Integer, half-integer and unquantized dc voltage generation in THz-driven semiconductor superlattices

E.H. Cannon[1], K.N. Alekseev[2,3*],F.V. Kusmartsev[4,5], and D.K. Campbell[6]
[1]Department of Electrical Engineering, University of Notre Dame, Notre Dame, IN 46556, USA
[2]Department of Physical Sciences, Box 3000, University of Oulu FIN-90014, Finland
[3]Theory of Nonlinear Processes Laboratory, Kirensky Institute of Physics, RAS, Krasnoyarsk 660036, Russia
[4]Department of Physics, Loughborough University, Loughborough LE11 3TU, UK
[5]Landau Institute of Theoretical Physics, RAS, Moscow 142432, Russia
[6]Departments of Electrical and Computer Engineering and Physics, Boston University, Boston, MA 02215, USA

We consider the spontaneous creation of a dc voltage across a strongly coupled semiconductor superlattice subjected to THz radiation. We show that the dc voltage may be approximately proportional either to an integer or to a half- integer multiple of the frequency of the applied ac field, depending on the ratio of the characteristic scattering rates of conducting electrons. For the case of an ac field frequency less than the characteristic scattering rates, we demonstrate the generation of an unquantized dc voltage.

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The theoretical analysis of nonlinear transport properties of semiconductor superlattices (SSLs) irradiated by a high-frequency electric field started in the mid 1970's [1]. Recent progress in creating powerful sources of THz radiation, the development of a coupling technique [2,3], and improvement in the fabrication technology of microstructures leading to very high carrier mobility [4], stimulated many new theoretical works on the problem. Recent interest in THz-field driven SSLs has focused on such strongly nonlinear effects as multistability [5], short pulse generation [6], chaos [7–9], and spontaneous generation of a dc voltage in a purely ac-driven SSL [10–13]. In this paper we address the last effect.

Two distinct mechanisms are known for the spontaneous generation of a dc bias in purely ac-driven SSLs [10,11,13]. Both of these nonlinear mechanisms work in SSLs with a high mobility and a relatively high level of doping, when the effects of a self-consistent electric field generated by an electron's motion become sufficient. The first mechanism arises if the ac field frequency, ω, is much greater than the plasma frequency, ω_{pl}, and is related [10,13] to an instability caused by absolute negative conductivity in the ac-driven SSL [14]. The effect has been attributed [10] to the phenomena of dynamical localization of electrons [15] and miniband collapse in a collisionless SSL [16]. It was reported that the generated dc bias is such that the "induced Bloch frequency" $\omega_B = eaE_{dc}/\hbar$ (E_{dc} is the spontaneously generated dc electric field and a is the SSL period) is approximately equal to the ac field frequency ω [10,13,17].

The second mechanism is responsible for dc bias generation when the ac field frequency is near the plasma resonance, $\omega \simeq \omega_{pl}$; it arises for a smaller ac field strength than is required in the previous situation [11]. The instability responsible for the dc bias generation in SSLs with strong enough electron scattering also results in chaotic motion in the case of small scattering rates or for a collisionless SSL [7,11]. The creation of a dc bias may be qualitatively explained [12] and classified [13] using the semiclassical theory of wave-mixing in SSLs [18].

In this paper we re-examine the problem of spontaneous dc voltage generation in an SSL subjected to a THz electric field. We show that, depending on the relative values of the scattering rates and the ac field frequency, a variety of different dc voltage states can exist – namely, both integer and half-integer quantized states, for which the induced Bloch frequency is approximately an integer or half-integer multiple of the ac field frequency, and completely unquantized states. In particular, if the electron velocity relaxation rate, γ_v, is sufficiently different from the electron energy relaxation rate, γ_ε, and $\omega \gtrsim \gamma_v$, we find integer states with $\omega_B \approx n\omega$ ($n = \pm 1, \pm 2, \ldots$); while for $\gamma_v = \gamma_\varepsilon$, the states are close to the half-integer states $\omega_B \approx n\omega/2$. In contrast, in the case of low frequency driving or high damping, $\omega < (\gamma_v, \gamma_\varepsilon)$, the dc voltage states are unquantized. We first reported the integer states in our previous papers [11], and a recent work claimed that the spontaneously generated bias should be a fractional state [13].

We study electron transport trough a single miniband, spatially homogeneous SSL with period a and miniband width Δ, which is subjected to an ac electric field $E(t) = E_0 \cos \omega t$ along the SSL axis. For the tight-binding energy-quasimomentum dispersion relation $\varepsilon(k) = (\Delta/2)\,[1 - \cos(ka)]$ (k is the electron wave vector along the axis of SSL), the dynamics of electrons is described by the superlattice balance equations [10,11,13]

*E-mail: Kirill.Alekseev@ oulu.fi

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\begin{aligned}
\dot{v} &= uw - \gamma_v v, \\
\dot{w} &= -uv - \gamma_\varepsilon (w - w_{eq}), \\
\dot{u} &= \omega_{pl}^2 v - \alpha u + I_{ext}(t),
\end{aligned}
\tag{1}
$$

where $v = m_0 \overline{V} a/\hbar$, $w = (\overline{\varepsilon} - \Delta/2)(\Delta/2)^{-1}$ and w_{eq} are a scaled electron velocity, a scaled electron energy, and an equilibrium value of scaled electron energy, respectively, and $m_0 = (2\hbar^2)/(\Delta a^2)$ is the effective mass at the bottom of miniband. The scaled variables $v(t)$ and $w(t)$ are proportional to the variables $\overline{V}(t)$ and $\overline{\varepsilon}(t)$, which are the electron velocity and energy averaged over the time-dependent distribution function satisfying the Boltzmann equation. The lower (upper) edge of the miniband corresponds to $w = -1$ ($w = +1$), and the value of w_{eq} is a function of the lattice temperature. The variable $u(t)$ is related to the electric field inside the SSL $E(t)$ as $u = eaE/\hbar$. In deriving Eqs. (1), we assumed that the electrical properties of an SSL of total length l and cross-section S can be modeled by an equivalent high-quality circuit which consists of a capacitor $C = (\epsilon_0 S)/(4\pi l)$ (ϵ_0 is the average dielectric constant for the SSL) driven by an ac current of the form $I_{ext} = -\omega_s \omega \sin \omega t$, $\omega_s = eE_0 a/\hbar$, in our scaled units. The degree of nonlinearity in Eqs. (1) is controlled by the value of miniband plasma frequency, $\omega_{pl} = \left(4\pi e^2 N/m_0 \epsilon_0\right)^{1/2}$, which is a function of the electron doping density N, while the parameter α determines the quality of the circuit ($\alpha \ll \omega_{pl}$).

The relaxation processes for miniband electrons are characterized by an average energy scattering rate, γ_ε, as well as by an average velocity scattering rate, $\gamma_v = \gamma_\varepsilon + \gamma_{el}$, where γ_{el} is an average rate of elastic collisions [3,10]. The scattering rates, γ_v and γ_ε, can have different values depending on the material, the doping density, the temperature, etc. In particular, for microstructures with modulation doping [19], the ionized impurities are spatially separated from electrons, which greatly reduces the elastic scattering rate γ_{el} so that $\gamma_v \approx \gamma_\varepsilon$. In contrast, for many vertical SSLs operating at room temperature, the scattering rate for electron velocity is about of order of magnitude greater than the characteristic scattering rate of electron energy, $\gamma_v/\gamma_\varepsilon \approx 10$ [3,10].

We solve the nonlinear balance Eqs. (1) numerically for the initial conditions $v(0) = 0$, $w(0) = w_{eq} = -1$, with the circuit damping rate $\alpha/\omega_{pl} = 0.01$, and for two typical sets of relaxation constants: (i) $\gamma_v/\gamma_\varepsilon = 10$, $\gamma_\varepsilon/\omega_{pl} = 0.01$, and (ii) $\gamma_v/\gamma_\varepsilon = 1$, $\gamma_\varepsilon/\omega_{pl} = 0.1$. After removing the transients, we calculate the time-average value $\langle u \rangle$, which gives the value of the Bloch frequency determined by the spontaneously generated dc bias E_{dc}: $\omega_B \equiv eaE_{dc}/\hbar = \langle u \rangle$. Figs. 1 and 2 present the results of computations of $\langle u \rangle$ for 201 value of ω_s equally distributed in the range $0 \le \omega_s \le 2\omega_{pl}$ for each driving frequency.

For the first set of relaxation rates and for $\omega > 2\gamma_v$, plateaus of integer-quantized states are clearly observable: $\omega_B \approx n\omega$, with $n = \pm 1, 2, 3, 4$ in Fig.1. However, at low frequencies, $\omega < 2\gamma_v$, the width of the $n = 1$ plateau is comparable to its spacing from the $n = 0$ plateau. The dependence of the induced Bloch frequency $\langle u \rangle$ on the ac field frequency ω for the second set of damping parameters is presented in Fig.2. Here, qualitative differences from Fig. 1 appear, including 1) the existence of half-integer states, specifically states with $n \approx \pm 1/2$, and 2) the nonzero width of the $n = 0$ plateau. Naively, one can expect the width of a plateau to be equal to the scattering rate, $0.1\omega_{pl}$ in this case. As a result, we have plotted the lines $n \pm 0.1\omega_{pl}/\omega$ for $n = 0, \pm 1, \pm 2, \pm 3, -4, -5$ in Fig.2. We found that, indeed, most points for a given plateau lie in the region demarcated by the two lines with same n. In contrast to Fig. 1, there are states with $\langle u \rangle/\omega < 0.1$, but $\langle u \rangle \neq 0$. Also, some points fall very near the line $\langle u \rangle = \pm 0.5\omega$. As an example we refer to the solution of Eqs. (1) for $\omega/\omega_{pl} = 0.6$ and $\omega_s/\omega_{pl} = 0.6$ (other parameters are same as in Fig.2), which in the phase space corresponds to a symmetry-broken limit cycle[1] with $\langle u \rangle = 0.289\omega_{pl}$. Note also that in some limiting cases and for $\gamma_v = \gamma_\varepsilon$, the existence of half-integer states can be proved analytically [20].

The wider plateaus of Fig.2, compared to Fig.1, may be explained by the larger scattering rates. In order to understand the transition from quantized to unquantized dc bias states as the scattering rates increase while maintaining $\gamma_v \gg \gamma_\varepsilon$, we present in Fig.3 the dependence of $\langle u \rangle$on ω for strong damping. As is evident from this figure, the strong damping destroys all quantized states; dc bias generation only persists for some unquantized states.

While the detailed mechanism of unquantized dc bias generation remains unclear, we offer the following qualitative explanation of the underlying physics: when the electron scattering rates are sufficiently small and the amplitude of the ac field is large enough, the SSL spontaneously creates a Wannier-Stark ladder with the spacing, ω_B, that makes multiphoton absorption of ac field most effective, *i.e.* $\omega_B \approx n\omega$. However, for larger damping, only a small bias can be generated in the SSL, hence the spacing of the Wannier-Stark ladder states is less than the ac field frequency except for very low frequencies $\omega \lesssim \gamma$. In this case, the broadening of the self-organized ladder levels is quite comparable

[1] This is a limit cycle whose projection on the $v - u$ plane is not symmetric about the origin, in contrast to a symmetric limit cycle (see [11]).

with their spacing; therefore it is practically impossible to achieve quantized values of the voltage.

We have performed a systematic study of the positions and widths of different plateaus and of the unquantized states for many values of the driving amplitude and frequency and several different initial conditions at different damping levels [21]; the results are in a qualitative agreement with situation described above. We now make several remarks on the results of this search. First at all, we didn't see any indication of the noninteger (fractional) dc states for $\gamma_v \neq \gamma_\varepsilon$. However, for $\gamma_v = \gamma_\varepsilon$, the 1/2-dc-states are quite common. Moreover, for weak enough damping, we additionally saw a few dc states which are very close to fractional states of the form n/m with n being an integer and m always *being an even integer*. Such kind of dc states are formed by the symmetry-broken limit cycles with a high even period. As examples, we refer to 3/2-states formed by period-12 and period-24 limit cycles, as well as to the 7/6-state formed by a period-12 limit cycle; both occur for $\gamma_v = \gamma_\varepsilon = 0.05\omega_{pl}$ and $\alpha/\omega_{pl} = 0.01$. We should note, however, that for weak damping, chaotic behavior is quite typical [7,11] and that both the stable limit cycles and the asymmetric chaotic attractors, which are responsible for the generation of *stable quantized dc voltage states* in the SSL, occupy only a small amount of parameter space of the system [21,20].

In summary, we have showed that a semiconductor superlattice irradiated by a high-frequency electric field can spontaneously generate a dc bias, which can be quantized in approximately integer or fractional ratios of the driving frequency, or completely unquantized. In this respect, the described effect in semiconductor superlattices is no less rich than its counterpart in Josephson junctions subjected to a microwave field, where the exactly integer and the exactly fractional dc voltage states ("phase-locked states") are known [22].

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[1] F.G. Bass and A.P. Tetervov, Phys. Rep. **140**, 237 (1986).
[2] K. Unterrainer, B.J. Keay, M.C. Wanke, *et al.*, Phys. Rev. Lett. **76**, 2973 (1996); S. Winnerl, E. Schomburg, S. Brandl, *et al.*, Appl. Phys. Lett. **77**, 1259 (2000).
[3] S. Winnerl, E. Schomburg, J. Grenzer, *et al.*, Phys. Rev. B **56**, 10303 (1997).
[4] H.L. Stormer, L.N. Pfeiffer, K.W. Baldwin, *et al.*, Appl. Phys. Lett. **58**, 726 (1991); A. Majumdar, L.P. Rokhinson, D.C. Tsui, *et al.*, Appl. Phys. Lett. **76**, 3600 (2000).
[5] A.W. Ghosh, A.V. Kuznetsov, and J.W. Wilkins, Phys. Rev. Lett. **79**, 3494 (1997); E.P. Dodin, A.A.Zharov, and A.A. Ignatov, Zh. Eksp. Teor. Fiz. **114**, 2246 (1998) [JETP **87**, 1226 (1998)]; A.W. Ghosh, M.C. Wanke, S.J. Allen, *et al.*, Appl. Phys. Lett. **74**, 2164 (1999); Yu.A. Romanov and Yu.Yu. Romanova, Fiz. Tekh. Polupr. **35**, 211 (2001) [Semiconductors **35**, 204 (2001)].
[6] A.A.Zharov, E.P. Dodin, and A.S. Raspopin, Pis'ma Zh. Eksp. Teor. Fiz. **72**, 653 (2000) [JETP Lett. **72**, 453 (2000)].
[7] K.N. Alekseev, G.P. Berman, D.K. Campbell, *et al.*, Phys. Rev. B **54**, 10625 (1996).
[8] J.C. Cao, H.C. Liu, and X.L. Lei, Phys. Rev. B **61**, 5546 (2000).
[9] O.M. Yevtushenko and K. Richter, Phys. Rev. B **57**, 14839 (1998); Physica E **4**, 256 (1999).
[10] A.A. Ignatov, E. Schomburg, J. Grenzer, *et al.*, Z. Phys. B **98**, 187 (1995).
[11] K.N. Alekseev, E.H. Cannon, J.C. McKinney, *et al.*, Phys. Rev. Lett. **80**, 2669 (1998); Physica D **113**, 129 (1998).
[12] K.N. Alekseev, M.V. Erementchouk, and F.V. Kusmartsev, Europhys. Lett. **47**, 595 (1999).
[13] Yu.A. Romanov and Yu.Yu. Romanova, Zh. Eksp. Teor. Fiz. **118**, 1193 (2000) [JETP **91**, 1033 (2000)].
[14] A.A. Ignatov and Yu.A. Romanov, Fiz. Tverd. Tela **17**, 3388 (1975) [Sov. Phys. Solid State **17**, 2216 (1975)]; Phys. Stat. Sol. (b) **73**, 327 (1976).
[15] D.H. Dunlap and V.M. Kenkre, Phys. Rev. B **34**, 3625 (1986).
[16] M. Holthaus, Phys. Rev. Lett. **69**, 351 (1992).
[17] D.H. Dunlap, V. Kovanis, R.V. Duncan, *et al.*, Phys. Rev. B **48**, 7975 (1993).
[18] L. Esaki and R. Tsu, Appl. Phys. Lett. **19**, 246 (1971); Yu. A. Romanov, Opt. Spektr. **33**, 917 (1972) [Sov. Phys. Opt. Spectr. **33**, 503 (1972)].
[19] R. Dingle, H. Stormer, A.C. Gossard, *et al.*, Appl. Phys. Lett. **19**, 246 (1971)
[20] K.N. Alekseev, *et al.*, 2001 (unpublished).
[21] E.H. Cannon, Ph. D. thesis, University of Illinois at Urbana-Champaign, 1999.
[22] R.L. Kautz, Rep. Prog. Phys. **59**, 935 (1996).

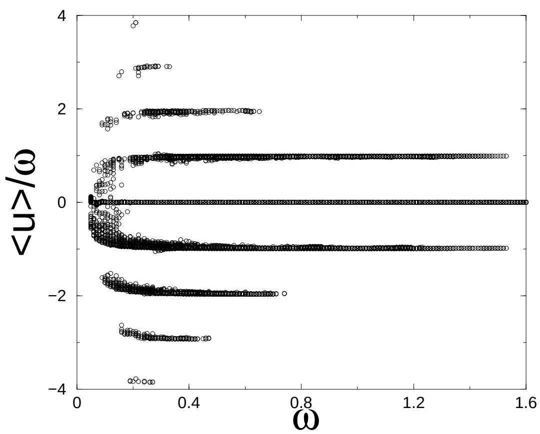

FIG. 1. The dependence of spontaneously generated dc bias $\langle u \rangle/\omega$ on ac frequency ω, scaled to the miniband plasma frequency ω_{pl}, and for $\gamma_v = 0.1\omega_{pl}$, $\gamma_\varepsilon = \alpha = 0.01\omega_{pl}$.

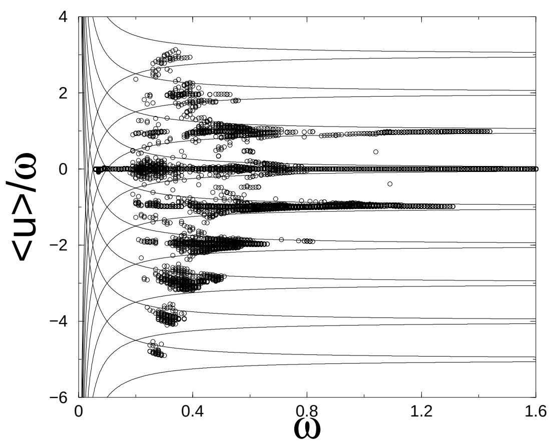

FIG. 2. Same as in Fig.1, but for $\gamma_v = \gamma_\varepsilon = 0.1\omega_{pl}$, $\alpha = 0.01\omega_{pl}$.

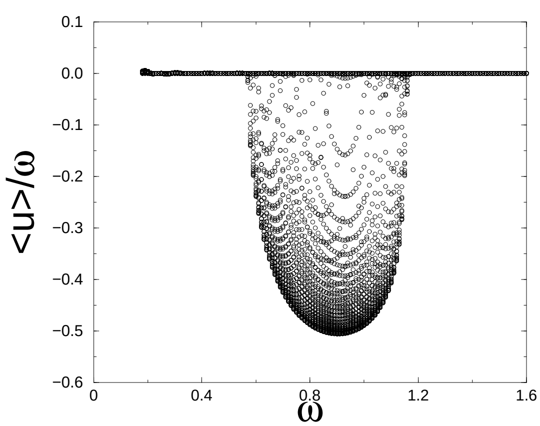

FIG. 3. Same as in Figs.1 and 2, but for strong damping: $\gamma_v = \omega_{pl}$, $\gamma_\varepsilon = 0.1\omega_{pl}$, $\alpha = 0.03\omega_{pl}$.