Game, Set and Match: Evaluating the Efficiency of Male Professional Tennis Players

Anthony J. Glass, Karligash Kenjegalieva[†] and Jason Taylor

School of Business and Economics, Loughborough University, Loughborough, Leicestershire, LE11 3TU, UK.

E-mails:A.J.Glass@lboro.ac.uk, K.A.Kenjegalieva@lboro.ac.uk , and J.L.Rave@lboro.ac.uk

Abstract

We exploit the natural distinction between the attacking and defensive aspects of tennis to get a better understanding of the origins of relative inefficiency. Attacking is simply when a player is serving and defending is when a player is returning serve. We use data envelopment analysis (DEA) to compute the attacking, defensive and overall efficiencies of the top 100 male professional players for the 2009 season. An analysis of the efficiency scores using non-parametric kernel smoothing suggests that there are four groups of players in the sample- those that are relatively efficient in attack and defence; those that are relatively inefficient in attack and defence; and those that are relatively efficient in attack or defence. Truncated regression equations for the technical and super attacking, defensive and overall efficiencies as a function of off-court variables (e.g. height, age, etc.) suggest that being a left-handed player has a significant positive effect on the overall efficiencies.

Keywords: Tennis; Data envelopment analysis; Super efficiency; Truncated regression

JEL Classification: C69; L83

[†] Corresponding author- School of Business and Economics, Loughborough University, Leicestershire, UK, LE11 3TU. E-mail: K.A.Kenjegalieva@lboro.ac.uk. Tel.: +44(0)1509222708; fax: +44(0)1509222701.

1. Introduction

Professional tennis players compete for large amounts of money. In 2007, for example, Roger Federer became the first player to win over $10,000,000 in a single season. In recent years there has also been a marked increase in the availability of statistics on individual facets of the game (e.g. average number of aces per match). Whilst these statistics are a valuable source of information for players, coaches, etc. on individual aspects of a player's game, these statistics do not include a measure of overall performance across all facets of the game. In this paper therefore, we estimate a tennis player's overall efficiency. One measure of a tennis player's overall efficiency which we employ refers to a player's ability to translate his on-court performance in all facets of the game into prize money.

The most recent quantitative analyses of tennis are primarily due to the work of Klaassen and Magnus (2001; 2003; 2009), and Magnus and Klaassen (1999a; 1999b; 1999c). Notwithstanding the seminal contribution of their work in this area, their work uses data on individual points at the Wimbledon Championships to tackle research questions which relate to particular aspects of the game (e.g. whether there is an advantage from serving with new balls) or individual matches (e.g. whether it is easier for an unseeded woman to beat a seeded player than it is for an unseeded man). We recognise, however, that there is scope for a broad frontier analysis of tennis. Therefore, the purpose of this paper is to obtain composite measures of performance for the top 100 male players on the ATP tour for the 2009 season.[1] Using data envelopment analysis (DEA), we calculate attacking, defensive and overall efficiencies which provide a more detailed view of a player's ability to achieve his aim of winning money, given the resources at his disposal.

In the DEA we firstly, use a single output and then secondly, dual outputs and in each case calculate three sets of technical and super efficiencies (attacking, defensive and overall). The key output and hence that which is used in the single output DEA is prize money.[2] This is because players set out in their careers to do the best that they can in the big tournaments where the prize money on offer is much larger. We recognise, however, that some players enter tournaments for strategic reasons in the dual outputs DEA, where the outputs are prize money and win percentage. For example, it is common for young players with a lot of potential to play in smaller tournaments which the top players do not enter to

[1]The Association of Tennis Professionals (ATP) tour is the tour which the top male professional players compete on.

[2]We thank an anonymous referee for the justification for prize money being the key output.

gain experience and confidence from winning rather than to win prize money.[3] A player returning from injury may enter smaller tournaments for the same reason. Such a strategy is taken into account by using win percentage as one of the outputs in the dual outputs DEA.

To calculate the attacking technical and super efficiencies we use the above output(s) and the attacking inputs. Examples of attacking inputs include average aces per match and the inverse of average double faults per match. To calculate the defensive technical and super efficiencies we use the above output(s) and the defensive inputs. Defensive inputs are, for example, percentage of break points won and percentage of points won returning the second serve. Finally, we calculate overall efficiencies by using the above output(s) and both sets of attacking and defensive inputs. Notwithstanding that both attacking and defensive inputs are used to produce the output(s), by using the 'dropping an input' approach employed by Fried *et al*. (2004) we find that the inputs which are important for technical attacking efficiency and technical defensive efficiency are not necessarily important for technical overall efficiency. To illustrate, we find that average aces per match and the inverse of average double faults per match are the most important inputs for technical attacking efficiency but they are not very important inputs for technical overall efficiency. The most important input for technical overall efficiency is percentage of points won on the second serve. Interestingly, this suggests that service performance dominates for technical attacking efficiency, whereas given that the second serve is far less likely to dominate a point than the first serve, we can conclude that it is a player's ability to win a point on his second serve when a rally commences which is most important for technical overall efficiency.

Two other key empirical findings emerge from this paper. The first relates to the attacking and defensive technical efficiencies, and the other is from the second stage truncated regressions of the technical and super attacking, defensive and overall efficiencies on a set of off-court variables (e.g. height, age, etc.). Firstly, the kernel density distributions of the attacking and defensive technical efficiencies with both a single output and dual outputs indicate that there are four groups of players in the sample- those that are relatively efficient in attack and defence; those that are relatively inefficient in attack and defence; and those that are relatively efficient in attack or defence. Secondly, although within tennis it is widely felt that taller players have faster serves and will therefore be more proficient attackers, we find that height can have a significant negative effect on attacking efficiency.

[3]From casual observation of the tournaments won by the top players in our sample throughout their careers to date, we can see that very often the first tournament which they won was a small tournament that they no longer enter.

The remainder of this paper is organised as follows. In Section 2, related studies are briefly reviewed with particular reference to the application of DEA to measure efficiency in team and individual sports. In Section 3, we firstly discuss the methodological issues involved in the application of our chosen DEA techniques to the case in hand and, secondly, we provide an overview of the truncated regression techniques which are used in the second-stage analysis of the efficiency scores. Section 4 describes the data set, focusing on the outputs, the on-court attacking and defensive inputs and the off-court explanatory variables for the second-stage regressions. In section 5, the efficiency scores and the fitted second-stage regression equations are presented and analysed. In section 6, we conclude by summarising the salient features of the paper.

2. A Brief Overview of Related Studies

DEA has been widely applied to measure efficiency in sports. The vast majority of the applications of DEA to sports have focused on teams. For example, the constant returns to scale (CRS) DEA model (Charnes *et al.*, 1978) and/or its extension to allow for variable returns to scale (VRS) (Banker *et al.*, 1984) have been used to measure the efficiency of: top flight professional football clubs in England (Hass, 2003; Barros and Leach, 2006) and Germany (Hass *et al*., 2004); Major League Baseball (MLB) teams (Einolf, 2004); National Football League (NFL) teams (Einolf, 2004); and nations at the summer Olympics (Lozano *et al*., 2002). In addition, various extensions of VRS DEA have been used to measure the technical efficiency of sports teams. Sexton and Lewis (2003), for example, develop a two-stage DEA model, which involves the output from the first stage being used as an input in the second stage, and they apply the model to measure the efficiency of MLB teams.

A number of studies have stratified team sports according to the role of individuals and used DEA to estimate efficiency scores for players and managers e.g. batters in MLB (Mazur, 1994; Anderson and Sharp, 1997), batters in Central League baseball in Japan (Sueyoshi *et al*., 1999), college basketball coaches in the U.S. (Fizel and D'itri, 1996) and playmakers, power forwards, guards, etc. in male professional basketball in Spain (Cooper *et al*., 2009). To the best of our knowledge, however, there is only one application of DEA to individual sports- the analysis of golf professionals by Fried *et al*. (2004). Using data for the 1998 season they assess the average performance of males, ladies and senior males on the three tours (Professional Golf Association, PGA, Ladies Professional Golf Association, LPGA, and the Senior Professional Golf Association, SPGA) using the VRS DEA model.

Various methods are used in Fried *et al*. (2004) to establish the relative contribution of an input to a player's earnings.[4] We adopt one of these approaches here- the 'dropping an input' approach. This approach is used to analyse the relative effect of an input on efficiency, which because of concerns about the curse of dimensionality we use to inform which inputs are of lesser importance and can therefore be omitted from the analysis. Interestingly, this approach indicates that some attacking (defensive) inputs are important for technical attacking (defensive) efficiency but not very important for technical overall efficiency and vice versa. Interestingly, we therefore proceed to use the same output(s) and attacking (defensive) inputs to calculate technical and super attacking (defensive) efficiencies which differ from the combination of attacking and defensive inputs which are used to calculate the technical and super overall efficiencies.

The most recent quantitative analyses of tennis are by Klaassen and Magnus (2001; 2003; 2009), Magnus and Klaassen (1999a; 1999b; 1999c), and Walker and Wooders (2001). These studies, however, analyse particular aspects of tennis or individual matches, rather than an entire season as we do here. For example, Klaassen and Magnus (1999c) focus on particular aspects of tennis using data on almost 90,000 points at the Wimbledon Championships over the period 1992-1995. Specifically, they test whether: (i) in men's singles the dominance of service is greater than in the women's singles; (ii) a player is only as good as the quality of his/her second serve; (iii) there is a psychological advantage from serving first in a set; and (iv) fewer breaks of serve occur in the first few games of a match. The study by Klaassen and Magnus (2009) is particularly relevant to our study because using the above data for the Wimbledon Championships they undertake an efficiency analysis of service strategy. In particular, they estimate the service efficiency of players relative to the optimum (i.e. absolute efficiency) by not allowing for the possibility that the best performers will lie on the frontier. They find that, on average, there is very little service inefficiency in terms of points lost which given the quality of the server could potentially have been won (inefficiency is on average just 1.1% for men across the first and second service and the corresponding estimate for ladies is 2.0%). They conclude that by serving

[4]For the men and the senior males there are six inputs: (i) average driving distance i.e. length of tee shot in yards; (ii) percentage of drives on the fairway i.e. percentage of tee shots which land on the fairway; (iii) percentage of greens hit in regulation i.e. percentage of greens reached in two shots under par; (iv) putts per green i.e. the number of putts per green hit in regulation; (v) scrambling percentage i.e. the percentage of pars per green reached in excess of regulation; and (vi) sand save percentage i.e. the percentage of pars achieved from a sand trap. For the ladies there are five inputs because there is no data on the scrambling percentage. Also, for ladies the putts per green is the number of putts per green irrespective of whether the green has been reached in regulation.

efficiently, on average, men and ladies can increase the probability of winning a point by 0.7% and 1.2%, respectively.

Notwithstanding the contribution of the most recent quantitative studies of tennis at the level of particular facets of tennis and individual matches, it would seem that there is scope for a broader analysis at the level of a season. More specifically, it would also appear that there is potential for a non-parametric study of professional tennis players to measure the efficiency of players relative to the top performer(s).

3. Empirical Methodology

3.1 *Estimation of the Technical and Super Efficiencies*

In this paper a VRS non-parametric model is used to construct a hypothetical efficiency frontier. To estimate efficiency measures for the top 100 players on the ATP tour, an output-orientated model is used so that each player takes the given set of inputs which he has at his disposal and aims to maximise the level of output(s) simultaneously. The linear-programming problem developed by Tone (2001), which extends the Charnes *et al.* (1985) additive VRS DEA model, is solved to compute each player's relative output-orientated efficiency.

Formally, the optimum level of inputs and outputs is given by the relevant frontier which represents the common technology, *T*, which players, $j = 1,\ldots, n$, use to transform inputs, X ($m \times n$), into outputs, Y ($s \times n$):

$$\hat{T} = \{(X,Y) \mid \quad X \quad can \quad produce \quad Y\}. \qquad (1)$$

It is assumed that $\hat{T}$ is a consistent estimator of the unobserved true technology set. The matrix of observed inputs, $X = (x_{ij}) \in R^{m \times n}$, is an $n \times m$ matrix and the matrix of observed outputs, $Y = (y_{hj}) \in R^{s \times n}$, is an $n \times s$ matrix. The elements of matrices *X* and *Y* are non-negative and have strictly positive row sums and column sums. Given these conditions, the individual output-oriented efficiency for player *0* is computed relative to the estimated frontier by solving the following linear programming problem:

$$\min \qquad \hat{\rho}(x_0, y_0 | T(x)) = 1 + \frac{1}{s}\sum_{k=1}^{s} S_{k0}^{+} / y_{k0}$$

$$\text{subject to} \qquad x_{i0} = \sum_{j=1}^{n} \lambda_j x_{ij} + S_{i0}^{-},\ i=1,2,\ldots,m, \qquad (2)$$

$$y_{k0} = \sum_{j=1}^{n} \lambda_j y_{kj} - S_{k0}^{+},\ k=1,2,\ldots,s,$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

and $\lambda_j \geq 0, \quad S_{i0}^{-} \geq 0, \quad S_{k0}^{+} \geq 0,$

where λ is the intensity variable and the vectors $S^{-} \in R^{m}$ and $S^{+} \in R^{s}$ represent the input excess and output deficit, respectively, such that the measure of efficiency captures the inefficiency of a player due to slacks.

In order to distinguish between the performance of the efficient players we apply the super efficiency methodology, which enables an efficient player to achieve an efficiency score greater than one. In particular, solving the linear program problem computes the distance to the frontier which is evaluated without the player under analysis. Despite several problems with the super efficiency methodology such as differences in weights, the possibility of excessively high super efficiency scores and the infeasibility issue, the super efficiency methodology is popular in the literature (Adler et al. 2002). In this paper, for players with an efficiency score of unity (i.e. $\hat{\rho}(x_0, y_0 \,|\, T \;\; (x)) = 1$), we estimate super-efficiency using the following output-oriented super efficiency model (Tone, 2002):

$$\min \qquad \hat{\delta}(x_0, y_0 \,|\, T_{*} \;(x)) = \left[\frac{1}{s} \sum_{k=1}^{s} \bar{y}_{k0} / y_{k0} \right]^{-1}$$

$$\text{subject to} \qquad \bar{x}_{i0} \geq \sum_{j=1,\neq 0}^{n} \lambda_j x_{ij} \,,\; i=1,2,\ldots,m, \tag{3}$$

$$\bar{y}_{k0} \geq \sum_{j=1,\neq 0}^{n} \lambda_j y_{kj} \,,\; k=1,2,\ldots,s,$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

and $\lambda_j \geq 0, \quad \bar{x}_{i0} = x_{i0}, \quad 0 \leq \bar{y}_{k0} \leq y_{k0},$

where the projected values of the inputs and output(s) of player *0* to the frontier calculated having omitted player *0* are $\bar{x}_{i0}$ and $\bar{y}_{k0}$, respectively.

Bivariate kernel density functions are constructed for pairwise combinations of efficiencies (e.g. attacking and defensive, overall and attacking, etc.). This is in order to simultaneously compare any two of the three sets of efficiencies. Let $d = (d_1, d_2)$ be a sample of bivariate random vectors drawn from the distribution described by the density function f and let $H = (h_1, h_2)$ be a vector of h_1 and h_2 bandwidths for each coordinate direction. The bivariate kernel density estimate is then defined to be (see Tortosa-Ausina, 2002):

$$\hat{f}(d,H)=\frac{1}{Nh_1h_2}\sum_{j=1}^{N}K\left(\frac{d_1-NES_j^{M_1}}{h_1},\frac{d_2-NES_j^{M_2}}{h_2}\right), \tag{4}$$

where NES are efficiency scores normalised relative to the mean efficiency, $NES_j^M=\rho_j^M \Big/ \left(1/N\sum_{j=1}^{N}\rho_j^M\right)$, and M denotes attacking, defensive or overall. The bandwidths h are calculated according to the solve-the-equation plug-in approach for the bivariate Gaussian kernel (see Wand and Jones, 1994).

3.2 *Second-stage Truncated Regressions*

We regress in turn the attacking, defensive and overall technical and super efficiency scores on a set of off-court variables such as height, weight, age, etc. using two types of truncated regression suggested by Simar and Wilson (2007), where the first type is for the technical efficiency scores and the second type is for the super efficiency scores and technical efficiencies otherwise when a player is not technically efficient. The first type of truncated regression is such that

$$0\le\rho_j=\psi(z_j,\beta)+\varepsilon_j\le 1, \tag{5}$$

where ρ_j is the true efficiency measure of the j-th player and not the biased estimate; z_j is a vector of the j-th player's off-court characteristics; ψ is a smooth continuous function; β is a vector of parameters; ε_j is a truncated random variable $N(0,\sigma_i^2)$ truncated at $-\psi(z_j,\eta)$ and $1-\psi(z_j,\eta)$.

The efficiencies calculated using program (2), $\hat{\rho}_j$, however, are biased estimates of the true efficiencies, ρ_j. We therefore use Algorithm 2 in Simar and Wilson (2007) with two truncation points to correct $\hat{\rho}_j$ for the bias from the market factors. Hence, we estimate $\hat{\rho}_j$ using program (2) and then obtain estimates, $\hat{\beta}$ and $\hat{\sigma}_\varepsilon$, using the truncated regression in equation (5) by using maximum likelihood to regress $\hat{\rho}_j$ on z_j. Then we estimate the L_1=100 bootstrap estimates for each $\hat{\rho}_j$ to provide n sets of bootstrap estimates, $B_j=\{\hat{\rho}_{jb}^*\}_{b=1}^{L_1}$. For each j =1,…, n, we draw ε_j from the distribution $N(0,\hat{\sigma}_\varepsilon^2)$ with left truncation at $-\psi(z_j,\eta)$ and right truncation at $1-\psi(z_j,\eta)$, and then compute $\hat{\rho}_j^*=z_j\hat{\beta}+\varepsilon_j$. To obtain draws from a normal distribution with left and right truncations the procedure in Appendix A.2 in Simar and Wilson (2007) is used with a left truncation at constant a and a right truncation at constant b, where $a' = a/\sigma$ and $b' = b/\sigma$. We also set $v' = \Phi(a') + [\Phi(b') - \Phi(a')]v$ where we generate v from a uniform distribution (0,1). The normal deviate with right and left truncations is equal to $u = \sigma\Phi^{-1}(v')$.

To obtain bias corrected efficiencies we use transformed output(s), y_j^*, for all j =1,…, n, where $y_j^* = y_j \hat{\rho}_j / \hat{\rho}_j^*$ and keep the input measures x_j unchanged. We then replace Y with $Y^* = [y_1^* ... y_n^*, y_j]$ and use the original vector of inputs $X = [x_1 ... x_n, x_j]$ to solve program (2). In other words, the frontier for player j is constructed with respect to X and Y^*, which contain the original inputs x_j and output(s) y_j. This is due to the reference-set dependence property of the SBM efficiency measure, $\hat{\rho}_j$ (Tone, 2001). As a result of this property $\hat{\rho}_j$ 'is not affected by values attributed to other DMUs not in the reference set'. Finally, the bias corrected efficiencies, $\hat{\hat{\rho}}_j$, for each j =1,…,n are computed, where $\hat{\hat{\rho}}_j = \hat{\rho}_j + BIAS(\hat{\rho}_j)$.

Once the bias corrected efficiency scores are obtained the following steps are undertaken:

1. Obtain estimates, $\hat{\beta}$ and $\hat{\sigma}_\varepsilon$, by fitting the truncated regression of $\hat{\hat{\rho}}_j$ on z_j along the lines of equation (5) using maximum likelihood.

2. Compute a set of L bootstrap estimates for β and σ_ε, $A = \{(\hat{\beta}^*, \hat{\sigma}_\varepsilon^*)_b\}_{b=1}^{L}$, where we set L equal to 1,000 replications. Specifically, for each j =1,…, m we draw ε_j from the normal distribution $N(0, \hat{\sigma}_\varepsilon^2)$ with left truncation at $-\psi(z_j, \eta)$ and right truncation at $1-\psi(z_j, \eta)$, and compute $\hat{\hat{\rho}}_j^* = z_j \hat{\beta}^* + \varepsilon_j$. We obtain the estimates, $\hat{\beta}^*$ and $\hat{\sigma}_\varepsilon^*$, by fitting the truncated regression of $\hat{\hat{\rho}}_j^*$ on z_j using maximum likelihood. Once the set of L bootstrap estimates for β and σ_ε have been obtained, the percentile bootstrap confidence intervals are constructed.

We analyse the 'off-court' determinants of the super efficiencies when a player is technically efficient and the technical efficiencies otherwise in a similar manner but using another type of truncated regression. Since the super efficiency and technical efficiency scores, $\hat{\delta}_j$, have only one boundary at zero we employ the original methodology of Simar and Wilson (2007), and change the value of the left truncation point to zero. In other words, the following truncated regression is estimated:

$$0 \leq \delta_j = \psi(z_j, \beta) + \varepsilon_j, \qquad (6)$$

where δ_j is the true efficiency measure of the *j*-th player; ψ is a smooth continuous function, β is a vector of parameters; ε_j is a truncated random variable $N(0, \sigma_i^2)$ truncated at $\psi(z_j, \eta)$. $\hat{\delta}_j$ is calculated using programs (2) or (3) and is an estimate of δ_j.

4. Data

All the data for the inputs and outputs are for virtually the entire 2009 season (up to and including the week ending 19/10/09) for the top 100 players and was downloaded from the ATP tour website.[5] We calculate two sets of attacking, defensive and overall efficiency scores. As we justified in the opening section of this paper, we draw a distinction between which of our two outputs is they key one and use this as a single output and then we see how our results are affected by assuming that there are two outputs. Specifically, one set of efficiencies is with the key output, season earnings (US$) to date, and the other set is with two outputs- the percentage of matches won to date in the season and season earnings (US$) to date.

The standard indicators of performance when a player is serving (returning serve) are the attacking (defensive) inputs. The seven attacking inputs are: (i) *percentage of first serves in* (number of serves which land in the service box on the first attempt as a proportion of the total number of serves); (ii) *percentage of points won on the first serve* (proportion of points won given the first serve lands in); (iii) *percentage of points won on the second serve* (proportion of points won given the second serve lands in); (iv) *average number of aces per match* (a player serves an ace when his serve lands in the service box and his opponent does not make any contact with the ball); (v) *the inverse of the average number of double faults per match* (a player double faults when neither his first serve or his second land in the service box); (vi) *percentage of break points saved* (break points saved are those which if the server wins the point the game continues and the server does not lose his service game); (vii) *percentage of service games won* (proportion of games won when the player is serving).

The five defensive inputs are: (i) *percentage of points won returning an opponent's first serve* (proportion of points won when the opponent's first serve lands in the service

[5]The input data is updated periodically on the tour website throughout a season. The last update for the 2009 season is used and corresponds to 89% of the tournaments in the season i.e. all but the final six tournaments.

box); (ii) *percentage of points won returning an opponent's second serve* (proportion of points won when the opponent's second serve lands in the service box); (iii) *percentage of break points won* (if the player returning serve wins the break point the game ends and the server loses his serve); (iv) *percentage of return games won* (proportion of games won returning serve); (v) *percentage of tie breaks won* (if the score in a set is six games each a tie-break is played). A tie break begins with a player serving for one point, after which his opponent serves for two points and from thereon on each player serves alternately for two points until a player wins the tie-break by reaching at least seven points and being two points clear of his opponent. The percentage of tie breaks won is classified as a defensive input because it is not possible to win a tie break without winning a point on your opponent's serve, which reflects how good a player is at returning serve.[6] The overall efficiency scores are calculated using both attacking and defensive inputs.

The descriptive statistics for the sample are presented in Table 1. The inputs with the highest and second highest range and standard deviation are the percentage of tie breaks won and the percentage of points won on the first serve, respectively. This suggests that *vis-à-vis* his contemporaries, there is greater scope for a player to improve the overall quality of the inputs at his disposal by channelling his efforts into increasing the percentage of tie breaks which he wins and the percentage of points won on his first serve. In contrast, there is very little difference between the standard deviations of the percentage of service games won and the percentage of return games won. This suggests that there is similar scope for a player to improve in these two aspects of the game. Finally, we note that prize money is the only variable where the standard deviation is greater than the mean. With reference to the input variables this suggests that there is a relatively large variation in prize money even though there is only a relatively small variation in the inputs which the players have at their disposal.

[Insert Table 1 about here]

A distinction is made between a player's on-court inputs such as percentage of first serves in and his off-court characteristics such as being right or left-handed, height, weight, age, etc., which are explanatory variables when estimating the truncated regression equations for the attacking, defensive and overall technical and super efficiencies. It is

[6]Within the game, winning a point on an opponent's serve in a tie break is often referred to as a "mini-break" of serve.

widely felt amongst tennis commentators and pundits that, other things being equal, left-handed players are more difficult to play against. This is primarily because players are more accustomed to facing a right-handed opponent and there are fundamental differences when facing a left-handed player. For example, a swinging serve by a left-handed player will swing in the opposite direction to a swinging serve from a right-handed player. A dummy variable which takes a value of 1 if the player is right-handed and 0 otherwise is a regressor in the truncated regression equations to test whether being left-handed has a significant effect on the attacking, defensive and overall technical and super efficiencies.

Furthermore, it is widely accepted in the game that taller players have an attacking advantage because taller players usually have faster serves. Height is therefore an explanatory variable in the truncated regressions for technical and super attacking, defensive and overall efficiencies. Since tennis involves regular explosive movements around the court, weight could also be an important factor in explaining technical and super attacking, defensive and overall efficiencies. In addition, dummy variables for the continent from which a player originates are also included as explanatory variables to capture the different approaches to the game. This is because one particular court surface tends to dominate across a continent which will have a big bearing on a player's style. In particular, consistent ground strokes are key on slow clay courts in South America and Continental Europe, whereas a fast and accurate serve is much more effective on the fast hard courts in North America and Australia.[7]

It is reasonable to think that the age of a player may also have a significant effect on the efficiency scores as age could reflect, among other things, fitness, hand-eye coordination, etc. Additionally, the experience of a player (2009 minus the year in which the player turned professional) is included as an explanatory variable in the truncated regressions. This is because it is generally felt that more experienced players cope better under pressure and make better decisions.

5. Results and Analysis

5.1 *Ranking the importance of tennis players' inputs*

To rank the importance of the inputs we follow Fried *et al*. (2004) and use the 'dropping an input' approach. The method is based on there being a greater change in a player's technical efficiency when a more important input is omitted from program (2).

[7]There are no African players in the sample so to avoid perfect collinearity when estimating the truncated regression equations the omitted continent is Asia.

Specifically, technical efficiency scores are calculated when each input is dropped in turn from the DEA. Then the difference between the average technical efficiency score from the partially specified DEA and the average technical efficiency from the full DEA are calculated. These differences are used to rank the importance of the input variables. In order to reduce the number of variables in the model specifications and to guard against overestimating player performance by including inputs which are relatively unimportant, we propose the following procedure to omit input variables. In particular, an input is deemed to be of relatively little importance and is therefore dropped from the efficiency analysis if the change in average technical efficiency associated with the dropped input is less than one standard deviation of the average technical efficiency changes from the partial DEA specifications. In summary, the difference between average technical efficiency from the partial DEA when one input is dropped and average technical efficiency from the full DEA when all inputs are included shows the extent to which the original technology set is affected by the omitted input. A relatively less important input will have a relatively small effect on average technical efficiency from the full DEA. We therefore assume that the technology set is adequately determined without the relatively less important input.

With both a single output and dual outputs we solve program (2) to calculate the reported technical attacking, defensive and overall efficiencies with four, three and six inputs, respectively. Table 2 presents the ranking of the inputs when each input is dropped in turn and indicates which inputs are excluded from the efficiency analysis. The table therefore allows us to observe how big a role each input plays in determining the average attacking, defensive and overall technical efficiencies. For the average technical defensive efficiencies, the percentage of tie breaks won is the most important input. This is not surprising because if a player loses a tie break he loses an entire set, which will significantly reduce the probability of him winning the match. The percentage of tie breaks won also remains an important input for the average technical overall efficiencies. Interestingly, for average technical overall efficiencies whilst the percentage of return games won is the second most important input, the percentage of service games won is not a key input. This suggests that, on average, to improve technical overall efficiency a player would be advised to concentrate on increasing the number of games he wins when returning serve rather than focusing on increasing the number of service games he wins. This is what we would expect to find given that breaks of serve are key to winning sets and thus a match.

[Insert Table 2 about here]

We can see for the average technical attacking efficiencies that the average number of aces per match and the inverse of the average number of double faults per match are the most important inputs, and the percentage of points won on the second serve is one of the least important inputs. For the average technical overall efficiencies, however, the most important input is the percentage of points won on the second serve. Furthermore, for the average technical overall efficiencies, average number of aces per match and the inverse of the average number of double of faults per match are of lesser importance. We can therefore conclude that service performance dominates for the average technical attacking efficiencies. Given that the second serve is far less likely to dominate a point than the first serve, we can conclude that it is a player's ability to win a point on his second serve when a rally is underway which is most important for average technical overall efficiencies.

5.2 *Efficiency Scores*

The efficiency scores for selected players are presented in Table 3.[8] As we expected a number of players are technically efficient in attack, defence and/or overall so super efficiency scores are calculated to enable the performance of technically efficient players to be compared. The top four ranked players in the world in the sample are Roger Federer (world ranking 1), Rafael Nadal (world ranking 2), Novak Djokovic (world ranking 3) and Andy Murray (world ranking 4), where Federer and Nadal are technically efficient in attack, defence and overall with both a single output and dual outputs.[9] Djokovic, however, is technically efficient in attack and overall with both a single output and dual outputs but is only technically efficient defensively with dual outputs. Interestingly, the technical efficiency scores for Murray indicate that there is some latent technical inefficiency in his attacking, defending and overall play with both single and dual outputs.

[Insert Table 3 about here]

The above findings have interesting implications for the competition to be the number one player in the world to which a great deal of prestige is attached. From the above overall technical efficiency scores with dual outputs we observe that even though Nadal and Djokovic are managing their tennis inputs at best practice, Federer is the world number one.

[8] The efficiency scores for all players are available from the corresponding author on request.

[9] Of the top four ranked players in the world in the sample, Murray has the lowest prize money ($3,142,632) and Djokovic has the lowest win percentage (78.57%).

However, as pointed out by an anonymous referee, if Federer's performance is held constant, Nadal or Djokovic could rise to the top of the world rankings by improving the defensive and/or offensive dimensions of their game. With a single output, the defensive technical efficiency scores show that the gap between Djokovic and Federer in the world rankings would narrow if Djokovic improved his technical defensive efficiency with a single output. However, even if Djokovic improved his technical defensive efficiency to 100%, he would not be able to displace Federer as world number one or catch up with Nadal. This is because Djokovic's prize money when he is projected onto the best practice defensive frontier with a single output is $4.3 million, which is $0.9 million and $1.9 million below Nadal and Federer's prize money, respectively.[10] Since there is technical inefficiency in Murray's attack, defence and overall play with both a single output and dual outputs, there is scope for Murray to narrow the gap in the world rankings between himself and Federer by becoming more technically efficient. The technical efficiency scores for Murray indicate that the best way for him to narrow the gap in the world rankings between himself and Federer would be for him to improve his attacking efficiency. However, even if Murray was to improve his attacking efficiency with a single output to 100%, he would not take over from Federer as world number one. This is because Murray's prize money when he is projected onto the attacking frontier with a single output is $5.4 million below Federer's prize money.

We now briefly discuss the super efficiency scores in Table 3. When computing the super efficiencies the relevant technically efficient player is omitted from the construction of the frontier. This is, however, potentially inconsistent with the technical efficiency scores. This is because if we omit a technically efficient player, the technical efficiencies of those players in the reference set of the omitted player will change. For example, when computing the technical attacking efficiencies with a single output, Nadal is in the reference set of 55 players. The weights attached to Nadal in the calculation of the technical attacking efficiencies of these players ranges from 0.9% (Andrey Golubev- world ranking 61)-70.1% (Juan Carlos Ferrero- world ranking 21). The efficiency scores of these 55 players ranges from 0.13 (Ivo Minar- world ranking 69)-0.75 (Andy Murray), where the weights attached to Nadal when calculating these lower and upper limit efficiencies are 19.2% and 10.7%, respectively. As an anonymous referee suspected our results are a little sensitive to an outlying performer such as Nadal. To illustrate, with a single output the average technical attacking efficiency score of the 55 players increases by 0.08 when Nadal is omitted from

[10] We thank an anonymous referee for suggesting that we use projected prize money in the analysis of the technical efficiency scores.

the analysis. Moreover, with a single output the technical attacking efficiency scores of the 55 players range from 0.14 (Ivo Minar)-0.91 (Albert Montanes- world ranking 40) when Nadal is omitted. In our preferred results which we report, however, Nadal is included in the analysis because he is a key member of the top 100 players.[11]

Turning our attention to the efficiency results for players further down the world rankings. The extreme case is Michael Russell who has the lowest world ranking in the sample but yet is technically efficient in attack, defence and overall with both a single output and dual outputs. Russell's prize money and win percentage are $78,259 and 33%, both of which are well below their respective sample means. We are therefore able to conclude from Russell's technical efficiency scores that his relatively low prize money and win percentage is due to the poor quality inputs at his disposal. To increase his prize money and win percentage Russell must therefore increase his inputs. In direct contrast, Robert Kendrick (world ranking 86), Robby Ginepri (world ranking 92) and Marcel Granollers (world ranking 78) are the most striking examples of players who have a low world ranking and exhibit technical inefficiency in their attacking, defensive and overall performance with both a single output and duals outputs. These players are seriously underperforming given their attacking, defensive and overall aptitude. For these players there is considerable scope to increase their outputs and hence move up the world rankings by making better use of the inputs which they have at their disposal.

Thus far we have discussed the performance of players with technical inefficiency in their attacking, defensive and overall play. There are, however, players whose relative technical inefficiency is in attack or defence. One such player is John Isner (world ranking 42) who is an interesting example because he is 6 ft 9 inches and renowned for this attacking because of his big serve, having recorded the eighth fastest serve in history at 149.9 mph. His technical attacking efficiency scores, however, are a meagre 0.20 and 0.12 with a single output and dual outputs, respectively. Given the serve is the key attacking shot in tennis, Isner's technical attacking efficiency scores suggest that he is making very inefficient use of his big serve. On the other hand, Isner's defensive play is technical efficient.

In direct contrast to Isner, with both a single output and dual outputs Fabio Fognini (world ranking 57) and Juan Ignacio Chela (world ranking 94) are technically efficient in attack and there is technical inefficiency in their defensive play. These results suggest that

[11]It should be noted that the players' super efficiency scores are not compared. Since super efficiency scores capture the extent to which the frontier moves inwards when an efficient unit is removed from the sample, as an anonymous referee pointed out, it is not therefore appropriate to compare super efficiency scores. This is because the comparisons would relate to different points on the frontier and therefore different input mixes.

both players could increase their single and dual outputs by making better use of their defensive inputs. That said, Fognini, Ignacio Chela and for all intents and purposes Isner are technically efficient overall with both a single output and dual outputs.[12] There are two interpretations of these efficiencies. The first is that the inefficiency in the defensive/attacking play of these players is more than offset by their technical efficiency in attack/defence. The second is that they are able to combine their defensive and attacking inputs efficiently. For these players it is not clear which interpretation best explains the observed technical efficiencies. We can, however, conclude that with dual outputs the former world number one Lleyton Hewitt (world ranking of 22) is efficient at combining his defensive and attacking inputs. This is evident because there is technical inefficiency in Hewitt's defensive and attacking play but he is technically efficient overall. One possible reason for this is because Hewitt is widely regarded as being a very gritty match player.

In Figure 1 we present three pairs of normalised technical efficiency scores with a single output, where a normalised efficiency score above (below) 1 indicates that the score is above (below) the sample mean. The three pairs of normalised technical efficiency scores with a single output in Figure 1 are similar to the corresponding contour plots for the normalised technical efficiencies with dual outputs.[13] We can see from Figure 1 panel (i) that there are four clear groups of players in the sample- those players who are highly efficient in attack and defence; those players who are highly efficient in either attack or defence; and those players who are not very efficient in either. It is also apparent from Figure 1 panel (i) that there are a lot more players who are not very efficient in either attack or defence than there are players who are highly efficient at both or who are highly efficient at defending or attacking. Moreover, Figure 1 panel (i) also indicates that there are more players who are highly efficient attackers and not very efficient defensively than there players who are highly efficient in defence and attack or highly efficient defensively but not very efficient in attack.

[Insert Figure 1 about here]

Interestingly, in Figure 1 panel (ii) and panel (iii) it is apparent that there are three clear groups of players in the sample, rather than the four groups in Figure 1 panel (i). Specifically, Figure 1 panel (ii) (panel (iii)) indicates that the sample is made up of: those

[12] Isner is essentially technically efficient overall with a single output as his score is 0.99.
[13] The corresponding contour plots of the normalised technical efficiencies with dual outputs are available from the corresponding author upon request.

players who are highly efficient in attack (defence) and not very efficient overall; those players who are highly efficient in attack (defence) and overall; and those players who are not very efficient in attack (defence) and overall. Of the three groups in Figure 1 panel (ii) (panel (iii)) the largest group is that where the players are not very efficient in attack (defence) and overall.

5.3. *Truncated Regression Results*

The estimation results for the bootstrapped truncated regressions of the attacking, defensive and overall technical efficiencies on a set of off-court variables are presented in Table 4. In Table 5 we present the estimation results for the bootstrapped truncated regressions of the super attacking, defensive and overall efficiencies and where this is not relevant the corresponding technical efficiencies on the same set of off-court variables. A striking result is the positive and significant parameter on the right-hand dummy variable in the four overall models, albeit only at the 10% level. Why might this be? It is likely to be because it is widely accepted within the game of tennis that left-handed players are harder to play against that their right-handed counterparts. One possible reason for this is because with only twelve left-handed players in the sample, players will become more accustomed to right-handed opponents.

[Insert Table 4 and 5 about here]

The right-hand parameter is not significant in any of the four attacking models. This is at odds with the widely held view that the biggest advantage left-handers have is their serve. Specifically, with a left-handed player, the natural curvature and angle of the serve is to the back-hand of the right-handed receiver, which makes it more difficult for the receiving player to hit powerful returns. Roger Federer describes the widely perceived advantage from being a left-handed server as follows:

> *"Left-handers always get the break points on their favourite side. With their swinging serves, it makes it extremely difficult, especially with a one-handed backhand. It is tough getting used to left-handers' serves early on."* (Buddell, 2012)

It is also widely believed that taller players are more proficient attackers because they tend to have faster serves. We, however, find that height has a negative effect on the efficiencies, which in three of the four truncated models is significant. Although taller players clearly have an advantage when serving because of their tendency to have faster serves, our results suggest that this advantage is outweighed by taller players being at a disadvantage in other attacking facets of the game. For example, a taller player may be relatively slow around the court which, other things being equal, will put him at a disadvantage in a rally.

Finally, we note that the coefficients on age and experience in the truncated model for attacking efficiency with a single output in Table 5 are significant, negative and positive, respectively, and large relative to the corresponding parameters in the other truncated models. Finding that age and experience have negative and positive effects may at first appear contradictory but this is not the case. This is because with more experience a player is able to select a better match play strategy. At the same time, a player must have the physical capabilities to effectively implement his match play strategy. Age, however, places obvious limitations on, for example, fitness and hand-eye coordination and thus a player's ability to effectively implement his match play strategy. On the basis of these results we conclude that a player should turn professional at a young age because it allow him to become more experienced while he is relatively young and still in good physical condition.

6. Concluding Remarks

Klaassen and Magnus (2009) estimate the service efficiencies of male and female professional tennis players at the Wimbledon Championships. They compute a player's absolute efficiency as their methodology does not allow for the possibility that the best performers will lie on the frontier. A viable alternative approach is to measure a player's technical efficiency using DEA, which is a relative measure of efficiency and allows for the possibility that the top performer can lie on the frontier. Moreover, rather than analyse player efficiency in one of the many facets of the game such as service efficiency we have undertaken a broader efficiency analysis over a season. In particular, we use the DEA model developed by Tone (2001, 2002) to estimate the relative efficiency of the top 100 male professional tennis players on the 2009 ATP tour.

Our application to tennis is interesting because of the logical categorisation of the inputs according to whether they are measures of attacking or defensive proficiency.

Splitting the inputs up in this way allows us to glean more information about the sources of relative inefficiency. Our results seem sensible as we find that the top two ranked players in the world in our sample are technically efficient in attack, defence and overall with both a single output and dual outputs. Klaassen and Magnus (2009) find that, on average, there is very little absolute inefficiency in service in terms of points lost, which given the quality of the server could potentially have been won. From the kernel density distributions of the attacking and defensive technical efficiencies with both a single output and dual outputs we observe four clear groups of players in the sample which provides clear support for the categorisation of the inputs into attacking and defensive inputs. The four groups are as follows: those players that are relatively efficient in both attack and defence; those players that are relatively inefficient in attack and defence; and those players that are relatively efficient in attack or defence. These findings also suggest that there may be some value in applications of DEA to other sports from a similar categorisation of the inputs.

In addition, a couple of findings from the fitted second-stage truncated regressions are at odds with conventional wisdom within tennis. Tennis commentators wax lyrical about the attacking prowess of tall players and left-handers. This is because tall players tend to have fast serves and it is widely felt that left-handed serves are more difficult to return because of their trajectory. We find, however, that height tends to have a significant and not significant negative effect on the attacking efficiencies. Furthermore, we find that being left-handed does not have a significant effect on the attacking efficiencies, or the defensive efficiencies for that matter. However, we do find that being left-handed has a positive and significant effect on the overall efficiencies. This suggests that it is the combined effect of the attacking and defensive capabilities of left-handed players *vis-à-vis* their right-handed counterparts which gives them an efficiency advantage.

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Table 1: Descriptive statistics

	Mean	Min.	Max.	Std. Deviation
Attacking Inputs				
First serves in (%)	60.9	52.0	74.0	4.5
Points won on first serve (%)	65.2	52.0	79.0	7.3
Points won on second serve (%)	51.1	43.0	58.0	3.0
Average aces per match	5.9	1.7	20.7	3.1
Average double faults per match	2.8	1.3	6.7	0.9
Break points saved (%)	60.8	49.0	74.0	4.8
Service games won (%)	78.4	66.0	92.0	5.7
Defensive Inputs				
Points won returning the first serve (%)	28.9	21.0	37.0	3.0
Points won returning the second serve (%)	49.5	42.0	58.0	3.0
Break points won (%)	39.7	32.0	48.0	3.8
Return games won (%)	22.3	11.0	36.0	4.9
Tie Breaks won (%)	50.2	0.0	100.0	18.0
Outputs				
Prize money (US$)	709,797	78,259	6,183,961	954,978
Matches won (%)	51.7	25.0	87.3	14.2

Table 2: The input dropping ranking

Inputs	**Attacking**		**Defensive**		**Overall**	
	Single output	Two outputs	Single output	Two outputs	Single output	Two outputs
First serves in (%)	***5***	***5***			***8***	***8***
Points won on first serve (%)	4	3			5	5
Points won on second serve (%)	***6***	***6***			1	1
Average aces per match	1	2			***11***	***10***
Inverse of average double faults per match	2	1			***9***	***9***
Break points saved (%)	3	4			4	***6***
Service games won (%)	***7***	***7***			***10***	***11***
Points won returning the first serve (%)			***4***	***5***	6	***7***
Points won returning the second serve (%)			2	3	***7***	4
Break points won (%)			3	2	***12***	***12***
Return games won (%)			***5***	***4***	2	2
Tie Breaks won (%)			1	1	3	3

Note: The most important input is ranked 1. The variables with a ranking in ***bold italics*** are excluded from the efficiency analysis.

Table 3: Selected efficiency scores

Player	World Ranking	Attacking		Defensive		Overall	
		Single output	Two outputs	Single output	Two outputs	Single output	Two outputs
Roger Federer	1	1.00	1.00	1.00	1.00	1.00	1.00
		(3.01)	(12.96)	(3.32)	(16.48)	(3.26)	(17.79)
Rafael Nadal	2	1.00	1.00	1.00	1.00	1.00	1.00
		(3.91)	(27.62)	(3.04)	(9.30)	(3.91)	(28.17)
Novak Djokovic	3	1.00	1.00	0.74	1.00	1.00	1.00
		(2.91)	(4.99)		(2.43)	(2.92)	(5.16)
Andy Murray	4	0.61	0.75	0.82	0.89	0.75	0.82
Fabio Fognini	57	1.00	1.00	0.13	0.22	1.00	1.00
		(1.00)	(1.00)			(1.00)	(1.04)
Juan Ignacio Chela	94	1.00	1.00	0.18	0.62	1.00	1.00
		(1.00)	(1.01)			(1.00)	(1.01)
Michael Russell	100	1.00	1.00	1.00	1.00	1.00	1.00
		(1.01)	(4.18)	(1.06)	(6.34)	(1.12)	(4.95)
Marcel Granollers	78	0.24	0.36	0.20	0.32	0.34	0.58
Robert Kendrick	86	0.16	0.26	0.37	0.51	0.26	0.32
Robby Ginepri	92	0.15	0.23	0.20	0.30	0.17	0.29
John Isner	42	0.12	0.20	1.00	1.00	0.99	1.00
				(1.02)	(1.02)		(1.03)
Lleyton Hewitt	22	0.50	0.33	0.47	0.28	0.59	1.00
							(1.00)

Note: Super efficiency scores for efficient players are given in parentheses.

Table 4: Results of the truncated regression with two truncations: SBM efficiency measures (Algorithm 2)

Variable	Attacking		Defensive		Overall	
	Single output	Two outputs	Single output	Two outputs	Single output	Two outputs
Right-handed	0.041	0.031	-0.052	-0.139	-0.235*	-0.414*
Continent of origin						
North America	0.151	-0.012	0.156	-0.160	-0.324	-0.348
South America	0.159	0.223	0.215	0.229	-0.049	-0.137
Australia	-0.110	-0.166	-0.021	0.417	0.177	-0.515
Europe	0.070	0.040	0.064	0.008	-0.404	-0.196
Height (cm)	-0.010***	-0.018	-0.004	-0.010	0.000	-0.009
Weight (lbs)	-0.001	0.003	0.003	0.007**	-0.003	0.005
Experience	-0.003	0.018	-0.027	-0.055**	-0.078	-0.037**
Age	0.008	-0.005	0.026	0.051**	0.057	0.042***
Constant	2.148***	3.244***	0.135	0.490	0.758	1.335
Sigma	0.250***	0.271***	0.276***	0.289***	0.307***	0.330***

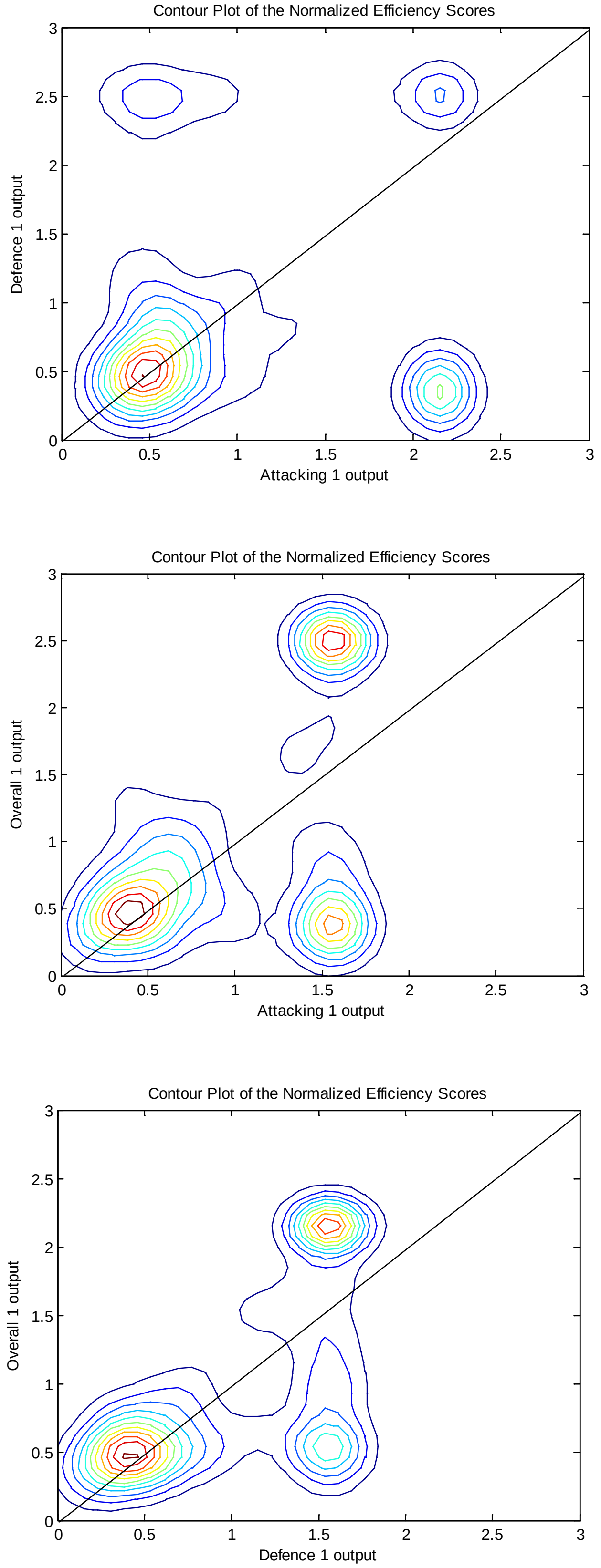
Notes: Statistical significance according to the bootstrap confidence intervals- *** denotes statistically significant at the 1% level; ** denotes statistically significant at the 5% level; * denotes statistically significant at the 10% level.

Table 5: Results of the truncated regression with one truncation: SBM super efficiency measures (Algorithm 2)

Variable	Attacking		Defensive		Overall	
	Single output	Two outputs	Single output	Two outputs	Single output	Two outputs
Right-handed	-1.642	-0.333	6.030	-0.221	-1.800*	-0.731*
Continent of origin						
North America	0.761	0.847	3.423	0.010	-0.654	0.597
South America	0.801	0.764	4.573	-0.180	0.979	1.206
Australia	0.519	0.096	6.042	-0.274	-0.043	-0.296
Europe	1.151	0.444	9.887	-0.345	1.016	0.486
Height (cm)	-0.122*	-0.032*	-0.794	0.000	-0.183	-0.020
Weight (lbs)	0.050	0.002	0.269	0.019**	0.070	-0.002
Experience	0.538**	0.085	-0.276	-0.054	0.123	-0.069
Age	-0.533**	-0.075	-0.999	0.111	-0.128	0.063
Constant	25.071***	7.377***	119.544	-4.571	26.572**	4.253
Sigma	3.078***	0.757***	36.597***	1.056***	3.491***	1.225***

Notes: Statistical significance according to the bootstrap confidence intervals- *** denotes statistically significant at the 1% level; ** denotes statistically significant at the 5% level; * denotes statistically significant at the 10% level.

Figure 1: Contour plots of normalised attacking, defensive and overall technical efficiency scores with a single output



(i)

(ii)

(iii)

Note: We use bivariate Gaussian kernels and the bandwidths are calculated according to the solve-the-equation plug-in approach (see Wand and Jones, 1994).