STOCHASTIC REPRESENTATION OF THE MECHANICAL PROPERTIES OF IRREGULAR MASONRY STRUCTURES

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ABSTRACT

A procedure for the stochastic characterization of the elastic moduli of plane irregular masonry structures is presented in this paper. It works in the field of the random composite materials by considering the masonry as a mixture of stones (or bricks) and mortars. Once that the elastic properties of each constituent are known (deterministically or stochastically), the definition of the overall masonry elastic properties requires the knowledge of the random field describing the irregular geometry distribution. This last one is obtained by a software, implemented ad hoc, that, starting from a colour digital photo of the masonry and using the instruments of the digital image processing techniques, gives the random features of this field in both the space and frequency domain. The definition of the stochastic properties of masonry structures may be very useful both for the application of the stochastic homogenization techniques and for the direct stochastic analysis of the structures.

Key words: irregular masonry panels, random fields, stochastic analysis, digital image processing, statistical descriptors.

1. INTRODUCTION

The study of masonry structures has attracted a considerable attention in the last years, especially in connection with reconstruction and rehabilitation of historical structures. These structures are often characterized by an irregular geometry of the masonry constituents (stones or bricks and mortars). The most used techniques for analyzing the masonry structures are the homogenization ones. The application of these approaches to masonry was proposed by many authors (see, for example, Pande et al. (1989), Maier et al. (1991), Pietruszczak and Niu (1992) and Anthoine (1995)). These approaches operate in the framework of micromechanics theory allowing one to model a heterogeneous material as an equivalent homogeneous one. In the framework of masonry structure, these techniques have been predominantly applied under the assumption of periodic microstructure that implies equal dimension of all the stones and mortars.

It is clear that the assumption of periodic microstructure often fails in historical structures, where the geometrical distribution of the constituents may be very irregular. In these cases, an useful way to take into account the irregular geometry of stones and mortars is through its stochastic characterization.

Recently, in the literature some works have been devoted to the Stochastic Homogenization that requires a suitable definition of the masonry random properties. They applied to the masonry structure field some stochastic homogenization procedures related to the studies of composite random materials (see, for example, Povirk, 1995). Among them, Sejnoha et al. (2004) showed a procedure based on the definition of a periodic unit cell that possesses statistical properties similar to the original material and that can be therefore considered a reasonable approximation. Gusella and Cluni (2006) improved the classical test-window method for the definition of the representative volume element (Baxter and Graham, 2000) by coupling a probabilistic convergence criterion (requiring a statistical description of the microstructural heterogeneity) with the classical mechanical convergence criterion. However the above cited works do not work in the random field theory, their goal being represented by the definition of the most representative volume element from a statistical point of view.

If the mechanical properties are defined through the random field theory (Vanmarcke, 1988), some alternative techniques based on the Stochastic Mechanics and, in particular, on the Stochastic Finite Element (Ghanem and Spanos, 1991) approaches could be applied. Even if these techniques have been applied in the framework of the composite materials (see, for example, Kaminski and Hien (1999), Khoroshun (2000) and references therein), no application is to the irregular masonry, at our knowledge.

In both the Stochastic Homogenization techniques and the Stochastic Mechanics approaches, the stochastic representation of a masonry structure, for example in terms of the random fields describing its mechanical properties, could be very useful.

In the present work a procedure able to give a suitable representation of the mechanical properties of irregular masonry through the theory of random fields is presented. This procedure starts from the analysis of an image of the masonry structure that is digitally reduced to a binary image through the techniques of the Digital Image Processing (Serra, 1982). Then, the characteristics of the random geometry are evaluated in both the space and frequency domain by the statistical descriptor approaches. Finally, by using the theory of composite materials, the random fields describing the mechanical properties of the masonry are defined by means of the evaluation of its fundamental quantities (for example, the correlation functions). This is made both in the case of deterministic values of the mechanical properties of each constituent and in the case of random mechanical properties.

It is important to note that, even if most of the theoretical concepts here reported are not original in the general field of composite materials, their application to the irregular masonry is a novelty. Hence, they have been here reported for those readers that are not familiar with them.

2. RANDOM FIELDS OF THE MASONRY ELASTIC MODULI

In a 2-D linear-elastic continuum, the plane-state constitutive equations are characterized by a stiffness matrix having the form:

$$\mathbf{C}(x,y)=\begin{pmatrix} C_{11}(x,y) & C_{12}(x,y) & 0 \\ C_{12}(x,y) & C_{22}(x,y) & 0 \\ 0 & 0 & C_{33}(x,y) \end{pmatrix} \tag{1}$$

In the field of composite materials made by two components, the stiffness matrix of a 2-D masonry can be expressed as:

$$\mathbf{C}(x,y)=\chi_s(x,y)\mathbf{C}^{(s)}+\left(1-\chi_s(x,y)\right)\mathbf{C}^{(m)} \tag{2}$$

where $\mathbf{C}^{(s)}$ and $\mathbf{C}^{(m)}$ are the stiffness matrices of the stone and the mortar, respectively, while $\chi_s(x,y)$ is the so-called characteristic function taking the values:

$$\chi_s(x,y)=\begin{cases}1 \Leftrightarrow (x,y)\in stone/brick\\ 0 \Leftrightarrow (x,y)\in mortar\end{cases} \tag{3}$$

In an irregular masonry the characteristic function cannot be defined deterministically: it has to be statistically defined through the knowledge of the so-called *n*-point correlation functions in the space domain, or through the Fourier spectra in the frequency domain. In particular, the *n*-point correlation functions give a measure of the probability of finding *n* points all lying in the region of the domain occupied by one of two constituent materials. For example, the one-point and two-point correlation functions are defined as follows:

$$\mathrm{S}_s^{(1)}(x,y)=\mathrm{P}\left(\chi_s(x,y)=1\right) \tag{4}$$

$$\mathrm{S}_s^{(2)}\left[(x_1,y_1),(x_2,y_2)\right]=\mathrm{P}\left(\chi_s(x_1,y_1)\,\chi_s(x_2,y_2)=1\right) \tag{5}$$

$\mathrm{P}(\bullet)$ being the probability of $(\bullet)$. If the ergodicity assumption is made, the function $\chi_s(x,y)$ can be considered as a random field whose statistical moments coincide with the corresponding *n*-point correlation functions that are:

$$\mathrm{E}\left[\chi_s(x_1,y_1)\,\chi_s(x_2,y_2)\cdots\chi_s(x_n,y_n)\right]\equiv\mathrm{S}_s^{(n)}\left[(x_1,y_1),(x_2,y_2),\cdots,(x_n,y_n)\right]\quad\forall n \tag{6}$$

$\mathrm{E}(\bullet)$ being the stochastic mean of $(\bullet)$. If the masonry under consideration has such characteristics that it can be considered as a statistically homogeneous field, then the *n*-th order moments of $\chi_s(x,y)$ do not depend on the position of the *n* points (x_i,y_i) (with $i=1,2,\cdots,n$), but on the $(n-1)$ relative positions (x_i-x_1,y_i-y_1) (with $i=2,3,\cdots,n$). This implies, for example, that the mean $\mathrm{E}\left[\chi_s\left(x,y\right)\right]$ is constant for any point $\left(x,y\right)$, while for the second order moment we can write:

$$\mathrm{E}\left[\chi_s\left(x_1,y_1\right)\chi_s\left(x_2,y_2\right)\right]\equiv \mathrm{S}_s^{(2)}\left[\left(x_2-x_1,y_2-y_1\right)\right] \tag{7}$$

Moreover it is not difficult to verify that the *n*-th moments at a single point have the following form:

$$\mathrm{E}\left[\chi_s^n(x,y)\right]=\gamma_s=\frac{\Omega_s}{\Omega}=\frac{\Omega-\Omega_m}{\Omega}=1-\gamma_m \tag{8}$$

γ_s and γ_m being the domain area fractions occupied by the stones and the mortar, respectively. Eqs.(7) and (8) imply that the second order correlation function of the field $\chi_s\left(x,y\right)$ is given by:

$$\mathrm{R}_{\chi_s}^{(2)}\left[x_2-x_1,y_2-y_1\right]=\mathrm{S}_s^{(2)}\left[\left(x_2-x_1,y_2-y_1\right)\right]-\gamma_s^2 \tag{9}$$

while its variance is:

$$\sigma_{\chi_s}^2=\mathrm{E}\left[\chi_s^2\right]-\mathrm{E}\left[\chi_s\right]^2=\gamma_s-\gamma_s^2=\gamma_s\gamma_m \tag{10}$$

Another important simplification is when the distance x_2-x_1 and/or y_2-y_1 tends to be very large. In fact, in this case, the corresponding two variables tend to be independent and, as consequence:

$$\mathrm{E}\left[\chi_s\left(x_1,y_1\right)\chi_s\left(x_2,y_2\right)\right]\equiv \mathrm{E}\left[\chi_s\left(x_1,y_1\right)\right]\mathrm{E}\left[\chi_s\left(x_2,y_2\right)\right]=\gamma_s^2 \quad \Rightarrow \quad \sigma_{\chi_s}^2=0 \tag{11 a,b}$$

2.1. Deterministic elastic moduli of constituents

Once that the stochastic field $\chi_s\left(x,y\right)$ is known through its moments or its correlation functions, the corresponding statistics of the elastic moduli $C_{ij}(x,y)$ can be obtained starting from Eq.(2). In particular, when the moduli of the two constituent are deterministically defined, the mean and the second order correlation are given by:

$$\mathrm{E}\left[C_{ij}(x,y)\right]=\gamma_s C_{ij}^{(s)}+\gamma_m C_{ij}^{(m)} \tag{12}$$

$$\mathrm{R}_{C_{ij}C_{kl}}^{(2)}\left(x_2-x_1,y_2-y_1\right)=\mathrm{R}_{\chi_s}^{(2)}\left(x_2-x_1,y_2-y_1\right)\left(C_{ij}^{(s)}-C_{ij}^{(m)}\right)\left(C_{kl}^{(s)}-C_{kl}^{(m)}\right) \tag{13}$$

The auto-correlation function of the component C_{ij} of the stiffness matrix can be obtained by setting $i\equiv k,\, j\equiv l$ in Eq.(13), that is:

$$\mathrm{R}_{C_{ij}}^{(2)}\left(x_2-x_1,y_2-y_1\right)=\mathrm{R}_{\chi_s}^{(2)}\left(x_2-x_1,y_2-y_1\right)\left(C_{ij}^{(s)}-C_{ij}^{(m)}\right)^2 \tag{14}$$

while the corresponding variance is:

$$\sigma_{C_{ij}}^2=\sigma_{\chi_s}^2\left(C_{ij}^{(s)}-C_{ij}^{(m)}\right)^2=\gamma_s\gamma_m\left(C_{ij}^{(s)}-C_{ij}^{(m)}\right)^2 \tag{15}$$

2.2. Stochastic elastic moduli

Now let us consider the case in which the moduli of the stones and the mortars are defined as random variables, statistically independent of the stochastic field $\chi_s\left(x,y\right)$ and characterized, for example, by the means $\mathrm{E}\left[C_{ij}^{(s)}\right]$ and $\mathrm{E}\left[C_{ij}^{(m)}\right]$ and by the second order moments $\mathrm{E}\left[C_{ij}^{(a)}C_{kl}^{(b)}\right]$ (with $a=s,m;\;\; b=s,m$). In this case it is possible to evaluate the first two statistics of $\mathbf{C}(x,y)$ in the form:

$$\mathrm{E}\left[C_{ij}(x,y)\right]=\gamma_s\mathrm{E}\left[C_{ij}^{(s)}\right]+\gamma_m\mathrm{E}\left[C_{ij}^{(m)}\right] \tag{16}$$

$$\begin{aligned}\mathrm{E}\left[C_{ij}(x_1,y_1)C_{kl}(x_2,y_2)\right]=\;&\mathrm{S}_s^{(2)}\left[\left(x_2-x_1,y_2-y_1\right)\right]\mathrm{E}\left[C_{ij}^{(s)}C_{kl}^{(s)}\right]+\left(\gamma_s-\mathrm{S}_s^{(2)}\left[x_2-x_1,y_2-y_1\right]\right)\\&\times\left(\mathrm{E}\left[C_{ij}^{(s)}C_{kl}^{(m)}\right]+\mathrm{E}\left[C_{ij}^{(m)}C_{kl}^{(s)}\right]\right)+\left(1-2\gamma_s+\mathrm{S}_s^{(2)}\left[x_2-x_1,y_2-y_1\right]\right)\mathrm{E}\left[C_{ij}^{(m)}C_{kl}^{(m)}\right]\end{aligned} \tag{17}$$

Particularizing Eq.(17) for $i \equiv k, j \equiv l$, the second order moment of the modulus C_{ij} is obtained in the form:

$$\begin{aligned}&\mathrm{E}\left[C_{ij}(x_1,y_1)C_{ij}(x_2,y_2)\right]=\mathrm{S}_s^{(2)}\left[\left(x_2-x_1,y_2-y_1\right)\right]\mathrm{E}\left[C_{ij}^{(s)2}\right]\\&+2\left(\gamma_s-\mathrm{S}_s^{(2)}\left[x_2-x_1,y_2-y_1\right]\right)\mathrm{E}\left[C_{ij}^{(s)}C_{ij}^{(m)}\right]+\left(1-2\gamma_s+\mathrm{S}_s^{(2)}\left[x_2-x_1,y_2-y_1\right]\right)\mathrm{E}\left[C_{ij}^{(m)2}\right]\end{aligned} \tag{18}$$

that, evaluated for $\left(x_1,y_1\right)\equiv\left(x_2,y_2\right)$, gives:

$$\mathrm{E}\left[C_{ij}^2\right]=\gamma_s\mathrm{E}\left[C_{ij}^{(s)2}\right]+\gamma_m\mathrm{E}\left[C_{ij}^{(m)2}\right] \tag{19}$$

The relationships reported in this section evidence that the stochastic characterization up to the second order of the stiffness matrix of the masonry requires only the knowledge of γ_s and $\mathrm{S}_s^{(2)}\left(x_2-x_1,y_2-y_1\right)$, besides of the elements of $\mathbf{C}^{(s)}$ and $\mathbf{C}^{(m)}$. These last ones may be known deterministically or stochastically. In the last case their mean and second order moments must be known.

As it will be seen in the next sections, γ_s and $\mathrm{S}_s^{(2)}\left(x_2-x_1,y_2-y_1\right)$ can be obtained by applying image processing techniques to a digital image of the masonry structure under consideration. To this purpose the assumptions of ergodicity and stochastic homogeneity of the geometry of the masonry portion represented in the photo will be made.

3. DIGITAL IMAGE PROCESSING

An image can be defined as a two-dimensional function $f\left(x,y\right)$, where the amplitude of f at any pair of coordinates $\left(x,y\right)$ is the so-called intensity of the image at that point. When the coordinates and the amplitude values of f are all finite and discrete quantities, then the image is a so-called digital image and the composing finite number of elements are called pixels. The total image size is then defined in units of pixel. Each pixel, designated by $\left(i,j\right)$, where $i=1,\ldots,N_x$ and $j=1,\ldots,N_y$, represents an integer coordinates in a 2D field.

In the first step a digital colour photo of the masonry panel is converted into a black and white image that typically contains 256 shades of grey with values ranging from 0 (black) to 255 (white). If the phases of the masonry are clearly delineated then they can be distinguished on the basis of grey scale levels.

Often the black and white photograph has most of its pixels clustered around central values of grey scale determining a bad contrast. To improve the contrast in an image it is necessary to spread the intensity values over the full range of the image, using a process called histogram equalization. This histogram provides most of the information needed to choose the threshold value required for generating the final digitized image in which each masonry constituent has to be distinguished by a particular grey level.

If the masonry of the panel under consideration is made by only one stone types, then the above cited histogram equalization must be calibrated in order to generate a binary image, that can be as a special kind of intensity image containing only black and white. Then, it is stored as two dimensional array of 0's and 1's.

In order to apply these digital image processing approaches, we implemented a software, working in Matlab and called STONES (STochastic masONry ElementS), whose first step deals with the generation of a binary image starting from a digital colour image of a one type stone masonry. In Figure 1 the graphical user interface related to this first operation is reported with reference to a masonry panel sample.

For more detailed information about the digital image processing the reader can be referred to Serra (1982).

4. STATISTICAL DESCRIPTOR EVALUATION

When the masonry structure is replaced by its binary image, a digital representation can be considered as a discretization of the characteristic function $\chi_s(x,y)$, usually presented in terms of a $N_x \times N_y$ bitmap.

Replacing the point coordinates (x,y) by the pixel (i,j) located in the *i*-th row and in the *j*-th column of the bitmap then the characteristic function is defined by the discrete values $\chi_s(i,j)$. As a consequence, the one-point and two-point correlation functions may be estimated, under the assumptions of ergodic and statistically homogeneous field, by using the following relationships:

$$\mathrm{S}_s^{(1)} = \frac{1}{N_x N_y}\sum_{i=1}^{N_x}\sum_{j=1}^{N_y}\chi_s\left(i,j\right) = \gamma_s \tag{20}$$

$$\mathrm{S}_s^{(2)}\left(m,n\right) = N_{x_1}N_{x_2}\sum_{i=i_m+1}^{i_M}\sum_{j=i_n+1}^{i_N}\chi_s\left(i,j\right)\chi_s\left(1+\left(i+m\right)\%N_{x_1},1+\left(j+n\right)\%N_{x_2}\right) \tag{21}$$

where *m* and *n* assume here the significance of pixel distances between two generic points and $\%$ denotes modulo. Hence, coming back to the $\left(x,y\right)$ coordinates, we can define the following correspondence:

$$\left(m,n\right)\Rightarrow\left(x_2-x_1,y_2-y_1\right)\qquad\forall\left(x_1,y_1\right),\left(x_2,y_2\right) \tag{22}$$

Observe that the evaluation of the function $\mathrm{S}_s^{(1)}$ requires $\left(N_x\times N_y\right)$ operations, while $\left(N_x\times N_y\right)^2$ operations are needed for the function $\mathrm{S}_s^{(2)}\left(m,n\right)$. The computational complexity of this approach involves very lengthy calculation, particularly for a large image (Zeman, Šejnoha et al., 2006). In fact the double-sum of product of pairs of pixels in Eq.(21) must be recalculated for each combination of $\left(m,n\right)$ corresponding to all possible translation of $\chi_s\left(i+m,j+n\right)$ respect to $\chi_s\left(i,j\right)$.

The required number of operation, however, can be reduced using two results of Fourier analysis: the correlation theorem and the Fast Fourier Transform (FFT) (Bracewell, 1986). The Fourier correlation theorem states that the correlation of two functions is equal to the inverse transform of the product of their Fourier transforms. That is, the correlation can be obtained by computing the transform of image, multiplying this for its complex conjugate, and then computing the inverse transform of the product. The result is identical to that obtained by the direct method. The FFT is an algorithm for rapidly computing the Fourier transform of a discrete array that needs only $\left(N_x\times N_y\right)\log\left(N_x\times N_y\right)$ operations.

Then, the two-point correlation function $\mathrm{S}_s^{(2)}\left(m,n\right)$ can be obtained from the relation

$$\mathrm{S}_s^{(2)}(m,n) = \frac{1}{N_x \cdot N_y} IFFT\left\{FFT\left[\chi_s(i,j)\right]\cdot\overline{FFT\left[\chi_s(i,j)\right]}\right\} \tag{23}$$

where $FFT[\bullet]$ and $IFFT[\bullet]$ stands for the direct and inverse Fourier transforms and $\overline{[\bullet]}$ denotes the complex conjugate.

Hence, the methods arising from the Fourier analysis of digitized media offer another, computationally more attractive, way to evaluate the desired statistics.

In Figure 2 the binary image of the previous masonry sample and the corresponding $\mathrm{S}_s^{(2)}(m,n)$ graph, evaluated by STONES, are reported together with its horizontal and vertical sections. These functions display some interesting characteristics. First, the value of $\mathrm{S}_s^{(2)}(m,n)$ for *m* and *n* equal to zero is the domain area fraction γ_s occupied by the stones ($\mathrm{S}_s^{(2)}(0,0) \equiv \mathrm{E}\left[\chi_s^2\right] = \gamma_s$). Second, these functions tend to assume the value γ_s^2 for large *m* and *n*. This is due to the fact that, as said before in section 3, $\chi_s(x_1, y_1)$ tends to be independent of $\chi_s(x_2, y_2)$ when the distance between the points becomes very large. Obviously the velocity of convergence towards this value increases by increasing the irregularity of the geometry.

If we are interested to other second order statistics, as the correlation function $\mathrm{R}_{\chi_s}^{(2)}(m,n)$ or the correlation coefficient $\rho_{\chi_s}(m,n)$, they can be easily obtained by the following relationships:

$$\mathrm{R}_{\chi_s}^{(2)}(m,n) = \mathrm{S}_s^{(2)}(m,n) - \gamma_s^2 \tag{24}$$

$$\rho_{\chi_s}(m,n) = \frac{\mathrm{R}_{\chi_s}^{(2)}(m,n)}{\sigma_{\chi_s}^2} = \frac{\mathrm{S}_s^{(2)}(m,n) - \gamma_s^2}{\gamma_s - \gamma_s^2} \tag{25}$$

At this point, the first two order statistics of the fields representing the elastic moduli of the random masonry may be evaluated by using the relationships shown in section 2.

5. ANALYSIS OF MASONRY TEXTURE IN THE FREQUENCY DOMAIN

The Fourier Transform is an important image processing tool which is used to decompose an image into its sine and cosine components. The output of transformation represents the image in the *frequency domain*, while the input image is the *spatial domain* equivalent. In the Fourier domain image, each point represents a particular frequency contained in the spatial domain image (Gonzales and Woods, 1993). In a wide range of applications the general concept of frequency is used and it represents the number of times that a periodic function repeats the same sequence of values during a unit variation of the independent variable. A related term used in this context is *spatial frequency*, which refers to the periodicity with which the image intensity values change. Image features with high spatial frequency (such as edges) are those that change greatly in intensity over short distances.

As we are only concerned with digital image, we use the Discrete Fourier Transform that for a square image of size $\left(N_x \times N_y\right)$ is given by:

$$F\left(u,v\right) = \frac{1}{N_x \cdot N_y} \sum_{x=1}^{N_x} \sum_{y=1}^{N_y} \chi\left(x,y\right) \cdot e^{-i2\pi\left(\frac{u\,x}{N_x} + \frac{vy}{N_y}\right)} \tag{26}$$

where $\chi\left(x,y\right)$ is the image in the spatial domain and i represents $\sqrt{-1}$. The frequency variables u and v are the inverse dimension of a distance and are often called wave numbers. The number of frequencies corresponds to the number of pixels in the spatial domain image, i.e. the image in the spatial and Fourier domain is of the same size.

In general, we see from Eq.(26) that the components of Fourier transform are complex quantities. In the analysis of complex numbers is convenient to express $F\left(u,v\right)$ in polar coordinates:

$$F\left(u,v\right) = M\left(u,v\right) \cdot e^{-i\,\phi\left(u,v\right)} \tag{27}$$

where

$$M(u,v) = |F(u,v)| = \left[\operatorname{Re}(u,v) + \operatorname{Im}(u,v)\right]^{1/2} \tag{28}$$

is called *magnitude* or *spectrum* of the Fourier transform, and

$$\phi(u,v) = \tan^{-1}\left[\frac{\operatorname{Im}(u,v)}{\operatorname{Re}(u,v)}\right] \tag{29}$$

is called the *phase angle* or *phase spectrum* of the transform. In Equ.(28) $\operatorname{Re}(u,v)$ and $\operatorname{Im}(u,v)$ are the real part and imaginary parts of $F(u,v)$, respectively.

In image processing often only the magnitude is displayed, as it contains most of the information of the geometric structure of the spatial domain image. The Fourier spectrum is useful for describing the directionality of periodic or almost periodic 2D patterns in an image. These global texture patterns are easily distinguishable as concentration of high-energy burst in the spectrum. Hence, we consider two features in plot of the magnitude $M(u,v)$ that are useful for the texture description of same masonry panels: a) prominent peaks in the spectrum give the principal direction of the texture patterns; b) the location of the peaks in the frequency plane gives the fundamental spatial period of the patterns.

Figure 3 illustrates the use of Eqs.(28) and (29) for the global texture description of masonry panel with periodic texture. The magnitude (Figure 3-b,d) of the two dimensional Fourier transform is represented with low frequencies in the centre of image and high frequencies are located towards the edges. It is showed that the slowest varying frequency components ($u = v = 0$) corresponds to the average grey level of the image, that is:

$$F(0,0) = \frac{1}{N_x \cdot N_y} \sum_{x=1}^{N_x} \sum_{y=1}^{N_y} \chi(x,y) \tag{30}$$

It is displayed in the centre of the image, as the spectrum is symmetric about the origin. Then every periodic pattern is associated with only one peak. Looking particular in Figure 3-d we can see prominent components in horizontal and vertical direction, which correspond to two principal periodic textures. In the vertical direction we can observe a single stripe of peaks with higher values than those in the horizontal direction, which confirm a strong periodic component in this path. In horizontal direction we can observe the two different patterns of texture that have the same values of magnitude but are shifted.

Other examples of analysis of periodic patterns are pointed out in the next section.

6. EXAMPLES

In this section the results of the applications of our software, STONES (STOchastic masoNry ElementS), to some masonry panel samples are showed.

In a first stage, STONES was applied to a chaotic masonry panel that is a masonry wall in which the blocks of stone have different dimensions and various shapes and their arrangement is disordered. Figure 4 shows the window of STONES for this masonry sample. The full image measures 120×120 pixels. In the binary representation wherein white areas represent the stones, while black areas represent the mortar matrix. The value of γ_s is reported in the window. For this sample it is $\gamma_s \approx 0.7$; this means that the domain area fraction occupied by the stones is nearly 70% of the total domain. In the same window the graph of the two-points correlation function $\mathrm{S}_s^{(2)}(m,n)$ and its horizontal and vertical sections are reported. It can be observed that $\mathrm{S}_s^{(2)}(m,n)$ is almost radial symmetric, then the panel is almost stochastically isotropic. We can observe (Figure 5) that in a sample of “chaotic” masonry the values of Fourier spectrum are closed around the centre and there is not any principal direction that evidences a regular texture how can be seen looking geometric structure of this sample in the three-dimensional domain image.

Figure 6 represents the STONES window related to the second sample here reported. We analysed a digital image (200×200 pixels) of a masonry panel made up of rectangular stones with various dimensions height, while the thickness of mortar joints is almost constant. The panel shows almost continuous mortar beds, while the vertical joints are very irregular. The behaviour of the function $\mathrm{S}_s^{(2)}(m,n)$, and, in particular, of its vertical and horizontal sections, remarks these characteristics . In fact, the first one shows many peaks corresponding to the beds; while the second one decreases slowly towards γ_s^2; this last behaviour is due to the relatively big horizontal dimension of the blocks.

At last, Figure 7 illustrates the results obtained by STONES for a sample of quasi-periodic masonry. This is characterized by the presence of continuous almost horizontal mortar bed and by more irregular vertical joints. These characteristics are evidenced by the $\mathrm{S}_s^{(2)}(m,n)$ graph and, in particular, by its section. In fact, the vertical one shows evident peaks of the same amplitude, while the horizontal one evidences a smoother behavior. From analysis in the Fourier domain (Figure 8) we can se that the image contains components of all frequencies, but their magnitude gets smaller for high frequencies. Hence, low frequencies contain more information than the higher ones. The transform image tell us that there are two dominating direction, one passing vertically and one passing horizontally through the centre. However, in the vertical direction we can observe most regularity than in the horizontal direction, due to the regular height of blocks and the homogeneous thick of mortar.

7. CONCLUSIONS

A procedure for the stochastic characterization of the elastic moduli of plane irregular masonry structures has been presented. It works in the field of composite materials by considering the masonry as a mixture of stones (or bricks) and mortar. Once that the elastic properties of the single constituents are known (deterministically or stochastically), the definition of the overall elastic properties requires information about the geometric distribution of the various components. For irregular masonry structure this information can be given only through the characterization of the geometry random field. At this purpose, a Matlab software has been implemented that, starting from a colour digital photo of the masonry under consideration, gives the statistics of corresponding geometry. At last, through the theory of random composites, it is possible to obtain the elastic moduli statistics. Up now, only two-phase irregular panels have been considered. This means that they are composed by one type stones, besides of the mortar. However, the extension to more numerous components appears to be straightforward.

The definition of the stochastic properties of masonry structures may be very useful both for the application of the stochastic homogenization techniques and for the direct stochastic analysis of the structures, through, for example, the use of the Stochastic Finite Element techniques. The opinion of these authors is that the second application is the most interesting and one of their future goal will be the coupling of the software here presented with a Stochastic Finite Element technique.

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LIST OF FIGURES

Figure 1: From colour to binary image

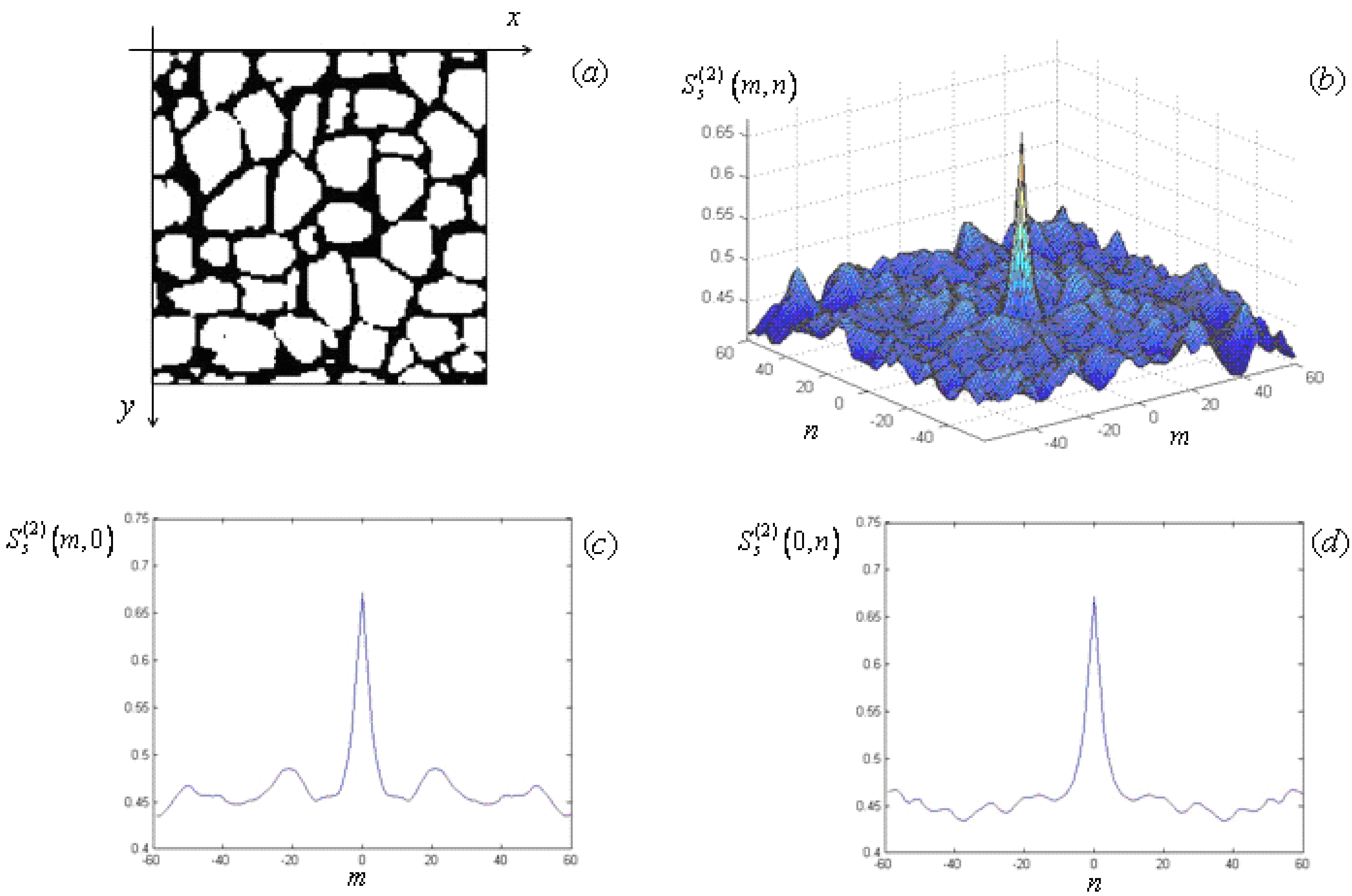

Figure 2: a) Binary image; b) Two-point correlation function; c,d) Horizontal and Vertical Section

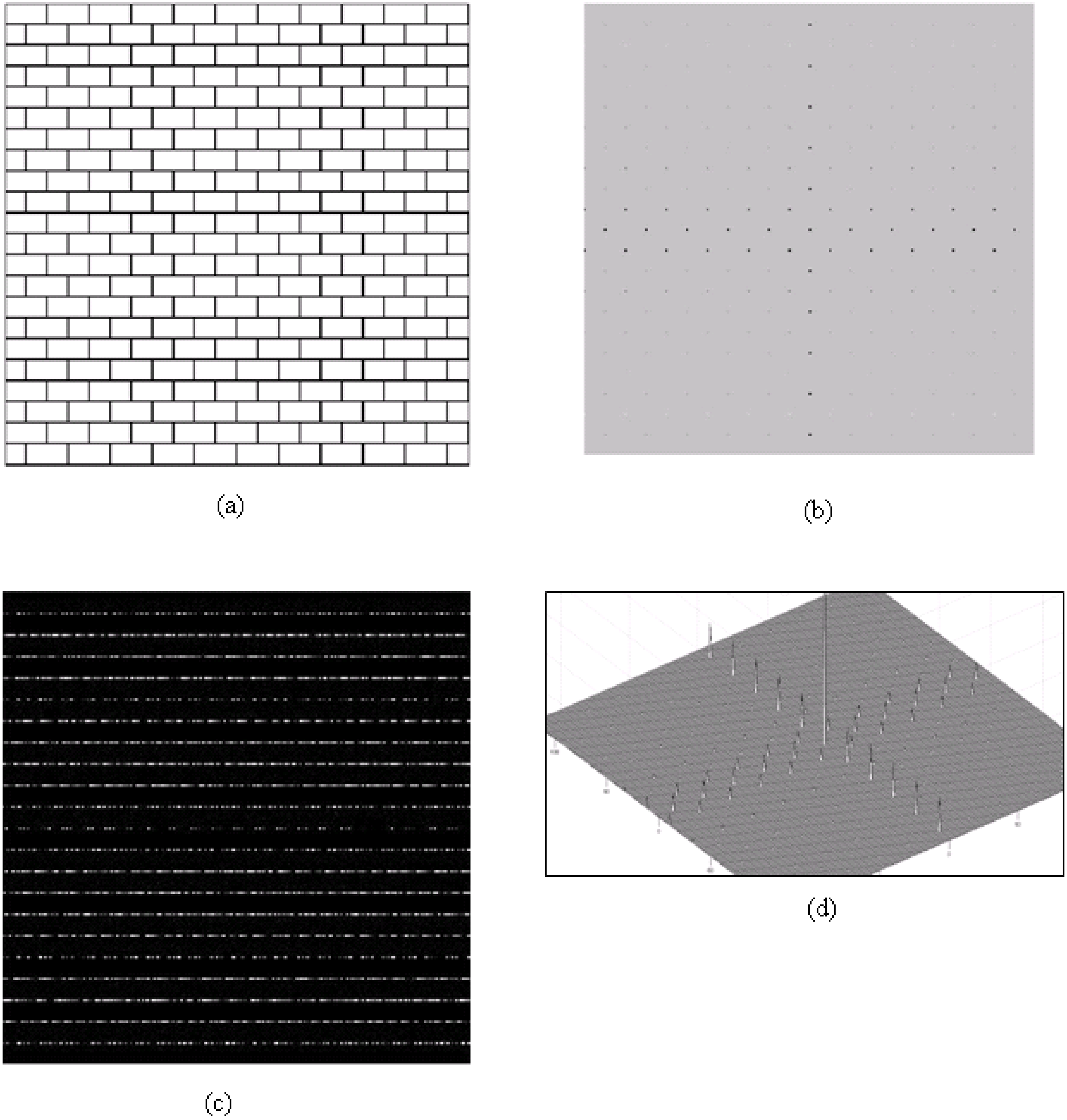

Figure 3: a) Sample of masonry panel with regular periodic texture; b) Fourier spectrum of a); c) Phase spectrum of a); d) Particular of Fourier spectrum in 3D representation.

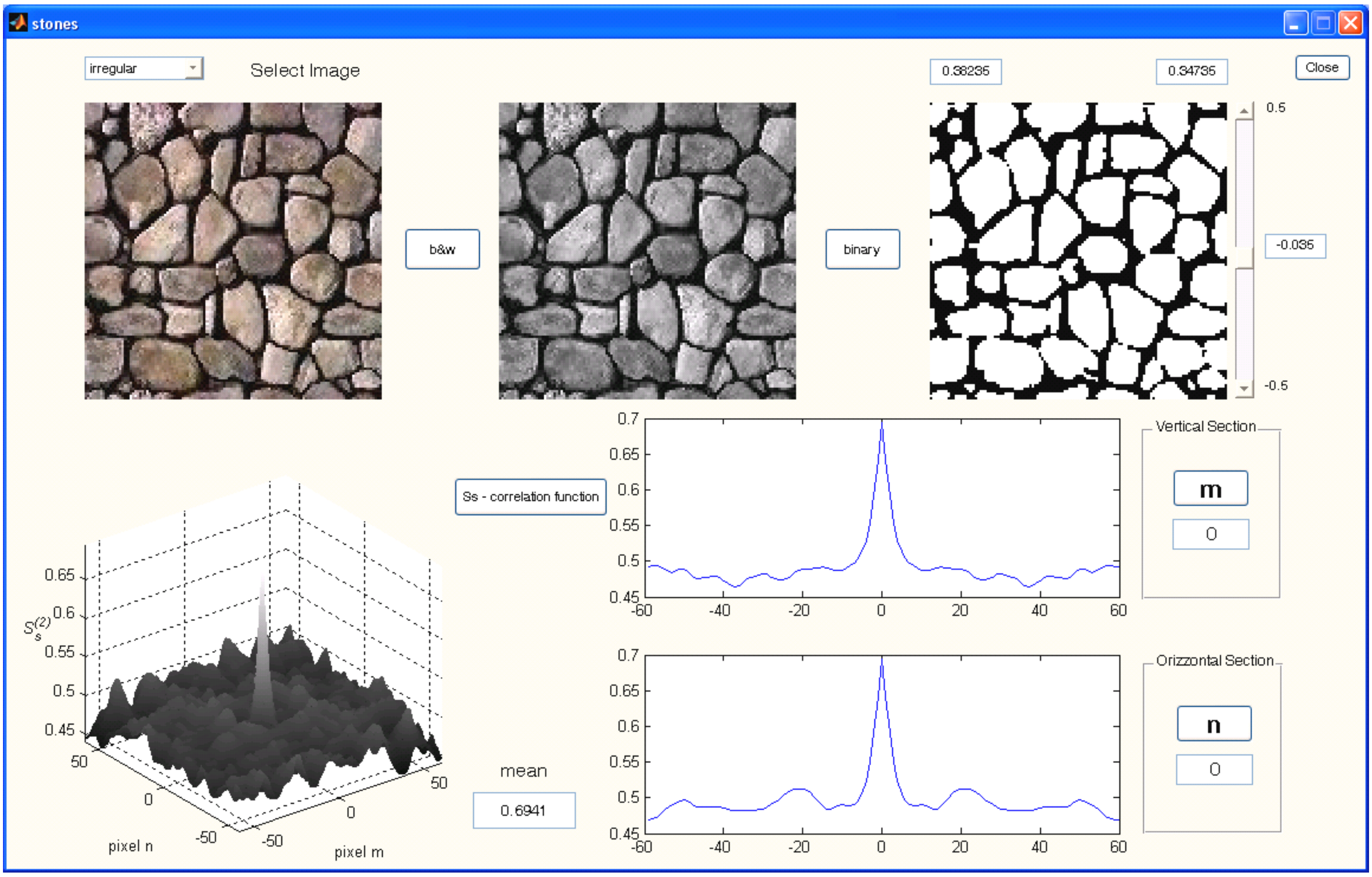

Figure 4 : Example of "chaotic" masonry.

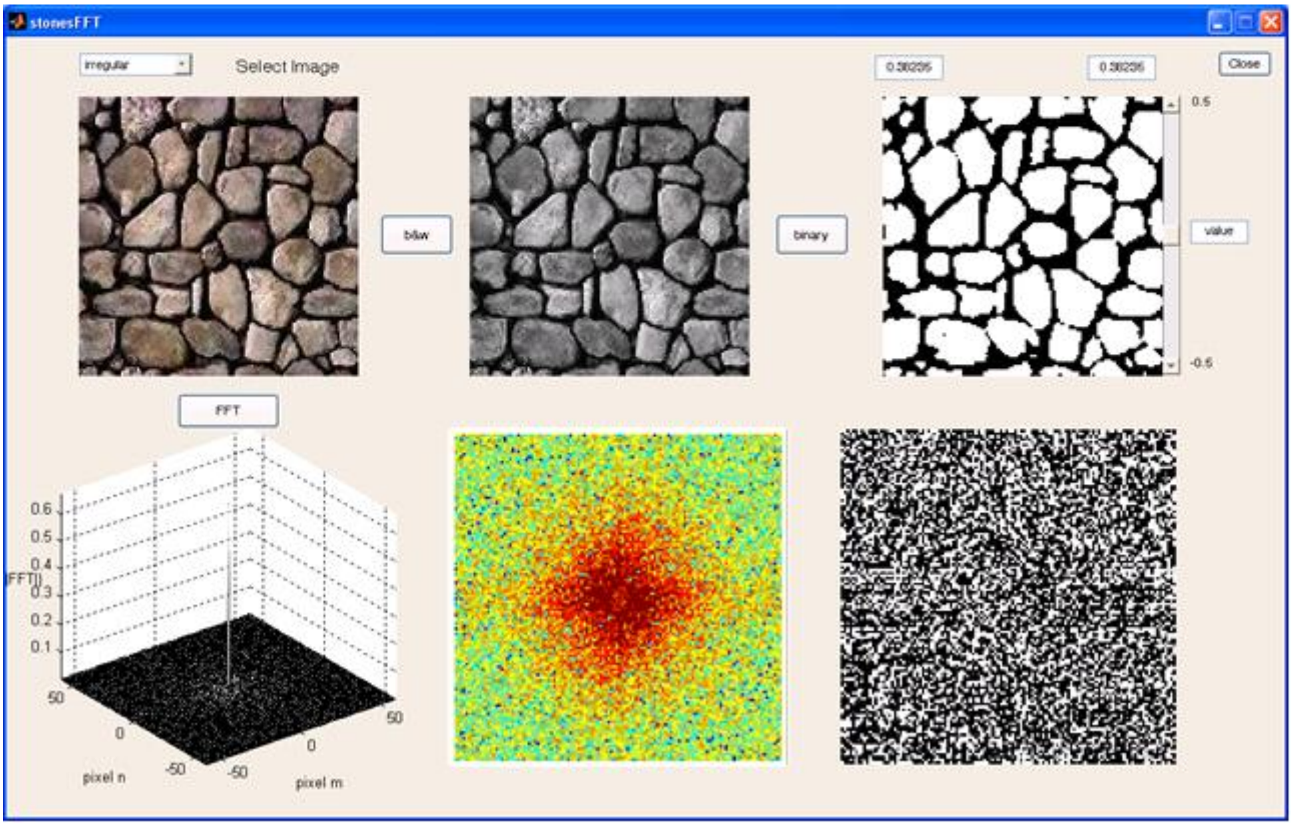

Figure 5 : Sample "chaotic" masonry and its Fourier spectrum.

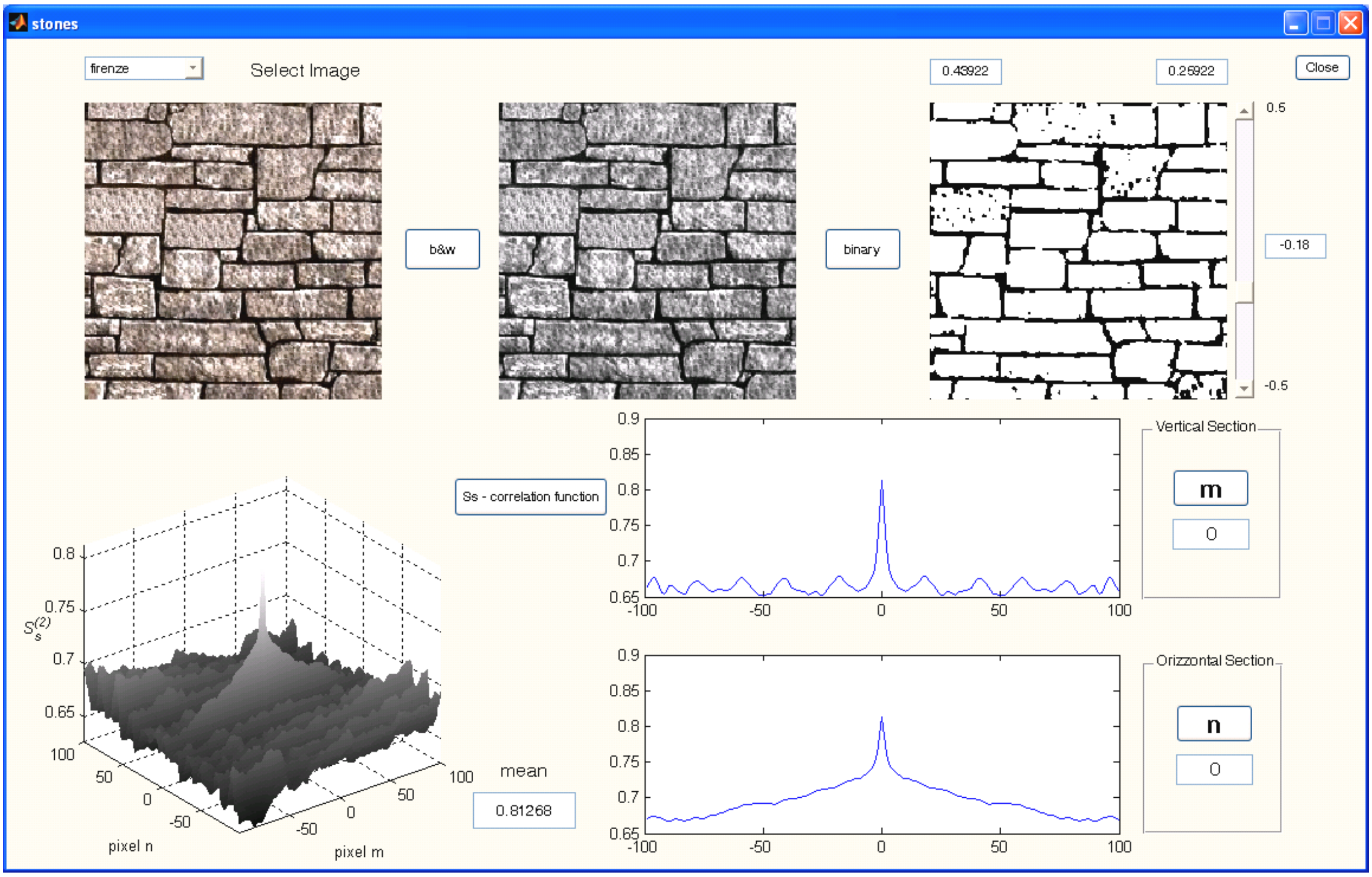

Figure 6 : Example of masonry with "irregular" dimension of blocks.

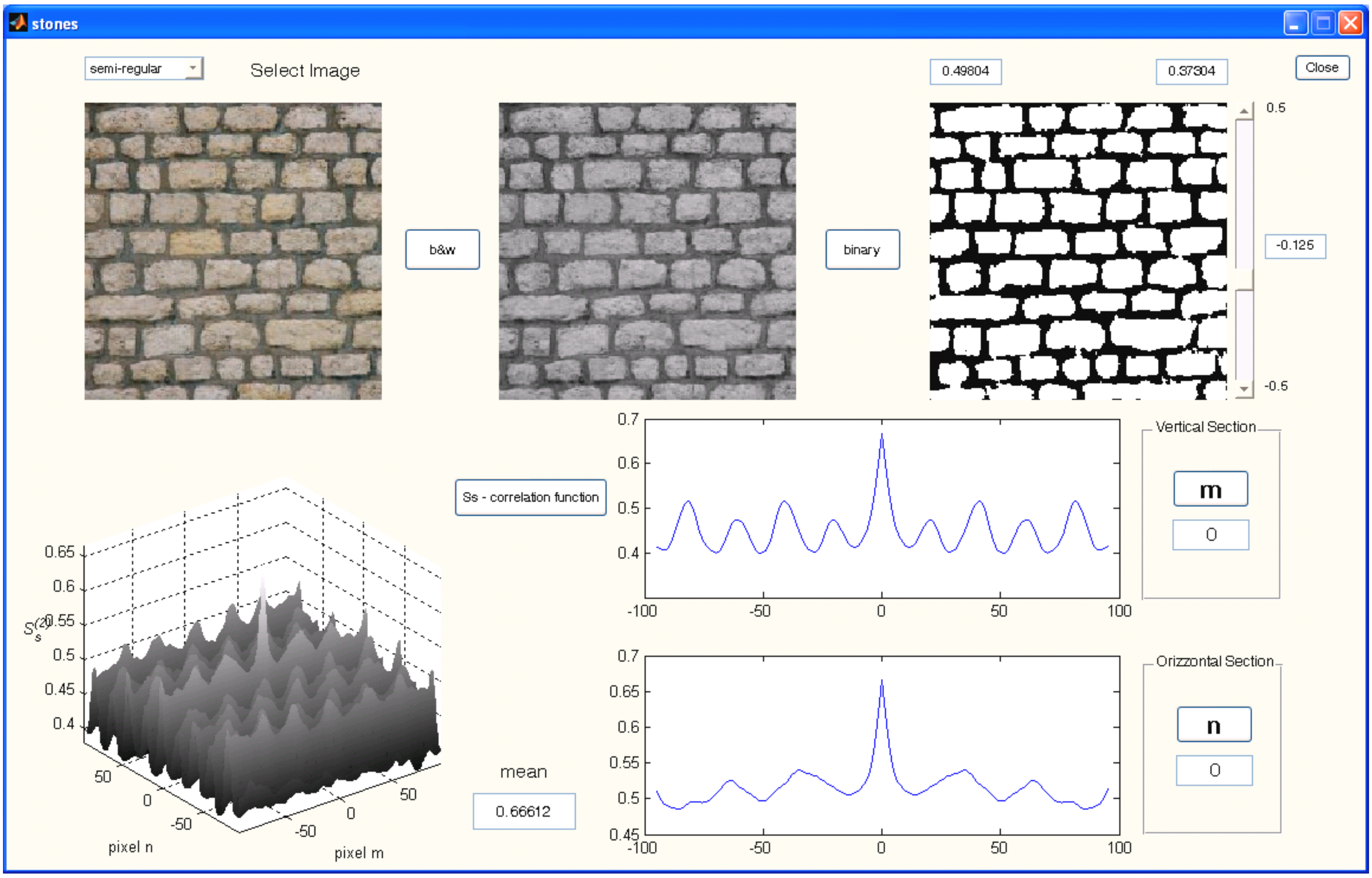

Figure 7 : Example of "quasi-periodic" masonry

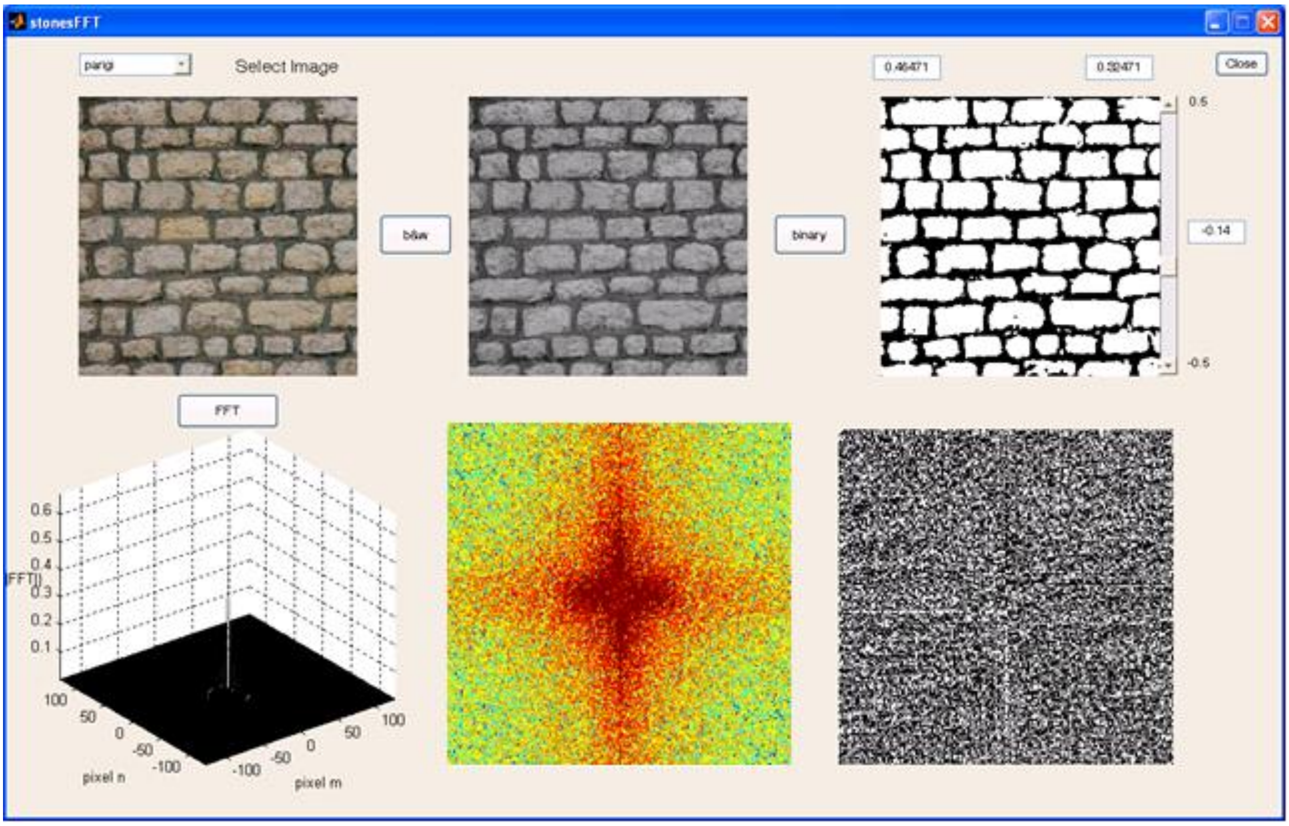

Figure 8 : Sample of "quasi-periodic" masonry and its Fourier spectrum.