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Power shrinkage—curvelet domain image denoising using a new scale-dependent shrinkage function

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Power Shrinkage – Curvelet Domain Image Denoising using a New Scale Dependent Shrinkage Function

Oussama Kadri · Zine-Eddine Baarir · Gerald Schaefer

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Abstract Image processing and analysis algorithms are at the heart of applications in various scientific fields, such as medical diagnosis, military imaging, and astronomy. However, images are typically exposed to noise contamination during their acquisition and transmission. In this paper, we explore recent advancements in image denoising using curvelet domain shrinkage and present a novel scale dependent shrinkage function, which we call *power shrinkage*, to enhance restored image quality. Experimental results confirm our proposed method to perform better than classical thresholding and to outperform recent state-of-the-art approaches in denoising different types of noise including speckle, Poisson and additive white Gaussian noise.

Keywords image noise · image quality · image denoising · curvelet shrinkage · power shrinkage

1 Introduction

Image denoising deals with the problem of recovering an original image from a noisy and corrupted observation. Whether during acquisition or transmission, images are always exposed to noise contamination, which is often assumed to have an additive Gaussian distribution. Therefore, effective image denoising techniques are essential to allow better information processing and interpretation. One of the main challenges in image de-

noising is to effectively remove noise while at the same time preserving image details.

Denoising methods based on transform domain shrinkage have received significant attention in recent years [13]. Among these transforms, the curvelet transform [6] is particularly interesting due to its high spatial and directional selectivity which allows better representation of image features such as edges and contours. Applications of curvelet-based image denoising include seismic exploration [12], remote sensing [16], and astronomical imaging [2].

The performance of image denoising using curvelet domain shrinkage methods relies mainly on the optimisation of the shrinkage function and threshold estimation procedure. The threshold is considered as the limit that divides the transform domain coefficients into two groups, with coefficients below the threshold considered mostly generated by image noise, while those above the threshold are associated mainly with the original noiseless image. Crucially, in natural images, valuable image details often fall in the region around the threshold, and a hard threshold is therefore not optimal. As an alternative, classical thresholding can be replaced by more sophisticated shrinkage functions to optimise the overall process [9, 14, 19].

In this paper, we propose a novel shrinkage function to enhance denoising performance and improve image quality. Compared to existing curvelet domain shrinkage methods, our proposed *power shrinkage* function has better denoising capabilities for images with higher concentrations of textures and geometric regularities. In a set of experiments, we demonstrate that it outperforms state-of-the-art methods for several types of noise, proving that better processing of curvelet domain coefficients near the threshold leads to improved image denoising performance.

Oussama Kadri · Zine-Eddine Baarir
LESIA Laboratory of research, Electrical Engineering Department, Mohammed Khider University, Biskra, Algeria
E-mail: kadrioussama86@gmail.com, zinedinebaarir@gmail.com

Gerald Schaefer
Department of Computer Science, Loughborough University, Loughborough, U.K. E-mail: gerald.schaefer@ieee.org

The remainder of the paper is organised as follows. Section 2 introduces the fast discrete curvelet transform and its main characteristics, while Section 3 gives an overview of the recent state-of-the-art of curvelet domain shrinkage functions. In Section 4, we present our new scale dependent power shrinkage function. Experimental results are presented and discussed in Section 5, and Section 6 concludes the paper.

2 Curvelet Transform

The curvelet transform is a multi-resolution and multi-directional transform that was developed to overcome the lack in directionality of classical wavelets. It is obtained by projecting the input signal on a set of analysing bases, indexed by scale, location and direction, with the width and length of their supports linked by a parabolic scaling law ($\text{width} \approx \text{length}^2$). The fast discrete curvelet transform (FDCTG2) [4] is constructed by tiling the frequency domain with concentric squares and shears. Curvelet coefficients are obtained by applying the inverse Fourier transform on the sheared grid indexed by scale, location and direction.

The curvelet transform provides a near optimal non-adaptive representation for objects with edges [7] due to the elongated needle-shaped form and high spatial and frequency selectivity of its basis. It also has two important features which are useful for image denoising. First, it transforms additive white Gaussian noise (AWGN) in an image into additive correlated Gaussian noise in each sub-band of the transform domain [17]. Second, the FDCTG2 corresponds to a tight frame, and hence the standard deviation can be estimated by calculating the redundancy factor of the transform.

Based on these, Donoho and Starck [17] proposed the first curvelet-based denoising approach in which they adopted a scale-dependent threshold given by

$$th = k\sigma\sigma_j, \quad (1)$$

where σ_j represents the individual variances calculated using Monte-Carlo simulations and k is a scale-dependent constant that is set to 4 for the finest scale and to 3 for the others. Individual variances are used to estimate the curvelet norm by evaluating the curvelet transform of a few standard white noise images.

3 Shrinkage functions for image denoising

The choice of an optimal shrinkage function that is able to exploit the characteristics of the curvelet transform in image denoising applications is still an open research problem. Various approaches have been presented in

the literature with the resulting functions having been developed under specific assumptions about the noise and threshold estimation procedure.

3.1 Hard thresholding

The hard thresholding function [17]

$$y_{hard}(x) = \begin{cases} x & \text{if } |x| \geq th \\ 0 & \text{if } |x| < th \end{cases} \quad (2)$$

simply sets all coefficients below a specified threshold th to 0. When the threshold is perfectly selected, and the signal and noise coefficients are totally separated, hard thresholding is the simplest and the most effective function. However, this is not the case for real world images. The overlap of noise and signal coefficients and the discontinuity of the thresholding function at the threshold value result in the generation of artefacts and the so-called Gipps effect around image edges.

3.2 Soft thresholding

The soft thresholding function [10], defined by

$$y_{soft}(x) = \begin{cases} \text{sign}(x)(|x| - th) & \text{if } |x| \geq th \\ 0 & \text{if } |x| < th \end{cases} \quad (3)$$

is continuous around the threshold value, and can for some cases perform better than hard thresholding [8, 18, 15]. However, it reduces the values above the threshold, which results in lower image contrast and modifies intensities of geometric regularities. This problem seems to be more pronounced in the curvelet than in the wavelet domain due to the higher sparsity of the curvelet transform where image information is more concentrated in high-amplitude coefficients.

3.3 Semi-soft thresholding

Semi-soft thresholding [3] introduces an intermediary region to solve the continuity problem of hard thresholding and eliminates the constant bias of soft thresholding. It is controlled by two thresholds, making threshold selection more challenging, and is defined by

$$y_{semi-soft}(x) = \begin{cases} x & \text{if } |x| > t_2 \\ \text{sign}(x) \frac{t_2(|x| - t_1)}{(t_2 - t_1)} & \text{if } t_1 < |x| \leq t_2 \\ 0 & \text{if } |x| \leq t_1 \end{cases} \quad (4)$$

3.4 Garrote thresholding

Garrote thresholding [11] is performed by

$$y_{garrote}(x) = \begin{cases} x - (th^2/x) & \text{if } |x| \geq th \\ 0 & \text{if } |x| < th \end{cases} \quad (5)$$

Steered by only one threshold, it is both continuous and less deteriorating to high-amplitude coefficients. It converges towards the identity function gradually and slowly, which may alter original noiseless coefficients near the threshold value.

3.5 Cubic thresholding

The cubic thresholding function is given by [1]

$$y_{cubic}(x) = \begin{cases} x(1 - |th/x|^3) & \text{if } |x| \geq th \\ 0 & \text{if } |x| < th \end{cases} \quad (6)$$

and converges faster to the identity function compared to garrote thresholding. It has been used with multiple scale-dependent local thresholds in the curvelet domain for removing AGWN to give better results compared to hard and soft thresholding.

3.6 Chi-squared-based thresholding

In [9], Cui *et al.* deduced, based on exploring the denoising performance of hard, soft and semi-soft thresholding functions, four characteristics of an optimal thresholding function: (1) continuity of the function and its derivatives to be suitable for gradient decent algorithms; (2) the function should estimate large curvelet coefficients as close as possible to the empirical coefficients; (3) fast convergence to 0 over the region of small curvelet coefficients; and (4) a good balance between detail preservation and noise removal when shrinking the curvelet coefficients near the threshold value.

They consequently proposed a new thresholding function based on the chi-squared distribution and given by

$$y_{\chi^2}(x) = \begin{cases} \chi^2cdf(|x| - th + n)x & \text{if } |x| \geq th \\ 0 & \text{if } |x| < th \end{cases} \quad (7)$$

where $\chi^2cdf()$ denotes the chi-squared CDF with n degrees of freedom defined as

$$\begin{cases} F(x | n) = \int_0^x \frac{p^{\frac{(n-2)}{2}} e^{-\frac{p}{2}}}{2^{\frac{n}{2}} \Gamma(n/2)} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

3.7 Swamy and Vani thresholding

Swamy and Vani [19] proposed a novel thresholding function in the curvelet domain for SAR speckle noise removal. Their function is given by

$$y_{SV}(x) = \begin{cases} \varepsilon x + (1 - \varepsilon) \text{sign}(x)(x - \frac{\beta th}{(\beta + |x| - th)}) & \text{if } |x| > th \\ \frac{|x|^5}{th^4} & \text{if } |x| \leq th \end{cases} \quad (9)$$

where $\varepsilon = 1 - e^{-\mu\sqrt{(|x| - th)}}$, $0 < \varepsilon < 1$, is an adjustment parameter, $\beta > 0$, and μ is a positive number. The function is controlled by two parameters selected so that it behaves between hard and soft thresholding.

Unlike the other functions, which set coefficients below the threshold to 0 and focus on enhancing the estimation of coefficients above the threshold, this approach shrinks the coefficients below the threshold value gradually towards 0.

4 Proposed Shrinkage Function

In this paper, we propose a novel shrinkage function called *power shrinkage* to yield an enhanced estimation of coefficients on both sides of the threshold. The proposed function is adaptive, flexible, and continuous around the threshold value.

We suppose that an ideal threshold function should be continuous around the threshold value and decrease smoothly and quickly towards 0 to provide a good balance between detail preservation and noise removal while shrinking the curvelet coefficients below the threshold. At the same time, it should preserve those coefficients above the threshold which are supposed to be noise free, avoiding the drawback of the constant bias involved in soft thresholding.

4.1 Function conception

Considering that natural images are very diverse and difficult to be modelled by a specific distribution, we define our function without any assumptions about image distribution. Instead, we establish our function on the strong sparsity of the curvelet transform where the coefficients decay quickly towards 0.

Our proposed shrinkage function is defined by

$$P_{shrink}(x) = th^{(1-k)}|x|^k, \quad (10)$$

where k is the compression factor and th is the threshold value.

It satisfies two principal requirements of curvelet domain denoising: continuity and fast decay around

the threshold. In order to ensure its continuity at the threshold value, the function must pass through two points: (th, th) and $(0, 0)$. We can easily verify this since

$$\begin{cases} \lim_{x \rightarrow 0} P_{shrink}(0) = 0 \\ \lim_{x \rightarrow th} P_{shrink}(x) = th^{(1-k)}th^k = th \end{cases} \quad (11)$$

Hence, the resulting thresholding function is

$$y_{P_{shrink}}(x) = \begin{cases} x & \text{if } |x| \geq th \\ \text{sign}(x)th^{(1-k)}|x|^k & \text{if } |x| \leq th \end{cases} \quad (12)$$

We notice that in the interval $[-th; th]$ the function proposed in [18] is a specific case of P_{shrink} when the compression factor k equals 5. Fig. 1 graphs the power shrinkage function for different values of k .

Fig. 1: Power shrinkage function for different values of k and $th = 8$.

4.2 Determination of compression factor k

The larger the curvelet transform scale, the sparser the signal. Therefore, small valued coefficients are more likely to be generated by irrelevant image information. Based on this, we propose a compression factor related to scale and variance, and defined by a logarithmical scale dependent function as

$$k = f(\sigma)\sigma \log s, \quad (13)$$

where σ is the noise variance and s is the scale.

We conducted a simulations on a number of test images by varying the compression factor k in steps of 0.01 for various noise levels. We then associate k values that maximise PSNR results to the corresponding noise variance, and employ linear regression using a least squares method to define an empirical formula,

$$f(\sigma) = (120 - \sigma)/500 \quad \text{with } f(\sigma) > 0, \quad (14)$$

to relate k to noise variance.

5 Experimental Results

In our experiments, we use a standard set of 512×512 grayscale images¹, namely *Barbara*, *Lenna*, *Goldhill*, *Boat*, and *Angiography* as shown in Fig. 2.

Fig. 2: Test images used in experiments: (from left to right) *Barbara*, *Lenna*, *Goldhill*, *Boat*, *Angiography*.

The curvelet domain image denoising algorithm using our proposed shrinkage function is developed in MatLab 7.11 and uses the FDCTG2 via wrapping [5] to obtain the curvelet coefficients from the noisy input image. All first scale coefficients are maintained, since they represent the continuous part of the image, while the coefficients of other scales are processed using the proposed power shrinkage function. The threshold is selected as in Equ. (1).

To put the obtained results into context, we compare our algorithm with other curvelet-based denoising approaches, in particular hard and soft thresholding (using the implementations provided in the CurveLab toolbox [5]), and semi-soft and chi-squared-based thresholding (based on our own implementations).

As evaluation metrics, we employ the peak signal-to-noise ratio (PSNR) and the structural similarity index (SSIM) [20], both of which are commonly used for evaluating image quality and denoising algorithms.

Although curvelet-based denoising methods are primarily evaluated on images contaminated by additive white Gaussian noise, we also perform experiments on speckle and on Poisson noise.

5.1 AWG noise removal

We applied AWGN with $\sigma = 10, 20, 30, 50$ to all test images and applied the various denoising algorithms.

Table 1 gives the results for all images and algorithms in terms of both PSNR and SSIM. As is evident from there, our proposed denoising method gives the best PSNR results for all images and all noise levels as well as the best SSIM results in all but two cases.

As we expected, using a shrinkage function instead of classical thresholding maintains a portion of the curvelet coefficients below the Donoho threshold, hence maintaining more image textures. Our proposed power shrink

¹ <http://decsai.ugr.es/cvg/CG/base.htm>

Table 1: Experimental results in terms of PSNR and SSIM for various AWGN levels and all algorithms.

image	σ	PSNR						SSIM					
		noisy	hard	soft	semi	chi ²	power	noisy	hard	soft	semi	chi ²	power
Barbara	10	28.16	31.80	27.97	30.60	31.26	33.06	0.913	0.957	0.911	0.946	0.953	0.964
	20	22.22	28.84	25.39	27.69	28.47	29.31	0.766	0.916	0.843	0.898	0.912	0.922
	30	18.83	26.99	23.90	25.92	26.71	27.26	0.639	0.875	0.782	0.850	0.871	0.883
	50	14.17	24.11	21.68	23.13	23.91	24.40	0.431	0.778	0.663	0.738	0.774	0.785
Lenna	10	28.14	34.01	30.78	33.14	33.61	34.67	0.873	0.954	0.924	0.947	0.952	0.956
	20	22.10	31.14	27.99	30.23	30.85	31.52	0.677	0.914	0.867	0.903	0.912	0.915
	30	18.61	29.35	26.35	28.42	29.11	29.63	0.530	0.874	0.817	0.860	0.873	0.873
	50	14.16	26.65	24.05	25.79	26.50	26.86	0.347	0.798	0.736	0.787	0.803	0.792
Goldhill	10	28.13	31.08	28.03	30.05	30.61	32.25	0.910	0.933	0.861	0.912	0.924	0.948
	20	22.15	28.70	25.79	27.79	28.41	29.24	0.745	0.865	0.767	0.833	0.855	0.883
	30	18.76	27.18	24.51	26.28	26.94	27.50	0.597	0.810	0.702	0.772	0.801	0.826
	50	14.68	25.05	22.87	24.13	24.75	25.29	0.404	0.724	0.610	0.678	0.716	0.740
Boat	10	28.09	31.91	28.51	30.84	31.42	32.96	0.879	0.954	0.914	0.946	0.952	0.956
	20	22.18	29.03	25.86	27.89	28.63	29.63	0.704	0.905	0.842	0.888	0.902	0.908
	30	18.74	27.14	24.42	26.20	26.89	27.58	0.572	0.862	0.789	0.841	0.860	0.865
	50	14.71	25.01	22.65	24.19	24.85	25.24	0.412	0.789	0.711	0.768	0.791	0.787
Angiography	10	28.34	28.85	26.40	27.94	28.42	30.18	0.929	0.914	0.826	0.886	0.902	0.937
	20	22.44	26.84	24.71	26.14	26.60	27.38	0.792	0.842	0.731	0.805	0.830	0.864
	30	19.04	25.51	23.54	24.85	25.34	25.85	0.665	0.776	0.650	0.730	0.764	0.798
	50	14.91	23.51	21.87	22.95	23.41	23.69	0.469	0.670	0.540	0.617	0.659	0.689

function yields a decisive improvement compared to all other methods including chi-squared shrinkage, giving an average (over all images and noise levels) PSNR/SSIM improvement of 0.54/0.009, 3.31/0.090, 1.47/0.034, and 0.84/0.014 compared to hard, soft, semi-soft and chi-squared thresholding, respectively. Also, we can observe more significant improvements for lower noise levels which are encountered more commonly in natural images, and smaller improvements for higher noise levels.

Fig. 3 shows the noisy images and the results of the various denoising algorithms for $\sigma = 20$. Looking e.g. at the results for *Lenna*, we can notice that our proposed method preserves more textures and edges as noticeable in the hair and hat details. In contrast, soft thresholding blurs the image and smoothes the edges due to the constant bias introduced and discarded small coefficients around the threshold. Hard thresholding performs, as was also found in [9,14,18] better but also introduces some artefacts around the edges and contour. While our algorithm is not superior in dealing with noise in flat regions, it gives visually pleasing results in regions that are dense with small textures and geometric regularities. This property is particularly important for denoising highly textured images such as *Barbara* or *Goldhill* as is also apparent from Fig. 3.

5.2 Poisson noise removal

Poisson noise is intensity dependent and typically generated by electronic devices. In Table 2, we report the obtained results for all algorithms and all images. As we can see from there, our proposed method clearly out-

performs previous approaches also for this type of noise. Compared to hard, soft, semi-soft, and chi-squared thresholding we obtain average (over all images) improvements of 0.97/0.007, 3.57/0.060, 1.73/0.020, and 1.39/0.013 respectively in terms of PSNR/SSIM. Examples of this performance are shown in Fig. 4 which gives the noisy and denoised versions of *Barbara*, *Lenna*, and *Angiography*. We can clearly observe from there that the images obtained using the proposed power shrink function yield the best image quality due to the ability to maintain coefficients with small values around the threshold.

5.3 Speckle noise removal

Speckle noise is a multiplicative noise that primarily affects edges and fine details of images. We generated noisy versions of the test images with different levels of speckle noise (0.001, 0.005, 0.010, and 0.020) and applied the various denoising strategies on these. The obtained results are given in Table 3 for all images and algorithms. From there, we can observe that for the vast majority of cases, our proposed algorithm delivers the best denoising results and that only in a few instances, mostly for *Lenna*, it performs slightly worse than the best of the other four methods. Over all images and noise levels, our approach gives a PSNR/SSIM improvement of 0.79/0.003, 3.83/0.046, 1.67/0.010, and 1.18/0.006 compared to hard, soft, semi-soft, and chi-squared thresholding, respectively.

Fig. 5 shows the *Barbara*, *Lenna* and *Boat* images corrupted with 0.010 speckle noise and the images obtained by the various denoising algorithms. As we can

Fig. 3: Denoising results for images corrupted by AWGN with $\sigma=20$: (left to right) noisy image, hard thresholding, soft thresholding, semi-soft thresholding, chi-squared shrinkage, power shrinkage.

see, our proposed algorithm yields very good denoising results and generally better image quality compared to the other approaches.

6 Conclusions

In this paper, we have introduced a new shrinkage method – power shrinkage – and applied it in a scale dependent manner to enhance the performance of curvelet domain shrinkage for image denoising. Experimental results on a standard set of benchmark images and with varying noise levels confirm that, compared to classical thresholding and recent shrinkage methods, our proposed approach yields significantly better image quality. It proves to be particularly effective for denoising

images corrupted by additive white Gaussian noise, being able to enhance the preservation of image edges and geometric regularities while yielding excellent results for images with dense small textures. This is due to the improved processing of small coefficients around the threshold value, leading to a better balance between noise removal and detail preservation. In addition, we have shown our proposed shrinkage function to also give very satisfactory results for dealing with two other types of noise, namely Poisson and speckle noise for which it also outperformed other curvelet-based approaches. Since our current approach uses an empirical method to determine the relationship between shrinkage parameter, curvelet scale, and noise variance, future work will focus on investigating theoretical foundations that can accurately describe these relationships.

Table 2: Experimental results in terms of PSNR and SSIM for Poisson noise and all algorithms.

image	PSNR						SSIM					
	noisy	hard	soft	semi	chi ²	power	noisy	hard	soft	semi	chi ²	power
Barbara	27.48	31.75	28.03	30.34	31.27	32.61	0.905	0.953	0.908	0.944	0.949	0.959
Lenna	27.23	33.73	30.59	32.45	33.40	33.94	0.846	0.949	0.919	0.943	0.947	0.946
Goldhill	27.70	31.43	28.64	30.12	31.02	32.28	0.903	0.929	0.857	0.909	0.919	0.942
Boat	26.84	29.17	28.28	30.44	28.68	30.87	0.847	0.946	0.907	0.939	0.944	0.941
Angiography	28.65	29.33	26.86	28.22	28.91	30.54	0.951	0.924	0.840	0.898	0.909	0.946

Fig. 4: Denoising results for images corrupted by Poisson noise: (left to right) noisy image, hard thresholding, soft thresholding, semi-soft thresholding, chi-squared shrinkage, power shrinkage.

Table 3: Experimental results in terms of PSNR and SSIM for various speckle noise levels and all algorithms.

image		PSNR						SSIM					
		noisy	hard	soft	semi	chi ²	power	noisy	hard	soft	semi	chi ²	power
Barbara	0.001	35.94	35.87	31.52	34.14	34.98	38.13	0.983	0.982	0.957	0.976	0.977	0.988
	0.005	29.48	32.67	29.05	31.52	32.26	33.13	0.929	0.961	0.928	0.955	0.958	0.962
	0.010	26.89	30.98	27.84	30.08	30.80	30.99	0.881	0.942	0.903	0.937	0.942	0.940
	0.020	24.36	29.22	26.54	28.67	29.12	28.98	0.813	0.912	0.872	0.912	0.916	0.907
Lenna	0.001	35.65	36.63	33.40	35.75	38.43	38.58	0.970	0.978	0.960	0.974	0.974	0.981
	0.005	29.21	34.31	30.94	33.61	33.79	34.23	0.882	0.954	0.931	0.952	0.954	0.948
	0.010	26.19	32.76	29.66	32.05	32.44	32.45	0.808	0.934	0.911	0.934	0.937	0.923
	0.020	23.94	31.03	28.41	30.56	30.85	30.53	0.712	0.903	0.884	0.911	0.911	0.886
Goldhill	0.001	36.11	34.21	30.68	33.00	33.00	37.33	0.983	0.975	0.932	0.965	0.964	0.986
	0.005	29.68	32.11	28.96	31.04	31.39	32.71	0.932	0.942	0.887	0.930	0.935	0.951
	0.010	27.05	30.67	28.01	30.10	30.47	30.78	0.885	0.913	0.852	0.900	0.908	0.922
	0.020	24.55	29.12	27.00	28.95	29.20	28.91	0.816	0.874	0.808	0.862	0.872	0.882
Boat	0.001	34.57	35.53	31.45	34.17	34.18	37.30	0.967	0.980	0.958	0.976	0.976	0.981
	0.005	27.91	32.23	28.75	31.09	31.53	32.75	0.873	0.952	0.919	0.948	0.951	0.943
	0.010	25.3	30.65	27.51	29.67	30.15	30.94	0.797	0.926	0.891	0.925	0.929	0.913
	0.020	22.81	29.18	26.25	28.39	28.89	29.30	0.708	0.890	0.854	0.894	0.897	0.872
Angiography	0.001	38.08	31.31	28.13	29.96	30.27	34.77	0.995	0.961	0.894	0.941	0.944	0.984
	0.005	31.65	29.95	27.36	28.98	29.34	31.54	0.975	0.941	0.867	0.919	0.926	0.964
	0.010	29.00	28.93	26.79	28.25	28.60	29.70	0.951	0.922	0.843	0.899	0.909	0.944
	0.020	26.53	27.75	26.12	27.46	27.72	27.87	0.909	0.896	0.811	0.872	0.885	0.914

Fig. 5: Denoising results for images corrupted by speckle noise with noise level 0.010: (left to right) noisy image, hard thresholding, soft thresholding, semi-soft thresholding, chi-squared shrinkage, power shrinkage.

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