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A NOTE ON BUSETTI-HARVEY TESTS FOR STATIONARITY IN SERIES WITH STRUCTURAL BREAKS

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Abstract. In this note we highlight a minor error in the asymptotic distribution of one of the Buseti and Harvey (2001) tests for stationarity in the presence of structural breaks, and provide corrected asymptotic critical values where relevant. In addition, we examine the extent to which finite sample critical values for the Buseti-Harvey tests are approximated by their asymptotic counterparts when the location of the break is determined endogenously.

Keywords. Unit roots; endogenous breaks.

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1. INTRODUCTION

Busetti and Harvey (2001)—henceforth BH—present tests of the null hypothesis of stationarity against a unit root alternative when the process contains a structural break in level and/or slope. Their methodology generalises tests by Nyblom and Mäkeläinen (1983) and Kwiatkowski *et al.* (1992), who present stationarity tests in the absence of such breaks. In this note, we highlight a minor error in the asymptotic distribution of one of the test statistics and present corrected asymptotic critical values where relevant. Further, for all the BH tests where the location of the break is determined endogenously, we examine how closely the asymptotic critical values approximate those in finite samples.

2. BUSETTI-HARVEY TESTS AND CORRECTION TO ASYMPTOTIC DISTRIBUTION

BH consider four models which permit different orders of deterministic components and structural breaks:

$$\text{Model 1} \quad y_t = \mu_t + \delta w_t + \varepsilon_t \quad (2.1)$$

$$\text{Model 2} \quad y_t = \mu_t + \beta t + \delta_\mu w_t + \delta_\beta (w_t t) + \varepsilon_t \quad (2.2)$$

$$\text{Model 2a} \quad y_t = \mu_t + \beta t + \delta_\mu w_t + \varepsilon_t \quad (2.3)$$

$$\text{Model 2b} \quad y_t = \mu_t + \beta t + \delta_\beta z_t + \varepsilon_t \quad (2.4)$$

where $\mu_t = \mu_{t-1} + \eta_t$ with $\eta_t \sim \text{NID}(0, \sigma_\eta^2)$, $\varepsilon_t \sim \text{IID}(0, \sigma^2)$, $w_t = 1(t > \tau)$ and $z_t = 1(t > \tau)(t - \tau)$, with $1(\cdot)$ being the indicator function and τ the break point.

Model 1 has no trend component and a break in level, Model 2 contains a time trend and is subject to a simultaneous break in level and slope, while Models 2a and 2b also contain trends, but have breaks in level only, and slope only, respectively.

Assuming that the timing of the break is known, the locally best invariant test of $H_0 : \sigma_\eta^2 = 0$ against a one-sided alternative is then

$$\xi_i(\lambda) = \frac{\sum_{t=1}^T (\sum_{s=1}^t e_s)^2}{T^2 \hat{\sigma}^2} \quad (i = 1, 2, 2a, 2b) \quad (2.5)$$

where e_t denotes the residuals from the appropriate regression among (2.1)-(2.4) above, $\hat{\sigma}^2 = T^{-1} \sum_{t=1}^T e_t^2$, and $\lambda = \tau/T$.

For each of the four models considered, BH present the asymptotic distribution for the test statistic $\xi_i(\lambda)$ under the null hypothesis. However, for Model 2a (where a trend

is included in the model but the break is in level only), there is a minor error in the distribution given. The corrected distribution is instead

$$\xi_{2a}(\lambda) \Rightarrow \int_0^1 \{B_{2a}(r, \lambda)\}^2 dr \quad (2.6)$$

where

$$B_{2a}(r, \lambda) = \begin{cases} W(r) - \frac{r}{\lambda}W(\lambda) - \frac{6}{1-3\lambda+3\lambda^2}r(r-\lambda) \\ \quad \times \left[\int_0^1 r dW(r) - \frac{\lambda}{2}W(\lambda) - \frac{1+\lambda}{2}\{W(1) - W(\lambda)\} \right] & \text{for } r \leq \lambda \\ \{W(r) - W(\lambda)\} - \frac{r-\lambda}{1-\lambda}\{W(1) - W(\lambda)\} - \frac{6}{1-3\lambda+3\lambda^2}(r-1)(r-\lambda) \\ \quad \times \left[\int_0^1 r dW(r) - \frac{\lambda}{2}W(\lambda) - \frac{1+\lambda}{2}\{W(1) - W(\lambda)\} \right] & \text{for } r > \lambda \end{cases} \quad (2.7)$$

where $W(\cdot)$ is a standard Wiener process. Although the lemmas and proof provided by BH are correct, the multiplicative factor $(r-1)$ in $B_{2a}(r, \lambda)$ when $r > \lambda$ in (2.7) is incorrectly reported as r when the final expression is given. Despite this error, the asymptotic critical values when λ is known, reported in Table I of their paper, are generated using the *correct* asymptotic distribution.

In contrast, however, when the location of the break is unknown, the *incorrect* distribution is used to generate the asymptotic critical values. When λ is to be determined endogenously, BH recommend an approach following Zivot and Andrews (1992) where the break point is selected to give the most favourable result for the null hypothesis using the $\xi_i(\lambda)$ statistic of (2.5), i.e.

$$\tilde{\xi}_i = \inf_{\lambda \in \Lambda} \xi_i(\lambda) \quad (i = 1, 2, 2a, 2b) \quad (2.8)$$

where Λ is a closed subset of the interval $(0, 1)$.

Under an assumption that the magnitude of the level breaks (δ, δ_μ) decreases with the sample size at a rate faster than $T^{-1/2}$, and the magnitude of the slope break (δ_β) decreases faster than $T^{-3/2}$, the asymptotic distribution of the test statistic $\tilde{\xi}_i$ under the null is given by

$$\tilde{\xi}_i \Rightarrow \inf_{\lambda \in \Lambda} \int_0^1 \{B_i(r, \lambda)\}^2 dr \quad (i = 1, 2, 2a, 2b) \quad (2.9)$$

where $B_i(r, \lambda)$ ($i = 1, 2, 2b$) are as defined in BH, and $B_{2a}(r, \lambda)$ is as defined in (2.7).

BH generate asymptotic critical values for these test statistics (reported in Table VI of their paper), but for Model 2a, the results make use of their incorrectly reported term $B_{2a}(r, \lambda)$, leading to exaggerated critical values. We generated the correct critical values

by Monte Carlo simulation of (2.9), using a sample size of $T = 500$ as in BH. The space of values for λ is restricted to a closed subset of $(0, 1)$; in our simulations here and in Section 3, we restricted λ to lie between the conventionally chosen points $(0.2, 0.8)$ and used 10000 replications. The results are given in Table I. The degree of exaggeration is quite considerable; for example, the correct 5% and 1% critical values are 0.051 and 0.068 respectively, whereas those reported by BH are 0.089 and 0.125. The table also contains results for Models 1, 2 and 2b using this methodology, which are of course close to those reported by BH.

3. FINITE SAMPLE CRITICAL VALUES

In addition to analysing the asymptotic distribution of the test statistics, it is interesting to analyse finite sample critical values, and examine how well approximated they are by their asymptotic counterparts. Finite sample critical values for the four BH models when the break point is unknown are also provided in Table I. These critical values were simulated by repeated application of the test statistic $\tilde{\xi}_i$ of (2.8) to generated stationary series without breaks of length T (for $T = 50, 100, 200$). In each case, the critical values at different finite sample sizes are almost identical to those derived using the asymptotic distribution, even those for $T = 50$, indicating that the asymptotics obtain very quickly and provide an excellent approximation of behaviour in finite samples.

4. SUMMARY

In summary, we have highlighted an error in the asymptotic distribution of the BH stationarity test when the model contains a trend but admits a break in level only, and presented corrected asymptotic critical values for this test when the timing of the break is determined endogenously. We have also simulated finite sample critical values of all the BH tests (again when the break point is unknown), and found that the asymptotic critical values closely approximate those in finite samples.

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TABLE I
ASYMPTOTIC AND FINITE SAMPLE CRITICAL VALUES FOR BH STATIONARITY TESTS
WHEN THE BREAK POINT IS DETERMINED ENDOGENOUSLY

	Model 1			Model 2		
	10%	5%	1%	10%	5%	1%
$T = 50$	0.088	0.107	0.160	0.033	0.038	0.049
$T = 100$	0.086	0.107	0.158	0.033	0.038	0.049
$T = 200$	0.086	0.108	0.159	0.033	0.038	0.050
$T = \infty$	0.087	0.108	0.166	0.033	0.038	0.048
	Model 2a			Model 2b		
	10%	5%	1%	10%	5%	1%
$T = 50$	0.045	0.051	0.067	0.055	0.064	0.089
$T = 100$	0.044	0.052	0.067	0.056	0.066	0.090
$T = 200$	0.044	0.052	0.069	0.055	0.067	0.091
$T = \infty$	0.044	0.051	0.068	0.055	0.065	0.089