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Book review of "Algebra teaching around the world" edited by Frederick Koon Shing Leung, Kyungmee Park, Derek Holton and David Clarke.

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Book Review: *Algebra Teaching Around the World*. (2014). Frederick Koon Shing Leung, Kyungmee Park, Derek Holton, & David Clarke (Eds.). Rotterdam: Sense Publishers. 272 pages. ISBN Paperback: 9789462097056 (\$ 54.00), ISBN Hardcover: 9789462097063 (\$ 99.00), ISBN E-Book: 9789462097070

Ian Jones

In *Algebra Teaching around the World*, 23 mathematics education researchers investigated the teaching and learning of algebra in eighth grade classrooms in various countries. The book comprises 13 chapters and an appendix. A key strength of the book is that the data were collected in a range of countries using identical methods, as part of the Learner's Perspective Study (LPS). Culture is included among the various influences on classroom practice and the book explores whether a division between Confucian-Heritage and Western cultures provides a useful explanatory distinction. The teachers observed and interviewed were those considered effective within their own countries. In order to investigate teaching, learning, classroom culture, and wider contexts, multiple sources of evidence were collected, including videos and field notes of lessons, interviews with teachers and students, and documentary sources including student work and achievement data. Several chapters provide extensive detail about the local education context, for example by including detailed curricular specifications, and authors offer a detailed connection between the data and conclusions drawn, including substantial transcript excerpts and meticulous interpretation. Recurrent themes, such as within-country differences and the mismatch between stated and enacted curricula, ensure that the book contributes important and thought-provoking insights to the field. In this review I will focus on international differences, within-country differences, theoretical differences, and cultural differences, respectively.

1 International differences

The book presents itself as an international comparison of algebra classrooms. However, the authors' selection of data, focus of interest, and methods of analysis vary, and the outcome is not intended to be a ranking or point-by-point overview of national differences. The editors contribute overarching observations, some of which are comparative, but do not intend to offer a systematic comparison or synthesis. (Systematic comparison was instead the focus of another book in the LPS series: *Volume 2: Making Connections: Comparing Mathematics Classrooms Around the World*, Clarke, Emanuelsson, Jablonka, & Mok, 2006). Rather, the underlying philosophy of the Learner's Perspective Study was to complement traditional comparisons with "an understanding of the motivations and meanings" (p. 243) of participants in active and complex classroom settings.

Commonly, international comparisons are undertaken by a single research team using the same theoretical approach and analysis techniques to produce directly comparable outcomes for each country or jurisdiction. For example, PISA and TIMSS use a common test approach to score and rank countries. Askew, Hodgen, Hossain, and Bretcher (2010) undertook a systematic literature review to investigate the characteristics of countries achieving high-performance in mathematics. In *Algebra Teaching around the World* two chapters adopt a direct comparison approach.

In chapter 12, Rongjin Huang and Yeping Li compared the lessons of a teacher in the U.S. and a teacher in China. In both classrooms they observed teacher exposition, whole class and peer discussion, and students working through problems. The U.S. teacher's scope was broad, encouraging students to make connections between ideas of linear and non-linear relationships, and to see the relevance to "real-world" problems. In contrast, the Chinese teacher focussed on the more specific concept of coordinates, and encouraged students to extend their understanding through working on interconnected problems.

In chapter 13, Johan Häggström considered classrooms in Sweden and in China to investigate variation in the presentation of systems of linear equations involving unknowns. He reported no obvious pattern in the presentation of linear equations in terms of country. For example, some teachers in China exclusively presented equations with two unknowns, other teachers in China also presented equations with three or more unknowns. The finding was consistent with one of the editors' overarching observations: classroom differences within countries can be as striking as differences between countries. Therefore, if culture is to be used as an explanatory variable, it will have to be interpreted in a more nuanced way than as a simple Confucian-Heritage versus Western dichotomy. Additionally, one classroom must not be allowed to represent an entire school system or cultural category. Indeed, the book's editors are sensitive to this danger.

2 Within-country differences

Such within-country differences arise in other chapters. For example, in chapter 4, Jarmila Novotná and Alena Hošpesová compared two teachers in secondary schools in the Czech Republic. Despite both teachers working from mandated national curriculum materials, their approaches to teaching equations were markedly different. One teacher appealed to students' existing knowledge by relating equations to "real-world" problems, and the other appealed to existing knowledge by relating equations to arithmetic processes. We might infer that both teachers were broadly constructivist in their views of learning; the difference lay in the references to existing student knowledge (real world versus arithmetic) that were used as the foundation for building algebraic knowledge.

Differences can arise even with a single lesson of a single classroom. In chapter 6, Kyungmee Park and Frederick Leung reported contrasting views between the teacher and the students regarding the purposes of a lesson in Korea. The teacher's purpose was for students to understand that two different methods of solving simultaneous linear equations can lead to the same results. However, for some students the purpose was to learn an algorithm for solving simultaneous equations, and in interviews they focussed on superficial lesson features such as the colour of chalk used and the presence of mathematical mistakes. Moreover, the teacher had a more positive view of the success of the lesson than did the students. The authors question whether their findings are "content-specific and country-specific, or ... general phenomena" (p. 111). The power of the Learner's Perspective's Study data is that this question can be informed by applying the same analysis to different lessons in Korea, or to linear equations lessons in another country. All video and audio data were transcribed and

translated into English, so systematic cross-country analyses by a single research team would be possible.

In chapter 9, by Zhongdan Huan, Jianhua Li, Ping Ma, and Li Fu, international differences are apparent within individual classrooms. They reported that teachers in China, who are embedded in a culture with “a long tradition of accepting the examination as the instrument for social selection” (p. 156), work to a national strategy that incorporates elements of Western pedagogical practice. However, teachers did not necessarily implement these elements as intended. For example, student exploration and peer discussion was strongly guided by teachers, and ostensibly interactive learning resembled transmissionist teaching. Such mismatch between stated curricular aims and their implementation in the classroom will be familiar to researchers in many countries. In this particular case, the possible intrusion of pedagogical elements from another culture provides an interesting and important variation on most discussions of curriculum implementation, and one with contemporary relevance for many school systems in the South-East Asian region.

3 Theoretical differences

Some authors apply different theoretical approaches to the same, or closely related, algebraic concepts, albeit to data from different countries. For example, two chapters analysed teachers’ use of balance scales to focus students on the meaning of the equals sign as part of teaching linear equations. In chapter 2, Glenda Anthony and Tim Burgess adopted a phenomenological approach to study a New Zealand teacher’s interactions with students; in chapter 3, Birgit Pepin, Ole Kristian Bergem, and Kirsti Klette adopted a semiotic approach to study the role of signs in a Norwegian teacher’s classroom. Both chapters provide a detailed description and analysis of classroom activity, reporting variation in how the teacher and different students interpreted the equals sign and its role in linear equations. Anthony and Burgess achieved this through consideration of the teacher’s reflections in interviews; Pepin et al. did so through close scrutiny of sign use and interpretation during a short lesson excerpt. This difference in theoretical approaches certainly adds variety to the book. However, the reader is left struggling to compare either the two teaching situations or the relative advantages and disadvantages of each theoretical approach. It is interesting to consider what the outcome might have been had the researchers swapped data and applied their approaches to both classrooms.

These two chapters exemplify an issue that runs through the book. Contributors employ a range of theoretical approaches to interpret the Learner’s Perspectives Study data, including semiotics, communities of practice, opportunities to learn, variation theory, and Construction Zone (an adaptation of Vygotsky’s Zone of Proximal Development). But does this multiplicity of theoretical framings enrich or obfuscate? In the absence of multiple contributors applying their approaches to the same data set (as was the case in a recent book edited by Bikner-Ahsbals & Prediger, 2014) it is difficult to discern empirical evidence from theoretical interpretation. Would different authors have drawn similar or contrasting interpretations using one another’s data? Without knowing this we cannot confidently determine the extent to which differences in teaching and learning apparent across chapters are inherent to the classrooms being observed, or inherent to the different research focuses adopted. However, as mentioned above, the quality, richness and standardisation of the

Learner's Perspective Study data means that such questions can be readily addressed by applying the same approach to the analysis of classrooms, teachers, and students in different countries.

4 Cultural differences

It might be countered that local theorising is appropriate for local data, even if the methods of collection were the same across different countries. Traditional international comparison studies conducted by a single research team are subject to the culturally-specific interpretations and biases of the researchers. For example, researchers based in a country where the national debate reflects anxiety about performance in examinations can be expected to take a different approach from researchers in a country where debate reflects anxiety about the perceived lack of creative problem-solving skills of school leavers. Nevertheless, if the focus and assumptions of the research team are clearly stated, and the methods adopted appropriate to the questions asked, we can have some confidence that any *differences* reported are empirically grounded.

Conversely, the collaboration of several research teams located in the countries in which the data were collected is a key strength of the book. The data are rich and multi-faceted, notably due to the use of videotaped lessons and interviews with teachers and students. This contrasts with many international comparison studies that rely solely on test results and official documentation. Accordingly, authors can observe and articulate nuances in classrooms, including from the voices of teachers and learners, that might otherwise be missed. Moreover, as the researchers are mostly situated in the country where the data were collected, and indeed were responsible for its collection, they can provide essential contextual information. For example, students in Confucian-Heritage Culture countries commonly attend private tuition out of school hours. The overarching national aims of mathematics education in a particular schooling system, such as the focus on conceptual understanding in Japan, might well be complemented by activities during private tuition, such as consolidating and practising problems. Without these contextualised insights a narrow focus on schooling alone might risk concluding, for example, that emphasising conceptual understanding also promotes procedural skill.

However, cognitive research suggests that the importance and impact of culture for learning mathematics goes deeper than differences in schooling systems. The structure of the spoken number system varies across languages, and this can impact on children's acquisition and representation of symbolic numbers (Bender & Beller, 2011). The direction of written text also varies culturally, and this may impact on numerical representations (Cutini, Scarpa, Scatturin, Dell'Acqua, & Zorzi, 2012). Differences in how cultures count using fingers might be another factor influential in how young children learn arithmetic (Bender & Beller, 2012). Young children's depositions towards number, which are likely to be influenced by culture and family, are also related to later mathematical learning (e.g. Hannula, Räsänen, & Lehtinen, 2007).

Most mathematical cognition research into cultural differences to date has tended to consider young children's development of number, but cultural differences might be expected to impact on other areas of mathematics. This is particularly the case for

lower secondary algebra given the key role that existing arithmetical knowledge can play on its development (Chesney et al., 2014). Further investigation of fundamental cultural and linguistic differences, and their impact on learning algebra, would strengthen our discipline. For example, Li, Ding, Capraro, and Capraro (2008) found that students in China demonstrated a more robust understanding of mathematical equivalence than their counterparts in the U.S., and tentatively proposed differences in textbooks in each country as causal. However, more deeply rooted cultural factors, beyond those directly related to schooling, might be expected to play a measurable role too. Identifying those factors would help us to understand differences in teaching, resources, and policies related to the teaching of algebra around the world.

5 Conclusion

The book provides fascinating insights into the teaching and learning of algebra across culturally diverse countries. Perhaps the similarities across classrooms are more striking than the differences: symbolic algebra tends to be introduced early in secondary schooling after years of learning arithmetic; many teachers take a broadly constructivist view of learning and so attempt, albeit in different ways, to build algebraic understanding on the foundations of students' existing knowledge; intended curricula often do not align with enacted curricula; teachers and students commonly perceive the purpose and content of lessons differently. In addition, the differences in contributors' theoretical perspectives are perhaps more striking than the reported differences in classroom practices. This is a reflection of the condition of mathematics education research, and other books have overtly embraced the multiplicity of scholarly perspectives on the teaching and learning of algebra (e.g., Cai & Knuth, 2011). It is important these perspectives are shared and debated, and *Algebra Teaching around the World* offers such a forum.

Nevertheless, the fruitfulness of inviting multiple perspectives on different subsets of a larger, standardised corpus of data might be questioned. It threatens the authors' ability to generalise findings and make strong claims about comparative differences. This could be easily remedied. As mentioned above, research teams might swap datasets and re-apply their analyses in order to better understand how contextual factors such as culture and lesson topic shaped their findings. This would allow readers to ascertain more clearly differences in the teaching of algebra, as well as the relative strengths and weaknesses of the different theoretical perspectives adopted. Furthermore, the corpus of data might be made available to all researchers via an open-access database. The free availability of high-quality international data, such as that meticulously collected for the Learner's Perspective Study, would help strengthen our understanding of the teaching and learning of algebra. Just as importantly it would enable researchers to compare and contrast alternative perspectives and therefore refine and advance how we theorise practice in mathematics classrooms.

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