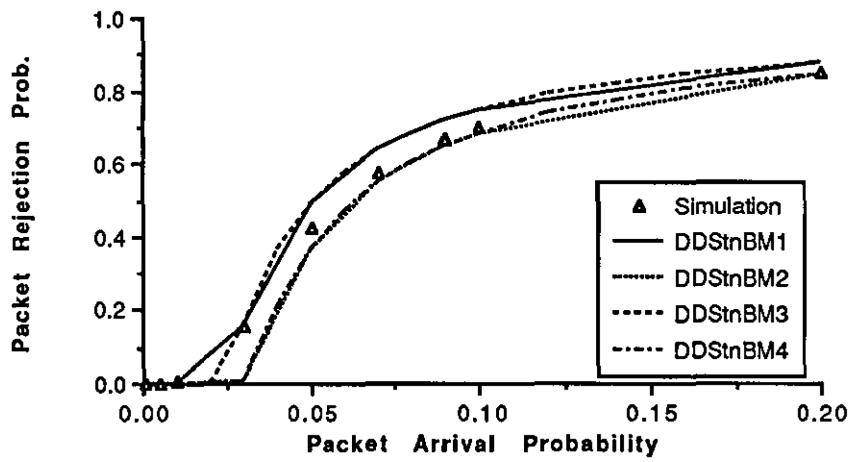


Fig. 4.5.29 Buffer Overflow Characteristics (N=64, M=1, Q=1)

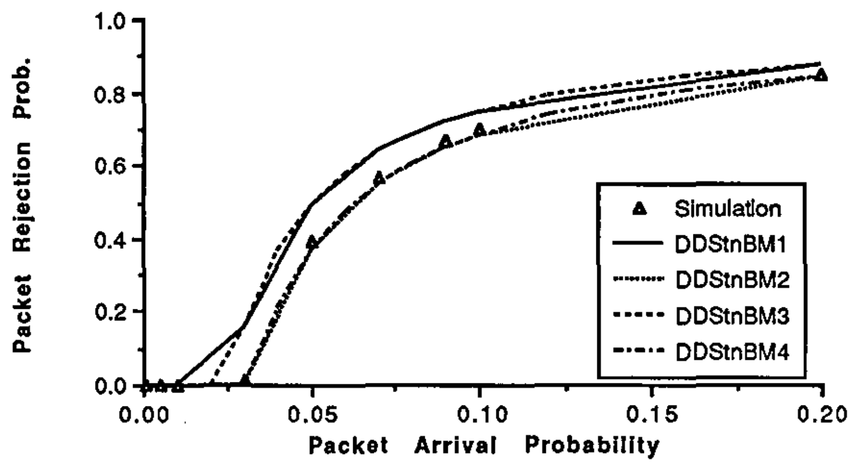


Fig. 4.5.30 Buffer Overflow Characteristics (N=64, M=1, Q=10)

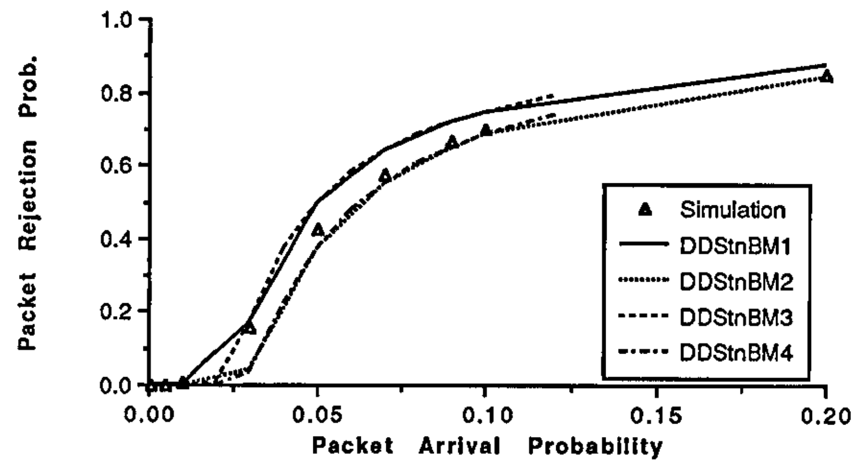


Fig. 4.5.31 Buffer Overflow Characteristics (N=64, M=8, Q=1)

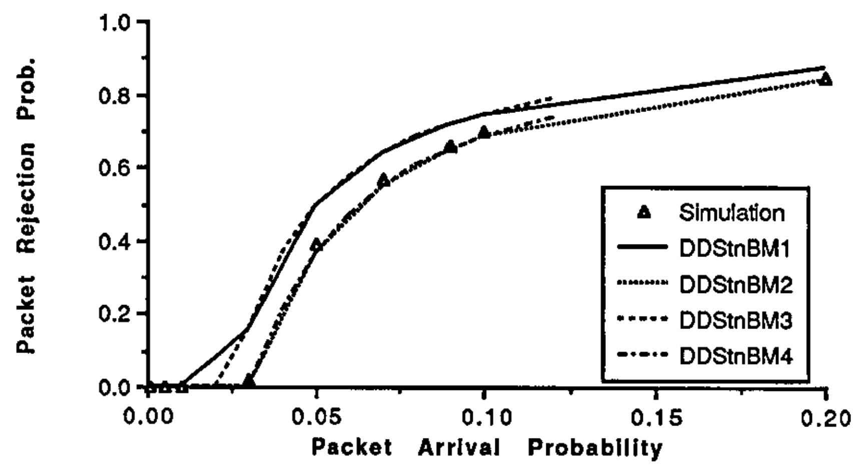


Fig. 4.5.32 Buffer Overflow Characteristics (N=64, M=8, Q=10)

CHAPTER 5

Slotted Ring, Destination Deletion Protocol : Slot Based Models

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5.0 Introduction

In this chapter, the destination deletion slotted ring network will be modelled based on observing the system from a slot. The two methods of considering the destination deletion probabilities mentioned in the previous chapter, i.e., constant and variable, will be used here as well, thus resulting in the models DDSlotBM1 and DDSlotBM2 respectively.

If any of the above two models are to be simplified as in DDStnBM3 or DDStnBM4, the result will only be valid for a packet arrival probability "p" of less than 1/(N-1), where p has the units of - packets per slot per station time. This limit, as it may be seen from the performance curves of the above models, is insufficient to cover even the standard operating range (non-saturated) of the network. Thus, such simplified models will not be considered.

The assumptions (2.0.1) to (2.0.5), (2.0.7), (2.0.10) to (2.0.12), (2.1.3), (3.1.1) to (3.1.3) and (4.0.1) to (4.0.3) will be common to both models in this chapter. As in the previous chapter, both models will be based on time intervals measured in station times.

5.1 Destination Deletion, Slot Based Model 1 (DDSlotBM1)

As mentioned under the introduction to this chapter, this model will assume a constant destination deletion probability for any given packet. Thus the assumption (4.1.1), in addition to those mentioned in section 5.0, will apply.

Figure 5.1.1 shows the Markov model of DDSlotBM1, all the transition probabilities except c_x, d_x and f_x ($1 \le x \le K$) having the same interpretation as those under SDSlotBM of chapter 3. The probabilities c_x, d_x and f_x model the effects of destination deletion, f_x indicating the probability of a packet in that particular slot reaching its destination address; d_x the probability of a packet arrival during this process, and c_x the probability of no packet arrival within the station time the slot reaches its destination. Q is the maximum queue per slot.

From assumption (4.0.2),

$$K = N-1$$

Considering the similarity between this model and DDStnBM1, it can be seen that the transition probabilities a_x, b_x, c_x, d_x, e_x and f_x remain identical (equations (4.1.1) to (4.1.6)). However, due to the different method of observing the network, the values of g and h change to -

$$g = 1-p$$
$$h = p$$

This is due to the fact that, whenever a slot is empty, it will be filled by the station against which its header is aligned provided the station has a packet to transmit.

The model can be solved using the matrix method described in sections 2.4 and 3.2, the network performance measures- throughput, average queue, average delay and the packet rejection probability being calculated in a similar manner to that in the section 3.1 (equations (3.1.7) to (3.1.11)).

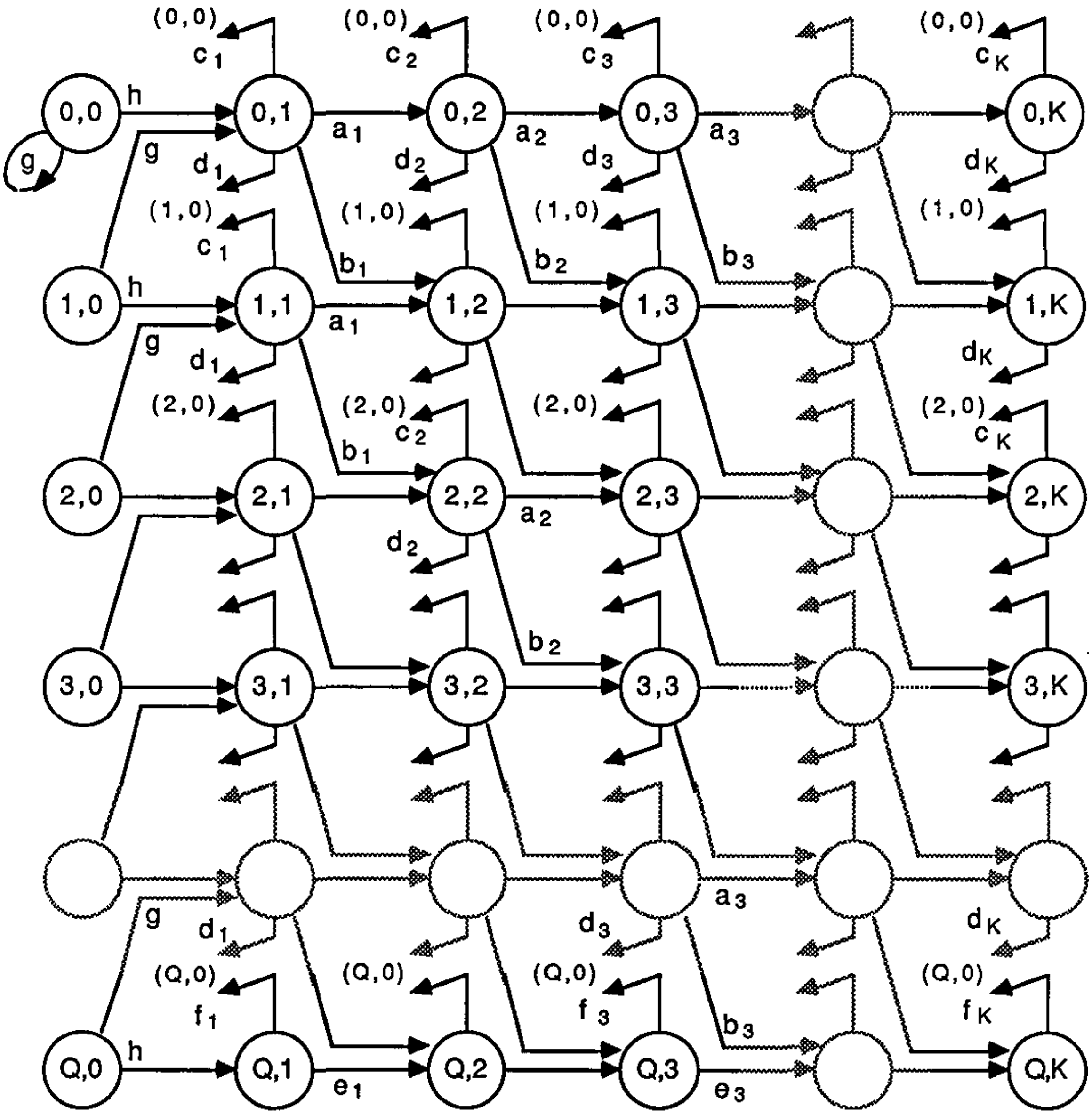


Fig 5.1.1 Discrete Time, Slot Based Markov Model for a Slotted Ring with a Destination Deletion Protocol (DDSlotBM)

5.2 Destination Deletion, Slot Based Model 2 (DDSlotBM2)

This method assumes a varying destination deletion probability for a packet when it moves across the network. Thus the assumption (4.2.1) is also applicable in addition to the ones mentioned in the introduction to this chapter.

All other factors remaining identical to DDSlotBM1, the Markov model remains unchanged to that in figure 5.1.1. However, the transition probabilities a_x, b_x, c_x, d_x, e_x and f_x change to become similar to those in model DDStnBM2 of section 4.2 (equations (4.2.1) to (4.2.6)). The values of g and h remain the same as those in DDSlotBM1.

This model too is solved using the matrix method, and the relevant network performance measures are again calculated using the equations (3.1.7) to (3.1.11).

5.3 Results

Results were obtained for all the usual combinations of system parameters, and will be considered in the analysis below. However, as in chapters 2 and 4, some of the resulting graphs will be omitted to limit the amount of graphs presented.

Throughput

As in all other previous models, the throughput tends follow the path of two intersecting straight lines with the increase of Q; these being -

$$Throughput = Np$$

and,

$$Throughput = \frac{1}{\frac{1}{2}+\frac{1}{N}}$$

DDSlotBM2 shows an excellent match to the simulation at all times, especially at the higher levels of Q; the DDSlotBM1, however, performs quite poorly at saturation.

Both models show very little dependency on M at higher values of Q, and no noticeable difference exits for all the curves when Q is increased from 10 to 50.

Average Queue

Similar to the other models, the average queue tends to show an increasing similarity to a step function with the increase of Q, its limits being 0 and Q.

Both models tend to reflect a higher value compared to the simulation, the disparity being highest in the transition region from a low average queue to its maximum. This difference between the models and the simulation appear to be increasing in only minute quantities compared to the increase in Q for a given p, and, as such, becomes negligible with subsequent increase of Q.

Even though the increase of Q sharpens the transition, the increase of M has the opposite effect.

Since a larger number of stations cause the network to saturate at a lower packet arrival rate per station, the increase of N also results in the curves giving a higher resemblance to a step function.

Average Delay

As in the case of the average queue above, here too, the curves tend to show an increasing similarity to a step function with the increase of Q or N, and its opposite effect with an increase in M.

Both models show an improved match with the increase in Q, the DDSlotBM2 resulting in an extremely close fit to the simulation at higher values of Q.

Compared to the simulation, the increase of M causes the DDSlotBM2 to attain its maximum value prematurely.

Packet Rejection Probability (PRP)

The values predicted by both models improve with the increase of Q, DDSlotBM2 again giving an excellent match to the simulation at higher values of Q.

The simulations indicate an independence from M, the models too showing this same effect at higher values of maximum buffer size.

The comparison with the simulation improves with the increase of N.

5.4 Summary

In this chapter, the destination deletion protocol for a slotted ring has been analysed based on observing the system from a slot. Two models, differing on the basis of how they consider their destination deletion probabilities have been presented.

Results show that the assumption of a variable destination deletion probability gives rise to an extremely satisfactory model. Also, for the destination deletion protocol, the DDSlotBM2 gives a superior performance compared to all the station based models.

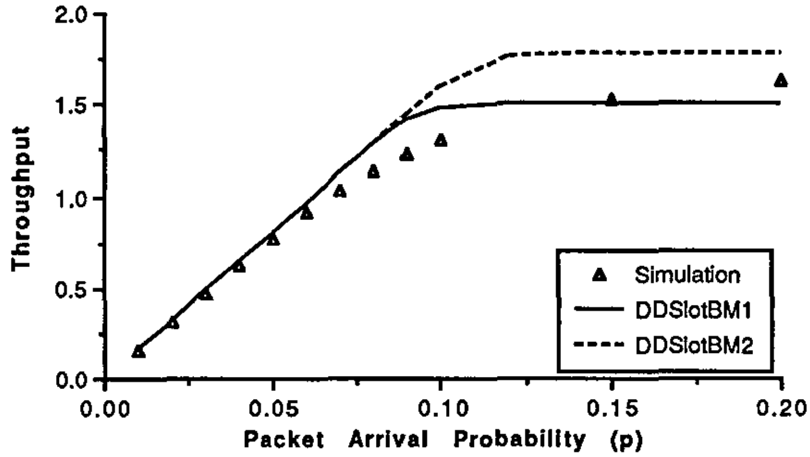


Fig. 5.3.1 Throughput Performance (N=16, M=1, Q=1)

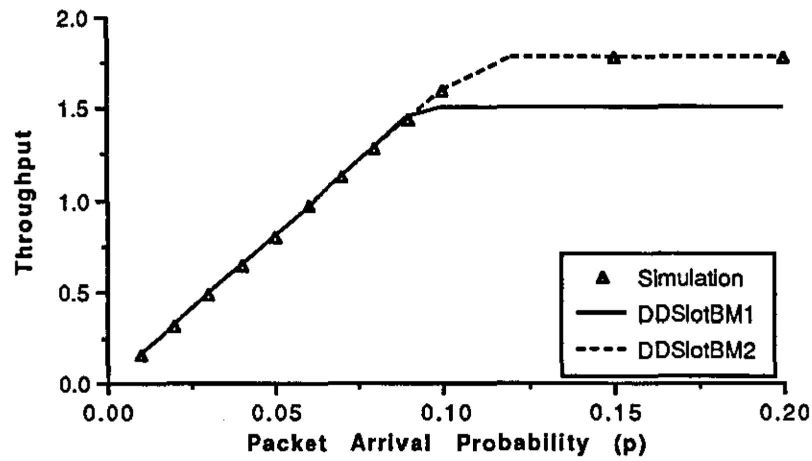


Fig. 5.3.2 Throughput Performance (N=16, M=1, Q=10)

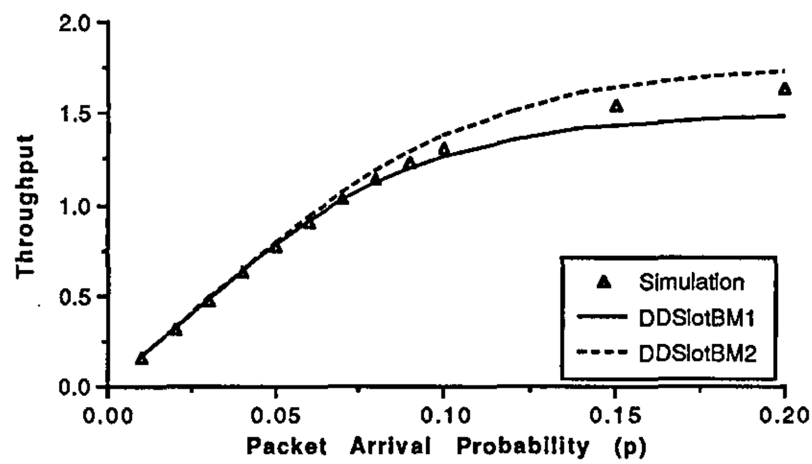


Fig. 5.3.3 Throughput Performance (N=16, M=8, Q=1)

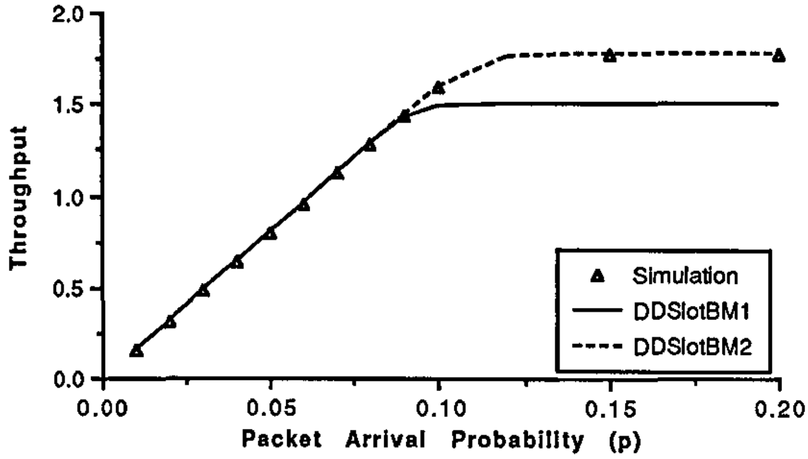


Fig. 5.3.4 Throughput Performance (N=16, M=8, Q=10)

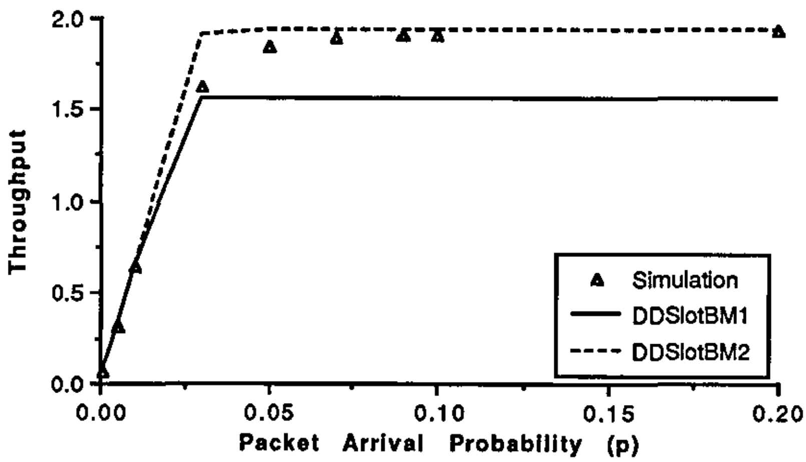


Fig. 5.3.5 Throughput Performance (N=64, M=1, Q=1)

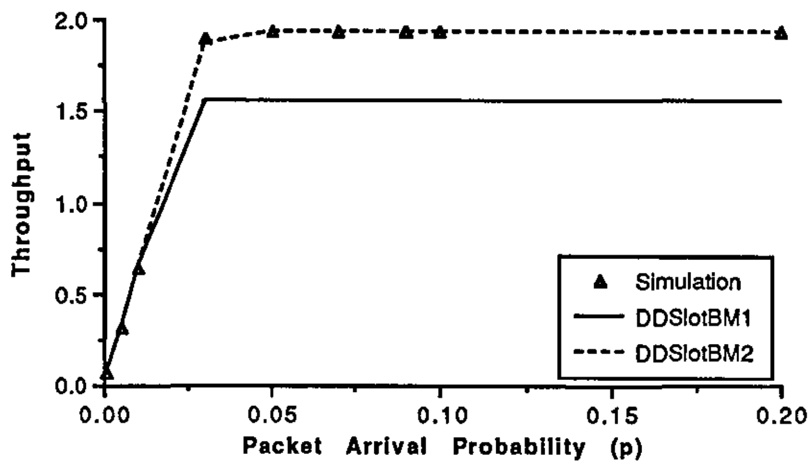


Fig. 5.3.6 Throughput Performance (N=64, M=1, Q=10)

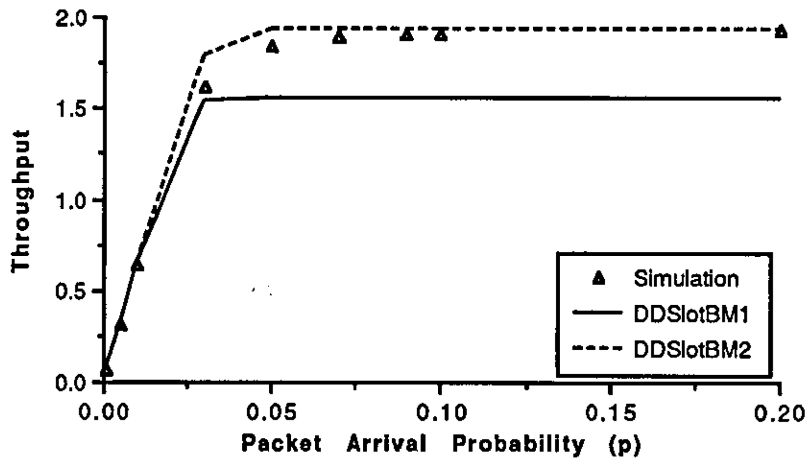


Fig. 5.3.7 Throughput Performance (N=64, M=8, Q=1)

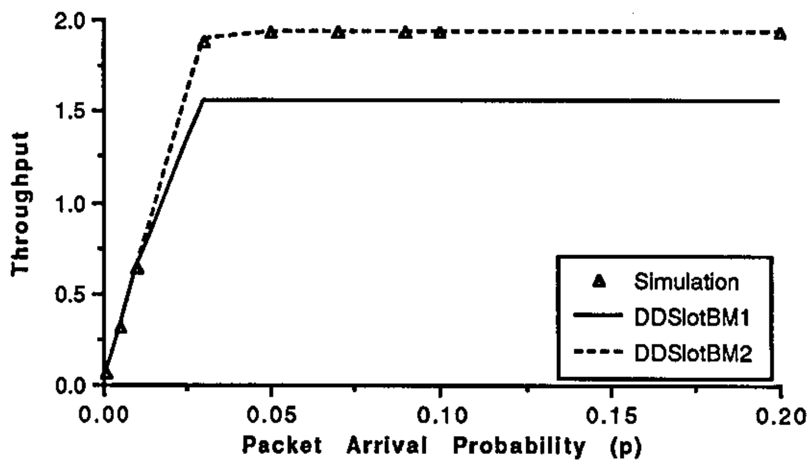


Fig. 5.3.8 Throughput Performance (N=64, M=8, Q=10)

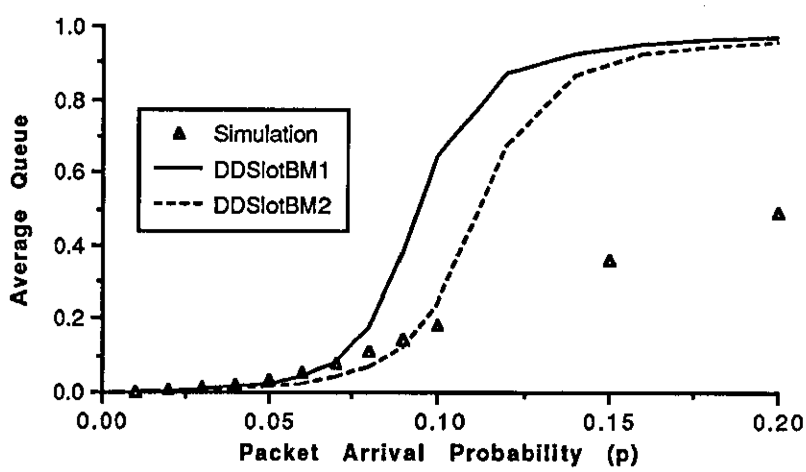


Fig. 5.3.9 Queuing Performance (N=16, M=1, Q=1)

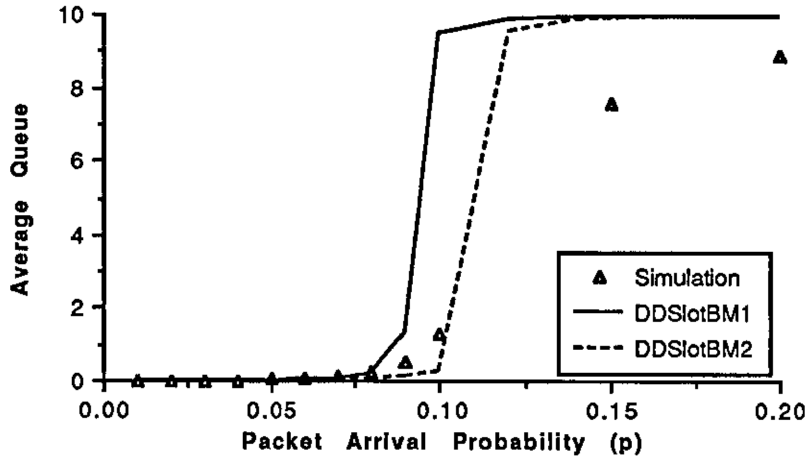


Fig. 5.3.10 Queuing Performance (N=16, M=1, Q=10)

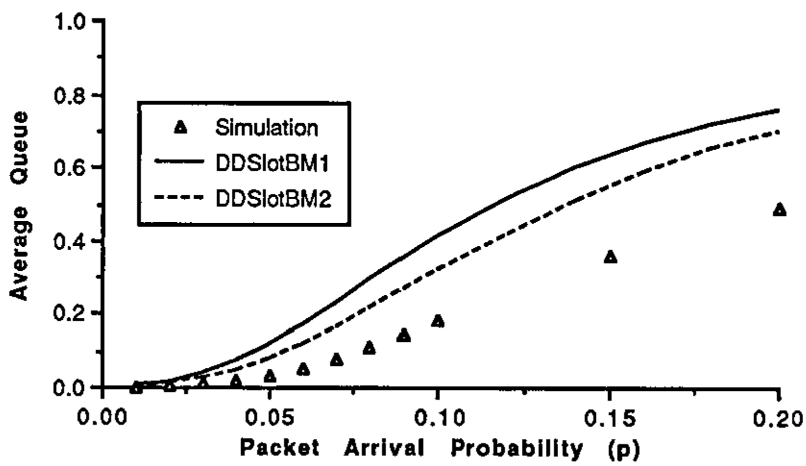


Fig. 5.3.11 Queuing Performance (N=16, M=8, Q=1)

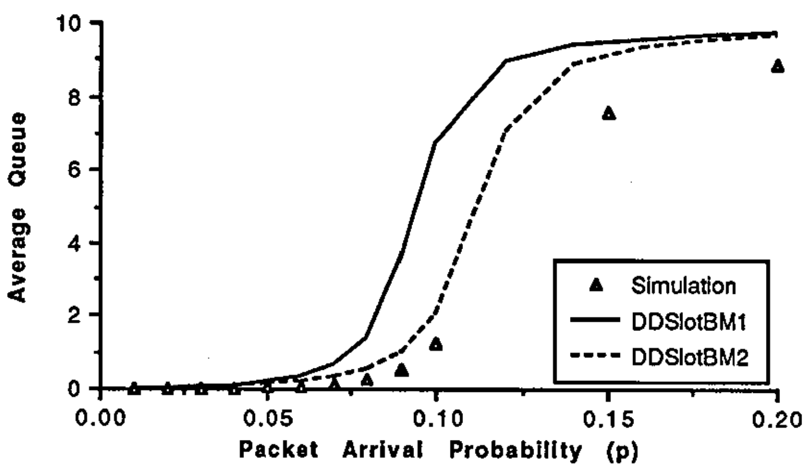


Fig. 5.3.12 Queuing Performance (N=16, M=8, Q=10)

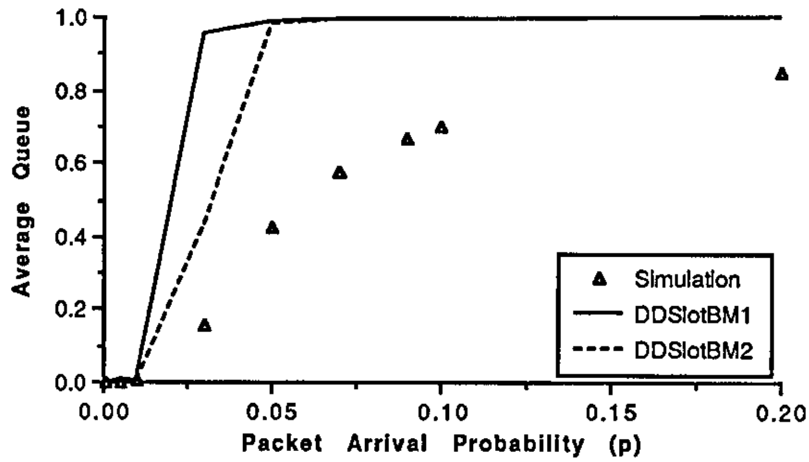


Fig. 5.3.13 Queuing Performance (N=64, M=1, Q=1)

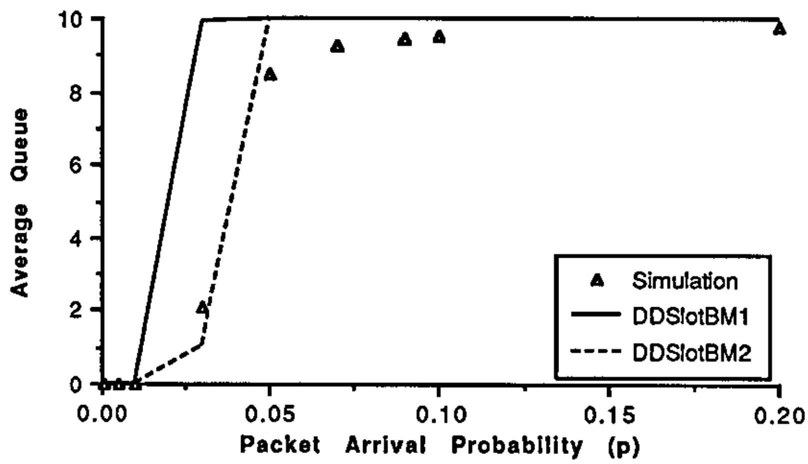


Fig. 5.3.14 Queuing Performance (N=64, M=1, Q=10)

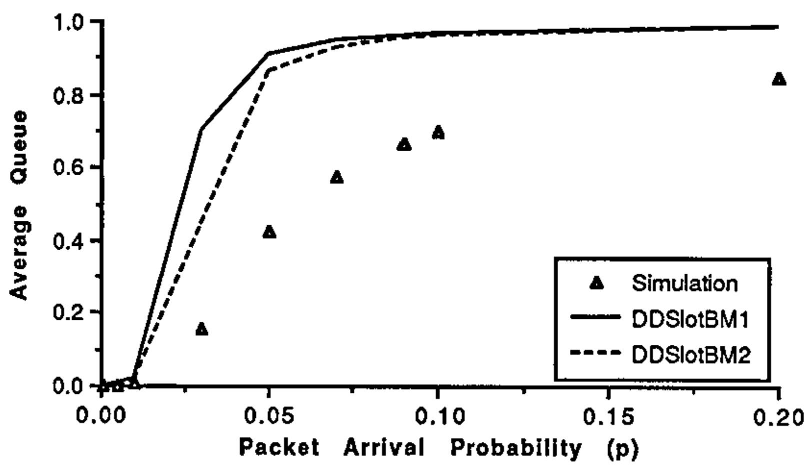


Fig. 5.3.15 Queuing Performance (N=64, M=8, Q=1)

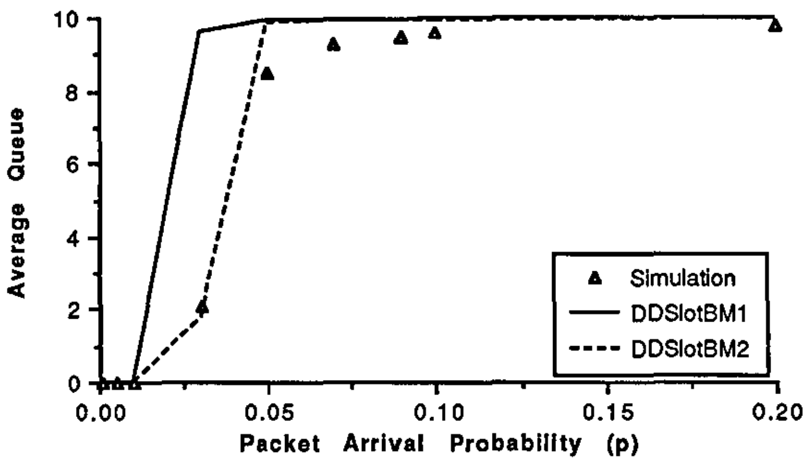


Fig. 5.3.16 Queuing Performance (N=64, M=8, Q=10)

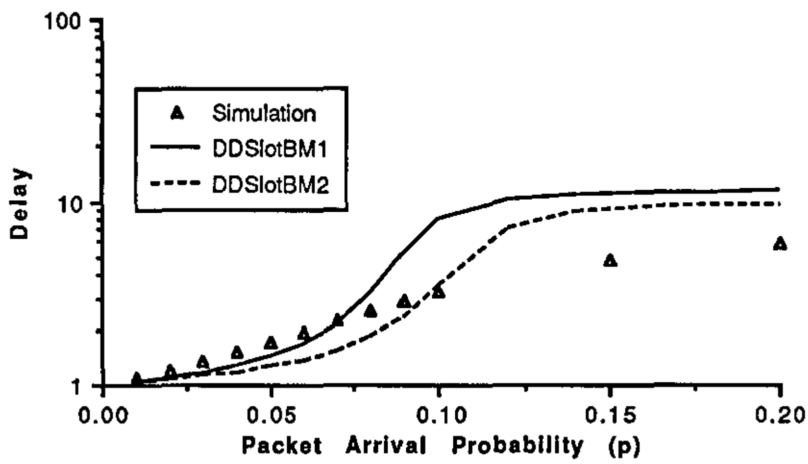


Fig. 5.3.17 Delay Performance (N=16, M=1, Q=1)

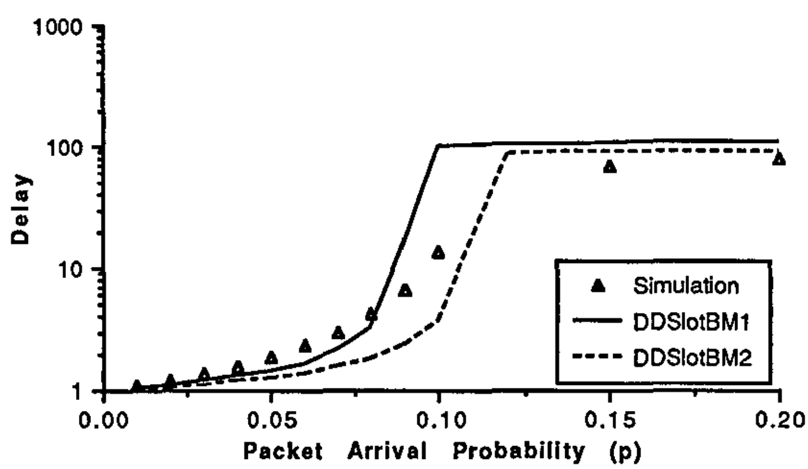


Fig. 5.3.18 Delay Performance (N=16, M=1, Q=10)

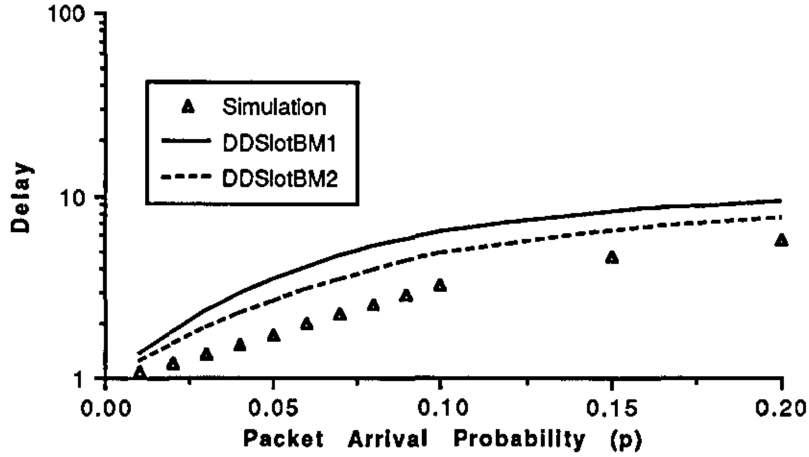


Fig. 5.3.19 Delay Performance (N=16, M=8, Q=1)

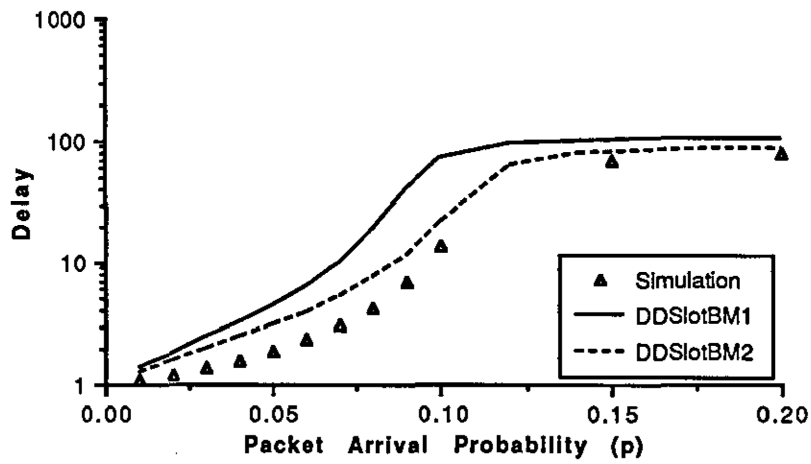


Fig. 5.3.20 Delay Performance (N=16, M=8, Q=10)

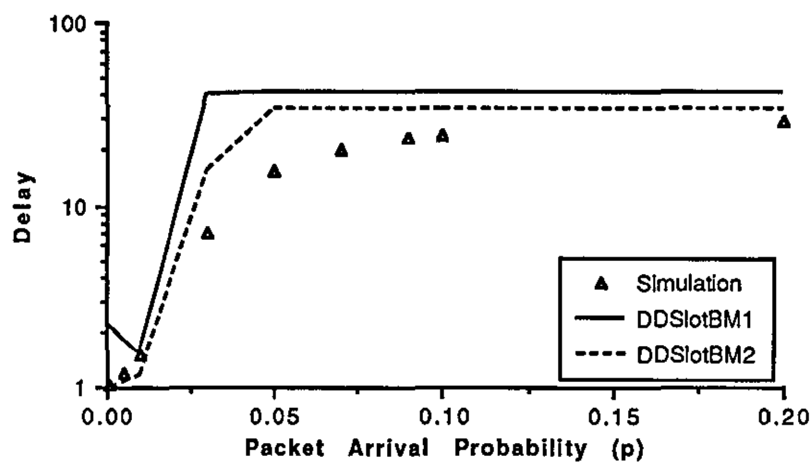


Fig. 5.3.21 Delay Performance (N=64, M=1, Q=1)

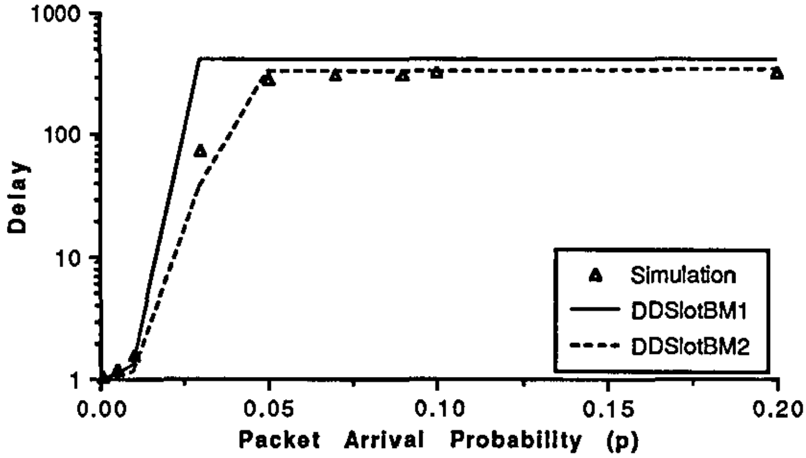


Fig. 5.3.22 Delay Performance (N=64, M=1, Q=10)

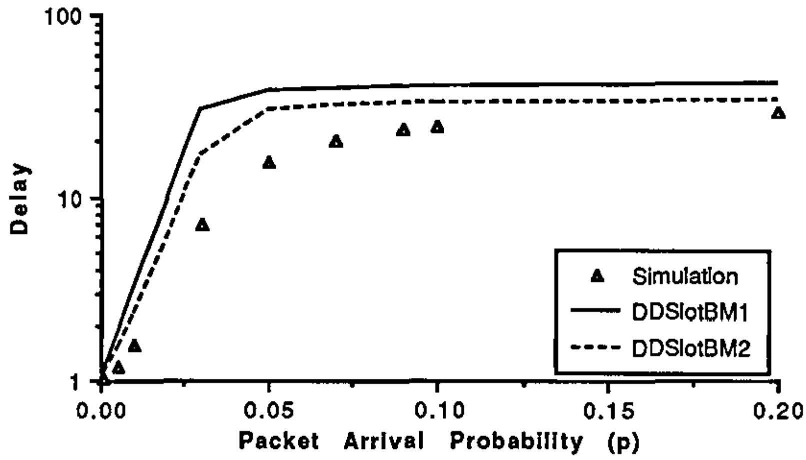


Fig. 5.3.23 Delay Performance (N=64, M=8, Q=1)

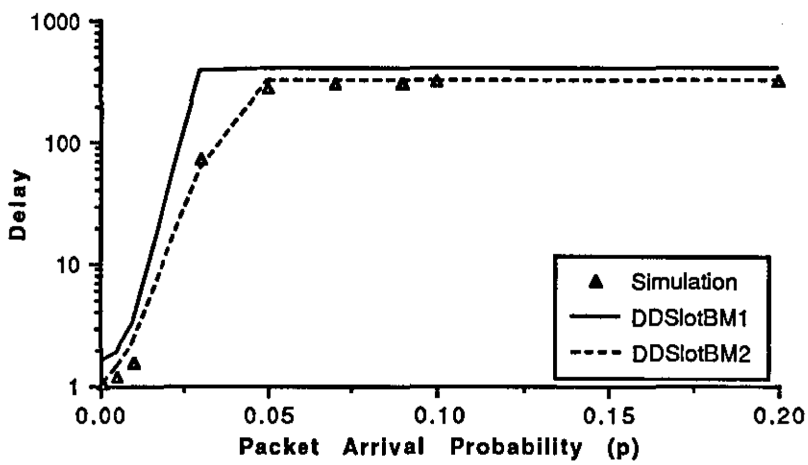


Fig. 5.3.24 Delay Performance (N=64, M=8, Q=10)

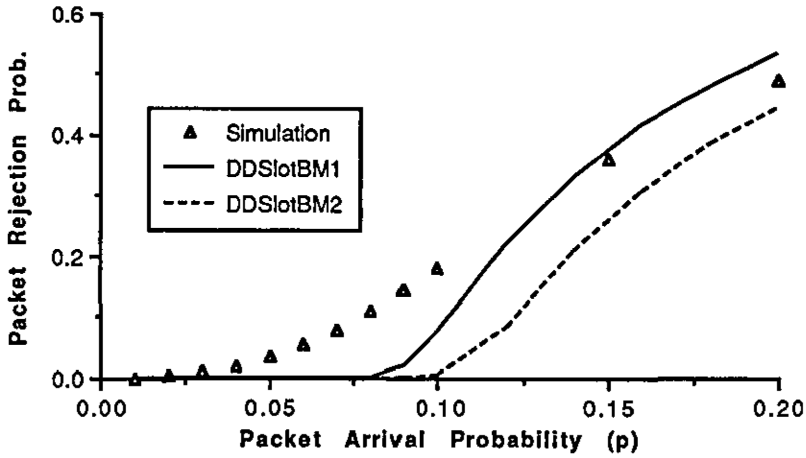


Fig. 5.3.25 Buffer Overflow Characteristics (N=16, M=1, Q=1)

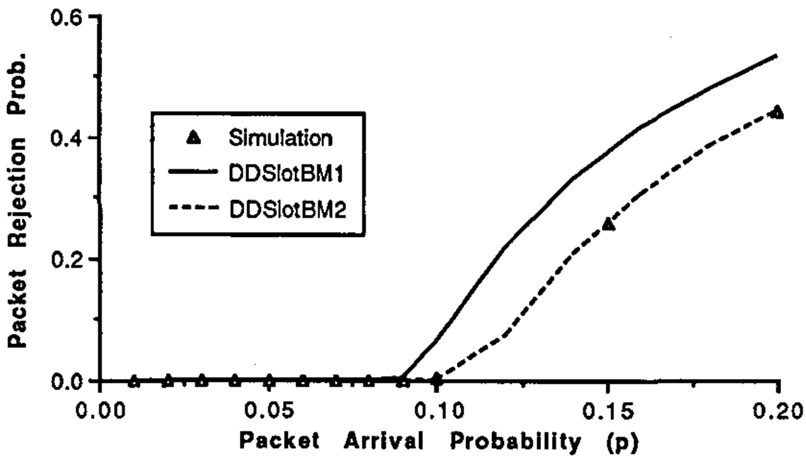


Fig. 5.3.26 Buffer Overflow Characteristics (N=16, M=1, Q=10)

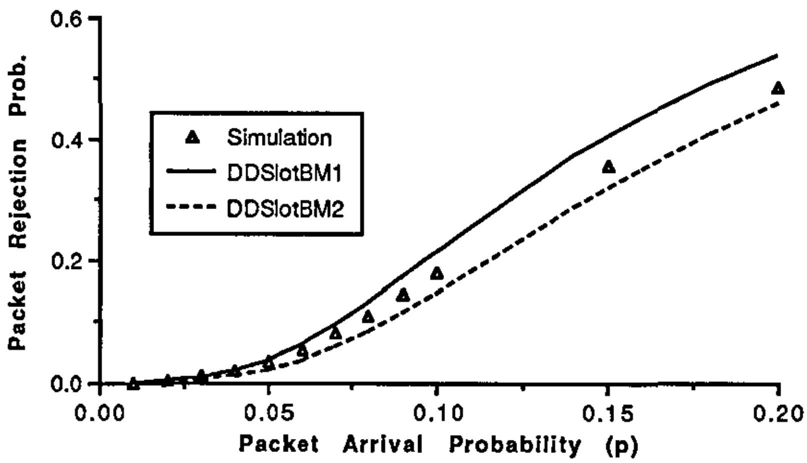


Fig. 5.3.27 Buffer Overflow Characteristics (N=16, M=8, Q=1)

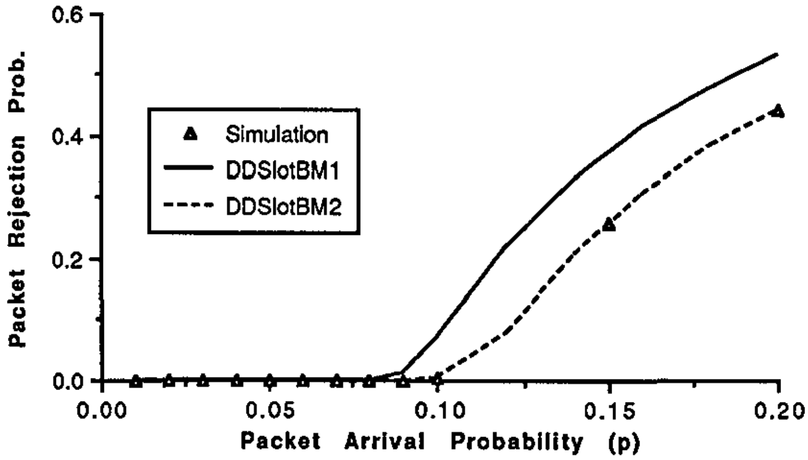


Fig. 5.3.28 Buffer Overflow Characteristics (N=16, M=8, Q=10)

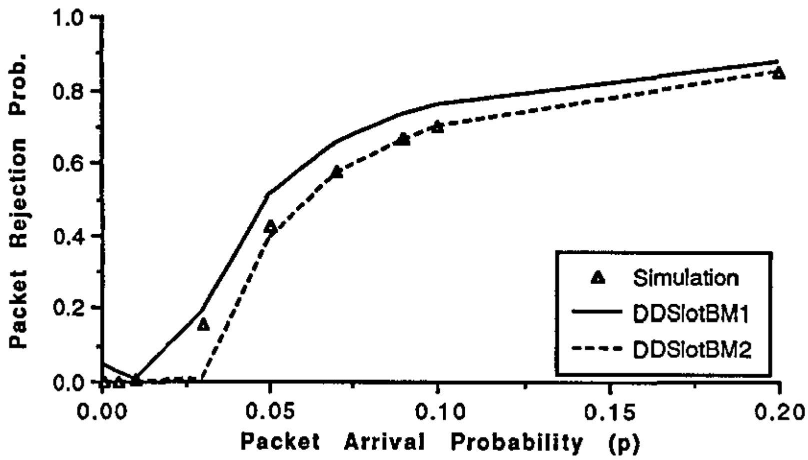


Fig. 5.3.29 Buffer Overflow Characteristics (N=64, M=1, Q=1)

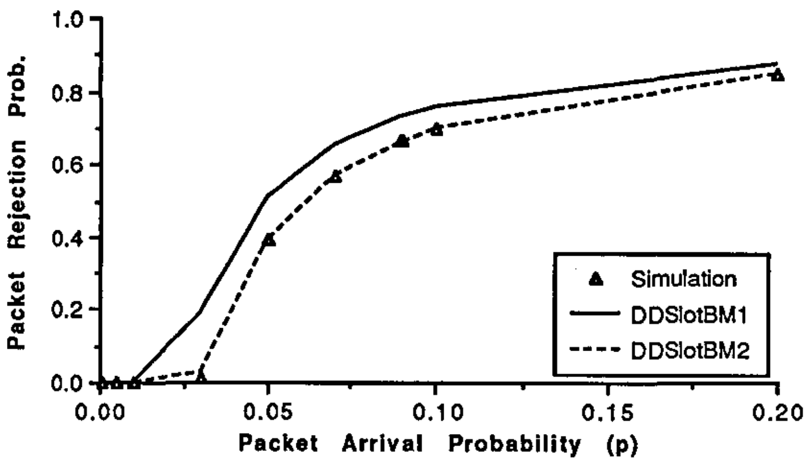


Fig. 5.3.30 Buffer Overflow Characteristics (N=64, M=1, Q=10)

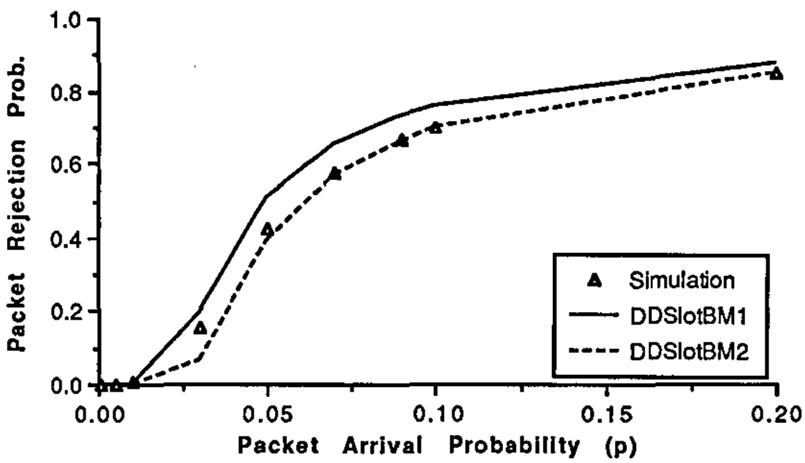


Fig. 5.3.31 Buffer Overflow Characteristics (N=64, M=8, Q=1)

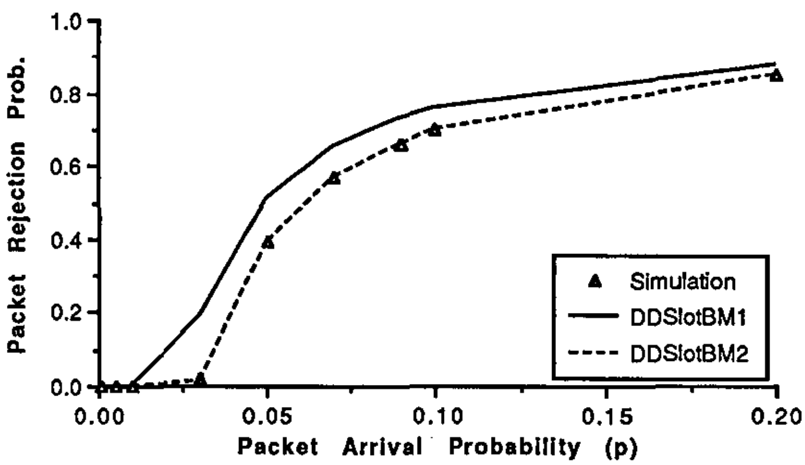


Fig. 5.3.32 Buffer Overflow Characteristics (N=64, M=8, Q=10)

CHAPTER 6

Orwell Protocol for the Slotted Ring

6.0 Introduction

With the advances made in optical fibre links and hardware in general, it was realised that slotted rings can provide extremely high levels of efficiency and performance (Bux 81). Orwell (Adams 84, Arem 90, Falconer 85a, Lee 91, Mitrani 86) is a protocol designed by the British Telecom in order for such a distributed control network to provide multiclass and delay sensitive data transmission capabilities. Under this protocol data is categorised into two classes: class 1 which is delay sensitive, and class 2 which is delay tolerant.

The protocol incorporates destination deletion of packets. To allow for time critical real time communications, counters are used at each station, so that when a packet is transmitted, the transmitting station will increment its counter. Upon reaching a locally agreed maximum "Di" value of the counter, that station will enter a state termed as the "Paused" state, which is a state where the station will not transmit any data before a network "Reset". This gives a opportunity for the other stations to transmit their packets up to their own Di limits.

A station in a Paused or "Idle" state is allowed to transmit "Trial" slots on empty slots. A trial consists of a special bit pattern in the slot header and is addressed to the trial originator. A station downstream that wishes to transmit may use these Trial slots.

Therefore, if a trial slot returns to its originator, it implies that no stations are in need for further transmissions due to lack of data, or that they are Paused. Thus, upon receiving a successful Trial slot, its originator converts it into a Reset slot, a slot having a special dedicated bit pattern in its header, which is then transmitted. A station that has a Reset slot under transmission is said to have a "Outstanding Reset".

Any station which may come across a Reset slot will reset its counters to zero, and, as such, the whole network will start afresh. The reset slots are converted into normal slots by any station with an Outstanding Reset.

When a request for a new connection with guaranteed bandwidth is received, the station will check the current reset rate, and accept the request if this call will not make the new expected reset time to exceed the maximum allowable reset time. The Di ceiling of that station will be accordingly incremented to accommodate the new connection.

In this manner, the number of links are controlled, such that, by making the network reset within a critical time (maximum allowable reset time), real time data can be transmitted at regular time intervals without giving rise to unacceptable delays.

In this chapter, two models for the Orwell protocol will be analyzed. The first one will be based on observing the system from a station (Orwell Station Based Model - OStnBM), and the second based on observing the system from a slot (Orwell Slot Based Model - OSlotBM).

The following assumptions are common to both the models in this chapter -

(6.0.1) Both class 1 and class 2 data packets will fit exactly into a slot.

(6.0.2) A packet, on average, will be transmitted to a station halfway around the ring.

(6.0.3) There are no gap bits between slots.

In addition, the assumptions (2.0.2) to (2.0.5), (2.0.7), (2.0.9) to (2.0.12), (2.1.3), (4.0.1) to (4.0.3) and (4.2.1) will be applicable in general.

The use of assumption (6.0.2) essentially implies that we have to use the variable destination deletion probabilities. A simple calculation can prove that only this method of the two discussed in the previous two chapters, can result in the condition specified by assumption (6.0.2). (See appendix B)

6.1 Orwell Station Based Model (OStnBM)

Figure 6.1.1 shows the Markov chain of the model for the Orwell protocol based on observing the network from a station.

In addition to the assumptions made in section 6.0, the following are assumed -

(6.1.1) A station with no packets in its buffer will not enter a Paused state.

Assumptions (2.0.8), (2.5.1) and (2.5.2) also apply.

As in all previous models, the nodes of the model show the states a given station may occupy. The first value inside a node indicates the queue size. The second value denotes the number of station times that have elapsed since the start of transmission (values between 0 and K), or the Paused state (indicated by P). Note that this (y,P) state also represents the state of station when it has a trial and/or reset slot(s) under transmission.

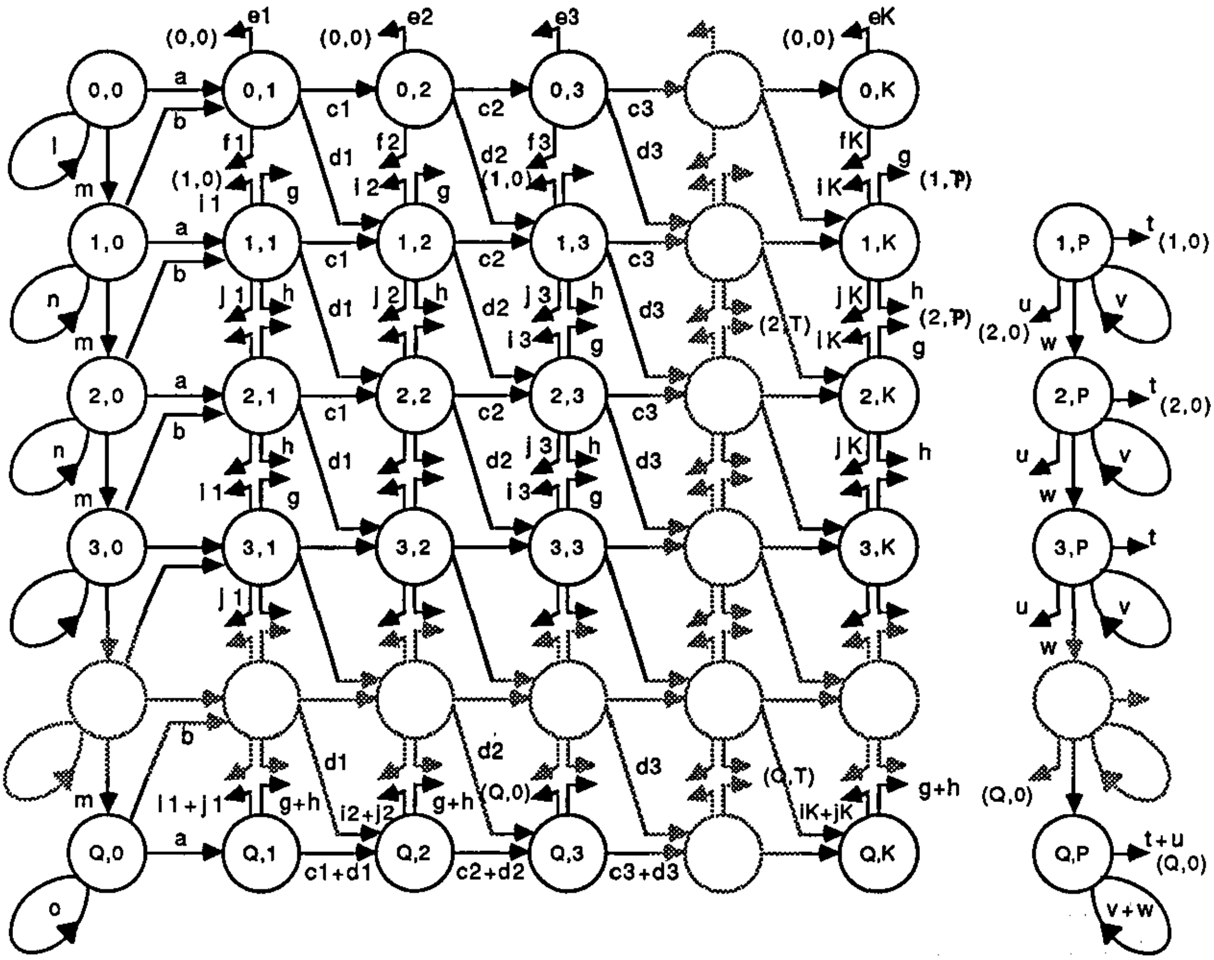


Fig 6.1.1 **Discrete Time, Station Based Markov Model for a Slotted Ring Operating Under The Orwell Protocol (OStnBM)**

As before, from assumption (4.0.2),

$$K = N-1$$

The transitions denoted by the probabilities a, b, c_x, d_x, e_x, f_x, i_x, j_x, l, m, n and o ($1 \le x \le K$) can be explained in a manner similar to that in DDStnBM2. The transition probabilities h and g indicate the chance of the station entering a Paused state, with and without the arrival of a packet at the same instant respectively.

Similarly, a station in a Paused state (y,P), where ($1 \le y < Q$), will move on to a state (y,0), (y+1,0) or (y+1,P) with a probability t, u or w respectively, or, remain in the same state with a probability v.

Considering steady state equilibrium, as before,

$$a = p(1-S)$$
$$b = (1-p)(1-S)$$
$$l = 1-p$$
$$m = pS$$
$$n = (1-p)S$$
$$o = S$$

From assumption (4.2.1),

$$\text{Probability that the slot has reached its destination} = \frac{1}{(K-x+1)}$$

Therefore,

$$e_x = \frac{(1-p)}{(K-x+1)} \tag{6.1.1}$$

$$f_x = \frac{p}{(K-x+1)} \tag{6.1.2}$$

$$c_x = \left\{1 - \frac{1}{(K-x+1)}\right\}(1-p) \tag{6.1.3}$$

$$d_x = \left\{1 - \frac{1}{(K-x+1)}\right\}p \tag{6.1.4}$$

Since all stations are statistically equivalent, let the maximum Di value of all stations be Dmax.

From assumption (6.0.2),

$$\begin{array}{l} \textit{Average time needed for a} \\ \textit{station in a transmitting} \\ \textit{state to transmit a packet} \end{array} = \frac{N}{2}$$

Thus,

$$\begin{array}{c} \textit{Average time needed to transmit} \\ \textit{Dmax packets of all stations when} \\ \textit{they are in a transmitting state} \end{array} = \frac{N}{2} \times \frac{N\,Dmax}{M}$$

Whenever the station is in a transmitting state with a non zero buffer, it will enter the Paused state only after transmitting its full quota of packets amounting to Dmax.

Therefore,

$$\begin{array}{c} \textit{Probability of entering the} \\ \textit{Paused state from a transmitting} \\ \textit{state in the next transition} \end{array} = \frac{2M}{(N^2\,Dmax)} \tag{6.1.5}$$

This leads to,

$$g = \frac{2M}{(N^2\,Dmax)}(1-p) \tag{6.1.6}$$

$$h = \frac{2M}{(N^2\,Dmax)}p \tag{6.1.7}$$

The following equation now applies,

$$c_x + d_x + g + h + i_x + j_x = 1$$

Substituting from (6.1.3), (6.1.4), (6.1.6) and (6.1.7),

$$i_x + j_x = 1 - (c_x + d_x) - (g + h)$$
$$= \frac{1}{(K-x+1)} - \frac{2M}{(N^2\,Dmax)}$$

Thus,

$$i_x = \left\{\frac{1}{(K-x+1)} - \frac{2M}{(N^2\,Dmax)}\right\}(1-p) \tag{6.1.8}$$

$$j_x = \left\{\frac{1}{(K-x+1)} - \frac{2M}{(N^2\,Dmax)}\right\}p \tag{6.1.9}$$

In Appendix C, it is shown that i_x and j_x are positive for all reasonable network parameter values.

When the stations are Paused, due to statistical equivalence of stations,

$$\textit{Time needed to Reset the network} = N + \frac{N}{M} \tag{6.1.10}$$

Here, the first term is the time needed for the trial; the second term being due to the average extra time needed for the Reset slot to reach the next station with an outstanding Reset.

Therefore,

$$\begin{array}{c}\textit{Probability of a Reset when the}\\ \textit{station is in a Paused state}\end{array} = \frac{1}{\left(N + \frac{N}{M}\right)}$$
$$= \frac{M}{(M+1)\,N}$$

$$t = \frac{M}{(M+1)N}(1-p) \tag{6.1.11}$$

$$u = \frac{M}{(M+1)N}\,p \tag{6.1.12}$$

$$v = \left\{1-\frac{M}{(M+1)N}\right\}(1-p) \tag{6.1.13}$$

$$w = \left\{1-\frac{M}{(M+1)N}\right\}p \tag{6.1.14}$$

Calculation of Dmax

Let,

g = Gap bits between slots

L = Total bits per slot

L_d = Class 1 data bits per slot

R = Transmission rate

R_1 = Class 1 data rate

Then,

$$\text{Propergation delay of the ring } (T) = \frac{M(g+L)}{R} \quad \text{(sec)}$$

From assumption (6.0.3), g=0. Thus,

$$T = \frac{ML}{R} \quad \text{(sec)} \tag{6.1.15}$$

$$\begin{array}{l}\text{Average time needed}\\ \text{to transmit one packet}\end{array} = \frac{N}{2}+1 \quad \textit{(station times)}$$

The +1 of the above equation represents the overhead involved at the receiving station in passing the slot to the next station.

Therefore,

$$\text{Time needed to transmit Dmax packets of all stations} = \left(\frac{N}{2}+1\right)\frac{N\,Dmax}{M}$$

Considering the fact that one station time is wasted after a slot is cleared from a Reset state to be passed to the next station before being filled up for transmission, and using equation (6.1.10),

$$\text{Total time needed to Reset when all stations are Paused} = N + \frac{N}{M} + 1 \qquad (6.1.16)$$

Therefore,

$$\text{Maximum time between Resets (sec)} = \left\{\left(\frac{N}{2}+1\right)\frac{N\,Dmax}{M} + \left(N + \frac{N}{M} + 1\right)\right\}\frac{T}{N}$$

$$= \frac{L_d}{R_1} \qquad (6.1.17)$$

since, at every L_d/R_1 interval, a new packet from the class 1 source will be awaiting transmission on the newly Reset ring.

Substituting for T from equation (6.1.15),

$$Dmax = \left\{\frac{L_d\,R}{R_1\,M\,L} - \left(1 + \frac{1}{M} + \frac{1}{N}\right)\right\}\frac{M}{\frac{N}{2}+1} \qquad (6.1.18)$$

Since Di is always a whole number, the decimals of Dmax are truncated.

The transition probabilities calculated above may be used to obtain the State Transition Matrix, which in turn can be used as described in section 2.4 to solve the Markov chain at equilibrium.

Once all the P(y,x) values are known, the performance measures can be obtained in the following manner.

$$S = \left(1-\sum_{y=0}^{Q} P(y,0)\right)\frac{(N-1)}{M} \tag{6.1.19}$$

Considering the number of transmissions per slot per cycle,

$$\mathit{Throughput} = \left\{\sum_{y=1}^{Q} P(y,0) + pP(0,0)\right\}\frac{(1-S)\,N^2}{M} \tag{6.1.20}$$

$$\mathit{Average\ Queue}\ (\overline{q}) = \sum_{y=1}^{Q}\sum_{x=0}^{T} yP(y,x) \tag{6.1.21}$$

$$\mathit{Average\ Delay} = \frac{\overline{q}\,N}{\mathit{Throughput}} + \frac{M}{N} \tag{6.1.22}$$

$$\begin{array}{l}\mathit{Packet\ Rejection}\\ \mathit{Probability\ given}\\ \mathit{packet\ arrival\ (PRP)}\end{array} = 1 - \frac{\mathit{Throughput}\ M}{p\,N^2} \tag{6.1.23}$$

$$\begin{array}{l}\mathit{Reset}\\ \mathit{Probability}\\ \mathit{per\ Cycle}\end{array} = \left((t+u)\sum_{y=1}^{Q} P(y,P) + \frac{P(0,0)\,(1-p)\,(1-S)}{\left(1+\frac{1}{M}+\frac{1}{N}\right)N}\right)N \tag{6.1.24}$$

Here the second term on the right hand side represents the probability of a Reset when a station is idle.

6.2 Orwell Slot Based Model (OSlotBM)

As in the case of previous protocols analysed, the differences between this model and OStnBM are minimal.

Since slots do not physically get paused, the state (y,P) $\{1 \le y \le Q\}$ indicates the probability that the station against which the slot header is aligned does not wish to transmit a data packet due to the fact that it is paused. However, the slot at this state may be converted into a trial slot or a reset slot by that station.

The following additional assumptions are made -

(6.2.1) A slot having an average of zero packets waiting to be transmitted (states represented by (0,x) where $0 \le x \le N-1$), will not enter the pseudo paused state mentioned above. Thus a state (0,P) does not exist.

Assumptions (3.1.1) to (3.1.3) also apply.

Figure 6.2.1 shows the Markov chain applicable to this model. Here,

$$Q = Average\ maximum\ buffer\ size\ per\ slot$$

and, as before from assumption (4.0.2),

$$K = N-1$$

The transition probabilities c_x, d_x, e_x, f_x, g, h, i_x, j_x, t, u, v and w $(1 \le x \le K)$, as well as Dmax remain the same as in the previous model, except that now, they are applied to the slot under consideration.

However, the values of a and b change to -

$$a = p$$
$$b = 1-p$$

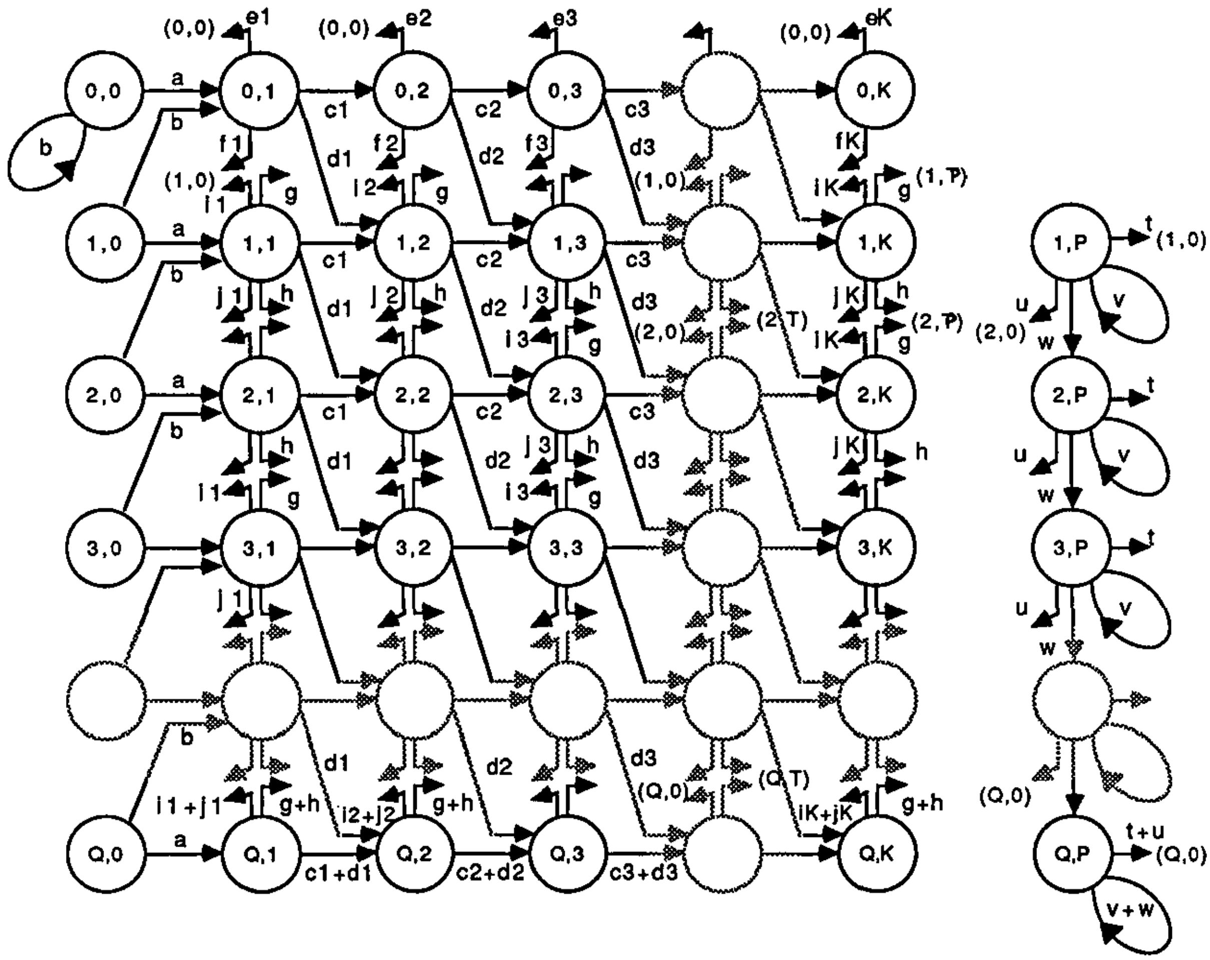


Fig 6.2.1 Discrete Time, Slot Based Markov Model for a Slotted Ring Operating Under The Orwell Protocol (OSlotBM)

Since all the transition probabilities are known, the model can be solved using the matrix method described in sections 2.4 and 3.2.

The network performance measures can then be calculated as follows -

$$Throughput = \begin{array}{c}Number\ of\ transmissions\\ per\ cycle\ per\ slot\end{array}$$

$$= N\left(P(0,0)p + \sum_{y=1}^{Q} P(y,0)\right) \tag{6.2.1}$$

$$\begin{array}{l}Probability\ of\ a\\ slot\ being\ full\end{array} = \sum_{y=0}^{Q}\sum_{x=1}^{K} P(y,x) \tag{6.2.2}$$

$$\begin{array}{l}Average\ Queue\ per\\ Station\ (\overline{q})\end{array} = \frac{M}{N}\sum_{y=0}^{Q}\sum_{x=0}^{P} yP(y,x) \tag{6.2.3}$$

$$Average\ Delay\ (\overline{d}) = \frac{\overline{q}N}{Throughput} + \frac{M}{N} \tag{6.2.4}$$

$$\begin{array}{l}Packet\ Rejection\\ Probability\ (PRP)\end{array} = 1 - \frac{Throughput}{pN} \tag{6.2.5}$$

$$\begin{array}{l}Reset\\ Probability\\ per\ Cycle\end{array} = \left((t+u)\sum_{y=1}^{Q} P(y,P) + \frac{P(0,0)\ (1-p)}{\left(1+\frac{1}{M}+\frac{1}{N}\right)N}\right)N \tag{6.2.6}$$

6.3 Simulation

As in the case of previous simulations, this too was done in two parts. Firstly, the network was allowed to reach a steady state. Then, in the second part, the performance measures were evaluated while in the steady state.

It is noted that, even though this second part of the simulation was done for a minimum of 10,000 slot times per station in each case, when compared to a real time situation, this amounts to less than even 1 second of operation of a ring with a transmission rate of 20 Mbaud and a slot size of 160 bits. Due to the large amount of computing time required to run the simulation, it was not feasible to lengthen the run time any further. Thus, it was assumed that -

(6.3.1) No existing calls terminate.
In other words, stations will not decrement their Di values and release bandwidth to the network.

Due to the lack of standards, the following protocol rules have also been assumed-

(6.3.2) Current Average Reset Interval = linear average of the last two Reset Intervals.
Here the "Reset Interval" refers to the amount of time between one network reset and the next. The average was used to smooth out any sudden variations that may occur within the short running time of the simulation.

(6.3.3) A constant value termed "TxTolerance" (having units of slot times), was used when giving permission for an increase in the Di value of a station.
This was done by firstly making sure that, from its current state, at least that amount of time was available before the network should

definitely reset (a check on the current status of that particular station). This was necessary due to the fact that, within a given network reset interval, more than one station may need extra bandwidth, but not be aware of this similar need of the other station(s).

The second check was to make sure that the extra time needed would not push the average current reset interval to an unacceptable limit (a check on the average network reset interval). The value of TxTolerance was selected on a trial and error basis so as to ensure that the network reset within the maximum allowable time, but did not do so too often when the load is high. For the cases 16 and 64 stations this value was kept constant at 75 and 100 respectively.

6.4 Results

The network performance measures for the models and the simulation were made for all the combinations of N (16 and 64), M (1, 8 and 16) and Q (1, 10 and 50), but the results containing M=16 or Q=50 excluded from the thesis for the usual reasons. When considering the maximum buffer size, the fact that there is very little difference in the performance measures between the cases where Q is 10 and 50 (throughput, PRP and reset rate), or where there is a difference (average queue and delay), the fact that it can be easily predicted by observing the similar case with Q=10 justifies this omission. However, as in the previous chapter, all results have been considered for the analysis of the performance measures.

Considering the proposed slot architecture (Falconer 85), the following network parameters were assumed constant throughout -

L = 160 bits per slot

L_d = 128 bits per slot

$R \quad = 20$ Mbaud

$R_l \quad = 64$ Kbaud

Throughput

As in the case of all previous throughput curves, with the increase of Q, these also tend to follow the path of two intersecting lines; the first part a line of type -

$$Throughput = Np$$

and the second part,

$$Throughput = Constant$$

Both models show a good match to the simulation in the linear region, the OSlotBM showing a better, but slightly higher match in the saturated region. The reason for this disparity is the fact that OSlotBM tend to show the ideal values obtainable.

An increase of M causes the saturation values of all the models to decrease, this effect being more pronounced in the OStnBM since all station based models are prone to such behaviour. The reason for this effect in the case of the slot based model can be explained by noting the behaviour of equation (A.19) with respect to M.

No apparent change can be detected in the comparison when N is increased, except the usual change in the gradient of the linear region and the improved saturation throughput of all curves.

Average Queue

The comparison shows the OStnBM giving a better match to the simulation. All three curves tend to become a step function with the increase of Q, the slot based model tending to predict the instance of this change in level more accurately.

The increase of M causes the station based model values to increase with respect to the simulation, this being due to the drop in the throughput for the same change in M. This increase becomes less in magnitude with the increase of N.

Average Delay

For the case of $Q=1$, the models show a relatively poor match to the simulation, but improve rapidly with the increase of Q, the OSlotBM resulting in a better match overall.

The increase of Q also gives rise to a more pronounced step function shape in both models as well as the simulation. The increase of M however has an opposite effect on the shape.

The delay of OStnBM tend to rise with the increase of M, the same effect being seen on the other two curves, although to a very minor extent.

A slightly improved match to the simulation can be seen on the OSlotBM with the increase of N.

Packet Rejection Probability (PRP)

Although the change in Q has little effect on both models, an increase of N results in an improved match to the simulation. An increase in M causes the values of OStnBM to increase with respect to the simulation.

In general, the OSlotBM shows a better overall match to the simulation than OStnBM.

Reset Rate

Here again, the OSlotBM results in a superior match to the simulation, especially when the network is in saturation. Both models improve greatly with the increase of N, and an increase of Q causes all the curves to transit more sharply into saturation. The effect of an increase in M is to increase the reset rate. This effect can be logically explained by the use of equations (A.12) and, (A.13) and (6.1.18), where an increase of M causes the two limits of the reset rate to increase.

Finally, it is noted that the simulation results for lower N/M ratios decrease at a lower rate, or, at times, even increase with the increase of the packet arrival rate before declining to follow the values predicted by the models. This effect can be explained as follows -

Assume that stations 2, 3 and 4 of figure 6.4.41 are idle at the instance shown. Slots A and C are successful trial slots of the stations 2 and 4 respectively, and thus will be duly converted into reset slots at the end of the current slot time. Slot B and D are full and addressed to a station beyond station 4 and to station 3 respectively. Slot E carries another trial of station 4.

Within the next five slot times, the status' of the slots are shown in the table below. Since station 4 will generate a trial on slot D at the end of the fourth slot time, and thus have an outstanding trial, it will convert slot E into a reset slot, thus creating another network reset within the same cycle.

As it can be seen, a lower N/M ratio with both N and M being considerably large, as well as a low and irregular packet arrival rate is needed for this effect to occur.

Slot Time	Slot A	Slot B	Slot C	Slot D	Slot E
1	Reset	Full, Ad>4	Reset	Full, Ad=3	Trial, Ad=4
2	Imm	Full, Ad>4	Reset	Full, Ad=3	Trial, Ad=4
3	Imm	Imm	Empty	Empty	Trial, Ad=4
4	Imm	Imm	Imm	Trial, Ad=4	Trial, Ad=4
5	Imm	Imm	Imm	Imm	Reset

where,

Ad=4 = Addressed to station 4

Imm = Immaterial

Figures (6.4.42) to (6.4.46) show the convergence properties of OStnBM when simple iteration, as mentioned in section 2.4, was used to calculate S.

At lower packet arrival probabilities "p", since the gradient with which the $S=f(S)$ curve intersect the $S=S$ line was quite low, a fast convergence to the solution was achieved.

However, as it may be observed from the above graphs, for higher values of p, the curve $S=f(S)$ intersect the $S=S$ with a gradient of over unity, thus diverging away from the solution.

This change of the gradient "m" at intersection from $|m|<1$ to $|m|\geq 1$ takes place near the saturation region, and under these circumstances, the bisection method was used to obtain a solution.

6.5 Summary

In this chapter, the Orwell protocol for the slotted ring was analysed using two models based on observing the system from a station and a slot respectively. The performance measures thus obtained were compared to simulation results.

All the results, in general, show a reasonably good match when compared with the simulation, especially when the maximum buffer size of a station is increased. The slot based model - OSlotBM, shows a superior comparison overall.

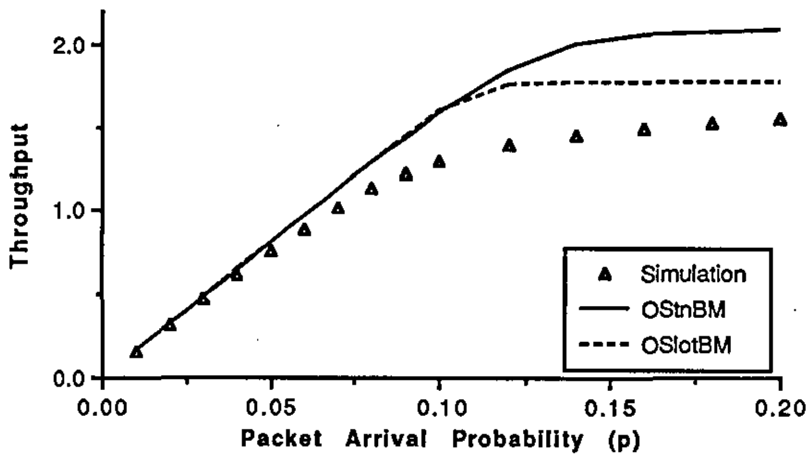


Fig. 6.4.1 Throughput Performance (N=16, M=1, Q=1)

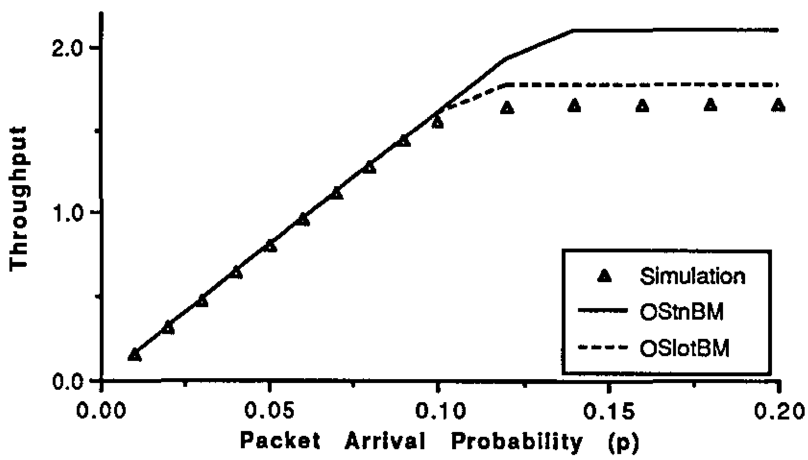


Fig. 6.4.2 Throughput Performance (N=16, M=1, Q=10)

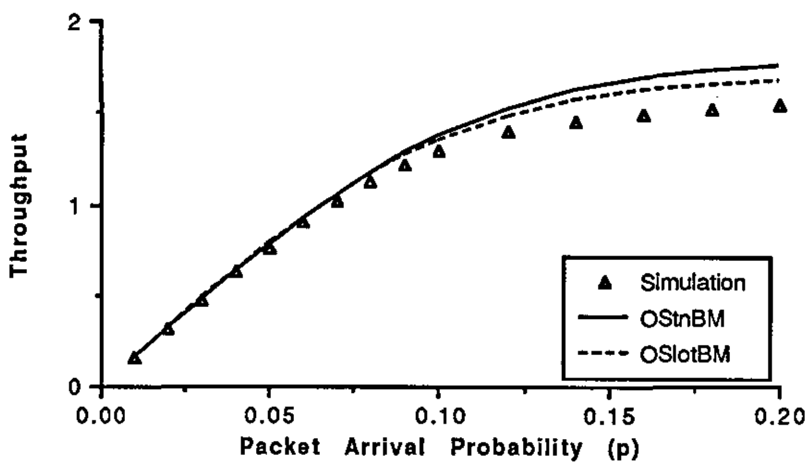


Fig. 6.4.3 Throughput Performance (N=16, M=8, Q=1)

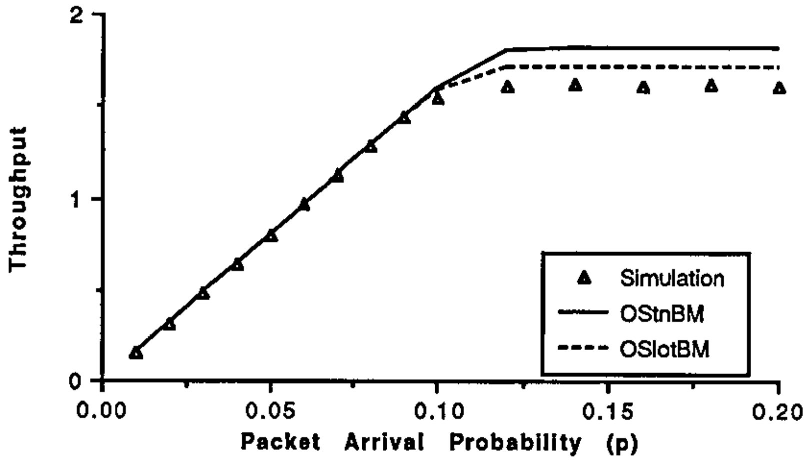


Fig. 6.4.4 Throughput Performance (N=16, M=8, Q=10)

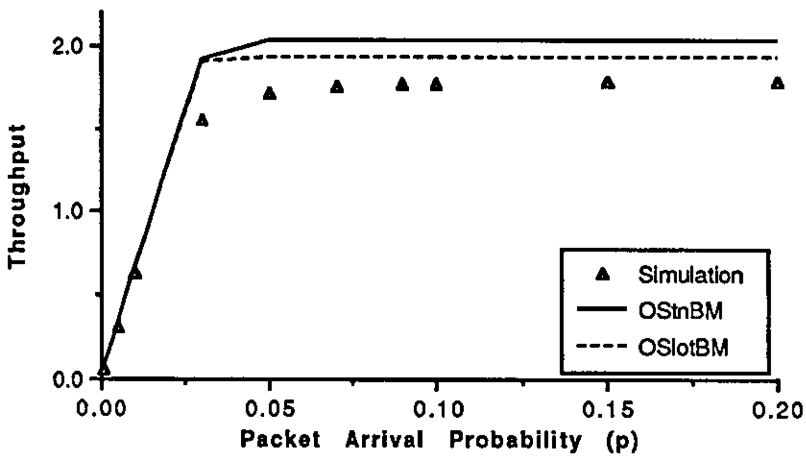


Fig. 6.4.5 Throughput Performance (N=64, M=1, Q=1)

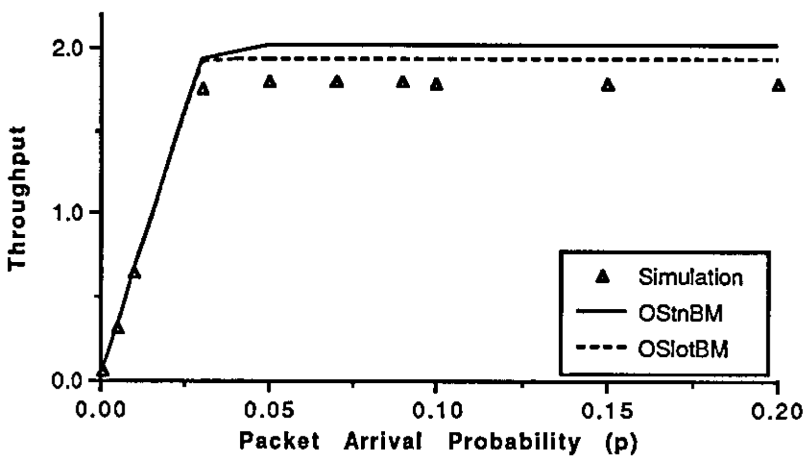


Fig. 6.4.6 Throughput Performance (N=64, M=1, Q=10)

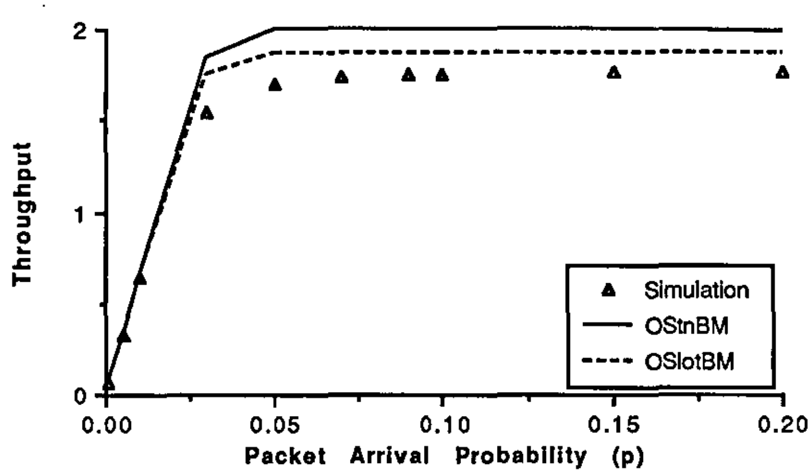


Fig. 6.4.7 Throughput Performance (N=64, M=8, Q=1)

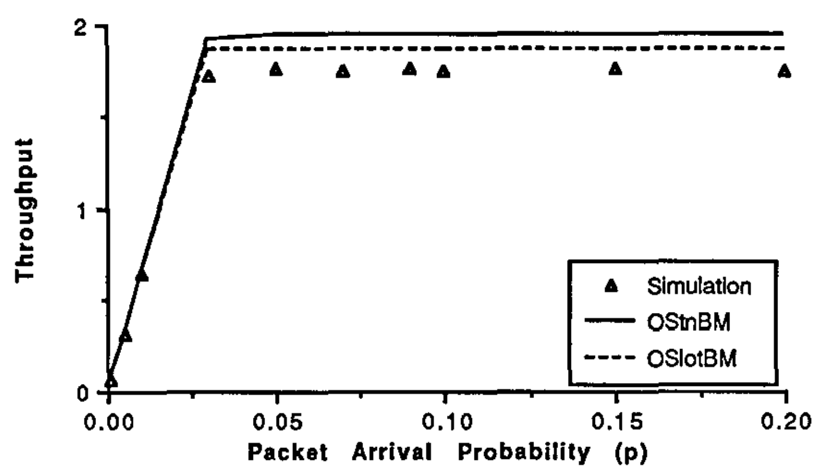


Fig. 6.4.8 Throughput Performance (N=64, M=8, Q=10)

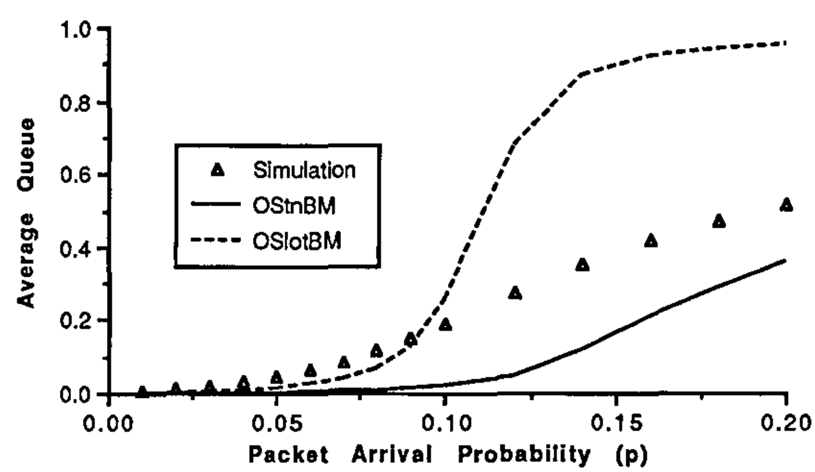


Fig. 6.4.9 Queueing Performance (N=16, M=1, Q=1)

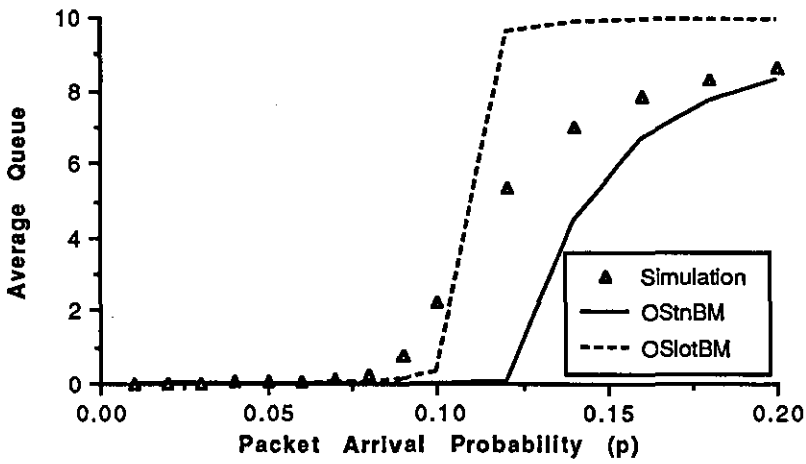


Fig. 6.4.10 Queueing Performance (N=16, M=1, Q=10)

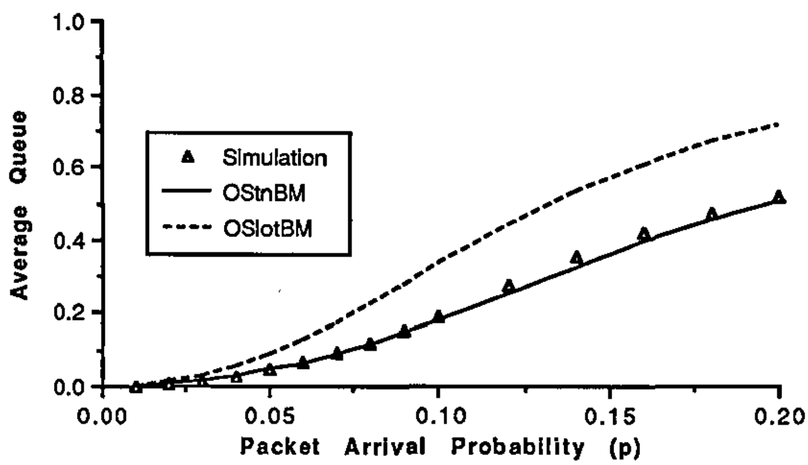


Fig. 6.4.11 Queueing Performance (N=16, M=8, Q=1)

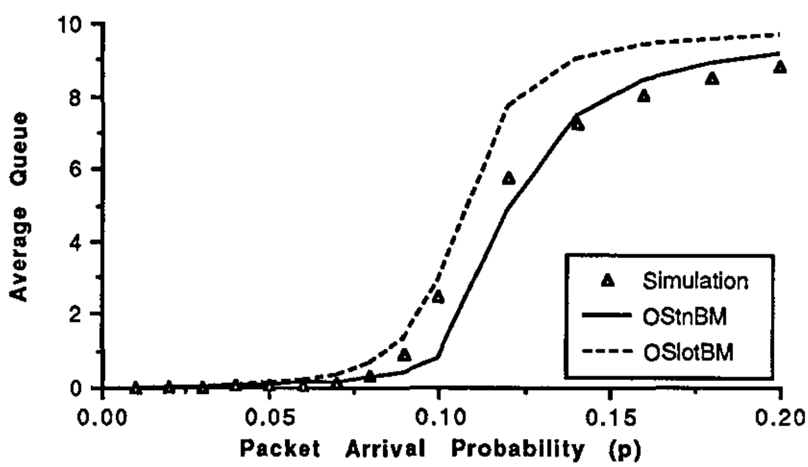


Fig. 6.4.12 Queueing Performance (N=16, M=8, Q=10)

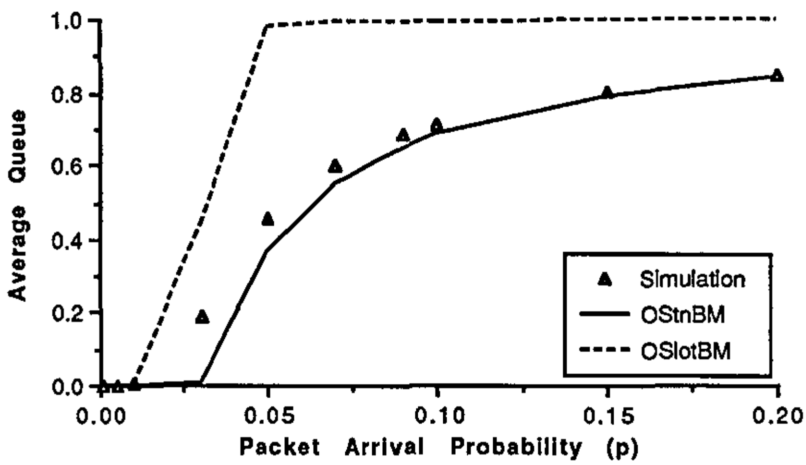


Fig. 6.4.13 Queueing Performance (N=64, M=1, Q=1)

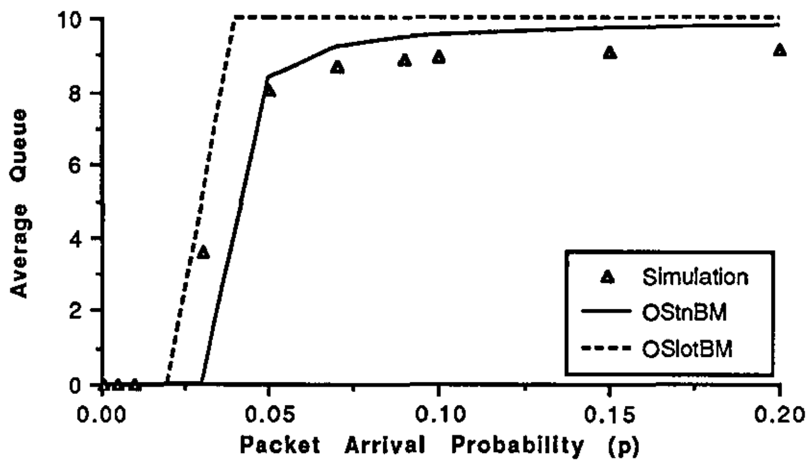


Fig. 6.4.14 Queueing Performance (N=64, M=1, Q=10)

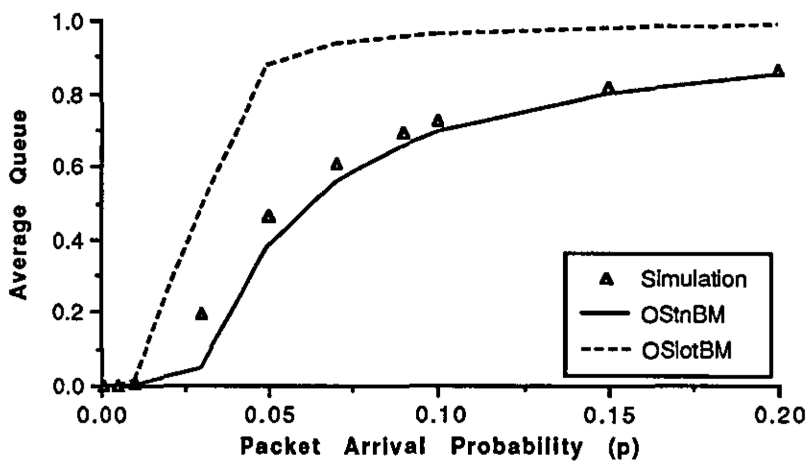


Fig. 6.4.15 Queueing Performance (N=64, M=8, Q=1)

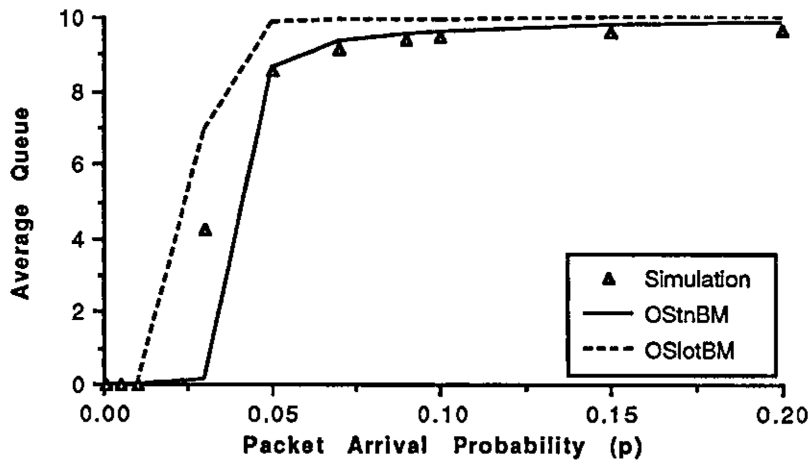


Fig. 6.4.16 Queueing Performance (N=64, M=8, Q=10)

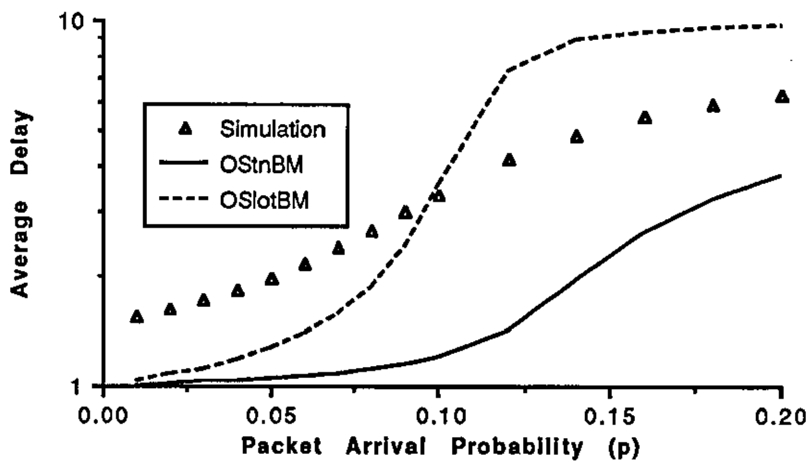


Fig. 6.4.17 Delay Performance (N=16, M=1, Q=1)

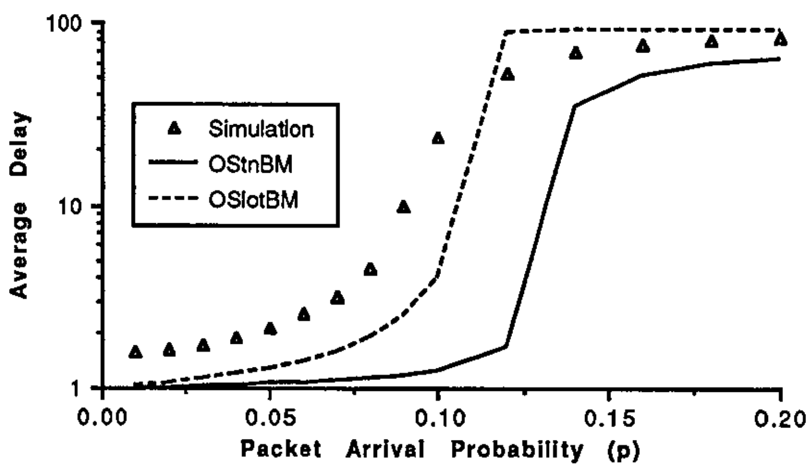


Fig. 6.4.18 Delay Performance (N=16, M=1, Q=10)

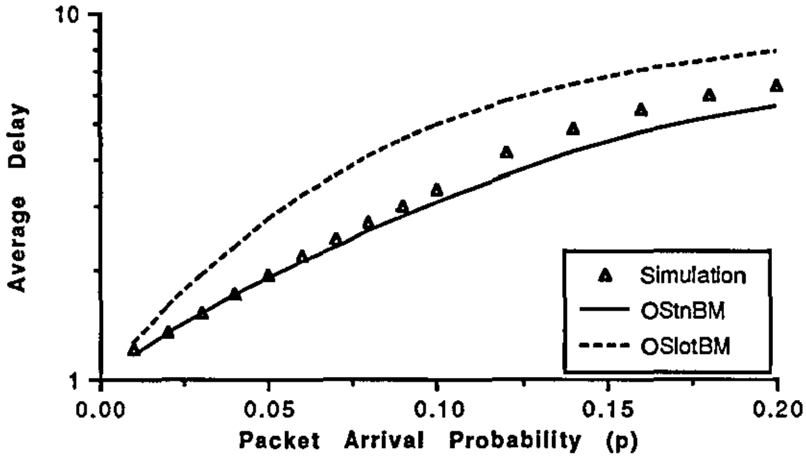


Fig. 6.4.19 Delay Performance (N=16, M=8, Q=1)

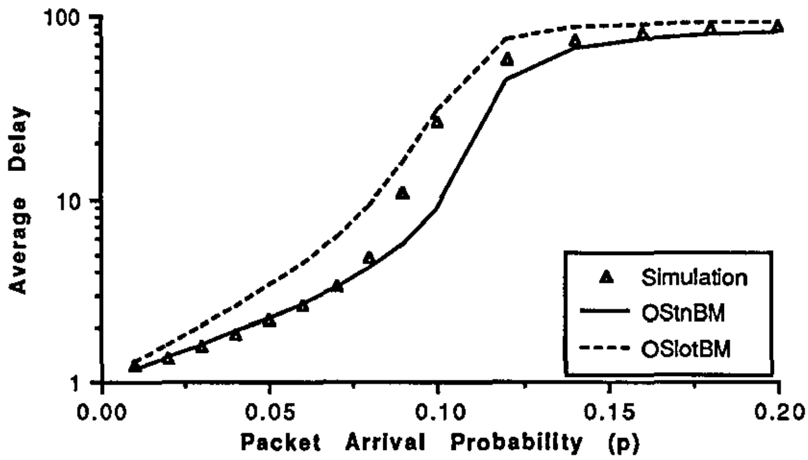


Fig. 6.4.20 Delay Performance (N=16, M=8, Q=10)

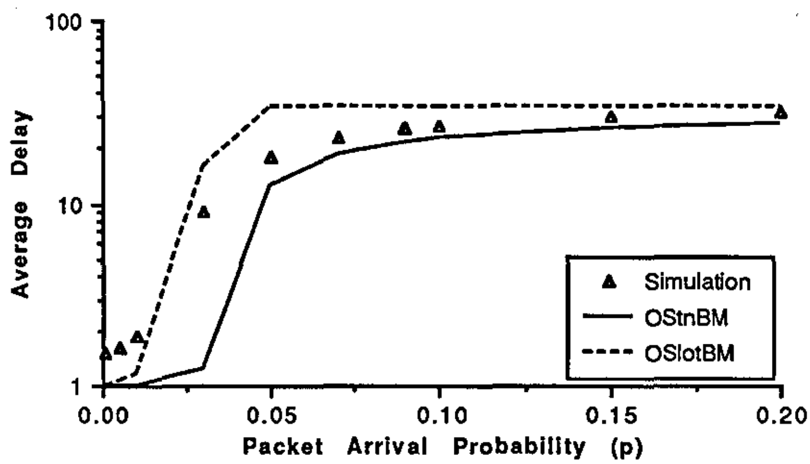


Fig. 6.4.21 Delay Performance (N=64, M=1, Q=1)

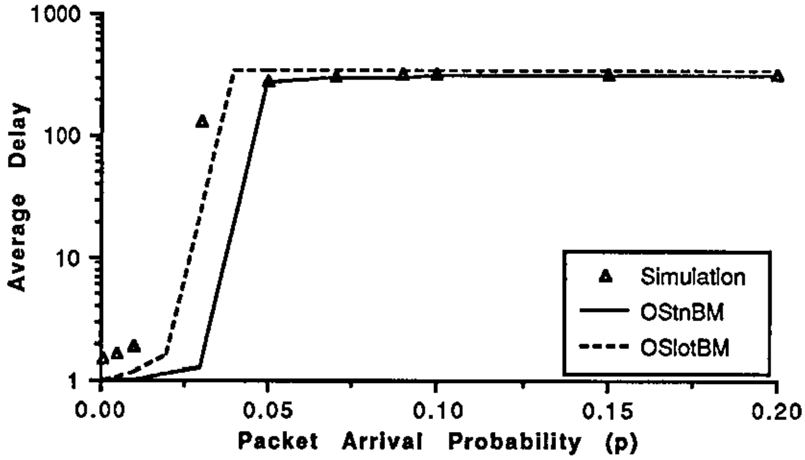


Fig. 6.4.22 Delay Performance (N=64, M=1, Q=10)

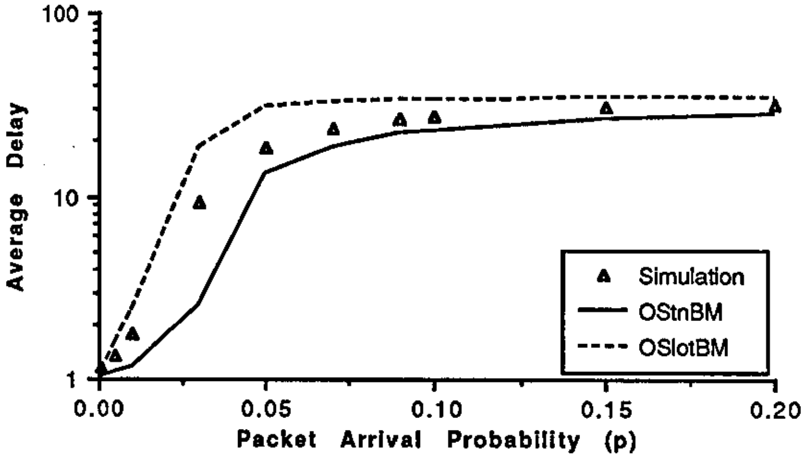


Fig. 6.4.23 Delay Performance (N=64, M=8, Q=1)

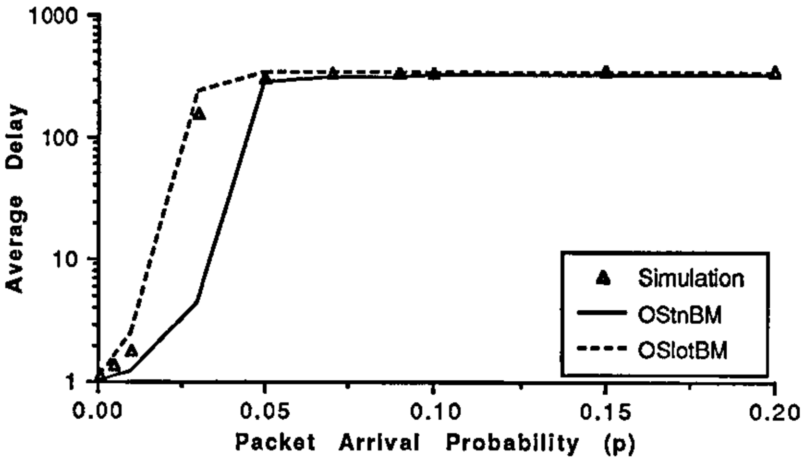


Fig. 6.4.24 Delay Performance (N=64, M=8, Q=10)

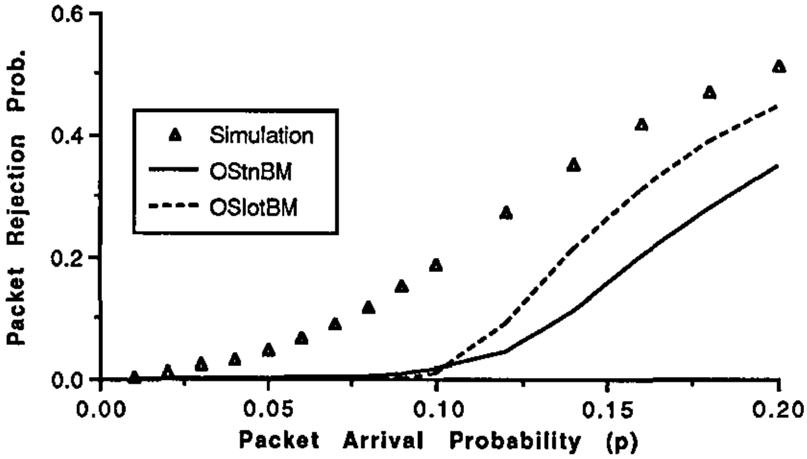


Fig. 6.4.25 Buffer Overflow Characteristics (N=16, M=1, Q=1)

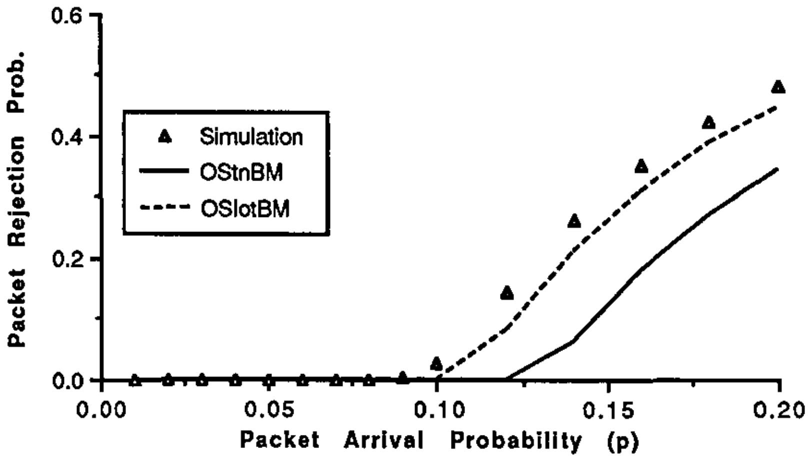


Fig. 6.4.26 Buffer Overflow Characteristics (N=16, M=1, Q=10)

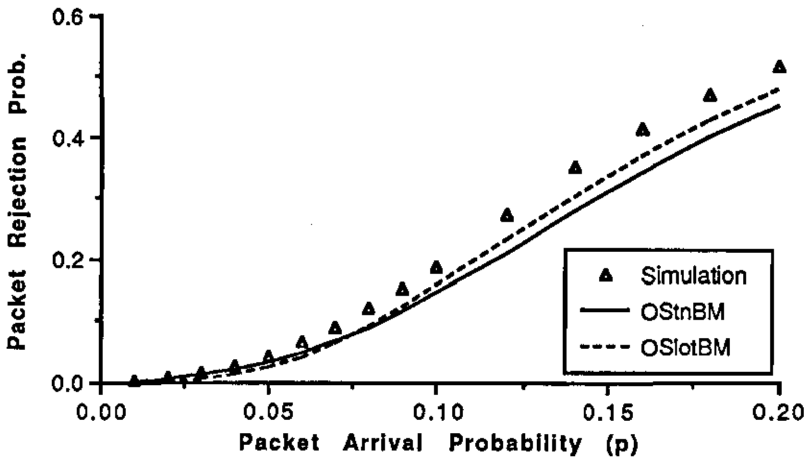


Fig. 6.4.27 Buffer Overflow Characteristics (N=16, M=8, Q=1)

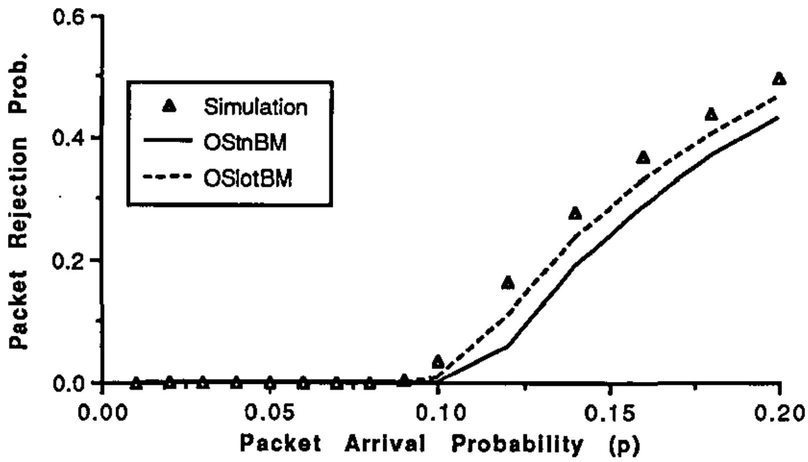


Fig. 6.4.28 Buffer Overflow Characteristics (N=16, M=8, Q=10)

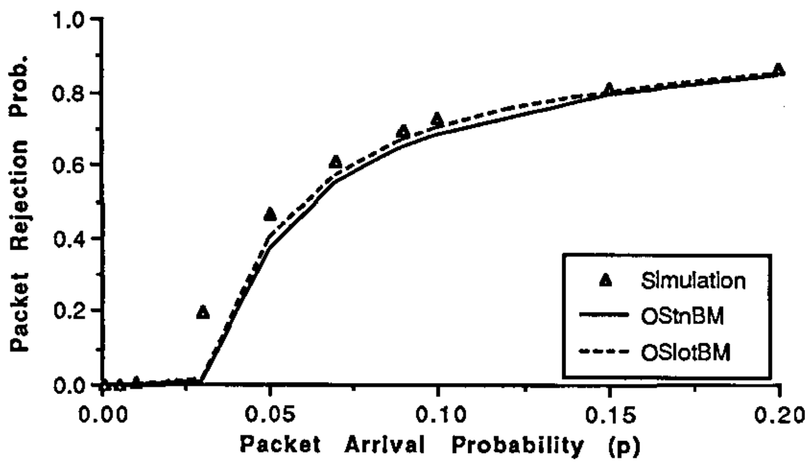


Fig. 6.4.29 Buffer Overflow Characteristics (N=64, M=1, Q=1)

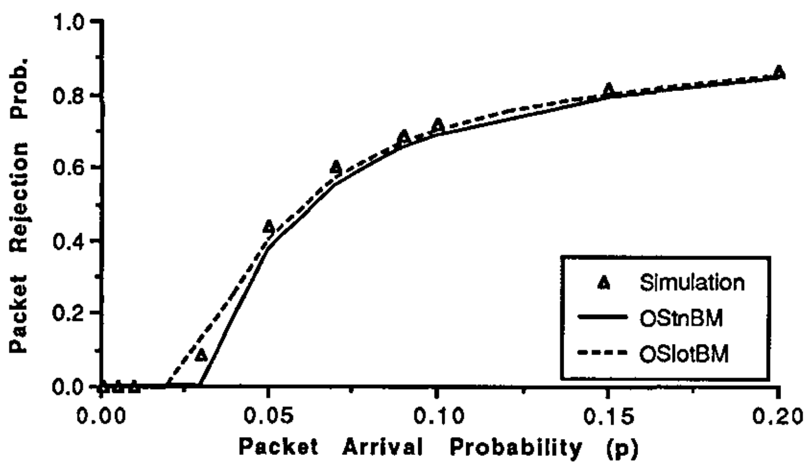


Fig. 6.4.30 Buffer Overflow Characteristics (N=64, M=1, Q=10)

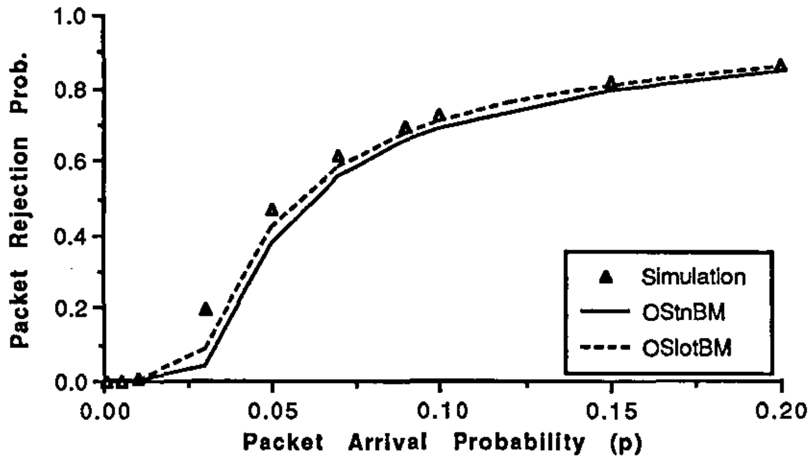


Fig. 6.4.31 Buffer Overflow Characteristics (N=64, M=8, Q=1)

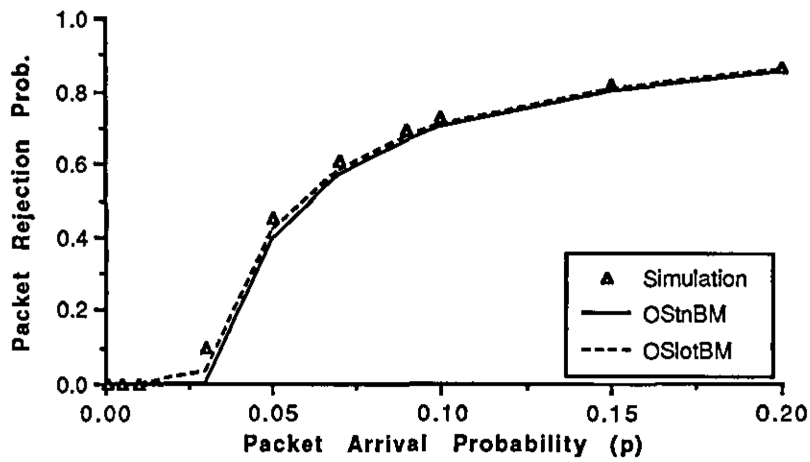


Fig. 6.4.32 Buffer Overflow Characteristics (N=64, M=8, Q=10)

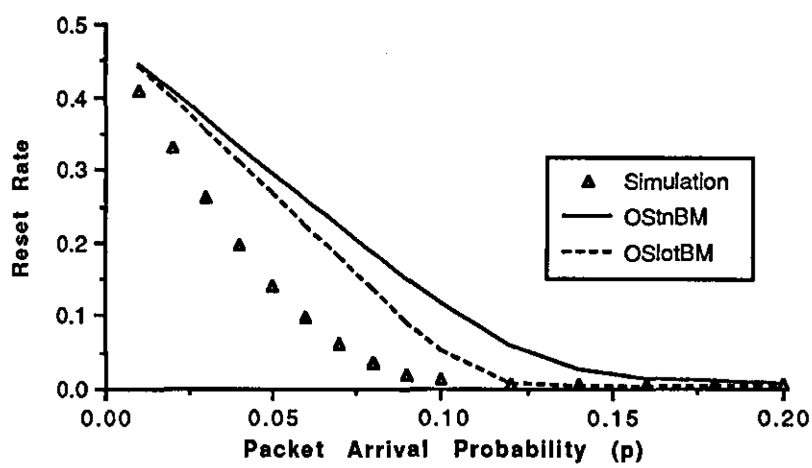


Fig. 6.4.33 Network Reset Characteristics (N=16, M=1, Q=1)

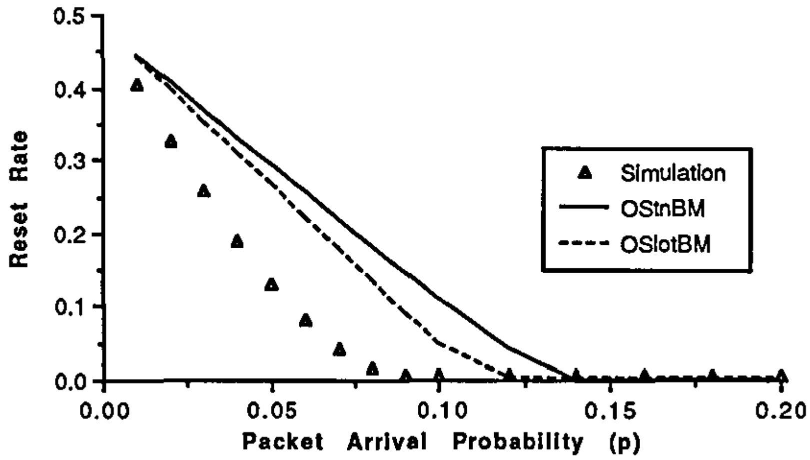


Fig. 6.4.34 Network Reset Characteristics (N=16, M=1, Q=10)

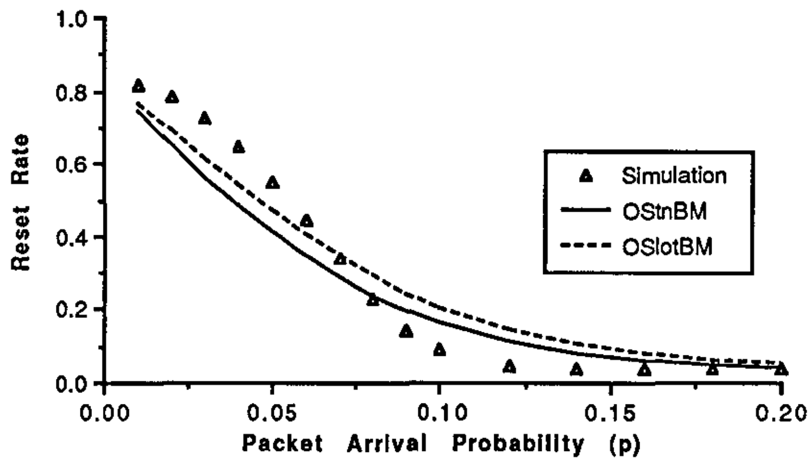


Fig. 6.4.35 Network Reset Characteristics (N=16, M=8, Q=1)

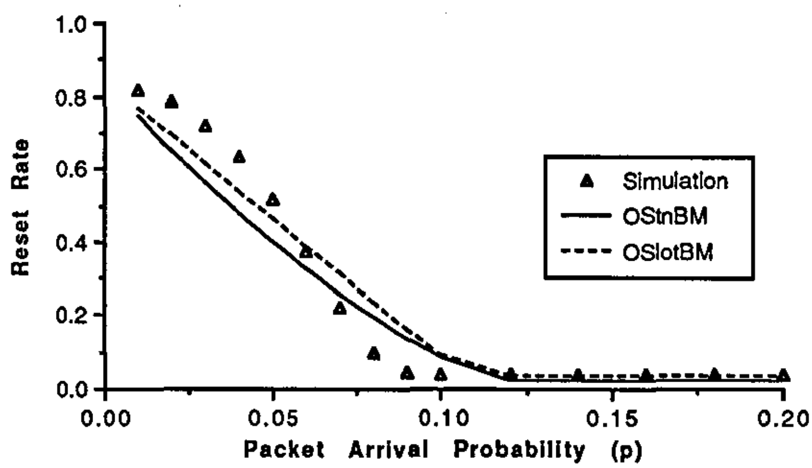


Fig. 6.4.36 Network Reset Characteristics (N=16, M=8, Q=10)

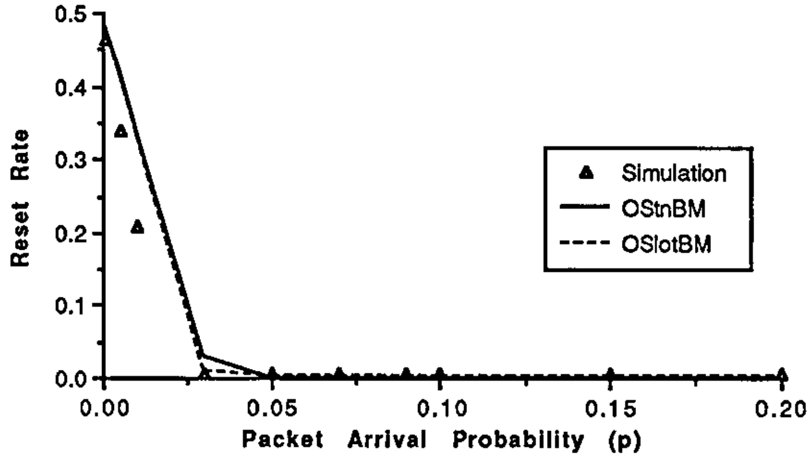


Fig. 6.4.37 Network Reset Characteristics (N=64, M=1, Q=1)

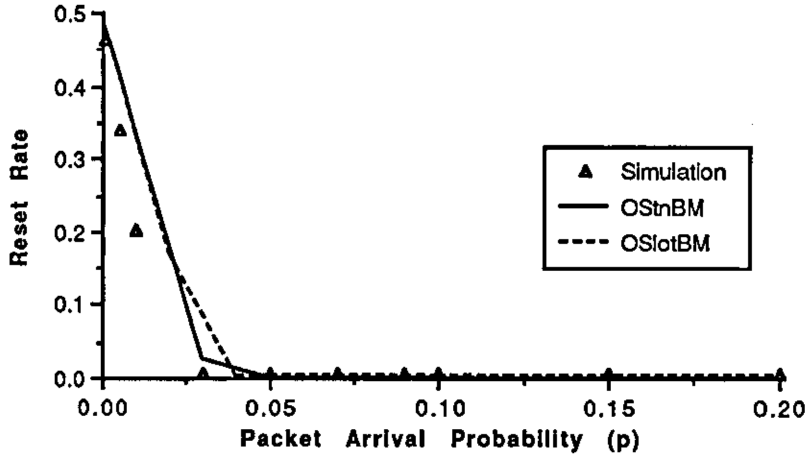


Fig. 6.4.38 Network Reset Characteristics (N=64, M=1, Q=10)

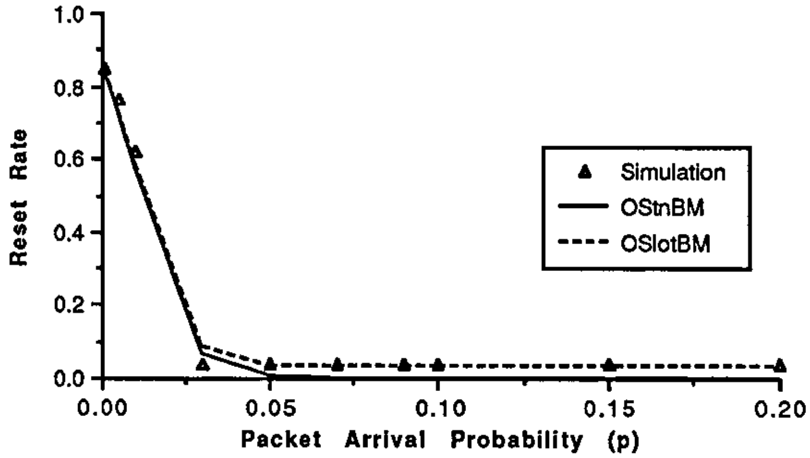


Fig. 6.4.39 Network Reset Characteristics (N=64, M=8, Q=1)

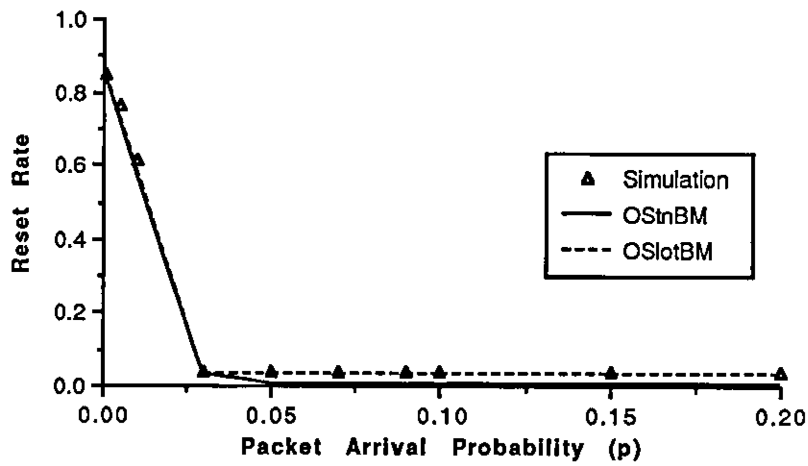


Fig. 6.4.40 Network Reset Characteristics (N=64, M=8, Q=10)

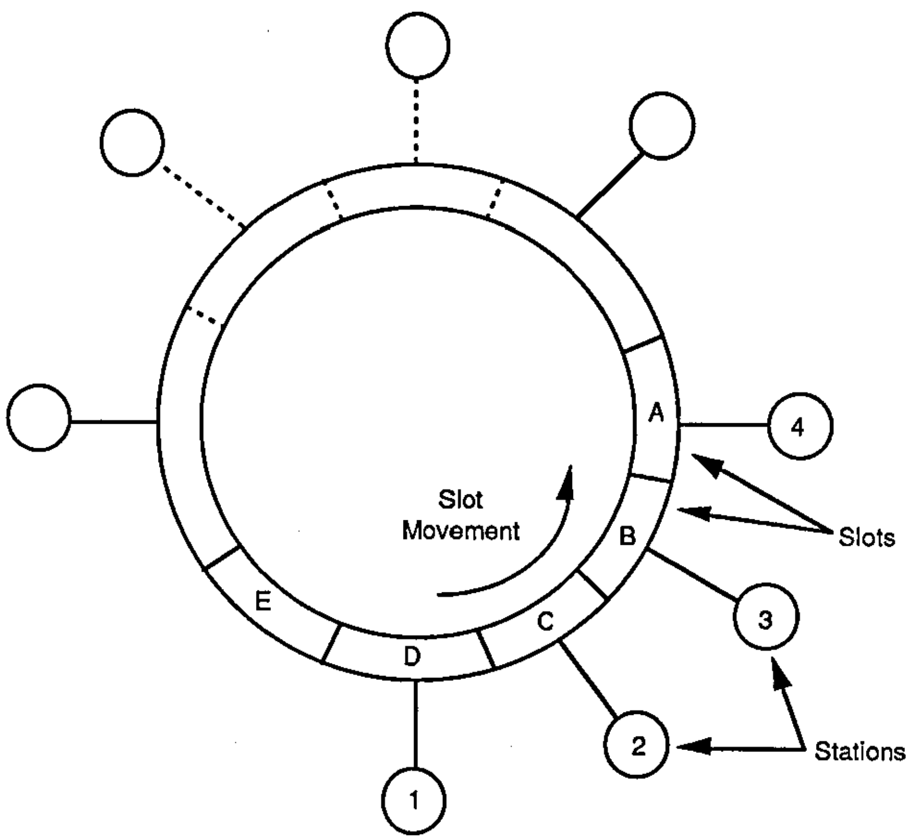


Fig. 6.4.41

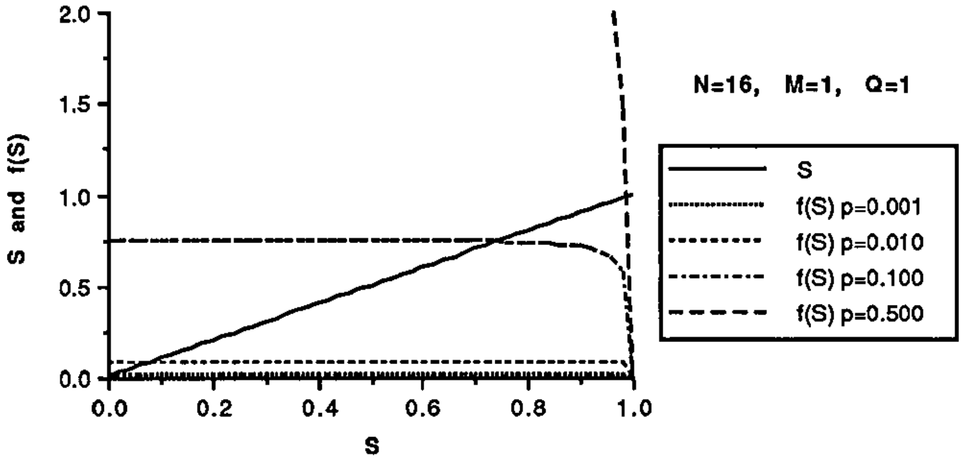


Fig. 6.4.42 Convergence Characteristics of OStnBM

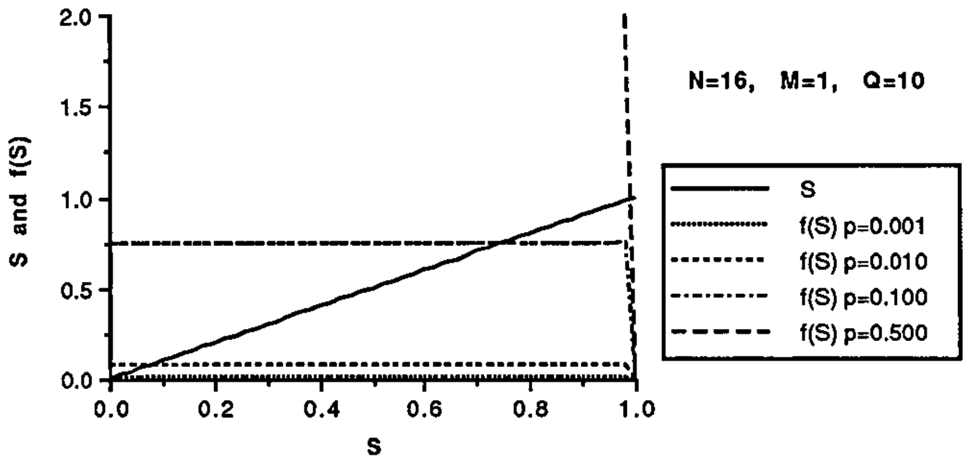


Fig. 6.4.43 Convergence Characteristics of OStnBM

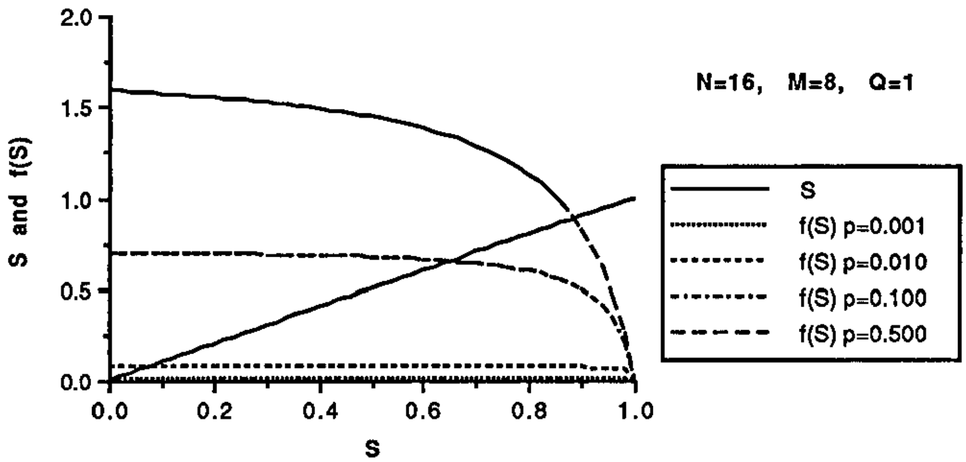


Fig. 6.4.44 Convergence Characteristics of OStnBM

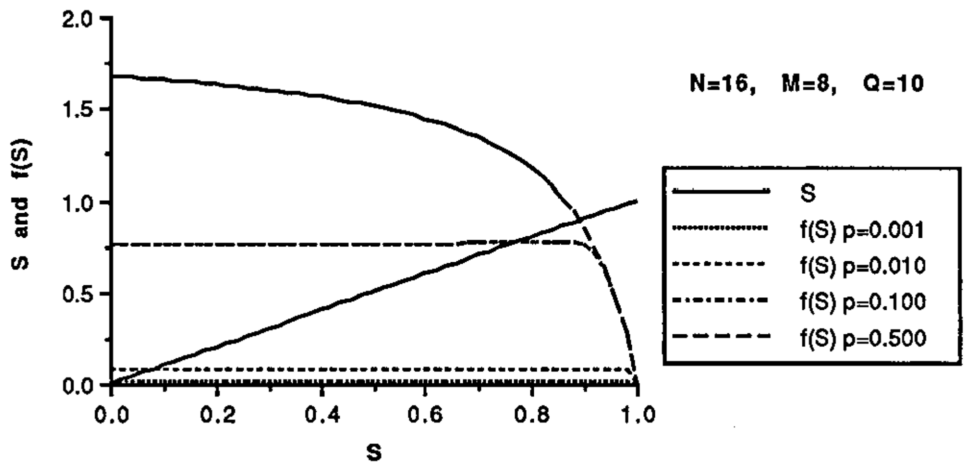


Fig. 6.4.45 Convergence Characteristics of OStnBM

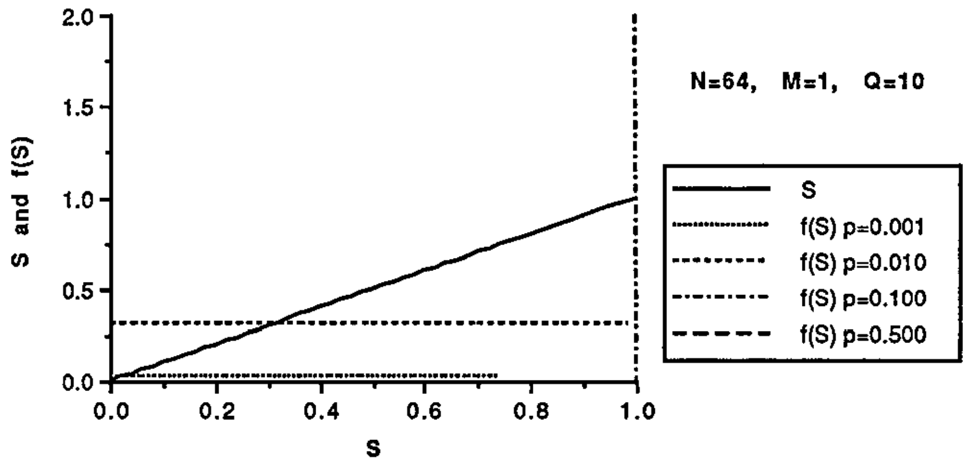


Fig. 6.4.46 Convergence Characteristics of OStnBM

CHAPTER 7

Modelling of Nonidentical, Independent Stations

7.0 Introduction

Although it has been demonstrated that the Equilibrium Point Analysis (EPA) method (Tasaka 86), widely used to model time slotted multiple access protocols including all the models in this thesis, can give excellent results when compared with simulations if the modelled protocol is stable, a major restriction associated with the method in the form presented by Tasaka is that the network's stations must be statistically identical. The purpose of this chapter is to show that, for station based models, by reformulating the EPA to reduce to a recursion instead of a fixed point equation, or by extending the equation which considers the effect the other stations have upon the station being modelled, the restriction of identical users can be relaxed without otherwise impairing the performance results to any noticeable extent.

In order to demonstrate the generality of the methods, they will be applied to the Slotted Ring (source deletion) network and the Slotted Aloha network in the sections 7.1, 7.4, 7.3 and 7.5. Section 7.2 is used to develop a station based model for the Slotted Aloha protocol which is utilized in the later sections to generate the nonidentical station modelling methods.

A general, more formal recursive model based on queuing networks is presented in Woodward 93.

7.1 Slotted Ring Recursive Model (SRRM)

In order to simplify the explanation, an unbuffered model based upon SDStnBM1 will be used. However, as it will become apparent, the method may be applied to any station based model with the stations having differing packet arrival rates, buffer sizes, etc.

As usual, consider a network of N stations with M slots, where $N \geq M$. Also, for a station i $\{1 \leq i \leq N\}$, let,

p_i = Packet arrival probability per slot time.

S'_i = Probability with which the next passing slot would appear full to station i when that station is not transmitting.

$P(x)_i$ = Steady state probability of the station occupying state "x".

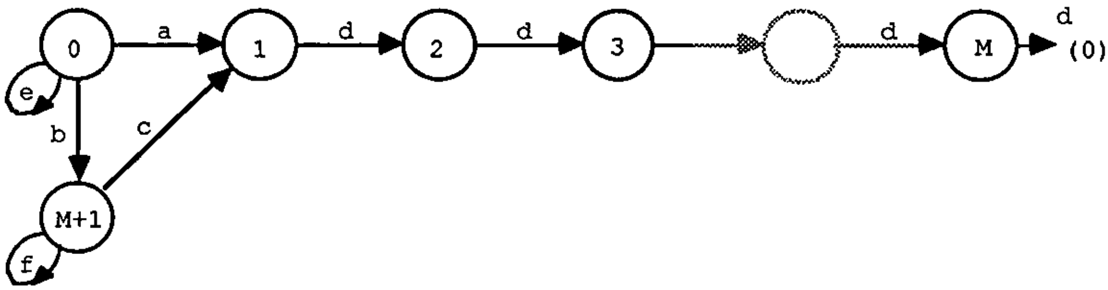
The assumptions (2.0.1), (2.0.2), (2.0.4), (2.0.6) to (2.0.12), (2.1.1) to (2.1.3) will be applicable for this model.

Then, the state transition diagram will be of the form Fig. 7.1.1.

Solving the system for steady state in the usual manner, the following equations may be obtained -

$$P(0)_i = \frac{1}{1 + Mp_i + \left(\frac{S'_i}{1-S'_i}\right)p_i} \tag{7.1.1}$$

$$P(1)_i = P(2)_i = \ . \ . \ . \ = P(M)_i = P(0)_i p_i \tag{7.1.2}$$



where - $a = p_i(1 - S'_i)$ $b = p_i S'_i$ $c = 1 - S'_i$

$d = 1$ $e = 1 - p_i$ $f = S'_i$

Fig 7.1.1 Discrete Time Markov model based on a Station for a Slotted Ring with a Source Deletion Protocol

$$P(M+1)_i = P(0)_i \, p_i \left(\frac{S_i'}{1-S_i'} \right) \tag{7.1.3}$$

Since station i will not see itself occupying a passing slot, such a slot can only contain packets from the other N-1 stations.

Thus, for a K station network, if

$$S_{1,2,\ldots K}(K)$$

is the throughput due to all the stations 1,2,...K, and,

$$S_{1,2,\ldots,K}^{(i)}(K)$$

the throughput contribution of station i, then,

$$S_i' = S_{1,2,\ldots,i-1,i+1,\ldots,K}(K-1) \tag{7.1.4}$$

and,

$$S_{1,2,\ldots,K}(K) = \sum_{i=1}^{K} S_{1,2,\ldots,K}^{(i)}(K) \tag{7.1.5}$$

Also,

$$\begin{aligned} \textit{total network throughput} &= S_{1,2,\ldots,N}(N) \\ &= \sum_{i=1}^{N} S_{1,2,\ldots N}^{(i)}(N) \end{aligned} \tag{7.1.6}$$

From equations (7.1.1) and (7.1.2),

$$\begin{aligned} S_{1,2,\ldots,K}^{(i)}(K) &= P(1)_i \\ &= \frac{1}{\frac{1}{p_i} + M + \frac{S_{1,2,\ldots,i-1,i+1,\ldots,K}(K-1)}{1-S_{1,2,\ldots,i-1,i+1,\ldots,K}(K-1)}} \end{aligned} \tag{7.1.7}$$

This is in recursive form; thus defining

$$S(0) = 0 \tag{7.1.8}$$

the following sequence may be used to calculate the throughput contribution of each station and the total throughput.

$S_i(1)$ for ${}^NC_1$ combinations of $i \in \{1,2,..N\}$

$S_{i,j}(2)$ for ${}^NC_2$ combinations of $i,j \in \{1,2,..N\}$

$S_{i,j,k}(3)$ for ${}^NC_3$ combinations of $i,j,k \in \{1,2,..N\}$

. . . . .

$S_{1,2,..N}(N)$ for the single ${}^NC_N = 1$ combination. (network throughput)

For example, let N=4. Then, using the above sequence, and equations (7.1.5), (7.1.7) and (7.1.8),

$$S_1(1) = \frac{1}{\frac{1}{p_1} + M}$$

$$S_2(1) = \frac{1}{\frac{1}{p_2} + M}$$

$$S_3(1) = \frac{1}{\frac{1}{p_3} + M}$$

$$S_4(1) = \frac{1}{\frac{1}{p_4} + M}$$

$$S_{1,2}(2) = \frac{1}{\frac{1}{p_1}+M+\frac{S_2(1)}{1-S_2(1)}} + \frac{1}{\frac{1}{p_2}+M+\frac{S_1(1)}{1-S_1(1)}}$$

$$S_{1,3}(2) = \frac{1}{\frac{1}{p_1}+M+\frac{S_3(1)}{1-S_3(1)}} + \frac{1}{\frac{1}{p_3}+M+\frac{S_1(1)}{1-S_1(1)}}$$

$$S_{2,4}(2) = \frac{1}{\frac{1}{p_2}+M+\frac{S_4(1)}{1-S_4(1)}} + \frac{1}{\frac{1}{p_4}+M+\frac{S_2(1)}{1-S_2(1)}}$$

$$S_{1,4}(2) = \ldots$$

$$S_{2,3}(2) = \ldots$$

$$S_{3,4}(2) = \ldots$$

$$S_{1,2,3}(3) = \frac{1}{\frac{1}{p_1}+M+\frac{S_{2,3}(2)}{1-S_{2,3}(2)}} + \frac{1}{\frac{1}{p_2}+M+\frac{S_{1,3}(2)}{1-S_{1,3}(2)}} + \frac{1}{\frac{1}{p_3}+M+\frac{S_{1,2}(2)}{1-S_{1,2}(2)}}$$

$$S_{1,2,4}(3) = \frac{1}{\frac{1}{p_1}+M+\frac{S_{2,4}(2)}{1-S_{2,4}(2)}} + \frac{1}{\frac{1}{p_2}+M+\frac{S_{1,4}(2)}{1-S_{1,4}(2)}} + \frac{1}{\frac{1}{p_4}+M+\frac{S_{1,2}(2)}{1-S_{1,2}(2)}}$$

$$S_{1,3,4}(3) = \ldots$$

$$S_{2,3,4}(3) = \ldots$$

$$S_{1,2,3,4}(4) = \frac{1}{\frac{1}{p_1}+M+\frac{S_{2,3,4}(3)}{1-S_{2,3,4}(3)}} + \frac{1}{\frac{1}{p_2}+M+\frac{S_{1,3,4}(3)}{1-S_{1,3,4}(3)}} + \frac{1}{\frac{1}{p_3}+M+\frac{S_{1,2,4}(3)}{1-S_{1,2,4}(3)}} + \frac{1}{\frac{1}{p_4}+M+\frac{S_{1,2,3}(3)}{1-S_{1,2,3}(3)}}$$

As it may be seen, as long as the throughput contribution of any station i to a K station network is calculated using the condition that, at the time of transmission it only sees the throughput of the other K-1 stations, it is correct to assume that the sum of all such partial throughputs represent the total utilization of the slots by those K stations.

Once the final throughput contribution of station $i \in \{1..N\}$,

$$(Throughput)_i = S^{(i)}_{1,2,..N}(N) \tag{7.1.9}$$

is known, the other performance measures of that station may be obtained as follows -
from equations (7.1.1) and (7.1.3),

$$P(M+1)_i = \frac{p_i\left(\dfrac{S_i^{'}}{1-S_i^{'}}\right)}{1 + Mp_i + \left(\dfrac{S_i^{'}}{1-S_i^{'}}\right)p_i}$$

$$= \frac{1}{\left(\dfrac{1}{p_i} + M\right)\left(\dfrac{1}{S_i^{'}} - 1\right) + 1}$$

$$\textit{Average queue of } i^{th} \textit{ station } (\overline{q}_i) = P(M+1)_i \tag{7.1.10}$$

$$\begin{matrix}\textit{Average delay for} \\ i^{th} \textit{ station } (\overline{d}_i)\end{matrix} = \frac{\overline{q}_i}{(Throughput)_i} \tag{7.1.11}$$

Using equation (7.1.2) and (7.1.7),

$$\begin{matrix}\textit{Packet Rejection Probability} \\ \textit{of station } i \ (PRP)_i\end{matrix} = \frac{Mp_i - M(Throughput)_i}{Mp_i} \tag{7.1.12}$$

$$= 1 - P(0)_i$$

the value of $P(0)_i$ being substituted from equation (7.1.1).

This iterative process may be carried out for any of the station based models to obtain their performance with regard to stations having varying parameters such as the buffer size, packet arrival probabilities, transmit back-off probabilities, etc.

The main difficulty that arises when carrying out the recursion in a normal (non parallel) computer is that, for large numbers of stations, the amount of combinations become far too excessive to give a result within a reasonable time. This becomes even more pronounced if a matrix solving method which considers a large number of states is to be used.

7.2 Slotted Aloha Station Based Model (SAStnBM)

In this section, the familiar model for the Slotted Aloha protocol by Tasaka (Tasaka 86) will be used to create a model based on observing the network from a station. The purpose of this exercise is to have a model that can be later modified to accommodate nonidentical stations. However, we first assume that all stations are identical.

To avoid complicating this with unnecessary parameters, the assumption is made of N identical unbuffered stations and a channel having zero propagation delay. This, as shown in figure 7.2.1, results in a simple two-node model for any given station in the network.

The nodes 1 and 2 of the figure respectively show the idle and the retransmission states a station may occupy. Thus, a station in state 1 can generate a new packet (which implies a transmission attempt) with probability σ per slot, with packets assumed to be generated at the beginning of a slot. A station in state 2 has a packet waiting for retransmission, and it will attempt this retransmission with

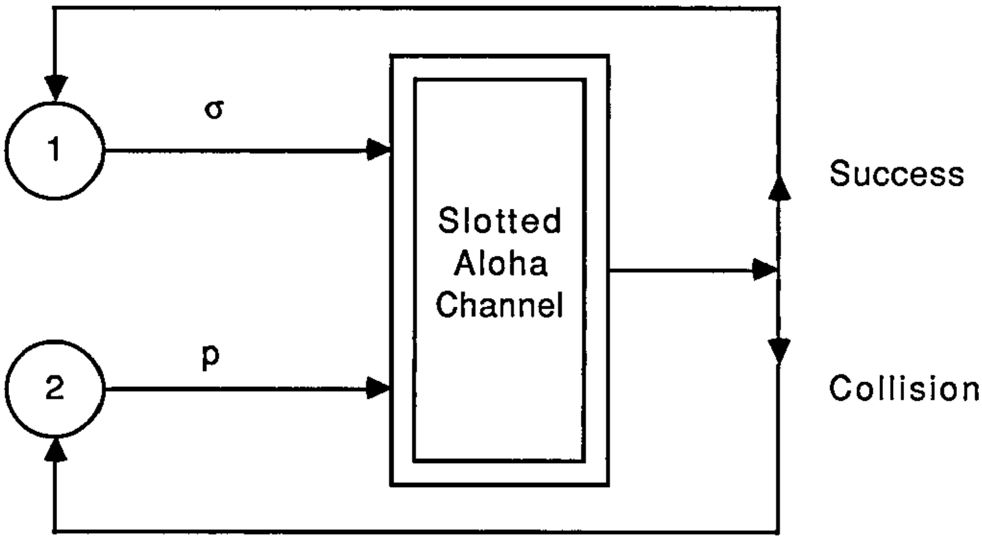


Fig .7.2.1 Discrete-time queueing network model based on a station for the Slotted Aloha

probability p per slot. If exactly one station attempts a transmission from node 1 or a retransmission from node 2 in a slot, this is successful, and the corresponding station jumps to node 1 at the end of the slot. If two or more transmissions or retransmissions are attempted in a slot, the corresponding users are involved in a collision and jump to node 2 at the end of the slot. If no transmissions or retransmissions are attempted in a slot, the users do not jump, but remain at their respective nodes.

If,

T = Probability that at least one of the other N-1 stations will attempt to transmit in the next slot,

m = Probability that the station is idle,

considering the steady state equilibrium of node 1 and using the fact that the station must always occupy one of the two nodes,

$$m\sigma = [m\sigma + (1-m)p]\,(1-T)$$

which simplifies to -

$$m = \frac{1}{\dfrac{\sigma T}{p(1-T)}+1} \qquad (7.2.1)$$

Since,

$$\mathit{Probability\ of\ a\ station\ not\ attempting\ to\ transmit} = 1 - m\sigma - (1-m)p$$

$$T = 1 - [1 - m\sigma - (1-m)p]^{N-1} \qquad (7.2.2)$$

The iteration of equation (7.2.1) may be solved for m by substituting for T from equation (7.2.2).

Then,

$$Throughput = m\sigma N \tag{7.2.3}$$

$$AverageQueue(\overline{q}) = 1-m \tag{7.2.4}$$

$$AverageDelay(\overline{d}) = \frac{1-m}{m\sigma}+1 \tag{7.2.5}$$

7.3 Slotted Aloha Recursive Model (SARM)

This model will make use of SAStnBM in developing the recursive algorithm that can take into consideration stations which are nonidentical, but independent.

Figure 7.2.1 is applicable for this model as well. In this case however, σ and p should be replaced by σ_i and p_i respectively, the subscript i indicating the relevant parameter of station i, where $1 \leq i \leq N$.

For a K station network, let,

$T_{1,2..K}(K)$ = Probability that at least one of the stations 1, 2,...K will attempt to transmit in the next slot,

$m^i_{1,2..K}(K)$ = Probability of station i being in the idle state,

$S^i_{1,2..K}(K)$ = Throughput contribution of station i.

Considering a single station of the network, since the probability of a station occupying nodes 1 or 2 is unity,

$$S^i_{1,2,\dots,K}(K) = \left\{m^i_{1,2,\dots,K}(K)\,\sigma_i + [1-m^i_{1,2,\dots,K}(K)]\,p_i\right\} * \left\{1-T_{1,2,\dots,i-1,i+1,\dots K}(K-1)\right\} \tag{7.3.1}$$

At steady state,

$$S^i_{1,2,..K}(K) = m^i_{1,2,..K}(K)\ \sigma_i \tag{7.3.2}$$

From equations (7.3.1) and (7.3.2),

$$m^i_{1,2,\ldots,K}(K) = \frac{1}{\left\{\dfrac{\sigma_i\ T_{1,2,..i-1,i+1,..K}(K-1)}{p_i\ (1-T_{1,2,..i-1,i+1,..K}(K-1)} + 1\right\}} \tag{7.3.3}$$

Also, for a K station network,

$$\begin{aligned}\begin{matrix}\textit{Probability of}\\ \textit{station i not}\\ \textit{transmitting}\end{matrix} &= 1 - m^i_{1,2,..K}(K)\ \sigma_i - \left[1 - m^i_{1,2,..K}(K)\right]p_i \\ &= 1 - m^i_{1,2,..K}(K)\left[\sigma_i - p_i\right] - p_i \end{aligned} \tag{7.3.4}$$

thus,

$$T_{1,2,..K}(K) = 1 - \prod_{i=1}^{K}\left\{1 - m^i_{1,2,..K}(K)\ (\sigma_i - p_i) - p_i\right\} \tag{7.3.5}$$

Equations (7.3.3) and (7.3.5) may now be alternately solved for increasing population levels by using the condition that -

$$T(0) = 0 \tag{7.3.6}$$

For example, consider three station network. Then, using equation (7.3.6),

$$m^1_1(1) = 1$$

$$m^2_2(1) = 1$$

$$m^3_3(1) = 1$$

$$T_1(1) = m_1^1(1)(\sigma_1 - p_1) + p_1$$

$$T_2(1) = m_2^2(1)(\sigma_2 - p_2) + p_2$$

$$T_3(1) = \ldots\ldots$$

$$m_{1,2}^1(2) = \frac{1}{\left\{\dfrac{\sigma_1\, T_2(1)}{p_1[1-T_2(1)]} + 1\right\}}$$

$$m_{1,2}^2(2) = \frac{1}{\left\{\dfrac{\sigma_2\, T_1(1)}{p_2[1-T_1(1)]} + 1\right\}}$$

$$m_{1,3}^1(2) = \ldots\ldots$$

$$m_{1,3}^3(2) = \ldots\ldots$$

$$m_{2,3}^2(2) = \ldots\ldots$$

$$m_{2,3}^3(2) = \ldots\ldots$$

$$T_{1,2}(2) = 1-\left\{1-m_{1,2}^1(2)\,[\sigma_1-p_1]-p_1\right\}\left\{1-m_{1,2}^2(2)\,[\sigma_2-p_2]-p_2\right\}$$

$$T_{1,3}(2) = 1-\left\{1-m_{1,3}^1(2)\,[\sigma_1-p_1]-p_1\right\}\left\{1-m_{1,3}^3(2)\,[\sigma_3-p_3]-p_3\right\}$$

$$T_{2,3}(2) = \ldots\ldots$$

$$m_{1,2,3}^1(3) = \frac{1}{\left\{\dfrac{\sigma_1\, T_{2,3}(2)}{p_1[1-T_{2,3}(2)]} + 1\right\}}$$

$$m_{1,2,3}^2(3) = \frac{1}{\left\{\dfrac{\sigma_2\, T_{1,3}(2)}{p_2[1-T_{1,3}(2)]} + 1\right\}}$$

$$m_{1,2,3}^3(3) = \ldots\ldots$$

Then, since,

$$\text{Throughput contribution of station } i = m^{i}_{1,2,..N}(N)\ \sigma_i$$

the total throughput is given by,

$$S_{1,2,..N}(N) = \sum_{i=1}^{N} m^{i}_{1,2,..N}(N)\ \sigma_i \qquad (7.3.7)$$

Also,

$$\text{Average queue of } i^{th} \text{ station } (\overline{q}_i) = 1 - m^{i}_{1,2,..N}(N) \qquad (7.3.8)$$

$$\text{Average delay for } i^{th} \text{ station } (\overline{d}_i) = \frac{\overline{q}_i}{(\text{Throughput})_i} + 1 \qquad (7.3.9)$$

$$\text{Packet Rejection Probability of } i^{th} \text{ station given packet arrival } (PRP)_i = \left[1 - m^{i}_{1,2,..N}(N)\right]\sigma_i \qquad (7.3.10)$$

7.4 Slotted Ring Extended Model (SREM)

In the previous chapters, all the models that were based on a station assumed that a station in a non transmitting state would see the next passing slot full with the probability -

$$S = \frac{N-1}{M} \times \text{Probability that the station is transmitting} \qquad (7.4.1)$$

This was due to the fact that, since all stations were considered identical, the throughput contribution of each station to the network is the same. Thus with the stations assumed to behave independently, the summation of the contributions of the other N-1 stations on a per slot basis resulted in the above equation.

However, if for a station i $\{1 \leq i \leq N\}$,

S_i = Probability that the station is occupying a given slot.

S'_i = Probability with which the next passing slot would appear full to station i when that station is not transmitting.

then, for a station r, the equation (7.4.1) can be extended to give -

$$S'_r = S_1 + S_2 + \ldots + S_{r-1} + S_{r+1} + \ldots + S_N \qquad (7.4.2)$$

This equation can now be converted into an iterative form as shown below.

Let $S_i(n)$ = The value of S_i after the n^{th} iteration.

Since

$$S_i = f(S'_i) \qquad (7.4.3)$$

which can be solved for the given model, by making initial guesses for $S_i(0)$ of all stations and using equations (7.4.2) and (7.4.3), the following sequence of calculations may be used to solve the system.

$$S_1(1) = f(S_2(0) + S_3(0) + \ldots + S_N(0))$$

$$S_2(1) = f(S_1(1) + S_3(0) + \ldots + S_N(0))$$

$$\ldots\ldots$$

$$S_r(1) = f(S_1(1) + S_2(1) + \ldots + S_{r-1}(1) + S_{r+1}(0) + \ldots + S_N(0))$$

$$S_1(2) = f(S_2(1) + S_3(1) + \ldots + S_N(1))$$

$$S_2(2) = f(S_1(2) + S_3(1) + \ldots + S_N(1))$$

$$\ldots\ldots$$

$$S_r(2) = f(S_1(2) + S_2(2) + \ldots + S_{r-1}(2) + S_{r+1}(1) + \ldots + S_N(1))$$

This procedure is continued until, for all stations,

$$|S_r(n) - S_r(n-1)| \leq Maximum\,Allowable\,Error \qquad \{1 \leq r \leq N\}$$

Once all the S_r values are known, the other performance measures may be computed in the usual manner.

7.5 Slotted Aloha Extended Model (SAEM)

In order to illustrate the generality of the method without complicating the model with added parameters, the basic model of the Slotted Aloha mentioned in section 7.3 will be used once again. Thus, all the similar notations in this model will have the same meaning.

Since this method considers the population level of the network to be N stations throughout the calculation, from equation (7.3.3),

$$m^i_{1,2,\ldots,K}(K) = \frac{1}{\left\{\dfrac{\sigma_i \; T_{1,2,\ldots i-1,i+1,\ldots K}(K-1)}{p_i \; (1-T_{1,2,\ldots i-1,i+1,\ldots K}(K-1)} + 1\right\}}$$

$$= m^i \qquad (say) \qquad (7.5.1)$$

But,

$$T_{1,2,\ldots,i-1,i+1,\ldots N}(N-1) = \begin{array}{l}\textit{Prob that at least one of} \\ \textit{the other } (N-1) \textit{ stations} \\ \textit{will attempt to transmit}\end{array}$$

Using equations (7.3.4),

$$T_{1,2,\ldots,i-1,i+1,\ldots N}(N-1) = 1 - \prod_{r=1,r\neq i}^{N} \{1 - m_{1,2,\ldots,N}^{r}(N)\,[\sigma_r - p_r] - p_r\}$$
$$= 1 - \prod_{r=1,r\neq i}^{N} \{1 - m^{r}[\sigma_r - p_r] - p_r\}$$
$$= T^{i} \quad (say) \tag{7.5.2}$$

To solve the model, an iteration similar to that done for SREM may be used as follows -

(1) For all stations guess initial values for m^i (say $m^i(0)$)

(2) For i=1 to N do

Calculate T^i by applying the latest values of m^i in equation (7.5.2).

Calculate new m^i using equation (7.5.1) and the new T^i.

(3) Repeat step (2) until -

$$|m^{i}(n) - m^{i}(n-1)| \leq Maximum\,Allowable\,Error \qquad i \in (1..N)$$

Once convergence is reached, the other performance measures may be calculated using the equations (7.3.2), (7.3.7) to (7.3.10).

7.6 Results

For both types of networks, initially, all stations will be assumed identical. The average results obtained from the recursive and extended models and the simulation will be compared with their respective equivalent station models using graphs.

Then, the performance measures obtained from the recursive and the extended models for each station having different packet arrival rates (and retransmission probabilities as well in the case of Slotted Aloha) will be compared in tabular form to simulation results. The column to the right of each model's performance measures show its percentage error with respect to simulation results.

Under this tabular comparison, large errors that arose due to a difference in two small numbers under consideration (e.g., 0.0001 and 0.0003 for throughput) will be ignored.

It is noted that, for the unbuffered models (Q=0), the queue size used to find the delay is considered to be the probability of a station being in a blocked or retransmission state for the models Slotted Ring and Slotted Aloha respectively. The queuing performance itself as well as the packet rejection probability have been omitted in the performance comparison to limit the amount of graphs and tables included in the thesis. However, they too provided an excellent match to the simulations whenever the delay and throughput, to which they are closely related, gave good predictions.

For the case of the extended models, two sets of results were obtained by initialising the iteration with the two possible extreme values. When the results were different, both sets have been presented. The basic models that consider identical stations have been solved by using a linear search and bisection method so as to detect any unstable equilibrium points.

Slotted Ring - Identical Station Case

Figures 7.6.1 to 7.6.12 show the relevant performance curves. SREM and SDStnBM1, as expected, give identical results for all performance measures. This is due to the fact that the extended model is equivalent to SDStnBM1 when all the stations are identical. Thus, all the observations made for SDStnBM1 with regard to the simulation in chapter 2 are applicable in this situation.

For N=16 and M=1, SRRM becomes unstable when the throughput reaches saturation, and thus, is inapplicable for higher packet arrival rates. Also, since the increase of buffer size causes the system to saturate at lower packet arrival rates, as can be expected, the instability too starts to occur at an earlier stage.

When compared to simulations, the SRRM, under stable conditions, generally show slightly improved results to those displayed by SREM.

Slotted Ring - Non Identical Stations Case

Here, the packet arrival rates were increased linearly across the stations for a given average packet arrival rate. Varying average packet arrival rates were obtained by multiplying this distribution by a constant so that this linearity was preserved. (Tables 7.6.1 to 7.6.3)

It is noted that, even though the model used to illustrate the two methods considered unbuffered stations, in this case, a constant maximum buffer size of 3 has been used, by utilizing the basic model SDStnBM1. The reason for this is that a buffered model is more representative of a real life situation and that buffered models for the Slotted Ring were readily available unlike for the Slotted Aloha.

The results obtained indicate that SRRM and SREM give excellent predictions for all performance measures when the average packet arrival rate is low. In the ranges of p and σ considered, the accuracy of SRRM (when stable) and SREM were within the range of 91% - 100% and 85% - 100% for throughput, and, 69% - 99% and 24% - 99% for delay respectively.

As in case where all stations were considered identical, here too, the SREM consistently show a higher saturation throughput to that given by the simulation, the value predicted by SRRM, when stable, generally resulting in a better comparison.

As before, the increase of packet arrival rates cause SRRM to become unstable for M=1, thus limiting the applicable range of the model for lower numbers of slots.

Slotted Aloha - Identical Station Case

To avoid having too many curves on a single graph, SAStnBM and SAEM will be initially compared to the model suggested by Tasaka (Tasaka 86). This latter model will be abbreviated as SATM (Slotted Aloha Tasaka's Model). (Figures 7.6.13 to 7.6.15 and 7.6.19 to 7.6.21)

Next, the average performance measure results of SAEM, SARM and simulations will be compared to each other. (Figures 7.6.16 to 7.6.18 and 7.6.22 to 7.6.24)

The curves show that SAStnBM and SAEM give the same results - this being due the fact that both the models become equivalent when the stations are identical. These models do not always give similar results to SATM. This is because, unlike the other models considered in this chapter, SATM is based on observing the network globally rather than from a particular station. However, both models do give identical results when the network is not in saturation, thus, conforming to a proof given in one of the references (Woodward 91a).

As it may be seen, all the models show bistable characteristics for certain values of packet arrival rates and retransmission probabilities. For the case of $\sigma=0.01$, the higher value of throughput in the bistable region is identical for all the models. SAStnBM, and SATM can also show a third (unstable) solution.

SARM always tend to predict similar or higher throughputs (i.e., similar or lower delays) with comparison to simulation results. For example, in the case where $\sigma=0.01$, SARM performance curves tend to merge with the locally stable equilibrium point curves of SAEM associated with the higher value of throughput (or lower delay). This, and the reason for unique solutions, is undoubtedly due to the fact that the starting population of all recursive models is always zero. One of the major disadvantages of SARM is its inability to predict the region over which the network is bistable.

In general, when compared with simulations, all models show an excellent match, especially for lower p and σ values.

Slotted Aloha - Non Identical Stations Case

The performance of the SARM and SAEM in the case of 16 stations, each having different p and σ values are shown in the tables 7.6.4 to 7.6.6. For the case of average packet arrival rate = 0.03, two columns have been provided for each performance parameter of SAEM to display the bistable solutions that occur within the region of retransmission probabilities considered. When left empty a unique solution is indicated.

These results are typical of the accuracy that can be achieved when a network is operating under stable conditions.

When compared to simulation in this region, the SARM, when stable, tend to give an accuracy within the range of 88% to 100% for delay and 91% to 99% for throughput. However, when the SARM tend not to follow the degradation of network performance under overload as shown in figures 7.6.16, 7.6.17, 7.6.22 and 7.6.23 the above accuracy is sharply reduced.

For SAEM, a similar comparison to above show typical accuracies of - 89% to 99% for delay, and 92% to 100% for throughput. The fact that one of the two possible results tends to follow the simulation, ensures that even under overload conditions, the method gives a reasonably good prediction provided the basic model upon which it is based also performs satisfactorily for similar average network parameters.

7.7 Summary

In this chapter, two methods for calculating the individual and overall performance of stations in a time slotted network were presented.

In the first method, the EPA method was reformulated into a recursion based upon the number of stations, which was then solved by initiating the recursion from a zero population level.

The second method consisted of replacing the total average network usage of the stations with the sum of individual network usages, which in turn allowed the effect of non identical stations to be taken into account in the form of an iteration.

One of the greatest disadvantages of the recursive model is its time requirements to reach the solution. For the case of the Slotted Ring with N stations, a total of

$$\sum_{r=1}^{N} {}^{N}C_{r}$$

combinations have to be solved. In SARM, due to the use of two operators, namely $m^{i}_{1,2,\ldots,K}(K)$ and $T_{1,2,\ldots,K}(K)$, this situation is worsened by the need to solve for a total of

$$\sum_{r=1}^{N} {}^{N}C_r + \sum_{r=1}^{N-1} {}^{N}C_r$$

combinations of the above two operators respectively.

Although the recursive algorithm has been presented in a form where each user (i.e., station) belongs to a different user class, it generalises in a rather obvious way to the case of C user classes, $1 <= C <= N$. If at a particular population level, for a given subset of users there is more than one user in the same class, then a calculation need only be carried out once for the class, since the results will be identical for all users in the same class. When C=1, then the algorithm will evaluate in a time proportional to O(N), where O(N) is a function such that O(N)/N converges to a positive number when $N \to \infty$. At the other extreme, when C=N, the time will be proportional to $O(2^N-1)$. In this worst-case situation, when using standard (non-parallel) computing facilities, the maximum number of users that can be handled has been found to be in the range of 20-25 for the given network parameters.

For the extended method, the time required to converge is proportional to the number of stations, the accuracy required and of course the rate of convergence. Under this method, both the networks reached their solution within a few (maximum 15) iterations for the network parameters considered, thus showing an improvement to the computation time upto several thousand times faster than the recursive model. Provided the algorithm converges in a finite number of iterations, time complexity in this case is O(N).

As it can be seen from the comparison of identical stations with Tasaka's model, the errors of the extended method were mainly due to the limitations of the basic models that were used rather than the algorithm itself. When the network was operating under stable conditions, this observation is also applicable to the recursive method.

It can be concluded that, when the network is stable, both algorithms give excellent predictions of performance for individual stations as well as the overall network.

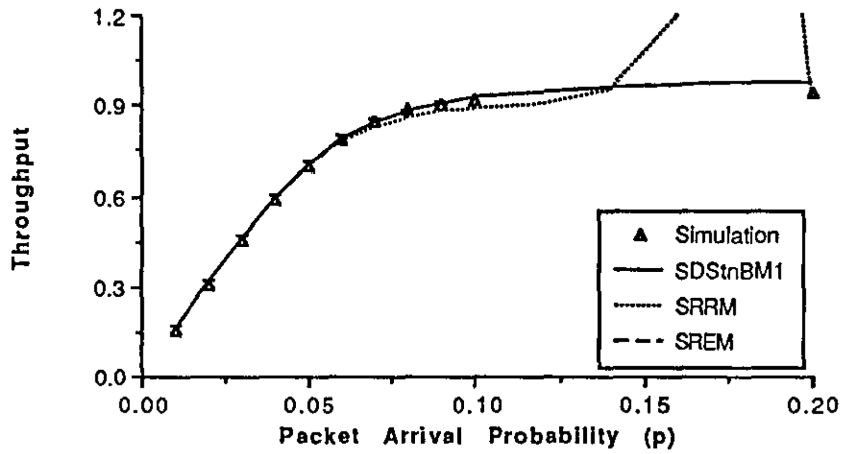


Fig. 7.6.1 Throughput Performance (N=16, M=1, Q=0)

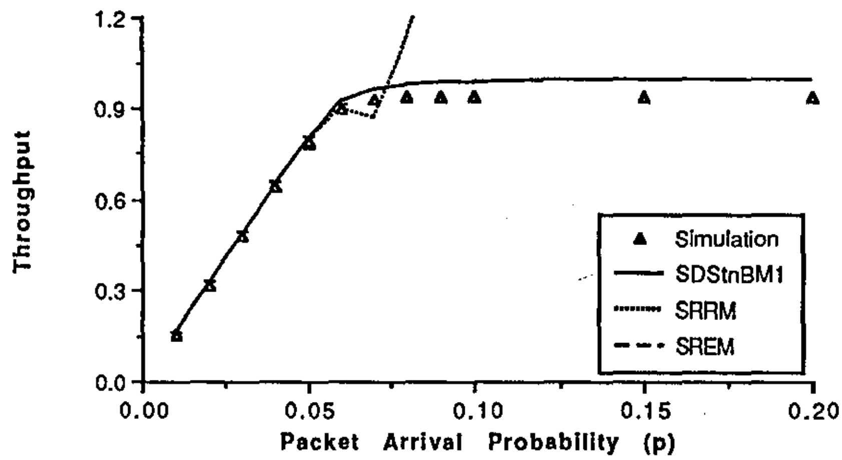


Fig. 7.6.2 Throughput Performance (N=16, M=1, Q=3)

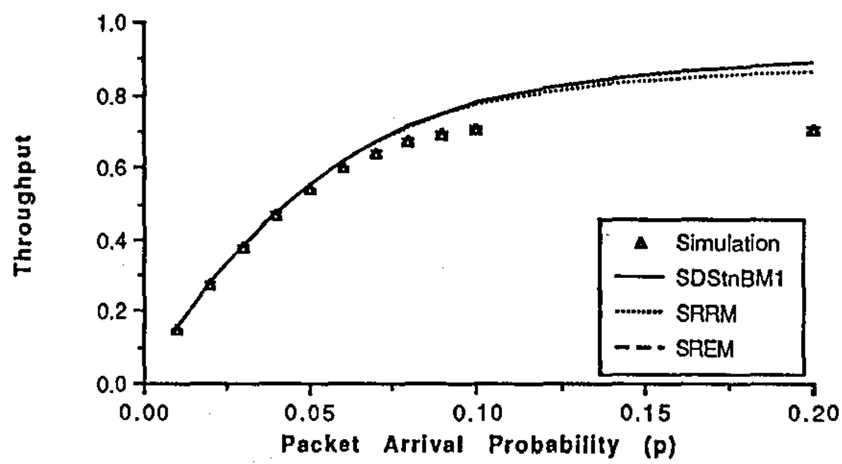


Fig. 7.6.3 Throughput Performance (N=16, M=8, Q=0)

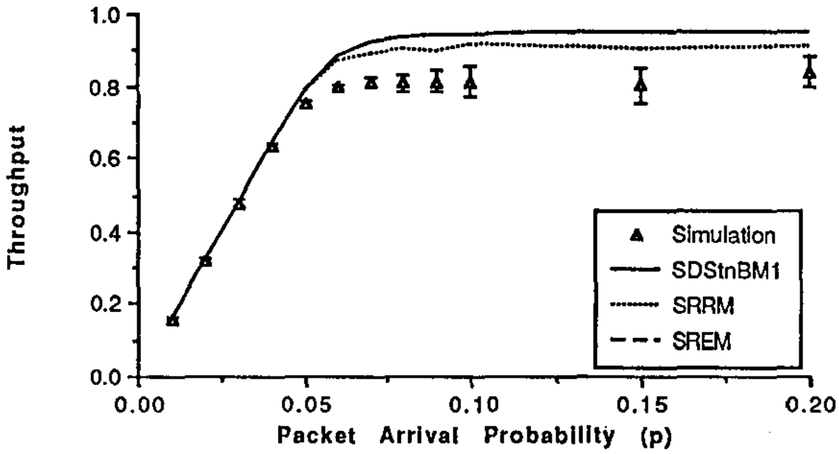


Fig. 7.6.4 Throughput Performance (N=16, M=8, Q=3)

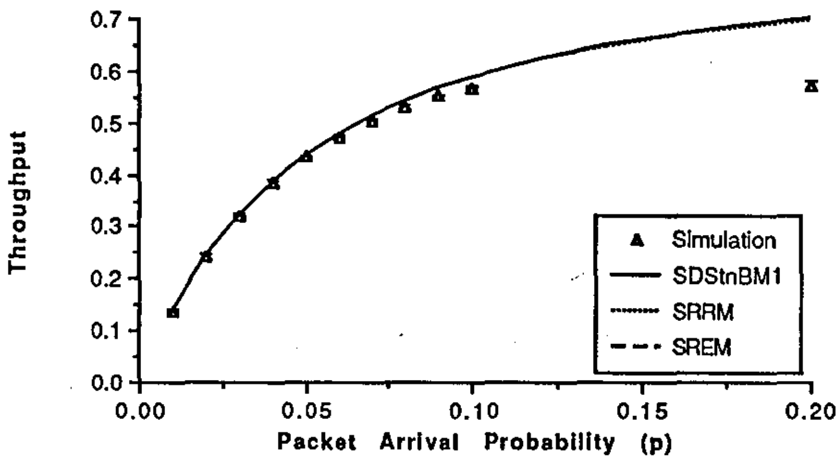


Fig. 7.6.5 Throughput Performance (N=16, M=16, Q=0)

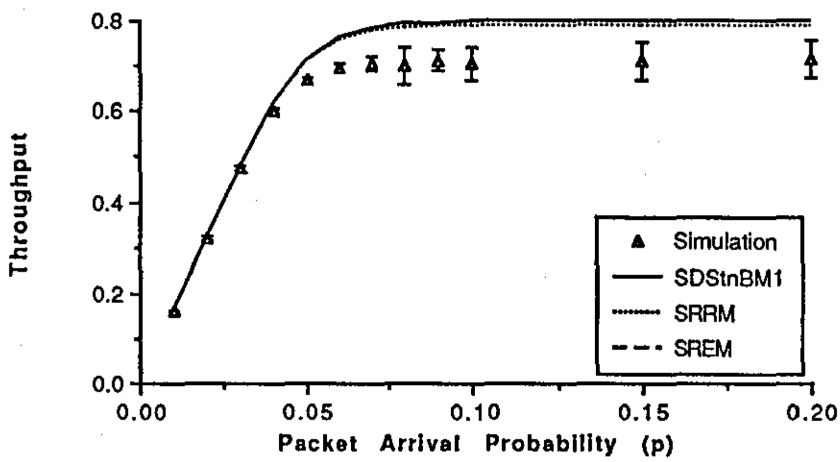


Fig. 7.6.6 Throughput Performance (N=16, M=16, Q=3)

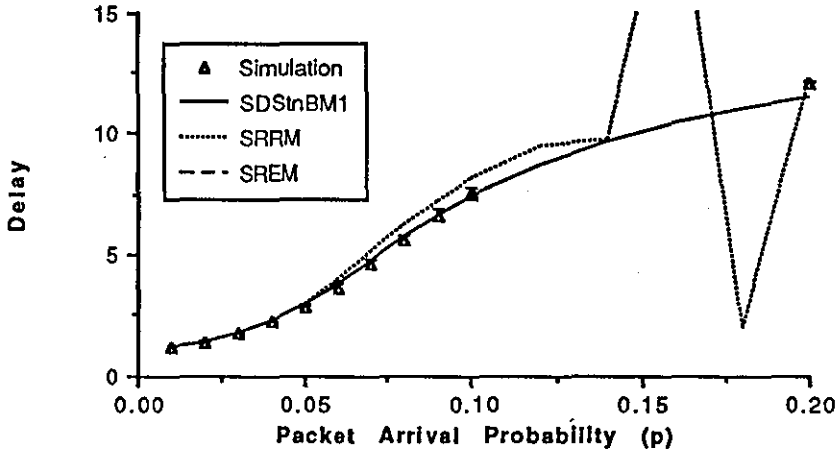


Fig. 7.6.7 Delay Performance (N=16, M=1, Q=0)

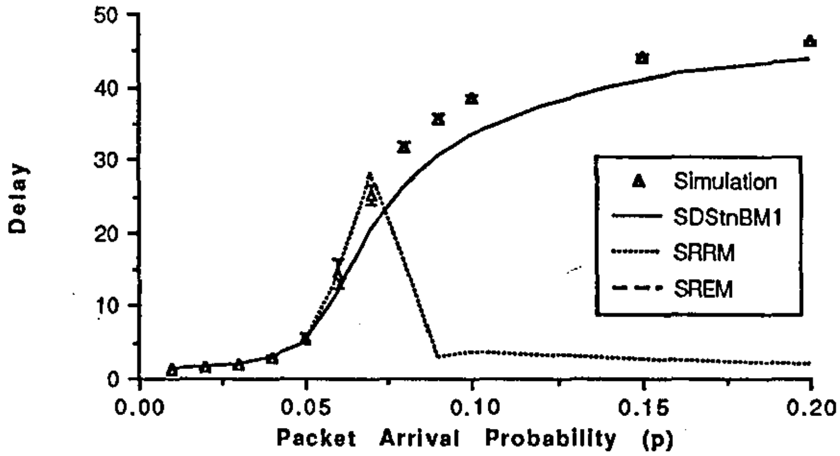


Fig. 7.6.8 Delay Performance (N=16, M=1, Q=3)

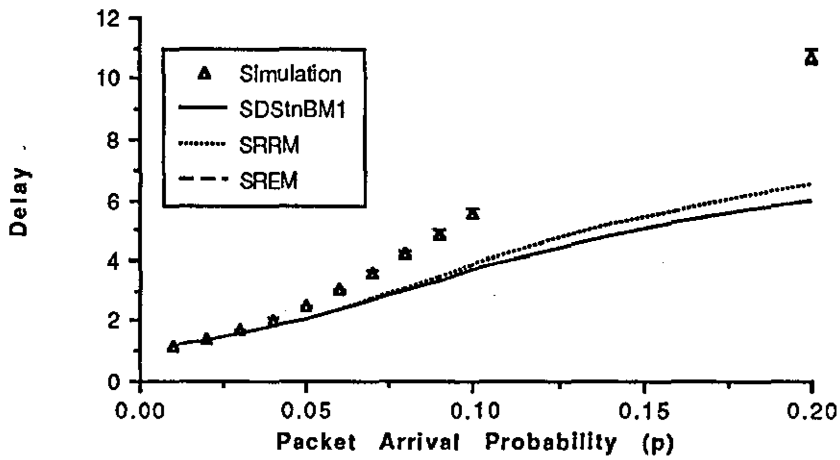


Fig. 7.6.9 Delay Performance (N=16, M=8, Q=0)

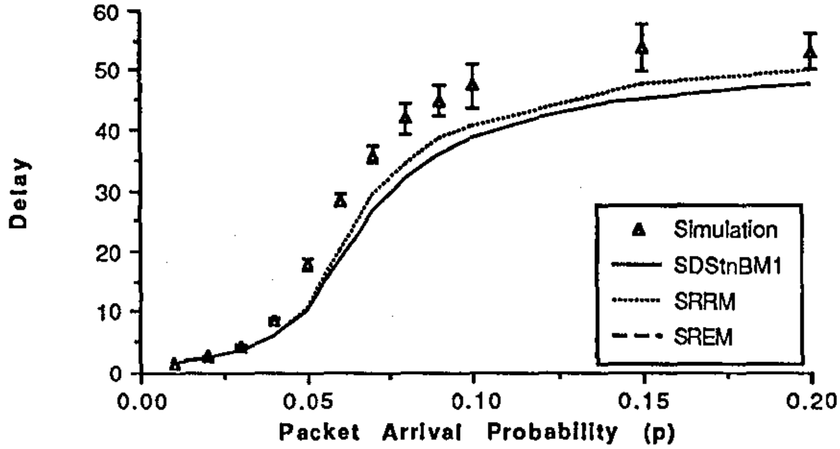


Fig. 7.6.10 Delay Performance (N=16, M=8, Q=3)

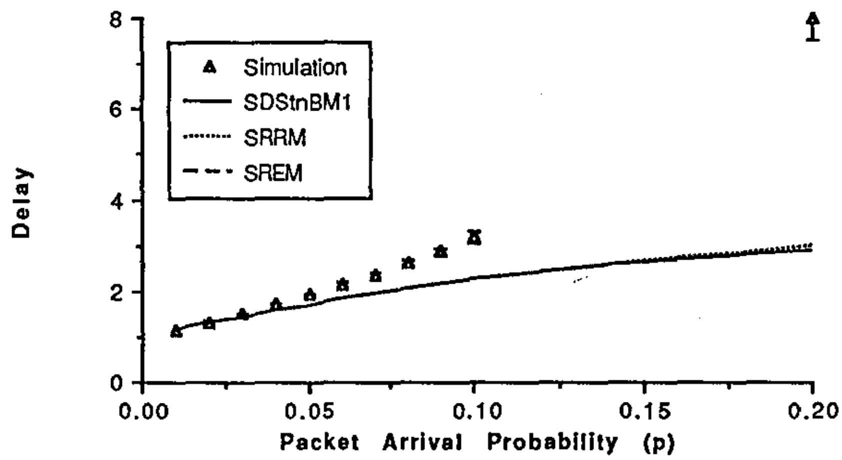


Fig. 7.6.11 Delay Performance (N=16, M=16, Q=0)

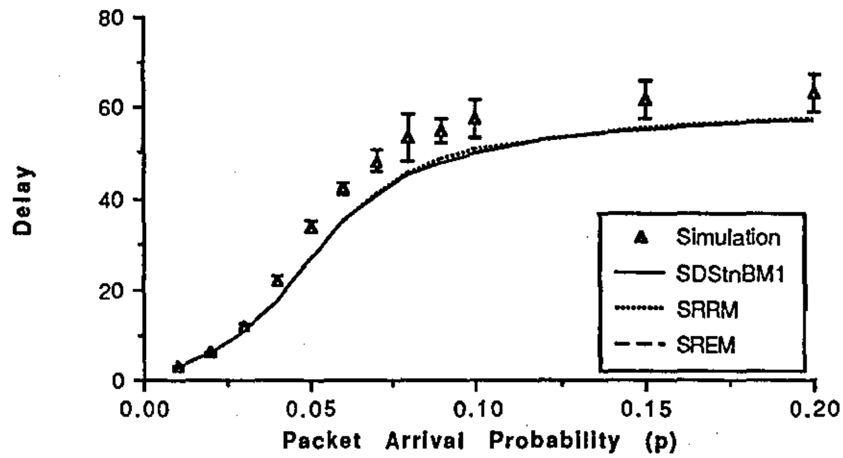


Fig. 7.6.12 Delay Performance (N=16, M=16, Q=3)

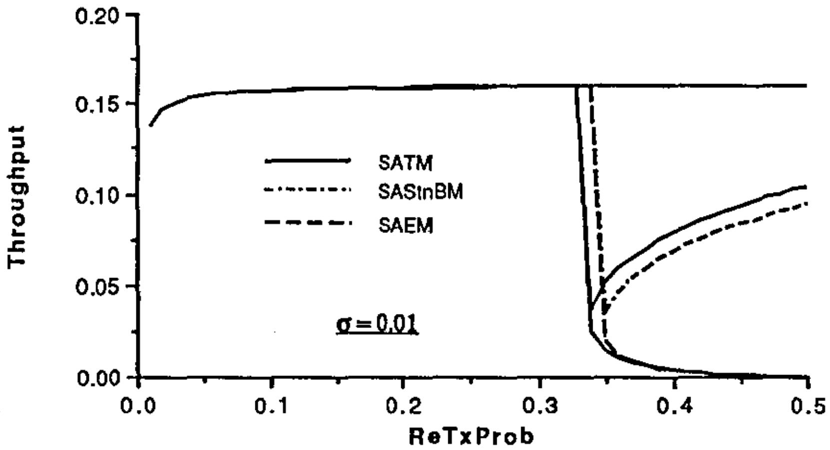


Fig 7.6.13 Throughput Performance (N=16, Q=0)

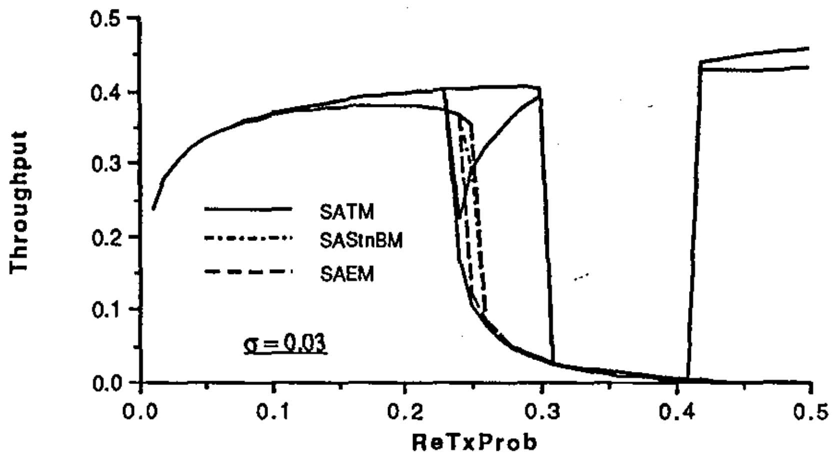


Fig 7.6.14 Throughput Performance (N=16, Q=0)

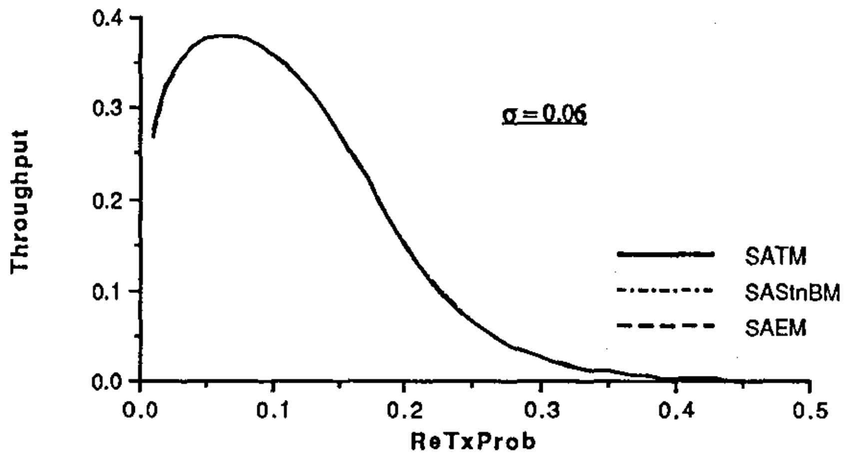


Fig 7.6.15 Throughput Performance (N=16, Q=0)

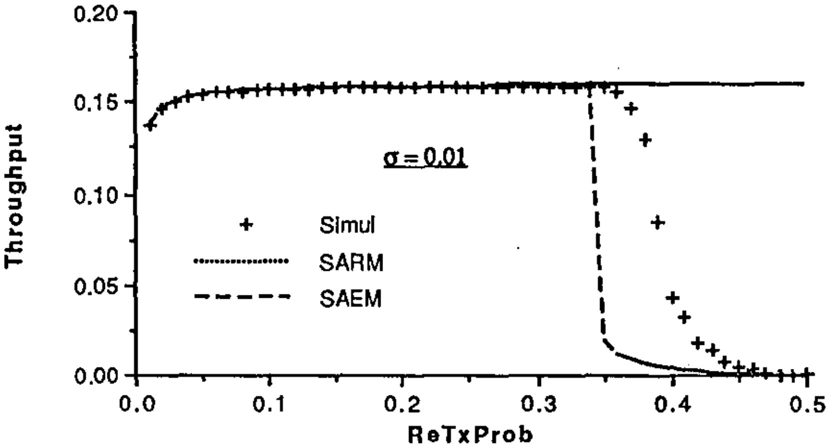


Fig 7.6.16 Throughput Performance (N=16, Q=0)

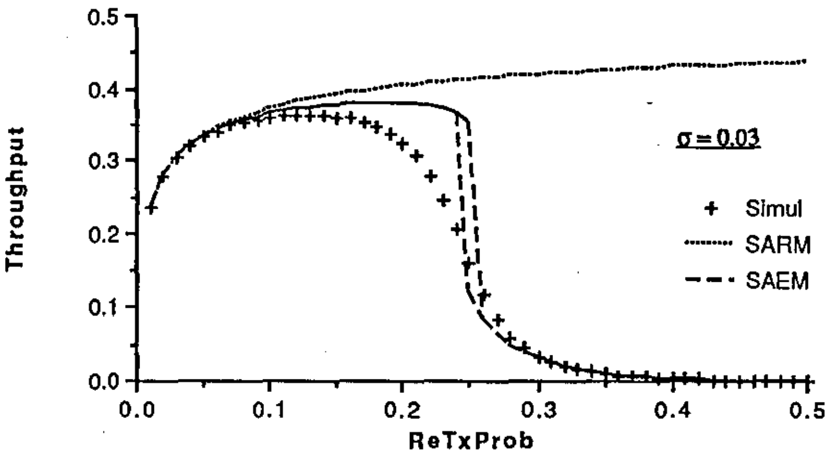


Fig 7.6.17 Throughput Performance (N=16, Q=0)

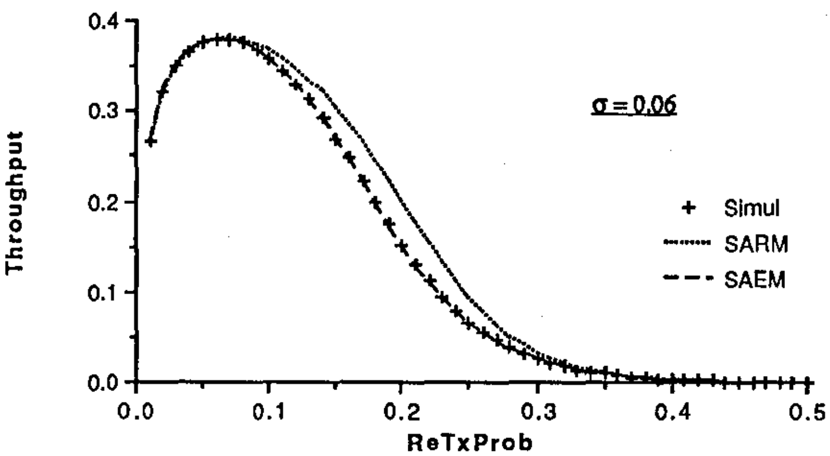


Fig 7.6.18 Throughput Performance (N=16, Q=0)

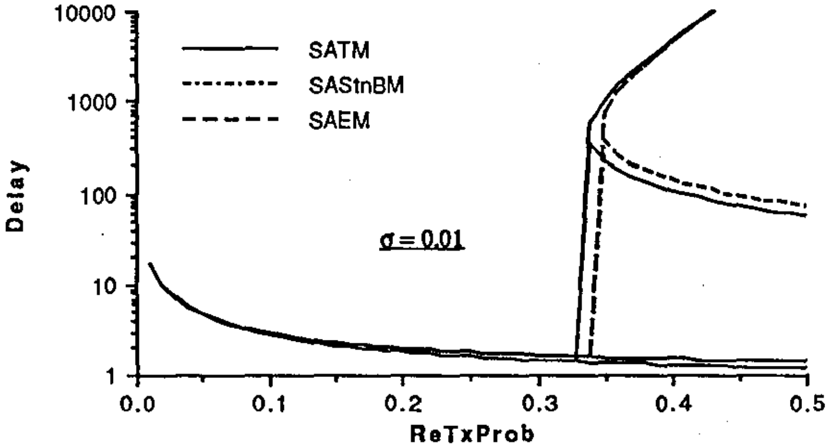


Fig 7.6.19 Delay Performance (N=16, Q=0)

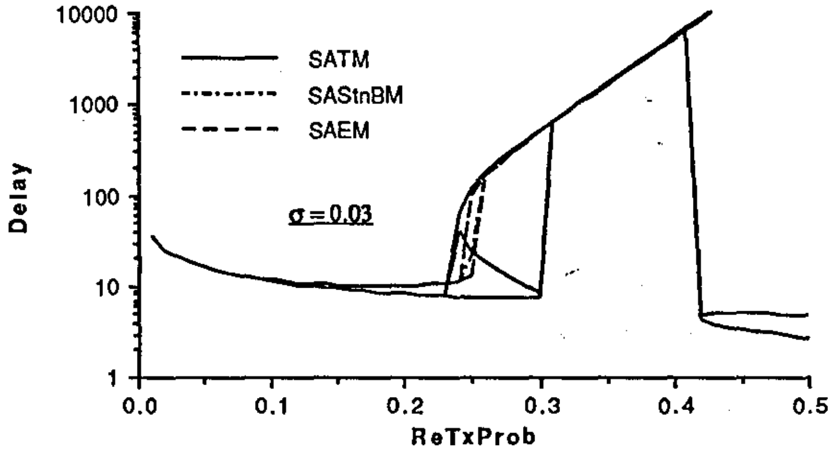


Fig 7.6.20 Delay Performance (N=16, Q=0)

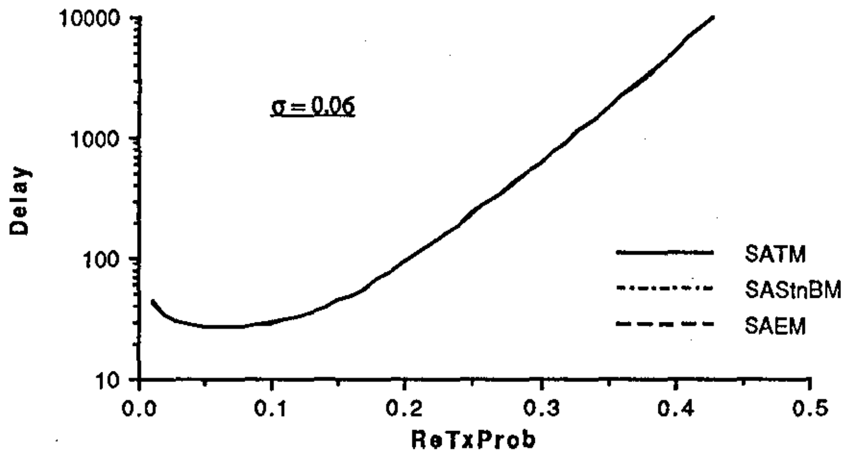


Fig 7.6.21 Delay Performance (N=16, Q=0)

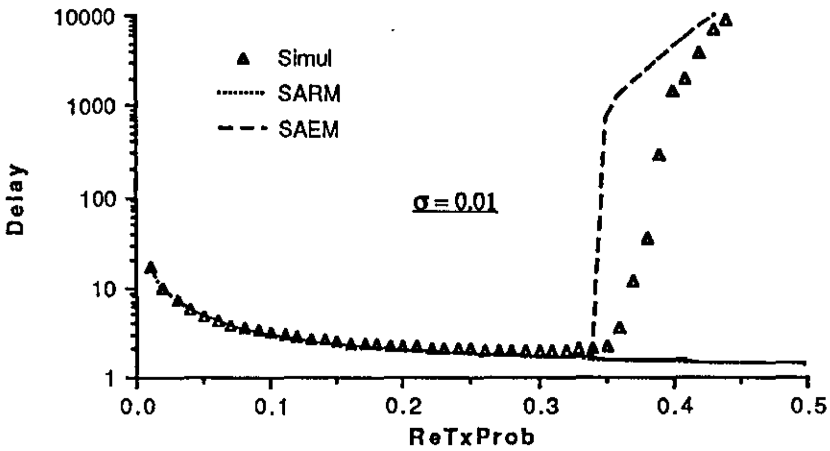


Fig 7.6.22 Delay Performance (N=16, Q=0)

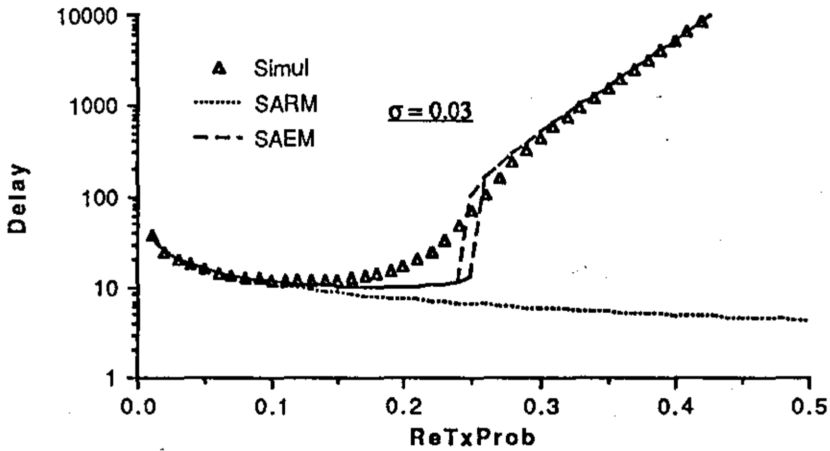


Fig 7.6.23 Delay Performance (N=16, Q=0)

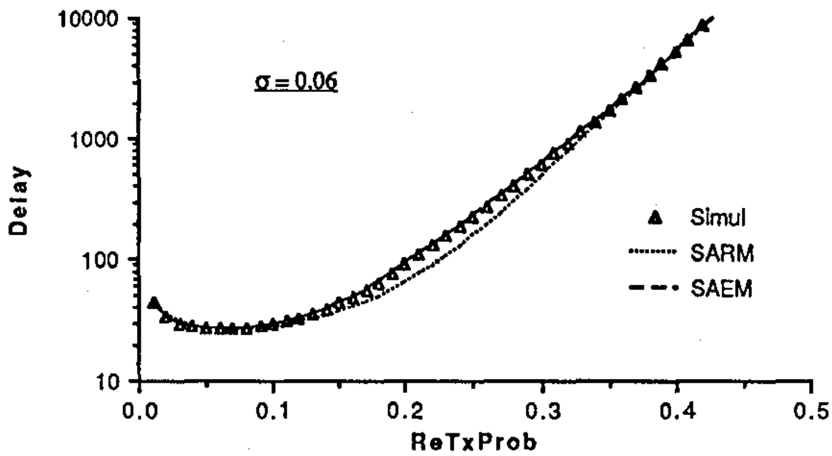


Fig 7.6.24 Delay Performance (N=16, Q=0)

Table (7.6.1) Slotted Ring (N=16, M=1, Q=3)

		Delay					Throughput				
Stn	p	Simul	SRRM	error	SREM	error	Simul	SRRM	error	SREM	error
1	0.0010	1.0742	1.1111	3.44%	1.1111	3.44%	0.0013	0.0010	-22.08%	0.0010	-22.08%
2	0.0017	1.1136	1.1111	-0.22%	1.1111	-0.22%	0.0018	0.0017	-6.42%	0.0017	-6.42%
3	0.0024	1.1412	1.1111	-2.63%	1.1111	-2.63%	0.0028	0.0024	-12.73%	0.0024	-12.73%
4	0.0031	1.1093	1.1111	0.17%	1.1111	0.17%	0.0032	0.0031	-1.59%	0.0031	-1.59%
5	0.0038	1.1413	1.1111	-2.64%	1.1111	-2.64%	0.0036	0.0038	6.54%	0.0038	6.54%
6	0.0045	1.1084	1.1111	0.25%	1.1111	0.25%	0.0043	0.0045	4.25%	0.0045	4.25%
7	0.0052	1.0914	1.1111	1.81%	1.1111	1.81%	0.0050	0.0052	4.00%	0.0052	4.00%
8	0.0059	1.1227	1.1112	-1.03%	1.1112	-1.03%	0.0051	0.0059	16.07%	0.0059	16.07%
9	0.0066	1.0930	1.1112	1.66%	1.1112	1.66%	0.0065	0.0066	1.28%	0.0066	1.28%
10	0.0073	1.1474	1.1112	-3.16%	1.1112	-3.16%	0.0072	0.0073	1.15%	0.0073	1.15%
11	0.0080	1.1108	1.1112	0.03%	1.1112	0.03%	0.0080	0.0080	0.00%	0.0080	0.00%
12	0.0087	1.1314	1.1112	-1.78%	1.1112	-1.78%	0.0078	0.0087	11.54%	0.0087	11.54%
13	0.0094	1.1024	1.1112	0.80%	1.1112	0.80%	0.0100	0.0094	-6.16%	0.0094	-6.16%
14	0.0101	1.1138	1.1112	-0.23%	1.1112	-0.23%	0.0094	0.0101	7.07%	0.0101	7.07%
15	0.0108	1.1439	1.1113	-2.85%	1.1113	-2.85%	0.0104	0.0108	4.35%	0.0108	4.35%
16	0.0115	1.1095	1.1113	0.16%	1.1113	0.16%	0.0120	0.0115	-4.43%	0.0115	-4.43%
	0.0063	1.1193	1.1112	-0.72%	1.1112	-0.72%	0.0983	0.1000	1.69%	0.1000	1.69%
1	0.0050	1.8403	2.0000	8.68%	2.0000	8.68%	0.0048	0.0050	4.53%	0.0050	4.53%
2	0.0085	1.9502	2.0002	2.57%	2.0002	2.57%	0.0086	0.0085	-0.97%	0.0085	-0.97%
3	0.0120	1.8377	2.0005	8.86%	2.0005	8.86%	0.0124	0.0120	-3.49%	0.0120	-3.49%
4	0.0155	2.0471	2.0009	-2.26%	2.0009	-2.26%	0.0160	0.0155	-3.02%	0.0155	-3.02%
5	0.0190	1.9089	2.0014	4.85%	2.0014	4.85%	0.0195	0.0190	-2.65%	0.0190	-2.65%
6	0.0225	1.9419	2.0020	3.09%	2.0020	3.09%	0.0213	0.0225	5.80%	0.0225	5.80%
7	0.0260	2.0393	2.0027	-1.80%	2.0026	-1.80%	0.0264	0.0260	-1.39%	0.0260	-1.39%
8	0.0295	1.9530	2.0034	2.58%	2.0034	2.58%	0.0298	0.0295	-0.90%	0.0295	-0.90%
9	0.0330	2.0229	2.0043	-0.92%	2.0043	-0.92%	0.0335	0.0330	-1.44%	0.0330	-1.44%
10	0.0365	2.0907	2.0053	-4.09%	2.0053	-4.09%	0.0354	0.0365	3.25%	0.0365	3.25%
11	0.0400	2.0710	2.0064	-3.12%	2.0064	-3.12%	0.0412	0.0400	-2.92%	0.0400	-2.92%
12	0.0435	2.1485	2.0076	-6.56%	2.0075	-6.56%	0.0425	0.0435	2.43%	0.0435	2.43%
13	0.0470	2.1063	2.0088	-4.63%	2.0088	-4.63%	0.0454	0.0470	3.48%	0.0470	3.48%
14	0.0505	2.1909	2.0102	-8.25%	2.0102	-8.25%	0.0505	0.0505	-0.04%	0.0505	-0.04%
15	0.0540	2.1075	2.0117	-4.55%	2.0117	-4.55%	0.0541	0.0540	-0.13%	0.0540	-0.13%
16	0.0575	2.1261	2.0133	-5.31%	2.0133	-5.31%	0.0556	0.0575	3.38%	0.0575	3.38%
	0.0312	2.0738	2.0049	-3.32%	2.0049	-3.32%	0.4968	0.5000	0.64%	0.5000	0.64%
1	0.0200	11.9738	-0.9298	-107.8%	70.0117	484.71%	0.0208	0.0200	-3.91%	0.0169	-18.69%
2	0.0340	14.2618	-0.9517	-106.7%	57.0267	299.86%	0.0330	0.0340	3.26%	0.0262	-20.49%
3	0.0480	19.1250	-1.3968	-107.3%	49.2602	157.57%	0.0460	0.0482	4.75%	0.0344	-25.22%
4	0.0620	23.6559	-0.7782	-103.3%	44.0994	86.42%	0.0552	0.0623	12.74%	0.0418	-24.30%
5	0.0760	27.5755	-0.7815	-102.8%	40.4507	46.69%	0.0597	0.0767	28.42%	0.0486	-18.66%
6	0.0900	30.8129	-0.8443	-102.7%	37.7730	22.59%	0.0633	0.0916	44.80%	0.0547	-13.50%
7	0.1040	33.9891	-0.9465	-102.8%	35.7650	5.22%	0.0647	0.1077	66.39%	0.0603	-6.82%
8	0.1180	35.6668	-0.8614	-102.4%	34.2438	-3.99%	0.0653	0.1241	90.12%	0.0653	0.06%
9	0.1320	37.3867	0.1004	-99.7%	33.0905	-11.49%	0.0656	0.1384	111.01%	0.0698	6.38%
10	0.1460	38.7092	-0.9715	-102.5%	32.2229	-16.76%	0.0659	0.1677	154.65%	0.0737	11.97%
11	0.1600	39.1210	0.5600	-98.6%	31.5813	-19.27%	0.0657	0.1600	143.59%	0.0772	17.51%
12	0.1740	39.9468	1.3958	-96.5%	31.1200	-22.10%	0.0658	0.1740	164.49%	0.0801	21.78%
13	0.1880	40.6352	1.2913	-96.8%	30.8029	-24.20%	0.0658	0.1880	185.56%	0.0826	25.47%
14	0.2020	40.9939	-2.8474	-106.9%	30.6004	-25.35%	0.0659	0.7301	1008.7%	0.0847	28.60%
15	0.2160	41.3441	1.0212	-97.5%	30.4878	-26.26%	0.0658	0.2160	228.10%	0.0864	31.23%
16	0.2300	41.7206	-5.7563	-113.8%	30.4447	-27.03%	0.0659	-1.0448	-1687.%	0.0878	33.31%
	0.1250	34.3506	-0.7935	-102.3%	38.6863	12.62%	0.9342	1.2938	38.49%	0.9905	6.02%

Note: The % error is based on the results expressed to 6 decimal places.

Table (7.6.2) Slotted Ring (N=16, M=8, Q=3)

		Delay					Throughput				
Stn	p	Simul	SRRM	error	SREM	error	Simul	SRRM	error	SREM	error
1	0.0010	1.1368	1.1472	0.92%	1.1472	0.92%	0.0011	0.0010	-5.14%	0.0010	-5.14%
2	0.0017	1.2084	1.1729	-2.94%	1.1729	-2.94%	0.0018	0.0017	-5.12%	0.0017	-5.12%
3	0.0024	1.2199	1.1989	-1.72%	1.1989	-1.72%	0.0025	0.0024	-2.04%	0.0024	-2.04%
4	0.0031	1.2512	1.2253	-2.07%	1.2253	-2.07%	0.0031	0.0031	-0.53%	0.0031	-0.53%
5	0.0038	1.2572	1.2520	-0.41%	1.2520	-0.41%	0.0039	0.0038	-2.04%	0.0038	-2.04%
6	0.0045	1.2798	1.2790	-0.06%	1.2790	-0.06%	0.0047	0.0045	-3.49%	0.0045	-3.49%
7	0.0052	1.2846	1.3064	1.70%	1.3064	1.70%	0.0053	0.0052	-1.65%	0.0052	-1.65%
8	0.0059	1.4585	1.3342	-8.52%	1.3342	-8.52%	0.0057	0.0059	2.68%	0.0059	2.68%
9	0.0066	1.3208	1.3624	3.15%	1.3624	3.15%	0.0067	0.0066	-1.31%	0.0066	-1.31%
10	0.0073	1.3804	1.3909	0.76%	1.3909	0.76%	0.0074	0.0073	-1.57%	0.0073	-1.57%
11	0.0080	1.3761	1.4199	3.18%	1.4199	3.18%	0.0078	0.0080	2.73%	0.0080	2.73%
12	0.0087	1.4541	1.4492	-0.34%	1.4492	-0.34%	0.0086	0.0087	1.31%	0.0087	1.31%
13	0.0094	1.4425	1.4790	2.53%	1.4790	2.53%	0.0094	0.0094	0.04%	0.0094	0.04%
14	0.0101	1.4890	1.5091	1.35%	1.5091	1.35%	0.0100	0.0101	1.38%	0.0101	1.38%
15	0.0108	1.5675	1.5397	-1.77%	1.5397	-1.77%	0.0109	0.0108	-1.26%	0.0108	-1.26%
16	0.0115	1.5299	1.5707	2.67%	1.5707	2.67%	0.0114	0.0115	0.91%	0.0115	0.91%
	0.0063	1.4175	1.3523	-4.60%	1.3523	-4.60%	0.1002	0.1000	-0.16%	0.1000	-0.16%
1	0.0050	2.5164	2.2138	-12.02%	2.2137	-12.03%	0.0049	0.0050	1.35%	0.0050	1.35%
2	0.0085	2.7044	2.3828	-11.89%	2.3826	-11.90%	0.0082	0.0085	3.92%	0.0085	3.92%
3	0.0120	2.8990	2.5650	-11.52%	2.5648	-11.53%	0.0119	0.0120	0.73%	0.0120	0.73%
4	0.0155	3.1701	2.7618	-12.88%	2.7615	-12.89%	0.0156	0.0155	-0.64%	0.0155	-0.64%
5	0.0190	3.5151	2.9745	-15.38%	2.9741	-15.39%	0.0186	0.0190	2.17%	0.0190	2.17%
6	0.0225	3.6899	3.2047	-13.15%	3.2043	-13.16%	0.0222	0.0225	1.36%	0.0225	1.36%
7	0.0260	4.0855	3.4539	-15.46%	3.4534	-15.47%	0.0264	0.0260	-1.62%	0.0260	-1.62%
8	0.0295	4.6400	3.7237	-19.75%	3.7230	-19.76%	0.0299	0.0295	-1.26%	0.0295	-1.26%
9	0.0330	4.8750	4.0155	-17.63%	4.0147	-17.65%	0.0330	0.0330	-0.12%	0.0330	-0.12%
10	0.0365	5.3881	4.3307	-19.62%	4.3299	-19.64%	0.0364	0.0365	0.13%	0.0365	0.13%
11	0.0400	5.8759	4.6708	-20.51%	4.6698	-20.53%	0.0397	0.0399	0.45%	0.0399	0.45%
12	0.0435	6.3640	5.0367	-20.86%	5.0357	-20.87%	0.0434	0.0434	0.02%	0.0434	0.02%
13	0.0470	7.0447	5.4295	-22.93%	5.4284	-22.94%	0.0468	0.0468	0.05%	0.0468	0.05%
14	0.0505	7.3194	5.8497	-20.08%	5.8485	-20.09%	0.0494	0.0502	1.64%	0.0502	1.64%
15	0.0540	7.8000	6.2976	-19.26%	6.2964	-19.28%	0.0531	0.0536	0.93%	0.0536	0.93%
16	0.0575	8.2027	6.7729	-17.43%	6.7718	-17.44%	0.0562	0.0569	1.24%	0.0569	1.24%
	0.0312	5.9545	4.1052	-31.06%	4.1045	-31.07%	0.4957	0.4982	0.50%	0.4982	0.50%
1	0.0200	12.1300	13.2657	9.36%	16.7287	37.91%	0.0203	0.0199	-1.93%	0.0198	-2.33%
2	0.0340	17.9377	17.4211	-2.88%	19.5267	8.86%	0.0329	0.0330	0.35%	0.0328	-0.44%
3	0.0480	24.2406	22.1260	-8.72%	22.6513	-6.56%	0.0431	0.0441	2.31%	0.0439	1.92%
4	0.0620	30.1515	26.7961	-11.13%	25.9529	-13.92%	0.0499	0.0521	4.56%	0.0526	5.50%
5	0.0760	35.4563	30.9597	-12.68%	29.1922	-17.67%	0.0532	0.0571	7.37%	0.0585	10.00%
6	0.0900	39.3815	34.3889	-12.68%	32.1202	-18.44%	0.0548	0.0599	9.30%	0.0621	13.37%
7	0.1040	42.2822	37.0743	-12.32%	34.5848	-18.20%	0.0554	0.0613	10.71%	0.0641	15.73%
8	0.1180	44.6440	39.1266	-12.36%	36.5621	-18.10%	0.0556	0.0620	11.42%	0.0651	16.99%
9	0.1320	45.9015	40.6880	-11.36%	38.1112	-16.97%	0.0557	0.0623	11.85%	0.0655	17.69%
10	0.1460	46.9988	41.8852	-10.88%	39.3187	-16.34%	0.0559	0.0624	11.78%	0.0658	17.76%
11	0.1600	47.8060	42.8169	-10.44%	40.2660	-15.77%	0.0558	0.0625	12.12%	0.0659	18.18%
12	0.1740	48.3596	43.5547	-9.94%	41.0187	-15.18%	0.0558	0.0626	12.11%	0.0660	18.21%
13	0.1880	48.9996	44.1499	-9.90%	41.6262	-15.05%	0.0557	0.0626	12.25%	0.0660	18.37%
14	0.2020	49.5079	44.6384	-9.84%	42.1244	-14.91%	0.0558	0.0626	12.23%	0.0660	18.35%
15	0.2160	49.9791	45.0460	-9.87%	42.5393	-14.89%	0.0558	0.0626	12.26%	0.0660	18.39%
16	0.2300	50.1807	45.3911	-9.54%	42.8900	-14.53%	0.0559	0.0626	12.05%	0.0660	18.17%
	0.1250	41.7590	35.5830	-14.79%	34.0758	-18.40%	0.8114	0.8896	9.64%	0.9261	14.14%

Note: The % error is based on the results expressed to 6 decimal places.

Table (7.6.3) Slotted Ring (N=16, M=16, Q=3)

		Delay					Throughput				
Stn	p	Simul	SRRM	error	SREM	error	Simul	SRRM	error	SREM	error
1	0.0010	1.2468	1.2502	0.27%	1.2502	0.27%	0.0010	0.0010	0.00%	0.0010	0.00%
2	0.0017	1.3402	1.3504	0.76%	1.3504	0.76%	0.0017	0.0017	0.00%	0.0017	0.00%
3	0.0024	1.3927	1.4532	4.34%	1.4532	4.34%	0.0024	0.0024	0.00%	0.0024	0.00%
4	0.0031	1.6578	1.5585	-5.99%	1.5585	-5.99%	0.0031	0.0031	0.00%	0.0031	0.00%
5	0.0038	1.7555	1.6665	-5.07%	1.6665	-5.07%	0.0037	0.0038	2.70%	0.0038	2.70%
6	0.0045	1.7876	1.7774	-0.57%	1.7774	-0.57%	0.0046	0.0045	-2.17%	0.0045	-2.17%
7	0.0052	1.9596	1.8911	-3.50%	1.8911	-3.50%	0.0050	0.0052	4.00%	0.0052	4.00%
8	0.0059	1.8868	2.0078	6.41%	2.0078	6.41%	0.0057	0.0059	3.51%	0.0059	3.51%
9	0.0066	2.1699	2.1276	-1.95%	2.1276	-1.95%	0.0067	0.0066	-1.49%	0.0066	-1.49%
10	0.0073	2.2115	2.2506	1.77%	2.2506	1.77%	0.0072	0.0073	1.39%	0.0073	1.39%
11	0.0080	2.4492	2.3769	-2.95%	2.3769	-2.95%	0.0081	0.0080	-1.24%	0.0080	-1.23%
12	0.0087	2.3891	2.5067	4.92%	2.5067	4.92%	0.0087	0.0087	0.00%	0.0087	0.00%
13	0.0094	2.5543	2.6400	3.36%	2.6400	3.36%	0.0094	0.0094	0.00%	0.0094	0.00%
14	0.0101	2.8600	2.7771	-2.90%	2.7771	-2.90%	0.0101	0.0101	0.00%	0.0101	0.00%
15	0.0108	2.9452	2.9179	-0.93%	2.9179	-0.93%	0.0110	0.0108	-1.82%	0.0108	-1.82%
16	0.0115	3.1529	3.0626	-2.86%	3.0626	-2.86%	0.0117	0.0115	-1.72%	0.0115	-1.71%
	0.0063	2.4194	2.1009	-13.16%	2.1009	-13.16%	0.1000	0.1000	0.00%	0.1000	0.00%
1	0.0050	2.9154	2.7173	-6.80%	2.4078	-17.41%	0.0049	0.0050	2.04%	0.0050	2.04%
2	0.0085	3.8507	3.3891	-11.99%	2.8712	-25.44%	0.0085	0.0085	0.00%	0.0084	-1.18%
3	0.0120	4.3713	4.1622	-4.78%	3.3209	-24.03%	0.0120	0.0120	-0.01%	0.0117	-2.50%
4	0.0155	5.3185	5.0537	-4.98%	3.7572	-29.36%	0.0153	0.0155	1.27%	0.0150	-1.96%
5	0.0190	6.5864	6.0811	-7.67%	4.1805	-36.53%	0.0182	0.0190	4.29%	0.0181	-0.55%
6	0.0225	7.9737	7.2597	-8.95%	4.5910	-42.42%	0.0225	0.0224	-0.23%	0.0211	-6.22%
7	0.0260	10.1041	8.6012	-14.87%	4.9890	-50.62%	0.0260	0.0259	-0.46%	0.0239	-8.08%
8	0.0295	11.1279	10.1114	-9.13%	5.3748	-51.70%	0.0290	0.0293	0.88%	0.0266	-8.28%
9	0.0330	13.3713	11.7877	-11.84%	5.7486	-57.01%	0.0329	0.0325	-1.10%	0.0291	-11.55%
10	0.0365	15.8164	13.6184	-13.90%	6.1109	-61.36%	0.0353	0.0357	1.12%	0.0314	-11.05%
11	0.0400	17.1286	15.5816	-9.03%	6.4619	-62.27%	0.0384	0.0387	0.73%	0.0336	-12.50%
12	0.0435	20.0126	17.6472	-11.82%	6.8018	-66.01%	0.0411	0.0414	0.85%	0.0356	-13.38%
13	0.0470	21.9408	19.7780	-9.86%	7.1309	-67.50%	0.0432	0.0440	1.78%	0.0375	-13.19%
14	0.0505	24.3537	21.9339	-9.94%	7.4496	-69.41%	0.0451	0.0462	2.46%	0.0392	-13.08%
15	0.0540	26.5320	24.0747	-9.26%	7.7580	-70.76%	0.0470	0.0482	2.47%	0.0408	-13.19%
16	0.0575	28.6384	26.1637	-8.64%	8.0566	-71.87%	0.0482	0.0498	3.36%	0.0422	-12.45%
	0.0312	17.6046	12.3726	-29.72%	5.4382	-69.11%	0.4674	0.4741	1.43%	0.4191	-10.33%
1	0.0200	12.2237	9.9684	-18.45%	6.1490	-49.70%	0.0199	0.0199	0.20%	0.0184	-7.54%
2	0.0340	21.5761	17.5353	-18.73%	7.6308	-64.63%	0.0326	0.0330	1.21%	0.0284	-12.88%
3	0.0480	33.2401	27.0072	-18.75%	8.9415	-73.10%	0.0411	0.0428	4.17%	0.0358	-12.90%
4	0.0620	42.0765	35.7249	-15.10%	10.0984	-76.00%	0.0447	0.0480	7.48%	0.0411	-8.05%
5	0.0760	47.8206	42.1449	-11.87%	11.1169	-76.75%	0.0458	0.0501	9.40%	0.0447	-2.40%
6	0.0900	52.1271	46.4093	-10.97%	12.0121	-76.96%	0.0462	0.0508	9.95%	0.0472	2.16%
7	0.1040	54.4443	49.2021	-9.63%	12.7981	-76.49%	0.0463	0.0510	10.19%	0.0489	5.62%
8	0.1180	56.2338	51.0876	-9.15%	13.4886	-76.01%	0.0464	0.0511	10.10%	0.0500	7.76%
9	0.1320	57.4720	52.4190	-8.79%	14.0956	-75.47%	0.0463	0.0511	10.39%	0.0508	9.72%
10	0.1460	58.4131	53.4024	-8.58%	14.6302	-74.95%	0.0464	0.0511	10.17%	0.0513	10.56%
11	0.1600	59.1532	54.1580	-8.44%	15.1020	-74.47%	0.0464	0.0511	10.17%	0.0516	11.21%
12	0.1740	59.7050	54.7581	-8.29%	15.5195	-74.01%	0.0463	0.0511	10.41%	0.0519	12.10%
13	0.1880	60.1651	55.2477	-8.17%	15.8900	-73.59%	0.0464	0.0511	10.18%	0.0520	12.07%
14	0.2020	60.6106	55.6560	-8.17%	16.2199	-73.24%	0.0463	0.0511	10.41%	0.0521	12.53%
15	0.2160	60.8931	56.0027	-8.03%	16.5145	-72.88%	0.0464	0.0511	10.18%	0.0522	12.50%
16	0.2300	61.2876	56.3015	-8.14%	16.7785	-72.62%	0.0463	0.0511	10.41%	0.0523	12.96%
	0.1250	51.9847	44.8141	-13.79%	12.9366	-75.11%	0.6938	0.7558	8.93%	0.7287	5.03%

Note: The % error is based on the results expressed to 6 decimal places.

Table (7.6.4) Slotted Aloha (N=16, Q=0)

StnPktArr		ReTx		Delay				Throughput				
	Rate	Prob	Simul	SARM	% err	SAEM	% err	Simul	SARM	% err	SAEM	% err
1	0.0070	0.0010	171.9180	164.600	-4.26%	164.6042	-4.25%	0.0033	0.0033	-0.74%	0.0033	-0.74%
2	0.0074	0.0022	78.6043	74.4600	-5.27%	74.4245	-5.32%	0.0047	0.0048	2.52%	0.0048	2.54%
3	0.0078	0.0034	47.7012	48.1800	1.00%	48.1501	0.94%	0.0057	0.0057	-0.08%	0.0057	-0.06%
4	0.0082	0.0046	36.1906	35.6800	-1.41%	35.6504	-1.49%	0.0064	0.0064	0.17%	0.0064	0.19%
5	0.0086	0.0058	28.4081	28.3700	-0.13%	28.3477	-0.21%	0.0070	0.0070	-0.13%	0.0070	-0.11%
6	0.0090	0.0070	23.0950	23.5900	2.14%	23.5602	2.01%	0.0075	0.0075	-0.66%	0.0075	-0.65%
7	0.0094	0.0082	20.3592	20.2000	-0.78%	20.1797	-0.88%	0.0080	0.0080	-0.29%	0.0080	-0.27%
8	0.0098	0.0094	17.8000	17.6900	-0.62%	17.6657	-0.75%	0.0084	0.0084	0.10%	0.0084	0.12%
9	0.0102	0.0106	15.9135	15.7500	-1.03%	15.7230	-1.20%	0.0088	0.0089	0.47%	0.0089	0.49%
10	0.0106	0.0118	14.6153	14.2000	-2.84%	14.1768	-3.00%	0.0093	0.0093	-0.05%	0.0093	-0.03%
11	0.0110	0.0130	12.9411	12.9400	-0.01%	12.9169	-0.19%	0.0098	0.0097	-0.49%	0.0097	-0.47%
12	0.0114	0.0142	11.8750	11.8900	0.13%	11.8706	-0.04%	0.0102	0.0101	-0.42%	0.0101	-0.39%
13	0.0118	0.0154	10.6013	11.0100	3.86%	10.9879	3.65%	0.0106	0.0106	-0.86%	0.0106	-0.80%
14	0.0122	0.0166	10.1221	10.2530	1.29%	10.2331	1.10%	0.0110	0.0110	-0.19%	0.0110	-0.15%
15	0.0126	0.0178	9.5507	9.6000	0.52%	9.5803	0.31%	0.0114	0.0114	0.06%	0.0114	0.06%
16	0.0130	0.0190	9.0067	9.0300	0.26%	9.0102	0.04%	0.0117	0.0118	0.29%	0.0118	0.32%
	0.0100	0.0100	32.4189	31.7200	-2.16%	31.6926	-2.24%	0.1337	0.1337	-0.04%	0.1337	-0.05%
1	0.0070	0.0100	21.0500	20.4400	-2.90%	20.6428	-1.93%	0.0061	0.0062	1.10%	0.0062	0.97%
2	0.0074	0.0220	9.8600	9.7780	-0.83%	9.8772	0.17%	0.0069	0.0069	0.33%	0.0069	0.26%
3	0.0078	0.0340	7.0893	6.6550	-6.13%	6.7221	-5.18%	0.0074	0.0075	0.74%	0.0075	0.69%
4	0.0082	0.0460	5.4149	5.1630	-4.65%	5.2151	-3.69%	0.0080	0.0079	-0.35%	0.0079	-0.38%
5	0.0086	0.0580	4.5118	4.2900	-4.92%	4.3324	-3.98%	0.0083	0.0084	0.73%	0.0084	0.70%
6	0.0090	0.0700	3.8534	3.7160	-3.57%	3.7525	-2.62%	0.0087	0.0088	1.08%	0.0088	1.05%
7	0.0094	0.0820	3.4402	3.3100	-3.78%	3.3425	-2.84%	0.0092	0.0092	0.18%	0.0092	0.15%
8	0.0098	0.0940	3.2326	3.0080	-6.95%	3.0373	-6.04%	0.0095	0.0096	0.74%	0.0096	0.71%
9	0.0102	0.1060	2.9132	2.7750	-4.74%	2.8012	-3.85%	0.0099	0.0100	0.73%	0.0100	0.69%
10	0.0106	0.1180	2.6637	2.5890	-2.80%	2.6131	-1.90%	0.0104	0.0104	0.40%	0.0104	0.41%
11	0.0110	0.1300	2.5530	2.4370	-4.54%	2.4598	-3.65%	0.0108	0.0108	0.04%	0.0108	0.00%
12	0.0114	0.1420	2.4393	2.3110	-5.26%	2.3324	-4.38%	0.0112	0.0112	0.53%	0.0112	0.53%
13	0.0118	0.1540	2.3571	2.2050	-6.45%	2.2249	-5.61%	0.0116	0.0116	0.03%	0.0116	0.04%
14	0.0122	0.1660	2.2200	2.1140	-4.77%	2.1330	-3.92%	0.0119	0.0120	0.97%	0.0120	0.92%
15	0.0126	0.1780	2.1351	2.0350	-4.69%	2.0534	-3.83%	0.0125	0.0124	-0.28%	0.0124	-0.32%
16	0.0130	0.1900	2.0761	1.9663	-5.29%	1.9839	-4.44%	0.0128	0.0128	0.51%	0.0128	0.48%
	0.0100	0.1000	4.8631	4.6740	-3.89%	4.7202	-2.94%	0.1552	0.1559	0.43%	0.1559	0.41%
1	0.0070	0.0300	7.8920	7.5920	-3.80%	7.6781	-2.71%	0.0066	0.0067	1.10%	0.0067	1.05%
2	0.0074	0.0660	4.4966	3.9820	-11.44%	4.0235	-10.52%	0.0073	0.0072	-0.79%	0.0072	-0.81%
3	0.0078	0.1020	3.3977	2.9230	-13.97%	2.9501	-13.17%	0.0076	0.0077	0.68%	0.0077	0.65%
4	0.0082	0.1380	2.7772	2.4160	-13.01%	2.4370	-12.25%	0.0081	0.0081	0.19%	0.0081	0.17%
5	0.0086	0.1740	2.4678	2.1190	-14.13%	2.1363	-13.43%	0.0086	0.0085	-1.08%	0.0085	-1.10%
6	0.0090	0.2100	2.2992	1.9238	-16.33%	1.9387	-15.68%	0.0089	0.0089	-0.19%	0.0089	-0.21%
7	0.0094	0.2460	2.1116	1.7859	-15.42%	1.7990	-14.80%	0.0094	0.0093	-0.37%	0.0093	-0.39%
8	0.0098	0.2820	2.0422	1.6832	-17.58%	1.6949	-17.01%	0.0097	0.0097	0.29%	0.0097	0.28%
9	0.0102	0.3180	1.9287	1.6037	-16.85%	1.6144	-16.29%	0.0101	0.0101	0.67%	0.0101	0.63%
10	0.0106	0.3540	1.8332	1.5404	-15.97%	1.5503	-15.43%	0.0105	0.0105	0.15%	0.0105	0.14%
11	0.0110	0.3900	1.7796	1.4888	-16.34%	1.4981	-15.82%	0.0109	0.0109	0.18%	0.0109	0.18%
12	0.0114	0.4260	1.7392	1.4460	-16.86%	1.4546	-16.36%	0.0113	0.0113	-0.08%	0.0113	-0.07%
13	0.0118	0.4620	1.6775	1.4098	-15.96%	1.4180	-15.47%	0.0117	0.0117	0.25%	0.0117	0.27%
14	0.0122	0.4980	1.6303	1.3788	-15.43%	1.3866	-14.95%	0.0121	0.0121	0.59%	0.0121	0.61%
15	0.0126	0.5340	1.5988	1.3520	-15.44%	1.3595	-14.97%	0.0126	0.0125	-0.21%	0.0125	-0.18%
16	0.0130	0.5700	1.5722	1.3287	-15.49%	1.3358	-15.04%	0.0129	0.0129	0.42%	0.0129	0.45%
	0.0100	0.3000	2.5777	2.2480	-12.79%	2.2672	-12.05%	0.1584	0.1586	0.15%	0.1585	0.11%

Note: The % error is based on the results expressed to 6 decimal places.

Table (7.6.5) Slotted Aloha (N=16, Q=0)

Stn	PktArr Rate	ReTx Prob	Delay Simul	SARM	% err	SAEM	% err	SAEM	% err	Throughput Simul	SARM	% err	SAEM	% err	SAEM	% err
1	0.0210	0.0010	402.23	357.70	-11%	358.04	-11%			0.0023	0.0025	8.1%	0.0025	8.0%		
2	0.0222	0.0022	165.95	161.60	-2.6%	161.29	-2.8%			0.0048	0.0049	1.6%	0.0049	1.7%		
3	0.0234	0.0034	103.55	104.10	0.5%	103.64	0.1%			0.0069	0.0069	-0.8%	0.0069	-0.5%		
4	0.0246	0.0046	77.937	76.640	-1.7%	76.172	-2.3%			0.0085	0.0086	0.8%	0.0086	1.2%		
5	0.0258	0.0058	61.638	60.600	-1.7%	60.126	-2.5%			0.0100	0.0102	1.2%	0.0102	1.7%		
6	0.0270	0.0070	49.417	50.090	1.4%	49.615	0.4%			0.0117	0.0116	-1.0%	0.0117	-0.4%		
7	0.0282	0.0082	42.575	42.680	0.2%	42.201	-0.9%			0.0130	0.0130	-0.6%	0.0130	0.0%		
8	0.0294	0.0094	36.917	37.170	0.7%	36.694	-0.6%			0.0143	0.0143	-0.5%	0.0143	0.2%		
9	0.0306	0.0106	32.789	32.910	0.4%	32.443	-1.1%			0.0155	0.0155	-0.4%	0.0156	0.4%		
10	0.0318	0.0118	29.524	29.520	0.0%	29.064	-1.6%			0.0167	0.0167	-0.3%	0.0168	0.4%		
11	0.0330	0.0130	26.159	26.770	2.3%	26.313	0.6%			0.0181	0.0178	-1.2%	0.0180	-0.4%		
12	0.0342	0.0142	24.144	24.480	1.4%	24.031	-0.5%			0.0191	0.0190	-0.7%	0.0191	0.2%		
13	0.0354	0.0154	21.666	22.550	4.1%	22.108	2.0%			0.0205	0.0201	-1.9%	0.0203	-1.0%		
14	0.0366	0.0166	20.453	20.900	2.2%	20.464	0.1%			0.0213	0.0212	-0.8%	0.0214	0.1%		
15	0.0378	0.0178	18.744	19.480	3.9%	19.044	1.6%			0.0226	0.0223	-1.4%	0.0225	-0.5%		
16	0.0390	0.0190	17.704	18.230	3.0%	17.805	0.6%			0.0235	0.0233	-0.9%	0.0236	0.1%		
	0.0300	0.0100	70.712	67.830	-4.1%	67.441	-4.6%			0.2290	0.2276	-0.6%	0.2293	0.1%		
1	0.0210	0.0100	102.16	97.56	-4.5%	101.89	-0.3%			0.0068	0.0069	2.7%	0.0067	-0.3%		
2	0.0222	0.0220	46.501	43.750	-5.9%	46.066	-0.9%			0.0111	0.0114	3.0%	0.0111	0.3%		
3	0.0234	0.0340	30.484	28.210	-7.5%	29.827	-2.2%			0.0138	0.0143	3.3%	0.0140	0.9%		
4	0.0246	0.0460	22.753	20.870	-8.3%	22.119	-2.8%			0.0160	0.0165	3.0%	0.0162	1.0%		
5	0.0258	0.0580	18.039	16.600	-8.0%	17.625	-2.3%			0.0180	0.0184	2.2%	0.0181	0.3%		
6	0.0270	0.0700	15.242	13.810	-9.4%	14.684	-3.7%			0.0195	0.0201	2.8%	0.0197	1.0%		
7	0.0282	0.0820	12.978	11.840	-8.8%	12.610	-2.8%			0.0211	0.0216	2.6%	0.0212	0.9%		
8	0.0294	0.0940	11.614	10.385	-11%	11.070	-4.7%			0.0223	0.0230	3.4%	0.0227	1.8%		
9	0.0306	0.1060	10.349	9.261	-11%	9.881	-4.5%			0.0238	0.0244	2.5%	0.0241	1.0%		
10	0.0318	0.1180	9.386	8.368	-11%	8.935	-4.8%			0.0251	0.0258	2.7%	0.0254	1.3%		
11	0.0330	0.1300	8.630	7.642	-11%	8.165	-5.4%			0.0264	0.0271	2.6%	0.0267	1.2%		
12	0.0342	0.1420	7.941	7.040	-11%	7.526	-5.2%			0.0275	0.0283	2.9%	0.0280	1.5%		
13	0.0354	0.1540	7.418	6.532	-12%	6.988	-5.8%			0.0288	0.0296	2.8%	0.0292	1.4%		
14	0.0366	0.1660	6.938	6.099	-12%	6.528	-5.9%			0.0299	0.0308	3.1%	0.0304	1.8%		
15	0.0378	0.1780	6.572	5.725	-13%	6.130	-6.7%			0.0313	0.0321	2.4%	0.0317	1.1%		
16	0.0390	0.1900	6.127	5.398	-12%	5.783	-5.6%			0.0325	0.0333	2.3%	0.0329	1.0%		
	0.0300	0.1000	20.196	18.690	-7.5%	19.739	-2.3%			0.3539	0.3637	2.8%	0.3580	1.1%		
1	0.0210	0.0300	3940.1	49.83	-99%	9529.0	142%	94.17	-98%	0.0003	0.0104	3784%	0.0001	-61%	0.0071	2560%
2	0.0222	0.0660	2005.9	22.37	-99%	4172.5	108%	42.39	-98%	0.0005	0.0151	2832%	0.0002	-54%	0.0116	2152%
3	0.0234	0.1020	1374.2	14.55	-99%	2598.2	89%	27.39	-98%	0.0007	0.0178	2363%	0.0004	-47%	0.0145	1905%
4	0.0246	0.1380	1082.9	10.86	-99%	1846.0	70%	20.29	-98%	0.0009	0.0198	2058%	0.0005	-42%	0.0167	1719%
5	0.0258	0.1740	911.7	8.72	-99%	1405.7	54%	16.16	-98%	0.0011	0.0215	1900%	0.0007	-36%	0.0185	1625%
6	0.0270	0.2100	708.7	7.33	-99%	1116.9	58%	13.45	-98%	0.0014	0.0231	1584%	0.0009	-37%	0.0202	1375%
7	0.0282	0.2460	624.5	6.34	-99%	913.1	46%	11.55	-98%	0.0015	0.0245	1500%	0.0011	-31%	0.0217	1319%
8	0.0294	0.2820	539.2	5.61	-99%	761.7	41%	10.14	-98%	0.0018	0.0259	1362%	0.0013	-29%	0.0232	1209%
9	0.0306	0.3180	470.3	5.05	-99%	644.9	37%	9.05	-98%	0.0020	0.0272	1251%	0.0015	-27%	0.0246	1118%
10	0.0318	0.3540	418.4	4.60	-99%	552.3	32%	8.18	-98%	0.0022	0.0285	1168%	0.0017	-24%	0.0259	1051%
11	0.0330	0.3900	365.6	4.24	-99%	477.1	30%	7.48	-98%	0.0026	0.0298	1067%	0.0020	-23%	0.0272	965%
12	0.0342	0.4260	325.5	3.94	-99%	415.0	27%	6.89	-98%	0.0029	0.0311	990%	0.0023	-21%	0.0285	898%
13	0.0354	0.4620	300.4	3.69	-99%	362.9	21%	6.40	-98%	0.0031	0.0323	952%	0.0026	-17%	0.0297	867%
14	0.0366	0.4980	266.1	3.47	-99%	318.8	20%	5.98	-98%	0.0034	0.0336	879%	0.0029	-16%	0.0310	803%
15	0.0378	0.5340	235.5	3.28	-99%	280.9	19%	5.62	-98%	0.0038	0.0348	805%	0.0033	-15%	0.0322	737%
16	0.0390	0.5700	210.0	3.12	-99%	248.2	18%	5.30	-97%	0.0043	0.0360	740%	0.0037	-15%	0.0334	679%
	0.0300	0.3000	861.2	9.81	-99%	1602.7	86%	18.15	-98%	0.0325	0.4113	1167%	0.0249	-23%	0.3658	1027%

Note: The % error is based on the results expressed to 6 decimal places.

Table (7.6.6) Slotted Aloha (N=16, Q=0)

StnPktArr		ReTx	Delay					Throughput				
	Rate	Prob	Simul	SARM	% err	SAEM	% err	Simul	SARM	% err	SAEM	% err
1	0.0420	0.0010	463.05	455.10	-1.72%	458.41	-1.00%	0.0021	0.0021	-1.24%	0.0021	-1.90%
2	0.0444	0.0022	200.85	206.00	2.56%	206.70	2.91%	0.0045	0.0044	-2.94%	0.0044	-3.25%
3	0.0468	0.0034	132.36	132.80	0.33%	132.77	0.31%	0.0066	0.0065	-0.90%	0.0065	-0.91%
4	0.0492	0.0046	96.23	97.77	1.60%	97.47	1.29%	0.0087	0.0085	-1.59%	0.0086	-1.34%
5	0.0516	0.0058	75.44	77.29	2.46%	76.82	1.83%	0.0107	0.0105	-2.02%	0.0105	-1.51%
6	0.0540	0.0070	62.65	63.84	1.90%	63.27	0.99%	0.0125	0.0123	-1.75%	0.0124	-1.04%
7	0.0564	0.0082	53.39	54.34	1.77%	53.70	0.57%	0.0143	0.0141	-1.40%	0.0142	-0.49%
8	0.0588	0.0094	47.20	47.28	0.16%	46.58	-1.32%	0.0159	0.0158	-0.37%	0.0160	0.75%
9	0.0612	0.0106	40.54	41.82	3.15%	41.09	1.35%	0.0179	0.0175	-2.35%	0.0177	-1.11%
10	0.0636	0.0118	36.93	37.47	1.46%	36.72	-0.59%	0.0194	0.0192	-1.01%	0.0194	0.44%
11	0.0660	0.0130	33.08	33.93	2.57%	33.16	0.23%	0.0212	0.0208	-2.02%	0.0211	-0.43%
12	0.0684	0.0142	30.25	30.99	2.44%	30.20	-0.17%	0.0228	0.0224	-1.79%	0.0228	-0.03%
13	0.0708	0.0154	28.13	28.51	1.35%	27.71	-1.50%	0.0242	0.0240	-0.92%	0.0245	1.02%
14	0.0732	0.0166	25.34	26.39	4.13%	25.58	0.93%	0.0263	0.0256	-2.69%	0.0262	-0.64%
15	0.0756	0.0178	23.91	24.56	2.70%	23.74	-0.74%	0.0277	0.0272	-1.96%	0.0278	0.26%
16	0.0780	0.0190	22.06	22.95	4.02%	22.13	0.31%	0.0295	0.0288	-2.63%	0.0295	-0.28%
	0.0600	0.0100	85.71	86.31	0.69%	86.00	0.34%	0.2644	0.2596	-1.81%	0.2636	-0.28%
1	0.0420	0.0100	280.66	275.00	-2.02%	281.68	0.36%	0.0033	0.0034	1.76%	0.0033	-0.45%
2	0.0444	0.0220	127.29	122.50	-3.76%	126.32	-0.76%	0.0067	0.0069	3.13%	0.0068	0.46%
3	0.0468	0.0340	79.75	77.91	-2.31%	80.81	1.32%	0.0100	0.0102	1.73%	0.0099	-1.13%
4	0.0492	0.0460	59.12	56.75	-4.01%	59.14	0.03%	0.0128	0.0131	2.99%	0.0127	-0.11%
5	0.0516	0.0580	46.18	44.44	-3.77%	46.50	0.68%	0.0155	0.0159	2.75%	0.0154	-0.52%
6	0.0540	0.0700	38.61	36.41	-5.70%	38.22	-1.00%	0.0178	0.0185	4.10%	0.0179	0.73%
7	0.0564	0.0820	32.41	30.77	-5.05%	32.40	-0.03%	0.0204	0.0211	3.34%	0.0204	-0.07%
8	0.0588	0.0940	28.00	26.60	-5.00%	28.07	0.26%	0.0227	0.0235	3.32%	0.0227	-0.14%
9	0.0612	0.1060	24.89	23.39	-6.03%	24.74	-0.60%	0.0248	0.0258	3.94%	0.0249	0.43%
10	0.0636	0.1180	22.18	20.85	-6.00%	22.10	-0.37%	0.0271	0.0281	3.68%	0.0272	0.17%
11	0.0660	0.1300	20.07	18.79	-6.36%	19.95	-0.58%	0.0293	0.0304	3.65%	0.0293	0.14%
12	0.0684	0.1420	18.38	17.09	-7.00%	18.17	-1.12%	0.0313	0.0326	4.13%	0.0315	0.60%
13	0.0708	0.1540	16.96	15.66	-7.67%	16.67	-1.71%	0.0332	0.0347	4.63%	0.0336	1.09%
14	0.0732	0.1660	15.67	14.45	-7.81%	15.39	-1.79%	0.0353	0.0369	4.62%	0.0356	1.09%
15	0.0756	0.1780	14.43	13.40	-7.16%	14.29	-0.99%	0.0375	0.0390	3.99%	0.0377	0.50%
16	0.0780	0.1900	13.50	12.49	-7.51%	13.33	-1.29%	0.0395	0.0411	4.14%	0.0398	0.65%
	0.0600	0.1000	52.38	50.41	-3.76%	52.36	-0.04%	0.3672	0.3812	3.82%	0.3686	0.40%
1	0.0420	0.0300	14670.90	7222.00	-50.77%	12978.2	-11.54%	0.0001	0.0001	69.67%	0.0001	-5.43%
2	0.0444	0.0660	6190.36	2768.00	-55.29%	5680.90	-8.23%	0.0002	0.0004	104.47%	0.0002	0.02%
3	0.0468	0.1020	3592.58	1538.00	-57.19%	3535.18	-1.60%	0.0003	0.0006	118.94%	0.0003	-4.01%
4	0.0492	0.1380	2524.20	989.70	-60.79%	2509.36	-0.59%	0.0004	0.0010	146.54%	0.0004	-1.63%
5	0.0516	0.1740	1910.27	690.40	-63.86%	1908.33	-0.10%	0.0005	0.0014	167.40%	0.0005	-1.64%
6	0.0540	0.2100	1560.35	507.60	-67.47%	1513.64	-2.99%	0.0006	0.0019	194.89%	0.0007	1.15%
7	0.0564	0.2460	1177.67	387.60	-67.09%	1234.69	4.84%	0.0008	0.0025	192.43%	0.0008	-5.51%
8	0.0588	0.2820	1013.62	304.50	-69.96%	1027.19	1.34%	0.0010	0.0031	218.91%	0.0010	-2.02%
9	0.0612	0.3180	885.09	244.80	-72.34%	866.88	-2.06%	0.0011	0.0038	241.49%	0.0011	0.70%
10	0.0636	0.3540	753.22	200.60	-73.37%	739.38	-1.84%	0.0013	0.0046	254.13%	0.0013	1.10%
11	0.0660	0.3900	620.82	166.90	-73.12%	635.62	2.38%	0.0016	0.0055	248.68%	0.0015	-2.82%
12	0.0684	0.4260	556.95	140.90	-74.70%	549.62	-1.32%	0.0018	0.0065	266.66%	0.0018	0.55%
13	0.0708	0.4620	477.10	120.20	-74.81%	477.24	0.03%	0.0020	0.0075	266.23%	0.0020	-0.39%
14	0.0732	0.4980	415.91	103.70	-75.07%	415.57	-0.08%	0.0023	0.0086	267.48%	0.0023	-0.13%
15	0.0756	0.5340	361.89	90.27	-75.06%	362.48	0.16%	0.0027	0.0098	263.52%	0.0027	-0.57%
16	0.0780	0.5700	318.42	79.21	-75.12%	316.38	-0.64%	0.0030	0.0110	261.79%	0.0030	0.31%
	0.0600	0.3000	2314.34	972.20	-57.99%	2171.92	-6.15%	0.0198	0.0684	244.50%	0.0197	-0.60%

Note: The % error is based on the results expressed to 6 decimal places.

CHAPTER 8

Discussion and Conclusions

8.0 Discussion

Even though Slotted Rings are still to be made popular and be standardised internationally, previous research has shown that they can perform exceptionally well in high speed, short packet length environments such as those required to transmit voice, video and computer data as integrated services (Zafirovic 88).

The common availability of high speed communication hardware and their ever increasing transmission rates are therefore likely to focus attention toward Slotted Rings as a network for future integrated services local area communication, thus generating the motivation for some of the research carried out in this thesis.

Most modelling studies carried out to date in the area of Slotted Rings (both basic slotted rings as well as its extensions such as the Orwell ring) only consider those cases where the packet buffer of a station is either unity or infinite (Arem 90, Harrus 85, King 87, Mitrani 86, etc.). Such models are quite limited in their ability to reflect practical networks having finite buffers and suffer from being unable to accurately predict one or more of the performance measures such as the average the packet queue, delay and the packet rejection probability; all of which are essential in designing real-life systems. In this research, an attempt has been made to rectify this deficiency.

Due to the reason mentioned above, except for the throughput which is less dependent on the maximum buffer size, no data or graphs of the other performance measures for models with a finite buffer were readily available, from different studies, for a direct comparison between the models in this thesis.

For the case of the throughput, such a comparison shows that all the models in this thesis, in general, give out as good, or better results, when matched to simulations.

The models developed in this thesis and their characteristics will be briefly summarised below.

8.0.1 Slotted Ring, Source Deletion, Station Based Models

In chapter 2, four models for the Slotted Ring with a source deletion protocol were analysed, all models being based upon observing the network from a station.

The second and third models, SDStnBM2 and SDStnBM3 respectively, were obtained by the simplification of the first model SDStnBM1 by using an additional assumption that no packets may arrive to a station while it is transmitting. Even though the degradation of results due to this assumption were minimal and was greatly outweighed by the extent of simplicity achieved, for increasing M/N ratios, it did however continue to limit the range of packet arrival probabilities applicable to those models.

SDStnBM3 only differed from SDStnBM2 by the fact that it assumed that the packet was retained in the station buffer until the slot in which it was being transmitted was emptied upon its return to the transmitting station. This model thus reflects a situation where an acknowledgement of reception is required.

SDStnBM4 was an extended model of SDStnBM1 in which the performance of the queue, when the station is not transmitting, was modelled in greater detail. However, the improvement from this exercise to the performance predictions were only marginal when compared to the increase in complexity it brought forth.

When Markov models become too complex to be solved using equations, an alternate option available is to convert the balance equations into a matrix form so that they may be solved numerically using a computer based method. Such a method, which suits models with state dependent transition probabilities as present in station based models, was introduced in chapter 2 and used to solve SDStnBM4.

This last model was also used to illustrate how a station based model may be easily converted into a model based on observing the network globally.

8.0.2 Slotted Ring, Source Deletion, Slot Based Model

An uncommon, but quite effective way of modelling a time slotted network is to model it by observing the network from a server (slot in this case). In chapter 3, such a model was developed for the slotted ring source deletion protocol.

Unlike for the SDStnBM1, equations were easily obtained to solve the slot based model (SDSlotBM) for a single variable, thus greatly simplifying the iterative procedure involved.

The results, as expected due to the assumption of independent stations and slots, showed the ideal performance characteristics (see Appendix A). Therefore, the best match to the simulation was when there was only one slot in the network, thus eliminating the occurrence of performance degrading quasi stable states.

In this chapter, an alternative method of solving slot based models using matrices was also introduced. Even though this method was simpler than the one suggested for the station based models, the large maximum buffer size per slot that was generated when the N/M ratio was high, created a large number of states and hence a large number of equations which had to be solved. Though this lead to a prohibitively high computation time, an often effective shortcut, applicable to all slot based models in this thesis when solved by an iterative sparse matrix routine, was used. It is noted that other models of time slotted networks based on the server may not always lend themselves to such a shortcut.

8.0.3 Slotted Ring, Destination Deletion, Station Based Models

As a prelude to modelling practical destination deletion slotted rings protocols such as the Orwell Ring, chapters 4 and 5 were dedicated to model and investigate the basic destination deletion mechanism based on a station and a slot respectively.

Two possible ways of calculating the destination deletion probability, viz constant or variable, were considered and their relative merits analysed for both complete and simplified models.

The results showed that the assumption of a variable destination deletion probability gave the best results. This is undoubtedly due to the fact that, as shown in appendix B, it is only under this method that a packet will be, on average, addressed to a station half way round the ring - a situation which will occur in a real life network with statistically identical stations.

The greatest shortcoming of the simplified models was their limitation of the maximum packet arrival rate, which, when the ratio M/N was large, did not even reach up to the network saturation levels. However, since this ratio is low in most real life networks, these models may be used effectively under normal operating (unsaturated) conditions.

8.0.4 Slotted Ring, Destination Deletion, Slot Based Models

In chapter 5, the basic destination deletion protocol for the slotted ring was modelled based upon observing the network from a slot. Both the earlier mentioned methods of calculating the destination deletion probability were analysed and it was found that the variable destination deletion probability gave an excellent prediction to the simulation results as well as giving the overall best match when considering all the basic destination deletion models in the thesis.

Due to the fact that the type of simplification done in the previous chapters would greatly reduce the range of validity of the packet arrival rate for all possible combinations of network parameters, such a simplification was not carried out for the two models in chapter 5.

8.0.5 Orwell Protocol For The Slotted Ring

Though still in an experimental stage, the Orwell Ring with its trial/reset mechanism has been found to be quite effective in carrying delay sensitive, real time data.

In chapter 6, two models based on a station and a slot respectively, were developed for the Orwell Protocol by extending the destination deletion models to incorporate the above mentioned trial/reset mechanism.

Performance measures obtained for the transmission of voice as class 1 data by using the network standards suggested by the designers - British Telecom, show a good match to the simulation results. Of the two models, the slot based one give superior predictions overall.

8.0.6 Modelling Of Nonidentical Stations

One of the major restrictions in the station based models of chapters 1 to 6 is that all stations should be statistically identical. In chapter 7, two algorithms which may be used to relax this constraint were introduced.

In the first algorithm, the method was reformulated into a recursion based upon the number of users (stations). To solve, this recursion was started with an initial population of zero stations and then built up to the required population level.

The second algorithm broadly consisted of replacing the variable of the EPA method which considers the total average usage of the network, with the sum of the individual station usages. This enabled the contribution of each station to be calculated in an iterative manner.

In order to show the generality of these methods, they were applied to both a Slotted Aloha and a Slotted Ring network. Validations made via simulations showed a good match in general, whenever the network and the model were stable. The accuracy of the results predicted were of course subject to the limitations of the basic models.

The major shortcoming of the recursive algorithm was that its worst case time complexity is of order $O(2^N-1)$ where N is the number of nonidentical stations. However, the second iterative algorithm had a much improved worst case time complexity of order $O(N)$.

8.1 Further Research

When the packet arrival rate increases, it becomes erroneous to assume that only a single packet may arrive at a station in a given time slot (slot-times or station-times). Some research should be done to examine the use of other arrival distributions, such as extended Bernoulli distributions for example (Pujolle 91), that allow for multiple packet arrivals in a time slot.

Under the analysis of nonidentical stations considered so far, the only variation between stations consisted of dissimilar packet arrival rates or retransmission probabilities. Since the algorithms suggested may be readily applied to the cases where there are variations in the maximum buffer size, preferential data sources (such as file servers) and targets (e.g. data gathering devices), etc., these algorithms could be used to analyse the performance of various time slotted networks under such different conditions.

The Orwell models and the simulation can be extended to cover effects such as data compression and talkspurt/silence characteristics which enable a greater amount of information to be transmitted in a given system. Details of the slot structure, transmission rate, line delays, etc. could be included so that they may be optimised in accordance with the accepted standards.

One of the important factors still to be standardised in the Orwell protocol is exactly when a station should stop incrementing its Di counter and reject any further requests for new connections. It is felt that this area could be investigated more thoroughly with the use of models and simulations.

8.2 Conclusion

This research attempted to model various medium access control protocols for Slotted Ring local area networks using fixed point approximation methods based on Markov chains. The models developed for the protocols - source deletion, destination deletion and the Orwell, were based on observing the network from a station as well as from a slot, and, provided satisfactory performance predictions. Where possible, various simplifying assumptions and their effect on the accuracy of the solution were examined.

Finally, two general algorithms which may be used to model nonidentical stations were presented. To demonstrate their generality, these were applied to both Slotted Ring and Slotted Aloha type networks which reflect two quite different access mechanisms, the latter being a satellite packet broadcast network.

APPENDIX A

In this section, a comparison of the network performance measures at the two possible limits of packet arrival probability, between the logically expected values and the ones obtained by some of the models, will be made. This comparison will not cover the station based models due to the fact that they always give rise to an iterative solution, rather than a closed form result.

Logical expected limits

Throughput

The throughput can be considered as the packet service rate, or, as the number of packets carried by a single average slot within one cycle.

At saturation, a slot is filled by the station downstream to the one that emptied it. (It is noted that in the case of the Source Deletion protocol, this occurs only under ideal conditions due to the quasi stable states. (Falconer 85)) Therefore, only one station time is wasted before the slot is filled with a new packet. Thus, considering the mean transmission time per packet,

$$\begin{matrix}\textit{Saturation throughput for source}\\ \textit{deletion slotted ring networks}\\ \textit{under ideal conditions}\end{matrix} = \frac{1}{1+\frac{1}{N}} \qquad (A.1)$$

where the denominator represents the time needed to transmit a single packet. Also,

$$\begin{array}{l}\textit{Saturation throughput for}\\ \textit{destination deletion}\\ \textit{slotted ring networks}\end{array} = \frac{1}{\frac{1}{2} + \frac{1}{N}} \qquad \text{(A.2)}$$

The Orwell protocol uses the destination deletion of packets, but adds its own overheads due to the trial/reset mechanism used. From equation (6.1.16), this amounts to -

$$\left(N + \frac{N}{M} + 1\right)\frac{1}{N} \qquad (\textit{cycles}) \qquad \text{(A.3)}$$

As we are considering the saturated values, the reset times are at their highest. Therefore from equations (6.1.17) and (A.3),

$$\begin{array}{l}\textit{Performance degredation}\\ \textit{due to trial/reset}\\ \textit{mechanism}\end{array} = \frac{Dmax\frac{N}{M}\left(\frac{1}{2}+\frac{1}{N}\right)}{Dmax\frac{N}{M}\left(\frac{1}{2}+\frac{1}{N}\right)+\left(1+\frac{1}{M}+\frac{1}{N}\right)} \qquad \text{(A.4)}$$

Since the destination deletion of slots is independent of this trial/reset mechanism, by multiplying the equations (A.2) and (A.4),

$$\begin{array}{l}\textit{Ideal saturated}\\ \textit{throughput of the}\\ \textit{Orwell network}\end{array} = \frac{Dmax\frac{N}{M}}{Dmax\frac{N}{M}\left(\frac{1}{2}+\frac{1}{N}\right)+\left(1+\frac{1}{M}+\frac{1}{N}\right)} \qquad \text{(A.5)}$$

Average Queue

At saturation, the average queue should obviously be equal to the maximum buffer size.

Average Delay

When the packet arrival rate is zero, this parameter is not defined. However, since any packet has to wait at least 1 slot time before transmission, minimum delay is unity.

Assuming a full buffer and an ideal throughput at saturation, using Little's law,

$$\begin{aligned}\textit{Average Delay at saturation for source deletion slotted ring} &= NQ\left(1+\frac{1}{N}\right)+1 \\ &= (N+1)\,Q+1\end{aligned} \tag{A.6}$$

$$\begin{aligned}\textit{Average Delay at saturation for destination deletion slotted ring} &= NQ\left(\frac{1}{2}+\frac{1}{N}\right)+1 \\ &= (N+2)\,\frac{Q}{2}+1\end{aligned} \tag{A.7}$$

$$\textit{Average Delay at saturation for Orwell ring} = \frac{NQ\left[Dmax\frac{N}{M}\left(\frac{1}{2}+\frac{1}{N}\right)+\left(1+\frac{1}{M}+\frac{1}{N}\right)\right]}{Dmax\frac{N}{M}}+1 \tag{A.8}$$

Packet Rejection Probability

Assuming ideal saturation throughputs,

$$\textit{Packet service rate at saturation for source deletion slotted rings} = \frac{1}{N+1} \quad \{\textit{per station time}\}$$

Therefore, when packet arrival rate is unity,

$$\text{Saturation PRP for source deletion slotted rings} = \frac{1 - \dfrac{1}{N+1}}{1}$$

$$= \frac{1}{1+\dfrac{1}{N}} \qquad (A.9)$$

Similarly,

$$\text{Packet service rate at saturation for destination deletion slotted rings} = \frac{1}{\dfrac{N}{2}+1} \quad \{\text{per station time}\}$$

$$\text{Saturation PRP for destination deletion slotted rings} = \frac{1 - \dfrac{1}{\left(\dfrac{N}{2}+1\right)}}{1}$$

$$= \frac{1}{1+\dfrac{2}{N}} \qquad (A.10)$$

$$\text{Saturation PRP for Orwell protocol} = 1 - \frac{\dfrac{Dmax}{M}}{Dmax\dfrac{N}{M}\left(\dfrac{1}{2}+\dfrac{1}{N}\right)+\left(1+\dfrac{1}{M}+\dfrac{1}{N}\right)} \qquad (A.11)$$

<u>Network Reset Rate (Orwell protocol only)</u>

When there are no packets arriving to the network, the reset rate is at its highest. Thus,

$$\textit{Reset Rate with no packet arrivals} = \frac{1}{1+\frac{1}{M}+\frac{1}{N}} \qquad \text{(A.12)}$$

However, at saturation, considering the time needed to transmit Dmax packets per station as well as the average time needed for the trial/reset mechanism,

$$\textit{Reset Rate at saturation} = \frac{1}{Dmax\frac{N}{M}\left(\frac{1}{2}+\frac{1}{N}\right) + \left(1+\frac{1}{M}+\frac{1}{N}\right)} \qquad \text{(A.13)}$$

SDSlotBM

At $p=1$, $P(y,x)=0$ $(0 \leq y < Q,\ 0 \leq x \leq N)$. Then the model simplifies to that shown in figure A.1.

Considering the steady state equilibrium,

$$P(Q,x) = P(Q,x-1) \qquad \{1 \leq x \leq N\}$$

Summing all the state probabilities,

$$\begin{aligned} 1 &= \sum_{y=0}^{Q}\sum_{x=0}^{N} P(y,x) \\ &= \sum_{x=0}^{N} P(Q,x) \\ &= (N+1)\ P(Q,0) \end{aligned}$$

$$P(Q,0) = \frac{1}{N+1}$$

Then,

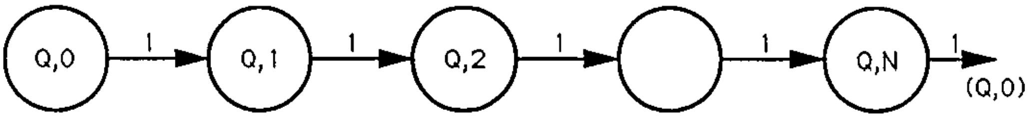


Fig. A.1

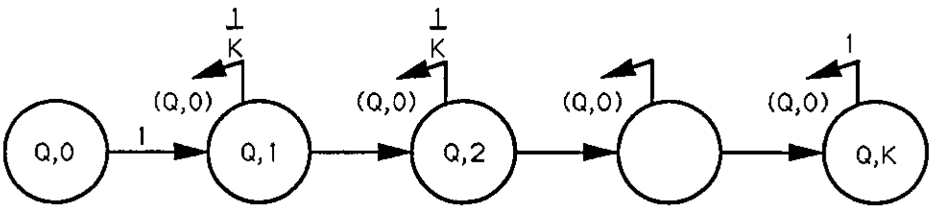


Fig. A.2

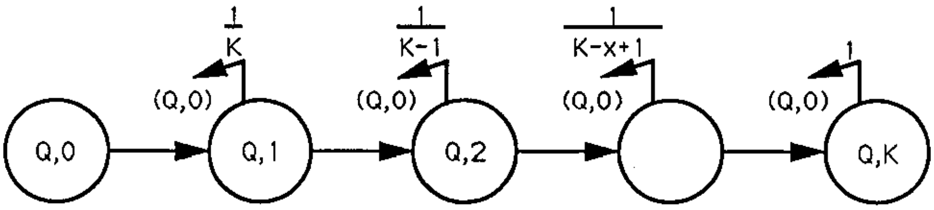


Fig. A.3

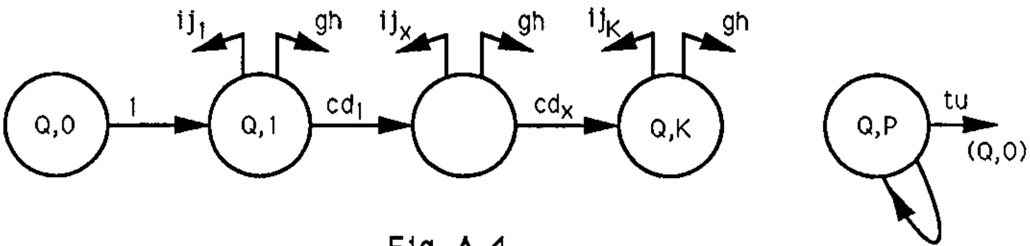


Fig. A.4

$$Throughput\ at\ saturation = P(Q,0)\,N$$

$$= \frac{N}{N+1} \tag{A.14}$$

which is identical to equation (A.1).

For p=0 and p=1, the average queue takes the values of zero and Q respectively. This result is quite obvious, but can be shown to be true by solving the model after substituting these values of p.

Considering the fact that at the two limits of possible packet arrival rates both the throughput and the average queue become equivalent to their logically expected values, the delay which is calculated using these two performance measures should also result in an ideal outcome as in equation (A.6).

As in the case of average delay, since the throughput shows the ideal values, the PRP which is dependent on this performance measure should result in an equation identical to (A.9).

DDSlotBM1

At p=1, P(y,x)=0 ($0 \le y < Q$, $0 \le x \le K$), where K=N-1. Thus the model reduces to that shown in figure A.2.
Then, as in equation (4.3.7),

$$P(Q,x) = P(Q,1)\,a^{x-1} \qquad \{1 \le x \le K\}$$

where,

$$a = 1 - \frac{1}{K}$$

Since

$$P(Q,0) = P(Q,1)$$

$$P(Q,x) = P(Q,0)\,a^{x-1} \qquad \{1 \le x \le K\}$$

Summing all the state probabilities,

$$\begin{aligned} 1 &= \sum_{y=0}^{Q} \sum_{x=0}^{K} P(y,x) \\ &= P(Q,0)\,[2+a+a^2+a^3+\dots+a^{K-1}] \\ &= P(Q,0)\,[1 + (1-a^{K})K] \end{aligned}$$

Thus,

$$\begin{aligned} \mathit{Throughput\ at\ saturation} &= P(Q,0)\,N \\ &= \frac{N}{[1+(1-a^{K})K]} \end{aligned} \qquad \text{(A.15)}$$

This equation is quite different to (A.2). Since the average delay and the PRP are functions of the throughput, their saturation values too will be different from their respective logical values.

DDSlotBM2

When p=1, the Markov diagram in figure 5.1.1 simplifies to that shown in figure A.3.

At steady state equilibrium,

$$P(Q,x) = \frac{(K-x+1)}{K}\,P(Q,1) \qquad \{1 \le x \le K\}$$

as in equation (4.4.7).

Since,

$$P(Q,0) = P(Q,1),$$

$$P(Q,x) = \frac{(K-x+1)}{K} P(Q,0) \qquad \{1 \le x \le K\}$$

As before, summing all the state probabilities,

$$\begin{aligned} 1 &= \sum_{x=0}^{K} P(Q,x) \\ &= P(Q,0)\,\frac{N+2}{2} \end{aligned}$$

Then,

$$\begin{aligned} Throughput &= P(Q,0)\,N \\ &= \frac{2}{N+2}\,N \end{aligned} \qquad \text{(A.16)}$$

This is equivalent to equation (A.2).

Since the average queue does vary between 0 and Q for p=0 and p=1, ideal limits are achieved.

Then,

$$\begin{aligned} Average\,delay\ (\bar{d}) &= \frac{NQ}{Throughput} + 1 \\ &= \frac{(N+2)\,Q}{2} + 1 \end{aligned} \qquad \text{(A.17)}$$

which is identical to equation (A.7), and,

$$PRP = 1 - \frac{Throughput}{N} = \frac{1}{1+\frac{2}{N}} \qquad (A.18)$$

identical to equation (A.10).

OSlotBM

As in the previous models, when p=1, the OSlotBM model simplifies to that shown in figure A.4. Using equations (6.1.3) and (6.1.4), (6.1.6) and (6.1.7), (6.1.8) and (6.1.9), and, (6.1.11) and (6.1.12), the following four equations may be obtained.

$$cd_x = c_x + d_x = \frac{K-x}{K-x+1}$$

$$gh = g + h = \frac{2M}{N^2\, Dmax}$$

$$ij_x = i_x + j_x = \frac{1}{K-x+1} - \frac{2M}{N^2\, Dmax}$$

$$tu = t + u = \frac{M}{N(M+1)}$$

As in DDSlotBM2, it can be shown that,

$$P(Q,x) = P(Q,0)\,\frac{(K-x+1)}{K} \qquad \{1 \le x \le K\}$$

At steady state equilibrium,

$$P(Q,T)\,tu = gh\sum_{x=1}^{K} P(Q,x)$$

which, using the above equations, simplify to -

$$P(Q,T) = P(Q,0)\,\frac{M+1}{Dmax}$$

Summing all the state probabilities,

$$1 = \sum_{x=0}^{K+1} P(Q,x)$$

$$= P(Q,0)\left[1 + \frac{N}{2} + \frac{M+1}{Dmax}\right]$$

Thus,

$$\begin{aligned} \begin{matrix}\textit{Throughput at}\\ \textit{saturation}\end{matrix} &= P(Q,0)\,N \\ &= \frac{N}{1 + \frac{N}{2} + \frac{M+1}{Dmax}} \\ &= \frac{\frac{N}{M}Dmax}{Dmax\frac{N}{M}\left(\frac{1}{N} + \frac{1}{2}\right) + \left(1 + \frac{1}{M}\right)} \end{aligned} \qquad \text{(A.19)}$$

When compared to (A.5), the only difference in the above equation is that the denominator of (A.19) lacks a term 1/N. The effect of this is minimal and with the increase of N, becomes negligible.

The average delay and the PRP, calculated in the usual way, too will show this small deficiency.

$$\begin{aligned}\text{Reset Rate at saturation} &= P(Q,T)\ tu\ N \\ &= \frac{1}{\left(\frac{1}{N}+\frac{1}{2}\right)Dmax\frac{N}{M}+\left(1+\frac{1}{M}\right)}\end{aligned} \tag{A.20}$$

which again, shows the lack of a 1/N term in the denominator when compared to (A.13).

At p=0, P(0,0)=1. Therefore, from equation (6.2.6),

$$\text{Reset Rate when no packets arrive} = \frac{1}{1+\frac{1}{M}+\frac{1}{N}} \tag{A.21}$$

which is identical to (A.12).

APPENDIX B

In this appendix it will be shown that, of the two destination deletion probability methods used, only the variable destination deletion probability will result in a packet being addressed, on average, to a station half way round the ring.

Constant Destination Deletion Probability

For ease of illustration and computation, consider a segment of the Markov chain where the buffer size remains constant (figure B.1). The packet arrivals during transmission will be ignored since it does not affect the result.

If P(x) denotes the probability of being in state x, then at steady state,

$$P(2) = P(1)\left(\frac{K-1}{K}\right)$$

$$P(3) = P(2)\left(\frac{K-1}{K}\right) = P(1)\left(\frac{K-1}{K}\right)^2$$

$$P(x) = P(1)\left(\frac{K-1}{K}\right)^{x-1} \qquad \{1 \le x \le K\}$$

where, as usual, K = N-1

$$\begin{matrix}\text{Mean distance}\\ \text{travelled}\\ \text{by a packet}\end{matrix} = \frac{\displaystyle\sum_{x=1}^{K}\left[\begin{matrix}\text{Distance of node}\\ \times\ \text{State probability of the node}\\ \times\ \text{Probability of leaving the node}\end{matrix}\right]}{\displaystyle\sum_{x=1}^{K}\left[\begin{matrix}\text{State probability of the node}\\ \times\ \text{Probability of leaving the node}\end{matrix}\right]} \qquad (B.1)$$

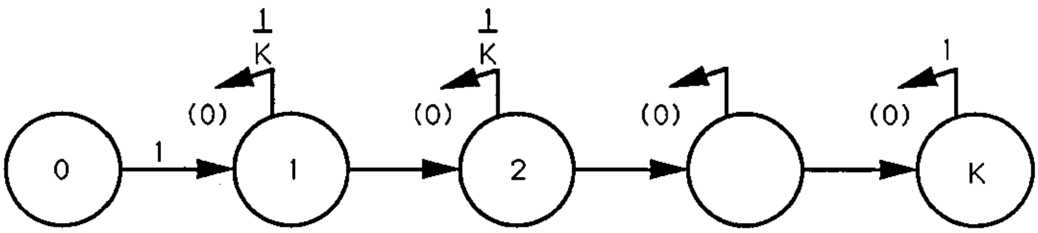


Fig. B.1

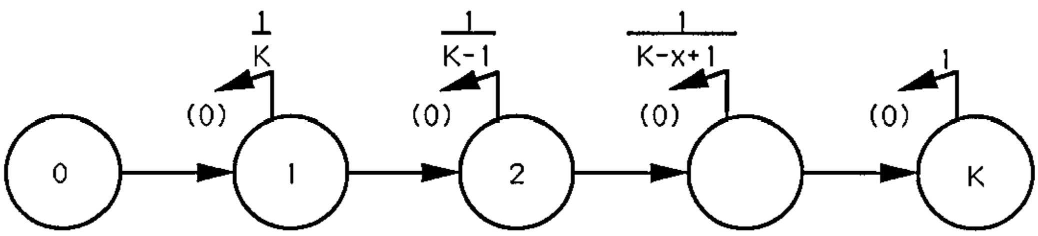


Fig. B.2

Thus,

$$\begin{matrix}\textit{Mean distance}\\ \textit{travelled}\\ \textit{by a packet}\end{matrix} = \frac{\displaystyle\sum_{x=1}^{K-1} xP(1)\left(\frac{K-1}{K}\right)^{x-1}\frac{1}{K} + KP(1)\left(\frac{K-1}{K}\right)^{K-1}1}{\displaystyle\sum_{x=1}^{K-1} P(1)\left(\frac{K-1}{K}\right)^{x-1}\frac{1}{K} + P(1)\left(\frac{K-1}{K}\right)^{K-1}1}$$

Summing up and simplifying this, it can be shown that,

$$\begin{aligned}\begin{matrix}\textit{Mean distance}\\ \textit{travelled}\\ \textit{by a packet}\end{matrix} &= (N-1)\left[1 - \left(\frac{N-2}{N-1}\right)^{N-2}\right] + \left(\frac{N-2}{N-1}\right)^{N-2}\\ &\neq \frac{N}{2}\end{aligned}$$

<u>**Variable Destination Deletion Probability**</u>

Figure B.2 illustrates the relevant Markov chain. Then,

$$\begin{aligned}P(2) &= P(1)\left(\frac{K-1}{K}\right)\\ P(3) &= P(2)\left(\frac{K-2}{K-1}\right)\\ &= P(1)\left(\frac{K-2}{K}\right)\\ P(x) &= P(1)\left(\frac{K-x+1}{K}\right) \qquad \{1 \le x \le K\}\end{aligned}$$

Applying in equation (B.1),

$$\begin{array}{l}\textit{Mean distance}\\ \textit{travelled}\\ \textit{by a packet}\end{array} = \frac{\displaystyle\sum_{x=1}^{K} xP(1)\left(\frac{K-x+1}{K}\right)\frac{1}{(K-x+1)}}{\displaystyle\sum_{x=1}^{K} P(1)\left(\frac{K-x+1}{K}\right)\frac{1}{(K-x+1)}}$$

$$= \frac{N}{2}$$

APPENDIX C

Here, it will be shown that the variables i_x and j_x $\{1 \le x \le K\}$ of Chapter 6 are non-negative for practical systems.

In order for the above condition to exist, from equations (6.1.8) and (6.1.9),

$$\frac{1}{K-x+1} \geq \frac{2M}{N^2\,Dmax}$$

Considering the minimum value of the L.H.S.,

$$\frac{1}{K} \geq \frac{2M}{N^2\,Dmax}$$

$$Dmax \geq \frac{2M(N-1)}{N^2}$$

Substituting for Dmax from equation (6.1.18),

$$\left[\frac{L_d R}{R_1 M L} - \left(1+\frac{1}{M}+\frac{1}{N}\right)\right]\frac{M}{\frac{N}{2}+1} \geq \frac{2M(N-1)}{N^2}$$

which simplifies to,

$$\left[\frac{L_d R}{R_1 L} - 1\right]\frac{1}{M} - 1 \geq \frac{N^2+2N-2}{N^2}$$

The R.H.S. of this equation can be shown to have a maximum value of 1.5 at N=2. Therefore,

$$\left[\frac{L_d R}{R_1 L} - 1\right]\frac{1}{2.5} \geq M$$

Considering the proposed slot architecture (Falconer 85),

L = 160 bits per slot

L_d = 128 bits per slot

If R_1 = 64 Kbaud, then

R (Mbaud)		M
20	$\leq$	99
100	$\leq$	499
500	$\leq$	2499

At the network transmission rates considered above, the practical numbers of slots that may be used are far below the maximum allowable by the restrictions of the model. Therefore, as we can see, the maximum value of M that can be applied to the model far exceeds the practical limits.

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