

Trendless Unemployment in a Model with Endogenous Technical Progress

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Abstract

An established stylized fact of OECD countries is trendless unemployment over the long run (e.g. over the last century). The present paper derives conditions for trendless unemployment in a model with endogenous technical progress. The results are consistent with earlier findings in a model with exogenous technical progress, but are in sharp contrast to the thesis in Karanassou, M. and D. Snower (2004), ‘Unemployment invariance’, *German Economic Review* 5(3), 297-317 in a reduced form model with endogenous technical progress.

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1 Introduction

While the rates of productivity growth and population growth differed over the last century, data on the unemployment rate over the 20th century show no long-run trend. According to important and influential work of Layard et al. (1991) (henceforth LNJ) this requires either the aggregate production function to be of Cobb-Douglas form (i.e. the elasticity of substitution

between labor and capital to be equal to unity) or the rate of capital accumulation to equal exactly the sum of the rate of labor-augmenting technical progress and the rate of population growth (see LNJ, p. 107). This requirement was formally derived in Rowthorn (1999) (henceforth simply Rowthorn). Karanassou and Snower (2004) (henceforth KS) criticize this argument with a figure just as the following Figure 1.

In Figure 1 the real wage (w) is measured on the vertical axis and employment (E) is measured on the horizontal axis. The figure shows a positively sloped wage-setting curve (WS). The wage-setting curve results from collective bargaining between unions and firms. Due to bargaining power of unions, unions and firms set the real wage as a mark-up over unions' real reservation wage (the real reservation wage is the real wage that makes workers indifferent between working for the firm or not). The wage-setting curve is positively sloped because unions' bargaining power rises with aggregate employment. Further, there is a negatively sloped labor demand curve (LD). Finally, there is a labor supply curve (LS) representing the labor force at a given real wage and for simplicity assumed to be vertical. The real wage w_1^* and the equilibrium employment level E^* are determined by the point of intersection of the wage-setting curve WS_1 with a labor demand curve LD_1 . There is equilibrium unemployment due to the mark-up of the real wage over the reservation wage. The equilibrium unemployment U^* equals the difference between equilibrium employment E^* and labor supply L^* . Following KS, capital accumulation and labor augmenting technical progress lead to an outwards shift of the labor demand curve to LD_2 . Hence, unchanged equilibrium unemployment in the long run requires an inward shift of the wage-setting curve to WS_2 by the same amount as the outward shift of the labor demand curve. In addition, an increase in the population size leads to an outwards shift of the labor supply curve. In this case unchanged equilibrium unemployment in the long run requires an outward shift of the wage-setting curve by the same amount as the outward shift of the labor supply curve. In econometric estimations of the wage-setting equation and the labor demand equation with medium run data of a few decades Layard and Nickell (1986) ensure a trendless unemployment rate by the cross-equation restriction the coefficients of the capital-labor ratio to be the same in the wage-setting equation and the labor demand equation. KS agree that unemployment is trendless in the long run, but criticize the restrictions of LNJ as to be too restrictive to be realistic.

However, Manning (1995) stresses that with a fixed unemployment benefit

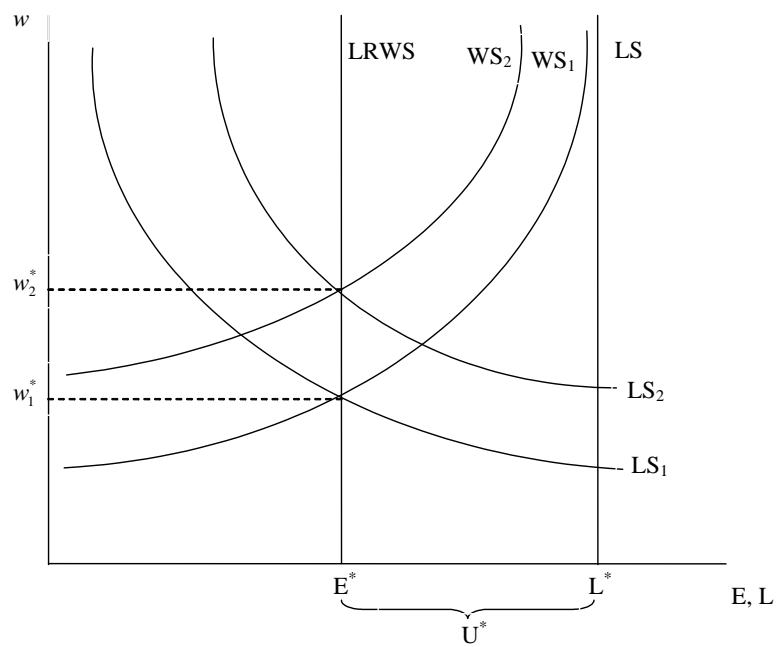


Figure 1: Trendless unemployment hypothesis

replacement ratio (i.e. a fixed ratio of unemployment benefits to unemployment contributions) the model of LNJ implies a vertical wage-setting curve. One can explain this with the fact that in this case the real unemployment benefits rise by the same proportion as the aggregate real wage. However, unemployment benefits are part of unions' reservation wage. For this reason, also the unions' real reservation wage rises by the same proportion as the aggregate real wage. In turn, this causes each union to demand in negotiations an increase in the real wage in its sector by the same proportion as the aggregate real wage and firms can only accede to this real wage increase by not increasing employment. Admittedly, in medium-run data of a few decades a vertical wage-setting curve cannot be confirmed empirically. However, LNJ and Blanchard and Katz (1997) explain this empirical failure with the thesis that unions' real reservation wage embodies aspirations as well as reality. In the medium run unions base their aspirations on past real wage changes. In the long run however unions adjust their aspirations to reality. This implies a positively sloped wage-setting curve in the medium run and a vertical long-run wage-setting curve as shown in Figure 1 and denoted there with *LRWS*.

Manning shows that the position of the long-run wage-setting curve is determined by the factor share of labor. With a Cobb-Douglas production function the factor share of labor is fixed and unaffected by productivity growth and population growth and hence the position of the long-run wage-setting curve is fixed, as well. As a consequence, capital accumulation and labor augmenting technical progress affect the real wage by shifting the labor demand curve, but leave equilibrium unemployment unaffected because the long-run wage-setting curve does not shift. For model consistency, the medium-run wage-setting curve *WS* must in the long run shift inward by the same amount as the outward shift of the labor demand curve to ensure that in the long run the medium-run wage-setting curve intersects the labor demand curve and the long-run wage-setting curve at the real wage in the long-run equilibrium. If the population size rises, the factor shares are initially changed and hence the long-run wage-setting curve has to shift outwards to re-establish the original fixed factor shares. The original fixed factor shares can only be re-established if the long-run wage-setting curve shifts by the same amount as the labor supply curve (which, as mentioned before, shift outwards when the population size increases). Hence, also in this case equilibrium unemployment is unchanged. Model consistency requires again that in the long run the medium-run wage-setting curve shifts such that it

intersects the labor demand curve and the long-run wage setting curve at the real wage in the long-run equilibrium. Hence, when one adds the concept of a vertical long-run wage-setting curve to the labor market, then the mechanisms in Figure 1 seem not to be too restrictive to be realistic. However, for the mechanisms of Figure 1 to take place the assumption of a Cobb-Douglas production function is crucial. If the aggregate production function is not of Cobb-Douglas form, then capital accumulation changes the factor share of labor. Hence, in this case it shifts the vertical long-run wage-setting curve, unless capital accumulation is matched by labor augmenting technical progress or population growth (or the sum of the two) by the same rate as the rate of capital accumulation. In the latter case the ratio of capital to effective labor is trendless and the factor share of labor does not change.

Although not all OECD countries' institutions imply a fixed unemployment benefit replacement ratio, an equilibrium in which real unemployment benefits are not raised along with real wages seems not to be politically feasible in the long run (see Pissarides (1998, p. 156)). Furthermore, Kaldor's stylized fact of constant shares of labor and capital in national income (Kaldor, 1963) are consistent with an aggregate production function of Cobb-Douglas form.¹ A constant ratio of capital to effective labor requires the effects from capital accumulation, technical progress, demographic change and labor force participation to have exactly offset each other by coincidence. This seems rather unlikely, but is consistent with the constant rate of economic growth in the 20th century in the US as shown in Jones (2002).² The result of LNJ is important because it implies that in the long run the unemployment rate cannot be influenced by policies that stimulate investment or increase the effective working-age population.

As mentioned before, KS criticize the restrictions of LNJ to ensure trendless unemployment as to be too restrictive to be realistic. In addition, they offer an alternative less restrictive restriction. In section 4 they present a model with endogenous technical progress, capital accumulation, a labor demand equation and a wage-setting equation. The labor demand equation and the wage-setting equation contain capital, efficiency of labor and labor, i.e.

¹Admittedly, Rowthorn argues that most econometric studies find estimates of the elasticity of substitution between labor and capital below unity. However, these estimates are controversial because of their inconsistency with Kaldor's stylized fact.

²However, Ben-David and Papell (1995) do not find evidence for a constant rate of economic growth in the 20th century for other advanced countries than the US and Canada, even after accounting for structural breaks.

all components of the ratio of capital to effective labor.³ The equations are *assumed* instead of *derived* from more primary assumptions. Because of this, the authors can assume the coefficients of the aforementioned components of the ratio of capital to effective labor to have any possible value. In turn, this enables the authors to derive less restrictive restrictions on these coefficients than those of LNJ to ensure trendless unemployment, i.e. the possibility of a trending ratio of capital to effective labor without the assumption of a Cobb-Douglas production function. In contrast to KS, the present paper derives the labor demand equation and the wage-setting equation in a similar model as the model in KS from more primary assumptions. It is shown that the model predicts the equilibrium unemployment rate according to the wage-setting equation to depend on the ratio of capital to effective labor. Further, it is shown that the model predicts the real wage according to the labor demand equation also to depend on the ratio of capital to effective labor. This in turn implies that the model result imposes the restrictions in the wage-setting equation and the labor demand equation the coefficient of capital to be equal to minus the coefficient of efficiency of labor and to be equal to minus the coefficient of labor. It can be shown that if these restrictions are imposed on the more general restriction for trendless unemployment of KS, then the ratio of capital to effective labor must be trendless or the production function must be of Cobb-Douglas form. Hence, the present paper confirms the restrictive restrictions of LNJ and it confirms the analytical results in Rowthorn's model with exogenous technical progress.

2 The Model

This section contains a model to derive the aforementioned results. The model marries the 'right-to-manage model' in LNJ with a model with endogenous labor augmenting technical progress influenced by Acemoglu (2002).

The economy consists of ϕN_t unskilled workers and $(1 - \phi) N_t$ skilled workers. Unions are assumed to bargain over the real wage of unskilled workers. This results in a real wage above unions' real reservation wage and therefore results in unemployment of some unskilled workers. The labor

³Actually, their wage-setting equation does not contain labor. However, since the authors do not give any justification for this, they must simply have forgotten to include it. (I come to this conclusion because in section 2, where the authors explain the restrictions in the model of LNJ, they include labor in the wage-setting equation).

market for skilled workers is assumed to be perfectly competitive. Hence, all skilled workers are employed.⁴ There is a larger number of intermediate goods. Influenced by Acemoglu (2002) I assume each intermediate goods producer i to produce intermediate goods output Y_{it} according to the CES-Production function

$$Y_{it} = \left\{ \alpha \left[\left(\int_0^{Q_t} x_{it}(j)^{1-\beta} dj \right) L_{uit}^\beta \right]^{\frac{\sigma-1}{\sigma}} + (1-\alpha) K_{it}^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}} \quad \sigma > 0, \quad (1)$$

where L_{uit} is employment of unskilled labor, $x_{it}(j)$ denotes the quantity of machine j . A range of horizontally differentiated machines produces together with unskilled labor the composite index between squared brackets in (1). The range of machines is denoted by Q_t . Further, K_{it} denotes capital and σ denotes the elasticity of substitution between the composite index between squared brackets and capital. Most importantly, the functional form of the composite index between squared brackets implies that σ is also the value of the elasticity of substitution between unskilled labor and capital (see Appendix A for a derivation). It can be shown that for $\sigma \rightarrow 1$ the CES-production function converges to the following Cobb-Douglas production function⁵

$$Y_{it} = \left[\left(\int_0^{Q_t} x_{it}(j)^{1-\beta} dj \right) L_{uit}^\beta \right]^\alpha K_{it}^{1-\alpha}.$$

The number of intermediate goods varieties, Q_t , is endogenously determined within the model. Similar to Jones (1995) and KS aggregate employment growth leads to expanding intermediate goods variety and therefore unskilled labor augmenting technical progress.

There is a single final good that can be consumed and transformed without costs into capital. The number of final goods firms is normalized to one and the market for the final good is perfectly competitive. Following Ethier (1982) I assume the final good to be produced by assembling the aforementioned intermediate goods according to the production function:

⁴The assumption of a unionized labor market for unskilled worker, but perfectly competitive labor market for skilled worker follows Lingens (2003).

⁵See similar on page 417 in Rowthorn.

$$Y_t = \left(\sum_{i=1}^m Y_{it}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \quad \eta > 1, \quad (2)$$

where m denotes the number of intermediate goods varieties, which is assumed to be constant, and η represents the elasticity of substitution between intermediate goods varieties. Eq. (2) implies that intermediate goods are horizontally differentiated. The price of the final good is normalized to one. Perfect competition in the final goods market causes the price of the final good to equal its unit cost. Therefore, one can derive from cost minimization that $1 = (\sum_{i=1}^m p_{yit}^{1-\eta})^{\frac{1}{1-\eta}}$, where p_{yit} denotes the price of each intermediate good variety i . Profit maximization of each final goods firm leads to the following aggregate demand for each intermediate goods variety i

$$Y_{it} = p_{yit}^{-\eta} Y_t. \quad (3)$$

Rearranging (3) yields $p_{yit} = Y_{i,t}^{-\frac{1}{\eta}} Y_t^{\frac{1}{\eta}}$. Substituting this expression for p_{yit} in the definition of $R_{yit} = p_{yit} Y_{it}$ gives⁶

$$R_{yit} = Y_{it}^{\kappa} Y_t^{\frac{1}{\eta}}, \quad \text{with } \kappa \equiv 1 - \frac{1}{\eta}. \quad (4)$$

The market for intermediate goods is assumed to be monopolistically competitive due to horizontal differentiation of intermediate goods. Profit is defined as $\Pi_{yit} = R_{yit} - \int_0^{Q_t} p_{Xit}(j) x_{it}(j) dj - w_{uit} L_{uit} - r K_{it}$, where $p_{Xit}(j)$ is the price of each machine variety j , w_{uit} denotes the wage for unskilled labor and r_t denotes the interest rate. Substituting (1) in (4) and substituting the resulting expression in the profit definition of each intermediate goods producer yields

$$\begin{aligned} \Pi_{yit} = & \left\{ \alpha \left[\left(\int_0^{Q_t} x_{it}(j)^{1-\beta} dj \right) L_{uit}^{\beta} \right]^{\frac{\sigma-1}{\sigma}} + (1-\alpha) K_{it}^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\kappa\sigma}{\sigma-1}} Y_t^{\frac{1}{\eta}} \\ & - \int_0^{Q_t} p_{Xit}(j) x_{it}(j) dj - w_{uit} L_{uit} - r K_{it}. \end{aligned} \quad (5)$$

⁶See Jerger and Michaelis (1999) and Bräuninger (2004).

Maximizing (5) upon optimal choice of L_{uit} and $x_{it}(j)$, $\forall j \in [0, Q_t]$ (and omitting for brevity the first order condition of K_{it}) gives the optimal factor shares of unskilled labor and each machine variety in each intermediate goods firm's revenue (after rearranging terms and use of (1) and (4)) as⁷

$$\frac{w_{uit}L_{uit}}{R_{yit}} = \kappa\beta\theta_{it}, \quad (6)$$

$$\frac{p_{xit}(j)x_{it}(j)}{R_{yit}} = \psi_{it}(j)\kappa(1-\beta)\theta_{it}, \quad \forall j \in [0, Q_t], \quad (7)$$

$$\text{with } \theta_{it} \equiv \alpha \left[\frac{\left(\int_0^{Q_t} x_{it}(j)^{1-\beta} dj \right) L_{uit}^\beta}{Y_{it}} \right]^{\frac{\sigma-1}{\sigma}} \quad (8)$$

$$\text{and } \psi_{it}(j) \equiv \frac{x_{it}(j)^{1-\beta} L_{uit}^\beta}{\left(\int_0^{Q_t} x_{it}(j)^{1-\beta} dj \right) L_{uit}^\beta} \quad \forall j \in [0, Q_t]. \quad (9)$$

Since $R_{yit} = p_{yit}Y_{it}$ and Y_{it}/L_{uit} measures the level of activity, (6) implies:

Result 1: The real wage rate, w_{uit}/p_{yit} , is according to the labor demand equation dependent on the level of activity, Y_{it}/L_{uit} , and $\beta\theta_{it}$, which will be shown to be the factor share of unskilled labor.

Unskilled workers of each intermediate goods variety are represented by their own union that bargains with its intermediate goods firm. Unskilled workers are assumed to be risk-neutral. Each union is assumed to maximizes the sum of its M_{it} members' utility, which is:⁸

$$\tilde{U}_{it} = L_{uit}w_{uit}(1-\tau_t) + (M_{it} - L_{uit})a_{it}, \quad (10)$$

where τ_t denotes a wage tax to finance unemployment benefits. Furthermore, a_{it} denotes the expected alternative income of members who are not employed by intermediate goods firm i . The L_{uit} members who are employed by firm i receive the net wage $w_{uit}(1-\tau_t)$, while the $(M_{it} - L_{uit})$ members

⁷Note that in the optimization problem each firm takes Y_t as given because it believes to be too small to have an influence on it.

⁸See Bräuninger (2004). In Appendix B I assume risk averse unskilled workers who avoid uncertain income by use of insurance contracts.

who are not employed by firm i receive expected alternative income a_{it} . I assume bargaining according to the ‘right-to-manage model’. According to this model the union and the firm are assumed to bargain over the real wage of unskilled workers. Each firm has the ‘right to manage’, i.e. it unilaterally sets the level of employment of unskilled workers L_{uit} that maximizes its profit, given the outcome of the wage bargain.⁹ It is assumed that the negotiators chooses the real wage that maximizes the Nash product Ω_{it} :¹⁰

$$\max_{w_{uit}} \Omega_{it} = U_{it}^\gamma \Pi_{yit}^{1-\gamma}, \quad (11)$$

where γ represents the relative bargaining power of the union in the bargaining. Further, $U_{it} = \tilde{U}_{it} - U_i^0$, where $U_i^0 = M_{it}a_{it}$ is the expected alternative income of all union members in intermediate goods sector i when no bargaining agreement is reached and therefore all members are not employed by firm i . Following, e.g. Pissarides (1998) and Koskela (2001) the expected alternative income is assumed to be

$$a_{it} = (1 - u_t)w_{ut}^e(1 - \tau_t) + u_t B_t, \quad (12)$$

where u_t denotes the aggregate unemployment rate (remember that only unskilled workers are unemployed), w_{ut}^e denotes the expected net wage of unskilled workers elsewhere than in intermediate goods sector i and B_t represents the real unemployment benefit. Optimization problem (11) yields

$$\frac{\partial \Omega_{it}}{\partial w_{uit}} = 0 \quad \Leftrightarrow \quad \gamma \frac{\partial U_{it}/\partial w_{uit}}{U_{it}} + (1 - \gamma) \frac{\partial \Pi_{it}/\partial w_{uit}}{\Pi_{it}} = 0 \quad (13)$$

Combining (13) with $U_{it} = \tilde{U}_{it} - U_i^0$, (10) and $U_i^0 = M_{it}a_{it}$ yields:^{11,12}

$$[w_{uit}(1 - \tau_t) - a_{it}] \left[\gamma \eta_{L_{ui}, w_{ui}} + (1 - \gamma) \eta_{\Pi_{yi}, w_{ui}} \right] - \gamma w_{uit}(1 - \tau_t) = 0, \quad (14)$$

⁹Alternatively, I could have assumed the ‘efficient bargaining model’ of McDonald and Solow (1981), in which the union and the firm bargain over the real wage and employment as well. However, most of the literature argues that bargaining over employment is rarely observed. Most importantly, the model results concerning trendless unemployment are unaffected by the choice between the ‘right-to-manage model’ and the ‘efficient bargaining model’.

¹⁰See, e.g. Koskela (2001).

¹¹Note that in the optimization problem $\partial L_{uit}/\partial w_{uit} \neq 0$.

¹²See similar in Manning (1993).

where $\eta_{L_{ui},w}$ denotes the (absolute value of the) wage elasticity of demand for unskilled labor and $\eta_{\Pi_{yi},w_{ui}}$ denotes the (absolute value of the) elasticity of profits with respect to the wage rate of unskilled workers. Rearranging (14) gives the net wage of unskilled workers as a mark-up μ_{it} over the expected alternative income according to

$$w_{uit}(1 - \tau_t) = \mu_{it}a_{it}, \quad (15)$$

$$\text{where } \mu_{it} \equiv \frac{\gamma\eta_{L_{ui},w_{ui}} + (1 - \gamma)\eta_{\Pi_{yi},w_{ui}}}{\gamma(\eta_{L_{ui},w_{ui}} - 1) + (1 - \gamma)\eta_{\Pi_{yi},w_{ui}}} \quad (16)$$

$$\text{with } \frac{\partial \mu_{it}}{\partial \eta_{L_{ui},w}} < 0 \quad \wedge \quad \frac{\partial \mu_{it}}{\partial \eta_{\Pi_{yi},w}} < 0, \quad \text{provided } \mu_{it} > 1.$$

Using in (12) the fact that due to symmetry $w_{uit} = w_{ut}^e = w_{ut}$ and $\mu_{it} = \mu_t$ and assuming a fixed unemployment benefit replacement ratio $B_t = bw_{ut}(1 - \tau_t)$ gives: $w_{ut}(1 - \tau_t) = \mu_t[(1 - u_t)w_{ut}(1 - \tau_t) + u_tbw_{ut}(1 - \tau_t)]$. Rearranging gives rise to the equilibrium unemployment rate as

$$u_t = \frac{\mu_t - 1}{\mu_t(1 - b)}, \quad (17)$$

$$\text{with } \frac{\partial u_t}{\partial \mu_t} > 0.$$

For the case of the CES-production function in (1) the (absolute value of the) wage elasticity of labor demand, $\eta_{L_{uit},w}$, is in Appendix A derived as

$$\eta_{L_{ui},w_{ui}} \equiv -\frac{\partial L_{uit}}{\partial w_{uit}} \frac{w_{uit}}{L_{uit}} = \eta_{L_{ut},w_u} = -\beta(1 - \sigma) + 1 + \beta\theta_t(\eta - \sigma), \quad (18)$$

where θ_t denotes "the aggregate share of expenditures for machines and unskilled labor in aggregate final goods output net of aggregate profits". I.e. $\theta_t = \left[\int_0^{Q_t} p_{Xt}(j)x_t(j)dj + w_{ut}L_{ut} \right] / [\kappa Y_t]$, where κY_t is in Appendix A shown to be aggregate final goods output net of aggregate profits. Most importantly, "the aggregate share of expenditures for *unskilled labor* in aggregate final goods output net of aggregate profits" (henceforth factor share of unskilled workers) is denoted by $\beta\theta_t$, i.e. $\beta\theta_t = w_{ut}L_{ut}/\kappa Y_t$ (where β is in Appendix A

derived to be "each intermediate goods firm's share of expenditures for unskilled labor in the sum of expenditures for machines and unskilled labor"). Similar to Hall and Nixon (2000, pp. 425-426), the (absolute value of the) elasticity of profits with respect to the wage rate of unskilled workers η_{Π_y, w_u} is in Appendix A derived as

$$\eta_{\Pi_{y_i}, w_{u_i}} \equiv -\frac{\partial \Pi_{y_{it}}}{\partial w_{uit}} \frac{w_{uit}}{\Pi_{y_{it}}} = \eta_{\Pi_y, w_u} = \beta \theta_t (\eta - 1), \quad (19)$$

Hence, under the plausible assumptions that $\eta > \sigma$ and $\eta > 1$, (18) and (19) imply that an increase the factor share of unskilled labor, $\beta \theta_t$, increases η_{L_u, w_u} and η_{Π_y, w_u} . In turn, increases in η_{L_u, w_u} and η_{Π_y, w_u} decrease the net wage mark-up, μ_t according to (16). Finally, a reduction in net wage mark-up decreases the equilibrium unemployment rate according to (17). As a consequence, the equilibrium unemployment rate is decreasing in the factor share of unskilled labor, $\beta \theta_t$. The latter result confirms the discussion to Figure 1 in the introduction. Eq. (17) represents the vertical long-run wage-setting curve in Figure 1 applied to unskilled labor. The long-run wage-setting curve determines a single unemployment rate for a given factor share of unskilled labor. A change of this factor share shifts the long-run wage-setting curve and hence changes unemployment. This gives rise to the following result:

Result 2: The equilibrium unemployment rate is according to the long-run wage-setting equation decreasing in the factor share of unskilled labor, $\beta \theta_{it}$.

In what follows I solve for a closed form solution of the factor share of unskilled labor. For this purpose it is first necessary to solve for a closed form solution of labor augmenting technical progress. I do so, by first noting that the market for machines is monopolistically competitive due to horizontally differentiated machines. This allows machine producers to make operating profits to finance fixed labor investments to enter the market for machines. From (7) and after aggregating over all m intermediate goods firms one can derive the indirect demand function that each machine producer j faces as

$$p_{X_t}(j) = \left[\frac{x_t(j)^{-\beta} L_{ut}^\beta}{\left(\int_0^{Q_t} x_t(j)^{1-\beta} dj \right) L_{ut}^\beta} \right] (1 - \beta) \kappa \theta_t R_{yt} \quad \forall j \in [0, Q_t]. \quad (20)$$

Production of machines requires skilled labor only. It is assumed that the total skilled labor requirement of each machine firm for producing $x_t(j)$ units equals $l_{xt}(j)$, with $l_{xt}(j) = \omega x_t(j) + F$. Here ω denotes the unit skilled labor requirement, while F represents fixed labor investment, which is due each period. The profit definition of each machine producer is $\Pi_{Xt}(j) = p_{Xt}(j)x_t(j) - w_{st}(\omega x_t(j) + F)$, where w_{st} denotes the wage of skilled workers. Substituting (20) in the profit definition $\Pi_{Xt}(j)$ and choosing the profit maximizing value of $x_t(j)$ gives the monopoly-pricing rule:¹³

$$p_{Xt}(j) = p_{Xt} = \left(\frac{1}{1 - \beta} \right) \omega w_{st} \quad \forall j \in [0, Q_t]. \quad (21)$$

Substituting (21) in the profit definition of each machine firm and imposing zero-profits, i.e $\Pi_{Xt}(j) = 0$ yields

$$x_t(j) = x = \left(\frac{1 - \beta}{\beta} \right) \left(\frac{F}{\omega} \right) \quad \forall j \in [0, Q_t]. \quad (22)$$

Utilizing in $\left(\int_0^{Q_t} x_t(j)^{1-\beta} dj \right) L_{ut}^\beta$ the fact that according to (22) $x_t(j) = x$ leads to

$$\left(\int_0^{Q_t} x_t(j)^{1-\beta} dj \right) L_{ut}^\beta = Q_t x^{1-\beta} L_{ut}^\beta. \quad (23)$$

Obviously we have $Q_t(\omega x_t(j) + F) = L_{st}$, where L_{st} is aggregate skilled labor. Substituting in this expression (22) for $x_t(j)$ yields $Q_t = \left(\frac{\beta}{F} \right) L_{st}$. Per definition $L_t = (1 - u_t)\phi N_t + (1 - \phi)N_t$, where N_t denotes the sum of unskilled and skilled persons. Hence, $N_t = \{1 / [(1 - u_t)\phi + (1 - \phi)]\} L_t$, which combined with $L_{ut} = (1 - u_t)\phi N_t$ and $L_{st} = (1 - \phi)N_t$ gives $L_{ut} = \{[(1 - u_t)\phi] / [(1 - u_t)\phi + (1 - \phi)]\} L_t$ and $L_{st} = \{[1 - \phi] / [(1 - u_t)\phi + (1 - \phi)]\} L_t$. Substituting the latter identity in $Q_t = \left(\frac{\beta}{F} \right) L_{st}$ gives rise to

$$Q_t = \left[\frac{\beta}{F} \right] \left[\frac{1 - \phi}{(1 - u_t)\phi + (1 - \phi)} \right] L_t. \quad (24)$$

Hence, eq. (24) shows that growth of aggregate employment (of unskilled and skilled workers) leads to expanding machine variety. Substituting (24), (22) and $L_{ut} = \{[(1 - u_t)\phi] / [(1 - u_t)\phi + (1 - \phi)]\} L_t$ in (23) yields

¹³Each intermediate goods producer takes θ_t and R_{yt} as given because it believes to be too small to affect these variables.

$$\left(\int_0^{Q_t} x_t(j)^{1-\beta} dj \right) L_{ut}^\beta = A_t L_t, \quad \text{with} \quad (25)$$

$$A_t \equiv \left\{ \left[\frac{1-\beta}{\omega} \right]^{1-\beta} \left[\frac{\beta}{F} \right]^\beta \left[\frac{1-\phi}{(1-u_t)\phi + (1-\phi)} \right] \left[\frac{(1-u_t)\phi}{(1-u_t)\phi + (1-\phi)} \right]^\beta \right\} L_t^\beta, \quad (26)$$

where A_t denotes endogenous labor augmenting technical progress. Eq. (26) reveals technical progress to be rising in growth of aggregate employment (of unskilled and skilled workers). Further, the term $\{.\}$ in eq. (26) in front of L_t^β is trendless if the unemployment rate is trendless.

Now that we have solved for endogenous labor augmenting technical progress, we can use this information in the labor demand equation of intermediate goods firms to derive from it a closed form solution of the factor share of unskilled labor. Substituting (25) in (1) yields

$$Y_{it} = \alpha \left[(A L_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) (K_t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \quad (27)$$

Upon substituting (25) and (27) in (8) and rearranging one gets

$$\beta \theta_t = \frac{\beta \alpha}{\alpha + (1-\alpha) \left(\tilde{k}_t \right)^{\frac{\sigma-1}{\sigma}}}, \quad (28)$$

$$\text{where } \tilde{k}_t \equiv \frac{K_t}{A_t L_t}. \quad (29)$$

From (28) it is straightforward to derive

$$\begin{aligned} \frac{\partial(\beta \theta_t)}{\partial \tilde{k}_t} &= \left(\frac{1-\sigma}{\sigma} \right) \beta \theta_t (1-\theta_t) \left(\frac{1}{\tilde{k}_t} \right) \\ \Rightarrow \frac{\partial(\beta \theta_t)}{\partial \tilde{k}_t} \begin{cases} > \\ = \\ < \end{cases} 0 &\Leftrightarrow \sigma \begin{cases} < \\ = \\ > \end{cases} 1. \end{aligned} \quad (30)$$

Note that, as explained in the introduction, the equilibrium unemployment rate is in the long run entirely determined by the long-run wage-setting curve and therefore labor demand determines in the long run only the real wage.

Further, as explained in Result 2, the equilibrium unemployment rate is according to the long-run wage-setting equation decreasing in the factor share of unskilled labor, $\beta\theta_{it}$. Hence, (30) implies together with Result 2 the following result:

Result 3: An increase in the ratio of capital to effective labor, \tilde{k}_t , decreases the equilibrium unemployment rate (through its influence on the factor share of unskilled labor, $\beta\theta_{it}$) if $\sigma < 1$ and increases it if $\sigma > 1$. The equilibrium unemployment is only trendless in the knife edge case $\sigma = 1$ or if \tilde{k}_t (and hence $\beta\theta_{it}$ as well) is trendless.

This result confirms the result of Rowthorn in a model with exogenous technical progress, but is in sharp contrast to the thesis in KS. Result 3 implies trendless unemployment to requires the intermediate goods production function to be of Cobb-Douglas form or the economy to be in the steady state.¹⁴

Appendix A: Intermediate Goods Firms' Cost Minimization problem

This appendix presents each intermediate goods firm's cost minimization problem to derive various elasticities and factor shares that were used in the text. To simplify the mathematics the cost minimization problem of each intermediate goods firm is split into two stages. In the first stage the costs to produce the composite index $D_{it} \equiv \left(\int_0^{Q_t} x_{it}(j)^{1-\beta} dj \right) L_{uit}^\beta$ in (1) are minimized by optimal choice of $x_{it}(j)$, $\forall j \in [0, Q_t]$ and L_{uit} . In the second stage the costs to produce $Y_{it} = \left[\alpha D_{it}^{\frac{\sigma-1}{\sigma}} + (1-\alpha) K_{it}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$ are minimized by optimal choice of D_{it} and K_{it} .

The first stage cost minimization problem is:

$$\min_{x_{it}(j), L_{uit}} \mathcal{L} = \int_0^{Q_t} p_{Xit}(j) x_{it}(j) dj - w_{uit} L_{uit} + \lambda \left[D_{it} - \left(\int_0^{Q_t} x_{it}(j)^{1-\beta} dj \right) L_{uit}^\beta \right],$$

which gives rise to the first order condition

¹⁴Appendix B shows briefly that with endogenous capital accumulation from endogenous saving \tilde{k}_t is upwards (downwards) trending for values of \tilde{k}_t below (above) its steady state value. Hence, \tilde{k}_t is only trendless in the steady state.

$$x_{it}(j) = \left(\frac{w_{uit}}{p_{Xit}(j)} \right) \psi_{it}(j) \left(\frac{1-\beta}{\beta} \right) L_{uit}, \forall j \in [0, Q_t] \quad (31)$$

Combining (31) with the cost definition $p_{Dit}D_{it} \equiv \int_0^{Q_t} p_{Xit}(j)x_{it}(j)dj + w_{uit}L_{uit}$ gives

$$x_{it}(j) = p_{Xit}(j)^{-1} \psi_{it}(j) (1-\beta) p_{Dit} D_{it}, \forall j \in [0, Q_t] \quad (32)$$

$$L_{uit} = w_{uit}^{-1} \beta p_{Dit} D_{it}, \quad (33)$$

where rearranging (33) and using the definition $p_{Dit}D_{it} \equiv \int_0^{Q_t} p_{Xit}(j)x_{it}(j)dj + w_{uit}L_{uit}$ confirms the claim in the text that β is "the share of expenditures for unskilled labor in the sum of expenditures for machines and unskilled labor". Substituting (32) and (33) in $D_{it} \equiv \left(\int_0^{Q_t} x_{it}(j)^{1-\beta} dj \right) L_{uit}^\beta$ and collecting terms yields

$$\begin{aligned} p_{Dit} &= Z_{it} \left(\int_0^{Q_t} p_{Xit}(j)^{1-\beta} dj \right) w_{uit}^\beta, \\ \text{with } Z_{it} &\equiv \left\{ \left(\int_0^{Q_t} \psi_{it}(j)^{1-\beta} \right) (1-\beta)^{1-\beta} \beta^\beta \right\}^{-1}. \end{aligned} \quad (34)$$

The second stage cost minimization problem is:

$$\min_{D_{it}, K_{it}} \mathcal{L}' = p_{Dit}D_{it} + r_t K_{it} + \lambda' \left\{ Y_{it} - \left[\alpha D_{it}^{\frac{\sigma-1}{\sigma}} + (1-\alpha) K_{it}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\},$$

where, p_{Dit} denotes the price index of D_{it} . The optimization problem leads to the first order condition

$$K_{it} = \left(\frac{1-\alpha}{\alpha} \right)^\sigma \left(\frac{p_{Dit}}{r_t} \right)^\sigma D_{it}. \quad (35)$$

Upon combining (35) with the cost definition $C_{it} = c_{it}Y_{it} \equiv p_{Dit}D_{it} + r_t K_{it}$ (where C_{it} represents total cost and c_{it} represents unit cost) one gets

$$D_{it} = \left[\frac{\alpha^\sigma p_{Dit}^{-\sigma}}{\alpha^\sigma p_{Dit}^{1-\sigma} + (1-\alpha)^\sigma r_t^{1-\sigma}} \right] c_{it} Y_{it}, \quad (36)$$

$$K_{it} = \left[\frac{(1-\alpha)^\sigma r_t^{-\sigma}}{\alpha^\sigma p_{Dit}^{1-\sigma} + (1-\alpha)^\sigma r_t^{1-\sigma}} \right] c_{it} Y_{it}. \quad (37)$$

Substituting (36) and (37) in $Y_{it} = \left[\alpha D_{it}^{\frac{\sigma-1}{\sigma}} + (1-\alpha) K_{it}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$ and collecting terms yields

$$c_{it} = \left[\alpha^\sigma p_{Dit}^{1-\sigma} + (1-\alpha)^\sigma r_t^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \quad (38)$$

Upon substituting (34) in (38) one gets

$$c_{it} = \left[\alpha^\sigma Z_{it} \left(\int_0^{Q_t} p_{Xit}(j)^{1-\beta} dj \right) w_{uit}^\beta + (1-\alpha)^\sigma r_t^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (39)$$

Following Hamermesh (1986) the Allen-Uzawa elasticity of substitution between unskilled labor and capital is defined as

$$\sigma_{L_{uit}, K_{it}} = \frac{c_{it} \left(\frac{\partial^2 c_{it}}{\partial w_{ut} \partial r_t} \right)}{\left(\frac{\partial c_{it}}{\partial w_{ut}} \right) \left(\frac{\partial c_{it}}{\partial r_t} \right)} \quad (40)$$

Application of (40) to (39) reveals that $\sigma_{L_{uit}, K_{it}} = \sigma$. Further, upon application of a formula of the wage elasticity of labor demand in the two input factor case in Hamermesh (1986) to our three input case we get the the wage elasticity of labor demand as

$$\eta_{L_{uit}, w_{uit}} \equiv -\frac{\partial L_{uit}}{\partial w_{uit}} \frac{w_{uit}}{L_{uit}} = +s_{L_{uit}} \xi_{it} - s_{L_{uit}} \sigma_{L_{uit}, L_{uit}} \equiv \beta \theta_{it} (\eta - \sigma_{L_{uit}, L_{uit}}), \quad (41)$$

where $s_{L_{uit}}$ denotes the factor share of unskilled labor, $\beta \theta_{it}$, $\xi_{it} \equiv \eta \equiv -\frac{\partial Y_{it}}{\partial p_{yit}} \frac{p_{yit}}{Y_{it}}$ denotes the price elasticity of demand for the intermediate good i and $\sigma_{L_{uit}, L_{uit}}$ denotes the own Allen-Uzawa elasticity of substitution, which is defined as

$$\sigma_{L_{uit}, L_{uit}} = \frac{c_{it} \left(\frac{\partial^2 c_{it}}{\partial w_{ut}^2} \right)}{\left(\frac{\partial c_{it}}{\partial w_{ut}} \right) \left(\frac{\partial c_{it}}{\partial w_{ut}} \right)}. \quad (42)$$

Application of (42) to (39) gives

$$\sigma_{L_{uit}, L_{uit}} = \left[\frac{\beta(1 - \sigma) - 1}{\beta\theta_{it}} \right] + \sigma. \quad (43)$$

Substituting (43) in (41) and noting that due to symmetry $\theta_{it} = \theta_t$, $\forall i = 1, \dots, m$ gives rise to (18) in the text.

Substituting (4) and the cost definition $p_{D_{it}}D_{it} + r_tK_{it} \equiv c_{it}Y_{it}$ in the profit definition $\Pi_{yit} = R_{yit} - (p_{D_{it}}D_{it} + r_tK_{it})$ gives $\Pi_{yit} = Y_{it}^\kappa Y_t^{\frac{1}{\eta}} - c_{it}Y_{it}$. Hence, profit maximization of each intermediate goods firm by optimal choice of Y_{it} leads to the monopoly pricing rule $\kappa p_{yit} = c_{it}$. Substituting the monopoly pricing rule for c_{it} in the profit definition $\Pi_{yit} = (p_{yit} - c_{it})Y_{it}$ yields

$$\Pi_{yit} = (1 - \kappa)p_{yit}Y_{it} = (1 - \kappa)R_{yit} \quad (44)$$

Upon using in the profit definition $\Pi_{yit} = R_{yit} - \int_0^{Q_t} p_{Xit}(j)x_{it}(j)dj - w_{uit}L_{uit}$ the envelope condition and use of (44) one gets

$$\eta_{\Pi_{yit}, w_{uit}} \equiv -\frac{\partial \Pi_{yit}}{\partial w_{uit}} \frac{w_{uit}}{\Pi_{yit}} = \frac{w_{uit}L_{uit}}{(1 - \kappa)R_{yit}} \quad (45)$$

Substituting (6) in (45), use of the definition $\kappa \equiv 1 - \frac{1}{\eta}$ and use of $\theta_{it} = \theta_t$, $\forall i = 1, \dots, m$ due to symmetry yields (19) in the text.

Since the price of the final good is normalized to one and the market for final goods is perfectly competitive, we have:

$$Y_t = mp_{yt}\bar{Y}_t = mR_{yt}, \quad (46)$$

where p_{yt} , \bar{Y}_t and R_{yt} denote the price, output and revenue of each intermediate goods firm in the symmetric equilibrium. Aggregating (6) and (7) over all m intermediate good varieties and combining with (46) yields: $\theta_t = \left[\int_0^{Q_t} p_{Xt}(j)x_t(j)dj + w_{ut}L_{ut} \right] / [\kappa Y_t]$. Combining (46) with (44) gives rise to $m\Pi_{yt} = (1 - \kappa)Y_t$, where Π_{yt} denotes the profit of each intermediate goods firm in the symmetric equilibrium. Hence, due to zero profits in the final goods and machine markets aggregate profits equal $m\Pi_{yt} = (1 - \kappa)Y_t$.

Appendix B: Endogenous Capital Accumulation

This appendix adds to the model of the text endogenous capital accumulation from endogenous saving. In each period t the economy consists of

N_t young persons and N_{t-1} old persons. Each young persons works one unit of time if employed. Of these young persons $(1 - u_t) \phi N_t$ work as unskilled workers, $u_t \phi N_t$ are unemployed and $(1 - \phi) N_t$ work as skilled workers. The number of young persons grows at the rate n . Each person born in period t receives when young the expected income $w_{u,t}(1 - \tau_t) + u_t B_t$ if unskilled or the certain income $w_{s,t}(1 - \tau_t)$ if skilled and allocates this income between consumption, c_t , and saving, s_{ut} if unskilled or s_{st} if skilled. Parents bequeath ownership of intermediate goods firms to their children. Because of this, each person born in period t receives when old the aggregate profit income per in period t old person $(1 - \kappa) Y_{t+1}/N_t$ (as was derived at the end of Appendix A) plus saving income, does not work and devotes all income to consumption. The intertemporal utility function of each person born in period t is assumed to be $U_t = \ln c_t + (1/(1 + \rho)) \ln c_{t+1}$, where c_t denotes consumption when young, c_{t+1} denotes consumption when old and ρ denotes the rate of time preference. Assuming for simplicity that unskilled persons avoid uncertain income from possible unemployment by use of insurance contracts, the optimization problem of each unskilled or skilled person born in period t is:

$$\begin{aligned} \max_{c_t, c_{t+1}} \mathcal{L} &= \ln c_t + \frac{\ln c_{t+1}}{1 + \rho} + \lambda \left[c_t + \frac{c_{t+1}}{1 + r_{t+1}} - I_{ht} - \frac{(1 - \kappa) Y_{t+1}}{N_t} \right], \\ \text{with } I_{ht} &= \begin{cases} w_{u,t}(1 - \tau_t) + u_t B_t & \text{if } h = \text{unskilled person} \\ w_{s,t}(1 - \tau_t) & \text{if } h = \text{skilled person} \end{cases}. \end{aligned}$$

which gives rise to the optimal saving rate

$$s_{ht} = \left(\frac{1}{2 + \rho} \right) \left[I_{ht} - (1 - \rho)(1 - \kappa) \frac{Y_{t+1}}{N_t} \right]. \quad (47)$$

Using in $\theta_t = \left[\int_0^{Q_t} p_{Xt}(j) x_t(j) dj + w_{ut} L_{ut} \right] / [\kappa Y_t]$ the fact that $p_{Xt}(j) = p_{Xt}$ according to (21) and $x_t(j) = x$ according to (22) yields $Q_t p_{Xt} x + w_{ut} L_{ut} = \kappa \theta_t Y_t$. Upon using in this expression the fact that due to zero profits in the machine market $Q_t p_{Xt} x = w_{st} L_{st}$, one gets $w_{st} L_{st} + w_{ut} L_{ut} = \kappa \theta_t Y_t$. Combining this expression with the fact that due to a balanced budget of unemployment benefits $\tau_t w_{ut}(1 - u_t) \phi N_t + \tau_t w_{st}(1 - \phi) N_t = B_t u_t \phi N_t$ gives

$$w_{ut}(1 - \tau_t)(1 - u_t) \phi N_t + B_t u_t \phi N_t + w_{st}(1 - \tau_t)(1 - \phi) N_t = \kappa \theta_t Y_t. \quad (48)$$

Combining (48) with (47) and using $K_{t+1} = s_t N_t$ (which assumes absence of capital depreciation) gives rise to

$$\begin{aligned}
K_{t+1} &= \left(\frac{1}{2+\rho} \right) \left[\kappa \theta_t Y_t - (1-\rho)(1-\kappa) \frac{Y_{t+1}}{N_t} \right] \quad \text{or} \\
\tilde{k}_{t+1} &= \left(\frac{1}{2+\rho} \right) \kappa \theta_t \left(\frac{1}{\hat{A}_t + 1} \right) \left(\frac{1}{\hat{v}_t + 1} \right) \left(\frac{1}{1+n} \right) \tilde{y}_t \\
&\quad - \left(\frac{1-\rho}{2+\rho} \right) (1-\kappa) \tilde{y}_{t+1}, \quad \text{with} \\
\hat{A}_t &\equiv \frac{A_{t+1}}{A_t} - 1, \quad \hat{v}_t \equiv \frac{(1-u_{t+1})\phi + (1-\phi)}{(1-u_t)\phi + (1-\phi)} - 1, \\
\tilde{y}_t &\equiv Y_t / (A_t L_t).
\end{aligned} \tag{49}$$

Total differentiation of (49) with respect to \tilde{k}_t , collecting terms yields, using $\eta_{\theta_t, \tilde{k}_t} \equiv \left(\partial \theta_t / \partial \tilde{k}_t \right) \left(\tilde{k}_t / \theta_t \right) = [(1-\sigma)/\sigma] (1-\theta_t)$, $\eta_{\hat{A}_t, \tilde{k}_t} \equiv \left(\partial \hat{A}_t / \partial \tilde{k}_t \right) \left(\tilde{k}_t / \hat{A}_t \right)$, $\eta_{\hat{v}_t, \tilde{k}_t} \equiv \left(\partial \hat{v}_t / \partial \tilde{k}_t \right) \left(\tilde{k}_t / \hat{v}_t \right)$ and $\eta_{\tilde{y}_t, \tilde{k}_t} \equiv \left(\partial \tilde{y}_t / \partial \tilde{k}_t \right) \left(\tilde{k}_t / \tilde{y}_t \right)$ and assuming that $\eta_{\hat{A}_t, \tilde{k}_t}$ and $\eta_{\hat{v}_t, \tilde{k}_t}$ have relatively small magnitudes if they should be negative (which in turn depends on the value of σ) yields

$$\frac{d\tilde{k}_{t+1}}{d\tilde{k}_t} = \frac{\left[\left(\frac{\kappa \theta_t \tilde{y}_t}{\tilde{k}_t} \right) \left(\frac{1}{\hat{A}_t + 1} \right) \left(\frac{1}{\hat{v}_t + 1} \right) \left(\frac{1}{1+n} \right) \right] \left[\left(\frac{1}{\sigma} \right) (1-\theta_t) + \eta_{\hat{A}_t, \tilde{k}_t} + \eta_{\hat{v}_t, \tilde{k}_t} \right]}{1 + \left(\frac{1-\rho}{2+\rho} \right) \left(\frac{(1-\kappa)(1-\theta_{t+1})\tilde{y}_{t+1}}{\tilde{k}_{t+1}} \right)} > 0. \tag{50}$$

Eq. (50) implies that \tilde{k}_t is upwards (downwards) trending for values of \tilde{k}_t below (above) its steady state value \tilde{k}^* . Hence, \tilde{k}_t is only trendless in the steady state. Local stability requires $-1 < \left(d\tilde{k}_{t+1}/d\tilde{k}_t \right) < 1$ evaluated in the steady state $\tilde{k}_{t+1} = \tilde{k}_t = \tilde{k}^*$. Hence, local stability is fulfilled if

$$\frac{\left[\left(\frac{\kappa \theta^* \tilde{y}^*}{\tilde{k}^*} \right) \left(\frac{1}{\hat{A}^* + 1} \right) \left(\frac{1}{\hat{v}^* + 1} \right) \left(\frac{1}{1+n} \right) \right] \left[\left(\frac{1}{\sigma} \right) (1-\theta^*) + \eta_{\hat{A}^*, \tilde{k}^*} + \eta_{\hat{v}^*, \tilde{k}^*} \right]}{1 + \left(\frac{1-\rho}{2+\rho} \right) \left(\frac{(1-\kappa)(1-\theta^*)\tilde{y}^*}{\tilde{k}^*} \right)} < 1. \tag{51}$$

Fulfillment of (51) can be confirmed by substituting (49) for $\tilde{k}_{t+1} = \tilde{k}_t = \tilde{k}^*$ in (51).

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