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Performance Analysis of Multi-Antenna Selection Policies Using the Golden Code in MIMO Systems

Lu Ge, Gaojie Chen, Yu Gong, and Jonathon Chambers

Abstract

In MIMO systems, multiple-antenna selection has been proposed as a practical scheme for improving signal transmission quality as well as reducing realisation cost due to minimising the number of radio frequency chains. In this paper, we investigate transmit antenna selection for MIMO systems with the Golden Code. Two antenna selection schemes are considered: maximum-minimum and maximum-sum approaches. The outage and pairwise error probability performance of the proposed approaches are analyzed. Simulations are also given to verify the analysis. The results show the proposed methods provide useful schemes for antenna selection.

Index Terms

Antenna selection, outage probability, pairwise error probability, the Golden Code

I. INTRODUCTION

Multiple-input and multiple-output (MIMO) wireless communications increases spectrum efficiency by spatial multiplexing and improves link reliability by antenna diversity [1] and [2]. Multiple antenna systems, however, need multiple radio-frequency (RF) chains associated with each antenna. A MIMO system, with N_t transmission and N_r receiving antennas, requires N_t and N_r RF chains respectively. Each RF chain includes an analog-to-digital converter, down converter and a low-noise amplifier. This leads to a considerable increase in the cost and complexity of implementing such systems and represents a major practical drawback.

Antenna selection selects a subset of antennas to feed to the RF chains. The selection algorithm is based on the power of the received signals. This benefits diversity but not spatial multiplexing. A reduced-complexity MIMO scheme that selects the L_r best available N_r antennas is proposed in [2]. In [3], a way is presented to achieve capacity gain by transmit antenna selection for high SNR with a potentially large number of transmission antennas. The impact of antenna selection on the pairwise error probability (PEP) for space-time code systems is approximately analysed in [4]. In [5], theoretical

performance analysis including PEP for multi-antenna systems with antenna selection is presented, as well as PEP. Outage probability analysis and a practical algorithm for antenna selection in MIMO wireless communication systems employing space-time block codes (STBC) is given in [6]. The outage probability of multiuser diversity (MUD) in a transmit antenna selection system is derived, where the exact closed form expression for Nakagami-m channels with an integer fading parameter is obtained in [7]. Both single transmit and single receive antenna selection are examined for flat Nakagami-m fading channels in [8]. Based on two-way relay networks, [9] proposed two strategies for transmit and receive antenna selection, and their outage probability results revealed that the joint relay and antenna selection strategies achieve significant diversity and array gains over those of their single relay counterparts. In [10], it presents a unified asymptotic framework for transmit antenna selection in MIMO multirelay networks with Rician, Nakagami-m, Weibull, and generalized-K fading channels and derives new closed-form expressions for the outage probability and symbol error rate (SER) of amplify and-forward (AF) relaying in MIMO multirelay networks with two distinct protocols: transmit antenna selection with receiver maximal-ratio combining and transmit antenna selection with receiver selection combining.

In this paper, we analyse the outage probability and PEP of transmit antenna selection schemes for the Golden Code network. The Golden Code matrix is a full-rate, full diversity and full rank linear 2×2 linear dispersion algebraic space-time code for two transmit antennas and two or more receive antenna MIMO system [11]. We focus on selecting the best two transmit antennas from N participating antenna by using maximum-minimum and maximum-sum selection strategies. Furthermore, we compare the outage probability of these two approaches. Then, we derive an upper bound for the PEP and show the diversity order. Finally, simulations are used to verify the theoretical analysis.

The remainder of this paper is organized as follows. Section II, introduces the system model and the Golden Code. Section III proposes two transmit antenna selection scenarios: maximum-minimum selection and maximum-sum selection; moreover, we analyze and derive outage probability expressions for these two schemes. Section IV analyzes the PEP of the proposed antenna selection system. Section V shows simulation results, and Section VI concludes the paper.

II. SYSTEM MODEL

We adopt a point-to-point MIMO network as a basic transmission scenario for the Golden Code, as shown in Fig. 1. We assume one base station (BS) as the transmitter and one user equipment (UE) as a receiver; moreover, we consider a wireless network with N transmit antennas at the BS, and two receive

antennas at the UE. Every antenna is half-duplex, so that it cannot transmit and receive simultaneously. We denote channels coefficients from BS to UE as $h_{A_n D_k}$, where n represents the n -th antenna at the BS and k represents the k -th antenna at the UE. The signals are encoded at the BS with the Golden Code. On the other hand, antenna selection is a low-cost low-complexity alternative to capture many of the advantages of MIMO systems. We choose the best two transmit antennas as transmitters for our system. The antennas are selected that provide the highest equivalent receive SNR. Transmit antenna selection, unlike receive selection, requires a feedback channel from the receiver to the transmitter. This feedback rate is rather small. In this paper, we assume perfect channel state information (CSI) knowledge at the transmitter. The resulting performances are thus the best potential performance that we can achieved through antenna selection, which provides a good insight in understanding the system. We assume that the channels are quasi-static Rayleigh flat fading, which are independent identically distributed (i.i.d.) complex Gaussian random variables with zero-mean and unit-variance, i.e. $h_{A_n D_k} \sim \mathcal{CN}(0, 1)$. The instantaneous signal-to-noise ratio (SNR) for channel $h_{A_n D_k}$ is $\gamma_{A_n D_k} = |h_{A_n D_k}|^2 E_s / N_0$, where E_s is the average power per symbol and N_0 is the noise variance. We assume E_s is unity and the noise variances are the same in all antennas at the destination. The total CSI is assumed known at the destination node. We next introduce the Golden Code matrix. In our system, the Golden Code is used to encode the 4-QAM (quadrature

Fig. 1. The system model of antenna selection at the transmitter, i.e. the i -th and j -th Antenna are selected.

amplitude modulation) signals for transmission. The Golden Code is a full-rate and full-diversity linear space-time code based on the Golden Number [12]. It is most appropriate for the 2×2 MIMO system. The properties of the Golden Code include non-vanishing determinant for increasing rate and achieving

the diversity and multiplexing tradeoff (DMT) [13]. The codeword is formed as

$$\mathbf{C} = \frac{1}{\sqrt{5}} \begin{bmatrix} \alpha(s_1 + s_2\theta) & \alpha(s_3 + s_4\theta) \\ \mu\bar{\alpha}(s_3 + s_4\bar{\theta}) & \bar{\alpha}(s_1 + s_2\bar{\theta}) \end{bmatrix} = \frac{1}{\sqrt{5}} \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}, \quad (1)$$

where s_1, s_2, s_3, s_4 describe the information symbol constellation; $\theta = \frac{1+\sqrt{5}}{2}$, $\bar{\theta} = \frac{1-\sqrt{5}}{2}$, $\alpha = 1 + \beta(1 - \theta)$ and $\bar{\alpha} = 1 + \beta(1 - \bar{\theta})$, and β is set as $\sqrt{-1}$, $|\mu|$ is set to unity to satisfy the non-vanishing determinant and ensures the same average power is transmitted [11]. The information bits are respectively encoded with the Golden Code matrix (1). Sending a Golden codeword needs two time slots. We list the transmission process as below if the best two transmit antennas are selected. The signal received at the UE k th antenna at the first time slot, $t = 1$,

$$d_k^1 = \sqrt{\frac{P}{2}}(h_{A_i D_k} x_1 + h_{A_j D_k} x_2) + n_k \quad (2)$$

and at the second time slot, $t = 2$

$$d_k^2 = \sqrt{\frac{P}{2}}(h_{A_i D_k} x_3 + h_{A_j D_k} x_4) + n_k \quad (3)$$

where $\frac{P}{2}$ is the transmitted energy at every antenna, n_k is additive white Gaussian noise (AWGN) with variances σ_k^2 . At the receiver, we use maximum-likelihood decoding to estimate the original signal

$$\arg \min_{\mathbf{x} \in \mathbf{S}_c} \{\|\mathbf{d} - \hat{\mathbf{H}}\mathbf{x}\|^2\}, \quad (4)$$

where $\mathbf{S}_c$ denotes the collection of possible symbol constellation points and $\|\cdot\|$ denotes the Euclidean norm. $\hat{\mathbf{H}}$ is the selected channel matrix which contains $h_{A_i D_k}$ and $h_{A_j D_k}$, for example

$$\hat{\mathbf{H}} = \begin{bmatrix} h_{A_1 D_1} & h_{A_2 D_1} & 0 & 0 \\ h_{A_1 D_2} & h_{A_2 D_2} & 0 & 0 \\ 0 & 0 & h_{A_1 D_1} & h_{A_2 D_1} \\ 0 & 0 & h_{A_1 D_2} & h_{A_2 D_2} \end{bmatrix} \quad (5)$$

$\mathbf{d}$ is the received signal vector which takes the form

$$\mathbf{d} = [d_1^1 \quad d_2^1 \quad d_1^2 \quad d_2^2]^T. \quad (6)$$

Next, we analyse the outage probability of the antenna selection scheme.

III. MULTI-ANTENNA SELECTION WITH OUTAGE PROBABILITY ANALYSIS

We perform outage probability analysis for the two antenna selection schemes. In the outage probability analysis of the direct link, the source is assumed to transmit directly its signal to the destination node; C_d denotes the channel capacity, and for a target rate R , $C_d < R$ and $P(C_d < R)$ denote the outage event and outage probability, respectively. The total end-to-end signal-to-noise ratio (SNR) at the UE in our system is

$$\gamma_d = \frac{|h_{A_i D_1}|^2 + |h_{A_i D_2}|^2 + |h_{A_j D_1}|^2 + |h_{A_j D_2}|^2}{\sigma_k^2}. \quad (7)$$

Therefore, the capacity between source and destination is given by

$$C_d = \log_2(1 + \gamma_d). \quad (8)$$

The multi-antenna selection chooses the best two antennas that maximize the SNR at the transmitter. Two antenna selection schemes are considered in this paper: the maximum-minimum and maximum-sum antenna selection, respectively.

A. Maximum-minimum selection

Because the Golden Code requires two antennas at the transmitter, the aim is to select the best two transmit antennas from N available antennas. From the traditional maximum-minimum selection, the best two links can be selected as:

$$(i, j) = \arg \max_{i \in N} \max_{j \in N-1} \{\min(\gamma_{A_n D_1}, \gamma_{A_n D_2})\}, \quad (9)$$

where i indexes the best antenna, and j the second best, $n \in (1, 2, \dots, N)$ is the participating channel index. The final SNR with the selected i and j are given by (7), which is clearly lower-bounded as

$$\gamma_d \geq 2 \max_{i \in N} \{\min(\gamma_{A_n D_1}, \gamma_{A_n D_2})\} + 2 \max_{j \in N-1} \{\min(\gamma_{A_n D_1}, \gamma_{A_n D_2})\}. \quad (10)$$

For all Rayleigh flat-fading channels, the probability density function (PDF) and the cumulative distribution function (CDF) of the SNR are given by

$$\begin{aligned} f_{\gamma_v}(\gamma) &= \frac{1}{\bar{\gamma}_v} e^{-\frac{\gamma}{\bar{\gamma}_v}} \\ F_{\gamma_v}(\gamma) &= 1 - e^{-\frac{\gamma}{\bar{\gamma}_v}}, \end{aligned} \quad (11)$$

where $\gamma_v \in (\gamma_{A_n D_1}, \gamma_{A_n D_2})$, $\gamma > 0$ and $\bar{\gamma}_v$ is the average mean SNR of all links. We assume the channels are i.i.d. and $\bar{\gamma}_{A_n D_1} = \bar{\gamma}_{A_n D_2} = \bar{\gamma}_v$. Thus, the CDF of $\min(\gamma_{A_n D_1}, \gamma_{A_n D_2})$ can be expressed as

$$\begin{aligned} F(\gamma) &= 1 - (P_r(\gamma_{A_n D_1} > \gamma))(P_r(\gamma_{A_n D_2} > \gamma)) \\ &= 1 - (1 - P_r(\gamma_{A_n D_1} \leq \gamma))(1 - P_r(\gamma_{A_n D_2} \leq \gamma)) \\ &= 1 - (1 - F_{\gamma_{A_n D_1}}(\gamma))(1 - F_{\gamma_{A_n D_2}}(\gamma)) \\ &= 1 - e^{-\frac{2\gamma}{\bar{\gamma}_v}}. \end{aligned} \quad (12)$$

Then, the PDF of γ_v can be obtained as

$$f(\gamma_v) = \frac{dF(\gamma)}{d(\gamma)} = \frac{2}{\gamma_v} e^{-\frac{2\gamma_v}{\bar{\gamma}_v}}. \quad (13)$$

In this approach, the best two antennas are selected with the largest two SNRs among all N available transmitted antennas. According to [14], the joint distribution of the L largest values from N candidates can be obtained as

$$f(x_1, x_2, \dots, x_L) = L! \binom{N}{L} [F(x_L)]^{N-L} \prod_{i=1}^L f(x_i), \quad (14)$$

where $x_1 \geq x_2 \geq \dots \geq x_L \geq \dots \geq x_N$. Substituting $L = 2$ into (14) given the joint PDF of the two largest SNRs as

$$f((\gamma_i, \gamma_j)) = N(N-1)F(\gamma_j)^{N-2}f(\gamma_i)f(\gamma_j), \quad (15)$$

Substituting (12) into (15), gives

$$f(\gamma_i, \gamma_j) = N(N-1) \sum_{u=0}^{N-2} \binom{N-2}{u} (-e^{-\frac{2\gamma_j}{\bar{\gamma}_v}})^u \frac{2}{\bar{\gamma}_v} e^{-\frac{\gamma_i}{\bar{\gamma}_v}} \frac{2}{\bar{\gamma}_v} e^{-\frac{\gamma_j}{\bar{\gamma}_v}}. \quad (16)$$

Let $\gamma_{low} = \gamma_i + \gamma_j$, the CDF $F_{\gamma_{low}}(\gamma)$ is obtained as

$$F_{\gamma_{low}}(\gamma) = Pr\{\gamma_i + \gamma_j \leq \gamma\} \\ = \int_0^{\frac{\gamma}{2}} \int_y^{\gamma-y} f(x, y) dx dy = 2N(N-1) \int_0^{\frac{\gamma}{2}} \frac{\left(-e^{\frac{2y}{\bar{\gamma}_v}} + 1\right)^{N-2} \left(\frac{1}{e^{\frac{4y}{\bar{\gamma}_v}}} - e^{-\frac{2\gamma}{\bar{\gamma}_v}}\right)}{\bar{\gamma}_v^2} dy. \quad (17)$$

According to the definition of outage probability, an outage occurs when the average end-to-end SNR falls below a certain threshold value γ_{th} , namely, according to (10), target SNR $\gamma_{th} = (2^R - 1)/2$. For any N and γ_v , (17) can be easily obtained numerically with, for example Matlab or Maple [15]. The outage probability can be expressed as

$$P_{out} = F_{\gamma_{low}}(\gamma_{th}). \quad (18)$$

B. Maximum-sum selection

In this section, we propose the maximum-sum selection strategy, and obtain the outage probability for each transmit antenna, we obtain the sum SNRs from the transmit antenna to the two receive antennas; and choose two antennas with the two largest sum SNRs.

$$(i, j) = \arg \max_{i \in N} \max_{j \in N-1} \{\gamma_{A_n D_1} + \gamma_{A_n D_2}\}. \quad (19)$$

The PDF of the sums $\gamma_\mu = \gamma_{A_n D_1} + \gamma_{A_n D_2}$ can be expressed as

$$f_{\gamma_\mu}(\gamma) = \frac{\gamma}{\bar{\gamma}_\mu^2} e^{-\frac{\gamma}{\bar{\gamma}_\mu}}, \quad (20)$$

where $\bar{\gamma}_\mu$ is the average mean SNR of $\gamma_{A_n D_1} + \gamma_{A_n D_2}$. Then the CDF of γ_μ is

$$F(\gamma) = 1 - \frac{\gamma}{\bar{\gamma}_\mu} e^{-\frac{\gamma}{\bar{\gamma}_\mu}} - e^{-\frac{\gamma}{\bar{\gamma}_\mu}} = 1 - e^{-\frac{\gamma}{\bar{\gamma}_\mu}} \left(1 + \frac{\gamma}{\bar{\gamma}_\mu}\right). \quad (21)$$

And substituting (20) and (21) to (15) gives the PDF of final SNR. Then the CDF of the final SNR can be obtained as

$$F_{\gamma_e}(\gamma) = \int_0^{\frac{\gamma}{2}} \int_y^{\gamma-y} N(N-1) \cdot \sum_{u=0}^{N-2} \binom{N-2}{u} (-1)^u e^{-\frac{yu}{\bar{\gamma}_\mu}} \sum_m^u \binom{u}{m} \cdot \left(\frac{y}{\bar{\gamma}_\mu}\right)^m \frac{xy}{\bar{\gamma}_\mu^4} e^{-\frac{x+y}{\bar{\gamma}_\mu}} dx dy \\ = \int_0^{\frac{\gamma}{2}} \frac{\xi \cdot \eta \cdot \varphi}{\omega} dy, \quad (22)$$

where

$$\begin{aligned}\xi &= N(N-1)y, \\ \eta &= \sum_{a=0}^N \binom{N}{a} (-1)^a e^{-\frac{ya}{\bar{\gamma}_\mu}} \cdot \sum_{b=0}^a \binom{a}{b} \left(\frac{y}{\bar{\gamma}_\mu}\right)^b, \\ \varphi &= -y - \bar{\gamma}_\mu + e^{\frac{2y-\gamma}{\bar{\gamma}_\mu}} (\bar{\gamma}_\mu + \gamma - y), \\ \omega &= -\bar{\gamma}_\mu y^2 - 2y\bar{\gamma}_\mu^2 + 2y\bar{\gamma}_\mu^2 e^{\frac{y}{\bar{\gamma}_\mu}} - \bar{\gamma}_\mu^3 + 2\bar{\gamma}_\mu^3 e^{\frac{y}{\bar{\gamma}_\mu}} - e^{\frac{2y}{\bar{\gamma}_\mu}} \bar{\gamma}_\mu^3.\end{aligned}$$

Finally, using the same target SNR $\gamma_{th} = 2^R - 1$, we can obtain the exact outage probability of the max-sum selection scheme. Similarly, for any N and γ_μ , the outage can be easily obtained by using a numerical method.

IV. PEP ANALYSIS OF TRANSMITTER ANTENNA SELECTION WITH THE GOLDEN CODE

In this section, we analyse the PEP over flat fading channels and investigate the diversity order of the Golden Code. For the purpose of better analysis, the $2 \times N$ channel is written in form of the following matrix

$$\mathbf{H} = \begin{bmatrix} h_{A_1 D_1} & h_{A_2 D_1} & \cdots & h_{A_N D_1} \\ h_{A_1 D_2} & h_{A_2 D_2} & \cdots & h_{A_N D_2} \end{bmatrix}. \quad (23)$$

The antenna selection can be described as choosing the two columns with the highest norm as the best transmitted antennas. First, we need to find the joint PDF of the largest two norms. In [5] the joint PDF of the selected $\tilde{\mathbf{H}}$ can be written as

$$f(\mathbf{h}_i, \mathbf{h}_j) = \frac{N!}{(N-2)!2} \left(\sum_{l=1}^2 (1 - e^{-\|\mathbf{h}_l\|^2} - e^{-\|\mathbf{h}_l\|^2} \|\mathbf{h}_l\|^2)^{N-2} I_{R_l}(\mathbf{h}_i, \mathbf{h}_j) \right) \frac{e^{-(\|\mathbf{h}_i\|^2 + \|\mathbf{h}_j\|^2)}}{\pi^4}, \quad (24)$$

where, $\mathbf{h}_i$ and $\mathbf{h}_j$ are the largest and the second largest norm, let $I(\mathbf{h}_i, \mathbf{h}_j)$ be the indicator function. If $(\mathbf{h}_i, \mathbf{h}_j) \in R_l$, $I_{R_l}(\mathbf{h}_i, \mathbf{h}_j)$ is 1 and else equals zero; l denotes the column in $\mathbf{h}_l$. R_l is a region $\{\mathbf{h}_1, \dots, \mathbf{h}_L : \|\mathbf{h}_l\| < \|\mathbf{h}_c\|, c = 1, \dots, l-1, l+1, \dots, L\}$. The PEP can be upper bounded by using the Chernoff bound [16],

$$P(\mathbf{X} \rightarrow \hat{\mathbf{X}}) \leq e^{-\frac{\rho}{8} \|\mathbf{H}\mathbf{E}\|^2}, \quad (25)$$

where $\mathbf{E} = \mathbf{X} - \hat{\mathbf{X}}$ is the code error matrix. $\mathbf{X}$ is the transmitted code matrix in (1) and $\hat{\mathbf{X}}$ is the expected receiving code matrix, $\|\cdot\|^2$ represents the sum of magnitude squares of all entries of a matrix, i.e. the squared Frobenius norm, ρ is the expected SNR at the receive antenna. Using the new selected channel

matrix $\tilde{\mathbf{H}}$ of matrix $\mathbf{H}$ with all channels, it follows that the averaged upper bound from (24) is

$$P(\mathbf{X} \rightarrow \hat{\mathbf{X}}) \leq \sum_{l=1}^2 \int \int_{R_l} e^{-\frac{\rho}{8} \|\tilde{\mathbf{H}}\mathbf{E}\|^2} \frac{N!}{(N-2)!2} (1 - e^{-\|\mathbf{h}_l\|^2} - e^{-\|\mathbf{h}_l\|^2} \|\mathbf{h}_l\|^2)^{N-2} \frac{e^{-\sum_{c=1}^{L=2} \|\mathbf{h}_c\|^2}}{\pi^4} d\mathbf{h}_i d\mathbf{h}_j. \quad (26)$$

One property of the Golden Code is a full rank space time code, so the eigenvalues of the matrix $\mathbf{E}\mathbf{E}^*$ are nonzero, where $(\cdot)^*$ is the Hermitian of matrix $\mathbf{E}$. We can simplify the PEP bound by using

$$\|\tilde{\mathbf{H}}\mathbf{E}\|^2 = \text{trace}((\tilde{\mathbf{H}}\mathbf{U})\mathbf{\Lambda}(\tilde{\mathbf{H}}\mathbf{U})^*) = \sum_{z=1}^2 \lambda_z \|\mathbf{h}_z\|^2, \quad (27)$$

where $\mathbf{U}$ is a unitary matrix and $\mathbf{\Lambda}$ is a diagonal matrix with eigenvalues of $\mathbf{E}\mathbf{E}^*$. We denote the minimum of λ_z as $\tilde{\lambda}$, where $z = 1, \dots, L$. The upper bound PEP can thus be rewritten as

$$P(\mathbf{X} \rightarrow \hat{\mathbf{X}}) \leq \sum_{l=1}^2 \int \int_{R_l} e^{-\frac{\rho}{8} \sum_{i=1}^2 \tilde{\lambda} \|\mathbf{h}_z\|^2} \frac{N!}{(N-2)!2} (1 - e^{-\|\mathbf{h}_l\|^2} - e^{-\|\mathbf{h}_l\|^2} \|\mathbf{h}_l\|^2)^{N-2} \frac{e^{-\sum_{c=1}^{L=2} \|\mathbf{h}_c\|^2}}{\pi^4} d\mathbf{h}_i d\mathbf{h}_j. \quad (28)$$

As in [5], for optimizing (28), we can use

$$g(v) = 1 - e^{-v} \sum_{n=0}^{N-1} \frac{v^n}{n!} \leq \frac{v^N}{N!} \quad (29)$$

where $v > 0$. Therefore, (28) yields the further simplified expression as

$$Q \leq \frac{N!}{(N-2)!2} \int_{R_l} e^{-\frac{\rho}{8} \sum_{i=1}^2 \tilde{\lambda} \|\mathbf{h}_z\|^2} \left(\frac{\|\mathbf{h}_l\|^4}{2} \right)^{N-2} \frac{e^{-(\|\mathbf{h}_i\|^2 + \|\mathbf{h}_j\|^2)}}{\pi^4} d\mathbf{h}_i d\mathbf{h}_j. \quad (30)$$

If we use an exponential form to represent h_{nl} , $h_{nl} = a_{nl}e^{b_{nl}}$, and set $c_{nl} = a_{nl}^2$, where $\|\mathbf{h}_l\|^2 = \sum_{n=1}^2 c_{nl}$, while $dh_{nl} = a_{nl}da_{nl}db_{nl}$ and take the integral with respect to db over $[0, 2\pi]$, therefore, we can obtain

$$Q \leq \frac{N!}{(N-2)!2} \int_0^\infty \dots \int_0^\infty e^{-\frac{\rho}{8} \tilde{\lambda} (c_{11}+c_{21}+c_{12}+c_{22})} \cdot \left[\frac{(c_{1l} + c_{2l})^2}{2} \right]^{N-2} e^{-(c_{11}+c_{21}+c_{12}+c_{22})} dc_{11}dc_{21}dc_{12}dc_{22} \quad (31)$$

A looser upper bound occurs during evaluating the integral throughout the whole space. We can rewrite the upper bound of Q as $Q \leq Q_{1^{(l)}} Q_{2^{(l)}}$, where

$$Q_{1^{(l)}} = \frac{N!}{(N-2)!2} \int_0^\infty \int_0^\infty e^{-\frac{\rho}{8} \tilde{\lambda} \sum_{i=1, i \neq l}^L \sum_{n=1}^N c_{ni}} \cdot e^{-(\sum_{i=1, i \neq l}^L \sum_{n=1}^N c_{ni})} \prod_{i=1, i \neq l}^L \prod_{n=1}^N dc_{ni} \quad (32)$$

and

$$Q_{2^{(l)}} = \int_0^\infty \int_0^\infty e^{-(\frac{\rho}{8} \tilde{\lambda} + 1)(c_{1l} + c_{2l})} \left[\frac{(c_{1l} + c_{2l})^2}{2} \right]^{N-2} \cdot dc_{1l}dc_{2l}.$$

According to $\int_0^\infty e^{-\varepsilon x} dx = \frac{1}{\varepsilon}$,

$$Q_{1(l)} = \frac{N!}{(N-2)!2} \left[\frac{1}{(1 + \frac{\rho\tilde{\lambda}}{4})(1 + \frac{\rho\tilde{\lambda}}{8})} \right]^2. \quad (33)$$

On the other hand, we can denote c_{nl} as p_n and then have

$$\left(\sum_{n=1}^2 p_n \right)^{2(N-2)} = \sum_{n_1=1}^2 \cdots \sum_{n_{2(N-2)}=1}^2 p_{n_1} \cdots p_{n_{2(N-2)}}, \quad (34)$$

where $p_{n_1} \cdots p_{n_{2(N-2)}} = \prod_{n=1}^2 (p_n)^{l_n}$. Hence, $\sum_{n=1}^2 l_n = 2(N-2)$. We can obtain

$$Q_{2(l)} = \left(\frac{1}{2} \right)^{N-2} \int_0^\infty \int_0^\infty e^{-\sum_{n=1}^2 (\frac{\rho\tilde{\lambda}}{8} + 1)p_n} \sum_{n_1=1}^2 \cdots \sum_{n_{2(N-2)}=1}^2 (p_1)^{l_1} (p_2)^{l_2} dp_1 dp_2. \quad (35)$$

Using $\int_0^\infty x^m e^{-ax} dx = \frac{m!}{a^{m+1}}$, we get

$$Q_{2(l)} = \left(\frac{1}{2} \right)^{N-2} \sum_{n_1=1}^2 \cdots \sum_{n_{2(N-2)}=1}^2 \frac{l_1! l_2!}{(\frac{\rho\tilde{\lambda}}{8} + 1)^{l_1+1} (\frac{\rho\tilde{\lambda}}{8} + 1)^{l_2+1}}. \quad (36)$$

At the high SNRs, therefore,

$$P(\mathbf{X} \rightarrow \hat{\mathbf{X}}) \leq \frac{N!}{(N-2)!2^{N-1}} \left(\frac{1}{\tilde{\lambda}^{2N}} \right) \left(\sum_{n_1=1}^2 \cdots \sum_{n_{2(N-2)}=1}^2 l_1! l_2! \right) \left(\frac{\rho}{8} \right)^{-2N}. \quad (37)$$

In general, the diversity order is $N \times M$, which M is the number of receive antennas. From (37), we know the diversity order is $2N$ for the full diversity system, and $\tilde{\lambda}$ is nonzero due to fact that the Golden Code is a full rank code. Thereby, maximizing $\tilde{\lambda}$ can design a code useful for transmit antenna selection.

V. SIMULATION RESULTS

In this section, in order to verify the above mathematical expressions, we simulate the outage events of the antenna selection scheme with the maximum-minimum and maximum-sum selection scenarios using the Golden Code. In addition, the simulation results of the PEP analysis are provided based on the antenna selection scheme. We assume 4, 6 and 8 participating transmitted antenna. Four different symbols would be transmitted in two time slots by the Golden Code. Therefore, we let the target rate equal to 2. All curves denote the simulation case and all points correspond to theoretical values.

Fig. 2 shows the simulation and theoretical upper bound of the outage probability based on antenna selection with maximum-minimum selection. It can be seen that with more participating transmit antennas, the outage probability becomes smaller, for example, with the total number of available antennas increased

Fig. 2. The outage probability of the best two transmit antennas with the maximum-minimum selection.

from 6 to 8, the target SNR of the best two antenna selection is decreased from approximately 7.6 dB to 6.8 dB when the probability is 0.1. When we choose the best two from 6 antennas, the outage probability upper bound is 0.189 at 7 dB target SNR, while in the simulation outage is 0.023 in the simulation.

Fig. 3 shows the outage probability of the best two antennas selection scheme with maximum-sum scenario. As is shown, when the number of participating transmit antennas is increased from 6 to 8, the outage probability still decreases. The mathematical analysis results well match the simulation results. It is also shown that the maximum-sum selection outperforms maximum-minimum selection.

Fig. 4 presents the upper bound PEP for the system with selecting the best two from 4 and 6 participating transmit antennas and fixed two receive antennas. The Golden Code provides full rank and full diversity. We observe that when $N = 4$ and $N = 6$, the diversity order is 8 and 12. The Chernoff upper bound and the simulated PEP are very tight at high SNR. These results verify that full diversity can be achieved when full rank codes are used.

VI. CONCLUSION

We have examined best two transmit antenna selection for the Golden Code in a MIMO system with instantaneous channel conditions by using maximum-minimum and maximum-sum selection. Mathematical derivation and analysis of the PDF and CDF of end-to-end SNR were performed for Rayleigh

Fig. 3. The outage probability of the best two transmit antennas using the maximum-sum selection.

fading channels. The numerical results presented the outage probability based on the different participating transmit antennas and the outage events of antenna selection for a MIMO system using maximum-sum selection is shown to outperform the maximum-minimum selection. We derived the PEP analysis for maximum-minimum transmit antenna selection within the Golden Code and obtained the diversity order. The result confirms that full diversity can be achieved by the full rank Golden Code.

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Fig. 4. Numerical results of PEP of the transmit antennas using the Golden Code.

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