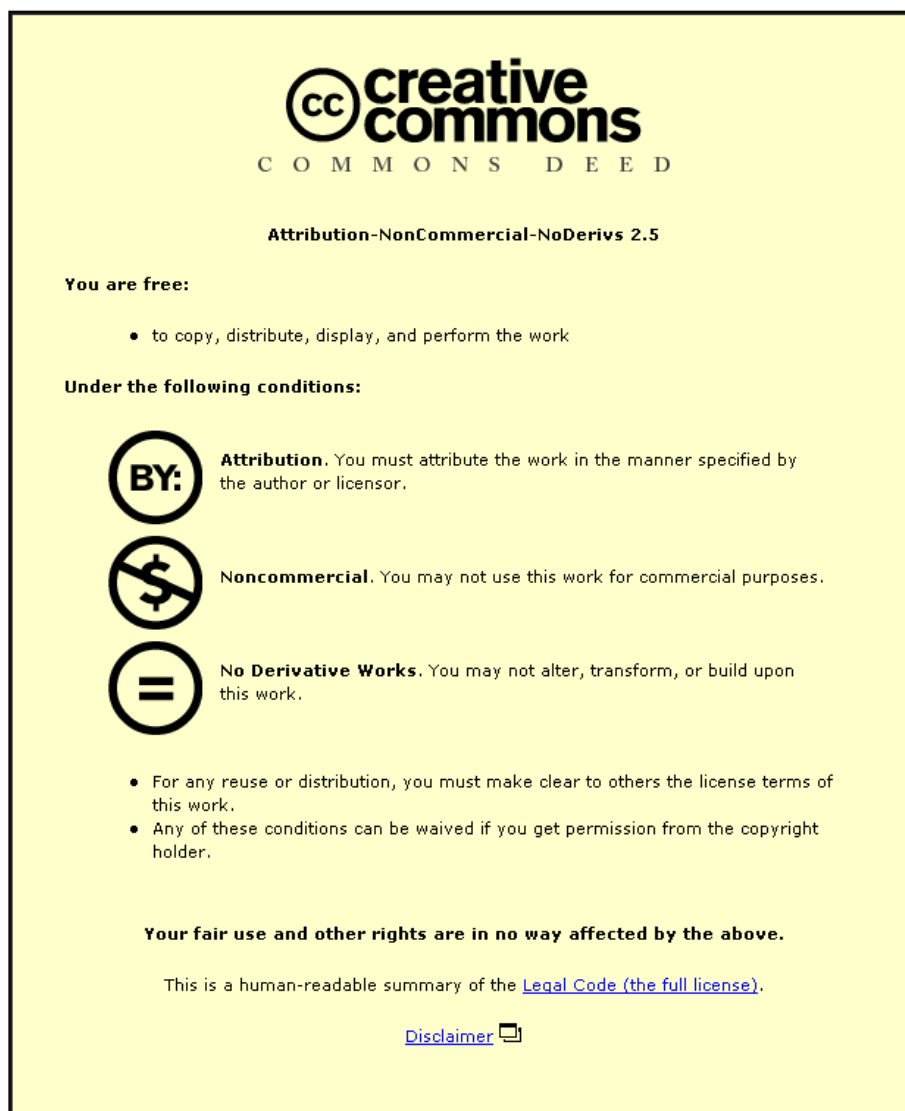


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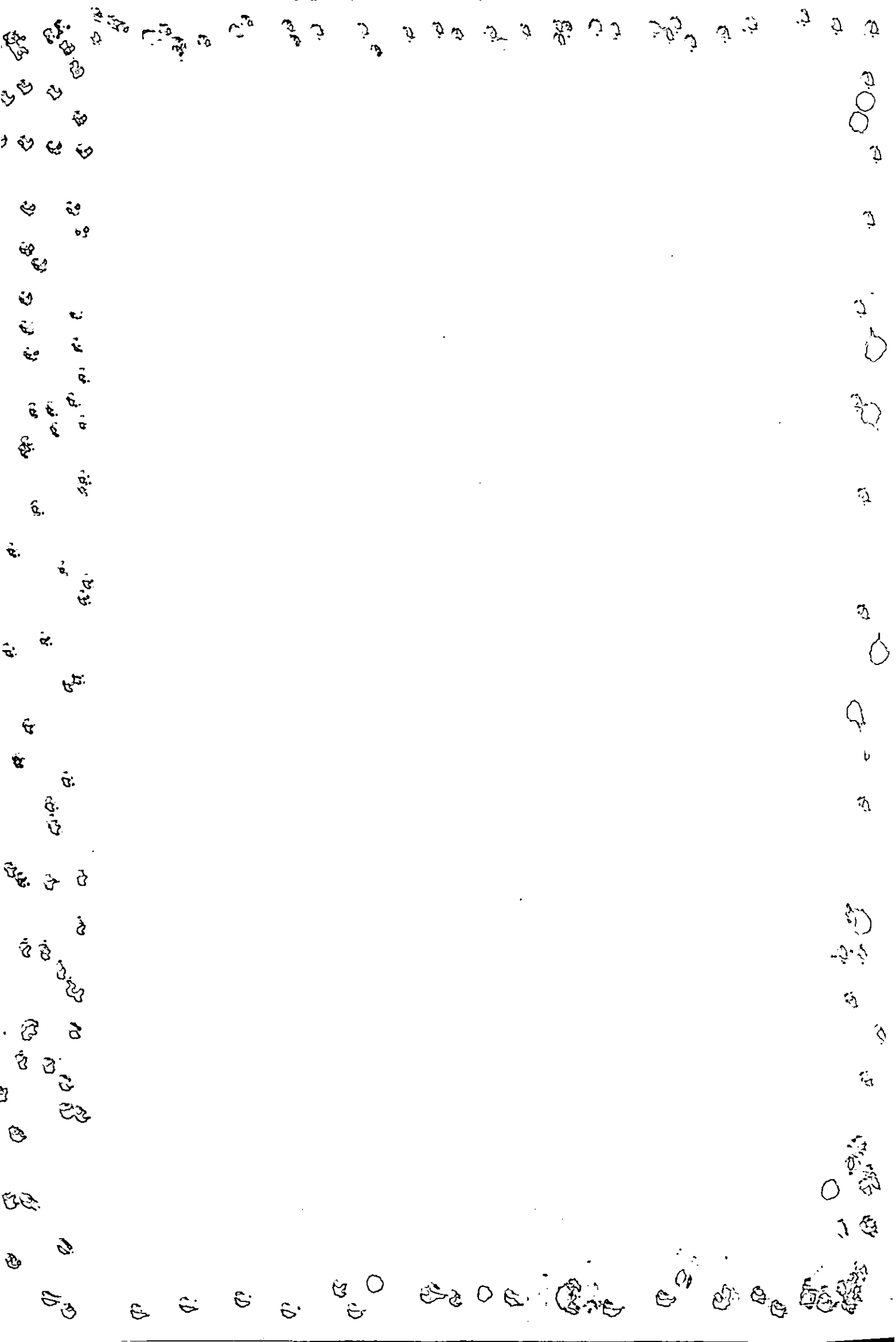
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Adaptive equalizer at the transmitter of  
a digital communication system.

by

MEHMET NUR SERINKEN

A Doctorial Thesis

Submitted in partial fulfilment of the  
requirements for the award of

Doctor of Philosophy of the  
Loughborough University of Technology

April 1975

Supervisor: R.Steele

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### Mathematical Notation

$\{x_j\}$	the set $x_1, x_2, \dots, x_j$ , where $j$ is given in the text
$X$	matrix notation
$ x $	magnitude of the scalar $x$
$ X $	length of the vector $X$
$X^T$	transpose of the matrix or vector $X$
$X^{-1}$	inverse of a matrix
$X(z)$	$z$ transform of a vector $X$
$P_e$	probability of error
$\{s_i\}$	the set of data elements
$t$	= time
$A(t)$	impulse response of an equivalent baseband channel
$R(t)$	the received signal at the output of the receiver filter
$W(t)$	noise waveform at the output of the receiver filter

## Abstract

This thesis deals with digital communication through linear dispersive channels. Topics related to the transmission channel are discussed initially then the equalization of a serial synchronous binary communication system is investigated.

An interesting nonlinear equalizer for use at the transmitter has recently been described in the published literature. The thesis presents a theoretical analysis of this system, together with the results of computer simulation tests, and proposes a simple modification to the equalizer that maximizes the tolerance of the data-transmission system to additive white Gaussian noise. It is shown that the original and modified equalizers are equivalent, to the conventional nonlinear equalizer used at the receiver, both in their basic methods of operation and in their tolerances to noise.

A simple error detector is described which operates on the received signal and does not require the presence of error detection codes. This detector gives powerful protection against errors when used with the original nonlinear equalizer at the transmitter. Further improvements are described by replacing the single element detector by a multi-element detector in the receiver.

Finally techniques for adaptive adjustment of the transmitter nonlinear equalizer tap gains are considered and computer simulation results presented of a technique which increments the gains of the nonlinear equalizer when transmitting data through time-varying channels.

## CHAPTER 1

### Introduction

## CHAPTER 1

### (1) Introduction

The most widely used communication channel is undoubtedly the voice-frequency channel designed for the transmission of speech. This channel passes a band of frequencies from approximately 300 to 3000 Hz. Almost all major towns and cities throughout the world are interconnected by the telephone network, using voice-frequency channels. With the rapidly increasing use of computers and the corresponding increase in the quantity of digital data transmitted to and from the larger computer installations, there is considerable interest in using the telephone network for the transmission of this data. It seems likely that over the coming years there will be a steady increase in the ratio of digital data to speech signals carried by the telephone network for communication between isolated locations. Over long distances, voice-frequency channels over high frequency radio links are often used. The data transmission systems employ channels other than voice-frequency channels such as microwavelinks, troposcatter links, wire communication links. Recent developments indicate that optical fibres can eventually be used for data transmission as well. The problems involved in transmitting digital data over all the channels are quite similar i.e. distortion, noise, time variation of the channels non-linearity etc; but with relative importance of each problem according to the channels in use.

In this thesis a brief description of voice-frequency channels and the problems involved in transmitting digital data over these channels, and the corresponding techniques for achieving satisfactory solution to these problems are presented. The limitation that is common to all the digital data transmission channels is the

distortion caused to the data signals. This thesis is concerned with the minimization of the effect of the channel distortion by equalization i.e. of the channel at the transmitter of a digital data communication system. The data communication system concerned with this research is the one which uses synchronous serial binary data transmission.

\* An interesting technique has recently been proposed for use in data transmission systems in which the data signal at the transmitter is fed through a feedback transversal filter that acts as an equalizer for the channel and includes a nonlinear network whose output signal is essentially the "modulo-n" of its input signal, ref. (1-3). The arrangement is sometimes referred to as the matched transmission technique, ref. ( 3 ), and it has been shown that it is an extension or generalization of the various correlative level coding and partial response techniques described in the published literature, ref. (3-19)

\* Recent investigation have shown that considerable advantages in tolerance to additive noise can be achieved by nonlinear equalizers compared to linear equalizers when the channel introduces significant amplitude distortion, ref. (20-37). It is therefore of interest to compare the performance of the new equalizer with the performance of the more conventional linear and nonlinear equalizers.

This thesis is not concerned with the details of the practical implementation of the equalizer but is primarily concerned with the basic principles of the equalizer and with a modification of the equalizer that optimizes the tolerance of the data-communication system to additive white Gaussian noise.

The method of operation of the equalizer, in its original and

modified forms is described, and the tolerances of the corresponding data-transmission systems to additive white Gaussian noise are estimated theoretically. Each form of the equalizer is compared with the corresponding conventional nonlinear equalizer used at the receiver. A simple error detector is then described which operates on the received signal, with no need for any redundant data for the purpose of error detection. Further improvements to error detector are investigated which involves the replacing of the single element detector by a multi element detector.

Finally the results of computer simulation tests of the system operating over 21 different channels in the presence of additive white Gaussian noise, are presented and compared with the theoretical estimates.

The techniques of maintaining the modulo-n feedback equalizer correctly matched to a time-varying channel are investigated. The performance of an iterative technique is tested by computer simulation and for different channels, graphs of convergence of this algorithm are presented.

The nonlinear equalizer in either of its two forms can be used over a very wide range of transmission rates, from a few bits per second to several megabits per second. Examples of practical channels that are suitable for the equalizer are telephone circuits, 600-ohm. pairs, coaxial-cable links, and fibre-optic links.



## CHAPTER 2

Properties of voice-frequency telephone channels

## CHAPTER 2

### (2-1) Transmission Properties of Telephone Circuits

A telephone circuit connecting one subscriber to another will normally be made up of several links connected in tandem. These links are usually of three distinct types unloaded or loaded audio, or a carrier link. Microwave, satellite and radio frequency links are sometimes used for long distance telephone circuits. A typical telephone circuit could be made up of the following arrangement of individual links: unloaded audio, loaded audio, carrier, loaded audio and unloaded audio.

Due to the several links involved the types of distortion to be expected over a telephone circuit will usually be a combination of the distortion of individual links. Unloaded audio links are generally short and have a delay distortion, and attenuation distortion, although negligible at low frequencies, which increase as the square root of the frequency. The links however have a high attenuation per mile that prevent the use of them for long distance communication. Loaded audio links may be very much longer, inductances are inserted at regular intervals to improve the high attenuation with distance characteristics of unloaded audio links. Their attenuation characteristics resemble that of a low pass filter, which increases rapidly with frequencies above 3000 Hz. Loaded audio links longer than few miles in length require repeaters (amplifiers) to offset the attenuation introduced by the line, and they are spaced at regular intervals. The amplifiers that are employed pass signals in one direction, two amplifiers are required, one for each direction. Thus a 4-wire line is needed to carry signals in both directions. Most subscribers are connected to the local exchange via single pair of wires, carrying signals in both directions. When a telephone circuit contains repeaters or carrier links, arrangements must be made to couple 2-wire lines to 4-wire lines. This is achieved by means of hybrid transformers. Hybrid transformers feed

signals from 2-wire lines to 4-wire lines, or vice versa. These transformers most often are mismatched and the result of this is that signals from go channels are passed to return channels. Signals can pass from one channel to another with steadily decreasing levels, causing signal to be dispersed. When a continuous sequence of signal elements is transmitted, due to dispersion effect, each received element can overlap the following elements and interfere with them. This is known as intersymbol interference.

Carrier links are in general the longest and use wideband channels for transmission. The wideband channel carries several signals in an arrangement of frequency division multiplexing. Due to the filters employed in modulation, demodulation and multiplexing process, these carrier links have the characteristic property of attenuating and introducing delay distortion at low frequencies of the transmission band. The main source of distortion introduced in a carrier link originates in the terminal equipment and is relatively independent of \* the length of the link. The carrier links also introduce a small and variable shift in the frequency location of the transmitted signal spectrum, ref. (49).

#### (2-2) Types of Telephone Circuits

Telephone circuits themselves may be divided into two groups, private and switched lines. A private line is one which is rented permanently or on a part time basis by the subscriber. It is not connected to the automatic switches in the exchange and also disconnected from the battery supplies which are used for d.c. signalling and various other purposes by the post office. A switched line is the circuit obtained when using an ordinary telephone to set up the call. This line is connected through transmission bridges, a number of switches, exchange battery supplies, and when compared to private lines has a

narrower bandwidth. The noise level over switched lines is in general very much higher than that over private lines.

(2-3) Attenuation and Group Delay Characteristics of Telephone Circuits.

For intelligible voice frequency, transmission frequencies above 3000 Hz and below 300 Hz are not needed. There is also gradually increasing attenuation at frequencies above 1100 Hz for circuits containing audio and carrier links. On some carrier links, to prevent the false operation of post office in band-voice frequency signalling equipment, frequency band from 450 to 900 Hz can not be used for data transmission. The available frequency band for data transmission of a switched line involving long audio and carrier links is limited to 900 Hz to 2100 Hz with a narrow band of 300 Hz to 450 Hz. This narrow frequency band is mainly used as a return line from the receiver to transmitter by the data transmission equipment, ref. (50).

Delay distortion causes time spreading of transmitted signal and restricts the transmission rate. The delay distortion varies with the type of line and circuits involving long audio and carrier links have the worst delay distortion. Private lines usually have better delay distortion characteristics than switched lines. Figs. (2-1a) and (2-1b) demonstrate the characteristics of telephone circuits which is a rising attenuation with frequency, for frequencies above about 1100 Hz. Fig. (2-1c) shows a typical group delay characteristic for a circuit containing both audio and carrier links. The group delay increases towards the lower and upper ends of the frequency band, these being a delay distortion of about one millisecond in the frequency band 600 - 2800 Hz. Fig. (2-1d) shows the group delay frequency characteristics of a poor circuit containing both loaded audio and carrier links. The characteristic shows a delay distortion of one

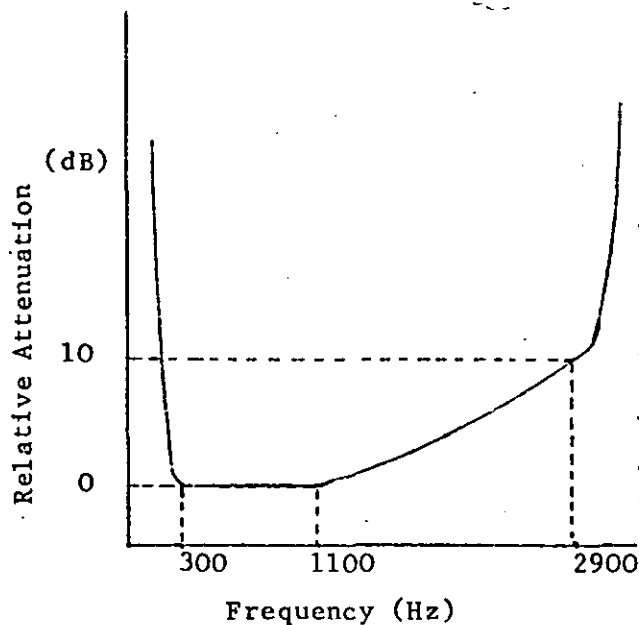


FIGURE (2-1a) TYPICAL ATTENUATION - FREQUENCY CHARACTERISTIC OF A TYPICAL CIRCUIT

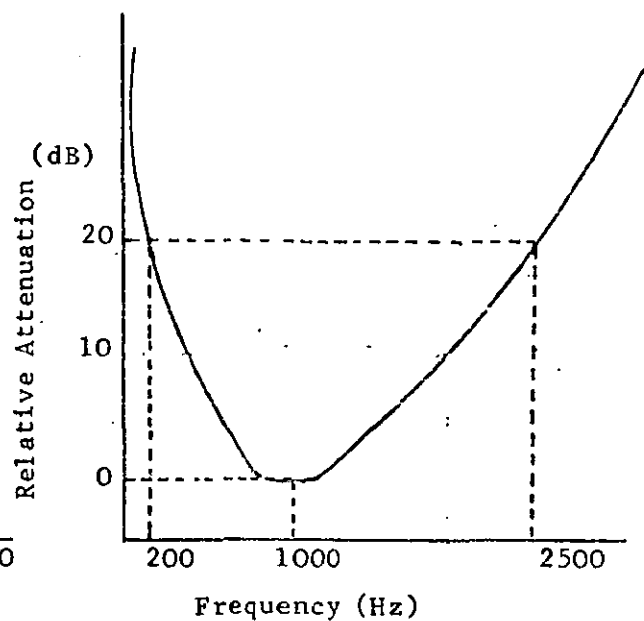


FIGURE (2-1b) ATTENUATION - FREQUENCY CHARACTERISTIC OF A POOR CIRCUIT

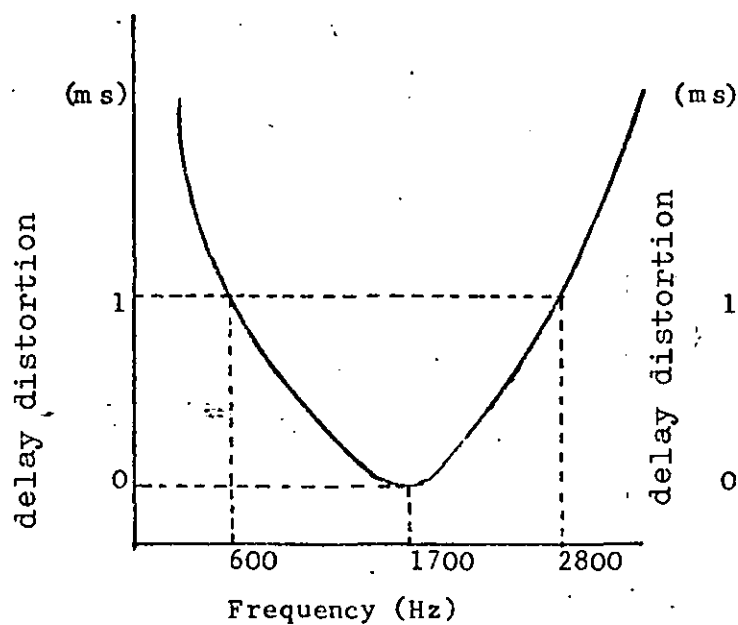


FIGURE (2-1c) TYPICAL GROUP-DELAY CHARACTERISTIC

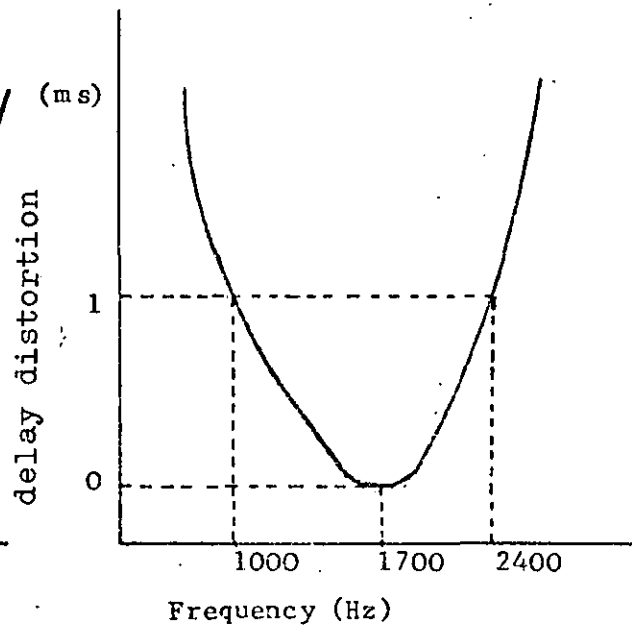


FIGURE (2-1d) GROUP-DELAY CHARACTERISTIC OF A POOR CIRCUIT

millisecond in the frequency band 1000 - 2200 Hz. Over the worst switched telephone lines, the delay distortion in this frequency band can be about 2 milliseconds, ref. (51).

#### (2-4) Characteristics of the Noise on Telephone Lines

The noise on telephone circuits can be classified into two groups, additive and multiplicative. Additive noise is where a noise waveform is added to the transmitted signal, and multiplicative noise is where a noise waveform modulates the amplitude and the frequency of the transmitted signal.

When the noise waveform reaches a sufficient level, its effect is to cause the receiver to interpret incorrectly the received signal, and to introduce errors at the output. Multiplicative noise causes the same number of errors regardless of the signal level, whereas additive noise causes less errors when the signal level is increased. The types of additive noise are crosstalk, impulsive noise, and white noise. Impulsive noise is the predominant type of noise over switched lines where its effects will often swamp those of the other types of noise. Crosstalk and white noise do not cause significant number of errors unless the signal level is very low, ref. (52).

The amplitude modulation effects of noise present over telephone circuits are modulation noise, transient interruptions and sudden level changes. Amplitude modulation of the signal by band limited white noise is the cause of large numbers of errors over circuits with carrier links. Transient interruptions appear as complete breaks in transmission lasting from 1 to 100 milliseconds and could cause a considerable number of errors over some of the longer and more complex private line telephone circuits. These transient interruptions are less likely to be important over shorter circuits, ref. (52).

Sudden signal level changes usually of the order of 1 or 2 dB, will be more serious and frequent over longer and complex circuits. The

frequency modulation effects are gradual frequency drift and sudden frequency fluctuation. These effects occur only over circuits containing carrier links and they shift the frequency location of received signal spectrum. Frequency drifts could be of the order of  $\pm 20$  Hz and sudden fluctuations could be of the order of  $\pm 100$  Hz, ref. (53).

A type of noise, other than Gaussian noise, which may be present over telephone circuits is impulse noise. This noise is characterized by long, quiet intervals followed by bursts of much higher amplitudes than would be predicted by a Gaussian distribution law. The reason for not including impulse noise in the models used go beyond the obvious considerations of proper characterization and mathematical tractability. Because of the long quiet intervals inherent to impulse noise, the speed limitation of the channel is determined by the Gaussian background noise rather than by the burst of impulse noise. Since the impulse noise causes errors with high probability whether the system operates at low or high speeds, the concern in efficient design is to protect against the ultimate limitation i.e. Gaussian noise, ref. (54).

Experimental and theoretical consideration have shown that (5) although the tolerance of a data transmission system to white Gaussian noise is not necessarily an accurate measure of its actual tolerance to the additive noise over telephone circuits, the relative tolerances to white Gaussian noise of different data transmission system are nevertheless a good measure of their relative tolerances to additive noise. Since Gaussian noise lends itself well to theoretical calculations and is also easily produced in the laboratory, the relative tolerances of different data transmission systems to Gaussian noise are very useful measure of their relative tolerances to the additive noise over telephone circuits.

## CHAPTER 3

### Basics of data transmission



### CHAPTER 3

#### (3-1) Maximum Transmission Rate

It has been shown by Nyquist in 1928 that the maximum signal element rate which may be transmitted over a bandwidth  $B$  Hz, for no intersymbol interference is  $2B$  elements per second (Bauds) and known as the Nyquist rate. In the complete absence of noise, distortion and assuming an infinitely sensitive receiver, each signal element can have one of many different levels as required, ref. (55). The information content of each element is  $\log_2 n$  bits, where  $n$  is the number of possible levels. The maximum rate of transmission of information is now  $2B \log_2 n$  bits per second. The information rate may be made as large as required. There is an absolute limit to the rate of signal elements over a given bandwidth, but there is no such limit to the rate of transmission of information over a given bandwidth, ref. (49).

In practice, because noise and distortion are always present, the number of different levels used for signal elements is limited and the maximum number which may satisfactorily be used is that which gives an adequate discrimination against the effects of noise and distortion.

#### (3-1-1) Transmission Rates

The preferred transmission rates for use over voice frequency channels are 600, 1200, 2400, 4800 and 9600 bits per second. The transmission rates of 600 and 1200 can be achieved easily even on poor channels. The rates 4800 and 9600 bits per second require the use of sophisticated techniques.

Binary signal elements are normally used at transmission rates of 600 and 1200 bits per second and 4-level (quarternary) signal elements and used at 2400 bits per second. At 4800 and 9600 bits per second 8- or 16- level signal elements may be used, ref. (49).

### (3-1-2) The Timing Information

A digital signal on its own has very little meaning in the absence of the corresponding timing information. A receiver normally has prior knowledge of the duration of an element, but does not have prior knowledge of positions of the element boundaries. Without this information, correct detection of the received data signal is impossible. The necessary timing information must therefore be transmitted with the data signal, and the receiver must always have a means of extracting the timing information from the received signal.

### (3-2) Baseband Signals

The simplest digital data signal contains a sequence of signal elements (units or pulses of the data signal) where each element is binary coded having the choice of two possible levels which correspond to the element values 0 and 1. Each signal element has the same duration of  $T$  seconds and follows immediately after the preceding element, so that the signal element rate is  $1/T$  elements per second.

When the element value 0 is represented by a signal value  $-k$  and the element value 1 is represented by a signal value  $+k$  it is known as bipolar baseband signal. If the element values 0 and 1 represented by signal values of 0 and  $2k$  it is known as unipolar baseband signal.

For these types of baseband signals frequency spectrum usually extends to zero frequency or to very low frequencies. If the baseband signal is an approximation to a square wave it has spectrum extending to high frequencies, when this waveform is suitably filtered to remove high frequencies it will have a narrower bandwidth and achieve a higher ratio of transmission rate to signal bandwidth.

### (3-3) The need for Modulation

The function of the data transmission equipment is to convert the

baseband signal into the form most suitable for transmission over telephone circuits. The information carrying baseband signal can not be transmitted over the telephone circuit directly as a large proportion of energy will be lost and such energy as does reach the other end of the telephone circuit will be severely distorted for proper detection. For this reason the information carrying baseband signal is used to modulate a sinewave carrier so that the energy is shifted from the lower frequencies to the available frequency band.

All the low frequencies can be transmitted with little or no distortion through short private lines which are not connected through a repeater or any other telephone carrier equipment. Under these conditions a suitably filtered form of baseband signal could be transmitted over the circuit without the need for modulation.

### (3-3-1) Modulation Systems

Two types of modulation that are used in practice for data transmission are linear and nonlinear modulation. Linear modulation techniques are well suited for high-speed data transmission because of their efficient use of available bandwidth. Nonlinear techniques such as frequency and phase modulation are not efficient from bandwidth point of view and therefore are not well suited for high-speed application.

A modulated sinewave carrier may be represented by the equation

$$y(t) = a(t) \sin \left( 2\pi \int_0^t f(\tau) d\tau + P(t) \right) \quad (3-1)$$

This waveform contains three different parameters which are variable quantities, and maybe used for information transmission. In a simple modulation process one of these parameters is made to vary with the modulating waveform. If the amplitude  $a(t)$ , or the frequency  $f(\tau)$  or phase  $p(t)$  is modulated by the baseband signal, the modulation systems are known as amplitude modulation (AM), frequency modulation (FM)

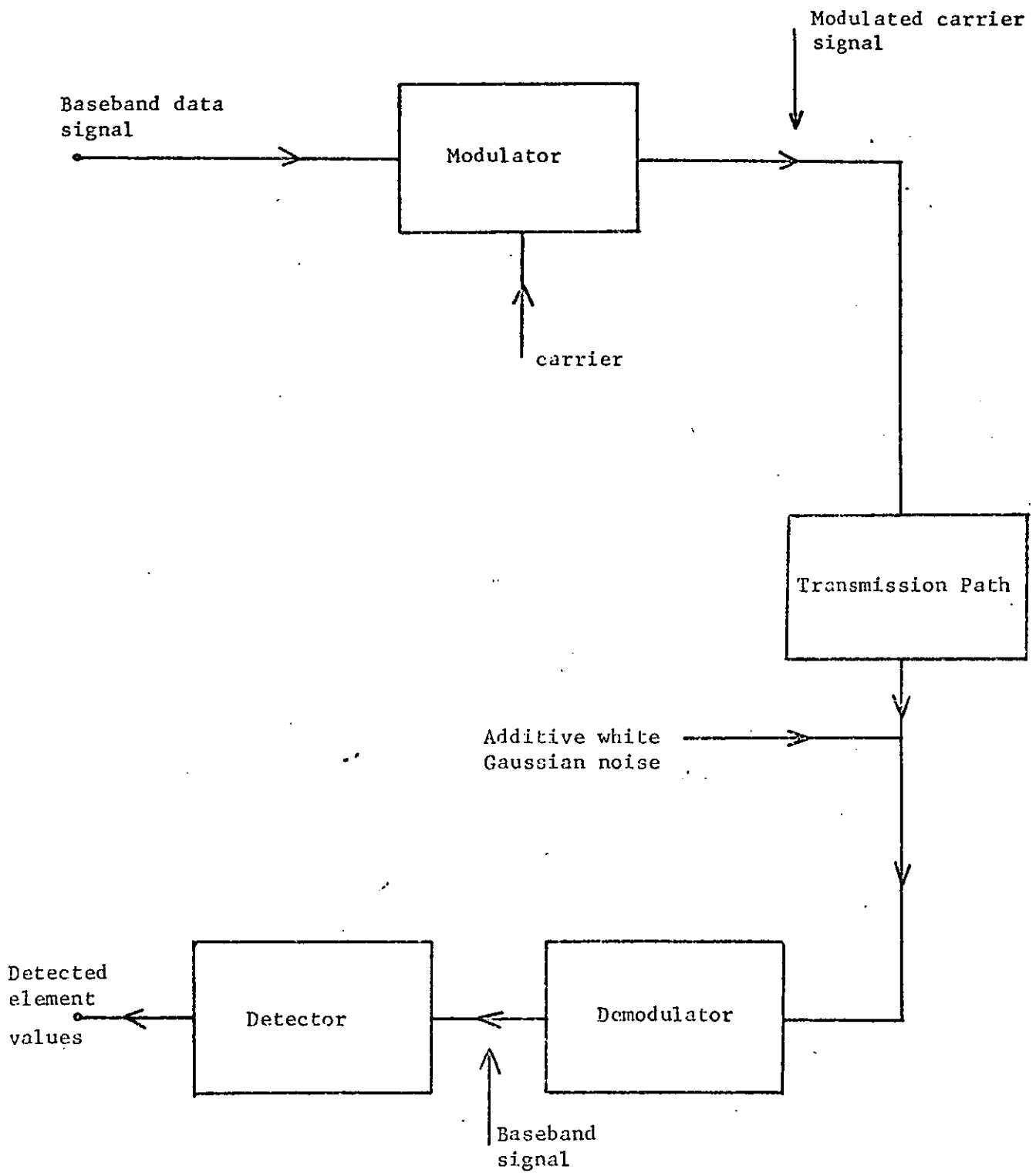


FIGURE (3-1) MODEL OF DATA TRANSMISSION WITH MODULATION

or phase modulation (PM).

The linear modulation (AM) may be produced by multiplying the information carrying signal by a sinusoidal carrier and processing the product by a linear filter. The nonlinear modulation techniques FM and PM use different carrier frequencies or carrier phase angles corresponding to the digital baseband signal.

The model of data transmission with modulation is shown in fig.(3-1). The baseband signal modulates a carrier in the transmitter. Then the modulated carrier is transmitted which has a spectrum that lies within the pass band of the transmission path. Additive white Gaussian noise is introduced during transmission. In the receiver the signal is suitably demodulated to recover the noise corrupted baseband signal which is then detected.

The most common technique for achieving the maximum available transmission rate over telephone circuits is to use a synchronous serial vestigial sideband suppressed carrier AM system, with coherent detection and adaptive equalization. The adaptive equalizer eliminates the signal distortion introduced by the voice-frequency channel.

### (3-3-2) Amplitude Modulation

Fig. (3-2) shows the frequency spectrum of an amplitude modulated carrier using rectangular baseband signal as a modulating waveform. The information carried in each sideband is the same due to this symmetry there is 50% redundant information in the two sidebands. The signal carrier conveys no useful information. With proper filtering, all the information could be transmitted in one sideband but the modulating waveform is usually concentrated towards the very low frequencies including d.c. due to low frequency content of the baseband signal, therefore single sideband transmission is not practical. It is possible to remove a good part of one sideband, then one sideband,

Power spectral  
density function

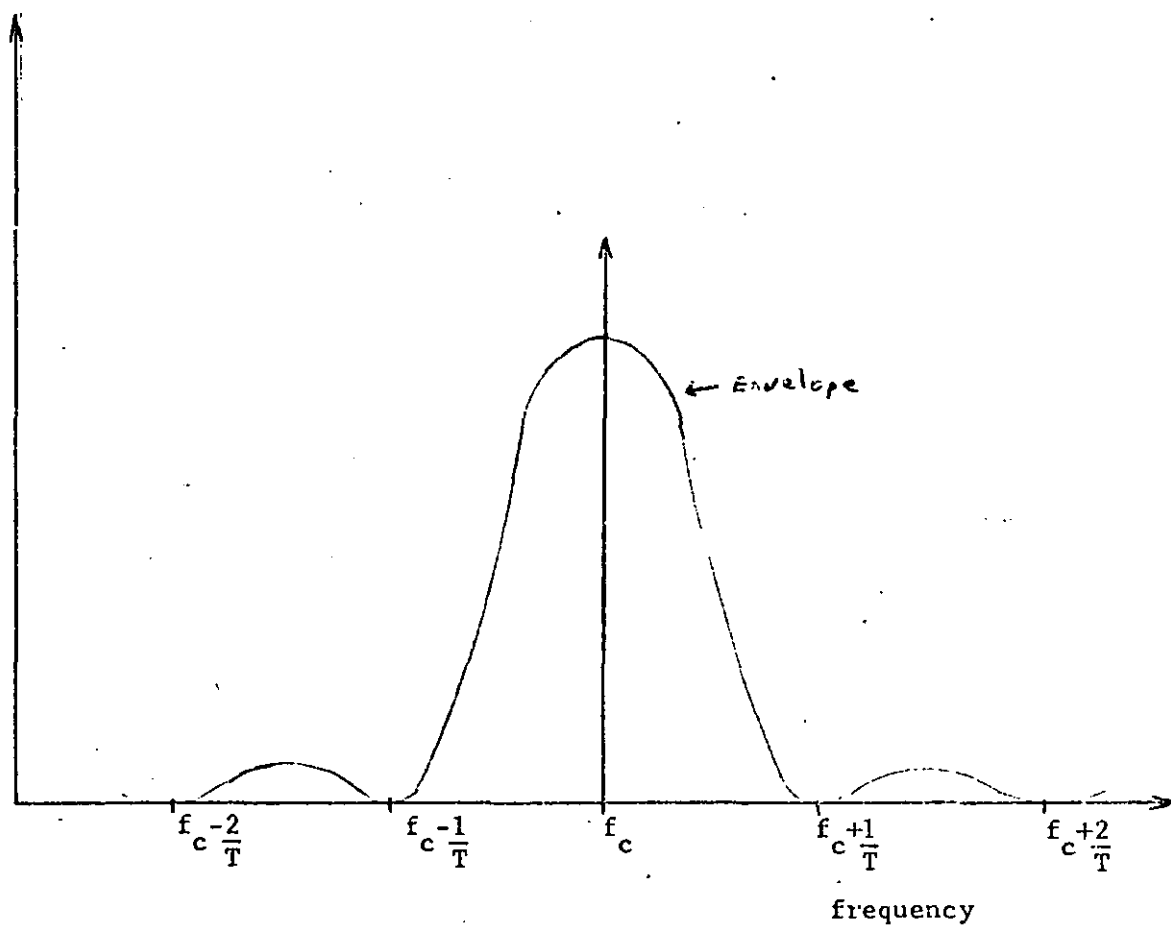


FIGURE (3-2) SPECTRUM OF AM SIGNAL

the carrier and a vestige of the other sideband are transmitted. The carrier can also be suppressed to give vestigial sideband suppressed carrier AM signal.

One undesirable effect in vestigial sideband signal is the presence of appreciable signal distortion, which appears as an additional sinewave at the carrier frequency but at 90 degrees with the main signal carrier. The effect of this distortion can be eliminated by using coherent detection at the receiver.

The element rate for a double sideband AM system is 1 baud per Hz of the frequency band. In the case of a vestigial sideband AM system the maximum element rate is about  $3/2$  bauds per Hz. The ideal would approach 2 bauds per Hz, practical systems use  $2/3$  bauds per Hz for double sideband AM and 1 baud per Hz for a vestigial sideband AM system.

### (3-3-3) Coherent Detection

Coherent detection of a digital signal at the receiver utilizes prior knowledge of the phase of the signal carrier in an element detection process.

For a binary AM signal with a rectangular modulating waveform, the receiver generates from the received signal a reference carrier having the same frequency and phase as the received signal carrier. The reference carrier is used to multiply the received signal and the resultant product is integrated over each element period to give at the end of this period a linear estimate of the value of the corresponding rectangular waveform. For a binary unipolar baseband modulating signal the output signal from the integrator has a value  $2k$  when a burst of carrier is received and it has the value zero when no signal is received.

At the end of an element period the output from the integrator is

fed to a comparator with a threshold level of  $k$ . If the signal level is greater than  $k$  binary value 1 is detected if the signal level is less than  $k$  binary value 0 is detected.

The reference carrier is usually generated by a phase locked oscillator which is held synchronized to the received signal carrier. The coherent detection gives the best result in the detection of vestigial sideband suppressed carrier AM signal.

#### (3-4) Automatic Gain Control

Due to the wide range of signal attenuation experienced over telephone circuits a means must be used to control the output signal level obtained from the input amplifier of any receiver, whatever modulation method is being used.

Automatic Gain Control (AGC) must always be used in an AM receiver and may also be used for FM or PM receivers. The signal attenuation over a typical voice-frequency channel is likely to have any value from 0 to 40 dB and may vary with time.



## CHAPTER 4

### Linear equalization techniques

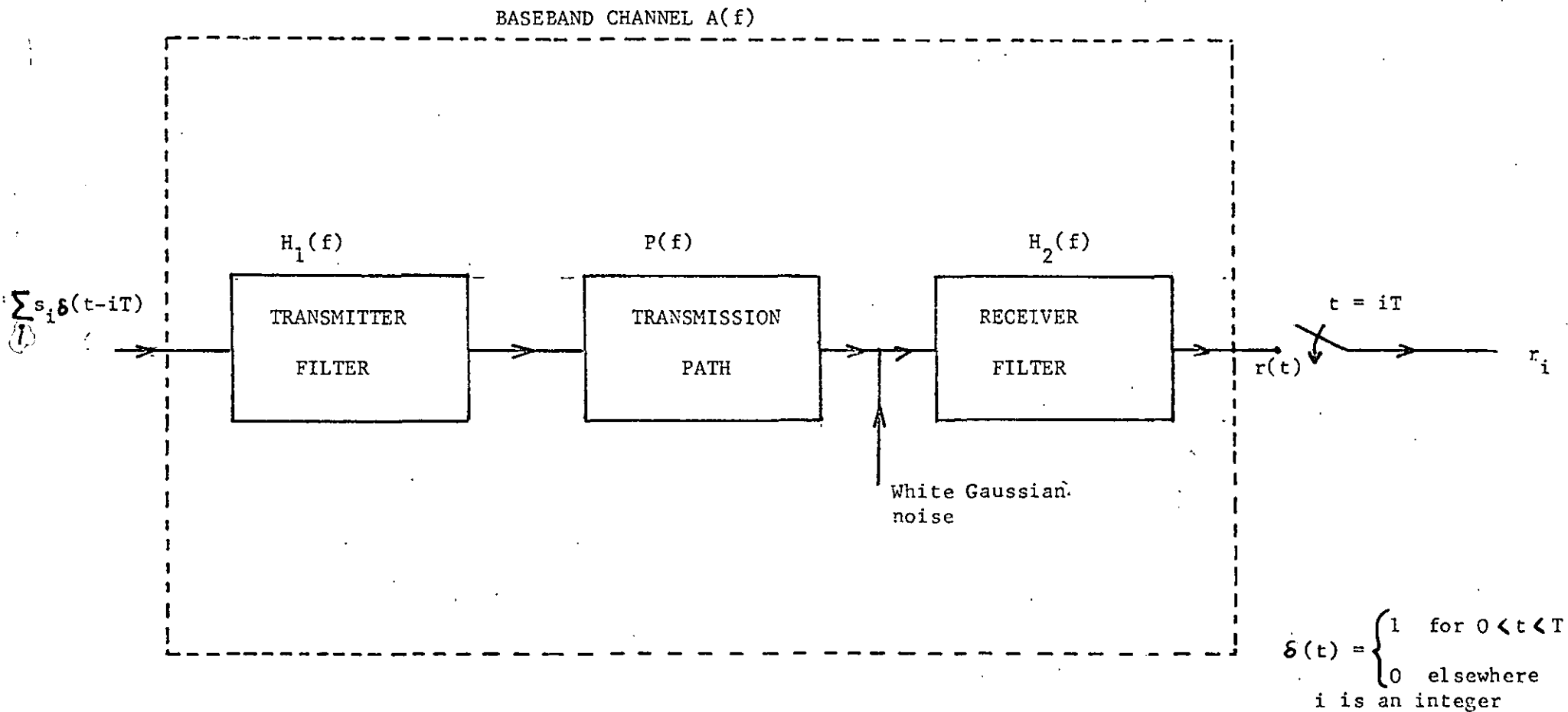


FIGURE (4-1) MODEL OF BASEBAND DATA TRANSMISSION

## CHAPTER 4

### (4-1) Basic Assumptions

The synchronous serial data-transmission system is shown in fig. (4-1). The signal at the input to the transmitter is a sequence of binary bipolar impulses, the impulse at time  $t = iT$  is  $s_i \delta(t - iT)$  and this has a value  $s_i$ , where  $s_i = \pm 1$ . The  $\{s_i\}$  are statistically independent and equally likely to have either binary value.

The transmission path could include a bandpass channel, such as a telephone circuit, a linear modulator at the transmitter and a linear demodulator at the receiver, all are considered to be part of the transmitter path itself. The transmitter filter, transmission path, and receiver filter together form a baseband channel with system transfer function of  $A(f)$ , ref. ( 54 ) where:

$$A(f) = H_1(f) P(f) H_2(f) \quad (4-1)$$

The corresponding impulse response  $A(t)$

$$A(t) = \int_{-\infty}^{\infty} A(f) \exp(j2\pi ft) df \quad (j = \sqrt{-1}) \quad (4-2)$$

The transfer function  $H_1(f)$  of the transmitter filter includes the transfer function needed to convert a sequence of impulses to a rectangular waveform.  $H_2(f)$  is the transfer function of the receiver filter, and  $H_1(f)$ ,  $H_2(f)$  must be chosen to minimize the error rate for a given signal/noise power ratio at the output of the transmission path.  $P(f)$  is the transfer function of the transmission path. In practice  $P(f)$  is not usually known or may vary with time. White Gaussian noise with zero mean and variance  $\sigma^2$  is added to the data signal at the output of the transmission path.  $W(t)$  is the corresponding noise waveform

at the output of the receiver filter and the resultant signal is:

$$R(t) = \sum_i s_i a(t - iT) + W(t) \quad (4-3)$$

The optimum choice of transmitter and receiver filter, which has a cascade transfer function of  $H(f)$  where,

$$H(f) = H_1(f) H_2(f) \quad (4-4)$$

was shown to be  $H_1(f) = H_2(f) = H^{\frac{1}{2}}(f)$  in ref. (56), in the presence of white Gaussian noise.

$$H(f) = \begin{cases} 1/2T (1 + \cos \pi f T) & -\frac{1}{T} < f < \frac{1}{T} \\ 0 & \text{elsewhere} \end{cases} \quad (4-5)$$

$H(f)$  given in (4-5) and shown in fig. (4-2a), is a common arrangement of baseband signal shaping filter, known as raised cosine filter,

$$H^{\frac{1}{2}}(f) = \begin{cases} T^{\frac{1}{2}} \cos(\pi f T / 2) & -\frac{1}{T} < f < \frac{1}{T} \\ 0 & \text{elsewhere} \end{cases} \quad (4-6)$$

for any value of  $P(f)$

$$A(t) = \int_{-\infty}^{\infty} P(f) H(f) \exp(j2\pi f t) df \quad (4-7)$$

when  $P(f) = 1$  no signal distortion is introduced by the transmission path.

$$A(t) = \int_{-\frac{1}{T}}^{\frac{1}{T}} \frac{1}{2} T (1 + \cos \pi f T) \exp(j2\pi f t) df \quad (4-8)$$

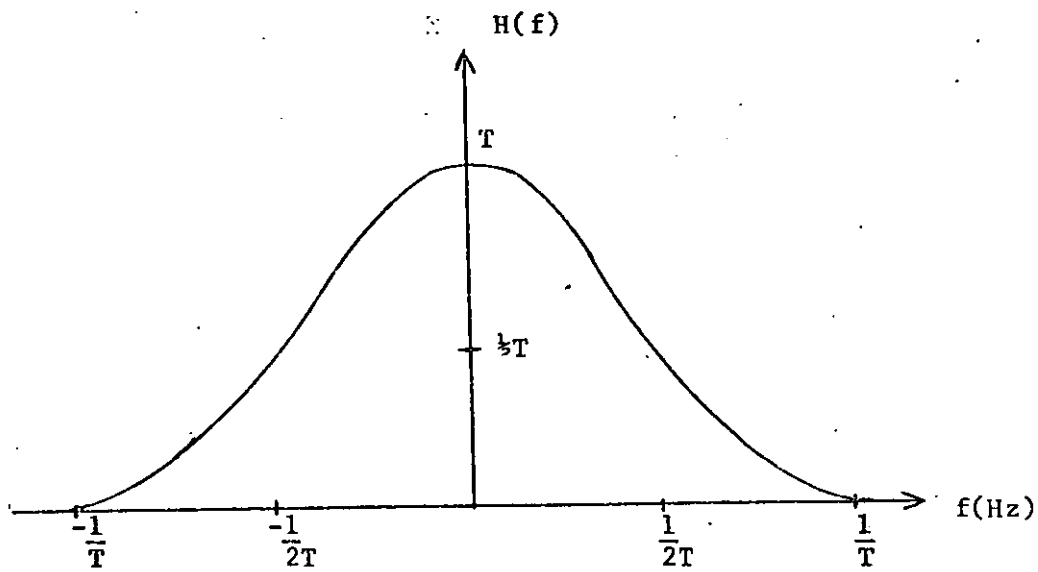


FIGURE (4-2a) AMPLITUDE - FREQUENCY CHARACTERISTIC OF  $H(f)$

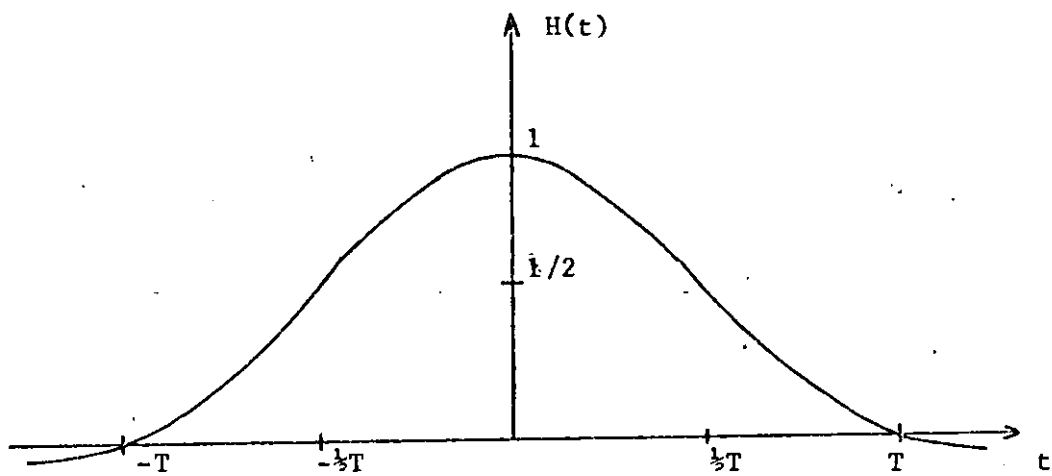


FIGURE (4-2b) IMPULSE RESPONSE OF  $H(t)$

$$A(t) = \frac{\sin \pi 2 t/T}{\pi 2 \frac{t}{T}} + \frac{1}{2} \frac{\sin \pi (2 \frac{t}{T} + 1)}{\pi (2 \frac{t}{T} + 1)} + \frac{1}{2} \frac{\sin \pi (2 \frac{t}{T} - 1)}{\pi (2 \frac{t}{T} - 1)} \quad (4-8)$$

The delay introduced by the transmitter and receiver filters has been neglected here,

$$A(0) = 1, \quad A(\pm \frac{1}{2} T) = \frac{1}{2} \quad (4-9)$$

and  $A(\pm \frac{1}{2} iT) = 0$

$A(t)$  is shown in fig. (4-2b) for  $P(f) = 1$  and  $H(f)$  as eqn. (4-5), for all values of the integer  $i$  other than 0 or  $\pm 1$ . With additive white Gaussian noise having a two sided power spectral density of  $\sigma^2$  at the input to the receiver filter, the noise power spectral density at the output of the receiver filter is

$$\sigma^2 |H(f)|^2 = \sigma^2 H(f) \quad (4-10)$$

from the Wiener-Khinchin theorem, the auto correlation function of the noise signal  $w(t)$  at the output of the receiver filter is:

$$d(t) = \int_{-\infty}^{\infty} \sigma^2 H(f) \exp(j2\pi fT) df \quad (4-11)$$

and has the values at:

$$\begin{aligned} d(0) &= \sigma^2 \\ d(iT) &= 0 \quad (\text{for any nonzero integer } i) \end{aligned} \quad (4-12)$$

also the energy of the  $i^{\text{th}}$  transmitted signal-element is  $s_i^2$  for the transmitter filter given in eqn. (4-6).

The received signal  $R(t)$ , at the output of the receiver filter, is sampled at time instants  $t = iT$ , for all integer values of  $i$ . Thus the  $i^{\text{th}}$  received element is sampled at time  $t = iT$  to give the sample value

$$r(iT) = s_i a(0) + w(iT)$$

or

(4-13)

$$r_i = s_i + w_i$$

where  $r_i = r(iT)$  and  $w_i = w(iT)$ .  $w_i$  is a sample value of statistically independent Gaussian random variable with zero mean and variance  $\sigma^2$ .

When  $P(f) \neq 1$ , signal distortion is introduced in the transmission path, there is usually intersymbol interference between the different received signal elements at the sampling instants  $t = iT$ . It is assumed that  $A(t)$  is for practical purposes of finite duration, not exceeding  $kT$  second where  $k$  is a positive integer and  $A(t)$  is time invariant. The sampled impulse response of the baseband channel is given by the  $(k + 1)$  component row vector.

$$A = a_0, a_1, a_2, \dots, a_k \quad (4-14)$$

and its  $z$  transform is ( see appendix A1),

$$A(z) = a_0 + a_1 z^{-1} + a_2 z^{-2} \dots a_k z^{-k} \quad (4-15)$$

the resultant signal at the output of the receiver filter has previously been given by eqn. (4-3). For no signal distortion  $A$  would be  $a_0 = 1$ ,  $a_j = 0$  for  $j = 1, 2, \dots, k$  but, the  $z$  transform of the  $i^{\text{th}}$  received signal element after sampling, is  $s_i z^{-i} A(z)$ . For an uninterrupted sequence of signal elements

$$r_i = \sum_{j=0}^k a_j s_{i-j} + w_i \quad (4-16)$$

thus if  $s_i$  is detected from  $r_i$  there is in addition to the noise component  $w_i$  an intersymbol interference component which is

$$\sum_{j=1}^k a_j s_{i-j}$$

added to the wanted signal  $a_0 s_i$ .

In the detection of  $s_i$  from  $r_i$  at high signal to noise ratios, the best tolerance to additive Gaussian noise is achieved through the effective elimination of all intersymbol interference, that is accurate equalization of the baseband channel, ref. ( 54 ).

#### (4-2) Linear Equalization

The most common linear equalization is the feedforward transversal filter in fig. (4-3) with  $m$  taps. This filter has a sampled impulse response

$$c = c_0, c_1 \dots \dots \dots c_{m-1} \quad (4-17)$$

and  $z$  transform

$$C(z) = c_0 + c_1 z^{-1} + c_2 z^{-2} \dots \dots \dots c_{m-1} z^{-m-1} \quad (4-18)$$

for accurate equalization of the channel. The  $z$  transform of the channel and equalizer is

$$A(z) C(z) \cong z^{-h} \quad (4-19)$$

where  $h$  is a positive integer in the range 0 to  $m + k - 1$ .

From eqn. (4-19) the  $z$  transform of the  $i^{th}$  received signal-element at the output of the equalizer is



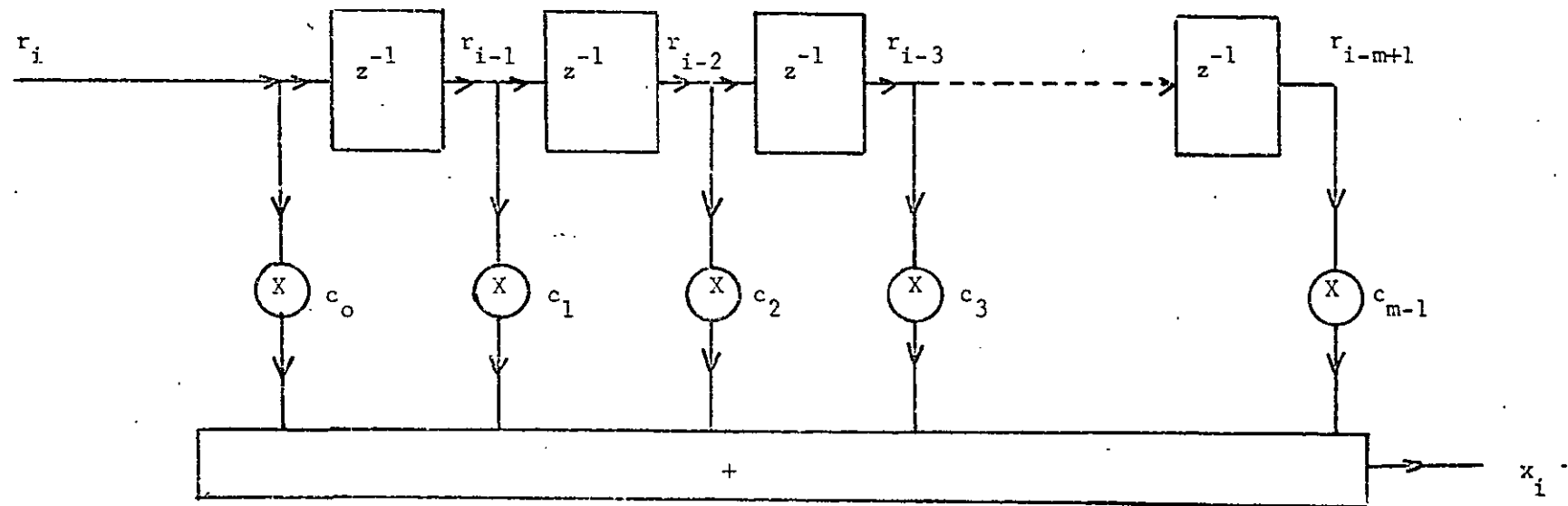


FIGURE (4-3) LINEAR FEEDFORWARD TRANSVERSAL FILTER

$$s_i z^{-i} A(z) C(z) \approx s_i z^{-i-h} \quad (4-20)$$

and this element is detected with negligible intersymbol interference from the sample value at the output of the equalizer at the time  $t = (i + h)T$ .

Let B the  $m \times (m + k)$  matrix whose  $i^{\text{th}}$  row is

$$B_{i-1} = \underbrace{0 \ 0 \dots 0}_{i-1} \underbrace{a_0 \ a_1 \dots a_k}_{k+1} \underbrace{0 \dots 0}_{m-i} \quad (4-21)$$

From eqn. (4-20) the sampled impulse response of channel and equalizer is

$$\sum_{i=0}^{m-1} C_i B_i = CB \approx U_h \quad (4-22)$$

$$\text{where } U_h = \underbrace{0 \dots 0}_h \ 1 \ \underbrace{0 \dots 0}_{m+k-h-1} \quad (4-23)$$

B is a convolution matrix and CB is the  $(m + k)$  component vector obtained by the convolution of A and C. If C is a finite feedforward transversal filter eqn. (4-22) will not be satisfied exactly, but will be,

$$CB = e_0 \ e_1 \dots e_{m-1+k} \quad (4-24)$$

Equalization with a finite feedforward transversal filter is possible provided that  $A(z)$  does not have zeros on or near the unit circle, ref. (1).

The peak distortion in the equalized signal, when C is a finite filter, is defined as:

$$D_{\text{peak}} = \frac{1}{e_h} \sum_{\substack{i=0 \\ i \neq h}}^{m+k-1} |e_i| \quad (4-25)$$

and the mean square distortion is defined as:

$$D_{rms} = \frac{1}{e_h} \sum_{\substack{i=0 \\ i \neq h}}^{m+k-1} e_i^2 \quad (4-26)$$

where  $e_h \approx 1$ , ref. (37). For a given equalizer it may be required to minimise either eqns. (4-25) or (4-26).

It has been shown, ref. (37), that a linear equalizer with either design criteria results <sup>essentially</sup> in the same performance in high signal/noise ratio. Minimization of the mean square distortion results in an equalizer which is correctly matched to a slowly time varying channel, ref. (20-37).

(4-2-1) Design of a Linear Equalization System whose Mean Square Distortion is Minimised.

$U_h$  is the ideal value of the sampled impulse response of the equalized channel, whereas CB given by eqn. (4-24) is the actual response. The mean square error of the sampled impulse response of the equalized channel is given by

$$\begin{aligned} (e_h - 1)^2 + \sum_{\substack{i=0 \\ i \neq h}}^{m+k-1} e_i^2 &= (U_h - CB) (U_h - CB)^T \\ &= |U_h - CB|^2 \end{aligned} \quad (4-27)$$

The vectors  $U_h$  and CB may be represented as points in an  $(m+k)$  dimensional Euclidian vector space, where the length of a vector is the distance from the origin to the corresponding point in the vector space.  $|U_h - CB|$  is the length of the vector  $U_h - CB$  and it is therefore the distance between the vectors  $U_h$  and CB.

It can be shown that a linear equalizer that minimises the mean square error in the sampled impulse-response of the equalized channel, also minimises the mean square distortion, ref. (37).

To minimise the mean square distortion it is necessary to minimise the distance between the vectors  $U_h$  and  $CB$ . It can be seen from eqn. (4-21) that the  $m$  vectors  $\{B_i\}$  are linearly independent, so that they span an  $m$ -dimensional subspace of  $(m + k)$  dimensional vector space. From eqn. (4-22)  $CB$  is a point in this subspace. Thus for the minimum mean square distortion,  $CB$  must be the point in the subspace at the minimum distance from  $U_h$ . By the projection theorem ref. (57),  $CB$  is the orthogonal projection of  $U_h$  onto the  $m$ -dimensional subspace. The  $m$  vectors  $\{B_i\}$  are now orthogonal to the error vector  $U_h - CB$  and

$$(U_h - CB) B^T = 0$$

or

$$C = U_h B^T (BB^T)^{-1}$$

(4-28)

To obtain the optimum equalizer,  $C$  must be determined for each value of  $h$  from 0 to  $m + k - 1$ , and the vector  $C$  for the required equalizer selected as that which gives the minimum value of  $|U_h - CB|$ .

The  $m$  tap coefficients of the linear feedforward transversal equalizer are chosen to be the vector  $C$ .

#### (4-2-2) Probability of Error

When the data signal is received in the presence of additive white Gaussian noise, the sample value  $r_i$  at the input to the equalizer at time  $t = iT$  is given by eqn. (4-16).

The equalizer eliminates the intersymbol interference components to give at time  $t = (i + h)T$ , the output signal

$$x_{i+h} \approx s_i + v_{i+h} \quad (4-29)$$

where

$$v_{i+h} = \sum_{j=0}^{m-1} c_j w_{i+h-j}$$

$v_{i+h}$  is a sample value of a Gaussian random variable with zero mean and whose variance is

$$E^2 = \sigma^2 \sum_{j=0}^{m-1} c_j^2 \quad (4-30)$$

and it can be shown that the probability of error in the detection of  $s_i$  is

$$P_1 = \int_{\frac{1}{E}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du \quad (4-31)$$

$$Q(y) = \int_y^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} u^2\right) du \quad (4-32)$$

$$P_1 = Q\left(\frac{1}{E}\right) \quad (4-33)$$

When the linear equalizer is located in the transmitter, the transmitted signal energy will be increased by an amount equal to  $k^2$ , where

$$k^2 = \sum_{j=0}^{m-1} c_j^2 \quad (4-34)$$

The normalized transmitted signal element power will be  $1/k^2$ , but the noise variance will be unchanged and equal to  $\sigma^2$ , thus the probability of error will be the same as the above value and there will be no change in eqn. (4-33) if the linear equalizer is located in the transmitter. This will increase equipment complexity for adaptive operation of linear equalizer for time varying channels which will require information about channel response to be transmitted via slow

speed return channel from receiver to the transmitter.

## CHAPTER 5

### Nonlinear equalization techniques

## CHAPTER 5

### (5-1) Nonlinear Equalization of the Baseband Channel by the Decision Directed Cancellation of Intersymbol Interference.

An alternative approach to linear equalization is to use a technique of decision directed signal cancellation in the receiver.

The received signal at the detector input is sampled at the time instants  $t = iT$ , for all integers  $i$ , the sample value at time  $t = iT$  is,

$$r_i = \sum_{j=0}^k a_j s_{i-j} + w_i \quad (5-1)$$

as in eqn. (4-13) where  $w_i$  is a sample value of a Gaussian random variable with zero mean and variance  $\sigma^2$ . It is assumed that the sampled impulse response  $A(z)$  of the channel are represented by eqns. (4-14) and (4-15). It is assumed that  $A(z)$  is known in the receiver. In the absence of delay and distortion,

$$\begin{aligned} a_0 &= 1 \\ \text{and } a_i &= 0 \text{ for } i = 1, 2, \dots, k \end{aligned} \quad (5-2)$$

under these conditions

$$r_i = a_i + w_i \quad (5-3)$$

so that the  $i^{\text{th}}$  received element is detected from the sample value  $r_i$  of the received signal at time  $t = iT$ . In the presence of distortion eqn. (5-2), is not applicable. Severe intersymbol interference maybe experienced in the detection of the  $i^{\text{th}}$  element  $s_i$  derived from the  $r_i$  received element. This is true for all received elements except the first whose binary value  $s_1$  is detected from  $r_1$  then

$$r_1 = s_1 a_0 + w_1 \quad (5-4)$$



and the detector uses the fact,

$$s_1 = \frac{r_1}{a_0} - \frac{w_1}{a_0} \quad (5-5)$$

provided that  $a_0 \neq 0$ , and  $w_1$  is of course unknown. Thus  $s_1$  is detected as +1 when  $r_1/a_0 \geq 0$  and as -1 when  $r_1/a_0 < 0$ . In this way the first element is detected from its first non-zero sample value. After the detection of first element, the intersymbol interference caused in the following  $k$  elements are cancelled, it is assumed that the receiver knows  $k + 1$  values of the sampled impulse response of the channel. The second element is detected in the same way and its intersymbol interference is eliminated in the following  $k$  elements. The process is repeated for each signal element. The eqn. (5-1) can be written as:

$$r_i = a_0 s_i + \sum_{j=1}^k a_j s_{i-j} + w_i$$

where

$$f_i = \sum_{j=1}^k a_j s_{i-j}$$

$f_i$  is the intersymbol interference term

$$s_i = \frac{1}{a_0} [r_i - f_i - w_i] \quad (5-6)$$

and  $f_i$  is cancelled from  $r_i$  before detection of  $s_i$  for all elements. It is clear that so long as the received signal elements are correctly detected their intersymbol interference in the following elements are eliminated and the channel is accurately equalized. The cancellation of intersymbol interference from the following elements depends on the decision made by the detector.

The nonlinear equalizer is implemented as a feedback transversal filter fed from the output of the detector as shown in fig. (5-1).

The output  $\{f_i\}$  of the transversal filter is subtracted from the sample value of the received signal which cancels the intersymbol interference

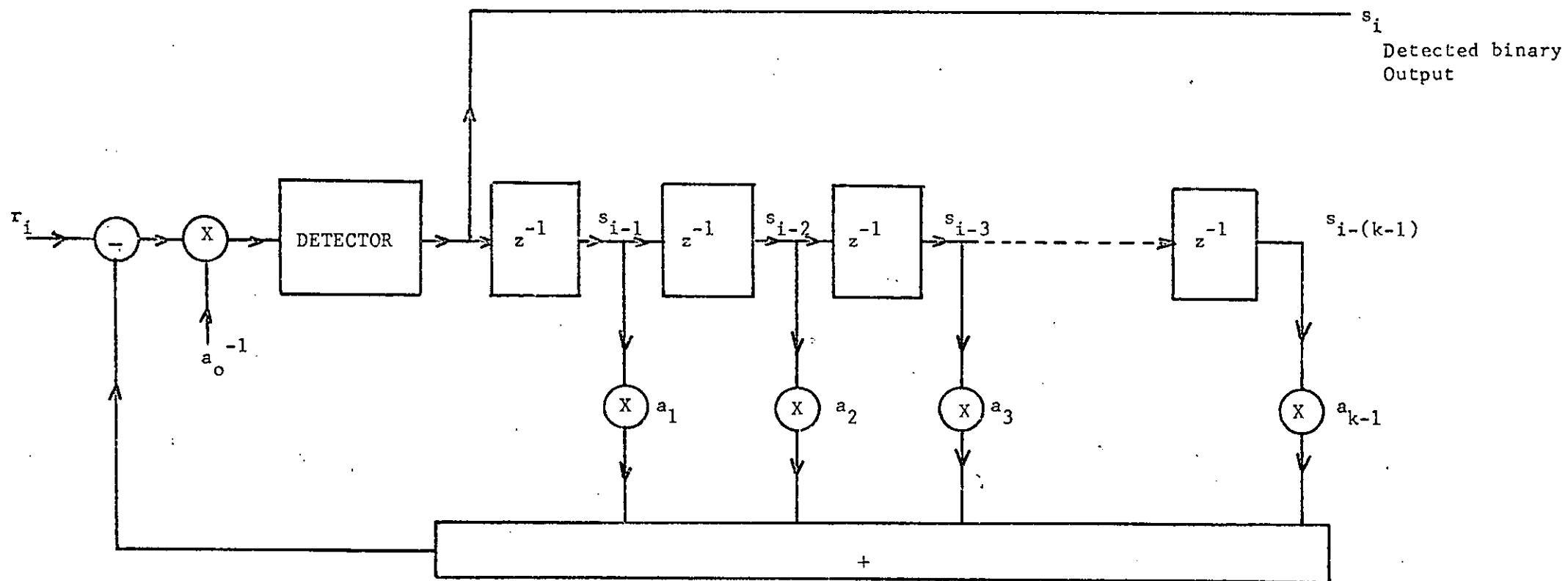


FIGURE (5-1) NONLINEAR EQUALIZER

provided that received signal elements are correctly detected.

To start this process of nonlinear equalization a known training sequence is transmitted which enables the receiver to extract the information about channel impulse response. With the aid of the training sequence the intersymbol interference is cancelled in the nonlinear filter without the detection of the received training sequence and uses its prior knowledge of the sequence. The channel is now correctly equalized and the following data elements are detected and cancelled as described, ref. (58).

#### (5-2) The Probability of Error

The received signal elements are detected from the sign of eqn. (5-6). The probability of error in the detection of a received signal-element, following the correct detection of the preceding  $k$  elements is

$$P_e = \int_{|a_o|}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{w^2}{2\sigma^2}\right) dw \quad (5-7)$$

$$= \int_{\frac{|a_o|}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{w^2}{2}\right) dw \quad (5-8)$$

$$= Q\left(\frac{|a_o|}{\sigma}\right) \quad (5-9)$$

When a received signal-element is incorrectly detected its intersymbol interference in the following elements is increased due to wrong cancellation. This increases the probability of error in their detection and errors tend to occur in bursts with the result that the system suffers from error-extension effects and the probability of error increases by about ten times. From computer simulation results at high signal/noise ratios the error extension effect reduce the

tolerance to additive Gaussian noise by about 1 dB.

An important advantage of nonlinear equalization technique over the corresponding linear transversal equalizer, is that it is always stable, whereas the linear equalizer is not, ref. (58).

When all the roots of  $A(z)$  lie inside the unit circle in the  $z$  plane,  $a_0$  is one of the larger components of  $A(z)$  and the nonlinear equalizer always gives a better tolerance to additive white Gaussian noise than a linear equalizer. However,  $A(z)$  frequently has roots outside the unit circle, such that  $a_0$  is one of its smaller components, and the nonlinear equalizer is now likely to have a lower tolerance to additive Gaussian noise than a linear equalizer, ref. (37).

A better performance is sometimes obtained with a nonlinear equalizer by detecting a received signal element not from the first non zero component of the element namely  $a_0 s_i$ , but from one of the following  $k$  sample values. Then the nonlinear filter can eliminate the intersymbol interference components of the signal-elements already detected. However, it can not eliminate the remaining intersymbol interference, thus the channel is only partially equalized. It is not possible to use this technique of equalization by decision directed cancellation in the transmitter of a digital data transmission system, due to the fact output of the filter is a binary sequence and the channel will introduce intersymbol interference regardless of the presence of the filter.

### (5-3) Combined Linear and Nonlinear Equalization

In this section a design technique for a nonlinear equalizer which is implemented by a combination of linear feedforward and decision feedback transversal filters is presented. The design minimizes the error probability in the detected binary bipolar signal elements, subject to the essentially accurate equalization of the channel.

The sampled impulse response of the channel is given by eqn. (4-14) and its  $z$  transform by eqn. (4-15). The  $A(z)$  can be split into two parts:

$$A(z) = A_1(z) A_2(z) \quad (5-10)$$

where all the roots of  $A_2(z)$  satisfy  $|z| \approx 1$ .

Both  $A_1(z)$  and  $A_2(z)$  are assumed to be known at the receiver. Let

$$A_1(z) = 1 + \alpha_1 z^{-1} + \alpha_2 z^{-2} \dots \dots \dots + \alpha_\theta z^{-\theta} \quad (5-11)$$

$$A_2(z) = \beta_0 + \beta_1 z^{-1} + \beta_2 z^{-2} \dots \dots \dots + \beta_\tau z^{-\tau} \quad (5-12)$$

where  $\theta + \tau = k \quad (5-13)$

$A_2(z)$  can not be equalized by means of a stable linear filter of limited length because the roots of eqn. (5-12) are on or near the unit circle. However,  $A_1(z)$  is approximately equalized by means of a transversal filter with  $z$  transform,  $C(z)$  is given by eqn. (4-18). The transversal filter has  $m$  taps and the equalized channel will be

$$A(z) C(z) \approx z^{-h} A_2(z) \quad (5-14)$$

$h$  is a positive integer.

The linear equalizer which performs a process of partial equalization is followed by nonlinear feedback transversal filter in fig. (4-1) which completes the equalization process. The nonlinear transversal filter uses decision directed cancellation of intersymbol interference described in section (5-1). The combined equalizers are implemented as in fig. (5-2).

The complete equalizer performs a separate process of linear and nonlinear equalization. The design objective is to determine the particular combination of these processes that minimize the probability of error in the detection of a signal element, ref. (29),

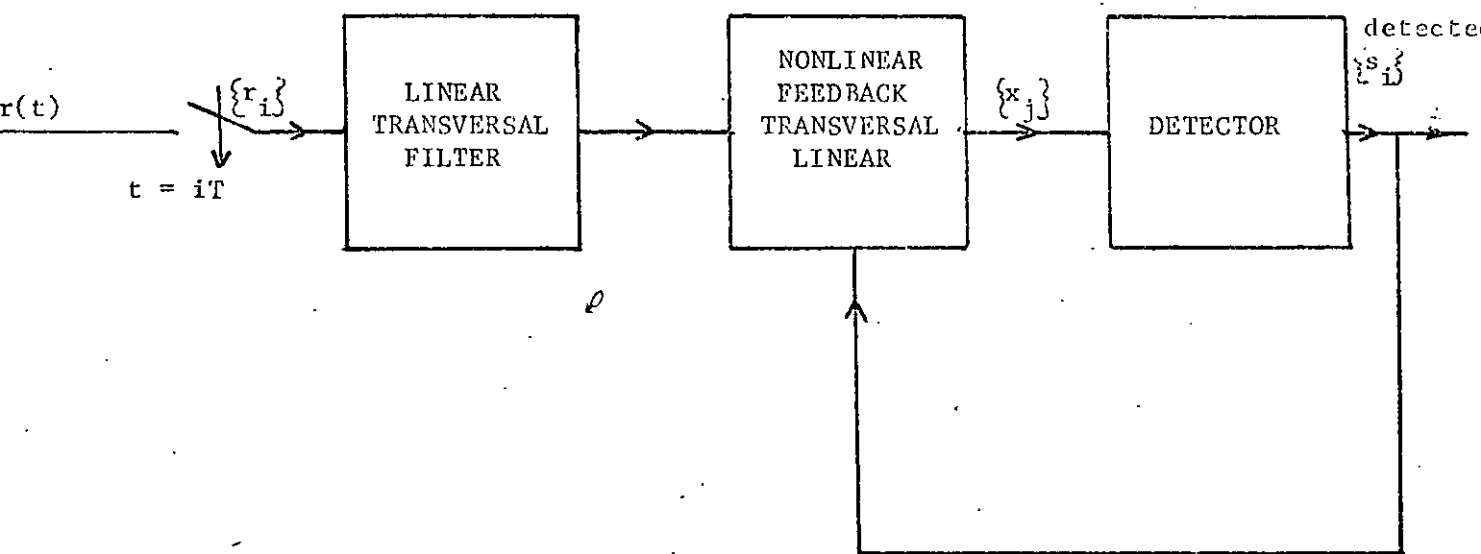


FIGURE (5-2) COMBINED LINEAR AND NONLINEAR EQUALIZER

subject to the essentially accurate equalization of the channel.

The linear transversal filter  $C(z)$  modified by a linear network having a pulse transfer function  $B(z)$  whose output has a transform  $D(z)$

$$B(z) C(z) = D(z) = d_0 + d_1 z^{-1} + \dots + d_{n-1} z^{-(n+1)} \quad (5-15)$$

so that the filter has  $n$  taps, also

$$B(z) = b_0 + b_1 z^{-1} + \dots + b_g z^{-g} \quad (5-16)$$

where

$$n = m + g \quad (5-17)$$

$m$  is the number of taps required by a linear transversal filter with a  $z$  transform  $C(z)$  for the satisfactory equalization of  $A_1(z)$  and  $n$  is the maximum number of taps acceptable for the linear transversal filter which precedes the nonlinear filter. For reasons which will become clear later  $b_0$  is constrained to satisfy

$$b_0 \beta_0 = 1 \quad (5-18)$$

the remaining coefficients of  $B(z)$  may be selected to have any real values. It is required to find the value of  $B(z)$  within the constraints of eqn. (5-17) and (5-18) that optimises the performance of the detector. The  $z$  transform of channel and linear filter is:

$$A(z) \tilde{B}(z) C(z) = z^{-h} B(z) A_2(z) \quad (5-19)$$

so that the  $z$  transform of the  $i^{\text{th}}$  received element, at the output of the linear filter is approximately  $s_i z^{-i-h} B(z) A_2(z)$  from eqns. (5-11) & (5-12) it can be seen that  $B(z) A(z)$  contain  $g + 1$  terms, so that an individual signal element at the output of the linear filter in fig. (5-2) has a sequence of nonzero sample values, from eqn. (5-18) and  $s_i$  having values of binary with unit magnitude.

In the detector and nonlinear filter, each signal element is detected from the sample values of the received signal containing the first nonzero component of that element, and its components in the following sample values are then cancelled thus eliminating the intersymbol interference of the signal element in the following elements.

The sample value  $x_{i+h}$  of the signal at the output of the nonlinear filter, at time  $t = (i + h)T$ , contains the first nonzero component of the  $i$ th received signal element at the output of the linear filter and from eqns. (5-18) and (5-19) has the value

$$x_{i+h} = s_i b_o \beta_o + u_{i+h} = s_i + u_{i+h} \quad (5-20)$$

assuming correct cancellation of the preceding signal elements.

Eqn. (5-18) normalises the level of the component of the  $i$ th signal element in  $x_{i+h}$ .  $u_{i+h}$  is a sample value of a Gaussian random variable with zero mean and variance  $\eta^2$

$$\eta^2 = \sum_{i=0}^{n-1} d_i^2 \sigma^2 = \sigma^2 DD^T \quad (5-21)$$

Where  $D$  is the  $n$ -component row vector

$$D = d_0 \ d_1 \ \dots \ d_{n-1} \quad (5-22)$$

$s_i$  is detected from sign of  $x_{i+h}$  in eqn. (5-20) and the probability of error in the detection of a received signal element is approximately



$$\begin{aligned}
 P_e &= \int_1^{\infty} \frac{1}{\sqrt{2\pi}\eta^2} \exp\left(\frac{-w^2}{2\eta^2}\right) dw \\
 P_e &= \int_{\frac{1}{\eta}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-w^2}{2}\right) dw \\
 P_e &= Q\left(\frac{1}{\eta}\right)
 \end{aligned} \tag{5-23}$$

when an element is incorrectly detected and therefore incorrectly cancelled, the probability of error in the detection of the following elements is greatly increased as described in section (5-2). To minimize the probability of error, the value of  $\eta$  must be minimized. Thus from eqn. (5-21) the tap gains  $\{d_i\}$  of the linear transversal filter must be adjusted, within the constraints imposed by eqn. (5-16) and (5-18) to minimize the value of  $DD^T$ . Let B be the  $(g+1)$  component row vector

$$B = b_0 \ b_1 \ \dots \ b_g \tag{5-24}$$

Further designate G as a  $(g+1) \times n$  matrix whose  $i$ th row is:

$$G_{i-1} = \underbrace{0 \ \dots \ 0}_{i-1} \ \underbrace{c_0 \ c_1 \ \dots \ c_{m-1}}_m \ \underbrace{0 \ \dots \ 0}_{g-i+1} \tag{5-25}$$

From eqn. (5-15)

$$D = BG = b_0 G_0 - LM = \frac{1}{\beta_0} G_0 - LM \tag{5-26}$$

Where

$$L = -(b_1 \ b_2 \ \dots \ b_g) \tag{5-27}$$

and M is the  $g \times n$  matrix whose  $i$ th row is  $G_i$ . M is completely determined by  $G_0$  for the given values of  $k$  and  $n$ .

In eqn. (5-26)  $(\frac{1}{\beta_0})G_0$  and LM are  $n$ -component row vectors, which may be represented as points in a  $n$ -dimensional Euclidean vector space.

From eqn. (5-21)

$$\begin{aligned}\gamma^2 &= \sigma^2 \left( \frac{1}{\beta_0} G_0 - LM \right) \left( \frac{1}{\beta_0} G_0 - LM \right)^T \\ &= \sigma^2 \left| \frac{1}{\beta_0} G_0 - LM \right|^2\end{aligned}\quad (5-28)$$

where  $\left| \frac{1}{\beta_0} G_0 - LM \right|$  is the distance between the vectors  $\frac{1}{\beta_0} G_0$  and LM in the vector space.

The error probability is a minimum when the distance between the vectors is a minimum. For given values of  $A(z)$ ,  $C(z)$  and  $n$  the values of  $\beta_0$ ,  $G_0$  and  $M$  are fixed, leaving  $L$  as the only variable in  $\left| \frac{1}{\beta_0} G_0 - LM \right|$ . Thus  $L$  must be chosen to minimize this quantity.

$$LM = \sum_{i=1}^g b_i G_i \quad (5-29)$$

where the  $g \{G_i\}$  are linearly independent. Thus LM is a point in the  $g$ -dimensional subspace spanned by the  $g \{G_i\}$ .  $L$  must be chosen such that LM is the point in the  $g$ -dimensional subspace at the minimum distance from  $(1/\beta_0)G_0$ . By the projection theorem the required vector LM is the orthogonal projection of  $(1/\beta_0)G_0$  onto the subspace. Thus the  $g$  vectors  $\{G_i\}$  given by the rows of  $M$  are orthogonal to the vector  $(1/\beta_0)G_0 - LM$ , so that

$$\left( \frac{1}{\beta_0} G_0 - LM \right) M^T = 0 \quad (5-30)$$

or

$$L = \frac{1}{\beta_0} G_0 M^T (MM^T)^{-1} \quad (5-31)$$

from eqns. (5-26) and (5-28)

$$D = \frac{1}{\beta_0} G_0 (I - M^T(MM^T)^{-1}M) \quad (5-32)$$

where  $I$  is an  $n \times n$  identity matrix. The  $n$  tap gains of the required linear transversal filter are given by the components of  $D$  in eqn. (5-26). After the detection of  $s_i$ , the components of the  $i$ th received signal element at the output of the linear filter are given by the coefficients of the powers of  $z$  in  $s_i B(z) A_2(z)$  and are therefore known at the receiver, these are cancelled before the detection of the following elements.

It can be seen that the design of the linear filter is unaffected by  $A_2(z)$ . Any intersymbol interference introduced by  $A_2(z)$  is eliminated by the nonlinear filter. The case where  $A(z) = A_2(z)$  the linear filter becomes an amplifier with a gain of  $1/\beta_0$  and the channel is equalized by the nonlinear filter. This linear filter can be located in the transmitter of data transmission system without affecting the results.

## CHAPTER 6

Equalization by modulo- $n$  feedback equalizers.

## Chapter 6

### (6-1) Correlative Coding Techniques

In high speed digital communications multi level signal transmission techniques have been used for some time, the common characteristics of existing multi level codes is the absence of correlation between the digits. New techniques were developed, refs. (5-19) for utilizing discrete signalling levels which would be correlated in the process of generating a multi level code, yet be independent in the detection process. Generation of a time wave consisting of correlated signal levels permits overall spectrum shaping in addition to individual pulse shaping. It is possible to redistribute the spectral energy so as to concentrate most of it at low frequencies or to introduce nulls in the power spectrum of transmitted data.

The correlative level coding (often called partial response) techniques have been developed for applications to digital data modems. The digital communication channel of fig. (4-1) with impulse response  $A(t)$  and  $z$  transform  $A(z)$ , a conventional digital communication system chooses digit spacing  $T$  large enough to avoid intersymbol interference, thus a linear system  $A(f)$  combined with the sampler is essentially a "memoryless" digital channel. If the sampling spacing and phase chosen as shown in fig. (6-1) and the  $z$  transform is given by the equation

$$A(z) = 1 + z^{-1} \quad (6-1)$$

Here it is assumed that  $A(t')$  at  $t' = iT'$  are virtually zero except for  $i = 0$  and  $1$ , if these conditions are not met additional channel shaping is necessary by either an analog filter or a transversal filter. With the changed sampling interval full amount of intersymbol interference has been introduced at  $t' = T'$ . Lender's, ref. (5), duobinary signalling is based on this principle. The binary element values  $\{s_i\}$  are fed to the transmitter at the rate of one every  $T$  second. The element value at time  $t = iT$  is  $s_i$ , where  $s_i$  has one of the two values  $\pm 1$ . The  $\{s_i\}$  are statistically independent and equally likely to have either binary value. The data transmission is best analysed in terms of the  $z$  transform of the sequences of element values  $\{s_i\}$  is

$$S(z) = \sum_i s_i z^{-i}$$

where  $z^{-i}$  represents at the time instant  $t = iT$  the  $z$  transform of  $B(z)$ ,  $C(z)$ ,  $X(z)$ ,  $R(z)$ ,  $H(z)$  and  $W(z)$  of the sequences  $\{b_i\}$ ,  $\{c_i\}$ ,  $\{x_i\}$ ,  $\{r_i\}$ ,  $\{h_i\}$ , and  $\{w_i\}$  respectively, which will appear later in this chapter.

Each signal-element  $b_i$  to be transmitted is sampled momentarily at time  $t = iT$  to give corresponding impulse  $b_i \delta(t - iT)$  of area  $b_i$ , which is fed to the transmitter filter in practice a rectangular or rounded waveform would normally be transmitted, with the appropriate change in the transmitter filter.

Binary data sequence  $\{s_i\}$  is first precoded into another binary sequence  $\{b_i\}$  according to the rule

$$b_i = b_{i-1} + s_i \quad (6-2)$$

The precoding prevents a possible propagation of errors and transfer function given by eqn. (6-2) and operates on input signal  $S(z)$  where  $S(z)$  is the polynomial representation of the sequence  $\{s_i\}$ . The addition done in eqn. (6-2) is according to the table below and for this section (6-1) all the + will be by the same rule.

modulo-2 addition

+	0	1
0	0	1
1	1	0

The precoder generates  $B(z)$  by,

$$B(z) = S(z) \left[ \frac{1}{1 + z^{-1}} \right] \text{ mod-2}$$

from eqn. (6-1)

$$B(z) = S(z) \left[ \frac{1}{A(z)} \right] \text{ mod-2} \quad (6-3)$$

The output signal  $B(z)$  from the precoder is transmitted through channel  $A(z)$  and the output from  $A(z)$  will be

$$C(z) = B(z) A(z) \quad (6-4)$$

$$C(z) = B(z) [1 + z^{-1}] \quad (6-5)$$

in the absence of noise  $s_i$  is recovered by

$$s_i = c_i \text{ (modulo-2)} \quad (6-6)$$

The accomplishment of this duobinary scheme is to transmit binary data at the Nyquist rate using realizable filters. Precoding operation of eqn. (6-3) is called duobinary technique and this is extended into polybinary signalling by choosing

$$A(z) = 1 + z^{-1} + \dots + z^{-N}, \quad (6-7)$$

or to polybipolar signalling which has no power at d.c.

$$A(z) = 1 + z^{-1} + \dots + z^{-(N+1)} - z^{-N} - \dots - z^{-(2N-1)} \quad (6-8)$$

and which generates multi level signals.

The case  $N = 1$  in eqns. (6-7) and (6-8) reduce to the duobinary or bipolar signalling techniques respectively.

Since the resulting signals are multi level with correlation among successive digits, this class of code transformation is called "correlative level coding". A communication channel with this type of signalling technique is often called a "partial response channel", since sample points are chosen at points halfway to a full response fig. (6-1).

A representation of correlative level coding or partial-response system with a precoder is given by fig. (6-2).

Among the general class of correlative level coding most frequently used  $A(z)$  is

$$A(z) = 1 - z^{-2} \quad (6-9)$$

which has spectral nulls at d.c. and at the Nyquist frequency. The spectral nulls at both ends of the transmission spectrum are desirable since they allow insertion of pilot tones which convey the modulating carrier phase and the data clock.

Because of many desirable features and simplicity of implementation of the duobinary signalling and correlative level coding schemes have been widely used in high speed data modems. However, the duobinary signal has three levels in the channel output which must



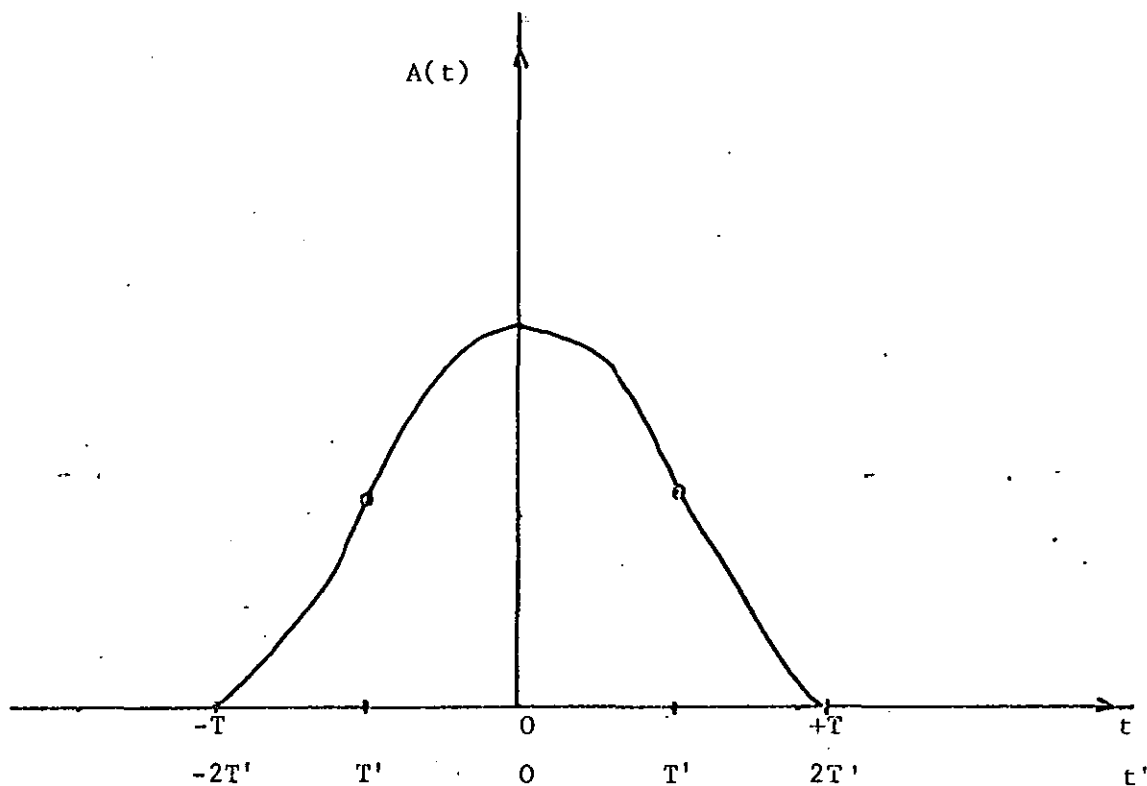


FIGURE (6-1) IMPULSE RESPONSE FUNCTION  $A(t)$

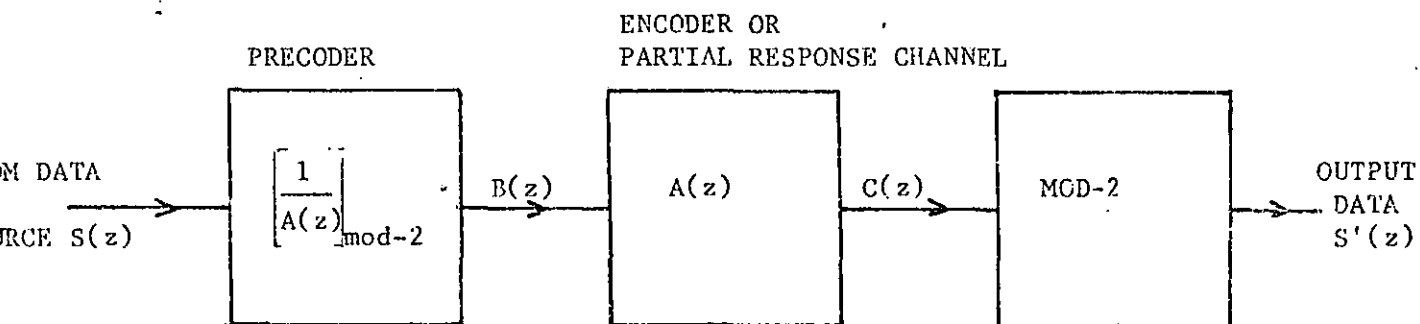


FIGURE (6-2) DISCRETE SYSTEM REPRESENTATION OF CORRELATIVE LEVEL CODING SYSTEM

be distinguished, it requires a higher signal power/noise ratio of about 3dB for equal performance than the ideal binary system without coding. The penalty is paid in exchange for the increase in data rate and the insensitivity to system perturbations.

In refs.(42-43) have shown that this apparent decrease in noise margin is not an inherent drawback of the correlative level coding, but is due to nonoptimality of the conventional bit-by-bit detection method. They clarified an analogy between a correlative levelcoder and a convolutional encoder, both systems are representable by finite state machines. This observation led them to develop a new type of decoding method which was Maximum Likelihood Decoding Algorithm (MLD). It is shown that a substantial performance gain is obtainable by this probabilistic decoding method, refs. (59-60). The discussion that has been made so far in the present section is in principle applicable to a general form of partial response channel  $A(z)$ , where

$$A(z) = a_0 + a_1 z^{-1} + \dots + a_n z^{-n} \quad (6-10)$$

this type of correlative coding techniques have the constraint that the  $\{a_i\}$  are a set of integers with their greatest common divisors equal to one, and  $a_0$  and  $n$  should be relatively prime. In refs.(61,3) and (1), independently proposed a scheme which can remove this constraint and is explained in the next section.

(6-2) Equalization with modulo-n feedback filters (system 1)

The correlative level coding technique in section (6-1) is limited to  $A(z)$  having certain integers. The equalization technique with modulo-n feedback filters is an extension of the correlative level coding scheme. The fig. (4-1) is the baseband channel to be equalized in the transmitter of a data transmission system. Binary bipolar data sequence  $\{s_i\}$  is fed to the equalization filter and the sequence  $\{b_i\}$  is the amplitude of signal pulses transmitted every  $T$  seconds. The linear baseband channel whose transfer function  $A(f)$  and the sampled impulse response  $A(z)$  are described in section (4-1).

It can be assumed that the interference is confined to  $k$  symbols, and without loss of generality  $A(z)$  is normalized so that  $a_0 = 1$  then the channel output is given by

$$r_i = \sum_{j=0}^k a_j b_{i-j} + w_i \quad (6-11)$$

$$= b_i + \sum_{j=1}^k a_j b_{i-j} + w_i \quad (6-12)$$

where  $f_i$

$$f_i = \sum_{j=1}^k a_j b_{i-j} \quad (6-13)$$

$$r_i = b_i + f_i + w_i \quad (6-14)$$

$w_i$  is the sample value of additive noise as described in section (4).  $f_i$  corresponds to the intersymbol interference, which is uniquely determined by the preceding transmitted symbols  $\{b_i\}$  (for  $j = 1, 2, \dots, k$ ). Therefore if  $b_i$  is chosen such that

$$b_i = s_i - f_i \quad (6-15)$$

The following equation is obtained from eqns. (6-12) and (6-13)

$$r_i = s_i + w_i \quad (6-16)$$

Eqn. (6-16) shows that  $\{s_i\}$  can be received without disturbance from intersymbol interference if  $b_i$  satisfies eqn. (6-15). An implementation corresponding to eqn. (6-15) is shown in fig. (6-3a), where the transfer function of the feedback equalizer is given by

$F(z) = 1 - A(z)$ , and can be constructed by a tapped delay line whose coefficients are  $(0, a_1, a_2, \dots, a_k)$ . The transfer function of the circuit shown in fig. (6-3a) is identical to the inverse of the sampled impulse response of the channel  $A^{-1}(z)$ , that is

$$\frac{1}{1 + F(z)} = \frac{1}{1 + [A(z) - 1]} = \frac{1}{A(z)} = A^{-1}(z) \quad (6-17)$$

This means that the scheme that transmits the  $b_i$  given by eqn. (6-15) is a simple linear equalization system with the inverse filter  $A^{-1}(z)$  of the channel inserted at the transmitting end. However, in this scheme another problem occurs, such that the peak value of transmitted signal elements  $\{b_i\}$  tend to increase or sometimes diverge infinitely, and the feedback transversal filter becomes unstable, whenever  $A(z)$  has any roots (zeros) outside the unit circle in the  $z$  plane. When all the roots of  $A(z)$  lie inside the unit circle, this arrangement is stable and is simply a linear equalizer for the channel. This difficulty of instability is solved by insertion of an additional nonlinear signal transformation so that the sequence  $\{b_i\}$  is peak limited, that is,

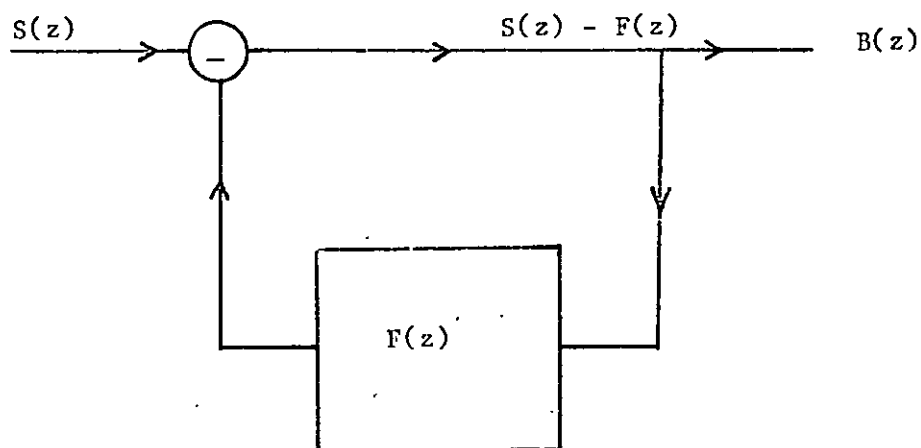


FIGURE (6-3a) BLOCK DIAGRAM OF EQUALIZER IN THE TRANSMITTER

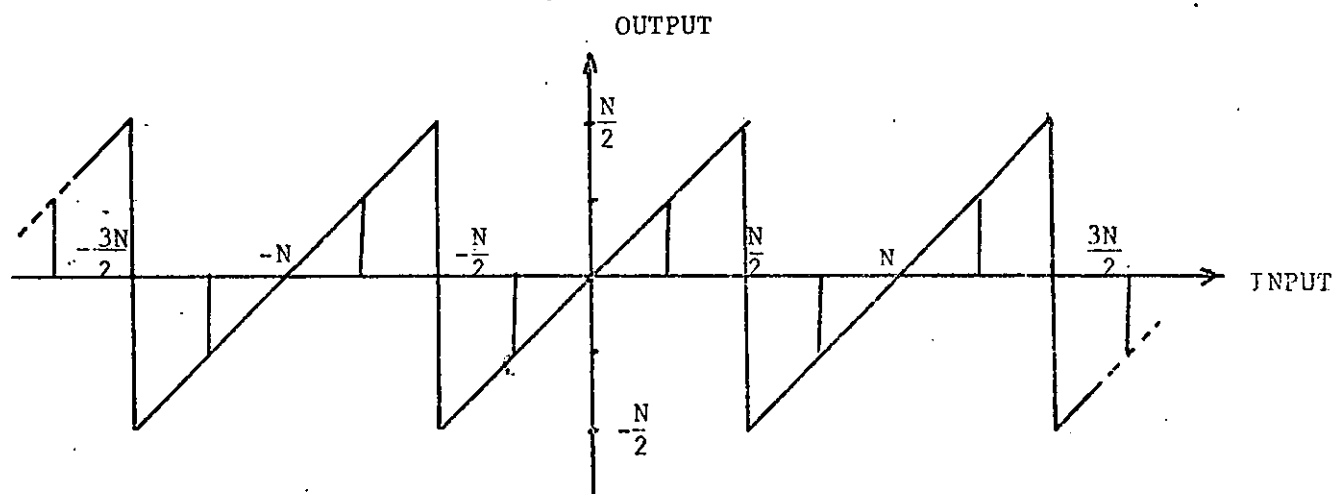


FIGURE (6-3b) INPUT-OUTPUT CHARACTERISTIC OF MODULO-N TRANSFORMATION.

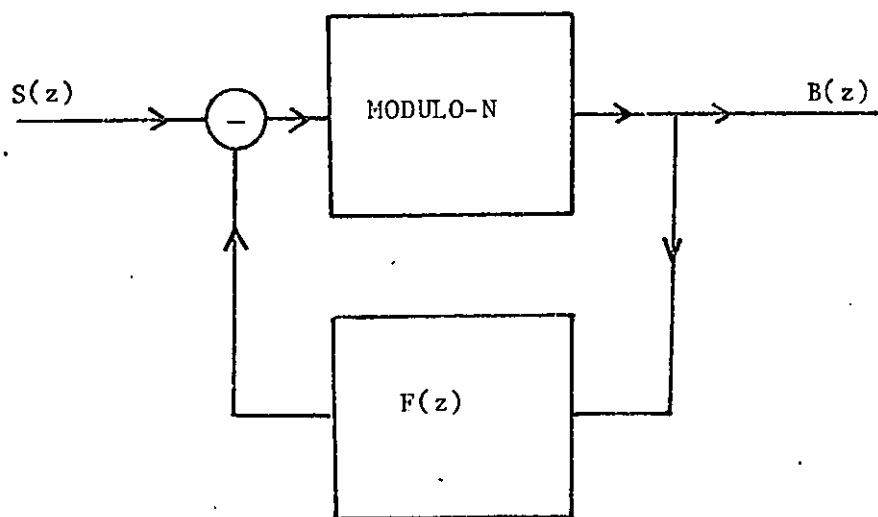


FIGURE (6-3c) EQUALIZER WITH MODULO-N TRANSFORMATION

$$b_{\min} \leq b_i \leq b_{\max} \quad (6-18)$$

for some  $b_{\min}$  and  $b_{\max}$ . The choice of this nonlinear transformation is such that no information will be lost passing through this transformation network. Such transformation have been proposed by refs. ( 1 ) and ( 3 ), and is known as modulo-n operation. Input-output characteristic of modulo-n transformation is shown in fig. (6-3b), this transformation operation subtracts a constant level  $n$  from the input, until the remainder is less than the magnitude of  $n$ . Then the remainder of this operation is the output signal. This operation does not have memory, and only operates on the sample value of the input signal element and there is only one output signal element for every input signal element.

If  $y$  has any positive or negative value an input signal  $y$  to the nonlinear network  $M$  (modulo- $n$ ) gives the output signal

$$M[y] = y(\text{modulo-}n) - n/2 \quad (6-19)$$

The subtraction of magnitude  $n/2$  is to obtain an output signal which can have positive or negative values. For  $n = 4$

$$M[y] = y(\text{modulo-}4) - 2 \quad (6-20)$$

where  $y(\text{modulo-}4) = y - 4j$  and  $j$  is the most positive integer such that

$$y - 4j \geq 0$$

Clearly,  $0 \leq y(\text{modulo-}4) < 4$

so that  $-2 \leq M[y] < 2$ ,

if  $-2 \leq y < 2$ ,

then  $M[y] = y$ .

In fig. (6-3c) shows the feedback equalizer with modulo-n feedback transformation in the feedback loop. The magnitude of  $n$  for the modulo-n transformation is chosen such that the transmission of digital data would be possible. For binary bipolar data signals  $s_i = \pm 1$ ,  $n$  should satisfy,

$$-n/2 < s_i < n/2. \quad (6-21)$$

The output from the modulo-n feedback filter  $\{b_i\}$  is obtained by

$$b_i = (a_i - f_i) - jn \quad (6-22)$$

$j$  is an integer, that is determined so as to limit the peak value of output sequence  $\{b_i\}$  between  $\pm n/2$ .

The model of system using modulo-n feedback equalizer is shown in fig. (6-4).

The binary input sequence to the modulo-n feedback equalizer is transformed into a sequence at the output whose  $z$  transform is  $B(z)$ . This signal is peak limited but distributed continuously between levels  $\pm n/2$  which are sampled, filtered then transmitted through the baseband channel  $A(z)$ .

The sampled signal element in the receiver in the absence of noise has value  $r_i$  where

$$r_i = \sum_{j=0}^k a_j b_{i-j}. \quad (6-23)$$

Modulo-n transformation in the receiver operates on the sample value of  $R(t)$  at time instant  $t = iT$  to give  $s_i$  at the output of the

modulo-n transformation circuit,  $s_i = r_i \text{ (modulo-n)}$ . The input-output characteristic of modulo-n operation in the receiver is identical to the operation done in the transmitter modulo-n feedback filter shown in fig. (6-3b). In the presence of additive white Gaussian noise with zero mean, variance  $\sigma^2$  and sample value  $w_i$

$$r_i = \sum_{j=0}^k a_j b_{i-j} + w_i \quad (6-24)$$

where  $r_i$  is the sample value at time  $t = iT$  at the input to the modulo-n operation. The detection circuit which follows modulo-n operation has a decision threshold of zero. If the magnitude of  $r_i \text{ (modulo-n)}$  is greater than zero, the output is a "1" and if the magnitude of  $r_i \text{ (modulo-n)}$  is less than zero the binary value "-1", is detected.  $s_i$  is detected after modulo-n transformation of one sample value of received signal element, every binary element is independently detected and a digital error in one element does not cause digital errors in other elements.

The equalization system can be best analysed by polynomial representation from fig. (6-4), and for any value of  $a_0$  other than  $a_0 = 0$ .

$$B(z) = M \left[ S(z) - (a_0^{-1} A(z) - 1) B(z) \right] \quad (6-25)$$

so that

$$M \left[ B(z) \right] = M \left[ S(z) + B(z) - a_0^{-1} A(z) B(z) \right] \quad (6-26)$$



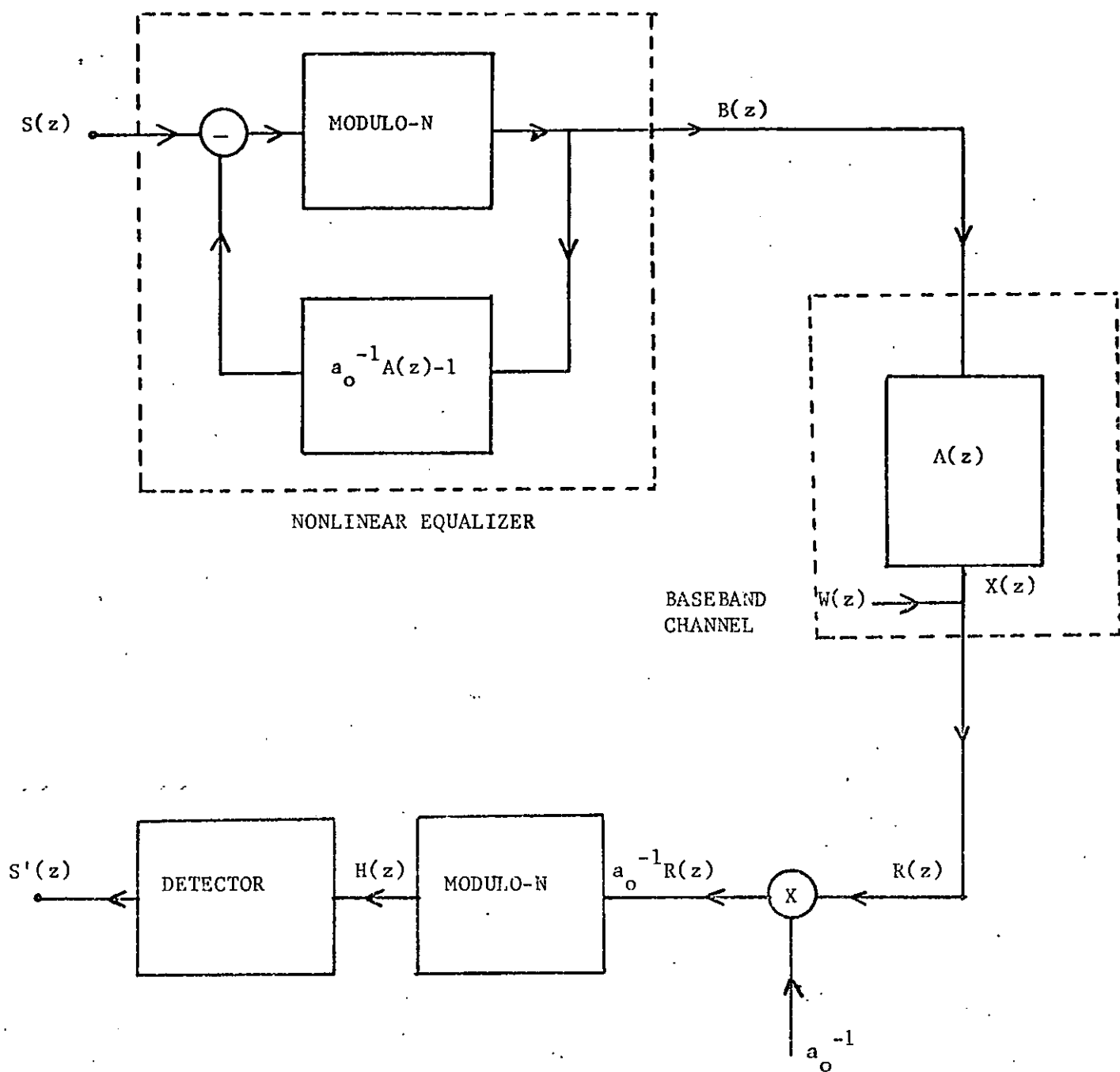


FIGURE (6-4) SYSTEM OF EQUALIZATION USING MODULO-N FEEDBACK EQUALIZER.

since of course,  $M \left[ B(z) \right] = B(z)$  from eqns. (5-19) and (6-25).

$$M \left[ a_o^{-1} A(z) B(z) \right] = M \left[ S(z) \right] \quad (6-27)$$

The  $z$  transform of the signal at the output of the nonlinear network  $M$  in the receiver of fig. (6-4) is

$$\begin{aligned} H(z) &= M \left[ a_o^{-1} R(z) \right] \\ &= M \left[ a_o^{-1} A(z) B(z) + a_o^{-1} W(z) \right] \\ &= M \left[ S(z) + a_o^{-1} W(z) \right] \end{aligned} \quad (6-28)$$

as can be seen from eqn. (6-27) that if  $H(z) = \sum h_i z^{-i}$  can be expressed in terms of the individual sample values at time  $t = iT$ , to give

$$h_i = M \left[ a_o^{-1} r_i \right] = M \left[ s_i + a_o^{-1} w_i \right] \quad (6-29)$$

in the detector  $\{s'_i\}$  will be detected as explained and the  $z$  transform of the sequence  $\{s'_i\}$  is designated  $S'(z)$ .

The channel  $A(z)$  and the modulo- $n$  operation in the receiver constitute an inverse of the modulo- $n$  feedback equalization of the transmitter. If the coefficients of  $A(z)$  are chosen to be a set of integers with greatest common divisor equal to one, the levels of the transmitted values reduce to discrete values. This special case is identical to the correlative level coding technique developed

by Lender ref. ( 7 ), for example,  $(a_0, a_1) = (1, 1)$  corresponds to the duobinary technique with modulo-n ( $n = 2$ ) for binary unipolar input signals.

(6-3) The Probability of Error

The magnitude of n for modulo-n feedback equalizer with binary bipolar data as input is chosen to be

$$n = 4 \left| s_1 \right|$$

for a given signal in the receiver the received signal is placed equidistant to all the decision boundaries to minimize the probability of error. The presence of modulo-n transformation in the receiver before the detection operation introduces one more decision boundary, degradation due to this effect has been calculated in the presence of additive white Gaussian noise zero mean and variance  $\sigma^2$ . For binary bipolar data signals as input to the modulo-n feedback equalizer at the transmitter and a channel that introduces no distortion, attenuation, or gain, (no distortion channel),  $A(z) = 1$ . For  $s_1 = \pm 1$  the average transmitted energy per signal element  $b_i^2 = 1$ . The boundaries for modulo-n transformation are at  $nk/2$ , all the integer values of k for  $n = 4$ . The error probability is calculated for modulo-n receiver, in eqn. (6-24)  $w_i$  is assumed to be a sample value of a Gaussian random variable with probability density function,

$$P(u) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-u^2}{2\sigma^2}\right). \quad (6-30)$$

If an element "1" is transmitted the probability of error in the detection of "1" after (modulo-4) transformation

$$P_{e_1} \approx \int_0^2 \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-1)^2}{2\sigma^2}\right) dx. \quad (6-31)$$

The probability of error of "-1" is the same as  $P_{e_1}$

$$P_{e_{-1}} \approx \int_{-2}^0 \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x+1)^2}{2\sigma^2}\right) dx$$

due to the symmetry of distribution given in eqn. (6-30),

$$P_{e_{-1}} \approx P_{e_1} \approx \frac{1}{\sqrt{2\pi\sigma^2}} \left[ \int_{-\infty}^{-1} \exp\left(-\frac{u^2}{2\sigma^2}\right) du + \int_1^{\infty} \exp\left(-\frac{u^2}{2\sigma^2}\right) du \right] \quad (6-32)$$

define  $Q(x)$  as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp\left(-\frac{y^2}{2}\right) dy \quad (6-33)$$

The average probability of element error

$$P_e = (P_{e_{-1}} + P_{e_1}) 2Q\left(\frac{1}{\sigma}\right)$$

where  $P_{e_{-1}} + P_{e_1} = 1$

$$P_e = 2Q\left(\frac{1}{\sigma}\right) \quad (6-34)$$

For binary bipolar data transmitted through no distortion channel in the presence of additive white Gaussian noise has an average element

error probability, for a receiver without modulo-n transformation given by ref. (54) and appendix A2

$$P_e = Q\left(\frac{1}{\sigma}\right) \quad (6-35)$$

The average element error probabilities are calculated for no distortion channel and average transmitted signal energy per element is assumed to be unity. From comparison of (6-34) and (6-35) the average error probability is increased twice due to presence of modulo-n transformation with respect to the no distortion channel without modulo-n transformation.

The case of channel which introduce intersymbol interference, i.e.  $A(z) \neq 1$ , from eqns. (6-19) and (6-29) that an error occurs in the detection of  $s_i$  whenever

$$4j - 3 < \left| a_o^{-1} w_i \right| < 4j - 1 \quad (6-36)$$

for all positive integers  $\{j\}$ .  $a_o^{-1} w_i$  is a sample value of a Gaussian random variable with zero mean and variance  $a_o^{-2} \sigma^2$ , and  $\left| a_o^{-1} w_i \right|$  is the magnitude of  $a_o^{-1} w_i$ .

For signal/noise ratios which frequently occur in practice the probability of error in the detection of  $s_i$  is approximately equal to the probability that  $\left| a_o^{-1} w_i \right| > 1$  and is therefore approximately

$$2 \int_1^{\infty} \frac{1}{\sqrt{2\pi a_o^{-2} \sigma^2}} \exp\left(-\frac{u^2}{2a_o^{-2} \sigma^2}\right) du \quad (6-37)$$

$$P_e \approx 2Q\left(\frac{|a_o|}{\sigma}\right)$$

at high signal/noise ratio, with error probabilities around 1 in  $10^7$  or 1 in  $10^6$ , the error probability in eqn. (6-37) can be taken to be

$$P_e = Q \left( \frac{|a_o|}{\sigma} \right) \quad (6-38)$$

with an inaccuracy of only a small fraction of 1dB in the corresponding signal/noise ratio.

The average transmitted energy per signal element  $\overline{b_i^2} = k^2$  clearly lies in the range 0 to 4, when the baseband channel introduces no distortion, so that  $A(z) = a_o$ ,  $k^2 = 1$ . To a first approximation this can also be taken to be the average transmitted energy per element for any channel and will be assumed here.

For distorting channels the average transmitted energy per signal element from the output of the modulo-n feedback equalizer is  $k^2$ . However, it is not possible to calculate  $k^2$  exactly from a formula due to nonlinear characteristic of modulo-n transformation in the feedback equalizer but  $k^2$  can be found by computer simulation of the equalizer or by practical experiments. When the baseband channel introduces severe signal distortion, an estimate of  $k^2$  is obtained by assuming that  $b_i$  is equally likely to have any value in the range -2 to 2. Now  $k^2 = 1.333$ , which is 1.25dB above the assumed value of unity.

It is convenient to compare systems in terms of decibel reduction, which is the increase in signal to noise ratio in decibels required to match the error rate of the optimum binary communication system signalling over the white Gaussian infinite bandwidth channel.

In fig. (6-4) the model is used for calculation of error probability for modulo-n feedback equalizer. In the receiver the factor  $a_o^{-1}$  is included to normalise the first component of  $A(z)$  to 1 and the channel vector is normalised to  $AA^T = 1$  not to introduce gain or attenuation of the signal transmitted.  $\sigma_A^2$  is

the variance of the additive white Gaussian noise for the required error probability. The normalised signal to noise ratio (SNR) in dB for the system of fig. (6-4)

$$(\text{SNR})_A = 20\log_{10} \left[ \frac{1}{|ka_o| \sigma_A} \right] \quad (6-39)$$

and for the same digit error rate, binary bipolar transmission over a no distortion channel SNR is given by

$$\text{SNR} = 20\log_{10} \left[ \frac{1}{\sigma} \right] \quad (6-40)$$

Reduction of dBs in tolerance to additive Gaussian noise at high signal/noise ratio, when the given channel replaces one that introduces no distortion or attenuation, as explained in this section,

$$\begin{aligned} \text{dB reduction} &= 20\log_{10} \left[ \frac{1}{\sigma} \right] - 20\log_{10} \left[ \frac{1}{|ka_o| \sigma_A} \right] \\ &= 20\log_{10} \frac{\sigma_A}{\sigma} + 20\log_{10} |k| + 20\log_{10} |a_o| \end{aligned} \quad (6-41)$$

For binary bipolar data transmission,  $\sigma^2$  is the noise variance for no distortion channel, and which would be found from required average probability of element error.  $\sigma_A^2$  is the noise variance that will result in same probability of error for a modulo-n receiver and this value can be found from eqn. (6-34). For high signal/noise ratio the presence of the modulo-n transformation in the

receiver increase the error probability in the detection of an element by two times. This corresponds to a reduction of less than 0.3dB in tolerance to additive Gaussian noise.

The magnitude of  $a_0$  is the main factor that effects eqn. (6-41). The smaller  $a_0$  the higher the reduction in tolerance to additive white Gaussian noise for a system that is using modulo-n feedback filter as equalizer at the transmitter. When all the roots of  $A(z)$  lie inside the unit circle in the  $z$ -plane,  $a_0$  is one of the larger components of  $A(z)$  and the modulo-n feedback equalizer always gives a better tolerance to additive white Gaussian noise than a linear equalizer does. This can be seen from the fact that the linear equalizer now has a gain of  $a_0^{-1}$  for its first tap and one or more other taps with non-zero gains. Thus the noise variance at the equalizer output must exceed  $a_0^2 \sigma^2$  from eqns. (4-30) and (4-33). The error probability here, in the detection of a received signal element, exceeds eqn. (6-38). However,  $A(z)$  frequently has roots outside the unit circle, such that  $a_0$  is one of its smaller components and modulo-n feedback equalizer is now likely to have a lower tolerance to additive white Gaussian noise than a linear equalizer does.

From eqns. (6-38) and (5-9), the error probabilities for modulo-n feedback equalizer and decision directed cancellation of intersymbol interference in the receiver are equal, so that both systems theoretically have the same tolerance to additive white Gaussian noise under the conditions assumed. Furthermore, at high signal/noise ratios, the reduction in tolerance to additive white Gaussian noise caused by error extension effects in decision directed equalization in the receiver is of the same order as the reduction in tolerance



to noise of modulo-n feedback equalization, resulting firstly from the fact that the actual error probability is  $2P_e$ , and secondly from the fact that the transmitted signal level generally exceeds the assumed value of unity. Thus, in practice, the two systems should have very similar tolerances to additive white Gaussian noise at high signal/noise ratios, and they can be considered as duals of each other, since they use the same equalization process which is located in one case at the transmitter and in the other at the receiver. Modulo-n feedback equalizer has the advantage that it does not have error extension effects of decision directed cancellation because the equalization operation done in the transmitter and detection is done bit by bit without reference to any other received signal element. Both equalization techniques have the advantage over linear equalizer that is they are always stable whereas the linear equalizer is not.

(6-4) Combined Modulo-n feedback equalizer with linear feedforward equalization (system 3)

Similarity of modulo-n feedback equalizer with detection and cancellation of intersymbol interference equalization techniques suggested that further improvements to tolerance to additive white Gaussian noise may be achieved by combination of linear transversal equalizer as shown in fig. (6-5), and named system 3.

The system of combined equalization shown in fig. (6-5) has the input binary sequence  $\{s_i\}$ , as explained in previous sections. The modulo-n feedback filter output is fed to a linear filter which has a z transform  $D(z)$ . The output from the linear filter  $\{b_i\}$  is a multi-valued sequence to be transmitted through the baseband channel

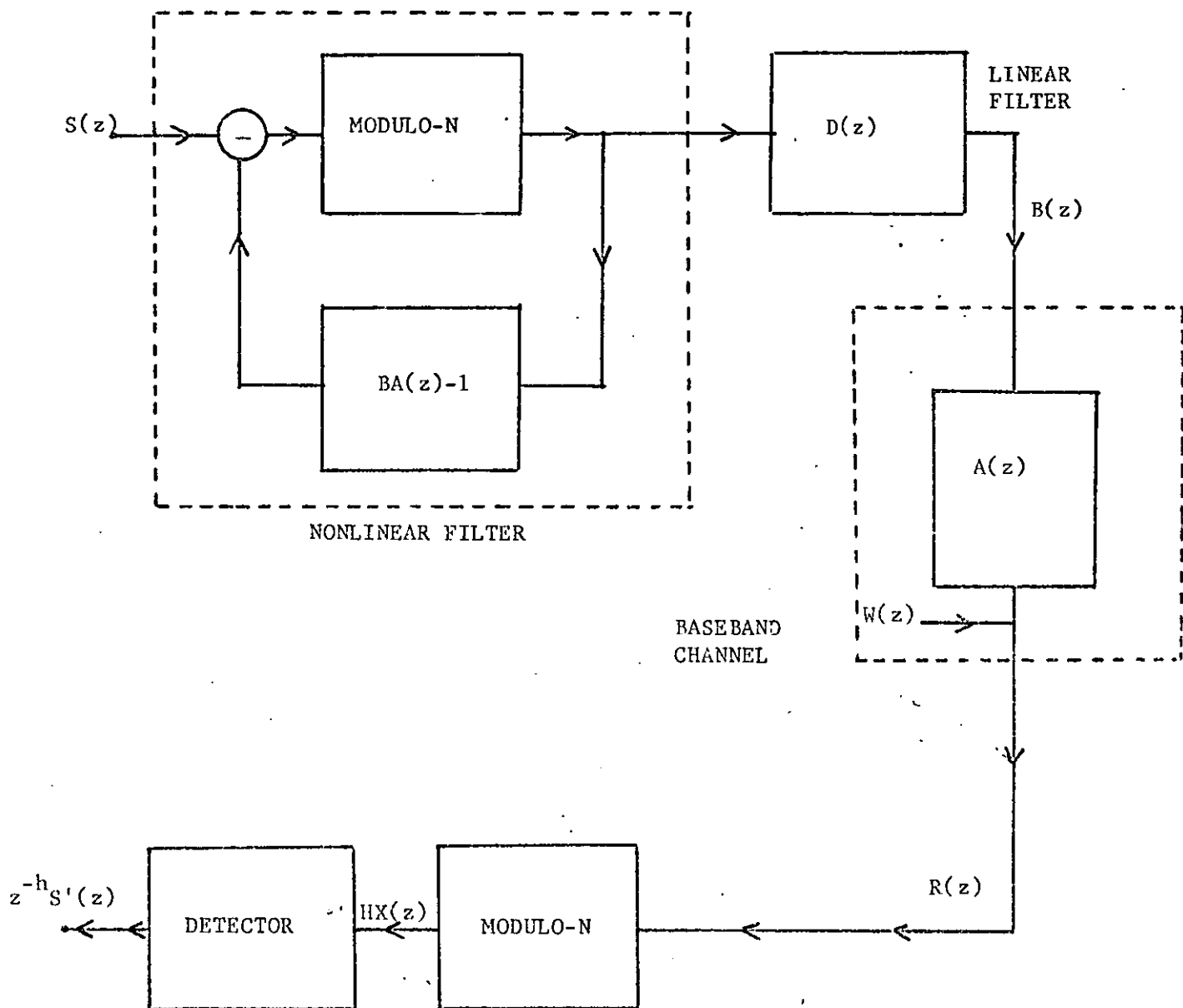


FIGURE (6-5) COMBINED SYSTEM OF MODULO-N FEEDBACK EQUALIZER AND LINEAR FILTER.

$A(z)$ , after sampling and wave shaping operation as previously described.

The optimum design of the linear filter is given in section (5-3) and the design equation by eqn. (5-32). The  $z$  transform of the linear filter and baseband channel is

$$A(z) D(z) \approx z^{-h} BA(z) \quad (6-42)$$

where  $h$  is a positive integer due to the delay introduced by the filter.  $BA(z)$  is the  $z$  transform of the linear filter and baseband channel, and is equalized by the nonlinear filter at the transmitter in a manner similar to that used for the equalization of the baseband channel by the modulo- $n$  feedback equalization technique (system 1). Thus the  $z$  transform of the transversal filter in the nonlinear equalizer is  $BA(z)-1$ . The constraint for the design of  $D(z)$  makes the first element of  $BA(z)$  to be unity, following the analysis of the modulo- $n$  feedback equalizer, it can be seen that the  $z$  transform of the signal at the input to the detector in fig. (6-5) is

$$HX(z) = M \left[ R(z) \right] = M \left[ z^{-h} S(z) + W(z) \right] \quad (6-43)$$

because of the delay of  $hT$  seconds introduced by the additional linear filter at the transmitter,  $s_i$  is detected at time  $t = (i + h)T$  from the sample value,

$$hx_{i+h} = M \left[ s_i + w_{i+h} \right] \quad (6-44)$$

which contains the first non-zero component of the  $i^{\text{th}}$  data element and the noise sample  $w_{i+h}$ . After the modulo-n operation on the received sample  $r_i$  (modulo-n) the resulting signal is fed to the detection circuit which for binary signal elements is a threshold detector, i.e. the decision threshold is set to zero level. The output from the detector is the binary bipolar data signal whose z transform is  $z^{-h}S(z)$ .

The advantage of combined equalizers to other types of equalization techniques is explained in chapter (5). The reduction in dBs in tolerance to additive Gaussian noise, for combined modulo-n feedback equalizer and linear equalizer at high signal/noise ratio when the given channel replaces the one which introduces no distortion or attenuation will be different to the ones given in eqn. (6-41) for modulo-n feedback equalizer. The average energy per signal element from the output of the modulo-n feedback filter will be modified by the linear filter at the transmitter by a factor equal to  $\eta^2 = DD^T$  (see eqn. (5-21)). The multiplier  $a_0^{-1}$  is not present in this system because the design for the combination of equalizers have a constraint to make the first component for the nonlinear equalizer unity. The channel response vector is normalised to have a unit length  $AA^T = 1$ , in order not to introduce gain or attenuation. In the receiver the modulo-n operation followed by detection is as before. The delay involved in detection does not effect the calculation for the tolerance to additive noise. Therefore with respect to a system where the transmission is through a no distortion channel and binary bipolar data as input in the presence of additive white Gaussian noise with zero mean and variance  $\sigma^2$  the reduction in tolerance to noise when

both systems have the same probability of error is

$$\begin{aligned} \text{dB reduction} &= 20\log_{10}\left(\frac{\sigma_A}{\sigma}\right) + 10\log_{10}(k^2 \eta^2) \\ &= 20\log_{10} \frac{\sigma_A}{\sigma} + 20\log_{10}|k| + 20\log_{10}|\eta|. \end{aligned} \quad (6-45)$$

When eqn. (6-45) compared to eqn. (6-41) the relation between  $\sigma_A$  and  $\sigma$  for the no distortion channel is not changed for the modulo-n receiver.  $\overline{b_i^2} = k^2$  the average transmitted energy per signal element at the output of the modulo-n feedback filter is estimated to be 1.333. This causes a 1.2<sup>5</sup> dB reduction in tolerance to additive white Gaussian noise. To a first approximation  $k^2$  can be taken as unity and doubling of error probability due to modulo-n transformation in the receiver can be neglected for high signal/noise ratios which is given by  $\sigma_A/\sigma$  and eqn. (6-45) reduces to

$$\text{The dB reduction} \approx 20\log_{10} \eta.$$

For the above approximation the probability of error will be the same as eqn. (5-23) and for this system

$$P_e = Q\left(\frac{1}{\eta}\right) \quad (6-46)$$

therefore both systems have the same tolerance to additive white Gaussian noise under the conditions assumed. They can be considered as duals of each other. For an alternative analysis of this section refer to appendix A4.

(6-5) Example of Linear equalization and the combination of Linear and modulo-n feedback equalization.

An example of a linear equalizer, then a linear equalizer for arrangement of the combined linear and modulo-4 equalizer, will be given in this section.

Consider the case where  $z$  transform of the sampled impulse response of the channel is

$$A(z) = 0.196 + 0.392 z^{-1} + 0.785 z^{-2} + 0.392 z^{-3} + 0.196 z^{-4} \quad (6-47)$$

The reason for the choice of this channel is that it is used for comparison of systems later in this chapter, and it introduces severe signal distortion. Also, it is required to minimize the probability of error in the detection of a signal element, subject to the essentially accurate equalization of the channel by a linear equalizer. The linear transversal filter is designed by minimum mean square distortion criteria, described in section (4-3), of the sampled impulse response of the equalized channel. The designed linear filter where the tap gains given by the  $m$ -component row vector  $C$  for  $m = 25$  are;

$$C = \begin{array}{cccc} 0.003 & -0.006 & -0.001 & 0.018 \\ -0.026 & -0.015 & 0.097 & -0.102 \\ -0.130 & 0.488 & -0.350 & -0.893 \\ 2.341 & -0.893 & -0.350 & 0.488 \\ -0.130 & -0.102 & 0.097 & -0.015 \\ -0.026 & 0.018 & -0.001 & -0.006 \\ 0.003 & & & \end{array} \quad (6-48)$$

When the linear equalizer is located in the transmitter (or the receiver)

the equalized signal elements from the channel (or receiver equalizer) output are applied to a detector with proper delay for the detection of binary signal values. The equalization is done completely by a linear equalizer.

For the combination of modulo-n feedback equalizer followed by a linear transversal feedforward equalizer  $D(z)$  the values of  $D(z)$  are determined by evaluation of eqn. (5-32) which leads to a row vector  $D$ , whose significant components are given by eqn. (6-49)

$$D = \begin{matrix} 0.001 & -0.001 & 0.000 & 0.003 \\ -0.003 & -0.004 & 0.013 & -0.009 \\ -0.025 & 0.064 & -0.023 & -0.150 \\ 0.290 & 0.003 & -0.845 & 1.248 \\ 0.592 & 0.608. & & \end{matrix} \quad (6-49)$$

$D(z)$  is the linear transversal filter for the system 3 of fig. (6-5) for the channel described by eqn. (6-47). The tap gains of modulo-n feedback filter is given by the product  $D(z) A(z)$ , the corresponding sampled impulse response is:

$$D(z) A(z) \approx z^{-18} (1.0 + 1.027z^{-1} + 0.954z^{-2} + 0.354z^{-3} + 0.119z^{-4}) \quad (6-50)$$

The modulo-4 feedback transversal filter has the tap gains obtained from eqn. (6-50) and this filter completes the equalization process in a manner similar to that for the equalization of the baseband channel in system 1. The significant components of residual intersymbol interference after linear equalization start after a delay of  $18T$  seconds for the above case. Therefore  $s_i$  is detected, after modulo-n operation in the receiver, from the sample value of received signal element at time  $t = (i + 18)T$ .

For the combined linear and nonlinear equalizer has less number

of taps than a linear equalizer. The design of the combined equalizer does not appear to be significantly affected by the choice of the length of linear filter  $C(z)$ .

(6-6) System Modifications

(6-6-1) Detection from second element for modulo-n feedback equalizer (system 6)

For the nonlinear equalizer described in chapter (5), decision directed cancellation of intersymbol interference the probability of error is a function " $a_0$ " of the sampled impulse response of the channel  $A(z)$ , where " $a_0$ " is the first sample of  $A(z)$ . Higher tolerance to additive white Gaussian noise can be achieved by choosing a component of  $A(z)$  which is larger than " $a_0$ " in the detection and cancellation of its intersymbol interference in the following elements. For this case due to the presence of error extension effects, if an element is incorrectly detected there is a considerable increase in the probability of error in the following elements.

The same technique is applied to a modulo-n feedback equalizer located in the transmitter by completely neglecting the first sample value " $a_0$ " of  $A(z)$  and adjusting the tap coefficients of the equalizer to equalize the sampled impulse response of the channel without " $a_0$ ". Now the received signal element  $a_1^{-1} r_1$  is detected after the modulo-n operation in the receiver at time  $t = (i + 1)T$  instead of  $t = iT$ .

This modification to system 1 is named as system 6 and computer simulation tests are carried out for 21 channels and results are presented in section (6-7). It was found that this technique is not a very successful equalization technique. This is because the majority of practical channel responses which are investigated had



value of " $a_0$ " which were comparable to " $a_1$ " of  $A(z)$ . Consequently detection based on " $a_1$ " rather than " $a_0$ " resulted in severe intersymbol interference as the signal transmitted is multi valued, and a theoretical calculation of worst case performance of this system will not be possible.

(6-6-2) The optimum division of the nonlinear equalizer between the transmitter and receiver (system 3a)

For the combined equalization technique of fig. (6-5) system 3, the linear filter  $D(z)$  can be shared between the transmitter and receiver for a better tolerance to additive noise. The splitting of  $D(z)$  can be done in many ways, one logical way is equal sharing between transmitter and receiver. In ref. (37) it was shown that no advantage could be gained by sharing the linear filter  $D(z)$  between the transmitter and receiver. A further attempt has been made to share the nonlinear equalizer between the transmitter and receiver. The arrangement proposed as in fig. (6-6) and it will be called system 3a. The combination of the channel  $A(z)$  and the linear filter  $D(z)$  is given by eqn. (6-42) where the unequalized part of the channel is  $BA(z)$  i.e. equalization of  $BA(z)$  completes the equalization process. The object is to split  $BA(z)$  between the transmitter and receiver by finding a polynomial  $BA(z)^{\frac{1}{2}}$  for the modulo-n feedback filter and feeding the output to the linear filter  $D(z)$  located at the transmitter. The signal elements are then transmitted through the distorting channel  $A(z)$ . In the receiver the remaining intersymbol interference due to  $BA(z)^{\frac{1}{2}}$  is cancelled by a nonlinear filter. The signal sample values from the nonlinear filter is operated on by modulo-n transformation and detection is done on

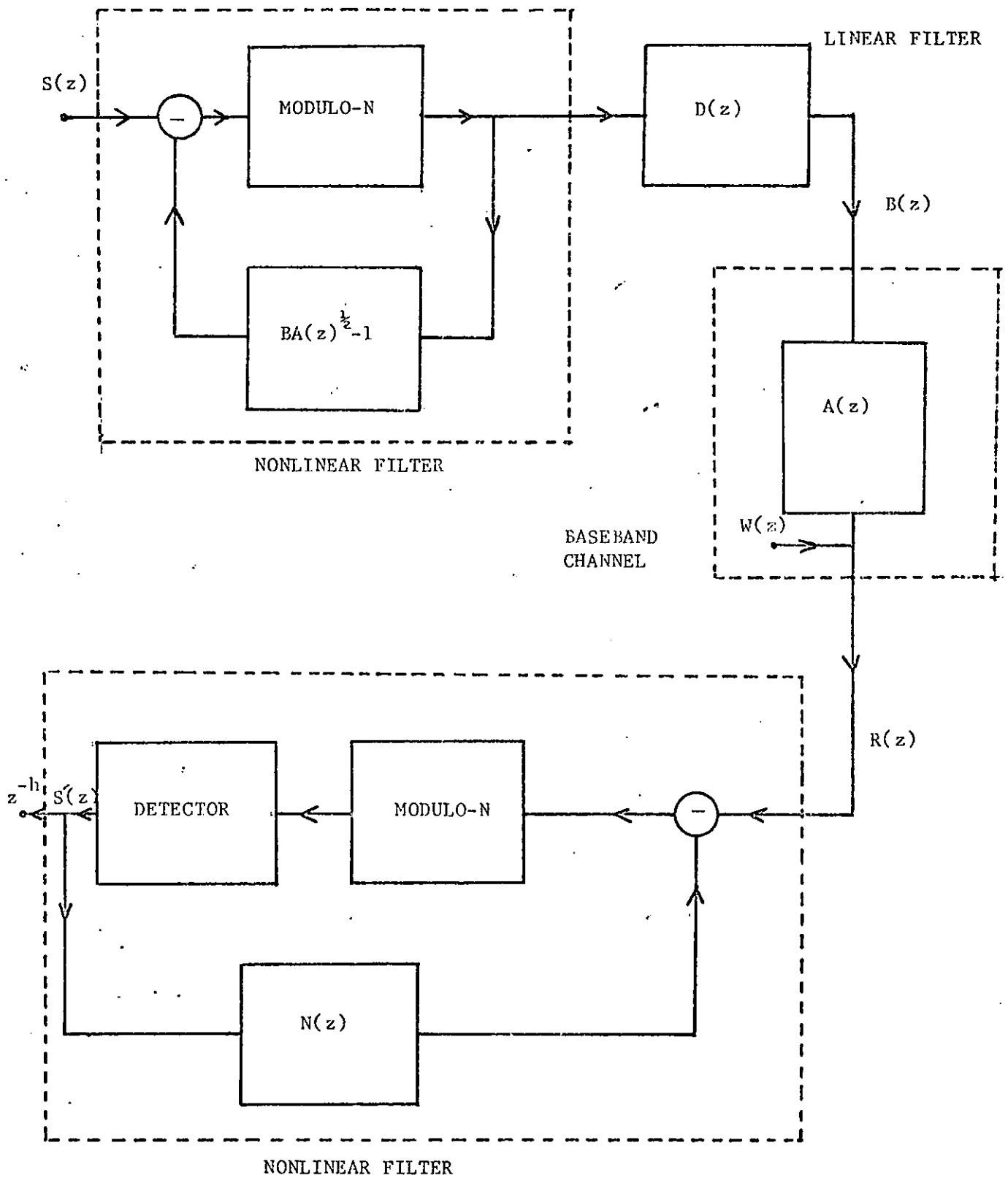


FIGURE (6-6) SHARED NONLINEAR EQUALIZER BETWEEN TRANSMITTER AND RECEIVER COMBINED WITH LINEAR FILTER.

the signal value from the output of modulo-n circuit. The output of the detector  $S'(z)$  is modified by the network  $N(z)$  in the nonlinear filter and cancels the intersymbol interference to complete the equalization process.

The equal sharing of  $BA(z)$  only changes the average transmitted element energy. The noise variance is not effected by the decision directed cancellation of intersymbol interference done for the other half of  $BA(z)^{\frac{1}{2}}$  in the receiver. To find the change in the average transmitted signal element energy for the shared nonlinear equalization process computer simulations for five different channel polynomials  $A(z)$  have been made. These channels are given in table (6-1), they are chosen such that for the combined linear and nonlinear equalization technique, the nonlinear part can be split into two equal polynomials. The polynomial for the modulo-n feedback transversal filter  $[BA(z)^{\frac{1}{2}} - 1]$  is determined by finding the roots of the polynomial  $BA(z)$  and then separating the roots into two equal groups. The polynomial  $BA(z)^{\frac{1}{2}}$  is formed from the one of the group of roots. In table (6-1) the first column lists the channel polynomials  $A(z)$  to be equalized, column two lists  $BA(z)$  the unequalized part from convolution of linear equalizer and the channel polynomial. Column three is the polynomial  $BA(z)^{\frac{1}{2}}$  formed by the above described method. The fourth column is the reduction in deciBells of average transmitted element energy of system 3a when compared to the case of combined linear and modulo-n feedback equalizer. For this comparison the same binary sequence of length 30000 is used for all simulations and the signal from the output of the linear equalizer is used to calculate the average transmitted signal element energy for both techniques.

In the proposed receiver after cancellation of intersymbol interference modulo-n operation is done on the signal element value. Then  $s'_i$  is detected from the modulo-n operated signal element at time  $t = (i + h)T$  where  $hT$  is the delay introduced by the linear filter at the transmitter. From table (6-1) the reduction of transmitted average signal energy was found to be negligible for transmitter equalizer of system 3a when compared to a transmitter modulo-n feedback equalizer followed by a linear feedforward filter (system 3). Further investigation of the receiver design for system 3a has not been extended due to the fact that there will be additional increase in probability of error by the decision directed cancellation of intersymbol interference operation in the receiver to complete the equalization process. Any gain made from the reduction of transmitted average signal energy will be lost because of error extension effects.

(6-7) Comparison of systems.

In table (6-2) the equalization techniques are given system numbers. The tolerances to additive white Gaussian noise of systems 1, 3, 5 and 6, when operating over 21 different channels, have been tested by computer simulation. Similar tests have also been carried out on a data transmission system derived from system 3 by removing the two modulo-n transformation networks and leaving a feedback transversal filter. The feedback filter without modulo-n operation is a stable network because the design of this filter is such that the magnitude of all the roots of the feedback filter in the  $z$  plane, (for the channels tested), were found to be less than unity ref. (Appendix A4).

Table (6-1) Results of Computer Simulation Tests for a comparison of sharing  $BA(z)$  between Receiver and Transmitter for combination of Nonlinear and Linear Equalizers.

Sampled Impulse Response of Channel $A(z)$					The part that has to be equalized by a nonlinear equalizer $BA(z)$					The split part of $BA(z)$ $BA(z)^{\frac{1}{2}}$			Reduction in average transmitted Energy per signal element in dB.
0.161	0.974	0.161	0	0	1.0	0.340	0.029	0	0	1.0	0.170	0	0.4
0.252	0.929	0.262	0	0	1.0	0.618	0.095	0	0	1.0	0.310	0	0.9
0.348	0.870	0.348	0	0	1.0	1.000	0.250	0	0	1.0	0.500	0	0.1
0.196	0.392	0.785	0.392	0.196	1.0	1.027	0.954	0.354	0.114	1.0	0.513	0.345	0.1
0.167	0.471	0.707	0.471	0.167	1.0	1.929	1.970	1.003	0.270	1.0	0.965	0.520	0.0

Table (6-2)

Meanings of System Numbers

- 1    Modulo-n feedback equalizer located at the transmitter section (6-2).
- 2    Nonlinear equalizer with the detection of a received signal element from the first nonzero component of that element section (5-1).
- 3    Optimum combination of linear and modulo-n feedback equalizers section (6-4) located at the transmitter.
- 4    Optimum combination of linear and nonlinear equalizers section (5-3) located in the receiver.
- 5    Theoretical results from linear filter designed by section (4-2). Computer simulation tests by removal of modulo-n transformation from system 3.
- 6    Modulo-n feedback equalizer which neglects " $a_0$ " and detects at time  $t = (i + 1)iT$  section (6-6-1).

Table (6-3) No. of dB reduction in tolerance to additive white Gaussian noise, at high signal/noise ratios, when the given channel replaces one that introduces no distortion or attenuation.

Channel No.	Sampled Impulse-Response of Channel					Results of theoretical analyses			Results of computer simulation tests			
						Systems 1 and 2	Systems 3 and 4	System 5	System 1	System 3	System 5	System 6
1	.161	.974	.161	0	0	15.9	0.5	1.0	17.6	1.5	1.1	1.4
2	.262	.929	.262	0	0	11.6	1.4	3.1	13.3	3.5	2.2	4.7
3	.348	.870	.348	0	0	9.2	3.1	7.9	10.6	4.8	8.7	8.6
4	.378	.845	.378	0	0	8.5	4.3	12.0	10.3	6.0	11.8	*
5	.219	.750	.625	0	0	13.2	4.1	11.4	15.9	5.8	11.6	9.4
6	.625	.750	.219	0	0	4.1	4.1	11.4	5.9	5.8	11.6	*
7	.575	.776	.259	0	0	4.8	4.8	14.5	6.6	6.6	14.7	*
8	.262	.928	-.262	0	0	11.6	0.0	0.0	13.5	0.6	0.2	4.7
9	.348	.870	-.348	0	0	9.2	0.1	0.1	10.8	0.7	0.2	9.2
10	.575	.776	-.259	0	0	4.8	0.6	1.5	6.4	2.0	1.6	*
11	.625	.750	-.219	0	0	4.1	0.9	2.5	5.7	2.6	2.6	*
12	.196	.392	.785	.392	.196	14.2	4.9	9.0	15.9	6.4	8.1	*
13	.167	.471	.707	.471	.167	15.5	9.9	20.4	17.4	11.7	20.4	14.4
14	.203	.339	.749	.406	.343	13.9	6.2	13.4	15.7	8.0	13.4	*
15	.137	.457	.684	.479	.274	17.3	8.3	15.2	19.1	10.1	15.2	14.5
16	.265	-.486	.728	-.368	.169	11.5	7.1	14.0	13.3	8.3	14.0	*
17	.152	-.429	.643	-.597	.152	16.4	7.7	15.1	18.2	9.5	15.2	15.8
18	.182	-.571	.642	-.428	.214	14.9	8.9	18.6	16.6	10.8	18.5	22.2
19	.273	.447	.682	-.455	-.161	11.3	0.4	0.8	13.1	1.3	0.9	*
20	.152	.597	.643	-.429	-.152	16.4	0.9	2.1	18.2	2.6	2.1	*
21	.307	.510	.702	-.314	-.234	10.3	1.0	2.1	11.9	2.6	2.1	*

The performance of this system is the same as that of a conventional data transmission system using a linear equalizer at the receiver, and the arrangement will be known as system 5. The theoretical results for system 5 is from the design of the linear filter as described in section (4-2-1). The computer programmes were written in Fortran 4 and run on ICL 1904A Computer, the binary data values are chosen from a uniform distribution and the noise samples are chosen from a normal distribution with zero mean and variance  $\sigma^2$ .

The 21 different channels used in the test were selected to represent a wide range of signal distortions, including many different combinations of amplitude and phase distortion. The sampled impulse-response of each channel was arranged so that  $AA^T = 1$ , this being the condition for no signal gain or attenuation in transmission. In each test, the average transmitted energy per signal element,  $k^2$ , was measured and the noise power spectral density,  $\sigma^2$ , was adjusted to give effectively 30 errors in the detection of 30,000 signal elements, i.e. an error rate of 1 in  $10^3$ . The tolerance to noise of each system, when operating over any of the 21 channel is measured by the number of dB reduction in the signal/noise ratio  $k^2/\sigma^2$  needed to maintain the error rate of 1 in  $10^3$ , when the given channel is replaced by one introducing no distortion or attenuation. The 95% confidence limits for each of the measured values of the order of  $\pm 0.2$  dB, ref. (appendix A3).

The results of the tests are given in table (6-3). Also shown in this table are the theoretically calculated performances of systems 1 to 5, derived from eqns. (6-38), (5-9), (6-46), (5-23) and (4-33) respectively.



It can be seen from table (6-3) that system 3 has in general a considerably better performance than system 1 and 5 particularly where there is severe signal distortion. System 1 has on balance a poorer performance than system 5. It is also clear that the tolerances to additive white Gaussian noise of system 1 and 3 are typically 1 dB below the theoretical calculated values. From theoretical considerations it appears that about 1.2 dB of this reduction is due to the fact that the transmitted signal level is greater than the assumed value, and the remaining 0.5 dB is due to the fact that the actual error probability is twice that assumed. It must be borne in mind here that the computer simulation tests were of necessity carried out at relatively high error probabilities, at which a doubling of the error probability has quite a noticeable effect on the signal/noise ratio. System 6 which is a modification of system 1 where " $a_0$ " of channel response in the modulo-n feedback filter has been neglected and detection in the receiver is performed from  $t = (i + 1)T$  sampling instant. Channels with a \* sign completely failed or resulted more than 8 dB below the poorest system. For a few channels when  $|a_0|$  is several times smaller than  $|a_1|$  data transmission is acceptable.

Computer simulation tests were also carried out on system 2, with channels 13, 15 and 17, these channels giving the most severe reduction in tolerance to noise of system 2. In each test a total of about 30 independent error bursts were counted. The 95% confidence limits of about  $\pm 0.2$  dB in the measured signal/noise ratios for a given error rate of  $4 \text{ in } 10^3$ . System 1 was also tested over these channels at the same error rate and to the same confidence limits.

In each case the measured tolerance to noise of system 2 was about 2.2 dB below the theoretical value given in table (6-3), whereas the measured tolerance to noise of system 1 was about 2.0 dB below the theoretical value. Thus the measured tolerance to noise of system 2 was about 0.2 dB below that of system 1. The relatively large discrepancy between the theoretical and measured performances of system 2 is due to the fact that at error probabilities around 4 in  $10^3$  causing error extension effects which produce an appreciable greater increase in the signal/noise ratio for the given error probability than the increase of about 1 dB to be expected at error probabilities around 1 in  $10^5$  or 1 in  $10^6$ . At these lower error probabilities, system 2 would have slightly better tolerance to Gaussian noise than system 1.

## CHAPTER 7

Detection and correction of errors with  
the aid of modulo- $n$  feedback equalizer;

## CHAPTER 7

### (7-1) Error detection and Correction

It is possible to reduce error rates in data transmission to as small a level as desired by appropriate encoding and decoding techniques. Binary codes are designed to detect and correct errors. There are many different techniques of coding, but the most common procedure is that of adding an additional number of digits to every sequence of  $k$  bits of data. The additional digits are called redundant or parity digits, and they are chosen such that it will be sufficient to correct one, two or more errors if desired in the  $k$ -bits of information digits. Error control is the calculated use of redundancy. Theoretically, near errorless transmission is possible, but in practice there is a trade-off between transmission reliability, efficiency and complexity of terminal equipment, ref. (62).

The parity digits are completely determined by the binary value of the associated information digits, the detected values of the parity digits may be used to detect various combination of errors in the received information and parity digits. The detected errors may then be corrected by retransmission. Alternatively, when the redundancy is sufficiently high, various combinations of errors may be corrected automatically in the receiver decoder. This is known as forward acting error correction. Sometimes a combination of the two arrangements may be used. There are basically two types of codes, block codes and convolutional codes.

In block codes, the information digits are divided into separate

groups, of  $k$  digits and to each group  $(n - k)$  parity digits are added to give a total of  $n$  digits per block. The  $n$  digits of a block form a code word. A convolutional code has the same basic structure as a block code, but the parity digits are here determined not only by the information digits in the same word, but also by information digits in the preceding  $m$  words, where  $m \geq 1$ .

The optimum decoder for a block code operates on each received word separately and its complexity is a function of the number of parity digits  $(n - k)$  and the word length  $n$ . The optimum decoder for a convolutional code, however, operates simultaneously on the stored values of all received code words, and this is clearly not a practical arrangement. Due to equipment complexity and information performance convolutional codes have not been popular as block codes. For detailed information on coding theory see ref. (63).

(7-2) Error detection and retransmission versus forward acting error Correction

The correction of errors by detection and retransmission has advantages over forward acting error-correction. A random error-correcting code, which can correct any combination up to  $d$  errors in a code word, can alternatively be used to detect up to  $2d$  errors. Even for small values of  $d$  this normally results in a very much smaller probability of undetected errors in a code word where errors are corrected by detection and retransmission. The reduction in information rate or effective transmission rate, caused by the retransmission of code words containing detected errors, is small for digit error probabilities of around 1 in  $10^5$ , even for large code words where the number of digits retransmitted is much

greater than the number of received digits which have been wrongly detected. It therefore follows that for a given effective transmission rate, a very much lower undetected rate can be achieved with error detection and retransmission than with forward acting error corrections.

The advantage of the error detection and retransmission method is that the detection of an error and retransmission requires only a "yes-no" decision, and does not therefore involve much equipment complexity. Forward acting error correction, however, requires the decoder not only to detect the presence of an error but also to locate the particular digits which are in error. This always requires considerably more complex equipment than error detection.

The advantage of forward acting error correction is of course that no return channel is required so that the arrangement <sup>λ</sup> be used over a one way link where error correction by retransmission cannot be used. The majority of practical communication systems do however make use of a return channel and do not therefore necessarily have to use forward acting error-correction.

Another advantage of forward acting error correction is that the delay involved in transmission is avoided. When working over a satellite circuit, with a value of about 1/2 second in the loop delay (total delay in forward and reverse channels) this advantage can be very significant.

Over most practical communication channels serious disruption of the transmitted signal can be caused by temporary loss of transmission (transient interruptions) which may last from one to several hundred milliseconds. Since interruptions will usually cause the loss of many digits, the only possible means of correcting the resultant

errors is by retransmission. A practical communication system must therefore have arrangements for detecting errors caused by loss of signal and to be retransmitted. It follows immediately that, with a practical communication channel a significant fraction of the detected errors can only be corrected by retransmission. If the remaining detected errors are here corrected by forward acting error-correction the average reduction in the transmission delay of digits containing detected errors is therefore unlikely to be sufficient to offset the serious disadvantage incurred with forward acting error correction.

(7-3) Error detection with modulo-n feedback equalizers

Various techniques have recently been proposed for detecting and correcting errors in a data transmission system, where this operates over a channel that introduces severe signal distortion, ref.(18, 39-48). When a nonlinear equalizer is used at the transmitter the received signal is particularly well suited to the appropriate arrangement for detecting errors.

In section (6) it was shown that the output of the modulo-n feedback equalizer has a correlative time function which has recurrent correlation properties extending over several digits. These properties are not utilised in the signal detection but can be employed to check pattern violations without introducing redundant digits into the original binary message. Such violations result in errors which are easily detected. Error detection is performed at the receiver by comparing the sample values of the received signal with the element values generated in the receiver from the detected

binary data  $S(z)$ ,  $B(z)$  and  $X(z)$  are the input transmitted and received signal polynomials which have the coefficients of corresponding sequences  $\{s_i\}$ ,  $\{b_i\}$  and  $\{x_i\}$ .  $A(z)$  is the channel response polynomial as described in previous sections and is assumed to be known in the transmitter and receiver.

From eqn. (6-27) the  $z$  transform of the signal at the output of the channel  $A(z)$  in the absence of noise is

$$X(z) = M \left[ S(z) \frac{1}{a_o^{-1} A(z)} \right] A(z) \quad (7-1)$$

but

$$R(z) = X(z) + W(z) \quad (7-2)$$

where  $W(z)$  is the polynomial which has coefficients of the sampled values of additive white Gaussian noise, with zero mean and variance  $\sigma^2$ . From fig. (6-4) at the receiver output,

$$S'(z) = M \left[ a_o^{-1} R(z) \right] \quad (7-3)$$

is obtained. The sample values of the output of the channel (after the multiplier  $a_o^{-1}$ ) is operated on by modulo- $n$  operation and the binary values are detected from these values. The noise term in eqn. (7-2) causes binary digit errors, causing the detected signal  $S'(z)$  to differ from input signal  $S(z)$ .

$$E(z) = \sum_i e_i z^{-i}$$

$S'(z) = S(z) + E(z)$	+1	-1	+ modulo-2 addition
$e_i = +1$ no error	+1	-1	
$e_i = -1$ error	-1	+1	

(7-4)



where  $E(z)$  is the binary error polynomial due to  $W(z)$  and  $E(z)$  is added to the transmitter equalizer binary input signal by this modulo-2 addition to give the received binary signal containing errors.

If the  $S'(z)$  in the receiver is fed to a coder which is identical to the nonlinear equalizer in the transmitter followed by a linear transversal filter with tap coefficients equal to  $A(z)$  that simulates the baseband channel, and finally a multiplier identical to that at the receiver input, then it is possible to generate with this coder a  $XD(z)$  from the knowledge of  $S'(z)$  and  $A(z)$  where

$$XD(z) = M \left[ S'(z) \frac{1}{a_o^{-1} A(z)} \right] A(z) a_o^{-1} \quad (7-5)$$

Notice that  $S'(z) = S(z)$  which is the case, in the absence of noise,  $XD(z) = a_o^{-1} X(z)$ . It has been found that with binary bipolar inputs to the modulo-4 feedback equalizer in the transmitter when the channel is equalized exactly and in the absence of noise, resulted in  $a_o^{-1} X(z)$  which always had odd integers as coefficients. These levels always had the same distance from modulo-n decision boundaries regardless of the binary input signal polarity in the transmitter. For example with modulo-4 a multi level sequence  $\{a_o^{-1} x_i\}$  is produced which can have amplitudes of  $[+1, +3, +5, +7, \dots]$  with the knowledge about the allowable amplitudes  $\{a_o^{-1} x_i\}$ , sample values of  $\{a_o^{-1} r_i\}$  can be quantized to the nearest level of  $\{a_o^{-1} x_i\}$  by a quantizer in the receiver. To determine the odd integer nearest to  $\{a_o^{-1} x_i\}$ , when noise is present  $\{a_o^{-1} r_i\}$ , is passed through a quantizer whose output signal  $\{q_i\}$  is the most positive odd integer

less than  $1 + a_0^{-1}r_1$ , and which is compared with  $\{xd_i\}$  generated in the receiver. From eqns. (7-5) and (7-4)

$$XD(z) = M \left[ (S(z) + E(z)) \frac{1}{a_0^{-1}A(z)} \right] A(z) a_0^{-1} \quad (7-6)$$

in eqn. (7-6) the error polynomial  $E(z)$  will cause  $XD(z) \neq Q(z)$ , where  $Q(z)$  is the polynomial with coefficients as  $\{q_i\}$ , this will happen when  $W(z)$  in eqn. (7-2) causes a binary digit error in the received sequence. The error detector relies on the fact that when there is severe signal distortion,  $xd_i$  and  $q_i$  can take on a wide range of odd integer values, from say -17 to 17. If now one or two of the  $\{s'_i\}$  are changed to incorrect values, not only will the corresponding  $\{xd_i\}$  be changed but also will the following  $\{xd_i\}$ , either indefinitely or over a very long sequence of these values. Following one error in the  $\{s'_i\}$ , the error detector in effect becomes unstable and continues to indicate errors because the sequence of the  $\{xd_i\}$  now follows a different path to the sequence of the  $\{q_i\}$ , this will be explained shortly. This practically ensures the detection of any likely pattern (or distribution) of errors. The arrangement of error detection is shown in fig. (7-2). The reason modulo-n feedback filters are not used as an equalizer in the receiver is because they cause errors to be propagated. However, this weakness has been used here as a strength enabling errors to be detected.

Following the detection of an error, when comparing  $xd_i$  and  $q_i$  at time  $t = iT$  the receiver calls for the retransmission of the data signal starting from  $s_{i-j}$  where  $j \geq 8$ . If an error causes discrepancies between the  $\{xd_i\}$  and the  $\{q_i\}$ , the first of these will normally

occur before the end of the detection of the following four elements.

Before the retransmission of the data signal, the transmitter sends a sequence of a few more than  $k$   $\{b_i\}$  all set to zero, where  $k$  is the number of delay elements of transversal filter. This is achieved by setting to zero all signals stored in the transversal filter at the transmitter, together with the following sequence of  $\{s_i\}$ . As soon as the loss of signal is detected at the receiver, this sets to zero all signals stored in the transversal filter in the coder, together with the  $\{s'_i\}$ , thus holding the  $\{x_{d_i}\}$  at zero. As soon as a received sample  $a_o^{-1}r_i$  of magnitude greater than  $1/2$  is detected due to the transmission of the first binary digit, i.e.

$$|s_i| = |b_i| = |a_o^{-1}r_i| = 1$$

the operation of the receiver is allowed to proceed normally and the correct operation of the error detector is restored. Alternatively, use can be made of the fact that the receiver knows or can determine the point in the received sequence of signal elements at which the retransmission of data commences. The data transmission system now operates as just described, except that the transmitter sets exactly  $k$  successive  $\{b_i\}$  to zero, and the receiver automatically resumes normal operation on the arrival of the first signal element of the retransmitted data, without the need to detect the arrival of this element. The receiver after the retransmission operation replaces the binary sequence which was previously detected to be in error with the retransmitted data sequence. During the retransmission process the receiver continues the error detection, due to the fact errors can also occur during the reception of retransmitted signal elements as before.

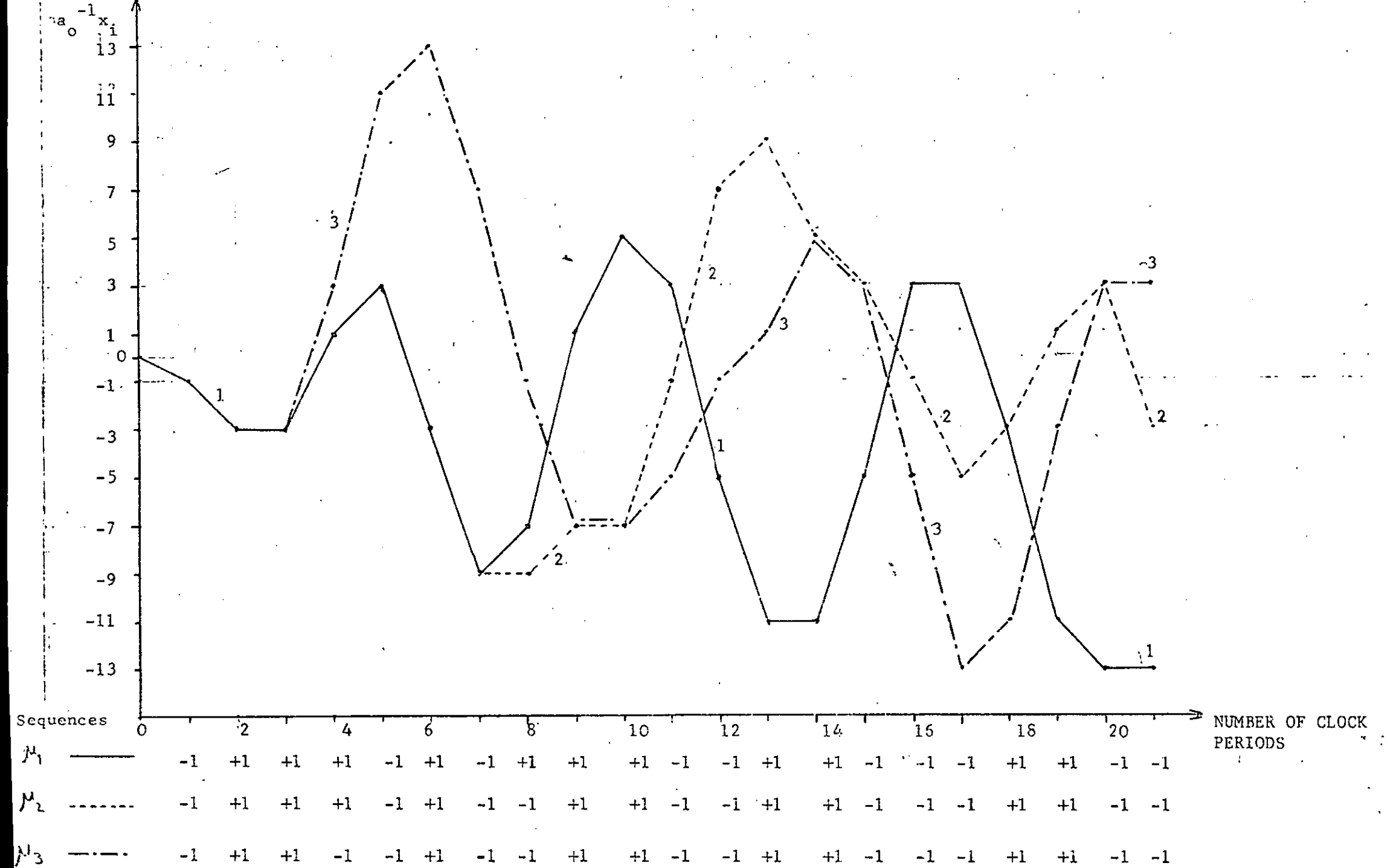


Figure (7-1) State diagram representing  $\{a_o^{-1}x_i\}$  for 3 different  $\{s_i\}$  for a given  $A(z)$  of channel No.13 from table (6-3).

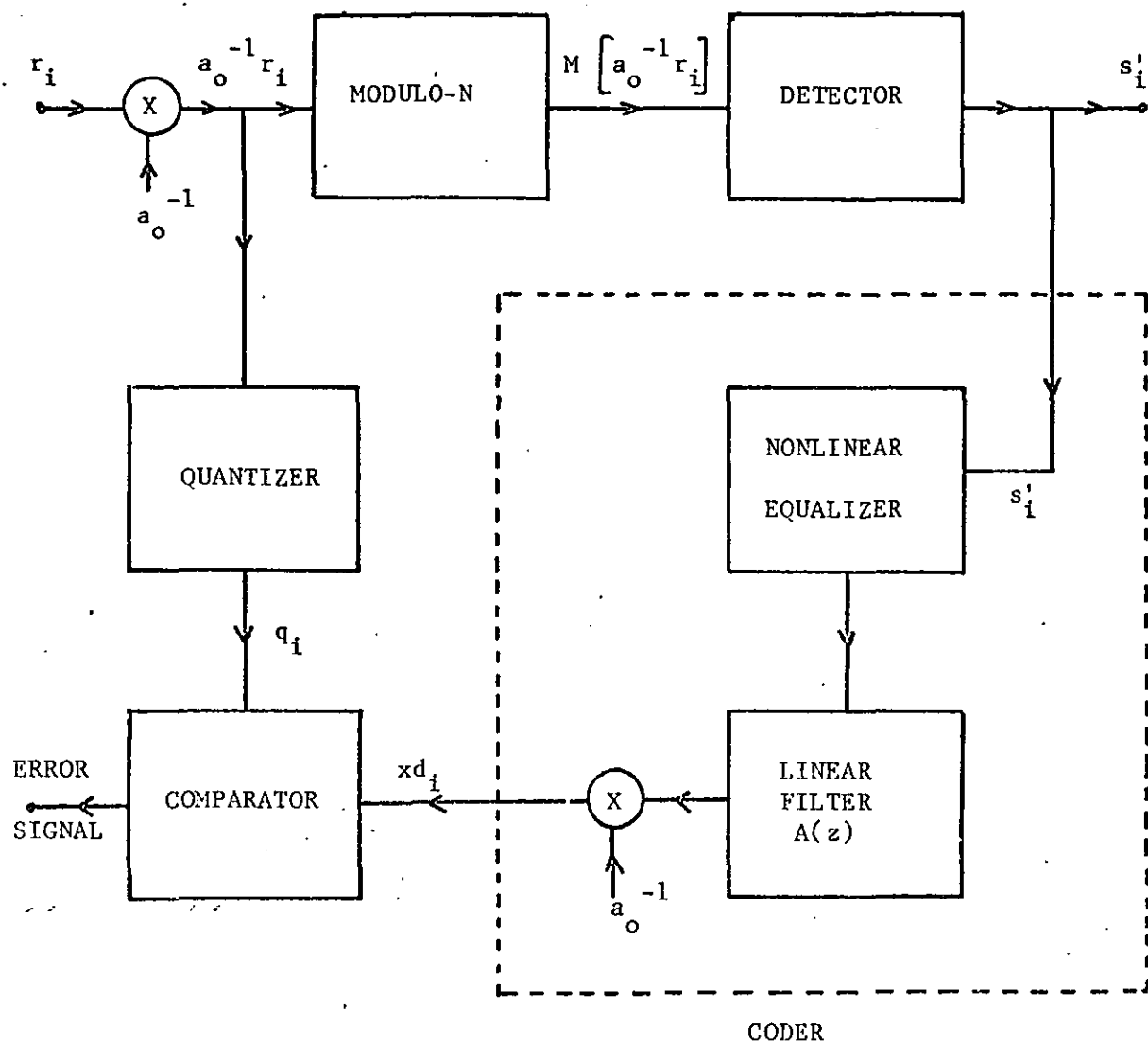


FIGURE (7-2) ERROR DETECTOR

Directly after the retransmission the normal transmission process continues.

In fig. (7-1) the output sequences  $\{a_o^{-1}x_i\}$  for a given channel  $A(z)$  and modulo-4 feedback equalizer is presented for three different binary sequences  $\{s_i\}$ . Observe that the sequence  $\{a_o^{-1}x_i\}$  is equal to the sequence  $\{a_o^{-1}r_i\}$  at the input to the modulo-4 transformation shown in fig. (7-2) when the channel noise sequence  $\{w_i\}$  is set to zero. Sequences  $\mu_1$ ,  $\mu_2$ , and  $\mu_3$  correspond to the curves 1, 2 and 3 in fig. (7-1) respectively. Curve 1 has amplitudes at sampling instants which are always odd and integer valued. In order to demonstrate how the error detector of fig. (7-2) functions, curve 2 which has been derived on the basis of zero error, will be assumed to be curve 1 when an error is introduced at the 8th clock instant. This error is such that the binary output will be -1 instead of the +1 for curve 1. Now

$$\{a_o^{-1}r_i\} = \{a_o^{-1}x_i\} + \{a_o^{-1}w_i\}$$

and at 8th clock instant,  $a_o^{-1}x_8 = -7$ . In order for curve 1 to be changed to curve 2,  $|a_o^{-1}w_8|$  must be greater than 1. Suppose  $-2 < a_o^{-1}w_8 < -1$  then  $a_o^{-1}r_8 < -8$  and when quantized  $q_8 = -9$ , from fig. (7-1). In addition the output of the modulo-4 transformation is  $M[a_o^{-1}r_i] < 0$  and is detected as  $s'_8 = -1$ . This  $s'_8$  is coded, see fig. (7-2) to give  $xd_8 = -9$ , since from curve 2 in fig. (7-1)  $a_o^{-1}x_8 = -9$ . The comparator inputs are both -9 and consequently no error signal is produced. Assume now that for every subsequent clock instant  $|a_o^{-1}w_i| < 1$ , i.e. a condition for no error. So  $a_o^{-1}x_9 = +1$

from curve 1 in fig. (7-1), and therefore  $q_9 = +1$ . Also  $0 < M [a_o^{-1} r_9] < 2$ , hence  $s'_9 = +1$ , and  $xd_9 = -7$ . The comparator deems an error if its output is not zero, and at this 9th instant  $q_9 > xd_9$ , hence an error is detected in the output sequence  $s'_i$ .

Before commenting on whether  $s'_9$  is in error, or the previous output data, the alternative condition of  $1 < a_o^{-1} w_8 < 2$  will be considered.

Now with this noise condition  $a_o^{-1} x_8 = -7$ , and hence  $a_o^{-1} r_8 > -6$ , hence  $q_8 = -5$ .  $s'_8 = -1$  as  $-6 < M [a_o^{-1} r_8] < -5$ , and  $xd_8 = -9$ . The output of the comparator therefore indicates an error.

It can now be seen that in this example the error in  $\{s'_i\}$  is detected at the instant when it occurred provided  $1 < a_o^{-1} w_8 < 2$ . However, when the noise contribution is negative,  $-2 < a_o^{-1} w_8 < -1$ , this same error is observed at the subsequent instant.

Notice that for just one error at the 8th clock instant, the output of the coder in fig. (7-2) is given by curve 2 the output of the quantizer  $q_i$  is given by curve 1 other than the instant of error occurring. Curve 3 in fig. (7-1) is generated from sequence  $\mu_3$ . This sequence  $\mu_3$  can be considered to be sequence  $\mu_1$  which has errors at the 4th and 8th clock instants. With these errors curve 3 is representative of  $xd_i$ . The curves in fig. (7-1) can be interpreted as  $xd_i$  for no errors, error at the 8th clock instant and two errors at the 4th and 8th instants, for curves 1, 2 and 3 respectively for the sequence  $\mu_1$ . Although these curves apply for a particular channel and sequence, in the general case where there is channel distortion and noise it is found that any error produces a unique  $\{xd_i\}$  which can be used to detect errors.

Another important property of the error detector is that in the presence of Gaussian noise, a false alarm indication is most

unlikely to occur, since this requires the noise component  $a_o^{-1}w_i$  to have a magnitude at least three times that needed for an error. The case of noise changing  $\{a_o^{-1}r_i\}$  to odd levels other than adjacent ones have negligible probability and does not cause a binary digit error when  $M[a_o^{-1}r_i]$  is detected due to alternating characteristics of modulo-n transformation see fig. (6-3b). However this causes false alarms and if false alarm prevention is needed then when the comparator is comparing  $xd_i$  with  $q_i$  and if the difference is equal to the magnitude of even number of levels then the comparator can be made to ignore this case and only detect odd number of level differences.

In fig. (7-2) the arrangement of error detection system is shown. In the transmitter the modulo-n feedback equalizer equalizes the input data sequence, the output from the equalizer is transmitted and in the receiver the sample values of signal elements are multiplied by  $a_o^{-1}$  then operated by the modulo-n transformation. The output of the decision circuit is the detected binary sequence  $\{s'_i\}$ . This sequence is fed to a coder in the receiver comprising a modulo-n feedback filter and followed by a linear transversal filter having the same sampled impulse response of the equivalent baseband channel  $A(z)$ . The output from this coder is  $xd_i$  generated from the receiver output sample  $s'_i$ .

The sampled value of  $a_o^{-1}r_i$  is also fed to the quantizer which has a staircase characteristic. The output levels of the quantizer are the allowable levels of  $a_o^{-1}x_i$ . The quantizer threshold levels are at  $[0, \pm 2, \pm 4, \pm 6, \dots]$  and the output levels are  $[\pm 1, \pm 3, \pm 5, \pm 7, \dots]$ . The quantizer output  $q_i$  and  $xd_i$  are fed to a comparator. If the

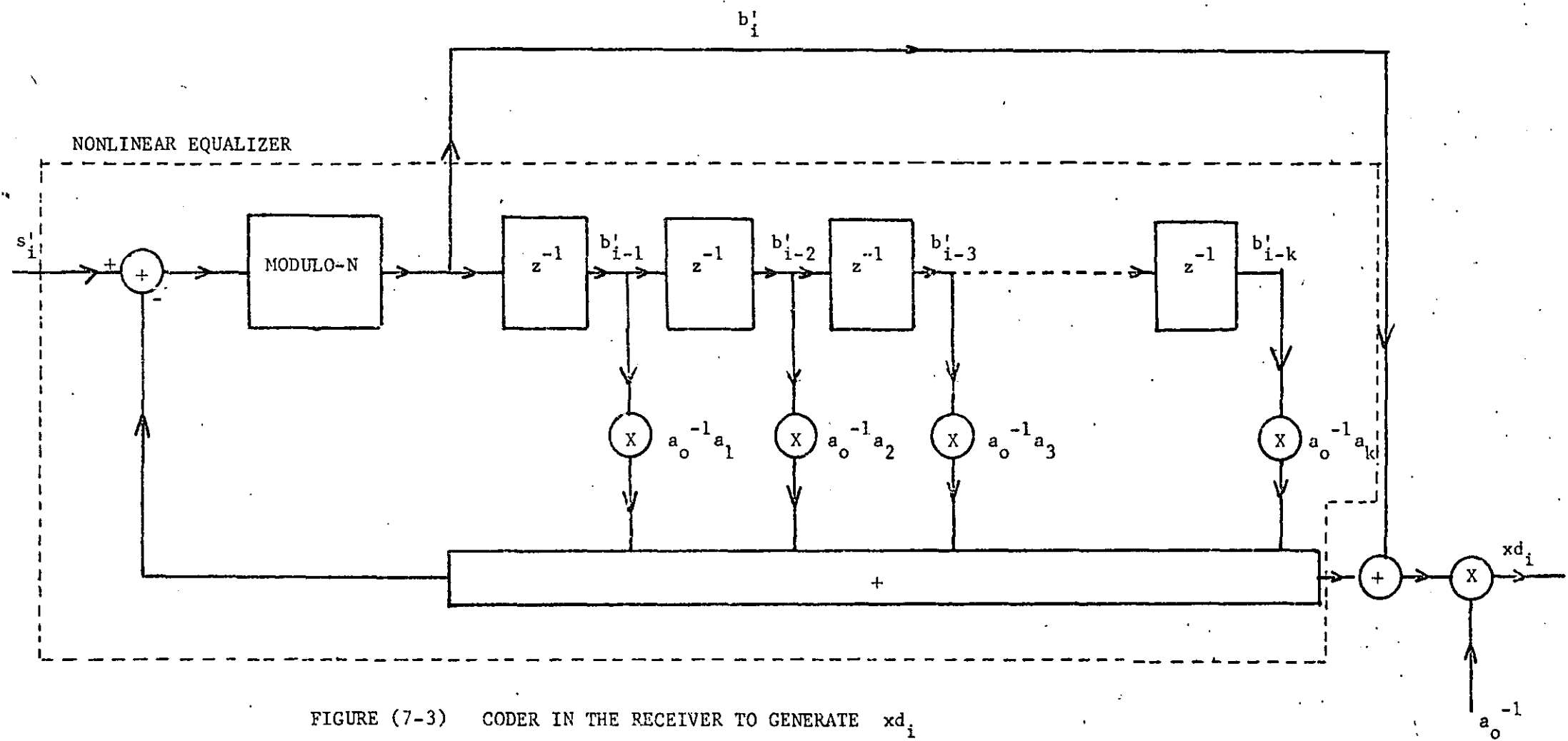


comparator detects any difference in the levels of the input signals, then retransmission of the data is requested via the return line.

An efficient way of implementing the coder shown in fig. (7-<sup>2</sup>) is the arrangement of the filter  $A(z)$ , representing the model of the channel, is simply produced from the nonlinear equalizer by adding the  $\{b'_i\}$  samples to the feedback samples. Such an arrangement is simple and inexpensive.

It can be seen that the error detector is likely to fail when the signal distortion is such that  $xd_i$ , for all or most of the time takes on only a few values, say -3 to 3. Since there is now a greater probability that  $\{xd_i\}$  maintains the same values as the  $\{q_i\}$  during and after a likely error pattern as the  $\{s'_i\}$ . Thus the error detector no longer necessarily goes unstable. This is the situation when " $a_0$ ", the first component of the sampled impulse response of the channel, is the largest component and for it is also the case when all roots of  $A(z)$  lie inside the unit circle in the  $z$  plane, even though  $a_0$  is not now necessarily the largest of the  $\{a_i\}$ . Of course, when there is no signal distortion,  $xd_i$  has the possible values  $\pm 1$  and the error detector fails completely.

The system explained in this section was first investigated as error detection and correction of the last detected binary digit  $s'_i$ . The error correction scheme had failed for 50% of the errors and the coder output  $\{xd_i\}$  continued to diverge from the quantizer output  $\{q_i\}$ . When the reasons for failure have been studied, it was found for some cases, that the error was due to the delay in the error detection also the need for proper resetting of coder memories after detection of an error. The error detector of this section has been



developed and successful operation is achieved. Error correction and improved detection schemes were investigated, and are described in section (7-4).

(7-3-1) Assessment of Error detection system

The error detector described in this section was tested with system 1 and system 3 over 21 different channels given in chapter 6. The binary digit error rate was chosen to be 1 in  $10^3$ , and the block length for retransmission is chosen to be 100. The first 4 bits are zero which are required to set the delay elements in modulo-n filters to zero for the given sampled channel impulse response. The remaining 96 bits are data elements. For the given binary error rate this block length of 100 elements resulted in 10% reduction of information rate due to retransmission, of data elements.

The error detection circuit detected all the errors that occurred in 30,000 transmitted binary signal elements for all the 21 channels listed in table (6-3) without missed errors and false alarms. It was found that in the case of system 1, all errors in the  $\{s_1^1\}$  were detected for each of the 21 channels. Since in these tests a total of some 30 errors occurred over each channel, it is clear that the error detector provides a most effective protection against errors when used in system 1. When the error detection system was tested for higher digit error rates as 1 in  $10^2$  all the errors were detected, but such error rates are unrealistic for high speed data transmission because of repeated retransmission the average information rate becomes very low due to the number of retransmitted data elements. It was found that on the average the system detects errors with a delay of

0 to 4 data elements, after the occurrence of binary digit error. If the loop delay is negligible the magnitude of retransmission block length can be chosen to be about 10 for the listed 21 channels in table (6-3), then the error detection and retransmission can operate successfully for high error rates without a big reduction in information rate.

The error detector can also be used with system 3, where the nonlinear equalizer at the transmitter equalizes the baseband channel and linear filter, whose resultant  $z$  transform is  $z^{-h}BA(z)$ . The coder for the error detection is arranged to be for  $BA(z)$ . In the case of system 3, however the error detector only gave complete protection for 7 of the 21 channels, namely, channels 4 - 7, 13, 15 and 18. The breakdown of error detection, that is, when the system fails to detect errors, in relation to channel response  $A(z)$  has been investigated. It has been found for channels where errors are not detected at the receiver all have roots whose magnitude  $< 0.7$ , in the  $z$  plane. Table (7-1) has been prepared to investigate the failure of error detection. It has been found for a wide range of polynomials whose roots magnitude is greater than unity, in the  $z$  plane, the error protection was complete. If the polynomial has all the roots whose magnitude is less than unity, in the  $z$  plane, the error detection system missed binary digit errors. Table (7-1) lists 21 impulse responses for system 3, for the nonlinear equalizer part, all these polynomials have roots with magnitude less than unity, in the  $z$  plane. For system 3 the first component of the sampled impulse-response of the channel and linear filter combination which is  $BA(z)$ , is always one of its larger components, and it is clear that the error

BA(z) Channel No. Nonlinear equalizer for system 3						A	B	C	D	-11	-9	-7	-5	-3	-1	1	3	5	7	9	11
1	1.0	0.340	0.029	0	0	54	30	0.55	0.17R						5069	4931					
2	1.0	0.618	0.095	0	0	67	50	0.74	0.34R					392	4732	4536	340				
3	1.0	1.000	0.250	0	0	66	62	0.93	0.50R					1258	3734	3736	1272				
4	1.0	1.237	0.382	0	0	50	50	1.00	0.62R					1531	3545	3513	1411				
5	1.0	1.200	0.350	0	0	70	70	1.00	0.69R												
6	1.0	1.200	0.350	0	0	51	51	1.00	0.69R					1597	3579	3389	1435				
7	1.0	1.350	0.451	0	0	65	65	1.00	0.75R				70	1623	3376	3342	1576	36			
8	1.0	0.000	-0.069	0	0	69	32	0.46	0.30R						5042	4958					
9	1.0	0.000	-0.123	0	0	68	33	0.48	0.35R						4924	5076					
10	1.0	0.338	-0.170	0	0	54	30	0.55	-0.62R					8	5011	4976	5				
11	1.0	0.450	-0.168	0	0	57	41	0.71	-0.71R					232	4711	4828	229				
12	1.0	1.027	0.954	0.354	0.119	73	67	0.91	0.58C				114	1410	3576	3460	1303	137			
13	1.0	1.929	1.970	1.003	0.270	66	66	1.00	0.76C	4	74	405	865	1575	1951	2081	1602	943	401	97	2
14	1.0	0.976	1.352	0.544	0.284	65	63	0.96	0.83C			5	368	1639	3092	3000	1552	343	1		
15	1.0	1.528	1.595	0.893	0.251	74	74	1.00	0.84C		2	155	833	1647	2394	2308	1723	773	160	5	
16	1.0	-1.311	1.395	-0.616	0.228	55	53	0.96	0.77C			53	474	1591	2887	2846	1632	466	51		
17	1.0	-1.603	1.366	-0.708	0.138	73	72	0.98	0.83R			121	645	1728	2468	2605	1636	682	114	1	
18	1.0	-1.768	1.600	-1.029	0.305	75	75	1.00	0.77C		22	278	836	1626	2225	2266	1601	865	251	30	
19	1.0	0.203	-0.106	-0.212	-0.050	64	36	0.56	0.64R						4969	5031					
20	1.0	0.369	-0.257	-0.182	-0.029	66	29	0.43	0.62R						5025	5975					
21	1.0	0.453	-0.361	-0.232	-0.191	39	31	0.79	0.61R						4981	5019					

A - Total number of errors in 60000 Samples

B - Total number of errors detected in 60000

C - Average of detected errors B/A

D - Magnitude of largest root, R = Real Root, C = Complex Root

Levels of  $xd_i$

Histogram for 10,000 samples of  $xd_i$

Table (7-1) Investigation for the failure of error detector of system 3.

detector will not operate as well as here as in the case of system

1. The number of binary digits transmitted for these tests is 60,000. The first column A represents total number of digital errors. Column B is the number of errors detected from the digit errors and corrected by retransmission of 96 data elements plus 4 zeros. Column D is the magnitude of largest root of  $BA(z)$  in the  $z$  plane. Also the histogram given in the same table for the corresponding polynomials  $BA(z)$  is for the values  $\{x_{d_i}\}$  in 10,000 trials. It can be seen that if  $\{x_{d_i}\}$  takes on more than two values greater protection from errors is obtained.

It has been found the error detection for  $A(z)$  of  $[1,1]$  is 100% effective. Combination of such a response with an  $A(z)$  which fails to detect all the errors result in a combined impulse response  $A(z)$  where the error detection system functions without missing errors when tested at given error rates.

The modification of channels  $BA(z)$  of table (7-1) which does not function effectively for error detection can when modified by an additional filter  $(1 + z^{-1})$  enable all errors to be detected. For such combinations  $A(z)$  of  $BA(z)$  with an additional linear filter impulse response  $(1 + z^{-1})$  does introduce an extra root in the  $z$  plane. This  $A(z)$  with a root magnitude of unity, then the linear filter and channel is now equalized by the modulo- $n$  feedback filter as in system 1 or system 3 with complete protection against errors. The combined filter  $A(z)$  results in about 3dB reduction in tolerance to additive white Gaussian noise when compared to same channel without the filter  $(1 + z^{-1})$ . Another solution for this problem would be to change the phase of the sampling clock to obtain entirely different

sampled impulse response of the channel. Such a technique has been suggested in ref. (64).

(7-4) Improved detection for modulo-n feedback equalizer

For the modulo-n feedback equalizer used in the transmitter to equalize a distorting channel and the detection in the receiver done on bit by bit basis gives the results obtained in chapter 6 which are substantiated by those given in ref. (18-19) and (46). This method of detection is not optimum and does not produce the least number of errors in the presence of additive Gaussian noise. As mentioned in chapter 6, modulo-n feedback equalizer is a special case of a correlative level coder which is also a convolutional encoder. It is possible to apply maximum likelihood detection techniques to modulo-n feedback equalizer.

Maximum likelihood detection algorithm can be viewed as a new solution to perform matched filter detection on sampled sequences, without requiring an unreasonable number of matched filters. The performance of the maximum likelihood decoding is much superior to bit by bit decoding, ref. (54).

If an assumption is made to view the distorting channel  $A(z)$  as a correlative level encoder in the transmitter for the modulo-n feedback equalizer then  $X(z)$  is the output from  $A(z)$ , and  $R(z)$  is as defined by eqn. (7-2).  $X(z)$  and  $S(z)$  are related through eqn. (7-1). Maximum likelihood sequence estimation is defined as the choice of that  $X(z)$  for which the conditional probability  $P \left[ R(z)/X(z) \right]$  is maximum.

The binary information sequence  $\{s_i\}$  is of length  $N$  i.e.  $1 \leq i \leq N$ , since the sequence  $\{s_i\}$  is mapped into  $\{x_i\}$  in one-to-one fashion

there are  $M = 2^N$  vectors which  $\{x_i\}$  can take on, and the noise samples  $\{w_i\}$  are samples of additive white Gaussian noise and are statistically independent. It is shown in ref. (65) that if all input sequences are equally likely the decoder which minimizes the error probability is one which compares the conditional probabilities (also called likelihood functions)  $P \left[ \{r_i\} / \{x_i^M\} \right]$  where  $\{r_i\}$  is the overall received sequence and  $\{x_i^M\}$  is one of the possible transmitted sequences. The decoder decides in favour of the particular sequence  $\{x_i\}$  which maximize the conditional probability, and this is called a maximum likelihood decoder. Generally it is more convenient to compare the quantities  $\log. P \left[ R(z)/X(z) \right]$ , called log-likelihood functions, and can be shown to be equivalent to the minimum distance decision rule based on the measure, ref. (59), (66, 67).

$$D(x^M) = \sum_i^N (r_i - x_i^M)^2. \quad (7-7)$$

The maximum likelihood solution is that  $\{x_i^M\}$  which minimizes  $D(x)$ , where  $\{x_i^M\}$  and  $\{s_i\}$  are related through eqn. (7-1), ref.(66). In the receiver it is possible to generate the sequence which minimizes eqn. (7-7) this sequence will be called  $\{x_{d,i}\}$  and corresponds to a binary sequence  $\{s'_i\}$  and will be the best estimation of binary data that was fed to the equalizer in the transmitter. But due to length of  $\{x_i\}$  number of sequential operations makes such a detection process impossible, algorithms are developed and analysed in refs. (46), (59), (32) to simplify the maximum likelihood detection operation to reasonable number of sequential



operations. A new detector that will asymptotically approach the performance of an MLD detector is explained in the next section.

(7-4-1) Proposed detector for modulo-n feedback equalizer

This proposal is derived from error detector of section (7-3) and shown in fig. (7-4) as improved detector for modulo-n feedback equalizer. The coder is as in fig. (7-2) but the detector operates in a fundamentally different manner. The element value  $s_i$  instead of being detected from  $a_o^{-1}r_i$  at time  $t = iT$ , is detected at time  $t = (i + j)T$  from the sample values

$$\{a_o^{-1}r_\ell\} = a_o^{-1}r_i, a_o^{-1}r_{i+1}, a_o^{-1}r_{i+2} \dots a_o^{-1}r_{i+j}$$

which are held in a buffer store.

The sequence generator generates in turn each of the  $2^{j+1}$  possible sequences of the  $(j + 1)\{s_\ell\}$ , where  $\ell = 0, 1, 2, \dots, j$ , and each of the sequences is converted by the coder into the corresponding sequence of the  $(j + 1)\{xd_\ell\}$ . During the generation of each sequence  $\{s_\ell\}$ , the coder is reset to its state immediately after feeding in the detected value  $s'_{i-1}$  and this is achieved by storing in the buffer store associated with the coder the sequence of signals held in the transversal filter  $A(z)$  and  $a_o^{-1}A(z)-1$  in the nonlinear equalizer in the coder, immediately after feeding in the detected value  $s'_{i-1}$ .

The detector now measures the mean square difference

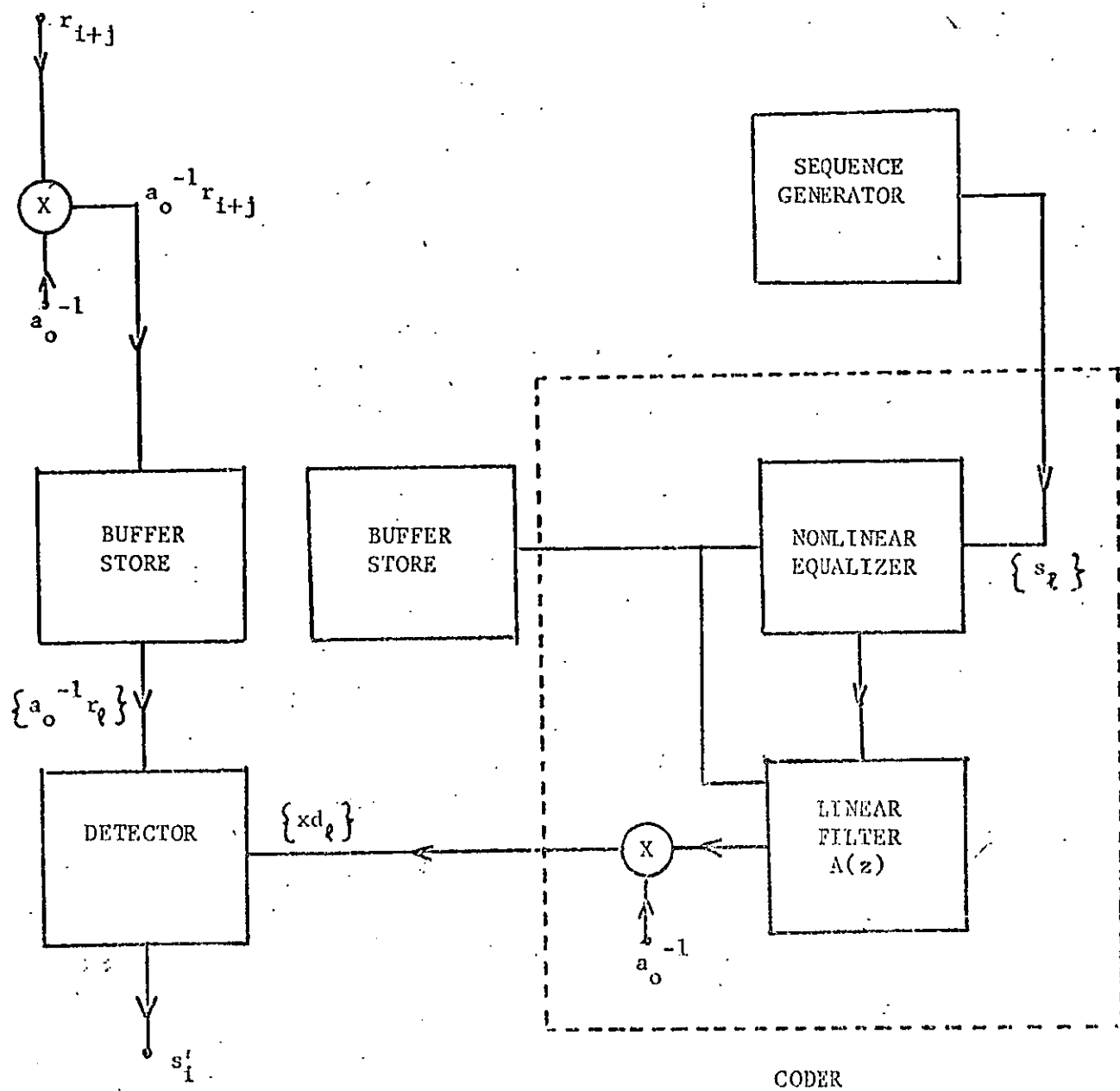


FIGURE (7-4) IMPROVED DETECTOR FOR MODULO-N EQUALIZER

$$\sum_{\ell=1}^{i+j} a_o^{-2} r_{\ell}^2 - x d_{\ell}^2 \quad (7-8)$$

between sequences  $\{a_o^{-1} r_{\ell}\}$  and  $\{x d_{\ell}\}$ , for each of the  $2^{j+1}$  possible sequences of the  $\{x d_{\ell}\}$ . The detector then selects  $s_i'$ , the detected value of  $s_i$ , as the value of  $s_i$  (i.e. the first value) in the sequence  $\{s_{\ell}\}$  which is used in the generation of the sequence  $\{x d_{\ell}\}$ , for which the mean square difference is minimum.

The whole detection process now repeats itself for the detection of  $s_{i+1}$  from the sample values

$$a_o^{-1} r_{i+1}, a_o^{-1} r_{i+2}, a_o^{-1} r_{i+3}, \dots, a_o^{-1} r_{i+j+1}$$

and so on. The reason why only  $s_i$  is used after the detection process is that every  $s_i$  should be detected from same number of received sample values. If whole of sequence  $\{s_{\ell}\}_m$  corresponds to the minimum distance and produces  $\{x d_{\ell}\}_m$ . If  $\{s_{\ell}\}_m$  is accepted as the detected binary sequence rather than just the first element in each  $\{s_{\ell}\}_m$  then each of the detected  $s_i'$  elements does not have same error probability. The last elements in the sequence  $\{s_{\ell}\}$  will have higher error probability than the first elements.

This proposal has not been tested due to available computer time that will allow the simulation of the system to obtain results within acceptable confidence limits. A hybrid system using error detection and correction utilizing a version of improved detector for modulo-n feedback equalizer has been investigated.

#### (7-5) Error Corrector for Modulo-n feedback equalizer

The arrangement of the error detector and corrector is shown in

fig. (7-5). In this figure detector A is a single element detector and detector B is a multi-element detector as previously explained. In the fig. (7-5) the switches, placed in position I in order to detect error in the incoming data, and move automatically to the error correct mode (position II) if an error has been detected and a search is made for  $\{s'_l\}$ , at the end of the search, the coder is reset to its correct state and  $\{s'_l\}$  replaces the sequence of length  $j + 1$  in the binary store. The error corrector then switches back to error detect mode.

The error correction technique of this section is derived from error detection of system (7-3) and improved detector of section (7-4-1) for modulo-n feedback equalizers. The detection is done, as explained in chapter 6, after modulo-n operation from  $M \begin{bmatrix} a_o^{-1} r_{i+j} \end{bmatrix}$  by a simple threshold detector and the detector will be called detector A, which is a single element detector. The error corrector functions by first detecting errors from  $a_o^{-1} r_{i+j}$ , by the system of fig. (7-2), by comparing  $q_{i+j}$  and  $xd_{i+j}$ . When a discrepancy between these two values is detected by the comparator then instead of requesting retransmission, the superior detector shown in fig. (7-4) is utilized.

The error corrector uses the same coder as the error detector with an associated buffer. When the error is detected the sequence generator generates in turn each of the  $2^{j+1}$  possible sequences of the  $j + 1$   $\{s'_l\}$  where  $l = 0, 1, 2, \dots, j$ , and each sequence is converted into the corresponding sequence of the  $j + 1$   $\{xd_l\}$  and resetting of the coder is done as in section (7-4-1). The  $\{xd_l\}$  is chosen that minimizes the eqn. (7-8). This selection is performed

by detector B, which is a multi-element detector. Detector B then outputs the optimum sequence  $\{s'_p\}$ . The coder is now reset to its correct state after the choice of  $\{s'_p\}$  has been made from the buffer store. Now the coder is ready for any further error detection operation starting at time  $t = (i+j+1)T$ . The sequence  $\{s'_p\}$  replaces the bit-by-bit detected  $\{s'_{i+j}\}$  in the binary data buffer store. This is done by replacing the last value of  $\{s'_p\}$  into  $s'_{i+j}$  and the first value of  $\{s'_p\}$  into  $s'_1$  and the values in between are put into their respective places in the binary data buffer store.

The error detector in fig. (7-5) accepts the complete sequence of  $\{s'_p\}$  instead of just its first element as in the proposed improved detector of fig. (7-4). After the error correction because this is what occurs, the error detector switches back to error detect mode and continues to check for errors, between the received signal-element  $a_o^{-1}r_{i+j+1}$  and regenerated  $xd_{i+j+1}$ . If the comparator still detects an error this error detection could be due to two causes. Firstly correction of errors at time  $t = (i + j)T$  was not satisfactory or a new error had occurred at time  $t = (i + j + 1)T$ , what ever the cause of the error that is detected at time  $t = (i + j + 1)T$  then the error detector switches back to error correction mode and another search is made for a new  $\{xd_p\}$  that minimizes the mean square difference between  $\{xd_p\}$  and

$$a_o^{-1}r_{i+1}, a_o^{-1}r_{i+2}, \dots, a_o^{-1}r_{i+j+1},$$

by the detector B. The detector then selects the sequence  $\{s'_p\}$  as the detected one used in the generation of  $\{xd_p\}$ . The sequence  $\{s'_p\}$  replaces the binary values which have been produced at the

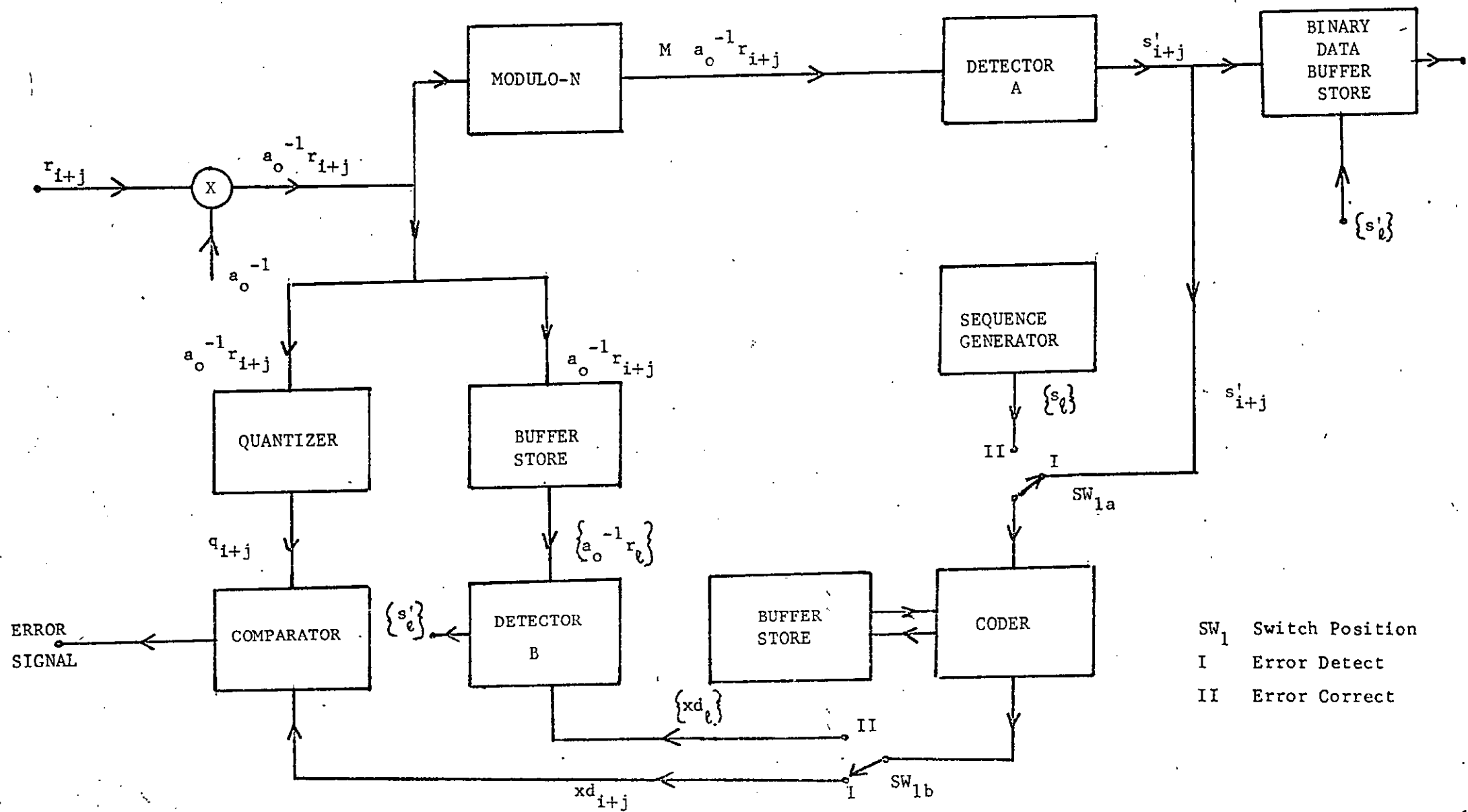


FIGURE (7-5) ERROR DETECTOR AND CORRECTOR

output of detector A and are currently stored in the binary databuffer store. Thus the sequence  $\{s_i^j\}$  of length  $j + 1$  elements replaces  $j + 1$  elements of binary data in the buffer store at time  $(i + j)T$ . Should the system of fig. (7-5) at the next sampling instant detect an error in mode I it again switches to mode II, and generates an optimum sequence  $\{s_i^j\}$ . This sequence again replaces  $j + 1$  elements in the binary data buffer store. Notice that at this sampling instant, namely  $(i + j + 1)T$  the detected element at  $iT$  is not altered. The size of binary data buffer store should be such that if a retransmission is requested it should allow correction of the complete block retransmitted, this is to be several times greater than  $j + 1$ .

#### (7-5-1) Assessment of System

The error corrector has been tested by computer simulation for a binary digit error rate of 1 in  $10^3$ . The value for  $j$  has been chosen to be 4 the reason for such a choice is that the error detector detected all the errors with a delay of 0 to 3 clock instants in previous tests. If the system is unable to correct a sequence of errors and consequently a safety measure is introduced to prevent the error correction operation being continuously applied. The safety procedure involves counting the number of times the error detector switched to error correct mode. If this is more than 5 times in the last 10 clock instants retransmission is requested. The retransmission process is explained in sections (7-3) and (7-3-1). Observe that this does not necessarily imply there have been 5 consecutive errors.

The error corrector corrected all the errors that occurred in 20,000 transmitted binary signal elements for 21 channels listed in table (6-3) with the exception of channels 9 and 10 which requested correction by retransmission once in these tests. A set of computer simulations for all 21 channels  $j = 5$ , the safety count of 6 in the last 10 clock instants and the error rate maintained at 1 in  $10^3$ . These tests yielded the following:

(i) error corrector successfully corrected all the errors that occurred in 20,000 transmitted binary signal elements for all the 21 channels. (ii) in the case of system 1 all the errors in the binary detected sequence from detector A were corrected for each of the 21 channels. For these tests in (i) and (ii) a total of some 20 errors occurred over each channel. The error corrector has not been tested for system 3 because the error detector does not give complete protection on all channels when used for this system.

An error rate  $> 1$  in  $10^3$  could not be tested for all the channels due to the fact that as the number of errors increase the error corrector is used more frequently which requires more computer time than bit by bit detection. A small number of channels have been tested with  $j = 5$  and increased noise power of 3 dB, giving an error rate greater than 1 in  $10^3$ , 3 dB increase in noise power results in about 363 errors in 20,000 bits. With channels 2 and 12 to 15, the error correction was found to be perfect for these tests.

The arrangement of the error corrector of fig. (7-5) can be implemented from fig. (7-2) by an addition to the error detector of few extra circuits and buffer stores, without a big increase in



cost. From the computer simulation tests it was observed that when an error is detected the error correction system is allowed to try consecutive searches and after each search it accepts the sequence  $\{s'_\ell\}$  which corresponds to sequence  $\{x_{d_\ell}\}$  which minimizes eqn. (7-8). If this operation is done continuously the error detector of this section becomes the improved detector of section (7-4-1), because after every search only first element of the sequence  $\{s'_\ell\}$  is accepted. Therefore the performance of the error detector and corrector can approach that of a maximum likelihood detector when the error correction operation is done continuously.

#### (7-6) Conclusion

The effective tolerance to additive white Gaussian noise of system 1 can be significantly improved by means of a suitable error detector at the receiver, which operates on the received data and requires no redundant data to be transmitted. The modulo-n feedback equalizer acts as a multi valued convolutional encoder.

Due to reasons given in previous section the error corrector of fig. (7-5) is a compromise approach to the improved detector. From computer simulation results it can be seen that there is improvement in the tolerance to additive white Gaussian noise when compared to bit-by-bit detector. The exact amount of improvement can not be quoted for all the channels due to the fact that each channel introduces a different magnitude of redundancy and it is this redundancy that is used for error detection and correction.

If the error correction used in system of fig. (7-5) is implemented

the error corrector in the error correction mode has to complete the search in one clock period,  $T$  seconds, and be ready for error detection at the end of this period. The hardware of the system will not be fully utilized during error detection if the speed of the error corrector is designed to be  $T$  seconds. The system can be redesigned as in fig. (7-6) the only difference between system of fig. (7-6) and the improved detector of fig. (7-4) is the addition of a circuit that checks for discrepancy between  $q_i$  (described in section (7-3)) and  $xd_i$ .  $xd_i$  is the element of sequence for  $\ell = i$  that corresponds to the minimum mean square distance between  $\{a_o^{-1} r_\ell\}$  and  $\{xd_\ell\}$ . If the comparator detects discrepancy between  $q_i$  and  $xd_i$  retransmission would then be requested.  $s'_i$  is detected as in the improved detector of fig. (7-4) by multi-element detector B shown in fig. (7-6). With this system the bit-by-bit detector is completely eliminated from the detection process. Observe that fig. (7-4) is solely a detector without the facility of calling for retransmission of data. The application of this system fig. (7-6) could be over channels which have a high error rate and large loop delay where a nonlinear equalization technique of detection and intersymbol interference cancelation in the receiver would fail due to error extension effects.

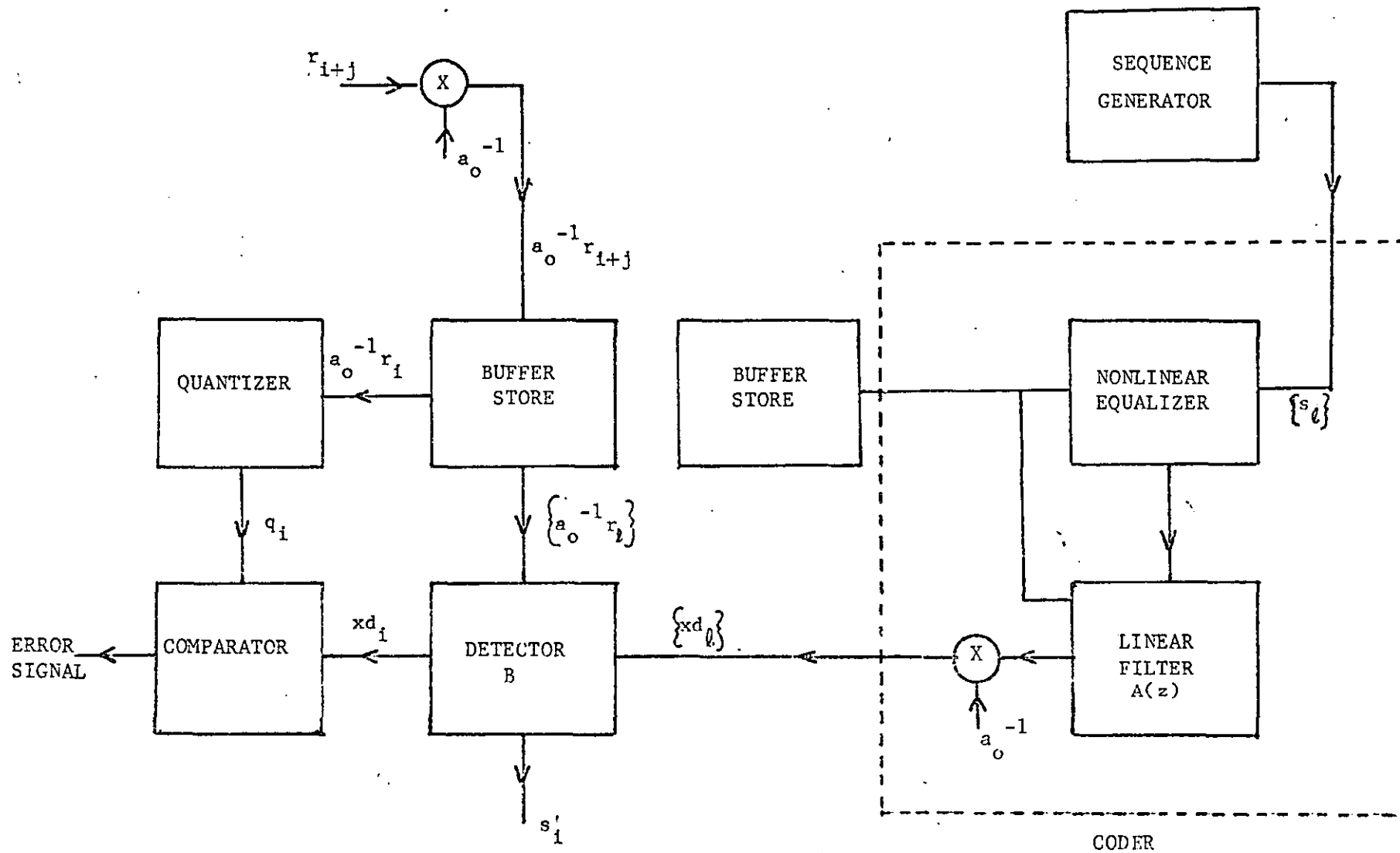


FIGURE (7-6) ERROR DETECTOR AND IMPROVED DETECTOR FOR MODULO-N EQUALIZER

## CHAPTER 8

Adaptive equalization of time varying channels

## CHAPTER 8

### (8-1) Introduction

Practical data transmission channels are time variant. The rate at which the impulse response  $A(t)$  of a telephone channel varies with time is slow. For example changes in  $A(t)$  will occur over a period of many minutes and are generally a small percentage. However, on HF channels significant changes can occur in less than a second. A useful model of these time varying channels is a time varying filter having an impulse response  $A(t, \tau)$  at whose output is added white Gaussian noise.

The use of tapped delay filter as an equalizer for  $A(t, \tau)$  has been investigated extensively and details of this history are reported by Rudin, ref. (68) and Ditoro, ref. (69). Recent work, ref. (68-80) has centred on use of this tapped delay filter for specific channels. Various algorithms for setting the tap gains automatically and use of various performance criterion have been described. Most notable is the work of Lucky ref. (70), who described a realization of an algorithm to force zeros in the impulse response of a system used in digital data transmission over telephone lines. Lucky and Rudin described an equalizer, ref. (81) ref. (68), for analog signal transmission which minimizes mean square error in response to a training signal. Niessen, ref. (75) in a short letter described an algorithm to minimize mean square error for data transmission in which the data itself is used as a training signal and adjustment of the tap gains continues throughout data transmission by use of decision directed feedback.

Some practical equalizers have been described in the literature refs. ( 82), Niessen and Willim, ref( 83), and Hirsch and Wolf, ref. ( 84). Recent interest has now centred on developing techniques to speed convergence of adaptive equalizers.

A number of papers deals with equalizer algorithms and techniques for faster convergence properties. Schonfeld and Schwartz, ref. (85), allow loop gains to be time varying throughout adaptation, so as to provide optimal convergence at the end of a specified duration. Richman and Schwartz, ref. ( 86) have used a dynamic programming approach to the adjustment of loop gains, while Walzman and Schwartz, ref. ( 87) have developed a discrete frequency domain technique to obtain faster convergence. In this later technique the use of the fast fourier transform allows the frequency domain properties to be utilised, where optimum gain constants are more readily available than in the time domain where a similar optimization would require determination of the eigenvalues of a matrix related to the system impulse response.

In the literature no definition of what constitutes convergence has been accepted. Furthermore the convergence time is a function of the particular channel selected. If the impulse response of the equivalent baseband channel  $A(t)$  is time variant  $A(t, \tau)$ , the best equalizer function is also a function of time. If, however, channel changes occur over a time interval much longer than the duration of the impulse response of the channel then it is also possible to measure  $A(t)$  and to construct the best equalizer in much less time than it takes the channel change. The problem of equalization of time varying channels is simplified by measuring a fixed channel

(not accurately known), determining  $A(t)$  and then constructing the best equalizer for that channel. A succession of measurements of  $A(t)$  will produce a succession of equalizer responses to track slowly changing channels. This is the approach generally used in the literature to investigate adaptive equalizers and for this reason  $A(t)$  will be used subsequently instead of  $A(t, \tau)$ .

### (8-2) Adaptive Equalizer Algorithms

Since each channel has a different type of distortion, the equalizer must be tailored to fit each particular channel. A scheme which is easy to implement was discovered by Lucky, ref. (88). This originally involved sending a series of test pulses prior to the transmission of data through the channel in order to train i.e. adjust the equalizer after which the equalizer settings were frozen. If the channel characteristics were to change during the message being transmitted, however, the equalization would no longer properly fit the channel. In a subsequent paper Lucky, ref. (70) arranged for the equalizer to be adjusted directly from the received data signal. In this way the equalizer continuously redesigns itself during the entire period that data is transmitted.

All the adaptive equalizer algorithms make use of a derived error signal  $e_1$  which is the difference between the actual equalized output and the distortion free, or reference signal. For adaptive operation, the reference signal can be approximated directly from the equalized output by taking advantage of the known format of a digital data signal, where one of a discrete number of levels is transmitted.

### (8-2-1) Zero forcing Algorithm

An algorithm was developed by Lucky, ref. (88 ), for setting the tap gains of a linear equalizer by minimizing the distortion factor  $D_{\text{peak}}$  defined in eqn. (4-25). This algorithm was called the zero forcing algorithm, and forces the equalized sampled impulse response of the channel to zero except at one main pulse. The number of zeros that can be forced by a linear transversal filter of length  $n$  is only  $(n - 1)$ . It can be shown that an estimate  $\{\hat{e}_j\}$  of the equalized sampled impulse response of the channel is given by the correlation between the received binary sequence  $\{s'_i\}$  and an error sequence  $\{e_i\}$ .

$$\hat{e}_j = \overline{s'_{i-j} e_i} \quad (8-1)$$

and the product of  $e_i$  and  $s'_{i-j}$  averaged over  $N$  samples. It is assumed that the input data is uncorrelated for this relation to hold. Lucky has shown that the distortion  $D_{\text{peak}}$  is convex with one global minimum, this allows iterative procedures to be used for incrementing the tap gains to their optimum values. The zero forcing algorithm adjusting the tap gain  $c_j$  of the linear feedforward equalizer in increments of  $\delta c_j$  such that  $\hat{e}_j$ ,  $j \neq h$ , becomes zero ( $e_j$ ,  $j = h$  which corresponds to the peak of the pulse). The tap increment is proportional and opposite in sign to the estimate  $e_j$  where

$$\delta c_j = -K \hat{e}_j \quad (8-2)$$

this algorithm can also be realised by only using sign function of



$e_i$  and incrementing by a fixed step size as explained in the ref. (70).

A disadvantage of zero-forcing algorithms is that even with an ideal reference training signal, convergence to the optimum solution can not be guaranteed. If the distortion is severe this algorithm may not converge at all, because the received data sequence and error sequence are decision-directed. Also the rate of convergence will be affected by the error rate during the initial phase of equalization. The zero forcing equalizer forces zeros in the equalized channel impulse response at  $\{e_j\}$ ,  $j \neq h$  and the main pulse at  $e_h$  being adjusted to give  $e_h = 1$ .

The algorithm for adaptive equalization of decision-directed intersymbol interference cancellation technique (see section 5-1, system 2) has the basic strategy for adjusting the tap gains as that for the zero-forcing algorithm in as much as it is based on the measurement of correlation between the message sequence  $\{s_i\}$  and the error sequence  $\{e_i\}$ . This results, as previously discussed, in each tap  $c_j$  forcing a zero in the equalized sampled impulse response. It was shown in ref. (28) for this type of equalization that convolution does not occur and there is no induced intersymbol interference outside the range of the equalizer. Furthermore, the action of forcing zeros ensures that the optimum solution i.e. minimization of the distortion factor  $D_{\text{peak}}$  has been achieved. Also with an ideal reference training signal, zero-forcing algorithm with this technique of equalization will enable the equalizer to converge to the optimum solution for all channel distortions. For a given length of equalizer, the residual intersymbol interference is in general less with decision directed equalization because it does not enhance the noise as linear equalizers do.

(8-2-2) Mean Square Error Algorithm

The criterion of performance for the mean square error algorithm is defined as the minimization of the mean-square error between the channel output and the transmitted message sequence  $\{s_i\}$ . This distortion is defined as  $D_{rms}$  in eqn. (4-26). It was shown in ref. (83) that an iterative procedure can be used to adjust the tap gains to the optimum solution by incrementing in steps proportional and opposite in sign of the cross-correlation function between channel output  $r_i$  and error signal  $e_i$ . Thus the  $j^{th}$  tap increment is given by

$$\partial c_j = - \overline{K r_{i-j} e_i} \quad (8-3)$$

The strategy for incrementing the tap gains involves the measurement of the cross correlation between the error signal and delayed channel output  $r_i$ . This requires complex correlator multipliers which are linear in both ports. However, it has been shown in ref. (84) that either or both of  $e_i$  and  $r_{i-j}$  can be replaced by their respective sign functions without significantly affecting residual equalized mean-square error, though with a penalty on the speed of convergence.

Zero-forcing and mean-square-error algorithms require random data, for convergence. Also equalized inputs are needed for the estimate of the error  $e_i$ . The main problem is that since the initial error estimates are made from the distorted signal and the estimated reference, the error signal is not reliable. One way of overcoming this problem is to transmit a known data pattern during a training period and to generate the same pattern in the receiver. Once the

locally generated pattern is synchronized with the received signal, it can be used as an ideal reference to replace the estimated reference signal. This estimated reference becomes identical to the ideal reference and the latter is no longer required. By switching over to the estimated reference the equalizer can operate adaptively, and the actual data can be transmitted.

(8-3) Estimation of channel response for Adaptive Operation of Modulo-n Feedback Equalizer (system 1)

The techniques of adaptive algorithms for equalizers located in the receiver and discussed in previous sections do not have the problems associated with modulo-n feedback equalizers. The two main problems are (i) that the knowledge about the output sequence  $\{b_i\}$  from the modulo-n feedback equalizer in the transmitter is needed for the adaptive operation its value is unavailable in the receiver so has to be assumed to be known. (ii) difficulty in transmitting the information about sampled impulse response of the channel back to the transmitter. An ideal return channel which introduces a small delay during the transmission is assumed. This information is transmitted as analog samples, but in practice they will be transmitted in digital format. Using assumptions (i) and (ii) the objective is to find a suitable algorithm and assess the system without several interacting factors which may influence the result.

(8-3-1) Estimation of  $A(z)$  by solution of linear simultaneous equations (Solution 1)

The input and output signals of the baseband channel whose  $z$  transform is  $A(z)$ , are linearly related. The samples of input signal elements into the channel are  $\{b_i\}$  the sampled impulse response

of the channel  $\{a_j\}$  for values of  $j = 0$  to  $k$ , and the channel output samples of signal elements will be related by

$$r_i = \sum_{j=0}^k b_{i-j} a_j \quad (8-4)$$

for positive integers of  $i$ , where it is assumed that until stated channel noise is absent. With the knowledge of  $\{b_i\}$  and  $\{r_i\}$  the sample values of channel impulse response  $\{a_j\}$  can be estimated by solving a system of  $k + 1$  linear simultaneous equations. For example for  $k = 2$  the equations will be

$$r_1 = b_1 a_0 \quad (i)$$

$$r_2 = b_2 a_0 + b_1 a_1 \quad (ii)$$

$$r_3 = b_3 a_0 + b_2 a_1 + b_1 a_2 \quad (iii) \quad (8-5)$$

The solution will be as follows, starting from  $r_1$  and solving for  $a_0$  by finding  $a_0 = r_1/b_1$ . Then substitute  $a_0$  into equation (ii) for  $r_2$ , to give  $a_1$ . By substituting  $a_0$  and  $a_1$  into equation (iii)  $a_2$  is evaluated. The next set of equations to determine  $\{a_j\}$  be from equations relating  $r_2$ ,  $r_3$  and  $r_4$  to  $b_2$ ,  $b_3$  and  $b_4$ , it being assumed that the  $\{a_j\}$  values are slowly varying with time. There will be intersymbol interference components in  $r_2$  and  $r_3$  due to  $b_1$  and these should be cancelled by the following operation before starting the solution of the next set of equations at time instant  $t = 4T$

$$r'_2 = r_2 - b_1 \bar{a}_1$$

$$r'_3 = r_3 - b_1 \bar{a}_2$$

$$r'_4 = r_4 = b_4 a_0 + b_3 a_1 + b_2 a_2 \quad (8-6)$$

and so on. The solution at time instant  $t = 4T$  will start by finding  $a_0$  from  $r'_2/b_2$ . The estimation operation will be repeated every sampling instant.  $\{\bar{a}_j\}$  represents the running average calculated after each solution due to time varying  $\{a_j\}$  and when additive noise is present in  $\{r_i\}$  the estimates of  $\{a_j\}$  will not be very accurate. Also  $\{b_i\}$  can be in error due to additive noise. The running average will then be updated after every solution by new estimates of  $\{a_j\}$  by the following operation

$$\bar{a}_j = a_j \Delta + (1 - \Delta) \bar{a}'_j. \quad j = 0, 1, 2, \dots, k \quad (8-7)$$

Where  $\bar{a}'_j$  is the last average, and  $\Delta$  is the weighting factor which can have a magnitude 0 to 1 depending on the noise variance and rate of change of  $\{a_j\}$  from sample to sample which can be optimized according to systems requirements.

This technique is well suited for adaptive equalization by decision directed cancellation of intersymbol interference for time varying channels (equalization system 2), because the equalizer uses the values of  $\{\bar{a}_j\}$  in the feedback transversal filter, therefore it can equalize slowly time varying channels from these estimates. This technique of estimating sampled impulse response of the channel for modulo-n feedback equalizer, (system 1) frequently resulted in inaccurate estimates of  $\{a_j\}$ . This is because  $\{b_i\}$  had values varying from -2 to 2 for (modulo-4), and when the value of  $b_i$  is close to zero there is difficulty in making the calculation  $a_0 = r'_1/b_i$ . Tests are carried out for different sampled impulse responses  $A(z)$  and the same failures were observed in the solution of linear simultaneous equations

once  $a_0$  was different from the actual value then all  $\{a_j\}$  sequences resulted in errors due to substituting the  $a_j$  in the next set of equations. Small changes in  $\{b_i\}$  and the accuracy of  $b_i$  when the latter is near zero also resulted in large variations in the solution of these simultaneous equations. This does not apply to system 2 where  $\{b_i\}$  is limited to  $\pm 1$  values only. This method of solution was found to be unreliable for modulo-n feedback equalizer.

### (8-3-2) Solution 2

The cross-convolution of the channel impulse response  $\{a_j\}$  and the sample values of transmitted signal elements  $\{b_i\}$  can be represented by matrix notation,

$$R = AB + W \quad (8-8)$$

where A is the row vector  $(a_0, a_1, \dots, a_k)$  whose components correspond to the sample values of the channel impulse response and B is the  $(k+1) \times (2k+1)$  matrix of rank  $k+1$  whose mth row is

$$\underbrace{0 \dots 0}_{m-1} \underbrace{b_i \dots b_{i+k-1} b_{i+k}}_{k+1} \underbrace{0 \dots 0}_{k+1-m}$$

(8-9)

where m can have integer values 1 to  $k+1$ . The row vector R has the  $k+1$  sample values of the received signal elements  $[r_i \dots r_{i+k-1} r_{i+k}]$ . W is a n-component row vector whose components are the sample values of additive white Gaussian noise with zero mean and variance  $\sigma^2$

$$[w_i \dots w_{i+k-1} w_{i+k}]$$

It was shown in ref. (8) that when the receiver knows the  $\{b_i\}$

but has no prior knowledge of the  $\{a_j\}$  or  $\sigma^2$  the best estimate it can make of A is the row vector U, where

$$\begin{aligned} U &= RB^T (BB^T)^{-1} \\ U &= (AB + W)B^T (BB^T)^{-1} \\ U &= ABB^T (BB^T)^{-1} + WB^T (BB^T)^{-1} \\ U &= A + WB^T (BB^T)^{-1} \end{aligned} \quad (8-10)$$

U has  $k + 1$  components which are estimates of the  $k + 1$  components of A respectively. The  $(k+1) \times (k+1)$  matrix  $BB^T$  is real, symmetric, and positive definite it can be shown that an inverse always exists.

After the first estimate the row vector R is modified by subtracting intersymbol interference components from all except the first component of R then shifting all  $k$  components to the left, and last component of R will be the new sample  $r_{i+k+1}$  for all second set of estimates. This cancellation is similar to the example given in eqn. (8-6). Also the averaging of estimates will be done as in eqn. (8-7). Now the matrix B and the row vector W will have the samples  $b_{i+1+k}$  and  $w_{i+1+k}$  at time instant  $t = (i+k+1)T$ . The estimation operation will be repeated every sampling instant to obtain new estimates of time varying channels.

This technique has been tested for modulo-n feedback equalizer and correct estimate of the sampled impulse response of the channel have been obtained in every estimate. Solution 2 and 1 was found suitable for system 2 but only solution 2 resulted in correct estimates for modulo-n feedback equalizer (system 1). However, the approach explained in this section is very complicated in terms of the number of sequential operations needed for an estimate when  $k$  takes

large values. Such operations become impractical to implement.

(8-3-3) Zeroforcing Algorithm applied to modulo-n feedback equalizer

Zeroforcing algorithm is applied to set the tap gains of modulo-n feedback equalizer such that to minimize distortion  $D_{\text{peak}}$  of eqn. (4-25). The adjustments to the tap gains of the feedback transversal filter are made periodically since the characteristics of normal telephone channels change very slowly and the speed of the return line available for such information transmission does not allow frequent changes. The error samples  $\{e_i\}$  are derived from finding the discrepancy between the desired and actual data levels which is the difference between input and output signals to the detector in the receiver. The error samples  $\{e_i\}$  are then correlated simultaneously with each element of the sequence at the output of the modulo-n feedback equalizer in the transmitter, namely  $b_{i-j}$  for  $j = 0, 1, \dots, k$ . To accomplish this the  $k + 1$  values of  $b_i$  are stored in a tapped delay line in the receiver. The output of the correlators (multipliers and accumulators) are sampled at  $N$  clock intervals and modifications are made to the tap gains in the feedback transversal filter of modulo-n equalizer. The  $j^{\text{th}}$  tap is incremented by an amount which is opposite in sign and proportional to  $\partial c_j$  where

$$\partial c_j = K \overline{e_i b_{i-j}} \quad (8-11)$$

For simplification of this correlation only the sign of  $b_i$  is used. Then eqn. (8-11) becomes,

$$\partial c_j = K \overline{e_i \text{ sign}(b_{i-j})} \quad (8-12)$$



In eqn. (8-12) sign  $b_{i-j}$  can only have  $\pm 1$  values but  $e_i$  can still have a range of values, when compared to eqn. (8-11), the algorithm requires multipliers which are linear at both inputs, and accumulators to average the product. Thus, the use of sign function simplifies the hardware requirements.

To test the convergence of the algorithm of eqn. (8-12) the following assumptions were made. The knowledge about the sampled impulse response of the channel is not exact and there is a small error in every sample value  $\{a_j\}$  used in the modulo-n feedback filters. The knowledge of the sequence  $\{b_i\}$  is not available in the receiver and it is generated by a coder from the detected binary values  $\{s_i\}$ . The feedback transversal filter in the coder has the same tap gains as the transmitter modulo-n feedback equalizer. To prevent the coder output  $\{b_i\}$  diverging from the actual  $\{b_i\}$  a resetting sequence of k zeros is transmitted periodically at intervals of 500 clock instants and both modulo-n filters reset to zero state during this interval. This operation is similar to the resetting of the coder described in chapter (7) in connection with error detection and retransmission. For these tests the channel impulse responses are chosen to represent a wide range of distortions. Simulations are carried out in the absence of noise in the data and return channels unless specified, and the return channel is assumed ideal i.e. it has no distortion, gain, or attenuation. For the information signal to be transmitted from the receiver to the transmitter a small delay of 50 clock instants is introduced between sampling the accumulators and incrementing the tap gains of modulo-n filters, and all the tap gains are incremented at the same instant. The adaptive modulo-n

feedback equalizer is shown in fig. (8-1), with switch  $Q_1$  in position 1 for eqn. (8-12) to apply. The accumulators are summing and memory devices, which are sampled and set to zero every 500 clock instants, i.e.  $N = 500$ . The sample values from the accumulators are averaged by dividing by the number of samples  $N$  and then scaling by a constant  $K$ . The magnitude of this scaling constant for the tap increments is found empirically. For channel responses with roots whose magnitude are less than unity (in the  $z$  plane)  $K = 1$ . For roots whose magnitude is greater than unity and also for the combination of roots with magnitude less than unity in the  $z$  plane  $K = 1/4$ , was found to give the fastest convergence.

First a sampled impulse response of the channel is chosen and an initial error is introduced into the tap gains of the modulo- $n$  feedback equalizer ( $A'(z) - 1$ ) and in the multiplier in the receiver, where  $A'(z) = A(z) + \partial A(z)$ .  $[\partial A(z)]$  is the error of the sampled impulse response of the channel at the start of the transmission for the equalizer. If the sampled values of  $\partial A(z)$  are chosen from a normal distribution, with zero mean and unit variance, then these samples are scaled to introduce a small error  $\partial A(z)$ ,  $\partial A(z)$  can be represented as a  $k + 1$  row vector  $\partial A$ . A measure of this distortion caused by the error vector is represented by its length. To investigate convergence the length of the error vector is given as a percentage with respect to the length of actual channel vector  $A$ , namely,

$$\text{Error \%} = \frac{|\partial A|}{|A|} \times 100.$$

When  $\partial A$  becomes zero then the error % also becomes zero, and the

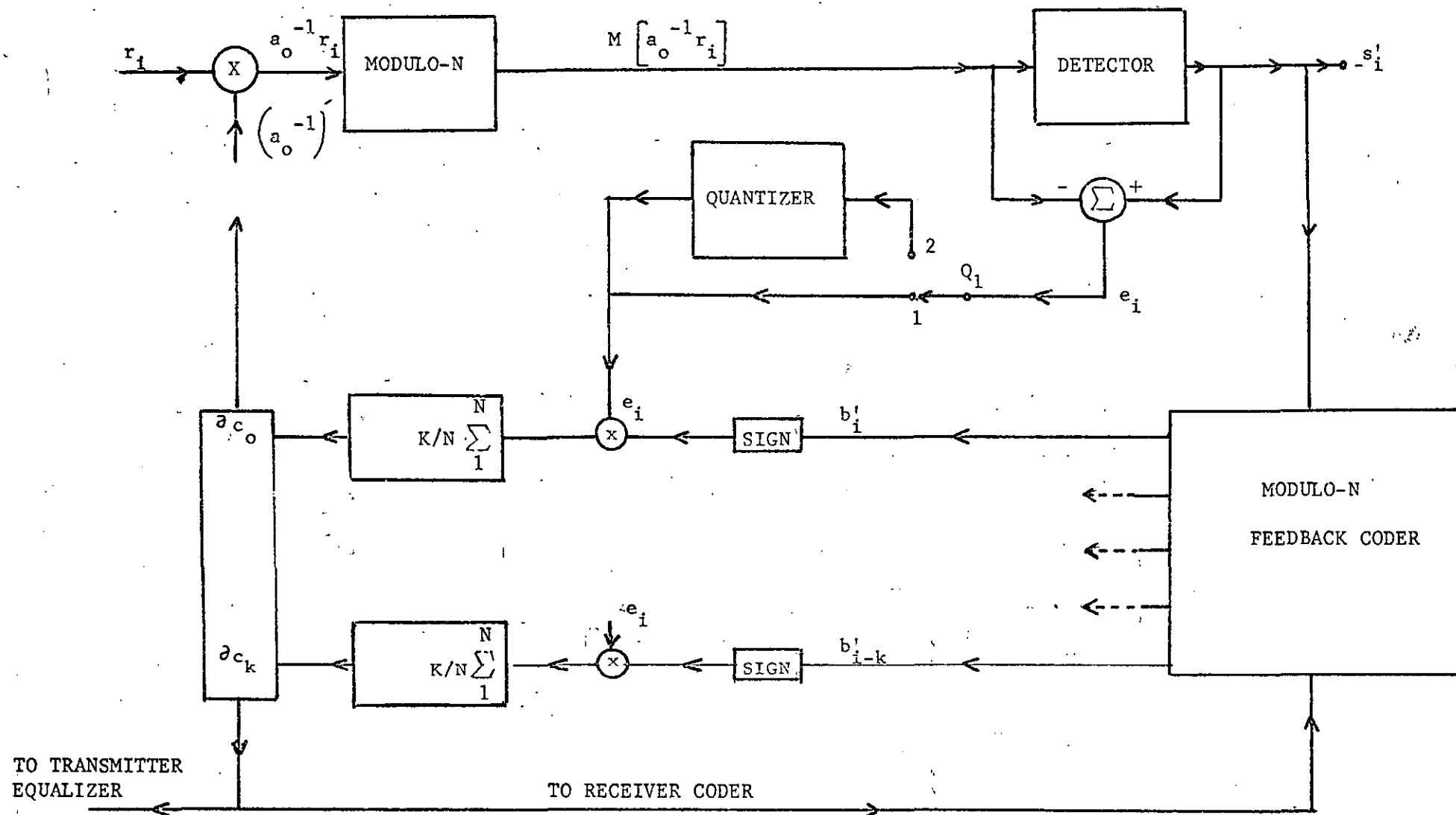


FIGURE (8-1) THE DERIVATION OF TAP INCREMENTS FOR MODULO-N FEEDBACK EQUALIZER

adaptation is completed. Tests were made for several channels given in table (8-1). The graphs displaying convergence are shown in fig. (8-2) to (8-9). The incrementation of tap gains is achieved by evoking eqn. (8-12), the same random binary input sequence is employed for all the tests. The horizontal axis of the graphs in figs. (8-2) to (8-18) represents time i.e. one bit transmitted per clock interval, whereas the vertical axis signifies the percentage error. This error is measured after incrementing the tap gains of the modulo-n feedback equalizer. To illustrate the choice of scaling factor K consider figs. (8-3) and (8-4) which are for scaling factor  $K = 1$  and  $K = 1/4$  for the same channel respectively. After observing the convergence of the adaptive equalizer further simplification to eqn. (8-12) has been done by quantizing the error  $e_i$  to three levels;  $(-\Delta, 0, \Delta)$ , by switch position  $Q_1$  at 2 in fig. (8-1),

If  $-\Delta/2 \leq e_i \leq \Delta/2$  then  $e_i$  quantized as 0  
 If  $e_i > \Delta/2$  then  $e_i$  quantized as  $\Delta$   
 If  $e_i < -\Delta/2$  then  $e_i$  quantized as  $-\Delta$

The jth tap increment is given by

$$\Delta c_j = K \text{ quantized } (e_i) \text{ sign } (b_{i-j}) \quad (8-13)$$

and is incremented by an amount proportional to  $\Delta c_j$  and of opposite polarity. The choice of the magnitude of  $\Delta$  depends on system requirements. A large value of  $\Delta$  will cause the system to converge faster but will have a large residual error after convergence, whereas a small  $\Delta$  will cause a slow convergence but will have a small residual error. Figs. (8-9) and (8-10) demonstrate this where the choice of  $\Delta$  is 0.09 and 0.045 respectively for the same  $A(z)$ .

Table (8-1)

The Sampled impulse-responses of the channel used in the computer simulation tests for the iteration techniques of the adaptive modulo-n equalizer.

Channel No.	The sampled impulse response of the Channel $A(z)$						The Characteristic of the magnitude of the roots of $A(z)$ in the z-plane.
A1	0.445	0.723	1.000	-0.723	0.445		mixed roots
A2	0.147	0.263	0.483	0.550	0.611		roots greater than unity
A3	0.776	-0.543	0.155	0.271	-0.078		roots less than unity
A4	0.167	0.471	0.707	0.471	0.167		mixed roots
A5	0.558	0.618	-0.502	-0.218	0.110		roots less than unity
A6	0.284	0.584	0.629	0.419	0.079		mixed roots
A7	0.262	0.457	-0.653	-0.522	0.153		mixed roots

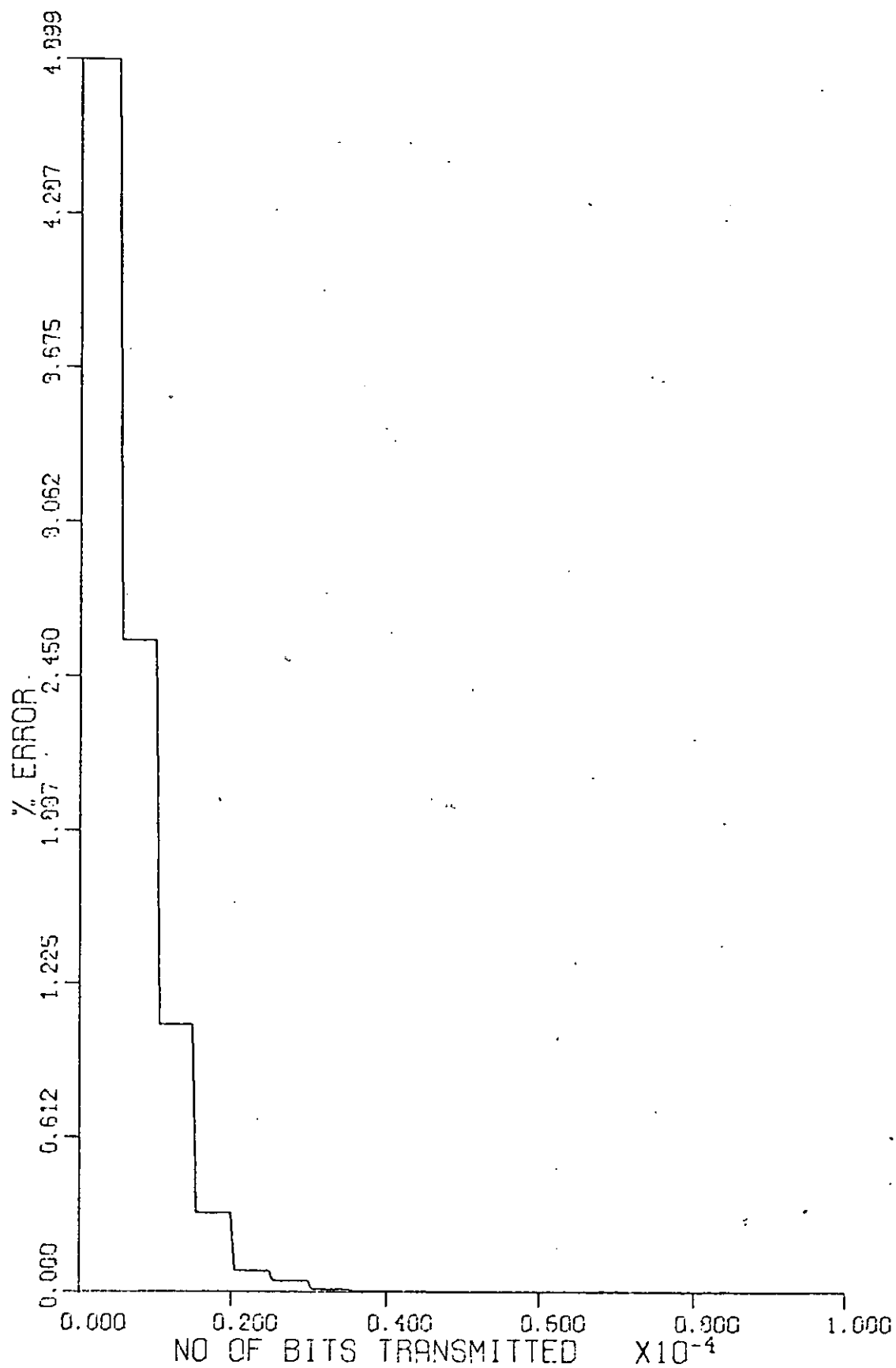


Figure (8-2) Incrementing technique of eqn. (8-12) for the equalizer of system 1, equalizing channel no. A2,  $K = 1/4$

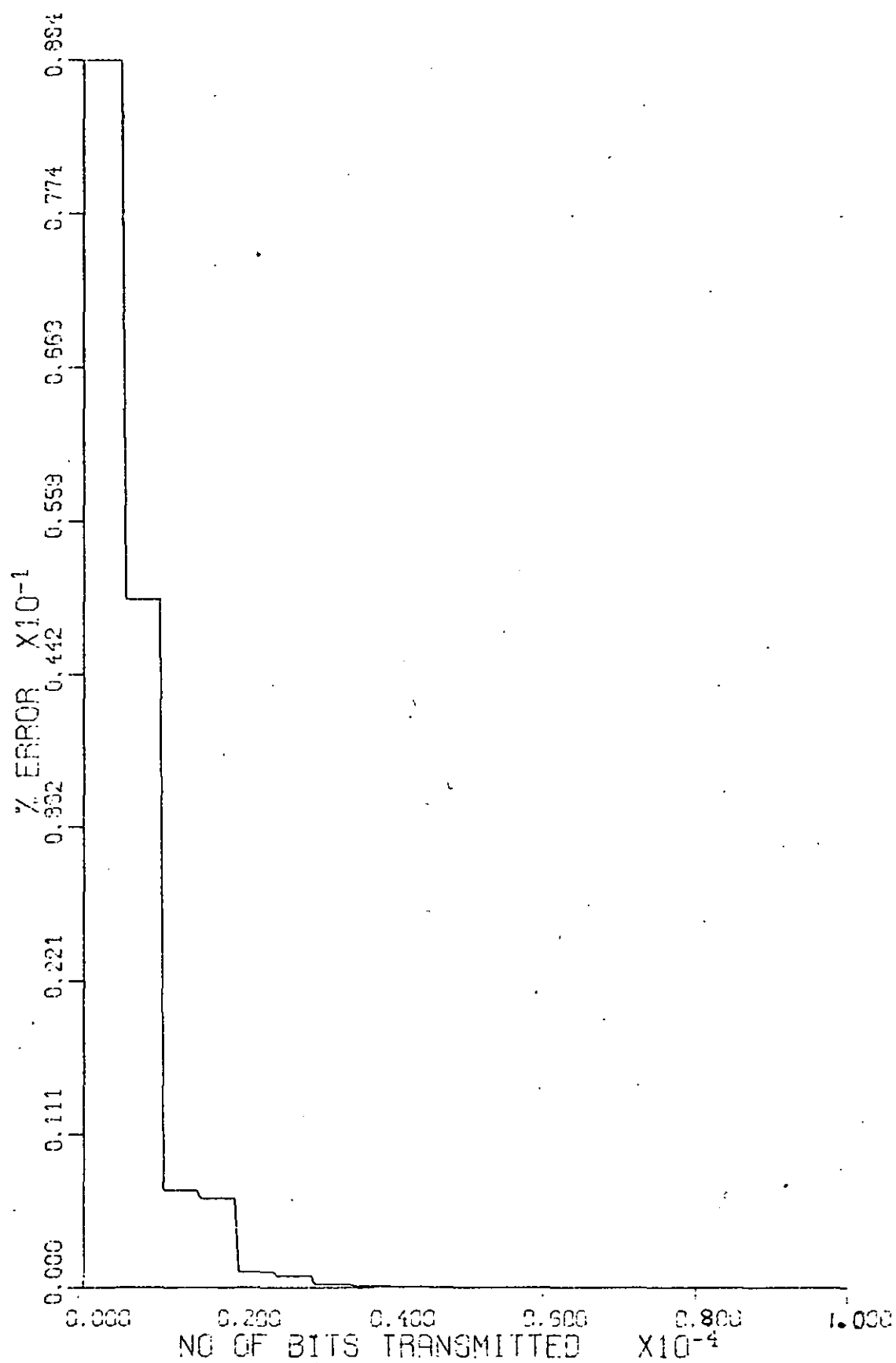


Figure (8-3) Incrementing technique of eqn.(8-12) for the equalizer of system 1, equalizing channel No. A3,  $K = 1$

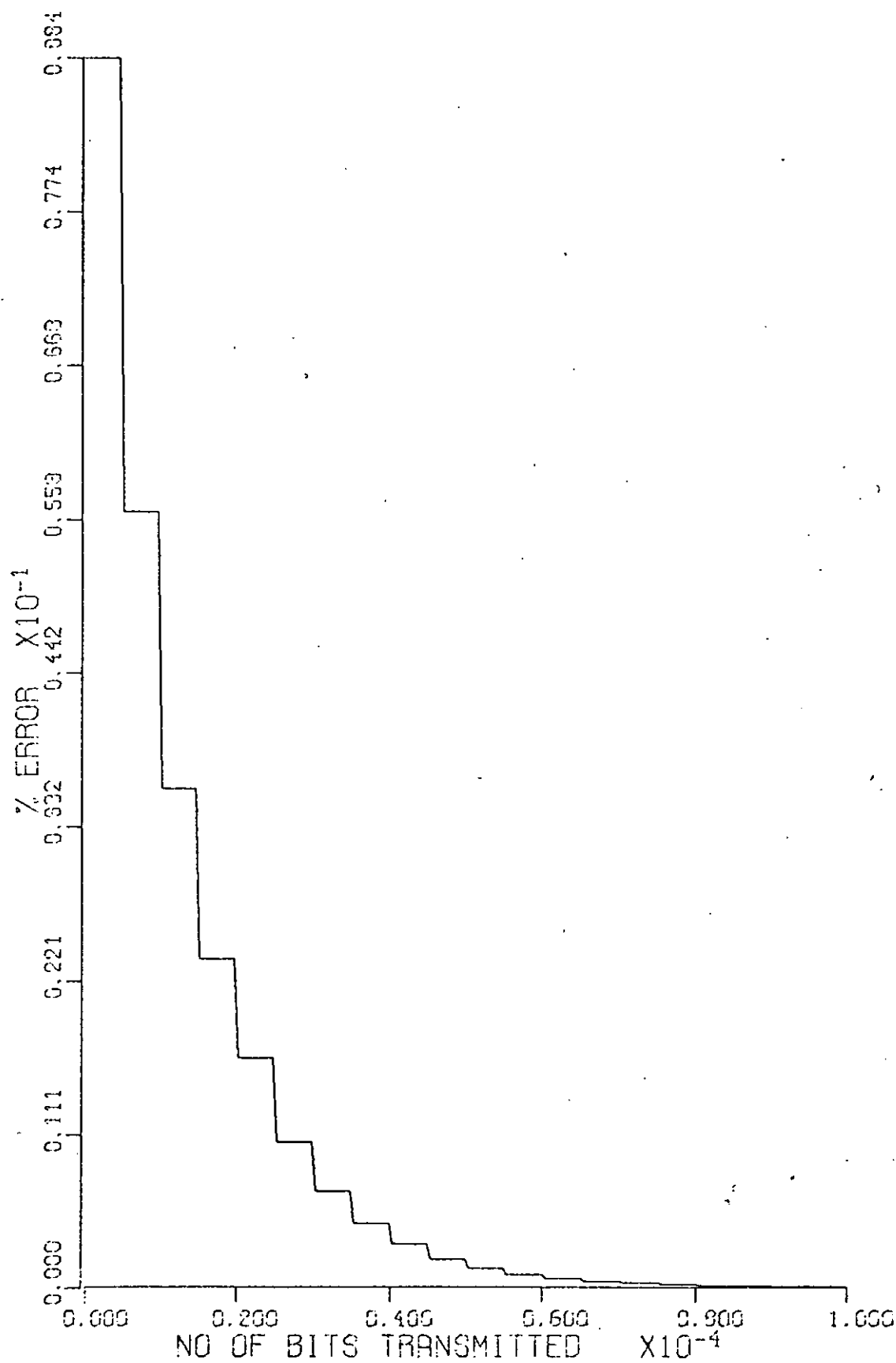


Figure (8-4) Incrementing technique of eqn. (8-12) for the equalizer of system 1, equalizing channel No. A3,  $K = 1/4$



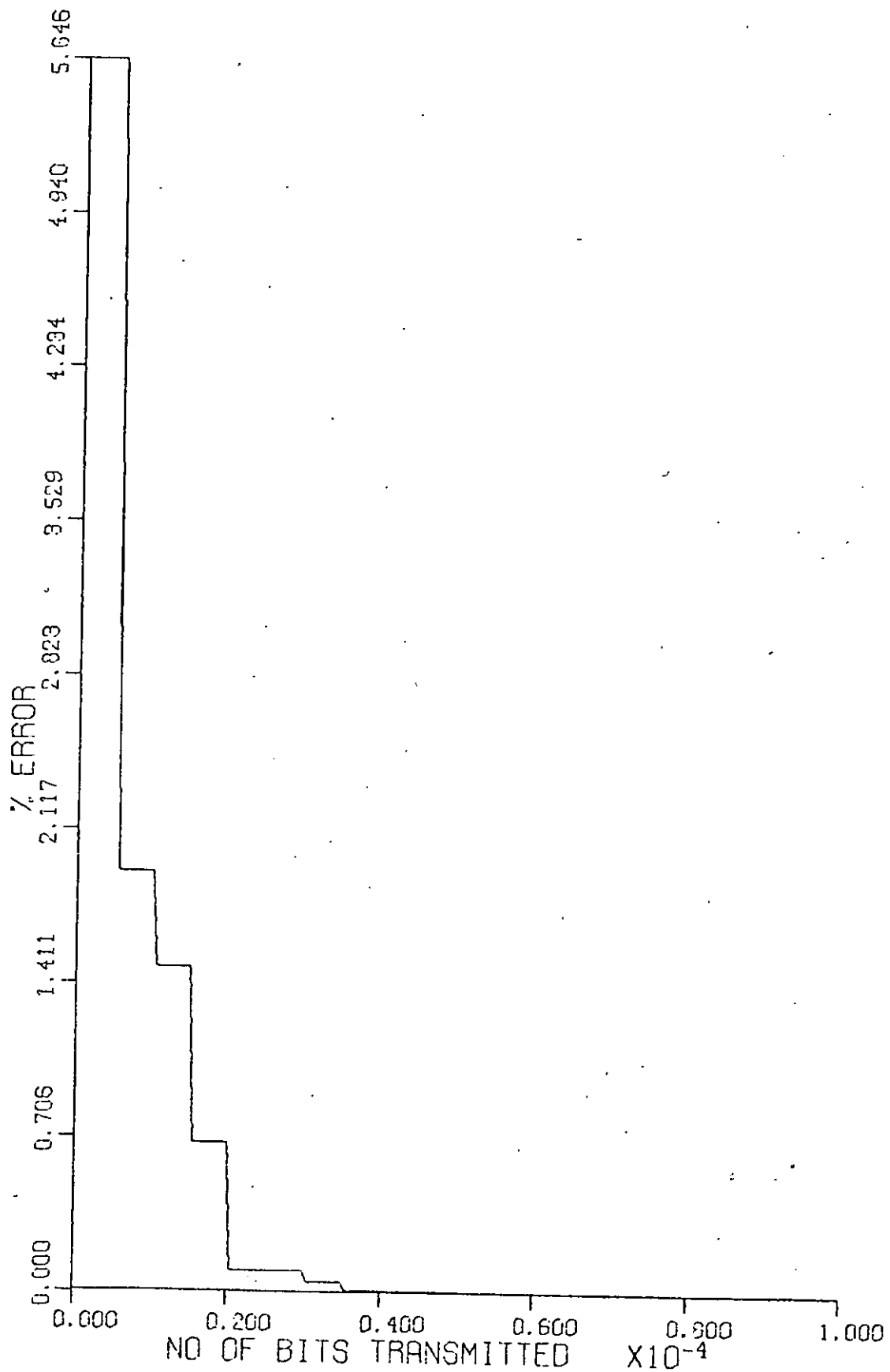


Figure (8-5) Incrementing technique of eqn. (8-12) for the equalizer of system 1, equalizing channel No. A4, K = 1/4

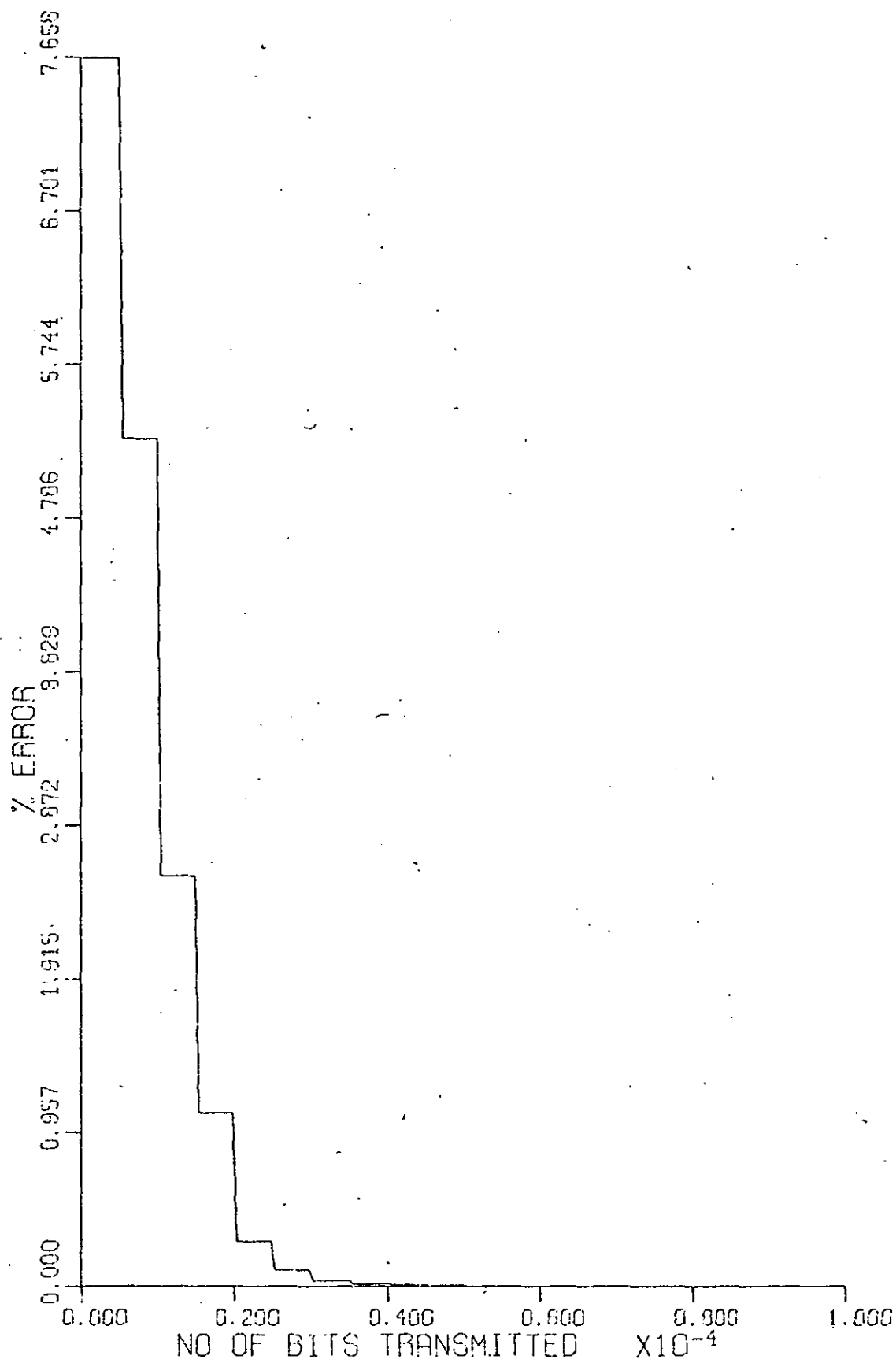


Figure (8-6) Incrementing technique of eqn. (8-12) for the equalizer of system 1, equalizing channel No. A5, K = 1

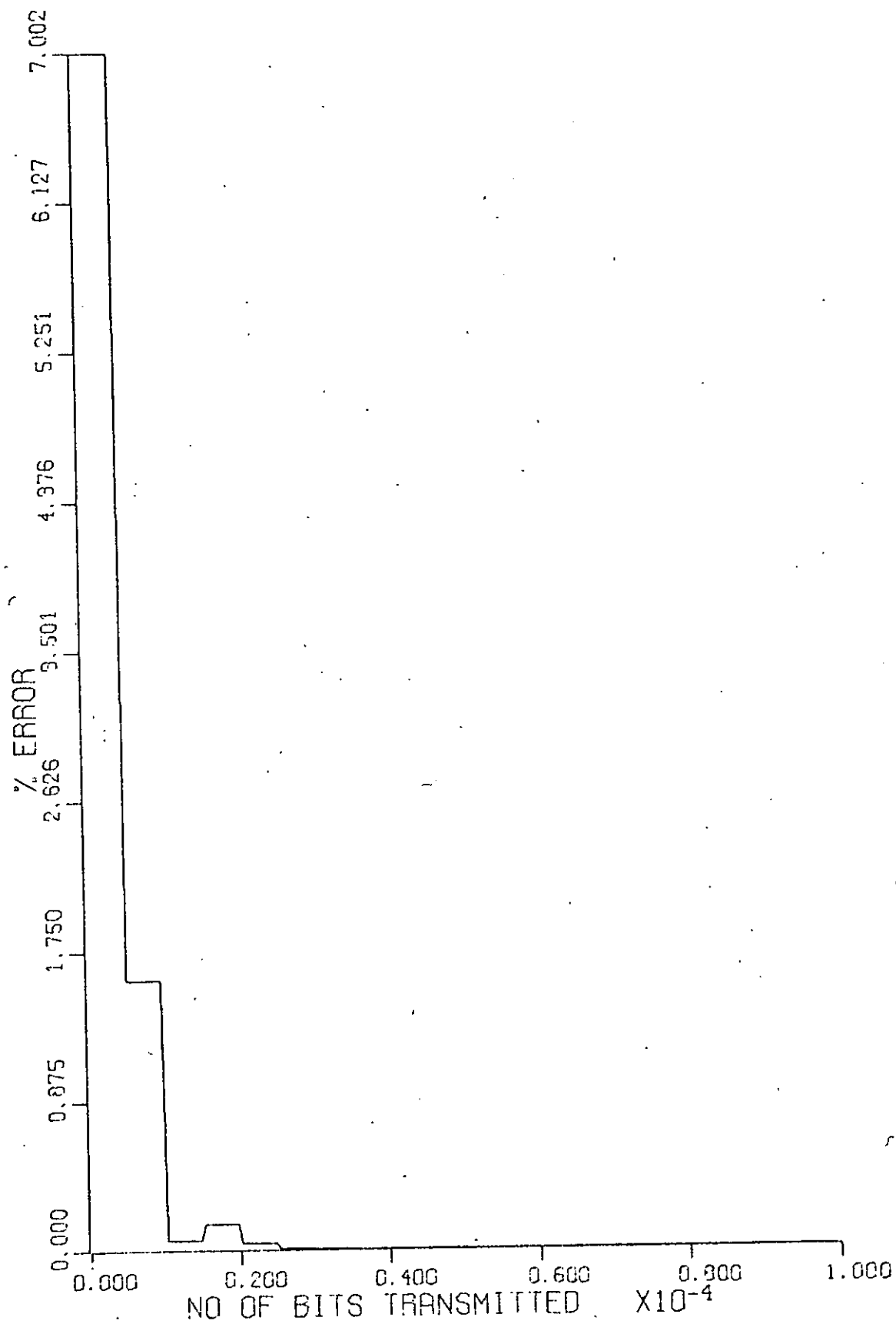


Figure (8-7) Incrementing technique of eqn. (8-12) for the equalizer of system 1, equalizing Channel No. A6,  $K = 1/4$

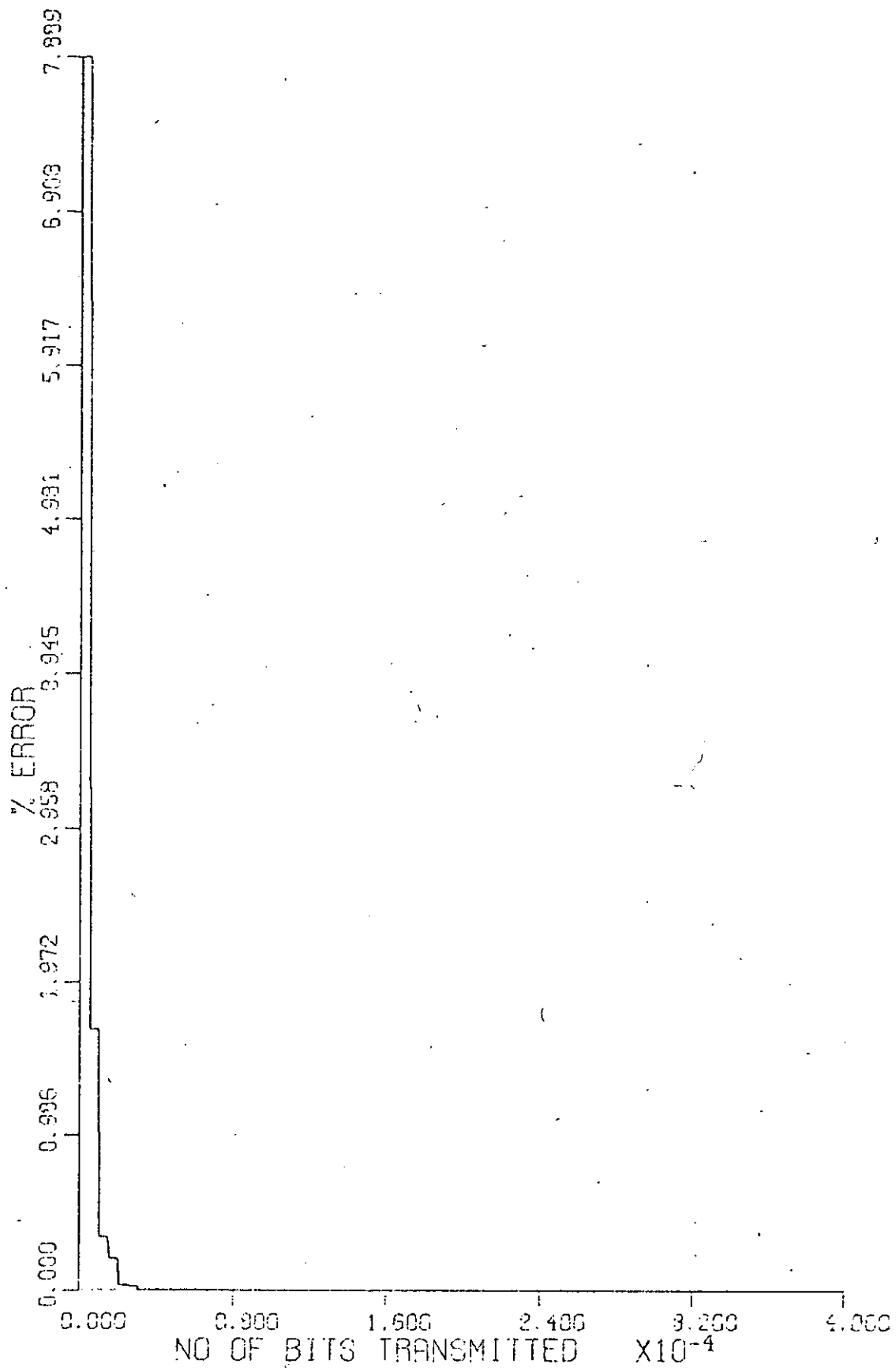


Figure (8-8) Incrementing technique of eqn. (8-12) for the equalizer of system 1, equalizing Channel No. A7,  $K = 1/4$

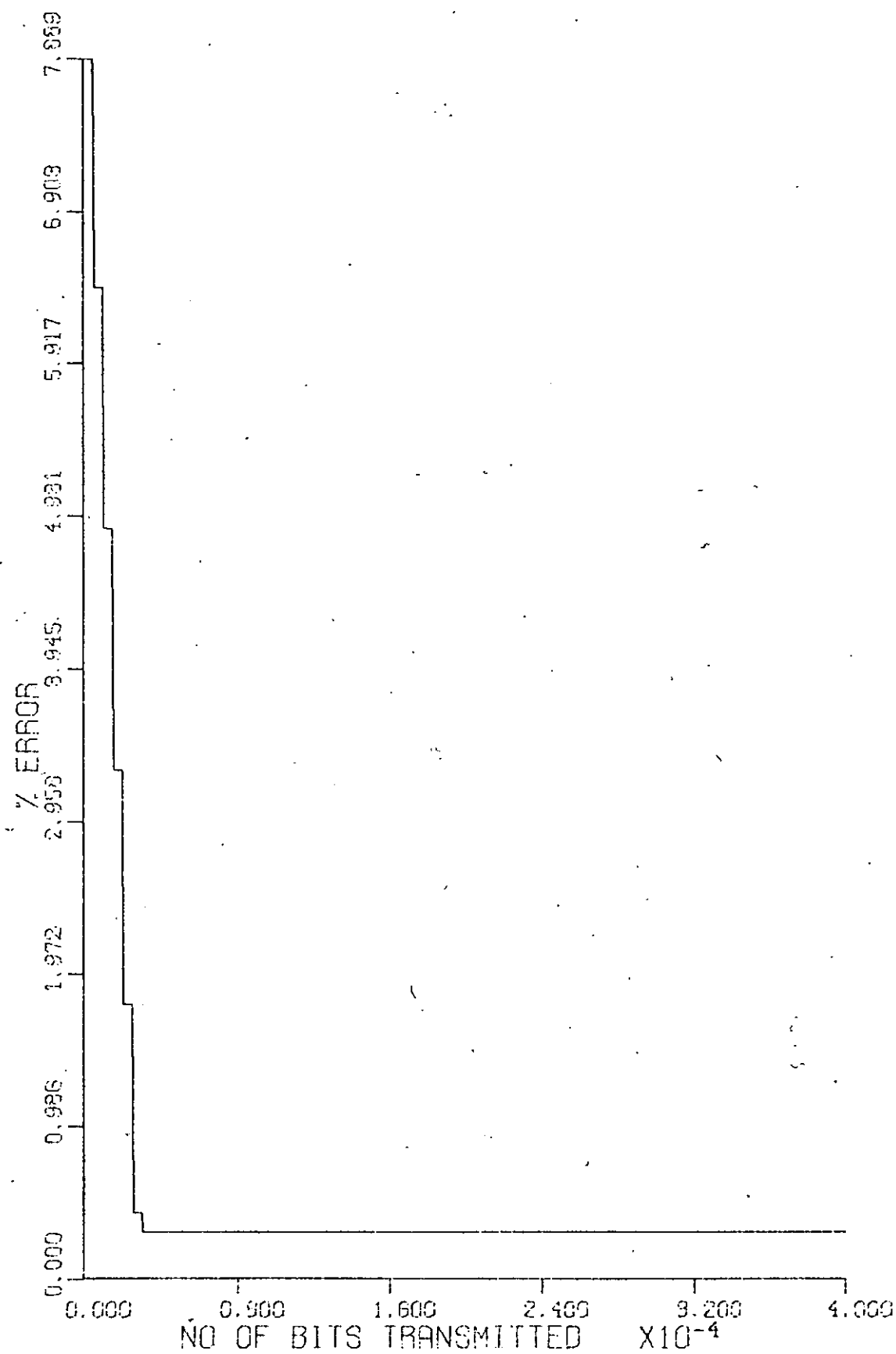


Figure (8-9) Incrementing technique of eqn. (8-13) for the equalizer of system 1, equalizing channel No. A7,  $K = 1/4$ ,  $|\Delta| = 0.09$

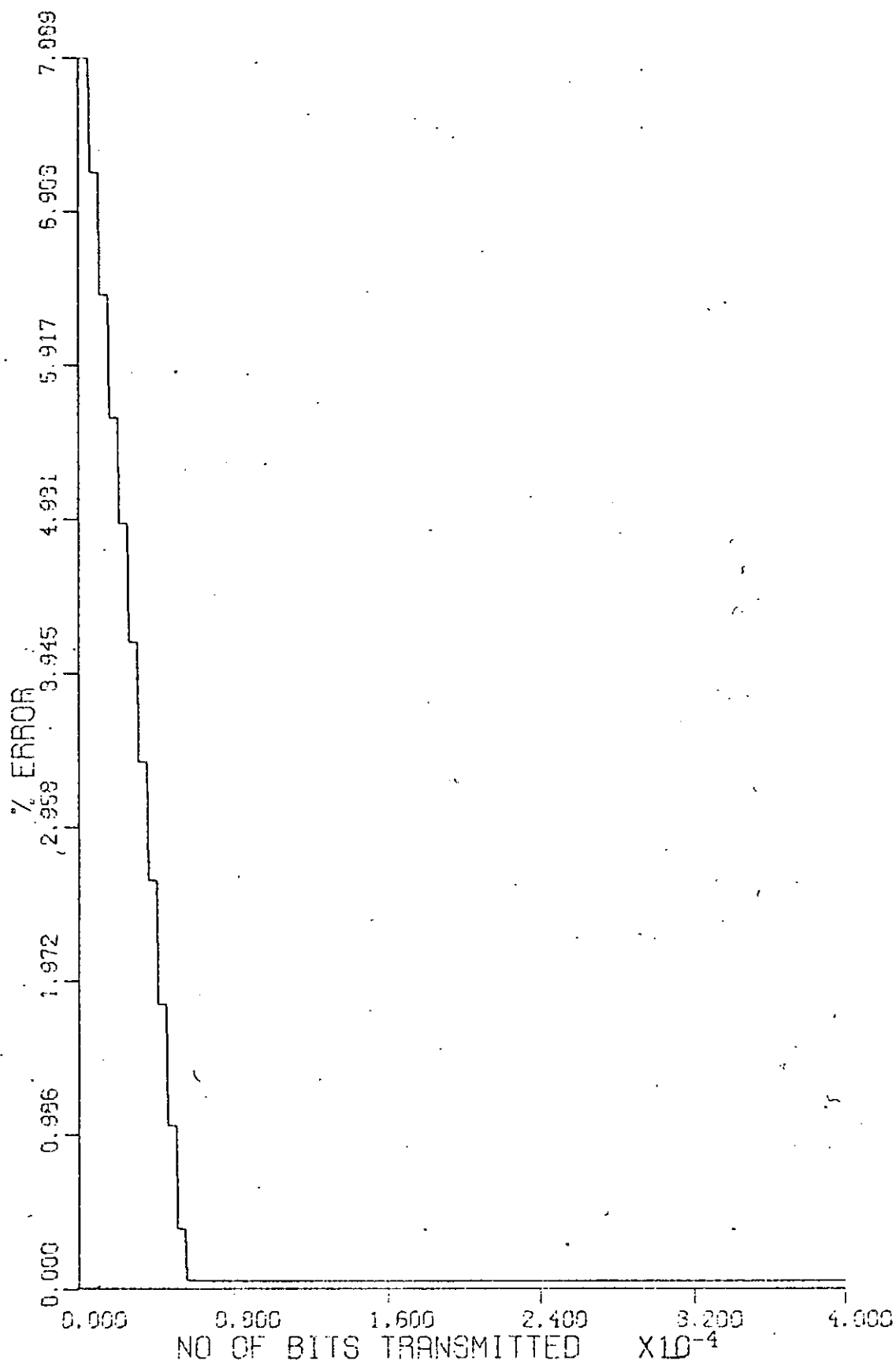


Figure (8-10) Incrementing technique of eqn. (8-13) for the equalizer of system 1, equalizing channel No. A7,  $K = 1/4$ ,  $|\Delta| = 0.045$

The effect of additive white Gaussian noise on the convergence of adaptive modulo-n equalizer is illustrated in figs. (8-11) to (8-13) for the same noise samples. The variance of noise distribution is constant throughout transmission and the return channel is assumed noiseless. The assumptions made for previous tests have been applied for these with the exception of noise in the data channel. The test was done for only one channel applying the techniques of eqns. (8-12) and (8-13). The incrementing technique of eqn. (8-12) is shown in fig. (8-11). A total of 5 binary digital errors occur during the first 500 clock periods, but there are no errors thereafter. The overall signal/noise ratio is 27.8 dB. Fig. (8-12) employs eqn. (8-13), where  $|\Delta| = 0.09$ . There are higher number of errors during the adaptation period, 10 binary errors occur in the first 2,500 bits transmitted, but no errors occurred after this period. For the choice of  $|\Delta| = 0.045$  and 9 binary errors occur in the first 1,000 bits, followed by 7 binary errors in the next 2,000 bits, but after 3,000 bits no errors occur in the subsequent 37,000 bits. This situation is illustrated in fig. (8-13). From these observations during the initial period of adaptation a high error rate results in a period clearly unsuitable for data transmission. A training signal is initially transmitted and the same known sequence is generated rather than detected in the receiver. This operation is used to estimate  $e_1$ . When the adaptation is completed i.e. the training period is over the system is switched to receive the data transmission. During the data transmission mode the adaptive algorithm keeps the equalizer tracking small changes in the sampled channel impulse response.

This adaptive modulo-n feedback equalizer has been tested for slowly

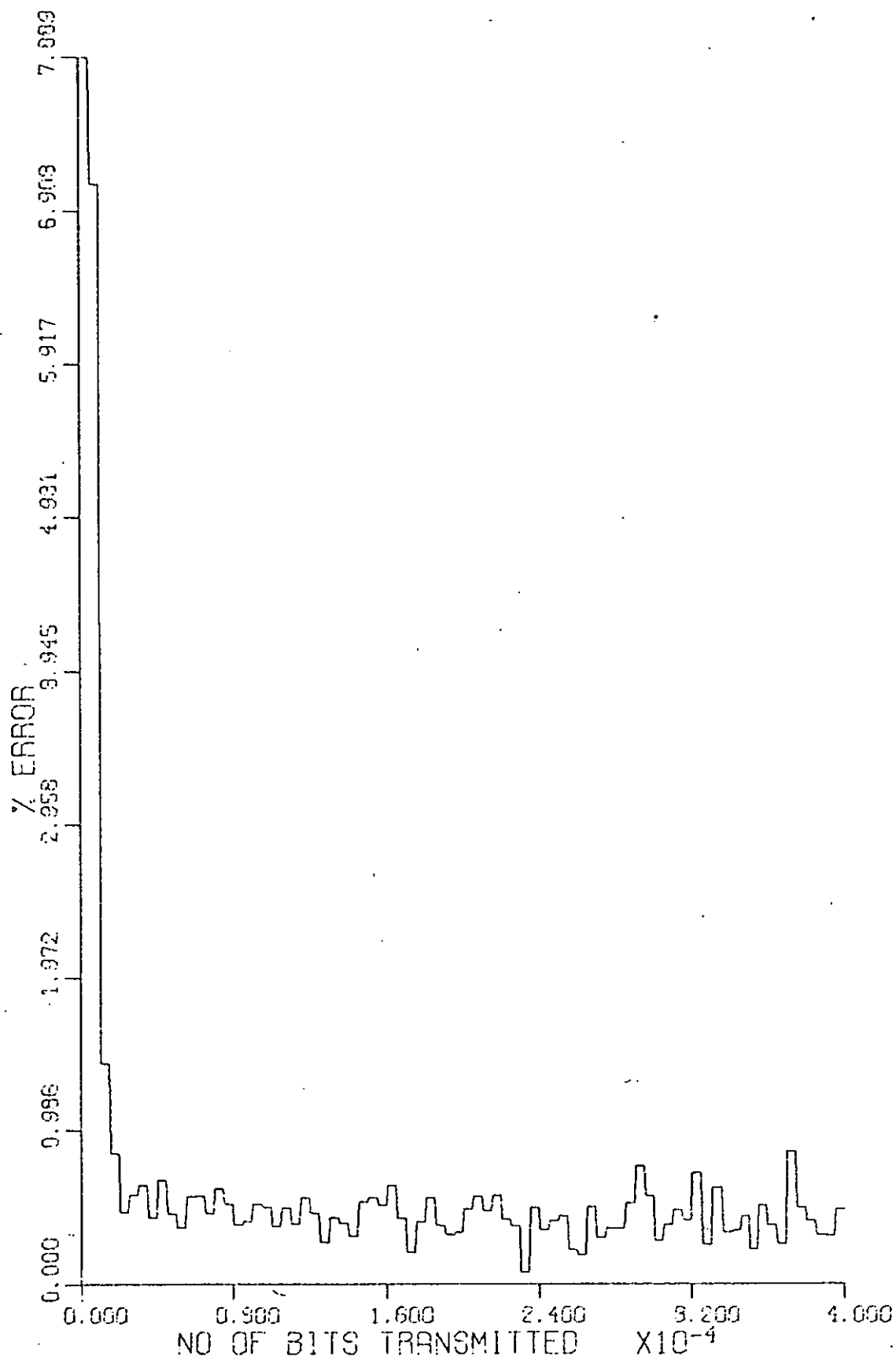


Figure (8-11) Incrementing technique of eqn. (8-12) for the equalizer of system 1, equalizing channel No. A7  $K = 1/4$ , in the presence of additive white Gaussian noise.



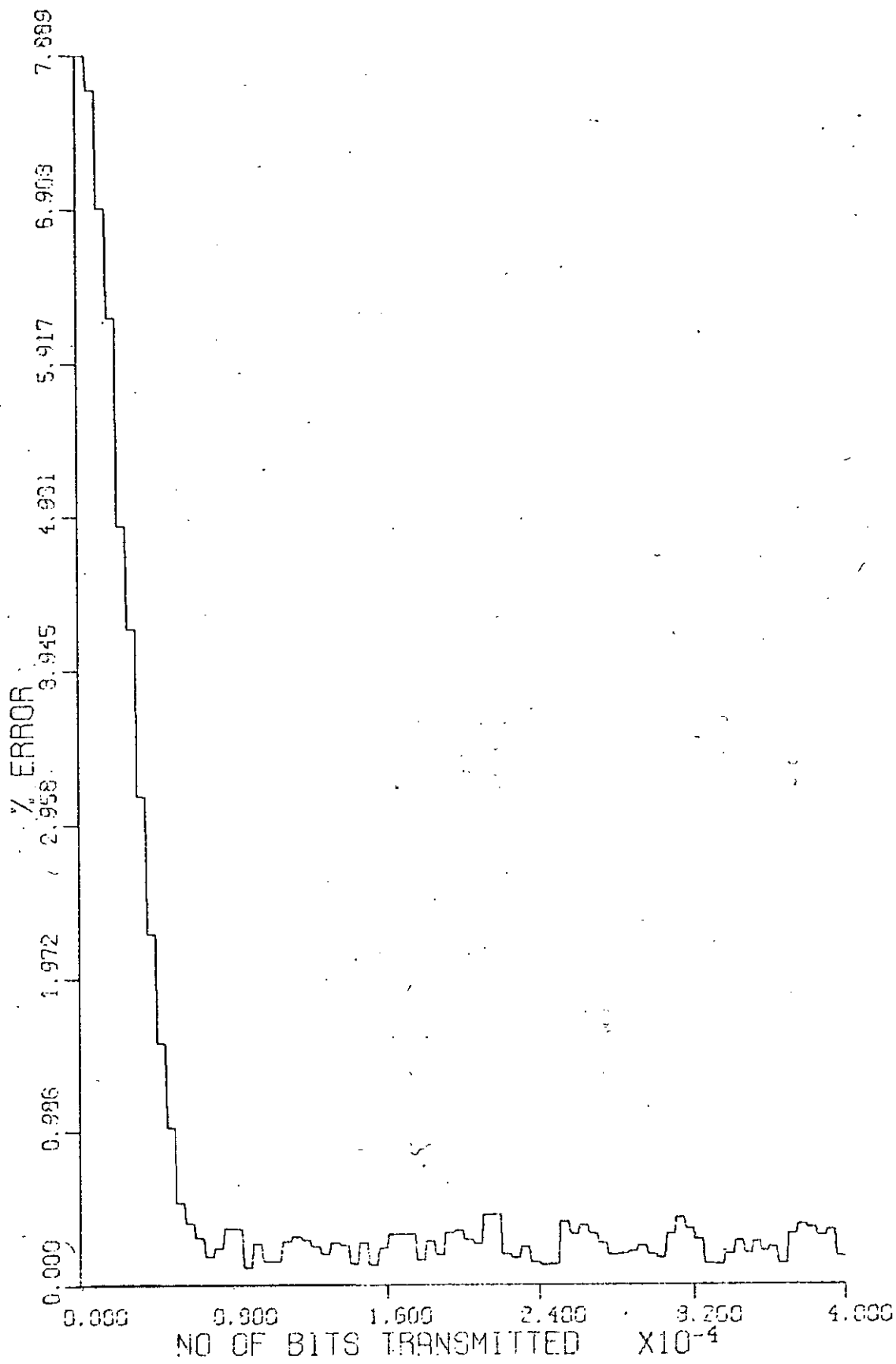


Figure (8-12) Incrementing technique of eqn. (8-13) for the equalizer of system 1, equalizing channel No. A7,  $K = 1/4$ ,  $|\Delta| = 0.09$ , and in the presence of additive white Gaussian noise.

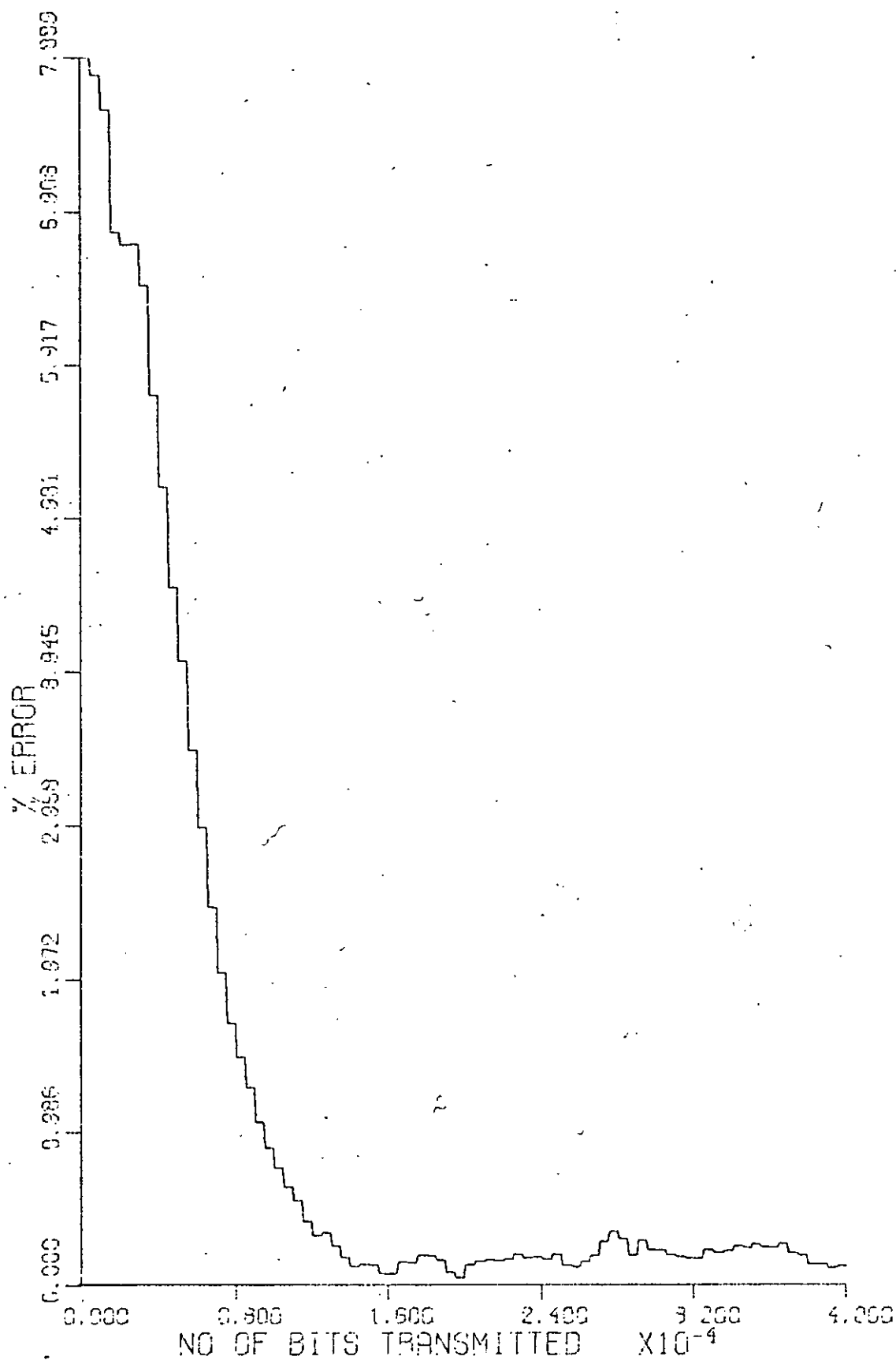


Figure (8-13) Incrementing technique of eqn. (8-13) for the equalizer of system 1, equalizing Channel No. A7,  $K = 1/4$ ,  $|\Delta| = 0.045$ , in the presence of additive white Gaussian noise.

time varying channel. The variation of the channel was generated by adding a small fixed increment (magnitude of 0.0005) to every sample value of the channel impulse response. The sampled channel impulse response is made to change slowly with time by adding an increment with a random polarity to every sample value. The polarity of each increment has equal probability and this incremental change is made every clock instant. The graph representing errors for this time variant channel is plotted in fig. (8-14). The first sampled channel impulse response at  $t = iT$   $i = 0$ , is taken as a reference and every 550 clock instants the sampled channel impulse response is compared with the reference channel and the percentage error is plotted for that instant. The modulo-n feedback equalizer follows the changes in the channel response by incrementing the tap gains of modulo-n feedback equalizer according to the relation given in eqn. (8-12). White noise is not introduced for this simulation and the updating of tap gains are implemented as previously explained. The graph of fig. (8-15) shows the error % determined by comparing the response that is known by the modulo-n feedback equalizer to the actual channel response at that instant. The two measurements are taken every 550 clock instants. One is taken before, and the second after updating the tap gains of the modulo-n feedback equalizer. In this graph the peaks represent the error % before correction and dips after correction at the same time instant. From this graph it can be seen that the algorithm keeps the modulo-n equalizer tracking the time varying channel response and results in a smaller error % when compared to an equalizer which does not have an adaptive operation, viz comparison between fig. (8-14) and (8-15). If the equalizer is a preset one after transmission of 10k bits of data the channel knowledge

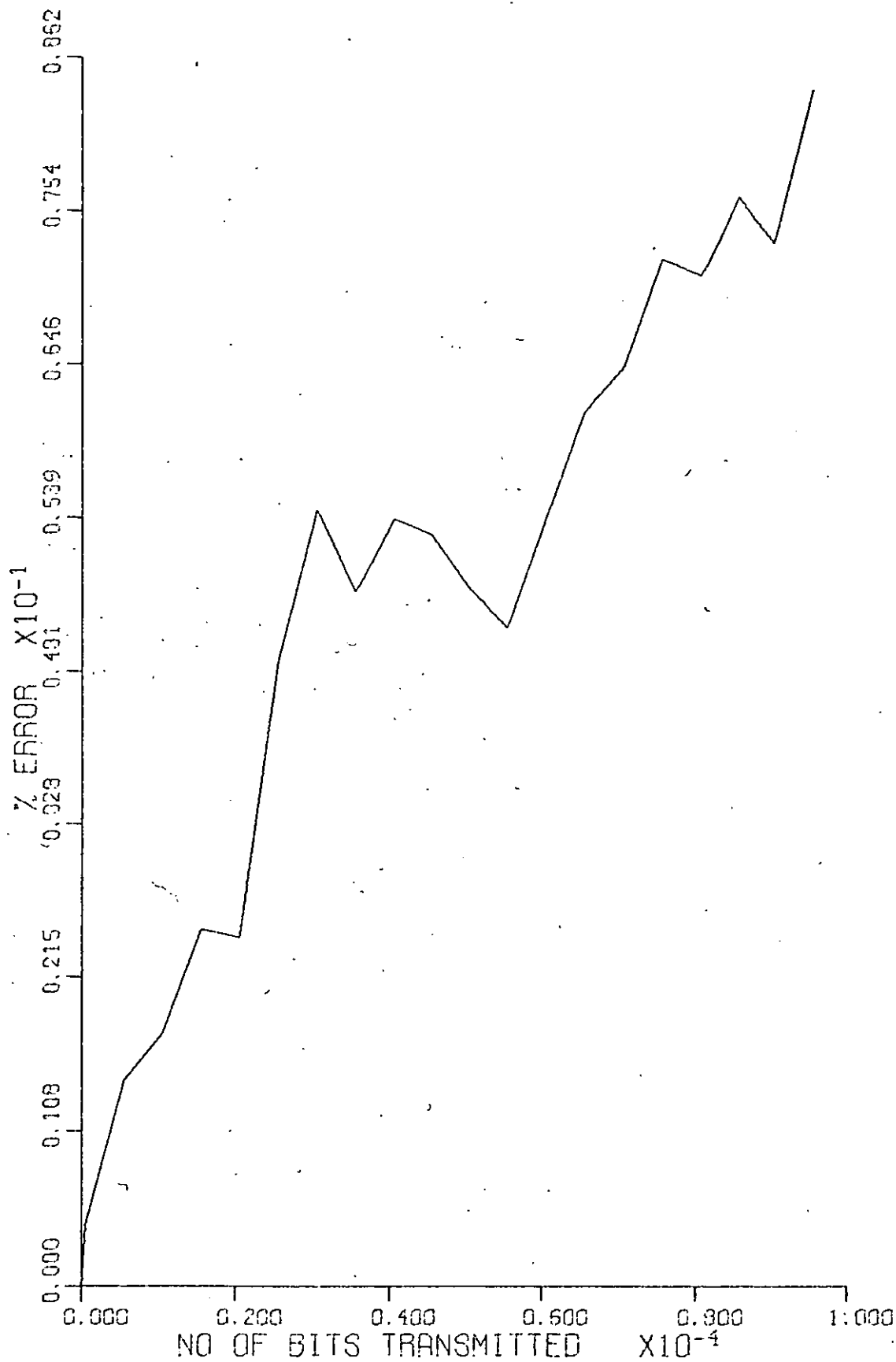


Figure (8-14) Simulated time varying channel by adding a small increment to the channel No. At every sampling instant.

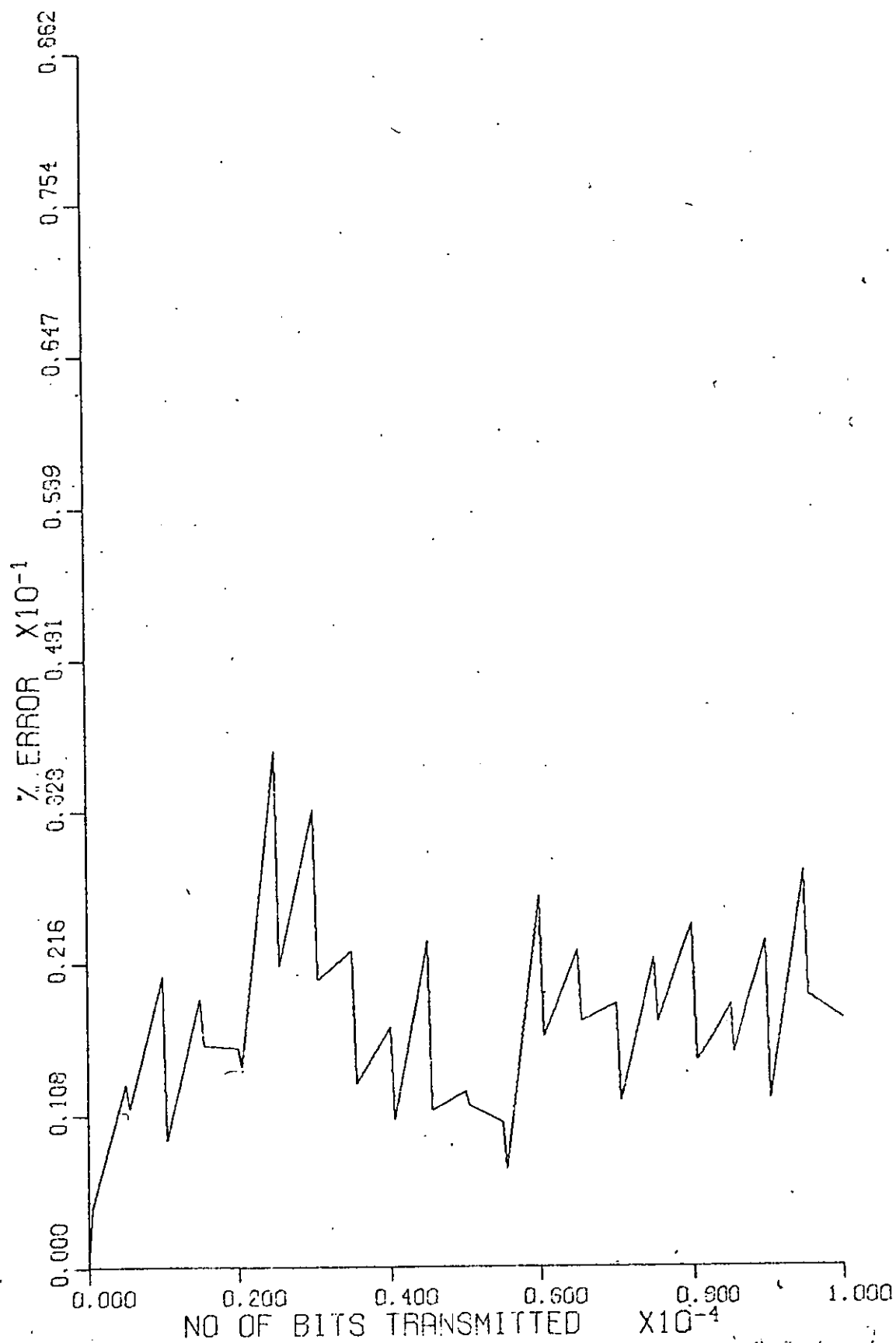


Figure (8-15) Adaptive equalization of channel No. A1 by the technique of eqn. (8-12).

would be different from the actual channel by some 3.6%, but with adaptive operation this discrepancy is less than 2% for the case of noiseless channel.

#### (8-3-4) Trials of different Iteration Algorithms (for system 1)

The adaptive algorithm described in section (8-2-2) utilizes the sample values of received signal elements  $\{r_i\}$  for the correlation with the error  $\{e_i\}$ . This technique has been tried for modulo-n feedback equalizer using the incrementing strategy of eqn. (8-3) and its modified forms. The reason for experimenting with this technique is that  $\{r_i\}$  is directly available in the receiver unlike the algorithm of section (8-3-3) which requires generation of sequence  $\{b_i\}$  by a coder. The error  $e_i$  is derived as described in previous sections. The tap gains are incremented opposite in sign and proportional to  $\partial c_j$  where  $\partial c_j$  is given by

$$\partial c_j = K \frac{e_i r_{i-j}}{r_{i-j}} \quad (8-14)$$

To investigate the convergence of this technique same assumptions were made as in the previous section and results for one sampled channel impulse response,  $A(z)$  will be presented. The incrementing of tap gains by eqn. (8-14) is shown in the graph of fig. (8-16). The initial error in the knowledge of  $A(z)$  is generated by the same error vector that is used for the graph of fig. (8-7). Observe from fig. (8-16) that incrementing the tap gains by eqn. (8-14) fails to converge to the actual channel response and hence minimize the error %.

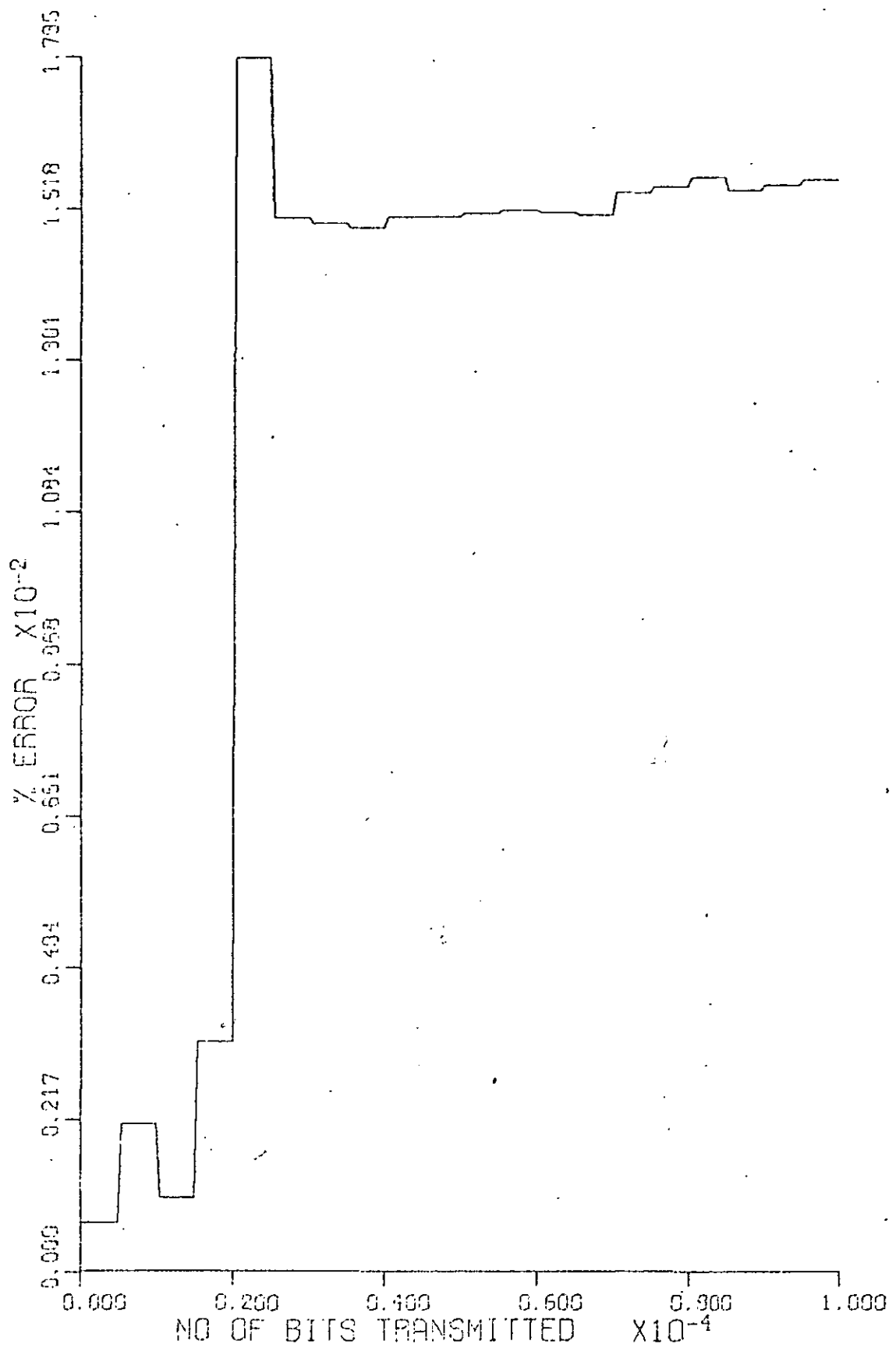


Figure (8-16) Incrementing technique of eqn. (8-14) for the equalizer of system 1 equalizing channel  
No. A6,  $X = 1/4$

Another trial uses  $a_o^{-1} r_i$  after modulo-n transformation to increment the tap gains and employs the relation given for  $\partial c_j$ , namely

$$\partial c_j = K e_i M \left[ a_o^{-1} r_{i-j} \right] \quad (8-15)$$

The result of this technique is shown in graph of fig. (8-17). Again the result is not acceptable as the error % is not reduced to a low level. A last try is made by quantizing the error to three levels  $-\Delta, 0, \Delta$  ( $\Delta = 0.09$ ) and using sign function of  $\{r_i\}$ . The tap increments which are opposite in sign and proportional to  $\partial c_j$  are given by

$$\partial c_j = K \frac{\text{quantized}(e_i) \text{ sign}(r_{i-j})}{\quad} \quad (8-16)$$

The result of this technique is shown in graph of fig. (8-18). Although there is a small reduction in error % the performance is far worse than that achieved using the algorithm of section (8-3-3) both for rate and accuracy of convergence.

Several other sampled channel impulse responses have been tried with techniques given in this section and the failure of convergence observed is similar to the graphs presented in this section.

#### (8-4) Conclusion

Several algorithms for linear and nonlinear equalization techniques have been investigated utilizing only the transmitted data sequence for adaptive equalization of modulo-n feedback equalizer. Adaptive equalization of linear equalizers is done by iterating the tap gains using the minimum mean square algorithm and results in positive convergence to an acceptable solution. This outcome is more likely



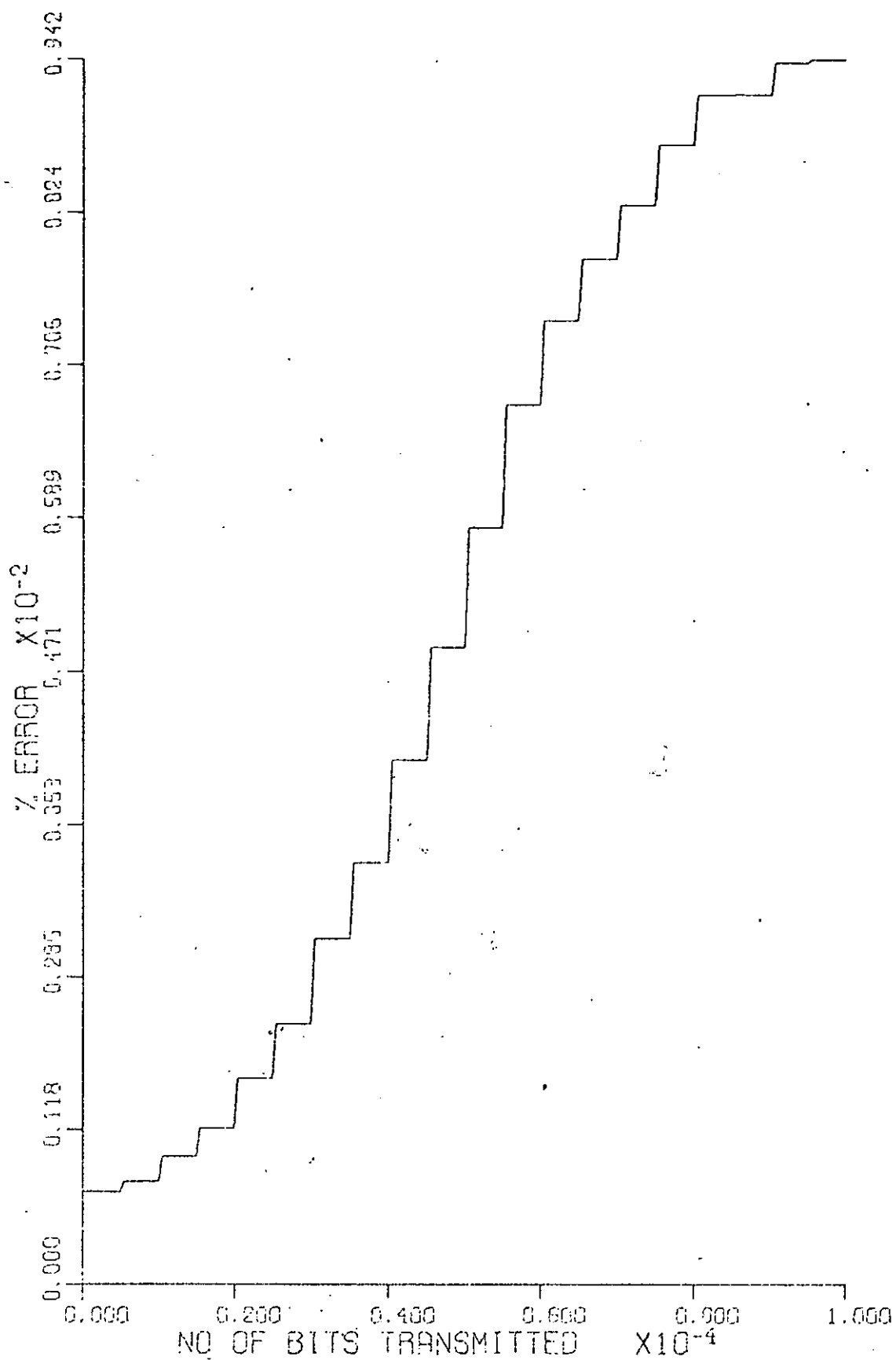


Figure (8-17) Incrementing technique of eqn. (8-15) for the equalizer of system 1 equalizing channel  
No. A6,  $K = 1/4$

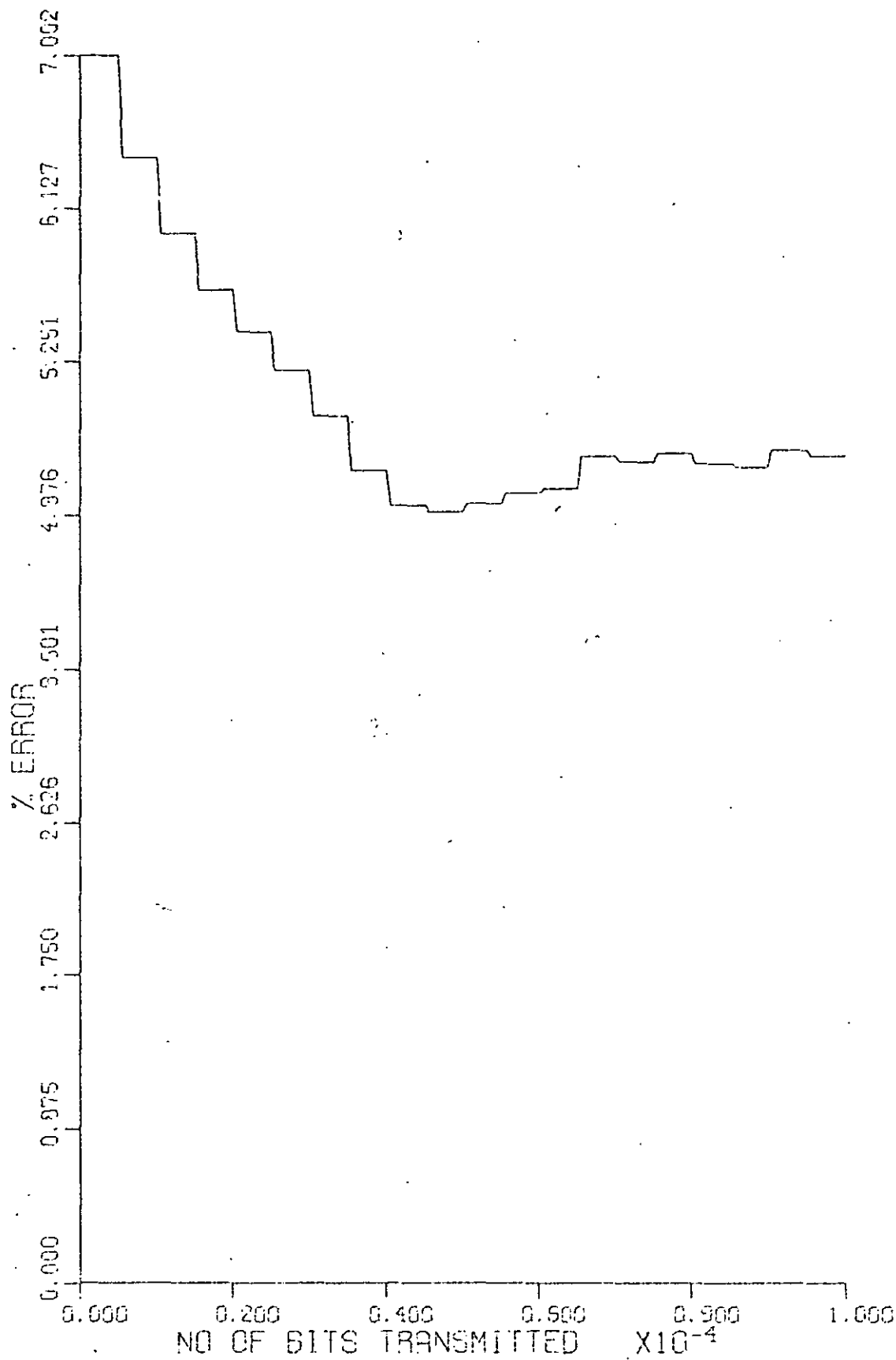


Figure (8-18) Incrementing technique of eqn. (8-16) for the equalizer of system 1 equalizing channel  
No. A6  $K = 1/4$ ,  $|\Delta| = 0.09$

to occur than it is with the zero forcing algorithm for a wide range of distortions. All the adaptive algorithms investigated for modulo-n feedback equalizer for time varying channels require a return line that will allow retransmission of information about the sampled impulse response of the channel from receiver to the transmitter. This is a disadvantage when compared to the equalization techniques that are located in the receiver, and such a requirement increases equipment complexity. Due to limitations of the return line, speed and accuracy of the convergence is impaired for modulo-n feedback equalizer, because the tap gains cannot be changed as frequently as an equalizer located in the receiver.

The techniques investigated in section (8-2) are suitable for linear equalizers. For adaptive decision directed intersymbol interference cancellation technique the algorithm described is the zero forcing type and the solution of linear equations given in section (8-3-1) (solution 1) is suitable for operation of this equalizer over time varying channels. The scheme described in section (8-3-2) (solution 2) results in a direct estimate of the sampled channel impulse response and the results are applicable for both nonlinear equalizers in system 1 and 2. However, it is a complex technique for practical realization. The algorithm investigated in section (8-3-3) for modulo-n feedback equalizer uses the cross-correlation between the error signal and the sign function of the sample value of the transmitted signal elements to increment the tap coefficients. This technique is successful for the types of distortion tested. Convergence of the equalizer is observed for the ideal case of noiseless channels and also for a channel with additive white

Gaussian noise. This algorithm is simplified by quantizing the error signal prior to the correlation operation, and it is found this quantization operation results in a small change in the rate of convergence and a residual error after converging. A scaling constant has been found experimentally for this algorithm which scales the tap increments and prevents the system becoming unstable. This iterative solution for adaptive operation of modulo-n feedback equalizer can be a practical solution if the adaptive operation of the equalizer is needed. Computer simulation tests have been presented using this algorithm with different channels, and adaptation of the equalizer starting with an approximate knowledge of the fixed channel response. A test for the adaptive modulo-n feedback equalizer tracking a slowly time varying channel is investigated, and the performance described. Further incrementing techniques for the tap gains of modulo-n feedback equalizer are given in section (8-3-4) which failed to adapt the equalizer to the sampled impulse-response of the channel which therefore cannot be employed in connection with time varying channels.

## CHAPTER 9

Equalization of the digital communication  
systems - the conclusions

## CHAPTER 9

### (9) Equalization of the Digital Communication Systems - the Conclusions

In this thesis the equalization techniques for serial synchronous binary data transmission systems are investigated. These techniques can be classified as linear, nonlinear or combination of both. The equalization systems can be located in either the transmitter or in the receiver. The main investigation is concerned with the development and assessment of modulo- $n$  feedback filters as equalizers and how their performances compare to other systems. From these comparisons a data transmission system with the original nonlinear equalizer using modulo arithmetic at the transmitter (system 1) has a tolerance to additive white Gaussian noise that is more often than not inferior to that of the corresponding system with a nonlinear equalizer at the receiver. The arrangement can however be modified by adding a linear filter to the nonlinear equalizer at the transmitter, and with the appropriate filter design (giving system 3) the tolerance to noise of the arrangement is greatly increased.

System 1 and system 3 are obtained when a nonlinear equalizer using decision-directed cancellation of intersymbol interference (as in system 2) and an optimum combination of linear and nonlinear equalizer (as in system 4), respectively, are transferred from the receiver to the transmitter. There is a negligible change in tolerance to additive white Gaussian noise in either case, so that system 3 achieves the best tolerance to additive white Gaussian noise which is obtainable with a conventional nonlinear transversal

equalizer provided the signal/noise ratios are large. The transfer of a nonlinear equalizer from the receiver to the transmitter is made possible by the use of two identical nonlinear networks, one at the transmitter and the other at the receiver, where the networks perform modulo- $n$  transformation on the respective input signals. Just as the feedback transversal filter in the receiver of system 2 or 4 removes the intersymbol interference by decision directed signal cancellation with no change in the signal/noise ratio, so also does the feedback transversal filter and nonlinear network in the transmitter of system 1 or 3 remove the intersymbol interference with essentially no change in the transmitted signal level and therefore again with no change in the signal/noise ratio.

The effective tolerance to additive white Gaussian noise of system 1 can be significantly improved by deploying a suitable error detector at the receiver which operates on the received data signal and requires no redundant data to be transmitted. With this error detection system further improvement can be made by an arrangement of a sequence generator and detector which detects the received signal from more than one sample value of received signal elements. This arrangement explained in section 7 of an improved detector combined with an error detector can improve the performance of system 1 considerably.

The systems 1 and 3 do not appear to be well suited to time-varying channels, because of the additional equipment complexity involved in transmitting information concerning the sampled impulse-response of the channel from the receiver to the transmitter. However, for channels that are time invariant and known, both system 1 with the error detector, and system 3 could be developed into cost-effective

systems. The techniques for adaptive operation of system 1 is investigated for time-varying channels. The technique that is best suited for this system utilizes the cross correlation between the error signal and sign function of the equalizer output sequence for incrementing the tap gains of the equalizer. The receiver uses a coder to generate the transmitter equalizer output sequence in the receiver. This coder is reset periodically to ensure that it operates correctly. If system 1 uses the error detection facilities the resetting would not be needed and the same coder could be used for error detection and also to derive information for adaptive operation of the system. In case of a binary error, resetting is done automatically by retransmission as described in section (7). System 1 when used with an error detector in an adaptive mode can achieve good performance over slowly time-varying channels.



### Suggestions for further work

Further investigation for system 1 can be carried out by evaluating the performance of the multi-element detector that is proposed in section 7 and is shown in its final arrangement in fig. (7-6). This detector detects the received signal from more than one element and also checks for errors. The system can be investigated for different types of distortions, possibly the channels listed in table (6-3).

Adaptive operation for time varying channel of system 3 and 4 can be investigated. The design technique of section (5-3) for these systems suggests that a double iteration will be required for the adaptive equalizer. Also the adaptive operation of the above proposed system with error detector can be investigated. The specific problems that will arise from practical application of system 1 and 3 are the use of return channel. This can be investigated by hardware implementation of the system in a data modem and the tests for the systems will be performed in realistic conditions, and this will complete the investigation centred around the proposed nonlinear equalizer.

Also, the error detector of section (7) can be applied to systems other than data transmission systems. Such systems could be analogue to digital feedback coders, coders like differential pulse code modulation or adaptive delta modulation systems. These systems suffer from channel errors. If this is the case then in the receiver, their decoders introduce error extension effects which degrades the performance of these coders. The transmitter coder can be utilised in a fashion similar to fig. (7-2) to detect errors introduced in the transmission path.

## Appendix A1

### The z transform and a polynomial representation

The z transform is a technique by which a unique polynomial can be assigned to a particular impulse response by assigning the coefficients of the polynomial to the sampled values of the response. The z transform of the samples  $a_0, a_1, a_2, \dots, a_n$  is the polynomial:

$$a_0 + a_1 z^{-1} + a_2 z^{-2} \dots \dots \dots + a_n z^{-n} \quad (A1-1)$$

This polynomial is useful for it enables processes such as convolution, deconvolution to be simply understood, and samples in particular time slots to be manipulated in general terms.

Operations on the polynomial, or conversely, the signal, have unique corresponding effects. Delaying the signal response by a time period is equivalent to multiplying the polynomial by  $z^{-1}$ . The coefficient of a power of  $z^{-1}$  is an analogue of the amplitude of a sample, and the particular power of  $z^{-1}$  is the relative time slot which the sample occurs. The z transform has two important properties.

1- Convolution in the time domain corresponds to polynomial multiplication in the z domain. With an input  $A(t)$  applied to a filter with impulse response  $K(t)$  the output  $Y(t)$  is given by

$$Y(z) = A(z) \cdot K(z) \quad (A1-2)$$

2- The function evaluated around the unit circle defined by  $z = e^{j 2 \pi f n T}$  in the complex  $z$  plane corresponds to the discrete fourier transform, thus the discrete fourier transform of  $Y(t)$  is

$$\sum_{n=0}^{\infty} y_n \exp (-j 2 \pi f n T) \quad (A1-3)$$

## Appendix A2

Calculation of noise variance for a given probability of error with no channel distortion and binary bipolar data transmission.

For additive white Gaussian noise with zero mean and variance  $\sigma^2$ , the required average error probability  $P_e$  is given by

$$P_e = 4 \times 10^{-3},$$

when there is no distortion introduced by the channel the following applies, the transmitted signal-elements have unit energy. The noise variance at the output of the receiver filter is equal to the noise power spectral density (two-sided) and equals  $\sigma^2$ ,

$$\begin{aligned} P &= \frac{1}{\sqrt{2\pi\sigma^2}} \int_1^{\infty} \exp(-W^2/2\sigma^2) dw \\ &= \frac{1}{\sqrt{2\pi}} \int_{1/\sigma}^{\infty} \exp(-W^2/2) dw \\ &= Q(1/\sigma) = Q(x) \end{aligned}$$

For  $P_e = 4 \times 10^{-3}$

$$P(x) = 0.996$$

$$Q(x) = 0.004$$

From standard mathematical tables this integral is tabulated and the value of  $x$  is 2.66 but  $x = 1/\sigma$ , hence  $\sigma = 0.376$ .

Thus for reference the systems tested for the above given error probability,  $\sigma = 0.376$  will be taken as 0dB and the systems will be compared with this level of  $\sigma$ . Expressed in dB

$$10 \log (1/\sigma^2) = -20 \log (\sigma) \text{ dB.}$$

For any other error probability the same procedure will be followed.

It is assumed that the signal-elements are +1 at the detector input.

### Appendix A3

#### Definition and calculation of the 95% confidence limits.

A problem that frequently arises in the engineering field is that of finding the probability of a failure or error in a given system. In many of the modern digital data transmission systems this probability is an extremely small number, and consequently it takes a long time to perform an adequate test to determine the error rate. Thus, the question naturally arises as to how long the test must be in order to insure that the measured error rate is within a specified range of actual rate, or to determine the limits over an interval within which a system error rate lies with a given "confidence". These limits will vary with respect to the number of errors observed in the test, ref. (90). It is possible to calculate the value of  $\sigma$  theoretically and compare this value experimental results. The limits for experimental results from ref. (91) where a total of 20 or more independent errors are observed in a test the following formula gives the 95% confidence limits of the probability of error. For this experiment,

$$P = P_e \pm \frac{2P_e}{\sqrt{n}}$$

where  $n$  = number of independent error. From this formula  $\sigma^2$  for upper and lower limits of probability from the Gaussian distribution can be found and compared to the ones found theoretically. These limits can also be expressed in  $\pm$  dB with respect to theoretical value.

# Appendix A4

## Alternative method for design and calculation of probability of error of system 3.

The modification of system 1 in which a linear feedforward transversal filter with  $n$  taps and  $z$  transform

$$D(z) = d_0 + d_1 z^{-1} + \dots + d_{n-1} z^{-n+1} \quad (A4-1)$$

is added at the output of the transmitter equalizer to give the data transmission system in fig. (A4).

It is assumed that

$$\sum_{i=0}^{n-1} d_i^2 = 1 \quad (A4-2)$$

so that the average transmitted energy per element at the input to the baseband channel in fig. (A4) is the same as the average energy per element at the output of the nonlinear filter, and as before for a first approximation as to have the value unity.

The  $z$  transform of the linear filter and baseband channel,  $A(z)$  is

$$A(z) D(z) \approx z^{-h} C(z) \quad (A4-3)$$

where

$$C(z) = C_0 + C_1 z^{-1} + \dots + C_m z^{-m} \quad (A4-4)$$

$h$  and  $m$  are appropriate positive integers. Clearly,  $z^{-h} C(z)$  is the

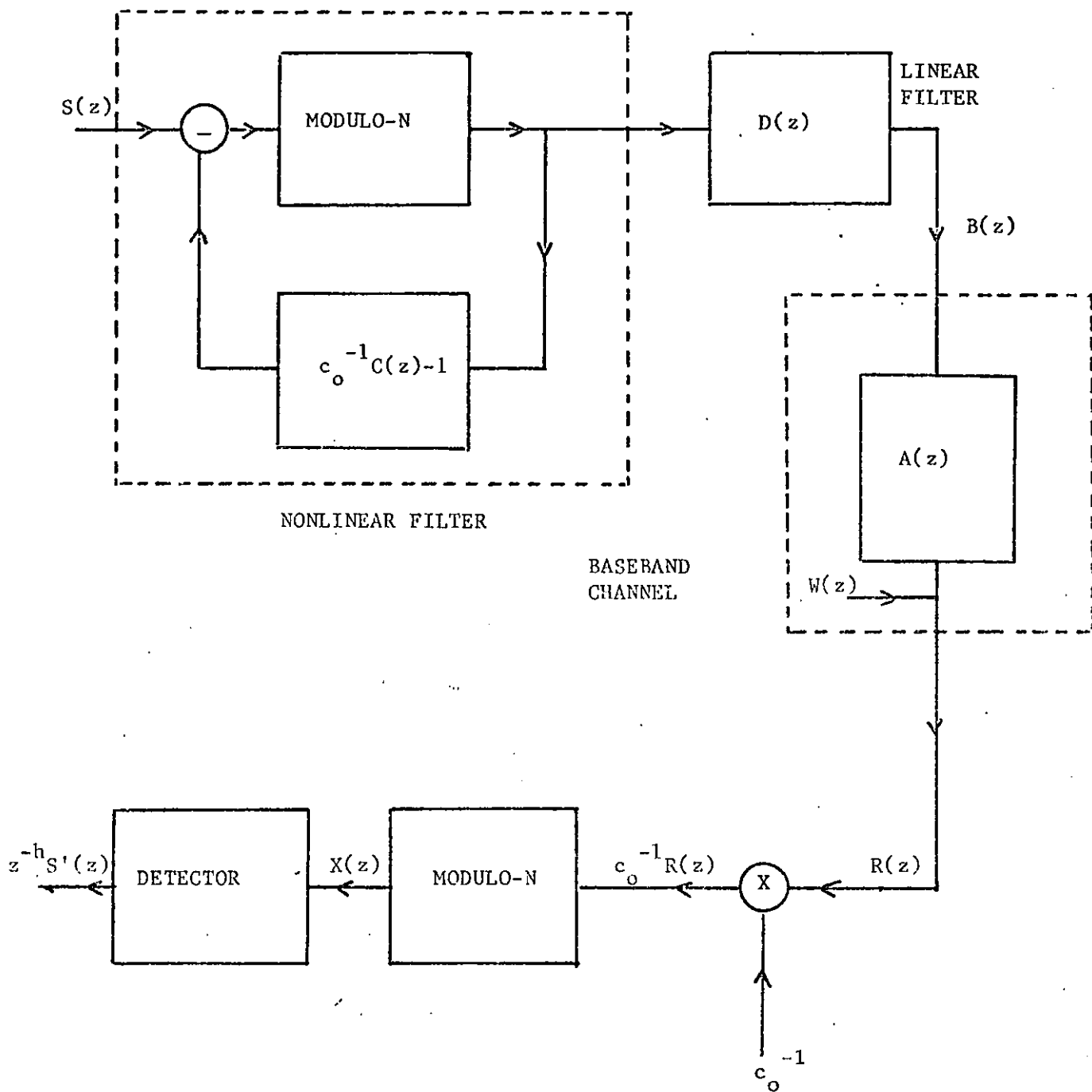


FIGURE (A4) SYSTEM 3.



z transform of the linear filter and baseband channel, and these are equalized by the nonlinear filter at the transmitter in fig. (A4), in a manner similar to that for the equalization of the baseband channel in system 1. Thus the z transform of the transversal filter that forms part of the nonlinear filter, is  $c_o^{-1} C(z) - 1$ , and, following the analysis of system 1, it can be seen that the z transform of the signal at the input to the detector in fig. (A4) is

$$X(z) = M \begin{bmatrix} c_o^{-1} R(z) \end{bmatrix} \approx \begin{bmatrix} z^{-h} S(z) + c_o^{-1} W(z) \end{bmatrix} \quad (A4-5)$$

because of the delay of  $hT$  seconds introduced by the additional linear filter at the transmitter,  $S(z)$  has  $z^{-h}$  delay in eqn. (A4-5) and  $s_i$  is detected at time instant  $t = (i + h)T$  from the sample value

$$x_{i+h} \approx M \begin{bmatrix} s_i + c_o^{-1} w_{i+h} \end{bmatrix} \quad (A4-6)$$

at the input to the detector. The probability of error in the detection of  $s_i$  can therefore be taken to be for system 3

$$P_3 = Q\left(\frac{|c_o|}{\sigma}\right) \quad (A4-7)$$

clearly, to minimize  $P_3$ ,  $|c_o|$  must be maximized and this can be achieved by the appropriate choice of  $D(z)$ .

The value of  $D(z)$  that maximizes  $|c_o|$ , given, eqn. (A4-2), is the same as the value of  $D(z)$  that minimizes

$$\sum_{i=0}^{n-1} d_i^2,$$

given  $c_0$ , and this has been determined in ref. (29, 37).

Suppose that

$$A(z) = A_1(z) A_2(z) \quad (A4-8)$$

where all the roots of  $A_1(z)$  lie inside the unit circle in the  $z$  plane and all the roots of  $A_2(z)$  lie outside the unit circle. Let

$$A_1(z) = 1 + f_1 z^{-1} + f_2 z^{-2} + \dots + f_{k-\ell} z^{-k+\ell} \quad (A4-9)$$

$$A_2(z) = g_0 + g_1 z^{-1} + g_2 z^{-2} + \dots + g_\ell z^{-\ell} \quad (A4-10)$$

$$A_3(z) = g_\ell + g_{\ell-1} z^{-1} + g_{\ell-2} z^{-2} + \dots + g_0 z^{-\ell} \quad (A4-11)$$

where  $\ell$  lies in the range 0 to  $k$ .

If  $A(z)$  contains one or more roots on the unit circle in the  $z$  plane, which implies that the baseband channel can not be equalized linearly, the corresponding factor of  $A(z)$  is included in  $A_1(z)$ . Such channels are not, however, considered further here.

The linear filter with  $z$  transform  $D(z)$  that maximizes  $|c_0|$ , given eqn. (A4-2), is such that ref. (37).

$$D(z) \simeq z^{-h} A^{-1}_2(z) A_3(z) \quad (A4-12)$$

so that the  $z$  transform of the filter and baseband channel is

$$A(z) D(z) \simeq z^{-h} A_1(z) A_3(z) \quad (A4-13)$$

$$\text{from (A4-3)} \quad C(z) = A_1(z) A_3(z) \quad (A4-14)$$

also,  $m = k$ ,  $c_0 = g_\ell$  and

$$P_3 = Q\left(\frac{|g_\ell|}{\sigma}\right) \quad (A4-15)$$

In practice, the average transmitted energy per signal element may be a little more than 1dB above the value unity assumed, giving the corresponding reduction in tolerance to noise. There is also a small reduction in tolerance to noise because of doubling of error probability due to modulo-n transformation at the receiver.

The linear filter with  $z$  transform  $D(z)$  is a pure phase equalizer that replaces all the roots of  $A(z)$  lying outside the unit circle in the  $z$  plane by their reciprocals. The linear filter, partially equalizes the channel, the equalization process being completed by the nonlinear filter that comprises the feedback transversal filter and the nonlinear network modulo-n. If this nonlinear network is removed the linear equalizer  $C(z)$  is stable since all the roots of  $C(z)$  lie inside the unit circle in the  $z$  plane (as in system 5), also the receiver modulo-n transformation can be removed.

The theoretical calculations of probability of error is done with eqn. (A4-15), and the results are found to be exactly the same as for system 3 given in table (6-3) for all the channels except channels no. 14, 15, and 19 were found to be 5.8dB, 8.4dB and 0.5dB respectively. This discrepancy could be due to computer algorithm for finding roots of  $A(z)$  for forming polynomials  $A_1(z)$  and  $A_2(z)$ .

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