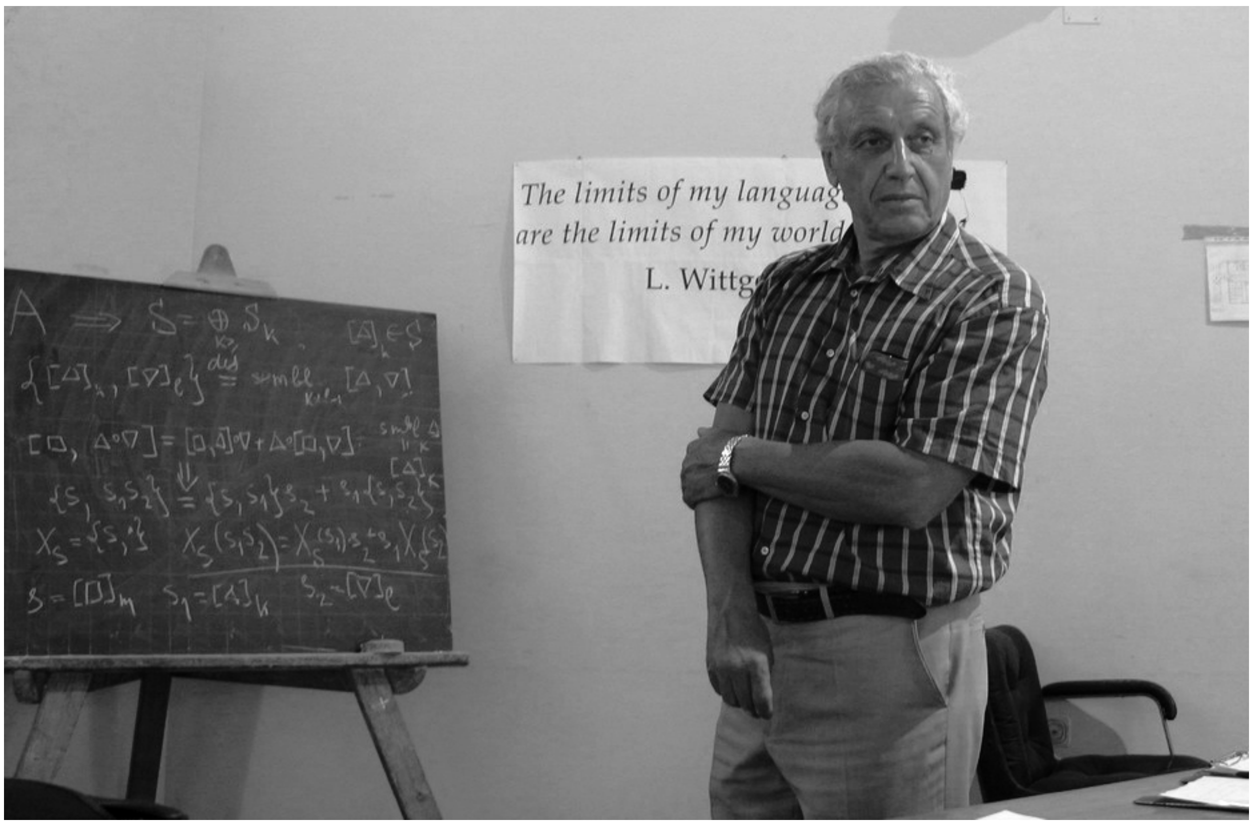
Alexandre Mikhailovich Vinogradov (obituary)

IN MEMORY OF A.M. VINOGRADOV

Alexandre Vinogradov, a remarkable mathematician and extraordinary man, our colleague, teacher, friend, passed away on September 20, 2019.



A.M. Vinogradov was born on February 18, 1938 in Novorossiysk. During the war, his family was evacuated to Kungur in the Perm Region. After the war, his parents moved to Kuntsevo, which was not part of Moscow at the time. In 1955 he became a student of *Mekhmat* (the Faculty of Mechanics and Mathematics) of Moscow State University and, in 1960, a graduate (PhD track) student there. After obtaining his PhD (*kandidatskaya dissertatsiya*) in 1964, he taught for a year at the Gornyi Institute, but was then invited by N.V. Efimov, the Dean of *Mekhmat*, to take a teaching position at the Chair of Higher Geometry and Topology (then headed by P.S. Alexandrov), which he held until he left the Soviet Union for Italy in 1990. He obtained the habilitation degree (*doktorskaya dissertatsiya*) in 1984. From 1993 to 2010, he held the position of professor at the University of Salerno in Italy.

While still a second year undergraduate student, Vinogradov published two works (with B.N. Delaunay and D.B. Fuchs) in number theory, but by the end of his undergraduate years his research interests changed: actively participating in the A.S. Schwarz seminar, he began working in algebraic topology. His PhD, for which V.G. Boltyansky was the (purely formal) adviser, was devoted to the homotopic properties of the embedding spaces of circles into the 2-sphere or the 3-disk. One of Vinogradov's first works was devoted to the Adams spectral sequence which, at the time, was justly regarded as the summit of algebraic topology. In a Doklady note in 1960 [1], Vinogradov announced the solution of J.F. Adams' problem concerning the relationship between the higher cohomological operations and the Adams filtration in the stable homotopy groups of spheres. Adams wrote a favorable review of that note. Vinogradov continued to work on problems of algebraic and differential topology until the beginning of the seventies. His research seminar, organized in 1967, was also devoted to the same subject matter.

For the second and last time, Vinogradov radically changed the direction of his research some time between the sixties and the seventies. Inspired by the ideas of Sophus Lie, he began to think about the foundations of the geometric theory of partial differential equations; having become familiar with the work of Spencer, Goldschmidt, and Quillen on formal solvability, he turned his attention to the algebraic (in particular, cohomological) component of that theory.

In 1972, the short note in Soviet Math. Doklady (publishing long texts in the Soviet Union at the time was very difficult) entitled "The logic algebra of the theory of linear differential operators" [2] introduced what Vinogradov himself called the main functors of the differential calculus in commutative algebras. On four pages of the journal, it was elegantly shown that for the definition and the study of such fundamental notions as vector field, differential form, jet, linear differential operator, etc., the category of modules over a commutative algebra with unit provides an appropriate setting, while the geometric prototypes of these notions occur when, for the algebra, one chooses the algebra of smooth functions on a manifold, and for the modules, the spaces of sections of vector bundles over the manifold. Later, an expanded version of this note became the first chapter of the book [3] and was partially included in [4]. More recently, a modernized version of this theory appeared in [5], while the example of its application to the construction of the algebraic model of Hamiltonian mechanics was presented in [6].

Here it is appropriate to note that Vinogradov was a natural "mathematical polyglot": he would easily switch from the language of algebra to the language of differential geometry, using a nontrivial "dictionary", translating terms from differential geometry to terms drawn from the geometry of infinitely prolonged differential equations. This multilingual attitude was a fruitful source of meaningful constructions, definitions, and statements [1]. He was always attracted to invariant, coordinate-free and therefore elegant expositions of Hamiltonian mechanics [8] or geometry [9].

Vinogradov's approach to nonlinear differential equations as geometric objects, with their general theory and applications, is developed in detail in the monographs [3] and [10], as well as in the articles [11, 12]. He combined infinitely prolonged differential equations into a category [13] whose objects, called diffieties (=differential varieties), are studied in the framework of what he called the secondary calculus [2] [14, 15]. One of the central parts of this theory is based on the $\mathcal{C}$-spectral sequence (now known as the Vinogradov spectral sequence), which was announced in [16] and later developed in detail in [17]. The term E_1 of this sequence gives a unified cohomological approach to many scattered concepts and statements, including the Lagrangian formalism with constraints, conservation laws, cosymmetries, the Noether theorems, and the Helmholtz criterion in the inverse problem of the calculus of variations (for arbitrary nonlinear differential operators), thereby allowing us to advance beyond these classical results. A particular case of the $\mathcal{C}$-spectral sequence (for an "empty" equation, i.e., for the space of infinite jets) is the variational bicomplex.

The results of the article [16] were later generalized by R.L. Bryant and P.A. Griffiths in [18] (using the language of exterior differential systems), by A.M. Verbovetsky in [19] (using the horizontal de Rham complex), and by T. Tsujishita in [20]. Essentially, the ideas underlying the construction of the $\mathcal{C}$-spectral sequence and the results naturally following from these ideas were the first decisive steps in the direction of what is now called "cohomological physics" (see, for example, the article by J. Stasheff [21]).

To the same field belong the important articles [22] and [23]. In the paper [22], Vinogradov introduced the construction of a new bracket on the graded algebra of linear trans-

[1] For example, this is how he came to the notion of a differential covering [7], which plays a central role in the nonlocal geometry of differential equations and turned out to be very important for the understanding of certain structures related to integrable systems.

[2] By analogy with "second quantization".

formations of a cochain complex. The Vinogradov bracket (which he himself called the L-commutator) is skew-symmetric and satisfies the Jacobi identity modulo a coboundary. Vinogradov's construction preceded the general concept of derived bracket on a differential Loday (or Leibniz) algebra introduced by Y. Kosmann-Schwarzbach in 1996 (see [24]). The Vinogradov bracket is a skew-symmetric version of the derived bracket generated by the coboundary operator. Derived brackets and their generalizations play an exceptionally important role in modern applications of homotopy Lie algebras, Lie algebroids, etc., and Vinogradov's paper [22] is one of the pioneering works in this direction. In particular, Vinogradov showed that the classical Schouten bracket (on multivector fields) and the Nijenhuis bracket (on vector fields with coefficients in differential forms) are restrictions of his bracket onto the corresponding subalgebras of (super)differential operators on the exterior algebra of differential forms. In his subsequent work [23], written with A. Cabras, these results were applied to Poisson geometry and new examples of differential-geometric derived brackets were constructed.

In modern applications, generalizations of Lie (super)algebras with "higher brackets" (i.e. those with $n > 2$ arguments) arise. Besides the strongly homotopy algebras (or L_∞-algebras) of Lada and Stasheff, among them are also the "Filippov algebras". The works [31, 32, 25] written by Vinogradov with coauthors were concerned with the analysis and comparison of such structures.

It should be noted that Vinogradov's research interests were always strongly motivated by serious and important problems in contemporary physics – from the dynamics of beams of sound waves [26] to the equations of magnetohydrodynamics (the Kadomtsev-Pogutse equations that appear in the stability theory of high-temperature plasma in tokamaks) [27]. Considerable attention to the mathematical understanding of the fundamental physical notion of observable is given in the book [4], written by him jointly with several participants in his seminar under the pen name of Jet Nestruev.

Vinogradov's published heritage consists of over a hundred articles and ten monographs. Whatever he worked on, be it the geometry of differential equations, the Schouten and Nijenhuis brackets [22, 23], mathematical questions of gravitation theory [28, 29, 30], n-ary generalizations of Lie algebras [31, 32, 25] or the structural analysis of the latter [33, 34], he produced work characterized by a very unorthodox approach, depth, and nontriviality of the obtained results.

The scientific activity of Vinogradov was not limited to the writing of books and articles. For many years he headed a research seminar at *Mekhmat* at Moscow State University; the seminar was in two parts – mathematical and physical – and became a notable phenomenon in Moscow's mathematical life between 1960 and 1980. He had numerous students (in Russia, Italy, Switzerland, and Poland), nineteen of whom obtained their PhD's under his guidance, six obtained the higher habilitation degree, and one became a corresponding member of the Russian Academy of Sciences. Vinogradov organized and headed Diffiety Schools in Italy, Russia, and Poland. He was the soul of a series of small conferences called "Current Geometry" that took place in Italy from 2000 to 2010, as well as of the large Moscow conference "Secondary Calculus and Cohomological Physics" (1997), whose proceedings were published in [15].

A.M. Vinogradov was one of the initial organizers of the Erwin Schrödinger International Institute for Mathematics and Physics in Vienna, as well as of the Journal of Differential Geometry and Applications, remaining one of the editors to his last days. In 1985 he created a department that studied various aspects of the geometry of differential equations at the Institute of Programming Systems in Pereslavl-Zalessky and headed it until he left for Italy. He was one of the organizers and first lecturers in the unofficial school for students who were not accepted to *Mekhmat* because they were ethnically Jewish. (He ironically called this school the "People's Friendship University".)

Alexandre Vinogradov was a versatile person – he played the violin, wrote poetry in Italian, played for the MSU water-polo team, was an enthusiastic football player. But the most important thing for him was, undoubtedly, mathematics.

Alexandre Mikhailovich Vinogradov continues to live in his work and in the memory of his students, his family, and his friends.

A.M. and I.V. Astashov
A.V. Bocharov
V.M. Buchstaber
V.N. Chetverikov
D.B. Fuchs
A.Ya. Helemskii
V.G. Kac
N.G. Khor'kova
Y. Kosmann-Schwarzbach
I.S. Krasil'shchik
I.M. Krichever
A.P. Krishchenko
S.K. Lando
V.V. Lychagin
M. Marvan
V.P. Maslov
A.S. Mishchenko
S.P. Novikov
V.N. Rubtsov
A.V. Samokhin
A.S. Schwarz
A.B. Sossinsky
J. Stasheff
V.A. Vassiliev
A.M. Verbovetsky
A.M. Vershik
A.P. Veselov
M.M. Vinogradov
L. Vitagliano
R.F. Vitolo
Th.Th. Voronov

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