

I would say that a vector in 2-dimensional space, such as $(1, 2)$, records a movement from a starting point to an ending point. We have a special point, which we call the origin, with coordinates $(0, 0)$. The vector $(1, 2)$ is like an arrow starting at the origin, and going to its end point, which is one unit right horizontally, and 2 units up vertically from the origin. Vectors are often used to describe the force applied to an object, such as the force being applied by gravity.

It is useful to add vectors together. Take $(3, -5)$ and $(-1, 2)$. Each of these corresponds to an arrow starting at the origin. To find the sum $(3, -4) + (-1, 2)$, we slide the $(-1, 2)$ arrow so that its starting point is now at the ending point of the $(3, -4)$ arrow (which changing its direction or length). We now look at the new ending point of the $(-1, 2)$ arrow, and we see that it is equal to $(2, -2)$, which is $(3 + (-1), (-4) + 2)$. One reason for adding vectors in this way is that it shows how forces compound on an object. So if gravity is applying a vector of $(3, -5)$ to an object, and an engine is applying a vector of $(-1, 2)$ to the same object, the combined force that the object feels is $(-2, 2)$.

We can also multiply vectors by numbers, so 2 times $(-1, 2)$ is just $(2 \times (-1), 2 \times 2) = (-2, 4)$. This corresponds to the force becoming twice as large.

When we work in an abstract vector space we are still thinking about vectors, addition, and multiplication. But now our vectors do not have explicit coordinates. So instead of talking about vectors with coordinates $(-1, 2)$ and $(3, 4)$, we might just have abstract vectors, which we label with something like u and v . Instead of adding and multiplying coordinates, we just have rules that tell us what the abstract vector $u + v$ is.

The reason we care about abstract vector spaces is that by studying them, we can study all concrete vector spaces simultaneously, and learn how they behave.