I would say that a vector space is just a collection of objects, together with a way to combine those objects, and a list of rules that govern how we combine them. We see examples of this from the moment we first learn about addition and multiplication of numbers. We learn how to add and multiply numbers, and then we learn that there are certain rules that addition and multiplication of numbers satisfy. Then we eventually learn about functions and that there are ways to combine those functions. We can add them, multiply them, or compose them. We can also multiply them by numbers. Then we learn that there are certain rules that these operations satisfy. Some of these are the same rules that we saw with addition and multiplication of numbers. We have also leaned about vectors and matrices, and that there are similar operations that we can do with these, such as addition and various forms of multiplication, including multiplying by numbers. Then we learned that these operations satisfy many of the same rules that we observed with operations on numbers and functions.

The purpose of the abstract notion of a vector space is to extract all of these common concepts, objects with operations that satisfy a certain set of rules, and then we use these rules as a foundation for proving other things, things that must be true for all objects with operations that satisfy this list of rules. Then we explore other things in mathematics and in the world around us that share these same features, objects with operations that satisfy the same list of rules. When we find something that matches these features, we know that they automatically must satisfy all of the extra properties and theorems that we proved about abstract vector spaces, without the need to prove them again in this specific instance.