

Agricultural Productivity Growth and Escape from the Malthusian Trap*

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Abstract

Industrialization allowed the industrialized world of today to escape from the Malthusian regime characterized by low economic and population growth and to enter the post-Malthusian regime of high economic and population growth. To explain the transition between these regimes, we construct a growth model with two consumption goods (an agricultural and a manufacturing good), endogenous fertility, and endogenous technological progress in the manufacturing sector. We show that with an exogenous increase in the growth of agricultural productivity our model is able to replicate stylized facts of the British industrial revolution. The paper concludes by illustrating that our proposed model framework can be extended to include the demographic transition, i.e., a regime in which economic growth is associated with falling fertility.

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1 Introduction

Our paper aims to explain the simultaneous take-off in economic growth and population growth in the industrialized world of today starting at the end of the eighteenth century with the British industrial revolution. We illustrate that our proposed model can be extended to explain the fertility decline (often referred to as the *demographic transition*) and the acceleration in economic growth which started in Western Europe at the end of the nineteenth century.

In his classic work Malthus (1798) argued that an increase in agricultural productivity results in increasing population size but no long-run improvement in living standards. The underlying assumptions are twofold. First, an increase in per capita income above a particular equilibrium level of consumption leads to increasing population size. Second, an increasing population size dilutes per capita resources and, as a consequence, consumption falls back to its equilibrium level. As a result, any given economy will be trapped in a situation with economic stagnation. This trap is commonly labeled *Malthusian trap* or *low level equilibrium trap* (see Nelson, 1956).

Malthus's view correctly describes the demography and economy of Western Europe before and during his lifetime. Up to this time there had been slow population growth and low per capita income growth - and population growth had a negative effect on per capita income growth. However, the industrial revolution later launched Western Europe into a new phase of human history, in which population growth ceased to prevent long-run growth in per capita income. One can refer to this new stage as the *take-off into self-sustained growth* (see Rostow, 1960). Moreover, population growth accelerated (see Maddison, 1991). We label this simultaneous take-off in economic and population growth *escape from the Malthusian trap*.

Subsequent to this *post-Malthusian regime* came the *modern growth regime*, in which the positive post-Malthusian relation between population and economic growth turned negative. During the modern growth regime economic growth continued to accelerate while population growth started to decline. As noted in Galor and Weil (2000, p. 809) "the key event that separates the

Malthusian and Post-Malthusian Regimes is the acceleration in the pace of technological progress, whereas the event that separates the Post-Malthusian and Modern Growth eras is the demographic transition that followed the industrial revolution.”

While the neoclassical growth model or various endogenous growth models are appropriate for explaining the regime of modern growth, they are inappropriate for modeling the escape from the Malthusian trap. For this reason, a recent strand in the growth literature calls on researchers to focus more on modeling the escape from the Malthusian trap and, more generally, on explaining economic development ranging from Malthusian stagnation to modern growth within unified models (cf. Galor and Weil, 1999 and 2000, and Lucas, 1998).¹

Our paper aims to contribute to this promising new research field. In particular, we propose a model which is able to replicate stylized facts of Great Britain and England and Wales for the time periods before the industrial revolution, shortly after the industrial revolution and during the demographic transition.

As suggested by economic historians (Crafts 1985) we emphasize in our model the role of demand factors in the industrialization of Western Europe. Crafts (1985) shows empirical evidence for income-inelastic demand for agricultural goods in pre-industrialized societies. This evidence implies that per capita income growth leads to labor re-allocation towards the non-agricultural sector. Based on this empirical evidence we propose an endogenous growth model with two consumption goods, an agricultural and a manufacturing good. We show that an exogenous increase in growth of total factor productivity (henceforth TFP) in the agricultural sector leads to population growth and, subsequently, to an acceleration in the growth of per capita manufacturing output.²

Hansen and Prescott (2001) use a two-sector model as well in order to explain the transition from stagnation to growth. However, in contrast to our model in Hansen and Prescott these two sectors produce the same consumption good. In their model the driving force is exogenous technological progress in a latent industrial technology that brings about the transition to sustained growth whereas in our model income-inelastic demand for agricultural goods as well as technological progress in the agricultural technology permits the take-off. Furthermore, Hansen and Prescott assume fertility to be an exogenous function of the standard of living. In contrast, we endogenize fertility by modeling it to be the result of utility-maximizing behavior

and incorporate the idea that population growth has a positive effect on technological progress in the manufacturing sector.³

Our model framework implies income-elastic demand for children. For that reason, fertility rises with a wage increase in our model. This is supported for pre-industrialized societies in important empirical work by Bailey and Chambers (1993) as well as in Lee (1997). For the sake of simplicity we ignore the impact of mortality decline on the escape from the Malthusian trap and focus exclusively on fertility.⁴ Our focus on fertility is justified by the empirical evidence presented in Dyson and Murphy (1985) and Coale and Treadway (1986) that Western Europe's take-off into self-sustained economic growth was accompanied by rising fertility up to the second half of the nineteenth century.

A further important aspect of our model is the incorporation of endogenous technological progress in the manufacturing sector. According to the recent literature in endogenous growth theory, endogenous technological progress increases with the rate of population growth. Our framework ensures that population growth will lead to an increase in labor productivity growth in the manufacturing sector, while it tends to reduce labor productivity growth in the agricultural sector (since the latter sector is characterised by decreasing returns to labor).

The analysis of our paper demonstrates that if demand for agricultural goods is income-inelastic then an *increase in agricultural productivity growth* leads to a re-allocation of labor to the manufacturing sector and to a post-Malthusian regime in which a growing population no longer depresses economic growth.

We close our paper by illustrating how our model can be extended to also capture the modern growth regime where population growth declined and economic growth accelerated. In particular, we show that an exogenous increase in the survival of children may cause parents to reduce their 'precautionary' demand for children, but increase their investment into the quality of their children. While the former result implies lower population growth, the latter decision will lead to an increase in human capital accumulation. This will foster economic growth and pave the way towards the modern growth regime.

2 Stylized facts about the British industrial revolution

Insert Table 1 about here

Table 1 summarises annual growth rates of selected economic and demographic indicators in Great Britain for the time intervals 1700-60 (i.e., before the industrial revolution) and 1820-40 (i.e., after the industrial revolution).

From these data one can infer the following stylized facts about the industrial revolution:

(i) *The growth rate of agricultural TFP, population and per capita income increased after the industrial revolution (cf. column 2-4).*

(ii) *The take-off in per capita income growth after the industrial revolution was entirely due to a rise in per capita manufacturing output growth. This follows from the fact that per capita income growth rose after the industrial revolution (cf. column 4), while per capita agricultural production growth did not. In fact, it actually fell (cf. column 5).*⁵

Insert Figure 1 about here

Figure 1 shows that the price of exported cotton piece goods in England and Wales relative to the price of wheat in Great Britain has been falling since the industrial revolution. Because of the importance of textiles for the British manufacturing sector and of wheat in agricultural goods production, Figure 1 gives rise to a third stylized fact:

(iii) *The price of manufacturing goods relative to agricultural goods has been falling since shortly after the industrial revolution.*

While the literature commonly assumes a one-sector model, these stylized facts make a strong case for modeling the escape from the Malthusian trap with a two-sector model. To allow for the emergence of a Malthusian trap before the industrial revolution, we assume decreasing returns to labor in the

agricultural sector. By further assuming that the growth rate of productivity in the manufacturing sector is rising with the population growth rate, our model implies a simultaneous take-off in population and economic growth after the industrial revolution.⁶

3 The basic model

3.1 Households

We assume that there are N_t individuals in each period and that each individual lives for one period (in what follows we use capital letters to denote aggregate variables and lower-case letters to denote per capita variables). Each individual is endowed with one unit of time that can be allocated between labor market participation and births. The time cost per birth is β (with $0 < \beta < 1$). The wage rate and full income of each household are both denoted by w_t . The full income of an individual is the income he would receive, if he devoted its entire time endowment to labor market participation. Each individual faces the constraint that he has to consume the subsistence level \tilde{a} of the agricultural good to survive.⁷ For a low level of full income this constraint is binding and each individual must cut back the quantities of his other choice variables below their utility-maximizing levels. Consistent with the empirical evidence in Crafts (1985), we assume the agricultural good to be a necessity (so that the income elasticity of demand is below one). For analytical convenience we assume that, in case the constraint is not binding, each household still demands inelastically the same quantity \tilde{a} .⁸ We normalize the agricultural good price to one.

Each individual derives utility from the consumption of a manufacturing good m_t and from births b_t , implying his choice variables to be m_t and b_t (the consumption of \tilde{a} does not provide any utility, but it is necessary for survival). For simplicity we assume a Cobb-Douglas utility function.

Given these implicit assumptions, each individual's static optimization problem is

$$\max u_t = m_t^{1-\alpha} b_t^\alpha \tag{1}$$

$$\begin{aligned} s.t. \quad & a_t + p_{M,t} m_t + w_t \beta b_t = w_t, \\ & a_t \geq \tilde{a}, \quad m_t \geq 0, \quad b_t \geq 0, \end{aligned}$$

with $0 < \alpha < 1$ and $p_{M,t}$ denoting the price of the manufacturing good.

In order to survive, each individual spends the amount \tilde{a} for consumption of the agricultural good and divides the rest of his full income between consumption of the manufacturing good and births, such that the Cobb-Douglas utility function is maximized. This gives rise to the following household demand functions:

$$a_t = \tilde{a}, \quad (2)$$

$$m_t = (1 - \alpha) \left(\frac{1}{p_{M,t}} \right) (w_t - \tilde{a}), \quad (3)$$

$$b_t = \left(\frac{\alpha}{\beta} \right) \left(1 - \frac{\tilde{a}}{w_t} \right). \quad (4)$$

3.2 The production of the agricultural good

The market for the agricultural good is perfectly competitive. Production of the agricultural good A_t requires labor $L_{A,t}$ and land T as inputs. We abstract in the entire paper from capital. In the agricultural sector there are constant returns to scale in both $L_{A,t}$ and T . Land is available for the economy in fixed quantity. This implies that for any given level of agricultural TFP $\Omega_{A,t}$ there are decreasing returns to labor in the agricultural sector. We assume the aggregate agricultural production function to have a Cobb-Douglas-form.

Given these implicit assumptions, the aggregate agricultural production function is

$$A_t = \Omega_{A,t} L_{A,t}^\gamma T^{1-\gamma} = \Omega_{A,t} L_{A,t}^\gamma, \quad \text{with } T \equiv 1, \quad (5)$$

where $0 < \gamma < 1$.

If labor in the agricultural sector were paid according to its factor productivity, the presence of decreasing returns to labor would mean positive profits for farmers. In order to avoid the complication of positive profits, we make the following Assumption 1, which seems realistic for primitive societies:⁹

ASSUMPTION 1: Wages are paid in the agricultural sector according to the rule

$$w_t = \frac{A_t}{L_{A,t}} = \Omega_{A,t} L_{A,t}^{\gamma-1}. \quad (6)$$

In each period there are N_t households which demand $N_t \tilde{a}$ units of the agricultural good. Setting demand equal to supply makes the equilibrium level of labor in the agricultural sector $L_{A,t} = (\frac{N_t \tilde{a}}{\Omega_{A,t}})^{\frac{1}{\gamma}}$. Substituting the equilibrium level of $L_{A,t}$ back into the definition of wage payment (6) results in an equilibrium wage of

$$w_t = (N_t \tilde{a})^{\frac{\gamma-1}{\gamma}} \Omega_{A,t}^{\frac{1}{\gamma}}. \quad (7)$$

Model consistency requires $w_t \geq \tilde{a}, \forall t$. Fulfillment of this condition can be ensured upon imposing the following parameter constellations:

ASSUMPTION 2:

- (i) $N_0 \leq (\frac{\Omega_{A,0}}{\tilde{a}})^{\frac{1}{1-\gamma}}$,
- (ii) $1 - \gamma \leq (\frac{\beta}{\alpha})(\hat{\Omega}_{A,t} + 1)^{\frac{1}{1-\gamma}} < 1$.

Assumption 2 (i) ensures $w_0 \geq \tilde{a}$. In Appendix B we show that the parameter constellation in Assumption 2 (ii) guarantees that $w_t \geq \tilde{a}, \forall t > 0$, provided Assumption 2 (i) is fulfilled.

3.3 The production of the manufacturing good

Following Ethier (1982) we assume the final manufacturing good to be produced by assembling a very large number of varieties of a horizontally differentiated intermediate good which are supplied in monopolistically competitive markets. The aggregate manufacturing production function is assumed to have the form¹⁰

$$M_t = \left(\int_0^{Q_t} X_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad \varepsilon > 1, \quad (8)$$

where M_t denotes aggregate final manufacturing good production, Q_t denotes the number of varieties of the intermediate good, $X_{i,t}$ denotes the quantity of the intermediate goods variety i , and ε denotes the elasticity of substitution between varieties of the intermediate good. Our assumption $\varepsilon > 1$ together with equation (8) implies that an expanding variety of the intermediate good leads to productivity gains.¹¹ Perfect competition in the market for the manufacturing good causes the price of the manufacturing good $p_{M,t}$ to equal its unit cost. Therefore, upon cost minimization, $p_{M,t}$ is derived as

$$p_{M,t} = \left(\int_0^{Q_t} p_{X_{i,t}}^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}, \quad (9)$$

where $p_{X_{i,t}}$ represents the price of variety i of the intermediate good. Profit maximization of manufacturing goods firms gives rise to the following aggregate demand for each intermediate goods variety i

$$X_{i,t} = \left(\frac{p_{M,t}}{p_{X_{i,t}}} \right)^\varepsilon M_t. \quad (10)$$

We assume that the total labor requirement of each intermediate goods firm for producing $X_{i,t}$ units equals $L_{X_{i,t}}$, with $L_{X_{i,t}} = F + \delta X_{i,t}$, where F denotes the fixed cost of each intermediate goods firm, which is due each period, and δ represents its unit labor requirement. Solving (10) for $p_{X_{i,t}}$, substituting the resulting expression in the profit definition for each intermediate goods firm i , $\Pi_{i,t} = p_{X_{i,t}} X_{i,t} - w_t(F + \delta X_{i,t})$ and maximizing $\Pi_{i,t}$ by the optimal choice of $X_{i,t}$ yields the well-known monopoly-pricing rule¹²

$$p_{X_{i,t}} \left(1 - \frac{1}{\varepsilon} \right) = w_t \delta \quad \text{or} \quad p_{X_{i,t}} = p_{\bar{X}_t} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) w_t \delta, \forall i \in [0, Q_t], \quad (11)$$

where $p_{\bar{X}_t}$ denotes the price of each intermediate goods variety.¹³ Note that, due to free labor mobility between agricultural goods producers and intermediate goods firms, the wage is the same in both sectors - it is determined in the agricultural good sector according to (7). In addition, there is free entry into the market for each intermediate goods variety until a zero-profit condition is fulfilled for each intermediate goods firm. Substituting (11) in the profit definition of each intermediate goods firm and setting $\Pi_{i,t} = 0$, we obtain

$$X_{i,t} = \bar{X} = \frac{(\varepsilon - 1)F}{\delta}, \forall i \in [0, Q_t]. \quad (12)$$

Equation (12) establishes that only a constant output level for each intermediate goods variety is consistent with zero profit and, due to symmetry, that this output level is the same for each intermediate goods variety. Utilizing this fact in (8) yields

$$M_t = Q_t^{\frac{\varepsilon}{\varepsilon-1}} \bar{X} = Q_t^{\frac{1}{\varepsilon-1}} (Q_t \bar{X}). \quad (13)$$

Obviously, we have $Q_t(F + \delta \bar{X}) = L_{X,t}$, where $L_{X,t}$ represents aggregate intermediate goods employment. Rewriting this identity and substituting (12) for \bar{X} gives

$$Q_t = \left(\frac{1}{\varepsilon F}\right) L_{X,t}. \quad (14)$$

Substitution of (12) and (14) in (13) yields:

$$M_t = \Omega_{M,t} L_{X,t}, \text{ with } \Omega_{M,t} \equiv \left[\varepsilon^{\frac{\varepsilon}{1-\varepsilon}} F^{\frac{1}{1-\varepsilon}} \left(\frac{\varepsilon - 1}{\delta}\right)\right] L_{X,t}^{\frac{1}{\varepsilon-1}}, \quad (15)$$

where $\Omega_{M,t}$ denotes manufacturing productivity.

It will later be shown that the level of $L_{X,t}$ rises with the population level. Therefore, (15) implies that manufacturing productivity growth is independent of the population level, but that it increases with the population growth rate.¹⁴ This result is compatible with the recent credo in the endogenous growth literature which argues that the balanced growth rate of per capita output should be independent of the population level (Jones, 1999). The idea that population has a positive effect on economic growth is empirically supported by findings in Kremer (1993) for human historical data from one million B.C. to 1990.¹⁵

To complete the picture, we must state that the economy is closed. By applying Walras's law, clearing of the commodity markets ensures fulfillment of the country's labor constraint $L_{A,t} + L_{X,t} = N_t(1 - \beta b_t)$.

3.4 Results

To ease our subsequent exposition we state the following definition, which follows the framework of Galor and Weil (2000):

DEFINITION:

- (i) In a *Malthusian steady state*, population and per capita manufacturing output are constant.
- (ii) In the *post-Malthusian balanced growth path*, population and per capita manufacturing output grow at constant, positive rates.

The following lemma explains the dynamic properties of the model (a “hat” on top of any variable denotes the growth rate of that variable):¹⁶

LEMMA: *Suppose total factor productivity in the agricultural good production $\Omega_{A,t}$ grows exogenously at the constant rate $\hat{\Omega}_A$ (which is possibly zero). Then, in a Malthusian steady state or post-Malthusian balanced growth path*

- (i) *the wage w is stationary, population growth \hat{N} is constant (with $\hat{N} = (\hat{\Omega}_{A,t} + 1)^{\frac{1}{1-\gamma}} - 1$) and the share of employment in the intermediate goods sector $L_{X,t}/N_t$ is constant.*
- (ii) *stability of the stationary state of w is guaranteed by Assumption 2 (ii).*

PROOF: See Appendix A.

Now we are prepared to state our proposition:

PROPOSITION: *If total factor productivity in agricultural good production $\Omega_{A,t}$ grows exogenously at the constant, positive rate $\hat{\Omega}_A$, there exists a post-Malthusian balanced growth path with:*

- (i) *A constant, positive population growth rate \hat{N} and a constant, positive growth rate of per capita manufacturing output \hat{m} .*
- (ii) *\hat{N} and \hat{m} are all dependent on the growth rate $\hat{\Omega}_A$, with the presence of multiplier effects from an increase of $\hat{\Omega}_A$ on \hat{N} and \hat{m} (the latter provided $(\frac{1/(\varepsilon-1)}{1-\gamma}) > 1$).*

PROOF: The assumption $\hat{\Omega}_A > 0$ and Lemma (i) imply

$$\hat{N} = (\hat{\Omega}_A + 1)^{\frac{1}{1-\gamma}} - 1 > 0 \text{ and } \partial \hat{N} / \partial \hat{\Omega}_A > 0. \quad (16)$$

Combining (16) with the constancy of $L_{X,t}/N_t$ in the balanced growth path (cf. Lemma (i)) we obtain

$$\hat{L}_X = \hat{N} > 0 \text{ and } \partial \hat{L}_X / \partial \hat{\Omega}_A > 0. \quad (17)$$

Dividing both sides of (15) by N_t , taking growth rates of the resulting expression (in a discrete mathematical framework), and using again constancy of $L_{X,t}/N_t$ in the balanced growth path yields

$$\hat{m} = (\hat{L}_X + 1)^{\frac{1}{\varepsilon-1}} - 1. \quad (18)$$

Substituting (16) in (17) and substituting the resulting expression in (18) yields

$$\hat{m} = (\hat{\Omega}_A + 1)^{\frac{1/(\varepsilon-1)}{1-\gamma}} - 1 > 0 \text{ and } \partial \hat{m} / \partial \hat{\Omega}_A > 0. \quad (19)$$

□

Note that a constant wage in the balanced growth path does not imply a constant standard of living. The reason is that productivity gains from the expanding variety of intermediate goods cause the price of the manufacturing good to fall.¹⁷ Since the agricultural price was normalized to one, this implies a rising purchasing power for the wage.¹⁸

The intuition behind the proposition is as follows. For a given population size, an increase in growth of agricultural TFP leads to an increase in labor productivity in the agricultural sector and, as a consequence, to an increase in the wage rate. As the demand for children is income-elastic, the population starts to grow. Population growth in turn leads to a growing demand for the agricultural good $N_t \tilde{a}$ and requires increasing labor input in the agricultural sector. Because of decreasing returns to labor in the agricultural sector, wage growth gradually peters out. The smaller the share of labor in agricultural production, γ , the more the returns to labor in agriculture will decrease

and, hence, the lower the level of the wage in the balanced growth path (cf. equation (23) in Appendix A). In turn, the balanced growth rates of population and per capita manufacturing output are positively related to the long-run wage. This follows from the fact that the demand for children is income-elastic and that the growth of per capita manufacturing output depends positively on the population growth rate since it affects positively the number of intermediate goods varieties. The larger $1/(\varepsilon - 1)$, the higher the productivity gains from expanding variety and the more likely it is that there is a multiplier effect on per capita manufacturing output growth.

COROLLARY: *If total factor productivity in agricultural good production $\Omega_{A,t}$ grows at the rate zero, there exists a Malthusian steady state with a constant population level N and a constant per capita level of manufacturing output m .*

This corollary implies that the positive impact on wages resulting from an increase in the level of agricultural TFP will, in the long run, be offset by the negative impact of an increase in the population size on labor productivity in the agricultural sector. Therefore, population and per capita manufacturing output growth cannot be sustained in the long run.

This result is in marked contrast to the two-sector growth model in Matsuyama (1992, Section 2), which shares with our model the assumption of non-homothetic preferences. Since Matsuyama assumes a constant population, the positive impact of an increase in the level of agricultural TFP on the wage is not offset by a negative impact of a rising population size. Due to income-elastic demand for the manufacturing good, the expenditure share of the manufacturing good increases in Matsuyama's model. Since the manufacturing sector is the sector with the highest productivity growth rate in his model, the rate of economic growth increases.

Insert Figure 2 about here

Figure 2 illustrates how our model can explain the transition from the Malthusian steady state to the post-Malthusian balanced growth path. The solid thin line plots the exogenous time path of agricultural TFP, the solid bold line population growth, and the dotted line per capita manufacturing output growth. (In the simulation we set the parameter values at $\tilde{a} = 0.1, \alpha =$

0.6, $\beta = 0.4$, $\gamma = 0.5$, and $\varepsilon = 1.75$. The initial size of the population N_0 was set at the Malthusian steady state value of N . Furthermore, we assumed $\hat{\Omega}_{A,t} = 0, \forall t = 0, \dots, 12$ and $\hat{\Omega}_{A,t} = 0.1, \forall t = 13, \dots, T$, with $T = 30$).

Figure 2 illustrates that an exogenous increase in agricultural TFP growth causes an increase in population growth and per capita manufacturing output growth (and that these growth rates approach constant, positive equilibrium values as the economy reaches the post-Malthusian balanced growth path). In addition, the time paths in Figure 2 illustrate the presence of multiplier effects. This can be seen from the fact that the growth rates of agricultural TFP, population, and per capita manufacturing output are all zero in the Malthusian steady state, while the post-Malthusian balanced growth rates of population and of per capita manufacturing output exceed the growth rate of agricultural TFP. (As is stated in our proposition, for a multiplier effect on \hat{m} to be present, a particular parameter constellation needs to be fulfilled. For the specific simulation in Figure 2, the parameter values were chosen such that this was the case).

4 A unified model

Insert Table 2 about here

In this section we illustrate how our model can be extended to explain the demographic transition and the subsequent modern growth regime that followed the post-Malthusian regime. In particular, we suggest an extension of our model that captures the following stylized facts for the time before and during the demographic transition in Great Britain and England and Wales respectively:

- (iv) *The death rate of children of age 1-4 increased slightly from 1841 to 1861 and fell strongly thereafter* (cf. Table 2, columns 2 and 3).
- (v) *Fertility increased slightly from 1841 to 1861 and after an adjustment lag it has fallen strongly since 1881* (cf. Table 2, column 4).
- (vi) *Investment in human capital increased constantly from 1841 to 1921* (cf. Table 2, column 5).

Furthermore, Galor and Weil (1999 and 2000) stress as another regularity the fact that

(vii) *while shortly after the industrial revolution population growth and economic growth were positively associated, after the demographic transition a reduction in population growth was associated with a spurt in economic growth* (see Galor and Weil 1999, p. 151).

To capture these stylized facts we need to extend our model so that it (a) can endogenously explain the demographic transition (i.e. a fall in the rate of population growth) and (b) guarantee sustained economic growth even when population growth starts to decline.

We follow the conventional wisdom in the field of demography and model the decline in fertility as caused by infant mortality decline. A model that can explain fertility decline as the optimal decision of individuals in response to a decrease in infant mortality has first been suggested by Kalemli-Ozcan (2001). Central to her work is the assumption of a stochastic optimization problem. Parents are uncertain about the number of their surviving children and, given that parents are risk averse, they will have a precautionary demand for children. This leads to a higher demand for children than in the case of the optimal choice in a certain world. An increase in the survival probability may then reduce the precautionary demand for children, and fertility will fall.

To allow for the continuation of economic growth when fertility starts to decline, we postulate that households care about not just the quantity, but also the quality of children.¹⁹ More specifically, we assume that each parent derives utility from the expected aggregate income of his children. Since parents face a quantity-quality trade-off, a reduction in infant mortality leads simultaneously to a fertility decline and an increase in human capital accumulation.²⁰

By adapting the household optimization problem as suggested in Kalemli-Ozcan (2001) our model is able to explain the demographic transition and the subsequent modern growth regime. (A brief summary of the optimization problem can be found in Appendix C). As compared to our baseline model, in the modern growth regime it is human capital accumulation rather than population growth that is the engine of economic growth. To allow for this, we replace raw labor by quality-adjusted labor input (i.e., raw labor multiplied by human capital per adult) in the production of the agricultural and the manufacturing good.

Insert Figure 3 about here

Figure 3 illustrates that our extended model can explain the transition from the Malthusian regime to the post-Malthusian regime, followed by the demographic transition and the modern growth regime (where the last regime is consistent with the empirical regularities (iv)-(vii)). The fine dotted line plots the exogenous time path of agricultural TFP, and the light grey line plots the exogenous time path of the probability of each child surviving childhood q_t . Moreover, the bold black line plots population growth, the bold dotted line growth of manufacturing output per adult, and the solid thin line investment in education for each child e_t . (In the simulation we set the parameter values at $\tilde{a} = 0.05, \alpha = 0.625, \beta = 0.05, \gamma = 0.5, \varepsilon = 15$ and $\tau = 3$. The initial size of the population N_0 was set at the Malthusian steady state value of N . Furthermore, we assumed $\hat{\Omega}_{A,t} = 0, \forall t = 0, \dots, 29, \hat{\Omega}_{A,t} = 0.05, \forall t = 30, \dots, T$ and q_t grows with the rate \hat{q}_t , where $q_0 = 0.7, \hat{q}_t = 0, \forall t = 0, \dots, 90, \hat{q}_t = 0.015$ for $t = 91$ and $\hat{q}_t = \hat{q}_{t-1}^{1.05}, \forall t = 92, \dots, T$ with $T = 120$. The simulations contain also a constant parameter τ that influences the productivity in the production of human capital.).

Similarly to Figure 2, Figure 3 illustrates that an exogenous increase in agricultural TFP growth causes an increase in population growth and in the growth of manufacturing output per adult. In addition, Figure 3 demonstrates that an exogenous decrease in infant mortality (which followed the post-Malthusian regime) may lead to a reduction in population growth and, therefore, to the demographic transition.²¹ Most importantly, the reduction in population growth is associated with a spurt in economic growth, as suggested by the modern growth regime. The time path of e_t reveals that the quantity-quality trade-off ensures that parents increase their investment in the education of each child simultaneously with a reduction in fertility. If the increase in human capital accumulation is strong enough to offset the decline in population growth, economic growth will accelerate, as suggested in Figure 3. Hence, increasing human capital accumulation replaces population growth as the engine of economic growth in the modern growth regime, which is in contrast to the post-Malthusian regime.

5 Conclusion

Our paper aims to explain the industrialized world's escape from a regime with low economic and population growth (the Malthusian regime) towards a regime with high economic and population growth (the post-Malthusian regime). To explain this transition we propose a model that replicates stylized facts about the British industrial revolution. These include the observation that the take-off in population and per capita income growth was entirely due to an increase in per capita manufacturing output growth and that the price for manufacturing goods fell relative to that of agricultural goods. These regularities make a strong case for modeling the escape from the Malthusian trap with a two-sector model, as we propose in our paper.

We set up a model that introduces demand factors into a growth model with two consumption goods (an agricultural and a manufacturing good) and model fertility as the outcome of utility-maximizing behavior. Furthermore, we assume exogenous growth in agricultural TFP growth and model technological progress in the manufacturing sector as an endogenous variable which is rising with population growth. While the former assumption guarantees positive population growth in the long run, the latter assumption supports sustained economic growth. More specifically, in the absence of agricultural TFP growth decreasing returns to labor in the agricultural good sector and income-elastic demand for children imply a stationary wage that is compatible with zero population growth (i.e., a situation labeled Malthusian trap). However, by introducing positive agricultural TFP growth, the level of the stationary wage rate is compatible with positive population growth and the economy can escape from the Malthusian trap.

In addition, we suggest how our model can be extended to also capture the modern growth regime in which fertility declined and economic growth accelerated. The driving force to generate the modern growth regime is the assumption that parents face a quantity-quality trade-off. In our set-up declining infant mortality leads to falling precautionary demand for children and - due to the quantity-quality trade-off - to rising human capital accumulation. For that reason, population growth declines and human capital accumulation replaces population growth as the engine of economic growth.

Appendix

A. Proof of Lemma

(i) Combining $N_t = b_{t-1}N_{t-1}$ with (7) yields after some manipulations

$$\hat{w}_t \equiv \frac{w_t - w_{t-1}}{w_{t-1}} = b_{t-1}^{\frac{\gamma-1}{\gamma}} (\hat{\Omega}_A + 1)^{\frac{1}{\gamma}} - 1. \quad (20)$$

Suppose we have in the (Malthusian) steady state or (post-Malthusian) balanced growth path $\hat{w}_t = 0$ (it will be shown below that this situation is stable). In this case, (20) implies

$$b = (\hat{\Omega}_A + 1)^{\frac{1}{1-\gamma}}. \quad (21)$$

Clearly, since $\hat{N} = b - 1$, this implies that we have

$$\hat{N} = (\hat{\Omega}_A + 1)^{\frac{1}{1-\gamma}} - 1 = \text{const}. \quad (22)$$

Substituting (21) into (4) for b and rewriting yields

$$w = \tilde{a} / [1 - (\frac{\beta}{\alpha})(\hat{\Omega}_A + 1)^{\frac{1}{1-\gamma}}] = \text{const}. \quad (23)$$

Zero profits for manufacturing goods firms implies $p_{M,t}M_t = p_{X,t}X_t$, where X_t denotes aggregate intermediate goods output. Moreover, zero profits for intermediate goods firms imply $p_{X,t}X_t = w_tL_{X,t}$. Combining both zero-profit conditions yields

$$L_{X,t} = \left(\frac{p_{M,t}}{w_t}\right)M_t. \quad (24)$$

Multiplying both sides of (3) with N_t , substituting the resulting expression in (24) for M_t and collecting terms gives $\frac{L_{X,t}}{N_t} = (1-\alpha)(1 - \frac{\tilde{a}}{w_t})$. Since $w = \text{const}$. according to (23), we therefore get

$$\frac{L_X}{N} = \text{const}. \quad (25)$$

(ii) To determine the stability of the stationary value of the wage we rewrite (20) to obtain

$$w_t = b_{t-1}^{\frac{\gamma-1}{\gamma}} (\hat{\Omega}_A + 1)^{\frac{1}{\gamma}} w_{t-1}. \quad (26)$$

Total differentiation of (26) w.r.t. w_{t-1} yields

$$\frac{dw_t}{dw_{t-1}} = b_{t-1}^{\frac{\gamma-1}{\gamma}} (\hat{\Omega}_A + 1)^{\frac{1}{\gamma}} \left[1 + w_{t-1} \left(\frac{\gamma-1}{\gamma} \right) b_{t-1}^{-1} \left(\frac{db_{t-1}}{dw_{t-1}} \right) \right]. \quad (27)$$

Upon differentiation of (4) w.r.t. w_{t-1} , substituting the resulting expression together with (21) in (27), and substituting (23) in the resulting expression we get

$$\frac{dw_t}{dw_{t-1}} \Big|_{w^{equilibrium}} = \left(\frac{1}{\gamma} \right) - \left(\frac{\alpha}{\beta} \right) \left(\frac{1-\gamma}{\gamma} \right) (\hat{\Omega}_A + 1)^{\frac{-1}{1-\gamma}}. \quad (28)$$

The stationary state of w_t is *locally stable* if $-1 \leq (dw_t/dw_{t-1}) \leq 1$. As can be verified from rearranging, the latter condition is fulfilled if $\left(\frac{\beta}{\alpha} \right) \left(\frac{1+\gamma}{1-\gamma} \right) \geq (\hat{\Omega}_A + 1)^{\frac{-1}{1-\gamma}} \geq \left(\frac{\beta}{\alpha} \right)$. In turn, since according to (21) in the steady state or balanced growth path $b = (\hat{\Omega}_A + 1)^{\frac{1}{1-\gamma}}$, a glance at (4) makes clear that $(\hat{\Omega}_A + 1)^{\frac{1}{1-\gamma}} \leq \left(\frac{\alpha}{\beta} \right)$ is always fulfilled. As a consequence, there is always fulfillment of $(\hat{\Omega}_A + 1)^{\frac{-1}{1-\gamma}} \geq \left(\frac{\beta}{\alpha} \right)$. Hence, the stationary state of w is stable, provided $(\hat{\Omega}_A + 1)^{\frac{-1}{1-\gamma}} \leq \left(\frac{\beta}{\alpha} \right) \left(\frac{1+\gamma}{1-\gamma} \right)$. However, fulfillment of the latter is ensured by Assumption 2 (ii).

Moreover, substituting $b_{t-1} = b(w_{t-1})$ from (4) lagged one period into (26) gives a function $w_t = f(w_{t-1})$. From the shape of this function it can be verified that the unique equilibrium is *globally stable*. More specifically, the function $w_t = f(w_{t-1})$ has an asymptote at $w_{t-1} = \tilde{a}$ and decreases between $w_{t-1} = \tilde{a}$ and its minimum value at $w_{t-1} = \tilde{a}/\gamma$. Since according to Assumption 2 (ii) $(1 + \hat{\Omega}_A) \geq \left(\left(\frac{\alpha}{\beta} \right) (1 - \gamma) \right)^{(1-\gamma)}$, it holds that $f(w_{t-1}) \geq w_{t-1}$ at $w_{t-1} = \tilde{a}/\gamma$. To the right of its minimum value the slope of the function $f(\cdot)$ is less than one implying that $f(\cdot)$ will cross the 45-degree line exactly once and the resulting equilibrium will be stable.

B. Proof that Assumption 2 (ii) guarantees that $w_t \geq \tilde{a}$, $\forall t > 0$, provided Assumption 2 (i) is fulfilled.

Monotonic convergence of w_t towards its equilibrium value is fulfilled if $0 \leq (dw_t/dw_{t-1}) \leq 1$. Rearranging (28) shows that fulfillment of this condition, and hence monotonic convergence of w_t , is guaranteed by fulfillment of $(\frac{\beta}{\alpha})(\hat{\Omega}_{A,t} + 1)^{\frac{1}{1-\gamma}} \geq 1 - \gamma$, i.e., the inequality on the left-hand side of Assumption 2 (ii).

Moreover, let us denote w^{eq} as the wage rate in the Malthusian steady state or the post-Malthusian balanced growth path.

(i) Suppose $w_0 < w^{eq}$, then monotony of w_t ensures that $w_t \geq \tilde{a}$, $\forall t > 0$, provided Assumption 2 (i) is fulfilled.

(ii) Suppose $w_0 > w^{eq}$, then monotony of w_t ensures $w_t \geq \tilde{a}$, $\forall t$ if $w^{eq} \geq \tilde{a}$. However, as a glance at (23) reveals, the latter is fulfilled with fulfillment of the inequality on the right-hand side of Assumption 2 (ii).

C. A brief summary of the optimization problem of each parent as taken from Kalemli-Ozcan (2001) and adapted to the framework of our model.

Each parent solves a two-stage optimization problem. In the first stage, the following problem is solved (note that in this appendix lower-case letters denote per adult variables)

$$\begin{aligned} \max u_t &= (m_t)^{1-\alpha}(v_t^e)^\alpha, \\ s.t. \quad w_t h_t &= p_{M,t} m_t + p_{v,t}^e v_t^e + a_t, \\ a_t &\geq \tilde{a} \wedge (m_t, v_t^e) \geq 0, \end{aligned}$$

where v_t^e denotes a quantity-quality composite good, that is, the expected utility of the quantity and quality of children (for simplicity we assume that each parent derives utility from the logarithm of the expected aggregate income of his children). Further, h_t denotes the human capital of each parent, and $p_{v,t}^e$ denotes the expected price of the quantity-quality composite good.

Analogously to the household decision problem in section 3.1, each parent devotes the amount \tilde{a} to consumption of the agricultural good and allocates

the rest of his full income between m_t and v_t^e , such that the Cobb-Douglas utility function is maximized. This gives rise to the first-order conditions

$$s_{v,t} \equiv \frac{p_{v,t}^e v_t^e}{w_t h_t} = \alpha \left(1 - \frac{\tilde{a}}{w_t h_t}\right), \quad (29)$$

$$m_t = (1 - \alpha) \left(\frac{1}{p_{M,t}}\right) (w_t h_t - \tilde{a}) \wedge a_t = \tilde{a},$$

where $s_{v,t}$ represents the budget share of v_t^e in full income $w_t h_t$.

In the second stage, each parent maximizes the quantity-quality composite good v_t^e subject to the second-stage budget constraint: $p_{v,t}^e v_t^e = w_t h_t (\beta + e_t) b_t$. The budget constraint implies that each child has a fixed cost β and an educational cost e_t , independently of whether or not the child survives (see Kalemli-Ozcan, 2001). The number of survivors is a random variable drawn from a binomial distribution. Kalemli-Ozcan shows that, using the Delta Method, one can rewrite the stochastic optimization problem as the maximization of a Taylor approximation of expected utility (for details of the derivation see Kalemli-Ozcan, 2001, Appendix A). Use of this method gives the second-stage optimization problem of each parent as

$$\begin{aligned} \max v_t^e &= \ln(b_t q_t w_{t+1} h_{t+1}) - \frac{1 - q_t}{2b_t q_t}, \\ \text{s.t. } p_{v,t}^e v_t^e &= w_t h_t (\beta + e_t) b_t, \end{aligned} \quad (30)$$

where q_t denotes the probability of each child to survive childhood and each parent's choice variables are e_t and b_t .

The production function of human capital for each child is assumed to be of the form $h_{t+1} = \tau e_t h_t$, where τ is a constant (for simplicity we assume a non-diminishing marginal return to e_t , while Kalemli-Ozcan assumes this return to be slightly diminishing). It is straightforward to show that the second-stage optimization problem leads to the first-order conditions

$$\begin{aligned} e_t &= 2\beta \left(\frac{q_t}{1 - q_t}\right) b_t, \\ b_t &= -\left(\frac{1}{4}\right) \left(\frac{1 - q_t}{q_t}\right) + \sqrt{\left(\frac{1}{16}\right) \left(\frac{1 - q_t}{q_t}\right)^2 + \left(\frac{1}{2}\right) \left(\frac{1 - q_t}{q_t}\right) \left(\frac{\alpha}{\beta}\right) \left(1 - \frac{\tilde{a}}{w_t h_t}\right)}, \end{aligned}$$

where b_t is the positive solution of a quadratic equation (in the derivation of this quadratic equation we made use of (29)).

Notes

1. Further closely related papers include Galor and Weil (1996), Galor and Moav (2001) and Jones (2001). Galor and Weil (1996, section 3B) generate a pattern of increasing and then decreasing fertility based on a transition from a traditional to a modern sector. In their set-up production in the traditional sector can take place at home and, therefore, can be combined with childrearing. In contrast, work in the modern sector cannot be combined with childrearing. Galor and Moav (2001) assume individuals with different preferences for quality (i.e., future earnings) of their children. Within a family this preference is transmitted from one generation to the next. For this reason, the authors are able to investigate the interplay between the take-off from economic stagnation to sustained growth and natural selection of the human species. Jones (2001) assumes an economy with a single consumption good with decreasing returns to labor but technological progress that rises with population growth. He shows that improvements in property rights for innovations can explain (and quantitatively account for) the escape from the Malthusian trap. Further, an elasticity of substitution between consumption and births which is larger than one enables Jones to generate the demographic transition.

2. Our model is closely related to that of Matsuyama (1992), who shows in a closed-economy model with two consumption goods (an agricultural good and a manufacturing good), with an income-inelastic demand for the agricultural good and a constant population, that an increase in the *level* of TFP in the agricultural sector unambiguously leads to a take-off in per capita income growth.

3. Another paper with a positive effect of population growth on industrialization is Goodfriend and McDermott (1995), which takes population growth to be exogenous, however.

4. As concerns the modeling of how the interaction between human capital accumulation and mortality decline might have caused an escape from the Malthusian trap, the reader is referred to Galor and Weil (1999), Lagerlöf (2001) and Steinmann, Prskawetz and Feichtinger (1998). Galor and Weil (1999) assume increasing returns to investment in human capital accumulation if mortality decreases. Lagerlöf (2001) assumes that the economy is hit by epidemic shocks throughout time. These shocks have a smaller impact on the economy once there has been a take-off in human capital accumulation. Steinmann, Prskawetz and Feichtinger (1998) assume that the rate of

depreciation of human capital decreases as mortality declines.

5. There also existed a service sector besides the agricultural manufacturing sector. Automatization in the service sector is very difficult, however, since labor is not only an input factor but also an “end-product” in the service factor (see Baumol, 1967). For this reason, it is unlikely that productivity growth in the service sector will figure prominently in an explanation of the data in Table 1.

6. According to Crafts (1985) the data for 1800-20 are not reliable. For this reason those data are not included in Table 1. Moreover, the corresponding data for 1760-1800 are not included in Table 1 (1760-1800 is the time period in which the British industrial revolution took place). This time period is characterized by a temporary contraction of agricultural TFP growth and per capita output growth and a temporary fall in real wages. (Ciccone (1997) explains this temporary fall in the real wage with relatively faster TFP growth in a more capital intensive industrial production sector than the less capital intensive “cottage” sector.) Yet, during that time period population growth continued to increase. Birdsall (1983) explains this with the fact that industrialization continued during that time period. In turn, industrialization caused an increase in fertility (despite a falling real wage) for three reasons: (i) Industrial earning opportunities increased, which fostered an increase in marriage rates and also marriages at earlier ages; (ii) Rising demand in the manufacturing sector for children as laborers (which reduced parents’ costs of children); (iii) Rising home-based female employment in the manufacturing sector, i.e., female employment of a type that was compatible with child care. Though these explanations support our claim that the manufacturing sector played a dominant role in the escape from the Malthusian trap, we abstract in our model from these features. Instead, we concentrate on the long-run positive association of wages and fertility between 1700-60 and 1820-40 (as is supported by the work of Wrigley and Schofield, 1981).

7. See Galor and Weil (2000) for a similar assumption concerning subsistence consumption.

8. As long as the income elasticity of demand for the agricultural good remains below one, all our results apply in a more general framework.

9. This assumption is borrowed from an earlier version of Jones (2001) (see also Lucas, 1998).

10. Goodfriend and McDermott (1995) assume a similar production function for the industrial revolution. In a model of modern growth Young (1998) enlarges the above production function with endogenous quality improvements

and vertical knowledge spillovers. While Young's framework is richer, we use the simpler framework as given by (8) in order to focus attention on the interactions between population growth and economic growth. As was already noted in Ethier (1982), one could alternatively interpret the intermediate goods varieties as successive stages. Such an interpretation would leave the implications of the model unaffected (see Edwards and Starr, 1987, for a formalization of this interpretation).

11. It will later be shown that $\varepsilon > 1$ is also a necessary condition for intermediate goods firms to be willing to engage in production.

12. Due to product differentiation, each intermediate goods firm enjoys monopoly power over the production of its respective variety. Hence, its choice of $X_{i,t}$ influences $p_{X_{i,t}}$. Nevertheless, it takes $p_{M,t}$, M_t and w_t as given because it believes that it is too small to have any influence on these variables.

13. In the production function of the manufacturing good in (8) ε represents the elasticity of substitution between intermediate goods varieties. In the pricing rule in (11), on the other hand, it represents the price elasticity of demand for each intermediate goods variety. Helpman and Krugman (1985, Ch. 6) established that for a large number of intermediate goods varieties the elasticity of substitution approaches the price elasticity. In the first equation in pricing rule (11), the left-hand side represents marginal revenue and the right-hand side marginal cost. As a consequence, our assumption $\varepsilon > 1$, which we made in (8), ensures positive marginal revenues of monopoly intermediate goods producers.

14. The dependence of manufacturing productivity growth on population growth can be found in most growth models with endogenous technological progress and a lack of scale effects - of course, in most of these models this is true for the economy's only good - (see Jones, 1999). To the best of our knowledge, the only growth model with endogenous technological progress and a balanced growth rate of per capita output that is independent of the population growth rate is Dalgaard and Hansen (2001).

15. As is argued in Jones (1999), there are two classes of non-scale economic growth models. The first class of models (labeled semi-endogenous growth models) implies that economic growth cannot be sustained in the long run if population growth is absent. In the second class of these models (labeled endogenous growth models), economic growth can be sustained in the long run even if population growth is absent. Our basic model belongs to the class of semi-endogenous growth models. Li (2000) shows that any non-scale growth model belongs to the class of semi-endogenous growth models unless

two very restrictive knife-edge conditions are fulfilled. In the latter case the model becomes a member of the class of endogenous growth models. Hence, semi-endogenous growth models, such as our basic model, appear to be more realistic due to their generality.

16. The existence of a Malthusian steady state or post-Malthusian balanced growth path depends on whether $\hat{\Omega}_A$ equals zero or is strictly positive. For the moment, we leave the value of $\hat{\Omega}_A$ unspecified.

17. This can be seen upon substitution of (11) in (9), which yields:

$$p_{M,t} = Q_t^{1/(1-\varepsilon)} \left(\frac{\varepsilon}{\varepsilon-1} \right) w_t \delta.$$

18. Alternatively this result can be deduced from the assumption that agricultural demand per person is inelastically set at \bar{a} , while per-capita manufacturing output grows. Therefore, there must be growth of a composite consumption index of the two goods together.

19. Up to now we have assumed that households care only about the number of children. Essentially, this assumption simplified the model since it ensured that growth in agricultural TFP unambiguously leads to a take-off in population growth.

20. Furthermore, Kalemli-Ozcan (2001) shows in a dynamic general equilibrium set-up that there exist two equilibria: One equilibrium mirrors our Malthusian steady state, and the other mirrors a steady state in the Modern regime, as in Galor and Weil (2000). As such, Kalemli-Ozcan (2001) complements our model and the work of Galor and Weil and others.

21. Note that the population growth rate is $\hat{N}_t = q_t b_{t-1} - 1$. Hence, when infant mortality falls, a more than proportional reduction in fertility is required in order for population growth to decline.

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Table 1: Per annum growth rates in Great Britain

Time period	Growth rate of			
	TFP in agriculture	population	per capita income	per capita output in agriculture
1700-60	0.50%	0.38%	0.31%	0.22%
1820-40	0.90%	1.34%	1.04%	0.04%

a. Source: Crafts (1985, p.158-160).

Table 2: Male and female death rate (per 1000 population) at age 1-4 in Great Britain, crude birth rate (per 1000 population) and children per teacher in primary school, England and Wales

Year	Death rate per 1000 population at age 1-4		Crude birth rate	Children per teacher
	male	female		
1841	33.2	32.2	32.2	NA
1861	37.3	36.2	34.6	99.88
1881	27.9	26.4	33.9	62.68
1901	21.2	20.4	28.5	40.35
1921	10.9	10.0	22.8	30.84

- a. Source: Office for National Statistics (1998, Figure 3.5, Figure 6.1 and Figure 6.15).
- b. Notes: NA means data are not available.
- c. Definition: The death rate is defined as the number of deaths among children aged 1-4 years divided by the population at risk of dying at age 1-4, multiplied by 1000.

Figure 1: Price of cotton piece goods (exported) in terms of wheat (1821-1840)

- a. Source: Mitchell (1988, Table 17 and 19).
- b. Note: Price of cotton piece goods is for England and Wales, while the wheat price is for Great Britain.

Figure 2: Transition from Malthusian steady state to post-Malthusian balanced growth path

a. Notation:

$\hat{\Omega}_{A,t}$ = growth of agricultural TFP,

\hat{N}_t = population growth,

\hat{m}_t = growth of per capita manufacturing output.

Figure 3: Transition from Malthusian regime to post-Malthusian regime and transition to modern growth regime

a. Notation:

$\hat{\Omega}_{A,t}$ = growth of agricultural TFP,

\hat{N}_t = population growth,

\hat{m}_t = growth of manufacturing output per adult,

e_t = level of investment in education of each child,

q_t = probability to survive childhood.

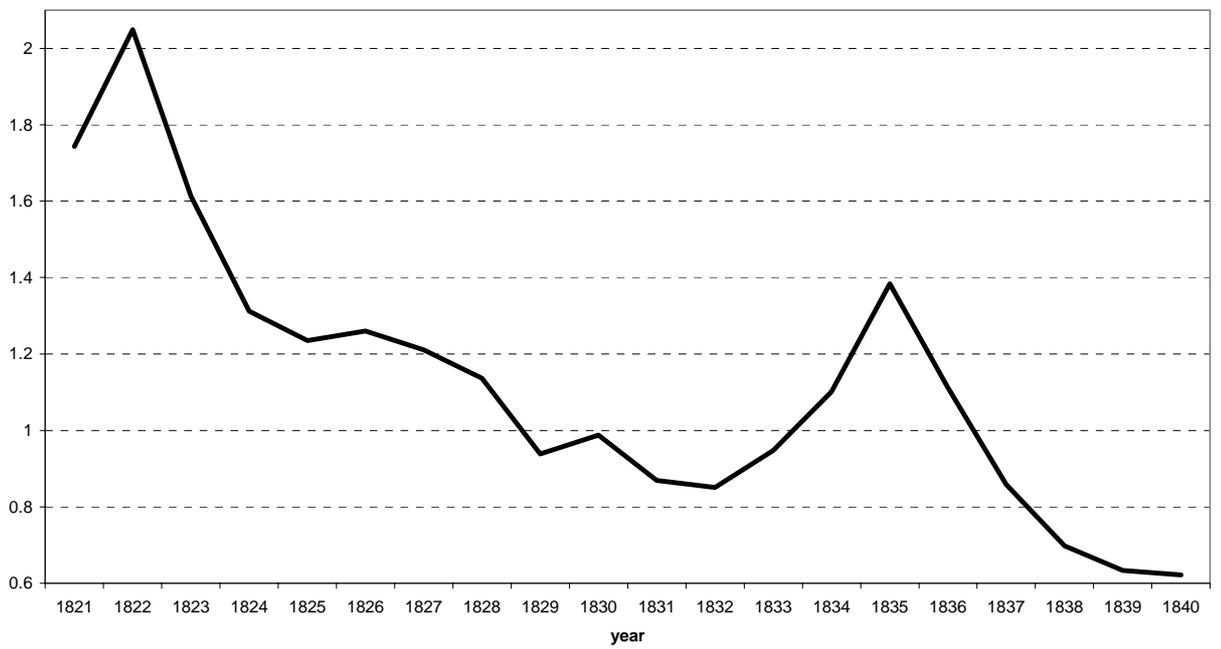


Figure 1:

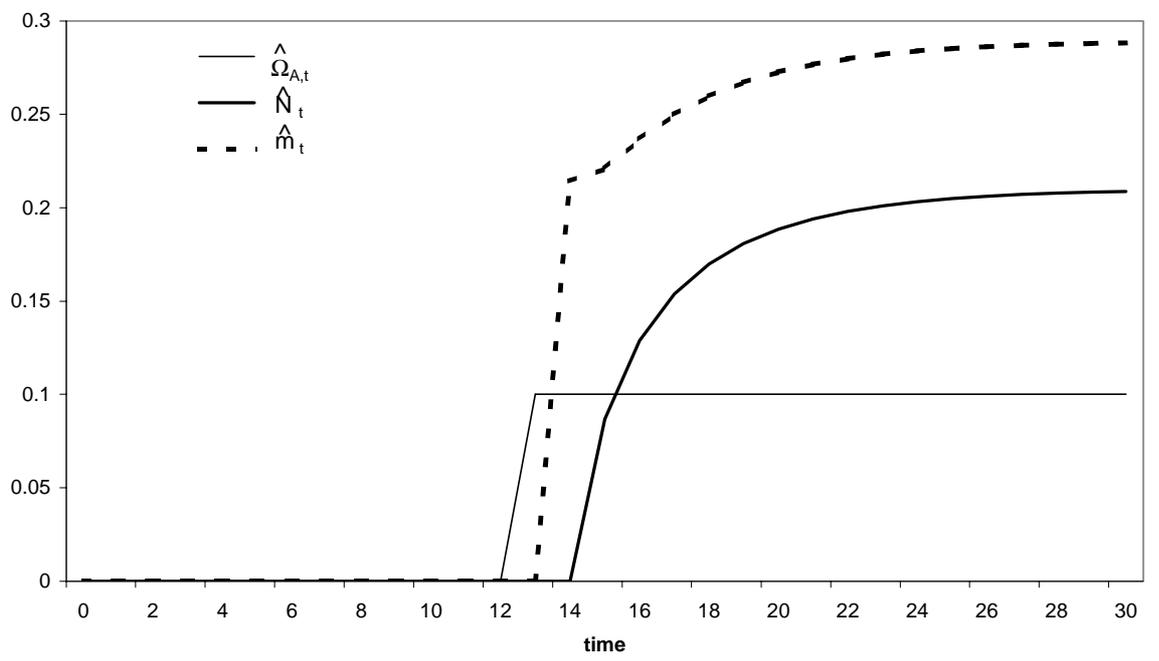


Figure 2:

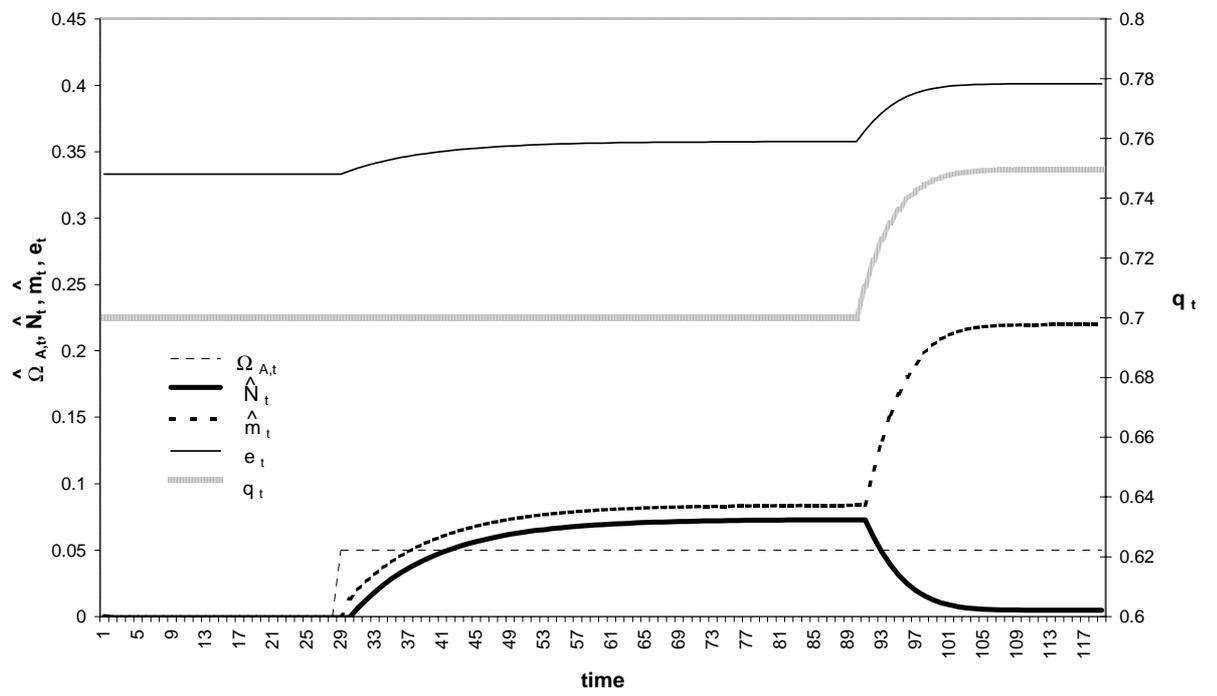


Figure 3: