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Sample selection bias and bank retail credit scoring

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Abstract

A number of previous studies have established that customer acceptance procedures contain information about credit quality biases the credit scoring of retail banks. This paper investigates the quantitative and business importance of this bias using a large data set on the performance of retail loans by a major UK retail bank. It uses a multi-process probit model in order to overcome this problem of non-random sample selection bias. There is significant correlation between the disturbance terms .

JEL: G21 G32 C35

Keywords: Credit scoring; Retail portfolio modelling; Sample selection bias; multi-process probit model

1. Introduction

Many studies (for example Heckman, 1979; Hausman and Wise, 1997; McFadden, 1984; Maddala, 1999) discuss the influence of non-random sampling in limited dependent variable econometrics. Failure to allow for sample selection can lead to substantial bias of parameter estimates.

A prominent example such bias is in bank credit scoring models. When a customer requests a loan, a bank first needs to decide whether to grant or reject the request based on the customer's background and relevant financial activity histories. Only *if* the request is accepted, does the bank then discover whether the customer defaults or instead performs satisfactorily. The samples available for estimating a credit scoring model have thus been pre-selected according to the bank's own assessment of credit worthiness.

This can be understood as a situation in which there are *two* limited dependent variable statistical processes, the credit granting (lend/ do not lend) and default (good/bad loan performance) processes. As Boyes *et al.*, (1989, p5) point out, there is '*an element of non-randomness to the loan granting process and ultimately the sample selection rule*'. As a result, the first credit granting process may be correlated with second loan performance process and this potentially biases the stand-alone estimates of default probabilities and default determinants based on the sample of bank customers, compared to the true model that correctly represents the total population of potential loan applicants.

This paper corrects for this sample selection bias using a multiple process credit scoring model, taking account of the correlation between customer selection and loan performance. Such multi-process modelling, allowing for correlations between the various

sample generating processes in order to overcome sample selection bias, was first proposed by Manski and Lerman (1977) and has been previously applied to the case of credit scoring of bank loans by Boyes, Hoffman and Law (1989), Lee and Zhang (2003) and Donkers, Franses, and Verhoef (2003). We improve on these studies by using a much larger personal loan database. The size of this database allows us to use robust bootstrap variable selection and to reserve a large proportion of our data sample in order to investigate the improvement in forecasting performance that is achieved by taking account of correlation between the acceptance and default processes.

There are good reasons for anticipating such correlation. As argued by Petersen and Rajan (1995), learning about borrower quality based on observation of customer performance over time, can help overcome problems of asymmetric information and be a source of bank profit. Banks can be expected to have additional knowledge about existing customers and therefore apply somewhat different criteria when making the lending decision, than applied to customers of other banks approaching them for a loan facility. This additional information can be of two kinds (i) recorded information that can potentially be used in credit scoring (frequency of breach of borrowing limits, volatility of bank balance, trends of income etc.). (ii) other unrecorded judgemental information (character, assessment of relationship with bank, etc.) that can influence the credit decision but cannot be used in credit scoring. Such unrecorded judgemental information appears as a component of the error term in both statistical models and generate a correlation between two and the estimation of the loan performance as a single statistical process leads to bias.

The organisation of this paper is as follows. In section 2 we look in more detail at the literature on credit scoring and the potential application of multi-process probit models

in this context. Section 3 states the two probit models, for customer approval and for loan performance. Section 4 introduces the data-set we use and explains our use of a bootstrapping technique to select from the large number of available explanatory variable. Section 5 presents the estimation results and conducts a forecasting exercise using a 'holdout' sample in order to quantify the magnitude of the resulting forecasting bias. Section 6 discusses the implications of these results and summarises our conclusions.

2. Literature

The issue of non-random sampling is widely recognised in literature (Vella (1998) provides a survey) and occurs in many applications of limited dependent variable econometrics. For example Anstine and Skidmore (2005) examine a non-random selection in the type of students that opt for the online MBA courses. Nawata and McAleer (2001) compare sample selection bias by conducting a Monte Carlo simulation and present an empirical example of labour supply model consisting of a labour participation equation and a labour supply equation. Lilliard and Panis (1996) consider the impact of the selection into and out of marriage on mortality rates.

A theme common to several studies is that the independence of the sample generating process from the population can not be guaranteed, i.e., substantial selection endogeneity is present (see for example, Villas-Boas and Winer 1999; Chintasguna 2001;). This selection endogeneity results in estimation bias. Chintasguna (2001) observes that ignoring endogenous can even have a greater great impact on the estimated coefficients than ignoring the effects of heterogeneity. This recent literature controls the sample

selection bias caused by endogeneity selection by allowing for correlated unobserved components among the sample generating process.

Need a paragraph or two summarising the previous treatment of this issue in the literature on bank credit scoring. Should summarise the contributions of Boyes, Hoffman and Law (1989), Lee and Zhang (2003) and Donkers, Franses, and Verhoef (2003), Jacobson and Roszbach 2003

The multi-process modelling strategy is particularly appropriate for our study, since this approach accounts for the presence of unobserved factors influencing both decision making and performance assessing processes. This allowance can both effectively eliminate the potential estimation bias and test the extent of the correlation between the two processes. If the correlation exists, it is expected that those customers whose credits have been granted at the first stage tend to have less chances being in default. As a result, the observed good customers will be over-represented for the second process estimation. As such it is appropriate to examine whether the loan decision process is related to the loan performing process in testing the following hypothesis:

This characterisation of the loan approval process is in some ways oversimplified. If banks classify their customers exclusively on the basis of the subset population, the banks may never know the characteristic information about those customers whose applications were rejected in the first process. It is in the interest of banks to gain full knowledge of whole customer information. This will put the banks in a better position in assessing borrowing risks than other competing banks (Greenbaum, et al. 1989; Sharpe 1990). Hence, as part of their competitive strategy, banks will sometimes accept loan applications that they would otherwise normally reject, in order to learn about the credit characteristics of

the high-risk borrower segments. Better modelling will in then allow banks to charge different customers appropriately so as to increase expected profits (Kanas 1987; Machauer and Weber 1998). Blochlinger and Leippold (2006) show that banks could deliberately accept low credit applications and then set the price of the credit according to the credit score. Our results indicate that the acceptance decision of these experimental acceptances should be modelled separately from that of other customers; but we are unable to do this because they are not separately identified in our dataset.

Another relevant issue is the impact of changing criteria for loan approval on the errors in a multiple process model of this kind. Credit scoring models are themselves an input into the credit process and can be expected to improve over time. Hence, when a sample over several years, is used to estimate a credit scoring model, the loan approval process can be expected to converge towards that of the modelled loan approval process (the “benefit of hindsight”). Such model errors can be expected to lead to a negative rather than positive correlation between the modelled loan approval and loan performance processes.

3. Model specification and estimation methodology

One binary outcome for the credit granting process can be observed; the bank either grants the loan request or rejects the loan request. Similarly a binary outcome for the loan performance is observed, either good performance or bad performance. This subsection states the statistical models for these two potentially correlated limited dependent variable processes. In both cases a probit specification is assumed, ~~thus the~~

The two limited dependent variables are as follows. For the approval process The credit decision depends on a latent variable which determines customer i ($i = 1, \dots, n$) credit quality Q_i^1 . A credit application is rejected if the credit quality is lower than a threshold q , saying $q = 0$, i.e., $Q_i^1 \leq 0$.

Let this is Y_{1i} with $Y_{1i} = 1$ if the loan request by customer i is rejected and $Y_{1i} = 0$ otherwise. Conditional on explanatory covariates X_1 , the rejection probability is given by a probit model, $P(Y_1 = reject | X_1) = \Phi(X_1 \beta')$ and thus the accepting probability is $\Phi(-(X_1 \beta'))$, where $\Phi(\cdot)$ denotes the cumulative normal distribution and β is a vector of estimated parameters for the stage of credit application. For those customers whose credit requests are accepted, we can also observe their repayment performances.

Similarly, for the loan performance process, we let $Y_{2i} = 1$ if customer repays the loan and $Y_{2i} = 0$ otherwise. The loan performance also depends on a latent variable which determines customer i ($i = 1, \dots, n$) performance quality Q_i^2 . Conditional on the explanatory covariates X_2 , the probabilities of good and bad are given by the probit model, $P(Y_2 = good | X_2) = \Phi(X_2 \gamma')$, where γ is a vector of estimated parameters for the stage of customer performance assessment. The explanatory covariates X_1 and X_2 for the two processes can have either the need not necessarily include the same or different variables.

Our estimation uses the general simultaneous equations model of Lillard (1993) for multi process events analysing the potential problem of choice endogeneity, for example

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the mortality studies of Panis and Lillard (1994, 1996). This general model is implemented in the software aML (Lillard and Panis 2003). aML is a flexible and generally applicable framework for modelling a variety of non-random sample selection problems. Many different statistical processes can be modelled in aML, including hazard rates, Poisson arrivals, (negative) binomial regression models etc.

In order to take into account this potential bias estimation, we insert two unobserved heterogeneity components, ε and η , for the two processes respectively, and let the two heterogeneity components be correlated with each other. The correlation between ε and η needs to be estimated. Next we estimate the probability of being rejected and the probability of non-default simultaneously within joint multi-process probit regressions. The overall model specification which consists of two probit models conditional on X_1 , ε and X_2 , η respectively:

$$probit(Y_1 = reject|X_1, \varepsilon) = X_1\beta' + \varepsilon \quad (1)$$

$$probit(Y_2 = good|X_2, \eta) = X_2\gamma' + \eta \quad (2)$$

The heterogeneity components, ε and η , are assumed to follow a joint normal distribution with mean zero and variance-covariance matrix as:

$$\begin{pmatrix} \varepsilon \\ \eta \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon\eta} \\ \sigma_{\eta\varepsilon} & \sigma_\eta^2 \end{pmatrix}\right) \quad (3)$$

Where $\sigma_{\varepsilon\eta} = \rho\sigma_\varepsilon\sigma_\eta$ and ρ is the correlation between ε and η . Since there is only one outcome observation for each individual customer for each process, the variance of the

heterogeneity terms can be identified by fixing them to be one, i.e., $\sigma_\varepsilon^2 = \sigma_\eta^2 = 1$. The conditional likelihood of the parameter set $\theta = (\beta, \gamma, \sigma_\eta^2, \rho)$ is

$$L(\theta|\varepsilon, \eta) = \prod_{k=1}^{N_1} L_{1k}(\varepsilon) \prod_{l=1}^{N_2} L_{2l}(\eta) \quad (4)$$

Where N_1 denotes the number of customers in the credit granting process; N_2 denotes the number of customers in the performance assessing process. The parameter estimation of interest θ is based on marginal likelihood, which is obtained by integrating out the unobserved heterogeneity components. The marginal or integrated likelihood function for θ is then

$$L(\theta) = \int_{\varepsilon \in \mathfrak{R}} \int_{\eta \in \mathfrak{R}} f(\varepsilon, \eta) \prod_{k=1}^{N_1} L_{1k}(\varepsilon) \prod_{l=1}^{N_2} L_{2l}(\eta) d\varepsilon d\eta \quad (5)$$

where $f(\varepsilon, \eta)$ is the joint normal probability density function for ε and η .

This model falls within the the general simultaneous equations model of Lillard (1993) for multi-process events.(for examples of other applications see the mortality studies of Panis and Lillard (1994, 1996)). This general model is implemented in the software aML (Lillard and Panis 2003).¹ The parameter set θ is estimated simultaneously by using full information maximum likelihood (FIML). The Gauss-Hermite quadrature numerical integration method is used to approximately calculate the double integrals (5) with user-decided integration support points following from the zeros of the associated J^{th} order Hermite polynomial.

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¹ aML is a flexible and generally applicable framework for modelling a variety of non-random sample selection problems. Many different statistical processes can be modelled in aML, including hazard rates, Poisson arrivals, (negative) binomial regression models etc.

4. Data and variable selection

The personal loans dataset is supplied by one of the biggest commercial banks in the UK between the years of 1995 and 2003. However, details on the types of personnel loans such as whether the loans are secured or unsecured personal lending, revolving or repay, ect., are not provided by the bank.

The data available to us excludes both mandatory accepts (where the internal credit score is sufficiently high that the loan is automatically approved) and automatic declines (where the internal credit score is so low that the loan is automatically rejected). The internal credit scores are based on a different scoring model from that reported in this paper. The data sample does not include the internal scores emerging from that model.

The dataset consists of three types of characteristics on 43,634 applicants together with a summary of the subsequent payment status. The first group of characteristics are in-house records detailing the customers' performance in the years prior to the loan application. There are 134 such characteristics given for each customer. The second group of characteristics are 43 variables obtained from on line credit bureaus which were obtained by a retrospective bureau search to provide the information that would have been available at that time of the loan decision. The third group consists of 44 Point of Sale application characteristics, taken from the loan application form. There are altogether potentially 221 recorded variables for each customer from the three characteristic groups. However the bank only provides 132 variables for each customer. For each customer in the dataset, we also have a variable of DECISION indicating whether or not the credit application is

accepted. Of the 43,634 customers in this decision making process, 2,934 applications were rejected.

Another variable, GBF, is created to distinguish the performance for those customers whose credit applications are granted. The performance assessment process consists of the 40,700 accepted customers, of which 36,070 are classified as good customers (GBF=1) and 4,360 are classified as bad customers (GBF=0). The definitions of the GBF variable, defined by the bank, are:

- **Good (GBF=1):** zero or one cycle delinquent (missed payments) over the outcome period of 18 months.
- **Bad (GBF=0):** Two or more delinquent cycles or written off over 18 months outcome.

One binary outcome for the credit granting process is observed, either grant the loan request or reject the loan request. Given such a large number of variable records, 132 variables for each customer, at our disposal, exploiting this fact in estimating probability default is difficult. Thus it is useful to build a parsimonious but still congruent model, encompassing the model using all variables. The traditional variable reduction approach uses only the single stepwise procedure to shrink the model size. However, one potential problem of using the stepwise procedure just in one-go is that noise variables can be included in the model.

To validate the multi-process probit model, we randomly split the whole dataset equally into training and holdout datasets. We use the training dataset for model

specification and hypotheses 1 and 2(i) test. We then use the holdout dataset to test the forecasting performance. It is especially important that we spilt the sample in this way in order to avoid problems of over fitting from the bootstrap variables selection procedure, described in the following paragraph.

We follow Zellner et al. (2004), using a “bootstrap bagging selection procedure” to prevent noise variables from entering the model.⁺ The bootstrap selection procedure can also overcome potential variable colinearity problems. We apply this approach to run the bootstrap simulation 1,000 times based on the training data sample. For each bootstrap simulation we use the size α of being 0.15 as the variable entry criterion. The final variables that are included in the multi-process probit model are those who enter more than 850 times among 1,000 runs. In this way, those noise variables can be prevented from entering the model while the authentic variables will be included in the model.

Starting from the 132 variables in the original training sample dataset and treating all these variables on an equal footing, the bootstrap selection procedure identifies 44 distinct explanatory variables, of which 22 variables are used for credit line granting process and 32 variables for performance assessing process. 8 explanatory variables appear in both processes among those 44 variables. The bootstrap bagging variable selection results suggest that the factors that determine credit granting process can be different from factors that determine customer performance process. For example, credit granting process may depend on customers’ past credit history, whereas the customer performance assessing process may depend on customer’s personal conditions. A statistical model should therefore jointly handle different variables for each process.

In Table 1, we provide key definitions and mean statistics for these 44 variables for the training data sample. The statistics are calculated using weighted data—as are all of the estimated presented in the remainder of this paper—where good customers were sampled 1 in 2 for the period November 95 to January 96 therefore they require a weight of 2 to represent the true population good rate.² The mean values are calculated based on event outcome (ie., accept and reject for credit granting process and good and bad for performance assessment process). The credit granting process variables can be categorized into two groups: customer relationship with the bank and customer credit information. For example, if customers have been at the bank more than 3 years (*tmcallyd*), their credit applicants are more likely to be accepted than those who have been at the bank for just over one year). Customers who have regular monthly salary (*Salfreq1*) are also easier to be accepted by the bank. The average total credits (*var9*) of accepted customers are £7078.11 versus £5010.08 for rejected customers. The average days of exceeding credit limit (*var30*) for accepted customers are 2.25 days for the last three months compared with 4.63 days for their rejected counterparts.

Comment [a1]: Need some discussion of this weighting

We next consider the sample of customers whose loan applications are accepted. We refer to this sample as the *process 2* sample. The process 2 sample consists of more customer personal information than *process 1* sample, such as customers' residential status (*Home* and *Tcurradd*), financial acorn status (*Mos93*), number of children (*Nkids*), employment history (*Tcurremp* and *Salfreq1*), and information on customer consuming behaviour (*Cstitem* and *Deposit*). The financial activity variables monitor customers' financial situations from various perspectives, such as the account balance, the kind of account, the duration of exceeding credit limit, the frequency of using ATM, the number of

delinquents on any products. As shown in Columns 4 and 5 in Table 1, the good and bad customers have quite different characteristics. For example, the total balance (*var208*) for good customers are most twice as more as their counterparts. Similarly to process 1 sample, the days of excess credit limit for good customers are half of bad customers for the past three months (2.11 days versus 4.34 days). In the next section, we present a more formal test to identify the causal effect of those characteristics on probability of default under different model specifications.

4. Estimation and holdout performance

As we have explained in our introduction, our objective is to quantify the sample selection bias arising from non-random sample selection. We first examine the effect of two error terms on credit default probability. We then present a detailed comparison of parameter estimates with and without the allowance of the sample selection bias. Finally we examine predicting performance on the holdout dataset.

Table 2 presents the estimated coefficients on all customer characteristics using training data sample. Columns 4 and 5 contain estimates for both credit granting and performance assessment processes, using multi-process probit regressions; and Columns 2 and 3 report estimates that exclude credit granting process (i.e., separate estimation equation (1) and (2) by imposing $\rho = 0$ and joint estimation equation (1) and (2) by imposing $\rho \neq 0$).

Consider first the ρ estimates in Column 2 of Table 2. The estimated correlation between the unobserved error components ε and η is positive and statistically significant $\rho = 0.4127$ with a standard error of 0.133. This positive correlation ρ means

that determined the choice of granting the credit to customers are positively correlated with the unobserved factors that ensure customers being good customers. In other words, to the extent that an increase in the customer credit quality Q_i^1 (lower rejection probabilities) tends to help increase customer loan performance quality Q_i^2 (higher performance probabilities). This suggests that there are unobserved factors influencing both loan granting and performance processes resulting in the non-independence of the loan performance process. Hypothesis 1 is supported in our analysis and as a result the estimates of loan performance coefficients are inconsistent if the loan granting process is ignored.

In both models, customer personal characteristics and financial resources are significantly associated with the loan performance process and conform to expectations. The negative coefficients on the *Home*, *Salfreq1* and *Nkids* variables indicate that those customers who are non-homeowner, non-monthly salary and having more kids are more likely to default on their loans. On the other hand, the longer the customer stays in current address (*Tcurradd*) and be employed (*Tcurremp*) the less chances of becoming default. It is interesting to note that students (*var201*) tend to be less risky customers. The effect of the relationship with the bank variables on loan performance suggests that if customers who have long history of check books (*var206*) and own various of banking cards (*Chqvisa1*, *Lloacs1*) tend to have lower probabilities of default. The effect of days with the bank (*var500*) is not statistically significant.

The estimation results suggest that the delinquent histories are significantly associated with the loan performance as well. The total number of delinquent (*cbvar28*) and the delinquent on any product for the last six months (*var527*) are strong indicators of

potential default risks both in terms of their negative signs and magnitudes compared to other variables. If a customer has been delinquent with any product for the last six months (*var527*), the chance of further delinquent behaviour over the next 18 months significantly increases. Similarly, the purchasing behaviours are good indicators for risk as well (*Deposit and Cstitem*). Customers who purchase less expensive items and with high deposit tend to honour loan repayment.

The empirical findings also show that credit hungriness over the short term is negatively and significantly associated with the loan performance. The number of credit searches within the last three months alone with cheque debit transactions (*cbvar37* and *var165*) can be used a measure of credit hungriness, the higher the credit eagerness over the short term the higher default probabilities that the customer will not honour the loan. This is especially for desperate customer using cheque debit transactions (*var165*), which tend to be charged very high interest rate for using this. Other short terms credit hungriness variables such as total number of free debits (*var165*), low balance for current account (*var24*) and days excess limit (*var30*) are also of expected sign and are associated with high default risks. The high values total balance (*var208*) can statistically reduce the number of delinquencies over the next 18 months.

A comparison of the estimation results (in Panel 2 of Table 2) between the joint estimation and the separate estimation models for the performance assessing process demonstrates substantial differences in parameter values for almost all variables. The coefficient values estimated by multi-process probit model are uniformly higher in absolute value than those estimated by single process probit model, i.e. negative coefficients are more negative and positive coefficients are more positive. Thus, one impact of adopting the

multi-process specification is to increase the probability of default for high-default probability observations and reduce the probability of default for low-default probability observations. The comparison with and without taking into account of loan granting process are critical to test Hypothesis 2(i), which suggests that default probabilities will be underestimated without considering loan granting process. The results are supportive of Hypothesis 2(i) and are consistent with prior empirical evidence (See for example, Rosch and Scheule 2005). This suggests the amount of economic capital to cover the unexpected loss will be insufficient when the loan granting process is not taken into account.

The gain of taking into account the loan granting process can be further investigated by an assessment of predicting accuracy of good and bad customers. We validate the predicted abilities to compare the predicted probabilities for the single-process and multi-process probit models. As discussed earlier, the validation is based on the holdout sample. We compare the predicting accuracy of multi-process probit model with that of single process probit model. The calculation of default probabilities for the validation dataset is based on the coefficients generated from multi-process probit model and single process model using training dataset estimation. The predicted probability of default for multi-process model is calculated using the multi-process estimated coefficients based on ε_i is small enough to grant the credit to customer i , such that,²

² The derivation of the probability of default for multi-process model is reported in appendix.

$$\begin{aligned}
prob(y_2^m = good) &= prob(\eta > -X_2\gamma' \mid \varepsilon < -X_1\beta') \\
&= \frac{\int_{-X_1\hat{\beta}}^{\infty} \frac{1}{2\pi} \exp(-0.5\eta^2) \Phi\left(\frac{X_2\gamma' + \hat{\rho}\eta}{\sqrt{1-\rho^2}}\right) d\eta}{\Phi\left(X_1\hat{\beta}\right)}
\end{aligned}$$

Where $\Phi(\bullet)$ is the standard normal cumulative density function.

The predicted probability of default for single process is straightforward. This is calculated using the single process estimated coefficients according to

$$prob(y_2^s = good) = prob(\eta > -X_2\hat{\gamma}) = \Phi\left(X_2\hat{\gamma}\right)$$

Customers in the validation dataset are ranked according to the predicted probability of default. A common way of estimating the predictive power of both models is to look at the receiver operating characteristic (ROC) curves. A ROC curve is a plot of the true positive rate against the false positive rate. This produces a curve going from (0,0) to (1,1). The nearer it gets to (0,1) the more accurate is the predicted ranking. Figure 1 shows the predicting results for the multi-process probit model are better than the single-process probit model. We can also calculate the area under the curve, in which the multi-process models is 0.8206 and the single-process probit model is 0.8004. This suggests that multi-process probit model predicts marginally better than single-process probit model, thus supporting hypothesis 2(ii).

6. Implications and conclusion

This study investigates the effect of sample selection bias on the probability of default for retail bank customers. In particular, this study has identified the existence of a

statistically significant relationship between credit decision-making and performance assessing processes. This strong relationship stems from the fact that banks use subjective judgements in deciding whether or not to grant credit applications at the stage of application. This finding provides evidence supporting the view that sample selection bias could have impacts on the model estimation results; in particular the probability of default is underestimated in modelling that does not consider the decision making process. This study suggests that sample selection bias can be overcome by estimating credit granting decision and performance assessing processes simultaneously. The findings of this paper also provide evidence of the need to account for the potential heterogeneity of customers when estimating the probability of default.

The results provide further implications for bank loan sales. The bank loan sales especially the securitisation of consumer loan continues to grow. Generally, there are three separate parties involved: the bank, the guarantor, the insurance, and the rating agency. All these parties need to price the loans precisely based on the correct estimation of the probability of default.

Finally, multi-process modelling is more desirable on conceptual grounds for the purpose of customer performance assessment, which is somehow related to the decision making at the time of credit application. A model without accounting for such a relationship could yield inconsistent estimates, which in turn may make banks wrongly categorise potential more default prone customers as less default prone customers. Thus, care must be taken in assessing the probability of default on retail bank lending. This can have implications for banks pricing loans more efficiently when facing different customers. If the pricing is based on the single-process model, banks then tend to charge potential high

risk customers less but charge potential good customers high. While the multi-process model is better in distinguishing good and customers by pricing good customers lower interest rate while pricing bad customers higher interest rate.

References:

- Anstine, J., Skidmore, M., 2005. A small sample study of traditional and online courses with sample selection adjustment. *Journal of Economic Education* 36, 107-112.
- Blochlinger, A., Leippold, M., 2006. Economic benefit of powerful credit scoring. *Journal of Banking and Finance* 30, 851-873.
- Boyes, W., Hoffman, D., Low, S., 1986. Lender reactions to information restrictions, The case of banks and the ECOA. *Journal of Money, Credit and Banking* 18, 211-219.
- Boyes, W., Hoffman, D., Low, S., 1989. An Econometric analysis of the bank credit scoring problem. *Journal of Econometrics* 40, 3-14.
- Chintagunta, P., 2001. Endogeneity and heterogeneity in a probit model: Estimation using aggregate data. *Marketing Science* 20, 442-456.
- Donkers, B., Francses, P., Verhoef, P., 2003. Selective sampling for binary choice models. *Journal of Marketing Research*, XL, 492-497.
- Greenbaum, S., Kanatas, G., Venezia, I., 1989. Equilibrium loan pricing under the bank-client relationship. *Journal of Banking and Finance* 13, 221-235.
- Heckman, J., 1979. Sample selection bias as a specification error. *Econometrica* 47, 153-162.
- Jacobson, T., Roszbach, K., 2003. Bank lending policy, credit scoring and value-at-risk. *Journal of Banking and Finance* 27, 615-633.
- Kanatas, G., 1987. Commercial paper, bank reserve requirements and the informational role of loan commitments. *Journal of Banking and Finance* 11, 425-448.
- Lee, T., and Zhang, M., 2003. Bias correction and statistical test for developing credit scoring model through logistic regression approach. *International Journal of Information Technology and decision making* 2, 299-311.
- Lillard, L., 1993. Simultaneous equations for hazards, Marriage duration and fertility timing. *Journal of Econometrics* 56, 189-217.
- Lillard, L., Panis C., 2003. aML multilevel multiprocess statistical software, Version 2.0. EconWare, Los Angeles, California.

- Machauer, A., Weber, M., 1998. Bank behaviour based on internal credit ratings of borrowers. *Journal of Banking and Finance* 22, 1355-1383.
- Manski, C., Lerman, S., 1977. The estimation of choice probabilities from choice based samples. *Econometrica* 45, 1977-1988.
- McFadden, D., 1984. Econometric analysis of qualitative response models. In *Handbook of Econometrics*, Z. Griliches and M. Intriligator (eds). Vol2. Amsterdam, North Holland.
- Nawata, K., McAleer, M., 2001. Size characteristics of tests for sample selection bias: A Monte Carlo comparison and empirical example. *Econometric Review* 20, 105-112.
- Panis, C., Lillard, L., 1994. Health inputs and child mortality, Malaysia. *Journal of Health Economics* 13, 455-489.
- Panis, C., Lillard, L., 1996. Marital status and mortality, The role of health. *Demography* 33, 313-327.
- Petersen, M., Rajan, R., 1995. The effect of credit market competition on lending relationships. *Quarterly Journal of Economics* 110, 407-443.
- Rosch, D., Scheule, H., 2005. A multifactor approach for systematic default and recovery risk. *Journal of Fixed Income* 14, 63-75.
- Sharp, S., 1990. Asymmetric information, bank lending, and implicit contracts, A stylized model of customer relationships. *Journal of Finance* 45, 1069-1087.
- Tobin, J., 1958. Estimation of relationships for limited dependent variables. *Econometrica* 26, 24-36.
- Vella, F., 1998. Estimating models with sample selection bias: A survey. *Journal of Human Resources* 33, 127-169.
- Villas_Boas, M., Winer, R., 1999. Endogeneity and brand choice models. *Management Science* 45, 1324-1338.
- Zellner, D., Keller, F., and Zellner, G., 2004. Variable selection in logistic regression models. *Communications in Statistics: Simulation and Computation* 33, 787-805.

Table 1. Sample mean and definitions for variables selected by bootstrap

Variable	Process1 Accept	Process1 Reject	Process2 Good	Process2 Bad	Definition
I. In-house Customer Performance					
Var9	£7078.11	£5010.08	---	---	Total Credits
Var10	£107.34	£103.69	---	---	Number credits plus debits this month
Var12	£100.07	£98.61	---	---	Number credits plus debits previous month
Var13	---	---	£-3125.2	£-3465	Minimum low balance
Var24	---	---	0.06	0.18	Minimum low balance for current account
Var30	2.25	4.63	2.11	4.34	Days in excess of limit (Last 3 months)
Var33	---	---	17.30	15.88	Days in credit per month per account (last 3 mths)
Var36	---	---	10.54	80.75	Cheque application score
Var42	£-40.51	£-74.90	---	---	Average credits – debits (Last 3 months)
Var47	668.21	643.96	---	---	Days since last debit (Unsecured loan)
Var49	---	---	996	973	Days since last debit (Secured loan)
Var53	---	---	0.30	0.60	Days since last bad or super bad (Last 12 months)
Var57	98.86	94.91	99.05	95.51	% No. of months credit information (Last 3 mths)
Var155	---	---	132.05	63.68	Max(months open over all chq)
Var165	0.06	0.12	0.05	0.14	Sum(No. of chq debit transactions across all chqs)
Var167	---	---	5.50	5.25	No. of ATM cash transactions
Var201	---	---	0.06	0.04	=1 if student indicator; 0 otherwise
Var206	123.67	62.26	126.16	73.67	Age of cheque account
Var208	£-746.61	£-1571	£-668.03	£-1601	Total balance
Var232	---	---	5.25	3.20	Total number of AUTO debits
Var233	---	---	3.19	3.68	Total number of free debits
Var500	56.85	33.27	57.37	39.52	Days since created on in-house
Var507	3.32	6.21	---	---	Number other debits this month
Var523	---	---	0.92	0.80	Credit taken not sec loan seg
Var526	0.53	0.76	---	---	Delinquent any product 1-3 months
Var527	---	---	0.64	0.79	Delinquent last 6 months any product
II. Credit Bureau					
Chvar28	---	---	0.02	0.04	=1 if delinquent and 0 otherwise
Chvar37	---	---	0.25	0.49	= 1 credit search happens last 3 mths; 0 otherwise
III. Point of Application					
balcurac	£-965.67	£-425.56	---	---	Balance current account

Chqvisa1	---	---	0.79	0.57	=1 if cheque / Visa card held; 0 if not
Cstitem	---	---	£3254.67	£3733.3	Cost of item customer purchasing
Deposit	---	---	6.03	3.38	% deposit customer is paying on item purchased
Home	---	---	0.42	0.69	=1 if customer NOT owned a home; 0 if owns
Fundln	£7425.39	£3360.80	---	---	Amount loan + additional borrowing bank willing offer
Grstcom	£275.33	£262	---	---	Monthly loan + Rent / Mortgage commitments
Lloacs1	---	---	0.25	0.14	=1 if Lloyd's access card held; 0 otherwise
Mos93	---	---	0.41	2.12	Mosaic code
Nkids	---	---	0.63	0.61	Number of children
Reptflg	0.005	0.05	---	---	If the loan be scored more than once on a different CSID
Salfreq1	0.30	0.48	0.28	0.47	=1 if salary is NOT on a mthly freq.; 0 otherwise
Tcurradd	---	---	739.14	551.02	Days at current address
Tcurremp	---	---	624.22	327.81	Days in current employment
tmcallyd	1070.78	432.70	---	---	Time customer has been at Lloyds's bank

Notes: --- denotes that the variable does not enter that particular process. Process 1 and 2 denote the loan granting process and customer performance assessment processes, respectively. The sample mean is calculated based on the training data sample.

Table 2. Coefficient estimates: Training Dataset

Variable	Separate Estimation		Joint Estimation	
	Coefficient	BHH-based	Coefficient	BHH-based
		Standard error		Standard error
Process 1: Credit granting process				
Constant	-2.0913*	0.0199	-2.9532*	0.0282
Var10	-1.0229*	0.1265	-1.4332*	0.1800
Var12	1.0452*	0.1263	1.4633*	0.1797
Var165	-0.1363	0.0319	-0.0204	0.0450
Var206	0.1505*	0.0286	0.2114*	0.0405
Var208	-0.3690*	0.0073	-0.5184*	0.0107
Var30	0.0394*	0.0026	0.0552*	0.0037
Var42	0.1205*	0.0046	0.1773*	0.0088
Var47	0.0304*	0.0123	0.0407*	0.0175
Var500	-0.4345*	0.0247	-0.6124*	0.0349
Var507	0.0007*	0.002	0.0009*	0.0003
Var9	0.1024*	0.0068	0.1463*	0.0100
balcurac	-0.1433*	0.0079	-0.2080*	0.0113
deposit	-0.0968*	0.0142	-0.1361*	0.0202
Fundln	-0.6443*	0.0076	-0.9126*	0.0108
Grstcomm	0.1579*	0.0063	0.2233*	0.0089
Reptflag	1.2198*	0.0637	1.7181*	0.0900
Temcalld	-0.1194*	0.01547	-0.1690*	0.0219
Salfrreq1	0.2820*	0.0256	0.3993*	0.0363
Log Likelihood	-7270.24		---	
Process 2: Performance assessment process				
Constant	1.5089*	0.1250	2.1640*	0.1772
Var13	0.0839*	0.0197	0.1148*	0.0280
Var155	0.1120*	0.0241	0.1530*	0.0341
Var165	-0.1122*	0.0291	-0.1559*	0.0413
Var167	0.0009*	0.0003	0.0013*	0.0004
Var201	0.2644*	0.0665	0.3768*	0.0939
Var206	0.0999*	0.0301	0.1544*	0.0428
Var208	0.0536*	0.0184	0.0550	0.0303

Var232	0.1557*	0.0192	0.2233*	0.0272
Var233	-0.0569*	0.0169	-0.0808*	0.0241
Var24	-0.1707*	0.0218	-0.2378*	0.0308
Var30	-0.0405*	0.0038	-0.0539*	0.0055
Var33	-0.0058*	0.0017	-0.0082*	0.0024
Var36	-0.0504*	0.0106	-0.0675*	0.0155
Var49	0.0630*	0.0229	0.0895*	0.0323
Var500	0.0050	0.0286	-0.0284*	0.0423
Var523	0.1127*	0.0505	0.1597*	0.0714
Var527	-0.2822*	0.0402	-0.4034*	0.0567
Var53	-0.0521*	0.0107	-0.0717*	0.0151
Var57	0.0081*	0.0012	0.0113*	0.0017
Cbvar28	-0.1364*	0.0312	-0.1918*	0.0441
Cbvar37	-0.0591	0.0566	-0.0890	0.0801
Cstitem	-0.1858*	0.0128	-0.2743*	0.0182
Deposit	0.1598*	0.0211	0.2297*	0.0300
Mos93	0.2030	0.1160	0.3321*	0.1647
Nkids	-0.0295*	0.0142	-0.0434*	0.0200
Tcurradd	0.0537*	0.0160	0.0735*	0.0225
Tcurremp	0.1080*	0.0169	0.1530*	0.0239
Chqvisal	0.2519*	0.0307	0.3530*	0.0433
Home	-0.1595*	0.0335	-0.2268*	0.0473
Lloacs1	0.1083*	0.0387	0.1543*	0.0546
Salfreq1	-0.3506*	0.0289	-0.4756*	0.0419
σ_{ε}^2	---	---	1.00	---
σ_{η}^2	1.00	---	1.00	---
ρ	---	---	0.4127*	0.1330
Log Likelihood	-4915.29		-12180.81	

Notes: Coefficient estimates are based on training sample. * Statistically significant at the 5% level.

Appendix

Considering equations (1), (2), and (3), the probability of being a good customer given that the customer's credit line request is accepted is

$$prob(Y_2 = good | Y_1 = accepted) = \frac{prob(\eta > -X_2\gamma', \varepsilon < -X_1\beta')}{prob(\varepsilon < -X_1\beta')}$$

Evaluating the numerator yields:

$$\begin{aligned} & prob(\eta > -X_2\gamma', \varepsilon < -X_1\beta') \\ &= \int_{-\infty}^{X_1\beta} \int_{-X_2\gamma'}^{\infty} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(\frac{\eta^2 - 2\rho\eta\varepsilon + \varepsilon^2}{-2\sqrt{1-\rho^2}}\right) d\eta d\varepsilon \\ &= \int_{-\infty}^{X_1\beta} \frac{1}{2\pi} \exp\left(\frac{\varepsilon^2}{-2}\right) \int_{-X_2\gamma'}^{\infty} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(\frac{(\eta^2 - \rho\varepsilon)^2}{-2(1-\rho^2)}\right) d\eta d\varepsilon \\ &= \int_{-\infty}^{X_1\beta} \frac{1}{2\pi} \exp(-0.5\varepsilon^2) \Phi\left(\frac{X_2\gamma' + \rho\varepsilon}{\sqrt{1-\rho^2}}\right) d\varepsilon \end{aligned}$$

The denominator is

$$prob(\varepsilon < -X_1\beta') = \Phi(X_1\beta)$$

¹ We acknowledge that the variable selection procedure even with the bootstrap bagging selection bears no theory to the choice of variables. However this is something unavoidable in conducting individual customer behaviour research. After all, this procedure is similar to procedures used in constructing credit scoring cards in retail banks.

² Such weighted data sample is provided by the bank.