

I would like to avail myself to very simple but illuminating examples, and highlight their similarities, despite the difference in mathematical context:

1. The real line, with the usual addition, and multiplication by a real constant.
2. The 2-dimensional Cartesian plane, with the usual vector addition, and multiplication by a real number.
3. Increase the dimension n , and talk about \mathbb{R}^n .
4. Let A be a given set. Consider the power set of A and the field $\mathbb{Z}_2 := \{0, 1\}$, with vector addition as disjoint union, and scalar multiplication defined by the two equations (1) $1 \cdot X := X$ and (2) $0 \cdot X := \emptyset$ for all subsets X of A . Then $(\mathcal{P}(A), \dot{\cup}, \cdot)$ forms a vector space.

The similarities to be highlighted include the (carrier) set V which is the vector space, the vector addition, and the scalar multiplication.

Then comes the set of axioms to be satisfied.