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Nonlinear waves in integrable and nonintegrable systems' by Jianke Yang (SIAM, Philadelphia, 2010) [book review].

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Nonlinear Waves in Integrable and Nonintegrable Systems

By Jianke Yang:

430pp., ?? (US\$ 85.00), ISBN 978-0-898717-05-1

(Society for Industrial and Applied Mathematics, Philadelphia, 2010).

Modern nonlinear wave theory is a rapidly developing area which includes a large variety of sophisticated ideas and powerful methods, as well as a vast number of important real-life applications. The book by Jianke Yang (2010) entitled ‘Nonlinear Waves in Integrable and Nonintegrable Systems’ will help an interested reader to discover many diverse aspects of nonlinear wave theory in a single book. The author is neither solely driven by the intrinsic beauty of the underlying mathematical theories, although he definitely admires them as he tells the story, nor by the efficiency of numerical methods, although he clearly appreciates them. It will be fair to say that the author is driven by the desire to develop all methods which can help us to understand, describe and predict physical phenomena, and this book is a reflection of the author’s own research path ‘from integrable to nonintegrable equations, from analysis to numerics, and from theory to experiments’. The physical phenomena to which this book is most relevant are nonlinear wave processes in optics and Bose-Einstein condensates, although the main equations considered in the book are of universal nature, and most of the approaches have counterparts relevant to many other applications.

The first chapter begins by introducing a very important nonlinear wave model, the Nonlinear Schrödinger (NLS) equation. The NLS equation is first derived from a Korteweg - de Vries (KdV) - type equation using asymptotic multiple - scales expansions. This powerful technique, introduced here in a very simple and instructive example and used in some consequent chapters, allows one to obtain the mathematical ‘DNA’ of many unmanageable problems - their reduced mathematical models amenable to analysis. The author then discusses, with useful references, the NLS and generalised NLS equations describing the nonlinear light propagation in a Kerr and non-Kerr medium, as well as the NLS equation with higher-order corrections and the coupled NLS equations, in the context of nonlinear waves in optics, and the Gross-Pitaevsky equation, in the context of waves in Bose-Einstein condensates. All these models are, in a sense, close relatives of the basic integrable model, the NLS equation. However, their solutions, and methods needed in order to study them, are different.

The second chapter is devoted to the integrability theory for the NLS equation by the Inverse Scattering Transform, the method which was first developed by Gardner, Green, Kruskal and Miura in 1967 for the KdV equation. The treatment of the NLS equation in the book is based on the Riemann - Hilbert formulation (Zakharov and Shabat, 1979; Zakharov, Manakov, Novikov, Pitaevskii, 1980). The integrability of the NLS equation was established by Zakharov and Shabat in 1971. Integrable models have many remarkable and beautiful mathematical properties, from the existence of Lax pairs, related integrable hierarchies, recursion operators and infinite numbers of conservation laws, to N -soliton solutions and connections between solutions of the linearised equations and squared eigenfunctions of the associated spectral problem, all of which are discussed in this chapter, in the context of the NLS equation and the AKNS hierarchy (Ablowitz, Kaup, Newell, and Segur (1974)). The purer aspects of integrable systems are left out of the scope of the book.

The third chapter is an extension of the integrability theory for a single NLS equation, associated with the second order Zakharov - Shabat spectral problem, to integrable models associated with the higher-order spectral problems, such as the vector NLS system. The models belong to one and the same integrable hierarchy generalising the AKNS hierarchy, and are treated together using the Riemann - Hilbert formulation for the hierarchy. The author highlights the existing similarities and differences in the treatment of the scalar and vector problems. The chapter includes the discussion of multisoliton solutions of the Manakov system, the coupled focusing - defocusing NLS system and the Sasa-Satsuma equation, three integrable models having a third-order spectral problem.

The fourth chapter deals with nonintegrable equations which constitute weak perturbations of integrable models. Such nearly - integrable equations naturally appear in the studies of various physical problems. In this chapter, soliton perturbation theory based on asymptotic multiple - scales expansions is developed for the perturbed NLS equation. The emerging linearised equations are solved using the squared eigenfunctions of the spectral problem associated with the unperturbed integrable equation. The perturbation theory is applied to study the evolution of the NLS soliton under the higher-order optical effects, including the Raman, self-steepening and third-order dispersion effects. The chapter also includes the studies of weak interactions of the NLS solitons, and soliton perturbation theory for the complex modified KdV equation. In the second case, the solitary wave solutions have new features; generically a solitary wave loses energy due to radiation of the Fourier mode which is in resonance with the solitary wave. Importantly, the amplitude of this oscillating ‘tail’ does not decrease at the fast time scale. Remarkably, among these nonlocal solitary wave solutions there exists a family of truly localised solutions, called ‘embedded solitons’ (Yang, Malomed and Kaup (1999)).

The fifth chapter moves further away from integrability, and deals with equations which can not be viewed as perturbations of integrable models. The chapter begins with the consideration of stability of solitary waves of the generalised NLS equation. The important Vakhitov - Kolokolov stability criterion is discussed for this equation and its multidimensional version, establishing connection between linear stability and the slope of the power curve of the positive solitary wave. The criterion is then generalised for the case of the multidimensional NLS equation with a more general form of nonlinearity and external potential, revealing that not only the slope of the power curve, but also the number of positive eigenvalues of one of the linear operators associated with the problem, determine the linear stability of the positive solitary waves. Stability of solitary waves which cross zeros is discussed as well, confirming the importance of the power curve for the study of the stability. The next large topic covered in this chapter is the exponential asymptotics technique for the description of the nonlocal waves in the NLS equation with a third-order dispersion perturbation. The amplitude of the tails of symmetric nonlocal waves is found and used to obtain the description of one-sided radiating solitary waves. The embedded solitons, their dynamics and stability properties are discussed in detail, in the contexts of the second harmonic generation when there exist semistable isolated embedded solitons, and some NLS-type equations, which possess continuous families of embedded solitons. Another topic discussed in this chapter is the fractal scattering in collisions of solitary waves, investigated by a combination of variational methods and numerical simulations. While such collisions in integrable systems result only in phase shifts, the outcomes of collisions in nonintegrable systems depend on initial conditions, and can be rather complicated. In particular, the exit-velocity versus collision velocity graphs can exhibit fractal structure. The fractal scattering can also occur in weak interactions of solitary waves, and this is discussed in the context of the generalised NLS equation. Two-wave interactions are shown to be governed by a second-order area-preserving map exhibiting a fractal structure. The discussion of the transverse instability of solitary waves and wave collapse in the two-dimensional NLS equation concludes this large chapter.

Nonlinear wave phenomena in periodic media are considered in the sixth chapter. The NLS-type model equations in this chapter have spatially periodic coefficients. The chapter begins with the introduction of Bloch modes. Two families of small-amplitude one-dimensional gap solitary waves bifurcating from edges of Bloch bands are analysed using asymptotic methods, and their linear stability is studied with the help of numerical methods. One-dimensional gap solitons not bifurcated from Bloch bands are also discussed. The chapter then continues with the study of two-dimensional gap solitons for a square potential. Several linearly independent Bloch modes can now coexist at a band edge, giving rise to more types of gap solitons bifurcating from band edges, again studied using a combination of asymptotic and numerical methods. Linear stability of two-dimensional on-site solitons is studied both analytically, by calculating asymptotic expansions for eigenvalues of solitons near band edges, and numerically. Two families of two-dimensional gap solitons not bifurcated from band edges, and their stability properties, are described next. The chapter concludes with an interesting and useful account of experimental observations of gap solitons in periodic media, both in nonlinear optics and Bose-Einstein condensates.

The last chapter is devoted to numerical methods used to study solutions of nonlinear wave equations, with an emphasis on evolution simulations, computations of solitary wave solutions and their linear-stability spectra. The methods have spectral spacial accuracy, and relevant sample MATLAB codes for the problems discussed in the book are included allowing the reader to quickly adopt the algorithms. Pseudospectral, split-step and integrating factor methods are used for evolution simulations. Petviashvili, accelerated imaginary-time evolution, squared-operator iteration and Newton conjugate-gradient methods are used for computations of solitary waves. Linear-stability spectra are studied using Fourier collocation method for the whole spectrum and Newton conjugate-gradient method for individual eigenvalues. Accuracy, numerical stability, convergence criteria and speeds for the methods are discussed.

To conclude, this book is a very useful addition to the existing collection of books on nonlinear waves. The many diverse topics covered in the book, some classical and some new, are discussed in the context of integrable and nonintegrable NLS-type models, with an emphasis on the new phenomena and techniques emerging in nonintegrable cases. Where it is possible, analytical and numerical results are linked with recent physical experiments and observations. This is an excellent book for researchers studying localised structures described by nonlinear wave equations, especially in nonlinear optics and Bose-Einstein condensates, including applied mathematicians, physicists and engineers, starting from postgraduate level and above.

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