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Output Selection Under Control and Fault Detectability Considerations

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Abstract—In a variety of practical engineering systems, i.e. aerospace, mechanical systems, railway vehicle systems, for a given requirement the range of possible locations for sensors is usually known, with the practical engineering issue of optimizing their location. Input-Output selection/placement for control systems has been widely researched in particular under fault-free conditions. In this paper we discuss on the feasibility of an (output) sensor selection scheme in a closed-loop framework based on both control performance and fault detectability metrics. The selection of sensors is based upon both closed-loop control and fault detection objectives by solving a mixed $\mathcal{H}_-/\mathcal{H}_\infty$ optimization problem for each group of sensors available via Linear Matrix Inequalities (LMI). The efficacy of the scheme is illustrated via a numerical example.

I. INTRODUCTION

Modern control and monitoring systems that involve a large number of actuators and sensors that are prone to failure are becoming rather complex and demanding in terms of maintenance. In a variety of practical engineering systems (aerospace, electro-mechanical systems, railway vehicle systems) for a given requirement the range of possible locations for sensors is usually known, the practical engineering issue is either to minimize the number of sensors to achieve a particular level of fault tolerance, or to optimise the location of such a number of sensors. The role of sensors in the overall process is undoubtedly very important. The signals provided by the sensory equipment comprise the data that is utilised by the control, monitoring and diagnostic algorithms. The choice of the number, location and type of sensors (and undoubtedly that of the actuators) has a significant impact on the performance, the complexity and the overall cost of the system. In this paper the primary focus is that of optimised sensor selection for efficient robustness properties of the system, assuming a consistent controller design, with relation to fault detectability issues.

The general problem of sensor/actuator placement (pairs) in the area of feedback control has received significant attention during the past two decades. The sensor/actuator location is optimised in order to increase, from a control perspective, the relative controllability and

observability of the important system modes. Moreover, an further survey on input/output selection methods can be found in [1] (and references within), where a review and some assessment issues are presented according to the desired properties relating to each selection method. However, the core of this and of other previous works is on fault-free environments.

In the area of fault detection and isolation there is some work on sensor placement with an early paper [2] discussing on optimal sensor location for monitoring eigen-structures of multivariable systems, as well as in other papers [3], [4] although still concentrating on probabilistic and statistical approaches. In addition, work on the integration of control and fault detection and fault tolerant feedback control can be found in [5], [6], although this concentrates on issues related to the solution of the control and fault tolerant subproblems rather than particular sensor selection.

In this paper we investigate a framework which incorporates sensor faults and determine an appropriate set of criteria for the optimal selection relating to fault detectability. An iterative approach is employed to lock on the sensor combination which provides the best mixed $\mathcal{H}_\infty$ performance for control and $\mathcal{H}_-/\mathcal{H}_\infty$ performance for fault detection. Each step in the iteration is solved by Linear Matrix Inequalities (LMIs). The $\mathcal{H}_\infty$ performance is employed to cover a worst case control purpose and the $\mathcal{H}_-/\mathcal{H}_\infty$ performance guarantees fault indication to be maximally insensitive to disturbances for a given minimum level of sensitivity to faults.

The paper is organised as follows: Section II gives a performance formulation of the output selection problem with consideration of fault detectability. Section III transforms the formulation into a state space framework under certain assumptions via LMI solutions and compares the performance index under different output combinations. Finally, a numerical example is given in Section IV while concluding remarks are made and future research directions are mapped out in Section V.

The notation we use is mostly standard and is summarized next for convenience. The set of real (complex) $n \times m$ matrices is denoted by $\mathcal{R}^{n \times m}$ ($\mathcal{C}^{n \times m}$). For $A \in \mathcal{C}^{n \times m}$ we use the notation A^T and A' to denote the transpose and complex conjugate transpose, respectively. For $A = A' \in \mathcal{C}^{n \times n}$, $A \geq 0$ denotes that A is positive semidefinite

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(that is, all the eigenvalues of A are greater than or equal to zero). For $A = A' \in \mathcal{C}^{n \times n}$, $\bar{\lambda}(A)$ denotes the largest and $\underline{\lambda}(A)$ the smallest eigenvalue of A , respectively. For $A \in \mathcal{C}^{n \times m}$, $\bar{\sigma}(A)$ denotes the largest, and $\underline{\sigma}(A)$ the smallest, singular values of A , respectively. The $n \times n$ identity matrix is denoted as I_n and the $n \times m$ null matrix is denoted as $0_{n,m}$ with the subscripts occasionally dropped if they can be inferred from context.

$\mathcal{R}(s)^{m \times p}$ denotes the set of all $m \times p$ real rational matrix functions of s . $\mathcal{L}_{\infty}^{m \times p}$ denotes the space of $m \times p$ matrix functions with entries bounded on the extended imaginary axis $j\mathcal{R}_e$. The subspace $\mathcal{H}_{\infty}^{m \times p} \subset \mathcal{L}_{\infty}^{m \times p}$ denotes matrix functions analytic in the closed right-half of the complex plane. A prefix $\mathcal{R}$ denotes a real rational function, so that $\mathcal{RH}_{\infty}^{m \times p}$ denotes the set of all $m \times p$ stable real rational matrix functions of s . For $G(s) \in \mathcal{RH}_{\infty}^{m \times p}$ we define

$$\|G\|_{\infty} = \sup_{\omega \in \mathcal{R}} \bar{\sigma}(G(j\omega)), \quad \|G\|_{-} = \inf_{\omega \in \mathcal{R}} \underline{\sigma}(G(j\omega)).$$

For $G(s) \in \mathcal{RH}_{\infty}^{m \times p}$, we define $G^{\sim}(s) = G(-s)^T$ to be the para-Hermitian complex conjugate transpose of $G(s)$. A square matrix function $G(s) \in \mathcal{RH}_{\infty}^{m \times m}$ is called inner if $G^{\sim}(s)G(s) = I_m$.

II. OUTPUT SELECTION PROBLEM FORMULATION

The proposed output selection scheme based upon a $\mathcal{H}_{\infty}$ control performance index of closed-loop system transfer functions, integrated with a fault detection filter as depicted in Fig. 1.

Fig. 1. Generalized regulator with fault detection filter

Note that $w(s)$ characterises any exogenous inputs entering the system, $u(s)$ is the fixed set of control inputs, $y(s)$ the measurements (their number varies depending on the scenario considered) and $z(s)$ is the vector of regulated outputs (these can be related to ∞ -norm or 2-norm or both types of the aforementioned norms). For the purposes of this work we consider only ∞ -norm regulation, i.e. $z_{\infty}(s)$, for the control objectives.

Next (and an important step towards fault tolerance) is to incorporate sensor fault detectability. The aim here is to cover additive sensor faults, Fig. 2, which affect the output directly as the dotted line in Fig. 1 indicates, i.e. $y(t) = y_R(t) + f_s(t)$.

Fig. 2. Additive sensor faults

Note that sensor faults can also have direct channels to state dynamics or can be transferred into pseudo-actuator faults when necessary [7], [8]. Hence, a generalized faulty system is given by

$$\dot{x}(t) = Ax(t) + B_1w(t) + B_2u(t) + B_f f(t) \quad (1)$$

$$y(t) = C_yx(t) + D_{y1}w(t) + D_f f(t) \quad (2)$$

where B_f and D_f are well-defined distribution matrices with appropriate dimensions.

Our objective is then to explore the channels by which a certain level of system performance is maintained and faults have most effect on the residual signal such that potential faults can be appropriately indicated. The selection of sensors can then be improved for post-fault configuration and fault tolerance.

First consider a faulty Linear Time Invariant (LTI) dynamic system subject to both disturbances and faults as follows

$$\dot{x}(t) = Ax(t) + B_1w(t) + B_2u(t) + B_f f(t) \quad (3)$$

$$z_{\infty}(t) = C_{\infty}x(t) + D_{\infty 1}w(t) + D_{\infty 2}u(t) \quad (4)$$

$$y(t) = C_yx(t) + D_{y1}w(t) + D_f f(t) \quad (5)$$

where $x(t) \in \mathcal{R}^{n_p}$, $u(t) \in \mathcal{R}^{n_{pu}}$ and $y(t) \in \mathcal{R}^{n_{py}}$ are the state, input and output vectors, respectively and $w(t) \in \mathcal{R}^{n_w}$ is the disturbance vector. The energy of the output signal $z_{\infty}(t) \in \mathcal{R}^{n_{p1}}$ is bounded for finite energy input signals by regulating the $\mathcal{H}_{\infty}$ norm of the system input-output gain (robustness metric). Here, $B_1 \in \mathcal{R}^{n_p \times n_w}$, $D_{\infty 1} \in \mathcal{R}^{n_{p1} \times n_w}$ and $D_{y1} \in \mathcal{R}^{n_{py} \times n_w}$ are the corresponding disturbance distribution matrices, and $B_2 \in \mathcal{R}^{n_p \times n_{pu}}$ and $D_{\infty 2} \in \mathcal{R}^{n_{p1} \times n_{pu}}$ are the corresponding control distribution matrices, respectively.

The objective now is to find an optimal sensor combination where there exists a stabilizing dynamic output-feedback controller $K(s)$ given by the following state-space expression

$$\dot{x}_c(t) = A_c x_c(t) + B_c y(t) \quad (6)$$

$$u(t) = C_c x_c(t) + D_c y(t) \quad (7)$$

and where there exists a fault detection filter $F(s)$ given by

$$\dot{x}_f(t) = A x_f(t) + B u(t) + L_f(y(t) - C x_f(t)), \quad (8)$$

$$z_f(t) = H_f(y(t) - C x_f(t)), \quad (9)$$

with filter gains L_f and H_f such that the following (RMS) performance index

$$\rho := \inf_{\substack{\lambda_{\infty} < \gamma_1 \\ \lambda_{\infty} < \gamma_0}} \sqrt{\frac{\gamma_1^2}{2} + \frac{\gamma_0^2}{2}} \quad (10)$$

with

$$\lambda^\infty = \|T_{z_\infty w}\|_\infty, \text{ and } \lambda_-^\infty = \frac{\|T_{z_f w}\|_\infty}{\|T_{z_f f}\|_-} \quad (11)$$

is obtained among all candidate sensor combinations, where $T_{z_\infty w}$ is the transfer function from w to z_∞ , $T_{z_f w}$ is the transfer function from w to the residual z_f and $T_{z_f f}$ is the transfer function from f to the residual z_f , respectively. Although a number of different forms for ρ can be chosen we adopt a root mean square approach, i.e. emphasizing the average of the magnitudes of γ_0, γ_1 quantities relating to control objective and fault detectability respectively.

Thus, the problem under consideration can be formulated as follows:

Problem 2.1: Let all variables be defined as above. Find an optimal sensor set $k \in \mathcal{S}$ with $\mathcal{S}$ representing the entire set of sensor combinations, such that $\rho_k \leq \rho_i$, $i = 1, 2, \dots, N$, where ρ_i is the mixed norm performance index selected as in (10) with the corresponding sensor set i deployed for a defined set of faults.

Remark 2.1: Note that the objective is in particular to find an appropriate sensor combination with the preferred control and fault detection properties, rather than directly to search for an optimal controller and an optimal fault detection filter for a given system. Our purposes is that the selected sensor configuration can be used as an effective basis prior to reconfiguration schemes ultimately leading to a complete fault tolerant system configuration. Undoubtedly the minimum possible set of sensors will be attractive in terms of reducing complexity in a practical system relative to maintenance as well as sensor equipment and installation costs.

Earlier work on similar concepts of (input)/output selection was mainly focused on evaluating a single performance index such as nominal performance, robust performance and robust stability applied to fault-free environment [9], [10]. In addition, work on the use of $\|\cdot\|_\infty$ and $\|\cdot\|_2$ for placing sensor/actuator pairs is addressed in [11] but in an open loop sense applied to flexible structures. Observer-based fault detection filters have been extensively exploited during the past ten years, by which a residual signal is generated to provide fault signatures [12]–[15]. The observer effectively cancels the process dynamics and is sensitive only to disturbances and faults. The filter design objective is then to reduce the sensitivity to disturbances while maintaining a given level of sensitivity to faults. It is hence of interest to incorporate fault detectability into initial system design and output selection. However, it should be noted that the term “fault detectability” we are referring to in this paper is the robust performance of the effect of faults in the residual in terms of norms, which is rather different from the term defined by [16], [17].

III. CONTROLLER AND FD FILTER SYNTHESIS VIA LMIS

An analytical solution of Problem 2.1 is not straightforward due to the difficulty of incorporating all available

sensor sets into one controller design setup. Here, we follow a tractable suboptimal solution using an iterative procedure to evaluate the performance index(es) for each chosen sensor combination.

Multi-objective optimizations problems in the area of robust control have been well studied via generalized LMI treatment [18]. However, in our case the problem is simple to solve since designing the controller and fault detection filter is separate (which is possible in the case where nominal models are assumed [6]).

We first consider designing the controller, i.e. referring to the control performance index γ_1 . With the plant P and controller K given in Section II, the closed-loop system has the realization in (12). By virtue of the Bounded Real Lemma [19], A_{cl} is stable and $\|T_{z_\infty w}\|_\infty < \gamma_1$ if and only if there exists a symmetric P with $P > 0$ and

$$\begin{bmatrix} PA_{cl} + A_{cl}^T P & \star & \star \\ B_{cl}^T P & -\gamma_1 I & \star \\ C_{cl\infty} & D_{cl\infty} & -\gamma_1 I \end{bmatrix} < 0 \quad (13)$$

where $\star$ denotes terms readily inferred from symmetry.

However, the matrix inequality in (13) cannot be solved directly by a convex optimization algorithm since nonlinear terms in the matrix inequalities will be encountered [18]. The following result gives a linearized formulation of the optimization problem of γ_1 , of an analytical and tractable manner.

Lemma 3.1: [18] Let all variables be defined as above, then a stabilizing dynamic controller exists such that $\|T_{z_\infty w}\|_\infty < \gamma_1$ is achieved if there exists $X, Y, \hat{A}, \hat{B}, \hat{C}$ and $\hat{D}$ such that (14) is true.

Then, the stabilizing dynamic controller is given by

$$\begin{aligned} D_c &= \hat{D}, \\ C_c &= (\hat{C} - D_c C_y X) M^{-T}, \\ B_c &= N^{-1}(\hat{B} - Y B_2 D_c), \\ A_c &= N^{-1}(\hat{A} - N B_c C_y X - Y B_2 C_c M^T \\ &\quad - Y(A + B_2 D_c C)X) M^{-T}, \end{aligned} \quad (15)$$

where square and nonsingular M and N should be chosen such that

$$MN^T = I - XY.$$

For the design of FD filter which should achieve minimal γ_0 , we adopt an optimal FD scheme which is maximally insensitive to disturbances for a given minimum level of sensitivity to faults. This so called mixed $\mathcal{H}_-/\mathcal{H}_\infty$ fault detection problem has been previously considered in [20]–[23], where partial solutions were given. Optimal solutions in frequency domain and in state space have been given in [7] and [24], respectively.

The next result gives an existence condition on finding minimal γ_0 such that $\frac{\|T_{z_f w}\|_\infty}{\|T_{z_f f}\|_-} < \gamma_0$ in an LMI formulation.

Theorem 3.1: [24] Let all variables be as defined above. Assume that $n_y \geq n_f$, (A, C_y) is detectable and that $G_f(s) := \begin{bmatrix} A & B_f \\ C_y & D_f \end{bmatrix}$ has no extended imaginary

$$T_{z_\infty w} := T_{cl\infty} \stackrel{s}{=} \left[\begin{array}{c|c} A_{cl} & B_{cl} \\ \hline C_{cl\infty} & D_{cl\infty} \end{array} \right] = \left[\begin{array}{cc|c} A + B_2 D_c C_y & B_2 C_c & B_1 + B_2 D_c D_{y1} \\ B_c C_y & A_c & B_c D_{y1} \\ \hline C_\infty + D_{\infty 2} D_c C_y & D_{\infty 2} C_c & D_{\infty 1} + D_{\infty 2} D_c D_{y1} \end{array} \right]. \quad (12)$$

$$\begin{bmatrix} AX + XA^T + B_2 \hat{C} + (B_2 \hat{C})^T & \star & \star & \star \\ \hat{A} + (A + B_2 \hat{D} C_y)^T & A^T Y + Y A + \hat{B} C_y + (\hat{B} C_y)^T & \star & \star \\ (B_1 + B_2 \hat{D} D_{y1})^T & (Y B_1 + \hat{B} D_{y1})^T & -\gamma_1 I & \star \\ C_\infty X + D_{\infty 2} \hat{C} & C_\infty + D_{\infty 2} \hat{D} C_y & D_{\infty 1} + D_{\infty 2} \hat{D} D_{y1} & -\gamma_1 I \end{bmatrix} < 0, \quad (14)$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0.$$

axis zeros. Then there exists

$$\begin{bmatrix} D_f^\dagger \\ D_f^\perp \end{bmatrix} D_f = \begin{bmatrix} I_{n_f} \\ 0 \end{bmatrix}, \quad \text{rank} \left(\begin{bmatrix} D_f^\dagger \\ D_f^\perp \end{bmatrix} \right) = n_y, \quad (16)$$

and

$$\gamma_o = \min \{ \gamma : Z \in \mathcal{R}^{n \times (n_y - n_f)}, \quad S \in \mathcal{R}^{n_f \times (n_y - n_f)}, \quad P = P^T \in \mathcal{R}^{n \times n},$$

$$\begin{bmatrix} P(A - B_f D_f^\dagger C_y) + Z D_f^\perp C_y + \star & \star & \star \\ (B_1 - B_f D_f^\dagger D_{y1})^T P + (D_f^\perp D_{y1})^T Z^T & -\gamma^2 I & \star \\ D_f^\dagger C_y + S D_f^\perp C_y & D_f^\dagger D_{y1} + S D_f^\perp D_{y1} & -I \end{bmatrix} < 0 \} =: \gamma. \quad (17)$$

Furthermore, for any $\gamma \geq \gamma_o$ there exists P , Z and S such that the inequality in (17) is satisfied and such that the equation $PR = Z$ has a solution for R .

The assumptions on the pair (A, C_y) and the zeros of $G_f(s)$ are necessary for $\|T_{zff}\|_- > 0$ and $A + L_f C_y$ stable.

By way of summarizing the results in [24] we give the following algorithm for the design of $\mathcal{H}_-/\mathcal{H}_\infty$ fault detection filter:

Algorithm 3.1:

- Define $D_f^\dagger$ and $D_f^\perp$ such that (16) is satisfied.
- Find γ_o , P , Z and S that solve the LMI optimization in (17).
- Solve the equation $Z = PR$ for R .
- Define L_f and H_f as in

$$L_f = -B_f D_f^\dagger + R D_f^\perp, \quad H_f = D_f^\dagger + S D_f^\perp. \quad (18)$$

Moreover, the following steps are important for sensor selection decision making:

Algorithm 3.2:

- 1) Define fault conditions for the problem setup.
- 2) Define sensor set.
- 3) Solve for stabilizing controller, i.e. find γ_1
- 4) Solve for FD filter as in Algorithm 3.1, i.e. find γ_o
- 5) Evaluate ρ given by (10).
- 6) Update and goto step 2 if fault conditions unchanged.
- 7) Update and goto step 1 if fault conditions change.

The following section illustrates the approach via a numerical example.

IV. ILLUSTRATIVE NUMERICAL EXAMPLE

We use a numerical example to illustrate the applicability of the proposed approach of sensor selection in dynamics systems. The system considered is a modified F16XL system [25], with the matrices of the linearized model given as follows

$$A = \begin{bmatrix} -0.0674 & 0.0430 & -0.8886 & -0.5587 \\ 0.0205 & -1.4666 & 16.5800 & -0.0299 \\ 0.1377 & -1.6788 & -0.6819 & 0 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix},$$

$$B_{de} = \begin{bmatrix} -0.1672 \\ -1.5179 \\ -9.7842 \\ 0 \end{bmatrix}, \quad B_{wg} = \begin{bmatrix} 0.0430 \\ -1.4666 \\ -1.6788 \\ 0 \end{bmatrix},$$

$$C = I_4, \quad D_f = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The state vector comprises four states, i.e. longitudinal velocity, normal velocity, pitch rate and pitch angle. The outputs are considered the same as the states, while the two disturbances affecting the system are wind gust and deflector perturbation with distribution matrices $B_1 = \begin{bmatrix} B_{wg} & B_{de} \end{bmatrix}$. The control input matrix is $B_2 = B_{de}$ from elevon deflector. We choose all four states as regulated signals in the generalised regulator framework for the whole procedure.

The control design objective is chosen as to ensure a worst case control performance via an $\mathcal{H}_\infty$ output-feedback controller. The additional design objective is to monitor two potential sensor faults: f_3 and f_4 from the pitch angle sensor, where no actuator faults are considered for simplicity ($B_f = 0$) so G_f is a static matrix. First, for the full sensor set (namely, 4 outputs), it can be verified that the conditions of Theorem 3.1 are satisfied and so (18) gives an $\mathcal{H}_-/\mathcal{H}_\infty$ fault detection filter with observer

gains L_f and H_f as

$$L_f = \begin{bmatrix} 16.1 & 10.4 & -4.8 & 0.5 \\ 404.4 & 169.5 & -131.9 & 6.1 \\ 1485.6 & 778.1 & -455.8 & 21.6 \\ -0.3 & -0.4 & -0.7 & -0.7 \end{bmatrix},$$

$$H_f = \begin{bmatrix} -1.4945 & -0.1374 & 1 & 0 \\ 0.1002 & -0.1598 & 0 & 1 \end{bmatrix},$$

and an optimal $\gamma_0 = 0.0666$.

The $\mathcal{H}_\infty$ controller is also given by (15) as

$$A_c = (1.0e+005) * \begin{bmatrix} -0.0164 & -0.0181 & 0.0707 & 0.0002 \\ -0.0144 & -0.0159 & 0.0622 & 0.0002 \\ 0.1787 & 0.1972 & -0.7713 & -0.0024 \\ 1.4345 & 1.6659 & -6.5447 & -0.1136 \end{bmatrix},$$

$$B_c = (1.0e+006) * \begin{bmatrix} -0.0005 & -0.0000 & -0.0010 & -0.0045 \\ 0.0190 & 0.0011 & 0.0027 & -0.0075 \\ 0.0018 & 0.0006 & -0.0442 & -0.0117 \\ 0.1325 & -2.3131 & -0.0272 & -0.0006 \end{bmatrix},$$

$$C_c = \begin{bmatrix} 178.3454 & 199.0380 & -779.1486 & -2.3767 \end{bmatrix},$$

$$D_c = 1.0e-005 * \begin{bmatrix} -0.0029 & 0.0253 & 0.1460 & -0.0184 \end{bmatrix},$$

and an optimal $\gamma_1 = 1.0000$. Therefore, the performance index for output selection under full sensor set is $\rho = 0.7087$.

Then, we investigate the remaining sensor sets following the same performance index. Given that pitch rate and angle sensors are essential for the fault scenario assumed, i.e. fault detection, we retain the corresponding measurements (y_3, y_4) in our output selection. The remaining part is to choose y_1, y_2 or neither. Note that if only fault f_3 or f_4 is assumed then it is essential to keep the pitch rate or pitch angle sensor respectively in the sensor set.

Now, assume that y_2 is removed from the full output set, we get the following fault detection filter as

$$L_f = \begin{bmatrix} -123.3 & 0.3 & 0 \\ -469.0 & -37.0 & -0.9 \\ -5559.2 & -60.6 & -1.5 \\ 23.9 & -1.7 & 0 \end{bmatrix},$$

$$H_f = \begin{bmatrix} -1.1999 & 1 & 0 \\ 0.5093 & 0 & 1 \end{bmatrix},$$

with an optimal $\gamma_0 = 0.3048$ and the $\mathcal{H}_\infty$ controller as

$$A_c = (1.0e+005) * \begin{bmatrix} 0.0030 & -0.1333 & -0.1159 & -0.0005 \\ 0.0237 & -1.0638 & -0.9250 & -0.0012 \\ 0.0908 & -4.0518 & -3.5229 & -0.0101 \\ 0.1218 & -5.3736 & -4.6713 & -0.0358 \end{bmatrix},$$

$$B_c = (1.0e+005) * \begin{bmatrix} 0.0542 & 0.0094 & 0.0332 \\ -0.2537 & -0.0963 & 0.2255 \\ 0.2068 & -1.1331 & -0.4612 \\ 5.8596 & -1.7592 & 1.9613 \end{bmatrix},$$

$$C_c = (1.0e+004) * \begin{bmatrix} 0.0801 & -3.5739 & -3.1074 & -0.0115 \end{bmatrix},$$

$$D_c = 1.0e-005 * \begin{bmatrix} 0.0031 & 0.1027 & 0.0036 \end{bmatrix},$$

with an optimal $\gamma_1 = 1.0017$. Therefore, the performance index for output selection under the sensor set y_1, y_3, y_4 is $\rho = 0.7404$. Note also that the controller is designed relative to the sensor set used, i.e. both the controller and filter are assumed to using the same sensor information.

Hence, we continue in a similar way relative to other sensor selections and summarize the results in Table I.

Note from Table I that rows 1-4 relate to both faults f_3, f_4 , while rows 5-6 and 7-8 only to fault f_3 and f_4 respectively. Thus, in a fault-free environment and for the

TABLE I
OUTPUT SELECTION PERFORMANCE INDEX CONSIDERING FAULTS
 f_3, f_4

Outputs	γ_1	γ_0	ρ
y_1, y_2, y_3, y_4	1.0000	0.0666	0.7087
y_1, y_3, y_4	1.0017	0.3048	0.7404
y_2, y_3, y_4	1.0015	8.9798	6.3889
y_3, y_4	1.0000	10.3811	7.3746
y_1, y_3	1.0000	0.3016	0.7386
y_2, y_3	72.6503	0.0729	51.3716
y_1, y_4	1.0028	0.0729	0.7175
y_2, y_4	1.0006	8.9830	6.3912

defined control problem formulation it is appropriate to choose a two-sensor set, apart from the set y_2, y_3 as it is still possible to have proper robustness properties to the disturbances affecting the aircraft (illustrated via the low values of γ_1). The situation changes once faults f_3, f_4 to monitor are considered. In such a case it is appropriate to use sensors y_1, y_3, y_4 rather than the full set as there is only a minimal effect on the value of ρ . Note that the aim is to choose the minimum set of sensors satisfying the objectives. This set can be used as a basis for further designs in a fault tolerant framework.

In addition, we consider a different set of potential sensor faults, i.e. f_1 and f_2 from the longitudinal and normal velocity sensors. By using the same procedure, the following Table II is drawn:

TABLE II
OUTPUT SELECTION PERFORMANCE INDEX CONSIDERING FAULTS
 f_1, f_2

Outputs	γ_1	γ_0	ρ
y_1, y_2, y_3, y_4	1.0000	1.0021	1.0010
y_1, y_2, y_3	1.0008	73.2363	51.7907
y_1, y_2, y_4	1.0005	1.1712	1.0892
y_1, y_2	1.0005	75.3057	53.2538
y_1, y_3	1.0000	73.1785	51.7498
y_1, y_4	1.0028	0.0880	0.7118
y_2, y_3	72.6503	1.0164	51.3766
y_2, y_4	1.0006	1.1740	1.0908

Similar to the previous case, in Table II rows 1-4 relate to both faults f_1, f_2 , while rows 5-6 and 7-8 only to fault f_1 and f_2 respectively. Similar decisions as in the case of Table I can be followed here, again noting that the fault scenario is rather important in the decision making. For example, in the case where both faults f_1, f_4 to monitor are considered, we can select sensors y_1, y_2, y_4 rather than the full set as again there is a minimal effect on the value of ρ .

Moreover, it is possible to follow a combinatorial decision making procedure if necessary. Thus, select different optimal sets of sensors corresponding to different (appropriate) fault considerations and utilise these as bases in the design of a re-configurable (e.g. switching between the different controllers) scheme for fault tolerant systems.

V. CONCLUSION

We discussed on a new setup to the output selection problem with consideration of fault detectability. The performance index for decision making investigated combines both an $\mathcal{H}_\infty$ controller design and an $\mathcal{H}_-/\mathcal{H}_\infty$ fault detection filter design. An iterative approach is then followed to testify different sensor sets under this performance index, with each of them solved by LMIs.

In this paper we emphasize the $\mathcal{H}_\infty$ -norm that relates to control performance measure and $\mathcal{H}_-/\mathcal{H}_\infty$ that relates to fault detection performance measure. It is envisaged that the selected sensor configuration can be used as an effective basis prior to reconfiguration schemes ultimately leading to a complete fault tolerant system configuration. Undoubtedly the minimum possible set of sensors, in practical systems e.g. aerospace or railway applications, will be attractive in terms of reducing complexity relative to maintenance as well as sensor equipment and installation costs.

While the proposed output selection algorithm takes account of disturbances, and hence has some robustness properties against additive plant uncertainties, it does not explicitly consider the issue of robustness against uncertainty. This may be an important issue since the formulated schemes are model-based and operate in a closed-loop framework. Current work investigates integrated design of the controller and filter via matrix inequalities under uncertainty conditions.

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