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A Basestation Beamforming Technique Using Multiple SINR Balancing Criteria

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Abstract

We propose a coordinated multicell beamforming technique for signal to interference plus noise ratio (SINR) balancing under multiple base station (BS) power constraints. Instead of balancing SINR of all users in all cells to the same level, we propose a new approach to balance SINR of users in various cells to different maximum possible values. This has the ability to allow users in cells with relatively more transmit power or better channel condition to achieve a higher balanced SINR than that achieved by users in the worst case cells. This multi-level SINR balancing problem is solved using SINR constraints based SINR balancing criterion and subgradient method. The simulation results support the optimality of the results through comparison to semidefinite programming (SDP) based optimization.

Index Terms-Beamforming, Spatial Diversity, Multipoint Techniques

I. INTRODUCTION

Spectrum efficiency is one of the important design goals for addressing high-demand for wireless services and ever increasing number of users. In cellular systems, this challenge is addressed by frequency reuse with a single-cell processing policy, which means that base stations (BSs) from different cells communicate with their respective remote terminals independently, treating signals from other BSs as

noise. Nonetheless, this approach leads to degradation of performance, in particular to users at the cell edge who suffer from both high signal attenuation and severe inter-cell interference.

Coordinated multicell processing provides an efficient solution to the inter-cell interference problem and substantially improves the network capacity by allowing cooperation among BSs. The main idea is that BSs coordinate their precoding or decoding operation based on the global channel state information and if possible, through exchanging user data information over backhaul links among several cells [1]–[5]. In [4] and [5] coordinated BSs have been considered as a single large array with distributed antenna elements and beamformers were design based on zero-forcing (ZF) and minimum mean square error (MMSE) criteria. These schemes achieve large performance gains, as inter-cell interference can be removed totally within the coordinated cells. However, full coordination requires a considerable backhaul overhead for information exchange and full phase coherence between the signals transmitted from various BSs. This motivated the work in [6]–[10], which proposed coordinated beamforming schemes without data sharing, however the precoding vectors are jointly designed across the BSs. The phase coherence between the BSs is not required, since each data stream is transmitted from a single BS node.

One common optimization approach for coordinated multi-cell beamforming and power allocation is the minimization of the transmit power across the BSs subject to signal to interference plus noise ratio (SINR) constraints for the users. This formulation is applicable to constant bit-rate applications with fixed quality-of-services (QoS) requirements. This downlink problem for the multicell system was first considered in [11], where an algorithm for iteratively optimizing the beamforming vectors and power allocations was proposed. The key idea is to consider a virtual dual uplink network with transmitters and receivers reversed in role and determining the optimal uplink receivers and iterate between the beamformer update steps and the power update steps to satisfy the target SINRs. This is motivated by the fact that the uplink beamforming problem is much easier to solve [12]. A similar approach within the context of CDMA was proposed in [13]. Beamformer design based on uplink-downlink duality, has been widely studied, for example [14]. When mobile terminals employ multiple antennas, other design criteria such as minimization of transmit power subject to MMSE or data rate targets can be employed [15]–[17]. A non-cooperative game theoretic approach has also been studied for multicell beamforming design in [18]. Even though, designs based on the minimization of transmit power subject to either SINR targets or data rate targets are attractive to real time users, beamformer designs based on these criteria may not

always be feasible. For example, if the transmit power is limited or if the channel conditions are bad, and if the target SINR is set very high, the optimization problem may turn out to be infeasible. In this case, the target SINR needs to be reduced until the optimization problem turns out to be feasible which results into considerable computational complexity. Another approach to circumvent this issue is to design beamformers to maximize the worst users' SINR subject to maximum available transmit power. This is known as SINR balancing beamformer design and it balances and maximizes SINR of all users, hence ensures user fairness as well. The SINR balancing based beamformer design has been widely studied in the literature for single cell [19], [20] as well as for coordinated multicells [21]–[25].

The works available in the literature on max-min SINR optimization aimed at maximizing minimal SINR of all users in all BSs. It means that the overall balanced SINR of all users in all cells is limited by the maximum possible balanced SINR of users in the worst case cell. As all users in all cells achieve the same SINR (or weighted SINR), the users in good cells may be disadvantaged, as the corresponding BS may not fully utilize its available transmit power and enhance the SINR of its users. Even though, this issue can be addressed to some extent by providing different weights to the SINR of users in various cells, it is not straightforward to determine appropriate set of weights a priori in order to ensure all BSs fully utilize their transmit powers and enhance the SINR of their users as much as possible. It should be noted that even if the BSs have identical maximum possible transmit power, due to different channel conditions, when the SINR of all users in all cells are balanced together, certain BSs may not fully utilize their transmit powers. The work in [26] proposed to allocate excess power available at any BS to its best performing user through inclusion of an additional beamformer component designed such a way not to introduce any additional interference to other users. However, this work also did not aim at balancing SINR of users in the BSs that have excess power. In contrast to all these works in the literature, we propose a new approach for multicell beamformer design. We propose a multilevel SINR balancing criteria and beamformer design technique so that the users in various BSs achieve their balanced SINR at different maximum possible levels. First, minimal SINR of all users in all cells are maximised. Certain BSs may not use all its transmit power. Hence, we propose an optimization technique to design beamformers to enable users in the BSs that have excess power to maximise their balanced SINR values while keeping the SINRs of users in other BSs unchanged. This problem is solved using SINR constraint based SINR balancing techniques with multiple linear power constraints.

Notation: Lowercase letters are used for scalars. Vectors and matrices are denoted by boldface lowercase and uppercase letters, respectively. We denote transpose and Hermitian transpose by $(\cdot)^T$ and $(\cdot)^H$. $E\{\cdot\}$ stands for statistical expectation. $[\cdot]_l$ and $[\cdot]_{nl}$ stand for the l th entry of a vector and (n, l) th entry of a matrix, respectively. $\text{blkdiag}\{\mathbf{S}_1, \dots, \mathbf{S}_M\}$ denotes a block-diagonal square matrix with $\mathbf{S}_1, \dots, \mathbf{S}_M$ as the diagonal square matrices. $\text{Diag}\{a_1, \dots, a_M\}$ is a diagonal matrix whose diagonal elements are $a_1, \dots, a_M$. $\mathbf{I}$ and $\mathbf{0}$ denote the identity and the all zero matrix, respectively. The vector $\mathbf{1}_K^T$ denotes a $K \times 1$ vector consisting of 1 as its elements. $\|\mathbf{x}\|$ denotes the Euclidean norm of a complex vector $\mathbf{x}$, while $|z|$ denotes the norm of a complex number z .

II. SIGNAL MODEL AND PROBLEM FORMULATION

We consider a multicell multiuser downlink (DL) beamforming problem with N cells that operate in the same frequency band. At each cell, a BS consisting of M antennas performs spatial multiplexing through beamforming and transmits signals simultaneously to K single-antenna terminals. Let $s_{i,j}$ denote the information symbol for the j th user in the i th cell, with $E\{|s_{i,j}|^2\} = 1$, $\mathbf{u}_{i,j} \in \mathbb{C}^{M \times 1}$ is the associated unity-norm beamformer vector (i.e., $\|\mathbf{u}_{i,j}\|_2 = 1, \forall i, j$) and $p_{i,j}$ is the power allocated to the j th user in the i th cell. Moreover, we introduce $\mathbf{p}_i = [p_{i1} \dots p_{iK}]^T$ and $\mathbf{p} = [\mathbf{p}_1^T \dots \mathbf{p}_N^T]^T$. Let $\mathbf{U}_i = [\mathbf{u}_{i,1} \dots \mathbf{u}_{i,K}] \in \mathbb{C}^{M \times K}$ denote the beamforming matrix for the users in the i th cell and $\mathbf{U} \in \{\mathbf{U}_1, \mathbf{U}_2, \dots, \mathbf{U}_N\}$ denotes the set of all beamforming matrices. The received signal at the j th user in the i th cell consists of the desired signal, intra-cell and inter-cell interference, as expressed by

$$\mathbf{y}_{i,j} = \sum_{k=1}^K \mathbf{h}_{i,i,j}^H \sqrt{p_{i,k}} \mathbf{u}_{i,k} s_{i,k} + \sum_{\substack{m=1 \\ m \neq i}}^N \sum_{l=1}^K \mathbf{h}_{m,i,j}^H \sqrt{p_{m,l}} \mathbf{u}_{m,l} s_{m,l} + n_{i,j}, \quad (1)$$

where $\mathbf{h}_{m,i,j}$ denotes the channel vector from the BS of the m th cell to the j th user in the i th cell and $n_{i,j}$ is the complex additive white Gaussian noise (AWGN) with zero mean and variance $\sigma_{i,j}^2$, $n_{i,j} \sim \mathcal{CN}(0, \sigma_{i,j}^2)$.

Let P_i^{max} be the maximum possible transmit power of the i th BS. Then it should hold that $\mathbf{1}_K^T \mathbf{p}_i \leq P_i^{max}$. By defining $\mathbf{R}_{i,i,j} \triangleq \mathbf{h}_{i,i,j} \mathbf{h}_{i,i,j}^H$, the downlink SINR for the j th user in the i th cell is expressed as

$$\Gamma_{i,j}^{DL} = \frac{p_{i,j} \mathbf{u}_{i,j}^H \mathbf{R}_{i,i,j} \mathbf{u}_{i,j}}{\sum_{\substack{l=1 \\ l \neq j}}^K p_{i,l} \mathbf{u}_{i,l}^H \mathbf{R}_{i,i,j} \mathbf{u}_{i,l} + \sum_{\substack{m=1 \\ m \neq i}}^N \sum_{l=1}^K p_{m,l} \mathbf{u}_{m,l}^H \mathbf{R}_{m,i,j} \mathbf{u}_{m,l} + \sigma_{i,j}^2}. \quad (2)$$

The goal is to jointly optimize the beamformers $\mathbf{U}$ and the power allocation vector $\mathbf{p}$ of all cells, so that the SINRs of users in each cell are balanced and maximised. Let t_i be the balanced SINR of the users in

the i th cell. The problem of interest can be stated as

$$\max_{\mathbf{U}, \mathbf{p}} \quad t_i, \forall i \in \{1, 2, \dots, N\} \quad (3a)$$

$$\text{s.t.} \quad \min_{1 \leq j \leq K} \frac{\Gamma_{i,j}^{DL}(\mathbf{U}, \mathbf{p})}{\rho_{i,j}} \geq t_i, \forall i \quad (3b)$$

$$\mathbf{1}_K^T \mathbf{p}_i \leq P_i^{max}, \forall i, \quad (3c)$$

where $\rho_{i,j}$ is a priority factor for user j in the i th cell. We wish to highlight that all available works in the literature so far considered $t_1 = t_2 = \dots = t_N = t$, i.e., SINRs of all users in all cells are balanced to an identical t value, however by considering different $t_i = 1, 2, \dots, N$, we aim to balance SINRs of all users in various cells to different possible maximum values.

III. SINR-BALANCING IN THE COORDINATED MULTICELL

The first stage of the proposed algorithm aims to balance the SINR of all users in all cells. This is achieved using the SINR balancing techniques proposed, for example, [21], [23] and [27]. For the purpose of completeness and to facilitate development of the proposed algorithms in Section IV, we summarise the SINR balancing technique for multicell beamforming in this section. The minimum SINR of users in any cell is maximised, subject to an upper bound on the transmit power as follows:

$$\max_{\mathbf{U}, \mathbf{p} \succeq 0} \min_{i,j} \quad \frac{\Gamma_{i,j}^{DL}(\mathbf{U}, \mathbf{p})}{\rho_{i,j}} \quad (4a)$$

$$\text{s.t.} \quad \mathbf{1}_K^T \mathbf{p}_i \leq P_i^{max}, \forall i. \quad (4b)$$

The multiple per-BS power constraints in (4b) can be put into a single linear constraint by introducing auxiliary variables $a_i \in \mathbb{R}^+$ as $\sum_{i=1}^N a_i (\mathbf{1}_K^T \mathbf{p}_i - P_i^{max}) \leq 0$. The auxiliary variables need to be adapted using a subgradient method as described in [26]. Defining a vector $\mathbf{a} = [a_1 \mathbf{1}_K^T \dots a_N \mathbf{1}_K^T]^T$ and a scalar $P^{max} \triangleq \sum_{i=1}^K a_i P_i^{max}$, the problem in (4) can be restated as

$$\max_{\mathbf{U}, \mathbf{p} \succeq 0} \min_{i,j} \quad \frac{\Gamma_{i,j}^{DL}(\mathbf{U}, \mathbf{p})}{\rho_{i,j}} \quad (5a)$$

$$\text{s.t.} \quad \mathbf{a}^T \mathbf{p} \leq P^{max}. \quad (5b)$$

A. Downlink Power Assignment for a given set of Beamformers

Let $\tilde{\mathbf{p}}$ be the optimal power allocation for the problem in (5). At the optimum, the inequality constraint should be satisfied with equality [19]. Hence for a given set of beamformers $\tilde{\mathbf{U}}$, the optimization in (5) should satisfy

$$\frac{\Gamma_{i,j}^{DL}(\tilde{\mathbf{U}}, \tilde{\mathbf{p}})}{\rho_{i,j}} = \gamma^{DL}(\tilde{\mathbf{U}}, P^{max}), \quad (6a)$$

$$\mathbf{a}^T \tilde{\mathbf{p}} = P^{max}, \quad (6b)$$

where $\gamma^{DL}(\tilde{\mathbf{U}}, P^{max})$ is the balanced SINR. Substituting the definition of SINR given in (2) and using the method proposed in [19], the following eigensystem can be constructed to determine optimum downlink power allocation:

$$\begin{bmatrix} \mathbf{D}\Psi & \mathbf{D}\sigma \\ \frac{1}{P^{max}}\mathbf{a}^T\mathbf{D}\Psi & \frac{1}{P^{max}}\mathbf{a}^T\mathbf{D}\sigma \end{bmatrix} \begin{pmatrix} \tilde{\mathbf{p}} \\ 1 \end{pmatrix} = \frac{1}{\gamma^{DL}(\tilde{\mathbf{U}}, P^{max})} \begin{pmatrix} \tilde{\mathbf{p}} \\ 1 \end{pmatrix}, \quad (7)$$

where $\mathbf{D} = \text{blkdiag}\{\mathbf{D}^1, \dots, \mathbf{D}^N\}$, $\mathbf{D}^n = \text{diag}[(\frac{\rho_{n,1}}{\tilde{\mathbf{u}}_{n1}^H \mathbf{R}_{n,n,1} \tilde{\mathbf{u}}_{n1}}) \dots (\frac{\rho_{n,K}}{\tilde{\mathbf{u}}_{nK}^H \mathbf{R}_{n,n,K} \tilde{\mathbf{u}}_{nK}})]$, $\sigma = [\sigma^1 \dots \sigma^N]^T$, $\sigma^n = [\sigma_{n,1}^2 \dots \sigma_{n,K}^2]^T$,

$$\Psi = \begin{bmatrix} \Psi^1 & \Psi^{2,1} & \dots & \Psi^{N,1} \\ \Psi^{1,2} & \Psi^2 & \dots & \Psi^{N,2} \\ \vdots & \vdots & \ddots & \vdots \\ \Psi^{1,N} & \Psi^{2,N} & \dots & \Psi^N \end{bmatrix}, \quad (8)$$

$$[\Psi^n]_{ki} = \begin{cases} \tilde{\mathbf{u}}_{ni}^H \mathbf{R}_{nnk} \tilde{\mathbf{u}}_{ni} & \text{if } i \neq k, i = 1, \dots, K, j = 1, \dots, K \\ 0 & \text{if } i = k \end{cases} \quad (9)$$

and

$$[\Psi^{l,n}]_{ki} = \begin{cases} \tilde{\mathbf{u}}_{li}^H \mathbf{R}_{lnk} \tilde{\mathbf{u}}_{li} & k = 1, \dots, K, i = 1, \dots, K, l \neq n \end{cases} \quad (10)$$

B. Uplink Power Assignment for a given set of Beamformers

Designing beamformers directly in the downlink is difficult due to coupling of all beamformer vectors and power allocation in the SINR equation of every user. Hence, SINR uplink-downlink duality has been proposed in [19], [26] which states that the same SINR achieved in the downlink can also be achieved in a

virtual uplink using the same set of beamformers under identical total power constraint. Using the results in [26], for the multiple linear constraints as well, it can be shown that for a given set of beamformers $\tilde{\mathbf{u}}_{i,j}$, the downlink optimization problem in (4) is equivalent to the following virtual uplink optimization problem:

$$\max_{\tilde{\mathbf{q}} \succeq 0} \min_{i,j} \frac{\Gamma_{i,j}^{UL}(\tilde{\mathbf{u}}_{i,j}, \tilde{\mathbf{q}})}{\rho_{i,j}} \quad (11a)$$

$$\text{s.t.} \quad \boldsymbol{\sigma}^T \tilde{\mathbf{q}} \leq P^{max}, \quad (11b)$$

where $\Gamma_{i,j}^{UL}$ is the SINR of user j in the i th cell given by

$$\Gamma_{i,j}^{UL} = \frac{\tilde{q}_{i,j} \tilde{\mathbf{u}}_{i,j}^H \mathbf{R}_{i,i,j} \tilde{\mathbf{u}}_{i,j}}{\tilde{\mathbf{u}}_{i,j}^H \left(\sum_{\substack{l=1 \\ l \neq j}}^K \tilde{q}_{i,l} \mathbf{R}_{i,i,l} + \sum_{\substack{m=1 \\ m \neq i}}^N \sum_{l=1}^K \tilde{q}_{m,l} \mathbf{R}_{i,m,l} + \boldsymbol{\Omega}_i \right) \tilde{\mathbf{u}}_{i,j}}, \quad (12)$$

and $\boldsymbol{\Omega}_i = a_i \mathbf{I}$ is the equivalent noise covariance matrix in the uplink, $q_{i,j}$ is the virtual uplink transmit power allocated to the j th user in the i th cell, $\tilde{\mathbf{q}}_i = [\tilde{q}_{i1}, \dots, \tilde{q}_{iK}]^T$ and $\tilde{\mathbf{q}} = [\tilde{\mathbf{q}}_1^T, \dots, \tilde{\mathbf{q}}_N^T]^T$. Similar to the downlink solution in (7), the uplink power allocation can be obtained by solving the following eigensystem:

$$\begin{bmatrix} \mathbf{D}\Psi^T & \mathbf{D}\mathbf{b} \\ \frac{1}{P_{max}} \boldsymbol{\sigma}^T \mathbf{D}\Psi^T & \frac{1}{P_{max}} \boldsymbol{\sigma}^T \mathbf{D}\mathbf{b} \end{bmatrix} \begin{pmatrix} \tilde{\mathbf{q}} \\ 1 \end{pmatrix} = \frac{1}{\gamma^{UL}(\tilde{\mathbf{U}}, P_{max})} \begin{pmatrix} \tilde{\mathbf{q}} \\ 1 \end{pmatrix}, \quad (13)$$

where $\mathbf{b} = [\mathbf{b}^1, \dots, \mathbf{b}^N]^T$ and $[\mathbf{b}^n]_i = \begin{cases} \tilde{\mathbf{u}}_{ni}^H \boldsymbol{\Omega}_i \tilde{\mathbf{u}}_{ni} & n = 1, \dots, N, i = 1, \dots, K. \end{cases}$

C. Beamformer Design for a given Power Allocation

For a given uplink power allocation $\tilde{\mathbf{q}}$, beamformers in the uplink are designed by solving the following generalized eigenvector problem

$$\tilde{\mathbf{u}}_{i,j} = \max_{\mathbf{u}_{i,j}} \frac{\mathbf{u}_{i,j}^H \mathbf{R}_{i,i,j} \mathbf{u}_{i,j}}{\mathbf{u}_{i,j}^H \mathbf{Q}_{ij} \mathbf{u}_{i,j}}, \quad \text{s.t.} \quad \|\mathbf{u}_{i,j}\|_2 = 1. \quad (14a)$$

where $\mathbf{Q}_{ij} = \sum_{\substack{l=1 \\ l \neq j}}^K \tilde{q}_{i,l} \mathbf{R}_{i,i,l} + \sum_{\substack{m=1 \\ m \neq i}}^N \sum_{l=1}^K \tilde{q}_{m,l} \mathbf{R}_{i,m,l} + \boldsymbol{\Omega}_i, \quad \forall i, \forall j$

TABLE I

Algorithm 1: Algorithmic solution of the SINR Balancing Problem (4)

-
- 1) **Initialize** $m \leftarrow 0, n \leftarrow 0, \mathbf{q}^{(0)}, \mathbf{a}^{(0)}, t, \epsilon$
 - 2) **Repeat**
 - 3) $m \leftarrow m + 1$
 - 4) **Repeat**
 - 5) $n \leftarrow n + 1$
 - 6) Solve (14) using $\mathbf{q}^{(n-1)}$ to obtain $\tilde{\mathbf{U}}^{(n-1)}$
 - 7) Compute $\mathbf{D}^{(n-1)}, \Psi^{(n-1)}, \mathbf{b}^{(n-1)}$, using $\tilde{\mathbf{U}}^{(n-1)}$
 - 8) Solve (13) and obtain $\gamma^{(n)}$ and $\tilde{\mathbf{q}}^{(n)}$
 - 9) **Until** $\gamma^{(n-1)} - \gamma^{(n)} \leq \epsilon$
 - 10) $\gamma^* = \gamma^{(n)}$ and $\tilde{\mathbf{U}}^* = \tilde{\mathbf{U}}^{(n-1)}$
 - 11) Obtain $\tilde{\mathbf{p}}^{(m)}$ using (7)
 - 12) Update auxiliary variables using (15)
 - 13) **Until** (16) is satisfied.
-

D. Iterative Solution

The optimization in (4) can be solved using an iterative algorithm. For a given set of auxiliary variables $\mathbf{a}$, we iteratively obtain the beamformers from (14) and the power allocation from (13) until convergence. From the uplink-downlink duality, the same SINR values can be achieved in both the uplink and the downlink with the same set of beamformers and total transmit power but with different power allocation. Hence, the beamformers $\mathbf{U}$ that have been computed for the uplink, are used in the downlink to obtain the downlink power allocation using (7). In order to update the auxiliary variables, we use a subgradient method [27], [28] as follows:

$$a_i^{(m+1)} = a_i^{(m)} + t(\mathbf{1}_K^T \mathbf{p}_i^{(m)} - P_i^{max}), \quad i = 1, 2, \dots, N, \quad (15)$$

where t denotes the step size of the subgradient method. The algorithm stops when the following criterion is satisfied

$$|a_i^{(m+1)}(\mathbf{1}_K^T \mathbf{p}_i^{(m)} - P_i^{max})| \leq \epsilon, \quad i = 1, 2, \dots, N, \quad (16)$$

where ϵ is a very small positive constant. The algorithm is summarized in Table I. The quantities that are associated with the n th iteration are denoted by the superscript (n) . We also use the shorthand notation $\gamma^{(n)}$, which is the inverse of the largest eigenvalue obtained as in (13).

IV. SINR-BALANCING WITH PER BASESTATION SINR-TARGET-CONSTRAINT

Once the SINR balancing is achieved as described in section III, at least one of the BS power constraints must be active, i.e., at least one of the BSs should be drawing the maximum possible transmit power. The rationale behind this argument is that suppose at the optimum, if all BSs do not use the maximum transmit power, the transmit power of all BSs can be scaled up by a factor $\alpha > 1$ until one of the BSs attains its full transmit power. Since SINR increases monotonically with increasing transmit power, the balanced SINR will be increased. As the aim was to maximise the balanced SINR, at least one of the BS power constraints must be active. Let us denote this BS as l . The corresponding balanced SINR γ_l obtained from Algorithm 1, is the maximum SINR that this BS can achieve. At this stage, other BSs may not have used all of their transmit power. Hence, we perform a second stage of optimization to enable other BSs to increase their balanced SINR using their remaining transmit powers while ensuring SINRs of all users' in the l th BS are kept at γ_l . Hence, our optimization problem is to maximize minimal SINR of all users in all BSs except the l th BS, while keeping all transmit power constraints and introducing an additional constraint that the users in the l th BS should achieve a target SINR γ_l as follows:

$$\max_{\mathbf{U}, \mathbf{p} \geq 0} \min_{i,j} \frac{\Gamma_{i,j}^{DL}(\mathbf{U}, \mathbf{p})}{\rho_{i,j}}, \quad \forall i, i \neq l, \forall j \quad (17a)$$

$$\text{s.t.} \quad \frac{\Gamma_{l,j}^{DL}(\mathbf{U}, \mathbf{p})}{\rho_{l,j}} \geq \gamma_l, \forall j \quad (17b)$$

$$\mathbf{1}_K^T \mathbf{p}_i \leq P_i^{max}, \forall i. \quad (17c)$$

The above optimization aims to maximise SINRs of users in the remaining $(N-1)$ BSs, while ensuring the balanced SINR of users in the l th BS is γ_l . At this stage, the BS l and possibly one of the remaining $(N-1)$ BSs will use the available transmit power fully. Let us denote the BS that uses its full transmit power from the remaining $(N-1)$ BS as l' and the corresponding balanced SINR as $\gamma_{l'}$. It should be noted that $\gamma_{l'} \geq \gamma_l$. This is because if $\gamma_{l'} < \gamma_l$, then BS l' should have used all of its transmit power at the first stage of the optimization. We repeat this target SINR based SINR balancing beamformer design until SINR of users in the last BS is maximised. At the end of this repeated optimization, users in various BSs should have achieved different levels of balanced SINR. Suppose we are at the N_1 th stage of this repeated optimization. Also, without loss of generality, assume the first N_1 BSs have achieved full use of their transmit power. We formulate the optimization as maximising the SINR of all users in all BSs

except the first N_1 BSs subject to the maximum transmit power constraints and SINR target constraint for the users in the first N_1 BSs as follows

$$\max_{\mathbf{U}, \mathbf{p} \succeq 0} \min_{i,j} \frac{\Gamma_{i,j}^{DL}(\mathbf{U}, \mathbf{p})}{\rho_{i,j}}, \quad i = N_1 + 1, \dots, N, \forall j \quad (18a)$$

$$\text{s.t.} \quad \frac{\Gamma_{i,j}^{DL}(\mathbf{U}, \mathbf{p})}{\rho_{i,j}} \geq \gamma_i, \quad i = 1, \dots, N_1, \forall j, \quad (18b)$$

$$\mathbf{1}_K^T \mathbf{p} \leq P_i^{max}, \forall i, \quad (18c)$$

where $\Gamma_{i,j}^{DL}(\mathbf{U}, \mathbf{p})$ is the SINR of the j th user at the i th cell in the downlink, which is given by (2) and γ_i is the balanced SINR achieved by the i th BS. First, we put the multiple linear constraints in (18c) into a single linear constraint by introducing the auxiliary variable $\mathbf{b} = [b_1 \mathbf{1}_K^T \dots b_N \mathbf{1}_K^T]$. These auxiliary variables can be updated based on the subgradient method described in [28]. Defining $P^{max} \triangleq \sum_{i=1}^K b_i P_i^{max}$, the problem in (18) can be written as

$$\max_{\mathbf{U}, \mathbf{p} \succeq 0} \min_{i,j} \frac{\Gamma_{i,j}^{DL}(\mathbf{U}, \mathbf{p})}{\rho_{i,j}}, \quad i = N_1 + 1, \dots, N, \forall j \quad (19a)$$

$$\text{s.t.} \quad \frac{\Gamma_{i,j}^{DL}(\mathbf{U}, \mathbf{p})}{\rho_{i,j}} \geq \gamma_i, \quad i = 1, \dots, N_1, \forall j \quad (19b)$$

$$\mathbf{b}^T \mathbf{p} \leq P^{max}. \quad (19c)$$

Based on the uplink-downlink duality [19], [26], for a given set of auxiliary variables, the dual uplink problem can be formulated as

$$\max_{\mathbf{U}, \mathbf{q} \succeq 0} \min_{i,j} \frac{\Gamma_{i,j}^{UL}(\mathbf{U}, \mathbf{q})}{\rho_{i,j}}, \quad i = N_1 + 1, \dots, N, \forall j \quad (20a)$$

$$\text{s.t.} \quad \frac{\Gamma_{i,j}^{UL}(\mathbf{U}, \mathbf{q})}{\rho_{i,j}} \geq \gamma_i, \quad i = 1, \dots, N_1, \forall j \quad (20b)$$

$$\boldsymbol{\sigma}^T \mathbf{q} \leq P^{max}, \quad (20c)$$

where $\Gamma_{i,j}^{UL}(\mathbf{U}, \mathbf{q})$ is the uplink SINR, given by (12).

A. Uplink Power Allocation for a given set of Beamformers

As the first step, for a given set of beamformers $\hat{\mathbf{u}}_{i,j} \forall i, j$, in the uplink, we determine the uplink power allocation $\hat{\mathbf{q}}$. The optimal power allocation should satisfy the following set of equations:

$$\frac{\Gamma_{i,j}^{UL}(\hat{\mathbf{u}}_{i,j}, \hat{\mathbf{q}})}{\rho_{i,j}} = \lambda(\hat{\mathbf{U}}, P^{max}), i = N_1 + 1, \dots, N, \forall j, \quad (21a)$$

$$\frac{\Gamma_{i,j}^{UL}(\hat{\mathbf{u}}_{i,j}, \hat{\mathbf{q}})}{\rho_{i,j}} = \gamma_i(\hat{\mathbf{U}}, P^{max}), i = 1, \dots, N_1, \forall j, \quad (21b)$$

$$\sigma^T \hat{\mathbf{q}} = P^{max}, \quad (21c)$$

where $\lambda(\hat{\mathbf{U}}, P^{max})$ is the balanced SINR of the users in the cells that have not used their full transmit power. We define $\hat{\mathbf{q}}_A = [\hat{\mathbf{q}}_1 \dots \hat{\mathbf{q}}_{N_1}]$ as the power allocation of users in the first N_1 cells and $\hat{\mathbf{q}}_B = [\hat{\mathbf{q}}_{N_1+1} \dots \hat{\mathbf{q}}_N]$ as the power allocation of users in the remaining cells and evolve our earlier approach [29] to write power allocation of a set of users in terms of power allocation of the remaining users and solve the uplink power allocation in (20). It can be shown from (12) and (19) that $\hat{\mathbf{q}}_A$ and $\hat{\mathbf{q}}_B$ satisfy

$$\hat{\mathbf{q}}_A = \mathbf{D}_A \Psi_A \hat{\mathbf{q}}_A + \mathbf{D}_A \Psi_B \hat{\mathbf{q}}_B + \mathbf{D}_A \mathbf{b}_A, \quad (22a)$$

$$\frac{1}{\lambda} \hat{\mathbf{q}}_B = \mathbf{D}_B \Psi_C \hat{\mathbf{q}}_A + \mathbf{D}_B \Psi_D \hat{\mathbf{q}}_B + \mathbf{D}_B \mathbf{b}_B, \quad (22b)$$

$$P^{max} = \sigma_A^T \hat{\mathbf{q}}_A + \sigma_B^T \hat{\mathbf{q}}_B, \quad (22c)$$

where $\sigma_A = [\sigma_1 \dots \sigma_{N_1}]^T$, $\sigma_B = [\sigma_{N_1+1} \dots \sigma_N]^T$, $\mathbf{D}_A = \text{blkdiag}\{\gamma_1 \mathbf{D}^1, \dots, \gamma_{N_1} \mathbf{D}^{N_1}\}$, $\mathbf{D}_B = \text{blkdiag}\{\mathbf{D}^{N_1+1}, \dots, \mathbf{D}^N\}$,

$$\Psi_A = \begin{bmatrix} \Psi^1 & \dots & \Psi^{1,N_1} \\ \vdots & \ddots & \vdots \\ \Psi^{N_1,1} & \dots & \Psi^{N_1} \end{bmatrix}, \quad (23)$$

$$\Psi_B = \begin{bmatrix} \Psi^{1,N_1+1} & \dots & \Psi^{1,N} \\ \vdots & \ddots & \vdots \\ \Psi^{N_1,N_1+1} & \dots & \Psi^{N_1,N} \end{bmatrix}, \quad (24)$$

$$\Psi_C = \begin{bmatrix} \Psi^{N_1+1,1} & \dots & \Psi^{N_1+1,N_1} \\ \vdots & \ddots & \vdots \\ \Psi^{N,1} & \dots & \Psi^{N,N_1} \end{bmatrix}, \quad (25)$$

$$\Psi_D = \begin{bmatrix} \Psi^{N_1+1} & \dots & \Psi^{N_1+1,N} \\ \vdots & \ddots & \vdots \\ \Psi^{N,N_1+1} & \dots & \Psi^N \end{bmatrix}, \quad (26)$$

$\mathbf{b}_A = [\mathbf{b}^1, \dots, \mathbf{b}^{N_1}]$ and $\mathbf{b}_B = [\mathbf{b}^{N_1+1}, \dots, \mathbf{b}^N]$. According to [29], $(\mathbf{I} - \mathbf{D}_A \Psi_A)$ is invertible and $(\mathbf{I} - \mathbf{D}_A \Psi_A)^{-1}$ is nonnegative. Therefore, $\hat{\mathbf{q}}_A$ can be written in terms of $\hat{\mathbf{q}}_B$ as

$$\hat{\mathbf{q}}_A = (\mathbf{I} - \mathbf{D}_A \Psi_A)^{-1} \mathbf{D}_A \Psi_B \hat{\mathbf{q}}_B + (\mathbf{I} - \mathbf{D}_A \Psi_A)^{-1} \mathbf{D}_A \mathbf{b}_A. \quad (27)$$

Using (27) in (22b), it follows that

$$\frac{1}{\lambda} \hat{\mathbf{q}}_B = \mathbf{B} \hat{\mathbf{q}}_B + \mathbf{t}, \quad (28)$$

where

$$\mathbf{B} = \mathbf{D}_B \Psi_C (\mathbf{I} - \mathbf{D}_A \Psi_A)^{-1} \mathbf{D}_A \Psi_B + \mathbf{D}_B \Psi_D \quad (29a)$$

$$\mathbf{t} = \mathbf{D}_B \Psi_C (\mathbf{I} - \mathbf{D}_A \Psi_A)^{-1} \mathbf{D}_A \mathbf{b}_A + \mathbf{D}_B \mathbf{b}_B \quad (29b)$$

By substituting (27) into (22c), we obtain the following

$$\mathbf{c}^T \hat{\mathbf{q}}_B = P^{max} - c, \quad (30)$$

where

$$\mathbf{c}^T = \sigma_A^T (\mathbf{I} - \mathbf{D}_A \Psi_A)^{-1} \mathbf{D}_A \Psi_A + \sigma_B^T \quad (31a)$$

$$c = \sigma_A^T (\mathbf{I} - \mathbf{D}_A \Psi_A)^{-1} \mathbf{D}_A \mathbf{b}_A. \quad (31b)$$

Multiplying both sides of (28) by $\mathbf{c}^T$ and using (30), we obtain

$$\frac{1}{\lambda} = \frac{1}{P^{max} - c} \mathbf{c}^T \mathbf{B} \hat{\mathbf{q}}_B + \frac{1}{P^{max} - c} \mathbf{c}^T \mathbf{t}. \quad (32)$$

From (28) and (32), $\hat{\mathbf{q}}_{ext} = [\hat{\mathbf{q}}_B^T \ 1]^T$ satisfies

$$\frac{1}{\lambda} \hat{\mathbf{q}}_{ext} = \begin{bmatrix} \mathbf{B} & \mathbf{t} \\ \frac{1}{P^{max} - c} \mathbf{c}^T \mathbf{B} & \frac{1}{P^{max} - c} \mathbf{c}^T \mathbf{t} \end{bmatrix} \hat{\mathbf{q}}_{ext}. \quad (33)$$

Let

$$\mathbf{W}(\hat{\mathbf{U}}) = \begin{bmatrix} \mathbf{B} & \mathbf{t} \\ \frac{1}{P^{max} - c} \mathbf{c}^T \mathbf{B} & \frac{1}{P^{max} - c} \mathbf{c}^T \mathbf{t} \end{bmatrix} \quad (34)$$

In [29], it was also shown that the conditions

$$\rho(\mathbf{D}_A \mathbf{\Psi}_A) \leq 1 \quad (35a)$$

$$c \leq P^{max} \quad (35b)$$

will imply that $\mathbf{W}(\hat{\mathbf{U}})$ is a nonnegative matrix.

According to the Perron-Frobenius theory, among all eigenvalues of $\mathbf{W}(\hat{\mathbf{U}})$, $\lambda_{max}(\mathbf{W}(\hat{\mathbf{U}}))$ is the unique eigenvalue of maximum modulus. It is real and positive and associated with a positive eigenvector $\hat{\mathbf{q}}_{ext}$. $\hat{\mathbf{q}}_B$ can be obtained from $\hat{\mathbf{q}}_{ext}$ by scaling the elements such that the last element is equal to one. Then $\hat{\mathbf{q}}_A$ can be obtained using (27).

B. Beamformer Design for a Given Power Allocation

For a given power allocation, the optimal beamformers for all the users in the virtual uplink are obtained by maximizing independently the SINR of each user in the uplink. Hence, we have to solve the same optimization problem as in (14).

C. Iterative Solution

From the uplink-downlink duality, the same SINR values can be achieved in both the uplink and the downlink with the same set of beamformers and total power constraints. Thus, the uplink beamformers $\hat{\mathbf{U}}$ obtained in the virtual uplink can be used to achieve the same SINR values in the downlink. Let

us denote the power allocation in the downlink by $\mathbf{p} = [\mathbf{p}_A^T \mathbf{p}_B^T]^T$, where $\mathbf{p}_A$ and $\mathbf{p}_B$ are the downlink power allocation vectors for the users belonging to the first N_1 BSs that have already used their powers fully and the remaining $(N - N_1)$ BSs, respectively.

Similar to (28), (29a) and (29b), we can write the following equations for the power allocation of the users in the downlink:

$$\frac{1}{\lambda} \hat{\mathbf{p}}_B = \mathbf{B}^{DL} \hat{\mathbf{p}}_B + \mathbf{t}^{DL} \quad (36)$$

where

$$\mathbf{B}^{DL} = \mathbf{D}_B \Psi_B^T (\mathbf{I} - \mathbf{D}_A \Psi_A^T)^{-1} \mathbf{D}_A \Psi_C^T + \mathbf{D}_B \Psi_D^T \quad (37a)$$

$$\mathbf{t}^{DL} = \mathbf{D}_B \Psi_B^T (\mathbf{I} - \mathbf{D}_A \Psi_A^T)^{-1} \mathbf{D}_A \boldsymbol{\sigma}_A + \mathbf{D}_B \boldsymbol{\sigma}_B \quad (37b)$$

We can show that $(\frac{1}{\lambda} \mathbf{I} - \mathbf{B}^{DL})$ is nonsingular and $(\frac{1}{\lambda} \mathbf{I} - \mathbf{B}^{DL})^{-1}$ is a nonnegative matrix. Using this assumption, the power allocation in the downlink is given by

$$\hat{\mathbf{p}}_B = (\frac{1}{\lambda} \mathbf{I} - \mathbf{B}^{DL})^{-1} \mathbf{t}^{DL} \quad (38a)$$

$$\begin{aligned} \hat{\mathbf{p}}_A &= (\mathbf{I} - \mathbf{D}_A \Psi_A^T)^{-1} \mathbf{D}_A \Psi_C^T \hat{\mathbf{p}}_B + \\ &(\mathbf{I} - \mathbf{D}_A \Psi_A^T)^{-1} \mathbf{D}_A \boldsymbol{\sigma}_A \end{aligned} \quad (38b)$$

Having obtained the beamformers and the power allocations for a given set of auxiliary variables $\mathbf{b}$, the auxiliary variables are updated according to the following subgradient method [27], [30].

$$b_i^{(m+1)} = b_i^{(m)} + t(\mathbf{1}_K^T \mathbf{p}_i^{(m)} - P^{max}), \quad i = 1, 2, \dots, N, \quad (39)$$

where t denotes the step size of the subgradient method. The algorithm stops when the following criterion is met

$$|b_i^{(m+1)}(\mathbf{1}_K^T \mathbf{p}_i^{(m)} - P^{max})| \leq \epsilon, \quad i = 1, 2, \dots, N. \quad (40)$$

The proposed algorithm is summarized in Table II.

TABLE II

Algorithm 2: Algorithmic solution of the mixed SINR balancing and SINR-target-constraints based beamformer design (18)

- 1) **Initialize** $m \leftarrow 0$, $n \leftarrow 0$, $\hat{\mathbf{q}}_A^{(0)}$, $\hat{\mathbf{q}}_B^{(0)}$, $\mathbf{b}^{(0)}$, t , ϵ
- 2) **Repeat**
- 3) $m \leftarrow m + 1$
- 4) **Repeat**
- 5) $n \leftarrow n + 1$
- 6) Solve (14) using $\hat{\mathbf{q}}_A^{(n-1)}$, $\hat{\mathbf{q}}_B^{(n-1)}$ to obtain $\hat{\mathbf{U}}^{(n-1)}$
- 7) Compute $\mathbf{D}_A^{(n-1)}$, $\mathbf{D}_B^{(n-1)}$, $\Psi_A^{(n-1)}$, $\Psi_B^{(n-1)}$, $\Psi_C^{(n-1)}$, $\Psi_D^{(n-1)}$, $\mathbf{b}_A^{(n-1)}$, $\mathbf{b}_B^{(n-1)}$ using $\hat{\mathbf{U}}^{(n-1)}$
- 8) Solve (33) and obtain $\lambda^{(n)}$ and $\hat{\mathbf{q}}_B^{(n)}$
- 9) Obtain $\hat{\mathbf{q}}_A^{(n)}$ from (27)
- 10) **Until** $\lambda^{(n-1)} - \lambda^{(n)} \leq \epsilon$
- 11) $\lambda^* = \lambda^{(n)}$ and $\hat{\mathbf{U}}^* = \hat{\mathbf{U}}^{(n-1)}$
- 12) Obtain $\hat{\mathbf{p}}_B^{(m)}$, $\hat{\mathbf{p}}_A^{(m)}$ using (38a) and (38b), respectively.
- 13) Update auxiliary variables using (39)
- 14) **Until** (40) is satisfied.

	Matrix Inversion	Eigenvalue Decomposition
Step 6	$nKN\mathcal{O}(M^3)$	$nKN\mathcal{O}[M^3 + (M\log^2 M)\log b]$
Step 8	–	$n\mathcal{O}\{(NK+1)^3 + [(NK+1)\log^2(NK+1)]\log b\}$
Step 11	–	$\mathcal{O}\{(NK+1)^3 + [(NK+1)\log^2(NK+1)]\log b\}$

TABLE III
COMPLEXITY OF THE 1ST STAGE OF THE ALGORITHM

V. COMPLEXITY AND CONVERGENCE ANALYSIS

Complexity Analysis

For a given set of auxiliary variables, the complexity of the proposed algorithm in Table I and Table II mainly depends on the complexities of the matrix inversion and the eigenvalue decomposition. For a given $n \times n$ matrix, the required arithmetic operations to determine its inverse and the eigenvectors are given by $\mathcal{O}(n^3)$ and $\mathcal{O}(n^3 + (n\log^2 n)\log b)$, respectively, where b is the relative error bound [31]. Based on this, the number of arithmetic operations required for every iteration of the algorithm in Table I and Table II are shown in Table III and Table IV, respectively.

	Matrix Inversion	Eigenvalue Decomposition
Step 6	$nKN\mathcal{O}(M^3)$	$nKN\mathcal{O}[M^3 + (M\log^2 M)\log b]$
Step 8	–	$n\mathcal{O}\{[(N-1)K+1]^3 + [(N-1)K+1]\log^2[(N-1)K+1]\log b\}$
Step 9	$n\mathcal{O}\{[N(K-1)+1]^3\}$	–
Step 12	$2\mathcal{O}\{[N(K-1)+1]^3\}$	–

TABLE IV
COMPLEXITY OF THE 2ND STAGE OF THE ALGORITHM

Only steps 6, 8 and 11 of Algorithm 1 and steps 6, 8, 9 and 12 of Algorithm 2 require matrix inversion or eigenvalue decomposition. Hence the total arithmetic operations required for each iteration is the summation of arithmetic operations needed for matrix inversion and the eigenvalue decomposition. In both tables, n denotes the number of iterations required to satisfy the stopping criterion, i.e., step 9 in Table I and step 10 in Table II. n depends on the adaptation step size t . For example, it was observed through simulation results that algorithm 1 requires approximately four to five iterations when $t = 0.05$.

Convergence Analysis

The convergence of the auxiliary variables is discussed. Assume that the i^{th} constraint (i.e., $\mathbf{1}_K^T \mathbf{p}_i \leq P_i^{\max}$) is not satisfied at the first iteration. Since $\mathbf{1}_K^T \mathbf{p}_i^{(1)} > P_i^{\max}$, the corresponding auxiliary variable a_i will be increased based on the subgradient method as follows:

$$a_i^{(m+1)} = a_i^{(m)} + t(\mathbf{1}_K^T \mathbf{p}_i^{(m)} - P_i^{\max}). \quad (41)$$

At the next iteration, the corresponding uplink noise covariance will be increased as $a_i^{(1)} > a_i^{(0)}$ (i.e., $\Omega_i = a_i \mathbf{I}$). This reduces the achievable SINR at the next iteration and yields into less power allocation (i.e., $\mathbf{1}_K^T \mathbf{p}_i^{(1)} \geq \mathbf{1}_K^T \mathbf{p}_i^{(2)}$). Hence, the corresponding auxiliary variable a_i increases with the iteration number n until the stopping criterion $|a_i^{(m+1)}(\mathbf{1}_K^T \mathbf{p}_i^{(m)} - P_i^{\max})| \leq \epsilon$ is satisfied as follows:

$$a_i^{(0)} \leq a_i^{(1)} \leq \dots \leq a_i^{(n)}. \quad (42)$$

The stopping criterium is satisfied when either a_i reaches a small value or the associated constraint is satisfied. Since, one of these conditions is achieved, the auxiliary variable adaptation converges. The similar argument holds for the scenario where the constraint is satisfied at the first iteration. In this case,

the corresponding auxiliary variable will be decreasing with the iteration. This monotonic convergence of the auxiliary variable can be observed in Fig. 2.

Having established the monotonic convergence of the auxiliary variables, we investigate SINR convergence of the inner-loop and outer-loop of the proposed algorithm. In the inner-loop, the beamformers and the power allocations are determined for a given set of auxiliary variables whereas the outer-loop updates the auxiliary variables through a subgradient method. In the inner-loop, the optimal beamformers and the power allocations are iteratively determined by maximizing the virtual uplink SINR of each user [19], [26]. Hence, the virtual uplink SINR increases at each iteration. However, the achievable uplink SINR is upper bounded, since the available transmit power is limited. Hence, the inner-loop converges to a fixed solution. In addition, the inner-loop provides the optimal solution for a given set of auxiliary variables [19], [26]. If the auxiliary variables are chosen to satisfy the constraints through the outer-loop, then the beamformers and power allocation obtained from the inner-loop is optimal solution for the original problem. As discussed earlier, the auxiliary variables are updated to satisfy the constraints and they either monotonically increase or decrease. Hence, the proposed iterative algorithms yield optimal solution which has been verified in the simulation section using the SDP formulation of the power minimization problem.

Beamformer Design based SDP

For a given set of target SINRs, beamformer weight vectors can be designed using the following optimization [32].

$$\begin{aligned}
& \min_{\mathbf{w}_{i,j}} \quad \sum_{i=1}^N \sum_{j=1}^K \|\mathbf{w}_{i,j}\|_2^2 \\
& \text{s.t.} \quad \frac{\mathbf{w}_{i,j}^H \mathbf{R}_{i,i,j} \mathbf{w}_{i,j}}{\sum_{l=1, l \neq j}^K \mathbf{w}_{i,l}^H \mathbf{R}_{i,i,j} \mathbf{w}_{i,l} + \sum_{m=1, m \neq i}^N \sum_{l=1}^K \mathbf{w}_{m,l}^H \mathbf{R}_{m,i,j} \mathbf{w}_{m,l} + \sigma_{i,j}^2} \geq \gamma_{i,j}, \quad \forall i, j,
\end{aligned} \tag{43}$$

The above optimization can be converted into convex form and solved using SDP [32], hence it provides optimum solution. We use this formulation to compare optimality of our results in the simulations section. However, it should be highlighted that the SINR balancing problem as considered in this paper can not be solved directly using the SDP approaches because the optimal balanced SINR values are not known a priori.

VI. SIMULATION RESULTS

First, we verify the performance of the SINR balancing algorithms without consideration to balancing SINR of each BS at different levels i.e., balancing SINR of all users in all BSs together but subject to individual BS power constraints. We have considered a scenario with two BSs, each serving two users. The BSs and the users consist of four antennas and single antenna, respectively. The noise power at each user terminal is set to 0.01. We have generated random channels for each user from both BSs using zero mean complex Gaussian random variables. The average channel gain from every BS to the corresponding users was set to one while the gain of channels from each BS to users in other BS (co-channel gain) was normalised to 0.1 so that the average power gain for the co-channels is 0.01. The maximum power limitations at the first and the second BSs were set to 0.1W and 10W, respectively. In all our simulations we have set the priority factor $\rho_{i,j} = 1$ for all users. We employed the SINR balancing algorithm described in section III. We set the stopping criterion threshold in (16) and (40) to 0.1 and adaptation step size t in (15) and (39) to 0.015. The adaptation of auxiliary variables for a given set of random channels is shown in Fig. 1. The adaptation is stopped when one of the auxiliary variables goes to zero or when there is insignificant change in subsequent values of both the auxiliary variables. In this example, the second auxiliary variable converges to zero. At the convergence, it was observed that the first BS uses all available power of 0.1W while the second BS uses 0.5425W. i.e., the second BS is not able to use all available power, hence the constraint associated with the second BS power is inactive confirming as to why the second auxiliary variable converges to zero.

After the convergence, we observed that for the random channels used, the SINR of users belonging to both the BS were balanced at 15.35. In order to confirm the optimality of this result, we have used a semidefinite programming (SDP) based beamformer design [32] as described in section V and set the SINR constraints for all four users to 15.35 and the power constraints for the two BSs at 0.1W and 10W, respectively as in the previous simulation. We observed that the SDP based design consumed the same power for BS1 and BS2, as obtained by our proposed algorithm i.e., 0.1W and 0.5425W. Also, we observed that the beamformers obtained using the proposed method and the SDP method are identical. When we have increased the SINR target of all four users to 15.35, the SDP confirmed that the problem is infeasible. This confirms that the balanced SINR obtained using our method is optimum. It should be noticed that for a given set of target SINRs, the beamformer design based on SDP is optimum,

however that the SDP method cannot be used directly to design beamformers for SINR balancing as the balanced SINR value is not known a priori and we used SDP method only to verify the optimum resource allocation for a given set of target SINRs which in our case is the balanced SINR obtained using our method.

Having confirmed the optimality of the first algorithm, we have used the second algorithm described in section IV to optimize the SINR of users with different balanced values for different BSs. For the set of random channels considered, as can be seen in the first row of Table V (i.e., channel 1) the second BS achieved a higher SINR of 34.54. We have obtained the power allocations and the balanced SINR values for each user at each cell for five different set of random channels, as depicted in Table V. We compared the results with the solutions obtained using the SDP approach in Table VI. The random channels used in both Tables are the same and the SINR targets in Table VI are the same SINR values achieved in Table V. For example, for random channel 2 in Table VI, the SINR targets for the two users in the first BS is set to [18.52 18.52] and for the two users in the second BS to [341.46 341.46] as obtained with our method in Table V. Comparing Table V and VI, for the same SINR targets, the power allocation obtained using the SDP approach and the proposed method is the same. In addition, we observed both methods provided the same set of beamformers. For all five random channels, for the verification of results using SDP, we have increased the SINR target of users slightly above the balanced SINR value obtained by our method and observed that SDP optimization turns out to be infeasible. Since the SDP method for the beamformer design for a given set of SINR targets is optimum [32], and since both our method and the SDP method provide the same result, as in Tables V and VI, the proposed method of balancing SINRs of users at different BSs to different levels is optimum.

Fig. 2 depicts the balanced SINRs (in dB) that the users achieve in the first and the second cells against the maximum transmit power of the second BS for a given set of random channels. We have set the transmit power of the first BS to 0.1 W and varied the maximum power of the 2nd BS. As observed in Fig. 2, the balanced SINR achieved from the users belonging to the second cell grows substantially as the maximum transmit power of the second BS increases, while the balanced SINR of the first cell remains at the same value.

For the results presented so far, we have used four antennas for each BS. Since there are only four users

altogether in both BSs, each BS has adequate degrees of freedom to mitigate interference satisfactorily, hence they are able to fully utilize the available transmit power to maximise the balanced SINR as high as possible. However, if the number of antennas are limited, while one of the BS uses its full transmit power (usually the one with lower maximum transmit power), the other BSs may not fully use its transmit power as otherwise it may introduce significant interference to the users in the first BS. For example, we have simulated a scenario as before for a set of random channels, but with three antennas for each BS. The first BS power was set to 1W, while the second BS power was set to 10W. The first BS used all its transmit power of 1W and achieved a balanced SINR of 57.81 for its users, however, the second BS used only 1.7279W and achieved a balanced SINR of 57.81 for its users. The second BS could not increase its transmit power beyond 1.7279W, because it may introduce interference to the first BS and it will result into lower balanced SINR for the first BS. However, a slight decrease in the SINR target for the first BS will increase the second BS transmit power utilization and will enhance the second BS users balanced SINR significantly. This tradeoff has been depicted in Fig. 3. Again, the proposed algorithm has the ability to first determine the maximum possible balanced SINR that the first BS could achieve and secondly to balance and maximise the SINR of the users in the second cell, while satisfying a wide range of target SINRs for the users in the first cell.

Finally, we compare the performance of the proposed algorithm with that of an extension of the work proposed in [24]. The work in [24] proposed to perform SINR balancing of all users in all the cells with multiple BS power constraints. Observing that after SINR balancing, certain BSs may not have used all the available transmit power as also described in our algorithm, the authors in [24] proposed to use any excess power available at these BSs to increase the SINR of the best performing users in those cells without affecting the SINR of other users. However, the work in [24] did not aim to balance the SINR of users in other BSs. Therefore, for the purpose of comparison, we extend the work in [24] to perform a second level of SINR balancing for the users in the BS that has not used all available power, by distributing the remaining power among all users in that BS such that the SINR of users in that BS is balanced and maximised. Assume, that the i^{th} BS used all its transmit power. Suppose if we wish to use all the excess transmit power of the j^{th} BS to its k^{th} user, according to the work in [24], we should design a new

beamformer for the k^{th} user as follows:

$$\mathbf{u}_{j,k}^{new} = \mathbf{u}_{j,k} + a_{j,k} e^{j\theta_k} \mathbf{v}_{j,k}^{ZF} \quad (44)$$

where $\theta_{j,k} = \angle(\mathbf{u}_{j,k}^H \mathbf{h}_{j,j,k})$ and $a_{j,k}$ is determined such that $\|\mathbf{u}_{j,k}^{new}\| = P_j^{max}$ and $\mathbf{v}_{j,k}^{ZF}$ is the projection of $\mathbf{h}_{j,j,k}$ onto the complement of the column space of $\tilde{\mathbf{H}}_{j,k} = [\mathbf{h}_{j,i,1} \cdots \mathbf{h}_{j,i,K} \mathbf{h}_{j,j,1} \cdots \mathbf{h}_{j,j,k-1}, \mathbf{h}_{j,j,k+1}, \cdots \mathbf{h}_{j,j,K}]$, i.e., $\mathbf{v}_{j,k}^{ZF} = (\mathbf{I} - \tilde{\mathbf{H}}_{j,k}(\tilde{\mathbf{H}}_{j,k}^H \tilde{\mathbf{H}}_{j,k})^{-1} \tilde{\mathbf{H}}_{j,k}^H) \mathbf{h}_{j,j,k}$. However, since our aim is to balance SINR of all users in the j^{th} cell, rather than allocating all the excess power to only one user, we modify the work in [24] and design extra beamformers for all the users and distribute the total excess power among all users in the j^{th} cell in such a way the SINRs of all users in the j^{th} cell are balanced. We have used bisection method to determine the optimum excess power allocation. We then compare the performance of our proposed algorithm with the above said extension of [24]. In order to compare our proposed method with the method described in [24], we have considered a scenario with 2 BSs and 3 users for each BS. The transmit power at the first and second BSs were set to 0.1W and 10W respectively. Five sets of different random channels were generated as shown in Table VII. As can be seen, for example in the first row of Table VII, the second BS achieved a higher SINR of 34.2 using our method as compared to the SINR of 25.89 achieved using the extended work of [24]. For all the set of random channels depicted in Table VII, the SINR values achieved using our method are higher than that achieved using the extended work of [24]. This is because, the work in [24] proposed to use the remaining power by using extra beamforming vector components that were obtained by projecting each user channel onto the complement of the column space of all other user channels. This can be viewed as zero forcing multiplexing transmit. Due to zero forcing there is loss of flexibility, however, our work does not impose this strict zero forcing condition, but aims to maximise minimal SINR directly using an optimization framework. Hence, it outperforms the extended work of [24].

Simulation Remarks: Here we summarize some additional key points that have been used to assist the simulation. If any of the BSs does not use all its transmit power, the associated auxiliary variable during the adaptation as in (15) will converge to zero. In this case, Ω_i in the SINR of the equivalent uplink in (12) is zero. Suppose, the number of antennas at the BS is greater than the number of interference components

Channels	Power Allocation				Total Power		Achieved SINR			
	BS1		BS2		BS1	BS2	BS1		BS2	
	User 1	User 2	User 1	User 2			User 1	User 2	User 1	User 2
Channel 1	0.0634	0.0366	7.0120	2.9868	0.1	10	15.35	15.35	34.54	34.54
Channel 2	0.0466	0.0534	3.2116	6.7878	0.1	10	18.52	18.52	341.46	341.46
Channel 3	0.0581	0.0419	3.9419	6.0579	0.1	10	22.37	22.37	806.82	806.82
Channel 4	0.0295	0.0705	3.2601	6.7399	0.1	10	18.95	18.95	659.56	659.56
Channel 5	0.0479	0.0521	1.4086	8.5912	0.1	10	15.08	15.08	280.01	280.01

TABLE V
POWER ALLOCATIONS AND THE ACHIEVED SINRS USING THE PROPOSED METHOD

shown in the denominator of the SINR in (12), the matrix $\mathbf{M} = (\sum_{\substack{l=1 \\ l \neq j}}^K q_{i,l} \mathbf{R}_{i,i,l} + \sum_{\substack{m=1 \\ m \neq i}}^N \sum_{l=1}^K q_{m,l} \mathbf{R}_{i,m,l} + \mathbf{\Omega}_i)$ will become rank deficient. Therefore, the beamformer for this case cannot be designed using the generalised singular value solution. For this case, we designed the beamformer as the vector that is in the null space of the above matrix $\mathbf{M}$. Also, when the number of antennas is higher than that of the users, the BS has the ability to manage interference if possible through steering appropriate nulls. In this case, the multiple BSs have the ability to perform SINR balancing efficiently and the BSs are likely to use the maximum available transmit power. However, with fewer antennas, if the whole network is severely interference limited, certain BSs may not have the flexibility to trade off the power between users and use all available transmit power. The auxiliary variables associated with these BS will be non-zero in this case.

VII. CONCLUSION

We have proposed coordinated multicell beamforming for multiple cells with various transmit power constraints. The aim was to balance and maximise the SINR of users in various cells to different maximum possible levels. This facilitates BSs with better channel conditions and more power to balance SINR of their serving users to a higher level than the balanced SINR of users attached to BSs with either low transmit power or bad channel conditions. The algorithm was solved sequentially by balancing SINR of users in various cells with constraints on transmit power and already achieved balanced SINRs. The results have been compared with SDP based beamformer design for optimality.

	Target SINR				Total Power		Power Allocation			
	BS1		BS2		BS1	BS2	BS1		BS2	
Channels	User 1	User 2	User 1	User 2			User 1	User 2	User 1	User 2
Channel 1	15.35	15.35	34.54	34.54	0.1	10	0.0634	0.0366	7.0120	2.9868
Channel 2	18.52	18.52	341.46	341.46	0.1	10	0.0466	0.0534	3.2116	6.7878
Channel 3	22.37	22.37	806.82	806.82	0.1	10	0.0581	0.0419	3.9419	6.0579
Channel 4	18.95	18.95	659.56	659.56	0.1	10	0.0295	0.0705	3.2601	6.7399
Channel 5	15.08	15.08	280.01	280.01	0.1	10	0.0479	0.0521	1.4086	8.5912

TABLE VI
TARGET SINRS AND THE USER POWER CONSUMPTIONS USING THE SDP-BASED METHOD

Fig. 1. Evolution of auxiliary variables alpha 1 and alpha 2 against adaptation number.

Fig. 2. Achieved SINR (dB) of the users in the 1st and 2nd cell versus the transmit power constraint of the 2nd cell

Fig. 3. Achieved SINR (dB) of the users in the 2nd cell for decreasing values of target SINRs (dB) in the 1st cell with $M=3$

Channels	Extended work of [24]		Our method	
	SINR of U1-U3 i.e., BS1	SINR of U4-U6 i.e., BS2	SINR of U1-U3 i.e., BS1	SINR of U4-U6 BS2
Channel 1	14.4	25.9	14.4	34.2
Channel 2	9.5	186.5	9.5	199.9
Channel 3	8.7	337.1	8.7	351.4
Channel 4	13.7	302.7	13.7	317.3
Channel 5	8.8	151.5	8.8	154.9

TABLE VII
THE ACHIEVED BALANCED SINRS USING THE EXTENDED WORK OF [24] AND OUR PROPOSED METHOD

REFERENCES

- [1] S. Shamai, B. Zaidel, and S. Verdu, "On information theoretic aspects of multi-cell wireless systems," *Proc. 4th International ITG Conference on Source and Channel Coding*, vol. 4, Berlin, Germany, Jan. 2002.
- [2] S. Jing, D. Tse, J. Soriaga, J. Hou, J. Smee, and R. Padovani, "Cochannel interference mitigation and cooperative processing in downlink multicell multiuser MIMO networks," *EURASIP J. Wireless Commun. and Networking*, vol. 2004, no. 2, pp. 222–235, Dec. 2004.
- [3] H. Zhang and H. Dai, "Cochannel interference mitigation and cooperative processing in downlink multicell multiuser MIMO networks," *EURASIP J. Wireless Commun. and Networking*, vol. 2004, Issue 2, pp. 222–235, Dec. 2004.
- [4] M. Karakayali, G. Foschini, and R. Valenzuela, "Network coordination for spectrally efficient communications in cellular systems," *IEEE Wireless Commun. Mag.*, vol. 3, no. 14, pp. 56–61, Aug. 2006.
- [5] H. Zhang, N. Mehta, A. Molish, J. Zhang, and H. Dai, "Asynchronous interference mitigation in cooperative base station systems," *IEEE Trans. Commun.*, vol. 7, no. 1, pp. 155–165, Jan. 2008.
- [6] H. Dahrouj and W. Yu, "Coordinated beamforming for the multi-cell multi-antenna wireless system," *IEEE Trans. Wireless Commun.*, vol. 9, no. 5, pp. 1748–1759, May 2010.

- [7] B. Lee, H. Je, I. Sohn, O.-S. Shin, and K. Lee, "Interference-aware decentralized precoding for multicell MIMO TDD systems," *IEEE Globecom*, New Orleans, LA, Nov. 30- Dec. 4-2008.
- [8] N. Hassanpour, J. Smee, J. Hou, and J. Soriaga, "Distributed beamforming based on signal-to-caused-interference ratio," *Proc. IEEE Int. Sym. Spread Spectrum Tech. Appl.*, pp. 405–410, Bologna, Italy, Aug. 25-28, 2008.
- [9] T. A. Le, S. Nasser, A. Zarrebini-Esfahani, and M. R. Nakhai, "Power-efficient downlink transmission in multicell networks with limited wireless backhaul," *IEEE Wireless Commun.*, vol. 18, no. 5, pp. 81–88, Oct. 2011.
- [10] H. Huh, H. Papadopoulos, and G. Caire, "Multiuser MIMO transmitter optimization for inter-cell interference mitigation," *preprint: arXiv:0909.1344v1*, 2009.
- [11] F. Rashid-Farrokhi, K. J. R. Liu, and L. Tassiulas, "Transmit beamforming and power control for cellular wireless systems," *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1437–1450, Oct. 1998.
- [12] F. Rashid-Farrokhi, L. Tassiulas, and K. J. R. Liu, "Joint optimal power control and beamforming in wireless networks using antenna arrays," *IEEE Trans. Commun.*, vol. 46, no. 10, pp. 1313–1324, Oct. 1998.
- [13] C. Botella, G. Piero, A. Gonzalez, and M. de Diego, "Coordination in a multi-cell multi-antenna multi-user W-CDMA system: A beamforming approach," *IEEE Trans. Wireless Commun.*, vol. 7, no. 11, pp. 4479–4485, Nov. 2008.
- [14] W. Yu and T. Lan, "Transmitter optimization for the multi-antenna downlink with per-antenna power constraints," *IEEE Trans. Signal Process.*, vol. 55, no. 6, pp. 2646–2660, Jun. 2007.
- [15] L. Venturino, N. Prasad, and X. Wang, "Coordinated linear beamforming in downlink multi-cell wireless networks," *IEEE Trans. Wireless Commun.*, vol. 9, no. 4, pp. 1451–1461, Apr. 2010.
- [16] J. Yang and D. K. Kim, "Multi-cell uplink-downlink beamforming throughput duality based on Lagrangian duality with per base station power constraints," *IEEE Commun. Lett.*, vol. 12, no. 4, pp. 277–279, Nov. 2009.
- [17] P. Marsch and G. Fettweis, "On downlink network MIMO under a constrained backhaul and imperfect channel knowledge," *Globecom*, Nov. 2009.
- [18] D. H. N. Nguyen and T. Le-Ngoc, "Multiuser downlink beamforming in multicell wireless systems: A game theoretical approach," *IEEE Trans. Signal Process.*, vol. 59, no. 7, pp. 3326–3338, Jul. 2011.
- [19] M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," *IEEE Trans. Vehicular Technology*, vol. 53, no. 1, pp. 18–28, Jan. 2004.
- [20] K. Cumanan, L. Musavian, S. Lambotharan, and A. B. Gershman, "SINR balancing technique for downlink beamforming in cognitive radio networks," *IEEE Signal Process. Letters*, vol. 17, pp. 133–136, 2010.
- [21] H. Shiwen, H. Yongming, Y. Luxi, A. Nallanathan, and L. Pingxiang, "A multi-cell beamforming design by uplink-downlink max-min SINR duality," *IEEE Trans. Wireless Commun.*, vol. 11, pp. 2858 – 2867, 2012.
- [22] Y. Huang, G. Zheng, M. Bengtsson, K.-K. Wong, L. Yang, and B. Ottersten, "Distributed multicell beamforming with limited intercell coordination," *IEEE Trans. Signal Process.*, vol. 59, no. 2, pp. 728–738, Feb. 2011.
- [23] D. H. N. Nguyen and T. Le-Ngoc, "Efficient coordinated multicell beamforming with per-base-station power constraints," *IEEE Globecom*, Dec. 2011.
- [24] Y. Huang, G. Zheng, M. Bengtsson, K.-K. Wong, L. Yang, and B. Ottersten, "Distributed multicell beamforming design approaching pareto boundary with max-min fairness," *IEEE Trans. Wireless Commun.*, vol. 11, no. 8, pp. 2921–2933, Aug. 2012.
- [25] D. W. H. Cai, T. Q. S. Quek, and T. Chee-wei, "A unified analysis of max-min weighted SINR for MIMO downlink system," *IEEE Trans. Signal Process.*, vol. 59, no. 8, pp. 3850–3862, Aug. 2011.

- [26] L. Zhang, R. Zhang, Y. Liang, Y. Xin, and H. Poor, "On Gaussian MIMO BC-MAC duality with multiple transmit covariance constraints," *IEEE Trans. Information Theory*, vol. 58, no. 4, pp. 2064–2078, April 2012.
- [27] S. Boyd, L. Xiao, and A. Mutapcic, "Subgradient methods," Oct. 2003.
- [28] W. Yu and R. Lui, "Dual methods for nonconvex spectrum optimization of multicarrier systems," *IEEE Trans. Commun.*, vol. 54, no. 7, pp. 1310–1322, Jul. 2006.
- [29] Y. Rahulamathavan, K. Cumanan, and S. Lambotharan, "A mixed SINR-balancing and SINR-target-constraints-based beamformer design technique for spectrum-sharing networks," *IEEE Trans. Vehicular Technology*, vol. 60, no. 9, pp. 4403 – 4414, Nov. 2011.
- [30] C. D. Meyer, "Matrix analysis and applied linear algebra," *Philadelphia, PA: SIAM*, 2000.
- [31] V. Y. Pan and Z. Q. Chen, "The complexity of the matrix eigenproblem," in *31st Ann. ACM Symp. on Theory of Computing*, *ACM Press*, vol. 1999, New York.
- [32] M. Bengtsson and B. Ottersten, "Optimal and suboptimal transmit beamforming," *Handbook of Antennas in Wireless Communications*, *L.C. Godara, Ed. Boca Raton, FL: CRC Press*, 2001.