UKPN Workshop, Sheffield 2012: Modelling in the Polar Sciences

Introduction to Modelling Glacier Hydrology

1. Linear reservoir approach

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Approaches to hydrological modelling

- *Statistical*
  - simply relies on statistical relationships
- *Physical*
  - attempts to represent system as realistically as possible, through physical equations
- *Conceptual*
  - represents system through simplified numerical relationships
- "...all models are wrong, but some are useful."
  Box (1987: en.wikiquote.org/wiki/George_E._P._Box)

Basis of linear reservoir approach

- Melt is (fairly) easily predicted, and this is fine for mass-balance studies (generally)
- *But…* water input is not instantaneously transformed into output: it has to pass through variable drainage pathways
- Most hydrological systems evolve different structures to accommodate water fluxes of different magnitudes
    - with a low-flux system which becomes a high-flux system when some threshold is crossed
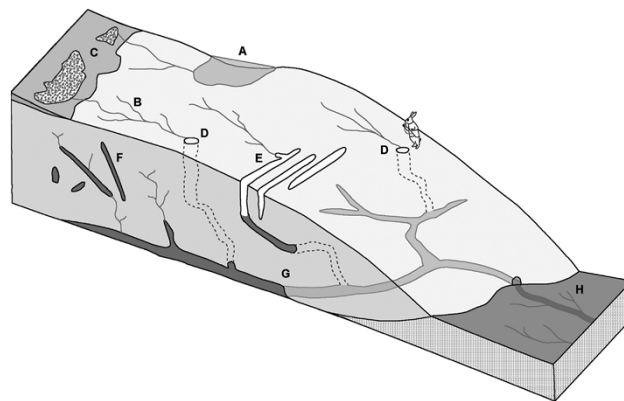

FIGURE 6.1 Cuffey & Paterson (2010)

Some elements of the glacier water system: (A) Supraglacial lake. (B) Surface streams. (C) Swamp zones near the edge of the firn. (D) Moulins, draining into subglacial tunnels (for scale, white rabbit is about 10m tall). (E) Crevasses receiving water. (F) Water-filled fractures. (G) Subglacial tunnels, which coalesce and emerge at the front. (H) Runoff in the glacier foreland, originating from tunnels and also from upwelling groundwater. Though not depicted here, water is also widely distributed on the bed in cavities, films, and sediment layers. Sediment and bedrock beneath the glacier contain groundwater.

Fast/slow drainage pathways

- Channels/ linked cavities
- Episodic icemelt/ diffuse snowmelt
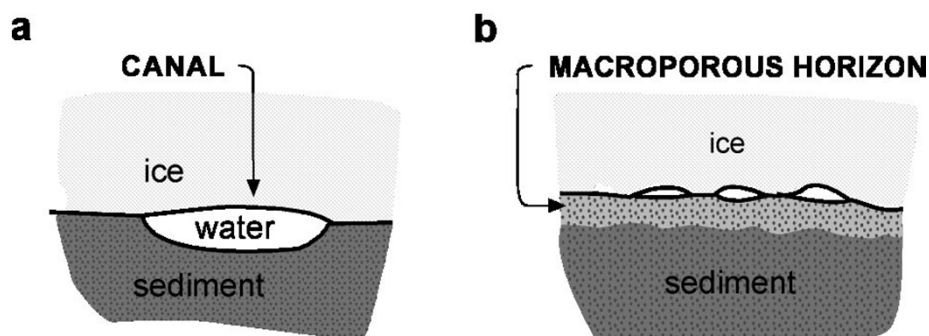- Flow at the ice-bed interface/ flow through a permeable substrate

a CANAL

ice

water

sediment

b MACROPOROUS HORIZON

ice

sediment

FIGURE 6.15 Cuffey & Paterson (2010)

Schematic depiction of water system elements on beds of sediment.

Relating input to output

- The storage equation is simply:

$$V_t = KQ_t \qquad \text{(Eq}^n\text{ 1)}$$

V is inflow (a volume flux, e.g. m^3 s^{-1}), K is a storage constant, Q is outflow (discharge/runoff), t denotes the timestep

- The continuity equation is then

$$\mathrm{d}V/\mathrm{d}t = I_t - Q_t \qquad \text{(Eq}^n\text{ 2)}$$

simply indicating that the rate of change of water storage equals the difference between the inflow and outflow rates

Water storage

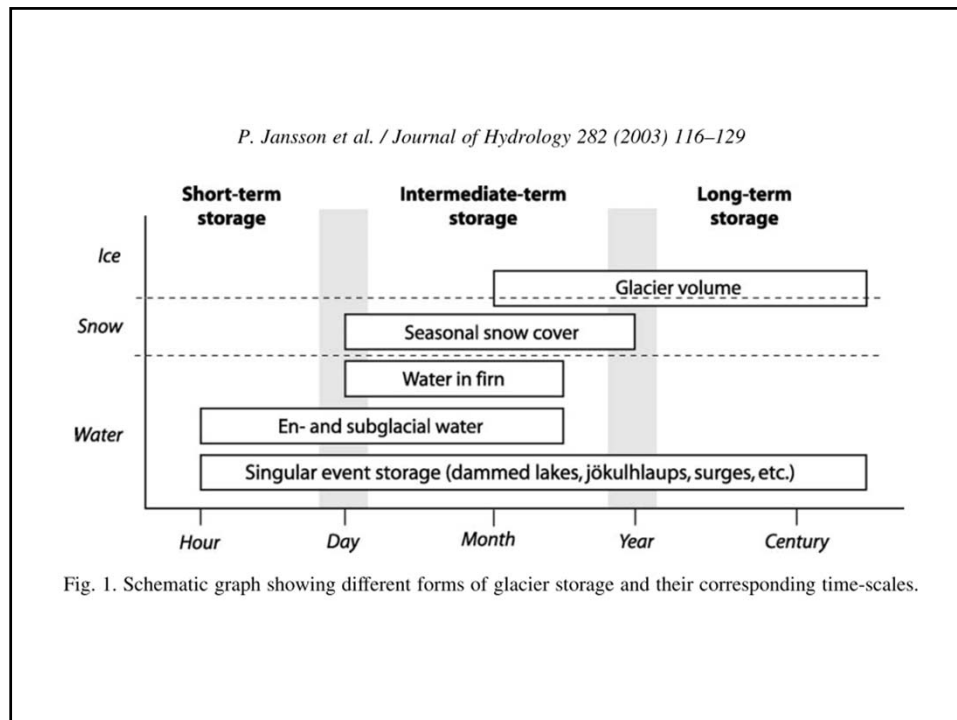- Water is effectively stored whenever input exceeds output, and can occur on a wide range of spatial and temporal scales. Combining Eq^{n}s 1&2 gives

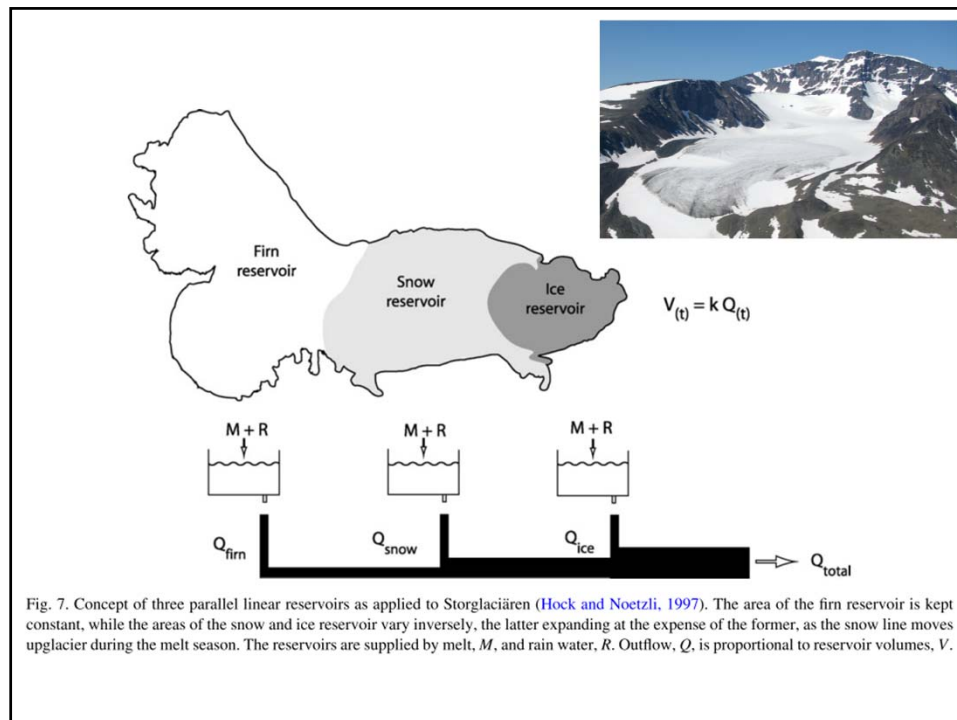$$K\,\mathrm{d}Q/\mathrm{d}t = I_t - Q_t \qquad \text{(Eq}^n\text{ 3)}$$

which simply rewrites storage change in terms of outflow and the storage constant

  - but integration of this equation gives expressions for *recession flow* and *recharge flow*

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P. Jansson et al. / Journal of Hydrology 282 (2003) 116–129

Fig. 1. Schematic graph showing different forms of glacier storage and their corresponding time-scales.

Model structure

- Most models assume two stores/pathways or *reservoirs*:
  - a fast one (accommodating high water fluxes)
    - typically icemelt drained through an efficient subglacial channel system
  - a slow one (accommodating low water fluxes)
    - typically snowmelt drained through an inefficient subglacial distributed system
- The system is broken down in a functional way, rather than thinking about the detailed, physical components
- While coarse, this implicitly links process, state and flux to retain the most important characteristics of the drainage pathways

Fig. 7. Concept of three parallel linear reservoirs as applied to Storglaciären (Hock and Noetzli, 1997). The area of the firn reservoir is kept constant, while the areas of the snow and ice reservoir vary inversely, the latter expanding at the expense of the former, as the snow line moves upglacier during the melt season. The reservoirs are supplied by melt, M, and rain water, R. Outflow, Q, is proportional to reservoir volumes, V.

Recession flow

- Need to determine recession/reservoir coefficients/storage constants
  - by tuning or recession analysis
- During periods when there is no recharge (fresh inflow) to the reservoir, the outflow (Q_t) at time t can be expressed as a function of the preceding flow (Q_0) at time t_0 and the storage constant K

$$Q_t = Q_0 e^{-(t-t_0)/K} \quad (\text{Eq}^n\ 4)$$

Eq^n 4 implies that during periods of recession flow, the value of *K* can be estimated from the slope of a semilogarithmic plot of discharge against time

Gurnell, A.M. 1993. How many reservoirs? An analysis of flow recessions from a glacier basin. *Journal of Glaciology* 39(132), 409–414.

Fig. 1. Examples of flow recessions.
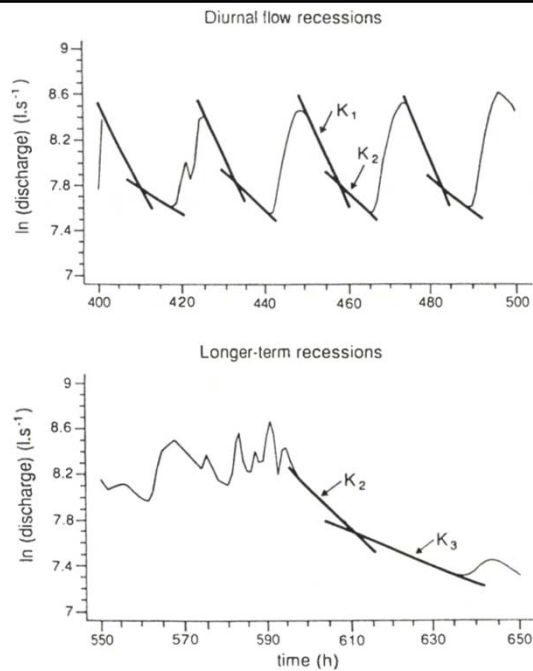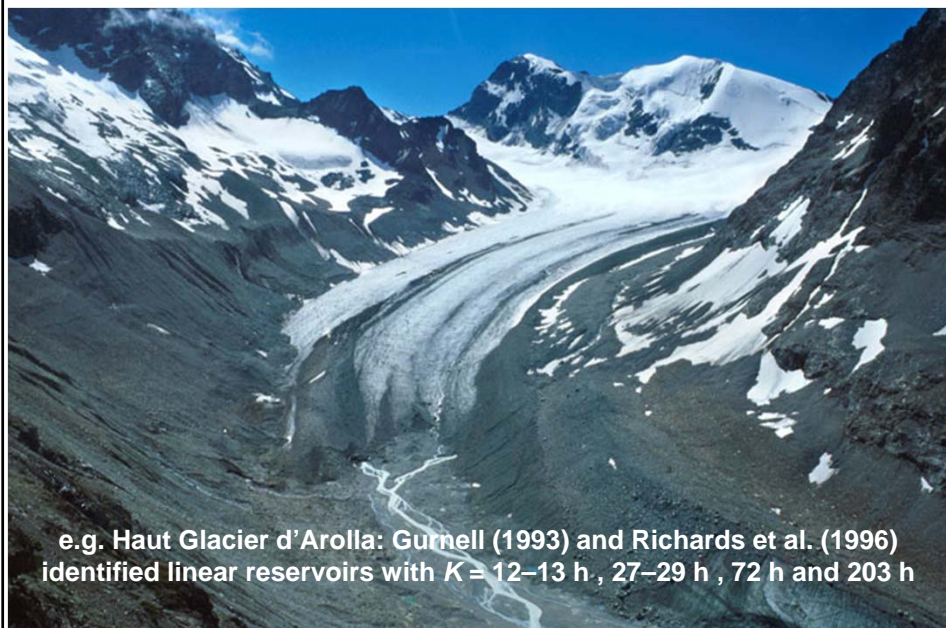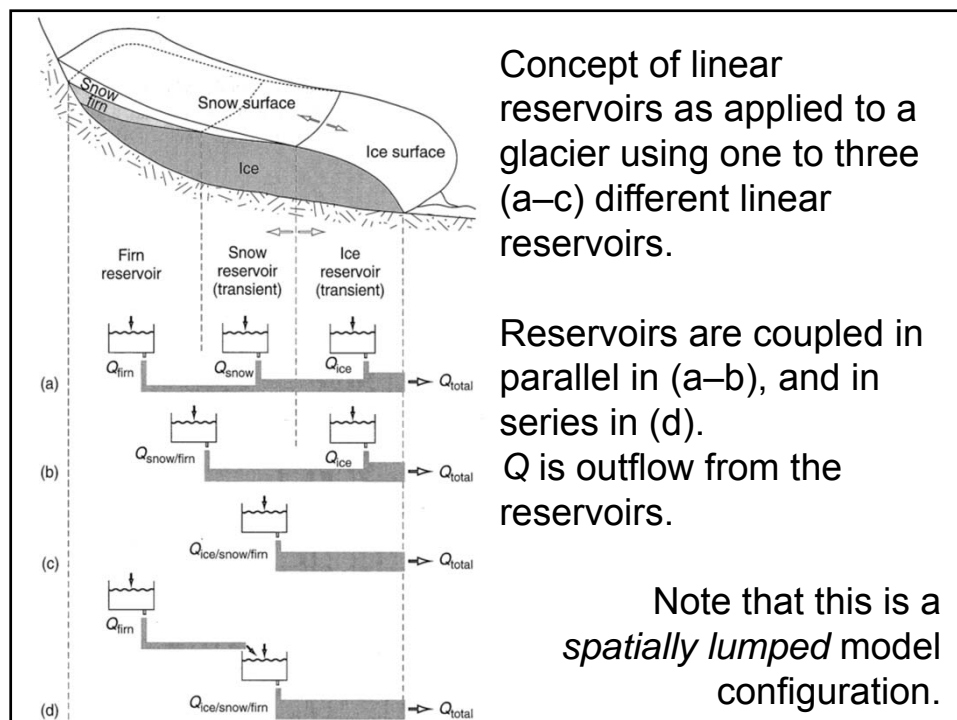

e.g. Haut Glacier d'Arolla: Gurnell (1993) and Richards et al. (1996) identified linear reservoirs with *K* = 12–13 h , 27–29 h , 72 h and 203 h

http://www.vaw.ethz.ch/people/gz/archive/gz_143_effects_of_climatic_fluctuations

Recharge flow

- Eq^n 4 defines the *recession flow*. If all input ceased, this would describe the runoff
- Actual runoff will consist of this recession flow, plus a *recharge flow* from continued water inputs:

$$Q_t = I_t\left(1 - e^{-(t-t_0)/K}\right) \quad \text{(Eq}^n\text{ 5)}$$

note that this has the same exponent as the recession flow, but depends on inflow at timestep t, whereas recession flow depends on outflow at the previous timestep

Concept of linear reservoirs as applied to a glacier using one to three (a–c) different linear reservoirs.

Reservoirs are coupled in parallel in (a–b), and in series in (d).
Q is outflow from the reservoirs.

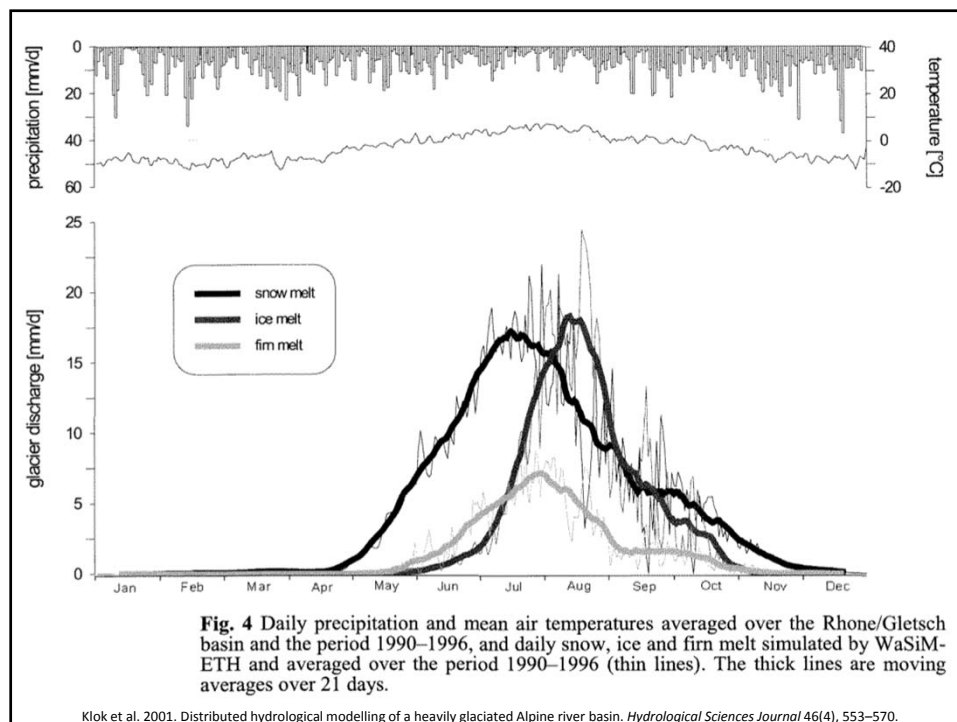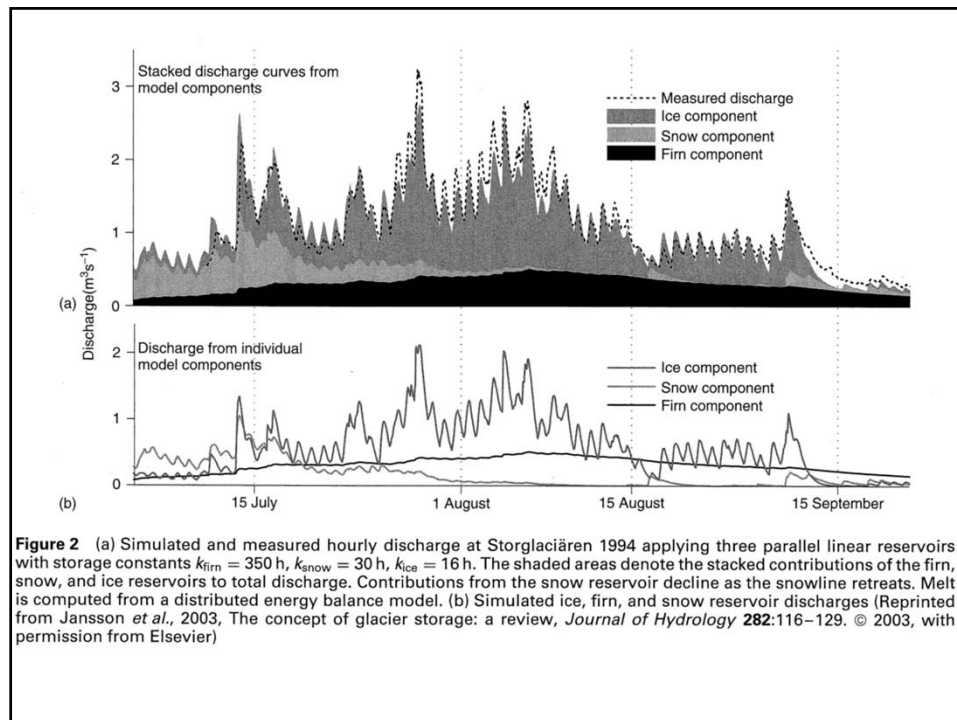Note that this is a *spatially lumped* model configuration.

Good advice from a good authority….

- "Everything should be made as simple as possible, but no simpler"
- "It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience"

  http://en.wikiquote.org/wiki/Albert_Einstein

Seasonal evolution

- Most studies assume constant reservoir coefficients
  - although drainage systems evolve seasonally
- But seasonal evolution *has* been taken into account by varying the *proportion* of the glacier drained by different reservoirs
  - Hock & Noetzli (1997), Klok et al. (2001) subdivided their study glaciers into firn, snow and ice reservoirs
  - as snowline retreats, more surface melt is routed to the faster-draining ice reservoir at the expense of the slower-draining snow reservoir
- So although this is a conceptual approach, it can represent physical processes

Figure 2 (a) Simulated and measured hourly discharge at Storglaciären 1994 applying three parallel linear reservoirs with storage constants $k_{firn} = 350\,h$, $k_{snow} = 30\,h$, $k_{ice} = 16\,h$. The shaded areas denote the stacked contributions of the firn, snow, and ice reservoirs to total discharge. Contributions from the snow reservoir decline as the snowline retreats. Melt is computed from a distributed energy balance model. (b) Simulated ice, firn, and snow reservoir discharges (Reprinted from Jansson *et al.*, 2003, The concept of glacier storage: a review, *Journal of Hydrology* **282**:116–129. © 2003, with permission from Elsevier)

Fig. 4 Daily precipitation and mean air temperatures averaged over the Rhone/Gletsch basin and the period 1990–1996, and daily snow, ice and firn melt simulated by WaSiM-ETH and averaged over the period 1990–1996 (thin lines). The thick lines are moving averages over 21 days.

Klok et al. 2001. Distributed hydrological modelling of a heavily glaciated Alpine river basin. *Hydrological Sciences Journal* 46(4), 553–570.

Fig. 3. Scatter plots of K against: a. Discharge at the commencement of flow recession. b. Day number.

Recession coefficients may vary with discharge and/or time

Hannah and Gurnell (2001) found values for a fast reservoir declined from 13 h to 5 h and for a slow reservoir from 45 h to 19 h over a melt season at Taillon Glacier, French Pyrénées.

Hock and Jansson (2005) defend the use of constant reservoir coefficients by suggesting that the difference between the fast reservoir and the slow reservoir is probably more important than seasonal drainage evolution.

Fig. 3. Sensitivity of r^2 values between measured and calculated discharge to the assumption on the storage constants k of firn, snow and ice in hours (Noetzli, 1996).

Models seem to have low sensitivity to large recession coefficients

Hock & Noetzli (1997)

Advantages/disadvantages

- The linear reservoir approach has the disadvantage of being a great simplification of complex, heteogeneous physical processes
- On the other hand, its has the advantage of being… a great simplification of complex, heteogeneous physical processes
- Offers modest data requirements and easy transportability

References

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Jansson, P., Näslund, J.-O., Rodhe, L. 2007. Ice Sheet Hydrology – a Review. *Svensk Kärnbränslehantering AB*, Technical Report TR-06-34. http://www.skb.se/upload/publications/pdf/TR-06-34webb.pdf.

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