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Performance-based seismic design of steel structures accounting for fuzziness in their joint flexibility

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1 Performance-based seismic design of steel structures 2 accounting for fuzziness in their joint flexibility

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6 **Abstract**

This paper presents a performance-based earthquake engineering framework to explicitly take into account fuzziness in the design parameters, with application to steel structures. Semi-rigidity of column-to-foundation and beam-to-column connections is considered as a relevant example of design parameters that can be properly modelled using fuzzy variables. Without lack of generality, their fixity factors are described by means of triangular membership functions, fully defined by lower and upper values of admissibility and their most likely value, i.e. their reference value. For demonstration purposes, the procedure is used to analyse two different case studies, namely a 5-storey single-bay plane frame and an industrial 3D modular structure. The analyses are performed accounting for the fuzziness of the connections, which is then propagated onto representative engineering demand parameters, within a general performance-based design (PBD) approach.

7 *Keywords:* Fuzzy variables, Modular structures, Performance-based design
8 (PBD), Semi-rigid connections, Steel structures

9 **1. Introduction**

10 In structural engineering design practice, steel connections are normally as-
11 sumed either as perfectly rigid or frictionless pinned, in order to speed up
12 and simplify the analyses. However, it is largely recognised that these ide-
13 alised behaviours are practically unattainable in most cases, as in general the

connections tend to function as semi-rigid joints [1]. Furthermore, many experiments have shown that nonlinearity plays an important role in the actual behaviour of steel connections under ultimate load scenarios, which in turn depends on the progressive yielding of their components [2]. For this and other reasons (e.g. geometric imperfections, residual stress due to welding, stress concentration, the effects of frame nonlinearity, etcetera), the problem of the connection design is much more complicated than typically assumed in the day-to-day design practice. Furthermore, it is affected by a high level of uncertainty, such that over-simplifications may lead to considerable inaccuracies in the prediction of the structural responses of interest [3]. It should also be noted that the actual connections are very often detailed by the steel fabricator, rather than being specified by the structural engineering team responsible for the overall design of the structure, which is therefore affected by inherent uncertainties.

Over the last 40 years, flexible connections have been thoroughly investigated, trying to establish models and procedures able to take into account their behaviour when subjected to both static and dynamic loads [4–12]. However, these studies consider deterministic models and do not take into account any uncertainty related to semi-rigid connections, which inevitably affect the overall stiffness and capacity of the steel frame. However, modelling their uncertainties as random variables could be problematic, as reliable statistics can hardly be available. In this scenario, a non-probabilistic approach, incorporating the concept of “fuzziness” (rather than “randomness”) is potentially an effective way to deal with uncertainties in the semi-rigid connections. Furthermore, this approach suits very well the common scenario in which the structural design has to be completed before the types of connections are specified, and sometimes even before the steel fabricator has been appointed. This means that only a form of expert judgement can be used to infer the “degree of belief” that a certain type of steel connection will be implemented. In this scenario, the stiffness and capacity of the connections cannot be effectively modelled as random variables, as neither the “frequentist” nor the “Bayesian” interpretation of probability (e.g. Ref. [13]) would be satisfactory. By contrast, fuzzy variables allow the designer to quantify, for instance, to what extent a nominal pin connection will result in certain values of rotational stiffness and bending capacity.

The fuzzy set theory was originally formalised in Zadeh’s seminal work

[14]. A fuzzy set is any set that allows its members to have different grades of membership in the interval $[0, 1]$. The latter are defined mathematically through a so-called membership function (MF). An extensive discussion on fuzzy theory and its definitions and properties can be found in Refs. [15–18]. In recent years, many researchers have investigated the applicability of fuzzy uncertainties in structural engineering, including fragility analyses [19–22]. Fuzzy variables are particularly effective in representing the effects of “epistemic” uncertainty, i.e. caused by lack of knowledge and data, inaccuracy in the measurements or the intrinsic limitations of the model used, rather than “aleatory” uncertainty, due to irreducible randomness of a given phenomenon [23]. Stochastic approaches such as the random vibration theory or the stochastic finite element method are more appropriate for this second type of uncertainties. Potential advantages of fuzzy models include: *i*) simplicity and flexibility of implementation; *ii*) ability to handle problems with imprecise and incomplete data sets; *iii*) possibility to model nonlinear functions of arbitrary complexity; *iv*) (relative) ease of development; *v*) lending themselves to task-parallelisation, which mitigates the time required to finalise the analyses.

It is worth mentioning here that various studies (e.g. [24–28]) have shown that the effects of epistemic uncertainty on structural models tend to be relatively small in comparison to the aleatory uncertainty in the seismic action, meaning that a deterministic structural model could be confidently adopted for design purposes. However, epistemic uncertainty might not always be negligible; this is the case, for instance, when the steel connections are detailed in a later structural design stage by a different design team, which is a customary practice for industrial modular structures [29].

In the present study, a performance-based procedure for the seismic analysis and design of steel structures with uncertain parameters is established, where the stiffness of beam-to-column and column-to-foundation connections is defined through MFs. This approach allows determining “defuzzified” design values of the selected engineering demand parameters (EDPs). These can be used to quantify rigorously the effects of this source of uncertainty in conjunction with the aleatory randomness of the seismic hazard, even with an affordable computational effort.

84 2. Performance-based design

85 The end of the 20th century has seen an increased research effort toward
86 improving earthquake engineering analysis and design, particularly through
87 procedures able to take into account the seismic hazards in the performance
88 assessment of a structure, balancing scientific rigour and engineering viability
89 in design practice.

90 One important reason that pushed engineers to look for alternatives to
91 prescriptive seismic design codes is that, although they appear to provide suf-
92 ficient protection against the no-collapse requirement, i.e. safeguarding the
93 users' life in case of events with a relatively high return period, the economic
94 losses caused by structural damage and from the loss of the use of facilities
95 in case of moderate events, comparatively with a lower return period, proved
96 often to be disproportionately high [30]. Indeed, the traditional prescriptive
97 codes of seismic design are primarily focused on structural resistance and, as
98 such, require a pre-defined minimum value for the demand-to-capacity ratio
99 (D/C), which ensures life safety and, as a by-product, damage control. Tak-
100 ing a completely different approach, the explicit goal of performance-based
101 design (PBD) is to achieve a desired level of performance that is directly
102 correlated to appropriate consequences and, ideally, can be agreed upon dis-
103 cussion with the client and the relevant stakeholders. Performance can then
104 be quantified in different ways, including monetary costs, considering for
105 instance both initial investments and likely maintenance costs [31].

106 Another important difference between PBD and traditional prescriptive
107 design consists of the steps that are required to approach the structural
108 problem. Whereas in traditional methods the level of seismic risk and the
109 acceptable level of damage are implicitly established by the design codes, in
110 PBD they are explicitly determined during the design process, taking into
111 account the desired performance levels [31], which in turn are inevitably
112 affected by any source of uncertainty in the design problem.

113 Since the early 2000s, the Pacific Earthquake Engineering Research (PEER)
114 centre started developing a new performance-based earthquake engineering
115 (PBEE) methodology. Building on the first PBD generation [32], the inno-
116 vative key feature of the PEER's PBEE approach is that the performance
117 is rigorously defined in a probabilistic manner. The framework consists of
118 four main stages that can be performed in cascade, namely: *i*) hazard, *ii*)

119 structural, *iii*) damage and *iv*) loss analysis. At the end of these, the ob-
 120 tained quantitative data allow decision makers to identify an “optimal” solu-
 121 tion, in whichever sense is most appropriate for each particular design. The
 122 framework is typically expressed mathematically through the following triple
 123 integral [33]:

$$p[DV|\{O, D\}] = \int \int \int p[DV|DM] \cdot p[DM|EDP] \cdot p[EDP|IM] \cdot p[IM|\{O, D\}] dIM dEDP dDM, \quad (1)$$

124 where $p[X]$ = probability density function (PDF) of the random variable X ;
 125 $p[X|Y]$ = conditional PDF (CPDF) of X given the event Y ; O = location
 126 of the structure; D = design of the structure; IM = IM of the earthquake;
 127 EDP = EDP, as a measure of the structural response; DM = measure of any
 128 physical damage; DV = decision variable, that is the performance parameter
 129 of interest.

130 If the structure is affected by fuzzy uncertainties, the random variable
 131 EDP in Eq. 1 is rigorously described by a CPDF with fuzzy statistical de-
 132 scriptors, and then this type of imprecise probability is propagated onto both
 133 DM and DV .

134 3. Semi-rigid connections

135 Beam-to-column and column-to-foundation connections are usually subjected
 136 to a combination of axial force, shear force and bending moment. However,
 137 since for the majority of them the axial and shear deformations are small com-
 138 pared to the flexural ones, only the rotational behaviour caused by flexural
 139 actions will be considered in what follows. In certain circumstances, how-
 140 ever, shear deformations can significantly affect the strength, stiffness and
 141 the ductility of a steel frame subjected to earthquake excitations, namely
 142 when the panel zone in some of the connections prove to be weak in shear
 143 (e.g. Refs. [34–36]).

144 The nonlinear behaviour of a connection can be shown in a moment-
 145 rotation ($M - \phi_c$) diagram, where ϕ_c is the rotation at the joint due to
 146 the inherent flexibility of the connection. Figure 1(a) represents typical
 147 $M - \phi_c$ curves for several common connections. The two extreme cases,

Figure 1: Typical $M - \phi_c$ curves for several common connections (adapted from [2])

148 ideally pinned and perfectly rigid, correspond to the horizontal and the ver-
 149 tical line, respectively [2]. For instance, the single-web connection represents
 150 an example of flexible joint, while T-stub connections, with their extended
 151 end plates, are rather stiff. Accordingly, to reach the same value of rotation
 152 ϕ_c , the former type of connection will require an end moment M significantly
 153 larger than the latter one. Different models can be used to predict the $M - \phi_c$
 154 curve of the joint behaviour. Ref. [37] summarises the most commonly used
 155 models, which can be grouped into: analytical (e.g. [7, 38, 39]), empiri-
 156 cal (e.g. [40–42]), experimental (e.g. [43–45]), mechanical (e.g. [38, 46]),
 157 numerical (e.g. [47–49]) and information-based models (e.g. [50–52]).

158 From a mathematical point of view, semi-rigid connections can be mod-
 159 elled through link elements ideally placed between beams and columns or
 160 at the base of the columns. The links act as rotational springs, which are
 161 typically used to model the effects of connection flexibility onto the overall
 162 stiffness matrix of the structure. In particular, the rotational stiffness k_c of
 163 a semi-rigid connection can be conveniently expressed as:

$$k_c(\nu) = \frac{3EI}{l} \frac{\nu}{1 - \nu}, \quad (2)$$

164 where E , I , l , ν are the Young's modulus, moment of inertia, length of
 165 the steel member (beam or column) and the dimensionless fixity factor, re-
 166 spectively. The latter can be defined as in Ref. [53, 54], and it is always

Figure 2: (a) Beam with rotational springs; (b) Column with base rotational spring

167 within the range $[0, 1]$. The two limiting cases, $\lim_{\nu \rightarrow 1} k_c(\nu) = +\infty$ and
 168 $\lim_{\nu \rightarrow 0} k_c(\nu) = 0$, represent a rigid connection (restraining rotation) and a
 169 pinned connection (permitting free rotation), respectively.

170 4. Fuzzification of the fixity factor

171 Recent years have seen an increasing interest among researchers and practi-
 172 tioners in the applications of non-probabilistic methods to engineering prob-
 173 lems affected by uncertainty [55–58]. Among them, fuzzy logic has a promi-
 174 nent role. Unlike randomness, fuzziness describes ambiguity in an event,
 175 attempting to measure the degree to which it occurs, not whether it occurs
 176 [59]. Even though fuzzy logic makes use of similar concepts as the probability
 177 theory, the final scope is different. As a matter of fact, probability theory
 178 deals with a collection of “well” defined events and make predictions on the
 179 chance of occurrence of each event, while fuzzy set theory deals with a collec-
 180 tion of “vague” events, assigning to them certain degrees of “belongingness”
 181 that are represented through the so-called “membership functions” (MFs)
 182 [60].

183 Considering a space of points X , with a generic element $x \in X$, the MF
 184 $\mu(x)$ associates x to a real number in the interval $[0, 1]$, which represents the
 185 “grade of membership” of x [14]. Obviously, the higher $\mu(x)$, the higher the

Figure 3: Triangular membership factor (MF) of a fixity factor

186 degree of truth for that particular value x .

187 In this paper, the fixity factors of beam-to-column (ν_{bc}) and column-to-
 188 foundation (ν_{cf}) connections are assumed to be uncertain and defined by
 189 means of fuzzy variables with triangular MFs, such as the one depicted in
 190 Figure 3. More complicated shapes can be used for the MFs of the input
 191 variables; however, this would require the availability of more information,
 192 which might be difficult to obtain in real-life design situations. For this
 193 reason, without affecting the generality and practical viability of the proposed
 194 procedure, only triangular MFs will be considered for ν_{bc} and ν_{cf} . That is,
 195 the MFs $\mu(\nu)$ for the fixity factors are built considering three values, namely
 196 ν_{min} , ν_{ref} and ν_{max} : the first and third values are, respectively, the lower and
 197 upper bound of the range of fixity factors values which are considered to be
 198 realistically possible, and they are associated to MF equal to zero; while the
 199 other value, ν_{ref} , is the reference value, e.g. the most likely one, for which the
 200 MF is taken equal to one. Clearly, as a consequence of the fuzzification of the
 201 semi-rigid connections, also the structural response in terms of EDPs, e.g.
 202 internal forces, absolute accelerations and displacements, are fuzzy variables,
 203 fully defined by their MFs.

204 As shown in Figure 3, in addition to the values ν_{min} , ν_{ref} and ν_{max} already
 205 mentioned above, there are other values resulting from a MF being cut at a
 206 given ordinate $\alpha \in [0, 1]$. The fuzzy set containing all elements with a MF
 207 of α and above is called the α -cut of the MF [61]. Obviously, one can make
 208 as many α -cuts as desired on the MF of the design variables, and then the
 209 corresponding α -cuts in the EDPs, DMs and DVs can be determined.

Figure 4: Pyramidal membership function (MF): (a) 3D view; (b) top view

210 The definition of triangular MFs for the two fixity factors ν_{bc} and ν_{cf}
 211 results into a pyramidal MF in the three-dimensional space $\{\nu_{bc}, \nu_{cf}, \mu\}$, as
 212 shown in Figure 4, where the α -cuts become horizontal planes characterised
 213 by the same value of MF. If $n_d \geq 3$ design parameters need to be described
 214 through fuzzy variables, then the overall MF will be represented mathe-
 215 matically by an $(n_d + 1)$ -dimensional hyperpyramid, and any α -cut will be
 216 described by an n_d -dimensional hyperplane orthogonal to the μ axis.

217 For the MF of Figure 4, adopting the so-called “vertex method” [62],
 218 each vertex $\{\nu_{cf}, \nu_{bc}\}$ derived from the combination of the values of the two
 219 fixity factors can be used to define a particular realisation of the structural
 220 model and therefore corresponds to a structural analysis. Importantly, the
 221 α -cut value of the MF of any EDP delivered by the structural analysis is
 222 the same as the value of the MF of the input fuzzy variables, i.e. input and
 223 output parameters have the same degree of membership. Once the largest
 224 and smallest values of each output parameter are calculated for each α -cut
 225 level, its MFs can be constructed.

226 It should be noted here that the vertex method provides a good approx-
 227 imation of the actual MF of the output parameters only if the input-output
 228 functional relationship is continuous and monotonic [62]. If these condi-
 229 tions are not met, other methods can be used, e.g. heuristic optimisation
 230 algorithms (such as genetic algorithms, particle swarm, ant colony, etcetera)
 231 or response surfaces. The procedure used to calculate the MF of the design
 232 quantities of interest will depend, in practical applications, on the complexity

Figure 5: Performance-based fuzzy design (Pbfd) framework

of the structural problem, the availability of data and the required accuracy.

5. Fuzzy analysis as a part of a fuzzified PBD framework

Once the fuzziness has been introduced into the design parameters, the classical PEER's framework for the PBD can be extended, introducing a fuzzy analysis as part of the structural analysis, as illustrated in Figure 5.

Aimed at demonstrating the practical applicability of the proposed approach as part of the day-to-day design practice, the seismic analysis of two case-study structures has been performed with the commercial structural analysis program SAP2000 [63], exploiting its OAPI (open application programming interface), which allows SAP2000 to be used in conjunction with other software, including a general-purpose numerical computing environment such as MATLAB [64]. The steps required by the proposed fuzzy seismic analysis are highlighted in the following paragraphs, and the numerical results are presented and discussed in detail in the Section 6.

247 In order to apply the fuzzified PBD approach, the first stage is the char-
 248 acterisation of the seismic hazard. This is typically done through the “hazard
 249 curve”, which gives the probability of exceedance (PoE) in N years of the
 250 chosen IM, with both N and the IM being chosen by the designer to fit the
 251 particular structural project being considered and the availability of data for
 252 the construction site. The hazard curve is then discretised in a certain num-
 253 ber of IM levels, n_{IM} , and n_{EQ} earthquake records are used to represent the
 254 seismic action for each of these levels. Importantly, the number and values
 255 of the IM levels $IM_1, IM_2 \dots, IM_{n_{IM}}$ must be carefully chosen to allow
 256 quantifying the effects of seismic events with a range of probabilities of oc-
 257 currence, while n_{EQ} should be large enough to provide a sufficient statistical
 258 variability for a given IM level. In total, a set of $n_{HAZ} = n_{IM} n_{EQ}$ earthquake
 259 records will be required to fully describe the seismic hazard, and typically
 260 $n_{HAZ} \geq 50$.

Once the set of earthquake records has been established, the proposed
 fuzzified version of the PBD requires that a time-history dynamic analysis
 is carried out for each of the n_{HAZ} earthquake records (which describe the
 aleatory variability of the seismic hazard) and each of the n_{STR} combination
 of the n_d fuzzy design parameters (which describe the epistemic uncertainty
 in the structural model). Considering that a n_d -dimensional hypercube has
 $n_{VER} = 2^{n_d}$ vertexes, the number of structural model combinations is:

$$n_{STR} = 1 + n_{VER} (n_\alpha - 1) , \quad (3a)$$

where n_α = number of α -cuts, including $\alpha = 0$ and $\alpha = 1$. Notably, each structural model variation corresponds to a combination of the input variables in which every one of them takes an extreme value, i.e. either the minimum or maximum value that the designer considers as realistically possible. Depending on the complexity of the structural problem, additional combinations could be considered for each α -cut level, e.g. one for each edge or each square in the n_d -dimensional hypercube defining the variability of the design variables. For instance, it can be shown that the number of edges is $n_{EDG} = n_d 2^{n_d-1}$ ($n_d \geq 2$) and the number of squares is $n_{SQR} = n_d (n_d - 1) 2^{n_d-3}$ ($n_d \geq 3$), so that the number of structural model combinations becomes, respectively:

$$n_{STR} = 1 + (n_{VER} + n_{EDG}) (n_\alpha - 1) ; \quad (3b)$$

$$n_{STR} = 1 + (n_{VER} + n_{SQR}) (n_\alpha - 1) . \quad (3c)$$

261 Once all the dynamic analyses have been executed, the whole set of values
 262 is obtained for the EDPs of interest, say $EDP_{ihj\ell k}$, where i denotes the i th
 263 EDP required for the subsequent stages of the PBD, i.e. damage and loss
 264 analyses; $j = 1, 2, \dots, n_{EQ}$ denotes the j th earthquake record for the h th
 265 level of the IM of the seismic risk (with $h = 1, 2, \dots, n_{IM}$); k denotes the
 266 k th combination of the fuzzy design variable for the ℓ th α -cut level.

267 It can be noted that, for a given level of the seismic hazard IM_h and within
 268 the theoretical framework of imprecise probabilities [65], $EDP_{ihj\ell k}$ represents
 269 the generic realisation of a random variable with fuzzy statistical parameters.
 270 As such, EDP_i is fully described by the IM -dependent membership functions
 271 of its statistical descriptors, such as its mean value, variance, higher-order
 272 cumulants, fractiles, etcetera.

273 Although appealing from a theoretical standpoint, this kind of repre-
 274 sentation is impractical in the everyday design practice. For this reason, a
 275 different approach is pursued here:

- 276 1. For each of the n_{STR} combinations $\{\ell, k\}$ of the fuzzy design variables,
 277 the CPDF $p[EDP_{i\ell k} | IM_h]$ is best fitted to the empirical set of n_{EQ}
 278 realisations $\{EDP_{ih1\ell k}, EDP_{ih2\ell k}, \dots, EDP_{ihn_{EQ}\ell k}\}$.
- 279 2. Said $\Pi_{ih\ell km}$ the m th statistical descriptor of $p[EDP_{i\ell k} | IM_h]$, with
 280 $m = 1, 2, \dots$ depending on the complexity of the model adopted for
 281 the CPDF, the n_{IM} pairs $\{IM_h, \Pi_{ih\ell km}\}$ are best fitted with a poly-

nomial function. In this way the statistical descriptor $\Pi_{ilm}(IM)$ can be evaluated for any value of the IM, not just the discrete values IM_h ; for instance, for each of the four performance levels (PLs) [66] known as “operational” (O), “immediate occupancy” (IO), “life safety” (LS) and “collapse prevention” (CP). In the following, the generic CCDF (conditional cumulative distribution function), defined as:

$$F[EDP_{ilk}|IM] = \int_{-\infty}^{EDP_{ilk}} p[EDP_{ilk}|IM] dEDP_{ilk}, \quad (4)$$

will be referred to as “response curve” of the specific i th EDP and structural model combination $\{\ell, k\}$ being considered.

3. Finally, the “design curve” for the i th EDP at a given level of seismic IM can be obtained by building the MF of the generic Y th fractile of the fuzzy random variable $EDP_i(IM)$, say $\mu_{EDP_{i,Y}(IM)}$, and extracting a “design value” from it, say $X = EDP_{i,Y}^*(IM)$, where the superscripted asterisk denotes here a defuzzified quantity. The parametric plot of the pair $\{X, Y\}$ for $0 \leq Y \leq 1$ defines the sought design curve. Importantly, although the actual design curve varies with the chosen method used to defuzzify the design variable, the overall framework does not depend on it.

6. Performance-based fuzzy design: numerical examples

For demonstration purposes, the proposed performance-based fuzzy design (Pbfd) framework has been applied to two different structures of increasing complexity, namely a planar frame and an industrial 3D modular structure. Hazard, structural and fuzzy analyses have been performed on both cases, whereas damage and loss analyses have not been carried out, as their practical implementation is very similar to the calculation of the EDPs. In both cases, the structures are assumed to be designed for a site in California, at latitude 37.8° North and longitude 122.417° West, corresponding to a site near San Francisco, that happens to be a class “B” (firm rock), in agreement with the classification map reported in Ref. [67].

Table 1: Geometrical properties of the steel members in the first numerical example

	$A \text{ [m}^2\text{]}$	$I \text{ [m}^4\text{]}$
Columns	254×10^{-3}	$1,367 \times 10^{-6}$
1st- and 2nd- storey beams	156×10^{-3}	921×10^{-6}
3rd- to 5th- storey beams	134×10^{-3}	671×10^{-6}

Figure 6: Structural model for the first numerical example

310 6.1. Case study #1: 5-storey frame structure

311 Figure 6 shows the first case-study model consisting of a 5-storey single-bay
312 frame adapted from [10, 54]. The material is steel, with Young's modulus
313 $E = 210 \text{ GPa}$. The geometrical properties are listed in Table 1. Each beam
314 element has lumped masses $M = 3.5 \text{ Mg}$ at its nodes, representing the effects
315 of dead, super-dead and imposed load.

316 The values $\nu_{cf} = 0.16$ and $\nu_{bc} = 0.84$ have been chosen as reference values
317 for the fixity factors of the two types of connections. In particular, $\nu_{bc} = 0.84$
318 could correspond to the fixity factor of either a T-stub or an extended end-
319 plate connection [2]. This choice might correspond to a scenario in which the
320 structural engineering design team has envisaged a steel frame with nearly-
321 pinned connections at the base of the columns and nearly-rigid connections
322 at the ends of the beams. The resulting fundamental period of vibration is
323 $T_1 = 0.90 \text{ s}$. The latter will be denoted in the following as the reference value
324 of the fundamental period of vibration, i.e. $T_{1,ref}$.

325 Triangular MFs have been built for the fixity factors in the range of $\pm 15\%$
326 with respect to the reference values. This means that the ratio between the
327 base of the triangle and the reference value of the MF (known as “amplitude
328 ratio”, AR) is always equal to 0.3. Although relatively high, this level of
329 fuzziness is realistic when one considers the uncertainty associated with the
330 detailing and fabrication of the connections. In practice, expert judgment
331 should be used in the design stage, e.g. based on previous projects involv-
332 ing various steel fabricators, to provide a more stringent definition of the
333 range of variability for the stiffness of the connections. Also, without precise
334 indications on the reference value for ν_{cf} and ν_{bc} , trapezoidal rather than
335 triangular MFs could be used instead.

336 As shown in Figure 7(a) and (b), only two α -cut levels have been consid-
337 ered in this numerical application, namely: $\alpha = 0$ and $\alpha = 1$. As a result,
338 nine structural model combinations were determined (Fig. 7(c)), considering
339 for $\alpha = 0$ one combination for each vertex and one further combination for
340 each edge (see Eq. (3b)). All the combinations of fixity factors used for the
341 structural analyses are listed in Table 2, along with the corresponding values
342 of the fuzzy fundamental period of vibration T_1 .

343 Figure 8 shows the MF of T_1 . As expected, the largest value of $T_1 =$
344 0.996 s is achieved when both fixity factors take the minimum values allowed
345 by their MFs (combination #3); similarly, the smallest value of $T_1 = 0.830$ s
346 occurs when the fixity factors are equal to their maximum permitted values
347 (combination #9). Since for T_1 the AR is equal to 0.184, one can conclude
348 that, compared to the input ARs, there is an uncertainty reduction equal to
349 $(0.300 - 0.184)/0.300 = 39\%$. This confirms the assumption that relatively
350 moderate variations can be expected for the value of T_1 , and thus the spectral
351 acceleration $S_a(T_{1,ref})$ appears as an effective choice for the IM of the seismic
352 hazard.

353 6.1.1. Hazard curve

354 The first stage in the application of the PBFD framework consists in the
355 definition of the probabilistic seismic hazard, $p[IM|\{O, D\}]$, considering all
356 the design parameters related to the location, including magnitude, faults
357 and soil conditions. The spectral acceleration at the period of the first mode,
358 $S_a(T_1)$, has been chosen as the IM of the seismic hazard, as this quantity

Figure 7: Membership functions for the first numerical example: (a) beam-to-column connections; (b) column-to-foundations connections; (c) top view of the pyramidal function

Figure 8: Membership function of the fundamental period T_1 for the first numerical example

Table 2: Combinations of the fixity factors in the first numerical example

Combination #	ν_{cf}	ν_{bc}	T_1 [s]
1	0.16	0.84	0.901
2	0.136	0.84	0.917
3	0.136	0.714	0.996
4	0.136	0.966	0.851
5	0.16	0.714	0.982
6	0.16	0.966	0.840
7	0.184	0.84	0.894
8	0.184	0.714	0.970
9	0.184	0.966	0.830

tends to be better correlated to the EDPs than the peak ground acceleration (PGA) (e.g. Ref. [68]). Additionally, since moderate variations are expected in the dependent fuzzy variable T_1 , the same sets of earthquake records can be used for all the time-history analyses, irrespective of any model variation due to the fuzzy design variables.

In this study, the hazard curve, expressed in the form of $S_a(T_{1,ref})$ against the PoE in 50 years, has been built with the OpenSHA software [69]. The hazard curve has then been divided into ten groups, each one characterised by 10% variations in the PoE (i.e. $n_{IM} = 10$), whose midpoints are marked with red thick dots in Fig. 9).

6.1.2. Ground motion data set

Once the hazard curve has been established, a database of 150 earthquake records has been created to be used for the nonlinear time-history analyses. The accelerograms, recorded at 63 different stations in California, all on firm rock, have been downloaded from the NGA-West2 PEER's ground motion database [70]. The 5%-damping response spectra of the earthquake records have been scaled with respect to the values of spectral acceleration $S_a(T_{1,ref})$ corresponding to each midpoint of the 10 PoE intervals previously defined for the hazard curve (see Table 3). The scale factors have been computed for all the 150 accelerograms and all the 10 IM levels, and only the best 7 with scale factors closer to 1 for each IM level have been used for the time-history

Figure 9: Hazard curve of the first numerical example

380 analyses, i.e. $n_{EQ} = 7$. The latter value has simply been chosen because
 381 international seismic codes [71–73] typically require a minimum number of 7
 382 time-history analyses for estimating the median of the structural response of
 383 interest although it should be noted here that, in contrast with the same code
 384 requirements, no compatibility rules and/or matching procedures have been
 385 applied to the earthquake spectra as part of the numerical examples). Fig-
 386 ure 10(a) shows the average scaled response spectra for each IM level, while
 387 Figure 10(b) demonstrates the variability of the response spectra for the ac-
 388 celerograms used to define the seismic hazard at a given IM level, namely the
 389 highest level, i.e. $S_a(T_{1,ref}) = 0.526$ g. Alternative and more sophisticated
 390 procedures exist, that could have been implemented for the selection and/or
 391 the artificial generation/modification of accelerograms (e.g. Refs. [74–78]),
 392 including compatibility with and/or matching to a given set of design spectra.
 393 Such procedures, however, do not directly affect the application of the pro-
 394 posed fuzzy version of the PEER’s PBEE framework, which is independent
 395 of the particular suite of earthquake records used for representing the seis-
 396 mic hazard. For the purposes of the present work, in particular, the adopted
 397 procedure appears to provide a sufficient level of record-to-record variability
 398 (as demonstrated by the response spectra of Figure 10(b), for instance).

(a)

(b)

Figure 10: Hazard analysis: (a) average response spectra for each IM level; (b) response spectra for IM_{10}

Table 3: IM levels for the hazard curve of the first numerical example

IM level	$S_a(T_{1,ref})$ [g]
I	0.0372
II	0.0586
III	0.0781
IV	0.0988
V	0.122
VI	0.151
VII	0.187
VIII	0.237
IX	0.314
X	0.526

Table 4: Probability of exceedance in 50 years for the four performance levels considered in the structural analysis, and corresponding spectral accelerations in the hazard curve for the first numerical example

Performance level	$PoE_{50}[\%]$	$S_a(T_{1,ref})$ [g]
O	50	0.14
IO	20	0.27
LS	10	0.40
CP	2.0	1.00

399 6.1.3. Structural analysis

400 Once the accelerograms were defined consistently with the hazard analysis,
401 the probabilistic characterisation of the structural response, $p[EDP|IM]$,
402 has been achieved for the $n_{STR} = 9$ structural model variations obtained
403 considering the different combinations of the fuzzy fixity factors. For illus-
404 tration purposes, EDPs belonging to two different damageable groups have
405 been considered, namely structural and non-structural components, i.e. the
406 maximum bending moment (MBM) of the beam at the 1st floor and the
407 peak absolute accelerations (PAA) and the peak displacement (PD) at the
408 top floor. Table 4 shows the damage level (DL) considered for the response
409 curves, with the corresponding values of the spectral acceleration, from 0.14
410 to 1.00 g.

Table 5: Lower bound, reference value and upper bound of the median of the maximum bending moment (MBM) in the first numerical example

Performance level	$MBM_{50,min}$ [kNm]	$MBM_{50,ref}$ [kNm]	$MBM_{50,max}$ [kNm]	AR
O	233.46	280.39	344.05	0.39.
IO	401.85.	460.17.	537.81.	0.30.
LS	556.24.	618.99.	702.61.	0.23.
CP	1186.66	1235.58	1310.15.	0.10.

Table 6: Lower bound, reference value and upper bound of the 90th fractile of the maximum bending moment (MBM) in the first numerical example

Performance level	$MBM_{90,min}$ [kNm]	$MBM_{90,ref}$ [kNm]	$MBM_{90,max}$ [kNm]	AR
O	374.97	396.32	506.02	0.33
IO	688.39	729.94	843.08	0.21
LS	1003	1046.8	1138.6	0.13
CP	2323	2437	2492	0.07

411 *6.1.3.1. Maximum bending moment.* After computing the 9 MBM response
 412 curves for each of the 9 structural model variations, the MF of their median
 413 and 90th fractile has been established. Although the analyses have been
 414 performed for all the DLs listed in Table 4, the results in terms of CDFs
 415 for the two EDPs are presented herein only for the performance levels of IO
 416 (i.e. PoE of 20% in 50 years) and CP (i.e. PoE of 2.0% in 50 years). Inter-
 417 estingly, in all the analyses conducted, the shape of the MF of the median
 418 always appears to be very close to an isosceles triangle, with the AR decreas-
 419 ing at higher levels of the IM. This is due to the fact that the larger the
 420 seismic forces, the more significant the importance of the yield moment of
 421 the steel members, which however have not been fuzzified and thus does not
 422 contribute to further enlarge the base of the MF. Different is the behaviour
 423 of the MF for the 90th fractile, which is always a scalene triangle, i.e. pro-
 424 nouncedly asymmetrical, meaning that in this case the centroid of the MF
 425 can be relatively distant from the reference value $MBM_{ref}(IM)$ for which
 426 $\mu_{MBM(IM)} = 1$, i.e. the deterministic case that would be obtained by neglecting
 427 the fuzziness in the steel connections.

Figure 11: IO (immediate occupancy) and CP (collapse prevention) performance levels for the maximum bending moment (MBM) in the first numerical example: (a) response curves; (b) membership functions of median and 90th percentile

6.1.3.2. *Peak absolute acceleration.* Differently from what has been seen for the MBM, the MFs of the median and 90th fractile of the PAA follow approximately the same trend with the variation of the IM. The only exception is the case of the CP performance level, as both MFs are right-angled triangles, but the vertical side corresponds to the upper bound for the median, i.e. $PAA_{50,min}(IM) = PAA_{50,ref}(IM)$, and to the lower bound for the 90th fractile, i.e. $PAA_{90,min}(IM) = PAA_{90,ref}(IM)$. This is indeed an interesting result, as it shows that the deterministic assessment of an EDP can either be under- or over-conservative. Obviously, more refined results could be achieved using: *i*) more earthquake records for a given value of the IM; *ii*) more α -cuts.

6.1.3.3. *Peak displacement.* For the sake of completeness, the MFs of the median and 90th fractile of the PD have also been established, which follow a very similar trend as the MFs of the PAA. In this case, however, only the MF of the 90th fractile is a right-angled triangle, with the vertical side corresponding to lower bound, i.e. $PD_{90,min}(IM) = PD_{90,ref}(IM)$. It is

Table 7: Lower bound, reference value and upper bound of the median of the peak absolute acceleration (PAA) in the first numerical example

Performance level	$PAA_{50,min}$ [g]	$PAA_{50,ref}$ [g]	$PAA_{50,max}$ [g]	AR
O	0.25	0.26	0.28	0.11
IO	0.43	0.46	0.48	0.097
LS	0.60	0.65	0.67	0.099
CP	1.29	1.55	1.55	0.16

Table 8: Lower bound, reference value and upper bound of the 90th fractile of the peak absolute acceleration (PAA) in the first numerical example

Performance level	$PAA_{90,min}$ [g]	$PAA_{90,ref}$ [g]	$PAA_{90,max}$ [g]	AR
O	0.41	0.45	0.49	0.19
IO	0.88	0.99	0.99	0.12
LS	1.52	1.54	1.71	0.12
CP	4.07	4.07	5.31	0.31

444 interesting to note here how different EPDs for the same structure give rise
 445 to MFs with different shapes, and this is something that must be accounted
 446 for if one wants to properly quantify the likelihood of structural and non-
 447 structural failures and their consequences (or, better, their degree of belief).
 448 For instance, the analysis of Figures 11, 12 and 13 clearly show that, for
 449 the structure under consideration, adopting the reference values for the con-
 450 nections' fixity factors leads to progressively less conservative estimates of
 451 both PAA and PD when considering seismic events of increasing intensity
 452 and higher values of the response fractiles. The MBM, on the contrary, is
 453 not affected by this trend.

454 6.1.4. Design curves

455 Once the MFs of MBM, PAA and PD have been obtained, design curves can
 456 be established, as described in Section 5. The defuzzification of the MFs can
 457 be achieved, for instance, as a given percentile under their area, e.g. 95%;
 458 that is, for the Y th fractile of the generic EDP at a certain IM level, the

Figure 12: IO (immediate occupancy) and CP (collapse prevention) performance levels for the peak absolute acceleration (PAA) in the first numerical example: (a) response curves; (b) membership functions of median and 90th percentile

459 design value $EPD_{Y,des}(IM)$ satisfies the condition:

$$\frac{\int_{EDP_{min}(IM)}^{EPD_{Y,des}(IM)} \mu_{EPD_Y(IM)}(s) ds}{\int_{EDP_{min}(IM)}^{EPD_{max}(IM)} \mu_{EPD_Y(IM)}(s) ds} = 0.95, \quad (5)$$

460 where s denotes the integration variable used for the MF $\mu_{EPD_Y(IM)}$ of the
 461 IM-dependent EDP at its Y th fractile.

462 Figure 14 shows the comparisons between the design curves (thick lines)
 463 of MBM, PAA and PD obtained for the performance levels of IO (red) and
 464 CP (blue), along with their envelope (shadowed grey areas), which visually
 465 demonstrates the effects of the uncertainty associated with the fuzzy fixity
 466 factors. Figure 15 depicts the design curves obtained for all the four perfor-
 467 mance levels considered as part of this numerical application. As expected,
 468 the performance level of CP is always characterised by design curves with
 469 both higher median and larger dispersion than the design curves of the other
 470 three performance levels.

Table 9: Lower bound, reference value and upper bound of the median of the peak displacement (PD) in the first numerical example

Performance level	$PD_{50,min}$ [m]	$PD_{50,ref}$ [m]	$PD_{50,max}$ [m]	AR
O	0.0243	0.0245	0.0261	0.07
IO	0.0624	0.0631	0.0652	0.04
LS	0.192	0.202	0.205	0.06
CP	0.509	0.562	0.566	0.10

Table 10: Lower bound, reference value and upper bound of the 90th fractile of the peak displacement (PD) in the first numerical example

Performance level	$PD_{90,min}$ [m]	$PD_{90,ref}$ [m]	$PD_{90,max}$ [m]	AR
O	0.0266	0.0310	0.0318	0.17
IO	0.118	0.129	0.132	0.11
LS	0.457	0.479	0.514	0.12
CP	1.47	1.47	2.16	0.47

471 6.2. Case study #2: Pre-assembled modular pipe-rack

472 In order to validate the proposed procedure also with a real case-study struc-
473 ture, the seismic performance of a steel pipe-rack adapted from an actual
474 modular steel frame designed for a petrochemical plant has been analysed
475 (the application of the conventional PEER's PBD framework for the same
476 case-study structure can be found in Ref. [79]). The structure consists of a
477 pre-assembled rack (PAR), which is 12 m long, 8 m wide and 10 m tall, and
478 it is used to support process pipes and electrical trays at different level of
479 elevation (EL) (Figure 16(b)). The structure is made of hot-rolled sections of
480 ASTM A572 grade 50 steel, with thick-plate girders, which make the struc-
481 ture quite stiff. ASCE/SEI 7-10 [72] and AISC 360-05 [80] are the main
482 codes that have been used to design it. Link elements have been inserted in
483 each column-to-foundation and beam-to-column joint, with $\nu_{cf} = 0.15$ and
484 $\nu_{bc} = 0.70$ being the reference values for their respective fixity factors. The
485 latter might correspond to an end-plate connection, with or without column
486 stiffeners [2]. The resulting fundamental period of vibration in the direction

Figure 13: IO (immediate occupancy) and CP (collapse prevention) performance levels for the peak displacement (PD) in the first numerical example: (a) response curves; (b) membership functions of median and 90th percentile

being analysed is $T_{1,ref} = 0.22$ s.

Similar to the case of the first numerical example, triangular MFs have been assumed for the fuzzy fixity factors, considering bounds of $\pm 15\%$ with respect to the reference values. As shown in Figure 17(a) and (b), three α -cuts have been considered in this case, namely $\alpha = 0$, $\alpha = 0.5$ and $\alpha = 1$. Thus, $n_{STR} = 17$ combinations of the $n_d = 2$ fuzzy variables have been analysed (see Fig. 17(c)), which are listed in Table 11 along with the corresponding values of T_1 . Figure 18 shows the resulting MF, whose AR of 0.12 is 60% less than the AR of the fixity factors. Also in this case, thus, the choice of $IM = S_a(T_{1,ref})$ appears justified.

The same analyses as for the first numerical example have been carried out for the industrial modular structure. In a first stage, the hazard curve of Figure 19 has been obtained, assuming the same location, and the values of the spectral acceleration for a PoE in 50 years of 5, 10, $\dots$, 95% are listed in Table 12. Due to the a lower value of $T_{1,ref}$, the spectral accelerations of the pipe rack are higher than in the case of the first numerical example (see

(a)

(b)

(c)

Figure 14: Design curves for the IO (immediate occupancy) and CP (collapse prevention) performance levels in the first numerical example: (a) maximum bending moment (MBM); (b) peak absolute acceleration (PAA); (c) peak displacement (PD)

(a)

(b)

(c)

Figure 15: Design curves for the four performance levels in the first numerical example: (a) maximum bending moment (MBM); (b) peak absolute acceleration (PAA) (c) peak displacement (PD)

Figure 16: Industrial modular structures used as second numerical example: (a) 3D view; (b) elevation

Figure 17: Membership functions for the second numerical example: (a) beam-to-column connections; (b) column-to-foundations connections; (c) top view of the pyramidal function

Table 11: Combinations of the fixity factors in the second numerical example

Combination #	ν_{cf}	ν_{bc}	T_1 [s]
1	0.15	0.70	0.222
2	0.1275	0.70	0.225
3	0.1275	0.5950	0.237
4	0.1275	0.8050	0.214
5	0.15	0.5950	0.234
6	0.15	0.8050	0.212
7	0.1725	0.70	0.219
8	0.1725	0.5950	0.231
9	0.1725	0.8050	0.210
10	0.1387	0.70	0.227
11	0.1387	0.6475	0.217
12	0.1387	0.7525	0.223
13	0.15	0.6475	0.221
14	0.15	0.7525	0.229
15	0.1613	0.70	0.218
16	0.1613	0.6475	0.226
17	0.1613	0.7525	0.216

503 Table 3).

504 In a second stage, two EDPs have been considered, namely the maximum
505 bending moment (MBM) of the first floor beams and the peak absolute ac-
506 celerations (PAA) of a single-degree-of-freedom (SDoF) oscillator of period
507 $T_{1,ref}$ attached to the free end of the cantilever beams supporting the pipes.
508 For each IM level of the seismic hazard, and for every combination of the
509 fuzzy variables at each α -cut level of the input MFs, each EDP has been
510 characterised probabilistically in terms of its CCDF, that is $F[EDP|IM]$,
511 obtained by best-fitting a lognormal model with the results of the seismic
512 analyses (in total, $n_{IM} \times n_{STR} \times n_{EQ} = 10 \times 17 \times 7 = 1,190$ nonlinear
513 time-history analyses have been carried out).

514 In a third stage, for each EDP and each structural model combination,
515 the least square method has been used to find the optimal regression curves

Figure 18: Membership function of the fundamental period T_1 for the second numerical example

Figure 19: Hazard curve of the second numerical example

Table 12: IM levels for the hazard curve of the second numerical example

IM level	$S_a(T_{1,ref})$ [g]
I	0.192
II	0.268
III	0.329
IV	0.386
V	0.449
VI	0.518
VII	0.603
VIII	0.719
IX	0.897
X	1.34

Table 13: Probability of exceedance in 50 years for the four performance levels considered in the structural analysis, and corresponding spectral accelerations in the hazard curve for the second numerical example

Performance level	PoE_{50} [%]	$S_a(T_{1,ref})$ [g]
O	50	0.48
IO	20	0.78
LS	10	1.05
CP	2.0	2.19

516 which approximate the variation with the IM of the position and dispersion
 517 parameters of the lognormal model, allowing then to define the lognormal
 518 distributions for the pre-defined performance levels (namely, O, IO, LS and
 519 CP). For illustration purposes, the $n_{STR} = 17$ CCDFs of MBM and PAA
 520 for immediate occupancy (IO, subplots (a)) and collapse preventions (CP,
 521 subplots (c)) are displayed in Figures 20 and 21, respectively, along with a
 522 3D visualisation of the MFs of the median and 90th fractile (subplots (b) and
 523 (d)). Contrary to what has been observed with the first numerical example,
 524 the effects of the fuzziness in the steel connections affects the MBM more
 525 than the PAA.

526 Finally, the design curves for both MBM and PAA have been obtained,
 527 considering the 95% percentile of the area under their MFs. The design

Figure 20: IO (top plots) and CP (bottom plots) performance levels for the maximum bending moment (MBM) in the second numerical example: response curves (left plots) and membership functions (right plots) of median and 90th percentile

Figure 21: IO (top plots) and CP (bottom plots) performance levels for the peak absolute acceleration (PAA) in the second numerical example: response curves (left plots) and membership functions (right plots) of median and 90th percentile

Figure 22: Design curves for the IO (immediate occupancy) performance level in the second numerical example: (a) MBM; (b) PAA

curves at the IO and CP performance levels are shown in Figures 22 and 23, respectively, where the shadowed grey areas visualise the envelopes of the CCDF, confirming that for this numerical application the uncertainty in the fixity factors of the connections affects more the MBM than the PAA.

7. Conclusions

In this paper, a new performance-based fuzzy design (Pbfd) procedure has been presented for steel moment-resisting frames, considering the effects of different sources of uncertainty, namely aleatory randomness on the seismic demand and epistemic uncertainty on the semi-rigidity of both column-to foundation and beam-to-column connections. In particular, the non-deterministic behaviour of the connections has been modelled by means of fuzzy variables with a triangular membership function (MF) for their fixity factors. The proposed framework is an extended version of the classical PEER's performance-based design (PBD) approach, in which an additional stage has been introduced as part of the structural analysis, namely the fuzzy analysis, which allows characterising the MF of the engineering demand parameters (EDPs) of interest.

Figure 23: Design curves for the CP (collapse prevention) performance level in the second numerical example: (a) MBM; (b) PAA

545 The proposed approach has been applied to a planar steel frame and to an
 546 industrial 3D modular structure, exploiting a commercial structural analysis
 547 programme (SAP2000) within a general numerical computing environment
 548 (MATLAB). The results demonstrate that the proposed PBFD procedure
 549 provides a deeper insight into the expected seismic performance of the struc-
 550 tures being analysed, particularly if the effects of epistemic uncertainties are
 551 significant. This is indeed the case for industrial steel structures, in which
 552 the actual flexibility of the connections is very often overlooked, and in fact
 553 their detailing is routinely left to the steel fabricators. As the structural en-
 554 gineering team responsible for the main structural design of the steel frame
 555 typically has little or no information about the connections details that will
 556 be specified and realised by the fabricators, the adoption of fuzzy variables
 557 for the stiffness of the connections appears particularly appropriate.

558 Interestingly, it has been shown that using the reference values for the
 559 fixity factors of the steel connections deterministically, i.e. those for which the
 560 triangular MF is assumed to be equal to 1, can either under- or over-estimate
 561 the majority of the results obtained by varying the values of the fuzzy design
 562 variables within their domains of definition (i.e. zero α -cuts). Potentially,
 563 this has huge consequences in terms of risk and resilience assessment, that
 564 can be properly quantified with the proposed formulation.

565 It should also be noted that, since fuzzy structural analysis can be eas-
566 ily task-parallelised, a significant advantage exists in that the probabilistic
567 characterisation of the EDPs, potentially cumbersome from a computational
568 point of view, can be achieved concurrently for the various combinations of
569 the fuzzy model parameters. This significantly reduces the overall time for
570 the completion of the analyses.

571 Based on the available results, further research will be required on vari-
572 ous aspects of the procedure, particularly the optimal number of earthquake
573 records for each level of the seismic intensity measure (IM), the optimal num-
574 ber of α -cuts for the fuzzy design variables and the defuzzification method
575 to extract the design values from the MFs of the EDPs.

576 Although the focus in this paper has been on seismic hazard and stiffness
577 of the connection, the proposed fuzzified PBD framework can be applied to
578 different sources of hazards, including multi-hazard scenarios, and to different
579 design parameters, e.g. the strength and ductility of the connections, the
580 properties of the foundation soil, etcetera.

581 **Appendix A. Notation**

582 In this paper, the following key symbols and acronyms have been used:

583 *List of symbols*

DM	=	Damage measure;
DV	=	Decision variable;
E	=	Young's modulus;
$EDP_{i,Y}$	=	Y th fractile of the i th engineering demand parameter;
$F[\cdot]$	=	Cumulative distribution function;
I	=	Second moment of area;
k_c	=	Rotational stiffness of the semi-rigid connection;
l	=	Length of the steel member;
IM_h	=	h th value of the intensity measure for the seismic hazard;
M	=	Bending moment;
n_d	=	Number of fuzzy design variables;

n_{EQ}	=	Number of earthquake records for each intensity level;
n_{STR}	=	Number of structural model variations in the analyses;
n_α	=	Number of α -cut levels;
$p[\cdot]$	=	Probability density function;
$S_a(\cdot)$	=	Elastic response spectrum in terms of pseudo-accelerations;
T_1	=	Fundamental period of vibration;
x_{max}	=	Upper bound of the fuzzy variable x ;
x_{min}	=	Lower bound of the fuzzy variable x ;
x_{ref}	=	Reference value of the fuzzy variable x , for which $\mu(x_{ref}) = 1$;
$\mu(\cdot)$	=	Membership function;
ν	=	Fixity factor;
Π_m	=	m th statistical descriptor of a given probability distribution;
ϕ_c	=	Rotation in the semi-rigid connection.

584 *List of acronyms*

AR	=	Amplitude ratio;
CP	=	Collapse prevention performance level;
IO	=	Immediate occupancy performance level;
LS	=	Life safety performance level;
MBM	=	Maximum bending moment;
MF	=	Membership function;
O	=	Operational performance level;
PAA	=	Peak absolute acceleration;
PBD	=	Performance based design;
PBEE	=	Performance based earthquake engineering;
PD	=	Peak displacement;
PGA	=	Peak ground acceleration;
PEER	=	Pacific earthquake engineering research;
PL	=	Performance level;
PoE	=	Probability of exceedance.

585 **References**

- 586 [1] J. Davison, P. Kirby, D. Nethercot, Rotational stiffness characteristics
587 of steel beam-to-column connections, *Journal of Constructional Steel*
588 *Research* 8 (1987) 17–54.
- 589 [2] W.-F. Chen, *Semi-Rigid Connections Handbook*, J. Ross Publishing,
590 2011.
- 591 [3] M. Hadianfard, R. Razani, Effects of semi-rigid behavior of connections
592 in the reliability of steel frames, *Structural Safety* 25 (2) (2003) 123–138.
- 593 [4] K. M. Romstad, C. V. Subramanian, Analysis of frames with partial
594 connection rigidity, *ASCE Journal of the Structural Division* 96 (11)
595 (1970) 2283–2300.
- 596 [5] M. Barakat, W.-F. Chen, Practical analysis of semi-rigid frames, *Engi-*
597 *neering Journal* 27 (2) (1990) 54–68.
- 598 [6] S. Kawashima, T. Fujimoto, Vibration analysis of frames with semi-rigid
599 connections, *Computers & Structures* 19 (1-2) (1984) 85–92.
- 600 [7] N. Kishi, W.-F. Chen, Moment-rotation relations of semirigid connec-
601 tions with angles, *Journal of Structural Engineering* 116 (7) (1990) 1813–
602 1834.
- 603 [8] P. D. Moncarz, K. H. Gerstl, Steel frames with nonlinear connections,
604 *ASCE Journal of the Structural Division* 107 (8) (1981) 1427–1441.
- 605 [9] M. Ivanyi, C. C. Baniotopoulos, Eds., *Semi-Rigid Joints in Structural*
606 *Steelwork*, Springer, 2000.
- 607 [10] M. Sekulovic, R. Salatic, M. Nefovska, Dynamic analysis of steel frames
608 with flexible connections, *Computers & Structures* 80 (11) (2002) 935–
609 955.
- 610 [11] E. Bayo, J. Cabrero, B. Gil, An effective component-based method to
611 model semi-rigid connections for the global analysis of steel and com-
612 posite structures, *Engineering Structures* 28 (1) (2006) 97–108.

- 613 [12] J. Cabrero, E. Bayo, Development of practical design methods for steel
614 structures with semi-rigid connections, *Engineering Structures* 27 (8)
615 (2005) 1125–1137.
- 616 [13] H. Kobayashi, B. L. Mark, W. Turin, *Probability, Random Processes,
617 and Statistical Analysis*, Cambridge University Press, 2011.
- 618 [14] L. A. Zadeh, Fuzzy sets, *Information and Control* 8 (3) (1965) 338–353.
- 619 [15] G. Klir, B. Yuan, *Fuzzy sets and fuzzy logic*, Vol. 4, Prentice Hall New
620 Jersey, 1995.
- 621 [16] H. Kwakernaak, Fuzzy random variables. Definitions and theorems, *In-
622 formation Sciences* 15 (1) (1978) 1–29.
- 623 [17] M. L. Puri, D. A. Ralescu, Fuzzy random variables, *Journal of Mathe-
624 matical Analysis and Applications* 114 (2) (1986) 409–422.
- 625 [18] G. Wang, Y. Zhang, The theory of fuzzy stochastic processes, *Fuzzy
626 Sets and Systems* 51 (2) (1992) 161–178.
- 627 [19] N. D. Lagaros, Fuzzy fragility analysis of structures with masonry infill
628 walls, *Open Construction and Building Technology Journal* 6 (2012)
629 291–305.
- 630 [20] F. Colangelo, A simple model to include fuzziness in the seismic fragility
631 curve and relevant effect compared with randomness, *Earthquake Engi-
632 neering & Structural Dynamics* 41 (5) (2012) 969–986.
- 633 [21] F. Colangelo, Probabilistic characterisation of an analytical fuzzy-
634 random model for seismic fragility computation, *Structural Safety* 40
635 (2013) 68–77.
- 636 [22] J.-R. Huo, H. H. Hwang, Incorporation of fuzzy damage states in seismic
637 fragility analysis, in: *Probabilistic Mechanics & Structural Reliability*,
638 ASCE, 1996, pp. 318–321.
- 639 [23] A. Der Kiureghian, O. Ditlevsen, Aleatory or epistemic? Does it mat-
640 ter?, *Structural Safety* 31 (2) (2009) 105–112.

- 641 [24] J. Song, B. R. Ellingwood, Seismic reliability of special moment steel
642 frames with welded connections: I, Journal of Structural Engineering
643 125 (4) (1999) 357–371.
- 644 [25] J. Song, B. R. Ellingwood, Seismic reliability of special moment steel
645 frames with welded connections: II, Journal of Structural Engineering
646 125 (4) (1999) 372–384.
- 647 [26] O.-S. Kwon, A. Elnashai, The effect of material and ground motion
648 uncertainty on the seismic vulnerability curves of RC structure, Engi-
649 neering Structures 28 (2) (2006) 289–303.
- 650 [27] A. Kazantzi, D. Vamvatsikos, D. Lignos, Seismic performance of a steel
651 moment-resisting frame subject to strength and ductility uncertainty,
652 Engineering Structures 78 (2014) 69–77.
- 653 [28] S. Kasinos, Seismic Response Analysis of Linear and Nonlinear Sec-
654 ondary Structures, Ph.D. thesis, Loughborough University (2018).
- 655 [29] M. Mancini, Ed., Advances in plant modularisation: From the state of
656 art to emerging challenges, ANIMP Servizi SRL, 2014.
- 657 [30] A. Ghobarah, Performance-based design in earthquake engineering:
658 state of development, Engineering Structures 23 (8) (2001) 878–884.
- 659 [31] M. Tang, E. Castro, F. Pedroni, A. Brzozowski, M. Ettouney,
660 Performance-based design with application to seismic hazard, Structure
661 Magazine 15 (6) (2008) 20–22.
- 662 [32] K. A. Porter, An overview of PEER’s performance-based earthquake en-
663 gineering methodology, in: 9th International Conference on Applications
664 of Statistics and Probability in Civil Engineering, 2003.
- 665 [33] T.-H. Lee, K. M. Mosalam, Probabilistic seismic evaluation of reinforced
666 concrete structural components and systems, Tech. Rep. PEER 2006/04,
667 Pacific Earthquake Engineering Research Center (2006).
- 668 [34] H. Krawinkler, S. Mohasseb, Effects of panel zone deformations on seis-
669 mic response, Journal of Constructional Steel Research 8 (C) (1987)
670 233–250.

- 671 [35] S. P. Schneider, A. Amidi, Seismic behavior of steel frames with de-
672 formable panel zones, *Journal of Structural Engineering* 124 (1) (1998)
673 35–42.
- 674 [36] C. Mao, J. Ricles, L.-W. Lu, J. Fisher, Seismic behavior of steel frames
675 with deformable panel zones, *Journal of Structural Engineering* 127 (9)
676 (2001) 1036–1044.
- 677 [37] C. Díaz, P. Martí, M. Victoria, O. M. Querin, Review on the modelling of
678 joint behaviour in steel frames, *Journal of Constructional Steel Research*
679 67 (5) (2011) 741–758.
- 680 [38] Y. L. Yee, R. E. Melchers, Moment-rotation curves for bolted connec-
681 tions, *Journal of Structural Engineering* 112 (3) (1986) 615–635.
- 682 [39] R. P. Johnson, C. Law, Semi-rigid joints for composite frames, in: *Inter-
683 national Conference on Joints in Structural Steelwork*, Pentech Press,
684 London, 1981, pp. 3–3.
- 685 [40] M. J. Frye, G. A. Morris, Analysis of flexibly connected steel frames,
686 *Canadian Journal of Civil Engineering* 2 (3) (1975) 280–291.
- 687 [41] N. Krishnamurthy, Analytical investigation of bolted stiffened tee stubs,
688 in: *Report No. CE-MBMA-1902*, Department of Civil Engineering, Van-
689 derbilt University Nashville, Tennessee, USA, 1978.
- 690 [42] A. Kukreti, T. Murray, A. Abolmaali, End-plate connection moment-
691 rotation relationship, *Journal of Constructional Steel Research* 8 (1987)
692 137–157.
- 693 [43] E. P. Popov, S. M. Takhirov, Bolted large seismic steel beam-to-column
694 connections part 1: experimental study, *Engineering Structures* 24 (12)
695 (2002) 1523–1534.
- 696 [44] A. M. G. Coelho, F. S. Bijlaard, L. S. da Silva, Experimental assess-
697 ment of the ductility of extended end plate connections, *Engineering
698 Structures* 26 (9) (2004) 1185–1206.
- 699 [45] A. M. G. Coelho, F. S. Bijlaard, N. Gresnigt, L. S. da Silva, Experimen-
700 tal assessment of the behaviour of bolted t-stub connections made up
701 of welded plates, *Journal of Constructional Steel research* 60 (2) (2004)
702 269–311.

- 703 [46] C. Faella, V. Piluso, G. Rizzano, Structural steel semirigid connections:
704 theory, design, and software, Vol. 21, CRC Press, 1999.
- 705 [47] X. Dai, Y. Wang, C. Bailey, Numerical modelling of structural fire be-
706 haviour of restrained steel beam-column assemblies using typical joint
707 types, Engineering Structures 32 (8) (2010) 2337–2351.
- 708 [48] M. E. Lemonis, C. J. Gantes, Mechanical modeling of the nonlinear
709 response of beam-to-column joints, Journal of Constructional Steel Re-
710 search 65 (4) (2009) 879–890.
- 711 [49] M. Mohamadi-Shooreh, M. Mofid, Parametric analyses on the initial
712 stiffness of flush end-plate splice connections using fem, Journal of Con-
713 structural Steel Research 64 (10) (2008) 1129–1141.
- 714 [50] M. N. Jadid, D. R. Fairbairn, Neural-network applications in predict-
715 ing moment-curvature parameters from experimental data, Engineering
716 Applications of Artificial Intelligence 9 (3) (1996) 309–319.
- 717 [51] A. Cevik, Genetic programming based formulation of rotation capacity
718 of wide flange beams, Journal of Constructional Steel Research 63 (7)
719 (2007) 884–893.
- 720 [52] E. Salajegheh, S. Gholizadeh, A. Pirmoz, Self-organizing parallel back
721 propagation neural networks for predicting the moment-rotation behav-
722 ior of bolted connections, Asian Journal of Civil Engineering 9 (6) (2008)
723 625–640.
- 724 [53] F. G. Al-Bermani, S. Kitipornchai, Elastoplastic nonlinear analysis of
725 flexibly jointed space frames, Journal of Structural Engineering 118 (1)
726 (1992) 108–127.
- 727 [54] S. Kasinos, A. Palmeri, S. Maheshwari, M. Lombardo, Dynamic analysis
728 of steel frames with uncertain semi-rigid connections, in: 12th Interna-
729 tional Conference on Structural Safety & Reliability, Vienna, Austria,
730 2017.
- 731 [55] F. Biondini, F. Bontempi, P. G. Malerba, Fuzzy reliability analysis of
732 concrete structures, Computers & Structures 82 (13) (2004) 1033–1052.

- 733 [56] Z. Kala, Stability problems of steel structures in the presence of stochastic
734 and fuzzy uncertainty, *Thin-Walled Structures* 45 (10) (2007) 861–
735 865.
- 736 [57] G. C. Marano, G. Quaranta, Fuzzy-based robust structural optimization,
737 *International Journal of Solids and Structures* 45 (11) (2008) 3544–
738 3557.
- 739 [58] G. C. Marano, G. Quaranta, M. Mezzina, Fuzzy time-dependent reliability
740 analysis of RC beams subject to pitting corrosion, *Journal of*
741 *Materials in Civil Engineering* 20 (9) (2008) 578–587.
- 742 [59] B. Kosko, Fuzziness vs. probability, *International Journal of General*
743 *System* 17 (2-3) (1990) 211–240.
- 744 [60] D. Dubois, H. M. Prade, *Fuzzy Sets and Systems: Theory and Appli-*
745 *cations (Mathematics in Science & Engineering)*, Vol. 144, Academic
746 Press, 1980.
- 747 [61] H. Li, V. C. Yen, *Fuzzy Sets and Fuzzy Decision-Making*, CRC Press,
748 1995.
- 749 [62] W. Dong, H. C. Shah, Vertex method for computing functions of fuzzy
750 variables, *Fuzzy Sets and Systems* 24 (1) (1987) 65–78.
- 751 [63] SAP2000, version 17.3.0, Computers and Structures Inc., Berkeley, Cal-
752 ifornia, 2010.
- 753 [64] MATLAB, release R2017a, The MathWorks Inc., Natick, Massachusetts,
754 2010.
- 755 [65] M. Beer, S. Ferson, V. Kreinovich, Imprecise probabilities in engineering
756 analyses, *Mechanical Systems and Signal Processing* 37 (1-2) (2013) 4–
757 29.
- 758 [66] SEAOC, SEAOC Vision 2000 Committee. Performance-based seismic
759 engineering, in: *13th World Conference on Earthquake Engineering*,
760 1995.
- 761 [67] C. J. Wills, M. Petersen, W. A. Bryant, M. Reichle, G. J. Saucedo,
762 S. Tan, G. Taylor, J. Treiman, A site-conditions map for California

- 763 based on geology and shear-wave velocity, Bulletin of the Seismological
764 Society of America 90 (6B) (2000) S187–S208.
- 765 [68] N. Shome, C. Cornell, Probabilistic Seismic Demand Analysis of Non-
766 linear Structures, Tech. Rep. RMS–35, Stanford University (1993).
- 767 [69] E. H. Field, T. H. Jordan, C. A. Cornell, OpenSHA: A developing
768 community-modeling environment for seismic hazard analysis, Seismo-
769 logical Research Letters 74 (4) (2003) 406–419.
- 770 [70] T. D. Ancheta, R. B. Darragh, J. P. Stewart, E. Seyhan, W. J. Silva,
771 B. S.-J. Chiou, K. E. Wooddell, R. W. Graves, A. R. Kottke, D. M.
772 Boore, T. Kishida, J. L. Donahue, NGA-West2 database, Earthquake
773 Spectra 30 (3) (2014) 989–1005.
- 774 [71] EN 1998-1:2004 – Eurocode 8: Design of Structures for Earthquake Re-
775 sistance – Part 1: General Rules, Seismic Actions and Rules for Build-
776 ings, Standard, European Committee for Standardization (CEN), Brus-
777 sels, Belgium (2004).
- 778 [72] ASCE/SEI 7–10, Minimum Design Loads for buildings and Other Struc-
779 tures, Standard, ASCE, Reston, Virginia (2000).
- 780 [73] IBC 2012, Standard, International Code Council (ICC), Falls Church,
781 Virginia (2012).
- 782 [74] P. Cacciola, P. Colajanni, G. Muscolino, Combination of modal re-
783 sponses consistent with seismic input representation, Journal of Struc-
784 tural Engineering 130 (1) (2004) 47–55.
- 785 [75] I. Iervolino, G. Maddaloni, E. Cosenza, Eurocode 8 compliant real record
786 sets for seismic analysis of structures, Journal of Earthquake Engineering
787 12 (1) (2008) 54–90.
- 788 [76] D. Cecini, A. Palmeri, Spectrum-compatible accelerograms with har-
789 monic wavelets, Computers & Structures 147 (2015) 26–35.
- 790 [77] G. Barone, F. L. Iacono, G. Navarra, A. Palmeri, A novel analytical
791 model of power spectral density function coherent with earthquake re-
792 sponse spectra, in: First ECCOMAS Thematic Conference on Uncer-
793 tainty Quantification in Computational Sciences and Engineering, 2015.

- 794 [78] E. I. Katsanos, A. G. Sextos, Reliable selection of earthquake ground
795 motions for performance-based design, in: First International Confer-
796 ence on Natural Hazards & Infrastructure, 2016.
- 797 [79] A. de Luca di Roseto, A. Palmeri, A. G. Gibb, Performance-based seis-
798 mic design of a modular pipe-rack, *Procedia Engineering* 199 (2017)
799 3564–3569.
- 800 [80] AISC 360–05, Specification for Structural Steel Buildings, Standard,
801 AISC, Chicago, Illinois (2005).