

I'd say that a vector space is first and foremost an Abelian group. It also has the action of a field. A vector space is an enrichment of an Abelian group. There are two ways to enrich an Abelian group $(V, +)$: one way is by making it a ring $(V, +, \cdot)$, another way is by making it a vector space over a field F . Both of those enrichments can be enriched even further to become an algebra over a ring.

