LOCALISED FLEXURAL WAVES IN IMMERSED STRUCTURES AS EFFICIENT MECHANISM OF AQUATIC PROPULSION

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1 INTRODUCTION

This paper describes the results of the experimental verification of the idea of wave-like aquatic propulsion of manned marine vessels first published by one of the present authors (V.V.K.) more than 10 year ago [1]. The idea is based on employing the unique type of localised flexural elastic waves propagating along edges of wedge-like structures immersed in water [1-4]. Such wedge-like structures supporting localised waves can be attached to a body of a small ship or a submarine like fish fins and used for aquatic propulsion (see Figure 1). The principle of using localised flexural waves as a source of aquatic propulsion is similar to that used in nature by some fish, e.g. stingrays, that utilise wave-like motions of their large pectoral fins (wings) for moving forward.

Note that there is a long history of human efforts to imitate fish swimming in man-made marine craft, especially in autonomous underwater vehicles (AUV) that are now being used in a wide variety of scientific investigations and surveillance operations [5-8]. Different types of fish swimming modes have been tried extensively with a limited success. For example, the beating of the caudal fin or tail for propulsion, which is used most extensively in nature, has been subjected to a great deal of engineering imitations using simple oscillators or smart actuators [5,6,8]. However, caudal fin type of propulsion, although applicable to AUV, is absolutely unsuitable for man-inhabited marine vessels, as the main body of the vessel would be rocked from side to side in reaction to such a propulsion, making onboard conditions unbearable for sailors and passengers.

For the above reason, the only mode of fish swimming which seems to be attractive for manned vessels is the undulatory wave-like motion seen in stingrays and skates. Stingrays and skates have large triangular pectoral fins (wings) that extend over the entire length of their body. The type of propulsion used by these fish is called 'rajiform' [6]. It is this specific undulatory swimming mode used by stingrays that is inspirational and the most close to the wave-like propulsion employing localised wedge elastic waves described in the present work. It is vitally important for the application of wedge elastic waves for propulsion of man-inhabited vessels that, in spite of the wings vibration, the main body of the craft remains virtually quiet because the energy of localised elastic waves is concentrated near the wings' tips [1].

The expected main advantages of the new wave-like propulsion method of marine craft over the existing ones, e.g. jets and propellers, are the following:

1. It is quiet, which is a particularly attractive feature for surveillance operations and for applications where minimal disturbance of wildlife is important;
2. It is expected to be energy-efficient since it follows nature (this, however, needs to be proven both theoretically and experimentally);
3. It is environmentally friendly and safe for people and wildlife.

Envisaged applications of wave like propulsion are small and medium research and pleasure man-inhabited ships and submarines. Obviously, it can be used also for AUV propulsion, in addition to existing methods. Another possible application is sailing. One of the problems associated with

sailing boats is that they are stranded in very calm wind conditions. This is usually overcome with outboard motors used in times of low wind. However, in addition to being dead weight when not in use, these motors are usually placed at the extreme aft of the hull and can therefore cause an undesirable pitch to the boat. The replacement of the outboard motor by a flexible keel providing wave-like propulsion, in place of the centre/dagger board, would have eliminated this problem. Apart from this, the propulsive keel (fin) would not interfere with the hydrodynamic characteristics of the hull or produce any additional drag as it simply replaces the already present component.

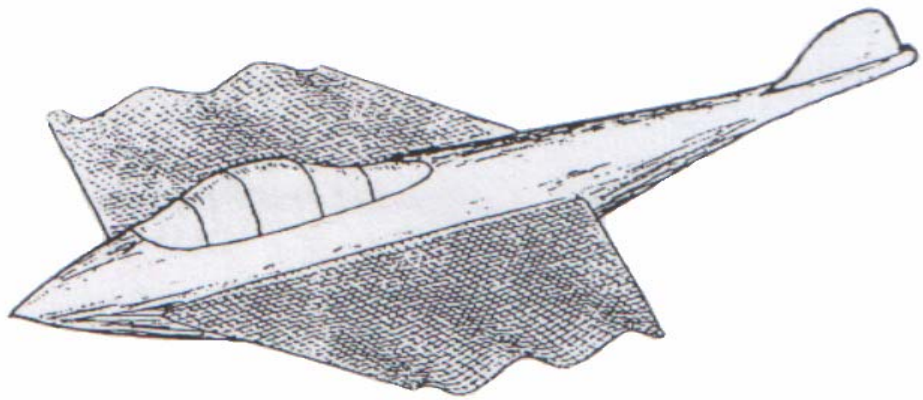

Figure 1. Artist's impression of the proposed use of localised elastic waves for propulsion of a small submarine [1]; the localised flexural waves propagate along the tips of the horizontal fins (wings), their energy being concentrated away from the main body.

In what follows, we report the results of the first experimental investigation of the above-mentioned idea of wave-like aquatic propulsion utilising localised flexural wave propagation in immersed elastic wedges and plates. To verify the idea, we have built and tested the first working prototype of a small catamaran employing the above-mentioned wave-like propulsion via the attached rubber keel. The tests have been carried out in two phases, in a water tank and then in open water. The test results have shown that the catamaran was propelled very efficiently and could achieve the speed of about one its body length per second (36 cm/s), thus demonstrating that the idea of wave-like propulsion of man-inhabited craft is viable. The reported proof of the viability of this idea may open new opportunities for marine craft propulsion, which can have far reaching implications.

2 SOME PROPERTIES OF LOCALISED FLEXURAL WAVES

The wave-like propulsion of man-inhabited marine vessels, which is the main focus of this paper, is based on employing the unique type of localised flexural waves propagating along tips of elastic wedges submerged in water. Such guided waves, that are often called water-loaded *wedge elastic waves*, have been first predicted theoretically by one of the present authors in the same paper [1] where the idea of using these waves for aquatic propulsion has been suggested. Further developments of the theory of water-loaded wedge waves have been published in Ref. 2, and their existence has been confirmed experimentally by independent researchers [3,4].

Because of their complex nature, wedge waves generally can be described only numerically even for the simplest case of wedges in vacuum which was first considered back in the 1970's (see e.g. Ref. 9 and references there for more detail). This is even more so for wedges in contact with water. However, for the important case of slender wedges the situation can be simplified in both cases by using the geometrical acoustics approximation [9-11]. Using this approximation, one can solve the equation for bending vibrations of a slender wedge considered as a thin plate of variable thickness,

thus obtaining relatively simple solutions for phase velocities of guided wedge waves and for their displacement amplitudes. Another significant advantage of the geometrical-acoustics approach is that it facilitates extending the analysis to more complex wedge shapes (in addition to the simplest case of the so-called 'linear' wedges that are formed by two intersecting planes [12,13]). As a result, one can obtain relatively simple and physically explicit solutions for localised guided waves propagating in wedges in contact with vacuum [9-13] as well as in wedges immersed in water [1,2,13].

For the purpose of aquatic propulsion, one can use wedge waves propagating in wedges of any shapes. The most suitable, however, appear to be quadratic wedges, which local thickness is described by the function $h(x) = \varepsilon x^2$, where x is the distance from the edge and ε is a constant. In such wedges, all modes of localised flexural waves are dispersive, i.e. their phase velocities depend on frequency. This would allow an operator of a marine vessel with wave-like propulsion to change wedge wave velocity by changing frequency, which may be very convenient for efficient start of the vessel from rest. Note in this connection that from the theory of swimming of slender fish, e.g. eels, it is known that the velocity of wave-like motion of a fish body at stationary conditions should be slightly higher than the velocity of swimming [14-17]. Apart from this, the advantage of quadratic wedges is that they utilise a larger proportion of their surfaces for localised wave propagation in comparison with linear wedges, which again is beneficial for aquatic propulsion.

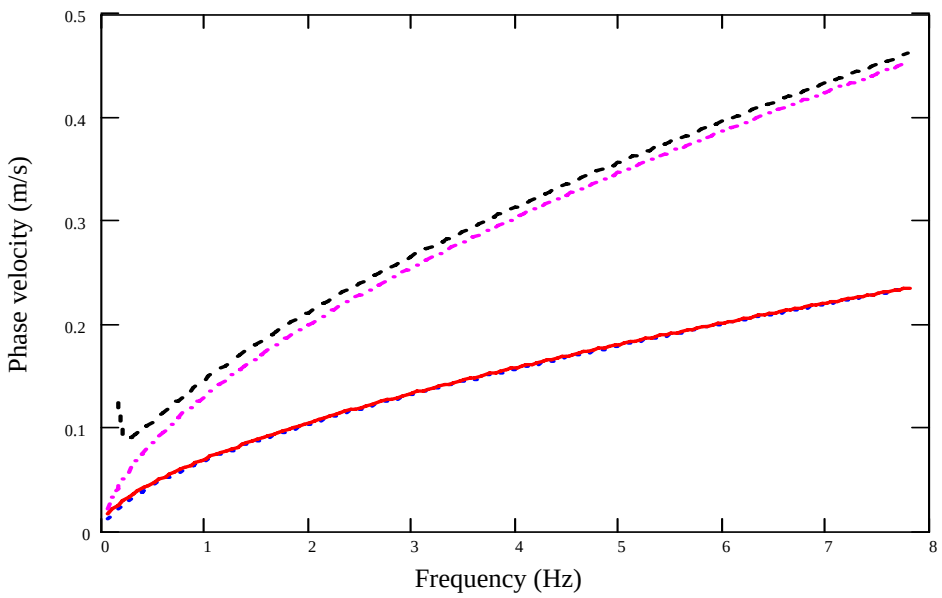

Figure 2. Phase velocities of the lowest order flexural mode in two 'clamped-free' water-loaded rubber strips as functions of frequency; both strips are of the same width $H = 150\ mm$, but have different values of thickness: $h = 1\ mm$ (solid curve) and $h = 3\ mm$ (dashed curve); for comparison, the velocities for infinite plates are shown for $h = 1\ mm$ (dotted curve, which practically coincides with the solid curve) and for $h = 3\ mm$ (dash-dotted curve).

Note that, although flexural waves in quadratic rubber wedges seem to be the most appropriate for aquatic propulsion of small man-inhabited craft, it was rather difficult to use such waves in the present experimental investigation due to difficulties in manufacturing of experimental wedges of quadratic shape. To pass by such difficulties in the preparation stage of these experiments, it was decided to avoid using localised flexural waves in quadratic wedges and to replace them by similar type of localised flexural waves propagating in simpler structures. In particular, one of the possibilities was to use the earlier established similarity between localised wave propagation in quadratic wedges and in the geometrically simpler systems comprising thin elastic ridges

embedded into an elastic half-space [12]. As was shown in Ref. 12, localised flexural waves in quadratic wedges and in systems comprising ridges embedded into an elastic half-space follow the same physical mechanisms of wave propagation and show similar dispersion behaviour.

From the point of view of flexural wave propagation, the latter systems in turn are similar to even more simple elastic structures - elastic strips with one free edge and with another edge being clamped. Note that all of the above-mentioned systems are characterised by similar dispersion behaviour of localised waves caused by physical similarity in wave-guiding properties of such systems. In particular, there are different modes of flexural waves in each system, which all have minima of phase velocity at certain frequencies. Therefore, for experimental purposes, it was more practical to use 'clamped-free' rubber plates instead of ideally suitable quadratic rubber wedges or elastic ridges embedded into an elastic half-space. One should keep in mind however that, in contrast to quadratic wedges, such ‘clamped-free’ plates do transmit vibrations to the main body of a vessel through the area of clamping. Therefore, although quite suitable for AUV, the aforementioned 'clamped-free' rubber plates can not be recommended for applications to real manned marine vessels. For the purpose of experimental investigations on a model vessel described in this paper they, however, are perfectly acceptable. Note that all of the above-mentioned localised flexural waves in contact with water are waves propagating in the subsonic regime of wave propagation (in respect of underwater sound). As it is well known, such waves ideally do not generate sound in the surrounding water. Therefore, the associated aquatic propulsion is virtually quiet, which is an attractive feature for both man-inhabited vessels and AUV.

Figure 2 shows the theoretically calculated frequency-dependent phase velocities (dispersion curves) of the lowest-order flexural mode in an immersed 'clamped-free' rubber strip. Calculations have been carried out using the geometrical acoustics approach [1,2] for the two values of strip thickness: 1 mm and 3 mm. The width of the strip was 150 mm in both cases. One can see that for a strip of 1 mm thickness the dispersion curves for a finite strip are almost indistinguishable from the dispersion curves for an infinite plate of the same thickness. For a 3 mm strip the phase velocity is slightly higher than in an infinite plate of the same thickness and has a minimum at frequency of around 0.3 Hz. Note that in the case under consideration the calculated phase velocities of flexural waves in immersed 'clamped-free' rubber strips are very low, from about 2 to 45 cm/s, depending on frequency (these are much lower than wave velocities in the same strips and plates in vacuum). Such low wave velocities have been specifically chosen to provide several wavelengths of flexural waves on the length of the propulsive fin, which was required to emulate the ‘rajiform’ wave-like motion associated with stingrays.

3 MODEL VESSEL AND EXPERIMENTAL RIG

According to the previous section, to produce wave-like propulsion of a small model of a marine vessel it was sufficient to use an elastic rubber plate clamped on one side, instead of using a quadratic rubber wedge. The plate was to be excited by given flexural displacement at a single point, with the resulting flexural waves propagating in the direction opposite to the direction of swimming. Although the main envisaged applications for the wave-like propulsion considered is for use on small ocean man-inhabited research submarines and AUV, at this early stage of the investigation the submarine design was not implemented. Instead, a simpler surface-boat-type vessel (a catamaran) and the associated experimental rig have been designed and built.

The basic rig comprised the central propulsive plate (fin) mounted on the two supporting pylons (30 cm each) positioned at 90 degrees to the fin (see Figure. 3). The pylons could then either be clamped above a water tank to study water flows generated by the vibrating propulsive fin or their ends could be attached to a pair of pontoons to form a catamaran. The propulsive rubber plates were clamped along one edge and were free on the other three sides, thus forming the aforementioned ‘clamped-free’ strips of finite length. Assuming that frequencies of wave excitation are in the range 2 - 10 Hz, the thickness of the basic rubber plate was chosen as 1 mm. The width (span) and the length (chord) of the propulsive rubber plate were chosen as 150 mm and 250 mm

respectively. The plates were excited at the tip by given flexural displacements of a mechanical pivoted arm that was attached to the leading edge of the propelling plate and driven by the electric motor. The motor was connected to the exciting pivoted arm via an additional mechanical arm and a disk with a set of 12 drilled holes placed at different distances from the centre of rotation. The drilled holes allowed for a change in amplitudes of the plate excitation by changing the pivotal position of the additional mechanical arm. These holes were designated by the symbols D1 - D12, from innermost to outermost ones, each of them being associated with the specific increased value of the displacement amplitude of the plate tip.

Figure 3. View of the assembled propulsion rig in a an empty water tank used for in-tank testing: the main propulsive plate made of white rubber is stiffened by two narrower plates of black rubber; holes drilled on the motor disk are used for changing amplitudes of plate vibration.

The experimental testing took place in two stages. The first stage had been carried out in a water tank, where the rig (the propulsive part of the model vessel) did not move. This testing in an enclosed and controlled environment allowed for easier and more accurate observations of the water-loaded flexural waves generated in an immersed propulsive fin and of the resulting water flows associated with the propulsion. The second stage took place in open water, with the rig mounted on a catamaran and moving due to the effect of the wave-like propulsion.

A 12-24 V dc motor with a gearbox was chosen to power the craft. This motor was selected for its small size and relatively high torque. The gearbox ratio could be adjusted between 4:1 and 4096:1. In particular, using a 64:1 gear ratio gave a suitable maximum rotational speed of roughly 300 rpm (or 5 Hz) at the maximum rated voltage of 24 V. The clamping of the propulsive rubber plate was realised by means of a hollowed aluminium bar that contained a set of nuts and bolts on both sides of the rubber plate.

The open-water stage of testing incorporated the same basic propulsive test rig as the water-tank testing, with the addition of buoyancy aids to the sides of the rig to assemble a catamaran craft. Two Styrofoam pontoons were placed between the two supporting plates in place of the sides of the water container. The pontoons were designed to maintain the fully immersed water depth of the fin. The rig was weighed, and the pontoons were sized accordingly. With the motor mass offset from centre in both the longitudinal and lateral directions, the right-hand pontoon needed to be longer than the left-hand (50 cm and 35 cm respectively), with both pontoons extending forward of the main deck. This configuration ensured a level deck of the vessel.

4 TESTING IN A WATER TANK

During the testing in a water tank the propulsive fin was placed vertically in the centre of the container and fully immersed. The flow speeds were measured by timing the movement of the small pieces of cork floated in water over a set distance. The scatter of the measured speeds was within 5%, therefore all measurements were performed three times, and the results were averaged.

To optimise the thickness of the propulsive fin, in addition to the basic plate of 1 mm thickness (see previous section), two thicker pieces of rubber of the same dimensions were tested. The thickest was of the order of 3 mm thick, and the thinner was approximately 2mm thick. The above-mentioned three rubber samples will be referred to as thin (1mm), medium (2mm) and thick (3mm). As expected, for operating frequencies from 2.5 to 4.5 Hz a complete flexural wavelength was not observed along the chord of the thick rubber plate, and the fin simply acted as a side-to-side paddle. On the other hand, the most flexible thin sample (1 mm) allowed for a number of wavelengths to be observed along the chord of the fin. The medium plate (2 mm) behaved similarly to the thin plate, but supported a fewer number of wavelengths along the chord for a given input frequency.

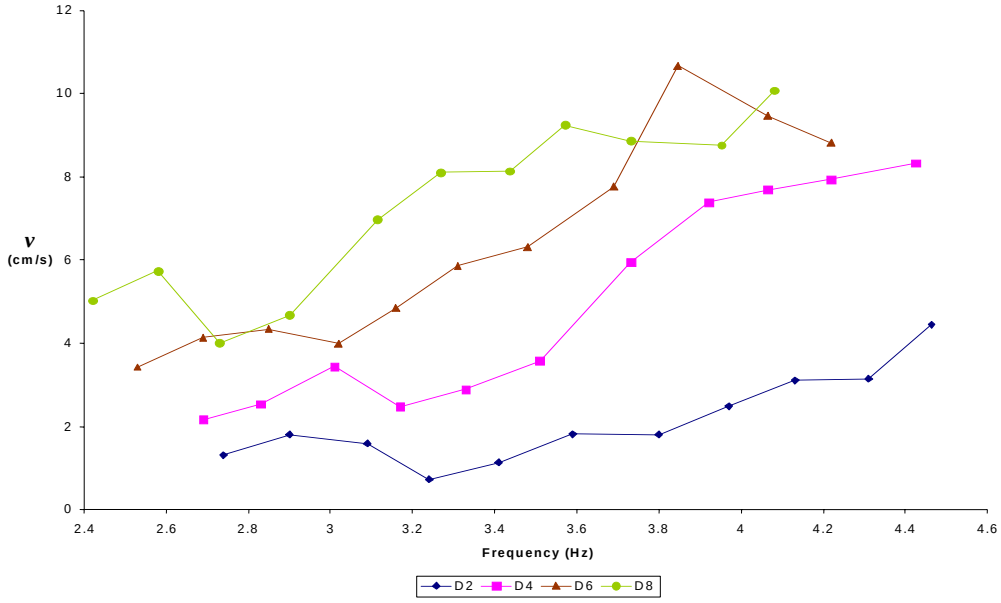

Figure 4. Recorded speed v of the flow generated in a water tank by a localised wedge elastic wave propagating along the rubber plate as function of frequency for different values of the wave displacement amplitudes; the amplitudes are identified by the locations of the holes on the motor disk: holes D2, D4, D6 and D8 correspond to the displacement amplitudes of 1.8, 2.2, 2.8 and 3.2 cm respectively.

The main part of the testing in a water tank included observation of the behaviour of the fin as a source of propulsion over a range of amplitudes and frequencies. The frequency (in Hz) was equal to a number of motor revolutions per second. The actual observed frequency depended on the loading and on the input electric power. The direct current power adapter was set up to allow the voltage to be user defined. The propulsive system was then preliminary tested in a tank at 1-volt intervals and the electric current was read off to determine the input electric power. Increasing the voltage increased the power and therefore increased the speed of the motor rotation and thus the frequency of the fin vibrations. It has been checked experimentally that the frequency of vibration versus power in from the motor was a straight line. In particular, over the whole range of power in used in the experiments, i.e. from 2 to 6 Watts, the frequency changed linearly from 2.5 to 4.5 Hz. The speed of the water flow generated by flexural waves propagating along the propulsive fin was

measured for different values of the input electric power and frequency. Results were recorded for 4 values of fin vibration amplitudes at 11 voltage settings, from 15V to 25V at 1-volt intervals, changing the frequency of fin vibration. In particular, the obtained dependence of the water flow speed on frequency is shown in Figure 4. It can be seen that the speed of the flow generally increased with increasing frequency and amplitudes of vibration. Note that the observed flow speeds in the water tank are comparable with the calculated phase velocities of localised flexural waves propagating along the propulsive rubber strips.

It is interesting to note from Figure 4 that there are minima on the plots, and for different vibration amplitudes these minima occur at different frequencies. Apparently, the observed minima of flow speeds can be associated with the plate resonant ('natural') frequencies at which the localised flexural waves fail to propagate and form partial standing waves along the chord, thus displacing some amount of water at 90 degrees from the desired flow direction. The first natural frequency of the plate operated at the disc amplitude setting D8, which corresponds to the highest value of the fin tip displacement amplitude U_0 = 3.2 cm, was at approximately 2.7 Hz. The same plate excited at D2 setting (i.e at U_0 = 1.8 cm) had its first natural frequency at 3.2 Hz. The observed dependence of the natural frequencies on vibration amplitudes can be explained by the role of non-linear effects for localised flexural waves considered. Indeed, typical displacement amplitudes of generated flexural waves (2.5 cm) were large enough in comparison with their typical wavelengths, thus resulting in high values of Mach number, so that non-linear effects must have been essential. Investigation of these non-linear effects, however, was out of the scope of this paper.

The input electric power readings were taken for each setting in order to estimate the size of battery required for testing in open water. Naturally, the best operating setting for sustained running of the model vessel in open water would be the one corresponding to the maximum efficiency of propulsion. Using the measured values of the input electric power $P_{in} = I\,V$ and of the flow speed v in a water tank, it was possible to estimate relative values of the propulsive efficiency of the fin at the various test conditions. Assuming that the combined drag produced by the fin, container and all other surfaces is proportional to the square of the generated flow speed [18], the efficiency η can be calculated using the following simple formula:

$$\eta = \frac{P_{out}}{P_{in}} = const \cdot \frac{v^3}{I \cdot V}, \qquad (1)$$

where P_{out} is the output power associated with the fin-generated water flow and calculated as the product of the associated thrust force produced by the fin and the flow velocity. Note that there is still an unknown constant in the equation (1) depending on the parameters of the rig. Therefore, equation (1) can be only used for relative comparison of the efficiencies for the same experimental rig, e.g. by normalising them with respect to the maximum efficiency achieved. The relative value of efficiency normalised in this way can be referred to as fraction of maximum efficiency.

The fraction of maximum efficiency as function of frequency has been calculated according to (1) using the measured values of v and I, V. In particular, it has been found that there is the maximum efficiency at around 3.8 Hz, with the amplitude setting D6 (U_0 = 2.8 cm). The frequency at which the maximum efficiency occurs corresponds to the input voltage of 23 V. To achieve the desired maximum efficiency for testing in open water the system would have to be pulling 5 W of electric power. For the open water testing, this was provided by the set of lead acid batteries. These were very heavy and required to be off the craft, with a wire feed.

5 OPEN WATER TESTS

The open-water testing was carried out in the experimental pool that could be considered as open water for the assembled small model catamaran with the wave-like propulsion. Two sets of tests were performed. The first was a timed run over a distance of 3m, the craft starting form rest. This was designed to measure averaged speeds of the craft over this distance. The second set of tests

was again a timed run over a distance of 2m, but with an initial speed achieved after passing the added 'acceleration' distance of 2m, an ample space to reach what appeared to be a stationary speed. This set of tests was obviously designed to measure stationary speeds. In addition to the black rubber plates tested in a water tank, a white rubber plate was added for open-water testing. This white rubber was of the same thickness as the thin black rubber, but was less stiff. As in the water-tank tests, a single rubber plate was used (1 sheet) with dimensions 250x150mm^2, or one could use a rubber plate of the same size supported by two smaller side rubber plates of the same thickness (3 sheets) with dimensions 70x250 mm^2.

When powered from the external batteries via a flexible cable, the catamaran demonstrated fast acceleration from rest to stationary speeds that were high enough in terms of its lengths per second (see Figure 5). The results of the two sets of tests for different configurations are presented in Tables 1 and 2. They have been arranged in time order, the fastest at the top, with an averaged time (and the resulting averaged speed) obtained over the three runs. The amplitude settings were D4 (U_0 = 2.2 cm), D6 (U_0 = 2.8 cm) and D8 (U_0 = 3.2 cm). The obtained results demonstrate that the wave-like aquatic propulsion considered is viable and efficient.

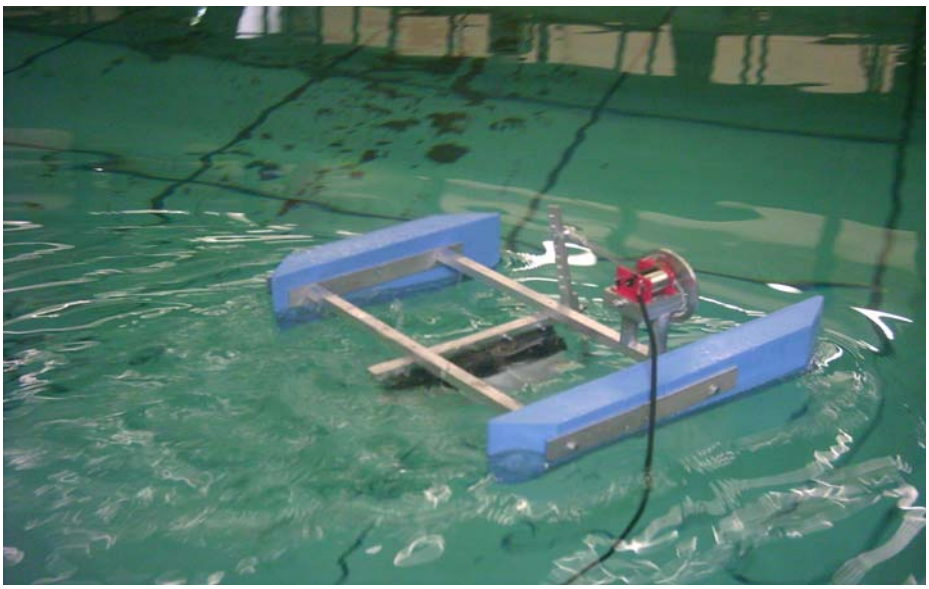

Figure 5. View of the moving catamaran with the wave propulsive system during its testing in open water.

Note that the propulsive thin plate performed more efficiently when flanked by additional two smaller rubber sheets. A possible explanation for this could be that a sided plate, being thicker, had a higher wave velocity, thus resulting in a higher stationary swim speed. The qualitative comparison of the measured steady craft speed in open water versus predicted phase velocities has shown that the achieved speeds were comparable with the wave speeds in the propulsive plate at the operating frequencies, around 3.8 Hz. Measuring actual wave velocities of localised flexural waves was out of the scope of this paper.

The comparison of the speeds of normal and heavier craft (with additional weight) has shown, as expected, that the normal craft performed better than the heavier one as it sat further out of the water and therefore produced less drag. As expected, the results on propulsion efficiency obtained from the in-tank testing agreed with those observed in the open-water test. For example, at the open-water test condition of 22 volts, the most efficient amplitude setting was D6, then D8, and then D4, which was in agreement with the in-tank testing.

Table 1. Open-water testing from rest

Number of Fin Sheets	Central Rubber Type	Voltage (V)	Amplitude setting	Weight conditions	Time (s)	Speed (cm/s)
3	White	22.1	D6	Normal	8.39	35.8
3	Thin	22.4	D6	Normal	8.74	34.3
3	Thin	22.2	D8	Normal	9.32	32.12
1	Thin	22.1	D8	Normal	9.68	31.0
3	Thin	22.0	D8	Heavy	10.52	28.5
3	Thin	22.3	D8	Heavy	10.54	28.5
3	Thin	22.2	D4	Normal	10.77	27.9
3	Thin	22.3	D8	Heavy	13.08	22.9

Table 2. Open-water testing with initial craft speed

Number of Fin Sheets	Central Rubber Type	Voltage (V)	Amplitude setting	Weight conditions	Time (s)	Speed (cm/s)
3	White	21.9	D6	Normal	5.73	34.9
3	White	22.0	D6	Heavy	7.28	27.5
3	White	22.0	D6	Heavy	7.39	27.1

It would be interesting to compare the efficiency of the described wave propulsion with the efficiency of a propeller. However, such a comparison was beyond the scope of this investigation concerned primarily with the feasibility studies. To make such a comparison meaningful one would require to optimise the mechanical design of the model vessel employing the proposed wave propulsion system. In the current design, with the motor disk and mechanical arms for flexural wave generation, a substantial amount of energy is being lost due to friction at the mechanical arm contact points. This reduces the energy efficiency of the system. In the future design, an improved version of the model vessel could be developed, e.g. using electro-active bending polymers (EAP) to directly generate flexural waves at a leading edge in a plate or wedge. This would reduce or even eliminate moving parts from the wave propulsive system, so that it would become more energy efficient and appropriate for a detailed comparison with traditional propeller systems.

6 CONCLUSIONS

The first and most important conclusion resulting from this work is that localised wedge or plate flexural elastic waves can indeed be used to generate wave-like aquatic propulsion and to propel a small marine craft. To propel a manned craft one should use propulsive fins made of quadratic elastic wedges that keep the wave vibration energy away from the main body of the vessel.

The speeds achieved by the small model catamaran in the open water testing are comparable with the wave speeds in the propulsive plate at the operating frequencies. This is in agreement with the well-known results following from the studies of fish swimming indicating that speed of swimming at stationary conditions should be slightly less than the velocity of wave-like motion of the fish body.

Although the current experimental rig used a rather complex mechanical construction to achieve the localised flexural wave excitation, it is expected that further investigations could lead to the development of a simpler and more efficient marine craft. In particular, this could include the use of electro-active bending polymers (EAP) to directly generate flexural waves at a leading edge in a plate or wedge. This would reduce or even eliminate any moving parts from a wave propulsion rig, giving it another advantage over a conventional propeller.

Further work is required to investigate the efficiency of wave propulsion in comparison with its main rival, a propeller. However, even if the efficiency of wave-like propulsion cannot be developed to surpass that of a propeller, there is still an unexplored niche for it. Namely, wave-like propulsion may have no rivals in cases where quiet and safe operation is paramount, in particular in the cases of small manned research submarines and autonomous underwater vehicles (AUV).

7 REFERENCES

1. V.V. Krylov: Propagation of wedge acoustic waves along wedges embedded in water. Proc IEEE Ultrasonics Symposium, Cannes, France, (1994) 793-796.
2. V.V. Krylov: On the velocities of localized vibration modes in immersed solid wedges. Journ. Acoust. Soc. Am., 103 (1998) 767-770.
3. J.R. Chamuel: Flexural edge waves along free and immersed elastic waveguides. Review of Progress in Quantitative Nondestructive Evaluation, vol.16, ed. D.O. Thompson and D.E. Chimenti (Proc. 16th Symp. Quant. Nondestruct. Eval., July 28-August 2, 1996, Brunswick, Maine), Plenum Press, New York, (1996) 129-136.
4. M. de Billy: On influence of loading on the velocity of guided acoustic waves propagating in linear elastic wedges. Journ. Acoust. Soc. Am., 100 (1996) 659-662.
5. I. Yamamoto, Y. Terada, T. Nagamatu and Y. Imaizumi: Propulsion system with flexible/rigid oscillating fin. IEEE Journal of Oceanic Engineering, 20 (1995) 23-30.
6. M. Sfakiotakis, D.M. Lane and J.B.C. Davies: Review of fish swimming modes for aquatic locomotion. IEEE Journal of Oceanic Engineering, 24 (1999) 237-252.
7. S. Guo, T. Fukuda and K. Asaka: A new type of fish-like underwater microrobot. IEEE/ASME Transactions on Mechatronics, 8 (2003) 136-141.
8. P.R. Bandyopadhyay: Trends in biorobotic autonomous undersea vehicles. IEEE Journal of Oceanic Engineering, 30 (2005) 109-139.
9. S.V. Biryukov, Yu.V. Gulyaev, V.V. Krylov and V.P. Plessky: Surface Acoustic Waves in Inhomogeneous Media. Springer-Verlag, Berlin, Heidelberg, 1995.
10. V.V. Krylov: Conditions for validity of the geometrical-acoustics approximation in application to waves in an acute-angle solid wedge. Soviet Physics - Acoustics, 35 (1989) 176-180.
11. V.V. Krylov: Geometrical-acoustics approach to the description of localized vibrational modes of an elastic solid wedge. Soviet Physics - Technical Physics, 35 (1990) 137-140.
12. V.V. Krylov: Localized acoustic modes of a quadratically-shaped solid wedge. Moscow University Physics Bulletin, 45(6) (1990) 65-69.
13. V.V. Krylov and A.L. Shuvalov: Propagation of localised flexural vibrations along plate edges described by a power law. Proceedings of the Institute of Acoustics, 22(2) (2000) 263-270.
14. M.J. Lighthill: Note on the swimming of slender fish. Journ. Fluid Mech., 9 (1960) 305-317.
15. M.J. Lighthill: Aquatic animal propulsion of high hydrodynamic efficiency. Journ. Fluid Mech., 44 (1970) 265-301.
16. M.J. Lighthill: Large amplitude elongated-body theory of fish locomotion. Proc. Roy. Soc. Lond. B, 179 (1971) 125-138.
17. J.Y. Cheng and R. Blikhan: Note on the calculation of propeller efficiency using elongated body theory. Journ. Exp. Biol., 192 (1994) 169-177.
18. G.K. Batchelor: An introduction to fluid dynamics. Cambridge University Press, Cambridge, 1994.