High-Order Mismatched Disturbance Compensation for Motion Control Systems via A Continuous Dynamic Sliding-Mode Approach

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***Abstract*—A new continuous dynamic sliding-mode control (CDSMC) method is proposed for high-order mismatched disturbance attenuation in motion control systems using a high-order sliding-mode differentiator. Firstly, a new dynamic sliding surface is developed by incorporating the information of the estimates of disturbances and their high-order derivatives. A CDSMC law is then designed for a general motion control system with both high-order matched and mismatched disturbances, which can attenuate the effects of disturbances from the system output. The proposed control method is finally applied for the airgap control of a MAGnetic LEViation (MAGLEV) suspension vehicle. Simulation results show that the proposed method exhibits promising control performance in the presence of high-order matched and mismatched disturbances.**

***Index Terms*—Motion control systems, high-order mismatched disturbances, dynamic sliding-mode control, MAGLEV Suspension vehicle.**

I. Introduction

In ALMOST all modern motion control systems, various uncertainties including parameter perturbations, unmodeled dynamics and external disturbances, always bring undesirable influence on the performance specification [1]. For example, see robot manipulator [2], [3], magnetic balance beam [4], MAGnetic LEViation (MAGLEV) suspension vehicle [5], [6], permanent magnet synchronous motor (PMSM) [7], [8], hard-disk drive [9]. Due to the growing interest in smart and high-precision motion devices, the development of disturbance rejection technique has received more and more attentions in motion controller design. Many elegant control approaches, such as H_2/H_∞ control [10], sliding model control [11], [12], adaptive control [13], robust control [14], [15] and backstepping control [16], [17], have been widely investigated in the literature for motion control systems. Although these methods have gained extensive applications and been proved to be efficient, they mainly focus on the stability (or robust stability) of uncertain systems [18], and in general, the robustness is achieved at a price of sacrificing the nominal performance [18], [19], [20]. In addition, most of those advanced feedback control approaches are designed without active feedforward disturbance compensation, and they can only attenuate the nonvanishing disturbances to a prescribed level rather than completely remove them from system [20], [21].

As a practical alternative approach, disturbance observer based control (DOBC) has been proved to be effective in compensating the effects of unknown external disturbances and model uncertainties in motion control systems [2], [7], [9], [23]. The major merit of the DOBC is that the robustness of the closed-loop system is obtained without sacrificing its nominal control performance [19], [20]. Another remarkable feature of DOBC lies in that it could completely remove the nonvanishing disturbances from system as long as they can be accurately estimated [22]. Despite the above excellent features, most of the existing DOBCs are only insensitive to matched disturbances but sensitive to mismatched ones. However, in many practical systems, the uncertainties would not rigorously satisfy the matching condition, for instance, see the MAGLEV suspension vehicle [23], the PMSM system [7], [14] and magnetically suspended balance beam system [4].

Due to the importance of compensating mismatched disturbances in both theory and engineering applications, several researchers have engaged in solving such a problem via DOBC, for example see [20]-[27]. In [24], the offset caused by mismatched disturbance is removed in the context of model predictive control by correcting the prediction error via a disturbance observer. An equivalent-input-disturbance based control framework is proposed for mismatched disturbance attenuation in [25]. By designing a specific disturbance compensation gain, a new DOBC framework was proposed to compensate mismatched disturbances for linear systems in [23] and also nonlinear systems in [20]. In [26] and [27], a new DOBC method was proposed to counteract the mismatched uncertainties in the system via designing a dynamic sliding surface incorporating the information of mismatched disturbances. When the sliding motion is realized, the prescribed specification including robust stability and disturbance attenuation can be achieved. However, it is noticed that, the mismatched disturbances in [20]-[27] are required to be a constant, which is not reasonably satisfied for many practical engineering issues [5], [28]. Taking the MAGLEV suspension vehicle as an example, the track input disturbance would not be

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a constant but fluctuates continuously with a high-order time-varying feature [5]. The presence of high-order mismatched disturbances will result in undesirable static and dynamic performance for methods in [20]-[27], which may constrain their applications in practical systems.

It is also noticed that in sliding mode control societies, much effort has been taken to the sliding surface design of systems under mismatched uncertainties (see e.g. Refs. [29]-[33] and references therein). However, the mismatched uncertainties therein considered should satisfy the condition of being H_2 norm-bounded, that is, the mismatched uncertainties are vanishing ones [26]. In addition, those methods suppress the mismatched uncertainties and disturbances in a robust way, that is, the ability against uncertainties is obtained at a price of sacrificing the nominal performance of control systems.

In this paper, a new continuous dynamic sliding-mode control (CDSMC) method is proposed to completely counteract the effects of both high-order matched and mismatched disturbances on the output of motion control systems. By fully taking into account the information of estimates of disturbances and their high-order derivatives, a new dynamic sliding surface is firstly designed which is insensitive to not only matched disturbances but also mismatched ones. As a result, the system output can be driven to the desired setpoint asymptotically by sliding motion along the new dynamic sliding surface even under both matched and mismatched disturbances. A continuous control law without any chattering is designed to drive the states to the designed dynamic sliding surface in finite time.

The proposed method exhibits the following attractive features. Firstly, the effects of both matched and mismatched disturbances are completely removed from the system output, where the disturbances are not constrained to be constant ones but could be high-order time-varying ones. Secondly, the nominal performance is retained with the proposed method, which means the proposed method acts the same as the baseline SMC in the absence of disturbances. Thirdly, the proposed control law is continuous without any chattering since the disturbances have been attenuated in finite time due to the finite-time convergence of the high-order sliding-mode differentiator and thus no switching control is required for disturbance rejection. The stability of the closed-loop system under the proposed method is addressed by means of Lyapunov stability method.

As a typical motion control system, recently, MAGLEV suspension vehicle has been attracting ever-increasing attention as a means of achieving noncontact transportation [34], [35] due to various advantages in practice including no direct environmental pollution and high safety and reliability [23]. As compared with the conventional wheel-on-rail ones, it does not have any mechanical contact with tracks, therefore, the friction, vibration, mechanical losses and acoustic noise are significantly reduced [5]. However, essentially, MAGLEV suspension vehicle is a nonlinear system subject to both external disturbances and parameter variations [23], which poses challenges to control designers. To this end, a great deal of elegant control methods for MAGLEV vehicles have been proposed in the past few decades including PI control [5], adaptive control [13], H-infinity control [10], sliding mode control [26], and so on. Simulation results of the MAGLEV suspension vehicle show that the proposed approach enables faster and higher-precision tracking performance as compared with other traditional control methods in the presence of high-order mismatched disturbances.

II. Motivations

Without loss of generality, the following second-order motion system subject to mismatched high-order disturbance is taken as an example to show the motivations of this paper

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 + d(t), \\ \dot{\eta}_2 &= a(\eta) + b(\eta)u, \\ y &= \eta_1, \end{aligned} \tag{1}$$

where η_1 and η_2 are states, u is the control input, $d(t)$ is the mismatched high-order disturbance, and y is the output. $a(\eta)$ and $b(\eta) \neq 0$ are smooth nonlinear functions in terms of η.

The sliding-mode control (SMC) and integral SMC (I-SMC) methods are taken as representatives to show how high-order mismatched disturbances affects the control performance of the closed-loop systems. The sliding surface as well as control law of baseline SMC are generally designed as [36]

$$\sigma = \eta_2 + c\eta_1, u = -b^{-1}(\eta)[a(\eta) + c\eta_2 + k\text{sign}(\sigma)]. \tag{2}$$

Combining (1) with (2) gives

$$\dot{\sigma} = -k\text{sign}(\sigma) + cd(t). \tag{3}$$

As shown in Eq. (3), the states in system (1) which are initially outside sliding surface $\sigma = 0$ will reach it in finite time if the switching gain in (2) is selected such that $k > \max\{|cd(t)|\}$. Considering the condition $\sigma = 0$ in (2), the sliding motion dynamics is governed by

$$\dot{\eta}_1 = -c\eta_1 + d(t). \tag{4}$$

It is noticed from Eq. (4) that, if the disturbance $d(t)$ is non-zero, then the output η_1 can not be driven to the desired equilibrium point.

An effective method to suppress the mismatched uncertainties would be I-SMC [36], which generally employs the following sliding surface

$$\sigma = \eta_2 + c_1\eta_1 + c_2\int \eta_1. \tag{5}$$

The control law of I-SMC is then designed as

$$u = -b^{-1}(\eta)[a(\eta) + c_1\eta_2 + c_2\eta_1 + k\text{sign}(\sigma)]. \tag{6}$$

Combining (1) with (5) and (6) yields

$$\dot{\sigma} = -k\text{sign}(\sigma) + c_1 d(t). \tag{7}$$

The states of system (1) will arrive the sliding surface $\sigma = 0$ in (5) in finite time if the switching gain in (6) is selected such that $k > \max\{|c_1 d(t)|\}$. Taking the condition $\sigma = 0$, we have

$$\ddot{\eta}_1 + c_1\dot{\eta}_1 + c_2\eta_1 = \dot{d}(t), \tag{8}$$

It can be derived from (8) that, the I-SMC method is efficient to eliminate the offset caused by mismatched disturbance with a constant steady-state value. However, in the presence of high-order disturbance ($\lim_{t\to\infty} \dot{d}(t) \neq 0$), the I-SMC method is unavailable for offset removal, i.e., the output η_1 can not be driven to the desired equilibrium point asymptotically. In addition, it is well known that the integral action of I-SMC method may bring some undesirable impacts to system performance, e.g., destroying its nominal control performance and introducing overshoot.

Another alternative approach to attenuate mismatched uncertainties is proposed in [26], [27], which is called enhanced SMC (ESMC) via a disturbance observer. The sliding surface in [26], [27] for system (1) is defined as

$$\sigma = \eta_2 + c_1\eta_1 + \hat{d}, \tag{9}$$

where $\hat{d}$ denotes the estimate of the disturbance d by a disturbance observer. The ESMC law could attenuate the mismatched disturbances without sacrificing its nominal performance, and the chattering problem can be relieved to some extent. However, the control law of ESMC is still discontinuous indicating that the chattering problem is unavailable.

In order to attenuate the effect of mismatched disturbance on the system output, the aforementioned control methods make some conservative assumptions on the mismatched disturbance. That is, the disturbance is required to be a vanishing one for traditional SMC method, and with a constant steady-state value for I-SMC and ESMC methods. However, the disturbance in practical applications may not satisfy those assumptions, which has been showed by MAGLEV suspension vehicle [26] and PMSM system [7], [14], etc. In those cases, the offset caused by high-order mismatched disturbance can not be eliminated effectively by the traditional SMC methods. In addition, when those methods are extended to control higher-dimension system, the high-order derivatives of the mismatched disturbances would have an adverse impact on both the dynamic and static performances. This motivates the research topic of this paper, that is, designing a new control law for nonlinear system with both high-order matched and mismatched disturbances such that the offset caused by disturbances is completely removed from the system output.

III. New Dynamic Sliding-Mode Control Design

Consider a single-input single-output motion control system with input relative degree (IRD) of ρ which is subject to high-order mismatched disturbances [37]:

$$\begin{aligned} &\dot{\eta}_i = \eta_{i+1} + d_i, 1 \leq i \leq \rho - 1,\\ &\dot{\eta}_\rho = a(\eta) + b(\eta)u + d_\rho,\\ &y = \eta_1, \end{aligned} \tag{10}$$

where $\eta = [\eta_1, \cdots, \eta_\rho]^T$ are the state vectors, u is the control input, y is the controlled output, d_i is the disturbance with at least $(\rho - i)$th order bounded derivatives. $a(\eta)$ and $b(\eta) \neq 0$ are smooth functions in terms of η.

The objective is to design a feedback controller for system (10), which could drive the control output y to the desired setpoint asymptotically in spite of the presence of both high-order matched and mismatched disturbances.

A. Controller Design

A new dynamic sliding surface for system (10) is designed as

$$\sigma = c_1\eta_1 + \sum_{i=2}^{\rho} c_i(\eta_i + \sum_{j=1}^{i-1} z_{i-j}^j) \tag{11}$$

where $c_i > 0, (i = 1, \cdots, \rho)$ are parameters to be designed, and z_{i-j}^j is the state of the following high-order sliding-mode differentiator

$$\begin{aligned} &\dot{z}_0^i = v_0^i + \eta_{i+1},\ \dot{z}_1^i = v_1^i,\ \cdots,\ \dot{z}_{r_i-1}^i = v_{r_i-1}^i,\ \dot{z}_{r_i}^i = v_{r_i}^i,\\ &v_0^i = -\lambda_0^i L_i^{\frac{1}{r_i+1}} |z_0^i - \eta_i|^{\frac{r_i}{r_i+1}} \mathrm{sign}(z_0^i - \eta_i) + z_1^i,\\ &v_1^i = -\lambda_1^i L_i^{\frac{1}{r_i}} |z_1^i - v_0^i|^{\frac{r_i-1}{r_i}} \mathrm{sign}(z_1^i - v_0^i) + z_2^i,\\ &\vdots\\ &v_{r_i-1}^i = -\lambda_{r_i-1}^i L_i^{\frac{1}{2}} |z_{r_i-1}^i - v_{r_i-2}^i|^{\frac{1}{2}} \mathrm{sign}(z_{r_i-1}^i - v_{r_i-2}^i) + z_{r_i}^i,\\ &v_{r_i}^i = -\lambda_{r_i}^i L_i \mathrm{sign}(z_{r_i}^i - v_{r_i-1}^i), \end{aligned} \tag{12}$$

where $\eta_{\rho+1}$ denotes $a(\eta) + b(\eta)u$ for the simplicity of expression, r_i is the order of differentiator, $\lambda_j^i > 0$ ($j = 0, 1, \ldots, r_i;\ i = 1, 2, \cdots, \rho - 1$) are the coefficients of the differentiator to be designed. Suppose that $d_i^{r_i}$ has a Lipshitz constant L_i.

Remark 1: The high-order sliding-mode differentiator (12) is referred to [38], where the only slight difference is design of the first equation $\dot{z}_0^i$. However, it will be shown next that the differentiator error system derived by (12) is the same as that in [38]. The order of ith differentiator is determined by $r_i = \rho - i + 1$, since only $d_i, \dot{d}_i, \cdots, d_i^{\rho-i}$ have an impact on the system output. In order to completely compensate the effects of disturbances from the output dynamics, we make an assumption that the disturbance d_i has $(\rho - i)$th order bounded derivatives. □

A continuous dynamic siding-mode control (CDSMC) law based on the dynamic sliding surface (11) for system (10) is designed as

$$\begin{aligned} u = -\tfrac{1}{c_\rho b(\eta)} \{ &c_1(\eta_2 + z_1^1) + \sum_{i=2}^{\rho-1} c_i(\eta_{i+1} + z_1^i + \sum_{j=1}^{i-1} v_{i-j}^j)\\ &+ c_\rho[a(\eta) + z_1^\rho + \sum_{j=1}^{\rho-1} v_{\rho-j}^j] + k\mathrm{sign}(\sigma)|\sigma|^\alpha \}, \end{aligned} \tag{13}$$

where $0 < \alpha < 1$ is a design parameter, z_1^i, v_i^{i-j} have been given in (12), $k > 0$ is a controller gain, and c_i are the parameters to be designed such that the polynomial

$$p_o(s) = c_\rho s^{\rho-1} + \cdots + c_2 s + c_1 = 0, \tag{14}$$

is Hurwitz. The block diagram of CDSMC is described by Fig. 1.

B. Stability Analysis

The stability of the closed-loop system is analyzed in this part based on the Lyapunov stability method. The following Lemma is firstly presented as an essential preliminary.

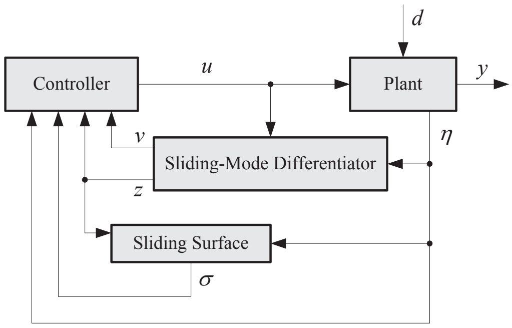

Fig. 1. Block diagram of CDSMC method.

Lemma 1: The differentiator error system governed by

$$\begin{array}{l}
\dot{e}_0^i = -\lambda_0^i L_i^{\frac{1}{r_i+1}} |e_0^i|^{\frac{r_i}{r_i+1}} \operatorname{sign}(e_0^i) + e_1^i, \\
\dot{e}_1^i = -\lambda_1^i L_i^{\frac{1}{r_i}} |e_1^i - \dot{e}_0^i|^{\frac{r_i-1}{r_i}} \operatorname{sign}(e_1^i - \dot{e}_0^i) + e_2^i, \\
\vdots \\
\dot{e}_{r_i-1}^i = -\lambda_{r_i-1}^i L_i^{\frac{1}{2}} |e_{r_i-1}^i - \dot{e}_{r_i-2}^i|^{\frac{1}{2}} \operatorname{sign}(e_{r_i-1}^i - \dot{e}_{r_i-2}^i) + e_{r_i}^i, \\
\dot{e}_{r_i}^i \in -\lambda_{r_i}^i L_i \operatorname{sign}(e_{r_i}^i - \dot{e}_{r_i-1}^i) + [-L_i, L_i],
\end{array} \tag{15}$$

where the errors are defined as $e_0^i = z_0^i - \eta_i$, $e_1^i = z_1^i - d_i$, $\cdots$, $e_{r_i-1}^i = z_{r_i-1}^i - d_i^{[r_i-2]}$, $e_r^i = z_{r_i}^i - d_i^{[r_i-1]}$, is finite-time stable [38], [39], that is, there exists a time constant $t_{if} > t_0$ such that $e_j^i(t) = 0$ $(j = 0, 1, \ldots, r_i)$ for $t \geq t_{if}$. The proof of this lemma can be followed from [38], which is omitted here for space. □

The main results of the paper are presented by the following theorem.

Theorem 1: For system (10) with the proposed dynamic sliding surface (11) under control law (13), the system output y will converge to the desired setpoint asymptotically and the states remain bounded.

Proof: For the proposed dynamic sliding surface (11), its derivative along the system dynamics (10) is

$$\dot{\sigma} = c_1(\eta_2 + d_1) + \sum_{i=2}^{\rho} c_i(\eta_{i+1} + d_i + \sum_{j=1}^{i-1} v_j^{i-j}). \tag{16}$$

Combining (10), (13) and (16) gives

$$\begin{array}{rcl}
\dot{\sigma} & = & \sum_{i=1}^{\rho} c_i(d_i - z_1^i) - k\operatorname{sign}(\sigma)|\sigma|^{\alpha} \\
& = & -k\operatorname{sign}(\sigma)|\sigma|^{\alpha} - e_l.
\end{array} \tag{17}$$

where $e_l = \sum_{i=1}^{\rho} c_i e_i^1$ is bounded due to the finite-time convergence of error system (15).

It will be shown that a bounded estimation error e_l will not drive the sliding variable to infinity in a finite time. To this end, define a finite-time bounded (FTB) function [40] $V(\sigma) = \frac{1}{2}\sigma^2$ for the sliding mode dynamics (17). Taking derivative of $V(\sigma)$ along (17) yields

$$\begin{array}{l}
\dot{V}_1(\sigma) = \sigma\dot{\sigma} = -k|\sigma|^{\alpha+1} - e_l\sigma \leq -e_l\sigma \\
\leq \frac{1}{2}\sigma^2 + \frac{1}{2}e_l^2 \leq K_{v_1}V_1(\sigma) + L_{v_1}
\end{array} \tag{18}$$

where $K_{v_1} = 1$ $L_{v_1} = \frac{1}{2}\max\{e_l^2\}$. Then, it can be obtained from (18) that $V(\sigma)$ and so σ are bounded in any finite time. Note that error system (15) is finite-time stable, which implies that e_l will converge to zero in a finite time t_e. In addition, it has been shown by (18) that the sliding variable σ will not be driven to infinity in the finite-time convergent process of differentiator. So after the finite-time stability of error system (15) is achieved, the sliding mode dynamics (17) will reduce to $\dot{\sigma} = -k\operatorname{sign}(\sigma)|\sigma|^{\alpha}$, which means the sliding variable σ will converge to zero in a finite time t_σ.

Defining $\tilde{\eta}_1 = \eta_1$, $\tilde{\eta}_i = \eta_i + \sum_{j=1}^{i-1} z_{i-j}^j$, $i = 2, \ldots, \rho$, the dynamics of states $\tilde{\eta}_i$ are obtained from (11), governed by

$$\begin{array}{l}
\dot{\tilde{\eta}}_i = \tilde{\eta}_{i+1} + \tilde{e}_i, \ i = 1, \ldots, \rho - 1, \\
\dot{\tilde{\eta}}_\rho = -\sum_{i=1}^{\rho} k_i\tilde{\eta}_i + \dot{\sigma},
\end{array} \tag{19}$$

where $\tilde{e}_1 = -e_1^1$, $\tilde{e}_i = \sum_{j=1}^{i-1}\left(\dot{e}_{i-j}^j - e_{i-j+1}^j\right) - e_1^i$ for $i = 2, \ldots, \rho - 1$. The dynamics of states (19) can be expressed in the following compact form

$$\dot{\tilde{\eta}} = A\tilde{\eta} + \tilde{u} \tag{20}$$

where $\tilde{\eta} = [\tilde{\eta}_1, \cdots, \tilde{\eta}_\rho]^T$, A is the companion matrix of the Hurwitz polynomial $p_o(s) = c_\rho s^{\rho-1} + \cdots + c_2 s + c_1$, $\tilde{u} = [\tilde{e}_1, \cdots, \tilde{e}_{\rho-1}, \dot{\sigma}]$ is the input vector with a bounded norm since both σ and e_j^i are bounded.

Next we will show the sliding surface dynamics (17) and the observer error dynamics (15) will not drive the state dynamics (19) to infinity in finite time. Define a FTB function $V_2(\tilde{\eta}) = \frac{1}{2}\tilde{\eta}^T\tilde{\eta}$ for system (20). Taking derivative of $V(\tilde{\eta})$ along dynamics (20), one obtains

$$\begin{array}{rcl}
\dot{V}_2(\tilde{\eta}) & = & \dot{\tilde{\eta}}^T\tilde{\eta} = \tilde{\eta}^T A\tilde{\eta} + \tilde{\eta}^T\tilde{u} \\
& \leq & \tilde{\eta}^T A\tilde{\eta} + \frac{1}{2}(\tilde{\eta}^T\tilde{\eta} + \tilde{u}^T\tilde{u}) \\
& \leq & (\lambda_{\max} + \frac{1}{2})\tilde{\eta}^T\tilde{\eta} + \frac{1}{2}\tilde{u}^T\tilde{u} \\
& = & K_{v_2}V_2(\tilde{\eta}) + L_{v_2},
\end{array} \tag{21}$$

where $K_{v_2} = 2\lambda_{\max} + 1$, $L_{v_2} = \frac{1}{2}\max\{||\tilde{u}||^2\}$, $\lambda_{\max}$ is the largest eigenvalue of matrix A.

It can be concluded from (21) that $V(\tilde{\eta})$ and so the state $\tilde{\eta}_i$ will not escape to infinity in finite time. This implies that the system dynamics (19) will reduce to the following system

$$\dot{\tilde{\eta}}_i = \tilde{\eta}_{i+1}, \ \dot{\tilde{\eta}}_\rho = -\sum_{i=1}^{\rho} k_i\tilde{\eta}_i \tag{22}$$

for $i = 1, \ldots, \rho - 1$ with a finite time after the stabilities of sliding surface dynamics (17) and observer error dynamics (15) are achieved in finite time. Since A is a Hurwitz matrix, system (22) is asymptotically stable, which implies that $y(t) = \eta_1(t)$ will converge to the desired setpoint asymptotically. □

Remark 2: In the absence of disturbances and uncertainties (that is, $d_i^1(t) = d_i^2(t) = \cdots = d_i^{[r_i]}(t) = 0$), if the initial states of the differentiator are selected as $z_0^i(t_0) = \eta_i(t_0)$ and $z_1^i(t_0) = z_2^i(t_0) = \cdots = z_{r_i}^i(t_0) = 0$, then it is obtained from the definition of e_j^i that $e_0^i(t_0) = e_1^i(t_0) = \cdots = e_{r_i}^i(t_0) = 0$. Note that the error system (15) is finite-time stable and the initial states are zeros, which implies that $e_0^i(t) = e_1^i(t) = \cdots = e_{r_i}^i(t) = 0$ holds for $t > t_0$. It is then obtained from the definition of errors e_j^i that $z_j^i(t) = 0, j = 1, \cdots, r_i$ for $t > t_0$, which implies that the proposed dynamic sliding surface (11) will reduce to the traditional sliding surface $\sigma = \sum_{i=1}^{\rho} c_i(\eta_i)$. In addition, $z_j^i(t) = 0, j = 1, \cdots, r_i$ for $t > t_0$ implies that $v_{i-j}^j = 0$ holds for $t > t_0$, which indicates that the CDSMC law (13) reduces to the traditional SMC law

$$u = -[c_\rho b(\eta)]^{-1}\{\sum_{i=1}^{\rho-1} c_i\eta_{i+1} + c_\rho a(\eta) + k\text{sign}(\sigma)|\sigma|^\alpha\} \tag{23}$$

This implies that the proposed CDSMC acts the same as the baseline SMC in the absence of uncertainties, that is, the nominal control performance of the proposed control method is retained. □

Remark 3: The CDSMC proposed in this paper can be seen as a nontrivial extension of the existing ESMC method in [26]. Actually, the CDSMC method reduces to ESMC in [26] under the assumption that the mismatched disturbance satisfies $\dot{d}_i = 0 (i = 1, \cdots, \rho)$. Since under the assumption that $\dot{d}_i = 0$, only the estimate of d_i is required to be included in the dynamic sliding surface (11), which means that the dynamic sliding surface (11) reduces to the sliding surface in ESMC

$$\sigma = c_1\eta_1 + \sum_{i=2}^{\rho} c_i(\eta_i + z_1^{i-1}) \tag{24}$$

where $z_1^{i-1} = \hat{d}_{i-1}$ is the estimation of d_{i-1}. However, if the assumption on mismatched disturbances $\dot{d}_i = 0$ does not hold, the system dynamics is then governed by

$$\sum_{i=1}^{\rho} c_i\eta_i^{[i]} = \sum_{i=2}^{\rho} c_i[(d_{i-1} - \hat{d}_{i-1}) + \sum_{j=1}^{i-2} d_j^{[i-1-j]}], \tag{25}$$

for ESMC, which means the derivatives and high-order derivatives of disturbances still have an undesirable effect on the system states even the sliding mode $\sigma = 0$ in (24) is realized. To eliminate the offset caused by the high-order derivatives of disturbances, not only the estimations of disturbances z_1^i but also the estimations of high-order derivatives of the disturbances z_i^j are incorporated in the dynamic sliding surface (11). As a result, when the dynamic sliding surface (11) is reached together with the convergence of the high-order sliding mode differentiator, the system dynamics will be governed by $\sum_{i=1}^{\rho} c_i\eta_i^{[i]} = 0$, which means that the offset caused by high-order derivatives of disturbances is completely removed from system output. □

IV. A MAGLEV Suspension Vehicle Design Example

A. Dynamic Model of MAGLEV Suspension Vehicle

1) Nonlinear Dynamic Model: The complete nonlinear model dynamics for a MAGLEV suspension vehicle are given by [5], [23],

$$B = K_b\frac{I}{G}, \tag{26}$$

$$F = K_f B^2, \tag{27}$$

$$\frac{dI}{dt} = \frac{V_c - IR_c + \frac{N_cA_pK_b}{G^2}(\frac{dz_t}{dt} - \frac{dZ}{dt})}{\frac{N_cA_pK_b}{G} + L_c}, \tag{28}$$

$$M_s\frac{d^2Z}{d^2t} = M_sg - F + d_{load}, \tag{29}$$

$$\frac{dG}{dt} = \frac{dz_t}{dt} - \frac{dZ}{dt}, \tag{30}$$

where variables I, Z, z_t, $\frac{dZ}{dt}$, $\frac{dz_t}{dt}$, G, F, B and V_c denote the current, the electromagnet position, the rail position, the electromagnet vertical velocity, the rail vertical velocity, the air gap, the force, the flux density and the coil voltage, respectively. A little difference from the model in [23] is that the load variation $d_{load} = m_sg$ caused by weight of passengers is explicitly included in the model. The rest symbols in Eqs. (26)-(30) represent system parameters as shown in Table I.

TABLE I
PARAMETERS OF MAGLEV SUSPENSION VEHICLE

Parameters	Meaning	Value
M_s	Vehicle mass	1000kg
K_b	Flux coefficient	0.0015T·m/A
K_f	Force coefficient	9810N/T^2
R_c	Coil's resistance	10Ω
g	Gravity constant	9.81m/s^2
L_c	Coil's inductance	0.1H
N_c	Number of turns	2000
A_p	Pole face area	0.01m^2

2) Model Linearization: In order to utilize the proposed control method, model linearization is required to transform the model of MAGLEV suspension system to meet the design formation as described in (10). The model here is linearized based on small perturbations around its operating point [5]. The following definitions are used, where the lower case letters denote a small variation around operating point while the subscript 'o' represents the operating condition

$$B = B_o + b, F = F_o + f, I = I_o + i,$$

$$G = G_o + (z_t - z), V_c = V_o + u_c.$$

The nominal values of MAGLEV vehicle in operating point are provided in Table II [5].

TABLE II
NOMINAL VALUES OF MAGLEV SUSPENSION VEHICLE

Parameters	Meaning	Value
B_o	Nominal flux density	1.0T
F_o	Nominal force	9810 N
I_o	Nominal current	10A
G_o	Nominal air gap	0.015m
V_o	Nominal voltage	100V

Then the linearilized dynamic model of MAGLEV suspension vehicle is obtained, which is depicted by

$$\begin{aligned} \dot{x} &= Ax + B_uu + B_dd(t) + \triangle Ax + O(x, u, d), \\ y &= Cx, \end{aligned} \tag{31}$$

where the states $x = [i, \dot{z}, (z_t - z)]^T$ represent variations of current, vertical velocity of electromagnet and air gap; the input $u = u_c$ denotes the voltage; the disturbances $d(t) = [\dot{z}_t, \frac{m_sg}{M_s}]^T$ are the vertical velocity of rail and the load variation; the controlled variable is air gap variation $y = z_t - z$; $\triangle A$ is the perturbation matrix; nonlinear function $O(x, u, d)$ represents the high order nonlinearities in terms of x, u and

d due to linearization. Suppose that the external disturbances $d(t)$ have at least twice order bounded derivatives. Here the system matrices in (31) are directly given as follows:

$$A = \begin{bmatrix} \frac{-R_c}{L_c+K_bN_c\frac{A_p}{G_o}} & \frac{-K_bN_cA_pI_o}{G_o^2(L_c+K_bN_c\frac{A_p}{G_o})} & 0 \\ -2K_f\frac{I_o}{M_sG_o^2} & 0 & 2K_f\frac{I_o^2}{M_sG_o^3} \\ 0 & -1 & 0 \end{bmatrix}, \tag{32}$$

$$B_u = \begin{bmatrix} \dfrac{1}{L_c+K_bN_c\frac{A_p}{G_o}} \\ 0 \\ 0 \end{bmatrix}, \tag{33}$$

$$B_d = \begin{bmatrix} \dfrac{K_bN_cA_pI_o}{G_o^2(L_c+K_bN_c\frac{A_p}{G_o})} & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \tag{34}$$

$$C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}. \tag{35}$$

The diagram of a MAGLEV vehicle is shown by Fig. 2.

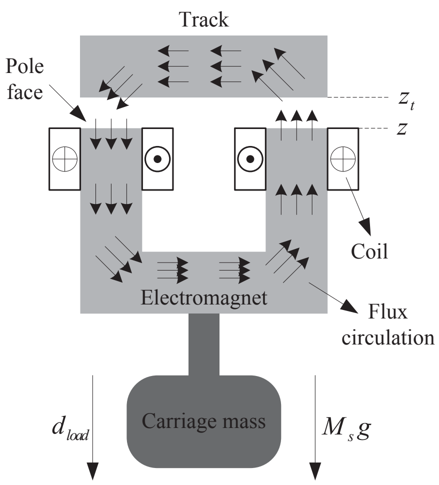

Fig. 2. Diagram of a MAGLEV vehicle.

The intended variations in the position of the track have to be followed by the MAGLEV suspension while the unintentional irregularities in the track position have to be rejected. There are generally two major disturbances including external disturbances and load variations. The first disturbances denote the track input to the suspension from vertical direction. The load variations can be considered as the second disturbance and modeled as a force in the vertical direction.

The control specifications for the MAGLEV system in the presence of the track input and load variation are provided in Table III [5].

TABLE III
CONTROL SPECIFICATIONS FOR MAGLEV VEHICLE

Constraints	Value
Maximum air gap deviation, $((z_t - z)_p)$	$\leq$0.0075m
Maximum input coil voltage, $((u_{coil})_p)$	$\leq$300V$(3I_oR_c)$
Settling time, (t_s)	$\leq$3s
Air gap steady state error, $((z_t - z)_{ess})$	=0

B. Controller Design

The high-order nonlinear term $O(x, u, d)$ in Eq. (31) is usually neglected in controller design due to its smaller magnitude as compared with the dominated linear dynamics. In this paper, however, such a nonlinear term is not neglected any more but handled as a part of lumped disturbances. As a result, the lumped disturbances include external disturbances, parameter variation and high-order nonlinearities, described as

$$d_l = B_d d + \Delta A x + O(x, u, d). \tag{36}$$

Remark 4: The lumped disturbances (36) in the MAGLEV system contain some state and input variables. It is well known that for disturbance estimator based control including extended state observer (ESO) based control [7], DOBC [2], [6], [22], [26], and equivalent input disturbance (EID) based control [25], it is not easy to verify the boundedness assumptions of the uncertainties $d_u = f(x, u, d) = \Delta A x + O(x, u, d)$ in lumped disturbances. In many practical engineering systems, however, the dominated dynamics have been stabilized by feedback control, while the uncertainties d_u in the lumped disturbance are usually very weak as compared with the dominated dynamics, which in general will not affect the stability of the closed-loop system. In this case, such uncertainties are generally reasonably regarded as a part of lumped disturbance and then handled by the proposed method. □

Substituting (36) into (31), the full dynamic model of the nonlinear MAGLEV suspension vehicle is described as

$$\begin{aligned} \dot{x} &= Ax + B_u u + B_l d_l, \\ y &= Cx, \end{aligned} \tag{37}$$

where $B_l = I$ is a 3×3 identity matrix.

To simplify the control design, the following transformation is employed [26], which can transform the original system into a system in Byrnes-Isidori normal form but subject to both matched and mismatched disturbances

$$\eta = Tx, \tag{38}$$

where

$$T = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}.$$

The MAGLEV system under such a coordinate transformation is then represented as

$$\dot{\eta} = \bar{A}\eta + \bar{B}_u u + \bar{B}_l d_l, \tag{39}$$

where $\bar{A} = TAT^{-1}$, $\bar{B}_u = TB_u$, and $\bar{B}_l = TB_l$.

Substituting (32)-(35) into (39) gives

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 + d_{l3}, \\ \dot{\eta}_2 &= \eta_3 - d_{l2}, \\ \dot{\eta}_3 &= CA^3T^{-1}\eta + CA^2B_u u + CA^2B_l d_l. \end{aligned} \tag{40}$$

where $d_l = [d_{l1}, d_{l2}, d_{l3}]^T$ in (40) is a vector with the dimension three, and $d_{li}(i = 1, 2, 3)$ denotes the lumped disturbances entering the ith channel.

Remark 5: It can be observed from (40) that the MAGLEV suspension vehicle is subject to both matched disturbance $CA^2B_l d_l$ and mismatched ones d_{l3} and d_{l2}. The matched disturbance $CA^2B_ld_l$ can be attenuated in conventional SMC by high-frequency switching control force, however, the mismatched disturbances d_{l3} and d_{l2} can not be attenuated by traditional SMC which have an adverse effect on the system output. □

Based on the above analysis, the CDSMC law (13) proposed in Section III can be directly applied to the control design of such a MAGLEV suspension vehicle in this section.

V. Simulation Results and Analysis

Simulation results are provided to validate the performance of the proposed method. To evaluate the efficiency of the proposed control method, the traditional SMC, the I-SMC and ESMC [26] methods are also employed in the simulations for the purpose of comparisons. The simulations are implemented for the full nonlinear dynamics of the MAGLEV system in a measurement noise environment for practicality. The control parameters of all the four control methods are listed in Table IV.

TABLE IV
Control parameters of the MAGLEV suspension vehicle

Controllers	Parameters
SMC	$c_1 = 100$, $c_2 = 20$, $c_3 = 1$, $k = 80$
I-SMC	$c_0 = 200$, $c_1 = 100$, $c_2 = 20$, $c_3 = 1$, $k = 80$
ESMC	$c_1 = 100$, $c_2 = 20$, $c_3 = 1$, $k = 30$, $\lambda(\eta) = 100\eta_1$
CDSMC	$c_1 = 100$, $c_2 = 20$, $c_3 = 1$, $k = 30$, $L_1 = L_2 = 20$ $\lambda_0^1 = 3$, $\lambda_1^1 = \lambda_0^2 = 2$, $\lambda_2^1 = \lambda_1^2 = 1.5\lambda_3^1 = \lambda_2^2 = 2.5$ $\alpha = 0.8$

A. External Disturbances Rejection

The track input components considered in this part are referred to [5] and shown in Fig. 3. They represent a gradient of 5% at a vehicle speed of 15 m/s, a vertical acceleration of 0.5 m/s^2 and a jerk level of 1 m/s^3. In practical applications, the track input disturbance would vary continuously due to the ups and downs of the rail. To this end, an additional time-varying track input disturbance $\dot{z}_t = 0.1\sin(\pi t)$ m/s is taken and imposed on the vehicle at $t = 4$ second to imitate the real engineering.

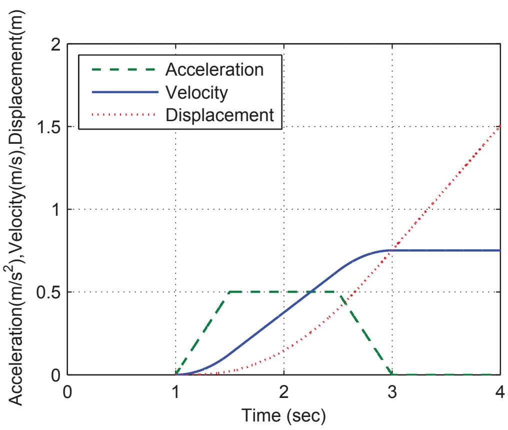

Fig. 3. Track input profile with a gradient of 5% at a vehicle speed of 15 m/s.

The initial states of MAGLEV suspension vehicle (31) are taken as $[i(0), \dot{z}(0), z_t(0)-z(0)]^T = [0, 0, 0.003]^T$. Simulation results are shown in Figs. 4-6.

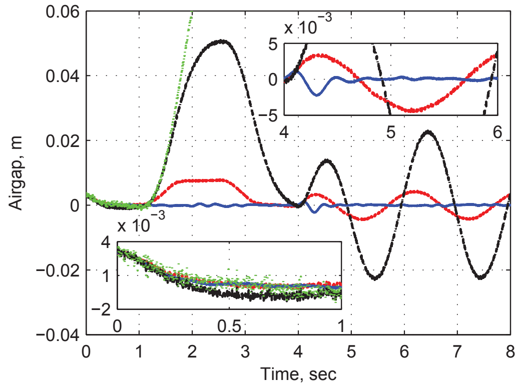

Fig. 4. Response curves of output $z_t - z$ of MAGLEV vehicle with external disturbances under four controllers: CDSMC (blue line), ESMC (red line), I-SMC (black line), SMC (green line).

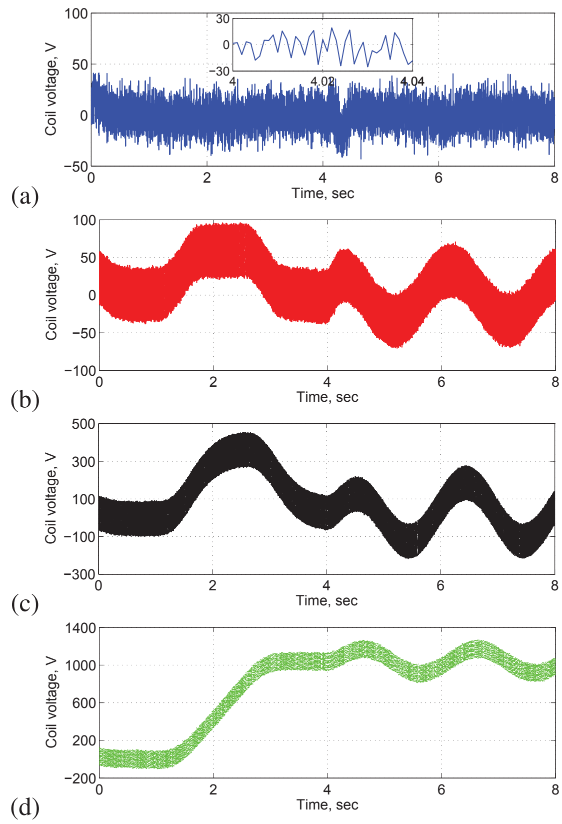

Fig. 5. Response curves of control input u_{coil} of MAGLEV vehicle with external disturbances under four controllers: (a) CDSMC; (b) ESMC; (c) I-SMC; (d) SMC.

As shown by Figs. 4 and 6, the proposed method has obtained the same response curves as those of the SMC method during the first sec when there is no disturbance in such a interval, which demonstrates the nominal performance recovery property of the proposed methods. In addition, Figs. 4 and 6 show that the conventional SMC severely suffers from

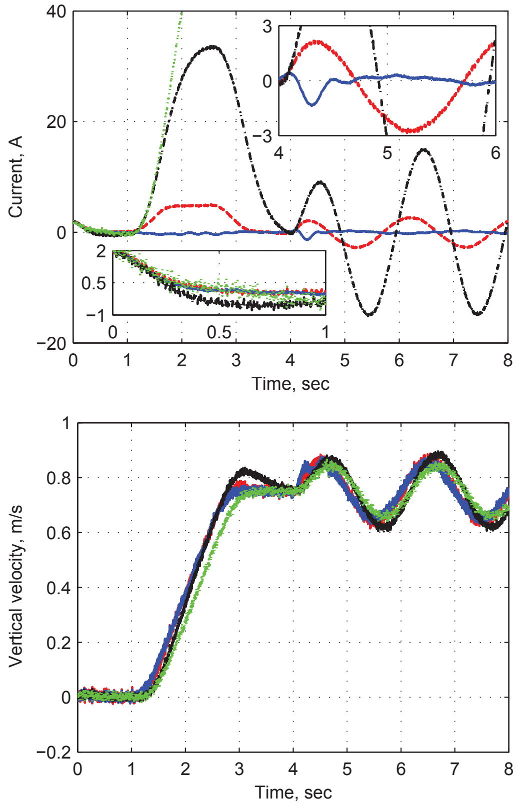

Fig. 6. Response curves of the states of MAGLEV vehicle with external disturbances under four controllers: CDSMC (blue line), ESMC (red line), I-SMC (black line), SMC (green line): (a) the current, i, (b) the vertical electromagnet velocity, $\dot{z}$.

the mismatched non-vanishing disturbances. The I-SMC and ESMC methods can remove the offset caused by track input disturbances represented in Fig. 3. However, the proposed CDSMC in this paper obtains better transient performance for disturbance rejection as compared with the I-SMC and ESMC. Also note that, in the presence of high-order time varying disturbances (for $t > 4$ second), both I-SMC and ESMC can not reject or compensate these disturbances effectively, while the proposed CDSMC achieves prominent disturbance compensation performance. Fig. 5 shows that the CDSMC does not lead to any chattering phenomenon due to the continuous control action, while traditional SMC, I-SMC and ESMC result in the chattering phenomenon due to the discontinuous sign function.

B. Robustness Against Load Variation

Simulation studies are performed to verify the robustness of the CDSMC against load variation in this part. The load variation under consideration is 40% of the total vehicle mass in 10 seconds, i.e., the disturbance load will vary from 0 kg (fully unladen vehicle) to 400 kg (fully laden vehicle). The load variation profile is shown in Fig. 7. The external disturbance in Fig. 3 is also considered and added on system at $t = 15$ sec. The initial states for MAGLEV suspension vehicle (31) are taken as $[i(0), \dot{z}(0), z_t(0) - z(0)]^T = [0, 0, 0]^T$. The control parameters for all the four control methods are the same as the ones in the case of external disturbance rejection.

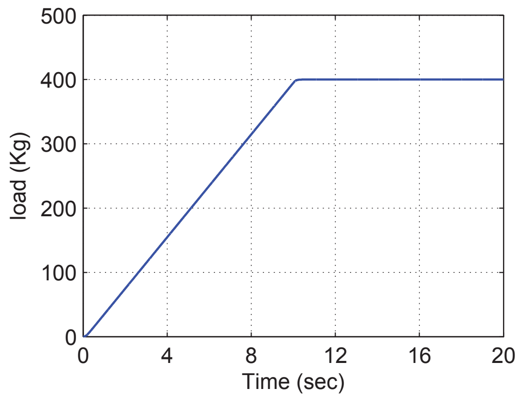

Fig. 7. Profile of the load variation.

The output and input response curves of the MAGLEV vehicle with load variation and external disturbance input under the four control methods are described by Figs. 8 and 9, respectively. The corresponding states response curves are shown in Fig. 10.

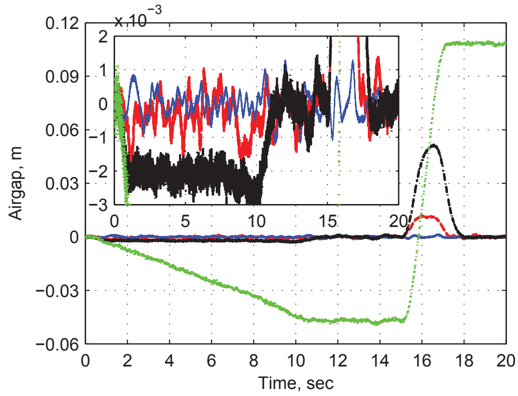

Fig. 8. Response curves of output $z_t - z$ of MAGLEV vehicle with load variation under four controllers: CDSMC (blue line), ESMC (red line), I-SMC (black line), SMC (green line).

As shown by Fig. 8, the proposed CDSMC obtains satisfying dynamic and static performance in the presence of both load variation and track input disturbances. The I-SMC and ESMC can obtain a satisfying static performance but quite poorer dynamic performance. In addition, it can be observed from Figs. 9 and 10 that the proposed CDSMC has a relatively lower control energy as compared with the rest control methods. In addition, the controller of the proposed CDSMC is continuous and no chattering phenomenon appears.

VI. Conclusions

The high-order mismatched disturbance compensation problem for motion control systems has been investigated in this paper. A new CDSMC approach based on a high-order sliding-mode differentiator has been proposed to attenuate the effects caused by high-order mismatched disturbances on the output

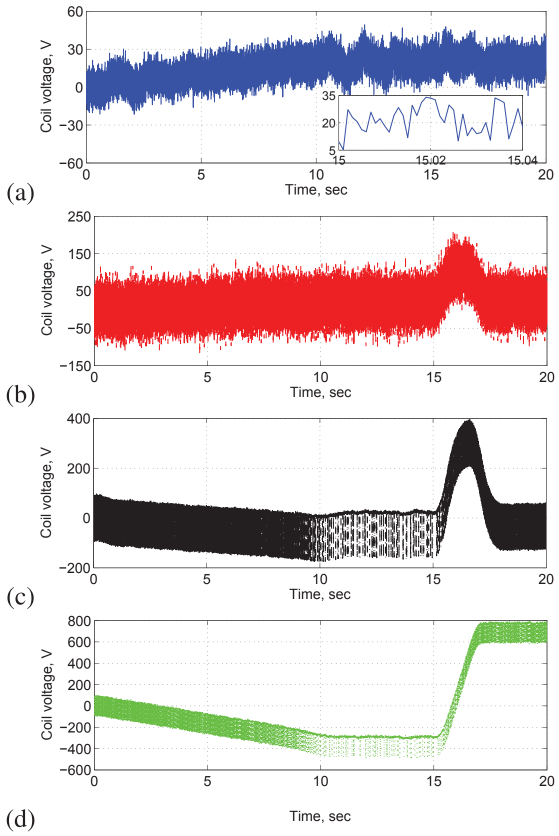

Fig. 9. Response curves of control input u_{coil} of MAGLEV vehicle with load variation under four controllers: (a) CDSMC; (b) ESMC; (c) I-SMC; (d) SMC.

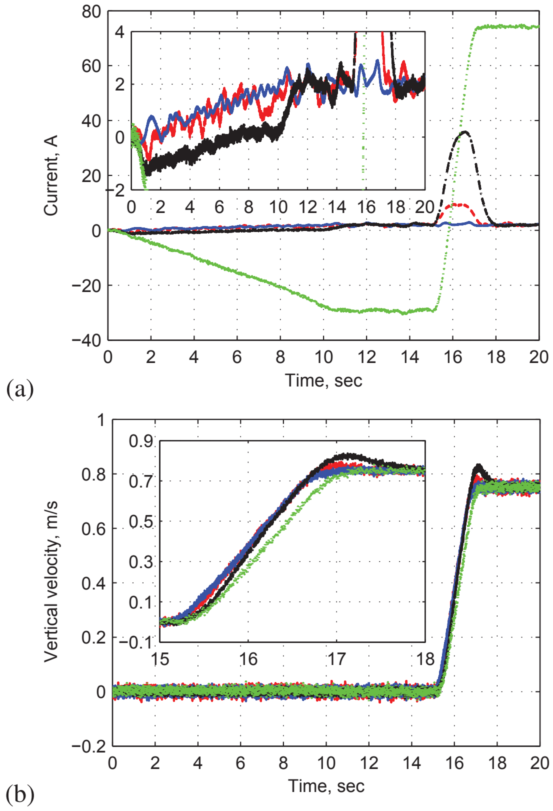

Fig. 10. Response curves of the states of MAGLEV vehicle with load variation under four controllers: CDSMC (blue line), ESMC (red line), I-SMC (black line), SMC (green line): (a) the current, i, (b) the vertical electromagnet velocity, $\dot{z}$.

in finite time. The main contribution here is to design a new dynamic sliding surface which incorporates the information of the estimations of disturbances and their high-order derivatives such that the sliding motion along the sliding surface can drive the system output to the desired equilibrium even in the presence of high-order mismatched disturbances. Simulation results of a MAGLEV suspension vehicle have demonstrated that the proposed method exhibits much better dynamic and static performances in the presence of high-order mismatched disturbances as compared with the traditional SMC, I-SMC and ESMC methods.

References

[1] A. Sabanovic, “Variable structure systems with sliding modes in motion control–A survey,” *IEEE Trans. Ind. Inform.*, vol. 7, no. 2, pp. 212-223, May. 2011.

[2] W.-H. Chen, D.J. Ballance, P.J. Gawthrop, and J. O’Reilly, “A nonlinear disturbance observer for robotic manipulators,” *IEEE Trans. Ind. Electron.*, vol. 47, no. 4, pp. 932-938, Aug. 2000.

[3] M.L. Corradini, V. Fossi, A. Giantomassi, G. Ippoliti, S. Longhi, and G. Orlando, “Mininal resource allocating networks for discrete time sliding mode control of robotic manipulators,” *IEEE Trans. Ind. Inform.*, vol. 8, no. 4, pp. 733-745, Nov. 2012.

[4] J.H. Lee, P.E. Allaire, G. Tao, and X.R. Zhang, “Integral sliding-mode control of a magnetically suspended balance beam: analysis, simulation, and experiment,” *IEEE/ASME Trans. Mechatron.*, vol. 6, no. 3, pp. 338-346, Sep. 2001.

[5] K. Michail, “Optimised configuration of sensing elements for control and fault tolerance applied to an elctro-magnetic suspension system,” Ph.D. thesis. Loughborough University. 2009. http://hdl.handle.net/2134/5806.

[6] J. Lee, S. Khoo, and Z. Wang, “DSP-Based Sliding Mode Control for Electromagnetic-Levitation Precise-Position System,” *IEEE Trans. Ind. Inform.*, vol. 9, no. 2, pp. 817-827, May 2013.

[7] H.X. Liu, and S.H. Li, “Speed control for PMSM servo system using predictive function control and extended state observer,” *IEEE Trans. Ind. Electron.*, vol. 59, no. 2, pp. 1171-1183, Feb. 2012.

[8] S.H. Li, and H. Gu, “Fuzzy adaptive internal model control schemes for PMSM speed-regulation system,” *IEEE Trans. Ind. Inform.*, vol. 8, no. 4, pp. 767-779, Nov. 2012.

[9] J.N. Teoh, C. Du, G. Guo, and L. Xie, “Rejecting high frequency disturbances with disturbance observer and phase stabilized control,” *Mechatronics*, vol. 18, no. 1, pp. 53-60, Feb. 2008.

[10] A. Bittar, R.M. Sales, “H-2 and H-infinity control for MagLev vehicles,” *IEEE Control System Magazine.*, vol. 18, no. 4, pp. 18-25, 1998.

[11] L. Yang, S. Yang, and B. Richard , “Modeling and robust discrete time sliding mode control design for a fluid power electrohyfraulic actuator (EHA) system,” *IEEE/ASME Trans. Mechatronics.*, vol. 18, no. 1, pp. 1-10, Jan. 2013.

[12] J. Yang, J. Su, S. Li, and X. Yu , “Continuous nonsingular terminal sliding mode control for systems with mismatched disturbances,” *Automatica*, vol. 49, no. 7, pp. 2287-2291, Jul. 2013.

[13] P.K. Sinha, A.N. Pechev, “Model reference adaptive control of a maglev system with stable maximum descent criterion,” *Automatica.*, vol. 35, no. 8, pp. 1457-1465, Aug. 1999.

[14] Y. Abdel-Rady and I. Mohamed, “Design and implementation of a robust current-control scheme for a PMSM vector drive with a simple adaptive disturbance observer,” *IEEE Trans. Ind. Electron.*, vol. 54, no. 4, pp. 1981-1988, Aug. 2007.

[15] S. Y. Khoo, L. H. Xie, and Z. H. Man, “Robust finite-time consensus tracking algorithm for multirobot systems,” *IEEE/ASME Trans. Mechatronics.*, vol. 14, no. 2, pp. 219-228, Jun. 2009.

[16] H.-J. Shieh and C.-H. Hsu, “An adaptive approximator-based backstepping control approach for piezoactuator-driven stages,” *IEEE Trans. Ind. Electron.*, vol. 55, no. 4, pp. 1729-1738, Apr. 2008.

[17] M. Morawiec, "The adaptive backstepping control of permanent magnet synchronous motor supplied by current source inverter," *IEEE Trans. Ind. Inform.*, vol. 8, no. 2, pp. 1047-1055, May 2013.
[18] J. Back, and H. Shim, "An inner-loop controller guaranteeing robust transient performance for uncertain MIMO nonlinear system," *IEEE Trans. Autom. Control.*, vol. 54, no. 7, pp. 1601-1607, Jul. 2009.
[19] J. Back, and H. Shim, "Adding robustness to nominal output-feedback controllers for uncertain nonlinear systems: A nonlinear version of disturbance observer," *Automatica*, vol. 44, no. 10, pp. 2528-2537, Oct. 2008.
[20] J. Yang, W.-H. Chen, and S.H. Li, "Non-linear disturbance observer-based robust control for systems with mismatched disturbances/uncertainties," *IET Contr. Theory Appl.*, vol. 5, no. 18, pp. 2053-2062, Dec. 2011.
[21] M. Krstic, and M. Bement, "Nonovershooting control of strict-feedback nonlinear systems," *IEEE Trans. Autom. Control.*, vol. 51, no. 12, pp. 1938-1943, Dec. 2006.
[22] W.-H. Chen, "Disturbance observer based control for nonlinear systems," *IEEE/ASME Trans. Mechatronics.*, vol. 9, no. 4, pp. 706-710, Dec. 2004.
[23] J. Yang, A. Zolotas, W.H. Chen, K. Michail, and S. Li, "Robust control of nonlinear MAGLEV suspension system with mismatched uncertainties via DOBC approach," *ISA Trans.*, vol. 50, no. 3, pp. 389-396, Jul. 2011.
[24] U. Maeder, and M. Morari, "Offset-free reference tracking with model predictive control", *Automatica*, vol. 46, no. 9, pp. 1469-1476, Sep. 2010.
[25] J.-H. She, X. Xin, and Y. Pan, "Equivalent-input-disturbance approach–analysis and application to disturbance rejection in dual-stage feed drive control system," *IEEE/ASME Trans. Mechatronics.*, vol. 16, no. 2, pp. 330-340, Apr. 2011.
[26] J. Yang, S.H. Li, and X.H. Yu, "Sliding-mode control for systems with mismatched uncertainties via a disturbance observer," *IEEE Trans. Ind. Electron.*, vol. 60, no. 1, pp.160-169, Jan. 2013.
[27] A.F.D. Loza, F.J. Bejarano, and L. Fridman, "Unmatched uncertainties compensation based on high-order sliding mode observation," *Int. J. Robust Nonlinear Control.*, vol. 23, no. 7, pp. 754-764, May 2013.
[28] K.-S. Kim, K.-H. Rew, and S. Kim, "Disturbance observer for estimating higher order disturbance in time series expansion," *IEEE Trans. Autom. Control.*, vol. 55, no. 8, pp. 1905-1911, Aug. 2010.
[29] X.H. Yu, B. Wang, and X.J. Li, "Computer-controlled variable structure systems: the state-of-the-art," *IEEE Trans. Ind. Inform.*, vol. 8, no. 2, pp. 197-205, May 2012.
[30] H.H. Choi, "LMI-based sliding surface design for integral sliding model control of mismatched uncertain systems," *IEEE Trans. Autom. Control.*, vol. 52, no. 4, pp. 736-742, Apr. 2007.
[31] J.M.A.D. Silva, C. Edwards, and S.K. Spurgeon, "Sliding-mode output-feedback control based on LMIs for plants with mismatched uncertainties," *IEEE Trans. Ind. Electron.*, vol. 56, no. 9, pp. 3675-3683, Sep. 2009.
[32] W.J. Cao, and J.X. Xu, "Nonlinear integral-type sliding surface for both matched and unmatched uncertain systems," *IEEE Trans. Autom. Control.*, vol. 49, no. 8, pp. 1355-1360, Aug. 2004.
[33] M. Rubagotti, A. Estrada, F. Castanos, A. Ferrara and L. Fridman, "Integral sliding mode control for nonlinear systems with matched and unmatched perturbations," *IEEE Trans. Autom. Control.*, vol. 56, no. 11, pp. 2699-2704, Nov. 2011.
[34] C. MacLeod and R.M. Goodall, "Frequency-shaping LQ control of Maglev suspension systems for optimal performance with deterministic and stochastic inputs," *IEE Proc-Control Theory Appl.*, vol. 143, no. 1, pp. 25-30, Jan. 1996.
[35] B. Peter and D. Michael, "Magnetic levitation over large translation and rotation ranges in all directions," *IEEE/ASME Trans. Mechatronics.*, vol. 18, no. 1, pp. 44-52, Feb. 2013.
[36] V.I. Utkin, "Variable structure systems with sliding modes," *IEEE Trans. Autom. Control.*, vol. 22, no. 2, pp. 212-222, Apr. 1977.
[37] A. Isidori, *Nonlinear Control System II*. Springer-Verlag: London, 1999.
[38] A. Levant, "Higher-order sliding modes, differentiation and output-feedback control," *Int. J. Control.*, vol. 76, no. 9/10, pp. 924-941, 2003.
[39] J. Yin, S. Khoo, Z. Man, and X. Yu, "Finite-time stability and instability of stochastic nonlinear systems," *Automatica*, vol. 47, no. 12, pp. 2671-2677, Dec. 2011.
[40] S.H. Li and Y.P. Tian, "Finite-time stability of cascaded time-varying systems," *Int. J. Control.*, vol. 80, no. 4, pp. 646-657, Feb. 2007.

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