UKPN Workshop, Sheffield 2012: Modelling in the Polar Sciences

Introduction to Modelling Glacier Hydrology

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Introduction to Modelling Glacier Hydrology: Linear Reservoir Approach

This session covers modelling *for hydrological purposes*: to estimate/interpolate/forecast fluxes of meltwater from a glacier drainage system for water resource issues, to quantify geomorphological or biogeochemical processes, to assess hydroecological status; to investigate drainage system structure, its seasonal change and the influence of that change on water storage and runoff patterns; to modulate estimates of runoff generated from melt models, since meltwater must of course traverse the glacier drainage system before being delivered to proglacial areas or fjords (this becomes increasingly important at greater spatial scales). This session does not cover the interaction of subglacial hydrology and ice dynamics, although it covers methods that potentially facilitate the interpretation of dynamic data.

We can estimate/predict snow- and icemelt (fairly) straightforwardly from energy-balance or temperature-index approaches. There is a huge literature devoted to melt modelling, but whatever approach is taken, if we are interested in runoff rather than mass balance, we have to allow for the fact that the meltwater that flows from a glacier is not equal to the melt at the same timestep: it has to travel through the glacier drainage system, in ways which may be rather simple, or somewhat complex. In the smallest, coldest glaciers dominated by supraglacial runoff from ice surfaces, there may only be a modest time lag before melt reaches the proglacial area. But in most cases, the surface meltwater is likely to penetrate the glacier at least to some extent and flow through it by variable pathways before emerging at the terminus.

Therefore, glacier hydrological modelling involves two steps: (i) melt prediction, (ii) water routing. This session covers step (ii).

Modelling water routing, rather than just the melt, requires us to specify the main components of drainage and their effects on the transformation of melt to runoff: this transformation essentially takes the form of a delay of some variable duration. Consequently, we have to make some choices about what form we expect the drainage system to take. Fortunately, there is much guidance to be found in the literature.

Fundamentally, most hydrological systems evolve different structures to accommodate water fluxes of different magnitudes, with a low-flux system which becomes a high-flux system when some threshold is crossed (and *vice versa*). Most systems feature a fast-flow, high-flux component and a slow-flow, low-flux component, and you may be able to think of many examples, both within the cryosphere and elsewhere. These include (fast-high component first, slow-low component second): river runoff/groundwater, overland flow/throughflow, episodic icemelt/ diffuse snowmelt, subglacial channels/linked cavities, flow at the ice-bed interface/flow through a permeable sediment bed.

At their most general level, hydrological models therefore need to relate the inflow to a store to its outflow. The storage equation is simply:

$$V_t = KQ_t \qquad \text{(Eq}^{\text{n}}\text{ 1)}$$

where V is inflow (usually a volume flux, e.g. $m^3\ s^{-1}$), K is a storage constant, Q is outflow (discharge/runoff), and t denotes the timestep. The continuity equation is then

$$dV/dt = I_t - Q_t \qquad \text{(Eq}^{\text{n}}\text{ 2)}$$

simply indicating that the rate of change of storage of water is equal to the difference between the rates of inflow and outflow: water is effectively stored whenever the former exceeds the

latter, and can occur on a wide range of spatial and temporal scales. Combining both equations above gives

$$K\,\mathrm{d}Q/\mathrm{d}t = I_t - Q_t \qquad \text{(Eq}^{\text{n}}\text{ 3)}$$

which simply rewrites storage change in terms of outflow and the storage constant, but integration of this equation gives the expressions for *recession flow* and *recharge flow*, which we shall be making use of below.

For the reasons and from the examples stated above, most models assume two stores/pathways or *reservoirs*: a fast one (which accommodates high water fluxes) and a slow one (which accommodates low water fluxes). In a glacier system, the former would typically represent icemelt drained through an efficient channelised system; the latter would typically represent snowmelt drained through an inefficient, distributed system. The flux of icemelt would be expected to be significantly larger than the flux of snowmelt. Note that we break the drainage system down in a functional way, rather than thinking about the detailed, physical components. While this implies a coarse approach to differentiating drainage, it implicitly links process, state and flux to retain the most important characteristics of the major drainage pathways (e.g. fast melting, fast-flow pathway, high-magnitude outflow: the cascade from melt to runoff is unbroken).

Having settled on an appropriate number of reservoirs, we need to specify the *storage constant* for each: this essentially describes how much of a delay each reservoir imposes on the generated melt (equivalent to the throughflow velocity); the combined effect of the number of reservoirs and their storage constants defines the form of the hydrograph. A range of storage constant values has been published, but there is considerable variation from glacier to glacier, so it is worthwhile attempting to derive constants for each new study, unless we are dealing with a particularly well-studied system. Constants are either obtained by tuning, that is, maximizing the agreement between modelled and measured glacier discharge (e.g. Hock and Noetzli, 1997; Klok et al., 2001) or by flow recession analysis (e.g. Gurnell, 1993; Hannah and Gurnell, 2001). When tuning, in this context as indeed in any other, it is important to bear in mind that good model fits can be achieved with unrealistic/implausible parameters. Flow recession analysis can be quite laborious, and does require an element of interpretation, but does derive an estimate of storage constants independent of the modelling procedure. As ever, both methods have their merits and limitations.

Let's have a look at flow recession analysis in a little more detail. Storage constants or *recession coefficients* (K, with units of hours) can be estimated from

$$K = -(t - t_0)/\ln(Q_t - Q_0) \qquad \text{(Eq}^{\text{n}}\text{ 4)}$$

where t_0 is the timestep preceding time t. This requires a knowledge of the hydrograph, so that the timing of and discharge at the onset and cessation of the flow recession can be defined. You may have been wondering why this method uses *linear* reservoirs, or else how it can be justified that something so inherently complex as a glacier drainage system can be approximated as linear. Flow recession analysis shows that this is indeed a reasonable approximation. During periods when there is no recharge (fresh inflow) to the reservoir, the outflow (Q_t) at time t can be expressed as a function of the preceding flow (Q_0) at time t_0 and the storage constant K

$$Q_t = Q_0 e^{-(t-t_0)/K} \qquad \text{(Eq}^{\text{n}}\text{ 5)}$$

This implies that during periods of recession flow, the value of K can be estimated from the slope of a semi-logarithmic plot of discharge against time (Figure 1), where recessions generated by outflow from a different reservoirs will plot as straight lines: hence linear reservoirs. Identification of more than one linear component is generally interpeted to

represent recessions from different reservoirs with different storage constants/recession coefficients.

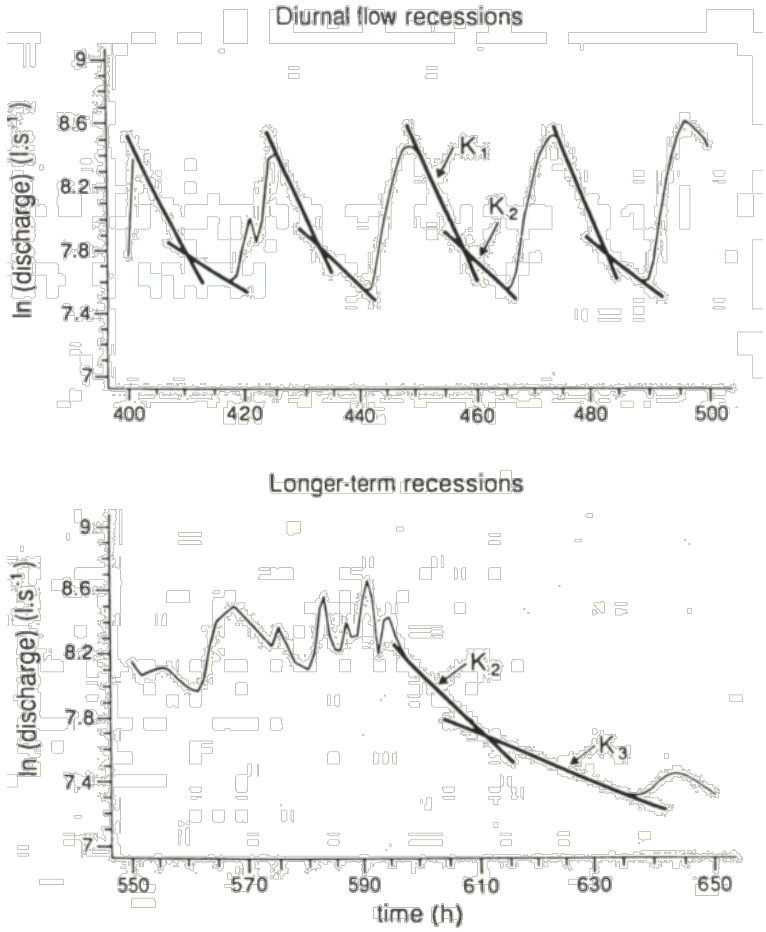

Figure 1: examples of flow recessions (Gurnell, 1993).

From analyses of several years' worth of runoff data from Haut Glacier d'Arolla, Gurnell (1993) and Richards et al. (1996) identified up to four linear reservoirs, with K = 12–13 h , 27–29 h , 72 h and 203 h.

Equation 5 defines what we shall call the *recession flow*. If all melting ceased, this would describe the runoff from the glacier. Actual runoff will consist of this recession flow, plus a *recharge flow* from continued inputs of melt and rainfall, defined as

$$Q_t = I_t\left(1 - e^{-(t-t_0)/K}\right) \qquad \text{(Eq}^{\text{n}}\text{ 6)}$$

note that this has the same exponent as the reservoir flow, but depends on inflow at timestep t, whereas reservoir flow depends on outflow at the previous timestep. We now have the components we need to define a simple, linear reservoir model of drainage:

$$Q_t = Q_0 e^{-(t-t_0)/K} + I_t\left(1 - e^{-(t-t_0)/K}\right) \qquad \text{(Eq}^{\text{n}}\text{ 7)}$$

which is of course the reservoir flow plus the recharge flow for a single reservoir; at least one more reservoir would typically be employed for a complete glacier model (Figure 2).

Typically, when applied to a glacier system, the reservoirs are arranged parallel, meaning they both contribute directly to runoff (Figure 2). Arranged in series, the slow reservoir could feed the fast reservoir (Figure 2), and this would make physical sense (Nienow et al., 1998), but in practice, the cumulative, parallel arrangement conceptually allows fast and slow reservoirs to co-exist, and there is evidence that this is also physically reasonable (Hubbard et al., 1995). Note that this is a spatially *lumped* approach: there is no differentiation of components beyond fast/slow, and there is no spatial distribution of components. The drainage system is

conceptualized as a set of reservoirs with predictable responses, without any attempt to simulate actual, physical components, the hydraulics of flow, or interaction between ice, water and substrate. The net influence of all of these factors is incorporated into *K*.

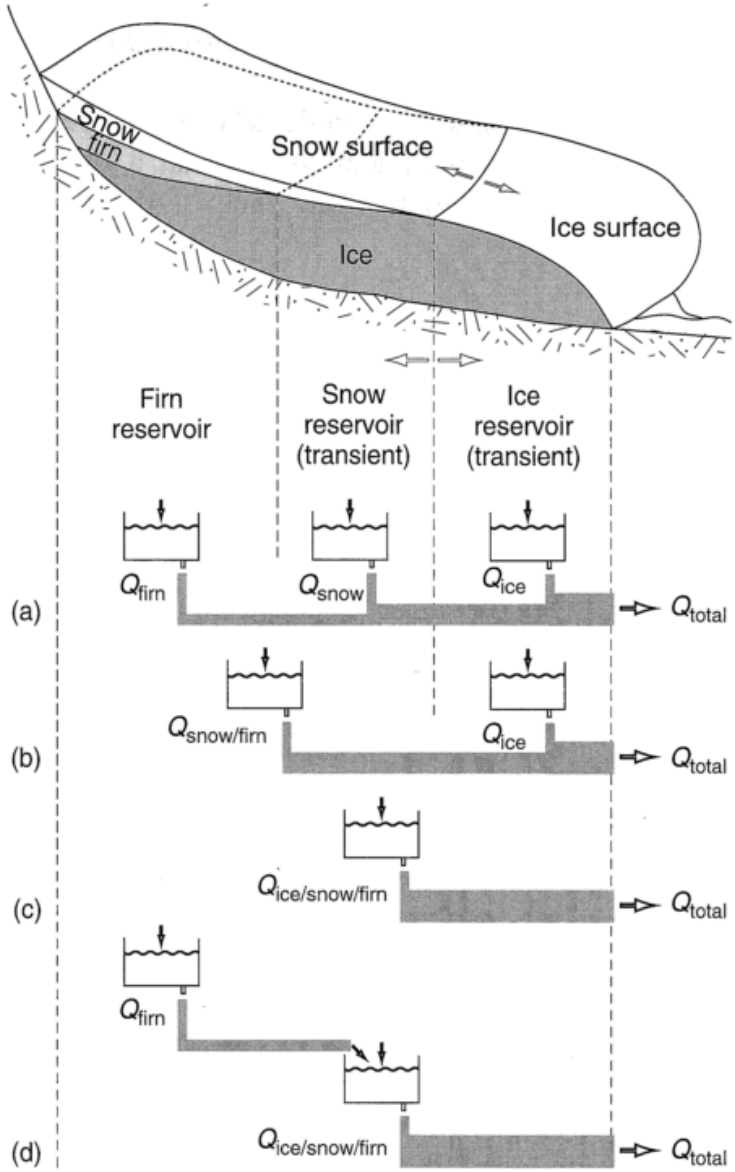

Figure 2. The concept of linear reservoirs as applied to glaciers using one to three different linear reservoirs. Multiple reservoirs are coupled in parallel in (a–b), and in series in (d). Q is outflow from the reservoirs (Hock and Jansson, 2005).

One of the most interesting aspects of glacier drainage systems is their seasonal evolution towards increasingly efficient states. However, most studies that have employed the linear reservoir approach have assumed constant reservoir coefficients. But drainage system evolution has been taken into account by varying the proportion of the modelled glacier drained by fast or slow reservoirs. For example, Hock and Noetzli (1997) and Klok et al. (2001) subdivided their study glaciers into reservoirs based on their surface characteristics: a firn reservoir above the previous year's equilibrium line, a (variable) snow reservoir defined as the snow-covered area outside the firn reservoir, and a (variable) ice reservoir, defined as the area of exposed ice. As the snowline retreats seasonally and more ice is exposed, more surface melt is routed to the faster-draining ice reservoir at the expense of the slower-draining snow reservoir, accouting for the seasonal evolution of the drainage system and (realistically) producing more peaked diurnal hydrographs. So although this is a conceptual

approach, it can represent physical processes in a realistic way without resorting to contrived parameters or empirical fixes.

Drainage evolution can also be indicated by flow recessions: Hannah and Gurnell (2001) found values for a fast reservoir declined from 13 h to 5 h and for a slow reservor from 45 h to 19 h over a melt season at Taillon Glacier, French Pyrénées. This underlines the value of careful scrutiny of data series: runoff needn't be regarded solely as calibration/validation fodder, it integrates the behaviour of the hydrological systems and so conveys useful information about processes taking place. This is likely to become even more important as we investigate larger glacier hydrological systems, such as those associated with the Greenland Ice Sheet. Gurnell (1993) found that *K* varied with discharge and with time during the melt season, indicating that reservoirs were actually not truly linear after all. She suggested that multiple linear reservoirs could be replaced with a single, nonlinear reservoir to achieve the most parsimonious and efficient model for purposes such as operational forecasting. However, while not perfectly linear, most drainage components identified by flow recession analysis or model tuning behave in a sufficiently linear fashion that the approach is not invalidated, especially bearing in mind the many assumptions, approximations and aggregations that characterize glacier modelling approaches of all complexities[1]. Similarly, Hock and Jansson (2005) defend the use of constant reservoir coefficients by suggesting that the difference between the fast reservoir and the slow reservoir in most studies is probably sufficiently pronounced that effects from seasonal evolution of drainage efficiency are masked.

Altogether, the linear reservoir approach has the disadvantage of being a great simplification of complex, heteogeneous physical processes. On the other hand, its principal advantage is that it is a… great simplification of complex, heteogeneous physical processes. As ever, the best solution is simplest one that actually gets the job done. In the specific context of glacier hydrology (because linear reservoir models are just as applicable to other areas of hydrology), this approach has the additional virtues of modest data requirements and easy transportability. Although not suited to coupling with ice-dynamics models, linear reservoir models are worth considering for most hydrological applications.

References

Gurnell, A. 1993. How many reservoirs? An analysis of flow recessions from a glacier basin. *Journal of Glaciology* 39, 409–414.

Hannah, D., Gurnell, A. 2001. A conceptual, linear reservoir runoff model to investigate melt season changes in cirque glacier hydrology. *Journal of Hydrology*, 246, 123–141.

Hock, R., Jansson, P. 2005. Modeling Glacier Hydrology. In Anderson, M.G. (Ed.), *Encyclopaedia of Hydrological Sciences*, Wiley.

Hock, R., Noetzli, C. 1997. Areal melt and discharge modelling of Storglaciären, Sweden. *Annals of Glaciology* 24, 211–216.

Hubbard, B., Sharp, M., Willis, I., et al. 1995. Borehole water level variations and the structure of the subglacial hydrological system of Haut Glacier d'Arolla, Switzerland. *Journal of Glaciology* 41, 572–583.

Klok, E., Jasper, A., Roelofsma, K., et al. 2001. Distributed hydrological modelling of a heavily glaciated Alpine river basin. *Hydrological Sciences Journal* 46, 553–570.

Nienow, P., Sharp, M., Willis, I. 1998. Seasonal changes in the morphology of the subglacial drainage system, Haut Glacier d'Arolla, Switzerland. *Earth Surface Processes and Landforms* 23, 825–843.

Richards, K., Sharp, M., Arnold, N., et al. 1996. An integrated approach to modelling hydrology and water quality in glacierised catchments. *Hydrological Processes* 10, 479–508.

[1]"The main conclusions are *first*, that truly linear reservoirs are not supported by recession-flow analysis, but *secondly* such an analysis does aid in the general understanding of glacier hydrology." (Gurnell, 1993:414).

Exercise: Getting a working linear reservoir model of glacier hydrology

Equation (7) above gives us the fundamental model:

$$Q_t = Q_0 e^{-(t-t_0)/K} + I_t\left(1 - e^{-(t-t_0)/K}\right)$$

This tells us that we can predict the runoff from a reservoir at any given time step if we know the runoff at the previous time step, the melt plus rainfall at the current time step, and the reservoir coefficient. Bear in mind that this calculation has to be performed for each reservoir, and most models use two or more.

The simplest, and in many ways the most efficient, way to put a working model together is to use a spreadsheet, so let's create a two-reservoir model in *Excel*. Let's break this down into three steps. First, consider what input data are needed (these are values that change with each time step); second, consider what model parameters need to be specified (these are values that describe some system component and remain constant); third, consider what calculations are necessary. The latter may seem obvious (but don't worry if not!), but the choice of how to aggregate or disaggregate calculations has a big impact on how efficient the modelling process will be. For a two-reservoir model based on Equation 7, we could break the actual calculations down into anything between one and seven steps. As a rule, the more steps you break down to, the simpler it is to troubleshoot and to make changes and see their effects. On the other hand, fewer steps can be more efficient, as you can soon find yourself dealing with dozens of columns and thousands of rows for a melt-season-length dataset. Note that we aren't going to tax the computer very much with this model: it's not at all computationally demanding (though it does help to have a big screen to view the speadsheet).

Required input data:

- Input flux at each time step, i.e. melt plus rainfall.
- Total runoff at the first time step (subsequent time steps use the modelled runoff as input)

That's all. We could build a more sophisticated model if we had more data, but this is the minimum required. The input flux could be measured or modelled. *Runoff could be measured or estimated.* Note that these data need to be in the same units, which will likely mean converting a melt rate in mm d^{-1} to a flux in $m^3\ s^{-1}$, which will require a knowledge of the glacier surface area.

Parameters that need to be specified:

- Number of reservoirs
- Storage constants for each reservoir
- Fraction of flow in each reservoir

Because we have two reservoirs, we need some kind of rule to prescribe the proportion of drainage which goes to each reservoir. If appropriate data were available, it would be reasonable to assume that flow generated above the snowline goes to a slow reservoir and flow generated below the snowline goes to a fast reservoir. We don't have information on snowline position, so we'll assume that 90% of flow goes to a fast reservoir and 10% goes to a slow reservoir. Storage constants could be estimated from the literature, from recession flow analysis, or from model tuning. The latter two methods would require a total glacier runoff time series: this would be useful in any case to validate the model.

What needs to be calculated:

- Recession flux and recharge flux for each reservoir
- Sum of the above four fluxes to give total glacier runoff

This is our output.

What to do with the spreadsheet:
Conventions used below: reservoir 1 or 2 is abbreviated to res1 or res2; Excel formulae are given in square brackets, e.g. [=1-G3].
Open up *Hydromodel_exercise.xlsx*. This contains data for a 10-day period in the late summer from the Svalbard glacier Finsterwalderbreen. You'll see that there are three date-time columns: Date, Hour and DecDay (which is day of year where January 1 = 1 and December 31 = 365 [in a non-leap-year], plus the hour of the day as a fraction of 24, i.e. 6 pm = 0.75). Input is total melt plus rainfall for the glacier, and output is the measured runoff: for most applications, runoff is what we'd be trying to determine, but it's included here for "training purposes." Of course, we do need the initial value of runoff. It might be useful to copy the data to a new tab in the workbook, so that you can revert to an original, unmodified version if you need to later.

We're going to start work on spreadsheet row 3 (DecDay 210.29), because we need that previous value of discharge (Q_0, in row 2 for DecDay 210.25) to get us started. Add 4 column headings after output: reservoir 1 constant, reservoir 1 fraction, reservoir 2 constant, reservoir 2 fraction (these are the "Parameters that need to be specified": recession coefficients and fractions of total drainage to each reservoir). Insert values into row 3 under each heading: 15 for reservoir 1 constant, 0.9 for reservoir 1 fraction, 50 for reservoir 2 constant; when it comes to the reservoir 2 fraction, make that equal to 1 – reservoir 1 fraction, i.e. type [=1-G3], then it will recalculate itself if you decide to change the fractions. Enter the four parameters this way – rather than in single reference cells – because when you drag/paste formulae for all the time steps, Excel will change the cell references to respect the structure of the initial equation, so it won't work if you just enter the values in one cell. Also, this enables you to vary the parameters with time later on, if you wish.

Now insert 5 headings at the tops of columns J–N: res1 recession, res1 recharge, res2 recession, res2 recharge, TOTAL (these are "what needs to be calculated.") For the first set of calculations, we're going to use the specified parameters, the discharge at time step 0 (cell E2 in this case, DecDay 210.25), and the input at time step 1 (row 3, DecDay 210.29). Below are the formulae; as you can see, they're simple sums which make use of the EXPONENT function (which returns *e* raised to the power of a given number). A simplification is that $(t - t_0)$ becomes 1, as we are working on a one-hour time step.

Recession flow is $Q_t = Q_0 e^{-(t-t_0)/K}$ (Eqn 5)
For res1 enter [=(E2*G3)*EXP(-1/F3)]

Recharge flow is $Q_t = I_t\left(1 - e^{-(t-t_0)/K}\right)$ (Eqn 6)
For res1 enter [=(D3*G3)*(1-EXP(-1/F3))]

(G3 here contains the value for the flow fraction we're assigning to res1). Then do the same for res2, making sure to use the appropriate cell references (of course). Under TOTAL, enter a sum function for the four components res1 recession, res 2 recharge, res 2 recsession, res 2 recharge [=SUM(J3:M3)]. Voilà, the first time step is calculated. If you haven't already, you may wish to tidy up the formatting. Make sure all the values are in "number" format (sounds obvious, but it's not the default). Then we're off, right? We can highlight the cells [F3:N3] and drag all the way to the end of the time series, to fill the remaining rows (= time steps) with the formulae necessary to determine the various flows. Not quite, unfortunately: we need to make a modification in the next row (row 4, DecDay 210.33). Remember we're only using the initial value of discharge to establish the model: discharge is usually what we're attempting to determine; for the purposes of this exercise, we'll assume we don't know any more values for now. So what we're going to do is slightly modify the row 3 formulae to use the calculated discharge from time step 1; then, when we fill the remaining cells to calculate the rest of the

time series, the formulae will take the discharge from the calculated total at the previous time step, which is what you'd do if you didn't know the actual discharge. In fact, we only need to modify the recession flows:

In line 4, for res1 enter [=(N3*G4)*EXP(-1/F4)]

And make the same modification for res2, using the value in N3 as the previous time-step discharge, instead of E2. *Now* you can highlight [F3:N3] and drag all the way to the end of the time series. Voilà (again), we have modeled discharge for the 10-day period.

Error metrics (validation)

The process of evaluating the performance of a model is known as *validation*. Essentially, we look at two aspects of performance: the fit of the modelled time series to the observed one, and the total error of the modelled time series. For these purposes, we assume that the observed series is the "true" value, although as a field measurement, it is still only an estimate of the discharge in reality. There is no single criterion that will confirm that the model is a "good" one, and whether the performance is acceptable or not should be assessed by a range of criteria, with reference to the purpose of the study. Sometimes, very good apparent model fits can be achieved from spuriously-tuned parameters, and typically, fit degenerates when the model is applied beyond the range of its calibration. It's best therefore to quantify fit and error in a range of ways, including visual inspection of the time series – if you look at the modelled/observed series in this exercise, you will notice that the fit is noticeably better in some parts of the series than others. When dealing with hydrological series in glacial environments, it's always wise to be mindful of the key timescales of change: seasonal and diurnal. So a good way of assessing model performance is to plot the residual series (residual = observed – modelled). Ideally, we'd want the magnitude of the residuals to be small compared to that of the modelled values, and we'd want them to be random in pattern: this is a theme that we take up in more detail when looking at time-series approaches in the next exercise. For now, here are three useful error metrics for hydrological time series:

Mean Error (*ME*) reflects the overall tendency of modeled runoff, Q^*, to underestimate (if *ME* is positive) or overestimate (if *ME* is negative) measured runoff, Q:

$$ME = \sum(Q - Q^*)/df \qquad \text{(Eq}^{\text{n}}\text{ 8)}$$

where *df* is degrees of freedom, determined as $N - P - 1$, where N is the number in the sample and P is the number of predictors. If you apply this to the data in this exercise, you'll see that *ME* is negative, and looking at a plot of the series, we do indeed generally overestimate runoff.

Root Mean Square Error (*RMSE*) provides the standardized, mean model error for runoff:

$$RMSE = \sqrt{\sum(Q - Q^*)^2/df} \qquad \text{(Eq}^{\text{n}}\text{ 9)}$$

Nash-Sutcliffe efficiency criterion, *E* provides an assessment of the goodness-of-fit of the modelled time series to the observed one:

$$E = 1 - \sum(Q - Q^*)^2/\sum(Q - \bar{Q})^2 \qquad \text{(Eq}^{\text{n}}\text{ 10)}$$

The range of E lies between 1.0 (perfect fit) and $-\infty$. An efficiency of lower than zero indicates that the mean value of the observed time series would have been a better predictor than the model (Krause et al., 2005).

Assuming you've preserved the positions of the various calculated values with respect to the rows and columns of the spreadsheet as suggested above, these error metrics can be calculated in one step from the following Excel formulae:

Mean error [=SUMPRODUCT(E3:E241-N3:N241)/(COUNT(E3:E241)-1-1)]

RMS error [=SQRT((SUMPRODUCT((E3:E241-N3:N241)^2))/(COUNT(E3:E241)-1-1))]

Nash-Sutcliffe efficiency [=1-SUMPRODUCT((E3:E241 - N3:N241)^2)/SUMPRODUCT((E3:E241 - Q3)^2)]

SUMPRODUCT is a handy command in this context because it allows us to perform simple calculations inline then sum the results, without having to generate new columns of intermediate-step data in the spreadsheet. For the Nash-Sutcliffe efficiency, caculate the mean of the observed discharge values (the "output" column) in cell Q3 (or put it where you prefer and change the cell refrence in the formula).

Reference: Krause, P., Boyle, D., Bäse, F. 2005. Comparison of different efficiency criteria for hydrological model assessment. *Advances in Geosciences*, 5, 89–97.

What to do now

Use your modelled runoff time series for the following:

- Plot the observed ("output") and modelled ("TOTAL") time series to see how well the model fits the temporal pattern of discharge.
- Plot the "input" and modelled time series to see how the drainage system modulates the melt and rainfall fluxes.
- Plot the res1 and res2 recession flows to see their relative magnitudes
- Determine the residuals from the modelled time series and plot them to see if there is any temporal pattern to the model errors.
- Calculate the cumulative modelled water flux over the 10-day period (you will of course need to change the time units).
- Calculate the Mean and RMS Error of the modelled series.
- Calculate the Nash-Sutcliffe efficiency criterion for the modelled series.
- You could look at the *sensitivity* of the model results to variations in the various parameters, such as initial discharge, different values of *K*, different flow fractions in each reservoir. These are all easy to change in the spreadsheet, but you will probably want to create copies of the model on new tabs, so that you can keep track of your work. As an example, Hock and Noetzli (1997) found that the linear reservoir model they applied was quite insensitive to the *K* value of the slow-flow snow and firn reservoirs (Figure 3).

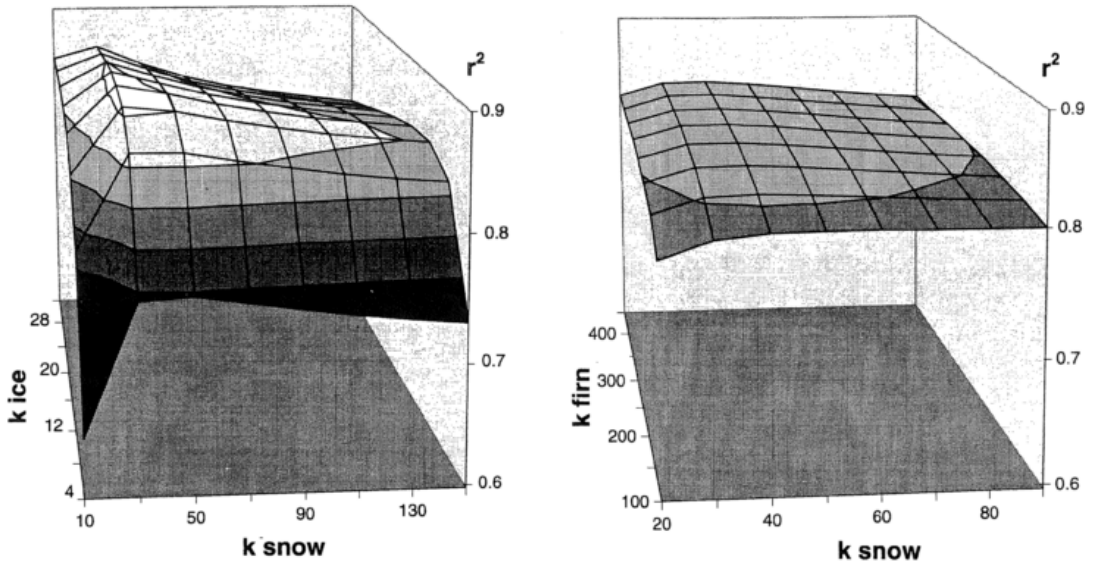

Figure 3. R^2 values mapped as a function of two model parameters to yield a response function: results are relatively insensitive to the choice of storage constants over a large range of *K* values (Hock and Noetzli, 1997)

Determining Recession Coefficients/Storage Constants by Recession Analysis

For the linear reservoir model exercise, we used recession coefficients/storage constants, *K*, with values of 15 h for the fast reservoir and 50 h for the slow reservoir. These are average seasonal values for the dataset from which the exercise data were derived, although there is some evidence that the values of *K* decline during the melt season, which may be a contributing factor to the general overestimation of low discharges (I'll tell you the main reason if you're curious, but it's mostly an artefact of the way the exercise was set up, rather than an actual property of the system!) As Hock and Noetzli (1997) found, changing the value of *K* for the slow reservoir has very little effect: to a large extent, this is due to the small proportional contribution of the slow reservoir in this case. On the other hand, the value for the fast reservoir has more impact, for the opposite reason: you could try changing *K* for the fast reservoir to about 12–13 h and see what the effect is.

You could obtain the values of *K* by model tuning if you knew both melt and rainfall inputs, and runoff from the system. Within Excel, this is a bit laborious, but can actually be achieved with the *Solver* tool, which can find solutions to formulae with two unknown values: clearly, you're not actually solving an equation for a particular parameter, you're finding pairs of parameter values that give the best match between observed and modelled outputs for an *instrumental* period. *Solver* is actually very powerful, but you would have to apply it one time step at a time. This may not be prohibitive: never underestimate the value of a day's getting on with a basic method versus a couple of weeks' thinking about a more sophisticated one; but as ever, what is most appropriate depends on your aims, circumstances etc. But if you actually have runoff data, you can conduct a recession analysis to determine *K* directly. An advantage of this approach is that *K* is derived independently from the model, which ought to make model results more robust. A disadvantage is that recession analysis is laborious: there's no simple way to automate it (if you find a way, I'd be grateful to hear about it).

Open up the file *Flow_recessions_exercise.xlsx*. Inside, on the "Data" tab, you'll find three days' runoff data from the same dataset as the linear reservoir exercise. These three days show nice flow recessions, so they're good examples to learn on. *A flow recession is any interval during which the runoff declines*; in a glacial environment, typically characterised by strong diurnal runoff cycling in late summer, we'd expect daily flow recessions as melt inputs decline from mid-afternoon on one day until mid-morning the next. Longer flow recessions may occur following large releases of water from storage. Clearly, we only want to be analysing significant recessions, so a cut-off is desirable: for these data, I suggest only recessions of 6 h or longer, which still occur most days. We don't want to look at runoff itself however, we want to look at the rate of change of runoff, so we substract the previous hour's vaue from the current one to obtain this: this has already been done in the spreadsheet, in column *Q change* (all runoff units are $m^3 s^{-1}$, it's obviously good practice to specify them whenever they occur, but in a spreadsheet, it becomes a bit of a waste of space, so we'll take the units for granted now). I've coloured in red the first significant flow recession, which starts at Decimal Day 212.792 (7 pm on 31 July). I've started here, although the *Q change* value at this time step is positive: what is important is the change from step-to-step, so regard this as the starting point for the negative change we obtain by the next time step, 212.833 (i.e. between 7 and 8 pm on 31 July, the runoff declines by 0.318 $m^3 s^{-1}$). The same reasoning means that we *don't* extend the length of the recession to the first positive change at the end of the recession (i.e. we cut the recession off at 213.208, following which discharge starts rising again). This gives us a 10-hour flow recession.

If you plot *Q* against time for this interval, you'll see that the runoff declines quickly initially, but less quickly towards the end. This is a strong indication that there is both a fast and a slow reservor contributing to the flow. Therefore, in the two columns following *Q change*, the Q data have been divided into fast- (*Res1 Q*) and slow- (*Res2 Q*) declining components: some trial-and-error is required here, though I prefer to think of it as approaching the problem heuristically… The scatter plot showing *Res1 Q* as blue and *Res 2 Q* as red is what I consider is the best separation of the components based on the break of slope. Sometimes this is a little ambiguous, but this example is fairly clear-cut. Note, very importantly, that the data in the *Res1/2 Q* columns have been natural-log transformed, using [=LN(*cellref*)]: this makes them more linear, and recall that the definition of a linear reservoir is a component from which drainage conforms to a straight-line on a semi-logarithmic plot of discharge against time. Linear regression lines have been used simply to help interpret the semi-log scatter plot (and hopefully you'll agree that both components *are* very linear), but the actual values of *K* are obtained from the original data:

$$K = -(t - t_0)/\ln(Q_t - Q_0) \qquad \text{(Eq}^{\text{n}}\text{ 4)}$$

To emphasize, Q_t and Q_0 are the original discharge values, we're not log-transforming data which are already log-transformed. Again, $t - t_0$ is 1 for an hourly time-step. Moving on to the "Stats" tab of the workbook, you'll see that a template has been set up, into which data from the "Data" tab can be entered, in order to calculate values of *K* (at the end, in red); it includes a few other descriptives to aid interpretation (e.g. Gurnell (1993) found that *K* varied with discharge at the onset of the recession). In order to determine *K*, you need to fill in the cells highlighted in yellow.

As you can therefore see, identifying and quantifying flow recessions involves an element of interpretation, although this interpretation is simple to justify by reference to the straight-line fits of the semi-log plot. ***Your task***, is to do for days 213 and 214 of the spreadsheet, what's already been done for day 212: look through the *Q change* data to identify the recession, separate the recession into two components using the semi-log plot to assist you, and when you're satisfied with your separation, enter the appropriate values in the "Stats" tab (you obviously have what's already been entered as a guide: click on the cells to see how it's done, there's no need to type everything out).

The data for these two exercises have been published in:

Cooper, R., Hodgkins, R., Wadham, J., Tranter, M. 2011. The hydrology of the proglacial zone of a High-Arctic glacier (Finsterwalderbreen, Svalbard): Sub-surface water fluxes and complete water budget. *Journal of Hydrology* 406, 88–96, doi:10.1016/j.jhydrol.2011.06.008.

Wadham, J.L., Tranter, M., Hodson, A.J., Hodgkins, R., Bottrell, S., Cooper, R., Raiswell, R.. 2010. Hydro-biogeochemical coupling beneath a large polythermal Arctic glacier: implications for sub-ice sheet biogeochemistry. *Journal of Geophysical Research (Earth Surface)* 115 (F04017), doi:10.1029/2009JF001602.

Hodgkins, R., Cooper, R., Wadham, J., Tranter, M. 2009. The hydrology of the proglacial zone of a High-Arctic glacier (Finsterwalderbreen, Svalbard): surface and atmospheric water fluxes. *Journal of Hydrology* 378, 150–160, doi:10.1016/j.jhydrol.2009.09.020.

Hodgkins, R., Cooper, R., Wadham, J. and Tranter, M. 2003. Suspended sediment fluxes in a High-Arctic glacierised catchment: implications for fluvial sediment storage. *Sedimentary Geology* 165, 105–117, doi:10.1016/S0037-0738(03)00218-5.

Cooper, R.J., Wadham, J.L., Tranter, M., Hodgkins, R. and Peters, N. 2002. Groundwater hydrochemistry in the active layer of the proglacial zone, Finsterwalderbreen, Svalbard. *Journal of Hydrology* 269, 208–223, doi:10.1016/S0022-1694(02)00279-2.