

TRANSLATIONAL VIBRATION MEASUREMENTS USING LASER VIBROMETRY: A THEORETICAL APPROACH FOR CONFIDENT DATA INTERPRETATION IN ADVANCED APPLICATIONS

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ABSTRACT

It is readily accepted that a Laser Vibrometer measures target velocity in the direction of the incident laser beam but it is essential that, for correct measurement interpretation, the target velocity be considered in terms of the various target motion components. This paper begins with a review of the theoretical description of the velocity sensed by a single laser beam incident in an arbitrary direction on a rotating target undergoing arbitrary motion. For continuous scanning Laser Doppler Vibrometry, the velocity sensitivity model is shown formulated in two useful ways. The first is in terms of the laser beam orientation angles, developing the original model to incorporate time dependency in the angles, whilst the second is expressed in terms of the scanning mirror angles, since it is these that the operator would seek to control in practice.

The origins of the additional components that occur in measured data due to instrument configuration and inevitable target misalignment are easily revealed using this revised velocity sensitivity model. Advanced applications such as those on rotating components and in scanning configurations are also those in which speckle noise issues are of significant interest and this is addressed in this paper. Scanning and tracking techniques are typically employed on targets with flexible cross-sections and the original theory is developed in this paper for the first time to include provision for such flexibility.

NOMENCLATURE

\hat{b}	Laser beam orientation unit vector
d_s	Laser beam deflection mirror perpendicular separation
O	Origin of the translating reference frame, xyz
P_0	Laser beam incidence point on undeformed shaft element
P	Laser beam incidence point on deformed shaft element
$\vec{r}_f, \dot{\vec{r}}_f$	Position, velocity vector of the deformed point relative to the undeformed point

$\vec{r}_O, \dot{\vec{r}}_O$	Position, velocity vector of O
$\vec{r}_P, \dot{\vec{r}}_P$	Position, velocity vector of the deformed point
$\vec{r}_{P_0/O}, \dot{\vec{r}}_{P_0/O}$	Position, velocity vector of the undeformed point relative to the translating reference frame
$\vec{r}_{P/O}$	Position vector of the deformed point relative to the translating reference frame
r_s	Circular scan radius
U_m	Measured vibrometer output signal
\vec{V}_f	Deformation vibration velocity
\vec{V}_O	Vibration velocity of O
\vec{V}_P	Vibration velocity of P
xyz	Translating reference frame
XYZ	Fixed reference frame
x, y, z	x, y, z direction target displacement
$\hat{x}, \hat{y}, \hat{z}$	x, y, z direction unit vector
x_0, y_0, z_0	x, y, z co-ordinate of the arbitrary known point
$\Delta x_0, \Delta x_0(t)$	Variation in known point x co-ordinate
x_{0m}, y_{0m}	x, y direction translational misalignment
$x_s, x_s(t)$	Probe laser beam x co-ordinate
$y_s, y_s(t)$	Probe laser beam y co-ordinate
$\dot{x}, \dot{y}, \dot{z}$	x, y, z , direction target vibration velocity
$\dot{x}_f(P)$	x vibration velocity component due to cross-section flexibility
$\dot{y}_f(P)$	y vibration velocity component due to cross-section flexibility
$\dot{z}_f(P)$	z vibration velocity component due to cross-section flexibility
$\dot{x}_r(P_0)$	Resultant of x vibration velocity components due to rigid body motion
$\dot{y}_r(P_0)$	Resultant of y vibration velocity components due to rigid body motion
$\dot{z}_r(P_0)$	Resultant of z vibration velocity components due to rigid body motion

$\alpha, \alpha(t)$	Laser beam orientation angle about the z axis
$\beta, \beta(t)$	Laser beam orientation angle about the y axis
$\delta(t)$	Time dependent component in α during scanning
$\varepsilon(t)$	Time dependent component in β during scanning
ϕ_S	Scan initial phase angle
$\theta_x, \dot{\theta}_x$	Angular vibration displacement, velocity about the x axis
$\theta_y, \dot{\theta}_y$	Angular vibration displacement, velocity about the y axis
$\dot{\theta}_z$	Angular vibration velocity about the z axis
θ_{xm}	Angular misalignment about the x axis
θ_{ym}	Angular misalignment about the y axis
$\theta_{Sx}, \theta_{Sx}(t)$	x deflection mirror scan angle
$\theta_{Sy}, \theta_{Sy}(t)$	y deflection mirror scan angle
Θ_{Sx}, Θ_{Sy}	x, y deflection mirror scan angle amplitude
$\bar{\omega}$	Angular velocity of P_0 about an instantaneous axis passing through O
Ω	Target rotational angular frequency
Ω_S	Scan rotational angular frequency

1. INTRODUCTION

The principle of Laser Doppler Vibrometry (LDV) relies on the detection of a Doppler shift in the frequency of coherent light scattered by a moving target, from which a time-resolved measurement of the target velocity is obtained. The Laser Vibrometer is now a well-established non-contact vibration transducer that is an effective alternative to the use of a traditional contacting transducer. The Laser Vibrometer can offer significant advantages over the contacting transducer and vibration measurements on hot, light or rotating components are often cited as important applications [1].

For such structures, when measurements are to be taken from several points, a non-contact vibration transducer capable of making a series of measurements across a component surface is desirable and LDV offers this possibility. A substantial reduction in test time can be realised by automating the "relocation" of the measurement transducer and the suitability of the Laser Vibrometer to such automation was recognised at an early stage in the development of the instrument [2]. Examples of the use of such scanning Laser Vibrometers include measurements on automotive [3] and turbomachinery [4] components and assemblies.

In addition to this point-by-point operation of the scanning LDV, it is possible to configure the instrument to function in a continuous scanning mode. Continuous scans are conveniently arranged for by driving the beam deflection mirrors with continuous time variant signals, enabling the target velocity profile along a pre-defined path to be determined in a single measurement. Post-processing of the Laser Vibrometer output signal results in a series of

coefficients that describe the operational deflection shape (ODS) or, where a frequency response function (FRF) is obtained, mode shape [5]. Straight-line, circular, small-scale circular and conical scans have all been proposed to measure various components of the vibration at various points on a target [5]. Continuous scanning is the particular focus of this paper. Throughout the remainder of this document, "scanning" LDV refers to operation in continuous scanning mode.

In rotating machinery, vibration measurement is typically performed from the earliest stages of design and development through to the condition monitoring of commissioned equipment [6]. The most common measurement is that of the vibration transmitted into a non-rotating component using a contacting transducer but a non-contact transducer capable of measuring directly from any location along the rotor is often desirable and LDV offers this possibility. One of the earliest reported applications of LDV was, indeed, for axial vibration measurement directly from a rotating turbine blade [7] and more recent and typical examples include the measurement of vibration in magnetic discs [8,9] and bladed discs [10,11]. Configuration of a continuous scanning Laser Vibrometer to scan a circular profile enables the measurement of axial vibration [5,12] and of mode shapes [13] in components such as axially flexible rotating discs. If the scan frequency is synchronised with the target rotation frequency, it is possible to perform a tracking Laser Vibrometer measurement in which the probe laser beam remains fixed on a particular point on the target [14].

This paper begins with a brief review of the theoretical description of the velocity sensed by a single laser beam incident in an arbitrary direction on a rotating target undergoing arbitrary motion. The totally general velocity sensitivity model illustrates that the measured velocity is dependent upon both the target velocity components and the orientation of the incident laser beam. In the original derivation, the illuminated section of the rotating target was assumed to be of rigid cross-section but, since Laser Vibrometer measurements are employed in applications where flexibility must be acknowledged, the first extension of the theory presented in this paper includes explicit provision for such flexibility.

The velocity sensitivity model is versatile enough to incorporate time dependent beam orientation and this is described with reference to a continuous scanning Laser Vibrometer measurement. The original derivation is developed to include time dependency in the beam orientation angles before being re-formulated to make use of the mirror scan angles, as it is these that the user would seek to control in practice. The advanced application of circular scanning on rotating targets is investigated as a means of illustrating the effectiveness of the model for the analysis of actual scan configurations. In particular, the origins of the additional components that occur in measured data due to instrument configuration are easily revealed using the revised velocity sensitivity model and an analysis and initial experimental validation of their influence is discussed in this paper.

For circular tracking measurements, in which the probe laser beam remains fixed on a single point on the target during

rotation, the noise generated by the laser speckle effect [15] should be at a minimum. The final experimental investigation presented in this paper examines the validity of this hypothesis.

2. VELOCITY SENSITIVITY ANALYSIS USING LASER BEAM ORIENTATION ANGLES

Previous research [16] resulted in the presentation of a theoretical analysis of the velocity sensed by a single laser beam incident on a target that is of substantial interest in engineering – a rotating shaft undergoing arbitrary six degree-of-freedom vibration. The full vibration velocity sensitivity is given as the sum of six terms, each the product of an inseparable combination of motion parameters – the “vibration sets”, and a combination of geometric parameters, relating to the arbitrary laser beam orientation.

The direction of the incident laser beam is described by the unit vector \hat{b} , which, if orientated according to the angles β and α as shown in Figure 1, is given by:

$$\hat{b} = [\cos \beta \cos \alpha] \hat{x} + [\cos \beta \sin \alpha] \hat{y} - [\sin \beta] \hat{z} \quad (1)$$

In the original derivation of the velocity sensitivity model, it was assumed that, although the shaft could be flexible, the illuminated rotor cross-section would not undergo changes in shape during the course of the measurement. Whilst this assumption is reasonable for some targets, Laser Vibrometer measurements are generally employed in applications where flexibility must be acknowledged and the first extension of the theory presented in this paper includes explicit provision for such flexibility.

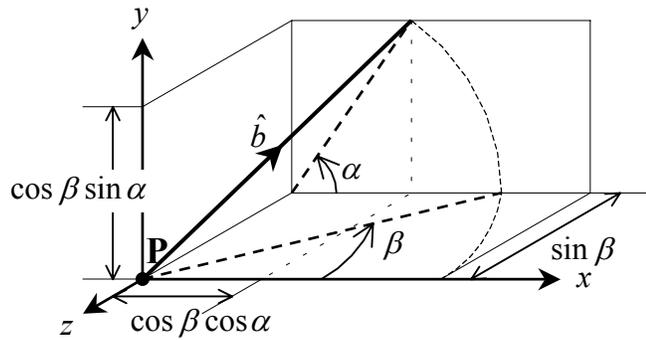


Figure 1 – Laser beam orientation, defining angles β and α

2.1. Structures with flexible cross-sections

The case considered is that of a rotating shaft in which the illuminated axial element has a flexible cross-section. As illustrated in Figure 2, P is the instantaneous point of incidence of the laser beam on the arbitrarily deformed shaft element, identified by the position vector $\vec{r}_{P/O}$, and P_0 defines the corresponding point on the displaced but undeformed shaft element, identified by $\vec{r}_{P_0/O}$. Clearly:

$$\vec{r}_P = \vec{r}_O + \vec{r}_{P/O} = \vec{r}_O + \vec{r}_{P_0/O} + \vec{r}_f \quad (2)$$

where \vec{r}_P identifies the position of P relative to the fixed reference frame XYZ , \vec{r}_O identifies the instantaneous position of the translating reference frame xyz , and \vec{r}_f represents the deformation.

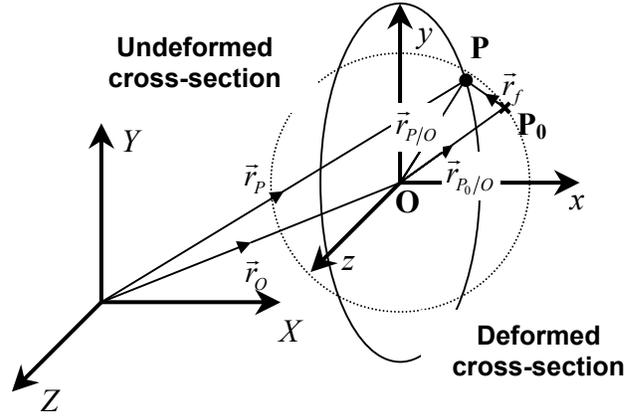


Figure 2 – Definition of axes and the points P and P_0 on a vibrating and rotating flexible shaft

The velocity of P, \vec{V}_P , is therefore given by:

$$\vec{V}_P = \dot{\vec{r}}_P = \dot{\vec{r}}_O + \dot{\vec{r}}_{P_0/O} + \dot{\vec{r}}_f = \vec{V}_O + (\vec{\omega} \times \vec{r}_{P_0/O}) + \vec{V}_f \quad (3)$$

where $\vec{\omega}$ is the angular velocity of P_0 about an instantaneous rotation axis passing through O. Equation (3) is similar to that which is obtained when considering the velocity of a point on a rotating shaft of rigid cross-section [16], the difference being the term \vec{V}_f , which represents the deformation vibration velocity of P due to cross-section flexibility and the velocity measured by the Laser Vibrometer, U_m , can be written:

$$U_m = \cos \beta \cos \alpha [\dot{x}_r(P_0) + \dot{x}_f(P)] + \cos \beta \sin \alpha [\dot{y}_r(P_0) + \dot{y}_f(P)] - \sin \beta [\dot{z}_r(P_0) + \dot{z}_f(P)] \quad (4)$$

where $\dot{x}_r(P_0)$, $\dot{y}_r(P_0)$ and $\dot{z}_r(P_0)$ are the resultant vibration velocity components in the x, y and z directions due to rigid body vibration, given by:

$$\dot{x}_r(P_0) = \dot{x} - (\dot{\theta}_z + \Omega)(y_0 - y) + (\dot{\theta}_y - \Omega \theta_x)(z_0 - z) \quad (5a)$$

$$\dot{y}_r(P_0) = \dot{y} + (\dot{\theta}_z + \Omega)(x_0 - x) - (\dot{\theta}_x + \Omega \theta_y)(z_0 - z) \quad (5b)$$

$$\dot{z}_r(P_0) = \dot{z} + (\dot{\theta}_x + \Omega \theta_y)(y_0 - y) - (\dot{\theta}_y - \Omega \theta_x)(x_0 - x) \quad (5c)$$

where \dot{x} , \dot{y} , \dot{z} and x , y , z are the translational vibration velocities and displacements of the origin, O, in the x , y , z directions, Ω is the total rotation speed of the axial shaft element (combining rotation speed and any torsional oscillation), θ_x , θ_y , $\dot{\theta}_x$, $\dot{\theta}_y$, $\dot{\theta}_z$ are the angular vibration displacements and velocities of the shaft around the x , y , z axes, referred to as pitch, yaw and roll, respectively, and (x_0, y_0, z_0) is the position of an arbitrary known point that lies along the line of the beam.

$\dot{x}_f(P)$, $\dot{y}_f(P)$, $\dot{z}_f(P)$ are the vibration velocity components in the x , y , z , directions due to cross-section flexibility, specific to point P. This shows that the rotor cross-section flexibility results in additional components due to the deformation velocities, which represent the difference between equation (4) and the original derivation of the model.

The development of equation (4) is significant since it can be conveniently employed to make the analysis of complex measurement configurations more straightforward. In particular, applications in which the laser beam is scanned can be investigated by considering a time dependent known point position. The depth of information offered by the

velocity sensitivity model will be demonstrated in this next section with reference to circular scanning LDV.

2.2. Circular scanning Laser Doppler Vibrometry

A circular scanning Laser Vibrometer measurement can be achieved by deflecting the laser beam through suitable angles around two orthogonal axes simultaneously, typically by using cosine and sine functions [5,13,14,17]. With reference to Figure 3, this laser beam deflection is performed in commercially available scanning Laser Vibrometers by the introduction of two orthogonally aligned mirrors, separated by some distance d_s , into the beam path.

The scanning system optical axis is defined as being the line along which the laser beam is directed towards the target when there is "zero" beam deflection. In this particular configuration, the scanning system and target reference frames are collinear and the scanning system optical axis lies on the z axis of the target reference frame. The two orthogonal axes about which the beam is deflected during scanning are chosen such that the resulting probe laser beam manipulation occurs in the x and y directions in the target plane.

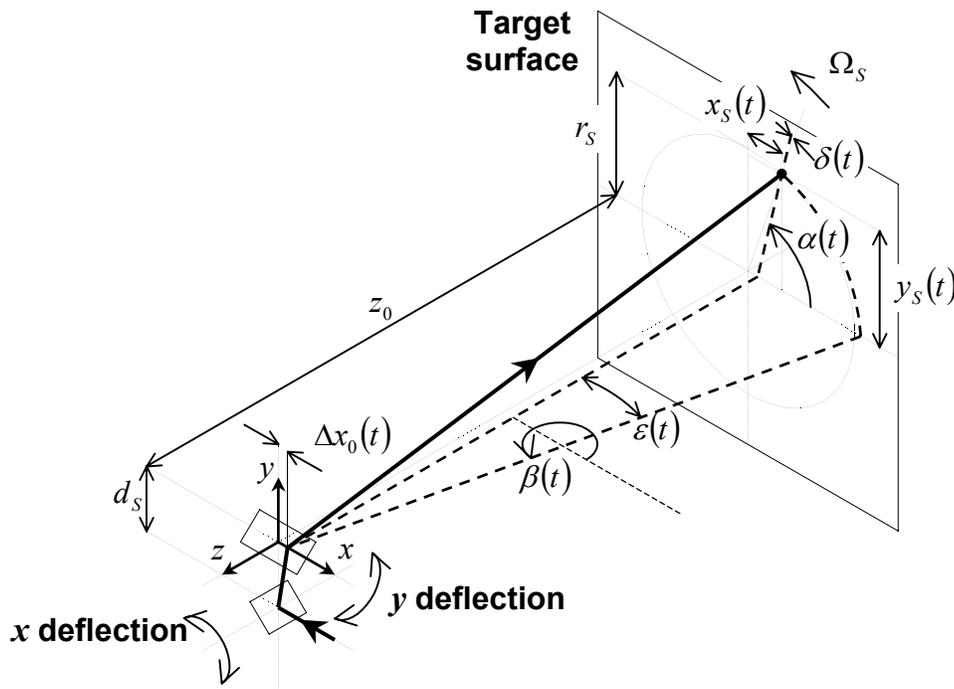


Figure 3 – The dual mirror scanning arrangement incorporating two orthogonally aligned mirrors, shown in terms of the laser beam orientation angles

With reference to Figure 3, it can be seen that when the laser beam is traced back there is no single point from which it appears to originate. The most convenient known point to choose is the incident point of the laser beam on the y deflection mirror, which scans back and forth along the mirror rotation axis. In addition, modulations in both β and α occur as a result of rotation of the x and y deflection mirrors, respectively. The velocity sensitivity model is sufficiently versatile to be able to account for this.

The time dependency in the chosen known point x coordinate, x_0 , is given by:

$$\Delta x_0(t) = \frac{r_s d_s}{z_0} \cos(\Omega_s t + \phi_s) \quad (6)$$

where r_s is the desired scan radius, z_0 is (in this case) the stand-off distance between the target and the Laser Vibrometer, Ω_s is the scan angular frequency and ϕ_s is the scan initial phase angle. It can be seen from Figure 4 that the laser beam orientation angles, β and α , necessary to scan a circle are given by:

$$\begin{aligned} \beta(t) &= \frac{3\pi}{2} - \varepsilon(t) \\ &= \frac{3\pi}{2} - \tan^{-1} \left(\frac{\sqrt{(x_s(t) - \Delta x_0(t))^2 + (y_s(t))^2}}{z_0} \right) \end{aligned} \quad (7a)$$

$$\begin{aligned} \alpha(t) &= \Omega_s t + \phi_s + \delta(t) \\ &= \Omega_s t + \phi_s + \tan^{-1} \left(\frac{x_s(t)}{y_s(t)} \right) - \tan^{-1} \left(\frac{x_s(t) - \Delta x_0(t)}{y_s(t)} \right) \end{aligned} \quad (7b)$$

where $\varepsilon(t)$ and $\delta(t)$ are readily seen as being directly related to the time dependency in x_0 .

Substituting for β and α in equation (4) using equations (7a&b) will immediately result in a full expression for the velocity measured during a circular scan on a rotating, flexible target undergoing 6 degree-of-freedom vibration but this expression is not particularly straightforward. Furthermore, it is the beam deflection mirror scan angles, not the laser beam orientation angles, that are controlled in real scanning systems and it is more appropriate, therefore, to re-express the velocity sensitivity model in terms of these.

3. VELOCITY SENSITIVITY ANALYSIS USING DEFLECTION MIRROR SCAN ANGLES

In order that the velocity sensitivity model can be re-expressed in terms of the mirror scan angles, it is necessary to recalculate the beam orientation unit vector, \hat{b} , in terms of the mirror scan angles and this is summarised in this next section.

3.1. Velocity measured by a dual mirror scanning Laser Vibrometer

With reference to Figure 4, the “zero” positions of the x and y deflection mirrors which result in deflection of the laser beam along the z axis are both 45° (to the y direction). The mirror scan angles, θ_{sx} and θ_{sy} , are defined as positive if anticlockwise about an axis in the z direction and the x axis respectively. Expressing the direction of each mirror surface in terms of a unit vector, it is possible to predict the direction of the laser beam after reflection and thus derive an equation for \hat{b} , in terms of θ_{sx} and θ_{sy} [18]:

$$\begin{aligned} \hat{b} &= [\sin 2\theta_{sx}] \hat{x} \\ &\quad - [\cos 2\theta_{sx} \sin 2\theta_{sy}] \hat{y} + [\cos 2\theta_{sx} \cos 2\theta_{sy}] \hat{z} \end{aligned} \quad (8)$$

Equation (8) is of great significance since it defines the incident laser beam direction for any combination of deflection mirror scan angles and is a direct alternative to equation (1). Evaluating the principal unit vector coefficients enables equation (4) to be re-expressed in terms of the deflection mirror scan angles:

$$\begin{aligned} U_m &= \sin 2\theta_{sx} [\dot{x}_r(P_0) + \dot{x}_f(P)] \\ &\quad - \cos 2\theta_{sx} \sin 2\theta_{sy} [\dot{y}_r(P_0) + \dot{y}_f(P)] \\ &\quad + \cos 2\theta_{sx} \cos 2\theta_{sy} [\dot{z}_r(P_0) + \dot{z}_f(P)] \end{aligned} \quad (9)$$

The known point x coordinate, x_0 , can be slightly redefined for convenience such that it excludes a component, Δx_0 , that is a function of the x deflection mirror angle, i.e.:

$$\Delta x_0 = -d_s \tan 2\theta_{sx} \quad (10a)$$

and equations (5b&c) are therefore re-formulated as follows:

$$\begin{aligned} \dot{y}_r(P_0) &= \dot{y} + (\dot{\theta}_z + \Omega)(x_0 - d_s \tan 2\theta_{sx} - x) \\ &\quad - (\dot{\theta}_x + \Omega\theta_y)(z_0 - z) \end{aligned} \quad (10b)$$

$$\begin{aligned} \dot{z}_r(P_0) &= \dot{z} + (\dot{\theta}_x + \Omega\theta_y)(y_0 - y) \\ &\quad - (\dot{\theta}_y - \Omega\theta_x)(x_0 - d_s \tan 2\theta_{sx} - x) \end{aligned} \quad (10c)$$

Derivation of equation (9) represents a significant development of the theoretical velocity sensitivity model as it allows the user to predict the sensitivity of a scanning Laser Vibrometer measurement for any combination of mirror scan angles on any target. It readily accommodates time dependent mirror scan angles where scanning profiles result and this will be discussed in the following sections for circular scanning measurements on rotating targets.

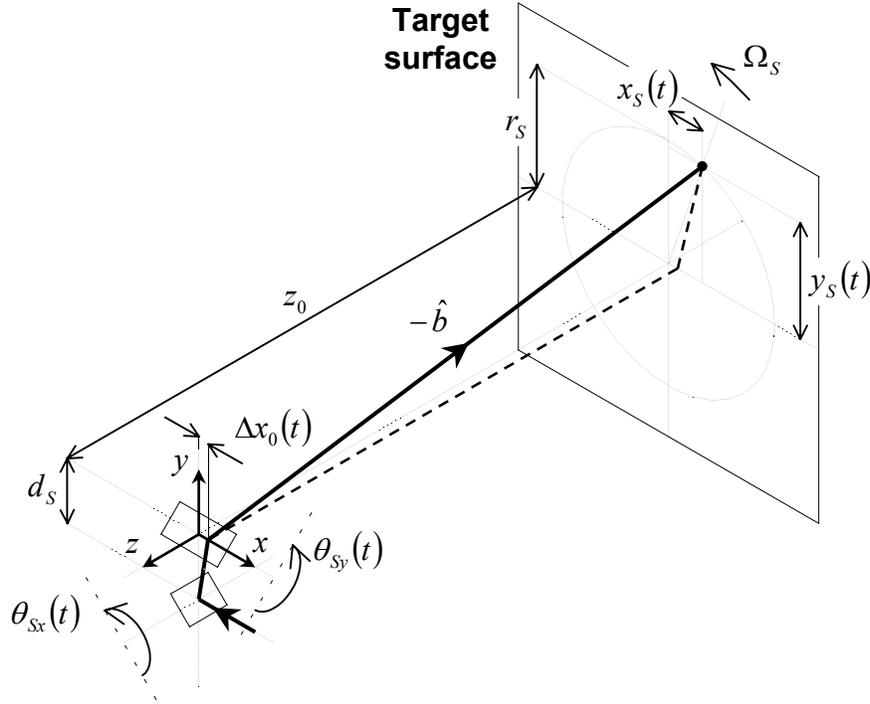


Figure 4 – The dual mirror scanning arrangement incorporating two orthogonally aligned mirrors, shown in terms of the laser beam deflection mirror scan angles

3.2. Deflection mirror scan angles for arbitrary scan profiles

With reference to Figure 4, the time dependent point of incidence of the laser beam on the target, \vec{r}_p , can be described by x_s and y_s (omitting the explicit declaration of time dependency for brevity in the equations):

$$\vec{r}_p = [x_s]\hat{x} + [y_s]\hat{y} \quad (11)$$

Consideration of the time dependent positions of the mirror incidence points and the target incidence point enables this to be defined in terms of the time dependent mirror scan angles θ_{sx} and θ_{sy} [18]:

$$x_s = -\tan 2\theta_{sx} \cdot \left(d_s + \frac{z_0}{\cos 2\theta_{sy}} \right) \quad (12a)$$

$$y_s = z_0 \tan 2\theta_{sy} \quad (12b)$$

Whilst equation (12b) can be rearranged such that the y deflection mirror scan angle can be obtained for any y_s , it can be seen from equation (12a) that x_s is not a simple function of the x deflection mirror scan angle. This is particularly important when attempting to obtain a circular scan profile via the simultaneous modulation of the x and y deflection mirror scan angles.

3.3. Circular scan profile analysis

As illustrated in Figure 4, a circular scan profile in the target plane, with radius r_s , scan angular frequency Ω_s and initial phase ϕ_s , requires that x_s and y_s are cosine and sine functions, respectively, such that equation (11) can be rewritten as:

$$\begin{aligned} \vec{r}_p &= [x_s]\hat{x} + [y_s]\hat{y} \\ &= [r_s \cos(\Omega_s t + \phi_s)]\hat{x} + [r_s \sin(\Omega_s t + \phi_s)]\hat{y} \end{aligned} \quad (13)$$

Substituting for x_s and y_s in equations (12a) and (12b) results in two equations which must be rearranged for the deflection mirror scan angles if such a scan profile is to be achieved. This rearrangement is not possible for equation (12a), the consequence of which is that a perfect circular scan cannot be achieved using basic functions to drive the deflection mirrors.

In real circular scanning LDV systems [5,14], cosine and sine functions of equal amplitude are used to perform a “circular” scan profile, i.e.:

$$\theta_{sx} = -\Theta_{sx} \cos(\Omega_s t + \phi_s) \quad (14a)$$

$$\theta_{sy} = \Theta_{sy} \sin(\Omega_s t + \phi_s) \quad (14b)$$

where

$$\Theta_{Sx} = \Theta_{Sy} = 0.5 \tan^{-1} \left(\frac{r_S}{z_0} \right) \quad (14c)$$

The slightly elliptical profile [18] which results can clearly be observed by substituting equations (14a,b&c) into equations (12a&b) and is shown, normalised to the desired scan radius, in Figure 5. Figure 6 shows the normalised actual scan radius as a function of scan angle.

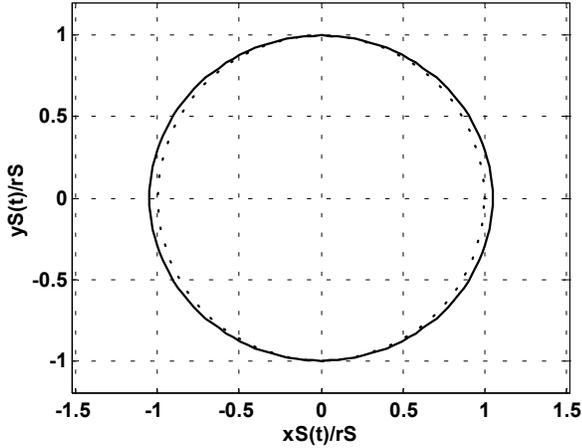


Figure 5 – Normalised scan profile which results from equal amplitude cosine and sine mirror drive signals
($d_S = 50\text{mm}$ and $z_0 = 1\text{m}$)

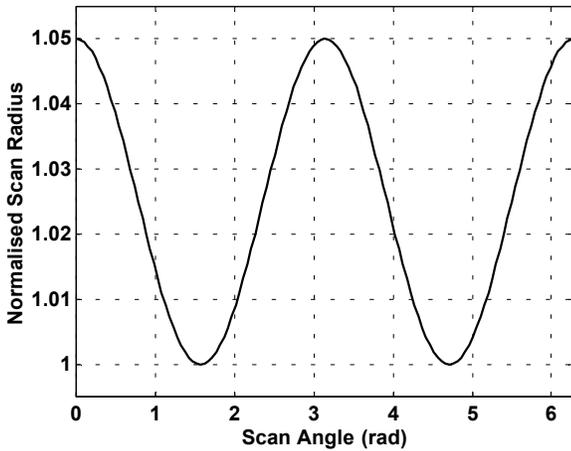


Figure 6 – Normalised scan radius vs. scan angle for equal amplitude cosine and sine mirror drive signals
($d_S = 50\text{mm}$ and $z_0 = 1\text{m}$)

3.4. Laser Vibrometer measurement analysis: dual mirror scanning system effects

In addition to this effect, and generally more important, is the influence that such variation in laser beam orientation has on the Laser Vibrometer measurement. Use of equation (9) allows prediction of the measured velocity in this particularly complex configuration with ease and it also shows how

additional components can occur when performing measurements on rotating targets.

These additional measurement components can be quantified by setting the flexible and rigid vibration components in equation (9) to zero. The system arrangement is as discussed earlier, i.e. the scanning system and target reference frames are collinear (no translational or angular misalignment), such that the measured Laser Vibrometer signal per unit rotation speed for this “no target vibration, no misalignment” case is given by:

$$\frac{U_m}{\Omega} = -d_S \sin 2\theta_{Sx} \sin 2\theta_{Sy} \quad (15)$$

Substituting for θ_{Sx} and θ_{Sy} using equations (14a,b&c), this becomes:

$$\frac{U_m}{\Omega} = -d_S \sin \left(\tan^{-1} \left(\frac{r_S}{z_0} \right) \cdot \cos(\Omega_S t + \phi_S) \right) \cdot \sin \left(\tan^{-1} \left(\frac{r_S}{z_0} \right) \cdot \sin(\Omega_S t + \phi_S) \right) \quad (16)$$

The additional information that exists in the measured Laser Vibrometer signal occurs at twice and six times the scan frequency, as shown in Figure 7. For typical rotation frequencies and scan radii, the level of the component at six times the scan frequency is well below the noise floor that results from the laser speckle effect, generally greater than 10^{-2}mm/s (10^{-4}mm/s/rad/s in Figure 7) and can therefore be considered insignificant. The component at 2x scan frequency is, however, of significance since typical levels are of the order of mm/s. This component has been observed previously [14] but without full explanation until recently [18].

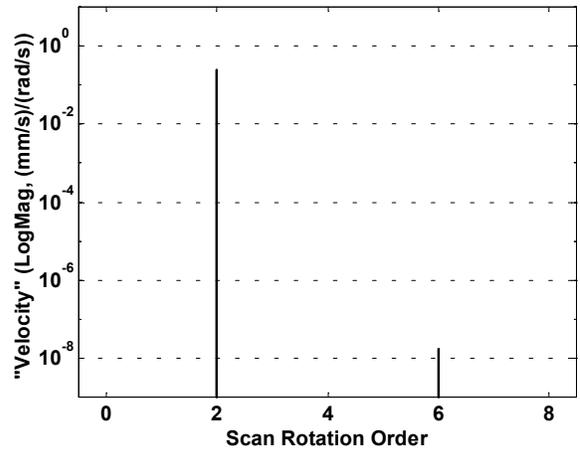


Figure 7 – Additional measurement components that occur in a dual-mirror circular scan when employing equal amplitude cosine and sine mirror drive signals
($r_S = 100\text{mm}$, $d_S = 50\text{mm}$ and $z_0 = 1\text{m}$)

It is due to additional measured “vibration” components such as this that care must be taken when interpreting vibration information obtained from such measurements. This issue demonstrates the value of the velocity sensitivity model very clearly – it enables the vibration engineer to make Laser Vibrometer measurements with confidence. Theoretical component amplitudes show good agreement with those that have been previously reported [14] and with those obtained from experimentation and experimental validation will be discussed in the final section of this paper.

Circular tracking measurements can be straightforwardly arranged for by setting the scan frequency equal to the target rotation frequency such that probe laser beam remains fixed on a single point on the target during rotation. The model continues to predict the additional components encountered which, in this case, occur at twice and six times rotation frequency.

3.5. Laser Vibrometer measurement analysis: target rotation and scanning system misalignment

The model can be used to predict the effect of translational or angular misalignment between the target and scanning system axes. Translational misalignment is accounted for in the model by including the constants x_{0m} and y_{0m} in the known point x and y parameters. Similarly, angular misalignment is represented by including θ_{xm} and θ_{ym} in the angular vibration displacement parameters. Again, setting the flexible and rigid vibration components to zero in equation (9) enables the measured velocity to be predicted for this “no target vibration, arbitrary misalignment” case. Making use of equations (5a) and (10b&c), equation (9) becomes:

$$\begin{aligned} \frac{U_m}{\Omega} = & \cos 2\theta_{Sx} \cos 2\theta_{Sy} [\theta_{xm} x_{0m} + \theta_{ym} y_{0m}] \\ & + \sin 2\theta_{Sx} [y_{0m} + \theta_{xm} (z_0 + d_S \cos 2\theta_{Sy})] \\ & - \cos 2\theta_{Sx} \sin 2\theta_{Sy} [x_{0m} - \theta_{ym} z_0] \\ & - d_S \sin 2\theta_{Sx} \sin 2\theta_{Sy} \end{aligned} \quad (17)$$

Substitution for θ_{Sx} and θ_{Sy} using equations (14a,b&c) immediately results in a full expression for the velocity measured but this will not be presented here in the interest of brevity. The additional information that exists in the measured Laser Vibrometer signal occurs at DC and harmonics of the scan frequency, as illustrated in Figure (10). Interestingly, it can be seen that the misalignment does not affect the component at 2x scan frequency, which is only due to the fourth term in equation (17) and is as given in equation (15).

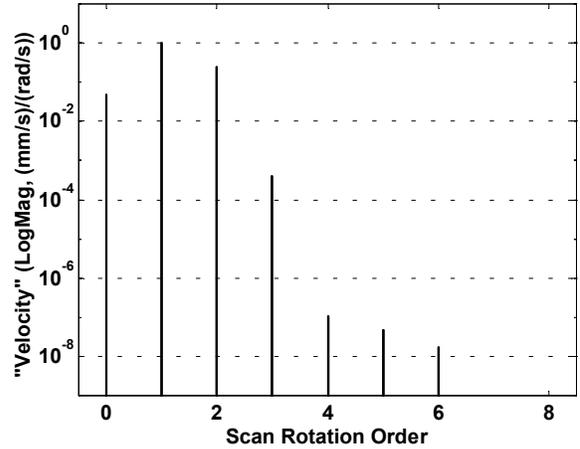


Figure 8 – Additional measurement components that occur due to misalignment between the (dual mirror) scanning system and target rotation axes ($r_s = 100\text{mm}$, $d_s = 50\text{mm}$, $z_0 = 1\text{m}$, $x_{0m}=y_{0m}= 5\text{mm}$ and $\theta_{xm}=\theta_{ym}= 5\text{mrad}$)

4. EXPERIMENTAL VALIDATION

Theoretical component amplitudes show good agreement with those obtained from experimentation and with those that have been previously reported [14] as discussed in this section.

4.1. Experimental arrangement

The scanning system used was custom built using a Polytec OFV323 Laser Vibrometer and a pair of GSI Lumonics M3 galvanometers. Each galvanometer can drive the mirror angular displacement by $\pm 15^\circ$ mechanical ($\pm 30^\circ$ optical). A two-channel signal generator was used to generate the cosine and sine wave necessary to perform a “circular” scan. The galvos are mounted relative to the Laser Vibrometer in similar fashion to that shown schematically in Figures 4 and 5. This is equivalent to the arrangement employed in both the Ometron Type 8330 and the Polytec PSV300.

The target used was a small ($\varnothing 30\text{mm} \times 5\text{mm}$), light aluminium disc of rigid cross-section mounted to a small DC motor. The target rotation frequency was controlled using a stable calibrated DC power supply and measured using a Polytec OFV400 Rotational Laser Vibrometer.

4.2. Dual mirror scanning system effects

Using small angle approximations to simplify equation (16):

$$\frac{U_m}{\Omega} = -\frac{d_S r_S^2}{2z_0^2} \sin 2(\Omega_S t + \phi_S) \quad (18)$$

it is evident that, for this “no target vibration, no misalignment” case, the component at 2x scan frequency dominates the measurement, as illustrated in Figure 7. Since the amplitude of this component is a function of the perpendicular mirror separation, d_S , as well as the scan

radius, r_s , and the stand-off distance, z_0 , the scanning system configuration included a facility that enabled variation of d_s between 30mm and 50mm.

Figure 9 shows a comparison between the predicted and measured amplitude of the additional component at twice scan frequency for a series of measurement configurations. Each solid line represents the theoretical prediction of U_m/Ω vs. r_s for a particular combination of d_s and z_0 , with the underlying points representing the corresponding measurement values. The theoretical prediction generally shows good correlation with the measured data when the amplitude is relatively high. When the amplitude is lower, the noise due to the speckle effect becomes more significant and the agreement is less good.

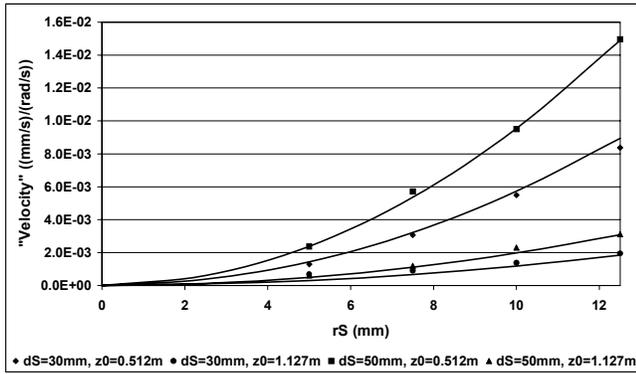


Figure 9 – Experimental validation of the additional measurement component at twice scan frequency ($\Omega_s=40\pi$ rad/s)

4.3. Target rotation and scanning system misalignment effects

Substituting for θ_{sx} and θ_{sy} using equations (14a,b&c) and using small angle approximations enables equation (17) to be re-written as:

$$\begin{aligned} \frac{U_m}{\Omega} = & \left[\theta_{xm} x_{0m} + \theta_{ym} y_{0m} \right] \\ & + \frac{r_s}{z_0} \left[y_{0m} + \theta_{xm} (z_0 + d_s) \right] \cos(\Omega_s t + \phi_s) \\ & - \frac{r_s}{z_0} \left[x_{0m} - \theta_{ym} z_0 \right] \sin(\Omega_s t + \phi_s) \\ & - \frac{d_s r_s^2}{2z_0^2} \sin 2(\Omega_s t + \phi_s) \end{aligned} \quad (19)$$

Clearly, for this “no target vibration, arbitrary misalignment” case, the components at DC, 1x and 2x scan frequency dominate the measurement, as illustrated in Figure (8). Equation (19) confirms that the component at 2x scan frequency is unaffected by misalignment, which mostly influences the DC and 1x components. Since spectrum analysers are typically AC coupled, the 1x component was used to validate the theory in this experiment and the

scanning system incorporated a facility that enabled the variation of x_{0m} , y_{0m} and θ_{ym} . Whilst the “no target vibration” condition is relatively straightforward to achieve in the laboratory by taking care with target selection, the “no misalignment” condition is not. It can be shown that small but inevitable initial misalignment between the scanning system and target rotation axes result in significant differences in the amplitude of the component at 1x scan frequency. Despite the fact that these initial misalignments are difficult to control, they can be accounted for in the model and an initial experimental validation carried out.

Figure 10 shows a comparison between the predicted and measured amplitude of the additional component at the scan frequency for a series of measurements in which the translational misalignment in the x direction, x_{0m} , was varied. The solid line represents the theoretical prediction (adjusted to include initial misalignments) of U_m/Ω vs. x_{0m} and the underlying points represent the corresponding measurement values. The theoretical prediction generally shows good correlation with the measured data, particularly when considering the number of parameters that must be controlled to perform this experimental validation.

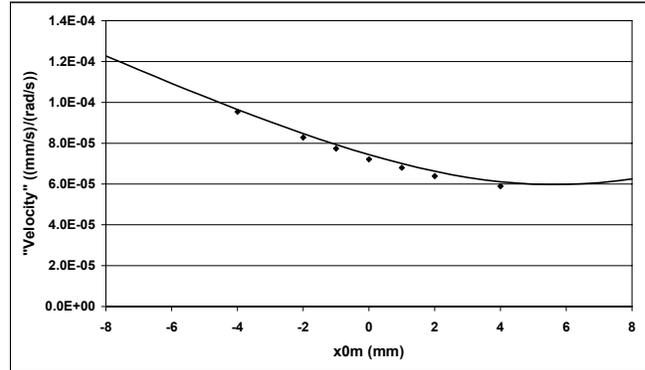


Figure 10 – Experimental validation of the additional measurement component at scan frequency ($r_s = 10$ mm, $d_s = 50$ mm, $z_0 = 1.272$ m, $\Omega_s=20\pi$ rad/s, $y_{0m} = 0$ mm and $\theta_{xm} = \theta_{ym} = 0$ rad)

4.4. Laser speckle effects

It has been suggested that configuring a scanning Laser Vibrometer to perform a tracking measurement (i.e. setting the scan frequency equal to the target rotation frequency) will lead to a significant reduction in the underlying noise level that is associated with the laser speckle effect. The series of measurements carried out to investigate this hypothesis involved the variation of the scan frequency whilst keeping all other parameters constant. Figure 11 shows the sum of the squares of the spectral components (minus the components at 1x and 2x scan frequency) vs. the scan frequency, with the target rotation frequency nominally 20Hz.

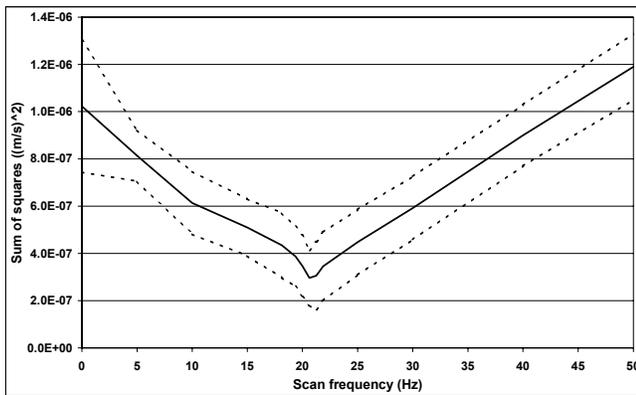


Figure 11 – Experimental validation of the influence of speckle noise in a scanning measurement

The solid line and broken lines represent the mean and standard deviation, respectively, for a number of tests in which various configuration parameters were changed. This data clearly shows that there is a significant reduction in the general noise level in a tracking Laser Vibrometer measurement.

CONCLUSIONS

The original derivation of the comprehensive velocity sensitivity model showed explicitly how the velocity sensed by an arbitrarily orientated laser beam incident on a rotating target, of rigid cross-section, undergoing arbitrary vibration, is dependent upon both the target velocity components and the direction of the laser beam. This was extended in this paper to include provision for targets with flexible cross-sections, since Laser Vibrometer measurements are employed in applications where flexibility must be acknowledged.

The use of Laser Vibrometers incorporating some form of manipulation of the laser beam orientation has become increasingly popular in recent years, in particular the operation of such scanning Laser Vibrometers in continuous scanning mode. This paper has investigated the application of the velocity sensitivity model to this particularly challenging measurement technique. Reformulation of the original model in terms of mirror scan angles, rather than laser beam orientation angles, allows easy formulation of measured velocity, revealing some important details in the measurement that were not apparent in previous research.

The revised velocity sensitivity model shows how the common use of a pair of orthogonally aligned scanning mirrors leads to an elliptical scan profile and a significant additional component in the Laser Vibrometer output at twice the scan frequency in circular scanning measurements on rotating targets. Furthermore, it has been shown how misalignment between the scanning system and the target rotation axes leads to significant components in the Laser Vibrometer output at DC and at the scan frequency. This paper has presented, for the first time, an experimental validation that confirms the validity of the velocity sensitivity model. This emphasises how, even for particularly complex measurement configurations, the model can be used to

readily predict the Laser Vibrometer output and enable the user to make measurements with confidence.

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