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Linear stochastic estimation of the coherent structures in internal combustion engine flow

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Linear stochastic estimation of the coherent structures in I.C. engines flow

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Abstract

A methodology for estimating the in-cylinder flow of an IC engine from a number of point velocity measurements (sensors) is presented. Particle image velocimetry (PIV) is used to provide reference velocity fields for the linear stochastic estimation (LSE) technique to investigate the number of point measurements required to provide a representative estimation of the flow field. A systematic iterative approach is taken, with sensor locations randomly generated in each iteration to negate sensor location effects. It was found that an overall velocity distribution accuracy of at least 75% may be achieved with 7 sensors and 95% with 35 sensors, with the potential for fewer if sensor locations are optimised. The accuracy of vortex centre location predictions are typically within 2-3 mm - suggesting that the presented technique could characterise individual cycle flow fields by indicating vortex locations, swirl magnitude or tumble for example. With this information on the current cycle a control system may be enabled to activate in-cycle adjustment of injection and/or ignition timing for example to minimise emissions.

Keywords

Linear stochastic estimation, In-cylinder flow, Cyclic variation, Velocity estimation, I.C. engine sensors, Coherent structures

Introduction

For the advancement internal combustion engine control strategies, knowledge of the real-time in-cylinder flow characteristics would be advantageous. Full field direct measurement of such flows often requires bespoke research engines; equipped with enhanced optical access via a transparent cylinder liner and/or a piston window^{1,2}. In addition, these measurements, can require extensive processing and produce large amounts of data; both of which make them unsuitable for online active control strategy. There has therefore been much research effort in the development of flow taxonomy from few measurement points, particularly with application to feedback flow control^{3,4}.

In I.C. engine design, there is an increasing application of active control strategies with regards to injection and ignition; particularly in GDI application where the particulate emissions (PN & PM) are known to be sensitive to injection timing due to the reduced time for mixing⁵. Large scale flow structures will impact the convective mixing and distribution of fuel and as these are subject to cyclic variation a single optimised injection calibration may not be suited to all flow conditions.

Whilst the application presented in this work is centred on the internal combustion engine, the problem of accurately estimating flows from few measurements exists in a bigger context (aerodynamics, mixing/blending etc).

In the wider field of turbulent fluid flows, Adrian^{6,7} introduced linear stochastic estimation (LSE) to approximate conditional averages of turbulent flow. Cole et. al. demonstrated that using the technique and hot-wire

anemometers, a velocity grid may be estimated from two-point statistics⁸. Since then, numerous groups have applied the technique to turbulent flows from unsteady jets⁹ to boundary layers¹⁰. Stochastic estimation, SE, has also been extended to higher orders known as quadratic stochastic estimation, QSE. For example, research by Druault et. al.¹¹ uses QSE to estimate far-field acoustic pressure from flow components.

It is common for LSE to be combined with a technique earlier developed by Lumley et. al.¹²; proper orthogonal decomposition (POD), also known as Karhunen-Loève decomposition. POD is widely used in the study of turbulent flows, particularly in cyclic variability of the internal combustion engine^{13–18}. Ukeiley et. al.¹⁹ and Bonnet et. al.²⁰ reported the combination of the two techniques to study the downstream region of a lobed mixer and jet shear layer respectively.

In a large number of studies, sensors used in the LSE technique have been in the form of wall pressure sensors^{3,4,10,21} or microphone-based⁹ as these quantities are often less intrusive to measure. Arnault et. al.²² describe the limitations in estimating smaller scales of turbulence using wall based pressure measurements. However, the study goes on to demonstrate that improvements may be made

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by considering sensor location optimisation according to an algorithm developed by Muradore et.al.²³. Arnault et. al.²² found improvement when the sensor locations were close to the extrema of POD modes which agrees with work by Cohen et. al.²⁴.

An alternative approach is to use velocity point measurements as sensors. Recent works by Tu et. al.²⁵ demonstrate how two-dimensional stereo PIV captured at a low repetition rate may be combined with a hot-wire rake measuring at 30 kHz using POD-LSE complimentary technique estimate highly temporally resolved velocity fields; analogous to time-resolved PIV (TRPIV). Such dynamic estimators are similar to work by Tu et. al.²⁵ who instead use Kalman filter techniques to achieve a similar goal.

The works discussed^{3,4,10,21,22,23,25} suggest that the LSE technique is suitable to estimate the cyclically variable coherent structures within the IC engine, using a number of point based velocity measurements as sensors. By using velocity components as sensors, one can expect a significant spatial correlation and therefore satisfactory measure of accuracy in the estimations generated with minimum sensor points.

In this work, planar in-cylinder flow velocities are measured in an optically accessible research engine using particle image velocimetry (PIV) and used as reference data for LSE. The reference velocity set provides a range of distinctly different coherent flow structures; a brief description of the measurement technique and velocity fields is given in the following section, for detail and further analysis of velocity fields see Butcher et. al.². A subset of velocity points are converted to their scalar (u,v) form and used as sensors in the estimation of the remaining velocity field.

Whilst full field measurements are required for the LSE correlations, subsequent estimations may be driven using sensor information from sparse, point velocity measurements which often require a lesser level of optical access (or even non-optical measurements), allowing for estimation of cycle condition.

The objective of the presented work is to assess the accuracy and representativeness of vector flow fields generated using a relatively small number of velocity point measurements. The impact of number of sensors required when combined with the complementary POD technique is also investigated. The work describes a methodology that may be used to inform sensor system design in active control systems that may use an ignition or start of injection timing optimised depending on the real-time flow condition.

The novelty of the work presented is the use of velocity components as sensors in the LSE technique with particular application to the I.C. engine environment in a manner which could provide the inputs to an adaptive control strategy; allowing for variations to engine timings to account for the inherent cyclic variation.

Reference velocity fields

Research engine and test conditions

The velocity field measurements used as reference in the presented work were carried out in the Lotus single cylinder

Figure 1. Lotus SCORE

Table 1. Engine specifications

Bore	88.0 mm
Stroke	82.1 mm
Swept volume	0.5 L
Compression ratio	10:1
Intake valve diameter	31 mm
Exhaust valve diameter	26 mm
Piston window diameter	60 mm (52 mm)
Intake open / close / lift	-15°/225°/9.35 mm
Exhaust open / close / lift	495°/15°/9.35 mm

optical research engine (SCORE; Figure 1). The engine allows full stroke optical access via a fused silica liner and additional sapphire piston window; allowing both tumble (vertical) and swirl (horizontal) planes to be measured. The engine is a four-valve, pent-roof design with a flat piston crown. The main engine specifications are given in Table 1, where 0° refers to top dead centre intake stroke. A schematic of reference field experiments is given in Figure 2a and the orientation of flow fields is depicted in Figure 2b with approximate valve and spark locations indicated.

Asymmetric intake valve lift strategy² is utilised to produce a range of coherent flow structures. These are realised by maintaining the valve timings set out in Table 1, but by scaling the maximum valve lift (MVL) by between 0 and 100% in 20% increments. This leads to distinctly different flow structure types to assess the robustness of the applied technique. For all tested conditions, the engine is motored at 2000 RPM with a manifold absolute pressure (MAP) of 450 mbar (+/- 15 mbar). The resulting in-cylinder maximum pressure was found to be 8.0 bar (+/- 0.1 bar) indicating the valve strategy had no impact on the trapped mass. Ambient conditions were not controlled but were monitored and found to be consistent throughout testing. All reference fields used in the presented work are at 75° CA, and there are a total of 795 instantaneous vector fields captured

Figure 2. Reference velocity field acquisition set-up

as reference; with a further 5 vector fields in each condition captured to be used for validation of estimations.

PIV measurements

A LaVision FlowMaster PIV system was used throughout this work for the velocity measurements. This comprised of a New Wave Solo Nd:YAG pulsed laser with sheet forming optics, a FlowMaster 3S CCD camera fitted with 60 mm Nikkor lens and the LaVision programmable timing unit (PTU9). The system was controlled by using LaVision DaVis 8.2 software and synchronised from a crank-shaft mounted optical encoder. A LaVision aerosol generator filled with olive oil; density approximately equal to 900 kg/m^3 provides flow seeding of diameter $1 \mu\text{m}$ directly to the engine intake plenum. This is shown by Melling²⁶ to be capable of following flows up to 10 kHz and is therefore suitable for the engine application presented.

The laser light source was operated at 84 mJ/pulse at the second harmonic frequency (532 nm, visible green). The 6 mm beam was shaped by 20 mm optics to provide a 1.5 mm thick sheet covering the cylinder bore. Whilst measurement was not carried out over the entire bore, delivering the light source in this manner ensured uniform light distribution within the measurement area. The height of the laser sheet and therefore measurement plane was 25 mm below the bottom of the pent roof.

The LaVision FlowMaster 3S camera and Nikkor 60 mm macro lens provided a field of view size of 86 mm x 69 mm. Given the 1280 x 1024 resolution sensor, the physical resolution is $69 \mu\text{m}/\text{pixel}$. The lens f-stop was set to 11, achieving a particle image diameter of 2.3 pixels (calculated according to Adrian and Westerweel²⁷). This was sufficient to ensure no peak-locking of the data; assessed during initial set-up tests within the DaVis software.

Image processing and vector calculation is carried out using LaVision DaVis v8.2 software. The images are pre-processed using a sliding background subtraction to remove

liner glare. Velocity vectors are calculated using a multi-pass, decreasing interrogation region with initial pass at 64×64 with 50% overlap followed by a further two passes at 32×32 with 87% overlap. An iterative median filter is applied to remove spurious vectors with greater than two standard deviations of surrounding vectors. Vectors with a Q ratio of less than 1.3 were also rejected. Across all of the data sets, each vector field contained less than 2% spurious vectors. Finally, the velocity vector fields were linearly down sampled to a final resolution of $1.1 \times 1.1 \text{ mm}^2$ to reduce computational time in the following steps. This was considered acceptable as the integral length scale is typically of the order of 4 mm^{28–30}.

It is not the intention of this work to analyse the measured velocity fields, for that the reader is referred to Butcher et. al.², however, it is important to present the key differences between the conditions. Figure 3 presents the ensemble average (of the 795 instantaneous reference) velocity fields for each of the valve conditions; represented by streamlines of the in-plane velocity components only. It is clear by comparing the six ensemble averages that each set of flow fields features different defining characteristics, ranging from a larger, central vortex in the case of Figures 3a & 3b through to counter-rotating vortex pairs such as those evidence in Figures 3d, 3e & 3f and to some extent Figure 3c. Such characteristics should be correctly recreated by any estimation technique applied.

Estimation methodology

The estimation of vector flow fields using linear stochastic estimation requires a set of reference (or slave) velocity fields that are used to generate a correlation matrix between sensor input(s) and the vector field. In the current work, as it is desirable for the estimation to correctly represent the significantly different flow features present in each of the MVL conditions all of the reference field sets (of 795) will be combined into a single set of 4770 vector fields, U_{ref} . It

Figure 3. Ensemble average for each of the reference sets; based on 795 instantaneous vector fields for each set

is important to note at this point, that a separate set of vector fields; 5 for each MVL condition was captured but not used for the generation of correlation matrices; instead these are only used for validation of the estimated fields, U_{val} .

In order to estimate only the coherent structures, another velocity field set, U_{coh} - based on cycles 1-796 (& $U_{coh, val}$ based on cycles 796-800) is generated by first POD filtering each reference set according to Equation 1 and the methodology described in earlier work² before combining. Figure 4 shows how an instantaneous velocity field; in this

case, a 0% MVL condition, may be decomposed to reveal only the coherent structures.

$$u(x, t_i) = \sum_{k=1}^M a_k(t_i) \phi_k(x) \quad i = 1, \dots, N \quad (1)$$

LSE requires each of the sensors to be correlated to each reference velocity field, generating a correlation matrix. As the intention of the work is to characterise the relationship between the selection and number of sensors required, an

(a) Raw velocity field

(b) POD filtered velocity field

Figure 4. Demonstration of POD filtering of an instantaneous reference field

iterative approach is taken, increasing the number of sensors by one each time from a minimum of 2 sensors through to 150 sensors; as this upper limit may be considered as not practical for a real time measurement application. In each iteration, a new set of sensor locations is chosen at random and the correlations, estimations and evaluation are carried out.

Sensor locations are chosen at random as literature discussed suggests that sensor location can impact the accuracy of resulting estimations. The purpose of this work is not to optimise sensor location selection (for that see ^{22,24}) but instead to provide a guide for the number of sensors required for an acceptable accuracy of estimation. Therefore, locations are randomised and the investigation is repeated a total of 10 times and the mean is taken to exclude the effects of sensor location. In reality the available sensor locations are likely to be constrained by the engine geometry; but the methodology presented here would be able to assess the capability of these locations.

For clarity, the notation to be used to define sensors will be p_r at each reference location, r . For all correlations and subsequent estimations, u_x and u_y components are considered as independent scalars, and as such only reference sensors from the $U_{ref,x}$ are used for estimation of $U_{est,x}$, similar for the y -component. Other approaches are possible,

for example using one velocity component from the sensor to drive estimation of both (or all 3 if available) velocity components in the field of interest. This would impact the number of sensors for a given accuracy requirement as it would also rely on the cross-component correlation which may not be as robust; this is not considered any further in the presented work for brevity. The correlations matrix, $a_r(x)$ is generated according to Equations 2 where p_q is the applicable component of velocity at sensor q and N_R is the total number of sensors.

$$\langle u(x, t) p_q(t) \rangle = \sum_{r=1}^{N_R} a_r(x) \langle p_r(t) p_q(t) \rangle \quad q = 1, \dots, N_r \quad (2)$$

Once the a matrices are generated, the sensor values at each of the same reference locations are taken from the U_{val} set and combined with the a matrices according to Equation 3 to produce a set of estimated vector fields, U_{est} . These are then compared to the relevant coherent validation set, $U_{coh, val}$ as described in the following section.

$$u_{est}(x, t) = \sum_{r=1}^{N_r} a_r(x) p_r(t) \quad (3)$$

The investigation strategy may be summarised as follows. Starting with 2 sensors ($N_R = 2$), 2 sensor locations are randomly generated from within the spatial domain of U , and the velocity at each of these locations in U_{coh} are extracted as scalar components, p_r . Equation 2 is applied to generate the correlation matrices, a_r . Using the same sensor locations, only the scalar components at these locations are extracted from the unfiltered reference fields U_{val} to simulate how the velocities may be collected in practice. Equation 3 is then applied to produce velocity fields U_{est} - this set presents the coherent estimate of all cycles in the validation velocity set $U_{coh, val}$, and as such a comparison may be carried out between $U_{coh, val}$ and U_{est} to assess the accuracy of the estimation. The estimations are then disregarded and N_R is incremented by 1 to a maximum of 150 sensors.

The whole process is repeated a total of 10 times in each case. It is the intention that by several repeats of the test and evaluation procedure and through spatially random sensor location selection that the effects of sensor location are negated. As discussed during the introduction section, it is possible to optimise sensor location, however, for real-world I.C. engine application, sensor location would likely be severely inhibited due to the difficult access in a production engine. Therefore, whilst optimal sensor location would potentially allow a reduced sensor count for a given accuracy, it is not likely that this situation would be achievable in real-application. Therefore for the purposes of this work 10 repeats of randomly distributed (spatial locations of) sensors is deemed sufficient to negate the effects of optimisation and reveal the inherent link between the number of sensors and achievable accuracy.

Assessment of estimations

It is necessary for the purposes of this work to assess the accuracy or representativeness of the estimated instantaneous

(a) Estimation of 20 MVL case

(b) Estimation of 100 MVL case

Figure 5. Estimated velocity fields based on 100 sensors

flow fields. Figure 5a and 5b demonstrate an estimation of MVL case 20% and 100% respectively. Qualitatively assessing the resulting flow fields, one can identify the key defining characteristics of each conditions described earlier, namely size, location and number of dominating vortices.

However, to study the effect of the number of sensors required to provide a representative estimation, a measure of estimation error is derived in the following section.

Quantifying estimation error

As each estimated velocity field generates a velocity vector at the same location as the validation fields, it is possible to calculate the spatial error at each location, x and therefore the distribution according to Equation 4.

$$E(x) = \frac{|u_{est}(x) - u_{coh}(x)|}{|u_{coh}(x)|} \quad (4)$$

Initially, this approach was considered and resulted in higher than expected error magnitude. A typical distribution is presented in Figure 6a showing a mean error of 53%. Inspecting the comparison of the validation and estimated velocity fields (Figure 6b) reveals the cause. In this case, it is clear that the estimated flow field is representative of the validation flow field, however, the vortex centre location at approximately $x = 15\text{mm}, y = 0\text{mm}$ is slightly shifted in the estimation by less than 2 mm in the positive x-direction

(a) Error distribution

(b) Velocity field comparison

Figure 6. Evaluation of estimation error method

which would lead to high level of correlation between the two fields, i.e. they have high similarity. Therefore, in the evaluation of the presented technique, vector field correlation, calculated according to Equation 5 is used to assess the representativeness of the estimated fields. This resulted in a correlation of 91% for this case. The accuracy of vortex centre location prediction is described in the vortex location estimation section.

$$R_{ii} = \frac{\overline{u_i(x)_{est} u_i(x)_{coh}}}{\sqrt{\overline{u_i^2(x)_{est}}} \sqrt{\overline{u_i^2(x)_{coh}}}} \quad (5)$$

As discussed in the previous section, the number of sensors between 2 and 150 is investigated using a total of 10 repeats for each evaluation. Figure 7 presents the mean correlation and vector error for each number of sensors. It is worth reiterating that the comparison drawn here is between the estimated fields and the validation coherent velocity fields, $U_{coh, val}$. However, the sensor input to the estimation is from the U_{ref} set, i.e. the raw signals; as would be the case in practical application. The mean vector error plot in Figure 7 has minimal value of approximately 25%, with little improvement observed above 100 sensors. This is due to

Figure 7. Estimation correlation and mean vector error. Shaded area represents standard deviation

Figure 8. Estimation using 7 sensors (100% MVL condition): Correlation = 80%

the issue described earlier where the vortex location may inaccurate by as little as 1 vector position and still cause a high apparent error. Conversely, the correlation shows how high correlation can be obtained using the presented methodology for a modest number of sensors.

For example, Figure 8 presents an estimation of the 100% MVL condition using 7 sensors which has a correlation of 80%. Note, this is slightly higher than the average for 7 sensors (76%) due to the random sensor location selection. Also presented in Figure 7 is the standard deviation around the correlation, showing the range of values that could be expected.

Despite the effects of sensor location being excluded due to repeated randomised sensor location selection; it is interesting to compare the distribution of the correlation matrix for an arbitrarily selected sensor location (Figure 9a) to a spatial correlation taken over the entire reference velocity field set about the same location, U_{ref} , shown in Figure 9b. As would be expected, the two are reasonably similar but subtly different due to the impact of other sensors and their correlation matrices. Nevertheless, the similarity suggests that sensor location may impact the accuracy of the estimation. Further, if the location is in or close to large coherent structures where higher magnitude integral length

(a) Correlation matrix for marked sensor. Other sensor locations also indicated

(b) Spatial correlation about point $x = 6.6$ mm, $y = 1.1$ mm (i.e. sensor 1 location)

Figure 9. Correlation matrix and spatial distribution for combined velocity set, U_{ref} for sensor 1

scales would be expected, then sensor count could be further optimised given prior flow knowledge.

In summary, increasing the number of sensors improves the accuracy of estimations, with 75% correlation achieved when more than 7 sensors are used. A lower number of sensors may be sufficient if the sensor location is also optimised. However, prior knowledge of the flow is required in this case; either the spatial correlation distributions as depicted in Figure 9b or the POD mode maxima as suggested in literature. Location optimisation may also be specific to particular flow structures, and therefore may not be practical over the range of in-cylinder flow structures (Figure 3). Figure 10 shows an example comparison for a typical cycle between the original (non-filtered) velocity field, its coherent part and the estimate generated from the presented technique.

Accuracy of vortex location estimation

To better illustrate the effectiveness of the estimation technique, we can consider derived values that may be useful to know in relation to a control strategy, such as vortex centre

Figure 10. Original velocity field and its respective coherent and estimation fields for a typical cycle

location. As discussed in the introduction, the I.C. engine coherent flow structures are subject cyclic variation. Taking the measured (U_{coh}) 100% MVL case as an example, the locations of the dominating vortices in the 795 instantaneous cycles may be determined and is depicted in Figure 11a. One may reasonably estimate the right-hand side (RHS) vortex has slightly higher variation and sits within a range of $x = 5 - 20$ mm, $y = -10 - 10$ mm; a range of 15-20 mm in each direction. This adds context to the accuracy estimations and the earlier described issue in vector accuracy magnitude. Figure 11b presents an example of the RHS vortex centre location in the estimated fields in relation to the measured vortex centre location. The maximum error magnitude is approximately 3.5 mm, suggesting that the estimation may provide valuable information about the individual cycle characteristics.

Impact of sensor count on computational time

As a potential application of the presented methodology could be real time assessment of flow characteristics, the computational effort required has been considered. Figure 12 presents the computational effort required to generate the a matrices and calculate the estimate velocity field; normalised by the minimal time required for an estimation. For reference, estimation time was approximately 0.1 - 0.4 sec and correlation time was between 8 sec - 10 min using a desktop workstation. An increase is observed in the a matrix generation time as one may expect as Equation 2 shows that the size of the $\langle p_r(t)p_q(t) \rangle$ term to be evaluated is proportional to N_r^2 , the number of sensors squared. Conversely, the computation time to generate the estimated velocity fields shows negligible impact as the number of sensors increases. This is significant as the exercise of generating the a matrices only needs to be carried out once, prior to the calculation of desired estimations.

Use of POD filtered sensor data

As discussed in the introduction, LSE and POD techniques are often used in combination, usually to predict the temporal coefficients for time resolved data. By decomposing the velocity fields into a limited number of spatial modes and temporal coefficients, the parameters required to describe the flow field is reduced. As LSE also allows the estimation of coherent fields by a limited number of parameters, care

should be taken when interpreting the effectiveness of the two combined technique.

In the presented work, the coherent velocity fields U_{coh} were used to generate the correlation matrices while the raw validation velocity fields, U_{val} were used to provide the sensor velocities. A coupling effect occurs if instead the sensor velocities are selected from the coherent validation fields, $U_{coh, val}$. Figure 13 shows how the average vector error drops to almost 0 once there are a greater number of sensors than POD modes used for the filtering of $U_{coh, val}$. Therefore when optimising sensor number or locations, it is important to consider this coupling effect.

It should be noted that the fidelity of the POD filtered coherent structures may be compromised if an insufficient number of spatial modes are used. In this work we have used 12 modes which the authors believe appropriate according to literature for this condition². Depending on the required flow field parameters, control strategies may be tolerant to further filtering (i.e. use of fewer POD spatial modes) and therefore would require fewer sensors in application.

Concluding remarks

In this study velocity point measurements are suggested in place of sensors used with linear stochastic estimation to estimate the coherent velocity fields in the internal combustion engine. A methodology for determining the number of velocity sensors required is introduced, as well as identification of opportunities for optimisation. Two-dimensional velocity measurements are made using PIV in an optical engine to provide reference velocity fields and to allow sensor values to be determined. The impact of the number of sensors is then investigated in a systematic manner.

It was found that reasonably high correlation (75% +) may be achieved from as few as 7 sensor locations with the potential for fewer sensors if location optimisation is investigated. However, optimising sensor location and subsequently reducing sensor count, may reduce the estimate accuracy if there is a wide range of in-cylinder flow structures. Therefore for the technique to be effective over a wide range of flow structures, determination of the number of sensors should be carried out according to the presented methodology. A maximum estimation accuracy of approximately 95% was identified, with diminishing improvements with more than 35 sensors. The number

(a) Distribution of vortex centres for U_{coh} (100 MVL condition, cycles 1-795) (b) U_{coh} vortex centre pdf (cycles 1-795), estimated vortex centres and their error overlaid (RHS only, 35 sensors, cycle 796)

Figure 11. Demonstration of vortex centre identification error

Figure 12. Impact of sensor count on the computational effort of LSE; normalised by the minimum estimation time

Figure 13. Effect of number of sensors in POD filtered data

of sensors was found to have minimal impact on the

computational effort required to generate the estimations; however, the effort for generation of correlation matrices is proportional to the square of the number of sensors.

Further, considering derived values, such as vortex centre location identification; the estimations are able to locate the vortex centre to within 3 mm (35 sensor case). This is useful in application to the I.C. engine where it is shown that vortices can be subject to cyclic variation, giving their locations a range of approximately 15-20 mm in each direction. This finding suggests the technique would allow a control system to adapt to cyclic variation.

An interesting coupling effect was found when the POD and LSE techniques are combined; if the sensors are also taken from the POD filtered velocity fields. An exact estimation is produced in the case where the number of sensors is greater than the number of POD modes used to filter the coherent structures. This indicates the LSE correlation matrix, a , is non-singular and offers evidence of robustness of this as an estimation approach. If a highly clustered sensor set is used it may be possible this robustness is reduced. Building on this, depending on the required derived parameters, this coupling suggests that small sensor sets could be practical.

The complexity of the I.C. engine may in reality inhibit full access (optical or otherwise), meaning that sensor placement is likely to be sub-optimal. The presented work therefore makes use of randomly selected (and therefore likely sub-optimal) sensor locations to assess the usefulness of this technique. It would be possible to replace the disperse velocity point measurements with other available measurement techniques and hardware, such as those that can provide a line of sight from the wall or spark plug. The presented findings and methodology may inform sensor design and provide an estimation of the accuracy of systems in-which prior flow information is available.

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