LOUGHBOROUGH UNIVERSITY

DOCTORAL THESIS

Mathematical Optimization and Game Theoretic Methods for Radar Networks

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CERTIFICATE OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this thesis, that the original work is my own except as specified in acknowledgements or in footnotes, and that neither the thesis nor the original work therein has been submitted to this or any other institution for a degree.

.... (Signed)

ANASTASIOS DELIGIANNIS (candidate)

I dedicate this Thesis to my loving family, my parents Evangelos and Dionysia Deligianni and my sister Penny.

Αυτή η διδακτορική διατριβή είναι αφιερωμένη στην αγαπημένη μου οικογένεια, στους γονείς μου Ευάγγελο και Διονυσία Δεληγιάννη και στην αδερφή μου Πένη.

Abstract

Radar systems are undoubtedly included in the hall of the most momentous discoveries of the previous century. Although radars were initially used for ship and aircraft detection, nowadays these systems are used in highly diverse fields, expanding from civil aviation, marine navigation and air-defence to ocean surveillance, meteorology and medicine. Recent advances in signal processing and the constant development of computational capabilities led to radar systems with impressive surveillance and tracking characteristics but on the other hand the continuous growth of distributed networks made them susceptible to multisource interference. This thesis aims at addressing vulnerabilities of modern radar networks and further improving their characteristics through the design of signal processing algorithms and by utilizing convex optimization and game theoretic methods. In particular, the problems of beamforming, power allocation, jammer avoidance and uncertainty within the context of multiple-input multiple-output (MIMO) radar networks are addressed.

In order to improve the beamforming performance of phased-array and MIMO radars employing two-dimensional arrays of antennas, a hybrid two-dimensional Phased-MIMO radar with fully overlapped subarrays is proposed. The work considers both adaptive (convex optimization, CAPON beamformer) and non-adaptive (conventional) beamforming techniques. The transmit, receive and overall beampatterns of the Phased-MIMO model are compared with the respective beampatterns of the phased-array and the MIMO schemes, proving that the hybrid model provides superior capabilities in beamforming.

By incorporating game theoretic techniques in the radar field, various vulnerabilities and problems can be investigated. Hence, a game theoretic power allocation scheme is proposed and a Nash equilibrium analysis for a multistatic MIMO network is performed. A network of radars is considered, organized into multiple clusters, whose primary objective is to minimize their transmission power, while satisfying a certain detection criterion. Since no communication between the clusters is assumed, non-cooperative game theoretic techniques and convex optimization methods are utilized to tackle the power adaptation problem. During the proof of the existence and the uniqueness of the solution, which is also presented, important contributions on the SINR performance and

the transmission power of the radars have been derived.

Game theory can also been applied to mitigate jammer interference in a radar network. Hence, a competitive power allocation problem for a MIMO radar system in the presence of multiple jammers is investigated. The main objective of the radar network is to minimize the total power emitted by the radars while achieving a specific detection criterion for each of the targets-jammers, while the intelligent jammers have the ability to observe the radar transmission power and consequently decide its jamming power to maximize the interference to the radar system. In this context, convex optimization methods, noncooperative game theoretic techniques and hypothesis testing are incorporated to identify the jammers and to determine the optimal power allocation. Furthermore, a proof of the existence and the uniqueness of the solution is presented.

Apart from resource allocation applications, game theory can also address distributed beamforming problems. More specifically, a distributed beamforming and power allocation technique for a radar system in the presence of multiple targets is considered. The primary goal of each radar is to minimize its transmission power while attaining an optimal beamforming strategy and satisfying a certain detection criterion for each of the targets. Initially, a strategic noncooperative game (SNG) is used, where there is no communication between the various radars of the system. Subsequently, a more coordinated game theoretic approach incorporating a pricing mechanism is adopted. Furthermore, a Stackelberg game is formulated by adding a surveillance radar to the system model, which will play the role of the leader, and thus the remaining radars will be the followers. For each one of these games, a proof of the existence and uniqueness of the solution is presented.

In the aforementioned game theoretic applications, the radars are considered to know the exact radar cross section (RCS) parameters of the targets and thus the exact channel gains of all players, which may not be feasible in a real system. Therefore, in the last part of this thesis, uncertainty regarding the channel gains among the radars and the targets is introduced, which originates from the RCS fluctuations of the targets. Bayesian game theory provides a framework to address such problems of incomplete information. Hence, a Bayesian game is proposed, where each radar egotistically maximizes its SINR, under a predefined power constraint.

Statement of Originality

The contributions of this thesis are mainly on the development and improvement of optimal beamforming and power allocation techniques within the context of multiple-input multiple-output (MIMO) radars, using convex optimization and game theoretic techniques. Additionally, novel game theoretic techniques for jammer cancelation and handling uncertainty are also presented. The novelty of the contributions is supported by the following international journal and conference publications:

- In Chapter 3, a new fully overlapped subaperturing technique for two-dimensional (2D) MIMO radar arrays is proposed. The transmit, waveform diversity and overall beampatterns for the resulting 2D Phased-MIMO radar are compared with the respective beampatterns for the phased array and MIMO only radar models. The proposed model offers substantial improvements in beamforming performance and accuracy as compared to the phased-array and MIMO only radar schemes. The study considers both adaptive and conventional beamforming techniques. The results of this work have been published in [4] and [5].

- The novel contribution of Chapter 4 is a rigorous mathematical Nash equilibrium analysis for a game theoretic power allocation model within the context of a multistatic MIMO radar network. In particular, the existence and uniqueness of the solution are proved, by exploiting the frameworks of standard functions, duality and Karush-Kuhn-Tucker (KKT) conditions. The mathematical analysis of the uniqueness of the solution leads to substantial results on the relation between the performance with respect to the detection criterion and the transmission power of the radars. Furthermore, a jammer cancelation technique based on hypothesis testing and noncooperative game theoretic resource allocation among a distributed radar network and multiple jammers is presented. The novelty of the Nash equilibrium analysis is supported by [2] and jammer-radar interaction is supported by [6].

- In Chapter 5, a novel Bayesian game theoretic joint SINR maximization and power allocation technique within a distributed radar network is proposed, by introducing multilevel channel gain uncertainty. The importance of the a priori belief of a player is highlighted to the outcome of the game and the proof of the existence and the uniqueness of the solution is presented. The novel results of this work will be submitted for journal publication [3].

- Chapter 6 presents a novel, broad game theoretic analysis for joint optimal beamforming and resource allocation within a distributed MIMO radar network with multiple targets. Convex optimization methods, non-cooperative, partially coordinated and Stackelberg game theoretic techniques have been applied to obtain the optimal beamformers and power allocation strategy and standard function and duality properties are exploited to prove the existence and uniqueness of the solution. This work has been accepted for journal publication [1].

Jounal Papers

1. A. Deligiannis, S. Lambotharan and J. A. Chambers, "Game Theoretic Analysis for MIMO Radars with Multiple Targets", accepted for publication in IEEE Transactions on Aerospace and Electronic Systems, 2016.

2. A. Deligiannis, A. Panoui, S. Lambotharan and J. A. Chambers, "Game Theoretic Power Allocation and the Nash Equilibrium Analysis for a Multistatic MIMO Radar Network", under review in IEEE Transactions on Signal Processing, 2016.

3. A. Deligiannis, S. Lambotharan and J. A. Chambers, "A Bayesian Game Theoretic Framework for Resource Allocation in Multistatic Radar Networks", to be submitted in IEEE Signal Processing letters, 2016.

Conference Papers

4. A. Deligiannis, J. A. Chambers and S. Lambotharan, "Transmit Beamforming Design for Two-Dimensional Phased-MIMO Radar with Fully-Overlapped Subarrays", Sensor Signal Processing for Defence (SSPD), Edinburgh, Sep. 2014.

5. A. Deligiannis, S. Lambotharan and J. A. Chambers, "Beamforming for Fully-Overlapped Two-Dimensional Phased-MIMO Radar", 2015 IEEE Radar Conference (RadarCon), Arlington, VA, USA, 2015.

6. A. Deligiannis, G. Rossetti, A. Panoui, S. Lambotharan and J. A. Chambers, "Power Allocation Game Between a Radar Network and Multiple Jammers", 2016 IEEE Radar Conference (RadarCon), Philadelphia, PA, USA, 2016.

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I could not close this section without highlighting my deepest and most sincere gratitude, thanks and appreciations to my beloved parents, Evangelo and Dionysia and to my sweet sister Penny, to whom I dedicate this work for their endless and unconditional love, trust and encouragement every moment in my life, including my PhD studies.

I wish I had more space to include everyone who added a bit in this 3 year effort, but I thank them from the bottoms of my heart for every nice memory.

Anastasios Deligiannis
June 2016

Contents

List of Symbols

$< \cdot >$	inner product operator
$\delta(\cdot)$	Kronecker delta function
$(\cdot)^*$	conjugate operator
$(\cdot)^T$	vector transpose operator
$(\cdot)^H$	Hermitian operator
$E[\cdot]$	statistical expectation operator
$\|.\|$	Euclidian norm
$\|.\|_F$	Frobenius norm
∇^2	Laplacian operator
$det(\cdot)$	determinant operator
$Tr(A)$	the trace of matrix A
$rank(A)$	the rank of matrix A
$diag(A)$	the diagonal of matrix A
$vec(A)$	stacks the columns of matrix A into one column vector
$\odot$	Hadamard product
$\otimes$	Kronecker product

Note, in this thesis, bold lowercase and bold uppercase letters represent vectors and matrices respectively.

List of Acronyms

2D	Two-Dimentional
AML	Approximate Maximum Likelihood
APES	Amplitude and Phase Estimation
AWGN	Additive White Gaussian Noise
B-FIM	Bayesian-Fisher Information Matrix
BNE	Bayesian Nash Equilibrium
BNG	Bayesian Nash Game
CSI	Channel State Information
DGC	Diagonal Growth Curve
DOA	Direction of Arrival
DSC	Diagonally Strict Concavity
GLRT	Generalized Likelihood Ratio Test
GNE	Generalized Nash Equilibrium
GNG	Generalized Nash Game
HMPAR	Hybrid MIMO Phased Array Radar
KKT	Karush-Kuhn-Tucket
LS	Least Squares
MIMO	Multiple-Input Multiple-Output
MTI	Moving Target Indication
MVDR	Minimum Variance Distortionless Response
NE	Nash Equilibrium
RCS	Radar Cross Section
SAR	Synthetic Aperture Radar
SDP	Semidefinite Programming
SDR	Signal-to-Disturbance Ratio
SINR	Signal-to-Interference plus Noise Ratio
SNG	Strategic Noncooperative Game
SNR	Signal-to-Noise Ratio
ULA	Uniform Linear Array
URA	Uniform Rectangular Array

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Chapter 1

Introduction

Radar is an object-detection system that uses electromagnetic energy to determine the range, altitude, direction, or speed of objects. Radar can be regarded as a way of extending human's sense of vision. Although radar is not able to resolve the characteristics of an object as detailed as the human eye, it has the ability to "see" a target under conditions such as darkness, fog, snowfall, rain or haze, and at such ranges that a human eye would fail. As a result, the importance of this breakthrough invention is colossal and the prospects of development vast.

The radar transmit dish or antenna emmits pulses of electromagnetic waves that bounce off any object in their path. The object returns a small part of the wave's energy to a receive dish or antenna that is usually located at the same site as the transmitter. Although the signal returned is usually very weak, the signal can be amplified. This enables radar to detect objects at ranges where other emissions, such as sound or visible light, would be too weak to detect. The range of the target is determined by measuring the time needed for the radar signal to travel to the target and back [1]. Regarding the angular position (direction) of the target, it is determined from the direction of arrival (DOA) of the reflected waveform. In order to distinguish moving targets from stationary objects, the shift in the carrier frequency of the reflected waveform (doppler effect) is used. Furthermore, doppler effect is used to calculate the target's relative velocity [2]. Modern radar technology applications are highly diverse, including military air-defence and antimissile systems, commercial air traffic control, meteorology, oceanology, police detection of speeding traffic, altimetry and medicine.

The fundamental principle of distance measurement exploits the fact that the radar signal pulse travels at the speed of light. By measuring the time needed for the signal emitted by the transmit antenna to arrive to the target and return to the receive antenna and assuming that the transmit and receive antennas are collocated, the distance R between the transmit antenna and the target is estimated as:

$$R = \frac{c \times t}{2}$$

where c is the speed of light (c=$3 * 10^8$ m/s) and t is the time needed for the pulse to travel to the target and arrive back. The factor of two in the equation comes from the observation that the radar pulse must travel to the target and return before detection, or covers twice the range, as shown in Fig.1.1.

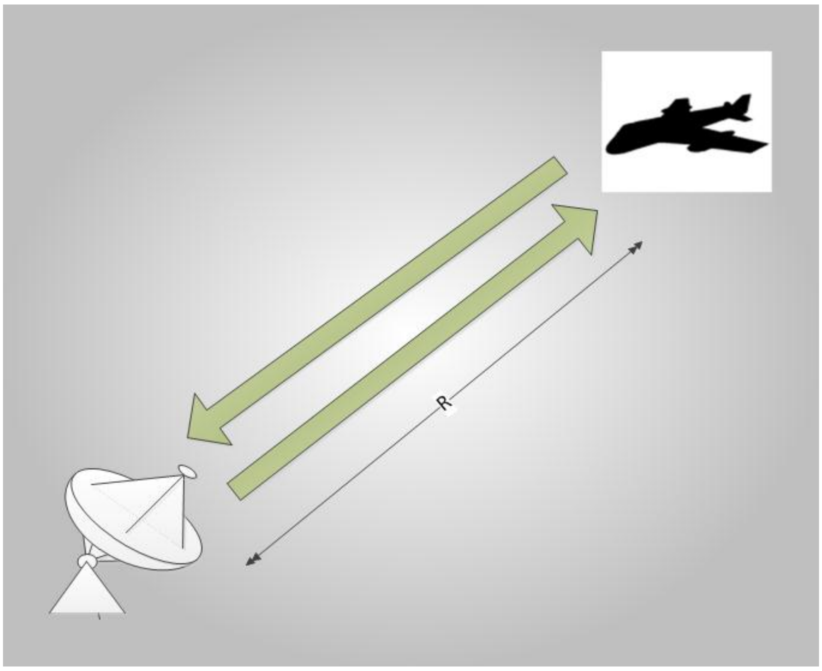


FIGURE 1.1: Basic Principle of Radar Detection

The vast field of radar research is far from young. The existence of electromagnetic waves and their ability to transmit through different types of materials and be reflected off metal surfaces were introduced in the late 19th century by Heinrich Hertz [3]. It was not before 1935 however, when Sir Robert Watson-Watt developed and patented a working radar system, that had all the necessary features of a functional pulsed radar.

During World War II, the capabilities and features of radars improved significantly. Sir Robert Watson-Watts radar technology was quickly reviewed, allowing the British Air Ministry to be the first to fully exploit radar as a defence against aircraft attack. This technology served as the basis for the Chain Home network of radars to defend Great Britain, which detected approaching German aircrafts in the Battle of Britain in 1940. This success led the radar to become pervasive by the military during the war. Furthermore, many of the radar projects currently implemented were developed during or just after the war, such as air defence, ground-approach radar, fire control and moving-target indication (MTI) radars.

After World War II, the development of radar technology continued, although not at the same rate. The next big step in the radar concept was the advent of synthetic aperture radar (SAR), which introduced a new active and unexplored area of research during the 1950s [4]. Moreover, in the 1960s, the development of phased-array antennas becomes a pioneer area of research, allowing radars to quickly change the probing beam direction. The introduction of digital signal processing, during the 1970s, was a gigantic breakthrough for the academic and research community worldwide. Naturally, this technology was also applied to radar signal processing, enabling adaptive array processing in modern radar system. Finally, the constant growth in computational capabilities allowed system engineers to introduce an emerging technology, known as Multiple-Input Multiple-Output (MIMO) radars.

MIMO radar innovative technology has raised expectations over the last decade that it will provide substantial improvements to the currently used radar systems. The superiority of a MIMO radar against other radar schemes lies in its waveform diversity, which in essence defines that a MIMO radar can simultaneously emit several diverse, possibly linearly independent waveforms via multiple antennas, in contrast to existing radar systems that transmit scaled versions of the same, predefined waveform [5, 6, 7](Fig.1.2). In particular, there are two principal types of MIMO radar, those that incorporate colocated antennas [8] and systems equipped with widely separated antennas (bistatic, multistatic) [9]. Radar systems with colocated antennas enjoy a substantially improved spatial resolution, as the matched filter receiving process extracts multiple channel information from all transmitting antennas to all receiving antennas. In other words, due to the phase differences induced by the different transmit and receive antennas, colocated MIMO radar system have the capability of forming long virtual receive arrays with a small number of antennas. MIMO radar with colocated antennas has been proved to offer better parameter identifiability [10], higher sensitivity to detect slowly moving targets, higher angular resolution, increased number of detectable targets, direct applicability of adaptive array and beamforming techniques [11] and exceptional clutter interference mitigation capabilities [12, 13]. On the other hand, distributed MIMO radar systems provide the ability to capture the target's geometrical characteristics through the radar cross section (RCS), since each widely separated radar captures a different aspect of the target [14, 15, 16]. In addition, multistatic MIMO radar system offers direct applicability of adaptive beamforming techniques, enhance the ability to combat signal scintillation and estimate precisely the parameters of fast moving targets.

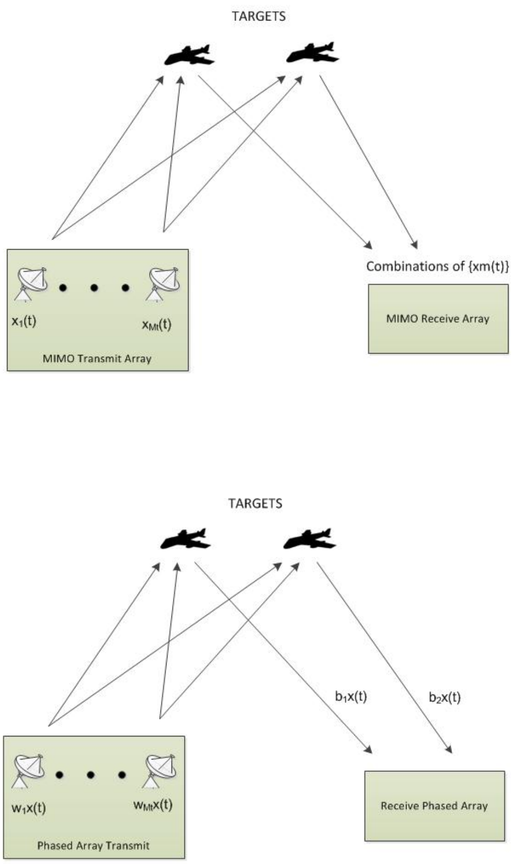


FIGURE 1.2: MIMO radar versus phased array radar.

Nevertheless, modern decentralized radar networks suffer from multiple source of interference imposed at the receivers of each radar, namely the cross channel interference among different radars in the same network, the clutter interference, the background noise and possible jamming interference. This interference seriously deteriorates the performance, the reliability and the tracking performance of a radar system. Hence, in order to mitigate this drawback the research community developed methods for optimized beamforming and power allocation, jammer suppression, physical and virtual array processing and uncertainty handling.

The term beamforming originates from the fact that early airborne radars incorporated parabolic type antennas, which formed pencil beams, in order to steer the beam at the direction of interest and to attenuate interfering signals from other directions [17]. Beamforming can be applied both at the transmitter and the receiver side of a radar system. Modern radars employ one dimensional or two dimensional arrays of antennas both at the transmitter and the receiver side and hence the transmit and receive beam steering is performed electronically by applying various signal processing techniques. In order to determine the appropriate beamformer, an estimate of the direction of interest is needed. Hence, in the absence of array calibration errors, several nonparametric adaptive techniques can be utilized such as the minimum variance distortionless response (MVDR or Capon) estimator [18], the amplitude and phase estimation (APES) approach or the MUSIC algorithm. After the acquisition of the target direction, the objective is to design accurate transmit and receive beamformers and thus one can use adaptive or non-adaptive techniques. Conventional non-adaptive beamforming is a simple technique, where the transmit and receive beamforming vectors are the normalized transmit and receive steering vectors at the direction of the target [19]. Furthermore, a receive beamformer that protects the target return signal and at the same time minimizes the sidelobe levels of the beampattern is the MVDR or Capon beamformer [18], [20]. Since 1990, convex optimization techniques have been widely exploited in engineering and more specifically in signal processing applications. Following the stream, convex approximation of a desired transmit or receive beampattern has been investigated, where the convex optimization problem minimizes the difference between a desired beampattern at the direction of the target and the actual beampattern produced by the array of antennas, under several power constraints [11], [21]. Convex optimization has been also used for power minimization beamforming and minimum sidelobe beampattern design [22],[11].

Apart from an optimal beamforming strategy, in order to combat multiple source interference in a radar field, while achieving high SINR, the system should apply an optimal power allocation strategy. Although uniform power allocation is currently used in many modern radar systems, it is proved that it is not necessarily the optimal solution for a specific power budget [23]. In communication theory, a centralized scheme is proposed for optimal resource allocation using convex optimization techniques [24]. Nevertheless, centralized control is impractical and computationally expensive to be implemented in a distributed radar network, as it requires large communications overhead, and thus an autonomous decentralized power allocation scheme is more appropriate. A natural and efficient tool to achieve an optimum interference mitigation is game theory, as it offers a mathematical framework of conflict and cooperation among intelligent, self-interested and rational players.

The increasing need for independent, autonomous and decentralized communication systems has sparked much interest in using game theoretic techniques in the communication literature [25]. More specifically, the aforementioned distributed, multistatic beamforming and resource allocation problem in radar systems can be compared to similar issues raised in multicell wireless systems in communication applications [26]-[27]. In [26], the authors introduced the idea of joint beamforming and power control, proposing an iterative algorithm to simultaneously obtain the optimal beamforming and power vectors. The incorporation of game theory in this context then rapidly became a focal point in communications research [28]-[29]. The majority of this literature considers the technique of strategic noncooperative games (SNG), where each player selfishly maximizes its payoff function, given the strategies of the other players. The authors of [28] exploited an iterative water-filling algorithm to reach the Nash equilibrium in a non-cooperative, distributed, multiuser power control problem. Since each player greedily optimizes its utility function, the equilibrium might not be the Pareto-optimal solution. Introducing pricing policies to the system resources leads to a more Pareto-efficient solution and increases the social welfare of the system. A pricing regime that is a linear function of the transmit power was studied in [30]. Another example of pricing the transmit power of each player is considered in [31], whereas in [32] and [33] the pricing policy is applied on the intercell interference among the players. In [29], the authors considered the optimization of a set of precoding matrices at each node of a multi-channel, multi-user cognitive radio MIMO network in order to minimize the total transmit power of the network, while applying a pricing scheme based on global information. Cooperative game theoretic techniques combined with a two-level Stackelberg game were utilized in [34] to address the problem of relay selection and power allocation without the knowledge of channel state information (CSI). Finally, the authors in [27] formulated a Stackelberg Bayesian game to obtain the optimal power allocation for a two-tier network, while applying an interference constraint at the leader and considering channel gain uncertainty. Recently, game theoretic techniques have been extensively explored within the radar research community to obtain optimal resource allocation. The authors in [35] and [36] addressed the power allocation problem by formulating a non-cooperative game with predefined SINR constraints. Since a radar in a distributed network can not obtain information regarding the transmission power of the remaining radars in the network, an SINR estimation technique was applied in [37], to extend the work in [35]. Furthermore, the authors in [38] exploited cooperative game theoretic techniques to encounter the resource allocation problem through maximizing the Bayesian-Fisher information matrix (B-FIM) and utilizing the Shapley value solution. A combination of a water filling algorithm and a Stackelberg game was used in [39] for optimal power distribution.

An adaptive defence system may also employ game theoretic methods when confronting smart targets equipped with jammers, that are able to damage the radar system and deteriorate its performance. Using several game theoretic techniques, such as non-cooperative games, when there is no communication among the players, cooperative games, when there is some coordination between the players, zero sum games, when each player's profit (or loss) of utility is exactly equal to the losses (or profits) of the remaining players, Stackelberg games, when there is hierarchy in the game considered, and Bayesian games, when there is some sort of uncertainty within the system, the radar engineer can model several scenarios and respond optimally to the jammer threat.

Handling uncertainty is a crucial problem in radar systems, especially within a decentralized and multistatic network, where no communication among the radars is considered. Most of the game theoretic radar literature employs non Bayesian and perfect information systems, where every player can obtain any information needed as common knowledge, such as other player's exact channel gains, the precise value of clutter channel gain or the target's RCS. However, this may not be feasible in a real distributed system. Therefore, uncertainty regarding the various channel gains or the RCS parameters is essential in order to obtain an optimal and realistic solution. Bayesian game theory provides the framework to address such problems of incomplete information. In a Bayesian game, the players are not considered uniform but they are characterized from their type, that depends on the uncertainty parameter and its distribution.

Overall, radar arrays subaperturing techniques, exploitation of waveform diversity, adaptive beamforming techniques, game theoretic beamforming and resource allocation methods, jammer suppression and uncertainty handling techniques provide promising ground to improve radar characteristics and mitigate multi-source interference in a radar network. This thesis presents and describes novel algorithms associated with the aforementioned approaches.

Thesis Outline

Interference attenuation, jammer suppresion, uncertainty handling and optimal beamforming and power allocation techniques are crucial challenges within the radar community, especially if one takes under consideration the increased demands of the modern air-defence and electronic warfare systems. The work in this thesis is focused on addressing the aforementioned challenges and improving the respective solutions. Therefore novel two-dimensional beamforming techniques and game theoretic techniques for beamforming, resource allocation, jammer suppression and uncertainty handling have been proposed for MIMO radar systems.

In Chapter 2, a thorough literature review is presented along with the necessary methodology and technical background. More specifically, the basic principles of the MIMO radar technology have been summarized. The chapter also includes adaptive and non-adaptive techniques for transmit and receive beamforming and array subaperturing. Furthermore, the basic frameworks for convex optimization and game theory are introduced and their applications in optimal beamforming, power allocation, SINR maximization and uncertainty handling are discussed.

In Chapter 3, a subaperturing technique for two-dimensional (2D) arrays within the context of MIMO radar is investigated. Initially, the performance of transmit beamforming using fully overlapped subarrays of a 2D transmit array is investigated. Then, the waveform diversity and overall transmit-receive beampatterns for the Phased-MIMO radar with fully overlapped subarrays are derived and compared with the respective beampatterns for the phased-array and mimo radar systems. As reported for one-dimensional linear arrays, fully overlapped subarrays offer substantially improved beamformers as compared with the phased-array and MIMO radar schemes. In order to obtain the optimal transmit-receive beamformers both adaptive (convex optimization, MVDR) and non-adaptive (conventional) techniques are considered.

In Chapter 4, a game theoretic power allocation scheme is investigated and a Nash equilibrium analysis for a MIMO radar network is performed. A network of radars is considered, organized into multiple clusters, whose primary objective is to minimize their transmission power, while satisfying a predefined SINR criterion. Since there is no communication between the distributed clusters, convex optimization methods and noncooperative game theoretic techniques based on the estimate of the SINR to tackle the power adaptation problem are utilized. Therefore, each cluster egotistically determines its optimal power allocation in a distributed way. Furthermore, the best response function of each cluster regarding this generalized Nash game (GNG) is proved to belong to the framework of standard functions. The standard function property together with the proof of the existence of solution for the game guarantees the uniqueness of the Nash equilibrium. The mathematical analysis of the uniqueness of the solution leads to two substantial contributions on the relation between the performance with respect to the detection criterion and the transmission power of the radars. In addition, a competitive power allocation problem for a MIMO radar system in the presence of multiple targets equipped with jammers is assumed. The main objective of the radar network is to minimize the total power emitted by the radars while achieving a specific detection criterion for each of the targets, while the intelligent jammers have the ability to observe the radar transmission power and consequently decide its jamming power to maximize the interference to the radars. In this context, convex optimization methods, noncooperative game theoretic techniques and hypothesis testing to identify jammers and to determine the optimal power allocation are incorporated. Finally, a proof of the existence and uniqueness of the solution is presented.

Chapter 5 investigates a Bayesian game theoretic SINR maximization scheme for a multistatic radar network. Specifically, a distributed network of radars is considered, whose primary goal is to maximize their signal-to-noise ratio (SINR), while satisfying a predefined power constraint. Moreover, no communication between the radars is assumed and hence a noncooperative approach is utilized. The channel gain between a radar and the target is assumed as private information and characterizes the type of the player, whereas the distribution of the channel gain is common knowledge to every player in the game. Subsequently, the examination and proof of the existence and the uniqueness of the Bayesian Nash equilibrium for the aforementioned game is presented.

Chapter 6 considers a distributed beamforming and resource allocation technique for a radar system in the presence of multiple targets. The primary objective of each radar is to minimize its transmission power while attaining an optimal beamforming strategy and satisfying a certain detection criterion for each of the targets. Therefore, convex optimization methods are utilized together with noncooperative and partially cooperative game theoretic approaches. Initially, a strategic noncooperative game (SNG) is considered, where there is no communication between the various radars of the system. Hence each radar selfishly determines its optimal beamforming and power allocation. Subsequently, a more coordinated game theoretic approach is assumed incorporating a pricing mechanism. Introducing a price in the utility function of each radar/player, enforces beamformers to minimize the interference induced to other radars and to increase the social fairness of the system. Furthermore, a Stackelberg game is formulated by adding a surveillance radar to the system model, which will play the role of the leader, and hence the remaining radars will be the followers. The leader applies a pricing policy of interference charged to the followers aiming at maximizing his profit while keeping the incoming interference under a certain threshold. Finally, the proof of the existence and uniqueness of the Nash Equilibrium (NE) in both the partially cooperative and noncooperative games is presented.

Concluding remarks and possible future research challenges are discussed in Chapter 7.

Chapter 2

Literature Review and Technical Background

2.1 Introduction

In this chapter, a comprehensive literature review and the respective technical and methodology background required for the contributing chapters are presented. Initially, the basic principle and the description of the main characteristics of the MIMO technology are provided. Then, adaptive and conventional beamforming techniques are presented and particular emphasis is placed on the adaptive convex beamforming methods. Furthermore, resource allocation techniques within the game theoretic framework are discussed and various game theoretic schemes are explained.

2.2 MIMO Radar Spatial Diversity Techniques

2.2.1 Virtual Array Concept

As mentioned in the introduction, emerging MIMO radars technology provides the ability for a radar system to transmit via its antennas multiple probing signals that may be correlated or uncorrelated, depending on the application requirements. Therefore, by matched filtering these signals at the receiver side, the radar system can exploit this waveform diversity and enable superior capabilities regarding target detection, higher resolution and better parameter identifiability. Another substantial benefit from waveform diversity is that the phase differences corresponding to different transmit antennas combined with the phase differences caused by different receive antennas can produce a virtual receive array, much longer than the actual one. Thus, the spatial resolution of the target and the mitigation of the interfering sources can be dramatically increased.

Consider a MIMO radar system with M_t antennas at the transmitter and M_r antennas at the receiver. The transmit and receive arrays are assumed to be colocated so that they see the targets at the far field at same directions. Moreover the i^{th} transmit antenna emits the i^{th} element of the predesigned independent waveform vector $\boldsymbol{\psi}(t) = [\psi_1(t), \ldots, \psi_{M_t}(t)]^T$ of size $M_t \times 1$, which satisfies the orthogonality condition $\int_{T_0} \boldsymbol{\psi}(t)\boldsymbol{\psi}^H(t)dt = \mathbf{I}_{M_t}$. Assuming that there is a target present at the far-field of the transmit and receive arrays at direction θ_t, the probing signal emitted at the target direction from the transmit array can be described as [5, 11, 21]:

$$x_{tr}(t) = \mathbf{a}^H(\theta_t)\boldsymbol{\psi}(t) \tag{2.1}$$

where $\mathbf{a}(\theta)$ denotes the transmit steering vector associated with direction θ and is given by:

$$\mathbf{a}(\theta) = [e^{j2\pi f_0 \tau_1(\theta)} \quad e^{j2\pi f_0 \tau_2(\theta)} \quad \ldots \quad e^{j2\pi f_0 \tau_{M_t}(\theta)}]^T \tag{2.2}$$

where f_0 is the carrier frequency of the radar and $\tau_m(\theta)$ is the amount of time needed by probing signal emitted from the m^{th} antenna to arrive at the target. The corresponding time delay can be given by:

$$\tau_m(\theta) = \frac{dsin(\theta)}{v} = \frac{dsin(\theta)}{f_0\lambda}$$

where d is the distance between adjacent antennas, v denotes the speed of light and λ the wavelength of the incident waves. Hence, the transmit steering vector can be restated as:

$$\mathbf{a}(\theta) = [1 \quad e^{\frac{j2\pi dsin(\theta)}{\lambda}} \quad \ldots \quad e^{\frac{j2\pi(M-1)dsin(\theta)}{\lambda}}]^T \tag{2.3}$$

The $M_r \times 1$ snapshot vector arriving at the receive array from the direction of the target θ_t can be modeled as:

$$\mathbf{x}_{rec}(t) = \beta_t(\mathbf{a}^H(\theta_t)\boldsymbol{\psi}(t))\mathbf{b}(\theta_t) + \mathbf{n}(t) \tag{2.4}$$

where β_t is the complex valued amplitude reflecting the RCS of the target and $\mathbf{b}(\theta)$ is the actual receive steering vector associated with direction θ and is given by:

$$\mathbf{b}(\theta) = [e^{j2\pi f_0 \bar{\tau}_1(\theta)} \quad e^{j2\pi f_0 \bar{\tau}_2(\theta)} \quad \ldots \quad e^{j2\pi f_0 \bar{\tau}_{M_r}(\theta)}]^T \tag{2.5}$$

where $\bar{\tau}_m(\theta)$ is the time needed for the signal reflected by the target located

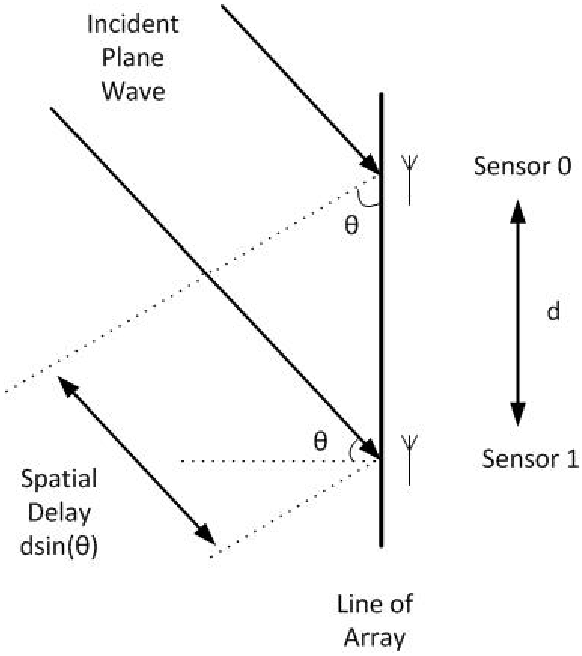


FIGURE 2.1: Spatial delay of the received signal.

at θ to arrive at the m^{th} receive antenna. Following the steps derived for the transmit steering vector above, the receive steering vector can be restated as (see Fig. 2.1):

$$\mathbf{b}(\theta) = [1 \quad e^{\frac{j2\pi d sin(\theta)}{\lambda}} \quad \dots \quad e^{\frac{j2\pi (M_r-1) d sin(\theta)}{\lambda}}]^T \tag{2.6}$$

The returns of the transmitted waveforms can be extracted by applying in each receive antenna matched-filtering to the received signal with each of the orthogonal waveforms $\psi_i(t)$, $i = 1, \dots, M_t$. Hence, the maximum number of recovered signals can be $M_t M_r$ and the $M_t M_r \times 1$ virtual receive data vector can be obtained as:

$$\mathbf{y} = [\mathbf{x}_1^T \dots \mathbf{x}_{M_t}^T]^T = \beta_t \mathbf{a}(\theta_t) \otimes \mathbf{b}(\theta_t) + \hat{\mathbf{n}} \tag{2.7}$$

where $\hat{\mathbf{n}} = \int_{T_0} \mathbf{n}(t)\psi_k^*(t)dt$ is the $M_t M_r \times 1$ noise term with covariance matrix $\mathbf{R}_n = \sigma_n^2 \mathbf{I}_{M_t M_r}$ (σ_n^2 is the noise variance) and $\mathbf{x}_i^T$ is given by:

$$\mathbf{x}_i = \int_{T_0} \mathbf{x}_{rec}(t)\psi_i^*(t)dt, \quad i = 1, \dots, M_t$$

The target signal part can be expressed as:

$$\mathbf{y}_t = \beta_t \mathbf{u}(\theta_t) \tag{2.8}$$

where $\mathbf{u}(\theta_t) = \mathbf{a}(\theta_t) \otimes \mathbf{b}(\theta_t)$ corresponds to the $M_t M_r \times 1$ virtual receive steering vector. It is straightforward that the phase differences are created by both the transmitting antennas and the receiving antennas at relevant locations. Therefore, the received target signal component in (2.8) is identical to the response received by a receive array with M_tM_r antennas. This M_tM_r element size array is known as MIMO virtual receive array (Fig.2.2) . Hence, a MIMO radar system with M_t transmit and M_r receive antennas can enjoy up to M_tM_r degrees of freedom by utilizing only $M_t + M_r$ physical sensors. The interested reader can find more detailed reviews regarding the virtual array concept in the literature [40, 6, 7, 41].

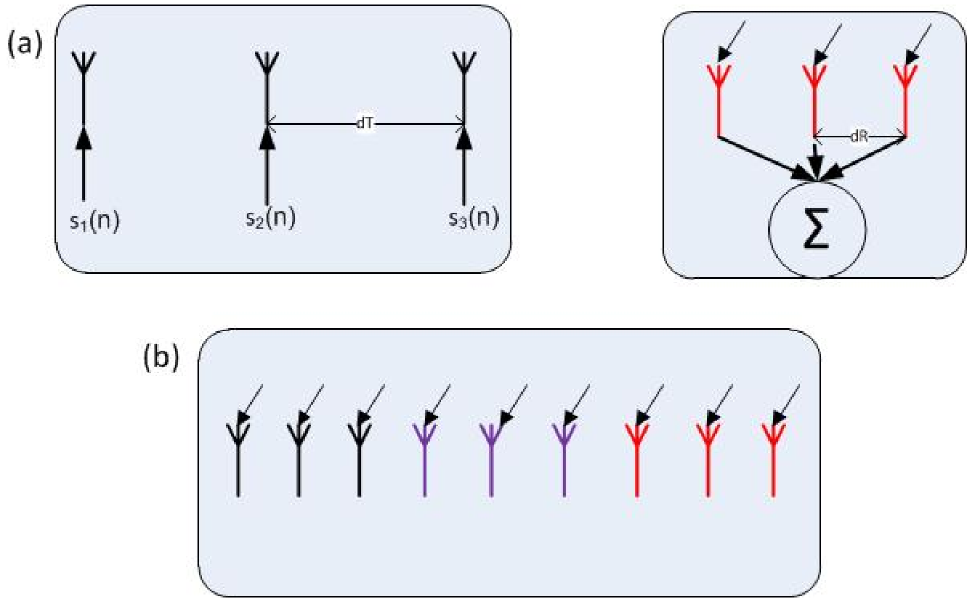


FIGURE 2.2: (a)Illustration of a ULA MIMO radar with $M_t = 3$ and $M_r = 3$ antennas.(b)Corresponding virtual receive array.

2.2.2 Target Estimation Adaptive Techniques

In this section, the ability of MIMO radars to use adaptive localization and detection techniques is discussed [42, 43]. This significant advantage of MIMO radars leads to much higher resolution and much more efficient interference rejection capability than phased-array radars. Furthermore, the direct application of adaptive techniques is made possible for MIMO radar systems without the use of secondary range bins or even range compression, since the probing signals reflected back from the targets are linearly independent and uncorrelated of each other [44, 45].

Under the assumption that the targets are points in space, the received data vector, when N samples of the continuous time are considered, can be expressed by the equation [14]:

$$\mathbf{y}(n) = \sum_{k=1}^{K} \beta_k \mathbf{b}^*(\theta_k)\mathbf{a}^H(\theta_k)\mathbf{x}(n) + \epsilon(n), \quad n = 1, \ldots, N \tag{2.9}$$

where n corresponds to the considered snapshot, K is the number of targets of interest, N is the total number of snapshots, β_k are the complex amplitudes proportional to the radar cross section (RCS) of the targets, θ_k are the location parameters of the targets, $\epsilon(n)$ expresses the interference plus noise term. The objective of Parameter Estimation is to estimate the unknown parameters β_k and θ_k.

Let

$$\bar{A} = [\beta_1 \mathbf{a}(\theta_1) \quad \beta_2 \mathbf{a}(\theta_2) \quad \ldots \beta_K \mathbf{a}(\theta_K)] \tag{2.10}$$

The sample covariance matrix of the target reflected waveforms is $\bar{A}^H R_{xx} \bar{A}$, where

$$\hat{R}_{xx} = \frac{1}{N} \sum_{n=1}^{N} \mathbf{x}(n)\mathbf{x}^H(n) \tag{2.11}$$

is the sample covariance matrix of the transmit signal vector. In the case of orthogonal waveform vectors used for MIMO transmit beampattern and $N \leq M_t$, then $\hat{R}_{xx}$ is a scaled identity matrix. As a result, $\bar{\mathbf{A}}^H \hat{\mathbf{R}}_{xx} \bar{\mathbf{A}}$ has full rank, if the columns of $\bar{\mathbf{A}}$ are linearly independent to each other, which requires that $K \leq M_t$. This fact implies that the target reflected waveforms are not completely correlated with each other. The fact that the target reflected signals are noncoherent allows the unconstrained application of many adaptive techniques for parameter estimation [44, 45].

It is assumed that θ is the direction of arrival (DOA) of the target, when the target is far away from the array of the receivers. The matrix form of the signal at the output of the receiving array can be expressed with the following equation:

$$\mathbf{Y} = \mathbf{b}^*(\theta)\beta(\theta)\mathbf{a}^H(\theta)\mathbf{X} + \mathbf{Z} \tag{2.12}$$

where $\mathbf{X} = [\mathbf{x}(1) \quad \mathbf{x}(2) \quad \ldots \quad \mathbf{x}(N)]$, the received data samples $\{\mathbf{y}(n)\}_{n=1}^{N}$ are the columns of $\mathbf{Y} \in \mathcal{C}^{M_r \times N}$, and $\beta(\theta) \in \mathcal{C}$ expresses the complex amplitude of the reflected signal from θ, which is analogous to the RCS of the location θ. Finally, the interferences from targets at different locations than θ or at other range bins, the intentional or not jamming, and the atmospheric noise are denoted by the matrix $\mathbf{Z} \in \mathcal{C}^{M_r \times N}$.

The objective of the target estimation techniques is to estimate $\beta(\theta)$ from the data matrix $\mathbf{Y}$. A spatial spectrum can then be created by the estimates of $\beta(\theta)$. As a result, the location of the targets can be decided by looking for the peaks in the aforementioned spatial spectrum.

Least Squares Estimation

The least-squares (LS) method is the simplest way of estimating $\beta(\theta)$. This method leads us to the following equation:

$$\beta_{LS}(\theta) = \frac{\mathbf{b}^T(\theta)\mathbf{Y}\mathbf{X}^H\mathbf{a}(\theta)}{N\|\mathbf{b}(\theta)\|^2[\mathbf{a}^H(\theta)\hat{\mathbf{R}}_{xx}\mathbf{a}(\theta)]} \tag{2.13}$$

Although it is a really fast method that requires minimum computational power, the LS method suffers from high sidelobes and displays low resolution results. Furthermore, when strong jamming and interference are present, this method fails to provide acceptable results.

Minimum Variance Distortionless Response (MVDR)

A much more efficient method of determining $\beta(\theta)$ is the Minimum Variance Distortionless Response beamformer, as it is well known [18, 20]. The MVDR beamformer can be designed as follows:

$$\min_{\mathbf{w}} \quad \mathbf{w}^H\hat{\mathbf{R}}_{yy}\mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H\mathbf{b}^*(\theta) = 1 \tag{2.14}$$

where $\mathbf{w} \in \mathcal{C}^{M_r \times 1}$ is the optimum set of weights that minimizes the mean square value of $\mathbf{y}(n)$, while suppressing noise, interference and jamming factors. $\mathbf{R}_{yy}$ is the correlation matrix of the received signal vector at the array of antennas:

$$\hat{\mathbf{R}}_{yy} = \frac{1}{N}\mathbf{Y}\mathbf{Y}^H \tag{2.15}$$

Using the method of Lagrangian multipliers to optimize (2.14), the solution for the weight vector can be obtained as:

$$\hat{\mathbf{w}}_{Capon} = \frac{\hat{\mathbf{R}}_{yy}^{-1}\mathbf{b}^*(\theta)}{\mathbf{b}^T(\theta)\hat{\mathbf{R}}_{yy}^{-1}\mathbf{b}^*(\theta)} \tag{2.16}$$

As a result, the output of the MVDR beamformer will be given by:

$$\frac{\mathbf{b}^T(\theta)\hat{\mathbf{R}}_{yy}^{-1}\mathbf{Y}}{\mathbf{b}^T(\theta)\hat{\mathbf{R}}_{yy}^{-1}\mathbf{b}^*(\theta)} \tag{2.17}$$

By substituting (2.12) to (2.17) the following stands:

$$\frac{\mathbf{b}^T(\theta)\hat{\mathbf{R}}_{yy}^{-1}\mathbf{Y}}{\mathbf{b}^T(\theta)\hat{\mathbf{R}}_{yy}^{-1}\mathbf{b}^*(\theta)} = \beta(\theta)\mathbf{a}^H(\theta)\mathbf{X} + \frac{\mathbf{b}^T(\theta)\hat{\mathbf{R}}_{yy}^{-1}\mathbf{Z}}{\mathbf{b}^T(\theta)\hat{\mathbf{R}}_{yy}^{-1}\mathbf{b}^*(\theta)} \tag{2.18}$$

Finally, in order to obtain $\beta(\theta)$, the LS method is applied to (2.18). As a result, the MVDR estimate of $\beta(\theta)$ will be given as:

$$\beta_{Capon}(\theta) = \frac{\mathbf{b}^T(\theta)\hat{\mathbf{R}}_{yy}^{-1}\mathbf{Y}\mathbf{X}^H\mathbf{a}(\theta)}{N[\mathbf{b}^T(\theta)\hat{\mathbf{R}}_{yy}^{-1}\mathbf{b}^*(\theta)][\mathbf{a}^H(\theta)\hat{\mathbf{R}}_{xx}\mathbf{a}(\theta)]} \tag{2.19}$$

Amplitude and Phase Estimation (APES)

An adaptive method that achieves better amplitude estimation accuracy is the Amplitude and Phase Estimation method (APES) [5, 46, 47]. Following [47] the APES method can be formulated as follows:

$$\min_{\mathbf{w},\beta} \|\mathbf{w}^H\mathbf{Y} - \beta(\theta)\mathbf{a}^H(\theta)\mathbf{X}\|^2 \quad \text{subject to} \quad \mathbf{w}^H\mathbf{b}^*(\theta) = 1 \tag{2.20}$$

where $\mathbf{w}$ denotes the same as in CAPON method. The translation of the aforementioned minimization problem is to acquire a beamformer that best approximates the waveform $\mathbf{a}(\theta)^H\mathbf{X}$.

Minimizing the objective function in (2.20) with respect to $\beta(\theta)$ yields:

$$\hat{\beta}_{APES}(\theta) = \frac{\mathbf{w}^H\mathbf{Y}\mathbf{X}^H\mathbf{a}(\theta)}{N\mathbf{a}^H(\theta)\hat{\mathbf{R}}_{xx}\mathbf{a}(\theta)} \tag{2.21}$$

As a result, the optimization problem (2.20) reduces to

$$\min_{\mathbf{w}} \mathbf{w}^H\hat{\mathbf{Q}}\mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H\mathbf{b}^*(\theta) = 1 \tag{2.22}$$

where $\hat{\mathbf{Q}}$ is the residual covariance estimate calculated as follows:

$$\hat{\mathbf{Q}} = \hat{\mathbf{R}}_{yy} - \frac{\mathbf{Y}\mathbf{X}^H\mathbf{a}(\theta)\mathbf{a}^H(\theta)\mathbf{X}\mathbf{Y}^H}{N^2\mathbf{a}^H(\theta)\hat{\mathbf{R}}_{xx}\mathbf{a}(\theta)} \tag{2.23}$$

Solving the optimization problem in (2.22), the APES method weight vector can be obtained as:

$$\mathbf{w}_{APES} = \frac{\hat{\mathbf{Q}}^{-1}\mathbf{b}^*(\theta)}{\mathbf{b}^T(\theta)\hat{\mathbf{Q}}^{-1}\mathbf{b}^*(\theta)} \tag{2.24}$$

By inserting (2.24) to (2.21), the estimation of APES method regarding $\beta(\theta)$ can be acquired as

$$\beta_{APES}(\theta) = \frac{\mathbf{b}^T(\theta)\hat{\mathbf{Q}}^{-1}\mathbf{Y}\mathbf{X}^H\mathbf{a}(\theta)}{N[\mathbf{b}^T(\theta)\hat{\mathbf{Q}}^{-1}\mathbf{b}^*(\theta)][\mathbf{a}^H(\theta)\hat{R}_{xx}\mathbf{a}(\theta)]} \tag{2.25}$$

It is observed that the two estimations of $\beta(\theta)$ from CAPON and APES are quite similar. In fact, at APES method the sample covariance matrix R_{yy} in (2.19) is replaced by the residual covariance estimate $\hat{\mathbf{Q}}$.

CAPES and CAML

Although the CAPON and APES methods seem similar, in fact there is a major difference between these two beamformers. This difference is a trade-off between high resolution and amplitude estimation. CAPON method provides high resolution, leading to accurate estimates of the targets location. However, its amplitude estimates are significantly biased downward. On the other hand, APES method manages to exactly approximate the amplitude, but trading-off resolution. In other words, CAPON beamformer is capable of resolving two targets, even if they are closely spaced, but lacks at the amplitude estimation. In contrast to this, APES method provides an exact estimation on the targets location, but needs greater minimum distance to resolve them.

To take advantage of the benefits of both CAPON and APES, a combination, known as CAPES, has been proposed [48]. This method initially estimates the target locations through the CAPON method and then estimates the amplitude at these locations using the APES estimator.

An improvement to the CAPES method has been proposed lately [49], referred to as CAML estimator. This method combines CAPON and the more recently proposed approximate maximum likelihood (AML) estimator, which is based on a diagonal growth curve (DGC) model. In fact, AML is substituting the procedure of APES at CAPES, in other words, it is used to estimate the target amplitudes after the target locations are acquired by CAPON. CAML is able to provide better amplitude estimation accuracy than CAPES, because AML, unlike APES, estimates the amplitudes of all targets at the same time rather than one at a time.

2.2.3 Transmitter Beamforming

It is assumed that the system is able to provide an estimate of the targets' locations through a surveillance radar. After acquiring an initial information on the target location coordinates from the aforementioned estimation techniques, then beamforming techniques are applied at the physical layer of the transmit array of the MIMO radar in order to control the directionality of the probing signal and focus the transmitted power at a desired sector of space, usually at the targets's direction [17]. Hence a beamformer coefficient should be designed for each transmit antenna to steer the transmit beam towards the desired sector in space. A typical linear transmit array is depicted in Fig.2.3, also showing the beamformer coefficients.

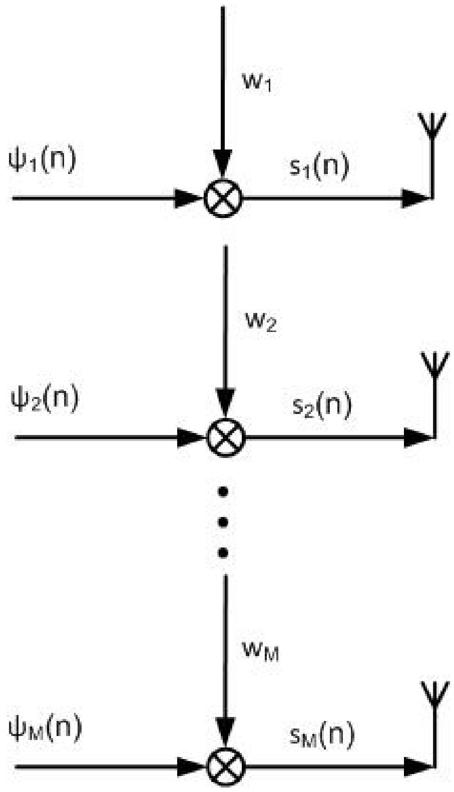


FIGURE 2.3: Linear transmit array with beamforming coefficients.

Transmit Matched Filtering Beamforming

The complex envelope of the signals at the output of the transmit array can be given as follows:

$$\mathbf{s}(t) = \mathbf{w} \odot \boldsymbol{\psi}(t) \tag{2.26}$$

where $\mathbf{w} \in C^{M\times 1}$ is the transmit weight vector regarding the linear transmit array, as shown in Fig.2.3, and $\boldsymbol{\psi}(t)$ is the predesigned independent waveform vector. The power of the probing signal emitted by the transmit array gives the transmit beampattern of the MIMO radar and can be modeled as:

$$P(\theta) = \mathbf{a}^H(\theta) \int\limits_{T_0} \mathbf{s}(t)\mathbf{s}^H(t)dt\mathbf{a}(\theta) = \mathbf{a}^H(\theta)\mathbf{w}\mathbf{w}^H\mathbf{a}(\theta)$$

In the absence of interfering or clutter sources, the non-adaptive matched filtering beamforming is the simplest technique to design the optimal transmit beampatterns. However, this method offers the highest possible output SINR gain only when a single target is observed in the background of white Gaussian noise [19]. By applying the matched filtering beamformer in a uniform linear array, the normalized transmit weight vector is obtained as

$$\mathbf{w} = \frac{\mathbf{a}_k(\theta_t)}{\|\mathbf{a}_k(\theta_t)\|} \tag{2.27}$$

and drives the transmit beampattern towards the direction of the single target.

Beampattern Matching Beamforming

In an actual radar field, the presence of more than one targets is probable and the absence of any interfering signals unrealistic. Thus, a more efficient beamforming technique than matched filtering is necessary. Having applied the target estimation adaptive techniques, the next step is to design a probing signal vector to approximate a desired transmit beampattern, containing the location information acquired by the aforementioned techniques. Transmit beampattern design is crucial in many fields including communications, defence and biomedical applications. By denoting as $\theta_1, \ldots, \theta_K$ the target locations, the desired beampattern is defined as follows:

$$\phi(\theta) = \begin{cases} 1, & \theta \in [\theta_k - \Delta, \theta_k + \Delta], \quad k = 1, \ldots, K \\ 0 & \text{otherwise} \end{cases} \tag{2.28}$$

where 2Δ is the chosen beamwidth for each target. The power of the probing signal at location θ is given by [21, 11, 50]:

$$P(\theta) = \mathbf{a}^H(\theta)\mathbf{R}\mathbf{a}(\theta) \tag{2.29}$$

where $\mathbf{R}$ is the covariance matrix of $\mathbf{x}(n)$:

$$\mathbf{R} = E\{\mathbf{x}(n)\mathbf{x}^H(n)\} \tag{2.30}$$

The equation (2.29), as a function of θ, is known as the *transmit beampattern*.

The primary objective is to determine the covariance matrix $\mathbf{R}$, which must be positive semidefinite, in order to minimize the difference between the desired transmit beampattern and the one obtained from (2.29). In other words, the desired $\mathbf{R}$ utilizes the available transmit power to maximize the probing signal power at the locations of the targets of interest and to minimize it anywhere else [11]. Furthermore, a second goal is that the covariance matrix $\mathbf{R}$ must minimize the cross-correlation between the probing signals at locations θ and θ_p ($\theta \neq \theta_p$), given by $\mathbf{a}^H(\theta)\mathbf{R}\mathbf{a}(\theta_p)$. This optimization problem is solved through convex optimization techniques [51].

Let $\{\mu_l\}_{l=1}^{L}$ be a fine grid of points that cover the radar far field area of interest and approximate the target locations $\{\theta_k\}_{k=1}^{K}$, where K denotes the number of the targets of interest. It is presumed that some of these grid points are acceptable approximations of the locations $\{\theta_k\}_{k=1}^{K}$ and these points are denoted as $\{\hat{\theta}_k\}_{k=1}^{K}$. Mathematically, the aforementioned problem of choosing $\mathbf{R}$, such that the transmit beampattern $\mathbf{a}^H(\theta)\mathbf{R}\mathbf{a}(\theta)$, best approximates the desired transmit beampattern $\phi(\theta)$ and furthermore such that the cross-correlation $\mathbf{a}^H(\theta)\mathbf{R}\mathbf{a}(\theta_p)$ is minimized over the acceptable approximations $\{\hat{\theta}_k\}_{k=1}^{K}$, can be formulated as:

$$\min_{a,\mathbf{R}} \left\{ \frac{1}{L}\sum_{l=1}^{L} w_l[\alpha\phi(\mu_l) - \mathbf{a}^H(\mu_l)\mathbf{R}\mathbf{a}(\mu_l)]^2 + \frac{2w_c}{K^2-K}\sum_{k=1}^{K-1}\sum_{p=k+1}^{K} |\mathbf{a}^H(\hat{\theta}_k)\mathbf{R}\mathbf{a}(\hat{\theta}_p)|^2 \right\}$$

$$s.t. \quad R_{mm} = \frac{P_{max}}{M_t}, \quad m = 1, \ldots, M_t$$

$$\mathbf{R} \geq 0 \tag{2.31}$$

where P_{max}/M_t is the transmitted power from each antenna and P_{max} is the total transmit power, which is given. The first constraint reflects the fact that transmit power from each antenna element should be the same, which allows the MIMO radar system to transmit at the maximum available power. Moreover, the second constraint ensures that $\mathbf{R}$ is a positive semidefinite matrix. At the minimization problem, α is an optimal scaling factor, introduced because our interest lies in approximating an appropriately scaled version of $\phi(\theta)$, w_l is the weight for the l^{th} gridpoint, and w_c is the weight for the cross-correlation term. A crucial advantage of this optimization problem is that it provides the freedom to choose the weights w_l and w_c. As a result, if the beampattern matching is considered more important than the minimization of the cross-correlation between the probing signals $w_l > w_c$ is set and vice versa.

Sidelobe Control

In beampattern design with sidelobe control, a method is formulated, which guarantees the distortionless response in the mainlobe domain, i.e. the directions of the targets, and in parallel minimizes the sidelobe level in a prescribed region. Within this sidelobe region, strong clutter could be present, which interferes with the radar network and deteriorates its performance.

This optimization problem can be formulated as a semidefinite program, which can be solved by convex optimization techniques [51], as follows:

$$min_{t,\mathbf{R}} \quad -t \tag{2.32}$$

$$s.t. \quad \mathbf{a}^H(\theta_0)\mathbf{R}\mathbf{a}(\theta_0) - \mathbf{a}^H(\mu_l)\mathbf{R}\mathbf{a}(\mu_l) \geq t, \quad \forall \mu_l \in \Omega$$

$$\mathbf{a}^H(\theta_1)\mathbf{R}\mathbf{a}(\theta_1) = 0.5\mathbf{a}^H(\theta_0)\mathbf{R}\mathbf{a}(\theta_0)$$

$$\mathbf{a}^H(\theta_2)\mathbf{R}\mathbf{a}(\theta_2) = 0.5\mathbf{a}^H(\theta_0)\mathbf{R}\mathbf{a}(\theta_0)$$

$$\mathbf{R} \geq 0$$

$$R_{mm} = \frac{P_{max}}{M_t}, \quad m = 1, \ldots, M_t$$

where θ_0 belongs to the desired beampattern mainlobe and $\theta_2 - \theta_1$ ($\theta_2 > \theta_0$ and $\theta_1 < \theta_0$) determines the necessary 3dB width of the mainbeam and Ω denotes the sidelobe region, where the clutter originates from.

Two relaxation techniques can be performed on the constraints in order to have a beampattern with lower sidelobe levels. The first one is applied on the 3dB width constraint and is implemented by replacing this constraint by

$$(0.5 - \delta)\mathbf{a}^H(\theta_0)\mathbf{R}\mathbf{a}(\theta_0) \leq \mathbf{a}^H(\theta_i)\mathbf{R}\mathbf{a}(\theta_i) \leq (0.5 + \delta)\mathbf{a}^H(\theta_0)\mathbf{R}\mathbf{a}(\theta_0)$$

, $i = 1, 2$, for some small δ. The second relaxation technique is implemented by introducing some flexibility in the elemental power constraint by allowing the elemental power to be within a certain range around P_{max}/M_t. Although this flexibility is allowed, the same total transmit power of P_{max} is maintained.

SINR Constraint Beamforming

It comes as a result from the previous section that adaptive techniques incorporating convex optimization methods offer substantial advantages at the transmit beamforming. Initially, following [52, 51], these convex problems can be efficiently solved using standard semidefinite programming algorithms with guaranteed convergence speed. Furthermore, by exploiting this method, new constraints, such as power allocation constraints, sidelobe control or SINR constraints can be added straightforwardly, without deteriorating the main objective, which is to focus the transmit power towards the targets' directions. In addition, convex optimization techniques can be used to increase the robustness of the system when facing channel gain uncertainty and estimation errors [22].

In this section, the main goal is to design the optimal transmit beampatterns, while minimizing the transmission power of the radar and satisfying a certain detection criterion for each of the targets, translated into SINR constraints. Hence, the following minimization problem arises [22, 53, 26, 54]:

$$\min_{\mathbf{w}_i} \quad \sum_{i=1}^{K} \|\mathbf{w}_i\|^2 \tag{2.33}$$

$$s.t. \quad \text{SINR}_i \geq \gamma_i, \forall i$$

where $\mathbf{w}_i$ is the transmit beamforming vector for target i and γ_i denotes the predefined SINR threshold. The optimization problem in (2.33) is a quadratically constrained non-convex problem. However, it can be efficiently solved by applying semidefinite programming (SDP) techniques through rank relaxation methods and using convex optimization toolboxes [51, 55].

Numerical Example

An example is presented to demonstrate the performance of the beampattern matching technique. Consider a MIMO radar system with a uniform linear array (ULA) constituted of $M_t = M_r = 10$ antennas with half-wavelength spacing between adjacent antennas. The same antennas are used for both transmitting and receiving signals. The total transmit power is set to $c_p = 1$.

The desired beampattern is given from (2.28). Suppose that the desired beampattern has one wide beam centered at $\theta_k = 0^0$ with a beamwidth of $50^0 (\Delta = 25^0)$, as shown in Fig.2.4. Using the beampattern matching design technique from (2.31) and choosing without loss of generality $w_l = 1$ and $w_c = 0$, the following optimization problem occurs:

$$min_{\alpha,\mathbf{R}} \quad \sum_{l=1}^{L} s_l$$

$$\begin{aligned} s.t. \quad \|\alpha\phi(\mu_l) - \mathbf{a}^H(\mu_l)\mathbf{R}\mathbf{a}(\mu_l)\| &\leq s_l \quad l = 1, \ldots, L \\ R_{mm} &= \frac{c_p}{M_t}, \quad m = 1, \ldots, M_t \\ \mathbf{R} &\geq 0 \end{aligned}$$

Solving this problem using the CVX Matlab Software [51], the following result is obtained as depicted in Fig.2.4.

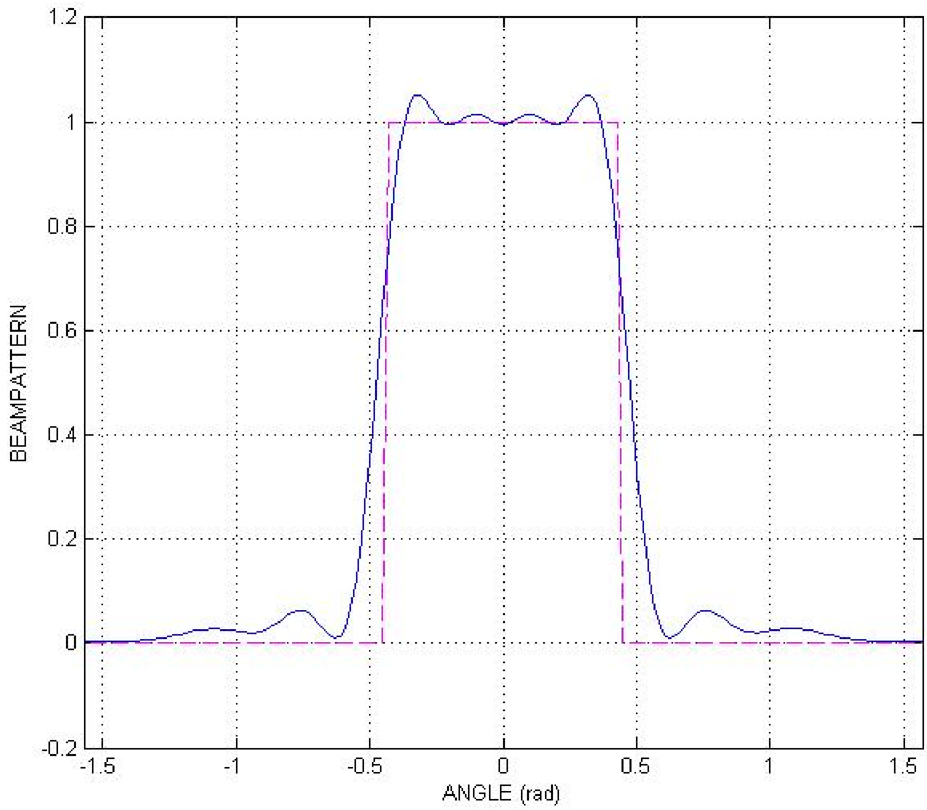


FIGURE 2.4: MIMO Beampattern matching design under the uniform elemental power constraint.

It is evident from Fig.2.4 that the power allocation of the transmit beampattern is concentrated in a specific sector, defined by the desired beampattern.

Furthermore, the sidelobe levels are very low and therefore high level of interference is prevented from undesirable targets, located outside the desired sector.

2.2.4 Receiver Beamforming

Besides the transmit array, it is also beneficial to apply beamforming techniques at the receive array of a MIMO radar system in order to enhance the performance in terms of the target parameters acquisition and the target tracking capabilities of the system. A typical receive beamformer scheme is shown in Fig.2.5.

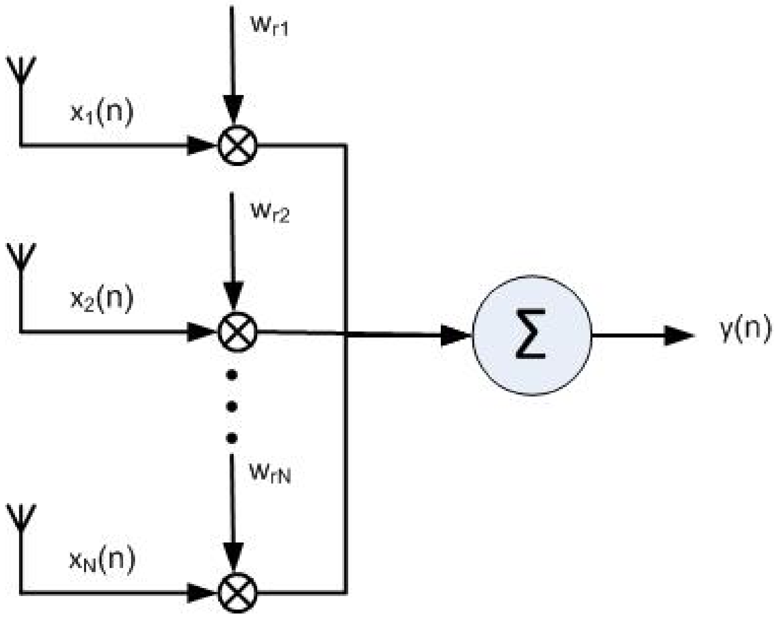


FIGURE 2.5: Linear receive array with beamforming coefficients.

Receive Matched Filtering Beamforming

Similar to the transmit matched filtering beamforming, when the system can obtain information regarding the target's direction and there is a single target in the absence of any other form of interference, then the optimal receive beamformer is the conventional, non-adaptive matched filtering at the recive array. Therefore, the signal return is coherently combined at the virtual receive array and the optimum matched filtering receive weight vector of size $MN \times 1$ can be defined as

$$\mathbf{w}_r = \mathbf{u}(\theta_t) \tag{2.34}$$

where $\mathbf{u}(\theta_t)$ denotes the virtual receive steering vector at the direction of the target.

Minimum Variance Distortionless Response (MVDR) Beamforming

Within an actual radar field except the desired target signal return, there are also interfering sources, that mitigate the performance of receive matched filtering beamforming. By considering L active interfering sources at locations $\{\theta_i\}_{i=1}^{L}$ and with reflection parameters $\{\beta_i\}_{i=1}^{L}$, then under the assumption of point sources, the $N \times 1$ receive data vector can be obtained as

$$\mathbf{y}(t) = \beta_t \mathbf{w}_t^H \mathbf{a}(\theta_t)\psi(t)\mathbf{b}(\theta_t) + \sum_{i=1}^{L} \beta_i \mathbf{w}_t^H \mathbf{a}(\theta_i)\psi(t)\mathbf{b}(\theta_i) + \mathbf{n}(t) \tag{2.35}$$

In this case, the premium goal is to secure the desired signal and mitigate the undesired interference. Thus, a popular beamforming method that satisfies both the steering capabilities whereby the target signal is always protected and the cancellation of undesired interference, so that the output SINR is maximized, is the Minimum Variance Distortionless Response (MVDR) beamformer [20, 18]. The main idea of the MVDR beamformer is to minimize the covariance of the beamformer output subject to a distortionless response towards the direction of the target. Hence, it can be formulated as the following optimization problem

$$\min_{\mathbf{w}_r} \mathbf{w_r^H} \hat{\mathbf{R}}_{yy} \mathbf{w_r} \quad \text{subject to} \quad \mathbf{w_r^H} \mathbf{u}(\theta_t) = 1 \tag{2.36}$$

where $\hat{\mathbf{R}}_{yy} = \sum_{n=1}^{N} \mathbf{y}(n)\mathbf{y}(n)^H$ is the sample covariance matrix of the observed data samples that can be collected from N different radar pulses. The solution to (2.36) is [20],

$$\mathbf{w_r} = \frac{\hat{\mathbf{R}}_{yy}^{-1}\mathbf{u}(\theta_t, \phi_t)}{\mathbf{u}^H(\theta_t, \phi_t)\hat{\mathbf{R}}_{yy}^{-1}\mathbf{u}(\theta_t, \phi_t)} \tag{2.37}$$

From (2.37) the MVDR or Capon receive beamformer is designed.

2.2.5 Phased-array and MIMO Radars: Merging the past with the future

There has been much conflict in recent years on whether or not MIMO radar will confirm its superiority as a breakthrough technology that will eventually substitute phased-array radar [56]. It is undeniable that compared to the phased-array radar, MIMO radar with colocated antennas provides higher angular resolution, improved target parameters identifiability, increase of the number of detectable targets, and flexibility for transmit beamforming design. However, the aforementioned benefits of MIMO radar arise at the cost of missing the transmit coherent processing gain offered by the phased-array radar [57]. As a result, MIMO radar suffers a substantial loss in signal-to-noise ratio (SNR) gain. Furthermore, the MIMO radar evinces a beam-shape loss which is the reason of performance decrease especially when the target's RCS is fading. This section stems from the belief that MIMO radar is not inevitably a disruptive technology that will outclass and supplant phased-array radar, but on the other hand it is a major factor of the evolution of phased-array radar. The innovative idea of combining the advantages of phased-array and MIMO radars has been reported recently in the literature.

Phased-MIMO Radar Analysis

The first attempt to jointly exploit the advantages of both the phased-array and MIMO radars is introduced in [44]. More specifically, a general antenna configuration is proposed where both the transmitter and receiver have multiple well-separated subarrays with each subarray containing closely spaced antennas and operating in phased-array mode. As a result, the resulting MIMO system achieves both coherent processing gain and spatial diversity gain. In [58, 59, 60], the Hybrid MIMO Phased Array Radar (HMPAR) is proposed, utilizing the idea of dividing a large number of transmit/receive elements into multiple disjoint subarrays, that are not allowed to overlap. Presenting a method for generating partially correlated signals, the authors have developed algorithms for MIMO signal sets to achieve arbitrary rectangular transmit beampatterns, while maintaining desirable temporal proerties. Moreover, the simulation results show that the height and width of the beampattern can be controlled by modifying the level of correlation between the signals. Finally, in [61] the authors presented a transmit subaperturing technique for MIMO radars, which results in a tunable system that offers a continuum of operating options from the phased array radar to the omnidirectional transmission based

MIMO radar, providing a tradeoff between the directional gain and interference rejection capability of the system.

Phased-MIMO Radar is a breakthrough notion in radar technology, introduced by A. Hassanien and S. A. Vorobyov [62]. The vantage point of the proposed technique against the aforementioned techniques is the partition of the transmit array into K $(1 \leq K \leq M)$ subarrays that are allowed to overlap, where M is the number of transmit elements. In particular, each subarray can be formed of any number of antennas from 1 to M, so no subarray is precisely the same as another one. The antennas of the k_{th} subarray of the transmit Uniform Linear Array (ULA) coherently emit the k_{th} element of the predesigned independent waveform vector $\boldsymbol{\phi}(t) = [\phi_1(t), \ldots, \phi_K(t)]^T$ of size $K \times 1$, which satisfies the orthogonality condition $\int_{T_0} \boldsymbol{\phi}(t)\boldsymbol{\phi}^H(t) = \mathbf{I}_K$, where T_0 is the radar pulse width, t refers to the time index within the radar pulse, $\mathbf{I}_K$ is the $K \times K$ identity matrix.

In order to define the k_{th} subarray, a $M \times 1$ selection vector $\mathbf{z}_k$ is used containing 0 and 1 entries. When the m^{th} entry equals 1 then the m^{th} element of the transmit array belongs to the k_{th} subarray, while 0 means that the element does not belong to the subarray. Using the selection vector $\mathbf{z}_k$, the $M \times 1$ steering vector associated with the k_{th} subarray can be derived as $\tilde{\mathbf{a}}_k(\theta) = \mathbf{z}_k \odot \mathbf{a}(\theta)$, where $\odot$ denotes the Hadamard product.

In one radar pulse, K transmit beams are formed, each of them is steered by the corresponding subarray. Primary objective is to focus the transmitted energy into the location sectors of interest, which approximate the locations of the targets. The complex envelope of the signals at the output of the k^{th} subarray can be designed by $\mathbf{s}_k(t) = \sqrt{\frac{M}{K}}\tilde{\mathbf{w}}_k^* \phi(t)$, where $\tilde{\mathbf{w}}_k \in C^{M \times 1}$ is the transmit weight vector, used to form the k^{th} transmit beam.

Assuming that a target is located at direction θ in the far-field of the collocated transmit/receive array, the signal reflected by this target can be modeled as:

$$r(t,\theta) = \sqrt{\frac{M}{K}}\beta(\theta)\sum_{k=1}^{K}\tilde{\mathbf{w}}_k^H\tilde{\mathbf{a}}_k(\theta)\phi(t) = \sqrt{\frac{M}{K}}\beta(\theta)\sum_{k=1}^{K}\mathbf{w}_k^H\mathbf{a}_k(\theta)e^{-j2\pi f_0\tau_k(\theta)}\phi(t) \tag{2.38}$$

where $\beta(\theta)$ are complex amplitudes proportional to the radar cross section (RCS) of the target, $\mathbf{w}_k$ is the weight vector containing only the elements corresponding to the active antennas of the k_{th} subarray, $\mathbf{a}_k(\theta)$ is the steering vector also containing only the active antennas elements of the k_{th} subarray, and $\tau_k(\theta)$ is the time needed for the wave to cross the distance between the first antenna of the transmit array and the first antenna of the k_{th} subarray.

In order to compare the transmit and waveform diversity, the authors of [62] have introduced the $K \times 1$ transmit coherent processing vector

$$\mathbf{c}(\theta) = [\mathbf{w}_1^H \mathbf{a}_1(\theta), \ldots, \mathbf{w}_K^H \mathbf{a}_K(\theta)]^T$$

and the $K \times 1$ waveform diversity vector

$$\mathbf{d}(\theta) = [e^{-j2\pi f_0 \tau_1(\theta)}, \ldots, e^{-j2\pi f_0 \tau_K(\theta)}]^T.$$

Incorporating these two vectors, the reflected signal (2.38) by a hypothetical target can be reformulated as

$$r(t,\theta) = \sqrt{\frac{M}{K}} \beta(\theta)(\mathbf{c}(\theta) \odot \mathbf{d}(\theta))^T \boldsymbol{\phi}(t). \tag{2.39}$$

Taking into account that there exist F interfering targets with reflection coefficients $\{\beta_i\}_{i=1}^F$ and directions $\{\theta_i\}_{i=1}^F$, the received data vector can be described by the equation

$$x(t) = r(t,\theta_s)\mathbf{b}(\theta_s) + \sum_{i=1}^{F} r(t,\theta_i)\mathbf{b}(\theta_i) + \mathbf{n}(t) \tag{2.40}$$

where $\mathbf{b}(\theta_s)$ and $\mathbf{b}(\theta_i)$ are the receiving steering vectors associated with the target and interference directions respectively, and $\mathbf{n}(t)$ is the noise term. By matched-filtering $x(t)$ to each of the waveforms $\{\phi_k\}_{k=1}^K$, the $KN \times 1$ virtual data vector is formulated, where N is the number of receiving antennas:

$$\mathbf{y} = \sqrt{\frac{M}{K}} \beta(\theta_s)\mathbf{u}(\theta_s) + \sum_{i=1}^{F} \sqrt{\frac{M}{K}} \beta(\theta_i)\mathbf{u}(\theta_i) + \tilde{\mathbf{n}} \tag{2.41}$$

where $\beta(\theta_s)$ and $\beta(\theta_i)$ are the reflection coefficients of the target and the interference respectively, and the $KN \times 1$ vector $\mathbf{u}(\theta)$ is defined as follows

$$\mathbf{u}(\theta) = (\mathbf{c}(\theta) \odot \mathbf{d}(\theta)) \otimes \mathbf{b}(\theta) \tag{2.42}$$

and stands for the virtual steering vector regarding direction θ, and $\tilde{\mathbf{n}}$ is the $KN \times 1$ noise vector.

At this point, the flexibility of the proposed model can be observed. If $K = 1$ is chosen, all the transmit antennas are considered as one subarray and the signal model (2.39) simplifies to the phased-array radar signal model, as only one waveform is emitted. At the other extreme, if $K = M$ is chosen, then the signal model transforms to the signal model for the MIMO radar without array partitioning at all. In between the two extremes, the choice of K offers a tradeoff between transmit coherent gain and waveform diversity gain.

Comparison of Phased-array, MIMO and Phased-MIMO radar

As suggested in [62], nonadaptive transmit/receive beamforming can be used to produce the transmit, waveform diversity, and overall beampatterns. Since all subarrays consist of the same number of antennas, the weight vectors for conventional uplink beamforming, are given by the equation:

$$\mathbf{w}_k = \frac{\mathbf{a}_k(\theta_s)}{\|\mathbf{a}_k(\theta_s)\|}, \quad k = 1, \ldots, K \tag{2.43}$$

At the receive array, the conventional beamformer is applied to the resulting virtual array. So, the receive beamformer weight vector is equal to the virtual steering vector associated with the direction of the target of interest θ_s:

$$\mathbf{w}_r = \mathbf{u}(\theta_s) = (\mathbf{c}(\theta_s) \odot \mathbf{d}(\theta_s)) \otimes \mathbf{b}(\theta_s) \tag{2.44}$$

As a result, the overall normalized radar beampattern is given by:

$$P(\theta) = \frac{|\mathbf{w}_r^H \mathbf{u}(\theta)|^2}{|\mathbf{w}_r^H \mathbf{u}(\theta_s)|^2} = \frac{|\mathbf{u}^H(\theta_s)\mathbf{u}(\theta)|^2}{\|\mathbf{u}(\theta_s)\|^4} \tag{2.45}$$

For the shake of the comparison, a ULA consisting of M=10 transmit antennas and a ULA of N=10 receiving antennas with half-wavelength spacing between adjacent elements are assumed. The direction of the target of interest is $\theta(s) = 10^o$ and two interfering targets are presumed at angles -30^o and -10^o. It is evident from Fig.2.6,2.7 that although the phased-array radar has the most efficient transmit conventional beampattern, it has no waveform diversity gain. On the other hand, MIMO radar has flat (0db) coherent transmitting gain, but it has the most efficient waveform diversity beampattern. However, it is clear from Fig.2.8 that the overall beampattern for the phased-MIMO radar is improved as compared to the phased-array and the MIMO overall beampatterns.

It is confirmed from the results that the phased-MIMO radar has a superior performance, as it combines the advantages of the phased-array and MIMO radars. More specifically, it exploits all advantages of MIMO radar such as detecting a higher number of targets, improving parameter identifiability, extending the array of antennas with virtual sensors, improving angular

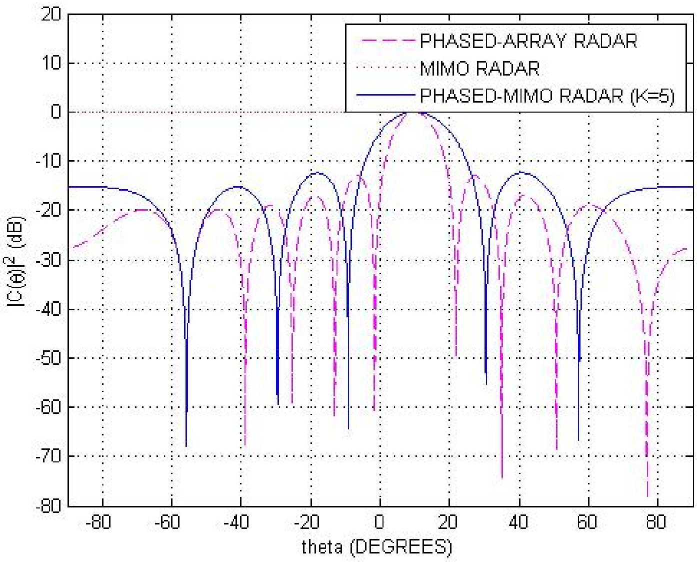


FIGURE 2.6: Transmit beampatterns.

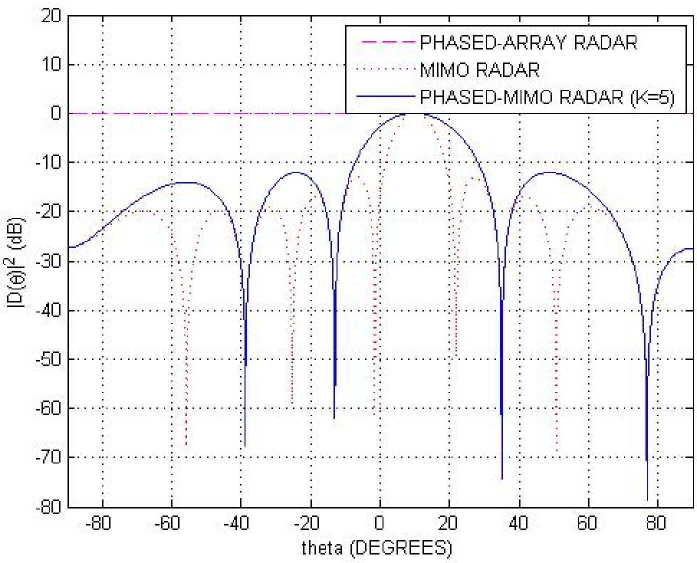


FIGURE 2.7: Waveform diversity beampatterns.

resolution, without losing the transmit coherent gain offered by the phased-array technology. Furthermore, it provides the possibility of designing the overall beampattern of the virtual array. Moreover, by choosing the number of subarrays, this model offers a tradeoff between robustness against beam-shape loss and angular resolution. Finally, it provides high robustness against powerful interference.

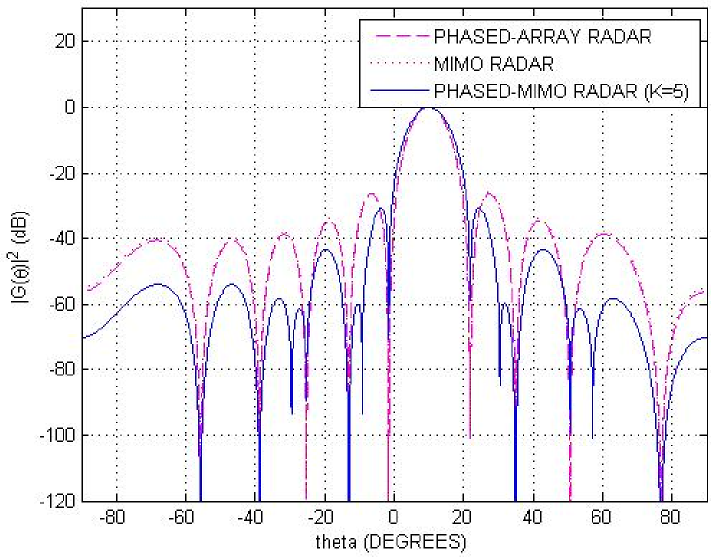


FIGURE 2.8: Overall transmit/receive beampatterns.

2.3 Game Theory in Radar Networks

Some of the advantages of the distributed radar networks were mentioned in Chapter 1. Furthermore, the superiority of decentralized systems is enhanced from the fact that they can operate independently and autonomously, without the demand of constant or central control. Nevertheless, within a modern distributed radar field, the dynamic multiple sources of interference deteriorates the performance of the system and constantly alternates the features and the constraints of the detection and tracking process. As a result, the system must automatically adopt to the new requirements autonomously. Game theory is a natural and effective tool for modeling this kind of interactions and provide a stable solution in a dynamic environment.

2.3.1 Fundamentals of Game Theory

Game theory is a study of strategic decision making and a natural and efficient tool to provide a formal mathematical framework for analyzing coordination and conflict between rational but selfish players. Initially, game theory provided pioneering results within the social sciences, especially in economics and politics. The foundations of the notion behind game theory were set from four innovative works [63, 64, 65, 66]. In [63], the author introduces the evolutionary notion of the best response, when a player adopts the best strategy given the actions of other players. Furthermore, the same work offers an initial intuitive approach to the solution of non-cooperative games, which more than

	Defect	Cooperate
Defect	(-5,-5)	(0,-20)
Cooperate	(-20,0)	(-1,-1)

TABLE 2.1: Prisoner's Dilemma payoffs.

a century later will be formally defined by John Nash [66, 67]. In particular, the Nash equilibrium refers to a stable solution of a game, where no player can benefit by unilaterally deviating its strategy and this strategy is a best response to all other strategies in that equilibrium. The author in [64] proposed the idea of competitive equilibria in a two-type economy framework. Finally, a stable mathematical background of non-cooperative games along with the definition of the Minimax Theorem were introduced in [65].

In general, a game is defined as the tuple:

$$\mathcal{G} =< \mathcal{N}, \{\mathcal{S}_i\}_{i\in\mathcal{N}}, \{u_i\}_{i\in\mathcal{N}} >$$

- The finite set of players is denoted as $\mathcal{N}$.
- The set of acceptable strategies for player i is denoted as $\{\mathcal{S}_i\}_{i\in\mathcal{N}}$.
- The utility function for player i is denoted as $\{u_i\}_{i\in\mathcal{N}}$.

A classical example to introduce the basic principles of game theory is the "Prisoner's dilemma" game. The game scenario involves two criminals arrested for a serious crime. However the police do not have enough evidence to convict either of them on the principal charge. Hence, the police interrogate them in separate cells and offer the following deal. If one criminal testifies against the other, he will be set free or charged with a minor crime and get a reduced sentence. It is important to notice that the two suspects are kept separated and have no information on each other's actions. Thus, this scenario can be modeled as a non-cooperative game, where the two suspects are the players, the strategy set is to cooperate with each other and stay silent or defect and confess against the other player and the utility function is the final sentence. As depicted in Table 2.1, if both players confess against each other (D, D), they get a reduced sentence of 5 years. If both of them choose to cooperate and stay silent (C, C), then they will be convicted for 1 year, as there is no evidence of the major crime. Finally, if only one of them confesses against the other and the other does not (D, U) or (U, D), the betrayer gets the best possible payoff as he walks free whereas the other player is imprisoned for 20 years, facing charges on the principal crime.

It is evident from Table 2.1 that if a player cooperates, the other player will choose to defect as it leads to a better payoff. If the first player chooses to defect then the other player will still defect as it again provides a better payoff. Therefore, regardless of the other player's action, a player in this non-cooperative game has an incentive to always defect and that is the equilibrium of the game (D, D), though mutual cooperation would provide a better utility for both players. This game also provides an insight of greediness and suspicion among rational and egoistic players.

2.3.2 Non-Cooperative Games

During the last decade, game theory has provided a robust mathematical and analytical framework to tackle various issues arisen in radar networks, such as resource allocation, beamforming, target detection and jammer suppression. Before the introduction of game theoretic methods in radar systems, game theory has already been studied and provided some impressive results in wireless communication networks. Many applications of game theory in wireless networks are used to solve similar problems that can also arise in radar networks, such as resource allocation and beamforming. One popular technique to tackle the aforementioned problems is the non-cooperative game theory. Non-cooperative game theory is considered to be a dominant branch of game theory regarding wireless and radar networks, since it studies and models competitive decision making among several egoistic but intelligent players, with no communication or coordination of strategic choices. Hence, the aforementioned properties highly resemble the interactions between widely separated stations or radars in a fully autonomous, distributed communication or radar network, respectively. In a non-cooperative game, each player attempts to optimize its utility function selfishly by selecting the most appropriate strategy, without any communication among the players, and this move has an impact on the utility functions of the other players. It is important to highlight at this point, that partial cooperation is feasible in non-cooperative games, bearing under consideration that any cooperation assumed in the system is self imposed to each player without any communication or coordination among the players.

Some pioneering studies of non-cooperative game theoretic techniques in communications networks are presented in [26, 28, 68, 69, 70]. The authors in [26] introduced the idea of joint beamforming and power control, designing an iterative algorithm to simultaneously obtain the optimal beamforming and power vectors. In [28], an iterative water-filling algorithm was utilized to obtain the Nash equilibrium in a non-cooperative, distributed, multiuser power control problem. A decentralized non-cooperative game was also proposed in [68] to perform sub-channel assignment, adaptive modulation, and power control by adding a referee to improve the results. Moreover, the authors in [69] perform a detailed literature review and concentrate some important results on distributed resource allocation game theoretic techniques, where a variety of utility functions and constraints is presented. Finally, a non-cooperative potential game method was exploited in [70] to obtain the optimal resource allocation both in the case of static and fluctuating channels.

Noncooperative game theoretic techniques have been also intensively studied lately within the radar research field to confront several problems and to improve and optimize various radar parameters. In particular, the authors in [35] and [36] formulated a non-cooperative game to address the power optimization problem with a predefined SINR constraint. Furthermore, to extend the study in [35], a signal-to-disturbance ratio (SDR) estimation technique was applied in [37]. A two-player, non-cooperative, zero-sum game was considered in [71] to investigate the interaction between a radar and a jammer. Furthermore, non-cooperative MIMO radar and jammer games were applied in [72], where the utility functions were formulated using the mutual information criterion. The authors in [73, 74] studied the problem of polarimetric waveform design by forming a zero-sum game between a target and a radar engineer. Finally, in [75] a novel noncooperative game model is considered for joint design of amplitudes and frequency-hoping waveforms.

An essential notion in game theory, that leads to the path of obtaining the final solution of a game, is that of the best response function and is defined as follows [76, 77]:

Definition: The best response function of player i when the remaining players of the game follow the profile of strategies $\mathbf{s}_{-i}$ is a set of strategies that satisfies the following equation:

$$BR_i(\mathbf{s}_{-i}) = \{\mathbf{s}_i \in \mathcal{S}_i \mid u_i(\mathbf{s}_{-i}, \mathbf{s}_i) \geq u_k(\mathbf{s}_{-i}, \mathbf{s}_i^{'}), \forall \mathbf{s}_i^{'} \in \mathcal{S}_i\}$$

where $\mathbf{s}_i$ is the strategy chosen by player i. The essence of the best response function is of great importance for game theory, as it provides a set of strategies for a player that produce the highest payoff when compared to different strategies that do not belong in $BR_i(\mathbf{s}_{-i})$ and when the other players are fixed at $\mathbf{s}_{-i}$. In other words, the best response of player i means that this player performs better by choosing a strategy from the set $BR_i(\mathbf{s}_{-i})$, when the rest of the players choose exactly the strategies $\mathbf{s}_{-i}$. The best response concept inspires one of the most challenging goals when designing a game, which is to prove that a solution exists and in some cases that this solution is unique. In non-cooperative games the most common solution notion is the Nash equilibrium, which describes a situation where no player can improve its payoff function by alternating its strategy, as aforementioned in Section 2.3.1 and is formally defined as [78]:

Definition: A pure strategy Nash equilibrium of a non-cooperative game $\mathcal{G} =< \mathcal{N}, \{\mathcal{S}_i\}_{i\in\mathcal{N}}, \{u_i\}_{i\in\mathcal{N}} >$ is a strategy selection $\mathbf{s}^* \in \mathcal{S}$ such that $\forall i \in \mathcal{N}$ the following stands:

$$u_i(\mathbf{s}^*_{-i}, \mathbf{s}^*_i) \geq u_i(\mathbf{s}^*_{-i}, \mathbf{s}_i), \quad \forall \mathbf{s}_i \in S_i(\mathbf{s}^*_{-i}), \forall i \in \mathcal{N}.$$

The investigation on the equilibria of a game, which correspond to possible stable solutions in cooperative or antagonistic games, is of high importance in radar networks. This is because it is crucial for the radar network designer to have the ability to foresee and even ensure the final, stable states at which the system offers the highest possible desired utility. Therefore, the research community heavily investigated the existence and uniqueness of such equilibria. Regarding the existence of an equilibrium, most game theorists used fixed point theorems, also exploiting the topological and geometrical properties of the strategy sets and the utility functions [67, 79, 80]. One of the most useful and popular existence theorems is defined as follows [81, 82, 83, 84]:

Theorem 2.1: Game $\mathcal{G} =< \mathcal{N}, \{\mathcal{S}_i\}_{i\in\mathcal{N}}, \{u_i\}_{i\in\mathcal{N}} >$ is assumed to be a strategic noncooperative game (SNG). If for every player $i \in \mathcal{N}$, the strategy set $\mathcal{S}_i$ is a compact and convex set, the payoff function $u_i(\mathbf{s}_{-i}, \mathbf{s}_i)$ is a continuous function in the profile of strategies $\mathbf{s} \in \mathcal{S}$ and quasi-concave in s_i, then the SNG $\mathcal{G}$ has at least one pure Nash equilibrium.

Securing the existence of the Nash Equilibrium is the first important step that is required to ensure its uniqueness. By guaranteeing the uniqueness of the Nash equilibrium, one can not only obtain an accurate prediction of the final state of the network when studying a radar game but may also transcend convergence issues. There are two important theorems that can be applied in radar network games to secure the uniqueness of the Nash equilibrium, after the existence is proved [85]. In the first theorem, Yates utilized the standard function property to prove the uniqueness of the solution [86]. In an earlier work, Rosen exploited the diagonally strict concavity (DSC) property to secure that an equilibrium is unique [87]. Before stating the two theorems, it is important to revisit the definition of a standard function:

Definition: A function $\mathbf{F}(\mathbf{x})$ is standard if for all $\mathbf{x} \geq 0$, the following properties hold:

- Positivity: $\mathbf{F}(\mathbf{x}) > 0$
- Monotonicity: If $\mathbf{x} \geq \mathbf{x}'$, then $\mathbf{F}(\mathbf{x}) \geq \mathbf{F}(\mathbf{x}')$
- Scalability: $\forall a \geq 1, a\mathbf{F}(\mathbf{x}) \geq \mathbf{F}(a\mathbf{x})$

Theorem 2.2: [86] If the best response functions of all players in a SNG $\mathcal{G}$ are standard, then the game admits a unique NE.

Theorem 2.3: [87] Assume a strategic game $\mathcal{G}$ where $\forall i \in \mathcal{N}$, the strategy set $\mathcal{S}_i$ is a compact and convex set and the utility function $u_i(\mathbf{s}_{-i}, \mathbf{s}_i)$ is a continuous function in the set of strategies $\mathbf{s} \in \mathcal{S}$ and concave in s_i. Consider $\mathbf{r} = (r_1, \ldots, r_N)$ be an arbitrary vector of fixed positive parameters. If the DSC property stands for some $\mathbf{r} > 0$

$$\forall \mathbf{s}, \mathbf{s}^{'} \in \mathcal{S}, \mathbf{s} \neq \mathbf{s}^{'} : (\mathbf{s} - \mathbf{s}^{'})(d(\mathbf{s}, \mathbf{r}) - d(\mathbf{s}^{'}, \mathbf{r})) > 0,$$

where $d(\mathbf{s}, \mathbf{r}) = [r_1 \frac{\partial u_1}{\partial s_1}, \ldots, r_N \frac{\partial u_N}{\partial s_N}]^T$, then the game $\mathcal{G}$ admits a unique pure Nash equilibrium.

Both Theorems provide a rigorous mathematical proof of the uniqueness of the solution and have been thoroughly used in wireless communication works. Theorem 2.3 is used in [88], where an optimal power allocation problem in fast fading multiple access channels is investigated, and in the water-filling game of [89]. The standard function Theorem 2.2 is exploited in [32] to prove the uniqueness of the solution regarding a joint optimal beamforming and resource allocation game in multicell wireless systems.

2.3.3 Stackelberg Games

In some non-cooperative scenarios, one or more players may be in a vantage position to select and impose their desired strategy before the other players act. Thus, these scenarios introduce some hierarchy in game theory, allowing the dominant players to enforce their strategies upon the remaining players in order to maximize their payoff. The dominant player that can apply its strategy first is called the leader of the game, whereas the rest of the players are known as followers. It is possible to model games with multiple leaders or multiple followers or both. Such scenarios in game theory are known as Stackelberg games.

The Stackelberg leadership model was initially described and applied in economics by Heinrich Freiherr von Stackelberg in 1934 [90]. In particular,

		Player 2	
		U	D
Player 1	L	(4,3)	(8,6)
	R	(5,5)	(9,2)

TABLE 2.2: Utility matrix for the Stackelberg example.

the Stackelberg model is a two part game, where two firms offer an identical product with known demands. In the first part of the game, the leader plays first and chooses its quantity of the product. The leading firm usually rises in the hierarchy of a market because of its size, brand name reputation, historical precedence in the market, research and information capabilities etc. In the second part of the game, the following firm chooses its quantity based on the quantity chosen by the leader and the selected quantity must agree with the best response function of the follower. In other words, depending on the quantity announced by the leader, the follower always chooses the best response strategy to maximize its utility. It is assumed that the leader has a priori knowledge of the best response function of the follower and strategically chooses its action to maximize its utility. Hence, the optimal strategy of the leader along with the corresponding best response of the follower, establish a Stackelberg equilibrium of the game [78, 77].

Definition: Let us assume a two-player Stackelberg game, where player 1 is the leader and player 2 is the follower. A strategy $s_1^* \in \mathcal{S}_1$ is a Stackelberg equilibrium strategy for the leader, when:

$$\min_{s_2 \in BR_2(s_1^*)} u_1(s_1^*, s_2) = \max_{s_1 \in \mathcal{S}_1} \min_{s_2 \in BR_2(s_1^*)} u_1(s_1, s_2) = u_1^*$$

It is obvious that u_1^* denotes the utility of the leader. In a Stackelberg game, the utility of the leader u_1^* at the Stackelberg equilibrium could impose a secured level of utility for the leader.

In order to illustrate the concepts around Stackelberg game theory, a game with utilities shown in Table 2.2 is considered. Consider that player 1 is the leader and player 2 the follower. If player 1 selects strategy L, player 2 would play strategy D, since it provides a higher payoff of 6 instead of 3 if he chose U. Hence, this leads to a payoff of 8 for the leader. On the other hand, if the leader plays strategy R, the follower would choose strategy U, as it offers a better utility of 5 instead of 2 if he played D. This case leads to a utility of 5 for the leader. As a result, the leader (player 1) will play strategy L, because it will provide a better payoff of 8 instead of 5 if he chooses strategy R. Thus, the Stackelberg equilibrium for this example is (L, D).

Stackelberg game theory can provide a highly efficient description of the hierarchy and interactions that arise in wireless communication or radar networks. The vast majority of the work that utilizes Stackelberg games in the aforementioned systems addresses the resource or power allocation problem. Concentrating on communications literature, the authors of [91] proposed a Stackelberg power allocation game in which the leader imposes a pricing policy for the available spectrum to the followers and subsequently the followers compete for the available bandwidth with primary objective to maximize their own utilities. In [92] a joint pricing and power allocation Stackelberg game for dynamic spectrum access networks is investigated, where the primary user (leader) determines both the power allocation and the interference price charged to the secondary user (follower) and the follower determines its resource demand with respect to the leader's action. The authors of [93] formulated a downlink power allocation problem as a Stackelberg game, where the macrocell base stations are the leaders and the femtocell access points are the followers. Primary target of every station is to maximize its capacity under power constraints. A price based resource allocation Stackelberg game for two-tier femtocell networks is formulated in [94], where the macrocell base station (leader) is self-protected by charging a price for interference to the femtocells (followers).

Apart from communication networks, Stackelberg game theory has been applied recently to encounter various issues in radar networks. The authors in [39, 95] proposed a water filling method for optimal power distribution by formulating a Stackelberg game, considering two optimization schemes, one with the radar as a leader and one with the target dominant in the presence of clutter. In [72] and [96] the interaction between a smart target and a smart MIMO radar is formulated as a Stackelberg game, where the utility functions are derived from the mutual information criterion. A Stackelberg game consisting of a single leader and multiple followers is investigated in Chapter 6 of this thesis. More specifically, a hybrid radar network is considered, where a single surveillance radar plays the role of the leader and multiple tracking radars are the followers. The leader applies a pricing policy of interference charged to the followers aiming at maximizing its profit while keeping the incoming interference under a certain threshold and the followers react to the leader's policy based on their best response function, which is a function of the leader's pricing charge and the strategies of the remaining followers.

2.3.4 Bayesian Games

The game theoretic techniques studied and analysed in the preceding sections were discussed under the ambiguous assumption that all players have common, complete information on the components that characterise the game, more specifically the player set, the strategy set and the players' utility functions. As a result, under the assumption of complete information games, each player $i \in \mathcal{N}$ chooses a strategy $s_i \in \mathcal{S}_i$ deterministically with probability 1 and this describes a pure strategy game. Nonetheless, in many realistic scenarios in a competitive environment, the players do not have complete information on the elements of the game. Bayesian game theory provides a rigorous mathematical framework to address situations of incomplete information [97, 98, 99].

Definition: A non-cooperative Bayesian game of incomplete information can be fully characterised as [82]:

$$\mathcal{G}_B = < \mathcal{N}, \mathcal{A}, \mathcal{T}, \Pi, \mathcal{S}, \mathcal{U} >$$

- The set of players: $i \in \mathcal{N} = 1, \ldots, N$.
- The acceptable action set for player i: $\mathcal{A}_i \, \forall \, i \in \mathcal{N}$, with $a_i \in \mathcal{A}_i$ denoting an instantaneous action of player i.
- The type set for each player is denoted as $\mathcal{T} = \mathcal{T}_1 \times \ldots \times \mathcal{T}_N$, with $t_i \in \mathcal{T}_i$ denoting a possible type for player i.
- The common prior or probability set is defined as $\Pi = \Pi_1 \times \ldots \times \Pi_N$, where Π_i is the probability distribution of the type for player i and it is common knowledge to every player.
- The strategy set for player i $\mathcal{S}_i$: $s_i \in \mathcal{S}_i$: $\mathcal{T} \rightarrow \mathcal{A}_i$, meaning that the strategy of player i s_i is a function of the action of player i a_i for each type t_i.
- The Bayesian game model is concluded by defining the utility function set as $\mathcal{U}(\mathbf{a}, \mathbf{t}) = \{u_1, \ldots, u_N\}$, where u_i represents the i^{th} player payoff function, $\mathbf{a} = (a_1, \ldots, a_N)$ and $\mathbf{t} = (t_1, \ldots, t_N)$. It is evident that the utility function is a function of actions and types of all players.

Before the initialization of a Bayesian game, nature selects the types of all players, based most of the times on a predefined distribution that is common knowledge to all players. Each player's type is solely known to itself as private information, as chosen by nature. It is assumed that the players choose their strategies simultaneously, with primary objective to optimize their utility function, based on the belief about the types of other players. After the completion of the game, each player receives its utility value. Similarly to the Nash equilibrium notion, one can define the Bayesian Nash equilibrium concept as the final solution of a Bayesian game, which is defined as the strategy $\mathbf{s}^* = (s_1^*, \ldots, s_N^*)$ and is given by:

$$s_i^*(t_i) = \max_{s_i \in \mathcal{S}_i} \sum_{t_{-i} \in \mathcal{T}_{-i}} \mathcal{U}_i(\mathbf{s}_{-i}^*, s_i; \mathbf{t}_{-i}, t_i) Pr(\mathbf{t}_{-i} | t_i)$$

In a wireless communication or radar network, the uncertainty could emerge from multiple sources, although in the majority of the literature it is translated into channel gain uncertainty. The authors of [100] modeled different jamming attack scenarios in various wireless networks exploiting Bayesian game theory, where uncertainty is assumed on user identity, traffic dynamics, channel gain or the costs and rewards of the users. In [101] a Baysesian game is formulated to design and analyze the power allocation problem in fading multiple access channels, where the players' main objective is to maximize their ergodic capacity with incomplete information about the channel gains. A joint Stackelberg Bayesian game was formulated in [27] for power allocation in two-tier networks, where the uncertainty comes from the channel gain incomplete information and characterizes the type of the players. A Bayesian game theoretic approach that extends the classical water-filling game [102, 89], is presented in [103] for the distributed resource allocation problem within the context of fading multiple access channels, where the uncertainty is associated with the channel state information of the players. A distributed power allocation problem is considered in [104] for a MIMO network utilizing Bayesian game theoretic techniques, where the local channel state information determines the type of each player. The authors of [105] proposed an interference aware MAC protocol by formulating a Bayesian channel-access game model, considering incomplete information on the channel gain. Finally, [106] attempts a broad analysis on the distributed resource allocation issue in wireless networks under uncertainty using different Bayesian game models.

Although Bayesian game theory can provide a realistic and accurate model of a multistatic radar network with incomplete information, its potential have not yet been fully exploited in radar literature. An initial attempt to include uncertainty in radars is performed in [107], where the interaction between a statistical MIMO radar and an intelligent target equipped with a jammer is investigated. The target has incomplete information regarding the radar cross section (RCS) and thus the channel gain of the target and the jammer does not know the radar receiver condition. The objective of the radar is obviously target detection by maximizing its signal to interference plus noise ratio (SINR) and on the contrary the objective of the jammer is to always attempt to prevent the target detection via the power allocation optimization. It is evident that Bayesian game theory in radar networks can be further exploited and provides a challenging area for further research. In particular, several target models (for example the Swerling target models [108]) could be utilized to define the distribution of the radar cross section in a radar network and subsequently define the distribution of the channel gains. As a result, it is feasible to obtain the distribution of the type set for every player and this distribution is common knowledge for all players. This knowledge can be exploited in order to address several problems in radar networks, as power allocation or SINR maximization issues.

2.4 Conclusion

This chapter presented a framework to set the basis for the following four contributing chapters. As aforementioned in this chapter, adaptive beamforming techniques (convex optimization techniques, MVDR beamformer) and noncooperative, Stackelberg and Bayesian game theoretic techniques provide significant potential to address several important issues in radar literature and substantially improve the currently used radar systems. This thesis aims to exploit those techniques and provide substantial results that improve the efficiency of MIMO radar networks and tackle critical problems, such as beamforming optimization, optimal power allocation and SINR maximization.

Chapter 3

2D Phased-MIMO radar

3.1 Introduction

The field of radar research is vast and has been endlessly developing since late 1930's. The gigantic breakthroughs in digital signal processing and the constant growth in computational capabilities have enabled the introduction of an emerging technology known as multiple-input multiple-output (MIMO) radar [5]. The essence of MIMO radar is the use of multiple antennas to simultaneously transmit diverse, possibly linearly independent waveforms, in contrast to a phased-array radar which transmits scaled versions of a single waveform. This waveform diversity offers superior capabilities as compared to the phased-array model. There are two fundamental regimes of operation investigated in the literature. In the first type, the transmit and receive antenna elements are widely spaced, whereas, in the second type, the antenna elements are closely spaced.

MIMO radar with colocated antennas [8] is known to offer higher sensitivity to detect slowly moving targets, higher angular resolution, increased number of detectable targets, direct applicability of adaptive array techniques and better parameter identifiability. On the other hand, MIMO radar with widely spaced antennas provides the ability to capture the spatial diversity of the target's radar cross section (RCS), enhance the ability to combat signal scintillation, estimate precisely the parameters of fast moving targets and improve the target detection performance, by taking advantage of the target's geometrical characteristics [9].

The substantial improvements offered by MIMO radar technology come at the cost of losing the transmit coherent processing gain offered by phased-array radar [56, 62]. This absence can lead to signal-to-noise ratio (SNR) decrease and beam-shape loss [56, 62]. The aforementioned disadvantages raise the dilemma of whether or not MIMO radar will meet the expectations that it will provide a colossal opportunity for improvements in the field of radar research. This work stems from the belief that MIMO radar is not a substitute technology that will totally outclass phased-array radars, but rather it provides the opportunity of jointly exploiting the benefits of both models, as reported recently in the literature [62, 58]. The authors of [58] proposed a radar model, utilizing the idea of dividing a large number of colocated transmit/receive elements into multiple subarrays, that are not allowed to overlap. Phased-MIMO radar is a breakthrough notion in radar technology, introduced in [62]. The vantage point of this technique is the partition of the transmit array into subarrays that are allowed to overlap. Earlier work in [109] investigated the application of this Phased-MIMO radar notion to 2D transmit arrays by designing the transmit beampattern through a convex optimization problem that minimizes the difference between a desired transmit beampattern and the actual one produced by the system [5].

The transmit beamforming design in MIMO radar with colocated transmit arrays has been extensively investigated in the literature [110, 21, 11]. In particular, most of the work is focused on a one dimensional ULA. It has been shown in [11] and [21] that convex optimization techniques can solve efficiently the problem of transmit beamforming in a ULA. The extension of this to two-dimensional transmit beamforming optimization is presented in [110].

In this chapter, transmit beamforming design for phased-MIMO radar with fully overlapped 2D transmit subarrays is investigated. Each subarray is programed to coherently transmit a waveform which is orthogonal to the waveforms transmitted by other subarrays. In order to achieve high coherent processing gain, a weight vector should be designed for each subarray to steer the transmit beam to a specific 2D sector in space, determined by a desired transmit beampattern. To accomplish this, an optimization problem is designed, that minimizes the difference between the desired transmit beampattern and the actual beampattern obtained by the array of antennas under the constraint of uniform power allocation across the transmit antennas. It is possible to add more constraints, such as minimum sidelobe level and uniform power distribution over each subarray. As the optimization problem in its original form is non-convex, it has been converted to convex form using semidefinite relaxation techniques. The simulation results highlight the advantage of the 2D subaperturing technique for MIMO radars.

Later in the chapter, transmit, waveform diversity and overall transmit-receive beamforming design is examined for Phased-MIMO radar with fully-overlapped 2D transmit subarrays. The Phased-MIMO beampatterns are obtained using both conventional and adaptive techniques and compared with the respective beampatterns of the phased-array and MIMO radars. Specifically, in order to design the adaptive transmit beampattern, a convex optimization problem is solved, that minimizes the difference between a desired transmit beampattern and the actual one produced by the system. Furthermore, the adaptive overall transmit-receive beampattern are designed by utilizing the Minimum Variance Distortionless Response (MVDR) Capon beamformer. The simulation results highlight the benefits provided by the 2D Phased-MIMO radar with fully overlapped subarrays.

3.2 Transmit Beamforming Design for Two-Dimensional Phased-MIMO Radar with Fully-Overlapped Subarrays

A subaperturing technique for two-dimensional (2D) transmit arrays within the context of multiple-input multiple-output (MIMO) radar is investigated. Specifically, the performance of transmit beamforming using fully overlapped subarrays of a 2D transmit array is studied. As reported for linear array of antennas, this 2D transmit array exploits the advantages of the MIMO radar technology without sacrificing the coherent processing gain at the transmit side provided by the phased-array concept. In order to achieve high coherent processing gain, a weight vector should be designed for each subarray to steer the transmit beam in certain 2D sector in space. This is achieved by solving a convex optimization problem that minimizes the difference between a desired transmit beampattern and the actual beampattern produced by the 2D array of antennas, under a constraint in terms of uniform power allocation across the transmit antennas.

3.2.1 System Model

In the proposed model, a radar system that has a uniform rectangular array (URA) at the transmit side is considered, which consists of $M_t \times N_t$ antennas, where M_t is the number of antennas in each column and N_t is the number of antennas in each row of the planar transmit array. The adjacent antenna elements at each column are assumed to be equidistant with spacing d_m and at each row also equidistant with spacing d_n. The main idea behind the system model is to partition the transmit 2D array into K subarrays ($1 \leq K \leq M_t \times N_t$) which are fully overlapped. An example of the fully overlapped

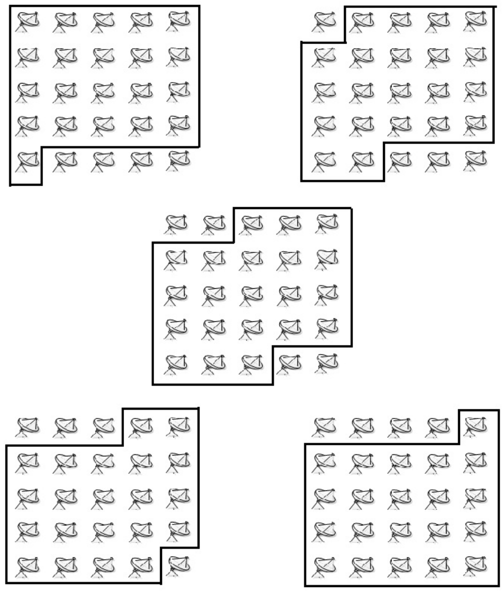


FIGURE 3.1: Fully overlapped subaperturing of a 5×5 uniform rectangular array(URA) when K=5.

subaperturing of a 5×5 URA into 5 subarrays is shown in Fig. 3.1. As described in Fig. 3.1, each subarray consists of $M_t \times N_t - K + 1$ antennas. In order to achieve this subaperturing, the selection matrix $\mathbf{Z}_k$ is introduced, which is an $M_t \times N_t$ matrix containing 0 and 1 entries. When the $(mn)^{th}$ entry equals 1 then the $(mn)^{th}$ element of the 2D array belongs to the k^{th} subarray, while 0 means that the element does not belong to the k^{th} subarray. As a result, the matrix $\mathbf{Z}_k$ defines the k^{th} subarray. The $M_t N_t \times 1$ steering vector associated with the k^{th} subarray can be denoted as:

$$\mathbf{a}_k(\theta, \phi) = vec(\mathbf{Z}_k \odot [\mathbf{u}(\theta, \phi)\mathbf{v}^T(\theta, \phi)]) \tag{3.1}$$

where $vec(\cdot)$ is the operator that stacks the columns of a matrix in one column vector, $\odot$ denotes the Hadamard product, $(\cdot)^T$ denotes the transpose, θ and ϕ denote the elevation and azimuth angles respectively. The vectors $\boldsymbol{\mu}(\theta, \phi) \in C^{M_t \times 1}$ and $\boldsymbol{\nu}(\theta, \phi) \in C^{N_t \times 1}$ are written as:

$$\boldsymbol{\mu}(\theta, \phi) = [1, e^{j2\pi d_m sin(\theta) cos(\phi)}, \ldots, e^{j2\pi(M_t-1)d_m sin(\theta)cos(\phi)}]^T$$

$$\boldsymbol{\nu}(\theta, \phi) = [1, e^{j2\pi d_n sin(\theta) sin(\phi)}, \ldots, e^{j2\pi(N_t-1)d_n sin(\theta)sin(\phi)}]^T$$

The k^{th} subarray of the transmit URA emits the k^{th} element of the pre-designed independent waveform vector $\boldsymbol{\psi}(t) = [\psi_1(t), \ldots, \psi_K(t)]^T$ of size $K \times 1$, which satisfies the orthogonality condition $\int_{T_0} \boldsymbol{\psi}(t)\boldsymbol{\psi}^H(t)dt = \mathbf{I}_K$, where T_0 is the radar pulse width, t refers to the time index within the radar pulse, $\mathbf{I}_K$ is the $K \times K$ identity matrix, and $(\cdot)^H$ denotes the Hermitian transpose.

The aim is to focus the energy of the transmit array into a 2D spatial sector defined by $\Theta = [\theta_1 \quad \theta_2]$ in the elevation domain and $\Phi = [\phi_1 \quad \phi_2]$ in the azimuth domain. Therefore, K transmit beams are formed, each of them is steered by the corresponding subarray. Then each of the orthogonal waveforms ψ_k is radiated over one beam. The complex envelope of the signals at the output of the k^{th} subarray can be designed by $\mathbf{s}_k(t) = \sqrt{\frac{M_tN_t}{K}}\mathbf{w}_k\psi_k(t)$, where $\mathbf{w}_k \in C^{M_tN_t\times 1}$ is the transmit weight vector, used to form the k^{th} transmit beam. The power of the probing signal emitted by the k^{th} subarray towards the direction (θ, ϕ) can be modeled as

$$P_k(\theta,\phi) = \mathbf{a}_k^H(\theta,\phi)E\{\mathbf{s}_k(t)\mathbf{s}_k^H(t)\}\mathbf{a}_k(\theta,\phi)$$

$$= \frac{M_tN_t}{K}\mathbf{a}_k^H(\theta,\phi)\mathbf{w}_k\mathbf{w}_k^H\mathbf{a}_k(\theta,\phi) \tag{3.2}$$

The array transmit beampattern is hence defined as

$$P(\theta,\phi) = \sum_{k=1}^{K}\frac{M_tN_t}{K}\mathbf{a}_k^H(\theta,\phi)\mathbf{w}_k\mathbf{w}_k^H\mathbf{a}_k(\theta,\phi) \tag{3.3}$$

The equation (3.3) of the total transmit power defines the array transmit beampattern.

3.2.2 Transmit Beamforming Design

In order to design the 2D transmit beamforming, the optimization problem of minimizing the maximum difference between the desired 2D transmit beampattern and the transmit beampattern of the system given by (3.3) is derived. The constraint of the optimization problem is the uniform power allocation across the transmit antennas. Therefore, similar to the work in [110] for URA without subaperturing, the following optimization problem is formulated:

$$\min_{\mathbf{w}_1,\dots,\mathbf{w}_K} \quad \max_{\theta,\phi} |P_d(\theta,\phi) - \sum_{k=1}^{K}\mathbf{w}_k^H\mathbf{a}_k(\theta,\phi)\mathbf{a}_k^H(\theta,\phi)\mathbf{w}_k| \tag{3.4}$$

$$s.t. \quad \sum_{k=1}^{K}|\mathbf{W}_{[lk]}|^2 = \frac{E}{M_tN_t-(K-1)}, \quad l = 1,\dots,M_tN_t \tag{3.5}$$

where $\mathbf{W} = [\mathbf{w}_1, ..., \mathbf{w}_K] \in C^{M_tN_t \times K}$ is the transmit beampattern weight matrix, $P_d(\theta, \phi)$ is the desired beampattern and E is the total available power. In the constraint (3.5), the total power is divided with $M_tN_t - (K-1)$ because there are $M_tN_t - (K-1)$ elements in each subarray space. It is possible to have additional constraints for this optimization problem, such as sidelobe control or uniform power distribution over each subarray. This optimization problem is in a non-convex form. Defining a matrix $\mathbf{X}_k = \mathbf{w}_k\mathbf{w}_k^H \in C^{M_tN_t \times M_tN_t}$, $k = 1, ..., K$, the optimization problem is formulated as:

$$\begin{aligned}
\min_{\mathbf{X}_1,...,\mathbf{X}_K} \quad & \max_{\theta,\phi} |P_d(\theta,\phi) - \sum_{k=1}^{K} Tr\{\mathbf{a}_k(\theta,\phi)\mathbf{a}_k^H(\theta,\phi)\mathbf{X}_k\}| \\
s.t. \quad & \sum_{k=1}^{K} diag\{\mathbf{X}_k\} = \frac{E}{M_tN_t - (K-1)}\mathbf{1}_{M_tN_t \times 1} \\
& \mathbf{X}_k \succeq 0, \quad k = 1, \ldots, K \\
& rank(\mathbf{X}_k) = 1, \quad k = 1, \ldots, K
\end{aligned} \tag{3.6}$$

where $Tr\{\cdot\}$ denotes the trace of a matrix, $diag\{\cdot\}$ denotes the diagonal of a square matrix, $\mathbf{1}_{M_tN_t}$ defines the $M_tN_t \times 1$ vector of ones, and $rank(...)$ denotes the rank of a matrix. The notation $\mathbf{X}_k \succeq 0, \quad k = 1, \ldots, K$ is used to indicate that $\mathbf{X}_k$ is positive semidefinite. The rank constraint maintains the optimization problem (3.6) as non-convex. Relaxing the rank constraint (semidefinite relaxation), the optimization problem is recasted as follows [111]:

$$\begin{aligned}
\min_{\mathbf{X}_1,...,\mathbf{X}_K} \quad & \max_{\theta,\phi} |P_d(\theta,\phi) - \sum_{k=1}^{K} Tr\{\mathbf{a}_k(\theta,\phi)\mathbf{a}_k^H(\theta,\phi)\mathbf{X}_k\}| \\
s.t. \quad & \sum_{k=1}^{K} diag\{\mathbf{X}_k\} = \frac{E}{M_tN_t - (K-1)}\mathbf{1}_{M_tN_t \times 1} \\
& \mathbf{X}_k \succeq 0, \quad k = 1, \ldots, K
\end{aligned} \tag{3.7}$$

After the rank relaxation, the optimization problem (3.7) is convex and it is solved using semidefinite programming (SDP). The next step is to extract the transmit weight vectors from the optimal solution of the optimization problem (3.7), denoted as $\mathbf{X}_k^*$, for $k = 1, \ldots, K$. There are two cases for deriving the optimal weight vectors $\mathbf{w}_k$. If the rank of $\mathbf{X}_k^*$ is one, which is the ideal case, the optimal $\mathbf{w}_k$ is obtained straightforwardly as the eigenvector of $\mathbf{X}_k^*$, corresponding to the principal eigenvalue, multiplied by the square root of the principal eigenvalue. On the other hand, it is still possible the rank of $\mathbf{X}_k^*$ is greater than one. In this case the use of randomisation techniques is needed to derive the optimal transmit weight vectors [110].

The following randomisation technique is applied. Initially, the eigenvalue decomposition of $\mathbf{X}_k^*$ is defined as $\mathbf{X}_k^* = \mathbf{U}_k \mathbf{L}_k \mathbf{U}_k^H$. Then Λ random vectors are produced, i.e. $\mathbf{r}_k^\lambda$, $\lambda = 1, ..., \Lambda$, with elements uniformly distributed on the unit circle of the complex plane, providing the Λ candidate transmit weight vectors as $\mathbf{w}_k^\lambda = \mathbf{U}_k \mathbf{L}_k^{(1/2)} \mathbf{r}_k^\lambda$. Then, the optimal weight vector is chosen $\mathbf{w}_{opt,k}$, as the one which minimizes the objective function of the optimization problem (3.7). Finally, the optimal weight vector $\mathbf{w}_{opt,k}$ is normalised as:

$$\mathbf{w}_{norm,k} = \mathbf{w}_{opt,k} \frac{||\mathbf{X}_k||_F}{||\mathbf{w}_{opt,k} \mathbf{w}_{opt,k}^H||_F} \tag{3.8}$$

where $|| \cdot ||_F$ denotes the Frobenius norm. Using the transmit weight vectors derived in (3.8) the transmit beampattern is designed for the system.

3.2.3 Simulation Results

In this section, simulation results of the proposed design model are presented. A 5×5 transmit URA is assumed with half-wavelength spacing between adjacent antennas ($d_m = d_n = \lambda/2$, where λ is the wavelength). In the first example, the transmit array is divided into 5 subarrays which are fully overlapped as described in Fig.3.1. Each subarray consists of 21 antennas. The desired beampattern has a mainlobe defined by the 2D sector $\Theta = [-40^o, -20^o]$ in the elevation domain and $\Phi = [50^o, 85^o]$ in the azimuth domain. Also, a transition zone is incorporated and defined by $\Theta = [-50^o, -40^o] \bigcup [-20^0, -10^o]$ and $\Phi = [40^o, 50^o] \bigcup [85^o, 95^o]$. Any error that occurs in this region is ignored in the beamforming design. The 2D transmit beampattern is obtained by solving the optimization problem (3.7) and it is shown in Fig.3.2. It is obvious that the power allocation of the transmit beampattern is concentrated in the desired 2D sector. Moreover, the sidelobe levels are very low and do not extend to the whole 2D space.

In the second example the same 5×5 transmit URA is considered, but the transmit array is divided into 7 subarrays which are fully overlapped. Each subarray consists of 19 antennas. In this simulation, the 2D sector of interest is defined by $\Theta = [15^o, 55^o]$ in the elevation domain and $\Phi = [110^o, 140^o]$ in the azimuth domain. Furthermore, a transition zone is incorporated and defined by $\Theta = [5^o, 15^o] \bigcup [55^o, 65^o]$ and $\Phi = [100^o, 110^o] \bigcup [140^o, 150^o]$. The resulting

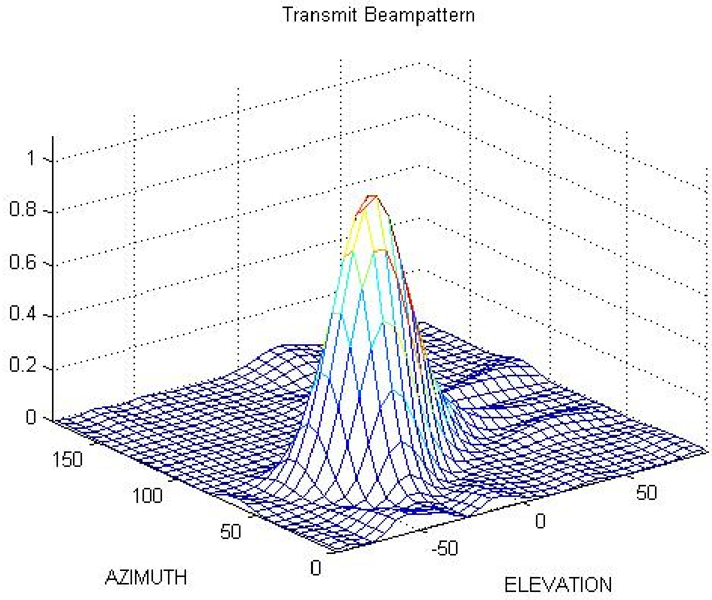


FIGURE 3.2: Transmit beampattern in the case of k=5 subarrays.

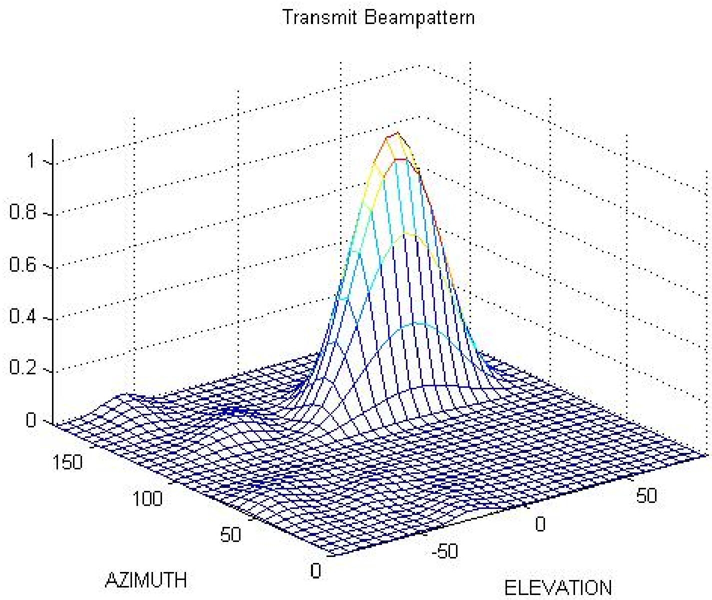


FIGURE 3.3: Transmit beampattern in the case of k=7 subarrays.

2D transmit beampattern is shown in Fig.3.3. It is clear from the two figures that in the case of 7 fully overlapped subarrays, the sidelobe levels are even lower than the case of 5 subarrays.

In the third example, the main objective is to compare the proposed subaperturing technique with the case when the URA uses all of its elements when

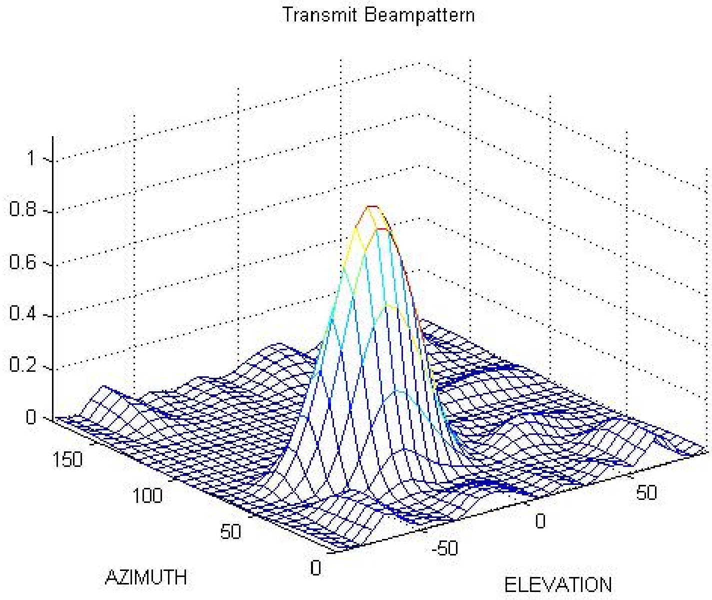
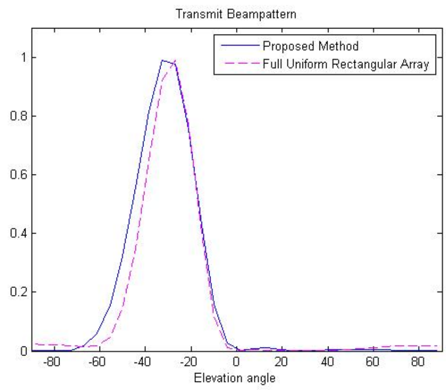
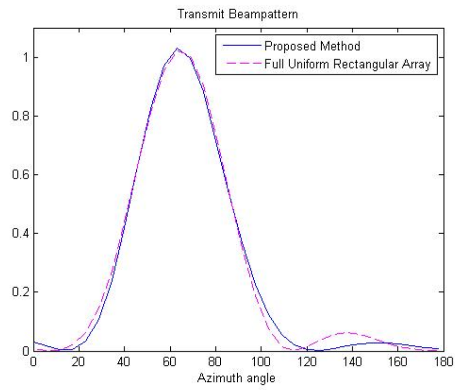


FIGURE 3.4: Transmit beampattern in the case of full URA.



(A) Elevation cross section



(B) Azimuth cross section

FIGURE 3.5: Cross sections of the transmit beampattern at $\phi = 63^o$ and $\theta = -27^o$, respectively.

transmitting the probing signal. Once again, a 5×5 transmit URA with half-wavelength spacing between adjacent antennas is assumed. The 2D sector of interest is defined as in the first example in order to facilitate the comparison. Five transmit beams are used to synthesize the 2D transmit beampattern. The resulting 2D transmit beampattern is shown in Fig.3.4. The results in Fig.3.5 show two cross sections of the transmit beampattern, incorporating both the proposed method and the full URA case. The first cross section is plotted against the elevation angle by keeping the azimuth angle constant at 63^o. Similarly, the second cross section is derived against the azimuth angle by holding the elevation angle constant at -27^o. It is worth noting that the sidelobe levels are clearly lower for the proposed method.

3.3 Beamforming for Fully-Overlapped Two-Dimensional Phased-MIMO Radar

In this section, the design of joint transmitter and receiver beamformer is investigated within the context of multiple-input multiple-output (MIMO) radar employing two-dimensional (2D) arrays of antennas. Specifically, the transmit, waveform diversity and overall transmit-receive beampatterns are derived for the Phased-MIMO radar with fully-overlapped subarrays and compared with the respective beampatterns for the Phased-array and MIMO radar only schemes. As reported for one-dimensional linear arrays, fully-overlapped 2D subarrays offer substantial improvements in performance as compared with the phased-array and MIMO only radar models. The work considers both the adaptive (convex optimization, CAPON beamformer) and non-adaptive (conventional) beamforming techniques. The simulation results demonstrate the superiority of the fully-overlapped subaperturing in both cases.

3.3.1 2D Phased-MIMO System Model

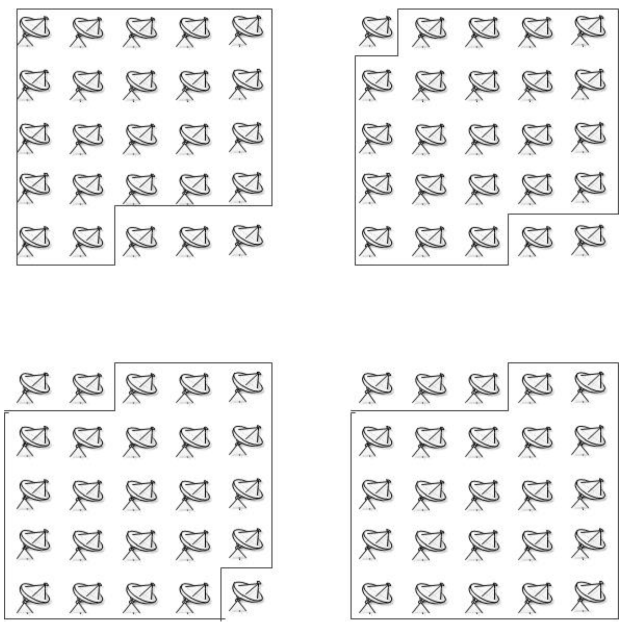


FIGURE 3.6: Fully overlapped subaperturing of a 5×5 uniform rectangular array (URA) when K=4.

In this work, a monostatic radar system is considered employing a uniform rectangular array (URA), which consist of $M_t \times N_t$ and $M_r \times N_r$ antennas at the transmitter and the receiver respectfully, where M_t and M_r are the number of elements in each column and N_t and N_r are the number of antennas in each row of the planar arrays. The 2D Phased-MIMO model is based on partitioning the transmit 2D array into K subarrays ($1 \leq K \leq M_t \times N_t$) that are fully overlapped [62], as depicted in Fig. 3.6, where a subaperturing of a 5×5 transmit URA into 4 subarrays is presented. Moreover the k^{th} subarray is composed of $M_t \times N_t - K + 1$ antennas and emits the k^{th} element of the predesigned independent waveform vector $\boldsymbol{\psi}(t) = [\psi_1(t), \ldots, \psi_K(t)]^T$ of size $K \times 1$, which satisfies the orthogonality condition $\int_{T_0} \boldsymbol{\psi}(t)\boldsymbol{\psi}^H(t)dt = \mathbf{I}_K$, where $(\cdot)^T$ denotes the transpose, t refers to the time index within the radar pulse, T_0 is the radar pulse width, $\mathbf{I}_K$ is the $K \times K$ identity matrix, and $(\cdot)^H$ denotes the Hermitian transpose.

In order to characterize the fully overlapped subaperturing of the 2D Phased-MIMO model mathematically, an $M_t \times N_t$ selection matrix is introduced $\mathbf{Z}_k$ [110]. When the $(mn)^{th}$ entry equals 1 then the $(mn)^{th}$ element of the 2D array belongs to the k^{th} subarray, while a 0 entry in $\mathbf{Z}_k$ means that the element is not a part of the k^{th} subarray. Thus, the matrix $\mathbf{Z}_k$ defines the structure of the k^{th} subarray. As a result, the $M_tN_t \times 1$ transmit steering vector related to the k^{th} subarray can be constructed as:

$$\mathbf{a}_k(\theta, \phi) = vec(\mathbf{Z}_k \odot [\boldsymbol{\mu}(\theta, \phi)\boldsymbol{\nu}^T(\theta, \phi)]) \tag{3.9}$$

where $vec(\cdot)$ is the operator that stacks the columns of a matrix into one column vector, $\odot$ denotes the Hadamard product, θ and ϕ denote the elevation and azimuth angles respectively. The auxiliary vectors $\boldsymbol{\mu}(\theta, \phi) \in C^{M_t \times 1}$ and $\boldsymbol{\nu}(\theta, \phi) \in C^{N_t \times 1}$ are derived from the array geometry and they are defined as follows:

$$\boldsymbol{\mu}(\theta, \phi) = [1, e^{j2\pi d_m sin(\theta)cos(\phi)}, \ldots, e^{j2\pi(M_t-1)d_m sin(\theta)cos(\phi)}]^T$$

$$\boldsymbol{\nu}(\theta, \phi) = [1, e^{j2\pi d_n sin(\theta)sin(\phi)}, \ldots, e^{j2\pi(N_t-1)d_n sin(\theta)sin(\phi)}]^T$$

where d_m and d_n are the distances between the adjacent antennas at each column and at each row respectively.

The primary objective of the work is to focus the transmit energy onto a certain 2D sector in space, determined by the direction of the target, and at the same time to achieve high transmit coherent processing gain. Hence, a weight vector should be designed for each of the K subarrays to steer the transmit beam in the desired spatial sector. The $M_tN_t \times 1$ vector which consists of the complex envelope of the signals at the output of the k^{th} subarray can be modeled as $\mathbf{s}_k(t) = \sqrt{\frac{M_t N_t}{K}}\mathbf{w}_k \psi_k(t)$, where $\mathbf{w}_k \in C^{M_t N_t \times 1}$ is the transmit beamformer weight vector, used to form the k^{th} transmit beam. The power of the emitted signal from the k^{th} subarray focused at a generic focal point with coordinates (θ, ϕ) is given by

$$P_k(\theta, \phi) = \mathbf{a}_k^H(\theta, \phi) E\{\mathbf{s}_k(t)\mathbf{s}_k^H(t)\}\mathbf{a}_k(\theta, \phi)$$

$$= \frac{M_t N_t}{K}\mathbf{a}_k^H(\theta, \phi)\mathbf{w}_k \mathbf{w}_k^H \mathbf{a}_k(\theta, \phi) \tag{3.10}$$

Using the far field assumption and adding the power of the probing signals emitted by all K subarrays, the 2D array transmit beampattern can be written as

$$P(\theta, \phi) = \sum_{k=1}^{K} \frac{M_t N_t}{K}\mathbf{a}_k^H(\theta, \phi)\mathbf{w}_k \mathbf{w}_k^H \mathbf{a}_k(\theta, \phi) \tag{3.11}$$

Assuming that there is a target present in the far-field of the transmit and receive arrays at direction θ_t in the elevation domain and ϕ_t in the azimuth domain, the signal reflected by the aforementioned target is modeled as

$$r(t, \theta_t, \phi_t) = \sqrt{\frac{M_t N_t}{K}}\beta_t \sum_{k=1}^{K} \mathbf{w}_k^H \mathbf{a}_k(\theta_t, \phi_t) e^{-j\tau_k(\theta_t, \phi_t)}\psi_k(t) \tag{3.12}$$

where β_t is the complex amplitude proportional to the radar cross section (RCS) of the target, and $\tau_k(\theta_t, \phi_t)$ is the time required for the signal to cover the distance between the first element of the transmit array and the first element of the k^{th} subarray.

If it is assumed that in addition to the desired target, there are L active interfering targets at locations $\{\theta_i\}_{i=1}^{L}$, $\{\phi_i\}_{i=1}^{L}$ and with reflection coefficients $\{\beta_i\}_{i=1}^{L}$, then under the simplifying assumption of point targets, the $M_r N_r \times 1$ received data vector can be described by the equation

$$\mathbf{x}(t) = r(t, \theta_t, \phi_t)\mathbf{b}(\theta_t, \phi_t) + \sum_{i=1}^{L} r(t, \theta_i, \phi_i)\mathbf{b}(\theta_i, \phi_i) + \mathbf{n}(t) \tag{3.13}$$

where $\mathbf{b}(\theta, \phi)$ is the $M_r N_r \times 1$ steering vector of the received array and $\mathbf{n}(t)$ is the noise component that is supposed to have zero mean. By applying matched filtering to the received data vector for each of the orthogonal waveforms $\psi_k(t)$, $k = 1, ..., K$, the $KM_r N_r \times 1$ virtual receive data vector can be constructed as

$$\mathbf{y} = \int_{T_0} \mathbf{x}(t)\psi_k^*(t)dt$$
$$= \sqrt{\frac{M_tN_t}{K}}\beta_t\mathbf{u}(\theta_t,\phi_t) + \sum_{i=1}^{L}\sqrt{\frac{M_tN_t}{K}}\beta_i\mathbf{u}(\theta_i,\phi_i) + \hat{\mathbf{n}} \tag{3.14}$$

where $\hat{\mathbf{n}} = \int_{T_0} \mathbf{n}(t)\psi_k^*(t)dt$ is the $KM_rN_r \times 1$ noise term with covariance matrix $\mathbf{R}_n = \sigma_n^2\mathbf{I}_{KM_rN_r}$ (σ_n^2 is the noise variance) and the $KM_rN_r \times 1$ vector

$$\mathbf{u}(\theta,\phi) = (\mathbf{c}(\theta,\phi) \odot \mathbf{d}(\theta,\phi)) \otimes \mathbf{b}(\theta,\phi) \tag{3.15}$$

is the virtual steering vector of the system. In order to derive the virtual steering vector, the $K \times 1$ transmit coherent processing vector is used

$$\mathbf{c}(\theta,\phi) = [\mathbf{w}_1^H\mathbf{a}_1(\theta,\phi),\ldots,\mathbf{w}_K^H\mathbf{a}_K(\theta,\phi)]^T \tag{3.16}$$

and the $K \times 1$ waveform diversity vector

$$\mathbf{d}(\theta,\phi) = [e^{-j\tau_1(\theta,\phi)},\ldots,e^{-j\tau_K(\theta,\phi)}]^T \tag{3.17}$$

In the case of the fully-overlapped partitioning of the 2D transmit array into K subarrays, the waveform diversity vector is equal to the K first elements of the transmit steering vector $\mathbf{a}(\theta,\phi) = vec(\boldsymbol{\mu}(\theta,\phi)\boldsymbol{\nu}^T(\theta,\phi))$.

At this point it is apparent that the 2D Phased-MIMO radar scheme exploits the benefits of both the phased-array and the MIMO radar model as a tradeoff between transmit coherent processing gain and higher angular resolution. This tradeoff is determined by the selection of the number of fully overlapped subarrays of the 2D transmit array. In particular, if $K = 1$ is chosen the radar model simplifies to the conventional phased-array scheme, since the whole transmit array forms the only subarray which emits only one waveform. However, if $K = M_tN_t$ is selected, the radar model simplifies to a MIMO radar.

3.3.2 Transmit-Receive Beamforming for the Phased-MIMO model

In this section, conventional and adaptive techniques are investigated to design the transmit and the overall transmit-receive beampattern of the Phased-array, Phased-MIMO and MIMO radar schemes.

Conventional Beampattern Design

Conventional non-adaptive beamforming is the simplest technique to design the transmit and overall beampatterns, however, it offers the highest possible output SNR gain only when a single target is observed in the background of white Gaussian noise [19]. By applying the conventional beamforming in the proposed 2D Phased-MIMO model, the normalized transmit weight vector for the k^{th} subarray can be obtained as

$$\mathbf{w}_k = \frac{\mathbf{a}_k(\theta_t, \phi_t)}{\|\mathbf{a}_k(\theta_t, \phi_t)\|}, \quad k = 1, \ldots, K \tag{3.18}$$

where $|| \cdot ||$ denotes the Euclidian norm. In order to derive the conventional transmit beampattern, (3.18) is substituted in (3.11). By enforcing the conventional beamformer at the virtual receive array, the $KM_rN_r \times 1$ receive weight vector is defined as

$$\mathbf{w}_r = \mathbf{u}(\theta_t, \phi_t) \tag{3.19}$$

The overall transmit-receive beampattern is given by

$$Q(\theta, \phi) = |\mathbf{w}_r^H \mathbf{u}(\theta, \phi)|^2 \tag{3.20}$$

Adaptive Beampattern Design

It is presumed that a surveillance radar is incorporated within the radar network, which detects incoming targets and provides an initial estimation regarding their coordinates. The technical analysis of the surveillance method goes beyond the scope of this thesis and is not investigated. After the target location coordinates are obtained from the detection scan of the surveillance radar as (θ_t,ϕ_t), the main goal is to focus the power of the next beam at a spatial sector around the target, defined by

$$\Theta = [\theta_t - \Delta_1, \quad \theta_t + \Delta_1] \tag{3.21}$$

$$\Phi = [\phi_t - \Delta_2, \quad \phi_t + \Delta_2] \tag{3.22}$$

in the elevation domain and the azimuth domain, where $2\Delta_1$ and $2\Delta_2$ are the chosen beamwidths for the target in the elevation and azimuth domain respectively (Δ_1 and Δ_2 should be greater than the expected error in θ_t and ϕ_t respectively). Following this approach, more accurate parameter identifiability is guaranteed for the target. The derivation of the transmit weight vector for each subarray is achieved by solving a convex optimization problem that minimizes the difference between the desired transmit beampattern and the beampattern produced by the 2D array of antennas, under a constraint in terms of uniform power allocation across the transmit antennas [110, 109]. In this work, strong clutter is considered, imposed by an obstacle within a certain 2D spatial sector, already estimated as $\Theta_c = [\theta_{c1} \quad \theta_{c2}]$ and $\Phi_c = [\phi_{c1} \quad \phi_{c2}]$ from training signals. The second constraint in the optimization problem is to restrain the sidelobe level in the prescribed region under a certain value δ, thus minimizing the clutter effect in the system. Hence, defining a matrix $\mathbf{X}_k = \mathbf{w}_k \mathbf{w}_k^H \in C^{M_t N_t \times M_t N_t}, k = 1, ..., K$, the optimization problem is formulated as:

$$\min_{\mathbf{X}_1,\ldots,\mathbf{X}_K} \quad \max_{\theta,\phi} |P_d(\theta,\phi) - \sum_{k=1}^{K} Tr\{\mathbf{a}_k(\theta,\phi)\mathbf{a}_k^H(\theta,\phi)\mathbf{X}_k\}|$$

$$s.t. \quad \sum_{k=1}^{K} diag\{\mathbf{X}_k\} = \frac{E}{M_t N_t - (K-1)} \mathbf{1}_{M_t N_t \times 1}$$

$$|\sum_{k=1}^{K} Tr\{\mathbf{a}_k(\theta_c,\phi_c)\mathbf{a}_k^H(\theta_c,\phi_c)\mathbf{X}_k\}| - \delta \leq 0, \quad \theta_c \in \Theta_c, \phi_c \in \Phi_c$$

$$\mathbf{X}_k \succeq 0, \quad k = 1, \ldots, K \tag{3.23}$$

where $P_d(\theta,\phi)$ is the desired beampattern, E is the total available power, $Tr\{\cdot\}$ denotes the trace of a matrix, $diag\{\cdot\}$ denotes the diagonal of a square matrix and $\mathbf{1}_{M_t N_t}$ defines the $M_t N_t \times 1$ vector of ones. The notation $\mathbf{X}_k \succeq 0, \quad k = 1, \ldots, K$ is used to indicate that $\mathbf{X}_k$ is positive semidefinite. The convex optimization problem (15) is solved using semidefinite programming (SDP) [51]. After obtaining the optimal solution, denoted as $\mathbf{X}_k^*$, the optimal transmit weight vectors $\mathbf{w}_k$ are derived. If $\mathbf{X}_k^*$ is of rank one, which is the ideal scenario, the optimal weight vector $\mathbf{w}_k$ is obtained straightforwardly as the principal eigenvector of $\mathbf{X}_k^*$ multiplied by the square root of the principal eigenvalue of $\mathbf{X}_k^*$. However, if the rank of $\mathbf{X}_k^*$ is greater than one, one must resort to randomisation techniques to obtain the optimal transmit weight vectors [109].

Besides the transmit array, it is also important to use adaptive techniques at the 2D receive array of the system in order to maximize the output signal to interference plus noise ratio (SINR). A beamformer that satisfies both the steering capabilities whereby the target signal is always protected and the cancelation of interference so that the output SINR is maximized, is the Minimum Variance Distortionless Response (MVDR) beamformer [20]. The main idea of the MVDR beamformer is to minimize the covariance of the beamformer output subject to a distortionless response towards the direction of the target. Hence, it can be formulated as the following optimization problem

$$\min_{\mathbf{w}_r} \mathbf{w_r^H}\hat{\mathbf{R}}_{yy}\mathbf{w_r} \quad \text{subject to} \quad \mathbf{w_r^H}\mathbf{u}(\theta_t, \phi_t) = 1 \tag{3.24}$$

where $\hat{\mathbf{R}}_{yy} = \frac{1}{N}\mathbf{y}\mathbf{y}^H$ is the sample covariance matrix of the observed data samples that can be collected from N different radar pulses. The solution to (3.24) is [20],

$$\mathbf{w_r} = \frac{\hat{\mathbf{R}}_{yy}^{-1}\mathbf{u}(\theta_t, \phi_t)}{\mathbf{u}^H(\theta_t, \phi_t)\hat{\mathbf{R}}_{yy}^{-1}\mathbf{u}(\theta_t, \phi_t)} \tag{3.25}$$

The receive weight vectors derived by (3.25) are employed to design the overall transmit-receive beampattern in the simulations.

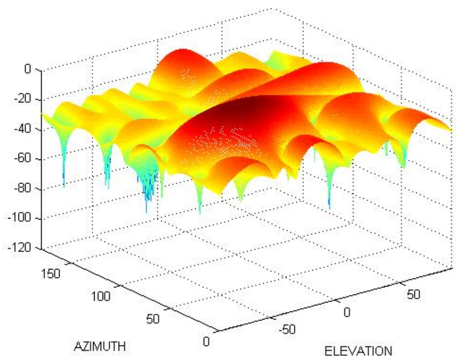
3.3.3 Simulation Results

The performance of the fully-overlapped 2D Phased-MIMO radar is compared to the phased-array and the conventional MIMO radar schemes. A 5×5 transmit-receive URA with half-wavelength spacing between adjoining antennas is assumed ($d_m = d_n = \lambda/2$, where λ is the wavelength). The emitted orthogonal baseband waveforms from each subarray are modeled as [112]:

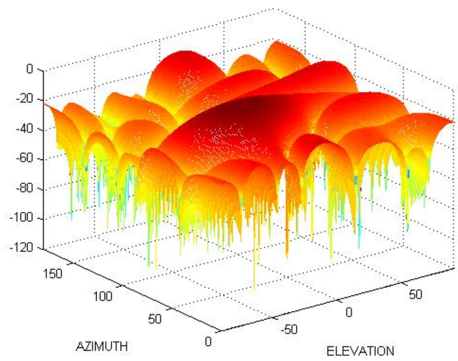
$$\psi_k(t) = \sqrt{\frac{1}{T_0}} e^{j2\pi(k/T_0)t}, \quad k = 1, \dots, K$$

The desired target is located at directions $\theta_t = -30^o$ and $\phi_t = 60^o$. Furthermore, one interfering target is assumed at directions $\theta_i = 30^o$ and $\phi_i = 90^o$. The 2D transmit array is divided into 5 subarrays that are fully overlapped and each of them consists of 21 antennas. The noise is considered as complex Gaussian with zero mean and variance 0.1. In order to derive the sample covariance matrix $N = 100$ data samples are used.

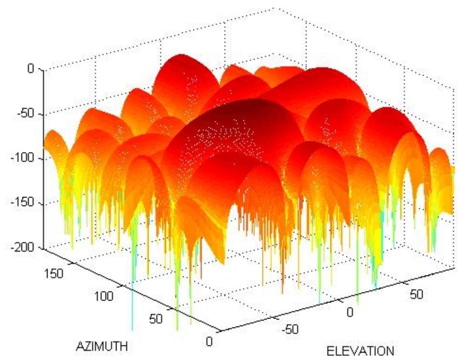
In the first example, the conventional non-adaptive beamformer is used to derive both the transmit and receive weight vectors. In order to obtain the waveform diversity beampattern, the waveform diversity vector obtained by (3.17) is considered as the weight vector. As a result, the transmit, the waveform diversity and the overall beampatterns for the 2D Phased-MIMO radar are depicted in Fig. 3.7. In Fig. 3.8, the same beampatterns are simulated for the phased-array radar model, by considering the whole 2D transmit array as the only subarray ($K = 1$). On the contrary, in order to simulate the conventional MIMO radar, $K = M_tN_t$ is set (each antenna of the transmit array is considered as a subarray) and the respective beampatterns are shown in Fig. 3.9. To facilitate the comparison between the three models, Figs. 3.10-3.12 show the cross section plotted against the elevation angle by keeping the azimuth angle constant at 60^o as well as the cross section plotted against the azimuth angle by holding the elevation angle at -30^o for all three schemes.



(A) Conventional transmit beampattern (dB).



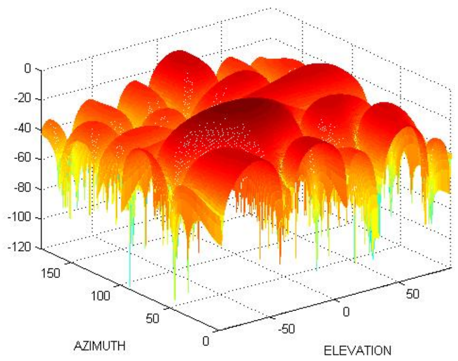
(B) Conventional waveform diversity beampattern (dB).



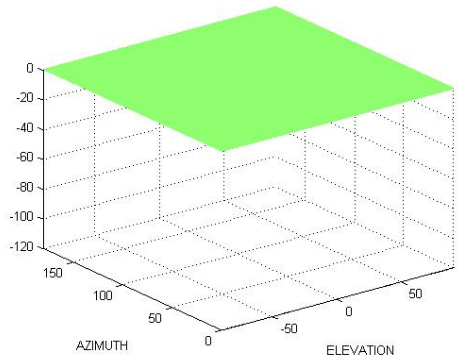
(C) Conventional overall beampattern (dB).

FIGURE 3.7: The beampatterns for the non-adaptive 2D Phased-MIMO radar.

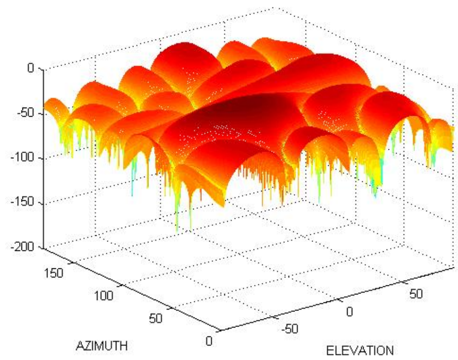
As reported for the case of the one-dimensional (1D) linear array in [62], for the 2D array also it is evident from Figs. 3.10 and 3.11 that although the phased-array radar has the most efficient transmit conventional beampattern due to its high transmit coherent processing gain, it has zero waveform diversity gain. On the other hand, the MIMO radar has flat ($0dB$) transmit beampattern, but it has the most accurate waveform diversity beampattern, because of the simultaneous emission of M_tN_t orthogonal waveforms. However, it is clear from Fig. 3.12 that the 2D Phased-MIMO radar remarkably outperforms the phased-array and MIMO radars in terms of the overall transmit-receive beampattern, as it has lower sidelobes and approximates better the desired



(A) Conventional transmit beampattern (dB).



(B) Conventional waveform diversity beampattern (dB).

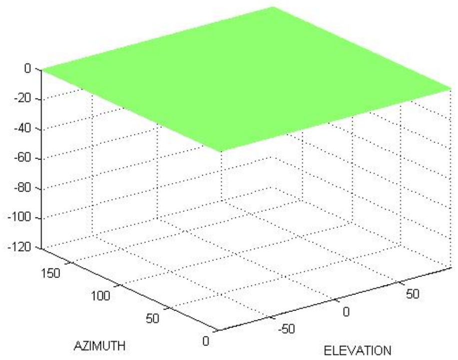


(C) Conventional overall beampattern (dB).

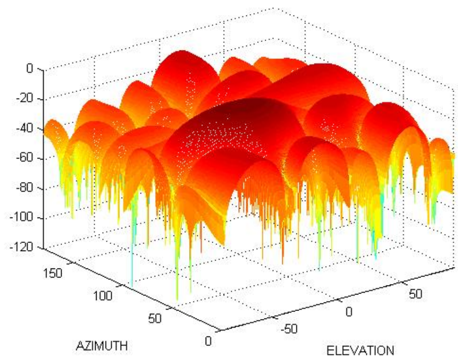
FIGURE 3.8: The beampatterns for the non-adaptive 2D phased-array radar.

target direction. Moreover, it is important to highlight that in the case of conventional beamforming the overall beampatterns of the phased-array and the MIMO radar are exactly the same.

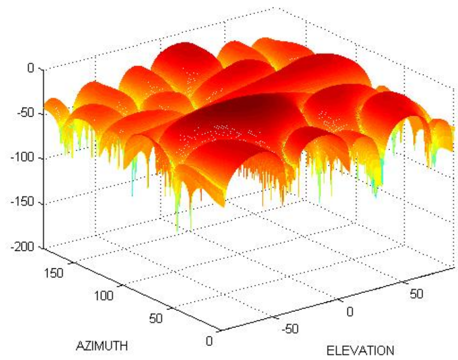
In the second example, adaptive beamforming techniques are employed to derive the transmit and receive beampatterns. In particular, convex optimization techniques are used to determine the transmit beamformer weight vectors and the MVDR (CAPON) based receiver beamformer for the receive weight vectors. In the simulations, strong clutter is assumed at the 2D spatial sector defined by $\Theta_c = [-90^o, -60^o]$ and $\Phi_c = [140^o, 180^o]$. It is considered that $\delta = 0.01$ (-20dB) to restrain the sidelobe level in the clutter region. The desired beampattern that it is wished to be approximated is given by (3.21) and (3.22) where $\Delta_1 = 10^o$ and $\Delta_2 = 20^o$. The total available power for the system is equal to one ($E = 1$) and the interference to noise ratio (INR) is fixed to 30dB. The 2D transmit beampattern for the Phased-MIMO radar is obtained by solving the optimization problem in (3.23) as shown in Fig. 3.13a. Similarly,



(A) Conventional transmit beampattern (dB).

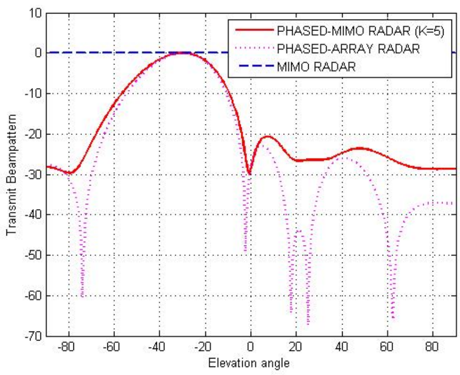


(B) Conventional waveform diversity beampattern (dB).

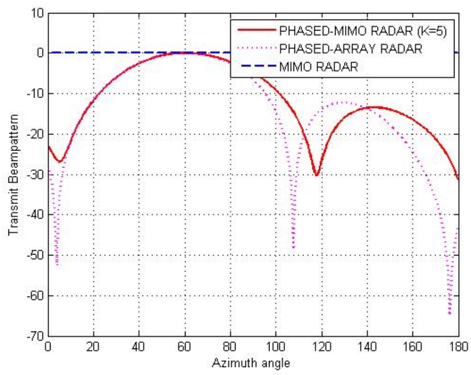


(C) Conventional overall beampattern (dB).

FIGURE 3.9: The beampatterns for the non-adaptive 2D MIMO radar.



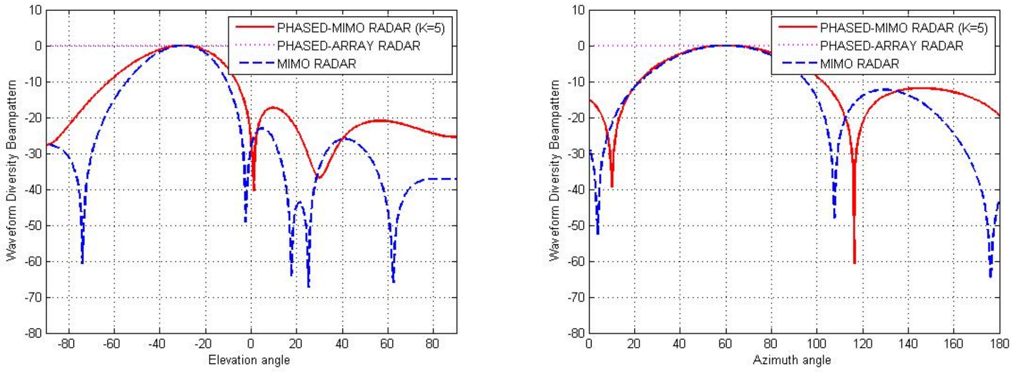
(A) Elevation cross section.



(B) Azimuth cross section.

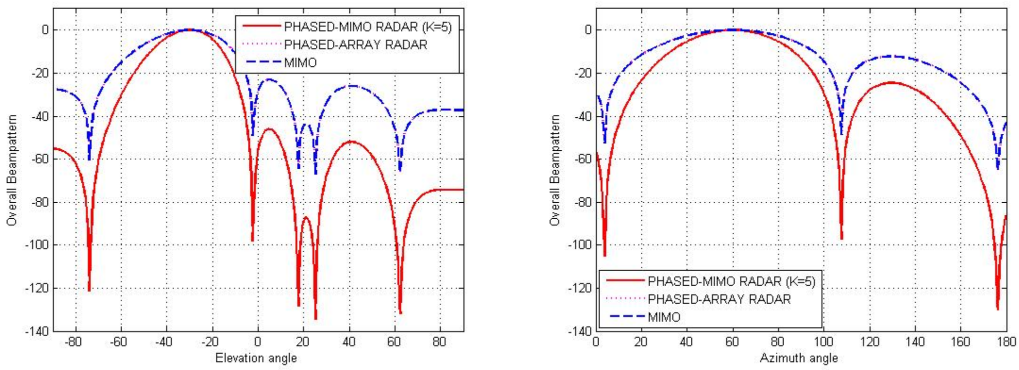
FIGURE 3.10: Cross sections of the transmit beampattern at $\phi = 60^o$ and $\theta = -30^o$, respectively.

by solving the same optimization problem considering the whole URA as one subarray ($K = 1$), the 2D transmit beampattern for the phased-array scheme is generated as shown in Fig. 3.13b. It is clear that the power allocation of



(A) Elevation cross section. (B) Azimuth cross section.

FIGURE 3.11: Cross sections of the waveform diversity beampattern at $\phi = 60^o$ and $\theta = -30^o$, respectively.

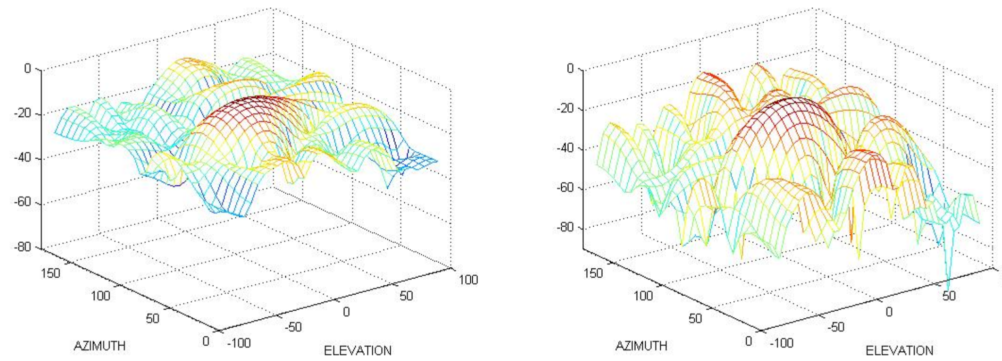


(A) Elevation cross section. (B) Azimuth cross section.

FIGURE 3.12: Cross sections of the overall beampattern at $\phi = 60^o$ and $\theta = -30^o$, respectively.

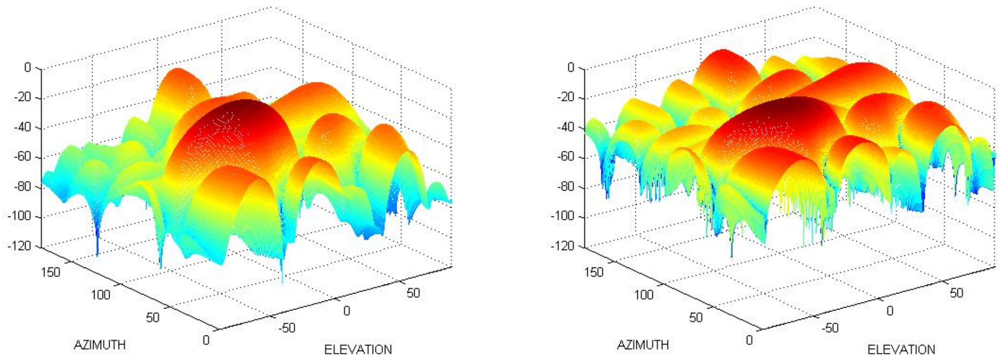
both beampatterns is concentrated in the desired space and the sidelobe level is very low, especially over the predefined clutter regions, where it has values lower than 20dB.

At the receive array, the MVDR beamformer is employed to derive the overall transmit-receive beampatterns for all radar schemes investigated, as shown in Fig. 3.14. Similar to the first example, Fig. 3.15 shows the cross sections of the overall beampatterns to help to facilitate the comparison between the three types of radar configurations. Corresponding to the results for conventional beamforming, it is clear from Fig. 3.15 that the 2D Phased-MIMO radar exploits the transmit superiority of the phased-array model and the waveform diversity of the MIMO scheme to result in a substantially improved overall beampattern.

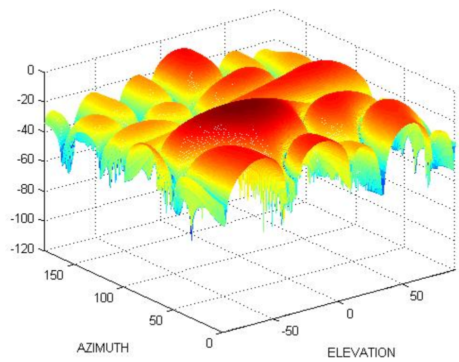


(A) 2D Phased-MIMO radar. (B) 2D Phased-array radar.

FIGURE 3.13: Transmit beampatterns using convex optimization(dB).



(A) 2D Phased-MIMO radar. (B) 2D phased-array radar.

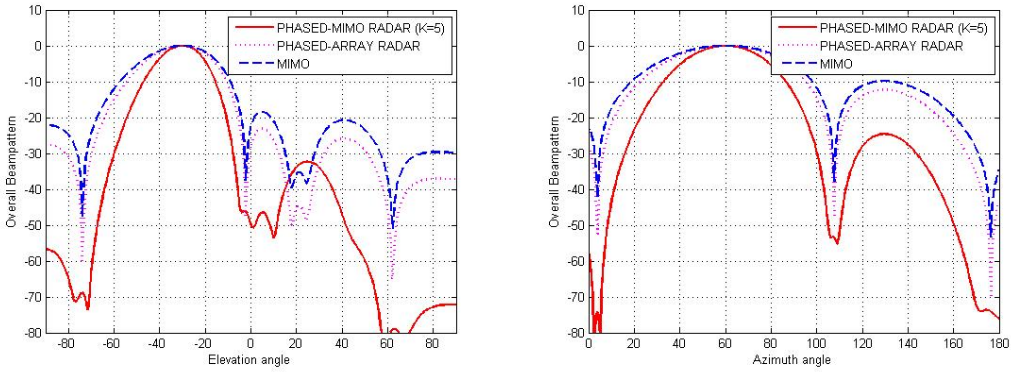


(C) 2D MIMO radar.

FIGURE 3.14: Adaptive Overall Beampatterns using MVDR beamformer (dB).

3.4 Conclusion

In this chapter, a new subaperturing technique for MIMO radars with planar URA at the transmit side was investigated. Specificaly, the problem of 2D



(A) Elevation cross section. (B) Azimuth cross section.

FIGURE 3.15: Cross sections of the overall beampattern at $\phi = 60^o$ and $\theta = -30^o$ (adaptive beamforming).

transmit beamforming design was considered for the MIMO radar with fully overlapped subarrays. The simulation results confirmed that the system transmit beampattern approximates the desired sector of space with high accuracy. Furthermore, the sidelobe levels are very low and are restricted in an area close to the mainlobe, without covering the whole 2D space. Moreover, it is apparent that as the number of subarrays increases the transmit beampattern produces lower sidelobe levels. Finally, a comparison was performed between the proposed method and the case when the transmit side consists of a full URA. It is shown that the concentration of the power within the desired 2D sector is more evident in the proposed method.

Furthermore, the performance of transmit/receive beamforming has been studied within the context of 2D Phased-MIMO radar with fully overlapped subarrays. The simulation results confirmed that there are substantial improvements of the overall transmit/receive beampattern of the 2D Phased-MIMO radar as compared to the phased-array and the conventional MIMO model. In particular, it was demonstrated that the Phased-MIMO scheme combines the transmit coherent processing gain of the phased-array radar and the waveform diversity of the MIMO model to produce a more efficient and accurate overall beampattern with very low sidelobe levels. This superiority is highlighted using both non-adaptive (conventional) and adaptive (convex optimization and MVDR) beamforming techniques.

Chapter 4

Resource Allocation Games and Nash Equilibrium Analysis

4.1 Game Theoretic Power Allocation and the Nash Equilibrium Analysis for a Multistatic MIMO Radar Network

In this section, a game theoretic power allocation scheme is investigated and a rigorous Nash equilibrium analysis for a multistatic multiple-input multiple-output (MIMO) radar network is provided. In particular, a network of radars is assumed, organized into multiple clusters, whose primary objective is to minimize their transmission power, while satisfying a certain detection criterion. Since there is no communication between the distributed clusters, convex optimization methods and noncooperative game theoretic techniques are incorporated based on the estimate of the signal to interference plus noise ratio (SINR) to tackle the power adaptation problem. Therefore, each cluster egotistically determines its optimal power allocation in a distributed scheme. Furthermore, it is proved that the best response function of each cluster regarding this generalized Nash game (GNG) belongs to the framework of standard functions. The standard function property together with the proof of the existence of solution for the game guarantees the uniqueness of the Nash equilibrium. The mathematical analysis of the uniqueness of the solution leads to substantial results on the relation between the performance with respect to the detection criterion and the transmission power of the radars. Finally, the simulation results confirm the convergence of the algorithm to the unique solution and demonstrate the distributed nature of the system.

4.1.1 Introduction

Recent advances in digital signal processing and the constant development of computational capabilities suggest that it may be feasible for next generation radar systems to incorporate multiple-input multiple-output (MIMO) technology. The superiority of a MIMO radar against other radar schemes lies in its waveform diversity, which in essence defines that a MIMO radar can simultaneously emit several diverse, possibly linearly independent waveforms via multiple antennas, in contrast to existing radar systems that transmit scaled versions of the same, predefined waveform [5]. In particular, there are two principal types of MIMO radar, those that incorporate colocated antennas [8] and systems equipped with widely separated antennas (bistatic, multistatic) [9]. MIMO radar technology provides direct applicability of adaptive beamforming [113], waveform design and power allocation, higher angular resolution, ability to acquire the target's geometrical characteristics through the radar cross section (RCS) and multiple target detection [5]. However, in order to combat multiple source interference in a radar field, while achieving high detection performance using minimum power consumption, the system should adopt an optimal resource allocation strategy. A centralised approach to resource allocation is possible using convex optimization techniques for example. Nevertheless, centralised control is impractical to be implemented in a multistatic radar network and thus it is preferred to consider an autonomous decentralised resource allocation scheme. A natural and efficient tool to achieve this is game theory, which provides a framework for analyzing coordination and conflict between rational but selfish players.

Recently, game theoretic techniques have been extensively explored within the radar research community to tackle several issues and to improve and optimize various radar parameters. Specifically, the authors in [35] and [36] formulated a non-cooperative game to address the power optimization problem with a predefined SINR constraint. Furthermore, to extend the study in [35], a signal-to-disturbance ratio (SDR) estimation technique was applied in [37]. A two-player, non-cooperative, zero-sum game was considered in [71] to investigate the interaction between a radar and a jammer. Non-cooperative MIMO radar and jammer games were also applied in [72], where the utility functions were formulated using the mutual information criterion. The authors in [74] studied the problem of polarimetric waveform design by forming a zero-sum game between a target and a radar engineer. Moreover, in [38], the power allocation problem of a distributed MIMO radar was tackled using a cooperative game approach through maximizing the Bayesian-Fisher information matrix

(B-FIM) and exploiting the Shapley value solution scheme. Potential game theory techniques were exploited in [114] for optimal waveform design and maximization of the detection performance. Finally, the authors in [39] proposed a water filling method for optimal power distribution using a Stackelberg game theoretic framework.

In this paper, motivated by the results in [35] and [37], the power allocation problem of a distributed, multistatic radar network is revisited, where multiple MIMO radars are organized into clusters. The primary goal of each cluster is to secure a certain detection criterion, in terms of signal-to-interference plus noise (SINR) ratio, while allocating the minimum possible power to each radar. An optimal power allocation is of great importance to a radar system that works on a specific power budget, i.e. portable radars operating with a battery, as by minimizing the power consumption the tracking time is extended. Furthermore, a minimized transmit power induces less interference to the receivers of other radars in the same field, belonging to the same organization. Hence, a generalized Nash game (GNG) is formulated, where there is no communication between the clusters of the network, despite the fact that they belong to the same organization. Such a scheme could be deployed in a scenario, where the opponent incorporates electronic warfare methods to intercept information about the location of the radars. In this case, in order to apply the game theoretic algorithm, an estimation of the SINR is required, as there is no coordination between the clusters and thus no information on the inter-cluster channel gains.

The main contribution of this work lies in the proof of the uniqueness of the Nash equilibrium of the game theoretic power allocation problem described above. Specifically, it is demonstrated that the best response function of each cluster in this GNG belongs to the family of standard functions by using convex optimization techniques and by analyzing the Lagrangian dual of the initial optimization problem. Moreover, through the game theoretic analysis, results were obtained on the behavior of the radars in a cluster. More specifically, the theoretical results showed that in a cluster, the number of radars that exactly achieve the desired SINR is equal to the number of radars that are actively transmitting. However, from the Lagrangian duality, this does not necessarily imply that the active radars are the ones that attain the target SINR. Furthermore, the simulation results confirm the convergence of the algorithm to the unique Nash equilibrium.

This Chapter is organized as follows. Section II introduces the decentralized radar network as the system model. In Section III the game theoretic formulation of the problem is presented and the definition of the generalized Nash game (GNG) considered in this paper. The SDR estimation technique utilized in this work is demonstrated in Section IV. The analysis on the existence and uniqueness of the Nash equilibrium is performed in Section V. Finally, the simulation results and the concluding remarks are presented in Sections VI and VII, respectively.

Notation: Bold lower-case letters and bold uppercase letters are used to denote column vectors and matrices, respectively. $\mathbf{a}^H$ gives the Hermitian of the vector $\mathbf{a}$ and $\mathbf{a}^T$ denotes its transpose. $\mathbf{A}(i,j)$ corresponds to the element located on the i^{th} row and j^{th} column of matrix $\mathbf{A}$. $\mathbf{I}_M$ stands for the $M \times M$ identity matrix. The Euclidean norm is denoted by $||\cdot||$. An $N \times 1$ vector of ones is indicated by $\mathbf{1}_N$. Finally, any inequalities among vectors are considered element-wise.

4.1.2 System Model

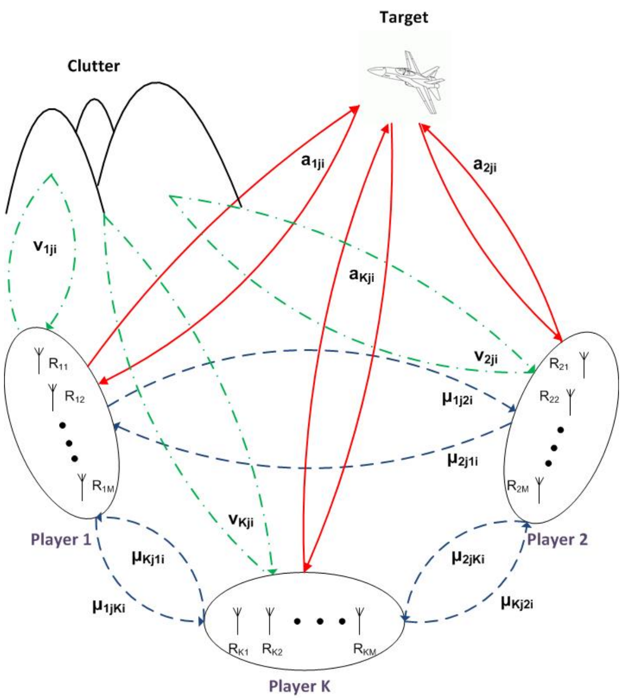


Figure 4.1: A distributed MIMO radar network with three radars and their corresponding channel gains.

We consider a decentralized, multistatic radar network that consists of K separate clusters $C = \{C_1, \ldots, C_K\}$ each consisting of M radars, i.e. $C_k = \{R_{k1}, \ldots, R_{kM}\}$ for all $k = 1, \ldots, K$. A target is considered in the far-field of the radars, as shown in Fig.1. The primary aim for each radar in every cluster is to attain a predefined detection criterion, consuming the minimum possible transmission power. In the considered framework of noncooperative games, each cluster performs the power minimization autonomously, since there is communication and coordination among the radars within the same cluster, whereas there is no coordination between different clusters in the network. Consequently, each cluster possesses full information regarding the channel gains of its respective radars, whereas it has no knowledge of the inter-cluster cross channel gains. Nevertheless, this scenario is not competitive and the radars should avoid causing interference to the rest of the clusters of the network intentionally, since they belong to the same organization.

In order to identify the desired target, each one of the M radars in the k^{th} cluster transmits the respective element of the independent, predesigned waveform vector $\boldsymbol{\psi}_k(t) = [\psi_{k1}(t), \ldots, \psi_{kM}(t)]^T$ of size $M \times 1$, which satisfies the orthogonality condition $\int_{T_0} \boldsymbol{\psi}_k(t)\boldsymbol{\psi}_k^H(t)dt = \mathbf{I}_M$, where T_0 is the radar pulse width and t refers to the time index within the radar pulse. Hence, we exploit the waveform diversity of the MIMO architecture, since the waveforms corresponding to different radars of the same cluster are orthogonal, i.e., $\int \boldsymbol{\psi}_{ki}(t)\boldsymbol{\psi}_{kj}(t)dt = 0$, where $i \neq j$. On the other hand, waveforms emitted from radars belonging to different clusters may not be orthogonal and thus could induce considerable inter-cluster interference. We assume that each cluster decides the presence of a target, by applying binary hypothesis testing on the received signal based on the generalized likelihood ratio test (GLRT) as proposed in [35]. The sampled pulses of the received signal for radar i in cluster k R_{ki}, under the two hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ of target being absent and target being present respectively, is written as the complex $N \times 1$ vector as:

$$\mathcal{H}_0 : \mathbf{x}_{ki} = \mathbf{i}_{ki} + \mathbf{d}_{ki} \quad \text{(no target present)} \tag{4.1}$$

$$\mathcal{H}_1 : \mathbf{x}_{ki} = \sum_{j=1}^{M} \alpha_{kji}\mathbf{s}_{kj} + \mathbf{i}_{ki} + \mathbf{d}_{ki} \quad \text{(target present)} \tag{4.2}$$

where $\mathbf{s}_{kj} = \sqrt{p_{kj}}\boldsymbol{\psi}_{kj}(n) \odot \mathbf{a}_{kj}$ denotes the received signal at radar R_{kj}, which cooperates the Doppler shift introduced by the target. The parameter α_{kji} denotes the channel gain, including including the radar cross section (RCS) of the target from radar R_{kj} to radar R_{ki}, $\mathbf{a}_{kj} = [1, e^{j2\pi f_{D,k,j}}, \ldots, e^{j2\pi(N-1)f_{D,k,j}}]^T$ is the Doppler steering vector of radar R_{kj} regarding the desired target, $f_{D,k,j}$ denotes the normalized Doppler shift at radar R_{kj} originating from the target's movement, N is the number of signal return samples that the radars receive at each time step of duration T_0 and p_{kj} stands for the transmission power of radar R_{kj}. The inter-cluster interference experienced by radar R_{ki} due to the emissions from radars belonging to all other clusters is denoted as

$$\mathbf{i}_{ki} = \sum_{\substack{\ell=1 \\ \ell\neq k}}^{K} \sum_{j=1}^{M} \beta_{\ell jki} \sqrt{p_{\ell j}} \boldsymbol{\psi}_{lj}(n) \odot \mathbf{1}_N$$

where $\beta_{\ell jki}$ describes the cross-channel gain from radar $R_{\ell j}$ to radar R_{ki}, which depends on the respective antennae characteristics and the distance between the radars. Since all the radars are considered stationary in the proposed model, there is no relative Doppler frequency regarding the cross-channel interference, hence the Doppler based steering vector associated with the waveform vector transmitted from the radars in clusters other than k is shown as an $N \times 1$ vector of all ones $\mathbf{1}_N$. The last components of the received signal in (4.2) are the clutter introduced by the waveforms transmitted by the radars in cluster k and the noise denoted by the parameter $\mathbf{d}_{ki} = \sum_{j=1}^{M} c_{kji} \sqrt{p_{kj}} \boldsymbol{\psi}_{kj}(n) \odot \mathbf{a}^c_{kj} + \mathbf{n})$, where c_{kji} includes the signal propagation loss at the direction of the clutter and the geometrical characteristics of the clutter, in other words its RCS, $\mathbf{a}^c_{kj} = [1, e^{j2\pi f^c_{D,k,j}}, \ldots, e^{j2\pi(N-1) f^c_{D,k,j}}]^T$ denotes the Doppler steering vector of radar R_{kj} associated with the clutter and $f^c_{D,k,j}$ denotes the normalized Doppler shift at radar R_{kj} originating from the clutter's movement and $\mathbf{n}$ is white Gaussian noise (WGN) with variance σ^2_n. Furthermore, in our work, we have included the clutter disturbance caused by the transmission of waveforms by all other radars than those in the k^{th} cluster in the noise term n.

The received signal $\mathbf{x}_{ki}$ is subsequently sent to a bank of matched filters, matching each of the orthogonal waveforms $\boldsymbol{\psi}_{ki}(n)$, $i = 1, \ldots, M$ and the corresponding energy at the output of the matched filter is accumulated. Hence, the expected energy of the signal originating from the target direction for radar R_{ki} can be given by:

$$\|y_{exp(ki)}\|^2 = \sum_{j=1}^{M} h_{kji} p_{kj} \tag{4.3}$$

where $\alpha_{kji} \sim \mathcal{CN}(0, h_{kji})$, hence h_{kji} denotes the variance of the desired channel gain, which includes the information on the target's RCS. As observed from Fig.1 and equation (4.2) the detection of a target is deteriorated by direct inter-cluster interference, in addition to the clutter effect and the noise power. Therefore, the expected power of the accumulated interfering and noise for radar R_{ki} can be modeled as:

$$\|y_{interf(ki)}\|^2 = \sum_{j=1}^{M} \nu_{kji} p_{kj} + \sum_{\substack{\ell=1 \\ \ell \neq k}}^{K} \sum_{j=1}^{M} \mu_{\ell jki} p_{\ell j} + \sigma_n^2 \tag{4.4}$$

where $c_{kji} \sim \mathcal{CN}(0, \nu_{kji})$ and ν_{kji} defines the accumulated clutter channel gains, embedding information on the clutter's RCS, $\beta_{\ell jki} \sim \mathcal{CN}(0, \mu_{\ell jki} \varrho_{\ell jki})$ and $\mu_{\ell jki} \varrho_{\ell jki}$ describes the accumulated cross-channel gain, incorporating a non-zero correlation factor $\varrho_{\ell jki}$ between the waveform vector emitted from radar $R_{\ell j}$ and the matched filtering waveform $\boldsymbol{\psi}_{ki}(n)$ and σ_n^2 denotes the noise power.

Based on the above definitions, the expected SINR for the i^{th} radar in the k^{th} cluster is written as

$$\text{SINR}_{ki} = \frac{\sum_{j=1}^{M} h_{kji} p_{kj}}{\sum_{j=1}^{M} \nu_{kji} p_{kj} + \sum_{\substack{\ell=1 \\ \ell \neq k}}^{K} \sum_{j=1}^{M} \mu_{\ell jki} p_{\ell j} + \sigma_n^2}. \tag{4.5}$$

In order to design an efficient detector for the hypothesis testing we utilize the GLRT. Assuming clutter and interference contribution is considered as Gaussian noise, the probability density functions of $\mathbf{x}_{ki}$ under hypothesis $\mathcal{H}_0$ and $\mathcal{H}_1$ respectively, can be given by:

$$f_{\mathcal{H}_0}(\mathbf{x}_{ki}; \boldsymbol{\sigma}_{\mathcal{H}_0}^2) = \frac{1}{(2\pi)^{N/2} \boldsymbol{\sigma}_{\mathcal{H}_0}^N} e^{-\frac{\|\mathbf{x}_{ki}\|}{2\boldsymbol{\sigma}_{\mathcal{H}_0}^2}} \tag{4.6}$$

$$f_{\mathcal{H}_1}(\mathbf{x}_{ki}; \boldsymbol{a}_{ki}, \boldsymbol{\sigma}_{\mathcal{H}_1}^2) = \frac{1}{(2\pi)^{N/2} \boldsymbol{\sigma}_{\mathcal{H}_1}^N} e^{-\frac{\|\mathbf{x}_{ki} - \sum_{j=1}^{M} \alpha_{kji} \mathbf{s}_{kj}\|^2}{2\boldsymbol{\sigma}_{\mathcal{H}_1}^2}} \tag{4.7}$$

where $\boldsymbol{a}_{ki} = [\alpha_{k1i}, \ldots, \alpha_{kMi}]^T$. The maximum likelihood (ML) estimate of noise variance under the hypothesis $\mathcal{H}_0$, when there is no target present, can be obtained by $\hat{\boldsymbol{\sigma}}_{\mathcal{H}_0}^2 = \|\mathbf{x}_{ki}\|^2/N$. Subsequently, by keeping $\boldsymbol{\sigma}_{\mathcal{H}_1}^2$ fixed, the ML estimate for α_{kji} $\forall i = 1, \ldots, M$ is given by $\hat{\alpha}_{kji} = \mathbf{s}_{kj}^H \mathbf{x}_{ki}/N$. After obtaining the ML estimate for α_{kji}, we substitute it in (4.7) and maximize with respect to $\boldsymbol{\sigma}_{\mathcal{H}_1}^2$ to derive the maximum likelihood estimate for $\boldsymbol{\sigma}_{\mathcal{H}_1}^2$ as:

$$\hat{\boldsymbol{\sigma}}_{\mathcal{H}_1}^2 = \frac{\|\mathbf{x}_{ki} - \sum_{j=1}^{M} \alpha_{kji} \mathbf{s}_{kj}\|^2}{N}$$

We assume that $\lambda_{ki} \in [0, 1]$ denotes the detection threshold for the hypothesis testing for each radar $i = 1, \ldots, M$ in cluster k and thus the GLRT can be reformulated as:

$$\frac{f_{\mathcal{H}_1}}{f_{\mathcal{H}_0}} = \frac{\sum_{j=1}^{M} |\alpha_{kji}\mathbf{s}_{kj}|^2}{\|\mathbf{x}_{ki}\|^2 N} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda_{ki} \tag{4.8}$$

The performance and efficiency of the hypothesis testing is evaluated by utilizing the probabilities of miss-detection P_{md} and false alarm P_{fa} for each radar. Following [36, 115, 116], the calculation of those probabilities can be derived from the following equations:

$$P_{fa}(\lambda_{ki}) = (1 - \lambda_{ki})^{N-1}$$

$$P_{md}(\text{SINR}_{ki}, \lambda_{ki}) = 1 - \left(1 + \frac{\lambda_{ki}}{1 - \lambda_{ki}} \frac{1}{1 + N\text{SINR}_{ki}}\right)^{1-N}$$

By setting an upper bound ε_{ki} on the sum of P_{md} and P_{fa}, we can decide the acceptable performance of the detection test:

$$P_{fa}(\lambda_{ki}) + P_{md}(\text{SINR}_{ki}, \lambda_{ki}) \leq \varepsilon_{ki} \tag{4.9}$$

As demonstrated in the analysis of [36], the optimum detection threshold can be obtained when (4.9) is satisfied with equality. The optimum SINR for each radar in cluster k is denoted by γ_{ki}^* and can be determined by exploiting the optimum detection threshold as shown below:

$$\gamma_{ki}^* = \min\{\text{SINR}_{ki} \mid \exists \lambda_{ki} \in [0, 1] \quad \text{s.t.} \; P_{md}(\lambda_{ki}) + P_{fa}(\text{SINR}_{ki}, \lambda_{ki}) \leq \varepsilon_{ki}\}. \tag{4.10}$$

The aforementioned technique allows us to obtain the optimum SINR for a desired P_{fa}, by appropriately deciding the design parameter ε_{ki} and the detection threshold $\lambda_{ki} \in [0, 1]$. The optimum SINR γ_{ki}^* for each radar in the system will be utilized within the game theoretic formulation for the proposed model, as presented in the next section.

4.1.3 Game Theoretic Formulation

As described in the previous sections, the main goal for each cluster is to decide the optimal power allocation to its respective radars, while attaining a specific detection criterion. It is observed from the SINR equation (4.5) that although increased power allocation at a specific cluster improves the detection performance, it induces higher interference to the environment and consequently to the remaining radars of the network. Therefore, by exploiting noncooperative game theoretic techniques, this interaction is modeled as a generalized Nash game. The set of clusters $C = \{C_1, \ldots, C_K\}$ are considered to be the players of the game. The action set of the k^{th} player is $\mathcal{P}_k = \mathcal{P}_{k1} \times \ldots \times \mathcal{P}_{kM}$ with

$$\mathcal{P}_{ki} = \{p_{ki} \in \mathbb{R}^+ \mid p_{ki} \in [0, \overline{p}_{ki}]\}, \quad \forall i \in \{1, \ldots, M\}$$

where $\overline{p}_{ki}$ denote the maximum available power for radar R_{ki}. The acceptable strategy set of the GNG depends both on the action of the k^{th} player $\mathcal{P}_k$ and the actions of all other players $\mathcal{P}_{-k}$ and is defined as

$$S_k(\mathbf{p}_{-k}) = \{\mathbf{p}_k \in \mathcal{P}_k \mid \mathrm{SINR}_{ki} \geq \gamma_k^*, \ \forall i = 1, \ldots, M\} \tag{4.11}$$

where $\mathbf{p}_{-k}$ denotes the power allocation adopted by all other players except player k. Let us also define $\mathbf{p}_k = [p_{k1}, \ldots, p_{kM}]^T$ as the power allocation vector of cluster k. It is evident from equation (4.5), that the SINR is a function of the power allocation of all K players. Thus, the interdependency of the admissible strategies is clearly stated through the constraints in (4.11). The game model is completed by defining the utility function as $u_k(\mathbf{p}_{-k}, \mathbf{p}_k) = \sum_{i=1}^{M} p_{ki}$, which represents the total transmission power of cluster k. At this point, the game may be summarized as:

$$\mathcal{G} =< C, (\mathcal{P}_k)_{k \in \{1,\ldots,K\}}, (S_k)_{k \in \{1,\ldots,K\}}, (u_k)_{k \in \{1,\ldots,K\}} >$$

In this GNG, player k greedily minimizes its transmission power, while all radars belonging to cluster k attain the target SINR, given the power allocation strategies of all the other players. Therefore, the best action for the k^{th} player is given by the following set, denoted by BR_k:

$$BR_k(\mathbf{p}_{-k}) = \{\mathbf{p}_k^* \in \mathcal{P}_k \mid$$

$$u_k(\mathbf{p}_{-k}, \mathbf{p}_k^*) \leq u_k(\mathbf{p}_{-k}, \mathbf{p}_k), \forall \mathbf{p}_k \in S_k(\mathbf{p}_{-k})\}$$

Recalling the action set of player k, the above set can be determined by solving the following convex optimization problem:

$$\min_{\mathbf{p}_k \in \mathcal{P}_k} u_k(\mathbf{p}_{-k}, \mathbf{p}_k) \tag{4.12}$$

$$\text{s.t.} \ \ \mathrm{SINR}_{ki} \geq \gamma_{ki}^*, \forall i = 1, \ldots, M$$

The most crucial part of a game theoretic analysis is to investigate whether the game $\mathcal{G}$ converges to a stable solution, where no player can benefit by unilaterally deviating its power allocation strategy. Such a solution defines the Nash Equilibrium and for the game $\mathcal{G}$ describes the strategy profile $(\mathbf{p}^*_{-k}, \mathbf{p}^*_k)$ when:

$$u_k(\mathbf{p}^*_{-k}, \mathbf{p}^*_k) \leq u_k(\mathbf{p}^*_{-k}, \mathbf{p}_k), \quad \forall \mathbf{p}_k \in S_k(\mathbf{p}^*_{-k}), \forall k \in C.$$

It is evident from the constraints of (4.12) and the definition of SINR (4.5), that each radar in cluster k requires the knowledge of the inter-cluster interference plus noise term, denoted as $r_{-ki} = \sum_{\substack{\ell=1\\ \ell\neq k}}^{K} \sum_{j=1}^{M} \mu_{\ell jki} p_{\ell j} + \sigma_n^2$, in order to decide its optimal power allocation. However, sinc no communication between the clusters is assumed, it is impossible to obtain the required information and thus this deficiency is tackled by using the estimate of the instantaneous SINR.

4.1.4 SINR Estimation

Each cluster, after receiving N signal return samples at each time step, decides its optimal power allocation by solving the convex optimization problem (4.12), that requires the value of the instantaneous SINR $\hat{\gamma}_{ki}$. However, direct calculation of $\hat{\gamma}_{ki}$ requires the knowledge of the inter-cluster cross-channel gains and hence, the transmission power from the radars in the remaining clusters of the system. Since we assume no coordination among different clusters, this information cannot be obtained. According to the model by [36], we can obtain an estimation of the SINR by taking the expected values of the numerator and the denominator of the SINR equation (4.5) with respect to the transmission power. Recalling that $h_{kji} p_{kj}$ is the variance of the parameter $\alpha_{kji}\sqrt{p_{kj}}$, similarly $\sum_{j=1}^{M} \nu_{kji} p_{kj} + \sigma_n^2$ denotes the variance of parameter d_{ki} and $\mu_{lkji} p_{lj}$ is the variance of β_{lkji} and considering only the resulting dominant terms, we arrive at the following equation for radar R_{ki}:

$$\hat{\gamma}_{ki} = \frac{\dfrac{\sum\limits_{j=1}^{M} |\mathbf{s}_{kj}^H \mathbf{x}_{ki}|^2}{N} - \dfrac{||\mathbf{x}_{ki}||^2}{N}}{||\mathbf{x}_{ki}||^2 - \dfrac{\sum\limits_{j=1}^{M} |\mathbf{s}_{kj}^H \mathbf{x}_{ki}|^2}{N}} \tag{4.13}$$

By substituting the received signal given by (4.2) in the numerator of (4.13) and expanding, we conclude that the dominant terms of the numerator of

(4.13) are

$$\frac{\sum_{j=1}^{M} |\mathbf{s}_{kj}^H \mathbf{x}_{ki}|^2}{N} \approx \sum_{j=1}^{M} |\alpha_{kji}|^2 N$$

$$\frac{||\mathbf{x}_{ki}||^2}{N} \approx \sum_{j=1}^{M} |\alpha_{kji}|^2$$

which yields an approximation of the numerator of the SINR in (4.5). Similarly, by substituting the received signal in the denominator of (4.13) and taking into account the low correlation among the transmitted signal and the interference terms, the dominant terms of the result are the terms in the denominator of the SINR equation (4.5).

4.1.5 Existence and Uniqueness of the Nash Equilibrium

Existence

The existence of a generalized Nash equilibrium (GNE) follows from the result by [117] on abstract economies. According to this result, a GNE exists if the following hold: for all players $k = 1, \ldots, K$ the set P_k is compact and convex, the utility function $u_k(p_{-k}, p_k)$ is continuous on $\mathcal{P}$ and quasi-convex in p_k. For every p_{-k} the set-valued function S_k is continuous with closed graph and for every p_{-k} the set $S_k(p_{-k})$ is non-empty and convex. For the studied problem, these requirements can be straightforwardly established using analytic notions, hence there exists a GNE for the game.

Uniqueness of the Solution through Duality Analysis

The main contribution of this paper lies in the analysis of the Nash equilibrium for the strategic noncooperative game $\mathcal{G}$ through the Lagrangian duality. According to the result in [86], the primary objective is to prove that the best response of each cluster is a standard function, which is a sufficient condition for the uniqueness of the solution. The definition of a standard function is given below [86]:

A function $\mathbf{F}(\mathbf{x})$ is standard if for all $\mathbf{x} \geq 0$, the following properties hold:

- Positivity: $\mathbf{F}(\mathbf{x}) > 0$
- Monotonicity: If $\mathbf{x} \geq \mathbf{x}'$, then $\mathbf{F}(\mathbf{x}) \geq \mathbf{F}(\mathbf{x}')$
- Scalability: $\forall a \geq 1, a\mathbf{F}(\mathbf{x}) \geq \mathbf{F}(a\mathbf{x})$

In order to prove that the best response function of each cluster is a standard function, the optimization problem of the k^{th} cluster will be considered as defined in (5). By rearranging the constraints in matrix form and explicitly imposing constraints for non negative radar power, the following minimization problem for the k^{th} cluster arises:

$$\min_{\mathbf{p}_k \in \mathcal{P}_k} \sum_{i=1}^{M} p_{ki} \tag{4.14}$$

$$s.t. \qquad \mathbf{G}\mathbf{p}_k + \mathbf{r}_{-k} \leq 0$$

$$-\mathbf{p}_k \leq 0$$

where $\mathbf{r}_{-k} = [r_{-k1}, \ldots, r_{-kM}]^T$ is the inter-cluster interference plus noise vector, which can be written as $\mathbf{r}_{-k} = \sum_{\ell \neq k}^{K} \mathbf{M}_\ell \mathbf{p}_\ell + \mathbf{1}\sigma_n^2$, where the cross-channel matrix $\mathbf{M}_i$ is given by:

$$\mathbf{M}_\ell = \begin{bmatrix} \mu_{\ell 1 k 1} & \ldots & \mu_{\ell M k 1} \\ \vdots & \ddots & \vdots \\ \mu_{\ell 1 k M} & \ldots & \mu_{\ell M k M} \end{bmatrix}$$

Let us also define the $M \times M$ matrix $\mathbf{G}$ as:

$$\mathbf{G} = - \begin{bmatrix} \frac{h_{k11}}{\hat{\gamma}_k} - \nu_{k11} & \ldots & \frac{h_{kM1}}{\hat{\gamma}_k} - \nu_{kM1} \\ \vdots & \ddots & \vdots \\ \frac{h_{k1M}}{\hat{\gamma}_k} - \nu_{k1M} & \ldots & \frac{h_{kMM}}{\hat{\gamma}_k} - \nu_{kMM} \end{bmatrix}$$

For the multi-static scenario considered in this paper, it is possible that not all radars in a cluster illuminate signals. There is a possibility that all radars in a cluster could satisfy the target SINR only using the signals illuminated by a subset of radars. Therefore, in order to optimize the transmission power certain radars in a cluster may opt to be silent but could use other radars' signal as signal of opportunity for target detection. When all radars are active, it is straightforward to establish uniqueness of GNE as will be shown in the forthcoming analysis, however, when at least one radar in a cluster is inactive, the establishment of Nash equilibrium requires further analysis in terms of the Karush-Kuhn-Tucket (KKT) conditions. Hence, the Lagrangian $\mathcal{L}$ associated with the problem (4.14) is defined as:

$$\mathcal{L}(\mathbf{p}_k, \boldsymbol{\lambda}_a, \boldsymbol{\lambda}_b) = \sum_{i=1}^{M} p_{ki} + \lambda_1 (G_{11} p_{k1} + \ldots + G_{1M} p_{kM} + r_{-k1}) +$$

$$\ldots$$

$$+\lambda_M (G_{M1} p_{k1} + \ldots + G_{MM} p_{kM} + r_{-kM})$$

$$-m_1 p_{k1} - m_2 p_{k2} - \ldots - m_M p_{kM}$$
$$= \boldsymbol{\lambda}_a^T \mathbf{r}_{-k} + (\mathbf{1} + \mathbf{G}\boldsymbol{\lambda}_a - \boldsymbol{\lambda}_b)^T \mathbf{p}_1 \quad (4.15)$$

where $\boldsymbol{\lambda}_a = [\lambda_1, \ldots, \lambda_M]^T$ and $\boldsymbol{\lambda}_b = [m_1, \ldots, m_M]^T$ are the Lagrange multipliers associated with the inequality constraints of (4.14). Let $(\mathbf{p}_k^*, \boldsymbol{\lambda}_a^*, \boldsymbol{\lambda}_b^*)$ be the primal and dual optimal points of (4.14). Then, the KKT conditions on convexity must be satisfied [51]. In particular one has:

$$\left.\begin{aligned} \frac{\partial \mathcal{L}}{\partial p_{k1}} &= 1 + \lambda_1 G_{11} + \ldots + \lambda_M G_{M1} - m_1 = 0 \\ &\ldots \\ \frac{\partial \mathcal{L}}{\partial p_{kM}} &= 1 + \lambda_1 G_{1M} + \ldots + \lambda_M G_{MM} - m_M = 0 \end{aligned}\right\} \quad (4.16)$$

$$\left.\begin{aligned} \lambda_1(G_{11}p_{k1} + \ldots + G_{1M}p_{kM} + r_{-k1}) &= 0 \\ &\ldots \\ \lambda_M(G_{M1}p_{k1} + \ldots + G_{MM}p_{kM} + r_{-kM}) &= 0 \end{aligned}\right\} \quad (4.17)$$

$$m_1 p_{k1} = 0, \ldots, m_M p_{kM} = 0 \quad (4.18)$$

In order to investigate all the potential outcomes of the game $\mathcal{G}$, all possible cases are considered with respect to the values of the Lagrange multipliers $\boldsymbol{\lambda}_a$, which correspond to the SINR constraints. In particular, firstly the case when all radars exactly achieve the SINR target and hence all the constraints in (4.14) are satisfied with equality is studied. In this case, the uniqueness is proved straightforwardly by exploiting the KKT conditions and the definition of the standard function. Secondly, the case when all the Lagrangian multipliers are zero is considered and it is shown that this case is infeasible. Finally, the case when at least one radar exactly achieves the SINR target and the remaining radars in the same cluster perform better than the SINR target is studied. When this happens, the analysis of the Lagrange dual problem and the derivation of the Lagrangian function and the KKT conditions for the dual optimization problem are exploited to conclude the proof of the GNE of the considered GNG. The mathematical analysis of the proof of the uniqueness of the solution for every possible case is demonstrated below.

Case 1: $\lambda_i \neq 0, \forall i = 1, \ldots, M$. In this case, the set of equalities (4.17) from KKT conditions implies that all the SINR inequality constraints are inactive and must be satisfied with equality. Hence, from (4.17):

$$G_{11}p_{k1} + \ldots + G_{1M}p_{kM} = -r_{-k1}$$

$$\dots$$

$$G_{M1}p_{k1} + \ldots + G_{MM}p_{kM} = -r_{-kM}$$

Reformulating the aforementioned equations in matrix form, one has $\mathbf{G}\mathbf{p}_k^* = -\mathbf{r}_{-k}$. Following Proposition 1 in [118] and Claim 2 in [32]. Assuming that the optimization problem (7) is always feasible $\forall \mathbf{r}_k > 0$, hence $\mathbf{G}$ must be invertible so $\mathbf{p}_k^* = -\mathbf{G}^{-1}\mathbf{r}_{-k}$. Since the elements of matrix $\mathbf{G}$ are non-zero by definition, this case corresponds to the scenario when all radars are active and actually transmit signal. As a result, by replacing the interference vector, the best response function can be stated as:

$$BR_k(\mathbf{p}_{-k}) = \mathbf{p}_k^* = -\mathbf{G}^{-1}\left(\sum_{\ell \neq k}^{K} \mathbf{M}_\ell \mathbf{p}_\ell + \mathbf{1}\sigma_n^2\right) \tag{4.19}$$

Lemma 1: The best response function (4.19) is a standard function.

Proof. Following [32], the best response strategy (4.19) satisfies the following necessary properties for all $\mathbf{p} \geq 0$:

a) Positivity: The best response of the k^{th} cluster $\mathbf{p}_k^*$ is always positive, as $\mathbf{r}_{-k} = \sum_{\ell \neq k}^{K} \mathbf{M}_\ell \mathbf{p}_\ell + \mathbf{1}\sigma_n^2 > 0$ and $\mathbf{p}_k^* = -\mathbf{G}^{-1}\mathbf{r}_{-k}$ is feasible $\forall \mathbf{r}_k > 0$.

b) Monotonicity: Let $\mathbf{p}, \mathbf{p}' \in \mathcal{P}_k$ with $\mathbf{p} \geq \mathbf{p}'$, then:

$$BR_k(\mathbf{p}) - BR_k(\mathbf{p}') = -\mathbf{G}^{-1}\left(\sum_{\ell \neq k}^{K} \mathbf{M}_\ell (\mathbf{p}_\ell - \mathbf{p}'_\ell)\right) \geq 0$$

c) Scalability: For all $a > 1$, $aBR_k(\mathbf{p}) > BR_k(a\mathbf{p})$. Indeed:

$$aBR_k(\mathbf{p}) - BR_k(a\mathbf{p}) = -(a-1)\mathbf{G}^{-1}\mathbf{1}_M\sigma_n^2 > 0.$$

□

This concludes the proof on the uniqueness for Case 1, where all the SINR constraints are satisfied with equality.

Case 2: The Lagrangian multipliers corresponding to the SINR constraints are zero, i.e. $\lambda_1 = \lambda_2 = \ldots = \lambda_M = 0$. It is proved that this case does not exist, as follows.

Assuming $\lambda_1 = \lambda_2 = \ldots = \lambda_M = 0$, then from (4.16) one has that $m_1 = \ldots = m_M = 1$. By substituting in (4.18), $p_{k1} = \ldots = p_{kM} = 0$ is obtained which indicates that all the radars in cluster k are inactive. Consequently, the constraints of the optimization problem (4.14) can be restated as:

$$r_{-k1}, \ldots, r_{-kM} \leq 0$$

which is a contradiction, since the inter-cluster interference plus noise terms are always positive, i.e. $r_{-k1}, \ldots, r_{-kM} > 0$. As a result, at least one radar in the cluster must be active in order for the optimization problem (4.14) to be feasible.

Case 3: Here, the case when at least one of the radars in the k^{th} cluster achieves the SINR target with equality is investigated and the remaining radars with inequality. Without loss of generality, suppose that the first n radars satisfy the SINR constraint with equality, hence from (4.17) $\lambda_1, \ldots, \lambda_n \neq 0$ and $\lambda_{n+1} = \ldots = \lambda_M = 0$. The Lagrangian function in this case becomes:

$$\begin{aligned}\tilde{\mathcal{L}}(\mathbf{p}_k, \boldsymbol{\lambda}_a, \boldsymbol{\lambda}_b) = &\sum_{i=1}^{M} p_{ki} + \lambda_1(G_{11}p_{k1} + \ldots + G_{1M}p_{kM} + r_{-k1}) \\ &\ldots \\ &+\lambda_n(G_{n1}p_{k1} + \ldots + G_{nM}p_{kM} + r_{-kn}) \\ &-m_1p_{k1} - m_2p_{k2} - \ldots - m_Mp_{kM} \\ &= \tilde{\boldsymbol{\lambda}}_a^T \tilde{\mathbf{r}}_{-k} + (\mathbf{1} + \tilde{\mathbf{G}}\tilde{\boldsymbol{\lambda}}_a - \boldsymbol{\lambda}_b)^T \mathbf{p}_k \end{aligned} \tag{4.20}$$

where $\tilde{\boldsymbol{\lambda}}_a = [\lambda_1, \ldots, \lambda_n, 0, \ldots, 0]^T, \tilde{\mathbf{r}}_{-k} = [r_{-k1}, \ldots, r_{-kn}, 0, \ldots, 0]^T$. In this case, the matrix $\tilde{\mathbf{G}}$ of size $M \times M$ is defined as:

$$\tilde{\mathbf{G}} = \begin{bmatrix} G_{11} & \ldots & G_{1M} \\ \vdots & \ddots & \vdots \\ G_{n1} & \ldots & G_{nM} \\ 0 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & 0 \end{bmatrix}$$

In order to investigate the interdependence among the number of radars that satisfy the detection criterion with equality and the number of the radars in cluster k that are active and actually illuminate waveforms, a critical analysis on the Lagrange multipliers $\boldsymbol{\lambda}_b$ is essential. Hence, the Lagrange dual function g is obtained as:

$$g(\tilde{\boldsymbol{\lambda}}_a, \boldsymbol{\lambda}_b) = \inf_{\mathbf{p}_k} \tilde{\mathcal{L}}(\mathbf{p}_k, \boldsymbol{\lambda}_a, \boldsymbol{\lambda}_b) = \tag{4.21}$$

$$= \tilde{\boldsymbol{\lambda}}_a^T \tilde{\mathbf{r}}_{-k} + \inf_{\mathbf{p}_k} (\mathbf{1} + \tilde{\mathbf{G}} \tilde{\boldsymbol{\lambda}}_a - \boldsymbol{\lambda}_b)^T \mathbf{p}_k$$

It is straightforward from (4.20) that the Lagrangian is an affine function of $\mathbf{p}_k$ and is bounded below only when $\mathbf{1} + \tilde{\mathbf{G}} \tilde{\boldsymbol{\lambda}}_a - \boldsymbol{\lambda}_b = 0$. Thus, it follows

$$g(\tilde{\boldsymbol{\lambda}}_a, \boldsymbol{\lambda}_b) = \begin{cases} \tilde{\boldsymbol{\lambda}}_a^T \tilde{\mathbf{r}}_{-k}, & \text{if} \quad \mathbf{1} + \tilde{\mathbf{G}} \tilde{\boldsymbol{\lambda}}_a - \boldsymbol{\lambda}_b = 0 \\ -\infty, & \text{otherwise} \end{cases} \tag{4.22}$$

Next, the Lagrange dual problem is formulated as:

$$\begin{aligned} \max \quad & g(\tilde{\boldsymbol{\lambda}}_a, \boldsymbol{\lambda}_b) \\ s.t. \quad & \tilde{\boldsymbol{\lambda}}_a \geq 0 \\ & \boldsymbol{\lambda}_b \geq 0 \end{aligned}$$

By excluding the case when g is infinite and changing the sign of the objective function and by exploiting the fact that from (4.22), $\boldsymbol{\lambda}_b = \mathbf{1} + \tilde{\mathbf{G}} \tilde{\boldsymbol{\lambda}}_a$, the aforementioned maximization problem can be rewritten as the following minimization problem:

$$\begin{aligned} \min \quad & -\tilde{\boldsymbol{\lambda}}_a^T \tilde{\mathbf{r}}_{-k} \\ s.t. \quad & \mathbf{1} + \tilde{\mathbf{G}} \tilde{\boldsymbol{\lambda}}_a \geq 0 \\ & \tilde{\boldsymbol{\lambda}}_a \geq 0 \end{aligned} \tag{4.23}$$

Subsequently, the Lagrangian of the dual optimization problem (4.23) is defined as:

$$\begin{aligned} \mathcal{L}_d(\tilde{\boldsymbol{\lambda}}_a, \mathbf{c}_a, \mathbf{c}_b) = & -\tilde{\boldsymbol{\lambda}}_a^T \tilde{\mathbf{r}}_{-k} - c_1(1 + G_{11}\lambda_1 + \ldots + G_{1n}\lambda_n) \\ & \ldots \\ & -c_n(1 + G_{n1}\lambda_1 + \ldots + G_{nn}\lambda_n + r_{-kn}) - c_{n+1} - \ldots - c_M \\ & -\delta_1\lambda_1 - \delta_2\lambda_2 - \ldots - \delta_n\lambda_n \end{aligned}$$

where $\mathbf{c}_a = [c_1, \ldots, c_M]^T$ and $\mathbf{c}_b = [\delta_1, \ldots, \delta_n]^T$. Let $(\boldsymbol{\lambda}_a^*, \mathbf{c}_a^*, \mathbf{c}_b^*)$ be the optimal points of (4.23). Then the KKT optimality conditions for the optimization problem (4.23) are satisfied and can be written as:

$$\left.\begin{aligned} \frac{\partial \mathcal{L}_d}{\partial \lambda_1} &= -r_{-11} + c_1 G_{11} + \ \ldots + c_n G_{n1} - \delta_1 = 0 \\ &\ldots \\ \frac{\partial \mathcal{L}_d}{\partial \lambda_n} &= -r_{-1n} + c_1 G_{1M} + \ldots + c_n G_{nn} - \delta_n = 0 \end{aligned}\right\} \tag{4.24}$$

$$\left.\begin{aligned} c_1(1 + G_{11}\lambda_1 + \ldots + G_{1n}\lambda_n) &= 0 \\ &\ldots \\ c_n(1 + G_{n1}\lambda_1 + \ldots + G_{nn}\lambda_n) &= 0 \\ c_{n+1} = 0, \ldots, c_M &= 0 \end{aligned}\right\} \tag{4.25}$$

$$\delta_1\lambda_1 = 0, \ldots, \delta_n\lambda_n = 0 \tag{4.26}$$

It is apparent from (4.25) that the Lagrangian multipliers vector $\mathbf{c}_a$ becomes $\mathbf{c}_a = [c_1, \ldots, c_n, 0, \ldots, 0]^T$. Next, all possible cases for the Lagrangian multipliers $\mathbf{c}_a$ are investigated:

Subcase I: $c_1 = \ldots = c_n = 0$. In this case, by following the KKT conditions (4.24), one has:

$$r_{-11} = -\delta_1, \ldots, r_{-1n} = -\delta_n$$

which is infeasible, as the inter-cluster interference plus noise terms are always strictly positive and $\mathbf{c}_b \geq 0$ element wise. As a result, this case is impossible.

Subcase II: $c_1, \ldots, c_n \neq 0$. When the Lagrange multipliers are strictly positive, one has from the KKT conditions of (4.25):

$$\mathbf{1}_n + \mathbf{G}_{\text{red}}\boldsymbol{\lambda}_a^{\text{red}} = 0. \tag{4.27}$$

where $\boldsymbol{\lambda}_a^{\text{red}} = [\lambda_1, \ldots, \lambda_n]$ and the reduced square $n \times n$ matrix $\mathbf{G}_{\text{red}}$ is defined as:

$$\mathbf{G}_{\text{red}} = \begin{bmatrix} G_{11} & \ldots & G_{1n} \\ \vdots & \ddots & \vdots \\ G_{n1} & \ldots & G_{nn} \end{bmatrix}$$

Subsequently, by revisiting (4.22) and omitting the case when the Lagrangian dual function is infinite, one has:

$$\boldsymbol{\lambda}_b = \mathbf{1}_M + \tilde{\mathbf{G}}\tilde{\boldsymbol{\lambda}}_a \tag{4.28}$$

By replacing (4.27) in (4.28), one has that the first n elements of the Lagrangian multipliers vector $\boldsymbol{\lambda}_b$ are equal to zero, i.e. $m_1 = \ldots = m_n = 0$ and the remaining $M - n$ elements are equal to one, i.e. $m_{n+1} = \ldots = m_M = 1$.

Subcase III: At least one of the Lagrange multipliers is equal to zero. Without loss of generality, it is assumed that $c_1 = 0$ and $c_2, \ldots, c_n \neq 0$. Hence the corresponding constraint to the Lagrange multiplier c_1 will become:

$$1 + G_{11}\lambda_1 + \ldots + G_{1n}\lambda_n \geq 0 \tag{4.29}$$

By inserting a non-negative value $\epsilon \geq 0$ into (4.29), one has:

$$G_{11}\lambda_1 + \ldots + G_{1n}\lambda_n = \epsilon - 1 \tag{4.30}$$

Since the elements of matrix $\mathbf{G}$ are straightforwardly negative by definition and the Lagrangian multipliers $\tilde{\boldsymbol{\lambda}}_a$ strictly positive from the assumption of **Case 3**, one has that $\epsilon - 1 < 0$ and $0 \leq \epsilon < 1$. Additionally, given the fact that from the objective function in (4.23) it is desired to acquire the maximum possible $\tilde{\boldsymbol{\lambda}}_a$, the value of ϵ becomes exactly zero (minimum possible value of $(\epsilon - 1)$) and thus equation (4.29) can be rewritten as:

$$1 + G_{11}\lambda_1 + \ldots + G_{1n}\lambda_n = 0$$

Thus, the same result for $\boldsymbol{\lambda}_b$ as in **Subcase II** is derived.

At this point, all the possible cases have been studied for the Lagrangian multipliers $\mathbf{c}_a$ and the following theorem can be explicitly derived, which constitutes one of the most important contributions of this paper.

Theorem 1: In the case when exactly n radars in cluster k achieve the detection criterion and satisfy the SINR constraints with equality, then at least $M - n$ radars in cluster k remain inactive and do not illuminate any signals.

Proof. By replacing the elements of $\boldsymbol{\lambda}_b$ from (4.28) in (4.18) one has that $p_{kn+1} = \ldots = p_{kM} = 0$. □

Corollary 1: The indices of the radars that are inactive in a cluster are determined only by the target and clutter channel characteristics of the corresponding cluster and the target SINR, and is independent of the actions of the other clusters and the corresponding cross-clutter channel interference.

Proof. It comes straightforwardly from the proof of Theorem 1 and equation (4.28) that the indices of the radars that remain silent in a cluster depend solely on the matrix $\tilde{\mathbf{G}}$, whose elements are functions of the channel gains and the target SINR of the corresponding cluster. □

The finding in Corollary 1 is very important for the Nash equilibrium analysis. When a subset of radars is inactive in a cluster, the action set in terms of the power allocation of a cluster is reduced to the power allocation of those radars that will eventually be active. In other words, determining indices of radars that are inactive is not part of the action set of the game as it will not be influenced by the action of other clusters. Hence the best response function for standard function analysis should include only the power allocation of active radars. Furthermore, the distributed nature of Corollary 1 strengthens the decentralized approach of the considered model.

By revisiting equations (4.17) from KKT conditions of the convex optimization problem (4.14), the SINR constraints corresponding to $\lambda_1, \dots, \lambda_n \neq 0$, are satisfied with equality. Thus, from (4.17) one has:

$$\left.\begin{aligned} G_{11}p_{k1} + \ldots + G_{1n}p_{kn} &= -r_{-k1} \\ &\ldots \\ G_{n1}p_{k1} + \ldots + G_{nn}p_{kn} &= -r_{-kn} \end{aligned}\right\} \tag{4.31}$$

Rewriting the above equations in matrix form, one has $\mathbf{G}_{\text{red}}\mathbf{q}_k^* = -\mathbf{r}_{-k}^{\text{red}}$, where $\mathbf{q}_k^* = [p_{11}, \dots, p_{1n}]^T$ and $\mathbf{r}_{-k}^{\text{red}} = [r_{-k1}, \dots, r_{-kn}]^T$. It is straightforward, that the solution of this set of n equations solely depends on matrix $\mathbf{G}_{\text{red}}$, which is determined from the channel gains regarding cluster k and from the target SINR. Hence, as the problem is always feasible (Claim 2 in [32]) $\forall \mathbf{r}_{-k}^{\text{red}} > 0$, $\mathbf{G}_{\text{red}}$ must be invertible and the best response function of cluster k in this case can be defined as:

$$BR_k(\mathbf{p}_{-k}) = \mathbf{q}_k^* = -\mathbf{G}_{\text{red}}^{-1}\mathbf{r}_{-k}^{\text{red}} \tag{4.32}$$

When $\mathbf{G}_{\text{red}}$ from equation (4.31) is full rank and when n radars in cluster k attain the SINR with equality, then exactly n radars in cluster k will be active and actually transmitting, whereas the remaining $(M - n)$ radars will remain inactive. However, it is possible theoretically to have certain channel gains, clutter gains and target SINR such that n radars could attain SINR with equality but with fewer than n radars being active. This happens when $\mathbf{G}_{\text{red}}$ is rank deficient or when any column of $\mathbf{G}_{\text{red}}$ is co-linear with $\mathbf{r}_{-k}^{\text{red}}$. In the latter case for example, all n radars may be achieving SINR with equality, however, only one radar will be transmitting. Although this may happen with almost zero probability, the following Lemma is still applicable to this scenario as well with a reduced size $\mathbf{G}_{\text{red}}$. Hence, without loss of generality, the case of full rank $\mathbf{G}_{\text{red}}$ is considered.

Lemma 2: The best response function (4.32) is a standard function.

Proof. Following **Lemma 1**, the best response strategy (4.32) satisfies the following necessary properties for all $\mathbf{p} \geq 0$:

a) Positivity: The best response of the k^{th} cluster $\mathbf{q}_k^*$ is always positive, as $\mathbf{r}_{-k}^{\text{red}} > 0$ and $\mathbf{q}_k^* = -\mathbf{G}_{\text{red}}^{-1}\mathbf{r}_{-k}^{\text{red}}$ is feasible $\forall \mathbf{r}_{-k}^{\text{red}} > 0$.

b) Monotonicity: for $\mathbf{p} \geq \mathbf{p}'$, one has from **Lemma 1** that $\mathbf{r}_{-k} \geq \mathbf{r}'_{-k}$ element wise and consequently $\mathbf{r}_{-k}^{\text{red}} \geq \mathbf{r}'^{\text{red}}_{-k}$. As a result:

$$BR_k(\mathbf{p}) - BR_k(\mathbf{p}') = -\mathbf{G}_{\text{red}}^{-1}\left(\mathbf{r}_{-k}^{\text{red}} - \mathbf{r}'^{\text{red}}_{-k}\right) \geq 0$$

c) Scalability: Using the same approach as **Lemma 1**, for all $a > 1$, it needs to be shown that $aBR_k(\mathbf{p}) > BR_k(\mathbf{ap})$. Indeed:

$$aBR_k(\mathbf{p}) - BR_k(a\mathbf{p}) = -(a-1)\mathbf{G}_{\text{red}}^{-1}\mathbf{1}_n\sigma_n^2 > 0.$$

□

Lemma 2 completes the uniqueness of the Nash equilibrium of the SNG $\mathcal{G}$, considering all possible cases. In the next section, simulation results are presented to support the mathematical analysis.

4.1.6 Simulation Results

In this section, simulation and numerical results are presented to illustrate the convergence of the algorithm to the unique solution and to demonstrate the distributed structure of the network. Initially, a network consisting of two clusters with six radars each is considered. In every time step, each radar receives $N = 32$ signal samples. The maximum number of iterations is set at $T = 30$ to investigate the convergence of the game. Before the initialization of the game, each radar determines the detection criterion by computing the optimum SINR using (4.10), which gives $\gamma_k^* = 2.1599$ for all radars when the design parameter is set to $\varepsilon_{ki} = 0.05$. For a predefined target channel gain h_{kji}, the values of the cross-channel and clutter gain are set as $\mu_{ljki} = {}^{h_{kji}}/_{20}$ and $\nu_{kji} = {}^{h_{kji}}/_{10}$. The channel gains for the simulations were chosen following a uniform distribution in the range $[0, 1]$. Finally, the Doppler shift is considered to be $f_{D,k,i} = 0.1$ for all $k = 1, \ldots, K$, $i = 1, \ldots, M$ and the noise power is set to $\sigma_n^2 = 0.01$.

In order to study the convergence of the GNG, Figures 4.2 and 4.3 demonstrate the power allocation update of all the radars in the network for two different initial power allocations of cluster 2. The channel gains remain the same in both simulations. First, it is obvious that the number of active radars in both clusters is the same in both examples, regardless the initial power allocation. Furthermore, both simulations converge to the same Nash equilibrium, as expected. The efficiency of the algorithm is evident, as the process converges to the optimal power allocation within 6 iterations. This result confirms Theorems 1 and 2, suggesting convergence to the unique Nash equilibrium regardless of the initial strategy.

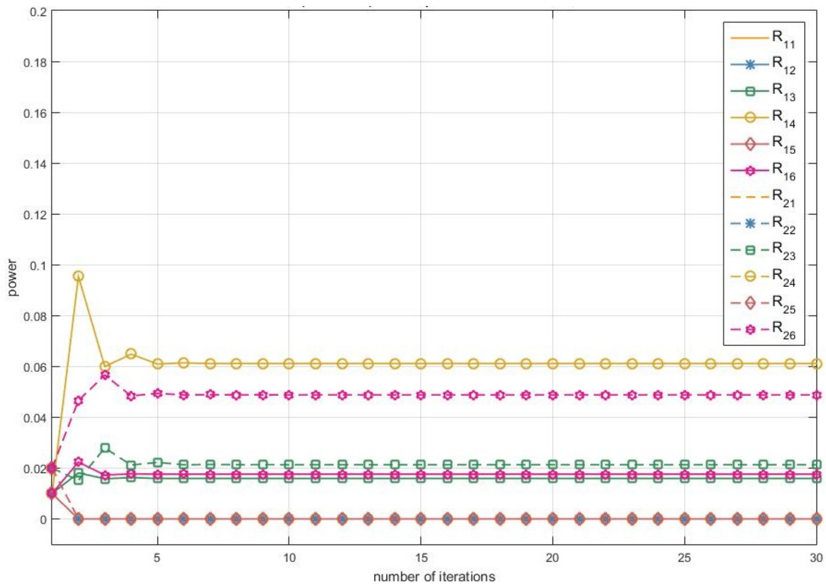


FIGURE 4.2: Power allocation of the network when $K = 2$ and $M = 6$ ($\mathbf{p}_1 = 0.01 \times \mathbf{1}_M$, $\mathbf{p}_2 = 0.02 \times \mathbf{1}_M$.)

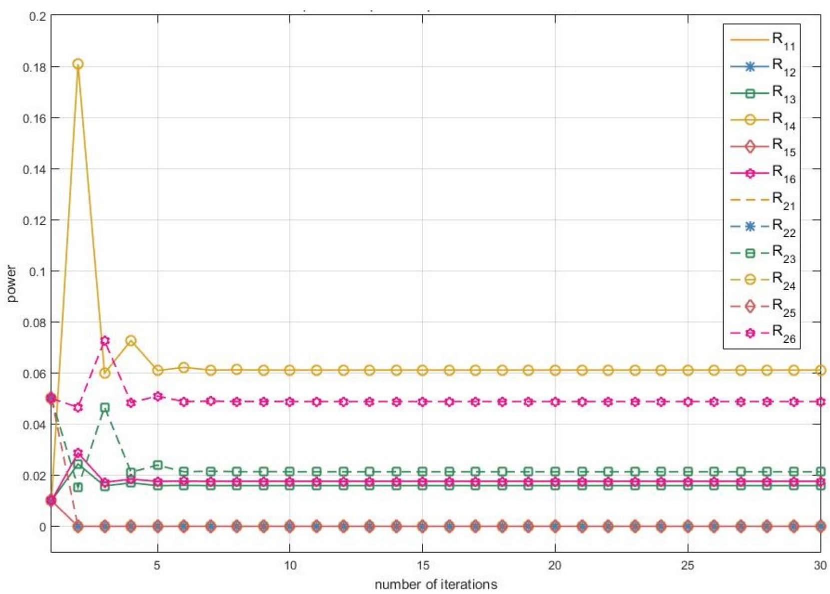


FIGURE 4.3: Power allocation of the network when $K = 2$ and $M = 6$ ($\mathbf{p}_1 = 0.01 \times \mathbf{1}_M$, $\mathbf{p}_2 = 0.05 \times \mathbf{1}_M$.)

In the second example a network of four clusters is considered. Each cluster consists of three radars. Figure 4.4 depicts the convergence to the optimal solution for player 1 for seven different initial strategies, when the rest of the players initialize the game with $\mathbf{p}_2 = [0.2855, 0.6874, 0.8295]$, $\mathbf{p}_3 = [0.3217, 0.4094, 0.4947]$ and $\mathbf{p}_4 = [0.7034, 0.0840, 0.2690]$. Similarly to this, in Figure 4.5 the same setup is considered, with the difference that the rest of the players begin the game with $\mathbf{p}_2 = [0.8080, 0.5531, 0.7784]$, $\mathbf{p}_3 = [0.8942, 0.7354, 0.9214]$ and $\mathbf{p}_4 = [0.7888, 0.6853, 0.7785]$. The results highlight that regardless of the starting point of the players, the game converges to the unique Nash equilibrium.

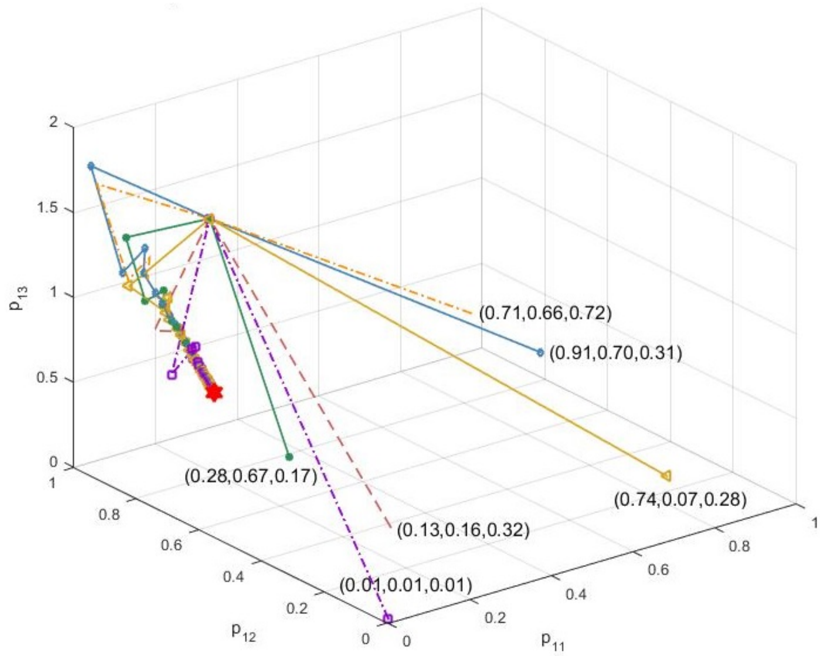


FIGURE 4.4: Convergence of power allocation of player 1 for different starting strategies when $K = 4$ and $M = 3$, first simulation (different linestyles correspond to different initial strategies for player 1).

In order to assess the efficiency of the proposed power allocation technique, the results of the proposed method are compared with the case when uniform power allocation is considered among the radars of the same cluster. Uniform power allocation has been studied in [14, 112] when a fixed system power budget is considered. By imposing an additional constraint at the optimization problem (4.12), which allocates uniformly the power among the radars in the same cluster, the resource allocation for the uniform power allocation GNG is obtained. To facilitate a fair comparison, the same SINR target in both cases is set and three different radar system scenarios are simulated, the first consisting of two clusters with two radars each, the second consisting of two

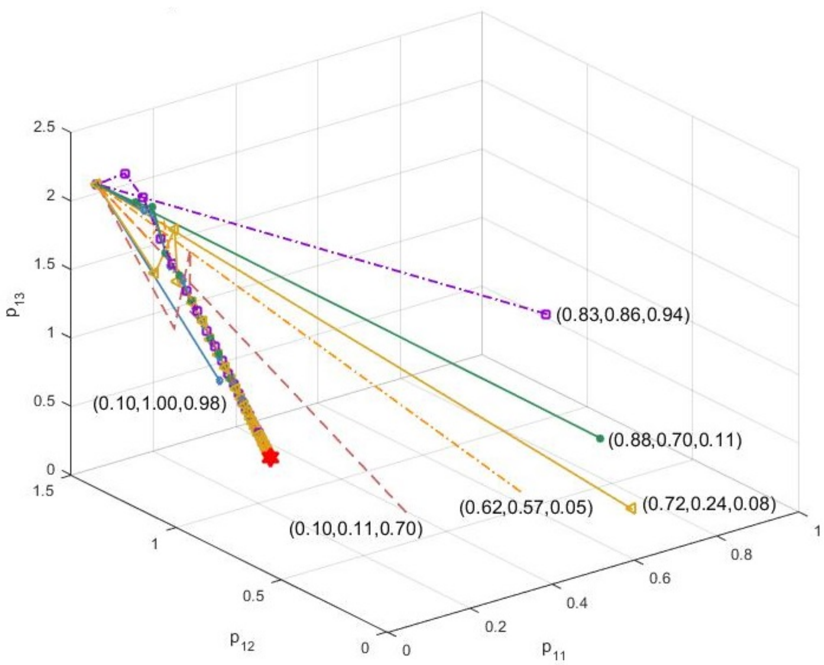


FIGURE 4.5: Convergence of power allocation of player 1 for different starting strategies when $K = 4$ and $M = 3$, second simulation (different linestyles correspond to different initial strategies for player 1).

clusters with six radars each and the last one considers three clusters consisting of three radars each. Table 4.1 presents the total power consumption in each cluster for each scenario comparing the proposed GNG with the uniform resource allocation case. It is obvious that the proposed game theoretic technique outperforms the uniform power allocation in all cases, as the total power consumption in each of the cluster is sufficiently lower in every scenario considered. In order to illustrate the aforementioned result, Fig.4.6 presents a histogram of the power consumption at cluster 1, comparing the two methods for the three different radar network cases. It is yet again evident, that the first cluster needs much less power to attain the SINR target in the proposed method for all system scenarios simulated, as compared to the uniform power allocation (UPA) technique.

For the third example, a network of two clusters is assumed, where each consists of four radars. Figure 4.7 shows the power allocation of the radars in the first cluster throughout the convergence process using the true and the estimated SINR from (4.13). It is evident that the estimation is sufficiently accurate and the convergence based on the estimation of the SINR follows the convergence trajectories of the power allocation game obtained using the true SINR values.

TABLE 4.1: Total power consumption in each cluster for three different system realizations considering the proposed GNG and the GNG with uniform power allocation.

	K=2, M=2		**K=2, M=6**		**K=3, M=3**		
Proposed GNG	0.0763	0.0418	0.1382	0.1398	0.0641	0.1190	0.0895
GNG with UPA	0.1186	0.0762	0.5993	0.6351	0.2918	0.3854	0.2375

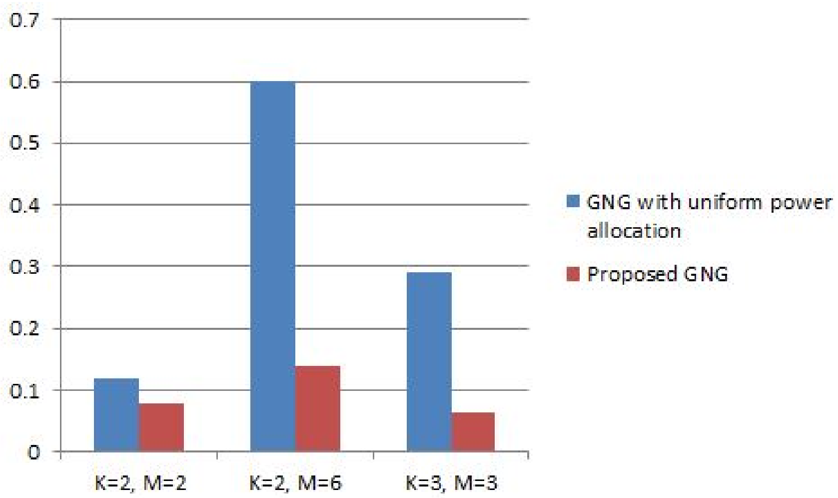


FIGURE 4.6: Total power consumption at cluster 1, comparing the proposed GNG and the uniform power allocation GNG, for different system scenarios.

4.1.7 Conclusion

This section has studied game theoretic power allocation for a distributed MIMO radar system. By defining a GNG and exploiting convex optimization techniques and duality properties, an extended Nash equilibrium analysis was presented, concluding with the proof of the existence and uniqueness of the solution. Through this analysis, important properties of the system were also derived. In particular, it was shown that the number of active radars in a cluster that actually transmit signals is exactly the same as the number of radars in the same cluster that satisfy the detection criterion with equality. In addition, the number of active radars and the optimal strategy of a cluster is dependent only upon the channel gains and the target SINR and is totally independent of the other players' power allocation. This contribution strengthens the decentralized and distributed nature of the system. Finally, the simulation results confirm the mathematical analysis of the convergence

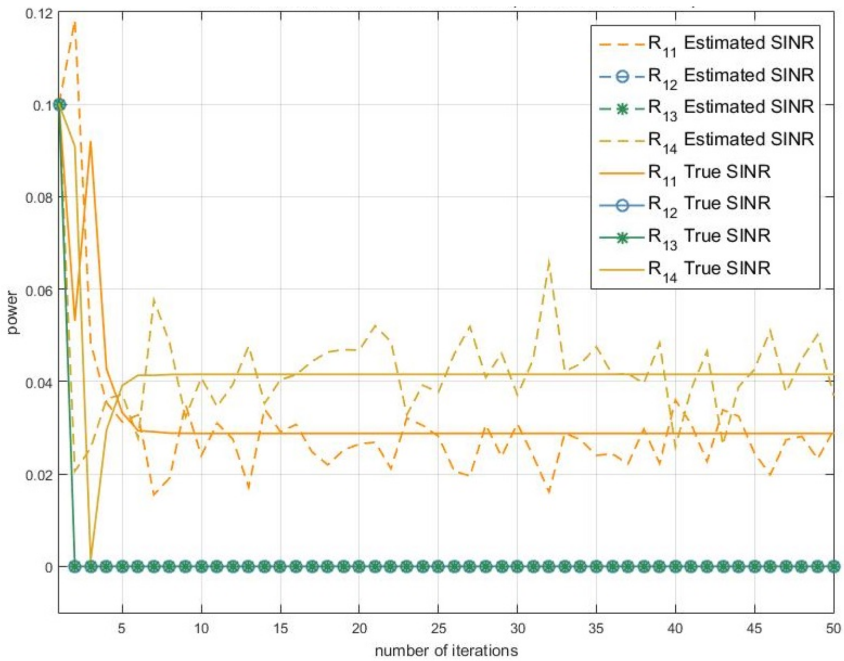


FIGURE 4.7: Power allocation in the second cluster using the true and the estimated value of the SINR when $K = 2$ and $M = 6$.

and the study of the existence and uniqueness of the Nash equilibrium.

4.2 Power Allocation Game Between a Radar Network and Multiple Jammers

In this section, a competitive power allocation problem for a MIMO radar system in the presence of multiple targets equipped with jammers is investigated. The main objective of the radar network is to minimize the total power emitted by the radars while achieving a specific detection criterion for each of the targets, while the intelligent jammers have the ability to observe the radar transmission power and consequently decide its jamming power to maximize the interference to the radars. In this context, convex optimization methods, noncooperative game theoretic techniques and hypothesis testing are incorporated to identify jammers and to determine the optimal power allocation. Furthermore, a proof is presented on the existence and uniqueness of the Nash Equilibrium (NE). The simulation results confirm the effectiveness of the proposed algorithm and demonstrate the convergence of the game.

4.2.1 Introduction

While a distributed radar network offers significant advantage in terms of diversity and accurate caption of target's radar cross section (RCS), a considerable disadvantage is the multiple source interference inflicted at the receivers of the system. Namely, the jammer interference, the inter-radar interference and the background noise yield substantial deterioration of both performance and detection capability. Thus, a successful strategy against a jamming attack based on the power allocation is critical in order to maintain a satisfactory detection performance while minimizing the power consumption of the network. Such an adaptive defence system is even more important when confronting smart targets with jammers, that are able to optimize their own jamming strategy.

A natural and efficient asset to address this kind of interactions is game theory, as it provides a framework for analyzing cooperation and confrontation among intelligent and egoistic players. In radar systems, game theory has been used to tackle various problems. Specifically, zero-sum games were used in [74] and [72] for polarimetric waveform design as an interaction between a MIMO radar and a smart jammer. Moreover, radar and jammer conflicts have been investigated in [71] and [107]. The authors in [36] and [35] incorporated game theory to tackle power allocation problems and highlighted the superiority of the game theoretic results in terms of signal to interference plus noise ratios (SINR). Finally, potential game theory was used in [114] to maximize the SINR by choosing appropriate coded waveforms.

In this section, the case when multiple aircrafts equipped with self-screening jammers attack a distributed radar network is investigated. It is assumed that the radars belong to the same organization, hence they can cooperate and be controlled centrally. Moreover, each radar in the network has the ability to identify the interfering jammer by applying directional beamforming and a hypothesis testing at its receiver. Furthermore, the intelligent targets/jammers can promptly estimate the transmission power of the radars and adapt their jamming power. Hence, a power allocation non-cooperative game (NCG) is developed between the centrally controlled radar regime and the jammers. Primary objective of the radar system is to attain a specific detection criterion using minimum possible power, while the jammers decide the optimal jamming power to maximize the damage induced to the radar system.

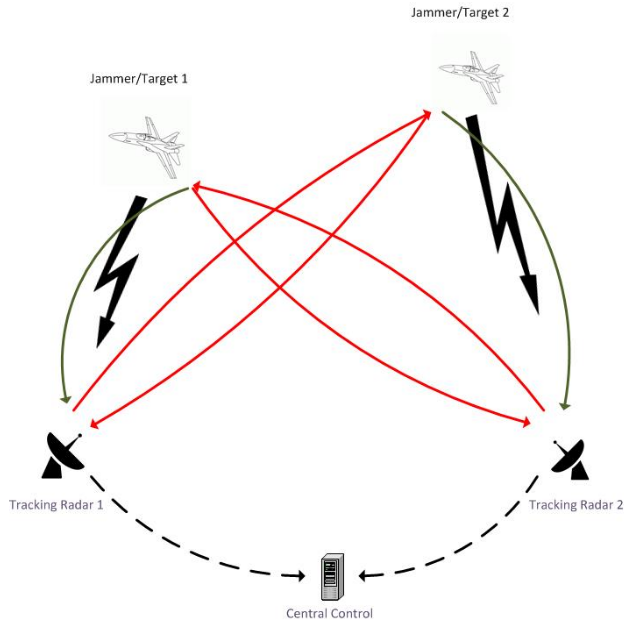


FIGURE 4.8: A multistatic radar network with a central controller, two radars and two jammers/targets.

4.2.2 System Model and Game Formulation

A multistatic air defence system is considered that consists of K separate radars and a central controller, that obtains the data from the radars and optimizes the transmission power for each radar. In order to complete the model, K aircrafts equipped with jammers approach the radars with primary objective to deteriorate or nullify the operation of the radar system. It is assumed that the opponent aircrafts have information about the approximate position of the radars and each jammer can attack only one radar, because the radars of the multistatic regime are installed too far apart from each other. Furthermore, each radar can identify the jammer attacking its receivers by applying a hypothesis testing based on the receiving power from the direction of each target, as follows:

$$\mathcal{H}_0 : x_{ki} = a_k p_{Rk} + \sigma_n^2 \quad \text{(normal target)}$$

$$\mathcal{H}_1 : x_{ki} = \beta_k p_{Jk} + a_k p_{Rk} + \sigma_n^2 \quad \text{(jammer present)}$$

$$x_k \underset{H_0}{\overset{H_1}{\gtrless}} \delta_k$$

where x_{ki} is the received signal for radar k from the direction of target i, a_k is the channel gain from the transmitters to the receivers of radar k including the effect of RCS, β_k is the channel gain from jammer k to the respective radar, p_{Rk} is the transmission power of radar k, p_{Jk} is the jamming power of jammer k, σ_n^2 denotes the noise power, δ_k is the detection threshold for radar k and is defined as $\delta_k = a_k p_{Rk} + \sigma_n^2 + \epsilon$ and ϵ is a positive design parameter that provides specific probabilities of missed detection and false alarm.

After identifying the respective jammer, each radar designs the transmit and receiver beampatterns applying a null at the direction of the attacker. In particular, the radar central control mechanism assigns a target for each radar, avoiding the interfering jammer. However, a residue of the interfering jammer's power will still affect the respective radar and this leakage power is denoted as $l_k p_{Jk}$, where l_k is the leakage gain for jammer k. Although an optimal hypothesis testing is considered, there is always a probability for a radar to misidentify an active jammer as a normal target and attempt to detect it, leading to severe SINR degradation or even missed detection. Following, a study on all possible outcomes of the hypothesis testing that could lead to missed detection is presented, considering a distributed radar network with a central controller, two radars and two jammers as depicted in Fig. 4.8, and it is assumed that jammer 1 attacks radar 1 and jammer 2 interferes with radar 2:

Case 1: Both radars identify the same target as jammer ($x_{11} > \delta_1$, $x_{12} < \delta_1$, $x_{21} > \delta_2$, $x_{22} < \delta_2$). In this case, since the fact that each jammer can affect only one target is common knowledge, the central control mechanism compares the receiver power from the supposed jammer for both radars and if ($x_{11} < x_{21}$) assigns erroneously the attacking jammer to the interfered radar.

Case 2: Both radars do not identify any target as jammer ($x_{11} < \delta_1$, $x_{12} < \delta_1$, $x_{21} < \delta_2$, $x_{22} < \delta_2$). In this case, the central controller compares the receive power from both targets for each radar and if (($x_{11} < x_{12}$) and ($x_{21} > x_{22}$) mistakenly assigns the wrong jammer to each radar.

Case 3: Both radars identify both targets as jammers ($x_{11} > \delta_1$, $x_{12} > \delta_1$, $x_{21} > \delta_2$, $x_{22} > \delta_2$). Similar to Case 2, there is potential missed detection if (($x_{11} < x_{12}$) and ($x_{21} > x_{22}$).

Case 4: Both radars identify as jammer the wrong target ($x_{11} < \delta_1$, $x_{12} > \delta_1$, $x_{21} > \delta_2$, $x_{22} < \delta_2$).

The investigation on the probabilities of missed detection and false alarm, within the context of the hypothesis testing, goes beyond the scope of this paper, which is focused on the game theoretic analysis of the model.

The interaction between the jammers and the radar system could be translated to a noncooperative game, where the players are the central radar controller and the jammers. The strategy set of the game is the transmission power of the radars $\mathbf{p}_R$ and of the jammers $\mathbf{p}_J$, respectively. Primary objective of the radar system is to minimize its total transmission power, while attaining a predefined signal to interference and noise ratio (SINR) for each of the targets. Thus, the best response strategy for the radar system is derived as the outcome of the following optimization:

$$\min_{\mathbf{p}_R} \quad \sum_{k=1}^{K} p_{Rk} \tag{4.33}$$

$$s.t. \quad \frac{a_k p_{Rk}}{\sum\limits_{\substack{j=1 \\ j \neq k}}^{K} m_{kj} p_{Rj} + l_k p_{Jk} + \sigma_n^2} \geq \gamma_k, \forall k$$

where $\mathbf{p}_R = [p_{R1}, \ldots, p_{RK}]^T$ and m_{ki} is the inter-radar channel gain between radar i and radar k.

It is clear from the constraint of the optimization problem (4.33) that the SINR for each radar is dependent on the power allocation of the other radars and the respective jamming power. Hence, the acceptable strategy set for the radar system is defined as $\mathcal{S}_R(\mathbf{p_J}) = \{\mathbf{p_R} \in \mathbb{R}_+^{K\times 1} | SINR_k \geq \gamma_k, \forall k\}$, where $\mathbf{p_J} = [p_{J1}, \ldots, p_{JK}]^T$. In order to complete the game theoretic framework, the definition of the utility function for all players is essential. Regarding the radar system, the utility function is the total power consumption of the radar system, defined as:

$$u_R(\mathbf{p_R}, \mathbf{p_J}) = \sum_{k=1}^{K} p_{Rk} \tag{4.34}$$

The utility function for jammer k is given by [119]:

$$u_{Jk}(\mathbf{p_R}, \mathbf{p_J}) = -\frac{a_k p_{Rk}}{\beta_k p_{Jk} + \sigma_n^2} - C_k p_{Jk} \tag{4.35}$$

where $C_k p_{Jk}$ is the cost function for jammer k and the best action for the jammer k is to attack the radar with transmission power given by:

$$p_{Jk}^{*} = arg \max_{p_{Jk}} u_{Jk}(\mathbf{p_R}, \mathbf{p_J}), \quad k = 1, \ldots, K$$

The solution of a noncooperative game is called Nash Equilibrium (NE). One of the primary objectives when designing a game is to investigate the existence and the uniqueness of the NE. In the game considered, the NE is a stable point, where no player can further profit by unilaterally changing its power allocation and is defined as the strategy set $\mathbf{p_R}^*$, $\mathbf{p_J}^*$ where:

$$u_R(\mathbf{p_R}^*, \mathbf{p_J}^*) \leq u_R(\mathbf{p_R}, \mathbf{p_J})$$

$$u_J(\mathbf{p_R}^*, \mathbf{p_J}^*) \geq u_J(\mathbf{p_R}, \mathbf{p_J}).$$

4.2.3 Existence and Uniqueness of the Nash Equilibrium

The existence of the solution for the noncooperative game defined in the previous section is guaranteed through the Arrow-Debreu theorem [117]. Since the existence of the NE is secured, the uniqueness of this NE is guaranteed by proving that the utility function of each jammer is strictly concave in $[0, \hat{p}]$ and that the best response function of the radar system is standard.

Lemma 1: The utility function of each jammer is a strictly concave function.

Proof: In order to prove that (4.35) is strictly concave, it needs to be shown that the second order partial derivative of $u_{Jk}(\mathbf{p_R}, \mathbf{p_J})$ with respect to p_{Jk} is negative:

$$\frac{\partial u_{Jk}(\mathbf{p_R}, \mathbf{p_J})}{\partial p_{Jk}} = \frac{a_k \beta_k p_{Rk}}{(\beta_k p_{Jk} + \sigma_n^2)^2} - C_k \tag{4.36}$$

$$\frac{\partial^2 u_{Jk}(\mathbf{p_R}, \mathbf{p_J})}{\partial^2 p_{Jk}} = -\frac{2 a_k \beta_k^2 p_{Rk}}{(\beta_k p_{Jk} + \sigma_n^2)^3} \tag{4.37}$$

This concludes the proof that the utility function of each jammer is strictly concave.

Lemma 2: The best response function of the radar system is a standard function.

Proof: Initially, it is proved in [32] that all the constraints in (4.33) must be satisfied with equality at the optimal power allocation. Thus, the constraints of the optimization problem can be written as:

$$\mathbf{G}\mathbf{p}_R = \mathbf{r}_J \tag{4.38}$$

where $\mathbf{G} \in \mathbb{R}^{K \times K}$ and is defined as $[\mathbf{G}]_{i,i} = (\frac{a_i}{\gamma_i})$ and $[\mathbf{G}]_{i,j} = (-m_{ij})$ and $\mathbf{r}_J$ denotes the total interference induced by the jammers to the radar system plus the additive white Gaussian noise (AWGN) from the environment vector and is defined as $\mathbf{r}_J = (\mathbf{G}_J \mathbf{p}_J + \mathbf{1}\sigma_n^2)$. The jammer interference matrix $\mathbf{G}_J \in \mathbb{R}^{K \times K}$

is diagonal and is defined as $[\mathbf{G}_J]_{i,i} = (\beta_i)$ and $[\mathbf{G}_J]_{i,j} = 0$. The optimal power allocation for the optimization problem (4.33) can be obtained from the solution of (6) and thus, the best response strategy for the radar system is given by $\mathbf{p}_R^* = \mathbf{G}^{-1}\mathbf{r}_J$. Furthermore, by replacing the interference vector $\mathbf{r}_J$, the best response strategy can be reformulated as:

$$BR_R(\mathbf{p_J}) = \mathbf{p}_R^* = \mathbf{G}^{-1}(\mathbf{G}_J\mathbf{p}_J + \mathbf{1}\sigma_n^2) \tag{4.39}$$

The best response strategy (4.39) must satisfy the following necessary properties in order to qualify as a standard function for all $\mathbf{p}_J \geq 0$:

a) Positivity: The best response strategy is strictly positive, $BR_R(\mathbf{p_J}) > 0$, as $\mathbf{G}^{-1}$ is a positive matrix straightforwardly from (4.39) and $\mathbf{G}_J$ is a positive matrix from its definition.

b) Monotonicity: Assuming $\mathbf{p}_J \geq \mathbf{p}'_J$, then:

$$BR_R(\mathbf{p_J}) - BR_R(\mathbf{p'_J}) = \mathbf{G}^{-1}\left(\mathbf{G}_J(\mathbf{p}_J - \mathbf{p}'_J)\right) \geq 0$$

c) Scalability: For all $a > 1$, one has:

$$aBR_R(\mathbf{p_J}) - BR_R(a\mathbf{p_J}) = (a-1)\mathbf{G}^{-1}\mathbf{1}\sigma_n^2 > 0.$$

This concludes the proof that the best response function of the radar system is a standard function.

Having proved Lemma 1 and Lemma 2, the uniqueness of the NE is secured.

4.2.4 Simulation Results

In this section, the convergence of the resource allocation noncooperative game between the radar system and the jammers to the unique Nash Equilibrium is illustrated. Thus, a bistatic MIMO radar network is assumed, where the two tracking MIMO radars are centrally controlled. Moreover, two intelligent targets are considered incorporated with jammers as depicted in Fig. 4.8. Due to the distance between the radars in the network, each jammer can only attack one radar. Furthermore, it is assumed that there is some communication between the jammers so they interfere against different radars. The environment noise is considered as AWGN with variance 0.5. The parameters representing the channel gains applied in this simulation are $a_1 = 0.75$, $a_2 = 0.71$, $m_1 = 0.0085$, $m_2 = 0.0064$, $l_1 = 0.0044$, $l_2 = 0.0062$, $\beta_1 = 0.73$, $\beta_2 = 0.52$. The cost parameter for both jammers is set equal to 2.4 ($C_1 = C_2 = 2.4$) and the detection criterion for both radars is considered equal to 6.5 ($\gamma_1 = \gamma_2 = 6.5$). Finally, the maximum number of game iterations is set at $T = 30$.

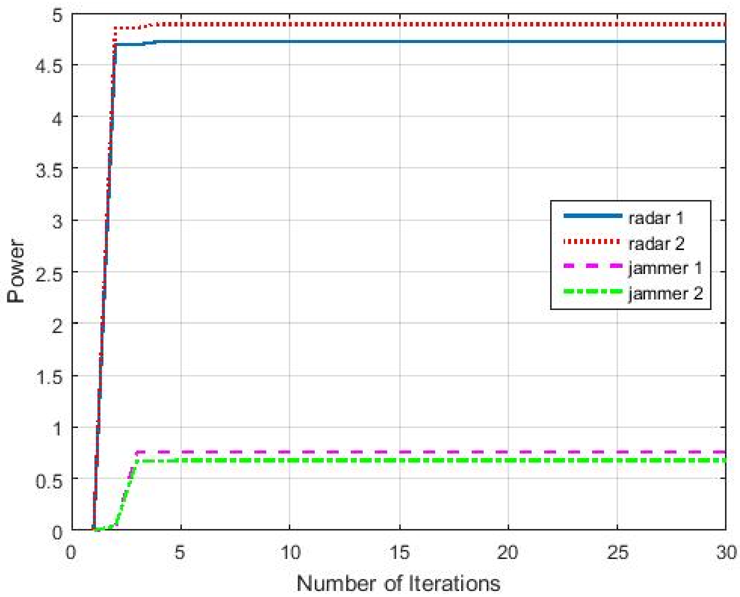


FIGURE 4.9: Power allocation convergence for the noncooperative game with jammer identification.

In order to verify the improved power allocation of the radar system when applying a hypothesis testing and identifying the interfering jammers, the results of the game with jammer suppression are compared to the outcome of the case, where the radars have no jammer identification technology. Under this assumption, the radars do not apply hypothesis testing for each of the targets and attempt to detect the target closer to them, which in the model is the interfering jammer. Hence, the jammer interference gain for the second game considered is equal to the actual gain β_k and not the leakage gain l_k as in the first game. The best response strategy of the radar system is reformulated as:

$$\min_{\mathbf{p}_R} \quad \sum_{k=1}^{K} p_{Rk} \tag{4.40}$$

$$s.t. \quad \frac{a_k p_{Rk}}{\sum\limits_{\substack{i=1 \\ i \neq k}}^{K} m_{ki} p_{Ri} + \beta_k p_{Jk} + \sigma_n^2} \geq \gamma_k, \forall k$$

In order to prove that the best response strategy in the second game (4.40) is a standard function and so there is a unique Nash equilibrium, Lemma 2 is followed. To facilitate the comparison, the same parameters representing the channel gains and set the same SINR targets as the first game are assumed

($\gamma_1 = \gamma_2 = 6.5$). Similar to the first game, the cost parameter for both jammers is set equal to 2.4 ($C_1 = C_2 = 2.4$). The number of iterations is set at $T = 60$ for the second game.

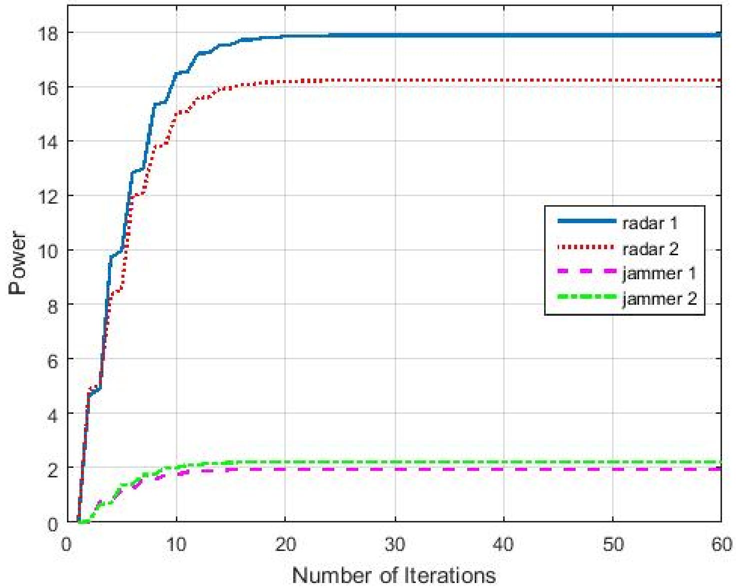


FIGURE 4.10: Power allocation convergence for the noncooperative game without jammer identification.

Fig. 4.9 and Fig. 4.10 depict the resource allocation convergence for the jammers and a radar network with jammer suppression and without, respectively. A first important observation is that in both cases the noncooperative game converges to its unique solution, as proved in section III. Moreover, a second crucial remark is that a radar network equipped with a jammer identification system can achieve the desired detection criterion consuming less power. This stems from the fact that the transmit and receive beamformers in the case of the game with jammer suppression apply a null at the direction of the jammer, whereas in the case of the game without jammer identification the beamformers are steered at the direction of the jammer and thus the jammer channel gain is much greater in the second case, i.e. $\beta_k > l_k$. Therefore, in the game without jammer identification the radars need to transmit more power to overcome the increased jammer interference. Furthermore, it is obvious that the game with jammer suppression converges faster to its unique solution. This is due to lower interference from the jammers to the radar network and thus the interaction between the players is less intense, leading to quicker convergence. Table 4.2 presents the interference induced by the jammers to the radar network for each of the two games. It highlights the substantial interference mitigation when a radar system incorporates a jammer suppression algorithm.

Table 4.2: Interference induced by the jammers to the radar system for each game.

Receiver / **Game played**	**Radar I**	**Radar II**	**Total**
With Jammer Suppression	0.0034	0.0042	0.0076
Without Jammer Identification	1.4239	1.1580	2.5819

4.2.5 Conclusion

A power allocation game theoretic problem between a radar system and multiple jammers was investigated. Initially, the interaction between the radar network and the jammers was modelled as a simultaneous noncooperative game and then a proof was presented on the uniqueness of the solution. Furthermore, the simulations highlighted the comparison between the case when the radars using a hypothesis testing to identify the interfering jammers and the case when the radars attempt to detect the target on the basis of proximity (i.e. distance), even if it is an interfering jammer. Finally, the simulation results confirm that the game with hypothesis testing provides a more Paretto-efficient Nash equilibrium than the game without jammer identification, as the radars utilize less resources to achieve the same SINR criterion.

Chapter 5

SINR Optimization and Resource Allocation for a Multistatic Radar Network: A Bayesian Game-Theoretic Approach

This chapter investigates a Bayesian game theoretic SINR maximization scheme for a multistatic radar network. A distributed network of radars is considered, whose primary goal is to maximize their signal-to-noise ratio (SINR), while attaining a predefined power constraint. Furthermore, no communication is assumed between the radars and hence a noncooperative approach is utilized. The channel gain between a radar and the target is assumed as private information and characterizes the type of the player, whereas the distribution of the channel gain is common knowledge to every player in the game. Subsequently, the existence and the uniqueness of the Bayesian Nash equilibrium for the aforementioned game is examined and proved. Finally, the simulation results confirm the convergence of the algorithm to the unique solution.

5.1 Introduction

Distributed radar networks benefit from many substantial advantages such as direct applicability of adaptive array techniques, capture of the geometrical characteristics of the target through the spatial diversity in the target's radar cross section (RCS), multiple targets detection, and slow moving targets tracking [9]. Nevertheless, multistatic radar networks suffer from multiple source interference imposed at the receivers of each radar, namely the cross channel interference induced by other radars in the same network and the clutter interference. This interference seriously deteriorates the performance and the tracking capabilities of the system and thus an optimal power allocation strategy that minimizes the interference and maximizes the detection performance is necessary. Game theory is an appropriate and efficient tool to address this issue, as it constitutes a mathematical framework of confrontation and coordination among selfish, intelligent and rational players.

Game theoretic techniques have been utilized recently to confront various radar problems. Especially optimal power allocation and distribution in radar networks motivated many authors to utilize different game theoretic techniques. The authors in [35] and [36] addressed the power allocation problem by formulating a non-cooperative game with predefined SINR constraints. Since a radar in a distributed network can not obtain information regarding the transmission power of the remaining radars in the network, an SINR estimation technique was applied in [37], to extend the work in [35]. The authors in [38] exploited cooperative game theoretic techniques to solve the resource allocation problem through maximizing the Bayesian-Fisher information matrix (B-FIM) and utilizing the Shapley value solution. A combination of a water filling algorithm and a Stackelberg game was used in [39] for optimal power distribution. In addition, a noncooperative power allocation game between a multistatic radar network and multiple jammers was presented in [120], together with the proof of the existence and uniqueness of the Nash equilibrium.

In the aforementioned radar literature, the radars have been assumed to have exact knowledge of the channel gain in terms of the RCS parameters of the targets and clutter, which may not be feasible in a real system. In this chapter, uncertainty is introduced on the channel gains associated with the radars and the targets, which arises due to the RCS fluctuations of the targets. Bayesian game theory provides a framework to address this problem of incomplete information. Therefore, a Bayesian game is considered, where each player egotistically maximizes its SINR, under a predefined power constraint. Within this framework, it is assumed that each radar/player exactly knows the channel gain between itself and the target as private information, however having uncertainty on the channel experienced by other radars in the network. Only the distribution of the channel gains is considered as common knowledge to every player. The distribution of the channel gains can be obtained by exploiting several target models, such as Swerling or extended-Swerling models, depending on the targets' type [108]. This problem is solved using a Bayesian game framework as proposed for communication application in [101]. The existence and uniqueness of the Bayesian Nash equilbrium is also proved.

5.2 System Model

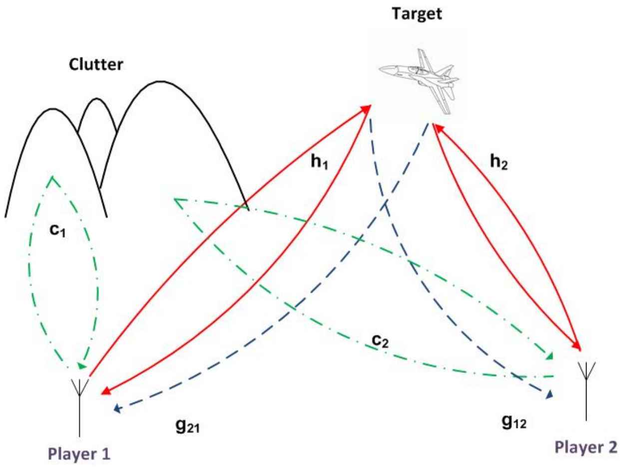


FIGURE 5.1: A multistatic MIMO radar network with two radars and one target.

The system model incorporates a multistatic radar network, that consists of K widely separated radars. In the far-field of the radars, a flying target is assumed. Hence, the primary objective of each radar is to achieve the highest possible SINR, while satisfying a maximum power constraint. In the noncooperative approach of the distributed radar network, each radar performs the optimization of the SINR selfishly and autonomously, having complete knowledge only for its own channel gain realization as private information. On the other hand, only the distribution of the inter-radar channel gains is available to every radar as common knowledge. In particular, this uncertainty on the cross channel gains is generated from the radar cross section (RCS) of the target, as only the distribution of the RCS is common information for every radar and not the exact instantaneous value of the RCS. Since it is assumed that all radars belong to the same organization, the game scenario is not competitive and there is no intentional interference among the radars.

In the presence of a target, the received signal for radar k is obtained by:

$$\mathbf{x}_k = h_k p_k \mathbf{s}_k + \sum_{\substack{j=1 \\ j\neq k}}^{K} g_{jk} p_j + c_k p_k + \hat{n} \tag{5.1}$$

where $\mathbf{s}_k = \psi_k \mathbf{a}_k$ describes the transmitted signal from radar k and $\mathbf{a}_k = [1, e^{j2\pi f_{D,k}}, \ldots, e^{j2\pi(N-1)f_{D,k}}]^T$ is the steering vector of radar k regarding the desired target, $f_{D,k}$ denotes the normalized Doppler shift at radar k, N is the number of signal return samples that the radars receive at each time step and ψ_k corresponds to the predesigned waveform transmitted from radar k. The parameter h_k denotes the desired channel gain at the direction of the target, p_k stands for the transmission power of radar k, g_{jk} describes the cross-channel gain among radars k and j, c_k and $\hat{n}$ denote the clutter channel gain and a zero-mean white Gaussian noise with variance σ_n^2. Hence, the SINR for the k^{th} radar is straightforwardly defined as:

$$\mathrm{SINR}_{ki} = \frac{h_k p_k}{c_k p_k + \sum\limits_{\substack{j=1 \\ j \neq k}}^{K} g_{jk} p_j + \sigma_n^2}. \tag{5.2}$$

In the next section, the Bayesian game theoretic formulation for the system is described.

5.3 Game Theoretic Formulation

In this section, the interactions between the K radars in the network as a Bayesian game are modeled, in which the main goal for each radar is to maximize its SINR for target detection under a power constraint and channel uncertainty. More specifically, the incomplete information in the considered system model reflects the inability of radar k to obtain the exact value of the cross channel gains, i.e. $[g_{1k}, g_{2k}, \ldots, g_{Kk}]$. Nevertheless, since each radar knows the type of the target, then the distribution of the RCS of the target and subsequently the distribution of the cross channel gain is common information. It is clear from the SINR equation (5.2) that although increased transmission power at a radar strengthens the desired signal, it induces higher cross interference to the remaining radars in the network. Thus, the aforementioned interaction is modeled as a noncooperative Bayesian game, which can be fully characterized as:

$$\mathcal{G} =< \mathcal{R}, \mathcal{T}, \mathcal{P}, \Pi, \mathcal{U} >$$

- The set of radars is considered to be the player set: $\mathcal{R} = \{R_1, \ldots, R_K\}$.
- The type set is denoted as $\mathcal{T} = \mathcal{T}_1 \times \ldots \times \mathcal{T}_K$, and corresponds to each player's channel gain, i.e. $\mathcal{T}_k = \{g_-, g_+\}$.
- The action set of the game is $\mathcal{P} = \mathcal{P}_1 \times \ldots \times \mathcal{P}_K$ with

  $$\mathcal{P}_k = \{p_k \in \mathbb{R}^+ \mid p_k \in [0, P_k^{max}]\}, \quad \forall i \in \{1, \ldots, K\}$$

  where P_k^{max} denotes the maximum available power for radar R_1.

- The common prior or probability set is defined as $\Pi = \Pi_1 \times \ldots \times \Pi_K$, where Π_k is the probability distribution of the channel gain for radar R_k and hence the distribution of the player's type and it is common knowledge to every player.

- The Bayesian game model is concluded by defining the utility function set as $\mathcal{U} = \{u_1, \ldots, u_K\}$, where u_k represents the k^{th} radar SINR as shown below:

$$u_k(p_1, \ldots, p_K) = \frac{h_k p_k}{c_k p_k + \sum\limits_{\substack{j=1 \\ j \neq k}}^{K} g_{jk} p_j + \sigma_n^2} \tag{5.3}$$

  It is evident from equation (5.3), that the utility function is a function of the power allocation of all K players.

In the considered Bayesian Nash Game (BNG), player k egotistically maximizes its SINR, while attaining a maximum power constraint, given the transmission power strategies of the remaining players. Therefore, the best response for player k can be determined by solving the following optimization problem:

$$\max_{p_k \in \mathcal{P}_k} \mathbb{E}[u_k(p_1, \ldots, p_K)] \tag{5.4}$$

$$\text{s.t.} \quad \mathbb{E}[p_k] \leq P_k^{max}$$

$$p_k > 0$$

The study on the convergence of the game $\mathcal{G}$ to a stable solution is the most critical part of the game theoretic analysis, as it provides the ability to predict the performance and the stability of the distributed radar system under channel uncertainty. This specific solution defines the Bayesian Nash equilibrium, where no player could benefit by unilaterally change its power allocation strategy. Hence, for the considered game $\mathcal{G}$ the Bayesian equilibrium describes the action profile $(\mathbf{p}^*_{-k}, p^*_k)$, where $\mathbf{p}_{-k}$ denotes the transmission power adopted by all other players except player k, when:

$$\bar{u}_k(\mathbf{p}^*_{-k}, p^*_k) \geq \bar{u}_k(\mathbf{p}^*_{-k}, p_k), \quad \forall \mathbf{p}_k \in \mathcal{P}_k, \forall k \in \mathcal{R}.$$

where $\bar{u}_k$ defines the expected utility for player k. The next section presents a rigorous mathematical analysis on the existence and the uniqueness of the Bayesian equilibrium.

5.4 Existence and Uniqueness of the Bayesian Equilibrium

Initially, it is important to underline that for a given set of opponent power strategies $\mathbf{p}_{-k}$, the optimization problem (5.4) is a convex optimization problem, since the objective and the constraint functions are quasiconcave and quasiconvex functions, respectively. Therefore, the maximization problem (5.4) can be reformulated to a standard form convex optimization problem, by changing the sign of the objective function, as follows:

$$\min_{p_k \in \mathcal{P}_k} -\mathbb{E}[u_k(p_1, \ldots, p_K)] \tag{5.5}$$

$$\text{s.t.} \quad \mathbb{E}[p_k] - P_k^{max} \leq 0$$

$$-p_k < 0$$

At this point, the Lagrangian $\mathcal{L}$ corresponding to the convex optimization problem (5.5) may be defined as:

$$\mathcal{L}(p_k, \lambda_1, \lambda_2) = \mathbb{E}\left[-\frac{h_k p_k}{c_k p_k + \sum_{\substack{j=1 \\ j\neq k}}^{K} g_{jk} p_j + \sigma_n^2}\right] + \lambda_1(p_k - P_k^{max}) - \lambda_2 p_k \tag{5.6}$$

where λ_1 and λ_2 are the Lagrange multipliers associated with the inequality constraints of (5.5). It is presumed that $(p_k^*, \lambda_1^*, \lambda_2^*)$ are the primal and dual optimal points of (5.5). Thus, the Karush-Kuhn-Tucker (KKT) conditions on convexity must be satisfied and one has:

$$\lambda_1^* = \mathbb{E}\left[\frac{h_k \sum_{\substack{j=1 \\ j\neq k}}^{K} g_{jk} p_j + h_k \sigma_n^2}{\left(c_k p_k + \sum_{\substack{j=1 \\ j\neq k}}^{K} g_{jk} p_j + \sigma_n^2\right)^2}\right] \tag{5.7}$$

$$\lambda_1^*(p_k^* - P_k^{max}) = 0 \tag{5.8}$$

From (5.7) it is straightforward that the optimal Lagrange multiplier λ_1 is strictly positive. Therefore, from (5.8) the optimal transmission power for radar k is equal to the maximum power constraint, i.e. $p_k^* = P_k^{max}$. However, it is evident from (5.7) that the optimal solution for radar k is a function of the transmission power of all K players, which is not common knowledge. Hence, in order for each player to obtain the optimal power allocation, each radar must optimize its transmission power based on an estimation of all the remaining radars' power allocation. The investigated Bayesian game theoretic framework models exactly this kind of interaction.

5.4.1 Existence

The existence of a Bayesian Nash equilibrium (BNE) follows from the result by [117] on abstract economies. According to this result, a BNE exists if the following hold: for all players $k = 1, \ldots, K$ the set P_k is compact, nonempty and convex, the utility function $u_k(p_{-k}, p_k)$ is continuous on $\mathcal{P}$ and quasi-convex in p_k. For every p_{-k} the set-valued function $\mathcal{P}_k$ is continuous with closed graph and for every p_{-k} the set $\mathcal{P}(p_{-k})$ is non-empty and convex. For the considered problem, these requirements can be straightforwardly established using analytic notions, hence there exists a BNE for the proposed game.

5.4.2 Uniqueness

In order to prove the uniqueness of the Bayesian equilibrium, it must be proved that the second derivative of the utility function of radar k is strictly concave with respect to its action set. Therefore, geometric programming techniques are utilized to prove the uniqueness of the solution [51], as the following Lemma suggests:

Lemma 1: The Bayesian game $\mathcal{G}$ has a unique solution.

Proof. Following [51], it is feasible to maximize a nonzero monomial utility function, by minimizing its inverse. Thus, the best response optimization problem (5.4) for player k following geometric programming techniques is restated as:

$$\min_{p_k \in \mathcal{P}_k} (h_k p_k)^{-1} \left(c_k p_k + \sum_{\substack{j=1 \\ j \neq k}}^{K} g_{jk} p_j + \sigma_n^2 \right) \tag{5.9}$$

$$\text{s.t.} \quad \mathbb{E}[p_k] - P_k^{max} \leq 0$$

$$-p_k < 0$$

At this point, by redefining the utility function as $u_k^{'}(\mathbf{p}_{-k}, p_k) = (h_k p_k)^{-1}(c_k p_k + \sum_{j=1, j\neq k}^{K} g_{jk} p_j + \sigma_n^2)$, game $\mathcal{G}$ becomes:

$$\mathcal{G}^{'} =< \mathcal{R}, \mathcal{T}, \mathcal{P}, \Pi, \mathcal{U}^{'} >$$

where $\mathcal{U}^{'} = \{u_1^{'}, \ldots, u_K^{'}\}$. Since it is shown from the KKT conditions (5.7) and (5.8) that the optimal transmission power is obtained when the power constraint is satisfied with equality, the transmission power when the channel gain is g_+ can be defined as $\pi_+ p_k(g_+) = P_k^{max} - \pi_- p_k(g_-)$, where π_+ and π_- correspond to the probability of high channel gain g_+ and low channel gain g_-, respectively. Hence the average utility function $\bar{u}_k^{'}$ is defined as a weighted sum function:

$$\bar{u}_k^{'}(\mathbf{p}_{-k}, p_k) = \sum_i \phi_i (h_k^i p_k)^{-1}(c_k p_k + \sum_{j=1, j\neq k}^{K} g_{jk}^i p_j + \sigma_n^2) \quad (5.10)$$

where i stands for the different jointly probability realizations of the channel gains, ϕ_i represents the respective probability for event i, h_k^i and g_{jk}^i denote the desired and cross-channel gains for event i. At this point, the first and the second derivative of the utility function of player k with respect to its strategy p_k can be derived, as shown below:

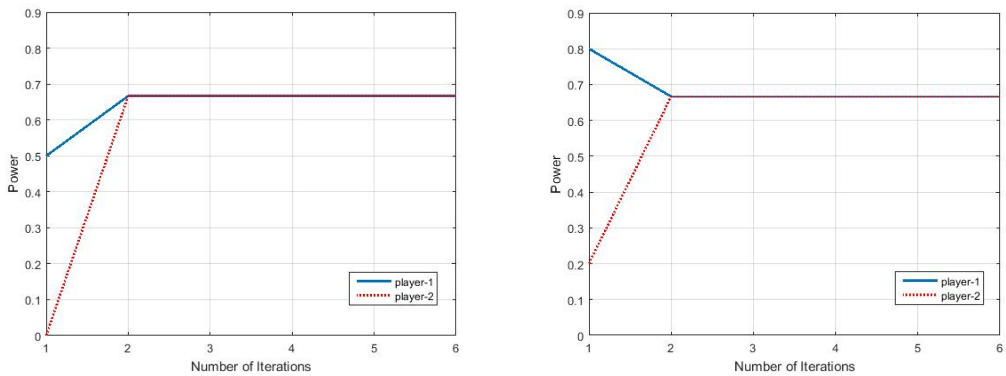
$$\frac{\partial \bar{u}_k^{'}(\mathbf{p}_{-k}, p_k)}{\partial p_k} = \sum_i \phi_i \left(-(h_k^i)^{-1} p_k^{-2} \sum_{j=1, j\neq k}^{K} g_{jk}^i p_j - (h_k^i)^{-1} p_k^{-2} \sigma_n^2 \right) \quad (5.11)$$

$$\frac{\partial^2 \bar{u}_k^{'}(\mathbf{p}_{-k}, p_k)}{\partial^2 p_k} = \sum_i \phi_i \left(2(h_k^i)^{-1} p_k^{-3} \sum_{j=1, j\neq k}^{K} g_{jk}^i p_j + 2(h_k^i)^{-1} p_k^{-3} \sigma_n^2 \right) \quad (5.12)$$

It is evident from (5.12) that the second derivative of the payoff function regarding the k^{th} player is strictly positive $\forall p_k > 0$ and hence the Bayesian game $\mathcal{G}^{'}$ has a unique solution. Consequently, the initial game $\mathcal{G}$ admits a unique Bayesian Nash equilibrium. □

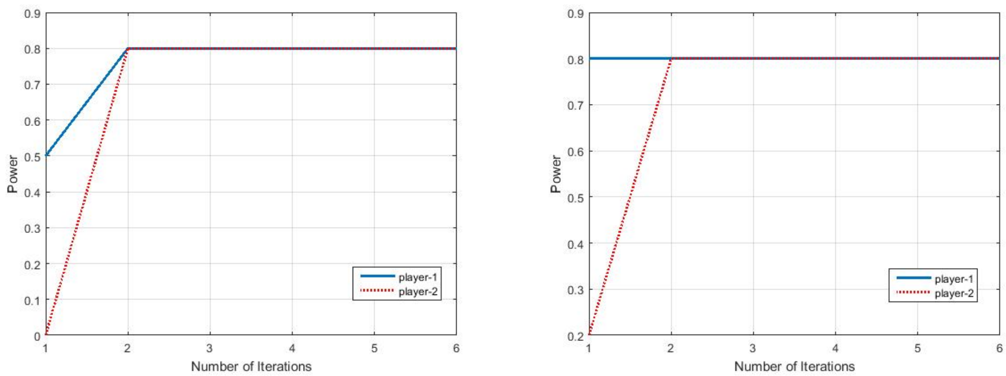
5.5 Simulation Results

In this section, simulation results are introduced to validate the theoretical background. A bistatic radar network is presumed consisting of two radars and two possible channel states $g_- = 1$ and $g_+ = 4$. There is also a target assumed at the far-field of the radars and substantial clutter, whose gain is set to $c_1 = 0.5$ and $c_2 = 0.3$. Initially, Fig. 5.2 displays the convergence of the power allocation to the unique solution for two different starting strategies when $\pi_- = \pi_+ = 0.5$. It is clear that the proposed Bayesian geometric programming game converges swiftly to the unique solution, regardless the initial strategy of the radars.



(A) $p_1(g_+) = 0.5$, $p_2(g_+) = 0.00001$. (B) $p_1(g_+) = 0.8$, $p_2(g_+) = 0.2$.

FIGURE 5.2: Convergence of the power allocation corresponding to g_+ for $\pi_- = \pi_+ = 0.5$ and different initial strategies.



(A) $p_1(g_+) = 0.5$, $p_2(g_+) = 0.00001$. (B) $p_1(g_+) = 0.8$, $p_2(g_+) = 0.2$.

FIGURE 5.3: Convergence of the power allocation corresponding to g_+ for $\pi_- = 0.25$ and $\pi_+ = 0.75$ and different initial strategies.

Fig. 5.3 confirms the convergence of the algorithm for different channel gain probabilities, hence $\pi_- = 0.25$ and $\pi_+ = 0.75$ is chosen. Similar to the first

TABLE 5.1: Bayesian equilibrium and SINRs for the two players for different values of π_+.

Probability π_+	**0.1**	**0.5**	**0.75**
Bayesian equilibrium	(0.5263,0.5263)	(0.6667,06667)	(0.8000,0.8000)
SINR 1	0.7830	0.8886	0.9009
SINR 2	0.8442	0.9493	0.9540

example, the convergence is secured, whatever the starting power allocation of the radars. In addition, one can observe that when the belief regarding the higher channel gain is stronger, both players allocate more power to the higher channel gain. This fact is further analyzed in Table 5.1, where the Bayesian equilibrium for different values of the probability π_+ is displayed along with the SINRs of the two radars. As expected, the higher the belief for g_+, the players transmit with increased power corresponding to the stronger channel and also the SINR for both players is increasing with respect to the confidence of the high channel gain π_+.

Fig. 5.4 highlights the importance of the prior belief of a player regarding the channel gains on the resulting power allocation. As the belief for a better channel gain gets more robust, the player is more confident of deciding a mixed strategy, where the transmission power is increased. On the other hand, when the aforementioned probability gets slimmer, the transmission power is restrained, as a worse channel gain is more probable.

5.6 Conclusion

This chapter investigated a Bayesian game theoretic SINR maximization and resource allocation technique within a distributed radar network, where the radars are considered to have private information only about their own channel gains. Initially, the interactions were modeled within the aforementioned multistatic network as a Bayesian game and then a proof of the existence and uniqueness of the Bayesian Nash equilibrium was presented. The simulation results validated the convergence to the unique solution, regardless the initial resource allocation strategy of the players. Furthermore, it was shown that the higher the confidence of a player regarding a better channel gain associated with the remaining players the higher the SINR and the transmission power of this player. In addition, the importance of the prior belief of a player was highlighted to the outcome of the game. A possible future extension would be

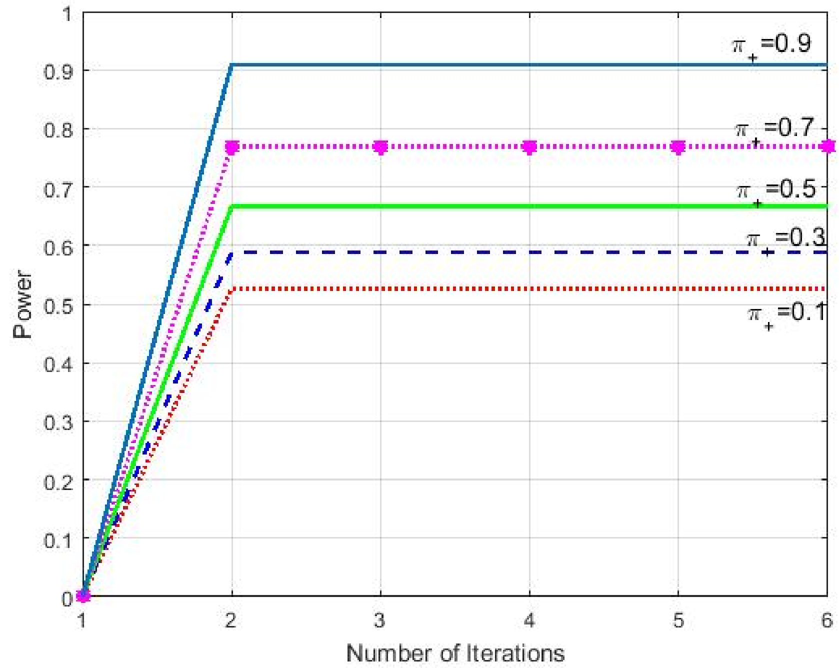


FIGURE 5.4: Transmission power (p_+) convergence for player 2 for different channel gain probabilities π_+.

the utilization of Swerling target models to model the uncertainty regarding the channel gains and analyze the SINR results.

Chapter 6

Game Theoretic Analysis for MIMO Radars with Multiple Targets

This chapter considers a distributed beamforming and resource allocation technique for a radar system in the presence of multiple targets. The primary objective of each radar is to minimize its transmission power while attaining an optimal beamforming strategy and satisfying a certain detection criterion for each of the targets. Therefore, convex optimization methods are used together with noncooperative and partially cooperative game theoretic approaches. Initially, a strategic noncooperative game (SNG) is considered, where there is no communication between the various radars of the system. Hence each radar selfishly determines its optimal beamforming and power allocation. Subsequently, a more coordinated game theoretic approach is assumed incorporating a pricing mechanism. Introducing a price in the utility function of each radar/player, enforces beamformers to minimize the interference induced to other radars and to increase the social fairness of the system. Furthermore, a Stackelberg game is formulated by adding a surveillance radar to the system model, which will play the role of the leader, and hence the remaining radars will be the followers. The leader applies a pricing policy of interference charged to the followers aiming at maximizing his profit while keeping the incoming interference under a certain threshold. A proof of the existence and uniqueness of the Nash Equilibrium (NE) is also presented in both the partially cooperative and noncooperative games. Finally, the simulation results confirm the convergence of the algorithm in all three cases.

6.1 Introduction

Multiple-input multiple-output (MIMO) radar is an innovative technology that has raised expectations over the last decade that it will provide substantial improvements to the currently used radar systems. The main characteristic that allows MIMO radar to offer superior capabilities as compared to other radar regimes is its waveform diversity, which implies that MIMO radar can use multiple antennas to simultaneously transmit several orthogonal waveforms and multiple antennas to receive the reflected signals from the targets [5]. There are two principal MIMO radar schemes considered in the literature, the systems incorporating colocated antennas and those that consist of widely separated antennas (bistatic, multistatic) [8], [9]. The leading fields of research within MIMO radar technology are beamformer and waveform design, detection optimization and radar imaging [58]-[121]. Succeeding the advances in those fields, the main advantages offered by MIMO radar are higher angular resolution, direct applicability of adaptive array techniques, multiple targets detection and the ability to obtain spatial diversity in the target's radar cross section (RCS). Nevertheless, one substantial drawback in a multiple target, distributed radar system, that has not yet been completely resolved, is the multiple source interference imposed at the receivers of each radar. More specifically, the inter-radar [1], the intra-radar [2] and the clutter interference lead to reduced efficiency and performance degradation of the radar system. Hence, an optimal beamforming and power allocation strategy is crucial as it minimizes the interference in between the radars of the same organization, while preserving a detection criterion. Game theory is a natural and effective tool for modeling this kind of interactions, as it offers a mathematical framework of conflict and cooperation between intelligent, self-interested and rational players.

The increasing need for independent, autonomous and decentralized communication systems has sparked much interest in using game theoretic techniques in the communication literature [25]. More specifically, the aforementioned distributed, multistatic beamforming and resource allocation problem in radar systems can be compared to similar issues raised in multicell wireless systems in communication applications [26]-[27]. In [26], the authors introduced the idea of joint beamforming and power control, proposing an iterative algorithm to simultaneously obtain the optimal beamforming and power vectors. The incorporation of game theory in this context then rapidly became a

[1] Cross channel (direct) and indirect interference induced among different radars.

[2] Interference imposed from the transmitters to the receivers of the same radar when detecting two or more different targets.

focal point in communications research [28]-[29]. The majority of this literature considers the technique of strategic noncooperative games (SNG), where each player selfishly maximizes its payoff function, given the strategies of the other players. The authors of [28] exploited an iterative water-filling algorithm to reach the Nash equilibrium in a non-cooperative, distributed, multiuser power control problem. Since each player greedily optimizes its utility function, the equilibrium might not be the Pareto-optimal solution. Introducing pricing policies to the system resources leads to a more Pareto-efficient solution and increases the social welfare of the system. A pricing regime that is a linear function of the transmit power was studied in [30]. Another example of pricing the transmit power of each player is considered in [31], whereas in [32] and [33] the pricing policy is applied on the intercell interference among the players. In [29], the authors consider the optimization of a set of precoding matrices at each node of a multi-channel, multi-user cognitive radio MIMO network in order to minimize the total transmit power of the network, while applying a pricing scheme based on global information. Cooperative game theoretic techniques combined with a two-level Stackelberg game were utilized in [34] to address the problem of relay selection and power allocation without the knowledge of channel state information (CSI). Finally, the authors in [27] formulated a Stackelberg Bayesian game to obtain the optimal power allocation for a two-tier network, while applying an interference constraint at the leader and considering channel gain uncertainty.

Game theory is also an efficient tool to overcome various problems that arise in radar systems. In particular, the authors in [74] approached the problem of polarimetric waveform design by considering a zero-sum game between an opponent and the radar system engineer. The zero-sum game was also used in [72] to investigate the interaction between a MIMO radar and an intelligent target, that applies jamming techniques. Potential game theory was exploited in [114] with the main objectives of optimal waveform design and maximization of the signal-to-interference plus noise ratio (SINR). A non-cooperative game theoretic per antenna power optimization based on signal-to-disturbance ratio (SDR) estimation with a desired SINR constraint was investigated in [37]. Non-cooperative game theory was also employed in [36] to facilitate the power control problem in a radar network. To address the power allocation problem the authors of [38] used a cooperative game approach and exploited the Shapley value solution scheme.

In this paper, inspired by the aforementioned game theoretic methods applied in communications [31], [32], [29], although reinvestigated to adapt to the radar case, a broad game theoretic analysis has been developed for the optimal beamforming and resource allocation problem in a MIMO tracking radar system with multiple targets. Initially, an SNG is considered, where each radar/player greedily optimizes the beamforming and power allocation vectors in two stages. In the first stage, the optimal transmit and receive beampatterns are designed by exploiting convex optimization techniques in a power minimization problem, while attaining a certain detection criterion. After designing the optimal beampatterns, the primary joint beamforming and resource allocation problem reduces to a power only minimization game. Thus, in the second stage of the game the best response strategy of a radar in an SNG setup is obtained and it is showed that it is a standard function [86], which proves the uniqueness of the Nash equilibrium, similar to the work in [32] for wireless communication applications.

The fact that each radar acts selfishly and does not take into account the damage it may inflict to other radars, through inter-radar interference, leads to a solution that may not be optimal from a social welfare point of view. Since it is presumed that the radars belong to the same organization, it is safe to consider some sort of cooperation and introduce a pricing policy to all players in order to minimize the interference induced to other radars. More specifically, the radars are encouraged to steer their beams in directions that cause less damage to other players, which results in a more Pareto-optimal solution.

In order to complete the radar model, a surveillance radar is incorporated as part of the previously studied MIMO tracking radar system. The main application of the surveillance radar is to continuously search the operating area for new incoming targets. By adding a surveillance radar, the hybrid radar system is capable of both acquiring new targets and tracking every target in an operating field. However, all radars operate simultaneously and hence the tracking radars interfere with the surveillance radar and increase the probability of false alarm. In order to secure the smooth operation of the system, a maximum limit of interference induced at the surveillance radar is set. In order to achieve both the target SINR and to guarantee the interference limit at the surveillance radar, a Stackelberg game approach is utilized. In particular, the surveillance radar is the leader and the MIMO tracking radars are the followers in the hierarchy of the game. Next, the system model is introduced.

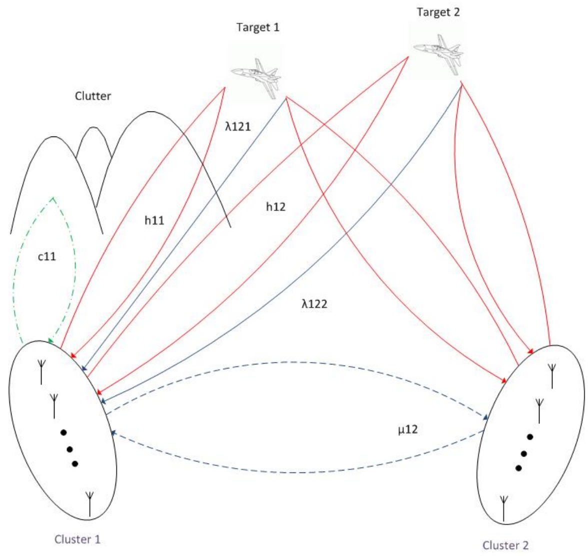


FIGURE 6.1: A multistatic MIMO radar network with two radars and two targets.

6.2 System Model

The system model considers a multistatic radar network that consists of K separate radars each consisting of M transmit/receive antennas. The set of radars is denoted by $C = \{1, \ldots, K\}$. In order to complete the model, L targets are assumed in the far-field of the radars, so that the main objective for each radar is to attain a specific detection performance for every target using the minimum possible transmission power. In the noncooperative design of the multistatic radar network, the radars try to minimize their transmission power independently, having full knowledge of the uplink and the downlink channels of their own radar, whereas they have no knowledge of the inter-radar channel gains. Since it is considered that the radars belong to the same organization, the design of the model is not competitive, as there is no deliberate interference between the radars. However, as no communication between radars is assumed, a noncooperative game is appropriate. An example of a multistatic radar network with two radars, two targets and clutter in the far-field is illustrated at Fig. 6.1.

In order to detect the l^{th} target, the transmit array of the k^{th} radar emits the l^{th} element of the independent, predesigned waveform vector $\boldsymbol{\psi}_k(t) = [\psi_{k1}(t), \ldots, \psi_{kL}(t)]^T$ of size $L \times 1$, which satisfies the orthogonality condition $\int_{T_0} \boldsymbol{\psi}_k(t)\boldsymbol{\psi}_k^H(t)dt = \mathbf{I}_L$, where $(\cdot)^T$ denotes the transpose operator, t refers to the time index within the radar pulse, T_0 is the radar pulse width, $\mathbf{I}_L$ is the $L \times L$ identity matrix, and $(\cdot)^H$ denotes the Hermitian transpose operator. Thus, the waveforms corresponding to different targets are not correlated, i.e.

$\int_{T_0} \boldsymbol{\psi}_{kl}(t)\boldsymbol{\psi}_{kl'}(t)dt = 0$, where $l \neq l'$. It is assumed that the waveform vector maintains the orthogonality condition for a set of acceptable time delays τ_a, $\tau_{a'}$ and Doppler frequency shifts f_{Da}, $f_{Da'}$, such as [112]:

$$\int_{T_0} \boldsymbol{\psi}_{ka}(t-\tau_a)\boldsymbol{\psi}_{ka'}(t-\tau_{a'})e^{j2\pi(f_{Da}-f_{Da'})t}dt \approx \begin{cases} 1, & \text{if } a = a' \\ 0, & \text{if } a \neq a' \end{cases}$$

However, if the waveforms arrive with considerable delays and Doppler shifts, nonzero correlation between waveforms may be expected. This correlation factor is denoted as:

$$\varrho_{k,l,l'}(\tau_{l,l'}) = \int_{T_0} \boldsymbol{\psi}_{kl}(t)\boldsymbol{\psi}_{kl'}(t+\tau_{l,l'})e^{j2\pi(\Delta f)t}dt$$

where $\tau_{l,l'}$ is the relevant delay of the waveform returned from the l^{th} target as compared to the delay of the waveform returned from l'^{th} target. The relative difference in Doppler frequency is given by $\Delta f = f_{Dl} - f_{Dl'}$. This introduces interference between the signals returning from different targets, as discussed later.

The $M \times 1$ vector which consists of the complex elements of the signal transmitted from the k^{th} radar and intended for the l^{th} target is of the form

$$\mathbf{x}_{kl}(t) = \mathbf{w}_{t(k,l)}\psi_{kl}(t)$$

where $\mathbf{w}_{t(k,l)}$ is the $M \times 1$ transmit beamforming vector from radar-k to target-l. Hence, the overall transmitted signal from radar-k is

$$\mathbf{x}_k(t) = \sum_{l=1}^{L} \mathbf{x}_{kl}(t) = \sum_{l=1}^{L} \mathbf{w}_{t(k,l)}\psi_{kl}(t)$$

As depicted in Fig. 6.1, $\mathbf{h}_{kl}$ is the channel gain vector from target-l to radar-k, $\mathbf{c}_{kl}$ denotes the interfering signal returns from the clutter when the k^{th} radar tags target-l. The cross-channel gain between radar-k and radar-i is denoted as $\boldsymbol{\mu}_{ki}$ and $\boldsymbol{\lambda}_{kij}$ represents the inter-radar interfering signal channel at the k^{th} radar echoing from the j^{th} target and emitted from the i^{th} radar. The uplink and downlink parts of the path gains can be obtained by the following equations with respect to the transmit beamforming vectors and the receive beamforming vectors respectively:

$$\mathbf{h}_{t(kl)} = \mathbf{b}(\theta_{kl})\mathbf{w}_{t(k,l)}^H\mathbf{a}(\theta_{kl})\beta_l$$

$$\mathbf{h}_{r(kl)} = \mathbf{b}(\theta_{kl})\mathbf{w}_{r(k,l)}^H\mathbf{a}(\theta_{kl})\beta_l$$

$$\mathbf{c}_{t(kl)} = \mathbf{b}(\theta_{cl(k)})\mathbf{w}^H_{t(k,l)}\mathbf{a}(\theta_{cl(k)})\beta_{cl}$$

$$\mathbf{c}_{r(kl)} = \mathbf{b}(\theta_{cl(k)})\mathbf{w}^H_{r(k,l)}\mathbf{a}(\theta_{cl(k)})\beta_{cl}$$

$$\boldsymbol{\mu}_{t(ki)} = \sum_{j=1}^{L}\mathbf{b}(\theta_{rad(k,i)})\mathbf{w}^H_{t(k,j)}\mathbf{a}(\theta_{rad(i,k)})$$

$$\boldsymbol{\mu}_{r(ki)} = \sum_{j=1}^{L}\mathbf{b}(\theta_{rad(k,i)})\mathbf{w}^H_{r(k,j)}\mathbf{a}(\theta_{rad(i,k)})$$

$$\boldsymbol{\lambda}_{t(kij)} = \mathbf{b}(\theta_{ki})\mathbf{w}^H_{t(i,j)}\mathbf{a}(\theta_{ij})\beta_j$$

$$\boldsymbol{\lambda}_{r(kij)} = \mathbf{b}(\theta_{ki})\mathbf{w}^H_{r(i,j)}\mathbf{a}(\theta_{ij})\beta_j$$

where $\mathbf{w}_{r(k,l)}$ is the $M \times 1$ receive weight vector for radar-k when aimed at target-l, β_l is the complex amplitude proportional to the radar cross section (RCS) of target-l, β_{cl} denotes the RCS amplitude of the clutter and $\mathbf{a}(\theta_{kl})$ and $\mathbf{b}(\theta_{kl})$ are the $M \times 1$ transmit and receive steering vectors for radar-k respectively as defined below:

$$\mathbf{a}(\theta_{kl}) = [1, e^{j\frac{2\pi}{\lambda}dsin(\theta_{kl})}, \ldots, e^{j\frac{2\pi}{\lambda}(M-1)dsin(\theta_{kl})}]^T$$

$$\mathbf{b}(\theta_{kl}) = [1, e^{j\frac{2\pi}{\lambda}dsin(\theta_{kl})}, \ldots, e^{j\frac{2\pi}{\lambda}(M-1)dsin(\theta_{kl})}]^T$$

where d is the distance between the adjacent antennas and is considered the same for all radars, θ_{kl} is the azimuth direction of target-l by considering radar-k as reference, $\theta_{cl(k)}$ is the direction of the clutter as seen from the k^{th} radar and $\theta_{rad(k,i)}$ is the direction of radar-i as observed from radar-k and λ is the wavelength of the transmitted signal. From the definition, it is apparent that the transmit and receive steering vectors are equal, as the uplink and downlink channels remain constant over the duration of a full game.

By matched-filtering at the receiver of radar-k each of the orthogonal waveforms $\psi_{kl}(t-\tau_l)e^{j2\pi f_{Dl}t}$, $l = 1, ..., L$, the desired received signal for the detection of target-l is obtained by

$$y_{des(kl)} = \mathbf{w}^H_{r(k,l)}\mathbf{h}_{t(kl)} \tag{6.1}$$

Considering a distributed, multistatic and multitarget radar scheme, the detection of a target is deteriorated by direct and collateral inter-radar interference, in addition to the interference induced by the signals intended for other targets by the same radar, the clutter effect and the noise power. As a result, the interference signal can be modeled as

$$y_{interf(kl)} = (\sum_{j \neq l}^{L} \mathbf{w}_{r(k,l)}^{H} \mathbf{h}_{t(kj)} \varrho_{k,l,j}(\tau_{l,j}) +$$
$$\sum_{m \neq k}^{K} \sum_{j=1}^{L} \mathbf{w}_{r(k,l)}^{H} \boldsymbol{\lambda}_{t(kmj)} \varrho_{k,l,m,j}(\tau_{l,j}) +$$
$$\sum_{m \neq k}^{K} \mathbf{w}_{r(k,l)}^{H} \boldsymbol{\mu}_{t(km)} + \sum_{i=1}^{L} \mathbf{w}_{r(k,l)}^{H} \mathbf{c}_{t(ki)} + \hat{n}) \tag{6.2}$$

where $\varrho_{k,l,m,j}(\tau_{l,j})$ denotes the correlation factor between the waveform emitted from the k^{th} radar and echoed by the l^{th} target and the waveform emitted from the m^{th} radar but echoed by the j^{th} target.

Since the desired and interfering signals for radar-k regarding target-l are defined in (6.1) and (6.2), the relevant SINR is straightforwardly defined as

$$SINR_{kl} = \frac{\|y_{des(kl)}\|^2}{\|y_{interf(kl)}\|^2} \tag{6.3}$$

where $||\cdot||$ denotes the Euclidian norm.

Using the above system model, the next section describes the game theoretic formulation of the proposed scheme.

6.3 Beamformer Design and Power Allocation Game

6.3.1 Game Theoretic Formulation

In order to determine the optimal transmit/receive beamformers and power allocation between the radars, an SNG is incorporated. The various radars are considered as players, and therefore the player set is denoted by $C = \{1, \ldots, K\}$. Consider the transmit beamforming weight vector matrix $\mathbf{W}_{t(k)} = \{\mathbf{w}_{t(k,1)}, \ldots, \mathbf{w}_{t(k,L)}\}$ as the strategy of player-k and the matrix $\mathbf{W}_{t(-k)}$ as the strategy chosen by the other players. Hence, the acceptable strategy set for radar-k is defined as

$$\mathcal{P}_k(\mathbf{W}_{t(-k)}) = \{\mathbf{W}_{t(k)} \in \mathbb{C}^{M \times L} \mid SINR_{kl} \geq \gamma_{kl}, \forall l\}$$

where γ_{kl} is the desired SINR for target-l when targeted from the antennas of radar-k. The decision on the desired SINR depends on the probabilities of misdetection P_{md} and false alarm P_{fa}, which are derived from the following equations [115, 116]:

$$P_{md}(\xi_{kl}) = (1 - \xi_{kl})^{N-1}$$

$$P_{fa}(\mathrm{SINR}_{kl}, \xi_{kl}) = 1 - \left(1 - \frac{\xi_{kl}}{1-\xi_{kl}} \frac{1}{1 + N\mathrm{SINR}_{kl}}\right)^{1-N}$$

where ξ_{kl} denotes the threshold of the generalized likelihood ratio test (GLRT), applied to determine if there is absence or presence of a target [116] and N is the number of samples used for the GLRT. A specific design parameter ε_{kl} is defined to set an upper bound on the tolerance regarding P_{md} and P_{fa}. Hence, the optimum SINR_{kl} for each radar regarding each target can be determined as [35, 36]:

$$\gamma^*_{kl} = \min\{\mathrm{SINR}_{kl} \mid \exists \xi_{kl} \in [0,1] \text{ s.t. } P_{md}(\xi_{kl}) + P_{fa}(\mathrm{SINR}_{kl}, \xi_{kl}) \leq \varepsilon_{kl}\}. \quad (6.4)$$

It is evident from (6.3) that the $SINR_{kl}$ for player-k is a function of the beamforming weight vectors (which include transmission power) of all players. Hence, the set of admissible strategies $\mathcal{P}_k(\mathbf{W}_{t(-k)})$ for radar-k depends on the beamforming weight matrix $\mathbf{W}_{t(-k)}$ of every other player (radar).

The last component required to complete the game is the utility function for each player, which is defined as $u_k(\mathbf{W}_{t(k)}) = \|\mathbf{W}_{t(k)}\|_F^2$ representing the transmit power of player-k, where $|| \cdot ||_F$ denotes the Frobenius norm. The game is summarized as

$$\mathcal{G} =< C, \{\mathcal{P}_k(\mathbf{W}_{t(-k)})\}_{k\in C}, \{u_k(\mathbf{W}_{t(k)})\}_{k\in C} >$$

In the SNG considered, given the beamforming strategies of the other players, each player selfishly minimizes its power allocation subject to a predefined detection criterion. As a result, the best response strategy for player-k is the result of the following optimization:

$$\min_{\mathbf{W}_{t(k)}} \quad \|\mathbf{W}_{t(k)}\|_F^2 \quad (6.5)$$

$$s.t. \frac{|\mathbf{w}_{t(k,l)}^H \mathbf{h}_{r(kl)}|^2}{\sum_{j\neq l}^{L} |\mathbf{w}_{t(k,l)}^H \mathbf{h}_{r(kj)}|^2 + \sum_{i=1}^{L} |\mathbf{w}_{t(k,l)}^H \mathbf{c}_{r(ki)}|^2 + r_{-kl}} \geq \gamma_{kl}, \forall l$$

where $\mathbf{r}_{-k} = [r_{-k1}, \ldots, r_{-kL}]^T$ is the total interference induced by all other radars except radar-k plus the additive white Gaussian noise (AWGN) from the environment vector. For target-l, it is defined as $r_{-kl} = \sum_{m\neq k}^{K} \sum_{j=1}^{L} |\mathbf{w}_{t(k,l)}^H \lambda_{r(kmj)}|^2 + \sum_{m\neq k}^{K} |\mathbf{w}_{t(k,l)}^H \mu_{r(km)}|^2 + \sigma_n^2$.

One of the main objectives of this work is to investigate whether the game $\mathcal{G}$ converges to a stable point, where no player can profit by unilaterally changing its beamforming strategy, as it will lead to higher power consumption to achieve the same SINR for every target. Such a point is a Nash Equilibrium (NE) and for the game considered, it is defined as the strategy set $\{\mathbf{W}_{t(1)}^*, \ldots, \mathbf{W}_{t(K)}^*\}$ where:

$$u_k(\mathbf{W}_{t(k)}^*) \leq u_k(\mathbf{W}_{t(k)}), \quad \forall \mathbf{W}_{t(k)} \in \mathcal{P}_k(\mathbf{W}_{t(-k)}^*), \forall k \in C$$

In the next section the optimal beampatterns will be determined and the best response strategy will be investigated. Also, the existence and uniqueness of the NE of the game $\mathcal{G}$ will be proved.

6.3.2 Convex Optimization Beamforming and the Best Response Strategy

Convex optimization has been widely utilized in the radar beamforming literature. Most of the work concentrates on designing the beamforming vectors in order to approximate a desired beampattern, decided by the target position [21, 122, 11, 113]. In the first stage of this analysis, the optimal beampattern for every radar is determined corresponding to each of the targets using convex optimization techniques. After securing the optimal beampatterns, each player should just allocate the minimum possible transmission power, while minimizing the inter-radar interference and achieving a certain detection performance.

The optimal transmit beampatterns for each radar can be designed by solving the following optimization problem:

$$\min_{\mathbf{W}_{t(k)}} \quad \sum_{l=1}^{L} \|\mathbf{w}_{t(k,l)}\|^2 \tag{6.6}$$

$$s.t. \frac{|\mathbf{w}_{t(k,l)}^H \mathbf{h}_{r(kl)}|^2}{\sum_{j \neq l}^{L} |\mathbf{w}_{t(k,l)}^H \mathbf{h}_{r(kj)}|^2 + \sum_{i=1}^{L} |\mathbf{w}_{t(k,l)}^H \mathbf{c}_{r(ki)}|^2 + r_{-kl}} \geq \gamma_{kl}, \forall l$$

The optimization in (6.6) can be converted to semidefinite programming (SDP) using the rank relaxation method and solved as in [22] and [111]. The optimal receive weight vectors can be found using generalised eigenvector techniques.

Claim 1: The optimal transmit and receive beampatterns are independent of the inter-radar interference $\mathbf{r}_{-k}$.

Proof: The proof can be found in Appendix A.

Hence, when the radars reallocate the power of transmission, the inter-radar interference plus noise vector $\mathbf{r}_{-k}$ is modified. From Claim 1, radar-k retains the optimal beampatterns derived from (6.6), however reallocates only its transmission power for each target, in order to achieve the detection criterion. This observation is similar to that considered in wireless communication applications [32], regardless of the appearance of additional clutter in the denominator of the SINR equation in (6.6). As a result, after obtaining the optimal transmit/receive beamforming vectors, the initial optimization problem (6.5) can be reformulated as a power minimization problem shown below:

$$\min_{p_{k1},\cdots,p_{kL}} \quad \sum_{l=1}^{L} p_{kl} \tag{6.7}$$

$$s.t. \frac{p_{kl}|\hat{\mathbf{w}}_{t(k,l)}^H \mathbf{h}_{r(kl)}|^2}{\sum_{j \neq l}^{L} p_{kj}|\hat{\mathbf{w}}_{t(k,l)}^H \mathbf{h}_{r(kj)}|^2 + \sum_{i=1}^{L} p_{ki}|\hat{\mathbf{w}}_{t(k,l)}^H \mathbf{c}_{r(ki)}|^2 + r_{-kl}} \geq \gamma_{kl}$$

where $\hat{\mathbf{w}}_{t(k,l)} = \frac{\mathbf{w}^*_{t(k,l)}}{\|\mathbf{w}^*_{t(k,l)}\|}$ is the normalised optimal transmit weight vector and p_{kl} is the power used by radar-k on the beam directed to target-l. At this point, by redefining the acceptable strategy as $\mathcal{P}'_k(\mathbf{p}_{-k}) = \{\mathbf{p}_k \in \mathbb{R}^L_+ \mid SINR_{kl} \geq \gamma_{kl}, \forall l\}$ and the utility function as $u'_k(\mathbf{p}_k) = \sum_{l=1}^{L} p_{kl}$, game $\mathcal{G}$ becomes a power allocation SNG:

$$\mathcal{G}' =< C, \{\mathcal{P}'_k(\mathbf{p}_{-k})\}_{k\in C}, \{u'_k(\mathbf{p}_k)\}_{k\in C} >$$

In order to prove the existence and the uniqueness of the NE of game G, it is needed to show that the best response strategy for every player is a standard function. It must be highlighted that all the constraints must be active at the optimal power allocation. As a result, the inequality in the constraints of (6.7) can be replaced by equality and can be written as:

$$\mathbf{G}_k \mathbf{p}_k^* = \mathbf{r}_{-k} \tag{6.8}$$

where $\mathbf{G}_k \in \mathbb{R}^{L \times L}$ and its elements are defined as $[\mathbf{G}_k]_{ii} = (\frac{|\hat{\mathbf{w}}_{t(k,i)}^H \mathbf{h}_{r(ki)}|^2}{\gamma_{ki}} - |\hat{\mathbf{w}}_{t(k,i)}^H c_{r(ki)}|^2)$ and $[\mathbf{G}_k]_{ij} = -|\hat{\mathbf{w}}_{t(k,i)}^H h_{r(kj)}|^2 - |\hat{\mathbf{w}}_{t(k,i)}^H c_{r(kj)}|^2$, for $i \neq j$. The solution of (6.8) provides the optimal power allocation for (6.7). Following Claim 2 in [32], the problem (6.7) is always feasible $\forall \mathbf{r}_{-k} > 0$ elementwise. As a result, the matrix $\mathbf{G}_k$ must be invertible so the best response strategy for the k^{th} cluster can be straightforwardly obtained as:

$$\mathbf{p}_k^* = \mathbf{G}_k^{-1} \mathbf{r}_{-k} \tag{6.9}$$

The existence of the solution is guaranteed through the Arrow-Debreu theorem [117]. Since the NE exists, the uniqueness of this NE is proved by establishing the best response function is standard [32]. The inter-cluster interference matrix from the m^{th} radar to the k^{th} radar is defined as $\mathbf{G}_{mk} \in \mathbb{R}^{L \times L}$ and $[\mathbf{G}_{mk}]_{i,j} = |\hat{\mathbf{w}}_{t(k,i)}^H \lambda_{r(kmj)}|^2 + |\hat{\mathbf{w}}_{t(k,i)}^H \mu_{r(km)}|^2$. Hence, by replacing the interference vector $\mathbf{r}_{-k}$, the best response strategy can be restated as:

$$BR_k(\mathbf{p}_{-q}) = \mathbf{p}_k^* = \mathbf{G}_k^{-1} \left(\sum_{m \neq k}^{K} \mathbf{G}_{mk} \mathbf{p}_m^* + \mathbf{1}_L \sigma_n^2 \right), \forall k \tag{6.10}$$

where $\mathbf{1}_L$ denotes the all ones vector of size $L \times 1$.

Lemma 1: The best response function (6.10) is a standard function.

Proof: The best response strategy (6.10) satisfies the following necessary properties for all $\mathbf{p} \geq 0$:

a) Positivity: $BR_k(\mathbf{p}) > 0$, as $\mathbf{G}_k^{-1}$ is a positive matrix straightforwardly from (6.9) and $\mathbf{G}_{mk}$ is a positive matrix from its definition.

b) Monotonicity: If $\mathbf{p} \geq \mathbf{p}'$, then:

$$BR_k(\mathbf{p}) - BR_k(\mathbf{p}') = \mathbf{G}_k^{-1} \left(\sum_{m \neq k}^{K} \mathbf{G}_{mk} (\mathbf{p}_m - \mathbf{p}'_m) \right) \geq 0$$

c) Scalability: For all $a > 1$, $aBR_k(\mathbf{p}) > BR_k(\mathbf{ap})$. Indeed:

$$aBR_k(\mathbf{p}) - BR_k(a\mathbf{p}) = (a-1)\mathbf{G}_k^{-1} \mathbf{1}_L \sigma_n^2 > 0.$$

By applying a pricing policy to each player some cooperation is introduced among them, which leads to a more Pareto efficient solution, as described in the next section.

6.4 Beamformer Design and Power Allocation Game with Pricing

6.4.1 Game Theoretical formulation

Since each radar optimizes its beamformers and power allocation greedily, the equilibrium point is not necessarily the best solution from a social fairness point of view. This is explained because each player ignores the direct path interference it induces on other players. In order to obtain a more Pareto efficient solution and to increase the social welfare of the SNG, a pricing scheme is introduced and applied to each radar's utility function. As a result, the players are encouraged to allocate their available resources more efficiently by minimizing the direct path interference induced to the other radars.

In order to achieve the aforementioned advantages, each radar/player needs to have information about the channel to the other radars in the system. Since the radars are presumed to belong to the same organization, the knowledge of the channels between the radars is justified, as each radar knows the exact position of the others. Hence, each radar performs the following optimization:

$$\min_{\mathbf{W}_{t(k)}} \quad \sum_{l=1}^{L} \|\mathbf{w}_{t(k,l)}\|^2 + \sum_{m\neq k}^{K} \sum_{i=1}^{L} \kappa_{kmi} \|\mathbf{w}_{t(k,i)} \boldsymbol{\mu}_{r(km)}\|^2 \tag{6.11}$$

$$s.t. \frac{|\mathbf{w}_{t(k,l)}^H \mathbf{h}_{r(kl)}|^2}{\sum_{j\neq l}^{L} |\mathbf{w}_{t(k,l)}^H \mathbf{h}_{r(kj)}|^2 + \sum_{i=1}^{L} |\mathbf{w}_{t(k,l)}^H \mathbf{c}_{r(ki)}|^2 + r_{-kl}} \geq \gamma_{kl}, \forall l$$

where κ_{kmi} is the price charged to radar k for the interference it induces to radar m when aiming at target i and $\|\mathbf{w}_{t(k,i)} \boldsymbol{\mu}_{r(km)}\|^2$ denotes the corresponding interference.

The aforementioned optimization encourages each player to adopt a more socially efficient power allocation strategy by steering its beampattern to the desired target, while keeping the sidelobes at the direction of the other players low and therefore causing less interference to other radars. As a result, the efficiency of the system as a whole is improved, yet the distributed nature of the game is preserved.

In order to reformulate the SNG $\mathcal{G}$ to a more cooperative game with pricing cosideration, it is just needed to redefine the utility function of radar k as

$\upsilon_k(\mathbf{W}_{t(k)}) = \|\mathbf{W}_{t(k)}\|_F^2 + \sum_{m\neq k}^{K}\sum_{i=1}^{L}\kappa_{kmi}\|\mathbf{w}_{t(k,i)}\boldsymbol{\mu}_{km}\|^2$. The mathematical form of the pricing game is:

$$\mathcal{G}_{pr} =< C, \{\mathcal{P}_k(\mathbf{W}_{t(-k)})\}_{k\in C}, \{\upsilon_k(\mathbf{W}_{t(k)})\}_{k\in C} >$$

6.4.2 Optimal Beamforming and the Best Response Strategy

In this section, the optimal transmit and receive beamformers and the best response strategy for each of the players are designed. Therefore, the fact that the optimization problem (6.11) can be reformulated as a convex optimization problem with second order cone (SOC) constraints [22] allows to obtain the optimal solution via duality. The Lagrangian associated with the optimization problem (6.11) can be written as:

$$\mathcal{L}(\mathbf{W}_{t(k)}, \boldsymbol{\lambda}_k) = \sum_{l=1}^{L}\|\mathbf{w}_{t(k,l)}\|^2$$

$$+\sum_{m\neq k}^{K}\sum_{i=1}^{L}\kappa_{kmi}\|\mathbf{w}_{t(k,i)}\boldsymbol{\mu}_{r(km)}\|^2$$

$$+\sum_{l=1}^{L}\lambda_{kl}\Bigg(\sum_{j\neq l}^{L}|\mathbf{w}_{t(k,l)}^H\mathbf{h}_{r(kj)}|^2 + \sum_{i=1}^{L}|\mathbf{w}_{t(k,l)}^H\mathbf{c}_{r(ki)}|^2 + r_{-kl}$$

$$-\frac{1}{\gamma_{kl}}|\mathbf{w}_{t(k,l)}^H\mathbf{h}_{r(kl)}|^2\Bigg)$$

where $\boldsymbol{\lambda}_k = [\lambda_{k1}, \ldots, \lambda_{kL}]^T$ is the $L \times 1$ vector of the Lagrangian multipliers associated with the SINR inequality constraints of the problem in (10). The Lagrangian can be reorganized as:

$$\mathcal{L}(\mathbf{W}_{t(k)}, \boldsymbol{\lambda}_k) = \sum_{l=1}^{L}\lambda_{kl}r_{-kl} + \sum_{l=1}^{L}\mathbf{w}_{t(k,l)}^H\Bigg(\boldsymbol{\Omega}_k(\kappa_{kml})$$

$$-\frac{\lambda_{kl}}{\gamma_{kl}}\mathbf{h}_{r(ki)}\mathbf{h}_{r(ki)}^H + \sum_{j\neq l}^{L}\lambda_{kj}\mathbf{h}_{r(kj)}\mathbf{h}_{r(kj)}^H$$

$$+\sum_{i=1}^{L}\lambda_{kl}\mathbf{c}_{r(ki)}\mathbf{c}_{r(ki)}^H\Bigg)\mathbf{w}_{t(k,l)}$$

where $\boldsymbol{\Omega}_k(\kappa_{kmi}) = \sum_{m\neq k}^{K}\sum_{i=1}^{L}\kappa_{kmi}\boldsymbol{\mu}_{r(km)}\boldsymbol{\mu}_{r(km)}^H + \mathbf{I}$. At this point, the Lagrange dual function is defined as the minimum value of the Lagrangian over $\mathbf{W}_{t(k)}$:

$$g_k(\boldsymbol{\lambda}_k) = \inf_{\mathbf{W}_{t(k)}} \mathcal{L}(\mathbf{W}_{t(k)}, \boldsymbol{\lambda}_k)$$

It is clear that if $\boldsymbol{\Omega}_k(\kappa_{kml}) - \frac{\lambda_{kl}}{\gamma_{kl}}\mathbf{h}_{r(ki)}\mathbf{h}_{r(ki)}^H + \sum_{j\neq l}^{L}\lambda_{kj}\mathbf{h}_{r(kj)}\mathbf{h}_{r(kj)}^H + \sum_{i=1}^{L}\lambda_{kl}\mathbf{c}_{r(ki)}\mathbf{c}_{r(ki)}^H$ is not positive semi-definite, the Lagrangian is unbounded below in $\mathbf{W}_{t(k)}$ and the dual function can take the value $-\infty$. Hence, the dual problem associated with (6.11) can be formulated as:

$$\max_{\lambda_{k1},\ldots,\lambda_{kL}} \quad \sum_{l=1}^{L}\lambda_{kl}r_{-kl} \tag{6.12}$$

$$s.t. \quad \sum_{i=1}^{L}\lambda_{kl}\mathbf{h}_{r(ki)}\mathbf{h}_{r(ki)}^H + \sum_{i=1}^{L}\lambda_{kl}\mathbf{c}_{r(ki)}\mathbf{c}_{r(ki)}^H + \boldsymbol{\Omega}_k(\kappa_{kml})$$
$$\succeq \left(1 + \frac{1}{\gamma_{kl}}\right)\lambda_{kl}\mathbf{h}_{r(kl)}\mathbf{h}_{r(kl)}^H, \quad \forall l$$

As mentioned in [118] and [32], where the authors investigate the downlink beamforming problem for communications application, the dual problem (6.12) is analogous to the following receive beamforming optimization problem:

$$\min_{\substack{\lambda_{k1},\ldots,\lambda_{kL}\\ \mathbf{w}_{r(k,1)},\ldots,\mathbf{w}_{r(k,L)}}} \quad \sum_{l=1}^{L}\lambda_{kl}r_{-kl} \tag{6.13}$$

$$s.t. \quad \frac{\lambda_{kl}|\mathbf{w}_{r(k,l)}^H\mathbf{h}_{t(kl)}|^2}{\sum_{j\neq l}^{L}\lambda_{kj}|\mathbf{w}_{r(k,l)}^H\mathbf{h}_{t(kj)}|^2 + \sum_{i=1}^{L}|\mathbf{w}_{r(k,l)}^H\mathbf{c}_{t(ki)}|^2 + \mathbf{w}_{r(k,l)}^H\boldsymbol{\Omega}_k(\kappa_{kmi})\mathbf{w}_{r(k,l)}} \geq \gamma_{kl}, \forall l$$

Since the constraints are satisfied with equality at optimality, the optimal Lagrangian multipliers can be obtained by applying the fixed point iteration [118], as shown below:

$$\lambda_{kl}^{(n+1)} = \frac{\gamma_{kl}}{1+\gamma_{kl}} \times \frac{1}{\mathbf{h}_{t(kl)}^H\left(\sum_{i=1}^{L}\lambda_{kl}^n\mathbf{h}_{t(ki)}\mathbf{h}_{t(ki)}^H + \sum_{i=1}^{L}\lambda_{kl}^n\mathbf{c}_{t(ki)}\mathbf{c}_{t(ki)}^H + \boldsymbol{\Omega}_k(\kappa_{kml})\right)^{-1}\mathbf{h}_{t(kl)}} \tag{6.14}$$

As proved in [118], the fixed point iteration described in (6.14) is shown to be a standard function and is guaranteed to converge to a unique solution, if the optimization problem (6.12) is feasible.

Subsequently, the optimal receive weight vector is the minimum mean-square error (MMSE) receiver, obtained as the following equation:

$$\mathbf{w}_{r(k,l)} = \left(\sum_{i=1}^{L} \lambda_{kl} \mathbf{h}_{t(ki)} \mathbf{h}_{t(ki)}^{H} + \sum_{i=1}^{L} \lambda_{kl} \mathbf{c}_{t(ki)} \mathbf{c}_{t(ki)}^{H} + \mathbf{\Omega}_{k}(\kappa_{kml}) \right)^{-1} \mathbf{h}_{t(kl)} \quad (6.15)$$

Following [123], the optimal transmit beamformer can be obtained as a scaled version of the receive weight vector, $\mathbf{w}_{t(k,l)} = \sqrt{\delta_{k,l}} \mathbf{w}_{r(k,l)}$, where $\delta_{k,l}$ is a scalar factor. The scaling factors $\delta_{k,l}$ can be found by exploiting the fact that the SINR constraints in (6.11) are met with equality at optimality. Hence by replacing $\mathbf{w}_{t(k,l)} = \sqrt{\delta_{k,l}} \mathbf{w}_{r(k,l)}$ into the SINR constraints, the scaling factors can be found from the following equation:

$$\boldsymbol{\delta_k} = \mathbf{F}^{-1} \mathbf{r}_{-k} \quad (6.16)$$

where $\boldsymbol{\delta_k} = [\delta_{k1}, \delta_{k2}, \ldots, \delta_{kL}]^T$ and $\mathbf{F} \in \mathbb{R}^{L \times L}$ and is defined as $[\mathbf{F}]_{ii} = (\frac{|\mathbf{w}_{r(k,i)}^{H} \mathbf{h}_{t(ki)}|^2}{\gamma_{ki}} - |\mathbf{w}_{r(k,i)}^{H} c_{t(ki)}|^2)$ and $[\mathbf{F}]_{ij} = -|\mathbf{w}_{r(k,i)}^{H} h_{t(kj)}|^2 - |\mathbf{w}_{r(k,i)}^{H} c_{t(kj)}|^2$, for $i \neq j$.

Having decided the optimal transmit and receive beamformers, the solution of problem (6.11) is concluded. Similar to the game without pricing consideration, the initial optimization problem (6.11) can be reformulated as a power minimization problem. Following the same analysis as in Section **III** and by denoting the power vector of radar k as $\boldsymbol{\pi}_k \in \mathbb{R}_{+}^{L}$, the best response strategy for the k^{th} radar can be obtained from the following equation:

$$\boldsymbol{\pi}_k^{*} = \mathbf{\Delta}_k^{-1} \mathbf{r}_{-k} \quad (6.17)$$

where $\mathbf{\Delta}_k \in \mathbb{R}^{L \times L}$ and is defined as $[\mathbf{\Delta}_k]_{ii} = (\frac{|\mathbf{w}_{t(k,i)}^{H} \mathbf{h}_{r(ki)}|^2}{\gamma_{ki}} - |\mathbf{w}_{t(k,i)}^{H} c_{r(ki)}|^2)$ and $[\mathbf{\Delta}_k]_{ij} = -|\mathbf{w}_{t(k,i)}^{H} h_{r(kj)}|^2 - |\mathbf{w}_{t(k,i)}^{H} c_{r(kj)}|^2$, for $i \neq j$. Moreover, the inter-radar interference matrix from the m^{th} radar to the k^{th} radar is denoted as $\mathbf{\Delta}_{mk} \in \mathbb{R}^{L \times L}$ and $[\mathbf{\Delta}_{mk}]_{i,j} = |\mathbf{w}_{t(k,i)}^{H} \lambda_{kmj}|^2 + |\mathbf{w}_{t(ki)}^{H} \mu_{r(km)}|^2$. Consequently, by replacing the interference vector $\mathbf{r}_{-k} = \sum_{m \neq k}^{K} \mathbf{\Delta}_{mk} \mathbf{p}_m^{*} + \mathbf{1}\sigma_n^2$ the best response strategy may be redefined as:

$$BR_k(\boldsymbol{\pi}_{-k}) = \boldsymbol{\pi}_k^{*} = \mathbf{\Delta}_k^{-1} \left(\sum_{m \neq k}^{K} \mathbf{\Delta}_{mk} \boldsymbol{\pi}_m^{*} + \mathbf{1}\sigma_n^2 \right), \forall k \quad (6.18)$$

Lemma 2: The best response function (6.17) of the game with pricing consideration is a standard function.

Proof: The proof is identical to that in Lemma 1.

In the next section a hierarchical strategic game is presented, known as Stackelberg game.

6.5 Stackelberg Game System Model

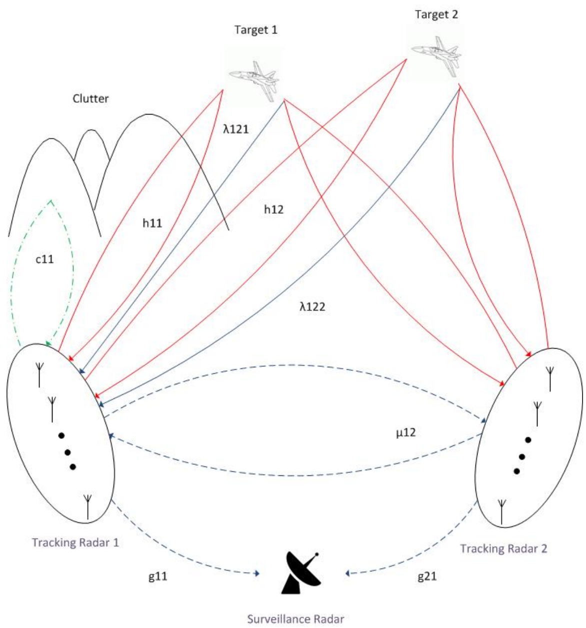


FIGURE 6.2: A hybrid distributed MIMO radar network with a surveillance radar, two tracking radars and two targets.

In this section, a hybrid MIMO network is considered. More specifically, in addition to the multistatic tracking radar network mentioned in Section 6.2, a surveillance radar is incorporated as part of the network, as seen in Fig. 6.2. It is presumed that all radars belong to the same organization and operate in the same field. As a result, the tracking radars may interfere with the surveillance radar and deteriorate its performance (increase the probability of false alarm). In order to guarantee the unimpeded operation of the system, the interference observed at the surveillance radar must not exceed a specific value, as shown below:

$$\sum_{k=1}^{K}\sum_{l=1}^{L}|q_{r(sur)}^{H}g_{kl}|^2 \leq I_{max} \tag{6.19}$$

where $g_{kl} = \mathbf{w}_{t(k,l)}^{H}\mathbf{a}(\theta_{sur(k)})$ denotes the interfering signal in the direction of the surveillance radar when the k^{th} tracking radar tags target l, $\theta_{sur(k)}$ is the direction of the surveillance radar as observed from the k^{th} tracking radar, and I_{max} is the maximum interference allowed. Since there is no transmit or receive beamformer at the surveillance radar, its receive filter $q_{r(sur)}$ is a complex scalar.

In order to guarantee constraint (6.19), an interference cost can be imposed on every tracking radar in order to minimize their effect on the surveillance radar. Thus, a similar pricing mechanism to the previous section can be applied to every radar with the main objective to minimize the direct path interference to the surveillance radar. Owing to the fact that all radars belong to the same organization, it can be safely assumed that the information of the inter-radar channels is given. Similarly to the previous section, each tracking radar performs the following optimization:

$$\min_{\mathbf{W}_{t(k)}} \quad \sum_{l=1}^{L} \|\mathbf{w}_{t(k,l)}\|^2 + \sum_{i=1}^{L} \kappa_{sur} \|\mathbf{w}_{t(k,i)}\mathbf{g}_{kl}\|^2 \tag{6.20}$$

$$s.t. \frac{|\mathbf{w}_{t(k,l)}^H \mathbf{h}_{r(kl)}|^2}{\sum_{j\neq l}^{L} |\mathbf{w}_{t(k,l)}^H \mathbf{h}_{r(kj)}|^2 + \sum_{i=1}^{L} |\mathbf{w}_{t(k,l)}^H \mathbf{c}_{r(ki)}|^2 + r_{-kl}} \geq \gamma_{kl}, \forall l$$

where κ_{sur} is the pricing factor of interference, which is equally imposed by the surveillance radar to all tracking radars.

This interaction between the radars can be translated to a power allocation Stackelberg game, where the surveillance radar is the leader and the tracking radars are the followers. The strategy of the leader is the price of interference charged to the followers and the leader's utility function is its profit, which is defined as:

$$s_{lead} = \sum_{k=1}^{K} \sum_{l=1}^{L} |w_{r(sur)}^H g_{kl}|^2 \kappa_{sur} \tag{6.21}$$

Based on the price imposed by the leader, the followers decide their best response strategy as the result of the optimization in (6.20).

6.5.1 Followers' Game

Since the followers know the price of interference announced by the leader, they decide their optimal beamformers and resource allocation by solving the optimization problem in (6.20). In order to formulate the followers' game, it is obvious that this game is similar to the game $\mathcal{G}_{pr}$, when the utility function of player k is redefined as $s_k(\mathbf{W}_{t(k)}) = \|\mathbf{W}_{t(k)}\|_F^2 + \sum_{i=1}^{L} \kappa_{sur}\|\mathbf{w}_{t(k,i)}\mathbf{g}_{kl}\|^2$. Hence the mathematical representation of the followers' game is:

$$\mathcal{G}_{fol} =< C, \{\mathcal{P}_k(\mathbf{W}_{t(-k)})\}_{k\in C}, \{s_k(\mathbf{W}_{t(k)})\}_{k\in C} >$$

Following the same analysis as for game $\mathcal{G}_{pr}$, the optimal beamforming vectors can be derived by exploiting the duality properties of the convex optimization problem (6.19). Hence, respectively to the receive weight vector optimization problem (6.13), the following optimization problem is addressed:

$$\min_{\substack{\bar{\lambda}_{k1},\ldots,\bar{\lambda}_{kL} \\ \bar{\mathbf{w}}_{r(k,1)},\ldots,\bar{\mathbf{w}}_{r(k,L)}}} \quad \sum_{l=1}^{L} \bar{\lambda}_{kl} r_{-kl} \tag{6.22}$$

$$s.t. \quad \frac{\bar{\lambda}_{kl}|\bar{\mathbf{w}}_{r(k,l)}^H \mathbf{h}_{t(kl)}|^2}{\sum_{j\neq l}^{L} \bar{\lambda}_{kj}|\bar{\mathbf{w}}_{r(k,l)}^H \mathbf{h}_{t(kj)}|^2 + \sum_{i=1}^{L} |\bar{\mathbf{w}}_{r(k,l)}^H \mathbf{c}_{t(ki)}|^2 + \bar{\mathbf{w}}_{r(k,l)}^H \mathbf{\Omega}_k(\kappa_{sur})\bar{\mathbf{w}}_{r(k,l)}} \geq \gamma_{kl}, \forall l$$

The vector of the Lagrangian multipliers associated with the inequality SINR constraints of problem (6.20) is denoted as the $L\times 1$ vector $\bar{\boldsymbol{\lambda}}_k = [\bar{\lambda}_{k1}, \ldots, \bar{\lambda}_{kL}]^T$, $\mathbf{\Omega}_k(\kappa_{sur}) = \sum_{i=1}^{L} \kappa_{sur}\mathbf{g}_{kl}\mathbf{g}_{kl}^H + \mathbf{I}$ and $\bar{\mathbf{w}}_{r(k,l)}$ denotes the $M \times 1$ receive weight vector for radar-k regarding target-l for the study of the Stackelberg game. Similar to (6.14), the optimal Lagrangian multipliers are obtained as shown below (the fixed point iteration below is a standard function and admits a unique solution [118]):

$$\bar{\lambda}_{kl}^{(n+1)} = \frac{\gamma_{kl}}{1+\gamma_{kl}} \times \frac{1}{\mathbf{h}_{t(kl)}^H \left(\sum_{i=1}^{L} \bar{\lambda}_{kl}^n \mathbf{h}_{t(ki)}\mathbf{h}_{t(ki)}^H + \sum_{i=1}^{L} \bar{\lambda}_{kl}^n \mathbf{c}_{t(ki)}\mathbf{c}_{t(ki)}^H + \mathbf{\Omega}_k(\kappa_{sur})\right)^{-1} \mathbf{h}_{kl}} \tag{6.23}$$

and the optimal receive beamformers through the MMSE receiver as:

$$\bar{\mathbf{w}}_{r(k,l)} = \left(\sum_{i=1}^{L} \bar{\lambda}_{kl}\mathbf{h}_{t(ki)}\mathbf{h}_{t(ki)}^H + \sum_{i=1}^{L} \bar{\lambda}_{kl}\mathbf{c}_{t(ki)}\mathbf{c}_{t(ki)}^H + \mathbf{\Omega}_k(\kappa_{sur})\right)^{-1} \mathbf{h}_{t(kl)} \tag{6.24}$$

The optimal transmit beamformers are scaled versions of the optimal receive weight vectors:

$$\bar{\mathbf{w}}_{t(k,l)} = \sqrt{\delta_{k,l}}\bar{\mathbf{w}}_{r(k,l)} \tag{6.25}$$

Correspondingly to the method of $\mathcal{G}_{pr}$ and by indicating the power vector of radar k as $\boldsymbol{\rho}_k \in \mathbb{R}_+^L$, the best response strategy for the k^{th} radar can be obtained from the following equation:

$$\boldsymbol{\rho}_k^* = \boldsymbol{\Xi}_k^{-1}\mathbf{r}_{-k} \tag{6.26}$$

where $\boldsymbol{\Xi}_k \in \mathbb{R}^{L\times L}$ and is defined as $[\boldsymbol{\Xi}_k]_{ii} = (\frac{|\bar{\mathbf{w}}_{t(k,i)}^H\mathbf{h}_{r(ki)}|^2}{\gamma_{ki}} - |\bar{\mathbf{w}}_{t(ki)}^H c_{r(ki)}|^2)$ and $[\boldsymbol{\Xi}_k]_{ij} = -|\bar{\mathbf{w}}_{t(ki)}^H h_{r(kj)}|^2 - |\bar{\mathbf{w}}_{t(ki)}^H c_{r(kj)}|^2$, for $i \neq j$. Furthermore, the inter-radar interference matrix from the m^{th} radar to the k^{th} radar is denoted as $\boldsymbol{\Xi}_{mk} \in \mathbb{R}^{L\times L}$ and $[\boldsymbol{\Xi}_{mk}]_{i,j} = |\bar{\mathbf{w}}_{t(ki)}^H \lambda_{r(kmj)}|^2 + |\bar{\mathbf{w}}_{t(ki)}^H \mu_{r(km)}|^2$. Consequently, by replacing the interference vector $\mathbf{r}_{-k} = \sum_{m\neq k}^K \boldsymbol{\Xi}_{mk}\mathbf{p}_m^* + \mathbf{1}\sigma_n^2$, the best response strategy can be redefined as:

$$BR_k(\boldsymbol{\rho}_{-k}) = \boldsymbol{\rho}_k^* = \boldsymbol{\Xi}_k^{-1}\left(\sum_{m\neq k}^K \boldsymbol{\Xi}_{mk}\boldsymbol{\rho}_m^* + \mathbf{1}\sigma_n^2\right), \forall k \tag{6.27}$$

The study on the existence and the uniqueness of the solution is similar to the one in Section **II**.

6.5.2 Leader's Game

From the definition of the Stackelberg game, the leader knows the best response strategy of the followers. Likewise in the considered model, the surveillance radar is aware of the existence of the tracking radars, as they belong to the same organization, and can determine the followers best response strategy. Hence, the leader's optimal strategy is extracted from the following optimization problem, where the leader's profit is maximized, while the interference is constrained under a maximum value to guarantee the efficient performance of the surveillance radar.

$$\max_{\kappa_{sur}} \sum_{k=1}^K \sum_{l=1}^L |w_{r(sur)}^H g_{kl}|^2 \kappa_{sur} \tag{6.28}$$

$$s.t. \quad \sum_{k=1}^K \sum_{l=1}^L |w_{r(sur)}^H g_{kl}|^2 \leq I_{max}$$

In order to determine the optimal price imposed by the leader to the tracking radars and solve the optimization problem (6.28), the learning algorithm for the leader is adopted as proposed in [27]. Initially, the price κ^*_{sur} is determined at the point where the constraint of the optimization problem (6.28) is met with equality:

$$\sum_{k=1}^{K}\sum_{l=1}^{L}|w^H_{r(sur)}g_{kl}|^2 = I_{max} \tag{6.29}$$

Hence, since the interference is a decreasing function of the price imposed by the leader, the constraint can be guaranteed when the price charged to the followers is not less than κ^*_{sur}, i.e. $\kappa_{sur} \geq \kappa^*_{sur}$. In Algorithm 1, it is presumed that α is the learning rate of the algorithm ($\alpha > 0$) and κ^t_{sur} is the price imposed by the leader at iteration t.

Algorithm 1: Learning algorithm for optimization problem (6.28)

Set an initial price $\kappa^1_{sur} = \kappa^*_{sur}$ determined at the equality of the constraint of optimization problem (27);
Determine an increment $\Delta\kappa_{sur}$ and set the second price value as: $\kappa^2_{sur} = \kappa^*_{sur} + \Delta\kappa_{sur}$
Set $t = 1$
while the convergence is not reached **do**:
  Obtain the best response strategies for the tracking radars, by playing the followers' game at price κ^t_{sur}
  Calculate the profit of the leader s_{lead} at price κ^t_{sur}
  Determine the new price from the following learning equation:

$$\kappa^{t+2}_{sur} = \max\left(1 + \alpha\frac{s^{t+1}_{lead} - s^t_{lead}}{\kappa^{t+1}_{sur} - \kappa^t_{sur}}\kappa^{t+1}_{sur}, \kappa^*_{sur}\right) \tag{6.30}$$

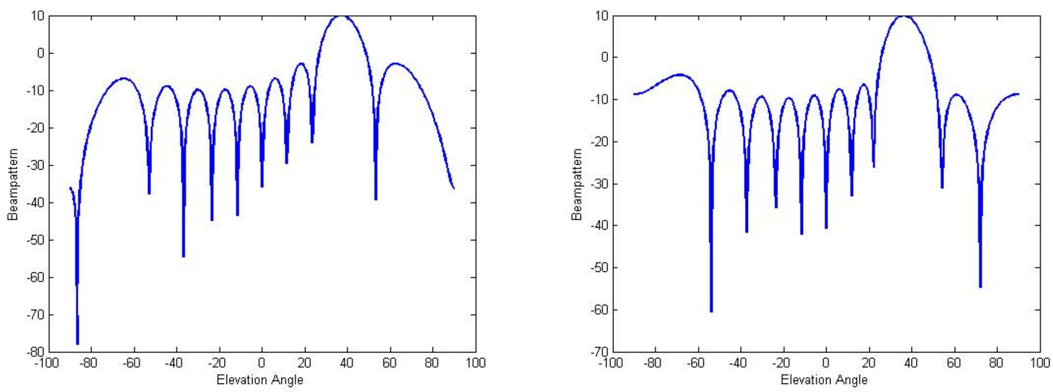
Set $t = t + 1$
end while

6.6 Simulation Results

In this section, some simulation results are presented to illustrate the performance of the beamformers and the convergence of the resource allocation methods for all three different games, which are the beamformer design and power allocation SNG, the beamformer design and power allocation game with pricing policy and the Stackelberg game. Thus, a bistatic network of two tracking MIMO radars is considered, where each one consists of 10 transmit/receive antennas with half-wavelength spacing between adjacent antennas. The referential direction of the second radar as seen from the first radar is $\theta_{rad(1,2)} = 72^o$ and $\theta_{rad(2,1)} = -75^o$ conversely. Moreover, two targets are assumed and placed at directions $\theta_{11} = 37^o$, $\theta_{12} = 22^o$ as observed from the first radar and $\theta_{21} = -38^o$, $\theta_{22} = -12^o$ using the second radar as reference. Furthermore, strong clutter is presumed as a focal point with directions $\theta_{cl(1)} = 52^o$ from radar-1 and $\theta_{cl(2)} = -54^o$ using the second radar as reference. The complex amplitudes of the targets and the clutter radar cross sections are equal to $\beta_1 = \beta_2 = \beta_{cl} = 1$. The background noise is considered as AWGN with variance 0.4 and the correlation factors between the waveforms for different targets $l \neq l'$ are fixed to be equal to 0.1 ($\varrho_{k,l,l'} = \varrho_{k,l,m,l'} = 0.1$).

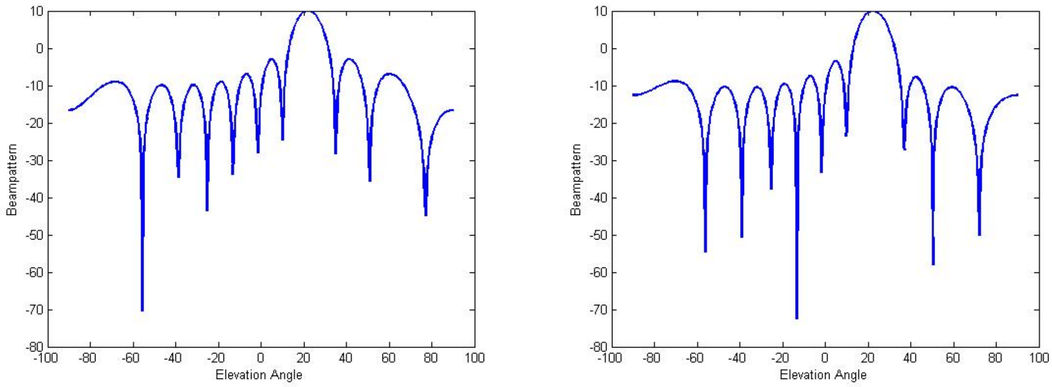
6.6.1 Comparison of the SNG and the coordinated game with pricing consideration

The first stage of the algorithm refers to the design of the optimal transmit and receive beamformers. In particular, for the SNG the aforementioned beamformers are obtained using convex semidefinite programming methods for the optimization problem (6.6), whereas for the coordinated game with pricing policy the duality properties of the optimization problem (6.11) are exploited and the transmit and receive weight vectors using the solution of the dual problem (6.15) are found. It is obvious that in both games the beampatterns are concentrated on the desired target by maintaining very low sidelobe levels in other directions. Figs. 6.3-6.6 clearly depict the tendency towards social welfare of the game with pricing consideration, since the beampatterns of the first player enforce deep nulls at the direction of the other player, minimizing the interference leakage.



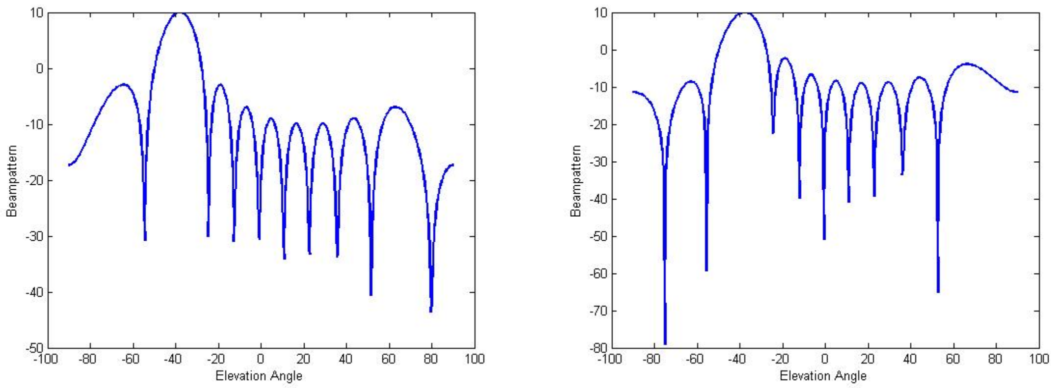
(A) Without pricing consideration. (B) With pricing consideration.

Figure 6.3: Comparison of the transmit beampatterns for player 1 aiming at target 1 (dB).



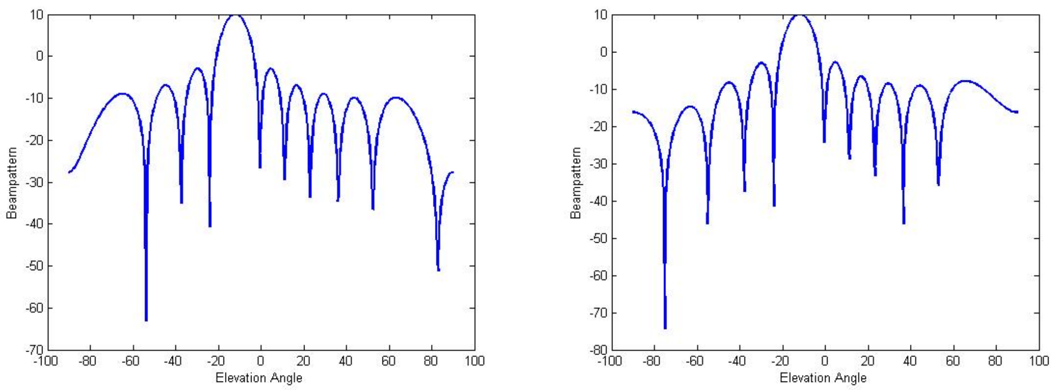
(A) Without pricing consideration. (B) With pricing consideration.

FIGURE 6.4: Comparison of the transmit beampatterns for player 1 aiming at target 2 (dB).



(A) Without pricing consideration. (B) With pricing consideration.

FIGURE 6.5: Comparison of the transmit beampatterns for player 2 aiming at target 1 (dB).



(A) Without pricing consideration. (B) With pricing consideration.

FIGURE 6.6: Comparison of the transmit beampatterns for player 2 aiming at target 2 (dB).

The resource allocation optimization is considered at the second stage of the algorithms for both games compared. Before the initialization of the games, the detection criterion for each player is decided by setting the SINR targets at 7 for radar 1 ($\gamma_{11} = \gamma_{12} = 7$) and 6.5 for radar 2 ($\gamma_{21} = \gamma_{22} = 6.5$) for both games. Moreover, the maximum number of game iterations is set at $T = 40$ to study the convergence of the algorithms. Figs. 6.8-6.9 depict the resource allocation update for each radar aiming each target. Power allocation using both methods clearly converges to a unique solution. Comparing Fig. 6.8 to Fig. 6.9 the advantages of the coordinated design with pricing are obvious, since the transmit power of each radar is lower compared to that of the SNG without pricing consideration. This result shows that due to the reduced interference among the radars using the coordinated design, as displayed in Fig.6.7, each player needs less power to attain the SINR target, and hence the resource allocation for this game is more efficient.

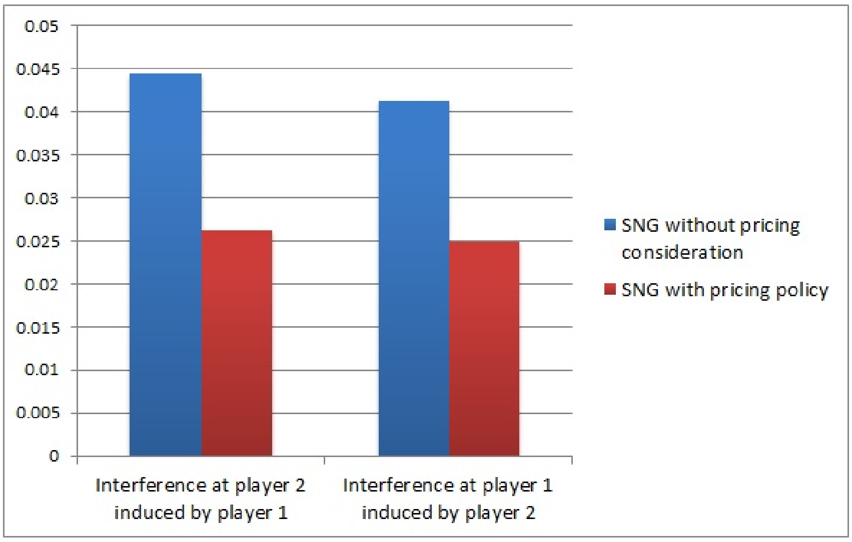


FIGURE 6.7: Interference among the MIMO tracking radars with and without pricing consideration.

6.6.2 Stackelberg Game

The surveillance radar is placed at direction $\theta_{sur(1)} = 65^o$ as observed from the first tracking radar and $\theta_{sur(2)} = -67^o$ using the second radar as reference. Based on the price announced by the leader, the followers decide their optimal beamformers and power allocation by following game $\mathcal{G}_{fol}$. The transmit weight vectors and the power allocation of the followers, when the price set by the leader is $\kappa_{sur} = 7.4$ are depicted in Figs. 6.10-6.11 and Fig. 6.12, respectively. It is clear that the beampatterns of both the followers are steered away from

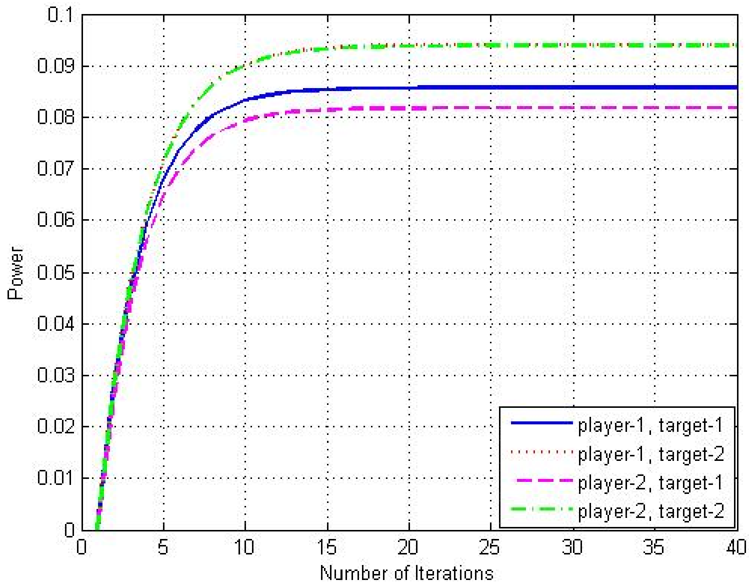


FIGURE 6.8: Power allocation convergence for the SNG without pricing consideration.

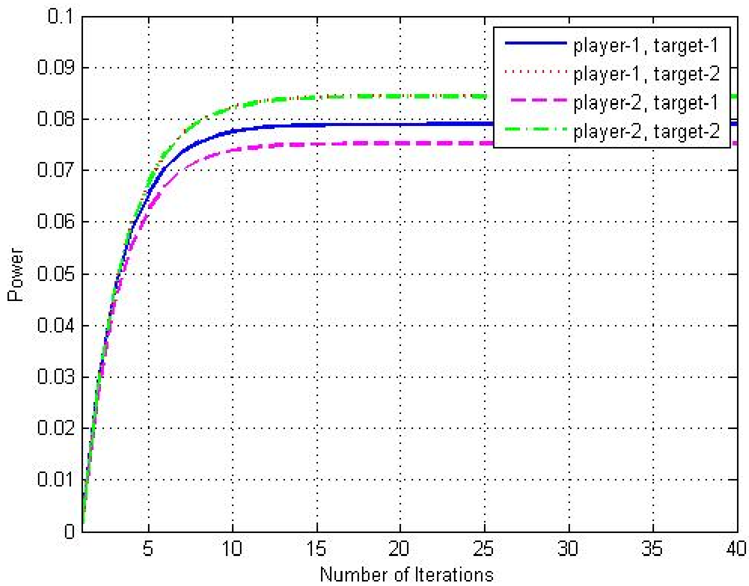
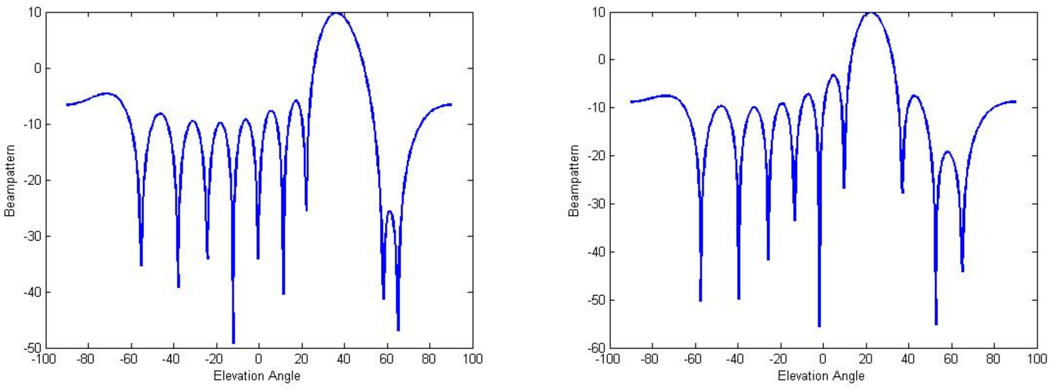


FIGURE 6.9: Power allocation convergence for the coordinated game with pricing policy.

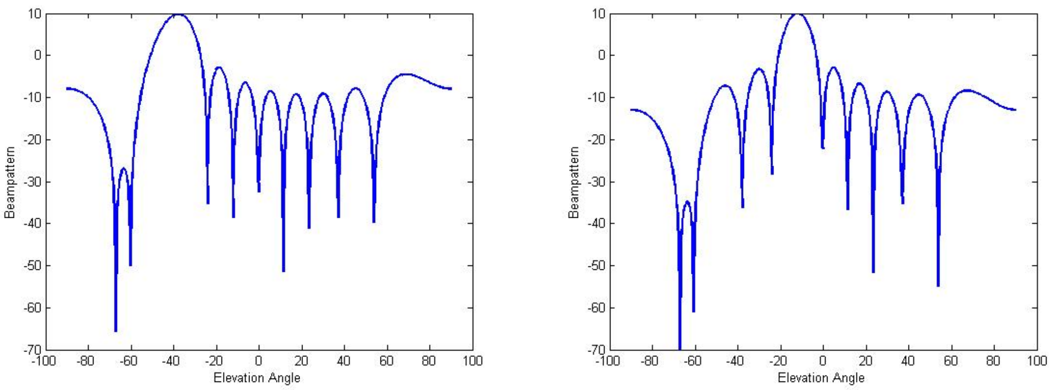
the direction of the leader and hence the interference leakage to the surveillance radar is minimized.

In order to find the optimal value of the price set by the leader, the optimization problem in (6.28) is solved incorporating the learning algorithm from Section **V**. The maximum interference allowed at the surveillance radar is set at $I_{max} = 0.0103$ and the learning rate at $\alpha = 0.2$. For this interference threshold, the corresponding price is determined as $\kappa_{sur} = 7.4$, which is considered as the initial price for the leader's game. The convergence of the price set by the leader is shown in Fig. 6.13. As expected, the algorithm rapidly converges to the starting price $\kappa_{sur} = 7.4$, which is the minimum price so that the leader's interference constraint is secured.



(A) Target 1 (dB). (B) Target 2 (dB).

Figure 6.10: Transmit beampatterns for player 1 aiming at targets 1 and 2 respectively (Stackelberg game).



(A) Target 1 (dB). (B) Target 2 (dB).

Figure 6.11: Transmit beampatterns for player 2 aiming at targets 1 and 2 respectively (Stackelberg game).

6.7 Conclusion

This chapter investigated a game theoretic approach to tackle the problem of joint beamforming and power allocation in a distributed radar network. At first, an SNG was studied, without any coordination among the radars/players.

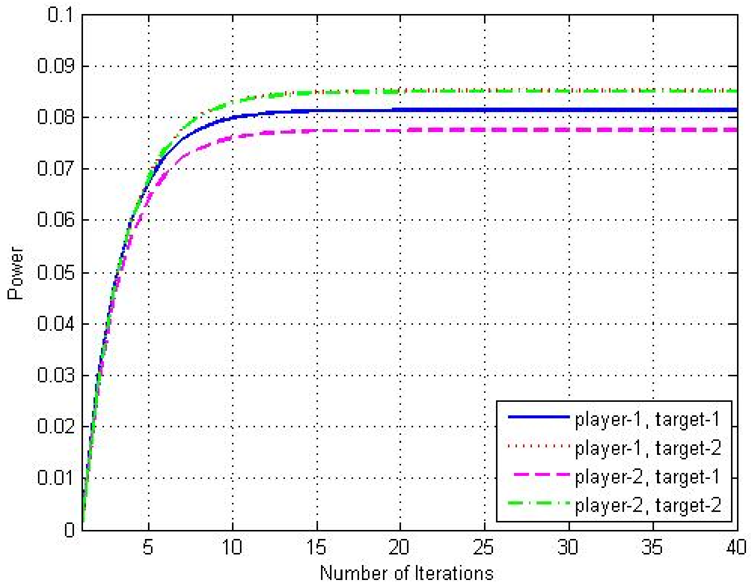


Figure 6.12: Power allocation convergence for the follower game when $\kappa_{sur} = 7.4$.

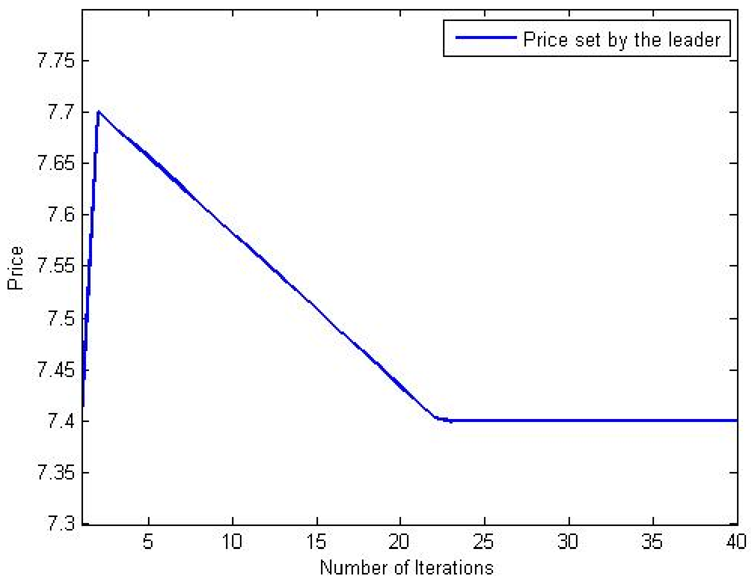


Figure 6.13: Convergence of the price imposed by the leader.

Thus each player greedily decides its optimal beamformers and power allocation. Furthermore, a pricing mechanism was incorporated to minimize the inter-radar interference and to improve the social welfare of the network. The simulation results confirmed that this partially coordinated game provides a more Paretto-efficient Nash equilibrium. Additionally, a Stackelberg game was formulated by introducing a surveillance radar within the network and studied the convergence of both the followers' and the leader's games. Finally, the proofs for the existence and the uniqueness of the solution for both the partially coordinated and the noncooperative games have also been presented.

Chapter 7

Summary, Conclusion and Future Work

In this chapter, the novel contributions of this thesis and the conclusions that can be drawn from them are summarised. Subsequently, a discussion on possible future work is also presented.

7.1 Summary and Conclusions

The research focus of this thesis has been on developing, analysing and validating signal processing algorithms, utilizing in particular convex optimization and game theoretic techniques in order to address various issues in radar networks, such as distributed power allocation, optimal beamforming, jammer avoidance and unceratinty handling. The improved beamforming capabilities of a two-dimensional Phased-MIMO radar with fully overlapped subarrays have been presented. It was shown that this hybrid model combines the benefits of both the phased array radar scheme and the MIMO only radar technology. Furthermore, noncooperative game theoretic techniques were applied to tackle power allocation problems in distributed radar networks and jamming scenarios, along with the studies on the existence and the uniqueness of the solution in each problem. Moreover, a Bayesian game theoretic study was performed to handle channel uncertainty in a multistatic radar system. Partially coordinating game theoretic techniques and Stackelberg games have also been applied to analyse the scenario of a distributed MIMO radar network with multiple targets. Analysing each chapter in more detail:

Chapter 1 introduced the basic principles of radar systems. Furthermore, a definition on what is a MIMO radar was given and the most crucial challenges in this technology were analysed.

In Chapter 2, a literature review on MIMO radar beamforming and game theory in wireless and radar networks was demonstrated. In particular, the virtual array concept and target estimation techniques were analysed. Moreover, the basic adaptive and conventional beamforming techniques for both the transmit and receive array were presented. An introduction in noncooperative, Bayesian and Stackelberg game theoretic techniques in wireless and radar systems completed Chapter 2.

The first contributing chapter "2D Phased-MIMO Radar" presents a novel two-dimensional fully overlapped MIMO transmit and receive arrays subaperturing technique. Specifically, the transmit, waveform diversity and overall transmit-receive beampatterns are derived for the 2D Phased-MIMO model and compared with the phased-array radar scheme and the MIMO radar model. The simulation results confirmed that the system beampatterns approximate efficiently the desired sector of space with high accuracy, restricting the sidelobe levels at low amplitude. Furthermore, there are substantial improvements of the overall transmit-receive beampattern of the proposed model as compared to the phased-array and the conventional MIMO scheme. In particular, it was demonstrated that the 2D Phased-MIMO regime combines the transmit coherent processing gain of the phased-array radar and the waveform diversity of the MIMO scheme to produce a more efficient and accurate overall beampattern. This superiority is highlighted using both conventional (matched-filtering) and adaptive (convex optimization, MVDR) beamforming techniques.

In Chapter 4, a game theoretic power allocation scheme for a distributed MIMO radar system is proposed. A novel and rigorous Nash equilibrium analysis is also presented, by defining a GNG and utilizing convex optimization techniques and duality properties of the system, concluding with the proof of the existence and the uniqueness of the solution. During this study, important properties of the system's resource allocation were also derived. More specifically, it was proved that the number of active radars in a cluster that actually transmit signals is exactly the same as the number of radars in the same cluster that satisfy the detection criterion with equality. In addition, the number of active radars and the optimal strategy of a cluster is dependent only upon the channel gains and the target SINR and is totally independent of the other players' power allocation. This contribution supports the decentralized and distributed nature of the system. Finally, the simulation results confirm the extended mathematical analysis of the convergence and the study of the existence and uniqueness of the Nash equilibrium. Later in the chapter, a noncooperative power allocation game between a radar system and multiple jammers was investigated and a proof was presented on the uniqueness of the solution. Furthermore, the simulations highlighted the comparison between the case when the radars using a hypothesis testing to identify the interfering jammers and the case when the radars attempt to detect the target on the basis of proximity (i.e. distance), even if it is an interfering jammer. Finally, the simulation results confirm that the game with hypothesis testing provides a more Paretto-efficient Nash equilibrium than the game without jammer identification, as the radars utilize less resources to achieve the same SINR criterion.

Chapter 5 investigated a Bayesian game theoretic SINR maximization and resource allocation technique within a distributed radar network, where uncertainty regarding the channel gains was introduced. Each radar is considered to have private information only about its own channel gain, whereas only the distribution of the remaining radars' channel gains is common knowledge. A proof of the existence and uniqueness of the Bayesian Nash equilibrium was also highlighted. The simulation results validated the convergence to the unique solution, regardless the initial power allocation strategy of the players. Moreover, it was shown that the higher the confidence of a player regarding a better channel gain associated with the remaining players the higher the SINR and the transmission power of this player. Also, the importance of the prior belief of player was highlighted to the outcome of the game.

The final contributing chapter proposed a game theoretic approach to tackle the problem of joint beamforming and power allocation in a distributed radar network. Initially, an SNG without any coordination among the players was designed. Hence, each player egotistically decides its optimal beamforming and resource allocation strategy, without considering the interference it imposes on the other players. This is however, not desirable, as it leads to socially unfair solutions, that are not close to Paretto optimality. Therefore, a pricing mechanism was incorporated to minimize the inter-radar interference and to improve the social welfare of the network. The simulation results confirmed that this partially coordinated game provides a more Paretto-efficient Nash equilibrium. Additionally, a Stackelberg game was formulated in a hybrid MIMO radar network, by introducing a surveillance radar within the network and studying the convergence of both the followers' and the leader's games. Finally, extended analysis for the existence and the uniqueness of the solution for both the partially coordinated and the noncooperative games has also been presented.

In summary, in this thesis initially a new 2D subaperturing method for MIMO arrays and the corresponding beamforming techniques were proposed. Furthermore, several game theoretic scenarios for optimal beamforming and resource allocation strategies have been investigated, within the context of distributed MIMO radar networks. Each game theoretic technique was followed by the proof of the existence and uniqueness of the Nash equilibrium, that guarantees convergence to a stable, unique state solution. Finally, game theory was also applied in the cases of jammers attacking a multistatic radar system and in the case when there is incomplete information about the channel gain, followed by the equilibrium existence and uniqueness analysis.

7.2 Future Work

The studies presented in this thesis can be extended towards several directions. First of all, multiple source uncertainty could be introduced in a radar system. In particular, uncertainty could originate from the clutter channel gain distribution, that could be considered Weibull or K-distribution [124]. Additionally, uncertainty can be introduced in a scenario where a radar network is attacked by a jammer and the type of the jammer is not common knowledge. The target's channel gain imperfect information could be addressed if Swerling target models are used to derive each target's RCS. Bayesian game theory could handle multiple source uncertainty and this is a challenging problem that should be studied in the future.

Another property of MIMO radar systems that is not yet fully exploited is the control of the sidelobe levels when designing the transmit beamformers. The sidelobe sector could provide the benefit of combining the radar operation with wireless communication between sensors of the same organisation [125]. More specifically, radar-comms systems could secure the detection of the target by designing an accurate transmit beampattern, where the mainlobe points at the direction of the desired target and simultaneously transmit encrypted data to trustworthy receivers by exploiting the sidelobe sector to the direction of the desired receivers and the waveform diversity of a MIMO system.

Appendix A

Proof of Claim 1

In order to prove the optimal beampatterns independence of the inter-radar interference, the dual problem of the optimization problem (6.6) is investigated. The Lagrangian associated with the aforementioned problem is given as:

$$\mathcal{L}(\mathbf{W}_{t(k)}, \boldsymbol{\lambda}_k) = \sum_{l=1}^{L} \|\mathbf{w}_{t(k,l)}\|^2 + \sum_{l=1}^{L} \lambda_{kl} \Bigg(\sum_{j \neq l}^{L} |\mathbf{w}_{t(k,l)}^H \mathbf{h}_{r(kj)}|^2$$
$$+ \sum_{i=1}^{L} |\mathbf{w}_{t(k,l)}^H \mathbf{c}_{r(ki)}|^2 + r_{-kl} - \frac{1}{\gamma_{kl}} |\mathbf{w}_{t(k,l)}^H \mathbf{h}_{r(kl)}|^2 \Bigg)$$

where $\boldsymbol{\lambda}_k = [\lambda_{k1}, \ldots, \lambda_{kL}]^T$ is the $L \times 1$ vector of the Lagrangian multipliers associated with the SINR inequality constraints of the problem in (6.11). The Lagrangian can be reformulated as:

$$\mathcal{L}(\mathbf{W}_{t(k)}, \boldsymbol{\lambda}_k) = \sum_{l=1}^{L} \lambda_{kl} r_{-kl} + \sum_{l=1}^{L} \mathbf{w}_{t(k,l)}^H \Bigg(\mathbf{I} - \frac{\lambda_{kl}}{\gamma_{kl}} \mathbf{h}_{r(ki)} \mathbf{h}_{r(ki)}^H$$
$$+ \sum_{j \neq l}^{L} \lambda_{kj} \mathbf{h}_{r(kj)} \mathbf{h}_{r(kj)}^H + \sum_{i=1}^{L} \lambda_{kl} \mathbf{c}_{r(ki)} \mathbf{c}_{r(ki)}^H \Bigg) \mathbf{w}_{t(k,l)}$$

Subsequently, the Lagrange dual function is written as:

$$g_k(\boldsymbol{\lambda}_k) = \inf_{\mathbf{W}_{t(k)}} \mathcal{L}(\mathbf{W}_{t(k)}, \boldsymbol{\lambda}_k)$$

It is clear that $\mathbf{I} - \frac{\lambda_{kl}}{\gamma_{kl}} \mathbf{h}_{r(ki)} \mathbf{h}_{r(ki)}^H + \sum_{j \neq l}^{L} \lambda_{kj} \mathbf{h}_{r(kj)} \mathbf{h}_{r(kj)}^H + \sum_{i=1}^{L} \lambda_{kl} \mathbf{c}_{r(ki)} \mathbf{c}_{r(ki)}^H$ must be positive semi-definite, for the dual problem to be feasible. Hence, the dual problem associated with (6.6) can be designed as:

$$\max_{\lambda_{k1}, \ldots, \lambda_{kL}} \quad \sum_{l=1}^{L} \lambda_{kl} r_{-kl} \tag{A.1}$$

$$s.t. \quad \sum_{i=1}^{L} \lambda_{kl} \mathbf{h}_{r(ki)} \mathbf{h}_{r(ki)}^H + \sum_{i=1}^{L} \lambda_{kl} \mathbf{c}_{r(ki)} \mathbf{c}_{r(ki)}^H + \mathbf{I}$$

$$\succeq \frac{\lambda_{kl}}{\gamma_{kl}} \mathbf{h}_{r(kl)} \mathbf{h}_{r(kl)}^H, \quad \forall l$$

Following [118] and [32], the dual problem (A.1) can be solved through the receive beamforming optimization problem below:

$$\min_{\substack{\lambda_{k1},\ldots,\lambda_{kL} \\ \mathbf{w}_{r(k,1)},\ldots,\mathbf{w}_{r(k,L)}}} \quad \sum_{l=1}^{L} \lambda_{kl} r_{-kl} \tag{A.2}$$

$$s.t. \quad \frac{\lambda_{kl} |\mathbf{w}_{r(k,l)}^H \mathbf{h}_{t(kl)}|^2}{\sum_{j \neq l}^{L} \lambda_{kj} |\mathbf{w}_{r(k,l)}^H \mathbf{h}_{t(kj)}|^2 + \sum_{i=1}^{L} |\mathbf{w}_{r(k,l)}^H \mathbf{c}_{t(ki)}|^2 + \mathbf{w}_{r(k,l)}^H \mathbf{w}_{r(k,l)}} \geq \gamma_{kl}, \forall l$$

Since the constraints are satisfied with equality at optimality, the optimal Lagrangian multipliers can be derived by applying the fixed point iteration method as shown below [126]:

$$\lambda_{kl}^{(n+1)} = \frac{\gamma_{kl}}{1+\gamma_{kl}} \times \frac{1}{\mathbf{h}_{t(kl)}^H \left(\sum_{i=1}^{L} \lambda_{kl}^n \mathbf{h}_{t(ki)} \mathbf{h}_{t(ki)}^H + \sum_{i=1}^{L} \lambda_{kl}^n \mathbf{c}_{t(ki)} \mathbf{c}_{t(ki)}^H + \mathbf{I} \right)^{-1} \mathbf{h}_{t(kl)}} \tag{A.3}$$

It is also shown in [126] that the fixed point iteration function in (A.3) belongs to the framework of standard functions. Thus, the aforementioned iteration process is guaranteed to converge to a unique solution, if the respective optimization problem is feasible.

The optimal receive weight vector is the minimum mean-square error (MMSE) receiver, obtained from the following equation:

$$\mathbf{w}_{r(k,l)} =$$

$$= \left(\sum_{i=1}^{L} \lambda_{kl} \mathbf{h}_{t(ki)} \mathbf{h}_{t(ki)}^H + \sum_{i=1}^{L} \lambda_{kl} \mathbf{c}_{t(ki)} \mathbf{c}_{t(ki)}^H + \mathbf{I} \right)^{-1} \mathbf{h}_{t(kl)} \tag{A.4}$$

Following [123], the optimal transmit beamformer can be obtained as a scaled version of the receive weight vector $\mathbf{w}_{r(k,l)}$. Thus, it is clear that the optimal transmit and receive beampatterns are independent of the inter-radar plus noise vector $\mathbf{r}_{-k}$.

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