NON-LINEAR CONTROL APPROACHES
FOR ACTIVE RAILWAY SUSPENSIONS

by

Hong Li

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In memory of my grandmother

ABSTRACT

This thesis studies various linear and non-linear control approaches for active railway suspensions. The aim of the study is to improve the system performance of active secondary suspensions in response to different track features. The primary motivation for active suspension on railway vehicles is to improve suspension performance and thereby run faster or provide a better ride quality. The problem of discriminating between the random track and deterministic track input is a fundamental problem for the design of active secondary suspensions on railway vehicle. The basic requirement of an active suspension system is to improve the ride quality without increasing the suspension deflection unacceptably when the vehicle negotiates on both straight track and deterministic track features.

This thesis presents and compares different control strategies of active suspension systems for railway vehicles. Firstly, a number of linear approaches for filtering the absolute velocity signal are theoretically examined in order to optimise the trade-off between the random and deterministic input requirements. What can be achieved with linear filters is initially determined. This is quantified by the degradation in the straight track ride quality needed to restrict the maximum deflection to an acceptable level as a vehicle traverses the transition to a typical railway gradient, and a range of filter types, frequencies and absolute damping rates are assessed in order to explore the boundary of what can be achieved through linear means. Secondly, some non-linear Kalman-Filter methods are investigated to further improve the suspension performance. Finally, a comparison between linear and non-linear strategies is studied.

ACKNOWLEDGEMENTS

PUBLICATIONS

1. Hong Li and Roger Goodall, "Distinguishing Between Random And Deterministic Track Inputs For Active Railway", 15th Symposium of Dynamics of Vehicles on Roads and Tracks (IAVSD), Aug. 1997, Hungary.

2. Hong Li and Roger Goodall, "Practical Implementation of Skyhook Damping for Active Railway Suspensions", 12th International Conference on System Engineering ICSE'97, pp. 425-430, Sept. 1997, Coventry, UK.

LIST OF SYMBOLS

A	state-space model matrix for vehicle dynamics (continuous system)	
A_{ctrl}	state-space model matrix for the controller	
A_d	state-space model matrix for vehicle dynamics (discrete system)	
A_k	state-space model matrix for Kalman filter	
A_r	track roughness	[m]
B	actuator force matrix	
B_{ctrl}	state-space model input matrix for the controller	
c	passive damping	[N / m s^{-1}]
c_s	skyhook damping	[N / m s^{-1}]
C	output matrix	
C_{ctrl}	output matrix for the controller	
C_k	output matrix for Kalman filter	
d	suspension deflection	[m]
d_{qs}	quasi-static deflection	[m]
D_B	actuator force matrix in the output equation	
D_{ctrl}	input matrix in the output equation for the controller	
f	frequency	[Hz]
f_i	filter cut-off frequency	[Hz]
f_s	spatial frequency	[cycle / m]
f_t	temporal frequency	[Hz]
G	state-space model matrix, track inputs (continuous system)	
G_d	state-space model matrix, track inputs (discrete system)	
k_s, k_d	spring stiffness	[N / m]
K,K1,K2	stationary gain matrix for Kalman filter	
m_b	vehicle body mass	[kg]

P_x	covariance matrix of the X state	
Q	covariance of track input	
Q_k, Q_{k1}, Q_{k2}	process noise for Kalman filter	
Q_w	white noise density	
$r1, r2$	threshold detector level	
$R, R2$	covariance of measurements noise	
S_y	output spectral density	
S_{zt}	spatial spectral density	$[m^2 / cycle\ m^{-1}]$
S_{z_t}'	single side vertical track velocity spectrum (frequency base)	$[m^2/Hz]$
t	time	
v	velocity	$[m / s]$
V	measurements noise	
w	track input	
X	state vector	
$\hat{X}$	state estimate	
X_{ctrl}	state vector for the controller	
Y	output vector	
$\hat{Y}$	output estimate	
Y_m	measured output	
z_b	vehicle body displacement	$[m]$
$\dot{z}_b$	vertical velocity of vehicle body	$[m / s]$
$\hat{\dot{z}}_b$	estimated vertical velocity of vehicle body	
$\ddot{z}_b$	vertical acceleration of vehicle body	$[m / s^2]$
$\hat{\ddot{z}}_b, \hat{\ddot{z}}_{b1}, \hat{\ddot{z}}_{b2}$	estimated vertical acceleration of vehicle body	
$\ddot{z}_{b_{rms}}$	root mean square (r.m.s.) of vertical body acceleration	
z_t	vertical track input displacement	$[m]$
z_1	suspension displacement	$[m]$

δ	dirac function [1 / time]	
ζ	damping ratio	
ξ_i	filter damping factor	
τ	time period	[s]
ω_0	natural frequency	[rad / s]
ω_i	filter cut-off frequency	[rad / s]

CONTENTS

CHAPTER 1

INTRODUCTION

A primitive form of railway existed as early as the end of the 15th century in the iron mines of Slovakia [Kalla-Bishop - 1972], and the world's first public railway was opened more than 170 years ago in 1825 [Allen - 1959]. Since then the railway industry has gone through tremendous changes and improvements in almost all its aspects, apart from that the basic form remains unchanged - vehicles running on a pair of steel rails. Much work has been done to improve its safety, speed, efficiency and ride quality etc. Nowadays, the railway is still one of the major transport systems widely used in the world, although it faces increasing challenges from other forms of transportation such as road vehicles and airplanes.

All railway vehicles are fitted with a suspension. The main tasks of such a suspension are: (1) to support the vehicle's weight and follow the track; (2) to isolate the vehicle body from the disturbances such as track irregularities and other external forces.

Depending on requirements, different railway vehicles may be fitted with different components. In general freight vehicles have less sophisticated suspensions than passenger vehicles or locomotives. Whilst passenger vehicles generally have both a primary suspension (between the wheelset and the bogie frame) and a secondary suspension (between the bogie frame and the vehicle body), freight vehicles often have only one layer of suspension.

The two tasks of suspensions mentioned above are in many senses contradictory, the first requiring a "stiff" suspension and the second a "soft" suspension, and there is

therefore a well known conflict between the two. For passenger vehicles this problem, to an extent, is tackled by using two levels of suspension: a stiff primary suspension for following the track and a soft secondary suspension for isolating the disturbances, but the design conflict still remains for the secondary suspension itself.

The railway vehicle suspensions are made up of a number of different components. These range from mechanically simple coil springs and friction dampers to sophisticated arrangements of air springs, levelling valves and reservoirs or active or semi-active components. According to the components used, the suspensions can be classified as:

1) Passive Suspensions, where only passive components are used

2) Active Suspensions, where some active components are involved.

1.1 Passive Suspensions

Conventional passive suspensions usually utilise coil or leaf springs, torsion bars, and viscous dampers; more recently there have been springs using rubber, liquids or gases or combinations of these media. Passive suspensions are fitted to nearly all current rolling stock, and for many applications the performance capability of a passive suspension is adequate.

The advantages of passive suspensions are obvious. They are relatively simpler and cheaper and do not require external power supplies. However, since passive suspensions can only store (i.e. a spring) or dissipate (i.e. a damper) energy, their performance is limited. A very fundamental limitation of passive elements is that its static deflection varies as the inverse square of the natural frequency (a more complete explanation and in depth study is given in Chapter 3); this limits the lower natural frequency to approximately 1 Hz with a corresponding static deflection of the order of 250 mm and causes large dynamic deflections when external loads of the same frequency are applied [Goodall - 1983]. In addition, the ground clearance and

the dynamic characteristics change with variations in static loading. Attempts have been made to counteract these deficiencies, mainly by using non-linear springs and also using self-levelling components (often air springs). However, even when vehicles using non-linear springs operate with light loads, rough road surfaces combined with high speed result in the suspensions operating in their non-linear range and they then behave as much stiffer systems, with resulting degradation of the vibration isolation characteristics.

1.2 Active Suspensions and Their Potential

Recently there has been a growing interest in the use of active suspensions on railway vehicles for its great potential improvements over passive suspensions. Many studies [Goodall - 1997] have been undertaken in connection with active suspensions for rail and road vehicles, and some of these have addressed the fundamental improvements which are possible through the use of active elements [Hedrick - 1981], [Thompson - 1976].

Generally, an active suspension consists of an external power source to inject as well as store and dissipate energy, a measurement device, an actuator and a feedback controller to provide control commands for the actuator. An active suspension may also contain passive elements in parallel with the active element, which is particularly useful with vertical active suspension where a spring can be used to support the vehicle, reducing power consumption and actuator size.

The potential advantages of such active suspensions are [Goodall - 1983]:
- low natural frequencies, which result in greater passenger comfort while still maintaining small static deflections
- low dynamic deflections
- suspension characteristics maintained regardless of loading
- high speed of response to any input

- high flexibility in the choice of dynamic response, especially between different modes of the vehicle

These advantages of active suspensions are derived from two basic features [Hedrick - 1975]:

- Active suspensions can continually supply and modulate the flow of energy whilst passive systems can only dissipate, temporarily store and later return energy to the system. Thus in an active system forces may be generated which do not depend upon energy previously stored by the suspension.
- An active system may generate forces which are a function of many variables, some of which may be remotely measured.

In consequence, an active system can adapt to various levels of external forces and track irregularities so that it simultaneously appears "soft" to irregularities and "hard" to external forces, providing a better compromise between a "soft" and a "hard" suspension than a passive suspension which always has fixed properties.

The principal disadvantages of active suspension are derived from their need for an external power source, their increased complexity and cost, and very importantly their increased maintenance requirements.

1.3 Literature Survey

Active suspensions for railway vehicles have now been under serious consideration at a theoretical or experimental level for around 30 years [Goodall - 1997]. It has been generally accepted that actively-controlled suspensions are able to offer improvements beyond what is possible passively, and consequently the use of active elements in railway suspension system is on the development agenda for a number of railway equipment manufacturers around the world. Over this period there have been a number of conceptual studies which have laid the theoretical foundations; also survey

papers which provided a review of techniques and developments. Some have given an overview of development [Goodall - 1983], [Hedrick - 1975], whereas others have endeavoured to question the rationale for their use [Karnopp - 1978a], [Goodall - 1990].

1.3.1 The Development of Active Suspensions

Although active suspensions have been under development since the 1930's, most of the significant developmental work has occurred since 1950; the development of active suspension systems has paralleled the development of high performance feedback control servo mechanisms. In the 1950's pneumo-mechanical suspensions for automotive-type vehicles were developed, as were low frequency load levelling suspensions for several types of vehicles. In the mid 1960's interest in active vehicle suspensions increased in the U.S. and a number of other countries as efforts to develop improved higher speed intercity transport systems capable of greater than 75 m/s were initiated [Hedrick - 1975]. Because of the greater speeds and high levels of performance required, research and development of advanced rail vehicles and new types of air cushion and magnetically levitated vehicles which employ active suspensions was initiated [Hedrick - 1975] and has continued to draw great attentions.

British Rail started its active suspension research in the late 1960's with work on Advanced Passenger Train (APT) tilting system [Goodall - 1997]. By tilting passenger vehicles while passing through curves the effect of lateral acceleration on the passengers is removed, leading to increased comfort and higher operating speeds. Though it is possible to tilt a vehicle by using pendulum action alone, it is usually necessary to include an active element to achieve sufficient speed of operation. Initially tilt systems were powered by hydraulic actuators, however recent work has shown that there are advantages to be gained by using electric-mechanical actuators [Williams - 1986].

Recently, active suspension technology for rail vehicles is developing rapidly. In Europe ABB have tested a semi-active lateral suspension for the X2000 trains [Roth - 1995]; this uses a novel semi-active damper which has been specially developed. ABB also offer an active lateral "hold-off" device, mainly for use on more conventional trains in the UK. GEC-Alsthom, the manufacturers of TGV vehicles, have studied a semi-active damper for lateral suspensions on a laboratory rig. And Bombardier Eurorail, SGP (Siemens), SIG and AEG also have on-going tests on active control. In Japan a wide range of studies are happening, partly with their WIN350 vehicle which is being used for extending Shinkansen operating speeds to 350 km/h, but also with a "Try-Z" test vehicle on which a number of active suspension concepts are being developed and tested [Goodall - 1997].

1.3.2 Degree of Control

It is possible to further subdivide "active suspension" into semi-active, semi-passive, and fully active categories [Hedrick - 1981]. Semi-active systems are considered to be derived from and closely related to the active systems. The semi-active system is one in which a linear absolute velocity damping force is provided by modulating an actuator so that it is "turned on" only when it is dissipating power. The concept requires sensor and control signal conditioning power but does not require extensive actuator power. Various studies have indicated that semi-active systems can achieve significant percentages of a fully active system's performance at greatly reduced power levels. The semi-active suspension concept proposed by [Karnopp - 1974] and applied to a rail vehicle [White - 1981] required the least power of all three concepts. In a more recent paper [Karnopp - 1990], Karnopp illustrated the simplicity of semi-active suspension and how this type of suspension could be implemented using hydraulic cylinders in conjunction with valve switching logic to implement the control law.

The semi-passive suspension is also known as adjustable passive and adaptive. Its characteristics are varied on the basis of a variable which is not influenced by the dynamic systems being controlled - a good example is varying the rate of the damper as a function of vehicle speed [Goodall - 1997]. Sensors and force actuators are required but a discrete control strategy is developed that switches between passive and active actions depending upon whether the train is on smooth or rough track.

The fully active suspension implies use of a power source, actuators, and sensors. The actuators operate with force transducers providing inner loop feedback signals to their controllers and are imagined to track faithfully a force demand signal determined by the control law. The control law may contain information of any kind obtained from anywhere in the system, and an important part of the active system design problem is the determination of the control law which will give a good system performance.

There is another aspect to the degree of control, primarily applicable to full-active suspensions, which can broadly be subdivided into low bandwidth or high bandwidth. In low bandwidth systems there will be passive elements which dictate the fundamental dynamic response, and the function of the active element is associated with some low frequency activity such as levelling or centring. This restriction enables some reduction in force and/or velocity requirement for the actuators. By contrast, high bandwidth active systems have a much enhanced capability, and the dynamic response will primarily be determined by the active control strategy, which will probably act throughout the frequency range which is relevant to the particular suspension function being controlled.

1.3.3 Technology of Control

Various actuators and sensors have been developed for active systems. The actuators in a fully active system can vary from devices that just augment the forces exerted by the passive components, to those that replace the conventional springs and dampers

with hydraulic struts, support the full weight of the vehicle and, using servo valves, have full authority over the suspension characteristics [Appleyard - 1995]. Actuator technologies which are possible for the active suspension applications are: servo-hydraulic, servo-pneumatic, electro-mechanical and electro-magnetic. Servo-hydraulic actuators themselves are compact and easy to fit, but the whole system tends to be bulky and inefficient, and there are important questions relating to maintainability. Pneumatic actuators are a possibility, particularly since the air springs fitted to many railway vehicles can form the basic actuator, but inefficiency and limited controllability are important limitations. Electro-mechanical actuators offer a technology with which railway is generally familiar, and the availability of high performance servo-motors and high efficiency power electronics are favourable indicators. However they tend to be less compact and reliability and life of the mechanical components needs careful consideration. Electro-magnetic actuators potentially offer an extremely high reliability and high performance solution, but they tend to be quiet bulky and have a somewhat limited travel [Goodall - 1994a].

The sensors available for a active suspension system would vary, but could comprise body and accelerometers, suspension deflection sensors, together with roll and yaw rate gyros [Appleyard - 1995].

1.3.4 Control Studies

The improvement of suspension by using feedback control has been studied for many years [Thompson - 1976], [Wilson - 1986], [Yue - 1989], [Williams - 1994], and strategies for control of active systems have been reviewed previously [Hedrick - 1981]. Two typical control strategies are known as linear optimal control and skyhook damping. Linear optimal control approach, in its simplest form, requires that all state variables are measured and the performance index to be minimised is of quadratic form [Kwakernaak - 1972]. Thompson proposed a performance index that uses a weighted sum of integral squares of body acceleration, dynamic tyre deflection and

relative body-to-axle displacement [Thompson - 1976], and then he modified usual optimal formulation by insertion of additional integrators to improve the low frequency performance [Thompson - 1988]. Also there is a quantity of material published relating to optimal active suspension: [Wilson -1986], [Karnopp - 1986], [Ulsoy - 1994]. Karnopp [Karnopp - 1974] developed a heuristic control scheme known as "skyhook damping" which has since been commonly considered for a particular type of active system. The term "skyhook damping" was coined to reflect the fact that absolute damping could in theory be applied by attaching a damper between the body and the sky. But in the practical skyhook damping approach, an actuator is included in the suspension whose actuation force is proportional to the absolute body velocity. It acts so as to reduce the body velocity caused by the track input. The result is to improve the low frequency response (which is associated with body resonance), without introducing high frequency noise [Appleyard - 1995]. Further studies of skyhook damping have demonstrated that when other suspension issues such as responses on curves and gradients are to be considered it does not provide an adequate form of suspension. Additional filtering is required to form a realistic design [Yue - 1989], [Williams - 1994].

Most papers have concentrated on the response to stochastic track inputs [Thompson - 1976], [Wilson - 1986], [Karnopp - 1978b], and it is relatively straight forward to show that an active secondary suspension can approximately halve the r.m.s. acceleration level. Also, there were some research programmes investigating lateral improvements [Sinha - 1978], [Celniker - 1982]. Rather fewer studies have included the response to deterministic inputs in their assessment. One recent paper shows an improvement in ride quality of nearly 40% with an active lateral suspension while the vehicle negotiates not only the straight track (random) inputs, but also the curves [Pratt - 1996]. There has been limited work on active vertical suspensions [Goodall - 1993] which attempts to reconcile the requirements relating to both random and deterministic inputs.

1.4 Problem Foundation

The primary motivation for active secondary suspension on railway vehicles is to improve suspension performance, and thereby run faster or provide a better ride quality. The problem of discriminating between the random track (track roughness) and deterministic track inputs (i.e. a gradient for the vertical suspension, a curve for the lateral suspension) is a fundamental issue for the design of active secondary suspensions on rail vehicles [Davis - 1988], [Goodall - 1993]. The difficulty comes from the use of absolute velocity damping (also known as "skyhook" damping). It is well accepted that the use of absolute velocity is a key feature of the active control law; e.g. on straight track this can typically deliver a 50% reduction in r.m.s. acceleration on the body of the vehicle even without softening the suspension in any way. However, the effect of skyhook damping can increase quite radically the suspension deflection which occurs when the vehicle negotiates a deterministic track feature, and it becomes necessary to modify the control law in some manner, for example by filtering the absolute velocity to remove the low frequency variations associated with the deterministic input. The net effect however is to reduce the improvements in straight track ride quality which can be achieved through active control.

This thesis describes the only comprehensive study of the random versus deterministic trade-off problem for vertical suspensions, and investigates and assesses performance of active systems using various linear and non-linear approaches in comparison with the basic passive suspension response. It first examines a number of approaches for filtering the absolute velocity signal in order to optimise the trade-off between the random and deterministic track input requirements. What can be achieved with linear filters is initially determined. This is quantified by the degradation in the straight track ride quality needed to restrict the maximum deflection to an acceptable level as a vehicle traverses the transition to a typical railway gradient, and a range of filter types, frequencies and absolute damping rates

are assessed in order to explore the boundary of what can be achieved through linear means. Then, some non-linear filtering strategies are also investigated to further improve the suspension performance.

The thesis concentrates upon the secondary vertical suspension, but many of the ideas apply equally to the lateral suspension.

1.5   Structure of the Thesis

Chapter 2 to 6 are organised as follows:

Chapter 2 - this chapter describes the features of the railway vehicle suspension inputs, suspension performance and the assessment methods. Three suspension performance analysis techniques are studied: frequency domain analysis, covariance analysis and time history analysis.

Chapter 3 - this chapter is concerned with the comparison of active and passive suspensions. And all three suspension performance analysis methods discussed in the previous chapter are used here for a comparison.

Chapter 4 - this chapter details the linear control strategies used for the active railway suspension. Three linear control strategies are: intuitive formulation control, complementary filter control and Kalman-Bucy filtering control.

Chapter 5 - in this chapter two non-linear control approaches are explored: dual Kalman filter control and single Kalman filter control. A comparison of linear and non-linear methods are given, and the advantages and disadvantages of the strategies are established.

Chapter 6 - this chapter summarises the results from the previous chapters. Conclusions from the findings of the research study are made and discussed, and recommendations for further work are made.

CHAPTER 2

METHODS OF SUSPENSION ANALYSIS

2.1 Features of the Track

A vehicle suspension system is subjected to various inputs, which may be subdivided into three types [Goodall - 1994b]: a) stochastic track inputs representing the irregular surface, b) deterministic track inputs representing intended features, such as gradients for the vertical suspension, curves for the lateral suspension, c) and also the force inputs which the suspension must react to.

2.1.1 Random Inputs

The random inputs represent the inaccuracies in laying track, the lack of straightness of the steel rail and the effects of fixtures. From the results of many studies [Spangler - 1966] [LaBarre - 1970], the vertical track level may be approximated as a random process with a power spectral density (referred to as a P.S.D.) of the vertical track input z_t. The general form of spatial spectral density of the track elevation $z_t(x)$ is as follows:

$$S_{z_t}(f_s) = \frac{A_r}{f_s^2} \qquad \left[m^2 \, / \, cycle \, m^{-1} \right] \qquad (2.1)$$

where f_s [cycle/m] is the spatial frequency, A_r is the track roughness factor. $A_r = 2.5 \times 10^{-7}$ is commonly taken to represent mainline track and is used throughout this thesis. In fact, a more complete representation has some higher order terms on the

denominator [Pratt - 1997], but for high speed vehicle this only affects the P.S.D. at higher frequencies which are not so important for a secondary suspension.

The dynamic modes of railway vehicle lie between 0.1 and 10 Hz, it will therefore be excited by track inputs in this frequency range. With this reason, equation 2.1 may be converted into a temporal form by using $vf_s=f_t$. Thus,

$$S_{z_t}(f_t) = \frac{A_r v^2}{f_t^2} \qquad [m^2 / cycle\ m^{-1}] \qquad (2.2)$$

where f_t [Hz] is the temporal frequency and v is the vehicle velocity.

The time-varying track displacement $z_t(t)$ is derived from traversing, at velocity v, a rigid road profile which has the form $z_t(x)$, where x indicates position in the direction of motion. At any instant, these two quantities $z_t(t)$ and $z_t(x)$ must be equal, because the position of the point of contact is given by $x=vt$. And the corresponding spectral densities are shown in equation 2.3 [Robson - 1979].

$$S_{z_t(t)}{'}(f_t) = \frac{S_{z_t(x)}(f_t)}{v} \qquad [m^2 / Hz] \qquad (2.3)$$

It is clear that the power spectral density $S_{z_t}(f_t)$ provides an invariant description of a road profile, from which the excitation spectral density $S_{z_t}{'}(f_t)$ can always be derived for any particular vehicle velocity.

By substituting the division shown in equation 2.3 into equation 2.2, a temporal frequency based spectrum is given in equation 2.4.

$$S_{z_t}{'}(f_t) = \frac{A_r v}{f_t^2} \qquad [m^2 / Hz] \qquad (2.4)$$

Since future calculations are greatly facilitated by the use of an input velocity spectrum, the power spectral density for dz_t/dt is then $\omega^2 S_{z_t}'$, i.e.:

$$S_{\dot{z}_t}'(f_t) = (2\pi)^2 A_r v \qquad \left[(m/s)^2 / Hz\right] \qquad (2.5)$$

This vertical random track spectrum is "flat" over all frequencies, and it is in essence "white noise" with a Gaussian distribution. The amplitude of this "flat" power spectrum of velocity is proportional to the vehicle speed.

2.1.2 Deterministic Inputs

Deterministic inputs are of low-frequency/long-wavelength features. These features are most conveniently accounted for by considering the transition when changing from level track onto a gradient, because it is necessary when designing these transitions to limit the superimposed vertical acceleration. If improperly specified the intended variations in the track inputs will cause discomfort to passengers.

Deterministic features of the track are effectively low frequency inputs and so the steady state suspension deflection must be considered. In this thesis, a typical railway gradient of 1 % is assumed with a superimposed acceleration limit of 5 %'g' (i.e. a/g *100%) and a 1.1 second transitional section. Figure 2.1 shows the deterministic inputs used to assess the effects of a vehicle transition onto a gradient. In the definition of the gradient a constant forward velocity of 55 m/s (200 km/hr) for the vehicle has been assumed, this being the typical speed of a modern high speed train.

Note that the superimposed acceleration value is used by civil engineers when they design the track geometry, and takes into account the intended vehicle speed for the

particular track section. Traversing the feature at lower speeds will of cause lower the superimposed acceleration perceived by the passengers.
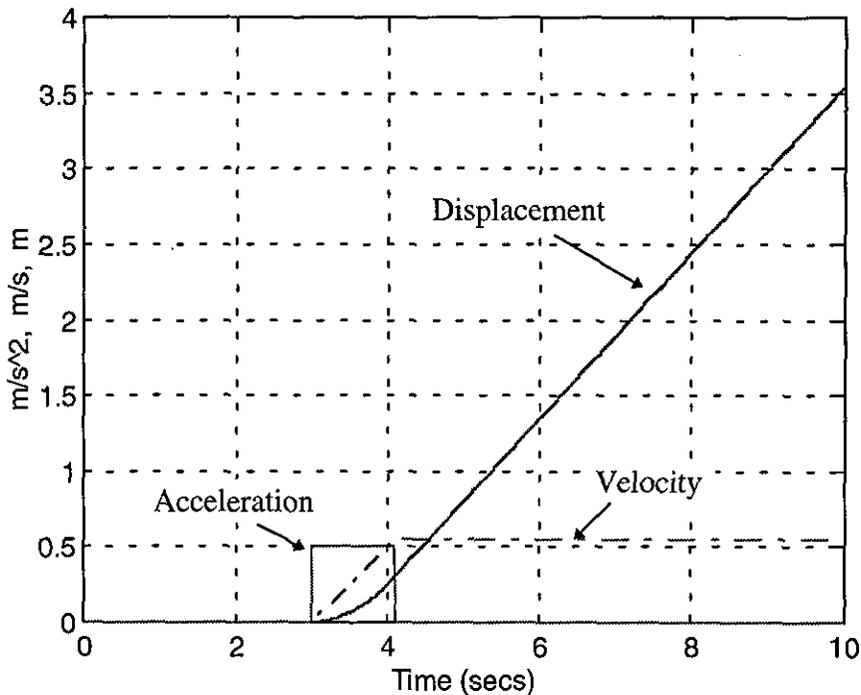
Figure 2.1   Definition of deterministic inputs, 1% gradient at 55 ms^{-1}

Also note that for the lateral direction (i.e. the transition onto a curve), the feature is more complex because it also includes a jerk limit.

2.1.3 Force Inputs

The force inputs to which the suspension must react are either changes in payload or external disturbances such as braking loads and aerodynamic effects. The character of the load variations will depend upon the application. For a small low-speed vehicle the payload variation may be 30-40 per cent from fully laden to empty. This is particularly significant for vehicles of lightweight construction. Another factor affecting smaller vehicles is that the load may change more rapidly as all the passengers can disembark much faster than for a larger vehicle. However, for a high

speed vehicle the load variation is smaller and slower and will not therefore be considered further in this thesis.

2.2 Suspension Performance

The performance of a suspension design must be assessed in terms of its ability to improve vehicle ride quality while maintaining an adequate suspension deflection. There are two main requirements in the suspension design.

The first and primary requirement is the quality of ride which it must deliver. This is a criterion of its effectiveness in providing isolation from the track roughness, and is quantified in terms of the root mean square or the r.m.s. acceleration. Strictly speaking the acceleration should be frequency weighted to reflect human sensitivity to vibration [Williams - 1986], but for a comparative study such as this it is not so important and has been neglected for simplicity. The level of the r.m.s acceleration can be determined from the acceleration spectrum, which is a combination of the transfer function of the vehicle's suspension and the track input spectrum. For a random input, the analysis of performance of a suspension relies on the calculation of accurate value for the ride quality. The detail of calculating the ride quality by different methods will be discussed in the following sections.

The second requirement is the suspension deflection when the vehicle negotiates intended track inputs (i.e. gradient for vertical suspensions). Strictly it should also take account of suspension deflection due to roughness as well as the deterministic input. A more accurate calculation to combine both two effects takes the maximum transient deflection on deterministic inputs plus three time the r.m.s. deflection due to irregularities [Pratt - 1996]. However, since the suspension deflection on random inputs is of zero mean value and relatively much smaller than on the deterministic features, it will be neglected in the future calculation and thus the term "suspension deflection" is used normally to refer to the displacement between vehicle body and

track input on the deterministic features. In addition if a combined input is used for time simulation, the random input superimposed on the deterministic feature can act to increase or decrease the overall suspension deflection due to its random nature, while the pure deterministic track input would give an average measure of performance of the linear or non-linear control strategies to be studied.

In suspension designs, the ride quality and the suspension deflection are always in conflict with each other and a balance or a compromise between the two must therefore be made.

2.3   Frequency Response Analysis to Stochastic Inputs

As discussed in the previous section, the ride quality is generally represented by the root mean square (r.m.s.) acceleration experienced by the passenger when the vehicle is excited by the roughness of the guideway. In this section one of the analysis methods known as the frequency domain analysis is discussed.

Frequency response is central to the classical analysis of control systems. It is also attractive from a heuristic viewpoint because the transfer function provides the engineer with a clear and concise description of the system behaviour. In a linearized analysis, the use of frequency response function has been the basis of the frequency response analysis. By using frequency response analysis, the r.m.s. acceleration level is determined from the acceleration spectrum, predicted from a combination of the transfer function of the vehicle's suspension and the track input spectrum.

The body acceleration power spectrum, $S_y(f)$, is equal to the square of the system transfer function multiplied by the input track power spectrum density $S_{z_t}'(f)$ given by equation 2.5 [Paddison - 1995];

$$S_y(f) = |H(2\pi f)|^2 S_{\dot{z}_t}'(f)$$
(2.6)

The mean square and the root mean square (r.m.s.) of body acceleration can then be represented in equations 2.7 and 2.8 respectively, where a single-sided power spectrum is used to make it consistent with the experimentally-derived spectrum described in section 2.1.1.

$$E[y^2] = \int_0^\infty |H(2\pi f)|^2 S_{\dot{z}_t}'(f) df$$
(2.7)

$$\ddot{z}_{b_{rms}} = \sqrt{E[y^2]} = \sqrt{\int_0^\infty |H(2\pi f)|^2 S_{\dot{z}_t}'(f) df}$$
(2.8)

2.4 Covariance Analysis to Stochastic Inputs

An alternative computational scheme to the frequency domain method for calculating the ride quality is known as covariance analysis technique. The covariance analysis approach is based upon the differential equation for the covariance matrix which is directly related to the state equations for the vehicle. Karnopp [Karnopp -1978b] and Müller [Müller -1979] describe this more specifically for time invariant vehicle dynamic systems. The method enables a system's response to a random input to be calculated directly without finding all necessary transfer functions.

The railway vehicle dynamics systems are substantially linear and can be described in the form:

$$\dot{X} = AX + Gw$$
(2.9)

$$Y = CX$$
(2.10)

where X, Y and *w* are functions of time, A is the system matrix, G is the input matrix, C is the output matrix, and *w* is the track input as being closely represented by a random white noise process described as :

$$E\left[w(t)w^T(t+\tau)\right] = Q_w(\tau) = 2\pi^2 A_r v\delta(\tau)$$ (2.11)

The system described in equation 2.9 is subjected to this random track input, and the covariance matrix of X is defined as follows:

$$P_x = E\{[X - \bar{X}][X - \bar{X}]^T\}$$ (2.12)

where E denotes expected value, $[\ ]^T$ denotes transposition and $\bar{X} = E\{X\}$ is the mean value.

For most vehicle dynamics work the models are formulated so that the mean value is zero, so equation 2.12 can be simplified to:

$$P_x = E\{XX^T\}$$ (2.13)

and the differential equation for the covariance is then:

$$\dot{P}_x = E\{\dot{X}X^T + X\dot{X}^T\}$$ (2.14)

Substituting equation 2.9 into equation 2.14 gives [Paddison - 1995]:

$$\dot{P}_x = AP_x + P_xA^T + GQ_wG^T$$ (2.15)

The system is driven by white noise, the process is stationary and the value at any instant is uncorrelated with the value at any other instant. Thus $\dot{P}_x = 0$ gives the equation:

$$AP_x + P_x A^T = -GQ_w G^T \tag{2.16}$$

This equation is classically referred as Lyapunov Equation. In the **MATLAB**™, the function **LYAP**$(A, GQ_w G^T)$ will give the solution (P_x) of the Lyapunov equation 2.16. Once the equation 2.16 has been solved, the diagonal elements of P_x give the mean square values of the state variables X.

For a system driven by random noise the expectation value of YY^T (mean square value of output Y) is:

$$E[YY^T] = P_{YY} = CP_x C^T \tag{2.17}$$

By substituting P_x into the equation 2.17, the r.m.s. value of output Y is :

$$[y_{1_{rms}} \; y_{2_{rms}} \; ... \;]^T = diag\left(\sqrt{E[YY^T]}\right) = diag\left(\sqrt{CP_x C^T}\right) \tag{2.18}$$

where in this case, y_1 is the vehicle body acceleration.

2.5 Time History Analysis to Stochastic Inputs

The two methods, frequency domain techniques and covariance analysis, are adequate for analysing linear systems. However, when non-linear controllers as to be investigated in Chapter 5 are used, the whole system will be non-linear and alternative analysis methods are needed. This section presents a time history analysis technique

which can be used to assess suspension performance of both linear and non-linear systems.

The principle of the time history analysis method is simple. The track history data, a set of random signals, is used as the input signal for a linear or non-linear model to simulate the system response. The output of the linear or non-linear model will naturally be the time history and the r.m.s. value of body acceleration can be derived as follows.

By using **MATLAB**™ to solve system differential / difference equations, the system output Y(t) or Y(i) can be easily obtained. As discussed in the Section 2.2, the ride quality is represented by the root mean square (r.m.s.) of the output body acceleration. In the time history analysis method, for the continuous-time system the calculation of the r.m.s value of output Y can be expressed as:

$$[y_{1_{rms}} \ y_{2_{rms}} \ \dots \ ]^T = diag\left(\sqrt{\frac{1}{T}\int_0^T Y(t)Y^T(t)dt} \right) \tag{2.19}$$

or equation 2.20 in the discrete-time system.

$$[y_{1_{rms}} \ y_{2_{rms}} \ \dots \ ]^T = diag\left(\sqrt{\frac{1}{n}\sum_{i=1}^n Y(i)Y^T(i)} \right) \tag{2.20}$$

The discrete-time representation of the vehicle dynamics can be described by the state difference equation:

$$X(i+1) = A_d X(i) + G_d w(i) \tag{2.21}$$

and output equation

$$Y(i) = CX(i) \qquad\qquad (2.22)$$

The relation between the continuous and discrete systems is:

$$A_d = e^{At_s} \qquad\qquad G_d = (\int_0^{t_s} e^{A\tau}\, d\tau\,)G \qquad\qquad (2.23)$$

where t_s is sampling interval.

The main zone of interest to the vehicle designer is at frequencies less than 10 Hz. It is in this frequency range that the vehicle rigid body modes are found and vibration of the vehicle body occurs thus affecting passenger comfort. At frequencies higher than this and up to 100 Hz resonance of the wheel/track system can occur and as a result much of the spectral component of the track force appears in this range [Hunt - 1986]. For this reason, a time sampling interval t_s=1 ms should be adequate.

2.6 Summary

This chapter has determined appropriate track inputs for railway suspensions, explained the main suspension performance requirements and identified the three analytical approaches which may be used. All three approaches can be applied in linear model, but for non-linear model only the time history analysis method can be used. These techniques will be further studied in Chapter 3.

CHAPTER 3

MODELLING AND COMPARISON OF ACTIVE AND PASSIVE SUSPENSIONS

This chapter contains a basic introduction to the requirements of a suspension, an explanation of the terminology "passive" and "active" suspensions, and a comparison between these two systems. The developed models represent the secondary suspension only, because a simplified model can reveal possibilities which are sometimes obscured by too complex a model, and the primary suspension is not directly related to the study. Also single wheel models are considered which represent one degree of freedom of motion and may be thought of as a unicycle model.

3.1   Suspension Models

3.1.1 Passive Suspension System

Figure 3.1 shows a simple passive suspension where the damping is applied between the vehicle and the track. The dynamics of the suspension can be described by the transfer function:

$$\frac{z_b}{z_t} = \frac{cs + k_s}{m_b s^2 + cs + k_s} \tag{3.1}$$

The natural frequency and damping ratio are given by equations 3.2 and 3.3:

$$\omega_0 = \sqrt{\frac{k_s}{m_b}} \tag{3.2}$$
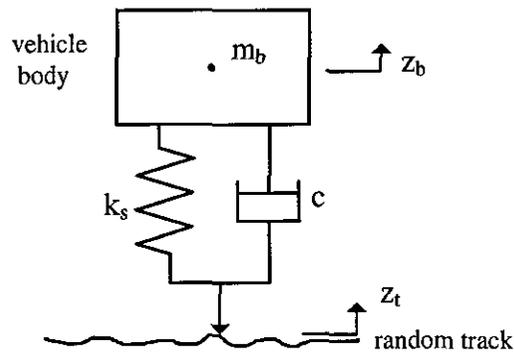
Figure 3.1   Simple passive suspension

$$\zeta = \frac{c}{2m_b\sqrt{\dfrac{k_s}{m_b}}}$$
(3.3)

Human anatomical response and the general suspension requirements dictate that the natural response of the suspension should be maintained around 0.5 ~ 1 Hz depending upon the speed of vehicles. If the natural frequency is set too low or to too high, the result in passengers feeling will be either "sea-sickness" or "harshness". For the vertical suspension discussed in this thesis, the natural frequency is normally around 1 Hz.

Figure 3.2 gives the frequency response of this simple passive suspension while increase the damping c, where the natural frequency is set to 0.8 Hz. The Figure shows the typical characteristic of a second order system: when the damping is increased the resonance peak is reduced, but the high frequency transmissibility is also increased. The choice of damping rate is a compromise between minimising the size of the resonant peak and the high frequency transmission. This "high frequency transmissibility" is perceived as harshness by the vehicle occupants; an undesirable feature. So, the designer must make a trade-off between resonance attenuation and reduction in the high frequency transmissibility.
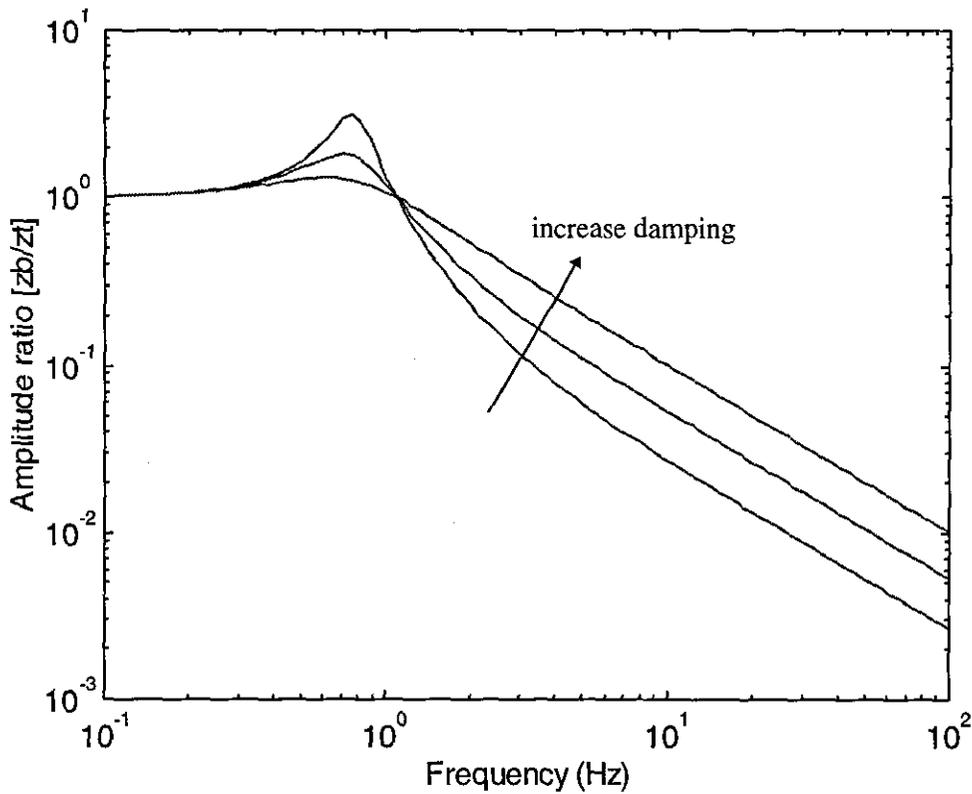
Figure 3.2   Frequency response

At higher frequency, i.e. $\omega \gg \omega_0$, the system is approximately a first order one as indicated in equation 3.4. Obviously the system is not very effective in isolating high frequency components.

$$\frac{z_b}{z_t} \approx \frac{c}{m_b s} \tag{3.4}$$

The capabilities of a realisable passive suspension can be better appreciated by considering an improved system shown in Figure 3.3. The addition of the spring in series with the damper improves the high frequency isolation while retaining the same static deflection. This configuration will be used through all the following chapters where a passive suspension system is considered.
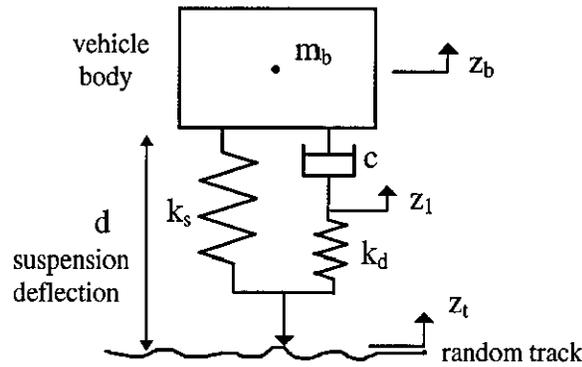
Figure 3.3   Improved Passive Suspension

The transfer function of this passive system is as follows:

$$\frac{z_b}{z_t} = \frac{(k_s + k_d)s + \dfrac{k_s k_d}{c}}{m_b s^3 + \dfrac{m_b k_d}{c}s^2 + (k_s + k_d)s + \dfrac{k_s k_d}{c}}$$ (3.5)

The frequency response plot of equation 3.5 is compared to the simple passive system of equation 3.1 for the same damping constant in Figure 3.4. Note that these two systems have the same static deflection.

The parameter values are given in Table 3.1 in section 3.1.3. In this particular case, the spring in series with the damper is relatively stiff, whereas often it is chosen to be rather lower. However this limits the maximum value of damping ratio which can be achieved [Karnopp - 1978a], and the optimisation of a passive suspension is an interesting problem which is not dealt with in this thesis.

Passive suspension systems are substantially linear, and it is useful to describe them by a set of linear first order differential (state-space) equations of the form:

$$\dot{X} = AX + Gw$$ (3.6)

$$Y = CX + Dw \tag{3.7}$$
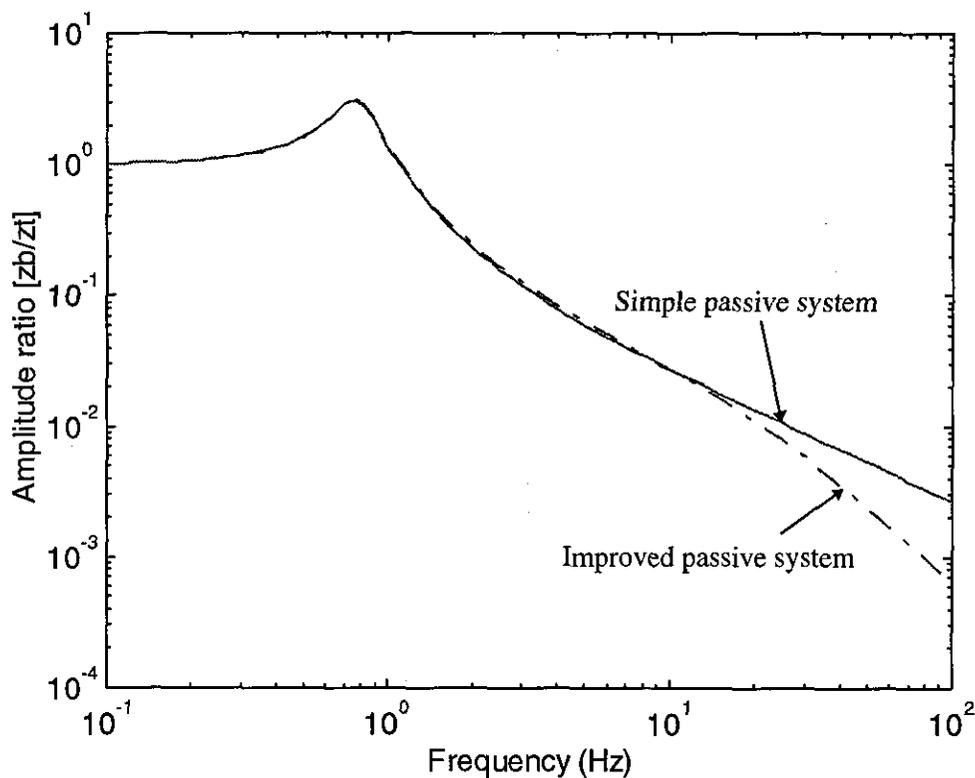
Figure 3.4   Frequency response

The differential equations of the passive system are most easily derived as follows:-

Applying Newton's law to the system shown in Figure 3.3 gives:

$$m\ddot{z}_b = c(\dot{z}_1 - \dot{z}_b) + k_s(z_t - z_b) \tag{3.8}$$

$$\dot{z}_1 = \dot{z}_b + \frac{k_d}{c}(z_t - z_1) \tag{3.9}$$

Let $z_t-z_1=d_1$ and $z_t-z_b=d$,

$$m\ddot{z}_b = c(\dot{d} - \dot{d}_1) + k_s d \tag{3.10}$$

$$\dot{d}_1 = \dot{d} - \frac{k_d}{c} d_1 \tag{3.11}$$

Thus the state-space equations are:

$$
\begin{bmatrix} \ddot{z}_b \\ \dot{d}_1 \\ \dot{d} \end{bmatrix} =
\begin{bmatrix} 0 & \dfrac{k_d}{m_b} & \dfrac{k_s}{m_b} \\ -1 & -\dfrac{k_d}{c} & 0 \\ -1 & 0 & 0 \end{bmatrix}
\begin{bmatrix} \dot{z}_b \\ d_1 \\ d \end{bmatrix} +
\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}
\begin{bmatrix} \dot{z}_t \end{bmatrix} \tag{3.12}
$$

where the input is the track velocity ($\dot{z}_t$) represented by a Gaussian white noise as discussed in Chapter 2. The output equation may be configured to whatever combination of state variables is required or is of interest. Here the outputs are body acceleration (used to assess ride quality) and suspension deflection.

$$
Y = \begin{bmatrix} \ddot{z}_b \\ d \end{bmatrix} =
\begin{bmatrix} 0 & \dfrac{k_d}{m_b} & \dfrac{k_s}{m_b} \\ 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} \dot{z}_b \\ d_1 \\ d \end{bmatrix} +
\begin{bmatrix} 0 \\ 0 \end{bmatrix}
\begin{bmatrix} \dot{z}_t \end{bmatrix} \tag{3.13}
$$

3.1.2 Skyhook Damping System

It is well known that active suspensions can provide suspension characteristics which cannot be realised with passive components alone [Hedrick - 1975]. The simplest form of an active suspension control method known as the "skyhook" damping scheme is shown in Figure 3.5.
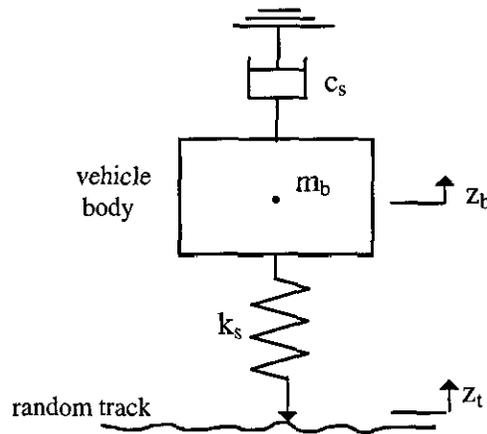
Figure 3.5   Skyhook damping system

For a vehicle, of course, this is impossible to achieve as there is nowhere to anchor the damper, but the benefits of this system will now be demonstrated. The transfer function of the "skyhook" damping system can be represented as:

$$\frac{z_b}{z_t} = \frac{k_s}{m_b s^2 + c_s s + k_s}$$

(3.14)

The high frequency transmissibility is then:

$$\frac{z_b}{z_t} \approx \frac{k_s}{m_b s^2}$$

(3.15)

which is independent of the damping rate.

The frequency response of this transfer function is shown in Figure 3.6. It appears that with "skyhook" damping the trade-off between the resonance attenuation and the high frequency transmissibility has been removed.
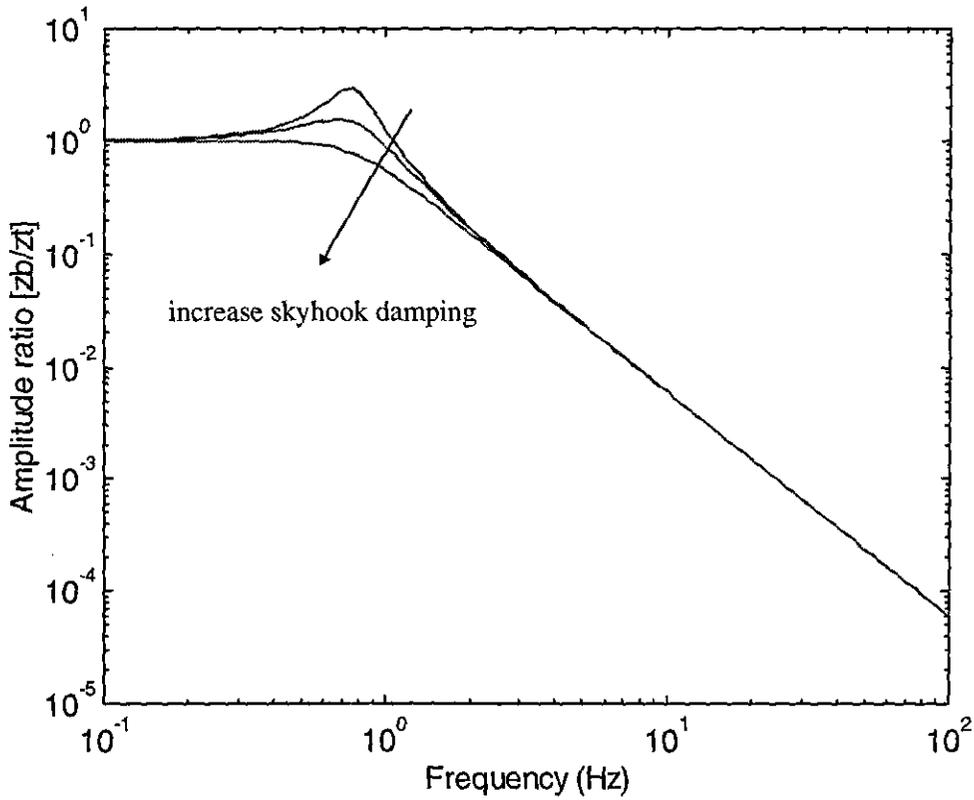
Figure 3.6   Frequency response of "skyhook" damping system

Since the motion of the "skyhook" damping system is given in equation 3.16,

$$m\ddot{z}_b = -c_s \dot{z}_b + k_s d \tag{3.16}$$

the state-space equation can be derived from it very easily. Equations 3.17 and 3.18 give the state-space and output equations respectively.

$$\begin{bmatrix} \ddot{z}_b \\ \dot{d} \end{bmatrix} = \begin{bmatrix} -\dfrac{c_s}{m_b} & \dfrac{k_s}{m_b} \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \dot{z}_b \\ d \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} \dot{z}_t \end{bmatrix} \tag{3.17}$$

$$\begin{bmatrix} \ddot{z}_b \\ d \end{bmatrix} = \begin{bmatrix} -\dfrac{c_s}{m_b} & \dfrac{k_s}{m_b} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{z}_b \\ d \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} \dot{z}_t \end{bmatrix} \tag{3.18}$$

Since the D matrix is zero for both passive and skyhook systems, it will be dropped from now onwards.

3.1.3 System Parameters

Table 3.1 gives the numerical data which will be used in the calculations for a complete vehicle.

<p align="center">Table 3.1   Complete vehicle model parameters</p>

Symbol	Value	Parameters
m_b	30000 (kg)	Vehicle body mass
k_s	700000 (N/m)	Spring stiffness
k_d	7000000 (N/m)	Spring stiffness
c	50000 (N/ms^{-1})	Passive damping
c_s	190000 (N/ms^{-1})	Skyhook damping

3.2   Response to Random Inputs

As mentioned earlier, the ride quality of a vehicle is generally represented by the r.m.s. acceleration experienced by passengers when the vehicle is excited by the roughness of the guideway and there are three methods to calculate the r.m.s. value. To study responses of different suspensions to the random track features, simulation models of passive and "skyhook" damping suspension systems are established using **MATLAB**™ and used in the analysis, and a comparison between the two systems will be discussed by using all three methods: frequency domain, time domain and covariance analysis.

3.2.1 Frequency Domain Response

It has received a great deal of attention in the literature survey that the calculations of r.m.s. value for vehicles traversing irregular surfaces are usually accomplished using frequency domain method. Figure 3.7 shows a comparison between the vehicle body acceleration P.S.D. for passive and "skyhook" damping systems. It is noticeable that the energy in the body acceleration spectrum at the bounce mode frequency is very significantly reduced by using the "skyhook" damping.

PSD for body acceleration
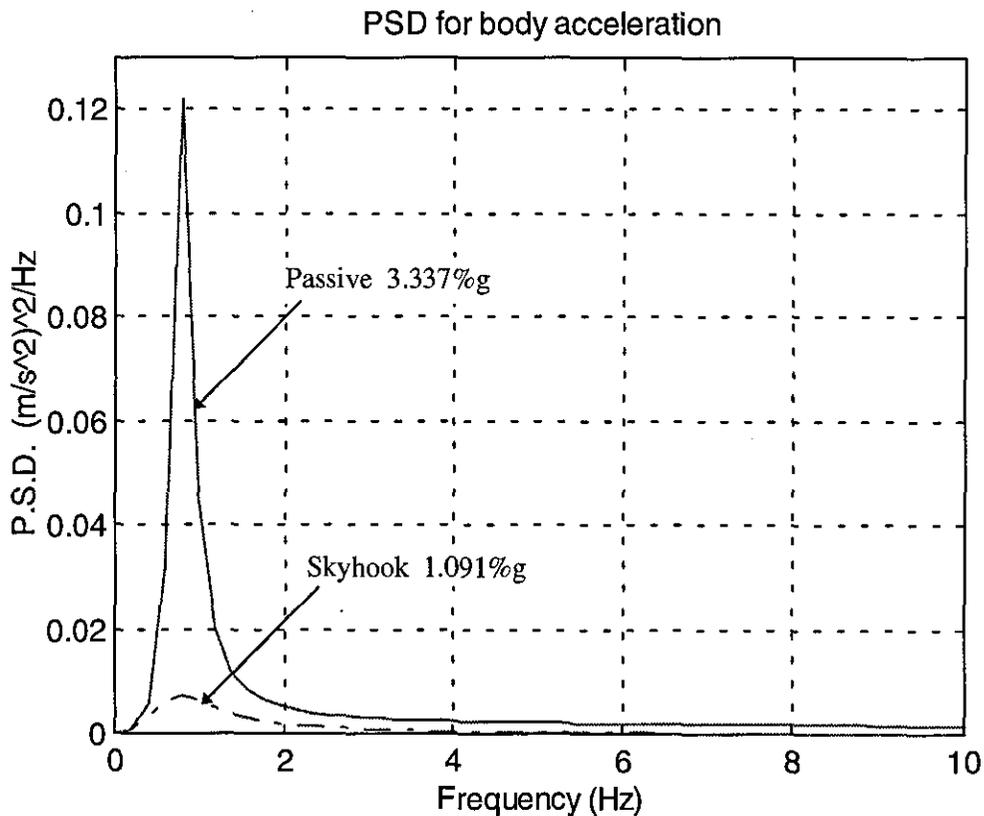
Figure 3.7   Random response of passive and skyhook systems

3.2.2 Time History Response

Figure 3.8 shows a time simulation result demonstrating how much the improvement on the ride quality can be achieved by the "skyhook" damping system. Excited by the

track input data, it is very clear that the body acceleration of the "skyhook" damping system is much smaller than the passive system, and it is particularly noticeable how much the high frequency vibrations are reduced.
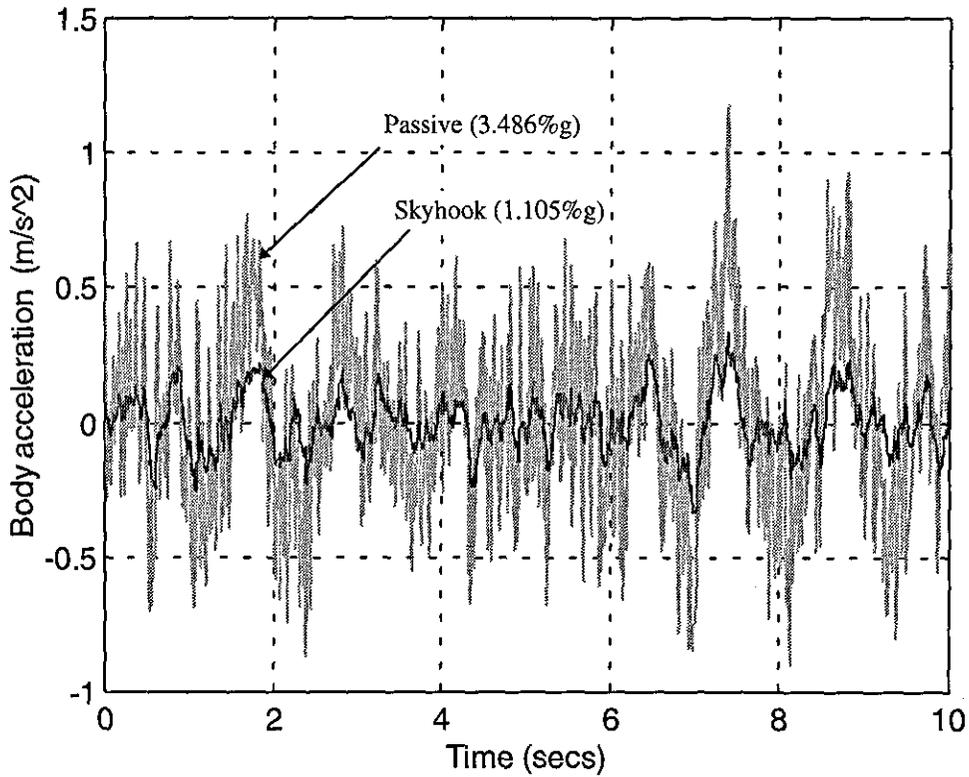
Figure 3.8   Passive vs. "skyhook" in the time domain response

3.2.3 Covariance Analysis

Chapter 2 described how the **MATLAB™** function **LYAP** could be used to give solutions for the Lyapunov equation. Table 3.2 shows the r.m.s. body acceleration of the two suspension systems by using covariance analysis. The results of other two methods are also given in the same table for comparison.

The table clearly shows that a very large improvement can be achieved in the r.m.s. acceleration value for the "skyhook" damping system. Using all three methods to calculate the r.m.s. body acceleration gives very similar results. It should be noted that

the covariance analysis is exact, whereas the accuracy of the frequency domain approach is governed by the parameters used for the numerical integration of the P.S.D. and so there is a small difference in the values. The time domain result is of course exact, but a finite time input sequence will never precisely give the required statistical parameter, and hence there is again a small difference. Nevertheless it is clear that the three methods are giving results which correspond very closely. The choice of analysis method for calculating the ride quality only depends on the systems. When the system is linear, any of them can be used, but with non-linear system, only time history analysis is appropriate since the other two approaches apply only for a linear system.

Table 3.2   R.M.S. body acceleration (%g)

	Covariance Analysis	Frequency Response	Time History
Passive	3.454	3.337	3.486
Skyhook	1.102	1.099	1.105

3.3   Response to Deterministic Inputs

It is obvious that the potential benefits of using "skyhook" damping system are a reduction in the suspension resonance without an adverse effect on the high frequency transmissibility. In addition to response to random inputs it is necessary to consider the vehicle's response to deterministic inputs.

Although the suspension deflection for any particular transition can always be determined by simulation, the transfer function representation provides an easy way of calculating quasi-static deflection corresponding to the maximum velocity and acceleration levels. The transfer function which determines the suspension deflection is :

$$\frac{d}{z_t} = \frac{z_t - z_b}{z_t} = 1 - \frac{z_b}{z_t} \qquad (3.19)$$

For the passive suspension system, by substituting the transfer function, equation 3.5 into equation 3.19, this gives

$$d = \frac{(\frac{m_b}{k_s}s^2 + \frac{m_b c}{k_s k_d}s^3)z_t}{1 + \frac{(k_s + k_d)c}{k_s k_d}s + \frac{m_b}{k_s}s^2 + \frac{m_b c}{k_s k_d}s^3} \qquad (3.20)$$

Also for the "skyhook" damping system, the suspension deflection is:

$$d = \frac{(\frac{c_s}{k_s}s + \frac{m_b}{k_s}s^2)z_t}{1 + \frac{c_s}{k_s}s + \frac{m_b}{k_s}s^2} \qquad (3.21)$$

Generally the "settling time" of the suspension's transient response will be somewhat shorter than the time period of the transition [Goodall - 1993], giving quasi-static deflections for the passive system in response to track inputs as follows:-

$$d_{qs} = \frac{m_b}{k_s} \ (track\ accel) \ + \ \frac{m_b c}{k_s k_d} \ (track\ jerk) \qquad (3.22)$$

and for the "skyhook" damping system,

$$d_{qs} = \frac{c_s}{k_s} \ (track\ vel) \ + \ \frac{m_b}{k_s} \ (track\ accel) \qquad (3.23)$$

Using the parameter values given in Table 3.1, these expressions enable us to estimate the deflections for the maximum velocity and accelerations inputs which are

approximately 0.0 m and 0.149 m for the passive and "skyhook" damping systems respectively. Table 3.3 gives the estimated quasi-static deflections components due to track input for both passive and "skyhook" systems, and it clearly shows that the "skyhook" damping system creates unacceptably large deflections on the gradient. Notice that the "skyhook" suspension response has a velocity-dependent term whereas the numerator of the passive third order transfer function means that its quasi-static response is primarily dependent upon acceleration..

Table 3.3   Quasi-static deflection components due to track inputs

	component due to vel=0.55 m/s	component due to accel=0.5 m/s^2
Passive	0.0 m	0.0214 m
Skyhook	0.149 m	0.0214 m
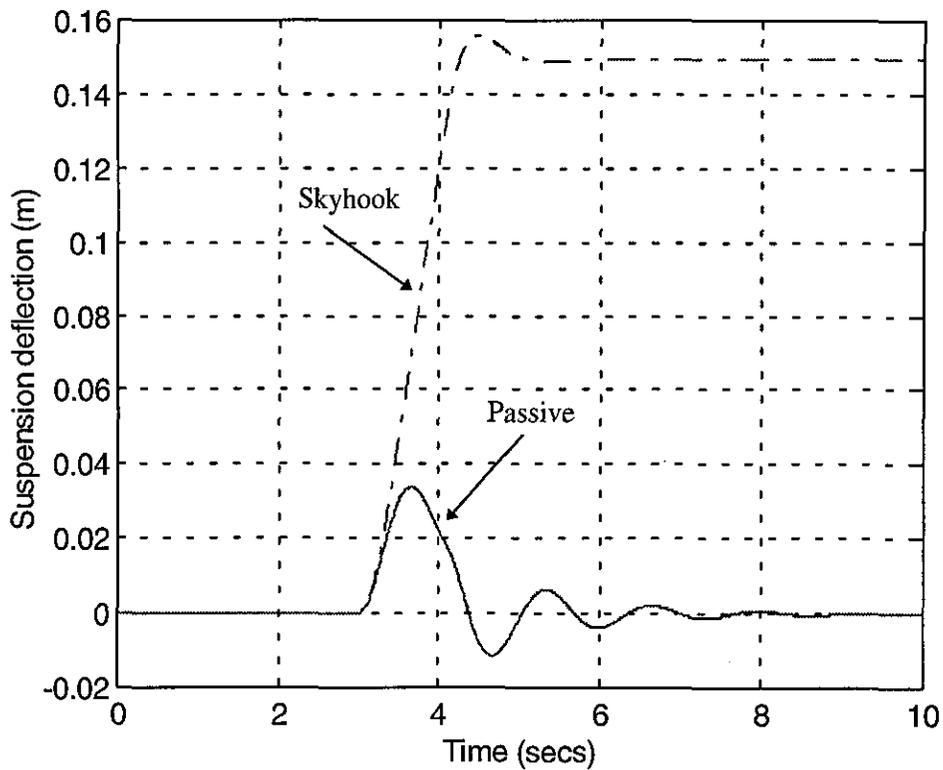
Figure 3.9   Response to deterministic inputs

The simulation results in Figure 3.9 for the transition to a gradient (see Section 2.1.2) confirm the above analysis, where the performances of the passive suspension and "skyhook" damping system in response to deterministic track inputs are given. The problem will have to be tackled in the development of any control strategies for active suspensions.

3.4   Summary

This chapter has presented the models of the passive and "skyhook" damping systems and determined the basic responses of the two systems to random track and deterministic input signals. Various analysis methods have been introduced such as frequency domain analysis, time history response and covariance analysis. It has also quantified the trade-off problem of the suspension responses to random and deterministic track features.

The next chapter will study three linear control strategies to provide a practical implementation of the "skyhook" damping as well as to improve its performance, especially to reduce the unacceptable level of the suspension deflection.

CHAPTER 4

LINEAR CONTROL STRATEGIES

Discussions in the previous chapter show that a suspension system with "skyhook" damping can make a significant improvement on the ride quality on straight tracks. However, apart from its problem of large suspension deflections, the "skyhook" damping is assumed to be attached to an absolute reference and in reality no such a connection is possible. Alternative methods to implement the system as well as to overcome its drawbacks have to be found if the idea of the "skyhook" damping is to be of any real use. It is well-known that linear systems are in general simpler in structure and easier to design and implement compared with their non-linear counterparts. In this chapter, three linear control strategies are studied to implement the principle of the "skyhook" damping and these are: intuitive formulation control, complementary filter control, and Kalman-Bucy filtering control.

4.1　Intuitive Formulation Control

An intuitive formulation control structure is presented in this section to implement the "skyhook damping" strategy in principle.

4.1.1　Vehicle Model

Figure 4.1 shows one degree of freedom system and Figure 4.2 gives a block diagram of the intuitive formulation control structure. The accelerometer output $(\ddot{z}_b)$ is fed through an integrator in order to derive the absolute velocity $(\dot{z}_b)$. The absolute

velocity is amplified by the skyhook damping (c_s) and then used to modulate an active force actuator, i.e.:

$$F = -c_s \dot{z}_b \qquad\qquad (4.1)$$
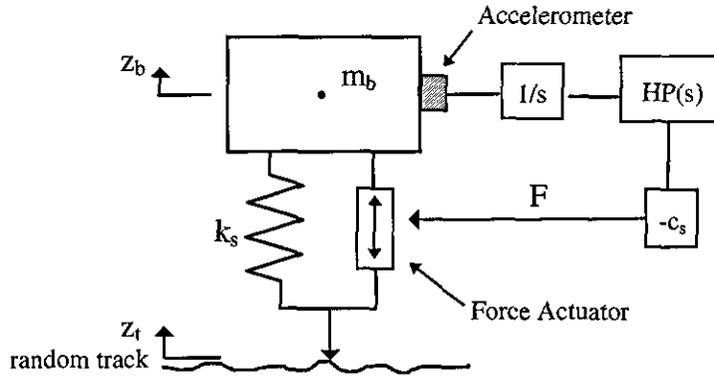
Figure 4.1   Intuitive formulation control configuration
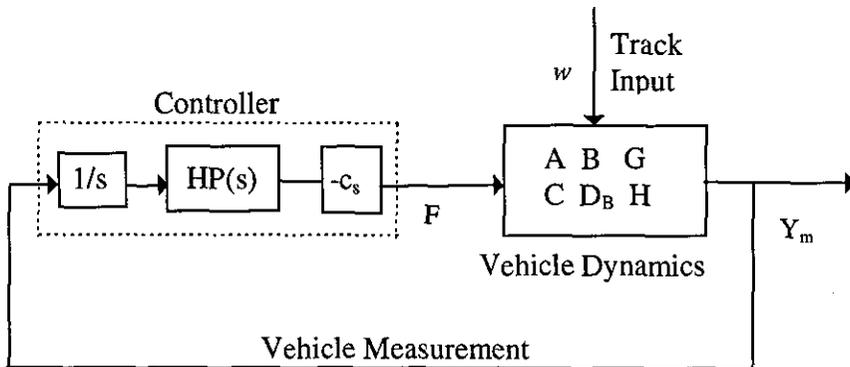
Figure 4.2   Control structure

In practice a high pass filter must be inserted between the "skyhook" damping (c_s) and the integrator (1/s), because there will always be an offset on the integrator output and the purpose of the filter is to eliminate its effect. Also, it will be seen that this high pass filter has the effect of reducing the low frequency velocity signal, which in turn reduces the suspension deflection for deterministic inputs.

4.1.2 State-space Expression of the Linear Model

The equation of motion of the vehicle can be represented by applying Newton's law:

$$m\ddot{z}_b = F + k_s d \tag{4.2}$$

And its state-space and output equations can be easily derived as:

$$\dot{X} = AX + BF + Gw \tag{4.3}$$

$$Y_m = CX + D_B F + HV \tag{4.4}$$

where $X = [\dot{z}_b \ d]^T$ is the state, $Y = [\ddot{z}_b \ d]^T$ is the measured output and $Y_m = Y + HV$, $w = \dot{z}_t$ is the track input, and $V = [v_1 \ v_2]^T$ is the measurements noise. The vehicle system matrices A, B, C, D_B, G and H are given as below:

$$A = \begin{bmatrix} 0 & \dfrac{k_s}{m_b} \\ -1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} \dfrac{1}{m_b} \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & \dfrac{k_s}{m_b} \\ 0 & 1 \end{bmatrix}$$

$$D_B = \begin{bmatrix} \dfrac{1}{m_b} \\ 0 \end{bmatrix} \qquad G = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Similarly, the state-space and output equations of the controller can be expressed as:

$$\dot{X}_{Ctrl} = A_{Ctrl} X_{Ctrl} + B_{Ctrl} Y_m \tag{4.5}$$

$$F = C_{Ctrl} X_{Ctrl} + D_{Ctrl} Y_m \tag{4.6}$$

where $X_{Ctrl} = [x_1 \, ... \, x_n]^T$, n is the order of the controller.

By substituting equation 4.6 into equations 4.3 and 4.4,

$$\dot{X} = AX + BC_{Ctrl}X_{Ctrl} + BD_{Ctrl}Y_m + Gw \tag{4.7}$$

$$Y_m = \left(I - D_B D_{Ctrl}\right)^{-1} CX + \left(I - D_B D_{Ctrl}\right)^{-1} D_B C_{Ctrl} X_{Ctrl} \\ + \left(I - D_B D_{Ctrl}\right)^{-1} HV \tag{4.8}$$

Substituting equations 4.8 into 4.7 and 4.5 gives

$$\dot{X} = \left[A + BD_{Ctrl}\left(I - D_B D_{Ctrl}\right)^{-1} C\right]X + \left[BC_{Ctrl} + BD_{Ctrl}\left(I - D_B D_{Ctrl}\right)^{-1} D_B C_{Ctrl}\right]X_{Ctrl} \\ + Gw + BD_{Ctrl}\left(I - D_B D_{Ctrl}\right)^{-1} HV \tag{4.9}$$

$$\dot{X}_{Ctrl} = B_{Ctrl}\left(I - D_B D_{Ctrl}\right)^{-1} CX + \left[A_{Ctrl} + B_{Ctrl}\left(I - D_B D_{Ctrl}\right)^{-1} D_B C_{Ctrl}\right]X_{Ctrl} \\ + B_{Ctrl}\left(I - D_B D_{Ctrl}\right)^{-1} HV \tag{4.10}$$

Thus, the closed loop state-space and output equations for this intuitive formulation control system are:

$$\begin{bmatrix} \dot{X} \\ \dot{X}_{Ctrl} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X \\ X_{Ctrl} \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} \\ 0 & B_{22} \end{bmatrix} \begin{bmatrix} w \\ V \end{bmatrix} \tag{4.11}$$

$$Y_m = \begin{bmatrix} C_{11} & C_{12} \end{bmatrix} \begin{bmatrix} X \\ X_{Ctrl} \end{bmatrix} + \begin{bmatrix} 0 & D_{12} \end{bmatrix} \begin{bmatrix} w \\ V \end{bmatrix} \tag{4.12}$$

where: $A_{11} = A + BD_{Ctrl}\left(I - D_B D_{Ctrl}\right)^{-1} C$ $A_{12} = BC_{Ctrl} + BD_{Ctrl}\left(I - D_B D_{Ctrl}\right)^{-1} D_B C_{Ctrl}$

$$A_{21} = B_{Ctrl}\left(I - D_B D_{Ctrl}\right)^{-1} C \qquad A_{22} = A_{Ctrl} + B_{Ctrl}\left(I - D_B D_{Ctrl}\right)^{-1} D_B C_{Ctrl}$$

$$B_{11} = G \qquad B_{12} = BD_{Ctrl}\left(I - D_B D_{Ctrl}\right)^{-1} H$$

$$B_{22} = B_{Ctrl}\left(I - D_B D_{Ctrl}\right)^{-1} H$$

$$C_{11} = \left(I - D_B D_{Ctrl}\right)^{-1} C \qquad C_{12} = \left(I - D_B D_{Ctrl}\right)^{-1} D_B C_{Ctrl}$$

$$D_{12} = \left(I - D_B D_{Ctrl}\right)^{-1} H$$

4.1.3 H.P. Filter

As indicated earlier, the high pass filter is introduced in the feedback control to eliminate the possible offset of the integrator. However this high pass filter compromises the improvement in ride quality, it is important to select the structure and parameters of a filter such that the best performance can be achieved with the least effect on ride quality. In this study, the first, second and third-order high pass filters HP(s) in Table 4.1 are considered, where the transfer functions combined with the integrator 1/s are also given for comparison. (N.B. an integrator on its own is impractical, which is why it is combined with the H.P. filter.)

In the table, ω_i is the cut-off frequency of the high pass filter. Standard Butterworth filter characteristics have been used in each case.

By varying cut-off frequencies of the three high pass filters their effect on the closed loop controlled suspension can be assessed.

Table 4.1   Filter transfer functions

	High-Pass (H.P.) filter	**Combined transfer function** $(1/s \times HP(s))$
1st	$HP_1(s) = \dfrac{\dfrac{1}{\omega_i}s}{1 + \dfrac{1}{\omega_i}s}$	$H_1(s) = \dfrac{\dfrac{1}{\omega_i}}{1 + \dfrac{1}{\omega_i}s}$
2nd	$HP_2(s) = \dfrac{\dfrac{1}{\omega_i^2}s^2}{1 + \dfrac{2\xi_i}{\omega_i}s + \dfrac{1}{\omega_i^2}s^2}$ (where $\xi_i = 0.7$)	$H_2(s) = \dfrac{\dfrac{1}{\omega_i^2}s}{1 + \dfrac{2\xi_i}{\omega_i}s + \dfrac{1}{\omega_i^2}s^2}$ (where $\xi_i = 0.7$)
3rd	$HP_3(s) = \dfrac{\dfrac{1}{\omega_i^3}s^3}{1 + \dfrac{2}{\omega_i}s + \dfrac{2}{\omega_i^2}s^2 + \dfrac{1}{\omega_i^3}s^3}$	$H_3(s) = \dfrac{\dfrac{1}{\omega_i^3}s^2}{1 + \dfrac{2}{\omega_i}s + \dfrac{2}{\omega_i^2}s^2 + \dfrac{1}{\omega_i^3}s^3}$

4.1.4   Response to Random and Deterministic Inputs

As mentioned earlier, one of the main drawbacks of using the simple active suspension is the conflict between ride quality and suspension deflection. By using the intuitive formulation control approach, a group of trade-off curves varying the cut-off frequency ω_i for different skyhook dampings are shown in Figure 4.3. Taking the extremes, a high pass filter frequency of zero would give pure skyhook damping; a large value of filter cut-off frequency will give the response of passive system. From the figure it can also be seen that decreasing the skyhook damping can significantly reduce the maximum suspension deflection, but worsen the ride quality.
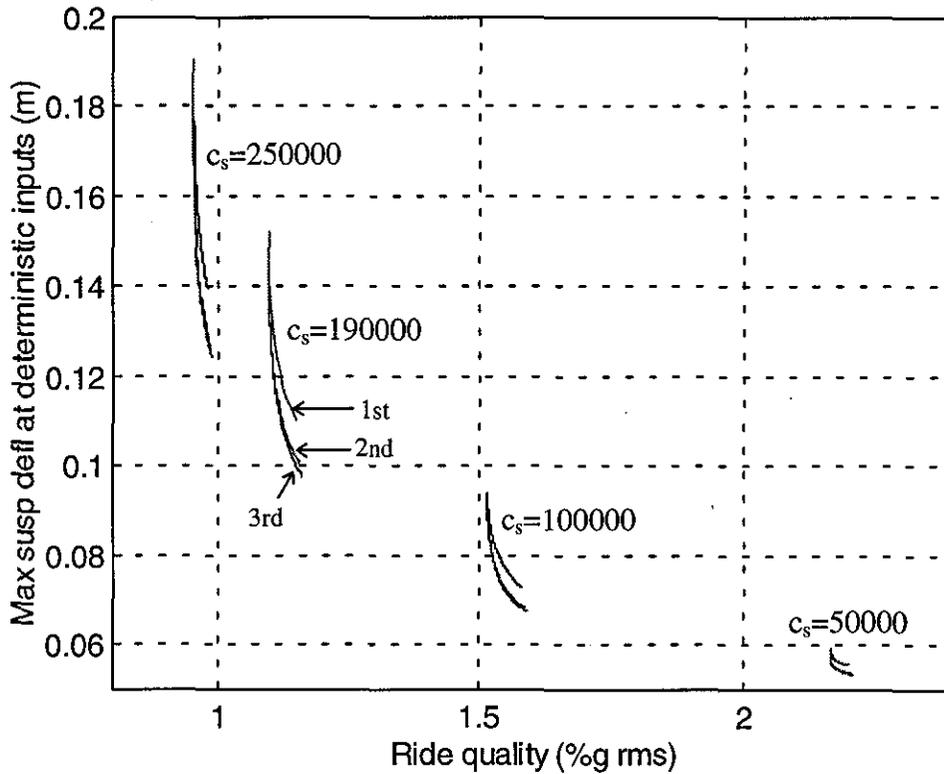
Figure 4.3   Trade-off curves with different skyhook dampings

The "skyhook" damping (c_s) and spring stiffness (k_s) parameters should always be adjusted to give a damping ratio $\zeta=0.707$ such that the best transient response can be achieved [Karnopp - 1978a]. For this reason $c_s=190000$ N/ms^{-1} is chosen here for further discussions and its detailed trade-off curves are shown in Figures 4.4 and 4.5, which demonstrate that the suspension has properties which can be characterised as varying between "stiff" and "soft".

In Figure 4.4, it is obvious that a significant improvement in the ride quality can be achieved with this simple control strategy on the random track inputs. Unfortunately, the strategy which gives the ride quality improvements also creates unacceptable large suspension deflections at the deterministic features. For a particular ride quality, it can be seen that the suspension deflection becomes smaller by increasing the order of the filter. But further increases of the filter order are unlikely to give much more benefit

and the first- or the second- order filter is normally chosen in practice. Increase of the filter cut-off frequency improves the maximum suspension deflection, but at the same time it increases the ride quality r.m.s. value.
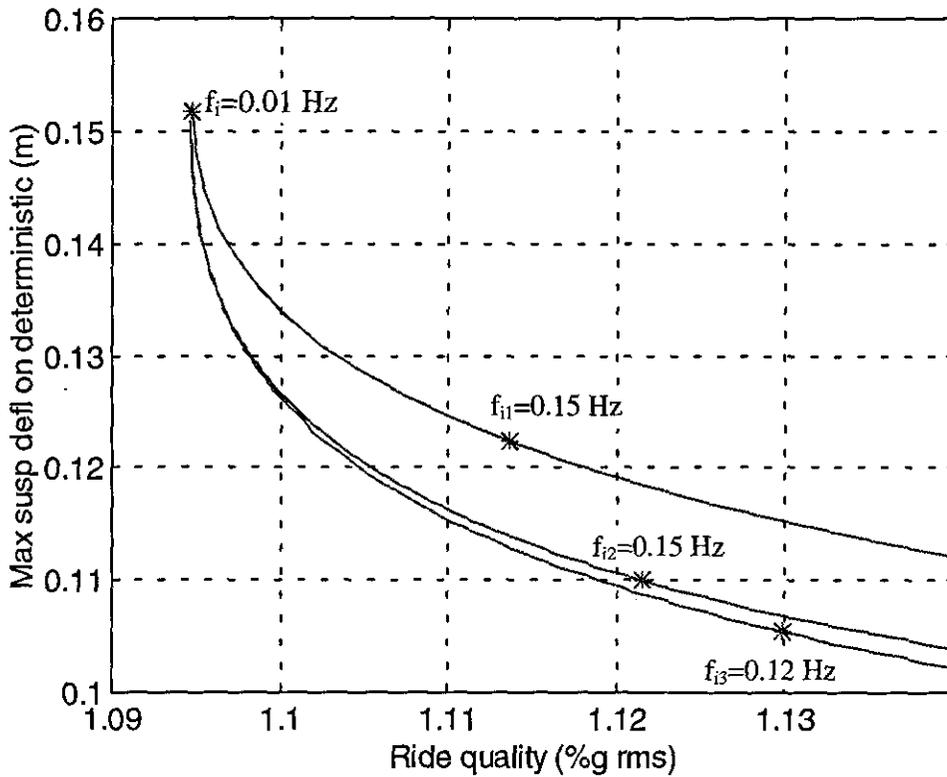
Figure 4.4   Trade-off curve (c_s=190000 N/ms^{-1}), ω_i varying

Further increasing the filter cut-off frequency ω_i is not to give any more benefit for the system with the 2nd- or 3rd- order HP filter, as the ride quality and maximum suspension deflection both become worse, as indicated in Figure 4.5.

One of the other problems is that the system may become unstable if ω_i is too large, as one pair of eigenvalues moves towards the right hand half on the s-plane with the increase of the cut-off frequency. Table 4.2 demonstrates this system instability for the 2nd order filter in the feedback. For the system with the first or third order filter in the feedback, the unstable point for the closed loop system is f_{i1}=18.326 Hz or f_{i3}=0.3031 Hz respectively.
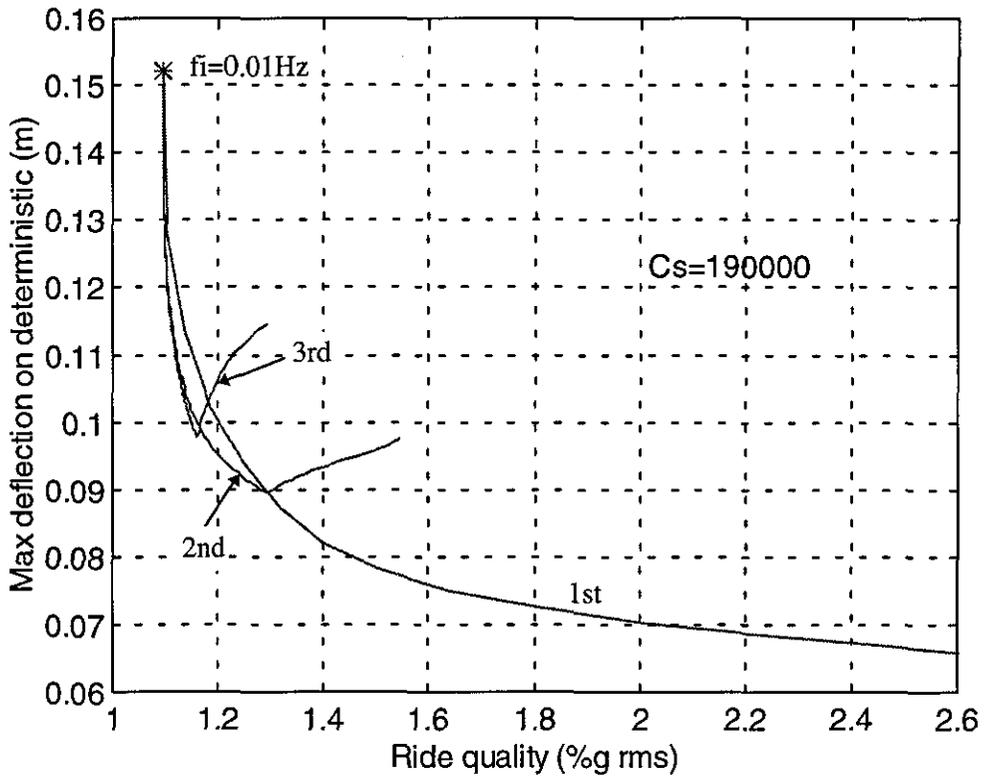
Figure 4.5   Trade-off curve, ω_i varying

Table 4.2   Eigenvalues vs. f_{i2}

f_{i2} (Hz)	eigenvalues	
0.0100	$-0.0445 \pm j\,0.0454$	$-3.1661 \pm j\,3.5700$
0.1058	$-0.5423 \pm j\,0.5767$	$-3.0896 \pm j\,2.6269$
0.2016	$-0.8821 \pm j\,1.7206$	$-3.3854,\ -2.9568$
0.2973	$-0.4565 \pm j\,2.4211$	$-5.6694,\ -2.3665$
0.3931	$-0.1834 \pm j\,2.7969$	$-6.7333,\ -2.6912$
0.4889	$0.0000 \pm j\,3.0719$	$-7.5389,\ -3.0950$
0.5000	$0.0173 \pm j\,3.0998$	$-7.6217,\ -3.1444$

The relatively small reduction in the maximum deterministic suspension deflection which is possible before the system goes unstable motivated a study of the two other linear control approaches - complementary filter controller and Kalman-Bucy filtering controller.

4.2   Complementary Filter Control

The system using the intuitive formulation control has the advantage of being very easy to design since it only uses a single feedback signal. However, there is a possible instability problem and the system still presents the maximum suspension deflections much greater than its passive counterpart, therefore this approach is not practically acceptable for the vertical suspension requirement.

As discussed previously, the fundamental suspension requirement can be described as the need to isolate the vehicle from the random track irregularities while following low frequency track design features (gradients), and hence avoiding excessively large suspension deflections. These two requirements occur at different frequencies and hence a control law capable of operating on those two frequency regions may be used to solve the problem. A new control strategy is developed in this section to avoid the instability and to allow low frequency stiffness or damping to be added to the system without detriment to the high frequency response. This method uses a complementary filter and thus is termed as a "complementary filter control" strategy.

4.2.1 System Model

The complementary filter control strategy has been studied by a number of researchers [Goodall - 1976], [Williams - 1994]. As its name suggests it is composed of a pair of filters, one providing a high-pass (HP) function, the other providing a low-pass (LP) function. These two filters are used to fulfil the fundamental suspension requirements. The low-pass filter acts to minimise suspension excursion which is predominantly a

low frequency effect, the high-pass filter provides the ride improvement function. The addition of normal (relative) damping at low frequencies has the effect of removing the instability problem, and it can easily be shown that if the filters are complementary (as described below) then the filter frequency ω_i can be varied freely.

An accelerometer and a displacement transducer will be used in practice. The accelerometer measures the vehicle body vertical acceleration $(\ddot{z}_b)$, which needs to be integrated as before; the displacement transducer measures the relative displacement between the track and the body $(z_b\text{-}z_t)$, and must therefore be differentiated to give the relative velocity. A block diagram of this strategy is shown in Figure 4.6.

The filters therefore give normal (relative) damping at low frequencies and the absolute damping at high frequencies. The improvement in the suspension deflection of this approach over the intuitive formulation control method is achieved through the low pass filter.
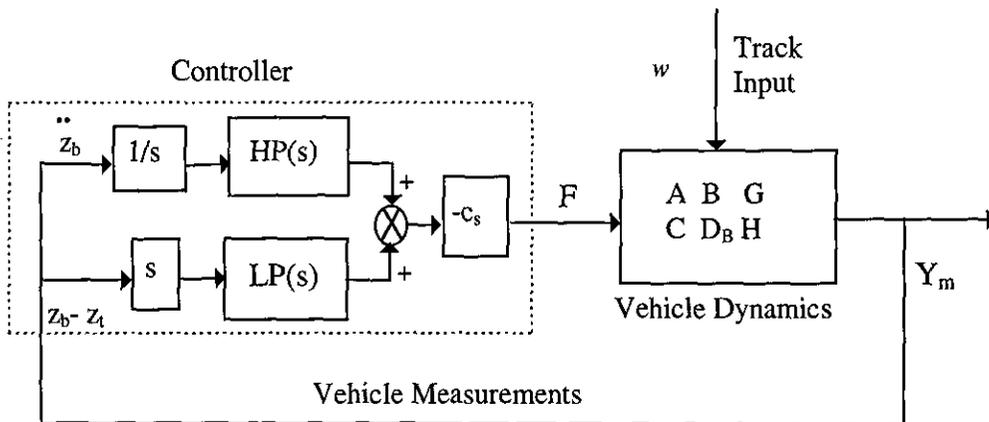
Figure 4.6   Complementary Filter Control

4.2.2 L.P. Filter

The low pass filter LP(s) can be easily derived from the high pass filter transfer function used previously:

$$LP(s) = 1 - HP(s) \tag{4.13}$$

Substituting the first-, second-, and third-order high pass filter transfer functions into equation 4.13, three low pass filter transfer functions are obtained, and their corresponding transfer functions after combined with the differentiator (s) are also given in Table 4.3, where ω_i is the cut-off frequency of the "low pass filter". (Notice again that a pure differentiator is not practically implementable, but is no problem when combined with the LP filter.)

<div align="center">Table 4.3   Filter transfer functions</div>

	Low-Pass (L.P.) filter	Combined transfer function (s x LP(s))
1st	$LP_1(s) = \dfrac{1}{1 + \dfrac{1}{\omega_i} s}$	$H_1(s) = \dfrac{s}{1 + \dfrac{1}{\omega_i} s}$
2nd	$LP_2(s) = \dfrac{1 + \dfrac{2\xi_i}{\omega_i} s}{1 + \dfrac{2\xi_i}{\omega_i} s + \dfrac{1}{\omega_i^2} s^2}$ (where $\xi_i = 0.7$)	$H_2(s) = \dfrac{s + \dfrac{2\xi_i}{\omega_i} s^2}{1 + \dfrac{2\xi_i}{\omega_i} s + \dfrac{1}{\omega_i^2} s^2}$ (where $\xi_i = 0.7$)
3rd	$LP_3(s) = \dfrac{1 + \dfrac{2}{\omega_i} s + \dfrac{2}{\omega_i^2} s^2}{1 + \dfrac{2}{\omega_i} s + \dfrac{2}{\omega_i^2} s^2 + \dfrac{1}{\omega_i^3} s^3}$	$H_3(s) = \dfrac{s + \dfrac{2}{\omega_i} s^2 + \dfrac{2}{\omega_i^2} s^3}{1 + \dfrac{2}{\omega_i} s + \dfrac{2}{\omega_i^2} s^2 + \dfrac{1}{\omega_i^3} s^3}$

4.2.3 Response to Random and Deterministic Features

The damping ratio $\zeta = 0.707$ is the optimum case [Karnopp - 1978a] and hence the value of skyhook damping $c_s = 190000$ N/ms^{-1} is used. As the pair of filters used are

complementary (i.e. add up to unity) it can easily be seen that the stability is not affected no matter what value of ω_i is chosen. Table 4.4 demonstrates this with the system using the second order filter. It is noticeable that when the cut-off frequency ω_i is increased, one pair of eigenvalues moves away from the y axis, and the other pair of eigenvalues keeps the same value for all cut-off frequencies.

Table 4.4   Eigenvalues vs. f_{i2}

f_{i2} (Hz)	eigenvalues		
0.010	-3.1667 ± j 3.6477	-0.0440 ± j 0.0449	-0.0440 ± j 0.0449
0.208	-3.1667 ± j 3.6477	-0.9148 ± j 0.9333	-0.9148 ± j 0.9333
0.406	-3.1667 ± j 3.6477	-1.7857 ± j 1.8218	-1.7857 ± j 1.8218
0.604	-3.1667 ± j 3.6477	-2.6565 ± j 2.7102	-2.6565 ± j 2.7102
0.802	-3.1667 ± j 3.6477	-3.5274 ± j 3.5986	-3.5274 ± j 3.5986
1.000	-3.1667 ± j 3.6477	-4.3982 ± j 4.4871	-4.3982 ± j 4.4871

A new trade-off curve varying the cut-off frequency ω_i is obtained as shown in Figure 4.7, where the ride quality and the maximum suspension deflection of the passive system are also given for comparison.

It is apparent that the system with whether the first, second or third order filter can improve both the ride quality and the maximum deflection. Two design points are examined in detail. The design point 1 is the point where the maximum suspension deflection of the active system is of the same value as the passive system and the design point 2 is where the ride quality is the same. For the first order filter, only a small improvement can be achieved when compared with the passive system. The maximum suspension deflection of the first order filter system is reduced from 0.0338 to 0.0327 (m) for the same ride quality. On the other hand, the r.m.s. value of the ride quality is improved from 3.486 to 3.3519 (%g).
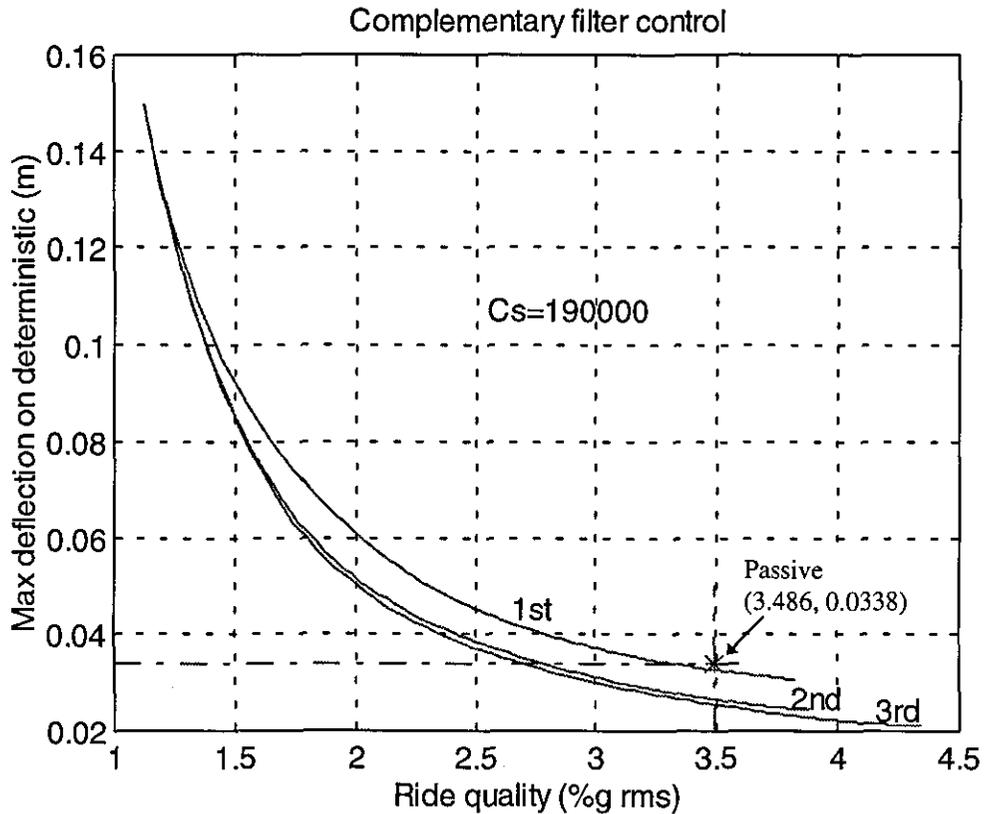
Figure 4.7   Trade-off curve for complementary filter control

The second and third order filters are more promising. The trade-off curves of the two systems are very close to each other. Compared with the passive suspension, the ride quality for the second order filter system can be improved by 19.8% if the deflection is the same as the passive. Alternatively the suspension deflection can be reduced by 21.6% if the same ride quality is maintained. For the third order filter system, the two figures are 22.3% and 24.6% respectively. Table 4.5 lists the filter cut-off frequencies corresponding to these design points.

The design point 1 is of most interest, because it gives the improvement in ride quality for the complementary filter control with the same suspension deflection as the passive one, but the other case (e.g. design point 2) has been included for completeness.

Table 4.5   Filter cut-off frequencies at two particular design points

	Design Point 1 deflection = passive (0.0338m)	**Design Point 2** ride quality = passive (3.486%g)
1st order	2.0826 (Hz)	2.2923 (Hz)
2nd order	0.6739 (Hz)	1.0901 (Hz)
3rd order	0.4400 (Hz)	0.7106 (Hz)
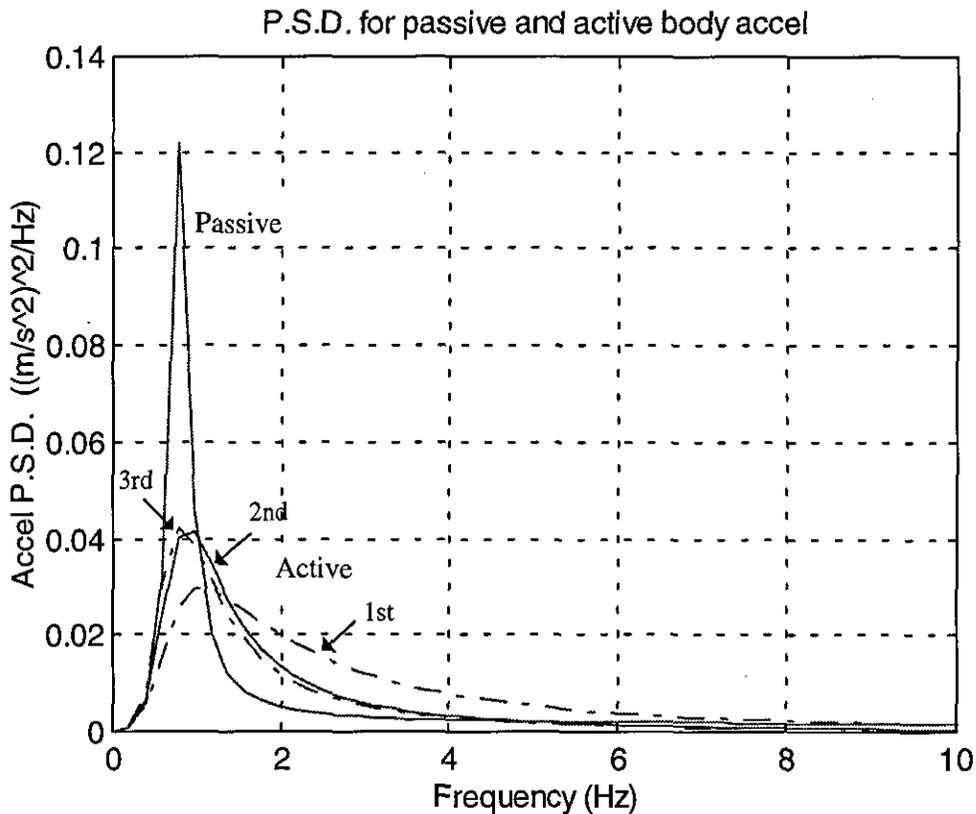
Figure 4.8   Acceleration P.S.D. at design point 1

Figure 4.8 gives the calculated acceleration P.S.D. values for the passive system and the active system using the first, second and third order complementary filters respectively, where the maximum suspension deflections of the active system are kept the same as the passive one for all three filters (design point 1). At the rigid body

frequency about 1 Hz, the acceleration P.S.D. values of the active system are less than half of that of the passive system with the first order filter being the lowest. Immediate above this frequency, the P.S.D. values of the active system are higher. With the increase of frequency, the P.S.D. values of the second and third order systems reduce rapidly to below the passive one again at the frequency about 5 Hz. From calculation, the r.m.s. values derived from the P.S.D. are also smaller. However the P.S.D. value of the first order system is above the passive one for a much wider frequency range, up to 9 Hz. As a result, the r.m.s. value for the first order filter system is only slightly smaller than the passive one.
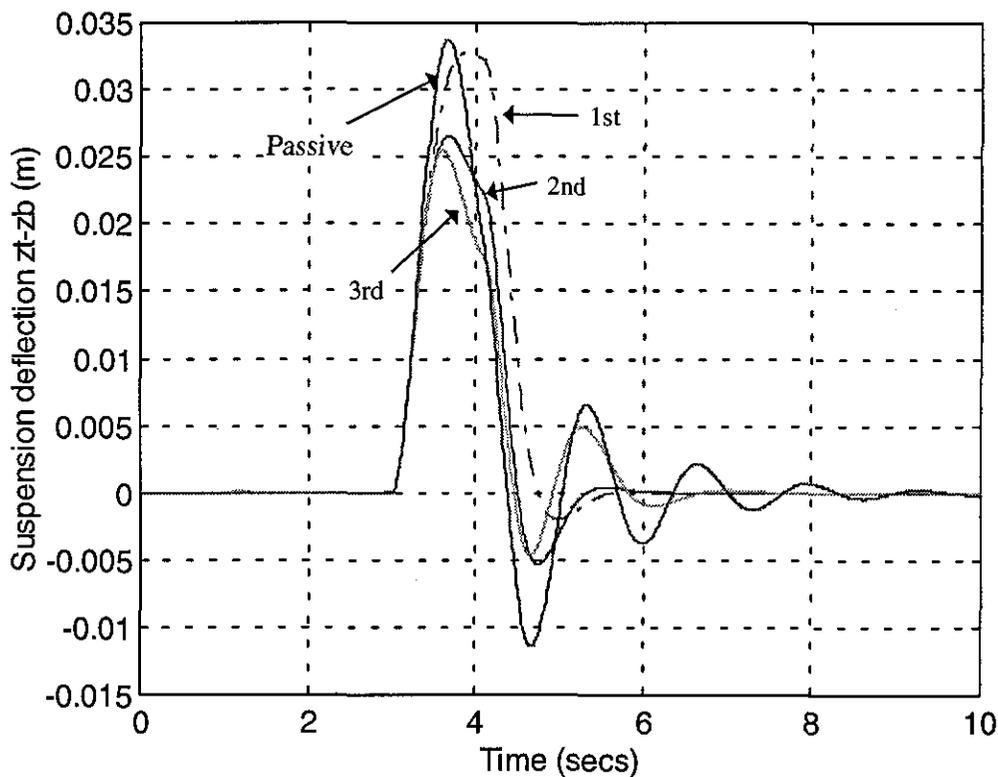
Figure 4.9   Suspension deflection at design point 2

Figure 4.9 gives the comparison for suspension deflections between complementary filter and passive one at design point 2. It shows that at this design point, the suspension deflections for the system with the second or third order filter are reduced

to around 0.025 m, but for the system with the first order filter in the feedback the reduction is not very obvious. So, the second order system is often used in practice.

4.3 Kalman-Bucy Filtering Control

In the intuitive formulation and the complementary filter controls the vertical velocity of the vehicle body is derived from the acceleration signals which can be easily measured through the accelerometer. An alternative to these strategies is to use a Kalman-Bucy filter, which enables a more rigorous estimation to be made on the basis of the system model and values for the process and measurement noises. By introducing the Kalman filter in the feedback loop to replace other controllers, the body velocity of the vehicle can be estimated by the filter as the velocity is one of the state variables of the system.

In this section an overview of the operation of the Kalman filter algorithm is presented, and then a Kalman-Bucy filter is introduced in the closed loop feedback system.

4.3.1 Principle of Kalman Filter

Kalman filtering is an optimal state estimation process applied to a dynamic system that involves random perturbations. More precisely, the Kalman filter gives a linear, unbiased, and minimum error variance recursive algorithm to optimally estimate the unknown state of a dynamic system from noisy data taken at discrete real-time intervals [Chui - 1987].

The Kalman filter is formulated using the state-space approach, in which a dynamical system is described by a set of variables called the state. The state contains all the necessary information about the behaviour of the system such that given the present

and future values of the input, the current and future state and output of the system may be calculated in mathematical terms.

The application of Kalman filter theory results in a set of difference equations, the solution of which can be computed recursively. In particular, each current estimate can be computed from the old estimate and new input data; hence only the most recent estimate needs to be stored. In addition to eliminating the need for storing the entire past observed data, the Kalman filter is more efficient than computing the estimates from the entire past observed data at each step. The Kalman filter is thus ideally suited for implementation on a digital computer. It has also been widely used in many areas of industrial and military applications such as video and laser tracking systems, satellite navigation, ballistic missile trajectory estimation, radar, and fire control. With the development of high-speed microprocessors and digital signal processors (DSP), the Kalman filter has become more useful even for very complicated real-time applications.

For the continuous-time analysis, a similar form known as the Kalman-Bucy filter can be used [Chui - 1987].

4.3.2 System Model

A Kalman-Bucy filter is used in the feedback loop of the suspension control system to estimate the absolute vertical velocity of the vehicle body. The velocity is then amplified by the "skyhook" damping (c_s) to apply the force (F) to the vehicle. The Kalman-Bucy filter has been derived from the ideal suspension model and its input signal is the measured body acceleration. Figure 4.10 gives the block diagram of the structure.
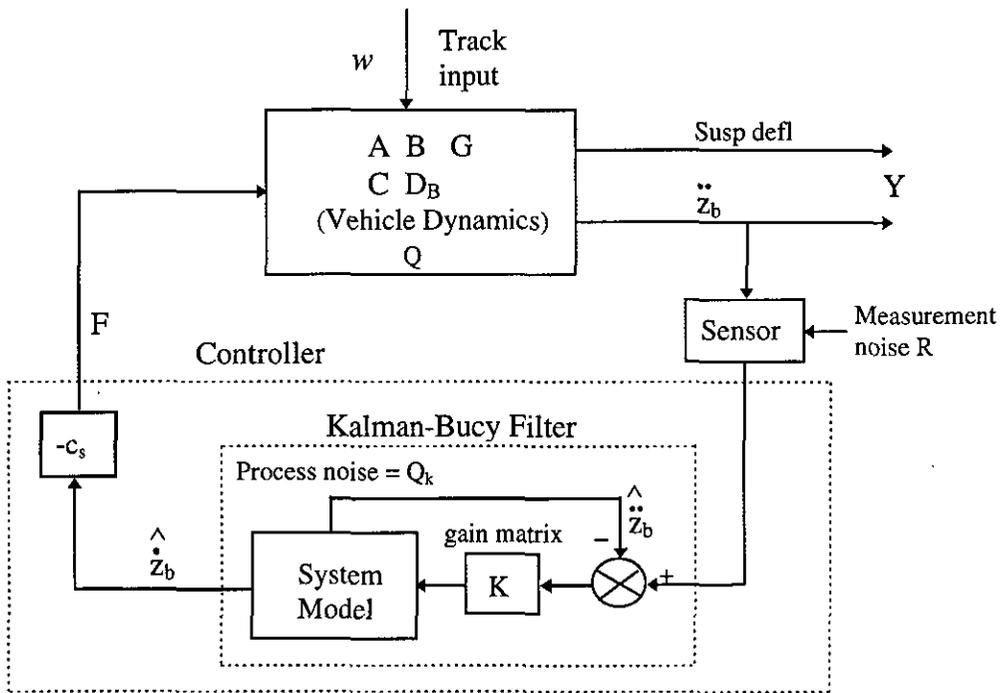
Figure 4.10   Kalman-Bucy filtering control

The state-space and output equations of the vehicle are given in equations 4.3 and 4.4. The track input w and the measurement noise v_1 can be treated as white noises and their covariances are expressed as:

$$E[ww^T] = Q = 2\pi^2 A_r v \qquad (4.14)$$

$$E[v_1 v_1^T] = R \qquad (4.15)$$

$$E[wv_1^T] = 0 \qquad (4.16)$$

$$E[w] = E[v_1] = 0 \qquad (4.17)$$

where the values of Q and R are constant.

Using the state-space and output equations of the vehicle body, the system model of the Kalman-Bucy filter can be derived as:

$$\dot{\hat{X}} = A_k \, \hat{X} + K(Y_m - \hat{Y}) \tag{4.18}$$

$$\hat{Y} = C_k \, \hat{X} \tag{4.19}$$

where $A_k = A - B * [c_s \; 0]$, $C_k = C - D_B * [c_s \; 0]$, the process and measurement noise covariances are Q_k and R, and $\hat{X}$ and $\hat{Y}$ are respectively the state and output estimates.

The Kalman-Bucy filter gain matrix K is fixed and designed off-line. In **MATLAB™**, this can be obtained by using the function **LQE** to produce the stationary gain matrix K, with which the filter will return the optimal estimate of the state X.

4.3.3 System Response to Random and Deterministic Features

A trade-off curve for the control structure shown in Figure 4.11 is obtained by adjusting the process noise for the Kalman-Bucy filter (Q_k). A typical expected maximum value for the vehicle body acceleration is 10 %g, and the r.m.s. value of the measurement noise R is chosen at a value corresponding to around 1 % of this expected maximum value. The process noise for the Kalman-Bucy filter Q_k is initially set equal to its expected value Q (covariance of the track), and varied downwards from that value. The ride quality and the maximum suspension deflection of the passive system are also shown in the figure for a comparison.
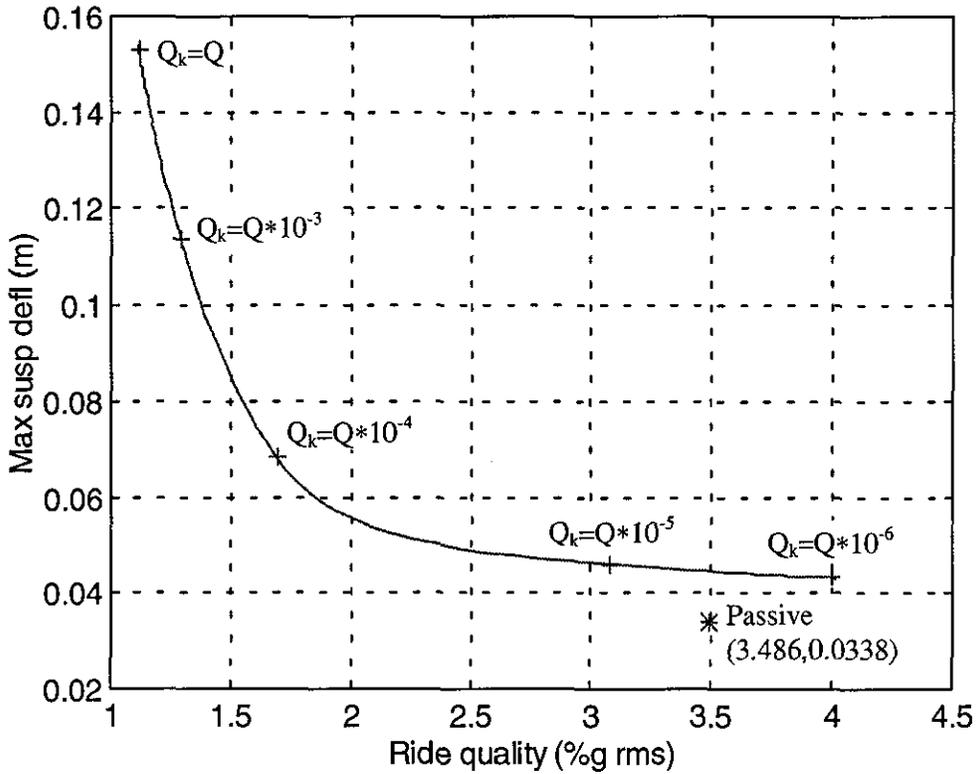
Figure 4.11   Trade-off curve for Kalman-Bucy filter control

It is apparent that when Q_k equals Q, the Kalman-Bucy filter gives a very good estimation for the body velocity of the vehicle, and for the random track a good ride quality is achieved, but for the deterministic feature it creates the largest suspension deflection. Decreasing Q_k reduces the maximum suspension deflection, but this is only achieved by compromising the ride quality. Figure 4.12 gives the details of the acceleration errors between measured and estimated accelerations of the Kalman-Bucy filter at two different Q_k values on the random track inputs, and Figure 4.13 shows the actual measured acceleration for the random inputs at $Q_k=Q*10^{-4}$ for reference. (Notice that measurement noise is always included in the measured acceleration.)
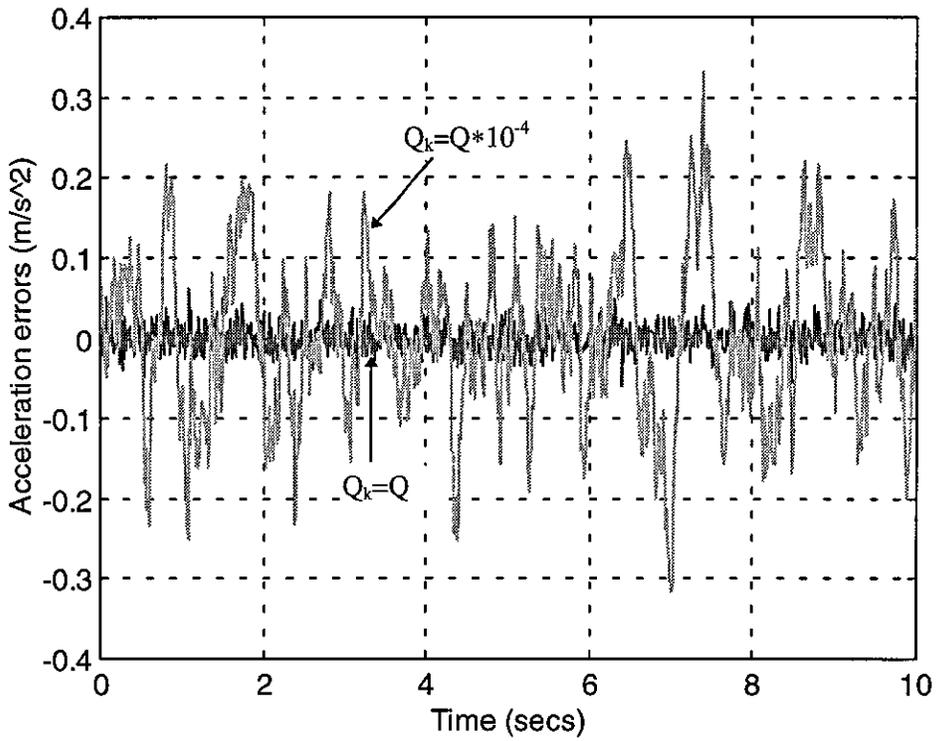
Figure 4.12   Acceleration errors at different Q_k for random inputs
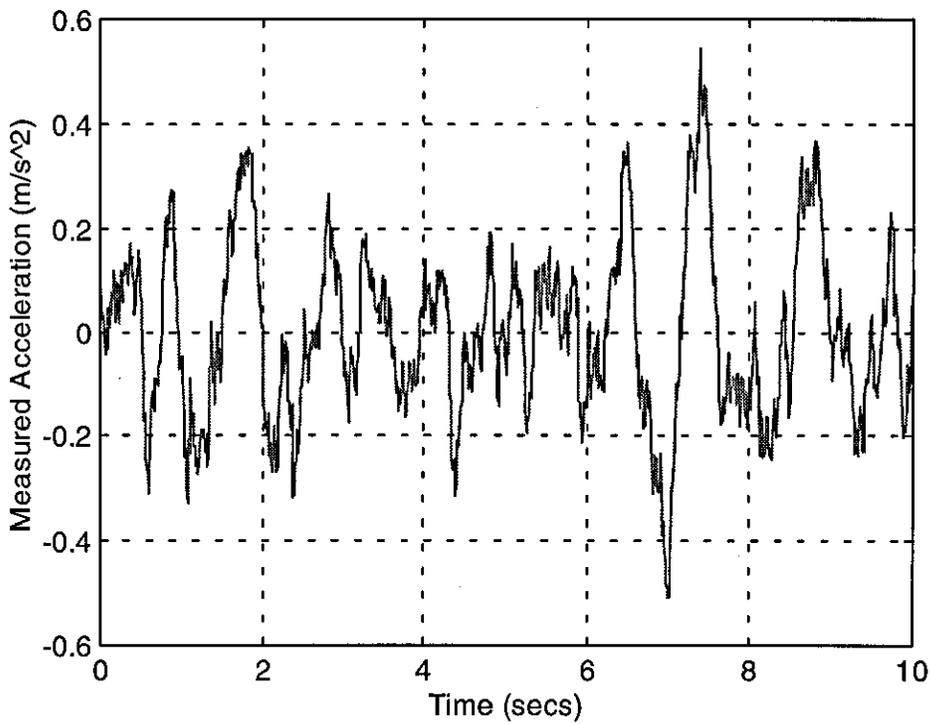
Figure 4.13   Measured acceleration at $Q_k=Q*10^{-4}$ for random inputs

It is clearly seen that when the process noise for the Kalman-Bucy filter Q_k equals Q the residual (acceleration error) between the measured and estimated accelerations is minimum, i.e. the Kalman-Bucy filter produces a very good estimation of the body velocity. For the random track input this will achieve the smallest r.m.s. ride quality value. When $Q_k=Q*10^{-4}$ the error signal between the measured and estimated accelerations is increased and larger than that at $Q_k=Q$, and this means that the Kalman-Bucy filter is less effective in estimating the effects of the random track input, as a consequence a larger r.m.s. ride quality value is generated.
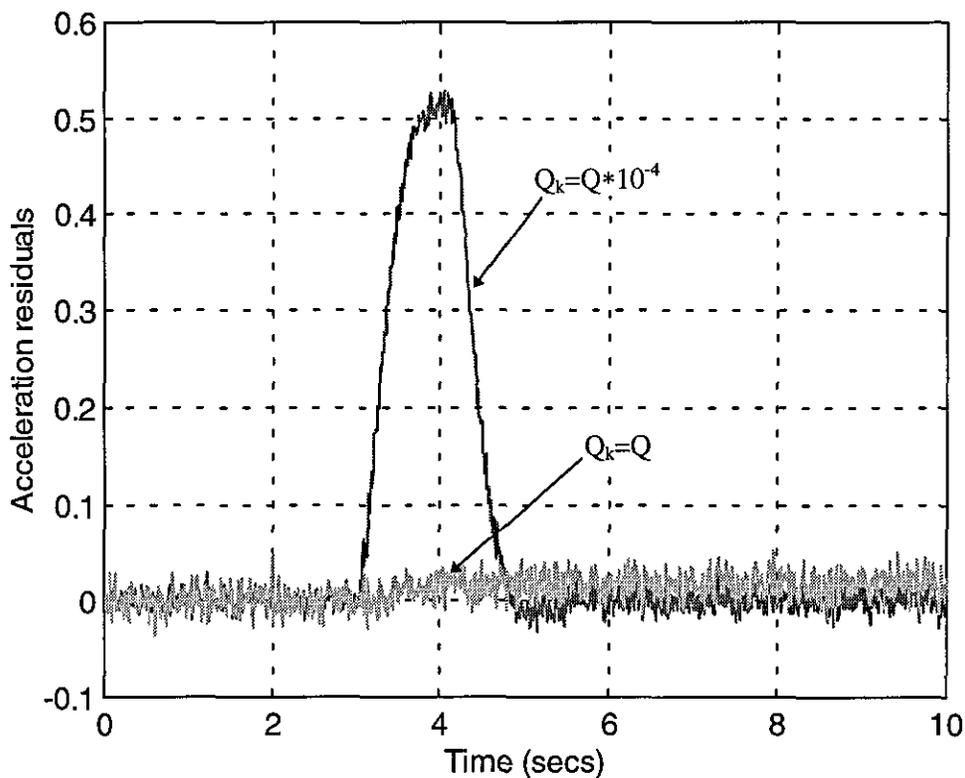
Figure 4.14   Acceleration errors at different Q_k for deterministic inputs

Figure 4.14 gives the acceleration errors at two different Q_k when the vehicle encounters the deterministic track features. It can be seen that when $Q_k=Q$ the error between the measured and estimated accelerations is also very small, and means that the output of the Kalman-Bucy filter will contain the effects of the deterministic track

inputs. Because the skyhook damper (c_s) is used in the feedback, a large suspension deflection is produced. Reducing the Q_k (e.g. $Q_k = Q*10^{-4}$) will make the Kalman-Bucy filter increase the weighting of its own estimation compared with the measurements, consequently the acceleration error is increased and less effects of the deterministic features will be contained in the estimated body velocity, thus the suspension deflection is reduced. Figure 4.15 shows the estimated body velocity at these two different Q_k values.
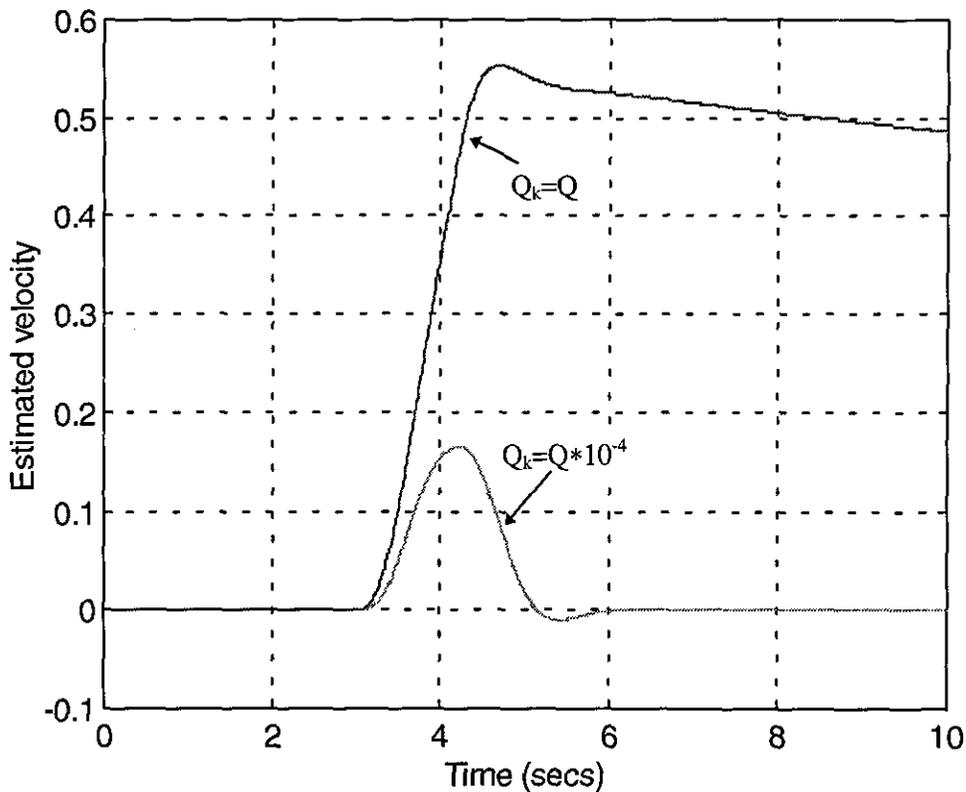
Figure 4.15   Estimated body velocity at different Q_k

Figures 4.16 and 4.17 give the comparison of two acceleration residuals responding to the random and deterministic track inputs at two different Q_k values respectively. When the process noise of Kalman-Bucy filter Q_k equals $Q*10^{-4}$, the two error signals (acceleration residuals) corresponding to the response of the random and deterministic track inputs are much easier to distinguish compared with that at $Q_k = Q$. As a consequence the maximum suspension deflection on the deterministic input is

reduced, but this is at the expense of the ride quality since the decreasing of Q_k will also make the acceleration error for the random track inputs larger. Further decreasing the Q_k to $Q*10^{-6}$ will make two error signals become difficult to separate again and the whole closed loop system starts to oscillate and eventually becomes unstable. This can be seen clearly through the acceleration error signals of the system, found in Figure 4.18.

It is noticeable that only when the process noise for the Kalman-Bucy Filter Q_k is set around $Q*10^{-4}$, the two acceleration residuals can be distinguished and separated very easily.
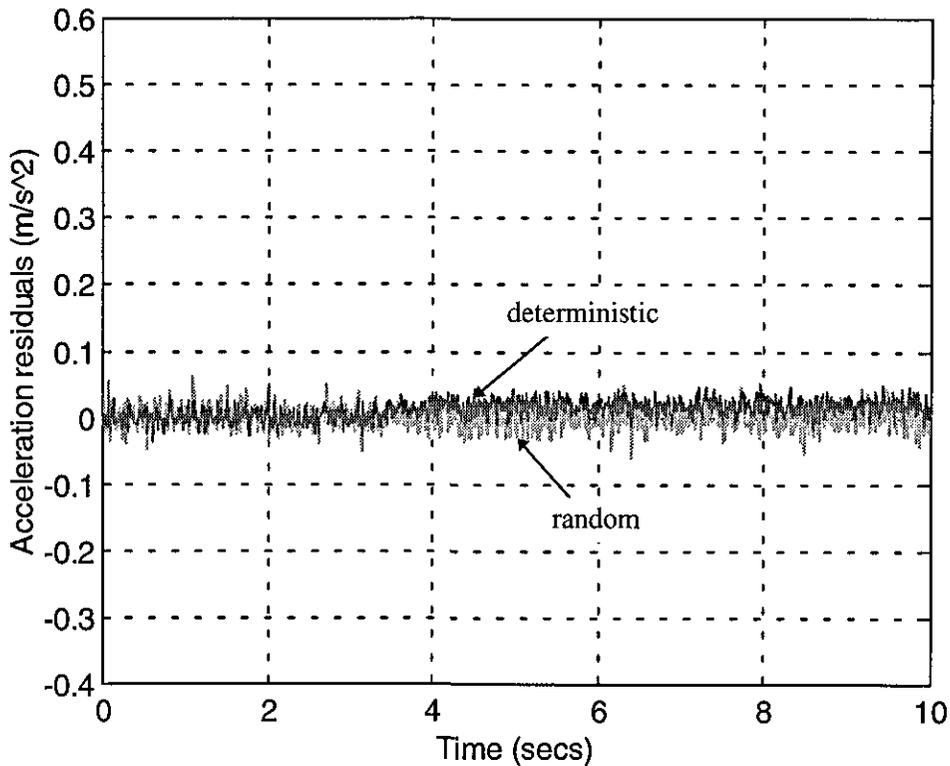
Figure 4.16   Acceleration residuals at $Q_k=Q$
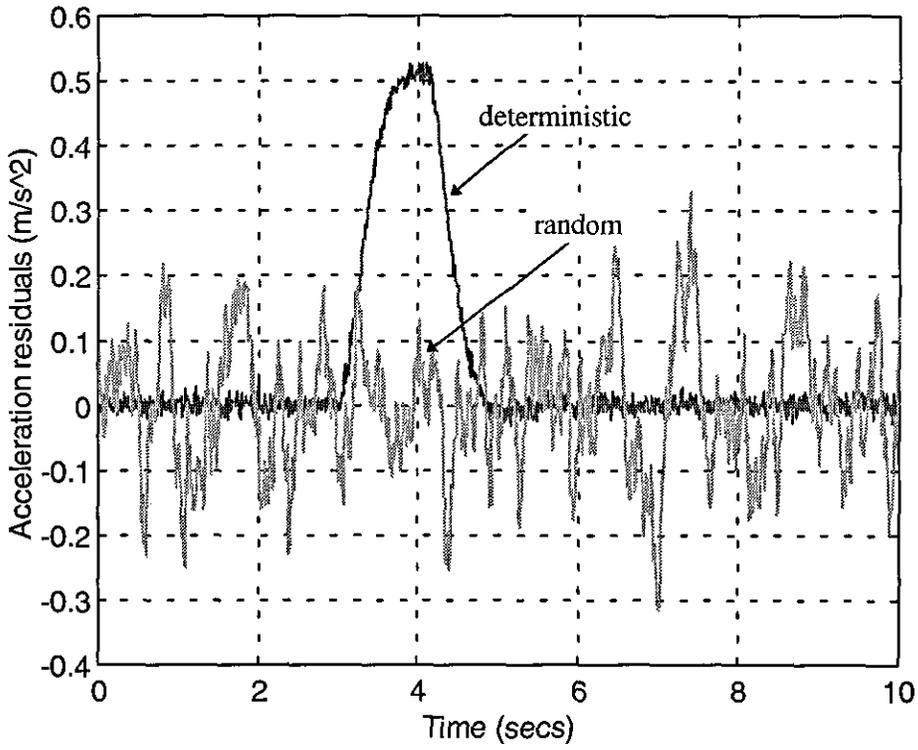
Figure 4.17   Acceleration residuals at $Q_k=Q*10^{-4}$
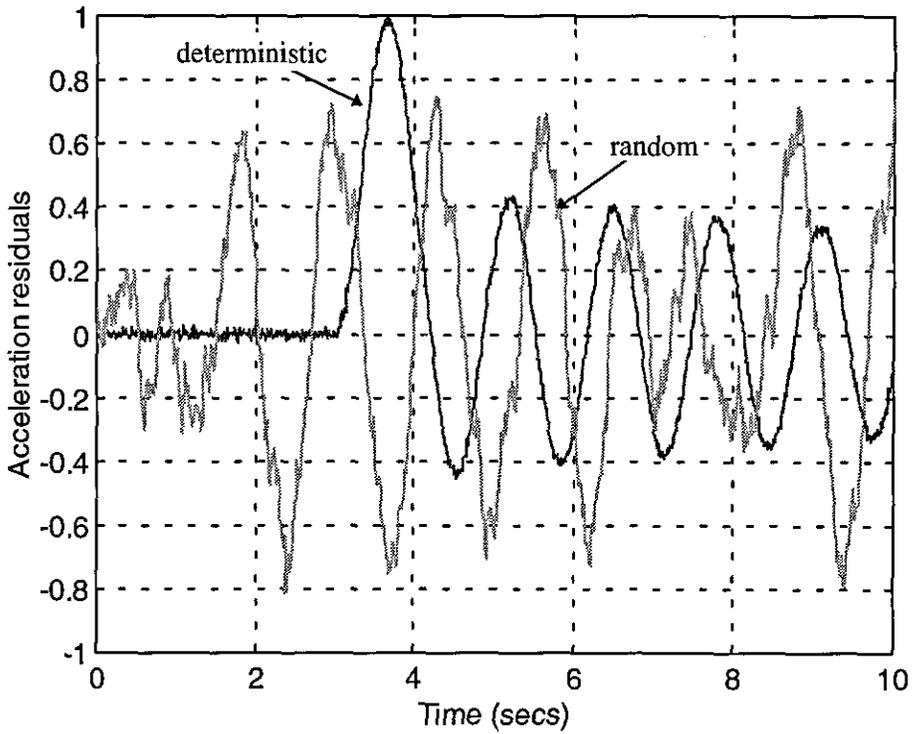
Figure 4.18   Acceleration residuals at $Q_k=Q*10^{-6}$

4.3.4 Other Options

Other options such as using both acceleration and suspension deflection measurements as the input signal of the Kalman-Bucy filter, and the Kalman-Bucy filter with time-varying gain matrix K have also been studied, but the simulation results have shown that the trade-off curves are very similar, and no further improvements can be achieved.
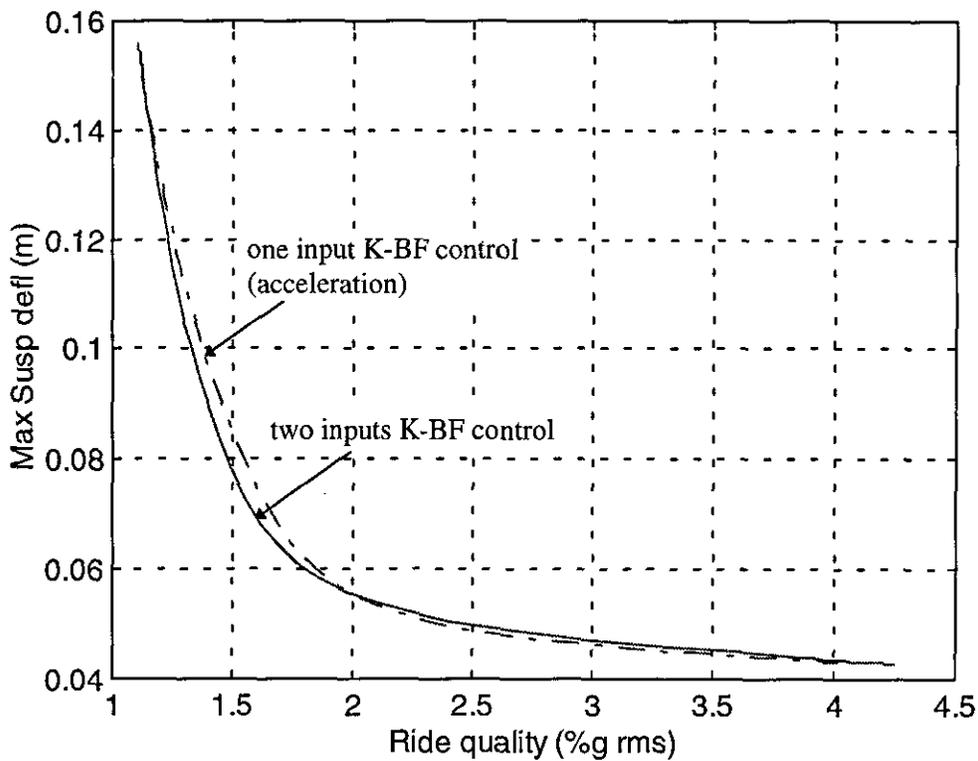
Figure 4.19   Trade-off curve for two inputs Kalman-Bucy filter control

Figure 4.19 compares the trade-off curve of the Kalman-Bucy filter control using two input signals with the trade-off curve of the single acceleration input Kalman-Bucy filter approach. These two curves are very similar, and the maximum suspension deflection in the two inputs Kalman-Bucy filter control is slightly smaller than the one input (acceleration) Kalman-Bucy filter at lower ride quality (less than 2 %g) and slightly larger at higher ride quality values.

Figure 4.20 shows the trade-off curves of the Kalman-Bucy filter using time-varying and fixed gain matrix (K), and it appears that two curves are exactly the same.
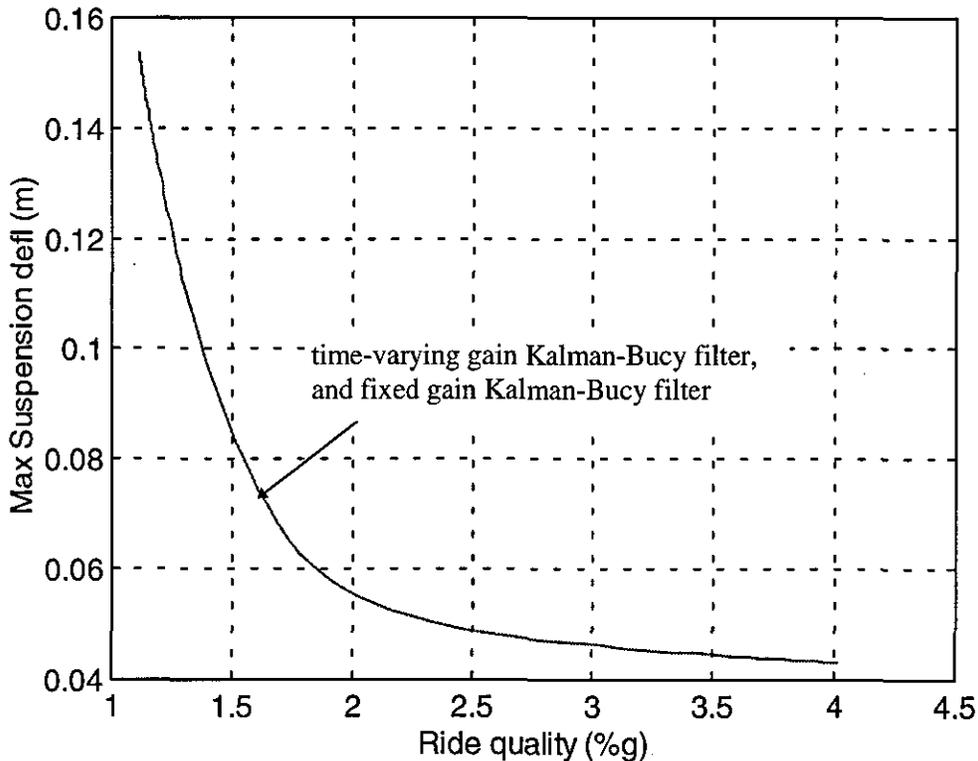
Figure 4.20   Trade-off curve for time varying gain Kalman-Bucy filter

4.4   Comparison Of All Three Approaches

The assessment of three linear control strategies suggests that when the constraint of the maximum suspension deflection at deterministic features is taken into consideration, the advantages of active control in terms of improving straight track ride quality is also reduced. The intuitive formulation control approach is the implementation of the "skyhook" damping strategy in principle and is very unlikely to be used in practice because the maximum suspension deflection is far beyond the acceptable range although the ride quality is significant improved compared with the passive suspension system. By contrast both the complementary filter control and Kalman-Bucy filtering control approaches can achieve a large improvement in the

ride quality while keeping the maximum suspension deflection near the passive one. Figure 4.21 plots the trade-off curves of these two methods in the same figure, and the ride quality and maximum suspension deflection of the passive system.

It can be seen that when the r.m.s. ride quality value is less than 1.7 %g the maximum suspension deflections for the Kalman-Bucy filter control and the complementary filter control with the second and third order are very similar, and they are all better than the first order complementary filter control method. When the r.m.s. ride quality value is larger than that, i.e. higher cut-off frequency for the complementary filter or lower Q_k for the Kalman-Bucy filter, the complementary filter control strategy performs better than the Kalman-Bucy filter control method in reducing the maximum suspension deflection.
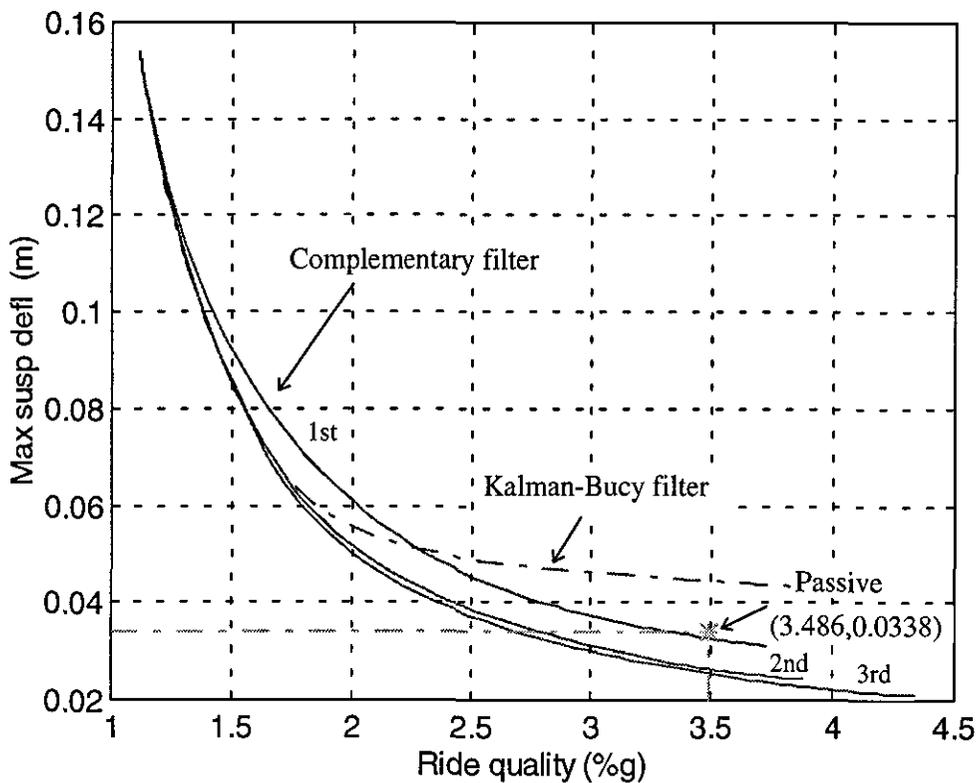
Figure 4.21   Comparison of Complementary and Kalman-Bucy filters

Overall, only the complementary filter control strategy can improve the ride quality while keeping the same maximum suspension deflection as the passive one, and the best figure can be achieved is 22.3%. On the other hand, the best point for reducing the suspension deflection is 24.6% if the same ride quality is maintained.

4.5   Summary

This chapter has presented a comprehensive study of three linear control strategies used to improve the performance of the vehicle suspension system. The intuitive formulation control is first introduced to implement the "skyhook" damping. This system has inherited the advantages as well as disadvantages of the "skyhook" damping, i.e. significant improvement in the ride quality but much increased suspension deflection on deterministic features. The complementary filter control can improve the ride quality and at the same time reduce the maximum suspension deflection. It also eliminates the possibility of the system becoming unstable when using the intuitive formulation control. The Kalman-Bucy filter control estimates the vertical velocity of the vehicle body and applies it as the feedback signal.

Although a large improvement has been achieved by using these linear control approaches, the full potential of the active suspension is not yet realized mainly because a compromise has to be made to ensure an acceptable suspension deflection. Further improvement may be possible if other control strategies in addition to the linear ones are to be used. This will be presented in the next chapter.

CHAPTER 5

NON-LINEAR CONTROL APPROACHES

The linear approaches discussed previously take no advantage of any knowledge of the deterministic features, which for a railway system are very well defined for reasons of passenger comfort. However it is possible to develop a non-linear system where such features can be used to improve the performance.

Non-linear approaches can often offer better solutions and thus better system performances and they become more and more acceptable in practical applications, as microprocessors and digital control techniques have been so advanced in recent years that even very sophisticated systems can be implemented with ease both technically and economically.

In this chapter two non-linear approaches are investigated, both based upon the use of Kalman Filters. The primary motivation for developing the strategy is to achieve further improvement in the trade-off between the maximum suspension deflection and ride quality.

5.1 Dual Kalman Filter Control

In the linear Kalman-Bucy filtering control approach, when the vehicle enters the deterministic track, it is noticed that as the process noise for the Kalman-Bucy filter Q_k is reduced from its expected value Q to $Q_k=Q*10^{-4}$, the residual between the measured and estimated accelerations is larger than that at $Q_k=Q$, and the two error signals (acceleration residuals) corresponding to the response of the random and

deterministic track inputs can be easily distinguished in the range around $Q*10^{-4}$. This observation leads to the development of a non-linear method presented in this section. The non-linear method is developed based upon the original linear Kalman-Bucy filter control approach and it uses a second Kalman Filter to "watch" the acceleration residual within the filter and to apply a non-linear means of correcting for the effects of the deterministic track inputs. The method is termed non-linear Dual Kalman Filter control because of the two filters used in the system.

5.1.1 System Model

A block diagram of the control strategy is shown in Figure 5.1, in which one Kalman Filter (KF1) provides the state estimate for applying skyhook damping, and the acceleration residual of the second filter (KF2) is used to provide a non-linear correction.

On a transition to the gradient which was defined previously the residual increases beyond the normal levels encountered on level track, and if it exceeds a threshold level then ± 0.5 m/s^2 is deducted from the measured acceleration going to KF1 because this is the superimposed acceleration level associated with the transition.

For Kalman Filter 1 (KF1) the process noise Q_{k1} is first set to the expected value Q, which is known to give good ride quality, but the process noise Q_{k2} for Kalman Filter 2 (KF2) and the threshold level (r1) are varied to separate the deterministic and the random track features and to give the smallest suspension deflection while keeping the ride quality the same as for the linear Kalman-Bucy Filter control method. The same procedure can be repeated with other values of Q_{k1} and a trade-off curve can be obtained.
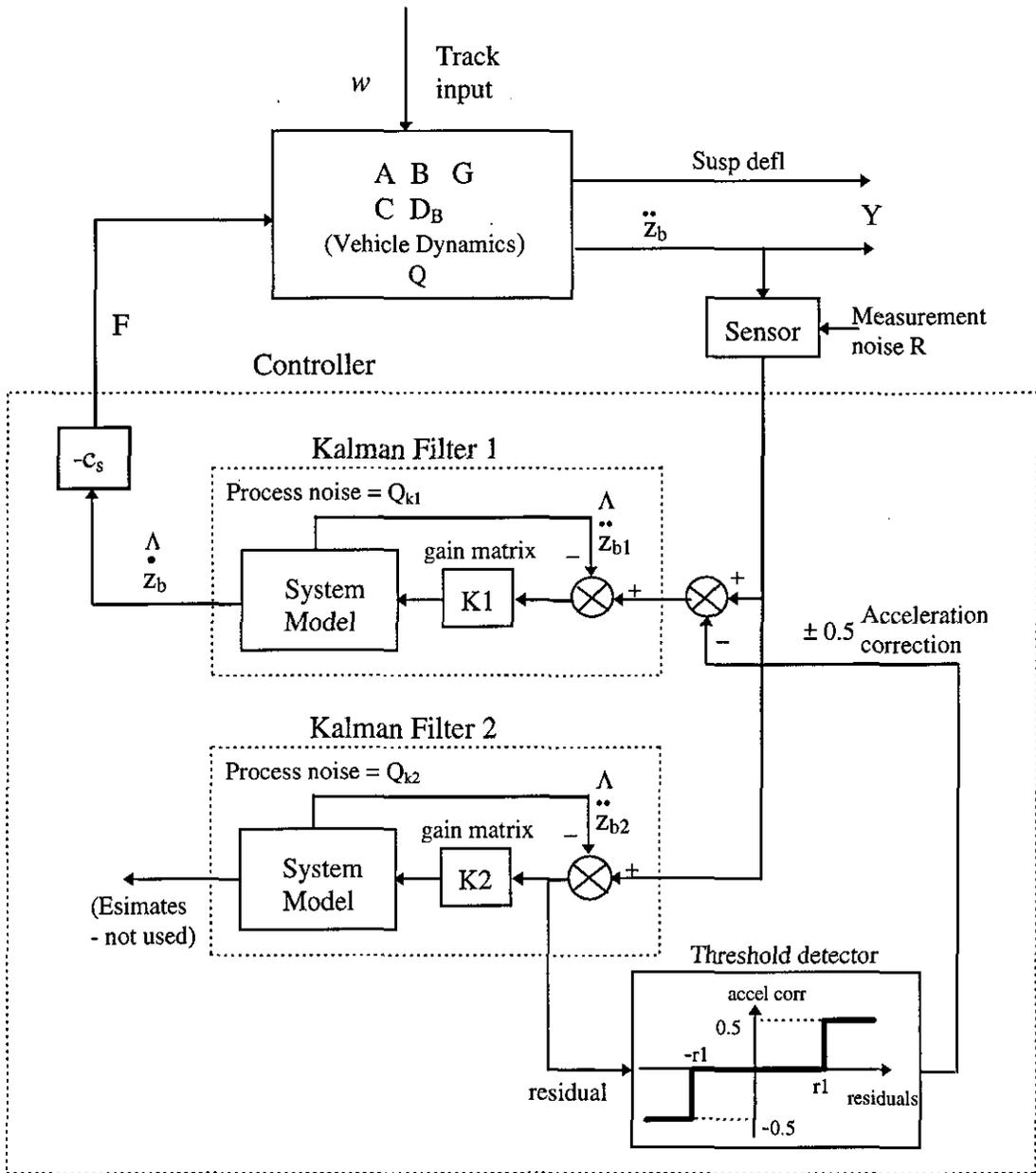
Figure 5.1   Dual Kalman Filter control structure

5.1.2 Threshold Detector

In this system the Kalman Filter 2 (KF2) is used to monitor the acceleration residual and the output of the filter is fed to a threshold detector. The threshold detector then separates the random and deterministic features of the input signals. The threshold value r1 is tuned on-line to obtain an optimal effect and it is first set to three times the standard deviation of the acceleration residual on the level track at $Q_{k2}=Q$ which can be calculated off-line. This initial setting is selected because it is very likely to fall between the normal levels of the residuals on the random track and the deterministic feature, and hence be able to separate the two. The value is then varied until a minimum suspension deflection is achieved and the r.m.s. ride quality value is maintained the same as for the linear case. This procedure is then repeated for different values of Q_{k2} to obtain the optimal setting for one Q_{k1} value.

5.1.3 System Response

By selecting different values Q_{k1} and tuning the Q_{k2} and the threshold level (r1), a trade-off curve is obtained. This is given in Figure 5.2, where the trade-off curve for the linear Kalman-Bucy filter control and the ride quality and maximum suspension deflection for the passive system are also given for comparison. Details of some points used to create the trade-off curve and their corresponding Q_{k1}, Q_{k2} and r1 values are given in Table 5.1.

It appears that when the process noise for Kalman Filter 1 (Q_{k1}) is set to Q, the ride quality can be significantly improved by around 68% compared with the passive system although the maximum suspension deflection is still high. Reducing the Q_{k1} will improve the maximum suspension deflection but worsen the ride quality at the same time.

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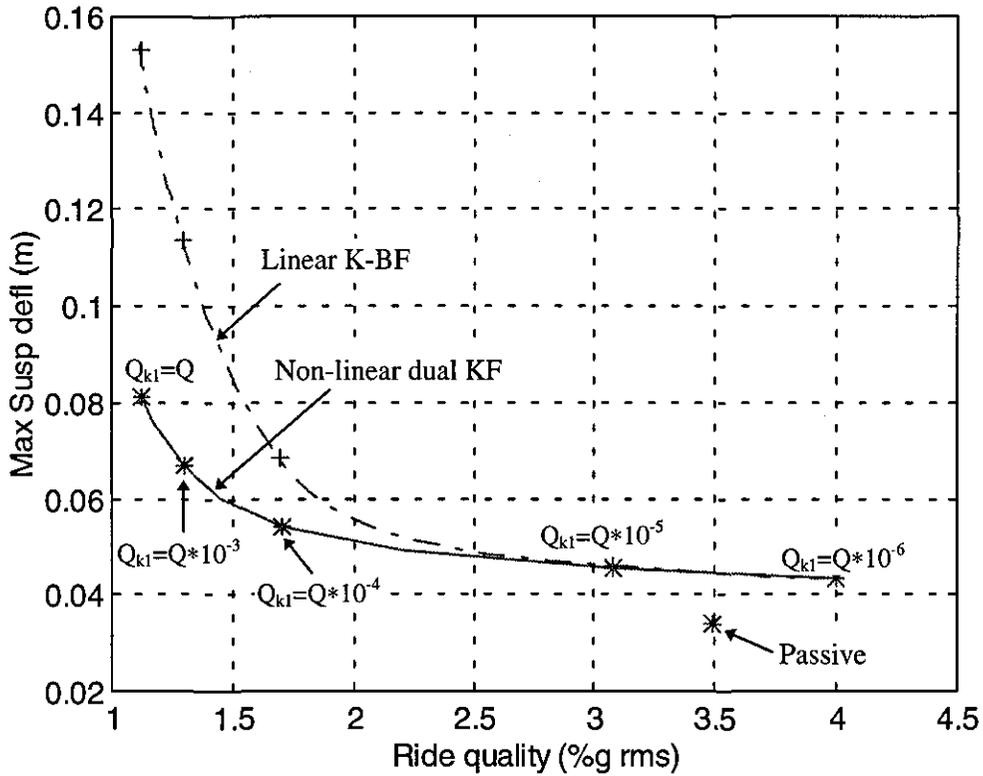
Figure 5.2   Non-linear Dual Kalman Filter control trade-off curve

Table 5.1   Various parameters values at design points for dual KF

Process noise for KF1: Q_{k1}	Process noise for KF2: Q_{k2}	Threshold level: $r1(m/s^2)$	Ride quality (%g)	Max susp defl (m)
Q	$Q*10^{-4}$	0.25	1.1186	0.0813
$Q*10^{-3}$	$Q*10^{-4}$	0.26	1.3010	0.0666
$Q*10^{-4}$	$Q*10^{-4}$	0.307	1.7033	0.0541
$Q*10^{-5}$	$Q*10^{-4}$	0.468	3.0795	0.0456
$Q*10^{-6}$	$Q*10^{-4}$	0.586	3.997	0.0432

The non-linear Dual Kalman Filter control strategy developed in this section is based upon the linear Kalman-Bucy Filter control approach and the aim is to achieve further improvement on the suspension performance. By comparing the two trade-off curves in the Figure 5.2, it is apparent that when the process noises (Q_{k1} for the non-linear

Dual Kalman Filter control and Q_k for the linear Kalman-Bucy Filter control) are equal to the expected value Q, a significant 47 % reduction in the suspension deflection can be achieved with the non-linear method while keeping the same ride quality as the linear one. This is because the linear approach takes no advantage of any knowledge of the deterministic features, and for the non-linear method the knowledge of deterministic feature which is detected from the threshold detector is added to the feedback loop such that the deterministic effects can be reduced when using the skyhook damping to generate the control force. When the process noise Q_{k1} is reduced to very small values, the effect of the deterministic features is reduced for reasons explained in the previous chapter and hence the improvement of the non-linear approach over the linear method becomes smaller, and the two curves in Figure 5.2 can be seen to converge at higher ride quality r.m.s. values.

The dual Kalman Filter control method using both acceleration and displacement inputs as the input signals for its two Kalman filters has also been studied, and the results show that no further improvement can be achieved.

5.2   Single Kalman Filter Control

The system using the non-linear Dual Kalman Filter control has the advantage of a significant reduction in the maximum suspension deflection compared with the linear Kalman-Bucy Filter control. However, there are two Kalman filters needed in the controller, and in practice the more complex the controller is, the more difficult to be used or be realised. Also the maximum suspension deflection is still higher than that of the passive system even though the ride quality can be improved by over 50%. A simpler control method using a Single Kalman Filter only is developed in this section primarily to make the system easier to implement. This is achieved by using an additional measurement feedback.

5.2.1   System Model

Figure 5.3 gives the block diagram of the Single Kalman Filter control method. In this case the measurement of the suspension deflection is used and the acceleration correction of a ± 0.5 m/s^2 simply comes from a threshold detector applied to the suspension deflection, with the threshold again being adjusted to obtain the smallest suspension deflection. The r.m.s. value of the measurement noise R2 is chosen at a value corresponding to around 1 % of the allowed maximum value of the suspension deflection, typically ± 25 mm.

The principle of this simple non-linear method is that, on a transition to a gradient, the suspension deflection increases beyond the normal levels encountered on level track which had already been demonstrated, and when it exceeds the threshold value (r2) then ± 0.5 m/s^2 is subtracted from the measured acceleration going to the Kalman Filter. For the Kalman Filter the process noise Q_k is initially set to Q, and the threshold value r2 is set to three times its standard deviation on level track, then the threshold value is adjusted to search for the smallest suspension deflection while keeping the same ride quality as the linear Kalman-Bucy Filter approach where the same Q_k value is used.
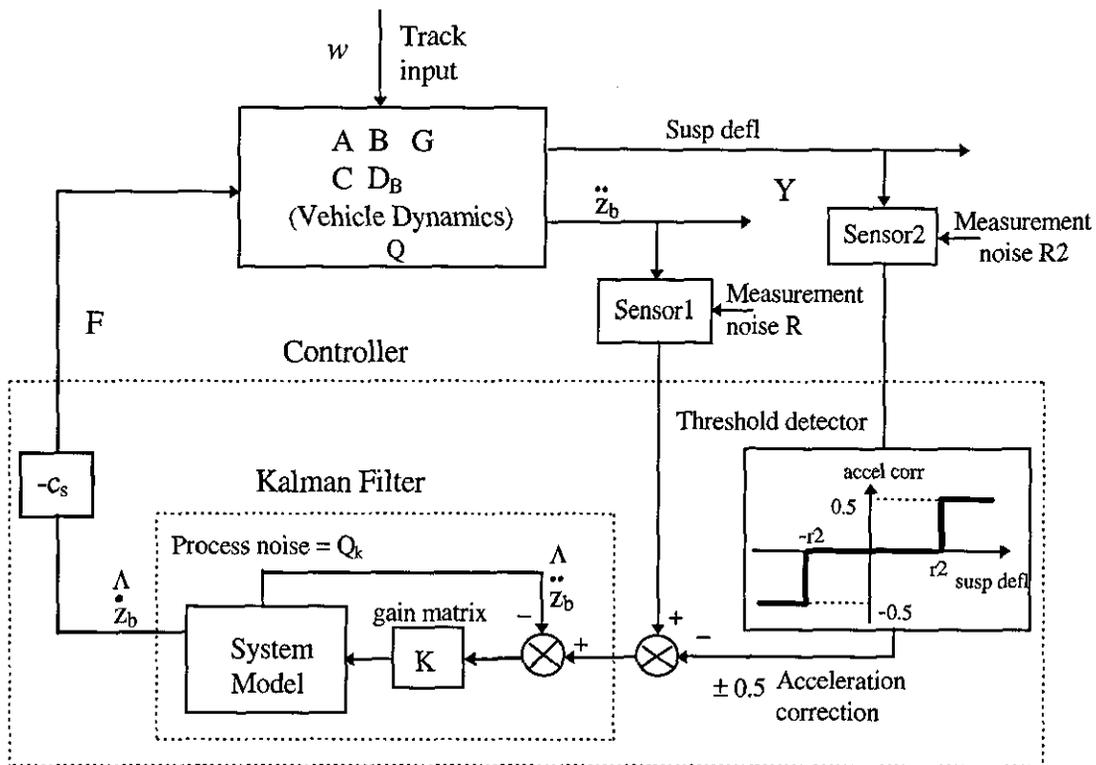
Figure 5.3   Single Kalman-Filter control structure

5.2.2 Threshold Detector

The aim of using a threshold detector is to separate the random and deterministic features. Figure 5.4 gives the suspension deflections of the linear Kalman-Bucy filter control strategy responding to the random track inputs at two different process noise values. When the Q_k is reduced from the expected value Q to $Q*10^{-4}$, there is only a slight increase in the suspension deflection compared with a much larger increase of the acceleration residuals shown in Figure 4.12. This means that for the random track inputs the suspension deflection is not very sensitive with the changing of the process noise Q_k in the Kalman-Bucy filter. Consequently, because the input signal of the threshold detector in the non-linear single Kalman Filter control is the suspension deflection, the threshold value r2 needs only to be adjusted in a small range around three times the standard deviation of the suspension deflection on the level track at $Q_k=Q$.
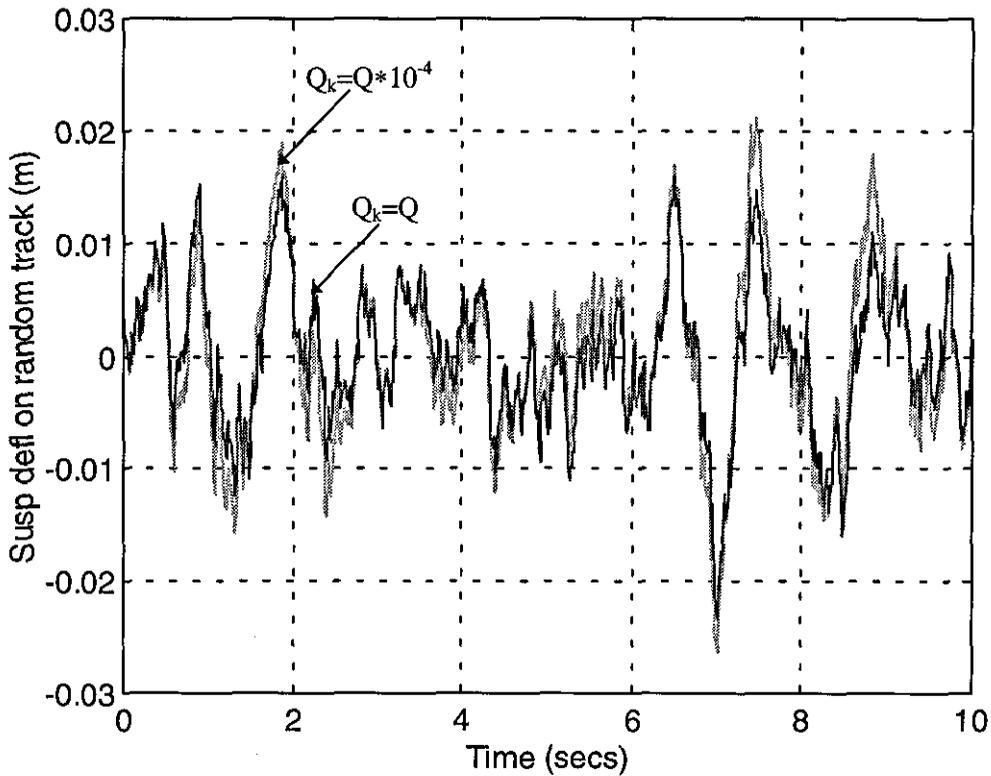
Figure 5.4   Suspension deflection on random track for linear K-BF at different Q_k

5.2.3   System Response

A trade-off curve with different Q_k values is obtained as shown in Figure 5.5, and the performance of the passive system is also given for comparison. Table 5.2 gives several points used to create this trade-off curve and their corresponding design values. It is found that when the $Q_k=Q$, a maximum 68% improvement can be achieved in the ride quality compared with the passive one, but the maximum suspension deflection is increased to 0.07 m. Reducing the Q_k will make the maximum suspension deflection smaller, but will also worsen the ride quality. Also the trade-off curve cannot reach the passive point even if the process noise is further reduced.
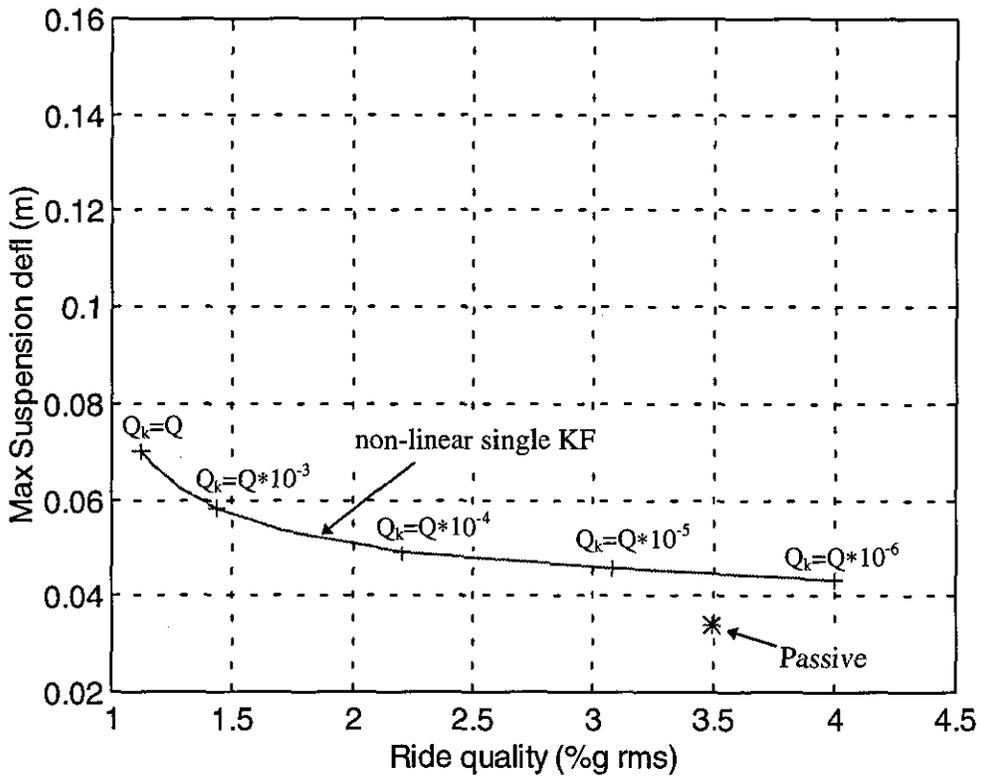
Figure 5.5   Trade-off Curve for non-linear Single Kalman Filter control

Table 5.2   Various parameters values at design points for single KF

Process noise for KF: Q_k	Threshold level: r2(m/s^2)	Ride quality (%g)	Max susp defl (m)
Q	0.024	1.1179	0.0702
$Q*10^{-3}$	0.024	1.2935	0.0622
$Q*10^{-4}$	0.026	1.6937	0.0540
$Q*10^{-5}$	0.036	3.0795	0.0457
$Q*10^{-6}$	0.04	3.997	0.0432

Figure 5.6 compares the trade-off curves for the two non-linear control methods. The results indicate that there is 14% reduction at the maximum suspension deflection in the single Kalman-Filter method compared with that of the dual Kalman-Filter control

while keeping the same ride quality, and the two curves converge at higher ride quality r.m.s. values.
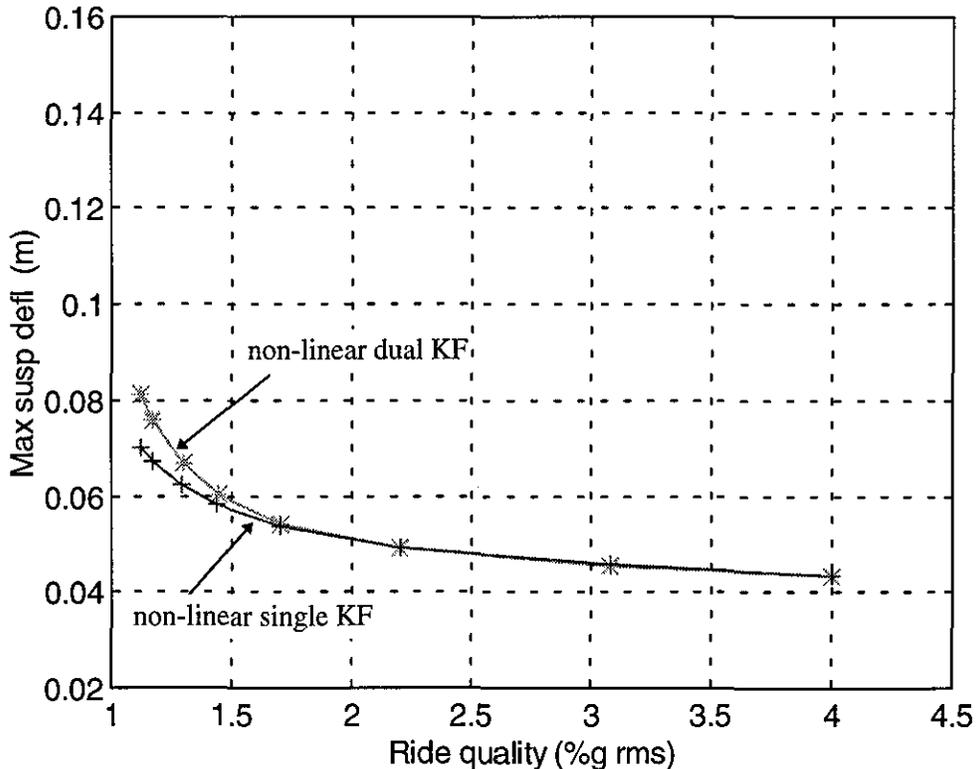
Figure 5.6   Single KF control vs. Dual KF control

5.3   Comparison of Linear and Non-linear "Trade-off" Curves

A comparison between all three linear approaches which have been discussed in Chapter 4 shows that the linear complementary filter control is the best one among the three. Also the comparison of the two non-linear methods shows that the non-linear single Kalman Filter control achieves better results. In this section, a comparison between the best linear and the best non-linear strategies is studied to examine their suspension performance.

Figure 5.7 gives the two trade-off curves corresponding to the two best control approaches, i.e. linear complementary filter control and non-linear single Kalman

filter control, and the ride quality and suspension deflection for the passive system are also given.
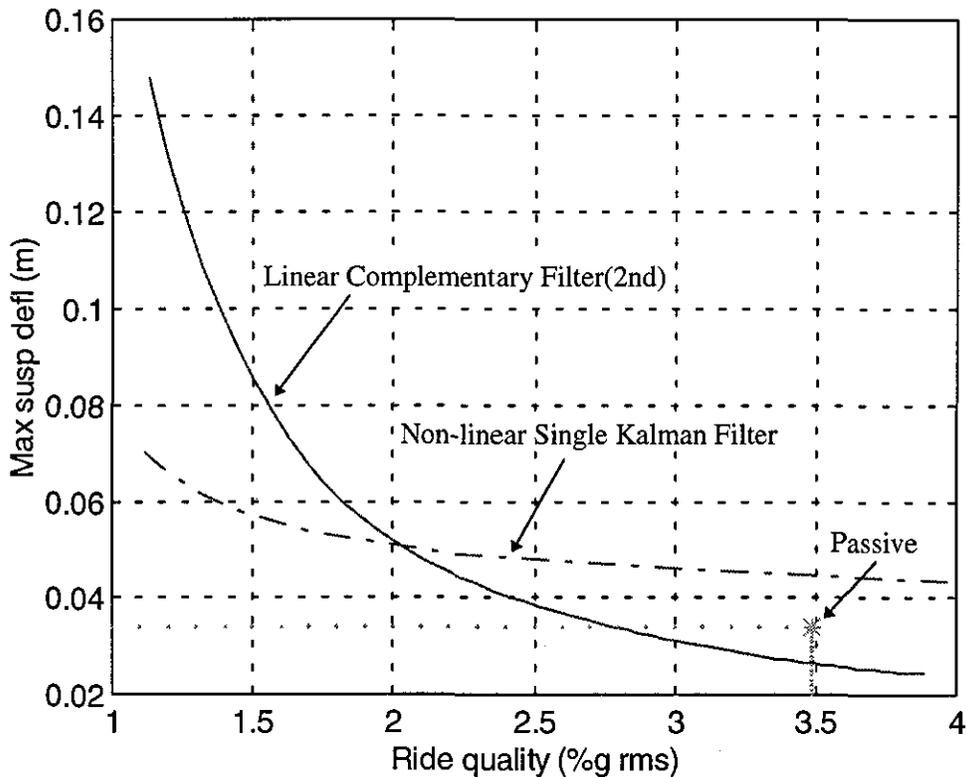
Figure 5.7   Linear CF control vs. non-linear single KF control

The prime requirement in the suspension design is the ride quality, and the purpose of modifying the linear and non-linear control strategies is to achieve an improvement in the ride quality without increasing the maximum suspension deflection. From the figure, it appears that the trade-off curve for the non-linear Single Kalman Filter control is flatter than that of the linear Complementary Filter control. When the ride quality value is better than 2 %g, a significant reduction in the maximum suspension deflections can be achieved by the non-linear approach while keeping the same ride quality as the linear complementary filter control (e.g. 54% reduction in deflection at 1.1 %g). Although the achievable suspension deflections are still higher than the passive one, the improvement in the ride quality is significant. When the ride quality is worse than 2 %g, the linear complementary filter control becomes better than the

non-linear one in reducing the maximum suspension deflection. The best improvement in the ride quality with the linear complementary filter control approach is 22.3% while maintaining the same maximum suspension deflection as the passive one.

It should be noticed that the both methods have their own advantages. For the linear complementary filter control approach, it can achieve 22.3% improvement in the ride quality while keeping the same low suspension deflection as the passive system. But for the non-linear single Kalman filter control strategy, a significant 68% improvement in the ride quality can be achieved, although the suspension deflection is larger than that for the passive case.

5.4   Effect of Under-speed Operation

All the non-linear control strategies carried out in this chapter have used a vehicle speed of 55 m/s. It would be interesting to examine the effect of those controllers when the vehicle is running below the intended speed. In this case the superimposed acceleration will be less than 0.5 m/s^2, but $\pm$0.5 m/s^2 will still be taken from the measured acceleration in the non-linear control (unless a revised speed-dependent strategy is used, but this will not be straight forward to implement). Tables 5.3 and 5.4 give the ride quality and maximum suspension deflection values of two design points for the two non-linear control strategies at three different speeds, and Table 5.5 shows the details of the same design points of the linear Kalman-Bucy filter control for comparison. It can be seen from these three tables that there is still an improvement in the maximum suspension deflection while keeping the same ride quality as the linear one even if the vehicle speed is reduced.

Table 5.3   Design points for non-linear dual KF control at different speeds

Q_k	55 m/s		45 m/s		30 m/s	
	rq	defl	rq	defl	rq	defl
$Q_{k1}= Q$ $Q_{k2}= Q*10^{-4}$	1.1186	0.0813	1.0125	0.0603	0.8273	0.0805
$Q_{k1}=Q*10^{-4}$ $Q_{k2}=Q*10^{-4}$	1.7033	0.0541	1.5893	0.0402	1.4132	0.0173

Table 5.4   Design points for non-linear single KF control

Q_k	55 m/s		45 m/s		30 m/s	
	rq	defl	rq	defl	rq	defl
$Q_k=Q$	1.1179	0.0702	1.0118	0.0440	0.8273	0.0238
$Q_k=Q*10^{-4}$	1.6937	0.0540	1.5888	0.0373	1.4132	0.0173

Table 5.5   Design points for linear K-BF control at different speeds

Q_k	55 m/s		45 m/s		30 m/s	
	rq	defl	rq	defl	rq	defl
$Q_k=Q$	1.1179	0.1532	1.0118	0.1239	0.8273	0.0805
$Q_k=Q*10^{-4}$	1.6916	0.0683	1.5888	0.0430	1.4132	0.0173

For the non-linear dual KF control, the threshold detector will stop switching between the random and deterministic track inputs when the speed is reduced to 30 m/s or lower, because the acceleration residuals caused by the two different input signals are no longer separable in the control. As a consequence the non-linear part of the system is no longer functional and the controller will work as same as the linear one. For the non-linear single KF control at $Q_k=Q$, because the threshold detector is fed directly from the measurement of the suspension deflection and the suspension deflection on the deterministic track is always larger than that on the random track, the threshold

detector will be functional even at very low vehicle speed. A significant improvement in the maximum suspension deflection can be achieved while the ride quality is kept the same as the linear K-BF control. When Q_k is reduced to $Q*10^{-4}$, the input signal in the threshold detector becomes difficult to separate when the vehicle speed is below about 30 m/s and the threshold detector will stop switching between the random and deterministic track inputs and consequently the non-linear single KF control will work as same as the linear one.

CHAPTER 6

CONCLUSIONS AND FUTURE WORK

The problem of discriminating between the random track and deterministic track inputs is a fundamental issue for the design of active secondary suspensions on railway vehicles. The difficulty comes from the use of absolute velocity ("skyhook") damping, which is excellent in terms of improving ride quality (i.e. the response to the random track inputs) but not good in terms of the large suspension deflection created when encountering deterministic track features. The aim of the study has been to achieve the best possible improvement of the system performance from an active secondary suspension on different track features.

6.1 Conclusions

This thesis has presented a comprehensive study of various linear and non-linear control approaches for a vertical secondary active suspension of railway vehicles, and has identified advantages and disadvantages of each controller developed in the study.

The thesis first compares performances of the passive system and skyhook damping system when the vehicle negotiates the random and deterministic track features. It has been shown that on straight track the skyhook damping system can halve the acceleration level transmitted onto the vehicle body and hence improve the ride quality, but the improvement cannot be sustained when the vehicle negotiates intended track features, because the strategy which gives the ride quality improvement also creates unacceptable large suspension deflection at deterministic features compared to its passive counterpart.

Secondly, three linear approaches are studied for filtering the absolute velocity signal and theoretically examined in order to optimise the trade-off between the random and deterministic input requirements. What can be achieved with linear filters is initially determined. This is quantified by the degradation in the straight track ride quality needed to restrict the maximum deflection to an acceptable level as a vehicle traverses the transition to a typical railway gradient. A range of filter types, frequencies and absolute damping rates are assessed in order to explore the boundary of what can be achieved through linear means. Trade-offs between the ride quality and deflection for all systems are examined in detail in this thesis. For the intuitive formulation control, the problem remains the same, i.e. it improves the ride quality, but the deflection is larger. The complementary filter control provides a much better solution, and its trade-off curve is below the passive at high ride qualities, i.e. it can improve the ride quality and reduce the maximum suspension deflection at the same time. For the Kalman-Bucy Filter control, the trade-off curve is very similar to the complementary filter control approach at lower ride qualities, but it is not possible to reduce the maximum suspension deflection as far as the complementary filter controller. The computer simulation results have indicated that the best performance can be achieved through the linear Complementary Filter control strategy is either a 23 % improvement in ride quality or a 25 % reduction in the maximum suspension deflection compared to the passive suspension.

Thirdly the thesis explores what can be achieved using non-linear filtering. Since the linear approaches take no advantages of any knowledge of the deterministic features, which for a railway system are very well defined for reasons of passenger comfort, two non-linear control strategies using this knowledge are developed and these are: dual Kalman Filter control and single Kalman Filter control. For the dual Kalman Filter control, one Kalman Filter (KF1) provides the state estimate for applying skyhook damping, and the acceleration residual of the second filter (KF2) is used to provide a non-linear correction. Simulation results have shown that a maximum 47%

reduction in the suspension deflection can be achieved at lower ride quality r.m.s. values compared with the linear Kalman-Bucy filter control, but there is no apparent improvement for higher ride quality r.m.s. values. For the single Kalman Filter control the non-linear acceleration correction simply comes from a threshold detector applied to the suspension deflection using an additional transducer, and the results have indicated that a slight improvement can be obtained in the maximum suspension deflection compared with the dual Kalman Filter control method. One of the obvious advantages of the Single Kalman Filter is the simplified control algorithm, however it does require an additional feedback sensor.

Finally a comparison between the linear and non-linear approaches is given. It has been shown that the Non-linear Single Kalman Filter control offers the best improvement in the ride quality whilst the Linear Complementary Filter control gives a more balanced performance between the ride quality and the maximum suspension deflection. With the linear complementary filter control approach a 23 % improvement in the ride quality can be achieved while keeping the same maximum suspension deflection as the passive system, or a 25 % reduction in the maximum suspension deflection if the same ride quality is maintained. If the designer is more interested in improving the ride quality, a significant 68 % improvement can be achieved by using the non-linear single Kalman-Filter control method, although the maximum suspension deflection will be somewhat worse than the passive one.

It had been expected that the increased system knowledge incorporated within the Kalman Filter would give a better system solution, even for the linear control strategies, and in general it is rather surprising that the greater sophistication of the Kalman Filter based methods does not give more wide-ranging improvements compared with the more classically based complementary filter approach.

6.2   Future Work

Further research would be to implement the control methods, which have been developed in the study, in order to fully explore their practical potentials. It is thought at this stage that both the linear complementary filter control and the non-linear Kalman Filter control approaches can be implemented and their performance compared in the "real" world. It would be particularly important to assess the robustness of the approaches developed, because mismatches between the Kalman Filter models and the system model have not been studied, although it is not believed that this will be a critical problem.

The basic non-linear concepts should also be applied to an active lateral suspension. The deterministic inputs in this case are quite different from those for a vertical suspension, because curves represent a sustained acceleration input, and the transition to the curve is designed to give a constant level of jerk. It is believed that the concepts will also be effective, but clearly the techniques will need to be modified to accommodate the very different input characteristics. Other non-linear techniques for use in the active railway suspension to further improve the suspension performance could be studied, and possible techniques are recommended as follows:-

- a heuristically derived filter using fuzzy logic designed to retain "pure" absolute damping on straight track but restrict the suspension deflection at the transition to the gradient

- using a set of rules which attempt to distinguish between straight track and the gradient, developed off-line using rule induction techniques with representative track data.

6.3   Original Contributions of the Research

The work described in the thesis has extended the knowledge of active suspension systems in two ways. Firstly it has carried out a comprehensive assessment of <u>linear</u> control approaches for practical implementations of skyhook damping, something which has not been carried out in this level of detail previously. Secondly it has indicated some of the improvements which can be achieved through <u>non-linear</u> control approaches, a feature which has not apparently been studied elsewhere and is therefore wholly original.

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APPENDIX I

Copy of IAVSD'97 paper:

DISTINGUISHING BETWEEN RANDOM AND DETERMINISTIC TRACK INPUTS FOR ACTIVE RAILWAY

Hong Li and Roger Goodall

Published in 15th Symposium of Dynamics of Vehicles on Roads and Tracks (IAVSD), Aug. 1997, Hungary.

DISTINGUISHING BETWEEN RANDOM AND DETERMINISTIC TRACK INPUTS FOR ACTIVE RAILWAY SUSPENSIONS

Hong LI and Roger GOODALL
Department of Electronic and Electrical Engineering
Loughborough University
Loughborough, Leics. LE11 3TU, UK

Abstract This paper presents and compares different control strategies for applying absolute or "skyhook" damping in active suspension systems for railway vehicles. It first examines a number of linear approaches for filtering the absolute velocity signal in order to optimise the trade-off between the random and deterministic track input requirements, and then what can be achieved using non-linear Kalman-Filter methods is explored. Studies in this paper show that an improvement of nearly 20% in ride quality can be achieved with a linear complementary filter control and over 50% by using non-linear Kalman filter methods.

1 INTRODUCTION

Railway vehicles are fitted with a suspension to support the vehicles' weight, to follow the low frequency features, and to isolate them from the high frequency irregularities of the surface over which they are travelling, thus minimising the acceleration experienced in the vehicle. It has been generally accepted that active-controlled suspensions are able to offer improvements beyond what is possible passively, and consequently the use of active elements in railway suspension systems is on the development agenda for a number of railway equipment manufactures around the world. Many studies have been undertaken in connection with active suspensions for rail and road vehicles [1], and some of these have addressed the fundamental improvements which are possible through the use of active elements [2,3].

This paper first examines theoretically a number of linear approaches for applying absolute velocity or "skyhook" damping in order to optimise the trade-off between the random and deterministic input requirements, and the analysis determines what can be achieved with linear filters. This trade-off is quantified by the degradation in the straight track ride quality needed to restrict the maximum deflection to an acceptable level as a vehicle traverses the transition to a typical railway gradient, and a range of filter types, frequencies and absolute damping rates are assessed in order to explore the boundary of what can be achieved through linear means. Then, some non-linear filtering strategies are investigated to further improve the suspension performance.

2 PROBLEM FOUNDATION

The primary motivation for active secondary suspension on railway vehicles is to improve suspension performance, and thereby run faster or provide a better ride quality. It is well accepted that the use of "skyhook" damping is a key feature of the active control law; e.g. on straight track this can deliver a 50% reduction in r.m.s. acceleration on the body of the vehicle even without reducing the spring rate of the suspension. However, skyhook damping can increase quite radically the suspension deflection which occurs when the vehicle negotiates a deterministic track feature [4,5], and it becomes necessary to modify the control

law in some manner, for example by filtering the absolute velocity to remove the low frequency variations associated with the deterministic input.

In this paper, a typical railway gradient of 1% is assumed with a superimposed acceleration limit of 0.5 ms^{-2} (5%g) and at a typical top speed of 55 m/s this corresponds to a 1.1 second transitional section. For a typical passive suspension system of a modern high speed (55 m/s) passenger vehicle [6], the r.m.s. body acceleration in straight track is 3.5%g based upon a mainline track quality with a power spectrum approximated by $2.5 \times 10^{-7}/f_t^2$ [5], and the maximum suspension deflection when traversing the transition which was defined previously is 34 mm.

3 LINEAR APPROACHES

Linear systems are in general simpler in structure and easier to design and implement compared with their non-linear counterparts. Three linear control strategies for the active suspensions are studied in this paper: intuitive formulation, complementary filter control, and Kalman-Bucy filtering control.

3.1   Intuitive formulation

An intuitive formulation of a control structure based upon integrating a measurement of acceleration is given in Figure 1, in which c_s is the skyhook damping rate.
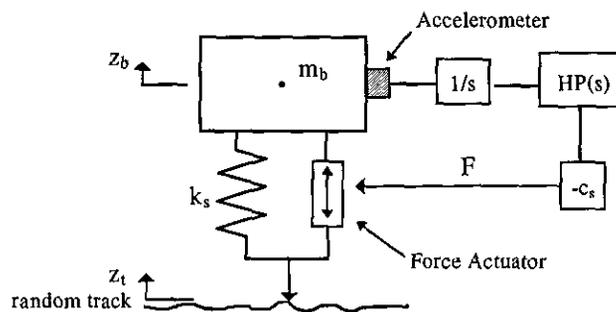
Figure 1   Intuitive Formulation Control Configuration

The high pass filter HP(s) is used to eliminate long term drift in the integrator, and in practice will be combined with it. Equation (1) gives the combined transfer function based upon second order Butterworth characteristics for the high pass filter, in which the cut-off frequency ω_i can be varied from a very low frequency. Computer simulations show that when the cut-off frequency is increased above a certain value, depending upon the order of the filter (0.49Hz for 2nd order filter), the system becomes unstable.

$$\frac{1}{s} * HP(s) = \frac{\dfrac{1}{\omega_i^2} s}{1 + \dfrac{\sqrt{2}}{\omega_i} s + \dfrac{1}{\omega_i^2} s^2} \tag{1}$$

3.2   Complementary filter

The instability problem with the previous strategy can be avoided by a "complementary filter" approach, in which the high-pass filtered absolute damping is replaced by normal "relative" damping at low frequencies [7] (see Figure 2), and if the pair of filters are

complementary (i.e. add up to unity) then it can easily be shown that stability is not affected no matter what value of ω_i is chosen.
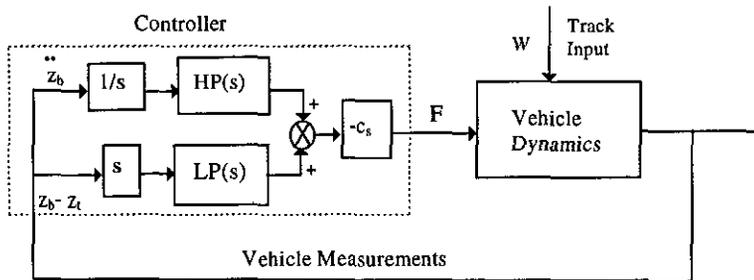
Figure 2   Complementary Filter Control

A trade-off curve between maximum suspension deflection and ride quality obtained with this approach is shown in Figure 3. Compared with the passive suspension, the ride quality can be improved by almost 20 % when the deflection is kept the same as for the passive case, or the deflection can be reduced by about 22 % if the same ride quality is maintained.
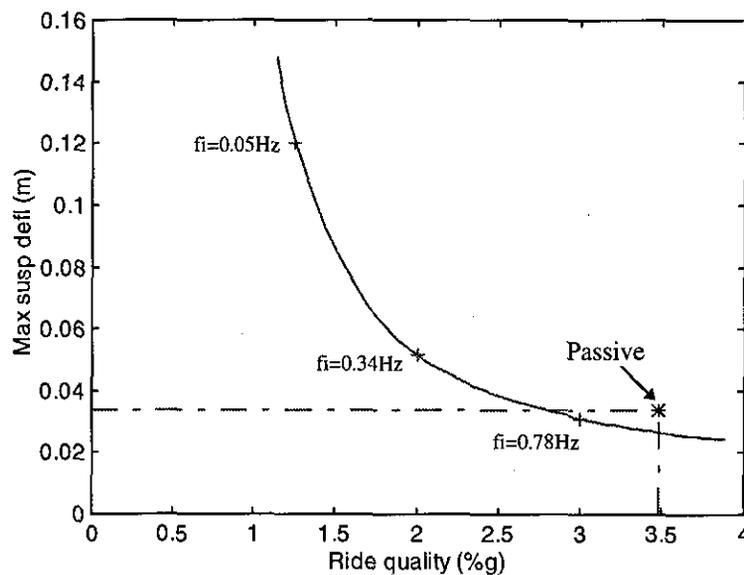
Figure 3   Trade-off Curve for Complementary Filter Control

3.3   Kalman-Bucy filtering

An alternative to the intuitive strategies is to use a Kalman-Bucy filter, which enables a more rigorous estimation to be made on the basis of the system model and values for the process and measurement noise [8]. The body velocity is one of the state estimates, and is processed directly by c_s to give the control force F. Figure 4 gives the block diagram of this control approach.
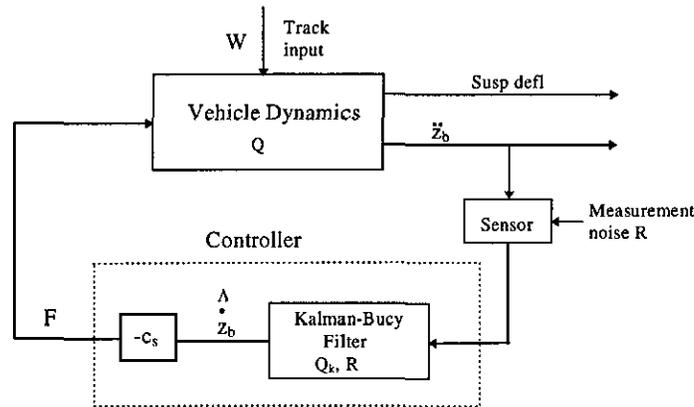
Figure 4   Kalman-Bucy Filter Control

A trade-off curve for this control structure is obtained as shown in Figure 5 by adjusting the covariance of the process noise in the Kalman-Bucy filter (Q_k). The measurement noise R was fixed at a value corresponding to around 1 % of the acceleration signal's expected maximum value. The process noise Q_k was initially set equal to its expected value Q (calculated from the track power spectrum), and varied (downwards) from that value. It is apparent that when Q_k equals Q, the Kalman-Bucy filter estimates the random track very well and achieves a good ride quality, but for the deterministic feature it creates a very large suspension deflection. Decreasing Q_k reduces the maximum suspension deflection, but the ride quality is worsened. It can be seen that initially the curve is very similar to the Complementary Filter approach, but it is not possible to reduce the maximum suspension deflection as far.
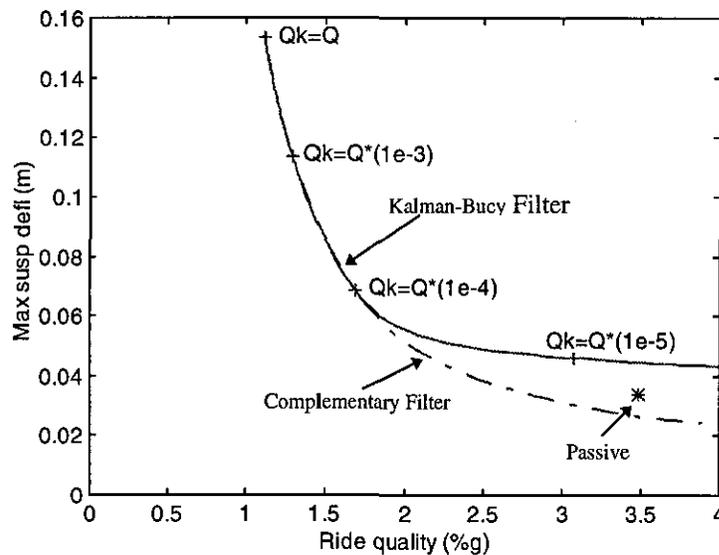
Figure 5   *Trade-off Curve for Kalman-Bucy Filter Control*

4 NON-LINEAR STRATEGIES

The linear approaches take no advantage of any knowledge of the deterministic features, which for a railway system are very well defined for reasons of passenger comfort, and two non-linear approaches using this knowledge are investigated in the studies: dual Kalman-Filter control and single Kalman-Filter control.

4.1 Dual Kalman-Filter method

A block diagram of this strategy is shown in Figure 6, in which the residual for one Kalman Filter (KF1) is used to provide a non-linear correction, and the other (KF2) provides the state estimate for applying skyhook damping. On a transition to a gradient the residual increases beyond the normal levels encountered on level track, and if it exceeds a threshold value then ± 0.5 ms^{-2} is taken from the measured acceleration going to KF2. For KF2 the process noise Q_{k2} is varied from the calculated value Q, which is known to give good ride quality, downwards. For each value of Q_{k2}, the process noise Q_{k1} and threshold level (r1) are both adjusted to get the smallest suspension deflection while keeping the same ride quality as the linear K-B Filter approach.

The trade-off curve in Figure 7 shows that significant reduction in suspension deflection can be achieved at lower ride qualities (e.g. 47% at 1.1%g), but the linear and non-linear curves can be seen to converge for higher ride qualities and can not achieve as good a trade-off as the Complementary Filter method.

4.2 Single Kalman-Filter method

A simpler method using a single Kalman-filter is shown in Figure 8. In this case the acceleration correction of ± 0.5 ms^{-2} simply comes from a threshold detector applied to the suspension deflection, with the threshold again being adjusted to get the smallest suspension deflection. A trade-off curve with varying Q_k is obtained as shown in Figure 9, and it was found that a slight improvement could be obtained compared with the dual Kalman Filter method, but again not able to compete with the Complementary Filter approach to achieve low deflection.
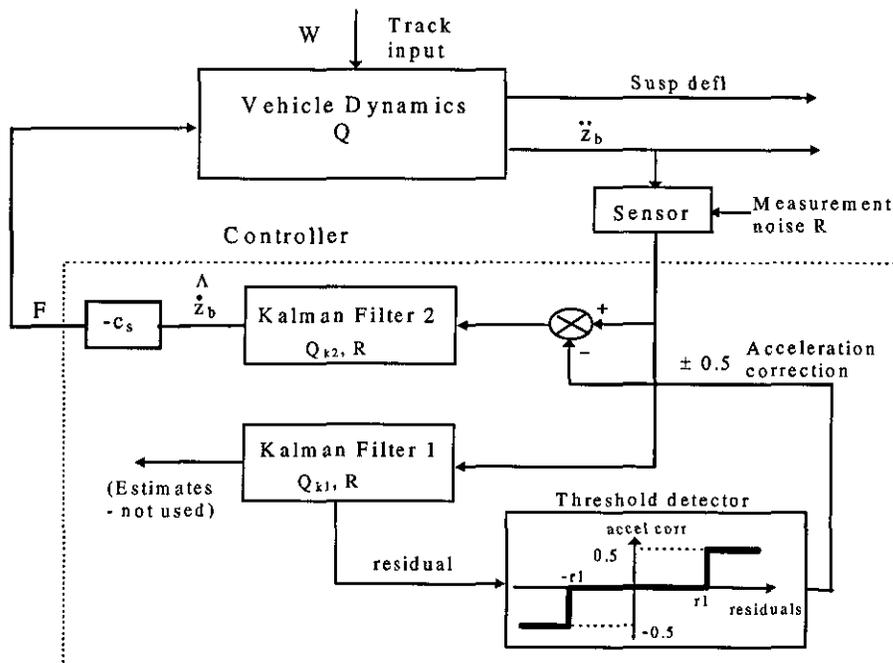
Figure 6   Non-linear Dual Kalman Filter Control
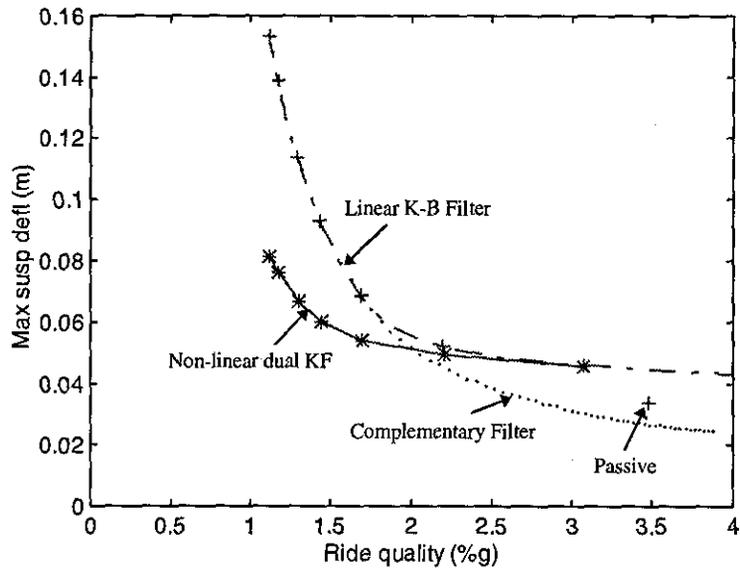
Figure 7   Non-linear Dual K-F Trade-off Curve
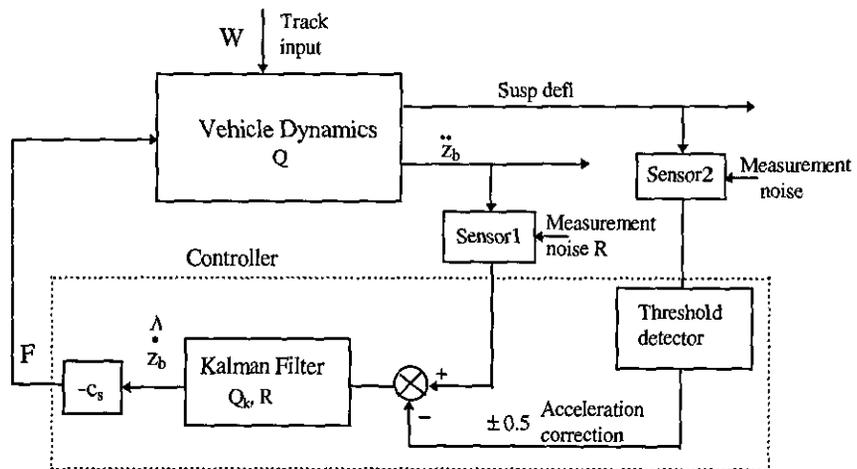
Figure 8   Non-linear Single Kalman Filter Control
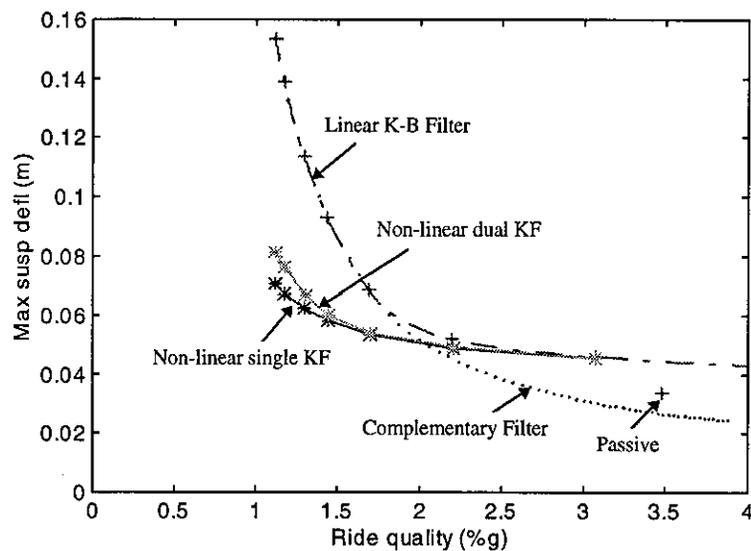
Figure 9 Trade-off Curve for Non-linear Single K-F

5 CONCLUSION

The paper has described a comprehensive study of three linear approaches and two non-linear strategies for a vertical secondary suspension of railway vehicles. The results indicate an improvement in ride quality of nearly 20% with an linear complementary filter control while keeping the same maximum deflection as a passive system. By using the non-linear Kalman-Filter methods a significant improvement in ride quality (over 50%) can be achieved, although only for suspension deflections rather larger than for the passive case. The models developed in this paper represent active vertical railway suspensions, but the techniques are also relevant to lateral railway suspensions.

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APPENDIX   II

Copy of ICSE'97 paper:

PRACTICAL IMPLEMENTATION OF SKYHOOK DAMPING FOR ACTIVE RAILWAY SUSPENSIONS

Hong Li and Roger Goodall

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PRACTICAL IMPLEMENTATIONS OF SKYHOOK DAMPING FOR ACTIVE RAILWAY SUSPENSIONS

Hong Li and Roger Goodall

Department of Electronic and Electrical Engineering
Loughborough University
Loughborough, Leics. LE11 3TU
Tel: 01509 228106
Fax: 01509 222854
Email: H.Li@lboro.ac.uk

Abstract

This paper addresses the design of control laws for active railway suspensions. It presents and compares three linear approaches for filtering the absolute velocity signal in order to provide a practical implementation of the concept "skyhook" damping, in particular to optimise the trade-off between the random and deterministic track inputs on a vertical secondary suspension. Studies in this paper show that an improvement of nearly 23% in ride quality can be achieved with a linear complementary filter control method.

1 Introduction

Active-controlled railway suspension systems offer a great potential for improvements in ride quality, and consequently the use of active elements in railway suspension systems is on the development agenda for a number of railway equipment manufacturers around the world. Many studies have been undertaken in connection with active suspensions for rail vehicles [1], and the fundamental improvements which are possible through the use of active elements has been recognised for many years [2,3].

It is well accepted that the use of "skyhook" damping (also known as absolute velocity damping) is a key feature of the active control law; e.g. on straight track this can deliver a 50% reduction in r.m.s. acceleration on the body of the vehicle even without reducing the spring rate of the suspension.

Unfortunately these improvements cannot be sustained when the vehicle also negotiates a deterministic track feature, because the strategy which gives the ride quality improvements also creates unacceptably large suspension deflections at deterministic features [4,5], and it is necessary to modify the control law to reduce the deflection, but at the expense of the ride quality.

This paper studies three linear control strategies for filtering the absolute velocity signal in order to implement the "skyhook" damping and to optimise the trade-off between the random and deterministic input requirements. This trade-off is quantified by the degradation in the straight track ride quality needed to restrict the maximum suspension deflection to an acceptable level as a vehicle traverses the transition to a typical railway gradient. A range of filter types, frequencies and absolute damping rates are assessed in order to explore the boundary of what can be achieved through the control methods developed.

2 Suspension modelling

All railway vehicles are fitted with a suspension. The main tasks of such a suspension are: (1) to support the vehicle's weight and follow the track; (2) to isolate the vehicle body from the disturbances such as track irregularities and other external forces. A railway vehicle normally has two distinct suspensions: the primary suspension between the wheel and the bogie, and the secondary suspension between the bogie and the body of the vehicle; in each case both the lateral and vertical directions have suspension components. This paper deals with full-active control applied to the vertical secondary suspension. The basic techniques

apply equally to a lateral secondary suspension, although the inputs in the lateral direction are substantially different and would therefore give different trade-off characteristics.

Figures 1 and 2 give the single wheel model of passive and skyhook damping systems which represent one degree of freedom of motion and may be thought of as a unicycle model. In the passive suspension system (Figure 1) the damper is fitted between the vehicle and the track, in this case with a spring in series as is common with railway suspensions. With an active suspension system it is possible to realise the "skyhook" damping system (Figure 2) in which the damper is effectively attached to an absolute reference.
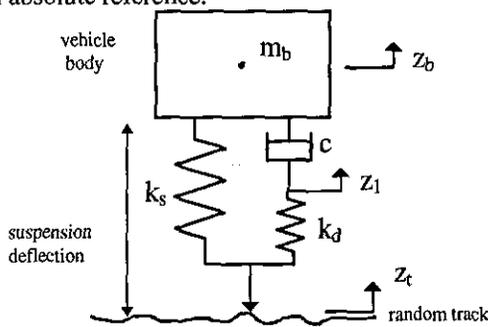
Figure 1. Passive suspension system
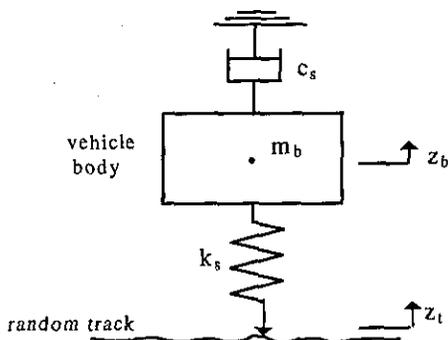
Figure 2. Skyhook damping system

Table 1 gives the numerical data which have been used in the calculations, these being typical values for a modern railway vehicle.

Symbol	Value	Parameters
m_b	30000 (kg)	Vehicle body mass
k_s	700000(N/m)	Spring stiffness
k_d	7000000 (N/m)	Spring stiffness
c	50000 (N/ms^{-1})	Passive damping
c_s	190000(N/ms^{-1})	Skyhook damping

Table 1. Complete vehicle model parameters

Table 2 and 3 give the eigenvalues with their natural frequency and damping factor of the passive and "skyhook" damping systems respectively. Notice that the passive suspension is third order, the consequence of having the spring in series with the damper; also that the damping is remarkably low, but this highlights the design conflict for passive systems: too low a damping value and the resonance is uncontrolled; too high and the high frequency transmission through the damper degrades the ride quality. However, for the active system using skyhook damping high levels can be introduced without degrading the ride quality.

Eigenvalue	Damping(%)	Freq. (Hz)
-0.84 +4.79i	17	0.77
-0.84 - 4.79i	17	0.77
-138.32	100	22.01

Table 2. Eigenvalues for the passive system

Eigenvalue	Damping(%)	Freq. (Hz)
-3.17+3.65i	65.6	0.77
-3.17- 3.65i	65.6	0.77

Table 3. Eigenvalues for the "skyhook" damping system

3 Suspension assessment

Secondary suspension design, whether passive or active, is a conflict between following the gradient effectively and isolating from the random high frequency track irregularities, and it is therefore necessary to quantify both these characteristics. The first and primary requirement for the suspension design is the quality of ride which it must deliver. This is a criterion of its effectiveness in providing isolation from the track roughness, and is quantified by the r.m.s. acceleration (usually frequency-weighted to allow for human susceptibility, although this is not incorporated in this study). The second is the suspension deflection when the vehicle negotiates intended track inputs, which must be less than the working space available, typically ±30-40 mm depending upon the particular design.

In this paper, a typical railway gradient of 1% is assumed with a superimposed acceleration limit of 0.5 ms^{-2} (5%g), and at a typical top speed of 55 m/s this

corresponds to a 1.1 second transitional section, - these values are likely represent the most severe deterministic input for a high-speed vehicle. The acceleration limit is chosen to ensure passenger comfort, and is used by the civil engineers in conjunction with the intended train speed to calculate the "design alignment" of the track.

The random inputs can be approximated by a power spectrum for the track position given by A_r/f_t^2 m^2/cycle m $^{-1}$, in which f_t is a spatial frequency (this can be converted to a temporal frequency using the train speed). A_r is a track roughness factor, commonly given a value of $2.5{\times}10^{-7}$ for mainline track [5].

Figure 3 shows a comparison between the acceleration power spectrum density (P.S.D.) for a typical passive and a "skyhook" damping suspension systems of a modern high speed (55 m/s) passenger vehicle [6], in which absolute damping of 190 kN/ms^{-1} has been applied to give 70% damping in the active case, as indicated in Table 3. The very large reduction in the peak around 1 Hz is clearly seen, and the r.m.s. body acceleration is reduced from 3.5 %g for the passive system to 1.1 %g for the "skyhook" damping system. Figure 4 gives the maximum suspension deflection for the two systems when entering the deterministic track feature. The maximum suspension deflection when traversing the transition is 34 mm for the passive and 156 mm for the "skyhook" system.

It can be seen very clearly from the two figures that the "skyhook" damping system make a significant improvement in the ride quality on straight track compared with the passive one, but the maximum suspension deflection is also increased radically and unacceptably when it encounters the deterministic track feature. Also the "skyhook" damping is assumed to be attached to an absolute reference and for vehicles no such connection is possible in reality. Therefore, three linear control strategies will be studied to implement the "skyhook" damping system in practice and to reduce the deflection at the same time. These are an intuitive formulation control, complementary filter control and Kalman-Bucy filtering control.
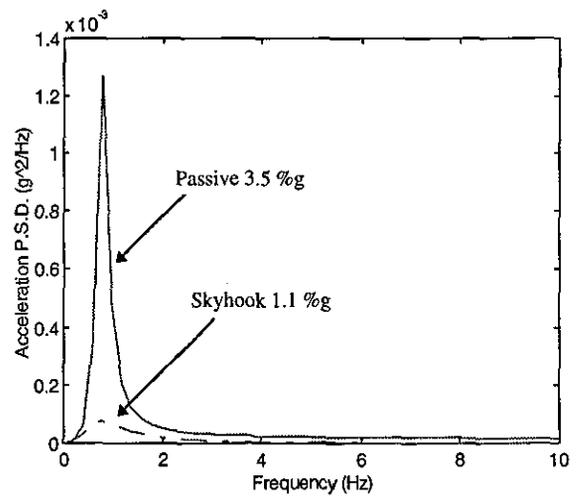
Figure 3. Response of passive and skyhook suspensions to the random input
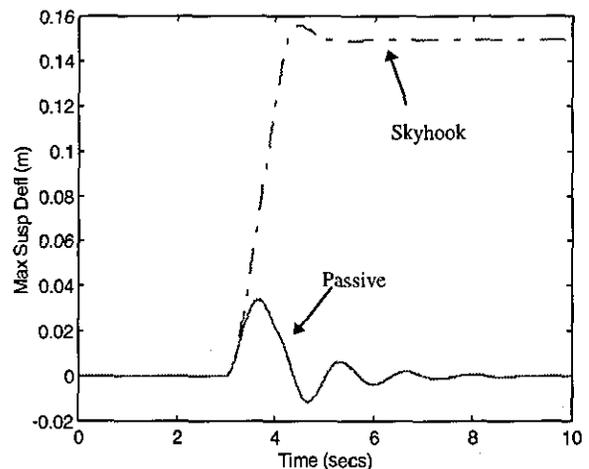
Figure 4. Response to the deterministic input

4 Intuitive formulation control

An intuitive formulation control approach based upon integrating a measurement of acceleration is presented in this section to provide a practical implementation of the "skyhook" damping.

4.1 System scheme

Figure 5 gives the structure of this method, in which c_s is the skyhook damping rate. It is important to realise that the absolute velocity of the vehicle is practically very difficult to measure directly, and must therefore be derived from a measurement of acceleration, which is fed through an integrator in

order to derive the absolute velocity. The absolute velocity is amplified by the "skyhook" damping (c_s) and then used to modulate an active force actuator (in this study assumed to be ideal).
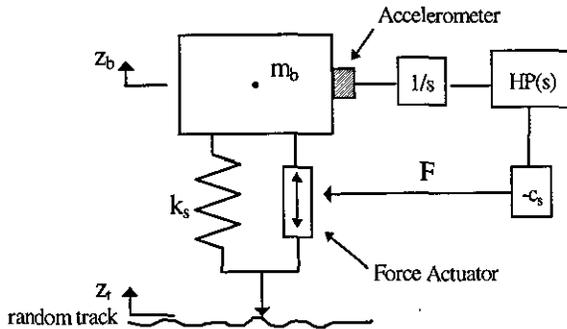
Figure 5. Intuitive formulation control configuration

The high pass filter HP(s) which is inserted between the "skyhook" damping (c_s) and the integrator (1/s) is used both to eliminate long term drift in the integrator and to reduce the size of the deterministic deflection. In practice these two are combined to give a single transfer function. In this study, first, second and third-order high pass filters HP(s) are considered, and Table 4 gives their combined transfer functions.

	Combined transfer function **(1/s x HP(s))**
1st-order	$$H_1(s) = \dfrac{\dfrac{1}{\omega_i}}{1 + \dfrac{1}{\omega_i}s}$$
2nd-order	$$H_2(s) = \dfrac{\dfrac{1}{\omega_i^2}s}{1 + \dfrac{2\xi_i}{\omega_i}s + \dfrac{1}{\omega_i^2}s^2}$$ (where ξ_i=0.7)
3rd-order	$$H_3(s) = \dfrac{\dfrac{1}{\omega_i^3}s^2}{1 + \dfrac{2}{\omega_i}s + \dfrac{2}{\omega_i^2}s^2 + \dfrac{1}{\omega_i^3}s^3}$$

Table 4. Filter transfer functions

In the table, ω_i is the cut-off frequency of the high pass filter. Standard Butterworth filter characteristics have been used in each case.

4.2 System response

The "skyhook" damper (c_s) should always be adjusted to give a damping ratio ξ=0.707 such that the best transient response can be achieved [7]. A trade-off curve between maximum suspension deflection and ride quality with varying cut-off frequency is obtained as shown in Figure 6.
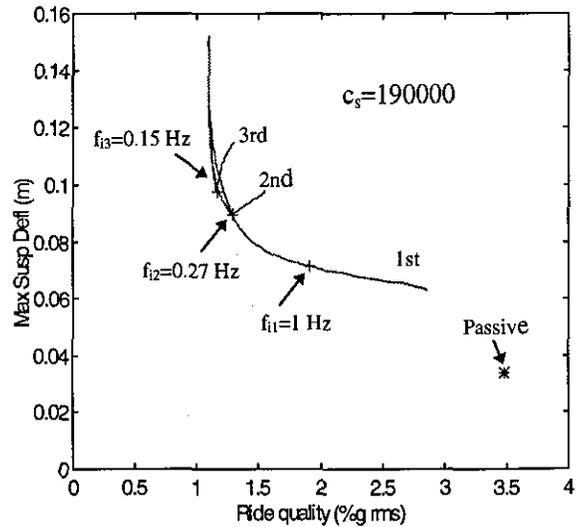
Figure 6. Trade-off curve for intuitive formulation control

The figure shows that a significant improvement in the ride quality can be achieved with the intuitive formulation control strategy on the random track input. However the strategy, although showing some improvement, still creates unacceptably large suspension deflections at the deterministic features. For a particular ride quality, it can be seen that the *suspension deflection becomes smaller by increasing* the order of the filter, although further increase is unlikely to give much more benefit and the first- or the second-order is normally chosen in practice. *Further increasing the filter cut-off frequency ω_i does* not give any more benefit either as the ride quality and maximum suspension deflection both become worse and the system may become unstable.

5 Complementary filter control

A new control strategy is developed in this section to overcome the instability problem with the previous strategy, and to allow the low frequency damping to be added to the system without detriment to the high frequency response. This method is termed a "complementary filter" approach.

5.1 Control scheme

A block diagram of the strategy is found in Figure 7, in which the high-pass filtered absolute damping is replaced by normal "relative" damping (i.e. a force proportional to the velocity across the suspension) at low frequencies [8], and if the pair of filters is complementary (i.e. add up to unity) then it can be easily shown that the stability is not affected no matter what value of ω_i is chosen.
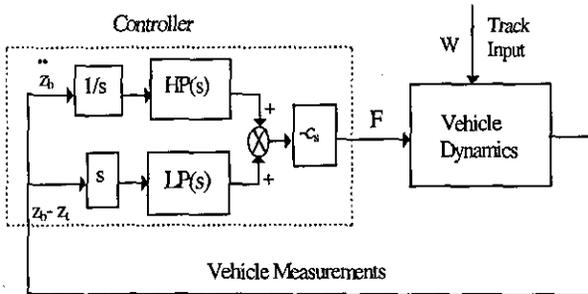
Figure 7. Complementary filter control

In effect the filter gives normal (relative) damping at low frequencies and absolute damping at high frequencies. The improvement in the suspension deflection compared with the intuitive formulation control is achieved through the low pass filter. The low pass filter LP(s) can be easily derived from the high pass filter transfer function used in previous section:

$$LP(s) = 1 - HP(s) \qquad (1)$$

Normally it will be suspension deflection rather than velocity which is measured; this necessitates a differentiator which in practice will be combined with LP(s). Substituting the first-, second- and third-order high pass filter transfer functions into equation (1), three combined transfer functions are obtained as shown in Table 5, where ω_i is the cut-off frequency as before.

	Combined transfer function (s x LP(s))
1st-order	$LP_i(s) = \dfrac{s}{1 + \dfrac{1}{\omega_i}s}$
2nd-order	$LP_2(s) = \dfrac{s + \dfrac{2\xi_i}{\omega_i}s^2}{1 + \dfrac{2\xi_i}{\omega_i}s + \dfrac{1}{\omega_i^2}s^2}$ (where $\xi_i=0.7$)

3rd-order	$LP_3(s) = \dfrac{s + \dfrac{2}{\omega_i}s^2 + \dfrac{2}{\omega_i^2}s^3}{1 + \dfrac{2}{\omega_i}s + \dfrac{2}{\omega_i^2}s^2 + \dfrac{1}{\omega_i^3}s^3}$

Table 5. Filter transfer functions

5.2 System response

A trade-off curve between the maximum suspension deflection and the ride quality obtained with this approach is shown in Figure 8. It is apparent that the system with the first, second or third-order filter can improve both the ride quality and the maximum deflection, and in fact the order of the filter is not a dominant factor in the trade-off. Compared with the passive suspension, the improvement in the ride quality is 23% when the deflection is kept the same as for the passive case, or the deflection can be reduced by about 25% if the same ride quality is maintained.
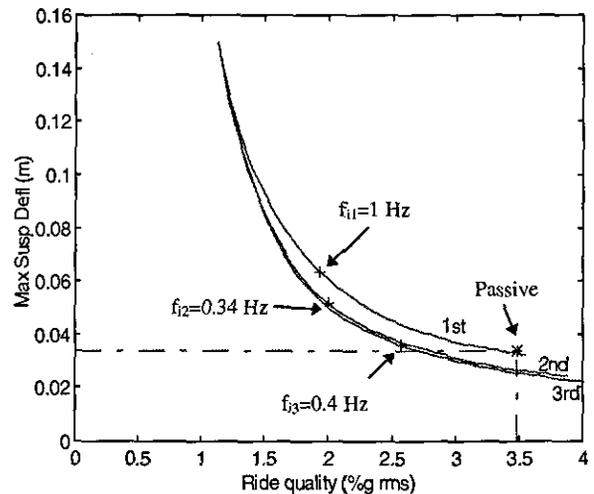
Figure 8. Trade-off Curve for Complementary Filter Control

6 Kalman-Bucy filtering

The aim of using the intuitive formulation and complementary filter control is to derive the vertical velocity of the vehicle body from the acceleration signals which can be easily measured through the accelerometer and to implement the "skyhook" damping. An alternative to the above strategies is to use a Kalman-Bucy filter, which enables a more rigorous estimation to be made on the basis of the system model and values for the process and measurement noise [9].

The body velocity is one of the state estimates, and is processed directly by c_s to give the control force F. The Kalman-Bucy filter is derived from the ideal suspension model and its input signal is the measured acceleration. Figure 9 shows the block diagram of this control approach.
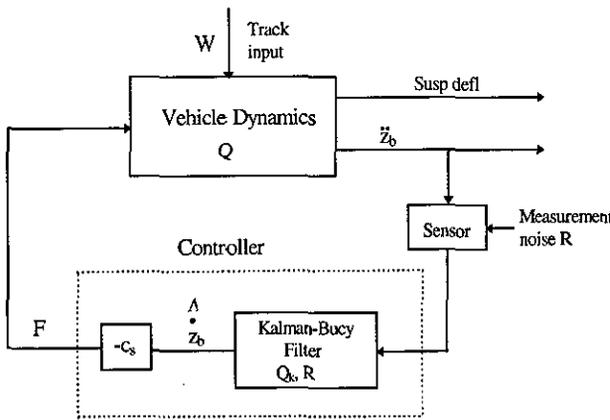
Figure 9. Kalman-Bucy Filter Control

A trade-off curve for this control structure is obtained as shown in Figure 10 by adjusting the covariance of the process noise in the Kalman-Bucy filter (Q_k). The measurement noise R was fixed at a value corresponding to around 1 % of the acceleration signal's expected maximum value. The process noise Q_k was initially set equal to its expected value Q (calculated from the track power spectrum), and varied downwards from that value. The trade-off curve for the complementary filter control and the ride quality and maximum deflection of the passive system are also shown in the figure for comparison.
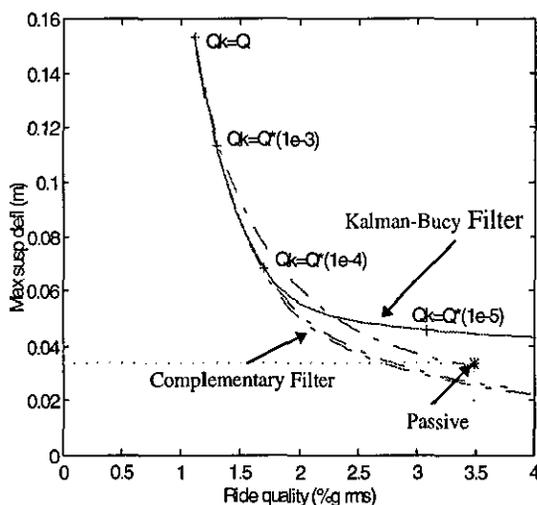
Figure 10. Trade-off Curve for Kalman-Bucy Filter Control

From the figure, it shows that when Q_k equals Q, the Kalman-Bucy filter estimates the random track very well and achieves a good ride quality, but for the deterministic feature it creates a very large suspension deflection, essentially the same result as for perfect skyhook damping. Decreasing Q_k reduces the maximum suspension deflection, but this is only achieved by compromising the ride quality. It also can be seen that initially the curve is very similar to the Complementary Filter approach, but it is not possible to reduce the maximum suspension deflection as far.

Other options using the measured suspension deflection or both acceleration and suspension deflection measurements as the input signal of the Kalman-Bucy filter have also been studied, but the simulation results have shown that the trade-off curves are very similar, and no further improvements can be achieved.

7 Conclusion

The paper has described a comprehensive study of three linear approaches for a vertical secondary suspension of railway vehicles. The obtained results have demonstrated the overall design trade-off which is possible between the ride quality and suspension deflection. It has also been indicated that an improvement of nearly 23% in the ride quality can be achieved with the linear complementary filter control strategy while keeping the same maximum deflection as a passive suspension when the vehicle runs onto a typical worst-case railway gradient. It had been expected that the greater design rigour and higher complexity of the Kalman filter approach would give benefits compared with the intuitively-developed formulation, but surprisingly this was not the case.

This paper has concentrated upon linear control laws, but the research has also been extended to consider non-linear control law formulations [10] which have been shown to yield further improvements in the trade-off characteristics, and the Kalman filter approach has formed the basis for developing these non-linear approaches.

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APPENDIX   III

System Models (Matlab™ Program)

Program 1: Vehicle secondary suspension model

```
function  [a,b,c,d,g,h] = moda1
%  FUNCTION [a,b,c,d,g,h] = moda1;
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%       Creat state-space of a 1 mass active secondary suspension vehicle model. %
%       (using relative position in state)                              %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%       Model Description:
%       input:                      states:   -    -
%                 w=[zt']                     Xv= | zb' |
%                                                 | d02|
%                 F=[-cs*xhat]                   -    -
%
%       output: -    -           measurements noise: -   -
%            Y= | zb" |                          V= | va |
%               |zt-zb|                             | vd |
%               -    -                             -    -
%   where    d02=zt-zb

%*** Vehicle suspension parameters ***
ks=700000;                  %spring factor
m=30000;                    %mass of the vehicle body[kg]
cs=190000;                  %skyhook damping

% *** State-space ***
a=[0  ks/m
   -1  0];
b=[1/m 0]';
g=[0  1]';
c=[0  ks/m
   0  1];
d=[1/m 0]';
h=[1  0
   0  1];
```

Program 2: Kalman-Bucy Filter system model

```
function  [ak,gk,ck] = modk1
%  FUNCTION [ak, gk,ck,] = modk1;
%
%       State-space matrix of Kalman-Bucy filter
%       (based on ideal skyhook damping system)

%*** Vehicle suspension parameters ***
ks=700000;                  %spring factor
m=30000;                    %mass of the vehicle body[kg]
cs=190000;                  %skyhook damping


% *** State-space ***
ak=[-cs/m  ks/m
      -1      0 ];
gk=[0  1]';

ck=[-cs/m  ks/m
      0      1];
```

APPENDIX VI

Simulation Programs for Control Strategies

Program 1: Complementary filter control: (2nd-order)

```
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                      %
%       MAIN PROGRAM (Complementary filter control)     %
%       Plot the trade-off between ride quality and max deflection    %
%                                                      %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%


%*** Vehicle suspension parameters ***
k=700000;              %spring factor
m=30000;               %mass of the vehicle body[kg]
cs=190000;             %skyhook damping

fn=1/(2*pi)*sqrt(k/m); %suspension frequency Hz

%*** Track parameters ***
grad=0.01;             %gradient(1%)
vs=55.;                %vehicle speed[m/s]
acl=0.5;               %acceleration limit(5%g) [m/s^2]
vv=grad*vs;            %vertical velocity
trat=vv/acl;           %transition time
Ar=2.5e-7;             %track roughness [m]

trksp=(2*pi)^2*Ar*vs;  %flat track spectrum (single side)
Q=2*pi^2*Ar*vs;        %covariance of white noise
R1=((1e-2)^2);         %covariance of measurement noise(accelerometer)
R2=((2.5e-4)^2);       %covariance of measurement noise(suspension defl)


%*** Create a deterministic (gradient) input ***
ts=0.001;              %sampling interval
fs=1/ts;               %sampling frequency
t=0:ts:10;
```

```
acc1=[zeros(size(find(t<3))),acl*ones(size(find(t>=3 & t<(3+trat)))),
       zeros(size(find(t>=(3+trat) & t<=10)))];
intacc=cumsum(acc1).*ts;


%*** Simulate the real track input(stochastic input) ***
vr1=randn(size(t));
u=sqrt(Q*fs)*vr1;              %random track data (velocity)


%*** Measurements noise ***
va1=randn(size(t));
vd1=randn(size(t));
va=sqrt(R1)*va1;              %noise in the accelerometer
vd=sqrt(R2)*vd1;             %noise in deflection


z1=[u' va' vd'];             %system input
z2=[(intacc)' va' vd'];      %system input


%***  Filter cut-off frequency range ***
fi1=linspace(0.01,2.9,100);
fi2=linspace(0.01,1.4,100);
fi3=linspace(0.01,1.15,100);


%*** Parameters initialised
rq2=zeros(1,100);
defl2=zeros(1,100);


%*** Simulation loop ***
for    i=1:100;
       pc=(i/100)*100;
       [rq2(i),defl2(i)]=lin_clo1(fi2(i),cs,trksp,z1,z2,t);

       if rem(i,20)==0,
       clc;
       disp(['Simulation ' num2str(pc) '% complete']);
       end
end


%*** Plot the trade-off curve ***
plot(rq2,defl2);
xlabel('Ride quality (%g rms)');
ylabel('Max deflection on deterministic (m)');
grid;
```

```
function [rq2,defl2] =lin_clo1(fi2,cs,trksp,z1,z2,t)
%        function [rq2,defl2]=lin_clo1(fi2,cs,trksp,z1,z2,t);
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%        This is a close loop control system used to calcuate the ride   %
%           quality and maximum suspension deflection                 %
%                 (second-order filter in the feedback)                %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%


%*** Obtain the closed loop state-space(second-order filter in the feedback) ***
[asys2,bsys2,csys2,dsys2]=close2(fi2,cs);

%*** Response for the deterministic input ***
[y2,x2]=lsim(asys2,bsys2,csys2,dsys2,z2,t);
defl2=max(abs(y2(:,4)));      %maximum suspension deflection (m)
%disp(['The max deflection on deterministic is (m)' num2str(defl1)]);

%*** The respone in the time domain for random track***
[ys2,xs2]=lsim(asys2,bsys2,csys2,dsys2,z1,t);
rq2=sqrt(mean(ys2(:,3).^2))*100/9.8;        %ride quality (%g)
%disp(['The R.M.S. of body acceleration is (%g)' num2str(rmsys1)]);
```

```
function [asys2,bsys2,csys2,dsys2]=close2(fi2,cs)
%        function [asys2,bsys2,csys2,dsys2,k,m]=close2(fi2);
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%        Creat the state-space of close loop control system (second-order filter)   %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%        Model Description:
%
%     input:  -    -              states: -    -
%             | w |                       | Xv  |
%             | V |                       |xfilt1|
%             -  -                        |xfilt2|
%                                         -    -
%     output: -    -
%             | Ym |
%             | Y  |
%             -   -
%


%*** Obtain 1mass model ***
[a,b,c,d,g,h]=moda1;

%*** Obtain second-order filter (1/s*HP(s)) state-space ***
[ha2,hb2,hc2,hd2]=hfilter2(fi2,cs);

%*** Obtain second-order filter (s*LP(s)) state-space ***
[la2,lb2,lc2,ld2]=lfilter2(fi2,cs);

% *** State-space for the close loop system***
I=eye(size(c,1));
a11=a+b*hd2*inv(I-d*hd2-d*ld2)*c+b*ld2*inv(I-d*hd2-d*ld2)*c;
a12=b*hc2+b*hd2*inv(I-d*hd2-d*ld2)*d*hc2+b*ld2*inv(I-d*hd2-d*ld2)*d*hc2;
a13=b*lc2+b*hd2*inv(I-d*hd2-d*ld2)*d*lc2+b*ld2*inv(I-d*hd2-d*ld2)*d*lc2;
a21=hb2*inv(I-d*hd2-d*ld2)*c;
a22=ha2+hb2*inv(I-d*hd2-d*ld2)*d*hc2;
a23=hb2*inv(I-d*hd2-d*ld2)*d*lc2;
a31=lb2*inv(I-d*hd2-d*ld2)*c;
a32=lb2*inv(I-d*hd2-d*ld2)*d*hc2;
a33=la2+lb2*inv(I-d*hd2-d*ld2)*d*lc2;

asys2=[a11   a12   a13
       a21   a22   a23
       a31   a32   a33];
```

```
b11=g;
b12=b*hd2*inv(I-d*hd2-d*ld2)*h+b*ld2*inv(I-d*hd2-d*ld2)*h;
b22=hb2*inv(I-d*hd2-d*ld2)*h;
b32=lb2*inv(I-d*hd2-d*ld2)*h;

bsys2=[       b11                          b12
        zeros(size(b22,1),size(b11,2))    b22
        zeros(size(b32,1),size(b11,2))    b32];

c11=inv(I-d*hd2-d*ld2)*c;
c12=inv(I-d*hd2-d*ld2)*d*hc2;
c13=inv(I-d*hd2-d*ld2)*d*lc2;
c21=c11;
c22=c12;
c23=c13;

csys2=[c11  c12  c13
       c21  c22  c23];

d12=inv(I-d*hd2-d*ld2)*h;
d22=zeros(size(d12));

dsys2=[zeros(size(d12,1),size(b11,2))    d12
       zeros(size(d12,1),size(b11,2))    d22];
```

```
function [ha2,hb2,hc2,hd2] = hfilter2(fi2,cs)
%        Function [ha2,hb2,hc2,hd2]=hfilter2(fi);
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%        Generate the state-space from  a second-order high-pass filter     %
%        and a integrator  (1/s*HP(s))                                       %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


%*** Filter parameter ***
wi=2*pi*fi2;
itai=0.7;                      %damping factor

num=[1/wi^2 0];
den=[1/wi^2 2*itai/wi 1];

%*** Transfer function to state-space conversion ***
[ha2,hb2,hc2,hd2]=tf2ss(num,den);
hb2=hb2*[-cs 0];               %generate the control force (F)
hd2=hd2*[-cs 0];               %generate the control force (F)
```

```
function [la2,lb2,lc2,ld2] = lfilter2(fi2,cs)
%        Function [la2,lb2,lc2,ld2]=lfilter2(fi);
%
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%        Generate the state-space from  a second-order low-pass filter      %
%        and a differentiator (s*LP(s))                                      %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


%*** Filter parameter ***
wi=2*pi*fi2;
itai=0.7;                          %damping factor

num=[2*itai/wi  1  0];
den=[1/wi^2  2*itai/wi 1];

%*** Transfer function to state-space conversion ***
[la2,lb2,lc2,ld2]=tf2ss(num,den);
lb2=lb2*[0 cs];                    %generate the control force (F)
ld2=ld2*[0 cs];                    %generate the control force (F)
```

Program 2: Linear Kalman-Bucy Filter control

```
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                              %
%       **** Linear Kalman-Bucy Filter control-- (varing  Qk)****    %
%                                              %
%       Active system with Kalman-Bucy filter in the feedback.       %
%       (The input of K-B filter is the measured body acceleration)  %
%                                              %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%       Model Description:
%
%       input: -      -              states: -     -
%              | zt' |                       | Xv  |
%              | va  |                       | Xkfit|
%              | vd  |                       -      -
%              -      -
%
%       output: -      -
%               | Ym  |
%               | zbe" |
%               |  Y   |
%               -      -
```

```
%*** Vehicle and track parameters ***
ks=700000;                 %spring factor
m=30000;                   %mass of the vehicle body[kg]
cs=190000;                 %skyhook damping
grad=0.01;                 %gradient(1%)
vs=55.;                    %vehicle speed[m/s]
acl=0.5;                   %acceleration limit(5%g) [m/s^2]
vv=grad*vs;                %vertical velocity
trat=vv/acl;               %transition time
Ar=2.5e-7;                 %track roughness [m]
trksp=(2*pi)^2*Ar*vs;      %flat track spectrum(single side)
Q=2*pi^2*Ar*vs;            %PSD of white noise(track input, double side)
R1=((1e-2)^2);             %covariance of measurements noise
R2=((2.5e-4)^2);           %covariance of measurements noise

ts=0.001;                  %sampling interval in seconds;
fs=1/ts;                   %sampling frequency in hertz
```

```
%*** Obtain 1mass vehicle model ***
[a,b,c,d,g,h]=moda1;

%*** Obtain Kalman filter ***
[ak,gk,ck]=modk1;

%*** Create a deterministic (gradient) input ***
t=0:ts:10;
acc=[zeros(size(find(t<3))),ac1*ones(size(find(t>=3 & t<(3+trat)))),
     zeros(size(find(t>=(3+trat) & t<=10)))];
intacc=cumsum(acc).*ts;

%*** Simulate the real track input(stochastic input) ***
vr1=randn(size(t));
vr=sqrt(Q*fs)*vr1;

%*** Measurements noise ***
va1=randn(size(t));
vd1=randn(size(t));
va=sqrt(R1)*va1;
vd=sqrt(R2)*vd1;

%*** System input ***
u1=[vr',va',vd'];
u2=[(intacc)',va',vd'];

Cs=[cs  0];
index=logspace(0,6,100);

%*** Simulation Loop (varing the process noise Qk)
for     i=1:100;
        pc=(i/100)*100;

        %*** Create  the Kalman filter ***
        R1k=R1;
        R2k=R2;
        Qk(i)=Q/index(i);
        Rk=[R1k 0;0 R2k];
        [L,P]=lqe(ak,gk,ck(1,:),Qk(i),R1k/fs);


        % *** Create state-space matrix for the system***
        asys=[a          -b*Cs
             L*c(1,:)  ak-L*d(1,:)*Cs-L*ck(1,:)];

        bsys=[ g        zeros(2,2)
```

```
        zeros(2,1)   L*h(1,:)]];


    % *** Output matrix for the system***
    csys=[c        -d*Cs
        zeros(1,2)  ck(1,:)
            c        -d*Cs];

    dsys=[zeros(2,1)     h
          zeros(1,1)  zeros(1,2)
          zeros(2,1)  zeros(2,2)]];


    % *** Ride quality (PSD) ***
    [y1,x1]=lsim(asys,bsys,csys,dsys,u1,t);
    rq(i)=sqrt(mean(y1(:,4).^2))*100/9.8;        %ride quality

    %*** Suspension Deflection ***
    [y,x]=lsim(asys,bsys,csys,dsys,u2,t);
    defl(i)=max(abs(y(:,5)));                     % max suspension deflection

    if rem(i,5)==0,
    clc;
    disp(['Simulation ' num2str(pc) '% complete']);
    end

end;


%*** Plot the trade-off curve for linear K-BF control ***
plot(rq,defl);
xlabel('Ride quality (%g)');
ylabel('Max Suspension defl (m)');
title('Kalman-bucy Filter');
grid;
```

Program 3: Non-linear single Kalman Filter control

```
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                        %
%          Non-linear single Kalman Filter control       %
%                                                        %
%          Active system with Kalman filter in the feedback.   %
%          (using measured suspension deflection as input signal of   %
%           threshold detector)                          %
%                                                        %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
```

```
%*** Vehicle and track parameters ***
ks=700000;                  %spring factor
m=30000;                    %mass of the vehicle body[kg]
Cs=190000;                  %skyhook damping
grad=0.01;                  %gradient(1%)
vs=55.;                     %vehicle speed[m/s]
acl=0.5;                    %acceleration limit(5%g) [m/s^2]
vv=grad*vs;                 %vertical velocity
trat=vv/acl;                %transition time
Ar=2.5e-7;                  %track roughness [m]
trksp=(2*pi)^2*Ar*vs;       %flat track spectrum(single side)
Q=2*pi^2*Ar*vs;             %PSD of white noise(track input, double side)
R1=((1e-2)^2);              %covariance of measurements noise (acceleration)
R2=((2.5e-4)^2);            %covariance of measurements noise (susp defl)

ts=0.001;                   %sampling interval in seconds;
fs=1/ts;                    %sampling frequency in hertz


%*** Obtain 1mass vehicle model ***
[a,b,c,d,g,h]=moda1;

%*** Obtain Kalman filter ***
[ak,gk,ck]=modk1;

%*** Create a deterministic (gradient) input ***
t=0:ts:10;
acc=[zeros(size(find(t<3))),acl*ones(size(find(t>=3 & t<(3+trat)))),
       zeros(size(find(t>=(3+trat) & t<=10)))];
intacc=cumsum(acc).*ts;
```

```
%*** Simulate the real track input(stochastic input) ***
vr1=randn(size(t));
vr=sqrt(Q*fs)*vr1;


%*** Measurements noise ***
va1=randn(size(t));
vd1=randn(size(t));
va=sqrt(R1)*va1;
vd=sqrt(R2)*vd1;


%*** Convert state-space matrix from continous to discrete ***
[Ad,Gd]=c2d(a,g,ts);
[Ad,Bd]=c2d(a,b,ts);
[Ad_k,Gd_k]=c2d(ak,gk,ts);


Cs=[cs 0]
R1k=R1;
R2k=R2;
Rk=[R1k 0;0 R2k];
Qk=Q*(1e-4);
Th=0.026;                    %Threshold detector level


%*** Steady state Kalman filter ***
[Ld,mm,Pd]=dlqe(Ad_k,Gd_k,ck(1,:),Qk*fs,R1k);



%----------------------------------------------------------------
%   Time domain simulation of the vehicle body response
%----------------------------------------------------------------
%
%*** Initialise variables for simulation ***
nt=max(size(t));
x0=zeros(2,1);
xe0=x0;
ymes=zeros(2,nt+1);
ytrue=zeros(2,nt+1);
xt=zeros(2,nt+1);
xhat=xt;                     %estimated state
yest=zeros(1,nt+1);
wnt=zeros(1,nt+1);
vnt=zeros(2,nt+1);
err=1e-8*zeros(1,nt+1);
nerr=err;
op=zeros(1,nt);
```

```
xd0=x0;
xed0=x0;
xdet=zeros(2,nt+1);
xhatd=xdet;
ymesd=zeros(2,nt+1);
ytrued=zeros(2,nt+1);
yestd=zeros(1,nt+1);
u1t=zeros(1,nt+1);
errd=1e-8*zeros(1,nt+1);
nerrd=errd;
opd=zeros(1,nt);

%*** Simulation loop ***
for n=1:nt,
        pc=(n/nt)*100;
 %------Ride quality calculation ------
        wn=vr(1,n);                    %random track
        vn=[va(1,n),vd(1,n)]';
        x=Ad*x0-Bd*Cs*xe0+Gd*wn;
        ymes(:,n+1)=c*x0-d*Cs*xe0+h*vn;
        ytrue(:,n+1)=c*x0-d*Cs*xe0;

        %  Kalman filter
        xe=Ad_k*xe0;
        yest(:,n+1)=ck(1,:)*xe;
        err(:,n+1)=ymes(1,n+1)-yest(:,n+1);
        nerr(:,n+1)=err(:,n+1);
        if ymes(2,n+1) >= Th
                op(n)=1;
                nerr(1,n+1)=nerr(1,n+1)-0.5;
        elseif ymes(2,n+1)<=-Th
                op(n)=-1;
                nerr(1,n+1)=nerr(1,n+1)+0.5;
        end
        xe=xe+Ld*nerr(:,n+1);
 %-------------------------------------

        %*** Save data ***
        x0=x;
        xe0=xe;
        xhat(:,n+1)=xe0;
        xt(:,n+1)=x0;
        wnt(:,n+1)=wn;
        vnt(:,n+1)=vn;

% Deterministic input
```

```
%----- Max deflection calculation ------
        u1=intacc(1,n);              %pure deterministic input
        xd=Ad*xd0-Bd*Cs*xed0+Gd*u1;
        ymesd(:,n+1)=c*xd0-d*Cs*xed0+h*vn;
        ytrued(:,n+1)=c*xd0-d*Cs*xed0;


        % K_B filter2
        xed=Ad_k*xed0;
        yestd(:,n+1)=ck(1,:)*xed;
        errd(:,n+1)=ymesd(1,n+1)-yestd(:,n+1);
        nerrd(:,n+1)=errd(:,n+1);
        if ymesd(2,n+1)>=Th
                opd(n)=1;
                nerrd(1,n+1)=nerrd(1,n+1)-0.5;
        elseif ymesd(2,n+1)<=-Th
                opd(n)=-1;
                nerrd(1,n+1)=nerrd(1,n+1)+0.5;
        end
        xed=xed+Ld*nerrd(:,n+1);
%-----------------------------------------


        %*** Save data ***
        xd0=xd;
        xed0=xed;
        xdet(:,n+1)=xd0;
        xhatd(:,n+1)=xed0;
        u1t(:,n+1)=u1;


        if rem(n,500)==0,
        clc
        disp(['Simulation ' num2str(pc) ' % complete']);
        end
end

%***Calculate the suspension performance for current Qk ***
rq=sqrt(mean(ytrue(1,:).^2))*100/9.8;
defl=max(abs(ytrued(2,:)));

%*** Plot the ride quality and max susp defl ***
plot(rq,defl,'*');
xlabel('Ride quality (%g)');
ylabel('Max Suspension defl (m)');
title('Kalman Filter');
grid;
```