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Guided notes for university mathematics and their impact on students' note taking behaviour

Paola Iannone · Dominic Miller

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Abstract This paper reports findings from a case study of the impact that using guided notes has on university mathematics students' note taking behaviour. Whereas previous research indicates that students do not appreciate the importance of lecturers' non-written comments and record in their notes only what is written on the board when taught with the traditional chalk and talk method, some students in our study recorded the non-written comments as well as some of their own links between sections of the lecture. We did not however find students' attitude towards those comments to be different from what previous research found. We conclude that guided notes can be an appropriate way of teaching university mathematics but on their own cannot make the pedagogical intentions of the lecturer clearer to the students. We also found that the educational environment plays a big part on all aspects of student learning, including on decisions related to note taking during lectures.

Keywords: Mathematics teaching, university, guided notes, note taking

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1 Introduction

Lecturing large classes is the prevailing mode of instruction in higher education across many subjects including mathematics, due to the increased number of students taking up university studies. While smaller classes and inquiry oriented approaches are seen to be more effective for mathematics learning (Rasmussen & Kwon, 2007) and some (Jaworski, Mali, & Petropoulou, 2017) associate lecturing large classes with a transmissionist view of learning, others see large class teaching as an opportunity to develop creative and less traditional teaching methods (Arvanitakis, 2014). Unlike other academic disciplines, mathematics teaching is rather uniform and in university mathematics classrooms the Definition-Theorem-Proof (DTP) format dominates, despite the fact that mathematicians are well aware that mathematics is not created in this way (Weber, 2004). The mode of delivery of such lectures in mathematics is also rather uniform, with lecturers¹ preferring to write on the board to using presentations in a style called often *chalk talk* (Artemeva & Fox, 2011). Indeed Artemeva and Fox (2011) show that this method and mathematicians' preference for it transcends cultures and countries and the practice of teaching university mathematics is dominated by cultures which have developed globally and transcend local ones. Despite this apparent uniformity which would suggest a traditionalism inspired by the old adage of *teach as you were taught* there is strong evidence that mathematics lecturers hold complex beliefs about teaching mathematics and those beliefs influence their teaching (T. P. Fukawa-Connelly, 2012; Weber, 2004). However their pedagogical intentions are often not clear to their students and lecturers' actions in the lecture room do not have the desired result in terms of student learning (Lew, Fukawa-Connelly, Mejia-Ramos, & Weber, 2016). In this paper we investigate whether using guided notes during lectures, a relatively new mode of teaching at least for university mathematics, can help making those pedagogical intentions clearer to the students therefore achieving some of their desired aims.

2 Research into university mathematics teaching practice

A typical description of a lecture in mathematics, one which links writing with the act of doing mathematics, can be found in Greiffenhagen (2014):

Lectures are a perspicuous way to exhibit the written nature of mathematics, since they involve a great deal of writing. It is not untypical for a lecturer to fill several blackboards during one lecture. Furthermore, lecturers typically write the definitions, theorems, and proofs out in full (i.e., do not just write a few keywords or a sketch on the board). Lecturers spend a lot time 'writing-talking' at the blackboard, i.e., writing things on the board while simultaneously saying them aloud. (p. 505)

¹ The term lecturer indicates university staff with teaching and research duties in the UK.

Greiffenhagen (2014) proceeds to analyse the use of the blackboard in mathematics lectures and the way in which the items that constitute the lecture (the definitions, proofs and theorems - but also crucially the examples) are placed on the board in order to establish the important interplay between writing mathematics and thinking mathematics, what he calls the *materiality* of mathematics. In his observations of mathematics lectures the author also reports the utterances made by the lecturer which are not written on the board. This can be seen for example in the following transcript.

... (referring to the use of a lemma in a proof and pointing to this on the board) So we've used all these bits straightaway, isn't that nice?
We have brought this toolbox and now we have used all of the tools right in the first -, huh, in the first half page of the proof. (p.513)

This utterance in this context (a postgraduate class in propositional logic) makes a comment on the way in which mathematics is organised and how all the hypotheses of a theorem are necessary and sufficient to its proof. At the receiving end of a lecture like this there are students who are taking notes. Will the comment above make its way into the students' notes? If not, will the student remember the comment and its meaning?

While research shows that lecturing in the DTP format is still dominant in advanced mathematics little is known of how mathematicians construct and deliver their lectures and the Speer, Smith, and Horvath (2010) noted that the teaching of mathematics remains largely an under explored practice. Practice here is not just intended as the act of teaching but includes also the planning, reflecting about and organising the act of teaching, as well as the impact that the teaching has on aspects of the student experience in the lecture. There is however strong evidence both from mathematicians' self report (e.g., Halmos, 1994) and from research studies focusing either on interviews with lecturers (e.g., Iannone & Nardi, 2005) or interviews and observation of practice (in small tutorial groups as well as in large lectures) that mathematicians are pre-occupied with the pedagogy of mathematics. Nardi, Jaworski, and Hegedus (2005) describe a pedagogical awareness spectrum derived by extensive interviews with mathematicians about their teaching (in this work mostly related to small group teaching) which elaborate the conceptualisation of first year undergraduate difficulties and the participants' pedagogical strategies to overcome those difficulties. Weber (2004) reports the case study of one professor and shows how his actions in the classroom are the result not only of his beliefs about mathematics but also of his beliefs about how mathematics should be taught. Since the review by Speer et al. (2010) the investigation of the practice of teaching mathematics at university level has gained momentum and several studies have been published to better illustrate and understand this practice. Examples include the study by Lai and Weber (2014) which aimed to bring to the fore the pedagogical considerations of participant mathematicians while teaching proof. The authors however also report the occasional mismatch between these consideration and the mathematicians' actions in the

classroom. With a similar focus the study of T. P. Fukawa-Connelly (2012) examines one lecturer's practice with respect to proof writing and her attempts to involve students in her lectures. While these studies describe and analyse mathematicians' pedagogical intentions and how these are enacted in their teaching practice, Lew et al. (2016) investigate how some of the actions of one mathematics lecturer motivated by his beliefs about teaching proof impact on his students' learning. Lew et al. (2016) found that while the lecturer placed great importance on his spoken comments (i.e., the words spoken but not written on the board while presenting a proof) as a means of guiding the students through the proofs, the students only recorded in their notes and recalled what was written at the board. The authors conjecture that this behaviour could be caused by the fact that the students ascribe little or no importance to the lecturer's spoken comments. Therefore this study finds a mismatch between the lecturer's pedagogical intentions and the students' actions in the classroom making some of the good intentions of the lecturer ineffective. Indeed in a follow up study on a larger sample of lectures and lecturers, T. Fukawa-Connelly, Weber, and Mejía-Ramos (2017) found that although oral comments (or informal comments as the authors call them) are very common in lectures, students do not record these comments in their notes, confirming the findings of Lew et al. (2016).

The studies reported above have investigated the *chalk talk* mode of lecturing, but there are some small changes to this mode of lecturing which are starting to appear in the teaching of mathematics while still retaining the link highlighted by Greiffenhagen (2014) between doing and writing mathematics. One such change involves the use of Guided Notes ². Guided Notes usually consist in pre-prepared sets of lecture notes with gaps which students fill in during the lectures. There is some evidence that this mode of teaching is used in science lecturing (e.g., Narjaikaew, Emarat, & Cowie, 2009) and in mathematics (Cardetti, Khamsemanan, & Orgnero, 2010) and it has shown to be very effective in teaching students with disabilities in secondary school in most subjects Suritsky and Hughes (1991). Indeed Cardetti et al. (2010) show in their study that the use of guided notes was linked to academic performance in mathematics. Below we summarise briefly the literature related to note taking and the effectiveness of students' own notes, guided notes and lecturer prepared notes.

3 Note taking and Guided Notes

There is strong evidence that students who take notes during instruction achieve better than students who don't (Einstein, Morris, & Smith, 1985) and that further improvement of performance depends on how students act on the notes taken (e.g., whether they review the notes taken or whether the students

² Guided Notes are also called skeleton notes or gappy notes. In this paper we will use the term guided notes, but in the interview extracts the terms skeleton notes or gappy notes may also be used.

feel that the act of taking notes equates to learning and therefore do not review them prior to testing, (Williams & Eggert, 2002)). Kiewra (1985b) shows in an experimental setting that although note taking during a lecture did not improve immediate performance when compared with not taking notes, after a time lapse of two weeks students who revised on their own notes performed significantly better than students who revised on lecturer produced notes. In a review of the effectiveness of students' own notes and lecturer's provided notes Kiewra (1985a) also found that students who took notes during lectures and revised on both their notes and on lecturer prepared notes performed better than students who revised only on one set of notes. The reasons why this is the case, according to this review, is that although organisation is an important factor in the effectiveness of lecture notes and students' own notes tend to be not as well organised as lecturer prepared notes, taking notes during lectures aids recall. Kiewra (1985a) concludes this review stating that it is therefore plausible to hypothesise that students who are given guided notes prior to the lecture and lecturer prepared notes afterward should achieve best as the guided notes help with the organisation of the student taken notes while also helping with recall. In a meta analysis of research on the effectiveness of guided notes Konrad, Joseph, and Eveleigh (2009) found only three studies conducted at post-secondary level and while the ones at secondary level showed the effectiveness of guided notes in enhancing attainment (especially for students with disabilities), the three studies at post-secondary level seemed to be inconclusive. A subsequent meta analysis by Larwin and Larwin (2013) specifically on research on guided notes at post-secondary level showed that guided notes have a moderate impact on student achievement and are linked to student engagement measured in terms of student response/questioning during lectures (Austin, Lee, Thibeault, Carr, & Bailey, 2002).

We discuss here what these results mean in terms of teaching mathematics at university level. Both Greiffenhagen (2014) and Artemeva and Fox (2011) found that in the *chalk talk* environment what is produced on the board is akin to what could be found in a textbook. Moreover, mathematicians prefer to use the *chalk talk* method also because this gives the students time to write accurately what is written on the board while the use of slides would make the pace of the lecture too fast for the students to either record the information accurately or to be able to follow the line of the argument presented (Artemeva & Fox, 2011). There is however the issue of the status of the spoken comments the lecturers make when they pause writing. Some research shows that these comments are indeed important as often arise from the lecturer's pedagogical beliefs about teaching mathematics and are part of the practice of teaching mathematics (Speer et al., 2010). These spoken comments often are about the importance of certain proving strategies (Lew et al., 2016) general connections between parts of the material covered or, as in the extract from Greiffenhagen (2014) we have reported earlier in this paper, about the fact that mathematics is an organised body of knowledge. These spoken comments however are not considered important by the students (Lew et al., 2016) who don't record them in their notes, do not see their importance and who are too busy trying

to keep up with what the lecturers writes on the board. In this way part of the practice of teaching mathematics goes entirely unnoticed and it is ineffective. It seems therefore plausible to hypothesise that if the students have a set of guided notes which in principle allows them to record smaller parts of writing and which appear, when paired with appropriate revision strategies, to be more effective in terms of learning, then students may have also time to focus on some of the verbal comments of the lecturer and record them, therefore realising some of the lecturer's pedagogical intentions. Hence our study asks the research questions:

RQ1: What are undergraduate mathematics students' attitudes towards the use of guided notes?

RQ2: To what extent does the use of guided notes impact on students' note-taking in mathematics? Does the use of guided notes help students to realise the importance ascribed by the lecturer to their spoken (but not written) comments during the lecture?

In order to answer these research questions we followed a methodology that was inspired by that of Lew et al. (2016) and in a context (a pure mathematics lecture course at the start of a mathematics degree) akin to the context of that study.

4 Methods

This study investigates the teaching practice of one lecturer of pure mathematics and the impact that using guided notes has on the note taking behaviour of his students in a university in the UK. The lecturer, Dr Nicholas, was asked to participate in the study as he had used guided notes in his teaching for many years. The module selected for the study was a compulsory beginning number theory module taught in year one of a mathematics degree. Although the teaching of this module still follows the DTP pattern typical of most pure mathematics modules, the lecturer uses guided notes instead of *chalk talk*. At the start of each lecture the students are given a print out of the notes which contain gaps and they can fill the gaps in as the lecture proceeds. The lecturer displays the same print out on the screen and writes in the gaps as he lectures using a visualiser.

Ethics approval for the research was obtained at the university where the first author works. Data collection was carried out in the second semester so that the students would have experienced some university teaching, both with the traditional *chalk talk* method and with guided notes. Participant students were recruited following an email to all students enrolled in this module, the data were collected by the second author of this paper. Eight students took part in the study, all enrolled in a year one mathematics degree. Of these there

were two male students (S2 and S5) and six female students (S1, S3, S4, S6, S7 and S8).

The lecturer was interviewed at the start of the study. The aim of this interview was to discuss the reason for using guided notes and what was the status he ascribed to the comments during lectures which were not written in the guided notes. The interview with the lecturer was audio recorded and fully transcribed. After this initial interview data collection with the students was organised in two cycles which we describe below.

4.1 Cycle 1

The first cycle of data collection consisted of lecture observations and subsequent interviews with the students. Table 1 summarises the data collected in this cycle.

Step	Data collected	Purpose
1	Observation Lecture 1	The lecture was observed and audio recorded. The field notes were taken on a copy of the guided notes to facilitate comparing them to the participants' notes.
2	Participant students' notes were photocopied	These were compared to the field notes taken during the lecture. This comparison will form the basis of the interviews.
3	Group Interview 1 (Students 2,3 and 7) Group Interview 2 (Students 1, 4 and 5) Interview student 6 Interview student 8	The interviews aimed to find out how students felt about working with guided notes rather than taking notes while the lecturer writes on the board and investigate the status they ascribed to the verbal comments made by the lecturer.

Table 1 Breakdown of data collected for Cycle 1

The lecture observed was roughly halfway through the module and the topic covered was co-prime numbers and congruencies. This lecture contained mostly definition/theorem/proof material. The students were interviewed in groups when possible, but two students were interviewed individually due to time constraints. The interviews were audio recorded and transcribed verbatim.

4.2 Cycle 2

The second cycle of data collection also consisted of lecture observations and subsequent interviews with students but the lecture observed was mostly constituted by examples and applications of theorems seen in the previous lecture. Preliminary analysis of the notes from cycle 2 taken by the participants showed that fewer comments and notes of the lecturer's verbal comments were recorded this time if compared to the previous cycle. It did not appear however from the audio of the lecture that the lecturer made fewer comments compared to the previous lecture observed. For this reason during the two group interviews

the students were asked to take part in a short stimulated recall exercise (Lyle, 2003). During the interviews the students were given back a copy of their lecture notes, were asked to listen again to a segment of the lecture audio and to add (if they thought necessary) any additional notes to their original notes taken during the lecture. The two segments of the lecture were two worked out examples (one of which is reported in figure 1). The aim of this exercise was to investigate the reason why the students' note taking appeared different from what was observed in the previous lecture. Table 2 summarises the data collected in this cycle. Also for this cycle the interviews were audio recorded and transcribed verbatim.

Step	Data collected	Purpose
1	Observation Lecture 2	The lecture was observed and audio recorded to have a record of the spoken parts. The field notes were taken on a copy of the guided notes to facilitate comparing them to the participants' notes. The focus was the spoken comments of the lecturer.
2	Participants' students notes were photocopied	These were compared to the field notes taken during the lecture. This comparison will form the basis of the interviews.
3	Group Interview 1 (Students 2, 3, 7 and 8) Group Interview 2 (Students 1, 4 5 and 6).	In this second interview we asked the student to do a small stimulated recall exercise by listening again to a section of the lecture

Table 2 Breakdown of data collected for Cycle 2

4.3 Data Analysis

The three main source of data, interviews with the students and the lecturer, students' lecture notes and lectures observations, were analysed separately and contributed to the narrative presenting the results of the case study reported below.

After each of the two lectures in Cycle 1 and Cycle 2 the second author of this paper collected the students' participant notes and photocopied them. These were compared to the field notes and the lecture audio to collect evidence of what the students wrote and whether this writing deviated from just reporting what the lecturer wrote in the gaps of the notes during the lecture. The aim of this part of the data collection was to investigate whether there was variation in the students' note taking behaviour across students and across lectures. In fact we found that students' note taking behaviour varied significantly even in this small sample of 8 and it varied also across the two lectures observed. These findings are illustrated in table 3 and table 4.

The interview with the lecture was coded to find evidence of pedagogical reflection on the lecture structure and the audio of the two lectures was analysed to look for examples to support the lecturer's statements. These are reported in the section about Dr Nicholas.

The interviews in Cycle 1 and Cycle 2 were analysed in order to find emergent themes. The transcript were initially coded with descriptive codes (Saldaña, 2015) by the second author of this paper, then the coding was revised and refined by the first author of this paper. Finally the codes were grouped in umbrella themes and these themes form the axis around which the narrative of the two cycles is presented. In Cycle 2, particular attention was given to codes illustrating the reactions of the students in the stimulated recall exercise.

5 Findings

This section is divided into three parts: the lecturer profile and the two cycles. We chose to organise the analysis in this way as the two cycles were designed for different purposes and the lecturer profile frames his pedagogical intentions and practice in the lectures.

5.1 The lecturer: Dr Nicholas

Dr Nicholas is a very experienced lecturer having taught at this university for over 20 years. His background is in applied mathematics and he has been teaching this number theory module for over 10 years. Five years ago he switched from a traditional *chalk talk* method of teaching followed by releasing a completed set of lecture notes to guided notes followed by releasing the full set of notes and he uses this method for all the modules he teaches (across the three years of the degree, both pure and applied mathematics). He provides the students with a print out of the notes with spaces for them to fill in during the lecture and he displays this same set of notes on the screen, writing in the empty spaces as the lecture proceeds. In the interview he states that this is his favourite way of teaching. Among the reasons given for this preference Dr Nicholas mentions that this mode allows him to complete the syllabus without having to lose time waiting for students to write up notes and crucially allows him to focus on important points that would otherwise be difficult to focus on. Above all Dr Nicholas believes that this method allows students to focus on the lecture and not on writing mechanically:

Dr N: I am concerned with the traditional method that they [*the students*] are spending too much time having to write things down possibly and not listening to the lecturer enough.

Typically Dr Nicholas' guided notes consist of printed statements of theorems and lemmas, definitions and statements of examples while the proofs of theorems, of lemmas and working for the examples are filled in during the lecture. He justifies this choice with the wish to get the students to think 'stage by stage' about the proofs and the way in which the examples illustrate the theory just seen.

When asked how he differentiates between the statements he prints in the guided notes, those he fills in during the lecture and the spoken statements that are not written in the lecture Dr Nicholas answers that the filled in guided notes need to constitute a coherent record of the lecture which can be read by itself (without having attended the lecture) while for his spoken comments:

Dr N: What I say but do not write down should, I hope, make it easier to understand. So for instance, a student who has been to the lecture and is then looking through the lecture notes afterwards, I suppose I hope that they may recall things that I've said and not written down and that will help them to understand as they're looking back through the notes.

In the transcript from the second lecture observed (figure 1) we can see just what Dr Nicholas explained. The part which was written in the guided notes during the lecture, on the left hand side, constitutes a correct, stand alone solution of the problem posed. The commentary, reported verbatim on the right hand side, contains explanations and recall of previous definitions (*So, divisible by 12 means you are going to get zero as a remainder modulo 12 . . . or on division by 12*), application of theorems (*and the theorem on multiplication up there essentially says it preserves congruencies*), further explanation of seen theorems (*wait we haven't got a theorem on subtraction but we don't need a theorem that deals with that because we know that subtracting 1 is like adding -1 so there is no problem with subtraction ...*).

From this short profile we can see that Dr Nicholas has strong beliefs about how to teach mathematics and these have guided the choice of the format he uses. His actions in the classroom are not dissimilar from those of other mathematicians as reported in the literature (T. P. Fukawa-Connelly, 2012; Weber, 2004) but he has chosen a method different from *chalk talk* to alleviate some of his concerns while still keeping the link between writing and doing mathematics.

5.2 Cycle 1

The first lecture observed included division by an integer number and prime factorisation and it was observed halfway through the second semester of the academic year. A first comparison between the field notes and the notes taken by the participants showed the great variation in terms of the recording of the verbal comments of the lecturer. We observed, for example, that S8 added a record of some of her own thoughts and links between sections as well as most of the verbal comments made by the lecturer while S6 recorded only what was written in the gaps of the notes by the lecturer.

The other students varied in their note taking behaviour between these two extremes. A summary of this variation for this lecture is in Table 3.

The interviews in this cycle aimed at finding out students' thoughts about using guided notes in comparison to other lecturing methods they had experienced and how their perceptions related to the pedagogical intentions of the

<p>Example Show that $(137 \times 89 - 1)$ is divisible by 12.</p> <p>(writing on the guided notes:)</p> <p>Since $137 \equiv 5 \pmod{12}$ and $89 \equiv 5 \pmod{12}$,</p> $\begin{aligned} 137 \times 89 - 1 &\equiv 5 \times 5 - 1 \pmod{12} \\ &\equiv 24 \\ &\equiv 0 \pmod{12} \end{aligned}$ <p>i.e. $12 \mid (137 \times 89 - 1)$</p>	<p>So a little numerical example of ... it would be easy to do this by calculator ... imagine you know your 12 times tables but you can't do long multiplication so you can't work out what this is and so you wouldn't want to divide 12 into that if you could do long multiplication. So how can we nevertheless show that this would be divisible by 12 just from our knowledge of the 12 times table?</p> <p>So let's see what those numbers are modulo 12. So, divisible by 12 means you are going to get zero as a remainder modulo 12 ... or on division by 12. So ... 137 ... you know that eleven 12s are 132 so that's congruent to 5 ... because 137 is 5 more than 132. What about 89? Well 89 is seven 12s plus 5 so again 89 also happens to be congruent to 5 so ...</p> <p>Now we do this multiplication and the theorem on multiplication up there essentially says it preserves congruencies so the result of 137 by 89 will be congruent to ... we have got -1 ... that is going to be congruent to 5 times 5 minus 1 ... wait we haven't got a theorem on subtraction but we don't need a theorem that deals with that because we know that subtracting 1 is like adding -1 so there is no problem with subtraction ... for modulo 12 still ...</p> <p>So we can do this calculation 5 times 5 minus 1 and that is two 12s which is the same as congruent to 0 modulus 12 if you want. So being congruent to zero modulus something means being divisible by this modulus so 12 divides divide 137 times 89 minus 1.</p>
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Figure 1 Transcript from the second lecture observed - Example 1

lecturer. Thematic analysis of the transcripts revealed two overarching themes: the *use* of the guided notes and the impact of this type of lecturing on student *engagement* in class.

Regarding the *use* of the notes most students recognised that it was much easier to revise on guided notes than on their own notes taken in class during *chalk talk* teaching:

S7: It helps with revision as well. I find it easier to revise with gappy notes than the notes that I've taken. Like with [*another lecturer*] when

Lecture 1 contents: 1 lemma, 2 theorems, 3 proofs, 1 definition		
	Description of student notes	Type of extra comments
S1	Slight variation to structure and content of original note	Personal annotations links to various parts of the lecture
S2	Very similar to lecturers' notes	Numerical example, conceptual ideas
S3	Substantial deviations from original notes	Generalisations of statements and ideas, Elaborations on theorems
S4	Very similar to lecturers' notes	Additional explanations of progressions between stages
S5	Verbatim copy of lecturers' notes	Minor variations from original lecturer's notes
S6	Verbatim copy of lecturers' notes	No variation from original lecturer's notes
S7	Verbatim copy of original notes with additional comments.	Minor calculations to clarify written work
S8	Very similar to lecturers' notes	Brief explanations and clarity points of lecture

Table 3 Students' note taking during lecture 1

I have to concentrate on writing things down and listening to what he is saying, as well as writing extra things it's a bit much.

They also mentioned that it was easier to catch up if a lecture was missed. Most participants preferred this method to just having notes posted online as it helped them in seeing what was done lecture by lecture. This organisational aspect is significant as students reported that the teaching of mathematics at university is substantially different from what they had experienced at school also because of the importance of independent study.

S2: I think in [*school*] you're so used to following like a format, you learn in the pattern, it's not necessarily Maths. It's just learning like a poem or something, and then repeating it, but when you come to uni it completely changes and its ...

S7: And the independence as well. You have to do a lot of independent work. Which I think that it should be taught from secondary school to start early to do independent work because that's something that I didn't learn until I came here and it was a bit of a struggle.

A set of well organised lecture notes may therefore be important at the revision stage as well as to organise independent study during the semester.

While all students wrote at least what the lecturer wrote in the gaps of the printed notes, there was consensus that this format would allow them to record not just what was written but also items the students thought may be helpful at the revisions stage. These may be links between parts of the lecture, between theory and examples or explanations of notation:

S4: I start off writing down what the lecturer will write down, and then if he says something that I didn't already know and he doesn't write it down then I'll write that down myself or if he says something that I

think is important and I'll need to remember it and he hasn't written it down then I'll write that down as well. But on the whole it will mainly be exactly what he writes plus me being a bit extra.

As we have noticed in table 3 there is much variation between students' note taking behaviour and this variation was also reflected in the interview responses. For example we can see in figure 2 the difference between the notes of S6 which report only what the lecturer writes and S8 who annotates the basic text heavily: her comments are mostly about linking ideas and explaining terms.

3 Foundations of Number Theory

3.1 Division

Lemma 3.1.1

For any $n \in \mathbb{Z}$ and $m \in \mathbb{N}$, the sets:

(i) $K := \{k \in \mathbb{N} : km > n\}$

(ii) $E := \{k \in \mathbb{N} : m^k > n\}$, with $m > 1$,

are non-empty (and so have a least member, by Theorem 1.6.6).

Proof for (i)

If $n \leq 0$, $k=1 \Rightarrow km > n \quad \forall m \in \mathbb{N}$.

If $n > 0$, $k=n+1 \Rightarrow km > n$

3 Foundations of Number Theory

3.1 Division

Lemma 3.1.1

Note: $\mathbb{N}$ is exactly the same as $\mathbb{Z}_+$

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(i) $K := \{k \in \mathbb{N} : km > n\}$

(ii) $E := \{k \in \mathbb{N} : m^k > n\}$, with $m > 1$, for 2 larger you can find a bigger #.

are non-empty (and so have a least member, by Theorem 1.6.6).

Proof for (i)

If $n \leq 0$, $k=1$ and m is a $\mathbb{Z}_+$ means a $\mathbb{N}$ $\Rightarrow km > n \quad \forall m \in \mathbb{N}$

If $n > 0$, $k=n+1 \Rightarrow km > n \quad \forall m \in \mathbb{N}$.

Integer Division:

$$\begin{array}{r} 02:3 \\ 4 \overline{) 11} \\ \underline{8} \\ 3 \end{array}$$

Figure 2 Extract from the first page of notes by S6 and S8 - Cycle 1

When we asked whether the students were expecting to leave the lecture having understood all the material all participants agreed that this was never the case. They stressed how for them it was normal to revise lectures and try to make sense of the parts that were not clear at the time of taking notes, which again places a lot of importance on a set of good notes and on the presence of that linking statements which are made by the lecturer.

I: So speaking about students in general, how important is it that students understand everything straight away in the lecture? How important is that?

S7: It's unlikely, especially when they're writing something new that you haven't ... it takes time, in order for the understanding to come together, Er ... I mean it's important to kind of know the general idea of the lecture or something, but I think it takes time, definitely takes time. Well, depending on if you do the work.

Or as Student 6 explains:

S6 Er ... so yeah sometimes I won't understand something in the lecture then it won't click until a few weeks before the exam when I'm going over it again. Sometimes it takes a fresh mind to go over it. ... So yeah it's important I suppose to understand the majority of what's going on but if it's only like 80% it's still, it's fine because you have time to go over it, you have people to talk to. And there's probably other people struggling with it, as well as you so, you can talk to them, and try and figure it out.

The second overarching theme in the interviews is the impact that using guided notes has on the students' *engagement* in class. Most participants cherished the opportunity to be able to stop writing and focus on what the lecturer is saying.

S1: I think I am probably more engaged in the ones [*lecturers*] that do give us gappy notes because you can kind of stop writing and actually listen to what the lecturer is saying. Rather than just having to copy down everything for an hour.

While many students agreed with this statement one participant disagreed: for him the fact that most of the material is already written is a stimulus for losing concentration. Some of the participants however were not very positive about lecturing altogether and reported losing concentration very quickly in a lecture room:

S5: I probably normally pay attention for the first 20 minutes, and then start zoning out after that, like, well, it depends on the lecturer still but, most of the time it's about after 20 minutes, my brain just shuts down and I don't know why. But, no, I can just keep on track with certain notes. I'll just be writing, I won't be registering what I'm writing, because I don't know why I'm writing, I'm just writing, for the sake of it.

Summarising, the students in our sample like using guided notes as this method allows them to take notice of some of the spoken comments of the lecturer and may even allow them to record their own thoughts and links which may become useful later. These notes are also well organised and useful

during revision for homework or for the exam, especially as the students are well aware that not all the material is understood during a lecture. However, when asked about the status of the oral comments made by the lecturer it appears that these are not considered *as important* as the written parts:

S5: I would say what he [*the lecturer*] writes down is the necessities for the exam and then if he doesn't write down its more just knowledge that you can just know in general so like if you are interested in it then you can probably go out further and research it a bit more but it's not going to be tested, so that's probably why I zone out a bit.

S4: I was just say ... just the same. The proper examinable he'll write it down, and then if he doesn't write it down I just assume we're not going to need to know it. Or it will just be like him just elaborating on what he's written down so he's just explaining it a bit further so then you don't necessarily need the full explanation for the exam so he doesn't need to write it down.

These two quotes are revealing of the students' perceptions and of the mismatch, observed already in the literature, between the intentions of the lecturer and the perceptions of the students. For Dr Nicholas, the spoken comments are the links and observations that will make the material easier to understand, especially at the revision stage. For the students, these comments are extra material, non examinable and therefore of less importance.

5.3 Cycle 2

Lecture 2 contents: 1 definition, 3 theorems, 3 proofs, 3 examples		
	Description of student notes	Type of extra comments
S1	Slight variation to structure and content of original note	Personal notations, links to various parts of the lecture, brief explanations of progressions
S2	Very similar to lecturers' notes	Explanations of steps and progressions of examples
S3	Substantial deviations from original notes	Generalisations of statements and ideas, elaborations on theorems
S4	Very similar to lecturers' notes	Additional explanations of progressions between stages of one example
S5	Verbatim copy of lecturers' notes	None
S6	Verbatim copy of lecturers' notes	None
S7	Absent from this lecture	-
S8	Very similar to lecturers' notes	Detailed explanations and clarity points throughout steps of an example. Explanations and comments on keywords

Table 4 Students' note taking during lecture 2

The second lecture observed contained more examples than the one observed in cycle one (a breakdown is in table 4) and we noticed that there was a different pattern of note taking. In this lecture most students wrote fewer comments than in the previous one. We asked in the second group interviews the reasons for this. Two of the students felt that the examples tended to be self-explanatory: having felt they understood the examples they did not feel the need of any additional comment. Another student commented on the difficulty in trying to understand examples whilst they are being presented in the lecture:

S7: I think too, sometimes . . . you just don't have enough time to really write all the extra stuff that they're [*the lecturer*] saying, when you're doing the examples . . . And sometimes, when you're trying and understand the example that he's given, then you don't really focus on the extra things that he's saying.

The second cycle of data collection had a stimulated recall exercise to help students explain how they take notes with particular focus on the lecturer's spoken comments. During the interviews the interviewer played two extracts from the lecture audio which contained two examples (the reproduction of what was written in the guided notes by the lecturer and the verbatim transcript of the audio of one of the two examples is reported in figure 1) and gave back the photocopied notes that each of the students had taken during that lecture. We focused on two extracts containing examples as we noticed that here there were fewer additional comments written by the students in their notes. The students were then asked to listen to the audio extracts and take additional notes (if they wished or felt it necessary) on their original notes.

After this exercise we found that S1, S2, S3 and S4 all wrote additional comments on their original notes after having re-listened to the lecture extracts. The students justified this by saying that the lecture did make useful additional verbal comments which they had not picked up during the lecture because of the demands of taking notes during the lecture.

S1: I found it really interesting because in the lecture you're kind of trying to keep up almost so you don't pick up on everything that they're verbally saying . . . Like I picked up on, the couple of bits that I didn't pick up on in the lecture . . . something I don't think I would pick up on if I hadn't just listened to it back.

This was also highlighted again by the students when asked why they may have written notes this time around as opposed to when in the lecture. Some of the students made reference to the different competing demands placed on them whilst in the lecture, such as recording the lecture notes, understanding the content, and listening out for any additional comments:

S6: Probably paying more attention to what the lecturer's actually saying because he did say a lot of little tips and tricks and they don't write down. Which I feel could be useful for me in the future whereas what

they write down just tends to be the actual front of the module but then it's the little tips that they give you that will make it easier for me so I'll probably focus more on what they're saying now.

S4: Yeah I guess like there's no harm in writing more than necessary because when you come back to it, it could be useful whereas is you choose not to write something then you come back to it and you've forgotten it then that's it its gone.

The results of this stimulated recall exercise are interesting. Upon re-listening to the lecture extract with their notes available some of the students recognised that they had missed some helpful points made by the lecture. A possible explanation for this is that there are both conflicting demands on student attention during lectures and that the status of the spoken comment is not clear. Indeed the students have expressed contradictory ideas about the lecturer's spoken comments: on the one hand they have been described as being something extra, not integral to the preparation for assessment, while on the other in the stimulated recall exercise these comments have been described as useful to understand, in this case, the example presented. We argue that the student position is to give priority to the written part of the lecture, especially it seems when the lecture is rich in examples, as this is deemed to be the most important. However, unlike previous results, the variation in additional comments made on the guided notes we have observed indicate that at least some students record the lecturer's spoken comment, even if they may not be aware of their importance or their usefulness at the time of recording them.

6 Discussion

Our study proposed two research questions. The first was to investigate mathematics university students' experiences and perceptions of using guided notes, against the backdrop of a fairly uniform way of lecturing in the subject. The second was to study the impact of guided notes on students' note taking behaviour. More specifically the focus of the second research question was to ascertain whether the use of guided notes helped the students to appreciate the importance of the lecturer's spoken comments during the lecture and facilitated their recording. By interviewing Dr Nicholas we found more evidence to support the findings that mathematicians hold beliefs about the teaching of mathematics that are not their beliefs about mathematics itself, as the studies by Weber (2004) and Lew et al. (2016) also showed. Indeed Dr Nicholas has chosen to use guided notes following a real concern that the fast pace of note taking in a *chalk talk* mode of lecturing would distract his students from pausing and appreciating some of the subtleties of the material. Dr Nicholas' concerns relate very closely to what the literature about note taking suggests. Indeed Kiewra (1985a) notices that notes taken during class are usually not well organised and that using guided notes helps student structure their notes

better which in turn facilitates revision and ultimately retention. Guided notes also maintain the link between doing mathematics and writing, highlighted as important for mathematicians both in the studies of Artemeva and Fox (2011) and Greiffenhagen (2014). Moreover Dr Nicholas ascribes similar value to his spoken comments to that found in previous studies (Lew et al., 2016; Weber, 2004) and describes those comments as the links that will help the students to see the connections between different parts of the lectures, part of the lectures and the examples. His reflection on the organisation of his lectures, stating that the written guided notes need to represent a stand alone record of the lectures that is meaningful even if read in isolation while his verbal comments are an aid for understanding of that stand alone material, resonate with what was found in Artemeva and Fox (2011) confirming that these teaching norms of mathematics are indeed global.

The students interviewed generally preferred to attend lectures supported by guided notes rather than by *chalk talk* for a variety of reasons including the possibility of taking a well structured set of notes. This reflects findings of the general literature on note-taking, which we have reviewed earlier. Resonating also with the findings in Austin, Lee, and Carr (2004) our participants felt that the notes taken in class on the guided notes were much better ordered, easier to navigate and more useful for revision than notes taken in a traditional *chalk talk* lecture. They also connected the use of guided notes to their engagement in the lecture. When the pressure on writing everything that is written on the board is taken away, the students felt they could engage more with what the lecturer was saying, again confirming some of the results found also in Austin et al. (2004). Therefore, with regards to our first research question our findings confirmed the findings of the general note-taking literature reviewed in Kiewra (1985a) and Konrad et al. (2009). While maintaining aspects of lecturing important for this subject and especially the link between writing and doing mathematics, using guided notes has the potential to allow students to engage in class by removing the pressure on the continuous recording of what is written at the board. In this respect, using guided notes seems to be suitable for teaching university mathematics.

For the second research question we note that the analysis of both the interviews with the students and the notes taken during lectures revealed a great variety of note taking behaviour, in contrast to what Lew et al. (2016) found. Recall that we carried out our investigation in a context similar to that of Lew et al. and adopted a similar methodology: therefore we consider our data and findings comparable. In the Lew et al. study almost no spoken comments of the lecture made their way in the students' notes, and indeed the students did not think that such comments were important, linking the lack of relevance to the fact that they were not recorded on the board. Some of the students in our sample did record such spoken comments, and some noted some of their own links within the material covered (see example of the notes taken by S8 in figure 2). Therefore our study shows that the use of guided notes can indeed leave at least some students time to record extra comments, either

comments spoken by the lecturers or their own links and other observations such as the explanation of mathematical terms.

As for the question of whether the students understood the importance of the spoken comments by the lecturer, we observed that indeed they didn't, just as Lew et al. (2016) conjectured. The participants in our study still believe that the important parts of the lecture are the written parts, no matter what additional comments the lecturer makes. They also equate *important* to *exam-inable* revealing a very assessment driven view of learning. The impact of summative assessment on students' engagement with the subject is well reported in the assessment literature (Gijbels & Dochy, 2006; Harlen & Deakin Crick, 2003), and here we see its impact on note taking. Through the stimulated recall exercise we noticed that for some students some of the spoken comments were appreciated and were recorded. The students however ascribed to these comments the status of tips or tricks and not that of material important for understanding. Therefore our study suggests that guided notes allow students to have some time to record the spoken comments of the lecturer and record some important links and general comments on the material covered which will be useful at the time of revising. However the use of guided notes does not improve the students' understanding of the pedagogical intentions of the lecturer or their ability to see the importance of such comments. Instead, an assessment driven approach to studying (Segers, Nijhuis, & Gijssels, 2006) shapes the engagement with learning of the participants in our study. We note here that this dimension of students' assessment driven approach to learning did not emerge from Lew et al.'s study. There may be two reasons why this is the case. The first is that while our study focused on students' note taking behaviour, Lew et al.'s study focused more specifically on note taking while teaching proof and this tighter focus may have driven their study to questioning mostly linked to learning proof. The second reason is contextual: higher education in the UK has undergone a process of marketisation in recent years (Lynch, 2006) which started with the introduction of fees for university studies in 2007. It could be the case that the strong emphasis on assessment we have observed in our data is linked to this relatively novel shift in higher education policy.

7 Concluding Remarks

In this paper we aimed to investigate the effects of a relatively novel practice in teaching mathematics at university: using guided notes. We focused on this relatively small change from the dominant *chalk talk* teaching as this method respects the link, important to mathematicians, between writing and doing mathematics (Artemeva & Fox, 2011) and it has been found to be more effective for learning than students' own notes when paired with appropriate revision strategies (Larwin & Larwin, 2013). We have seen how Dr Nicholas, the lecturer in our study, had very precise pedagogical reasons for choosing to

use guided notes which are linked to his beliefs about teaching mathematics. We have also seen how using guided notes can potentially impact on the note taking behaviour of students allowing them to record lecturer's comments that otherwise would go unnoticed. Crucially however we have seen that the use of this teaching method does not necessarily help students to realise that the verbal comments made by the lecturer are important. Above all we have seen how studying the practice of teaching university mathematics (Speer et al., 2010) is complex. Unexpectedly, from the analysis of the student data, it emerged just how strong the impact of recent changes to the UK higher education system is in terms of engagement with learning (see also Molesworth, Nixon, & Scullion, 2009).

Of course our study has all the limitations of a small qualitative study. It involved one lecturer and a small number of students, in one research intensive university in the UK. It also did not address the issue of note taking and achievement, which was beyond its scope, and focused instead on students' note taking behaviour. We do not have data to say whether the students who took richer notes achieved better than the students who just wrote down what the lecturer wrote down. This is certainly a question that needs addressing in future studies.

There are several direction that future research could take. One is to collect more cases in a variety of universities and a variety of countries to help distinguish between local cultures and global cultures. The other direction is to bring the institutional context of research to the fore and to examine teaching practices in a variety of higher education institutions. Could it be the case that student note taking behaviour varies from research intensive universities to teaching focused universities in the UK? This is a question that is still open yet important to investigate the practice of teaching mathematics at university level.

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