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Flux-coherent series SQUID array magnetometers operating above 77 K with superior white flux noise than single-SQUIDs at 4.2 K

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A very promising direction to improve the sensitivity of magnetometers based on superconducting quantum interference devices (SQUIDs) is to build a series-array of N non-interacting SQUIDs operating flux-coherently, because in this case their voltage modulation depth, ΔV , linearly scales with N whereas the white flux noise $S_\Phi^{1/2}$ decreases as $1/N^{1/2}$. Here, we report the realization of both these improvements in an advanced layout of very large SQUID arrays made of $\text{YBa}_2\text{Cu}_3\text{O}_7$. Specially designed with large area narrow flux focusers for increased field sensitivity and improved flux-coherency, our arrays have extremely low values for $S_\Phi^{1/2}$ between (0.25 and 0.44) $\mu\Phi_0/\text{Hz}^{1/2}$ for temperatures in the range (77–83) K. In this respect, they outperform niobium/aluminium tri-layer technology-based single-SQUIDs operating at 4.2 K. Moreover, with values for ΔV and transimpedance in the range of (10–17) mV and (0.3–2.5) k Ω , respectively, a direct connection to a low-noise room temperature amplifier is allowed, while matching for such readout is simplified and the available bandwidth is greatly increased. These landmark performances suggest such series SQUID arrays are ideal candidates to replace single-SQUIDs operating at 4.2 K in many applications. © 2015 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4932969>]

With recent advances in the fabrication of superconducting quantum interference devices (SQUIDs) made of high temperature superconductors (HTS), the increasing demands of complex designs required in many SQUIDs applications are now starting to be fulfilled. Indeed, while HTS SQUID fabrication in bicrystal¹ or multi-crystal² grain boundary technology reached maturity more than a decade ago, other fabrication methods are recently gaining momentum.^{3–5} Also, significant is the progress reported in the area of nanofabrication.^{6–8} At present, however, due to their superior flux noise performances in the vast majority of applications, SQUIDs made of low temperature superconductors (LTS) and operating at 4.2 K are being used. This is despite several significant advantages HTS SQUIDs operating above 77 K offer: low cost and user-friendly cooling procedures (at a time, when the price of liquid He is increasing significantly) and potential superiority as magnetic imaging devices due to a reduced separation between the sensors and the room temperature object under study (because of the decreased thermal insulation demand). Recently, it was shown^{9–12} that these advantages are particularly significant in magnetoencephalography. Therefore, improving noise properties and sensitivity of HTS SQUIDs has become more relevant than ever.

A very promising direction for improving a magnetometer's sensitivity is to build a series SQUID array (SSA) of N non-interacting SQUIDs operating flux-coherently, because in this case^{13–15} ΔV linearly scales with N , their dynamic range increases as $N^{1/2}$, whereas $S_\Phi^{1/2}$ decreases as $1/N^{1/2}$. Consequently, not only the noise properties of a SSA are superior to a single-SQUID but also a much larger ΔV

means their matching for room temperature readout is greatly simplified. Moreover, SSAs have the potential of also improving the bandwidth and the impedance matching as their impedance is N times larger than that of a single SQUID. All these predictions have been confirmed by pioneering research reported in Refs. 13–18 where large N SSAs have been developed in LTS technology. Such LTS SSAs were operated at 4.2 K and typically N is in the range 100–200, but devices with N as large as 7104 have also been designed and tested.¹⁸ The primary requirements to achieve coherent flux operation and, consequently, a linear increase of ΔV with N were found to be: (a) the SQUIDs should be sufficiently identical in critical current I_c and loop inductances L ; (b) the mutual inductance M should be approximately the same between all SQUIDs and the modulation line; (c) there has to be little or no random flux offset between the individual SQUIDs; (d) the mutual coupling between SQUIDs has to be negligible. So far these requirements have proved to be very challenging to achieve in large N SSAs made of HTS and operated at 77 K and above. Consequently, flux-coherent operation at 77 K and above has only been achieved in relatively small N SSAs (N in the range 5–30).^{19,20} Due to the relatively small value of N , their superiority over SQUIDs is less spectacular, and also they could never compete in terms of noise performances with single LTS SQUIDs operated at 4.2 K. Earlier first attempts to operate large N (N in the range 35–130) HTS SSAs did not show the expected improvements in the magnetic sensitivity due to the flux-coherent mode not being achieved throughout the entire array.^{21,22} Here, we report on the design, fabrication, and testing of an advanced layout of very large ($N = 484$ and $N = 770$) non-interacting SSA made of $\text{YBa}_2\text{Cu}_3\text{O}_7$ (YBCO) and operating flux-coherently. Consequently, in the temperature range (40–83) K, they have very large values for ΔV

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and superior white flux noise values than optimized single LTS SQUIDs operating at 4.2 K.²³

The SSAs were fabricated by depositing high quality epitaxial, 100 nm thick *c*-axis oriented YBCO films on $10 \times 10 \text{ mm}^2$, 24° and 45° symmetric [001] tilt SrTiO₃ bicrystals by pulsed laser deposition (the films were deposited by Ceraco ceramic coating GmbH). A 200 nm thick Au layer was deposited *in situ* on top of the YBCO film to facilitate fabrication of high quality electrical contacts for electric transport measurements. The films, with a critical temperature T_c of 89 K, were subsequently patterned by optical lithography and etched by an Ar ion beam to form large SSAs. In this report, results obtained from two samples are presented: a 484 SSA fabricated on a 45° symmetric [001] tilt SrTiO₃ bicrystal and a 770 SSA fabricated on a 24° symmetric [001] tilt SrTiO₃ bicrystal. In choosing designs parameters, we have been guided by theory²³ and previous reports of optimal HTS SQUIDs.²⁴ For both samples, all Josephson junctions are $3 \mu\text{m}$ wide. The 484 SSA consists of 44 identical sets of 11 SQUIDs each. Within each set of 11 SQUIDs, the length of the SQUIDs' loops varies monotonically from $13 \mu\text{m}$ to $8 \mu\text{m}$, while their width is constant at $3 \mu\text{m}$. Optical micrographs of small parts of the 484 SSA are shown in Figs. 1(a) and 1(b). The 770 SSA consists of 770 identical SQUIDs characterized by identical SQUID loops of length $13 \mu\text{m}$ and width $3 \mu\text{m}$ each (see Fig. 1(c)). For both samples, the SQUIDs are connected in series in a serpentine path along the bicrystal boundary.

The reason we choose to vary the SQUID loop areas artificially by 30% in the 484 SSA design was to investigate to what extent large variations in the SQUID inductances L impact on the degree of flux-coherency and voltage modulation depth in large N SSAs. This fundamental aspect indeed requires further investigations, since previous attempts^{21,22} to operate coherently large N SSAs at 77 K failed presumably because of incoherent modulation due to unequal fluxes $\Phi_{\text{SQUID}} = LB$ threading the SQUIDs. Here, B is the magnetic

field to be measured applied perpendicular to the planar array's structure via an input coil current. To increase field sensitivity, SQUIDs²⁵ and SSAs²⁰ are usually connected to large square/rectangular flux-focusers with both their dimensions much larger than the SQUID width (total width of SQUID hole and two junctions). In order not to compromise the number of SQUIDs, we could integrate on a standard $10 \times 10 \text{ mm}^2$ bicrystal substrate while still implementing flux-focusers for enhanced sensitivity, we developed *large area narrow flux-focusers*. Their width is identical to the SQUID's width, while only their length is much larger. For the 484 SSA, their dimensions are $88 \times 9 \mu\text{m}^2$ while for the 770 SSA they are $160 \times 9 \mu\text{m}^2$. Importantly, our results showed that the larger the area of such narrow flux focusers the higher the degree of flux coherence in the operation of SSAs.

Families of current-voltage characteristics (*IVC*'s) were measured by a 4 terminal method at various temperatures between 10 K and 89 K and for various B in the range $(-40, 40) \mu\text{T}$ with a resolution of $0.067 \mu\text{T}$ (the case of 484 SSA) or $0.27 \mu\text{T}$ (the case of 770 SSA). V can be measured along the entire array or parts of it (in multiple sets of 44 SQUIDs). From such families of *IVC*'s scanned over B , $V(B)$ for both positive and negative bias currents could be constructed. The *IV* of a single SQUID fabricated on the same chip showed the concave resistively R_N shunted junction (RSJ) model-like hyperbolic shape.²³ On the other hand, the shape of the *IV*s for the SQUID arrays near I_c is convex, which we attribute to the spread of critical current in the array. Apart from this region in the vicinity of I_c , the *IVC*s are well described within the RSJ model. The value of I_c for an array was determined by extrapolating the normal state resistance of the array to zero-voltage on the *IV*. The intercept point of the current represents the average critical current for the array. A family of 130 consecutive *IVC*'s measured at 77 K for the 770 SQUID array is plotted in Fig. 2. Average values across the SSAs of the individual SQUID's main parameters I_c , R_N ,

FIG. 1. (a) Optical micrograph of a small part of the 484 SSA showing one set of 11 SQUIDs; (b) high resolution optical micrograph of only two such SQUIDs connected in series via a narrow flux focuser and its equivalent electrical circuit (the Josephson junctions are represented by crosses, while L_{FF} is the flux focuser's inductance); (c) optical micrograph of a small part of the 770 SSA showing 8 SQUIDs connected in series. Josephson junctions can be seen as narrow bridges crossing the bicrystal boundary shown with dotted line. In both designs, each pair of SQUIDs is directly connected via a large area narrow flux focuser (like the one highlighted in red in (a) and (b)). During operation, the dc current bias I meanders along the series SQUID arrays as shown schematically with continuous line in (a) and vertical arrows in (b).

FIG. 2. Electric transport measurements of the 770 SSA at 77 K. A set of 130 consecutive IVC 's recorded at 77 K for various values of the applied magnetic field B in the range $(-24, 10.7)$ μT , with B changed in steps of 0.27 μT . Left inset: $V(B)$ for 10 different bias currents I between $(-770, -680)$ μA , with I changed in steps of 10 μA . Right inset: dI/dV versus V of a small part of the 770 SSA, namely, a single set of 44 SQUIDs; the multi-peaks are due to switching of individual SQUIDs from the superconducting state into the resistive state associated with different I_c values of the 44 SQUIDs. A histogram of peak positions (I_c values) with a calculated Gaussian distribution (in red) is also shown. For this particular case, the average $I_c = 0.65$ mA and $\sigma = 10\%$.

$I_c R_N$, L , and β are shown in Table I. Here, $\beta = 2LI_c/\Phi_0$ with Φ_0 being the flux quanta and R_{array} is the array's resistance. Individual SQUID inductances L were estimated by calculating the SQUID hole perimeter and considering that typically for such bicrystal SQUIDs $1 \mu\text{m}$ corresponds to 1 pH. For the 484 SSA, individual SQUID inductances were designed to vary monotonically between $(20, 30)$ pH within each set of 11 SQUIDs which gives an average value for $L = 25$ pH.

The dependence of voltage V modulation versus B as well as peak-to-peak voltage modulation ΔV for both SSAs was also measured in the temperature range $(10-89)$ K. Results for several temperatures are shown on Fig. 2 (left inset), Figs. 3(a) and 3(b), as well as Table I (T_{max} is the temperature, where ΔV reaches maximum). It is important to stress that both $V(B)$ and $\Delta V(\beta)$ behaviours differ significantly from the case of single-SQUIDs.^{23,24} Thus, below 70 K, no SQUID oscillations were observed for the 770 SSA due to the fact that at this temperature $\beta = 20$ is quite large and appears to be above a threshold value at which SSA's flux coherency is lost. Then, $\Delta V(\beta)$ does not reach its maximum at around $\beta = 0.5-1$. Finally, unlike single-SQUIDs, $V(B)$ is amplitude modulated and is suppressed to nearly zero within $7\Phi_0$ periods for the 484 SSA and $16\Phi_0$ periods for the 770 SSA. Such an amplitude modulated behaviour is well understood¹⁷ as being a consequence of a significant variation in the periods of individual SQUIDs along the array due to either variation in the SQUID holes or/and non-uniform magnetic field coupling across the length of the array. This affects the coherency of the array at large applied fields. A similar behaviour has been previously observed for both LTS SSA's^{13,16-18} and HTS SSAs.^{21,22} By comparing Fig. 3(a) with Fig. 3(b), one can see that, as expected,¹⁷ the loss of flux-coherency is more pronounced for the 484 SSA

FIG. 3. (a) $V(B)$ of the 484 SSA at 40 K for various current biases in the range $(-60, -30)$ μA and $(30, 60)$ μA with I changing in steps of 5 μA ; (b) $V(B)$ of the 770 SSA at 83 K for 11 different current biases I in the range $(-212, -172)$ μA with I changing in steps of 4 μA .

than for the 770 SSA. It is remarkable, however, that the 484 SSA shows a significant degree of flux-coherency considering that variations of L within the array as large as 30% have been artificially implemented. The left hand side inset of Fig. 2 shows the $V(B)$ periodic SQUID oscillations for 10 different current biases. A similar set of curves were obtained for positive current biases I as well. Figs. 3(a) and 3(b) show $V(B)$ periodic SQUID oscillations of the 484 SSA at 40 K and 770 SSA at 83 K, respectively, recorded at different current biases I . Here, the periodicity of $V(B)$ with B is significantly smaller for the 484 SSA because in this case the input coil was located much closer. The maximum value of

TABLE I. ΔV , R_{array} , and average values across the SSAs of the individual SQUID's main parameters I_c , R_N , $I_c R_N$, L , β at various temperatures.

Array	484 SSA		770 SSA	
ΔV	10.8 mV at 10 K	0.7 mV at 77 K	10.1 mV at 77 K	2.2 mV at 86 K
β	0.75 at 10 K	0.075 at 77 K	9.9 at 77 K	0.7 at 86 K
T_{max}	40 K		77 K	
at T_{max}	ΔV , R_{array} I_c , R_N , $I_c R_N$ L , β		ΔV , R_{array} I_c , R_N , $I_c R_N$ L , β	
	17 mV, 2.5 k Ω 14.5 μ A, 10.3 Ω , 150 μ V 25 pH, 0.35		10.1 mV, 0.31 k Ω 340 μ A, 0.81 Ω , 275 μ V 30 pH, 9.9	

$\Delta V = 10.1$ mV reached with the 770 SSA at 77 K for a bias of ± 730 μ A (see left inset of Fig. 2) is more than 200 times larger than for the best HTS single-SQUID operating at 77 K. Both $\Delta V = 10.1$ mV value for the 770 SSA at 77 K and $\Delta V = 17$ mV value reached with the 484 SSA at 40 K for a bias of ± 50 μ A (see Fig. 3(a)) are the highest ever reported for HTS SSA.^{19–22} Such large values for ΔV allow a direct connection of the SSAs to a low-noise room temperature amplifier, while matching for such a room temperature readout is simplified due to their large impedances R_{array} in the range (0.3–2.5) k Ω . The use of a magnetic shielding did not lead to a significant increase in ΔV suggesting that SSAs can operate in an unshielded environment.

By performing ultra-high resolution IVC 's of only a part of the 770 SSA, namely, individual sets of 44 successive SQUIDs, the I_c values of the majority of 44 SQUIDs could be identified as multi-peaks in the derivatives dI/dV that correspond to switching of individual SQUIDs from the superconducting state into the resistive state. One example is shown in the right inset of Fig. 2 where 36 peaks have been identified out of the expected 44. The difference can be associated to accidental overlapping of I_c values of 8 individual SQUIDs. Previous such studies in large SSAs could not resolve multipeak structures at temperatures above 30 K.^{26,27} Such investigations are important to figure out how the spread of I_c at 77 K compares with that at 4.2 K and understand the impact of it on achieving flux-coherence and optimum flux sensitivity in SSAs. Histograms of I_c were constructed with a Gaussian fitting that produced an average standard deviation of $\sigma = 10.1\%$ for sets of 44 SQUIDs. σ rises to about 12.2% for the entire array. It is important to note that σ is significantly smaller at 77 K than at 4.2 K, where $\sigma = (23.5–25.5)\%$ were found for similar large arrays fabricated using bicrystal technology.^{26,27} The spread of critical currents in our array, although significantly larger than in the LTS junctions (typically around $\sigma = (2–3)\%$ for 3 μ m wide junctions arrays¹³), appears not to play a critical role in achieving flux-coherency in large arrays. ΔV on the other hand is expected to decrease rapidly with increasing σ .

TABLE II. Theoretical and experimental values for $S_V^{1/2}$ and $S_\Phi^{1/2}$ of the SSAs.

Array, T		484 SSA, 40 K	770 SSA, 77 K
$S_V^{1/2}$	Theory: $N^{1/2}(16kTR_N)^{1/2}$	6.6 nV/Hz ^{1/2}	3.2 nV/Hz ^{1/2}
	Measured	40 nV/Hz ^{1/2}	8 nV/Hz ^{1/2}
$S_\Phi^{1/2}$	Theory: $S_V^{1/2}/N(\pi\Delta V_{\text{SQUID}}/\Phi_0)$	0.15 $\mu\Phi_0$ /Hz ^{1/2}	0.045 $\mu\Phi_0$ /Hz ^{1/2}
	Experiment: $S_V^{1/2}/(\pi\Delta V/\Phi_0)$	0.75 $\mu\Phi_0$ /Hz ^{1/2}	0.25 $\mu\Phi_0$ /Hz ^{1/2}

The noise of the arrays was measured by directly coupling the output of the SSAs to a low noise preamplifier at room temperature, the output of which was connected to a spectrum analyzer. The arrays were biased by a battery powered current source and operated in a small signal amplifier mode. The output voltage noise V_N of the arrays was measured at the point of maximum responsivity, i.e., where the array's flux-to-voltage transfer function ($\partial V/\partial \Phi$) was maximum. If we assume that the noise sources in the SQUIDs are uncorrelated, the theoretical values for the white noise voltage spectral density $S_V^{1/2}$ of an N junction SSA should be $N^{1/2}$ larger than for single SQUIDs,¹⁴ i.e., $S_V^{1/2} = N^{1/2} (16kTR_N)^{1/2}$. That gives $S_V^{1/2} = 6.6$ nV/Hz^{1/2} for the 484 SSA operating at 40 K and $S_V^{1/2} = 3.25$ nV/Hz^{1/2} for the 770 SSA operating at 77 K (see Table II). Experimentally, however, typical values for $S_V^{1/2}$ measured in the white region at 1 kHz were a factor of 3–6 times larger: $S_V^{1/2} = 40$ nV/Hz^{1/2} for the 484 SSA at 40 K and $S_V^{1/2}$ between (8 and 9) nV/Hz^{1/2} for the 770 SSA in the temperature range (77–83) K (such excess noise is not unusual for YBCO SQUIDs). These values correspond to white flux noises $S_\Phi^{1/2} = S_V^{1/2}/(\partial V/\partial \Phi) = S_V^{1/2}/(\pi\Delta V/\Phi_0)$ of 0.75 $\mu\Phi_0$ /Hz^{1/2} for the 484 SSA at 40 K and between (0.25 and 0.44) $\mu\Phi_0$ /Hz^{1/2} for the 770 SSA in the temperature range (77–83) K. These values are superior to those of optimized LTS single-SQUIDs operating at 4.2 K which typically have $S_\Phi^{1/2}$ around 1 $\mu\Phi_0$ /Hz^{1/2} (see Fig. 4). Then, the value of $S_\Phi^{1/2} = 0.25$ $\mu\Phi_0$ /Hz^{1/2} for the 770

FIG. 4. Flux density noise $S_\Phi^{1/2}$ at 1 kHz and maximum peak-to-peak voltage modulation ΔV (inset) versus the number of SQUIDs integrated in the SSAs. The discontinuous lines indicate $1/N^{1/2}$ dependence. Typical flux noise levels of optimized HTS SQUIDs at 77 K,²⁴ nano-HTS SQUIDs at 4.2 K,⁸ and LTS SQUIDs operating at 4.2 K (Ref. 23) are also shown as references.

SSA operating at 77 K is about 9 times lower than that of best reported values for optimized single HTS-SQUID operating at 77 K.²⁴ It is important to stress that the arrays investigated here were not optimized in terms of their geometrical and electric transport properties, including $S_\Phi^{1/2}$. Indeed, theoretical estimations for $S_\Phi^{1/2}$ of optimized SSAs assuming a typical value of $\Delta V_{\text{SQUID}} = 30 \mu\text{V}$ for a single SQUID are significantly lower (see Table II). This is very promising and it should stimulate research for further improvements. To a good approximation, the voltage modulation depth ΔV linearly increased with N , whereas the white flux noise $S_\Phi^{1/2}$ decreased as $1/N^{1/2}$ (see Fig. 4). That strongly suggests that both flux-coherency and SQUIDS non-interactivity were achieved in our large N arrays and at high temperatures. For both arrays, $S_V^{1/2}$ increases for frequencies below 1 kHz, a behaviour known to arise from critical current fluctuations. This $1/f$ low frequency noise can be eliminated by proper modulations techniques (i.e., bias reversal).

In summary, large N SSAs made of YBCO and operating flux-coherently have been fabricated and tested. Their voltage modulation depth and white flux-noise performances in the temperature range (40–83) K are much better than single optimized HTS-SQUIDS operating at similar temperatures and even outperformed single LTS-SQUIDS operating at 4.2 K. HTS SSAs are therefore ideal candidates to replace single-SQUIDS in many applications.

¹H. Hilgenkamp and J. Mannhart, *Rev. Mod. Phys.* **74**, 485 (2002).

²R. R. Schulz, B. Chesca, B. Goetz, C. W. Schneider, A. Schmehl, H. Bielefeldt, H. Hilgenkamp, J. Mannhart, and C. C. Tsuei, *Appl. Phys. Lett.* **76**, 912–914 (2000).

³S. A. Cybart, S. M. Wu, S. M. Anton, I. Siddigi, J. Clarke, and R. C. Dynes, *Appl. Phys. Lett.* **93**, 182502 (2008).

⁴J. Du, J. Y. Lazar, S. K. H. Lam, E. E. Mitchell, and C. P. Foley, *Supercond. Sci. Technol.* **27**, 095005 (2014).

⁵E. Y. Cho, M. K. Ma, C. Huynh, K. Pratt, D. N. Paulson, V. N. Glyantsev, R. C. Dynes, and S. A. Cybart, *Appl. Phys. Lett.* **106**, 252601 (2015).

⁶C. H. Wu, Y. T. Chou, W. C. Kuo, J. H. Chen, L. M. Wang, J. C. Chen, K. L. Chen, U. C. Sou, H. C. Yang, and J. T. Jeng, *Nanotechnology* **19**, 315304 (2008).

⁷C. P. Foley and H. Hilgenkamp, *Supercond. Sci. Technol.* **22**, 064001 (2009).

⁸T. Schwarz, J. Nagel, R. Wolbing, M. Kemmler, R. Kleiner, and D. Koelle, *ACS Nano* **7**, 844 (2013).

⁹M. I. Faley, U. Poppe, R. E. Dunin-Borkowski, M. Schiek, F. Boers, H. Chocholacs, J. Dammers, E. Eich, N. J. Shah, A. B. Ermakov *et al.*, *IEEE Trans. Appl. Supercond.* **23**, 1600705 (2013).

¹⁰F. Oisjoen, J. F. Schneidermann, G. A. Figueras, M. L. Chukharkin, A. Kalabukhov, A. Hedstrom, M. Elam, and D. Winkler, *Appl. Phys. Lett.* **100**, 132601 (2012).

¹¹J. Schneiderman, *J. Neurosci. Methods* **222**, 42 (2014).

¹²M. Xie, J. Schneiderman, M. L. Chukharkin, A. Kalabukhov, S. Whitmarsh, D. Lundqvist, and D. Winkler, *IEEE Trans. Appl. Supercond.* **25**, 1601905 (2015).

¹³R. P. Welty and J. M. Martinis, *IEEE Trans. Magn.* **27**, 2924 (1991).

¹⁴R. P. Welty and J. M. Martinis, *IEEE Trans. Appl. Supercond.* **3**, 2605 (1993).

¹⁵K. G. Stawiasz and M. B. Ketchen, *IEEE Trans. Appl. Supercond.* **3**, 1808 (1993).

¹⁶V. Foglietti, K. G. Stawiasz, M. B. Ketchen, and R. H. Koch, *IEEE Trans. Appl. Supercond.* **3**, 3061 (1993).

¹⁷K. Li and S. P. Hubbell, *IEEE Trans. Appl. Supercond.* **5**, 3255 (1995).

¹⁸K. Li and S. P. Hubbell, *IEEE Trans. Appl. Supercond.* **7**, 3217 (1997).

¹⁹K. Li, S. P. Hubbell, R. Cantor, and M. Teepe, *IEEE Trans. Appl. Supercond.* **9**, 4420 (1999).

²⁰S. H. Wu, M. H. Hsu, K. L. Chen, J. C. Chen, J. T. Jeng, T. S. Lai, H. R. Horng, and H. C. Yang, *Supercond. Sci. Technol.* **19**, S246 (2006).

²¹S. G. Lee, Y. Huh, G. S. Park, I. S. Kim, Y. K. Park, and J. C. Park, *IEEE Trans. Appl. Supercond.* **7**, 3347 (1997).

²²C. H. Wu, M. J. Chen, J. C. Chen, K. L. Chen, H. C. Yang, M. S. Hsu, T. S. Lai, Y. S. Tsai, H. E. Horng, J. H. Chen *et al.*, *Rev. Sci. Instrum.* **77**, 033901 (2006).

²³B. Chesca, D. Koelle, and R. Kleiner, “SQUID theory,” in *The SQUID Handbook*, edited by J. Clarke and A. I. Braginski (Wiley-VCH, Weinheim, 2004), p. 29.

²⁴D. Koelle, R. Kleiner, F. Ludwig, E. Dansker, and J. Clarke, *Rev. Mod. Phys.* **71**, 631 (1999).

²⁵J. M. Jaycox and M. B. Ketchen, *IEEE Trans. Magn.* **17**, 400 (1981).

²⁶Y. Tarutani, H. Hasegawa, T. Fukazawa, and K. Takagi, *J. Appl. Phys.* **83**, 5000 (1998).

²⁷H. Burkhardt, O. Brugmann, A. Rauther, F. Schnell, and M. Shilling, *IEEE Trans. Appl. Supercond.* **9**, 3153 (1999).