AN INVESTIGATION INTO THE ENGAGEMENT CHARACTERISTICS OF AN AUTOMOTIVE DRY FRICTION CLUTCH

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A DOCTORAL THESIS

Submitted in fulfilment of the requirements for the award of Doctor of Philosophy of the Loughborough University of Technology.

November 1974

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SUMMARY

The object of this research was to investigate the period of time during which the clutch plates of a motor vehicle are engaged to effect a start from rest. The problem was approached from two sides, firstly a mathematical model has been produced in the form of a digital computer program. Secondly, practical investigations were undertaken in order to shed more light upon certain problem areas highlighted whilst developing the mathematical model. Finally the two types of investigations were combined to give a more realistic model of the engagement period than any that had previously been published.

The mathematical model allowed parametric studies of the clutch to be made giving details of slip time, speeds, clamp load, friction level, heat generated and the one dimensional temperature distribution through one half of the clutch assembly. Two methods for the solution of the temperature distribution problem were used in order to give a cross check during the development of the mathematical model. Stability of the two methods has been investigated and limits suggested which if adhered to prevent instability errors occurring.

Practical work on a test rig was carried out in order to investigate the variation of friction levels during the engagement period and also the temperatures reached. A test vehicle was instrumented to find out how the general public operated the clutch in service and how their operation varied with test conditions i.e. gradients and traffic conditions.

Finally a computer controlled test rig comprising of an engine, clutch with electro-hydraulic ram operation and inertia flywheel was developed. This rig was completely automatic and could be used to simulate different engagement rates, gradients and, if inertia were added or subtracted,

different vehicles.

The result has been that a useful tool, in the form of an easily used computer program has been developed, the limitations of which have been investigated. Also, a test facility has been developed which can be programmed to simulate in vehicle use. The model and test rig produced lend themselves to extension to further work in the area of clutch and transmission vibrations, especially clutch judder.

CONTENTS

NOTATION

A	Area of frictional contact	m^2
A,B,C,D,θ	Matrices defined in the text	
a	Arbitrary constant	
B_I, B_Q	Constants defined in the text	
B_o	Biot number	
Cp	Specific heat	$J/Kg/^oC$
F_o	Fourier number	
h_f	Coefficient of forced convection	$W/m^2/^oC$
h_I	Coefficient of heat transfer across Interface thermal resistance	$W/m^2/^oC$
I_e	Engine Inertia	$Kg\ m^2$
I_v	Equivalent vehicle inertia	$Kg\ m^2$
k	Thermal conductivity	$W/m/^oC$
N	Number of friction faces on the clutch	
$n1$	Number of finite elements in friction material	
$n2$	Number of finite elements in metal member	
P	Clamp Load	N
Q	Heat flow	J
Q_f	Heat flow out of the back face of the metal member	W
Q_g	Heat generated by friction	W/m^3
Q_o	Heat flow across geometrical centre of centre plate	W
R_g	First gear ratio	

R1	Clutch outside radius	m
R2	Clutch inside radius	m
R	Mean clutch radius	m
T_c	Clutch torque	N.m
T_d	Drag torque	N.m
T_e	Engine torque	N.m
t	Time	secs.
V	Elemental volume	m^3
v	Linear slip speed at mean radius	m/secs
x	Distance into material	m
Z	Slip speed	rads/sec
α	Thermal diffusivity	m^2/sec.
θ	Temperature	^oC
θ_{fl}	Temperature of surrounding air	^oC
θ_w	Temperature at back face of metal member	^oC
θ'	Temperature at a fictitious mesh point	^oC
ω_e	Engine speed	rads/sec
ω_v	Propeller shaft speed	rads/sec
λ	Coefficient of distribution of heat	
ρ	Density	Kg/m
ΔP	Inlet Manifold Depression	mm.Hg.
Δt	Small time interval	secs
Δx	Small distance interval	m
μ	Coefficient of friction	

μ_s'	Static coefficient of friction
μ_∞	Coefficient of friction at high slip velocities

Suffices

1	Friction material
2	Metal member
T	Total heat generated
i	Mesh point, distance co-ordinate
j	Mesh point, time co-ordinate

ACKNOWLEDGEMENTS

I should like to thank the staff of the Transport Technology Department for their co-operation throughout the duration of this project especially:

Dr. G.G. Lucas - Supervisor

Mr. J. Richardson - Technician.

The sponsors of this project were Science Research Council and Mintex Ltd. I should like to thank Mintex Ltd. for information gained by their experience which was freely passed on.

I should also like to mention the following companies for materials supplied and discussions held.

Ferodo Ltd.,

Ford Motor Company

Borg & Beck Ltd.

In this thesis the tables, figures, graphs and plates are presented respectively at the end of the chapter to which they refer. Numbers in brackets refer to the references listed at the end of the thesis. The numbering system used translates as, the number before the decimal point referring to the chapter, the number after referring to the sequence.

CHAPTER 1

INTRODUCTION

An automotive single plate clutch comprises of a centre plate, a diaphragm spring pressure plate assemble, Plate No. 1.1, and the engine flywheel. When assembled the diaphragm spring assembly is bolted to the flywheel with the centre plate sandwiched in between. The job of the clutch assembly is to provide an infinitely variable speed ratio between the vehicle and the engine, this duty being performed by a slipping action between the clamping faces of the flywheel and pressure plate on the centre plate. The majority of the energy dissipated is in the form of heat, which if the assembly has not been designed correctly causes eventual failure of the unit. The problem of thermal failure in clutch assemblies is not new but because of the improvement in facing materials which allow smaller diameter facings to transmit the same torque levels and also due to the higher average speeds of modern engines, this type of failure is becoming all too frequent.

Broadly speaking the problems encountered in practice on automotive clutch assemblies can be catagorised under one of the following headings:

1. Thermal Failure.
2. Vibration
3. Mechanical Failure.

These are not necessarily in order of importance and some problems can and do fall into more than one catagory, in that a mechanical failure of, say, the release bearing can induce clutch slip thus generating excess heat, causing distortion of the assembly which in turn produces vibration during the engagement. Some examples of mechanical and thermal failure are shown in the plates and described in the following text.

Plate No. 1.2 shows a purely mechanical failure, the driving splines have been sheared out of the centre plate. Plate No. 1.3 is of a clutch assembly where the release bearing "seized" causing excessive wear of the fingers of the diaphragm spring resulting eventually in the break-up of the centre plate. Plate No. 1.4 presents another common mechanical failure, that of the cushion spring becoming over stressed and working loose causing excessive "rattle" during drive. Plate No. 1.5 shows a pressure plate that has been subject to excessive heat causing thermal cracking of the cast iron. Plate No. 1.6 shows a very common mode of failure, usually due to excessive slip, where the lining material has "denatured" and parted from the centre plate.

Purely mechanical failure or any combination of the three induced by a mechanical failure must be expected where mass production and statistical quality control is used. But the first two can be avoided, providing that a designer is given enough data concerning the causes and a reliable design procedure to follow. Both topics require a separate detailed study but have one thing in common and that is, that they need a proven model of the engagement period. A model of a mathematical nature but one that is known to be representative of the real situation.

Such a model has been developed during the course of this research and has been proven against practical tests. In the beginning it was decided that the calculations involved in an analytic solution only, would limit the models versatility and also make its use unattractive to designers who might wish to use it. Further, it was considered that the thermal problems are of greater concern to the industry at the present time especially in the light of the proposed European regulations governing the sale of vehicles within the E.E.C. The proposed regulations, with respect to clutches state that a fully laden vehicle with a trailer attached of weight approximately 50% that of the vehicle must complete five starts on a 16% gradient at one minute intervals and still be capable of further use. These tests are quite severe and expensive to carry out (especially if a high probability of success cannot be guaranteed).

There are different severities of thermal failure, Plate No. 1.1 shows a before and after situation, on the left is a new pressure plate and centre plate and on the right a used pair. These components came from the test vehicle used in this research, after approximately twenty subjects had been tested with a total mileage of somewhere in the region of four hundred miles. Plate No. 1.7 is a close up of the lining material, which is an asbestos yarn based material with zinc wire inclusions, coiled into an annulus, and bound with resin. The deposits in the rivet holes are zinc which has been "melted" out from the lining due to excessive heat. The zinc can also be seen deposited upon the pressure plate, Plate No. 1.8, an idea of the magnitude of temperatures encountered can be obtained from the fact that the surface of the metal had been tempered and that melting point of zinc is 420°C.

Plate No. 1.9 shows a microscopic view of the pressure plate material structure, the pressure plate in this case seemed, visually, to have been unaffected by heat, the structure comprising of random graphite in a matrix of pearlite, estimated temperatures of around 350°C had been encountered. Plate No. 1.10 however was of a pressure plate whose surface was similar to that shown in Plate No. 1.1 and the metals structure had been greatly affected, estimated temperatures of well in excess of 700°C must have been encountered to cause this structural change. Thus showing that even under normal running conditions the temperature encountered in the clutch, at the interface, can be quite severe. Therefore, if the causes and effects of overheating can be studied while a vehicle is still in the design stage the manufacturer could be saved a great deal of trouble.

A clutch on the face of it is a simple mechanical system but when at attempt is made to model this system mathematically numerous practical difficulties are encountered. These problems have long resisted analytical solution but if the problem is approached considering a digital computer solution of the equations and an empirical solution of the practical restrictions a model describing the engagement period can be developed. Looking at the problem as a whole it becomes a control exercise, man releases the clutch pedal (input), clutch plates transmit torque (output) and the vehicle accelerates (feed-back). This would be a simple feed-back loop except that a human being is involved and no two people are alike. The dynamic equations of the system can be readily solved if the input to the system can be defined i.e. man. In an attempt to define the input a test vehicle was used to measure how a driver operates a clutch during the engagement period, and in what manner this varies with road conditions. At first, it was thought that a relationship between type of driver and the manner in which the clutch was operated could be formed but this proved to be the subject of very complex research. Driver behaviour has been studied by Quenault (29, 30) and driver aggression by Parry (23) but no simple or reliable technique has as yet been evolved for general use. It was also found that no data was available on the distribution of drivers, with respect to age, employment, sex, number of miles driven, etc., and so a number of subjects were chosen at random for the tests.

Given the mathematical model of the engagement period and the practical results, the next step was to consider the heat flow into the components of the clutch. Again, digital techniques were employed, which enabled them to be easily combined with the original model and two methods were used to solve the heat conduction problem. Using two methods had the advantage that a cross check on the results was always possible throughout the development of the model. Heat flow was considered only in one dimension i.e. normal to the friction surfaces, this is an idealized situation and it was necessary to check the validity of this assumption. Numerous thermocouples were inserted into a clutch lining, flywheel and pressure plate, the assembly being fitted to a test rig which enabled part of the assembly to remain stationary thus facilitating the measurement of the temperatures. The model was also used to simulate the rig and predict the temperatures obtained during engagements. Using this rig it was also possible to describe the variation in the friction coefficient due to temperature and slip speed, using an empirical technique.

Having obtained a mathematical model and empirically described the driver's affect on the engagement it was necessary to produce a test rig upon which tests could be carried out in an attempt to marry the two together to produce a complete solution. This rig had to be able to simulate the driver's actions on the pedals and the various road conditions encountered on the tests, which meant automatic control of the clutch and rapid data acquisition. This obviated the need for computer control of the rig, which was then built with this in mind, resulting in a versatile and powerful research tool which could be programmed in the same way as the computer mathematical model.

Throughout the research the original aims of producing a design tool and useful information for the designer have been kept in mind, especially in the complexity of the model, a designer will not use a design aid, if it is more economic to build and test the component. The mathematical model produced allows parametric studies to be carried out before the building of a vehicle but it still provides a basis, from which further research work could be continued along avenues that will be outlined later in this thesis. Especially, where the problem of clutch judder and other transmission vibrations is concerned, this phenomenon will be seen from the test results to have occurred during both the vehicle and the rig tests.

The work of this thesis therefore concerns:

(a) Experimental work on the manner in which a driver operates a motor car clutch.

(b) Experimental work on the measurement of the coefficient of friction of a clutch assembly.

(c) A mathematical model of clutch engagement.

(d) Two models predicting the unsteady, one-dimensional heat conduction through the component parts of a clutch.

(e) Experimental tests of clutch engagement on a rig to prove the models.

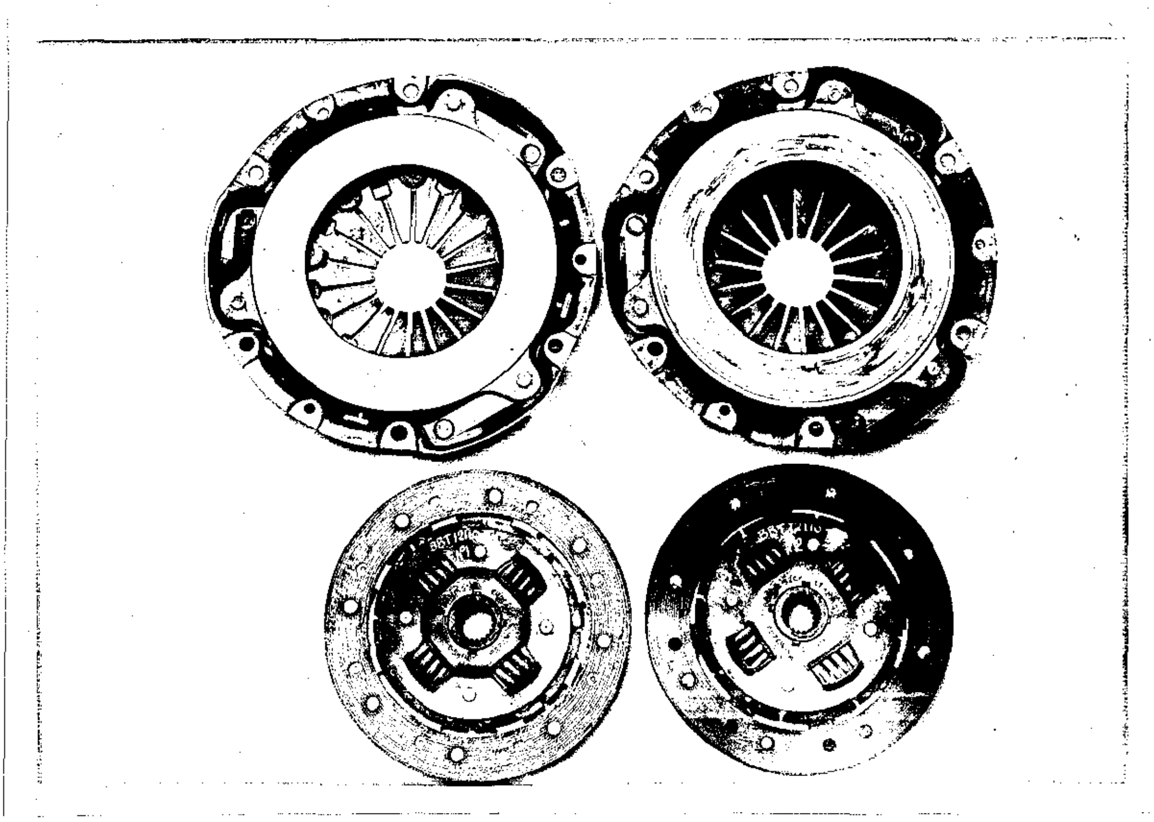
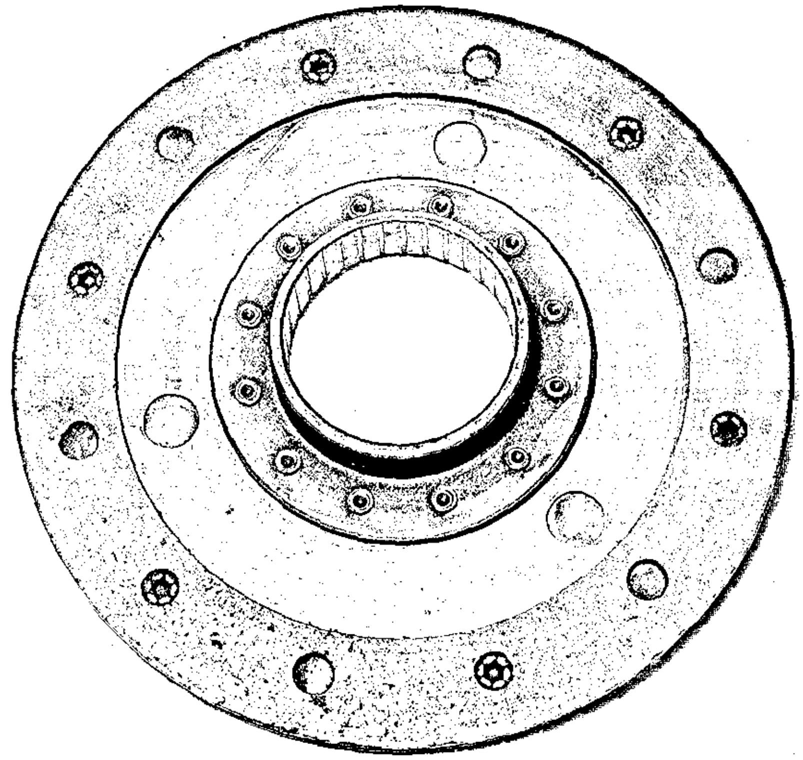


PLATE NO 1·1



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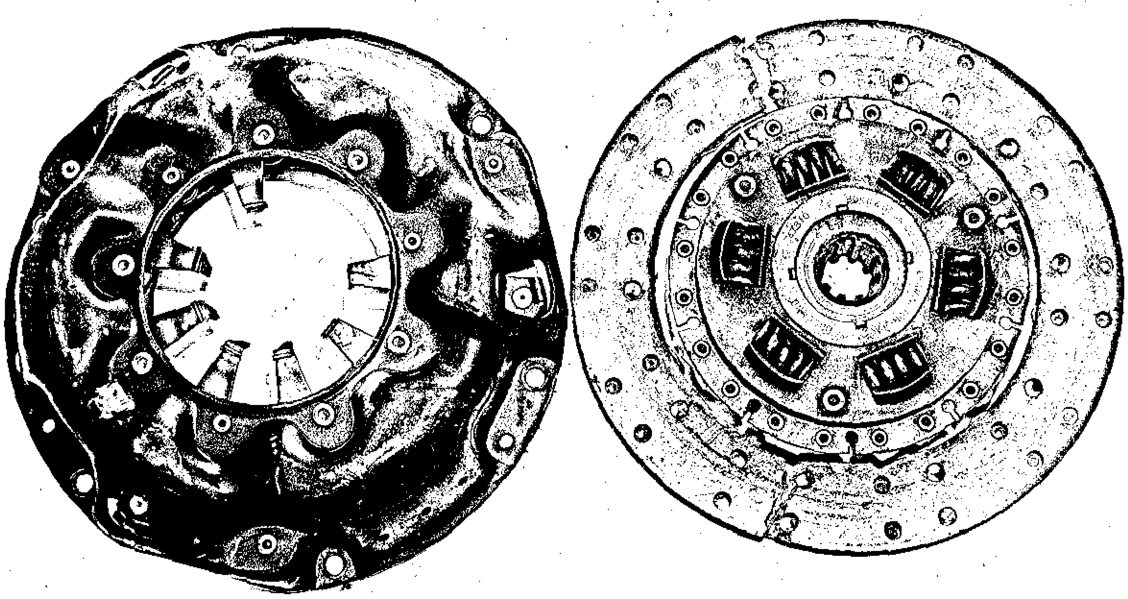

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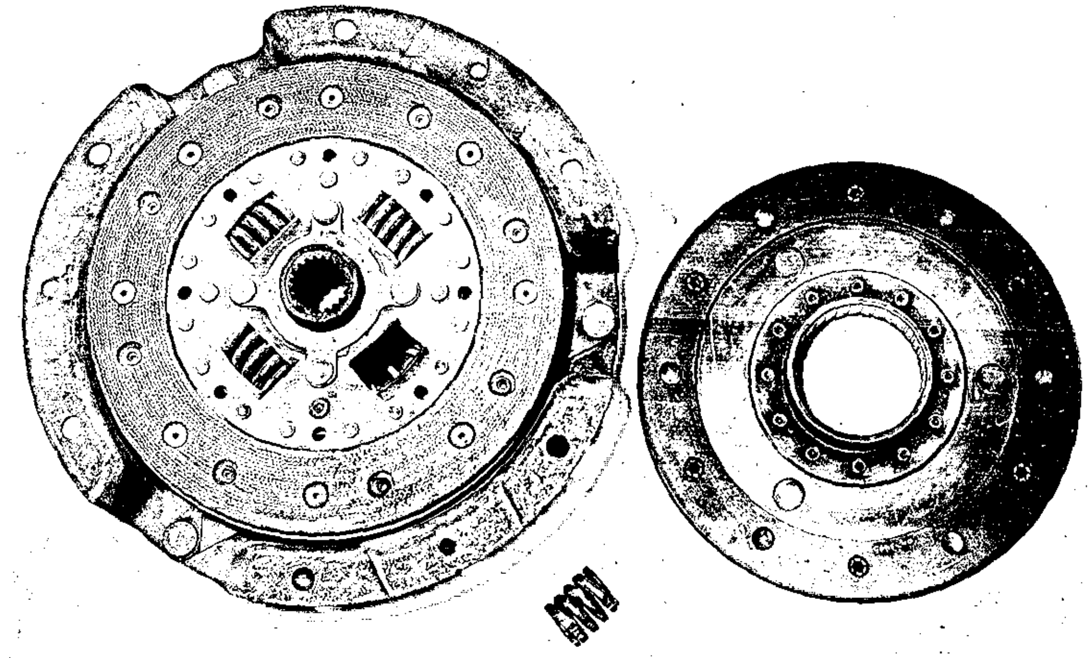

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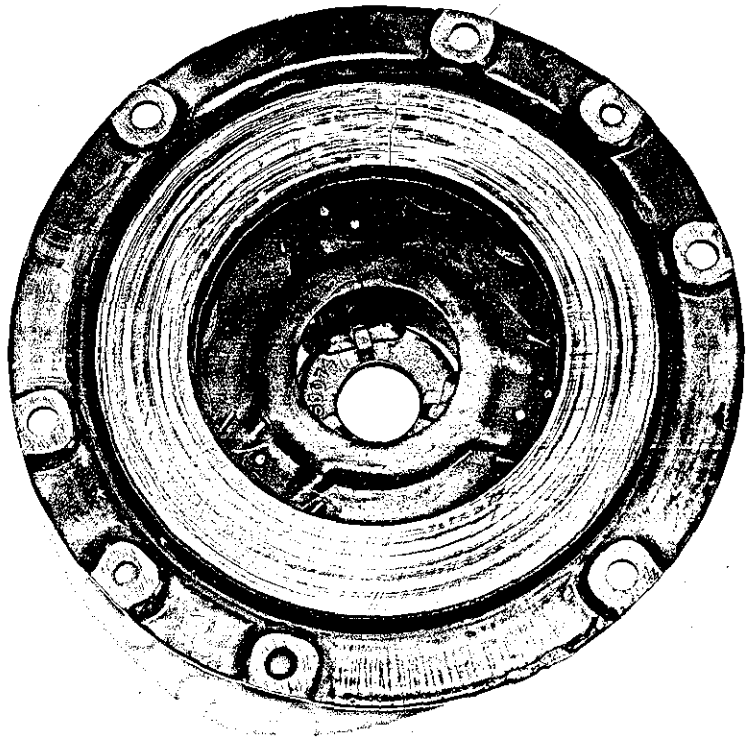

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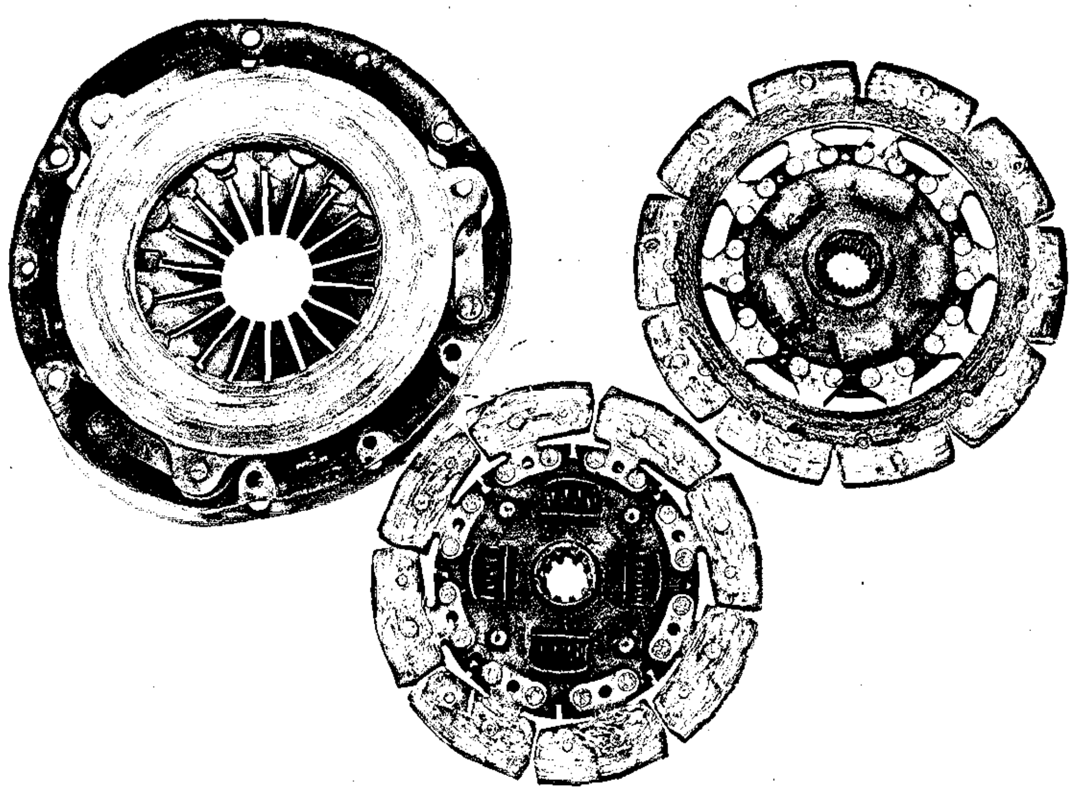

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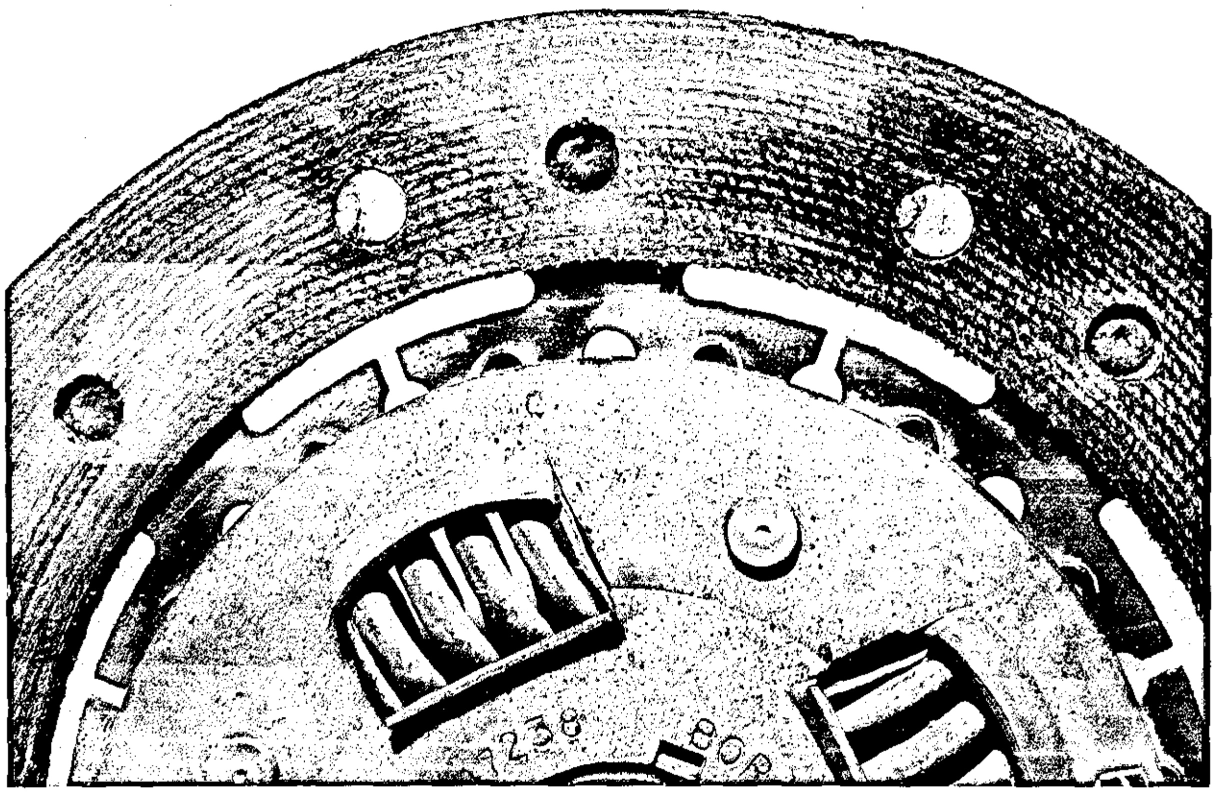

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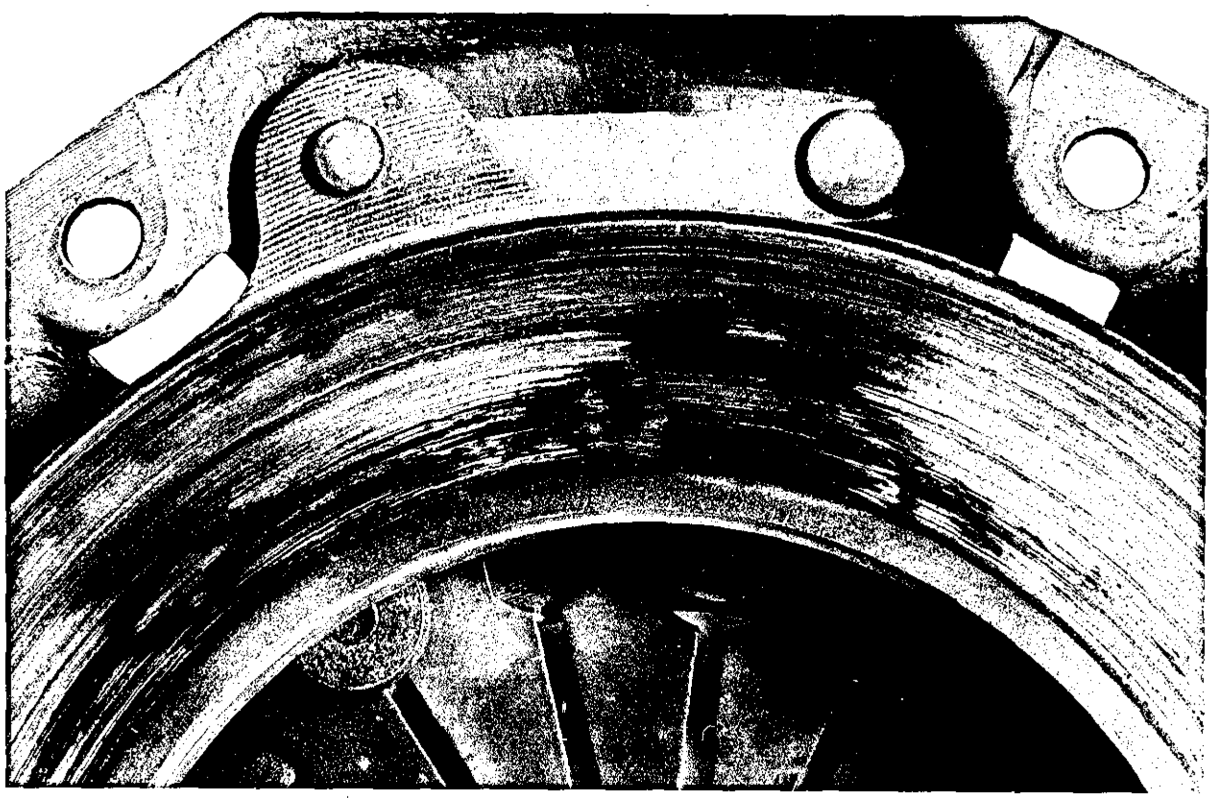


PLATE NO 1·8

PLATE NO 1·9

PLATE NO 1·10

CHAPTER 2

LITERATURE SURVEY

The engagement of a dry friction clutch requires two separate but related approaches to effect an analytical solution. First, the dynamics of the system, Figure No. 2.1, requires solution to obtain the rate of heat generation during the engagement period and second, the dispersion of this heat, by conduction, through the clutch components and also the resulting temperature rise needs investigation. Heat is generated in a clutch by the rubbing of two bodies together under pressure, the mechanism being called frictional heating. The heat is generated both on the surface and in the near surface of the rubbing bodies, from whence it is conducted away.

This survey covers the development of the analytical approach to the solution of the above mentioned problem dealing chronologically with the published literature. In the early days people were concerned only with the conduction of the generated heat away from the interface and workers produced some complex equations for its solution. One or two equations will be shown in the following text but these are only meant as a guide to indicate to what lengths it is necessary to go in order to effect a reasonable analytical solution. The survey shows how the subject has branched out from this very narrow beginning to what is now, a complex system.

As the mathematical models were developed more areas requiring further and more detailed analysis came to light, noticeably, the problem of understanding and predicting the mechanism of dry friction. Although much work has been carried out, no satisfactory method for its solution has been evolved and so a purely theoretical approach is not considered within the scope of this research and an empirical technique is considered instead. Also, the behaviour of the driver affects the theoretical modelling and again opens a wide avenue of research, down which little work has been carried out, but again in this research only the surface has been uncovered.

This survey therefore deals with both the analytical approach of the early mathematicians and the applied empirical techniques of the engineers who followed.

Jaeger (1) *set out to calculate the temperatures in a semi-infinite* solid on which another body slides, he began by taking the solution, proposed earlier by Carslaw (1A), for the following conditions. The temperature at a point (x,y,z) at time t in an infinite solid, which was initially at zero temperature, due to a quantity of heat Q being released instantaneously at a point (x'.y',z') at t = 0 is:

$$\frac{Q\alpha}{8k(\pi\alpha t)^{3/2}} \exp - \left[\frac{(x-x')^2 + (y-y')^2 + (z-z')^2}{4\alpha t} \right]$$

He proposed replacing Q by Qdy' and integrating with respect to y', between $-\infty$ and ∞ the solutions thus obtained applying to an instantaneous line source:

$$\frac{Q}{4\pi kt} \exp - \left[\frac{(x-x')^2 + (z-z')^2}{4\alpha t} \right]$$

Jaeger progressed using these basic equations, through solutions in which he considered, both band and rectangular sources of heat, at steady state and the case of a stationary source, to sources that vary over their area and where the velocity is not constant; which would be the case in practice. He also examined the case of one body rubbing on another, considering the two cases shown in Figure No. 2.2, and discussed the solution of temperatures in the steady state. This work was the first systematic study of the problem of moving heat sources and has formed the starting point for much of the work carried out in this area. The equations obtained however suffer from the assumptions that infinite or semi-infinite solids are being dealt with.

Bowden and Tabor (2) and Kragelskii (3) contain comprehensive chapters covering the topic of frictional heating and the steady state temperatures obtained as well as discussions of flash temperatures. Practical work, however has always been carried out on "ideal" substances rubbing together e.g. steel against steel, diamond against diamond. That is they considered homogenious materials of similar physical properties, of which reasonably accurate measurement of physical properties has been possible. But, when considering a non-homogenious material, such as an asbestos based friction material, which may contain upwards of twenty different constituents set in a phenolic resin, accurate data are not readily available. This has meant that a purely theoretical analysis has always been inhibited by physical constraints and most of the work in the field, has therefore been approached from the practical aspect.

Newcomb (4) considered the flow of heat in a parallel faced infinite solid, although this had been investigated by Odier and Leutard (7, 8) and Banister (9) using Fourier series solutions and the method of images, respectively. Newcomb approached the solution of the problem using Laplace transform theory. The problem considered was one where a finite solid was bounded by two infinite solids so that at $x = o$ in the finite solid there was no heat flow perpendicular to the plane and at the other boundary there was a linearly decreasing heat flux. In mathematical terms this became:

$$\text{Heat Flux} = Q(1 - at) \quad \text{where "a" is a constant.}$$

$$\text{at } x = 0 \qquad \frac{\partial \theta}{\partial x} = 0$$

$$\text{at } x = 1 \qquad k\frac{\partial \theta}{\partial x} = Q(1-at)$$

these form the required boundary conditions for the one dimensional heat conduction equation:

$$\frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial x^2}$$

The solution of which, using Laplace transforms, Newcomb found to be:

$$\frac{\alpha\theta}{Qk^{\frac{1}{2}}} = 2\,t^{\frac{1}{2}} \sum_{n=o}^{\infty} \left[i\ \mathrm{erfc}\ \frac{(2n+1)\ell - x}{2(kt)^{\frac{1}{2}}} + i\ \mathrm{erfc}\ \frac{(2n+1)\ell + x}{2(kt)^{\frac{1}{2}}} \right]$$

$$- a8t^{3/2} \sum_{n=o}^{\infty} \left[i\ \mathrm{erfc}\ \frac{(2n+1)\ell - x}{2(kt)^{\frac{1}{2}}} + i^3\ \mathrm{erfc}\ \frac{(2n+1)\ell + x}{2(kt)^{\frac{1}{2}}} \right]$$

where

$$i^n \operatorname{erfc} x = \int_x^\infty i^{n-1} \operatorname{erfc} y \, dy \quad (n = 1,2,3 \ldots)$$

and

$$i^o \operatorname{erfc} x = \operatorname{erfc} x = \frac{2}{\pi^{\frac{1}{2}}} \int_x^\infty \exp(-y^2) dy$$

Newcomb then went on to apply the above solutions to a brake lining sliding against a cast iron shoe and compared the results with practical values obtained which showed agreement within 10%. In a later paper (5) Newcomb considered the case of rubbing elements of finite thickness, thus increasing the complexity of the equations to be solved. As would be expected the resulting solution was very much more complex than the one above, requiring much tedious and repetative computation to effect a solution for values of temperature.

He applied these equations to practical tests in a paper (6), where he also compared the results obtained with the previously mentioned work of Odier, Leutard and Banister. In conclusion Newcomb suggested the following more easily managed equation which gave results within ± 10% of practical applications:

$$\Theta = \text{Const. } F.u \left[\frac{2t^{\frac{1}{2}}}{\pi^{\frac{1}{2}}} \left(1 - \frac{2}{3}\frac{t}{ts}\right) + 4t^{\frac{1}{2}} \sum_{n=1}^{\infty} i \operatorname{erfc} 2n\lambda - \frac{16t^{3/2}}{ts} \sum_{n=1}^{\infty} i^3 \operatorname{erfc} 2n\lambda \right]$$

where $$\lambda = \frac{1}{2(\alpha t)^{\frac{1}{2}}}$$

F is the friction force

u is the linear speed.

In this paper Newcomb investigated the effect of rhythmically repeated engagements in which it was found that the bulk temperature built up to a level and remained at that level until the rate of application was varied. The theory considered so far has been applied to either idealized test specimens or to the braking of a motor vehicle, which does the job of a friction clutch in reverse.

As with brakes the first investigations into clutches were approached from the practical aspect and Jania (10) in a series of four papers discussed the following points:

1. Factors affecting performance.
2. Analysis of the system.
3. Thermal aspects.
4. How to cope with the temperatures.

In the first article the author dealt mainly with the vibrational tendencies of a clutch system, suggesting that if the relevant shaft stiffnesses were choosen correctly the problem of transmission vibrations could be reduced. He also derived a formula for the torque capacity of a clutch assuming a constant pressure engagement and a rather more limiting assumption that coefficient of friction decreases linearly with velocity.

In the second article the author considered a system similar to that of, Figure No. 1, writing the equation of motion as:

$$Ie \frac{dw_e}{dt} = T_e - Tc \quad \text{and} \quad Iv \frac{dw_v}{dt} = T_c - Tv$$

which upon integration and substitution gave the rate of energy dissipation in the clutch as:

$$q(t) = T_e \left[\Omega_1 - \Omega_2 - T_c \left(\frac{I_1 + I_2}{I_1 \quad I_2} \right) t \right]$$

The above formula was derived with the following assumptions.

1. Coefficient of friction is a constant.
2. Compliance of the system zero.
3. No external torques.

These observations were pointed out by the author:

1. Slip time is proportional to the initial slip velocity.
2. Maximum rate of energy dissipation occurs at $t = 0$.
3. Energy dissipated does not depend on clutch torque or duration of slip.

But these points only apply to the system as the author constrained it and are not necessarily so in a practical installation.

The article went on to discuss other types of clutches and the application of the formula to them.

The third in the series of articles dealt with the thermal capacity and limitations of a clutch, the author suggested that two important values must be obtained.

1. Safe permissible maximum surface temperature.

2. Maximum surface temperature likely to occur under known conditions of loading.

To define these values at the design stage the author solved the one dimensional heat conduction equation applying similar boundary conditions, as to Newcomb, but the result was in the form of a series solution i.e. assuming heat flow = at + b

$$Q(t) = b_1 (\rho.cp.k_1)^{\frac{1}{2}} \left[\frac{t}{l_1 \alpha_1^{\frac{1}{2}}} + \frac{l_1 \alpha^{\frac{1}{2}}}{3} - \frac{2l_1 \alpha_1^{\frac{1}{2}}}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(\frac{n^2 \pi^2 t}{l_1^2 \alpha_1}\right)\right] - a_1(\rho_1 cp_1 k_1)^{\frac{1}{2}} \left[\frac{t^2}{2l_1 \alpha_1^{\frac{1}{2}}} + \frac{l_1 \alpha^{\frac{1}{2}} t}{3} - \frac{l_1^3 \alpha_1^{3/2}}{45} + \frac{2l_1^3 \alpha_1^{\frac{1}{2}}}{\pi^4} \sum_{n=1}^{\infty} \frac{1}{n^4} \exp\left(\frac{n^2 \pi^2 t}{l_1^2 \alpha_1}\right)\right]$$

But for $l \to \infty$ i.e. the plate is very thick the expression above is reduced to

$$\theta_1(t) = 2\, b_1(\rho_1 cp_1 k_1)^{\frac{1}{2}} \left(\frac{t}{\pi}\right)^{\frac{1}{2}} - \frac{4}{3}\left(\frac{a_1(\rho_1 cp_1 k_1)^{\frac{1}{2}}\, t^{3/2}}{\pi}\right)$$

which Jania suggested using for ease of calculation. He then continued to apply the formula to practical examples and in the final article discussed the bulk temperatures obtained and suggested ways for reduction of the occurence of flash temperatures. This series of articles gave a semi-practical, semi-theoretical treatise of the problems encountered in the design of clutch systems and although encompassing most of the problems encountered and drawing conclusions via theoretical analysis, no finite design tool emerged.

Newcomb (11) took the theoretical analysis of the system, Figure No. 2.1, further and in more detail than Jania, assuming:

1. Torque remains sensible constant throughout the engagement.
2. Drag torque remains constant.

He investigated the affects of temperature and slip speed upon coefficient of friction and from practical tests arrived at the following formula:

$$\mu = \mu\infty + (\mu_s - \mu\infty).\ \exp\ (-aZ)$$

Using these assumptions he derived formulae for the heat generated and slip time, also he applied equations previously derived in (4), (5), (12) and (13) to cases of single and multiple engagements. As stated, in the above analysis the author assumed constant torque throughout the engagement in order to facilitate an analytical solution of the dynamic equations but in practice this is not the case. Newcomb in his paper (14) considers the case where torque is assumed to vary with time:

1. Torque increasing linearly with time.
2. Torque increasing parabolically with time.

The former being a reasonable approximation to that which occurs in a single plate clutch and the latter to a multi-plate clutch, both cases are considered in detail and the resulting formulae are of similar complexity and size to the ones already given. After some simplification of these equations the author discusses the affect of making the constant torque assumption on the temperatures calculated. He concludes, that providing the clutch has the same slip torque capacity, the slip time is doubled when a linearly increasing torque is considered and also maximum temperature is 0.806 the value attained under the constant torque assumption and occurs at $(5/8)^{\frac{1}{2}}$ of the total slip time. In the case of parabolically increasing torque the slipping period was $1\frac{1}{2}$ times longer, the maximum temperature 0.9 that of the constant torque assumption. Thus seriously limiting the simplified solution and suggesting that the more complex solution of a linearly increasing torque is still not the full picture.

Ramachandra Rao (15) considering the dynamics of commercial vehicle drive-lines under severe operating conditions and developed a more comprehensive model than before, but the equations that resulted were larger and more complex. He carried out a number of tests on commerical vehicles towing a trailer loaded with 1360 Kg, with the handbrake applied, in order to simulate the severe operating conditions encountered in practice. The torque reaction of the rear axle was measured and graphs of this during the engagement period and over a period after the engagement were reproduced in his paper. These graphs showed torque increasing approximately linearly during the engagement period, although as the author pointed out this depends to a larger extent on the type of driver. In the period that follows synchronization of the clutch plates, called the stabilising period by the author, the torque reaction took quite a long time to settle down to steady state. Figure No. 2.3.

Newcomb (16) has also examined extreme clutch usage, namely:

1. Sudden engagements.

2. Vehicle held stationary upon a hill with clutch slipping.

Maximum engine speed was assumed in both cases, the author admitted the severity of the tests, attributing the first case to bad driving and the second to road conditions, such as, a stop on a steep minor road at a junction with a major road. Two types of vehicle were investigated, a small saloon car and a larger saloon and results in the form of graphs of temperature variation with time, under continuous slipping and also the affects of different gradients presented with some sample calculations. The analysis was discussed and conclusions drawn:

1. Heat generated depended on engine speed.
2. Larger saloon generated higher temperatures under the same loading conditions i.e. on a 1 in 4 gradient after 16 seconds slip gave 66^{o}C for the small saloon against 88^{o}C for the large saloon.

The author also pointed out that the temperatures attained under slipping conditions were much higher than if the clutch were engaged in the quickest possible time and should therefore be considered when designing the clutch. This statement is in conflict with the theoretical analysis of Jania (10) but as pointed out then Jania's system was constrained by the simplifying assumptions made and did not fully describe what happens in practice.

Haviland (17) used a test rig comprising engine, automatic gearbox, inertia flywheel and dynamometer, to study the affects of operating parameters on clutch surface temperature, during gear changing. He inserted thermocouples into the steel clutch plate of a multi-plate wet clutch, the signal being extracted from the rotating components using silver slip rings and brushes, which gave an overall estimated accuracy of ± 5^{o}C. From his experiments the author found that dynamometer load had little effect on surface temperature for a wide range of oil sump temperatures but that as, engine throttle setting was increased the surface temperature increased linearly up to an oil sump temperature of 120°C and there after exponentially, resulting from excessive slip. Which agrees in principle with the conclusion Newcomb came to for dry clutches, that the higher the engine speed the higher the temperatures attained. Haviland also compared calculated values of energy dissipated during the engagement period and found good agreement with practical results. Clutch plate temperatures in the regions 50 - 400°C were measured during the tests even though the oil sump temperature was constrained in the range 33 - 155°C.

In a computer orientated approach to the problem of thermal failure of clutches, in "off road" application of heavy vehicles, Dundore and Schneider (18) produced a computer program which predicted the rate of energy dissipation during the slipping period. Account was taken of engine transients, the torque convertor, power transmission and vehicle load, the clutch being a multi-plate wet type. Using this data the authors solved numerically the one dimensional unsteady state heat conduction equation:

$$\rho . c_p \frac{\partial \theta}{\partial t} = k \frac{\partial^2 \theta}{\partial x^2}$$

assuming the following boundary equations to apply:

$$\frac{\partial \theta}{\partial x} = - \frac{q}{kA} \quad \text{at the interface}$$

and $$\frac{\partial\theta}{\partial x} = 0 \text{ at the mid-point}$$

The first boundary condition stating that the heat generated is conducted away from the interface and the second that heat input from each interface is the same i.e. the temperature distribution in the plate is symmetrical about the mid-point.

The numerical technique they employed was a finite element one, which readily accepted the data from the other program. Results obtained for interface temperature were correlated with results from different test rigs upon which various rates of energy input could be simulated. From these correlations the authors were able to plot graphs from which, it could be seen above what temperature, failure of the clutch was likely to occur. They suggested that these results would enable designers to acquire a fuller understanding of the thermal aspects of multi-plate wet clutches in automatic transmissions.

Hermans (19) in a technical analysis of heavy duty two plate dry friction clutches examined torque capacity and energy absorption and correlated the analysis with vehicle test data. He considered clutches as fitted to commercial vehicles with engines of 190 KW plus used in on and off road applications. A vehicle (52600 Kg GVW) was instrumented to record engine speed, torque and output speed, from tests carried out on level roads he calculated a bulk temperature rise of 4.3°C. By comparison a theoretical study of a start on a severe gradient, assuming 20 seconds slip time and average engine output of 210 KW but all other data constant, produced a bulk temperature rise of 200°C. Thus illustrating that with an initial temperature of 60°C (normal running temperature) the resulting overall temperature would be high enough to cause severe deterioration in the organic facing materials used on these types of clutches, especially when considering that the temperatures reached on the surface would be much greater. In clutch design the author advocated the use of a parameter, called Start Ability, defined as:

$$S.A = \frac{T \times R \times E \times 1200}{Rad \times GVW} - R_{rg}$$

where	T	=	Engine torque at 800 revs/min in ft.lbf.
	R	=	Transmission overall gear ratio
	E	=	Transmission efficiency (85%)
	Rad	=	Rolling Radius
	GVW	=	Gross Vehicle Weight lbf.
	R_{rg}	=	Rolling resistance (1%)

Tests were carried out on two vehicles of different G.V.W in the first four gears and gave a range of start ability from 5.0% to 18.5%. A graph of energy absorbed by the clutch plotted against start ability was presented and showed that the start ability decreased, non-linearly, with increasing energy absorption. From experience a start ability of 10% was considered to be a good compromise which resulted in what was called a comfortable start.

The author concluded with a general discussion of lining materials and the factors affecting the smooth operation of clutches, suggesting that high energy level materials may be used to increase life and reduce pedal efforts.

Kulev (20) produced an analytical solution to the dynamic model,

Figure No. 2.1, and developed the following equations. For the work done during an engagement.

$$W.D.tot = \int_{o}^{ts} T_e \, \omega_e \, dt - \int_{o}^{ts} T_c \, \omega_v \, dt$$

In order that the integration could be performed the variation of the parameters T_e, ω_e and ω_v, with time had to be known, from practical tests the following equations were obtained:

$$T_c = T_{c_{min}} \left(\frac{t}{ts}\right)^m$$

$$\omega_e = \omega_{e_{max}} \left[1 - p. \left(\frac{t}{ts}\right)^L \right]$$

where $$P = \left[1 - \frac{\omega_e \text{ at}_{synch}}{\omega_{e_{max}}} \right]$$

and ω_r obtained from the integration of the dynamic equation governing the acceleration of the vehicle

$$\frac{T_c - T_v}{I_v} = \frac{d\omega_v}{dt}$$

Initially the author considered the simple cases of $m = L = 1$ but developed the analysis to the more general case of $0 < L < \infty$ and $0 < m < \infty$. He developed empirical formulae from a large number of practical results in which the parameters T_e and ω_e were defined.

$$\text{Work done} = t_s \, T_{c_{max}} \left((0.68-0.183P)\omega_{e_{max}}\right.$$

$$\left. - \frac{t_s}{I_v}(0.226T_{c_{max}} - 0.405T_v)\right)$$

where $$t_s = \frac{I_v \, \omega_{e_{max}} (1 - P)}{0.68 \, T_{c_{max}} - T_v}$$

As the empirical formulae were obtained for a particular vehicle the author recommended the use of the more complex solution where L and m are considered to be in the range $0 \rightarrow \infty$.

Mean surface temperatures were also calculated by the author using:

$$\theta_{1,2} = \frac{\text{W.D.} \times 9.808 \, \alpha}{t_s \times A_{1,2} \times \left(\dfrac{K_1}{t_1 \theta^*_1} + \dfrac{K_2}{t_2 \theta_2^*}\right)}$$

where $$\theta^*_{1,2} = \frac{1}{3} t_n + F_{01,2} t_A$$

t_a and t_n are time constants of power and frictional work done in slip derived under the assumptions:

1. No heat transfer from the free surface to ambient.
2. Heat flow linear in axial direction normal to friction surface.
3. Ambient temperature constant and zero.

In conclusion the author demonstrated the accuracy of the analysis against experimental data and suggested that allowance should be made for flash temperatures and proceeded to carry out the necessary analysis. He said that the formulae derived were sufficiently accurate to show the nature of temperature change and that flash temperatures are about 30% of the general temperature.

The proceeding papers have demonstrated the development of theoretical analysis related to dry friction clutches, from a beginning of a purely analytical approach to the problem by Newcomb and to the empirical techniques used by Kulev. These authors recognised the enormous practical problems in performing the theoretical analysis of the system, Kulev using measurement of the time varying parameters in order to reduce the need for simplifying assumptions and Newcomb examining the affect these assumptions have, thus knowing the limitations of the theory. The model, figure 2.1, gives two first order differential equations the analytical solution of which, providing the time varying parameters are known, can be performed, as some of the above authors have shown, but one of the major stumbling blocks is the variation of clutch torque during the engagement period. The classical formula for the torque transmitted by a single plate friction clutch is:

$$T_c = 2\mu.P.\bar{R}$$

$\overline{R}$ is the effective radius i.e. the radius at which the friction torque is assumed to act. Examining this formula and relating it to the practical situation it will be found that during the engagement period the coefficient of friction μ and the axial load P vary, thus opening two avenues for consideration, the first into a theoretical treatries of coefficient of friction and the second into clutch pedal manipulation by the driver. In the second case no one has published the results of a detailed study into driver behaviour during the engagement period although some of the above authors have carried out simple tests in order to define variation in torque transmitted.

Newcomb (11) in his theoretical analysis of clutch engagement, observed from rig tests that for many materials the variation of coefficient of friction with slip speed could be approximated by the following relationship.

$$\mu = \mu_{\infty} + (\mu_s - \mu_{\infty}) \exp (- a.z)$$

Where a is an experimentally determined constant for a particular material; but the interface temperature of the rubbing bodies also affects the coefficient of friction.

The classical laws of friction are:

1. Coefficient of friction is independent of area.
2. Coefficient of friction is independent of load.

These relationships were observed initially by Leonardo da Vinci and later by Amontons and are usually given the name Amontons Laws. These laws take no account of sliding speed or temperature affects but have formed a base from which many researchers have started, notably Bowdon, Tabor and Kragelskii. Consider the diagram, Figure No. 2.4, which represents an exaggerated view of the contact between two smooth materials, when a load is applied the contact points come under greater pressure, usually large enough to create elastic and plastic deformations of the materials. When sliding occurs these bonds are forceably torn apart and as sliding continues the bonds are being continually formed and broken, in addition when sliding the asperities plough into each other especially where one surface is harder than the other. This phenomenon does not occur solely on a molecular scale, the junctions can be quite large and the materials are affected to an appreciable depth. A purely theoretical analysis of this mechanism would be very complex indeed and fraught with practical problems especially in the determination of physical characteristics of a friction material.

The complexity of the theoretical analysis (27) has led researchers into this field, applied to friction materials, to revert to extensive testing and empirical techniques. Papers by Jenkins, Newcomb and Parker (21), Muzechuk (22) and Hatch and Goddard (23) have discussed test machines and methods of testing friction materials in the form of test pieces rather than as full sized components. The correlations between theoretical results, results from test samples and in service use have been built up for design purposes but in order to do this an enormous amount of test work was needed.

Heap (24) has considered the problem of coefficient of friction and wear of two bodies rubbing and in order to reduce the mathematical complexity he suggested a procedure for describing the coefficient of friction and its variation using a purely empirical technique. He proposed the splitting of the dynamic coefficient of friction into a number of coefficients of which all except the static, were normalized to their respective reference points, thus making it possible to write the dynamic coefficient of friction as follows:

$$\mu(P,Z,A,\theta\ldots) = \frac{[\mu(P)] \times [\mu(Z)].[\mu(A)].[\mu(\theta)]\ldots}{\mu_s\ \mu_a\ \mu_\theta\ \ldots\ldots}$$

Each of the coefficients being determined experimentally and described by a polynomial i.e.

$$\mu(P) = \mu_o + \mu_1 P + \mu_2 P^2 + \ldots\ldots$$

$$\mu(Z) = \mu_s + C_1 Z + C_2 Z^2 + \ldots\ldots$$

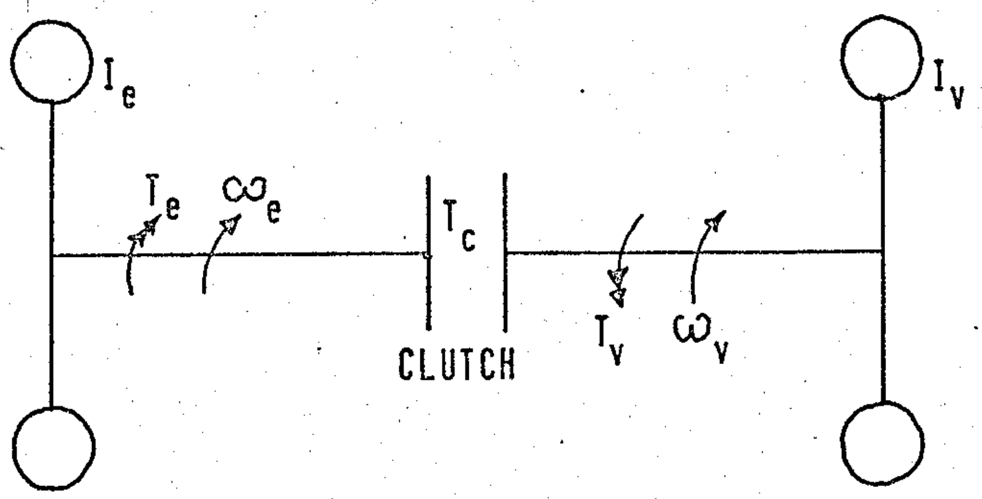
The author suggested that this method of presenting frictional data would benefit the designer who would then have a knowledge of the manner in which the coefficient of friction varied.

Herscovici (25) put forward a method for determining the static and dynamic coefficient of friction using a test rig comprising of a clutch assembly, where one shaft was fixed and the other connected to a torque arm. The torque was applied by allowing a weight to free fall through a known distance before applying a load to the torque arm. The torque was measured by the strain gauged fixed shaft which meant the only variable was the coefficient of friction which could be calculated using the formula for torque transmitted by a clutch. This rig has one major disadvantage in that it is not capable of measuring the coefficient of friction at higher velocities but the author says this method of determining static and dynamic coefficients of friction is useful for comparison of different materials and in the drawing up of operating characteristics of friction materials for designers.

Bunda, Fujikawa and Yokoi (26) studied the details of the frictional characteristics of friction materials with particular attention to the low speed area. They discovered the presence of a thin visco-elastic film at the rubbing surface which greatly affected the coefficient of friction. This film was very susceptible to changes in temperature, humidity, previous history of use and also to momentum. This film could help to explain the none repeatability of friction test results.

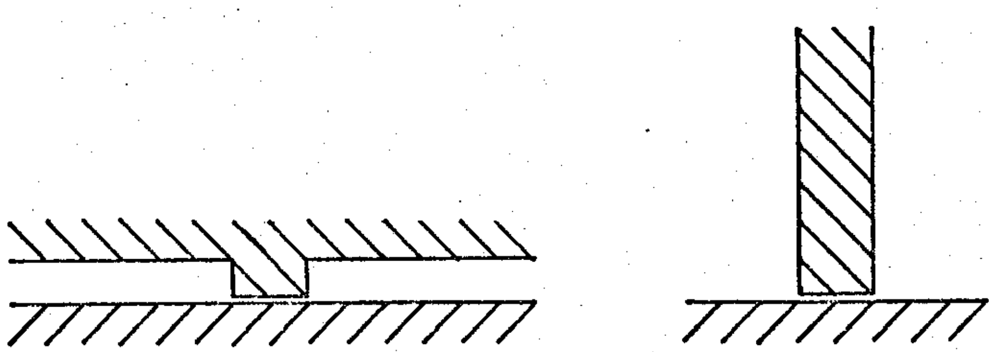
Garg and Rabins (28) also noted the appearance of a film on the friction surfaces, which was attributed to the softer material being worn and the wear particles deposited on the harder material. After extensive tests they suggested that fade as experienced under heavy usage of either brakes or clutches was due to this film breaking down. They proposed this explanation rather than that of charring of the lining surfaces causing the fade and its eventual removal by wear allowing recovery. They found that if during low work load slipping, carbon tetrachloride was poured over the rubbing surfaces the coefficient of friction was reduced rapidly but that it slowly recovered as slip was continued.

The proceeding papers indicate a need for a comprehensive study of the problem of clutch engagement to be made, including a theoretical approach and practical tests which would enable realistic studies of expected temperature at the design stage. Account should be taken of different driving techniques and different operating conditions.



MATHEMATICAL MODEL OF A VEHICLE

FIGURE NO. 2.1



MOVING HEAT SOURCES

FIGURE NO.2.2

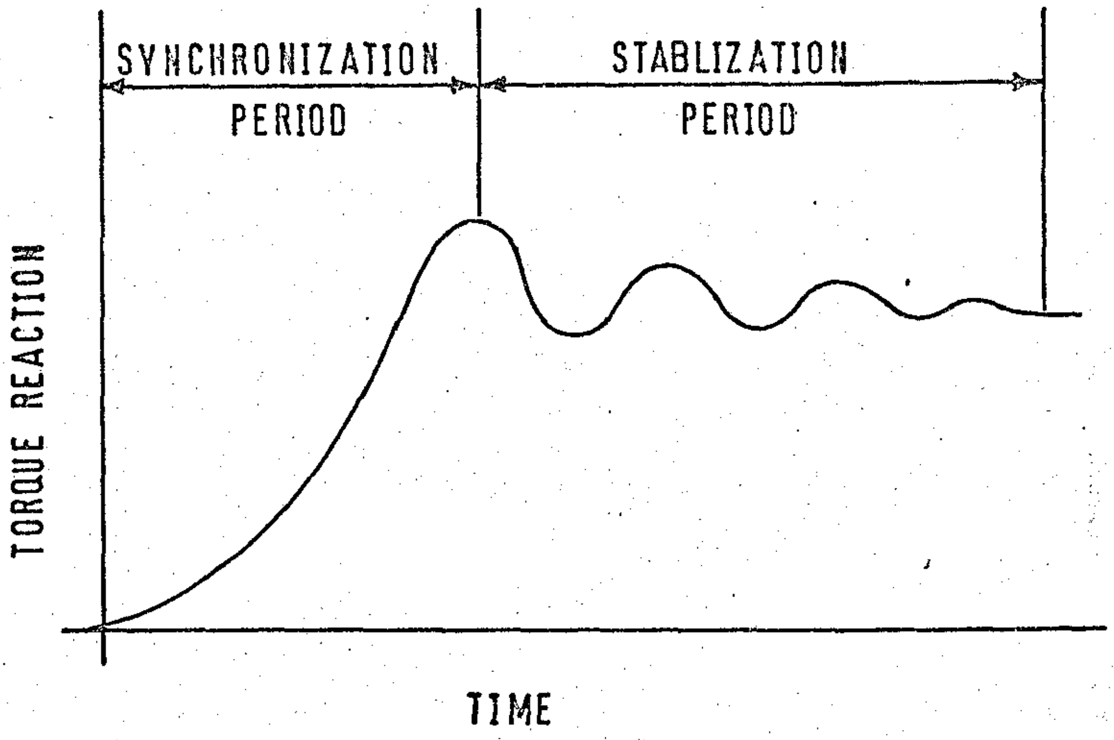
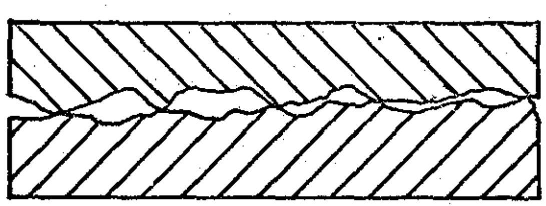


FIGURE NO. 2.3



MAGNIFIED CONTACT INTERFACE

FIGURE NO. 2.4

CHAPTER 3

MATHEMATICAL MODEL OF ENGAGEMENT

INTRODUCTION

Previous work in this field has highlighted the many problems encountered during the analytical solution of the dynamical behaviour of the clutch system. Figure No. 1 shows the mathematical model adopted for this computer orientated approach to the problem. The choice of a digital computer model was arrived at by considering the defects in the analytical techniques i.e.

1. Versatility
2. Ease of use
3. Application for further use.

Versatility; if a complex analytical equation were evolved for a particular set of conditions and assumptions a small change in the laws governing the approach i.e. a change of assumption to make the model more valid would require a major re-think on the model. By using a digital technique all that would be required is the insertion of a subroutine to take care of the assumption.

Ease of use; one of the aims of this work was to produce a useful tool, a digital computer program kept on computer file may be accessed readily by an engineer who has little knowledge of the mathematics of the program and still less of computers.

Application for further use; the end product was required to be capable of extension into a much broader study of the vehicle drive-line. All the above points could have been covered using a sufficiently large analogue computer, had one been available, but the solution of temperature distributions would have necessitated the use of a hybrid computer system or of splitting the model into two separate programs. The following approach therefore was aimed at the production of a digital computer program which initially suffered from some of the simplifying assumptions as did the analytical approach, but as the research progressed these assumptions were investigated and the model improved.

THEORETICAL APPROACH

Dynamic Equations

Figure No. 3.1 shows the model considered, comprising an engine delivering a torque, dependent on the rotational speed.

$$T_e = f(\omega_e)$$

The engine, of inertia I_e, is directly coupled to a clutch, considered as having only two rubbing surfaces instead of four. The output side of the clutch is coupled through a gearbox to a vehicle of inertia I_v. The vehicle inertia and drag torque are referred through the rear axle ratio for ease of presentation.

At any instant in time the engine inertia and the vehicle inertia will be accelerating (or decelerating), therefore the following differential equations are applicable.

$$\frac{\partial \omega_e}{\partial t} = \frac{T_e - T_c}{I_e} \qquad \text{.............................3.1}$$

$$\frac{\partial \omega_v}{\partial t} = \frac{T_c . R_g - T_d}{I_v} \qquad \text{..........................3.2}$$

$$= R_g . \frac{\partial \omega_v}{\partial t} + \omega_v . \frac{\partial R_g}{\partial t}$$

in the case of an infinitely variable gearbox.

where

$T_e \quad = \quad f(\omega_e)$

$T_d \quad = \quad f(\omega_v, I_v)$

$T_c \quad = \quad f(P, \mu, \text{clutch geometry})$

Initially the engine torque delivered was assumed constant but the drag torque was calculated,see Lucas (34), and the clutch torque calculated using the formulae developed assuming the constant wear hypothesis.

$$T_c = \mu.N.P. (R1 + R2)/2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 3.3$$

Where P is the clamp load at the instant in time considered and which has previously been considered to be full clamp load throughout the engagement, which means that the clutch pedal was released in zero time. The coefficient of friction μ has also been assumed constant previously except by Newcomb who after practical investigation gave the following pattern of variation, which was initially adopted in this model.

$$\mu = \mu_\infty + (\mu_s - \mu_\infty) \exp (-a.Z) \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.4$$

The model was developed using the assumption that the clamp load was applied in zero time but the option was given to change this by using the following differential equation

$$\frac{\partial P}{\partial t} = f(t) \qquad \text{.............................3.5}$$

The heat generated at the instant of time under consideration was obtained from:

$$\frac{\partial Q}{\partial t} = T_c.(\omega_e - \omega_v.R_g) \qquad \text{.......................3.6}$$

Equations 3.1, 3.2, 3.5 and 3.6 describe the dynamic system of Figure No. 3.1, and a digital computer program was written to integrate the four differential equations.

This model gave the heat generated in a dry friction clutch as a function of time and the next problem was to model the dissipation of this heat through the components of the clutch assembly and to obtain the resulting dynamic temperature distribution and bulk temperatures attained in the steady state.

Heat Dissipation

Heat is generated over a large surface area in the clutch, therefore the quantity of heat generated during an engagement was assumed to flow only in one direction i.e. axially into the components. Thus the one dimensional unsteady state heat conduction equation could be applied to the problem:

$$\rho.cp \frac{\partial \theta}{\partial t} = k \frac{\partial^2 \theta}{\partial x^2} \qquad \text{.........................3.7}$$

assuming also that the thermal conductivity of the materials remain sensibly constant in the temperature range under consideration.

In order to effect a solution of the above equation certain boundary conditions were required and as the clutch assembly comprises of two friction faces under similar loading conditions the heat generated at each surface was assumed the same. As the heat generated was the same at both interfaces the model only need be concerned with half of the assembly. The mechanism whereby the heat is generated has not been fully explained, and in this treatise a model advocated by Schaff (31) has been used. Figure No. 3.2 shows a diagramatic representation of the rubbing interface in accordance with the sections in Bowden and Tabor (2) and Kragelskii (3) the heat is generated on or at the interface, although some doubt exists as to how this generation takes place. The two rubbing surfaces, because they are not smooth, are separated by a thermal resistance, comprising of air, wear particles, etc. Hence the heat is able to flow in two directions:

1. By conduction, into the adjoining material.
2. By some mixture of conduction, convection across the interface resistance.

Therefore:

$$\text{Heat flow into the material} = k.A. \frac{\partial \theta}{\partial x} \qquad \ldots\ldots\ldots\ldots 3.8$$

$$\text{Heat flow across the interface} = +h_I \frac{\partial \theta}{\partial x} \qquad \ldots\ldots\ldots\ldots 3.9$$

The magnitude of the two sources depends upon the thermal constants of the two rubbing materials Kragelskii (3) and is defined as:

$$Q_{g2} = Q_{gT} (1 - \lambda)$$

$$Q_{g1} = Q_{gT}\lambda$$

where

$$\lambda = \frac{k_1}{k_2}\sqrt{\frac{\alpha_2}{\alpha_1}}$$

Therefore the total heat flow into the materials surface by conduction is:

$$Q_{g1,2} + h_I \frac{\partial\theta}{\partial x}$$

Considering the back face of the pressure plate or flywheel, heat is lost to the atomosphere by forced convection, therefore:

$$\text{Heat to the atmosphere} = -h_f.A. (\theta_w - \theta_{f1}) \quad \text{............3.10}$$

To complete the solution, the boundary condition at the geometrical centre of the centre plate is required. The two anulii of friction material are separated by thin leaves of steel, called cushion segments, which help reduce wear and violent clutch engagement. As the coefficient of heat conduction of the friction material is always in the region of 1/40th of that of steel and because the thickness of these slivers of steel is 1/5th that of the friction material the dynamic temperature distribution in the springs can be ignored in comparison to the friction material. Thus, the only two components which require consideration are the pressure plate or flywheel, and its associated friction material partner. As the heat flowing from each interface was assumed the same, there is no heat transfer across the geometric centre of the centre plate and thus the following equation holds:

$$Q = 0 = k.A. \frac{\partial \theta}{\partial x}$$

or

$$\frac{\partial \theta}{\partial x} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 3.11$$

Using the above boundary conditions equation (3.7) may be solved providing that the total rate of heat generation at the interface is known. This being readily available from the solution of the dynamic equations outlined in the first section of this chapter.

The decision having been made to opt for a digital computer solution to the problem left two techniques available for solution of the partial differential equation describing heat conduction:

1. The splitting of the partial differential into a number of first order ordinary differential equations using finite difference techniques and solving using similar numerical techniques to those used to solve the dynamic problem. This

suffered from the disadvantage that the solution of a large number of ordinary differential equations was required and if extensions to further dimensions were considered, the number of equations required increased by the square of the number of mesh points considered.

2. The method chosen was the splitting of the partial differential equation into a series of linear simultaneous equations, again using finite difference techniques. This method being logically extended to two or more dimensions if required and allowing the number of mesh points to be easily varied.

Finite Difference Techniques

When a study of the types of F.D. techniques available was made the object was to find a method of solution which was economical to use and was also easily modifiable. With this in mind two different approaches were used:

1. An explicit technique
2. An implicit technique

A finite difference technique relies on the approximation of differentials by gradients (Taylor Series).

Therefore:

$$\left.\frac{\partial \theta}{\partial x}\right|_{t=j} = \frac{\theta_{i+1,j} - \theta_{i,j}}{\Delta x}$$

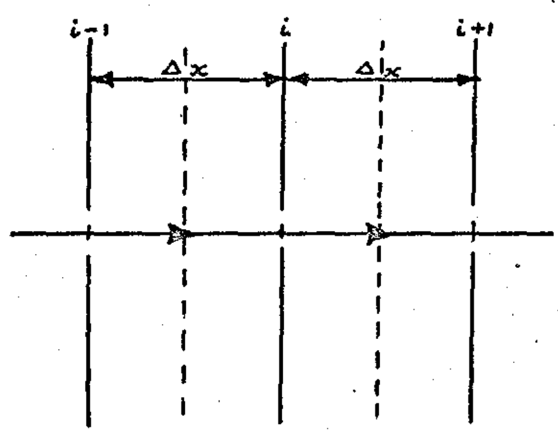
$$\text{and} \qquad \left.\frac{\partial^2\theta}{\partial x^2}\right|_{t=j} = \frac{\dfrac{\theta_{i+1,j} - \theta_{i,j}}{\Delta x} + \dfrac{\theta_{i-1,j} - \theta_{i,j}}{\Delta x}}{\Delta x}$$

$$= \frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i,j}}{\Delta x^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.12$$

The differences between these two techniques can be explained with reference to Figure 3.3, which is a diagram of a finite difference mesh with time progressing along the y-axis and distance into the material along the x-axis. The temperatures at the intersection of the mesh lines are those required for solution of the equations 3.12.

Explicit Technique

The explicit method requires the temperatures at stations (1,1), (1,2), (1,3) to effect a solution at station (1,2). To derive the general equation for heat conduction in a solid consider the elements shown



from Heat flow $= -k.A. \dfrac{\partial\theta}{\partial x}$ using finite differences

$$\text{Heat flow into element } i = -\frac{A.k.(\theta_{i,j} - \theta_{i-1,j})}{\Delta x}$$

$$\text{Heat flow out of element } i = +\frac{A.k.(\theta_{i,j} - \theta_{i+1,j})}{\Delta x}$$

Heat stored in the ith element in time t is

$$= \frac{\Delta x.A.\rho.cp.(\theta_{i,j+1} - \theta_{i,j})}{\Delta t}$$

but assuming no losses, Ht. stored in ith element in time Δt =

Ht inflow - Ht outflow

or

$$\frac{\rho.cp.}{\Delta t}(\theta_{i,j+1} - \theta_{i,j}) = -\frac{k}{\Delta x^2}(\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j})$$

which simplifying gives:

$$\underline{\underline{\theta_{i,j+1} = F_o.\theta_{i+1,j} + (1 + F_o)\theta_{i,j} - F_o.\theta_{i-1,j}}}$$

where $$F_o = \frac{\Delta t.k}{\rho.cp.\Delta x^2}$$

This equation can be expanded to cover each mesh point in turn inside of the boundaries but on the boundary requires modifying due to the relevant boundary equation previously mentioned. The full derivation of the equations is explained in Appendix 1. This technique has the advantage of giving the temperatures at the next time interval directly without very much calculation and also, all that is needed to start the method is the previous temperature history and the boundary equations. The technique does however suffer from mathematical instability if the following condition is not met during solution.

$$1 \geqslant \frac{\Delta t . k}{\rho . cp . \Delta x^2}$$

or $$\alpha \Delta t \leqslant \Delta x^2 \quad \text{where} \quad \alpha = \frac{k}{\rho . cp}$$

and is constant for a given material.

Hence the time interval and mesh size need to be carefully chosen.

Implicit Technique

The implicit method requires the temperatures at stations (1,1), (1,2), (1,3), (2,1) and (2,3) to effect a solution at (2,2). If equation 3.7 the one dimensional unsteady state heat conduction equation is approximated using finite differences it becomes:

$$\rho . cp \; \frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t} =$$

$$\frac{k}{2} \cdot \frac{\theta_{i+1,j+1} - 2\theta_{i,j+1} + \theta_{i-1,j+1} + \theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{\Delta x^2}$$

Which in words is the heat stored in the element in time Δt is the average of, the heat being conducted into minus the heat being conducted out of the element at time "t" and "t + Δt" and reduces to:

$$-F_o\theta_{i+1,j+1} + 2(1 + F_o)\theta_{i,j+1} - F_o\theta_{i-1,j+1}$$

$$= F_o\theta_{i+1,j} + 2(1 - F_o)\theta_{i,j} + F_o\theta_{i-1,j}$$

Again the equation can be applied at each mesh point in turn with application of the boundary conditions where relevant, a full derivation is provided in Appendix 1.

Thus giving "n" simultaneous linear equations, in "n" unknown where "n" is the number of mesh points chosen. This method is theoretically stable for all values of F_o although inaccuracy is kept to a minimum if F_o remains low.

Two programs were written using the two techniques and the same values for the temperatures reached during the engagement period were obtained. Table No. 3.1 shows various values obtained under different engagement conditions The analytical theory used here was that of Newcomb (12) and the first two rows of figures show that when the digital simulation was constrained similarly to the analytical theory the results were in agreement. These results were obtained assuming the engine was delivering maximum torque during the engagement and that the clutch clamp load was engaged in zero time. As this was not the case in practice, modifications were made to the program in order that the application of the clamp load could be made over a period of time, the method of application following a linear ramp. The next three rows of figures show the affect of varying this ramp time i.e. the longer the linear ramp the more heat generated and hence the higher the interface temperature.

However, the engine cannot deliver the maximum torque during the engagement but only a proportion of it, the second half of the table shows the effect of halving the torque delivered by the engine. The slip times were reduced and so were the heat generated and the resulting temperature rise. This suggested that the theory was not complete and that certain areas needed practical investigation.

These areas were:

1. Driver control of the clutch pedal and throttle
2. Coefficient of friction variation with operating conditions.

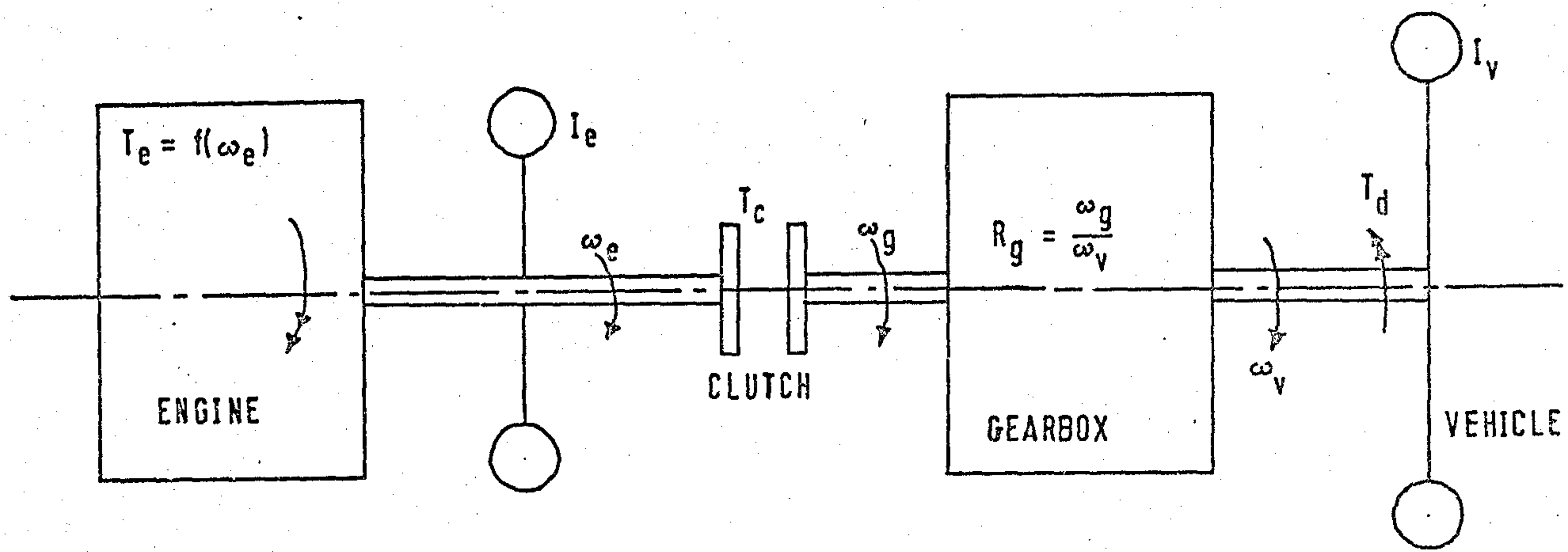
ENGINE TORQUE ASSUMED CONSTANT AT 100 NM

APPLICATION	SLIP TIME (Secs)	HEAT GENERATED (J)	MAX TEMP (Deg C)
Analytical Theory	0.352	5.87	59.0
Digital Simulation	0.350	5.86	57.0
RAMP TIME 1.0 SECS	1.17	19.26	118.0
RAMP TIME 0.5 SECS	0.79	12.00	90.0
RAMP TIME 0.2 SECS	0.52	8.24	72.0

ENGINE TORQUE ASSUMED CONSTANT AT 50 NM

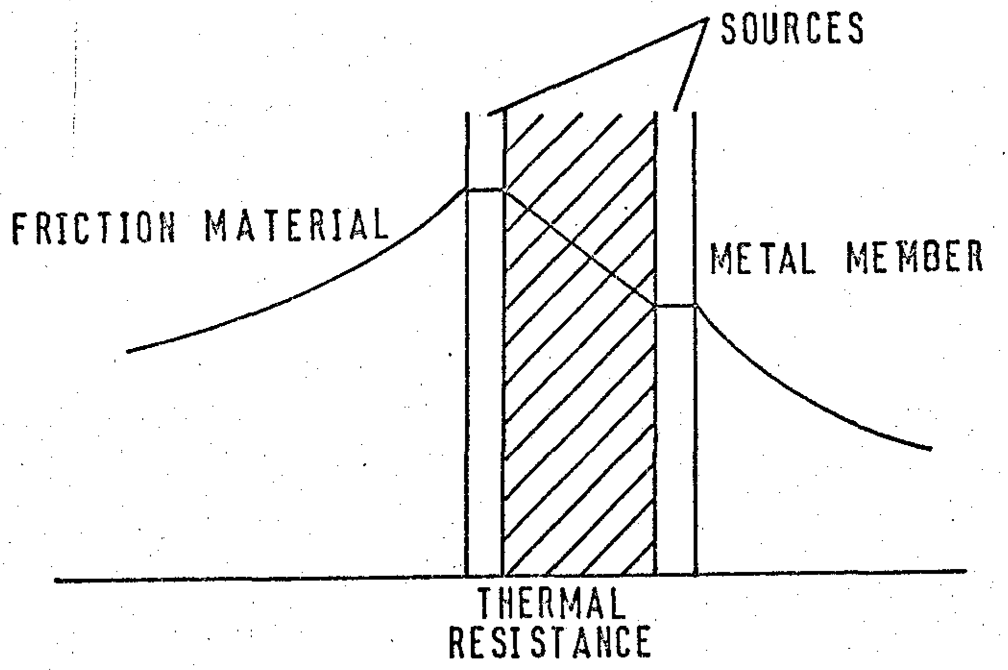
Analytical Theory	0.26	4.33	51.0
Digital Simulation	0.26	4.34	47.0
RAMP TIME 1.0 SECS	0.86	8.0	57.0
RAMP TIME 0.5 SECS	0.56	6.31	54.0
RAMP TIME 0.2 SECS	0.37	5.17	49.0

TABLE NO. 3.1



MATHEMATICAL MODEL OF CLUTCH ENGAGEMENT

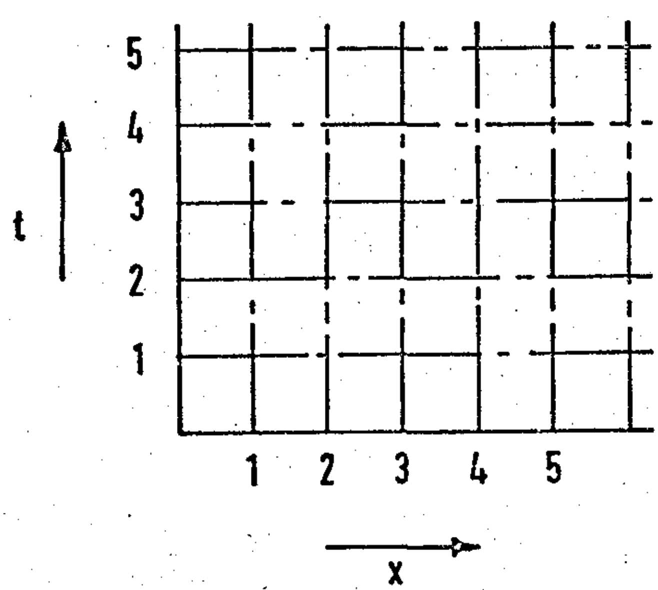
FIGURE NO.3.1



DIRECTION OF POSITIVE HEAT FLOW

MODEL OF THE INTERFACE

FIGURE NO. 3.2



FINITE DIFFERENCE MESH

FIGURE NO. 3.3

CHAPTER 4

PRACTICAL INVESTIGATION INTO THE MATHEMATICAL MODEL

In the theoretical appraisal of the mechanism of clutch take-up, two areas requiring more practical investigation were highlighted. Firstly, the variation in coefficient of friction during the engagement period and secondly, the clutch operating characteristics of a driver. This chapter deals with the practical test work carried out in order to gain an insight into these rather murky areas of the mathematical model. During the investigations it soon become apparent that two seemingly endless avenues of research had opened. The measurement of coefficient of friction and its variation with operating conditions has hampered workers for a long time and no analytical technique has succeeded in predicting levels of friction accurately. In fact, within the industry the non-repeatability of tests carried out on friction materials is well known though not fully understood. This fact, added to the problem that in service the clutch is operated by a human being means that, "in the vehicle", tests are used purely as comparators between materials. The human variant being minimized by using trained test drivers, but when dealing with the general public their behaviour appears to be random. It was necessary therefore, to call a halt at an appropriate point, even though the work was not fully completed, but suggestions will be made in a later chapter with respect to extensions along these avenues.

Variation of Coefficient of Friction during clutch operation

The manufacturers usually publish data sheets for their materials which contain graphs of variation in coefficient of friction with respect to relevant parameters but these graphs are only meant as a guide to the designer. The data being obtained from test work carried out on small specimens with specialised test machines and the results obtained although repeatable often bear no resemblance to results experienced in service. As, from the onset of this research a mathematical model simulating an "in service" condition was envisaged it was deemed necessary to obtain values of coefficient of friction from the full size specimen i.e. the actual type of clutch used in the test vehicle. The results had to be in a form readily usable by the digital computer program simulation i.e. as polynomials defining the lines on a graph.

As the operation of a clutch in practice takes, under normal operating conditions, in the region of two to three seconds, a rapid method of data acquisition was required when testing under operating conditions. Throughout the test work performed on this rig a mini computer operating as a high speed data logger was used, but this will be explained fully in connection with the more sophisticated vehicle simulation rig described in the next chapter.

Tests for variation in coefficient of friction

As these tests required the interface temperatures during the engagement period a number of thermocouples were inserted in the lining material in addition to one in the pressure plate and one in the flywheel, both at the surface and on the geometrical mean radius of the lining materials. In this way a comparison of measured and calculated temperatures could be made. The test rig used was as outlined schematically in Fig. 4.1 and was a modification of an existing test rig normally used for brake testing. The rig consisted of an electric motor directly connected to a flywheel, the inertia of which was larger than that of the vehicle, but the only affect this had was to prolong the slip times. The flywheel of a clutch assembly i.e. clutch input, was fixed to the end of the shaft from the electric motor and the centre plate i.e. output of the clutch was joined to a stationary shaft connected to ground through a strain ring (used for measurement of output torque).

Fig. 4.2 shows the cushion segments which separated the two lining materials on the centre plate and which transmitted the friction torque to the hub of the centre plate, from the lining materials. As the clamp load was applied the cushion segments deflected until at full clamp load the segments became flat. The segments, affect the engagement characteristics as indicated by Fig. 4.3. This is a graph of clamp load against pressure plate deflection and shows that the step in the graph is flattened due to the action of the cushion segments thus a smoother engagement can be achieved and less lining material wear experienced in practice. Before full clamp load was applied to the segments the pressure over the area of lining material covering the segment varied, which meant that the friction torque transmitted was done so by a smaller area than normally assumed which also suggested that the temperatures would vary greatly according to whether the pressure was high or low at the point of contact. As the centre plate in this test rig was fixed to a stationary output shaft it was possible to insert numerous thermocouples into the lining materials to try and measure the surface temperatures and their variation across the surface of the cushion segments. In addition to these temperatures, the surface temperatures of the flywheel and pressure plate were also measured, but as these components were rotating, during a test, it was necessary to bring the signals out using a different mechanism than for the centre plate.

A system of telemetry was developed in order to obtain the thermocouple signals from the rotating shaft. The signal from a thermocouple was D.C. amplified, and then converted into a frequency within the audio range, this frequency was then used to modulate

70 mega HZ carrier frequency, which was then transmitted from the rotating shaft. A modified F.M. radio receiver was used to monitor the transmitted signal, which was subsequently decoded, fed through a high speed frequency to voltage convertor to the data logger.

The rotational speed of the motor was measured by an inductive pick-up sensing the pulses caused by a 60 toothed wheel fixed to the shaft of the electric motor, the pulses were fed through a translation circuit to give an analogue voltage proportional to speed. The applied clamp load was measured by a strain gauged link in the clutch operating mechanism, the strain in this link having been calibrated statically against a load cell inserted between the clamping faces, in a modified centre plate.

To summarise, it was possible on the rig to record the following parameters during a test.

1. Clutch Torque
2. Clamp pressure
3. Flywheel and pressure plate surface temperature
4. Slip Speed
5. Two centre plate temperatures.

Test Objective

From a number of tests the temperature distribution through the clutch and its variation with repeated engagements could be observed. But the main purpose of the rig was to study the variation of coefficient of friction with clamp pressure, surface temperature and slip velocity and because the aim of these tests was to produce data for use in the mathematical model the results had to be in graphical form which could be described by polynomials. The method suggested by Heap (24) was used to describe the variation of coefficient of friction, the equation being in the form:

$$\mu = \frac{\mu_o . \mu(p) . \mu(\theta) . \mu(v)}{\mu v . \mu\theta . \mu p} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 4.1$$

where $\mu(p)$, $\mu(\theta)$ and $\mu(v)$ are polynomials describing the variation of coefficient of friction with clamp pressure, temperature and velocity respectively, each applying when the other two variables remain constant.

Test Procedure

In order to obtain the way in which coefficient of friction varied with respect to one parameter as the other two were kept constant; on a test rig where it was only possible to keep one parameter constant, i.e. clamp pressure. It was necessary to allow the other two, to vary over a wide range and interpolate from the graphical results, to obtain the results in the required format.

The tests consisted of setting the manual stop on the clutch release mechanism so that the clamp pressure was at the desired level, the clutch was disengaged and the electric motor run up to speed. When running at full speed the motor was switched off, the clutch engaged rapidly by hand and the data acquisition commenced. This was repeated at predetermined time intervals until the temperature of the lining attained around 150^{o}C. Above this temperature the lining material would be permanently damaged, thus conditioning the material and affecting following friction levels considerably. During the cooling of the assembly the static coefficient of friction at set temperatures was measured by means of a torque arm and weights, the torque arm being rigidly fixed to the engine flywheel of the clutch assembly and weights being progressively hung on the end until slip occurred. This procedure being repeated at various levels of clamp pressure, after allowing the whole assembly to cool to ambient temperature again. From these tests resulted a series of graphs similar to Graphs No. 4.1 and 4.2, at the various clamp pressure used throughout the tests and also the variation of static coefficient of friction with pressure and temperature was noted. The next stage was to reduce these graphs into the required form for use in equation 4.1.

Reduction of Results and Results

By cross plotting Graphs Nos. 4.1 and 4.2, a graph of coefficient of friction against surface temperature for different constant slip speeds was obtained, Graph No. 4.2. Also shown on this graph is the variation of static coefficient of friction with temperature.

From Graphs Nos. 4.1 and 4.2, by similar means, the variation of coefficient of friction with slip speed for constant temperatures was obtained, Graph No. 4.4, this procedure being repeated for each test pressure.

The following arbitrary reference points were chosen:

Temperature	$30^{o}C$
Velocity	2m/s
Pressure	15 KN/m^2

Graphs of variation in coefficient of friction with temperature, Graph No. 4.5, with velocity, Graph No. 4.6, and with pressure Graph No. 4.7, where obtained by cross plotting. In each case the other two parameters were kept constant at their respective reference points. Each one of these graphs was represented by a polynomial:

$$\mu(\theta) = A_o + A_1\theta + A_2\theta^2 + \ldots \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.2$$

$$\mu(v) = B_o + B_1 v + B_2 v^2 + \ldots \quad \ldots\ldots 4.3$$

$$\mu(P) = C_o + C_1 P + C_2 P^2 + \ldots \quad \ldots\ldots 4.4$$

μ_o is the static coefficient of friction at the reference conditions and μo, μv and $\mu\theta$ are the normalising coefficients for the polynomials 4.2, 4.3 and 4.4.

$$\mu_o = 0.412 \text{ at } P = 15\ KN/m^2 \theta = 30^oC \text{ and } v = 2m/s$$

$$\mu_P = 0.362 \text{ at } P = 15\ KN/m^2$$

$$\mu_\theta = 0.354 \text{ at } \theta = 30^oC$$

$$\mu_v = 0.361 \text{ at } v = 2m/s$$

These results are shown in Graphs Nos. 4.8 and 4.9.

Therefore equation 4.1 becomes:

$$\underline{\underline{\mu(P.v.\theta) = \frac{0.412.\mu(\theta).\mu(v).\mu(P)}{0.0475}}}$$

where

$$\mu(\theta) = 0.3993 - 0.2025 \times 10^{-2}.\theta + 0.6375 \times 10^{-4}.\theta^2$$

$$- 0.1749 \times 10^{-5}.\theta^3 + 0.2159 \times 10^{-7}.\theta^4 - 0.9295 \times 10^{-10}.\theta^5$$

$$\mu(v) = 0.3748 - 0.8818 \times 10^{-2}.v + 0.12027 \times 10^{-2}.v^2 -$$

$$0.1438 \times 10^{-3}.v^3 + 0.1021 \times 10^{-4}.v^4 - 0.2828 \times 10^{-6}.v^5$$

$$\mu(P) = 0.3333 + 0.1218 \times 10^{-2}.P - 0.1359 \times 10^{-4}.P^2 +$$

$$0.6834 \times 10^{-7}.P^3 - 0.1552 \times 10^{-9}.P^4 + 0.1303 \times 10^{-2}.P^5$$

Which is in a convenient form for use in the digital simulation program, The effect of using this description of the variation of coefficient of friction against what is normally assumed will be examined in a later chapter where the validity of the mathematical model is discussed.

Temperature Measurement

Graphs 4.10 and 4.11 show the flywheel and pressure plate surface temperatures respectively for ten engagements, carried out at five minute intervals, using full clamp load instantaneously applied. The graphs show that the peak temperatures on both surfaces were approximately the same but the pressure plate bulk temperature was higher than that of the flywheel, at the end of the ten engagements, this was attributed to the greater thermal mass of the flywheel conducting the heat generated away more rapidly.

Graphs 4.12 and 4.13 show the temperatures measured by the thermocouples that had been inserted in the lining material at position 1.2, see

Figure No. 4.2 which was an area of lining under medium pressure during light clamp load engagements due to the shape of the cushion segments. Only graphs at this position have been included because on other graphs the values for surface temperature measured varied greatly due to the effects of heat spotting. Although, at position 1.2 the results observed from Graphs Nos. 4.10 and 4.11 were mirrored i.e. the bulk temperature of the flywheel side was lower than that of the pressure plate side, it was felt that the results were clouded by the effects of heat spotting and that only the metal surface temperature gave reliable results.

Under full clamp load engagements there seemed little variation in temperatures measured at the different positions although the incidence of heat spotting was greater at the higher pressures. From the results it was decided to concentrate the temperature measurement on only the metal members surface, in regards to the envisaged work on the vehicle simulation test rig. The computer program simulation of the above test rig produced temperatures which were higher than the measured ones, but the discussion of this discrepancy will be left until a later chapter.

Engagement Characteristics of the Human Operator

Because clutches are operated by human beings it was assumed that the method by which the clutch was manipulated during the engagement period would vary greatly between people of different temperament, age, sex, employment, etc. But in order to make an assessment of these parameters especially the first, a very detailed questionnaire would have been required, and for the small sample study envisaged in this research it was not considered worthwhile. Instead a short questionnaire, Figure No. 4.4, was designed to obtain relevant information about the subjects from which trends may have been clear and to supply information to enable a reasonably average cross section of subjects to be taken. No attempt was made during this research to obtain the perfect "mister average" used by the statistician. Not only were the operating characteristics assumed to vary with people but also with the road conditions i.e. gradient and in addition to this, most human beings, although being quickly adaptable, are not able to change vehicles and continue immediately to drive as they normally would a familiar vehicle. Therefore a test route was choosen which gave the driver time to become familiar with the test vehicle. The test route covered twenty miles,the first ten miles of which was the settling period for the driver, within this period engagements upon hills, level roads and difficult road junctions were encountered which enabled most drivers, by the time the first test site was reached, to have become comfortable with the vehicle. One point which could not be masked was that of the idea of being tested, that is the effect of the subject knowing that he was being tested and driving accordingly. To try and alleviate the effect, the subjects were not informed as to the nature of the measurements until after the test was complete.

Five different road conditions were tested for each subject, these were:

a. A level road engagement under normal traffic conditions.

b. A level road engagement under congested traffic condition, where the su~~o~~bject was asked to propel the vehicle away from rest in the shortest possible time.

c. Slight gradient engagement approximately 1 : 28 under normal traffic conditions.

d. Medium gradient engagement approximately 1 : 17 under normal traffic conditions

e. Severe gradient engagement approximately 1 : 5 under normal operating conditions.

As pointed out in the section on the mathematical model, the method by which the clutch was engaged required defining for the road conditions outlined above and in order to do this a number of parameters required recording during the short time of the engagement period.

<u>Measurement of Required Parameters</u>

When this program of research commenced it was realised that a considerable amount of equipment was required for measuring and recording the data and so a reasonably sized test vehicle was required in order to carry it. In order not to put too great a strain on the test subjects learning capability, it was necessary to choose a popular vehicle which was easy to drive, hence a medium sized (1.5 litre) British estate car was purchased.

As with the test rig previously mentioned the information was transmitted very rapidly, less than 10 secs., thus it was necessary to record all the signals on magnetic tape using a 14 channel F.M. magnetic tape recorder. This is shown in Plate No. 4.1 which views the vehicle interior looking through the tailgate of the vehicle showing the layout of the equipment. In the foreground are two 12 volt heavy duty car batteries connected in series to give a 24 volts output and used to power the recorder, shown directly behind them. To the right is a two channel D.C. amplifier and its battery power supply. All this equipment was secured firmly to the vehicle to prevent shifting during the tests.

Plate No. 4.2 shows the method of measuring propeller shaft speed. A six toothed sheet metal disc was bolted to the differential input shaft flange; looking at the teeth was a photocell. The series of pulses obtained from this system were electronically shaped to a square wave which was recorded and integrated to give an analogue signal, also recorded. The first spike from the square wave was originally intended to trigger the data logging system, used later to reduce the results. This method was found unsatisfactory and replaced by fixing a clutch pedal operated micro switch to the steering column. Switching, discharged a capacitor, giving a 1 volt pulse, each time the clutch pedal activated the switch, Plate No. 4.2. Also shown in this plate is the linear displacement transducer which was fixed between the bulk head and the pedal, to monitor clutch pedal displacement. Engine speed was measured by conditioning the ignition pulses through a similar type of circuit used for the propeller shaft speed but in this case only the analogue voltage was recorded.

Plate No. 4.4 shows the strain gauged first motion shaft used to measure clutch torque. To the left of the strain gauges are the four copper slip rings, two feeding power into the gauges and the other two bringing the strain signal out. Shown below the shaft are the four silver graphite brushes, these were lightly spring loaded to ensure continual contact with the slip rings whilst the latter were rotating. Shown above the shaft is the modified release bearing guid, containing the brush holders with power and signal leads attached. Plate No. 4.5 shows the components as assembled in the bell housing with the release mechanism also in place. Before installation it was necessary to calibrate the system and this was achieved using the apparatus shown in Plate No. 4.6. This consisted of an internal spline into which the external spline of the first motion shaft fitted, the other end being supported by a plain pilot bearing. The internal spline from a clutch centre plate i.e. the hub, was modified and bolted to the torque arm, the complete assembly could be bolted onto a bench thus enabling weights to be gradually added to the arm.

For a number of tests this method of torque measurement was not available and so in order to obtain an estimate of the energy dissipated during the engagement another measure of torque was required. This was accomplished by recording inlet manifold depressions and from performance curves previously obtained for the vehicles engine, a correlation between engine speed, inlet manifold depression and engine torque was formulated. Plate No. 4.7 shows the system of measuring inlet manifold depression, a tee-piece was inserted into the manifold containing an orifice designed to damp out the cyclic pulsations caused by the difference in cylinder pressure between the compression stroke and induction stroke of the engine. The tee-piece was connected to an inductive type pressure transducer, the output of which was fed through one of the two D.C. amplifiers and the resulting analogue voltage was fed directly to the tape recorder.

Plate No. 4.8 is a view looking upwards at the bell housing and gearbox, in situ in the vehicle; the input and output leads of the strain gauged first motion shaft can be seen emerging from the bell housing. Behind those is the clutch release arm which was operated hydraulically through a rod from a slave cylinder. This rod was strain gauged and after suitable calibration provided a measure of clamp load used during the engagement. Also to be seen in this plate are the thermocouple wires used to monitor the bell housing temperature during the test.

To summarise, the following parameters were recorded during the tests:

1. Engine Speed - clutch input speed
2. Propellor shaft speed - clutch output speed
3. First motion shaft torque - clutch torque
4. Clamp load
5. Clutch pedal position
6. Manifold depression
7. Trigger Spike

In addition the bell housing temperature was observed during the test.

Test Procedure

Each test subject was asked to sit in the vehicle and make himself comfortable before driving to the site of the level road test. Under each road condition the subject was asked to perform five engagements stopping the vehicle between each. The tape recorder was left running during each set of five engagements and a voice commentary used to add any observations. Before and after each complete test static calibrations on all channels were performed to ensure that all the equipment was functioning correctly. After completion of the test, the subject was required to complete the questionnaire, shown in Figure 4.4.

Each test resulted in twenty five engagements, each engagement comprising of six channels of data. As forty subjects were to be tested a quick and compact method was required for reducing the results.

Data Reduction

A computer with a magnetic tape mass storage device was used to convert all the analogue signals recorded by the magnetic tape recorder to a digital form. A program was then written to perform all the calibrations on the signals and to curve fit each signal using sixth order polynomials. The coefficients of these polynomials were punched out on paper tape and in addition the program also printed out the total time for the engagement i.e. slip time, on the computer's teletype. Because of the limitations of the small computer used for this part of the analysis, the rest of the reduction was carried out on a bigger machine. A sample printout is shown, Appendix II, of the results from the program used to complete the reduction. Graph No. 4.14 shows the results for one engagement, the upper graph is of clutch torque and the lower graph of clutch input speed and output speed.

To perform this type of analysis even by computer for all the engagements carried out, would be tedious and so these graphs were reduced to a series of parameters as can be seen in the printout, Appendix II, which were then examined statistically. A break down of all the subjects tested is presented in Table No. 4.1.

Results

Graph No. 4.14 is a typical level road engagement, the shape of the curves shown varied little with the type of engagement but the magnitude of the measured parameters did. Table No. 4.2 shows the arithmetic means of the relevant parameters which can be used to define the engagement. From this table it can be seen that the values for slip time and heat generated calculated by the theory in chapter 3 were grossly in error, i.e. the slip times encountered in practice are much longer than predicted and consequently the heat generated is higher. Also noticeable, especially on the gradient engagements, is that time is spent with the vehicle stationary but the clutch slipping thus generating heat before the actual engagement. As the gradient increased the magnitude of the average engine speed increased as could be expected because the subject was trying to transmit more torque. The magnitude of the clamp load used during the engagement period is also shown in the table and as can be seen, even under severe gradient test conditions it was unnecessary to use more than 63% of the total available clamp load and under normal level road conditions only 30% of the clamp load was used. Thus, proving the assumption that maximum clamp load is applied in zero time to be not only invalid but also to give erroneous results.

By visually inspecting graphs of the test subjects' engagements it was found that they fell into three catagories, depicted by Graph No. 4.15 which shows graphs of engine speed and propellor shaft speed for the three conditions. These are actual engagements, the first one is of a driver who is over zealous with the accelerator pedal and creates more slip than is necessary to start the vehicle from rest. Graph number two shows a subject who feeds the clutch pedal out, and depressesthe accelerator in an attempt to keep the engine speed approximately constant at his initial setting. Finally, this test subject allows the engine speed to fall as the propellor shaft speed rises to meet it.

The first engagement resulted in 16.1 KJ of heat being generated in 3.0 seconds or a heat generation rate of 5.37 KW., the second a rate of 5.4 KW and the third a rate of 2.32 KW. Which indicates that although it is slower, the third type of engagement is preferable with regards to the rate of heat generation during the engagement period, but a great deal depends upon the magnitude of the engine speed.

From this series of vehicle tests the following points emerge:

1. The driver deliberately slips the clutch for quite a

long period before the vehicle begins to move and also that the total engagement period even on a level road is in the region of 3.0 seconds.

2. Under normal operating conditions drivers rarely, if ever, use maximum clamp load when moving the vehicle away from rest.

3. If a driver wishes to move the vehicle away on a gradient or at a faster pace than normal he will use a combination of higher engine speeds and more clamp load, thus generating more heat.

4. It is better from a rate of heat generation condition to allow the engine speed to fall off during the engagement even though a longer slip time results.

The next phase in this research was to build a test rig, upon which the vehicle could be simulated and measurements of temperature carried out. Also the theory required modifying in the light of the knowledge obtained from the above tests.

BREAK DOWN OF TEST SUBJECTS

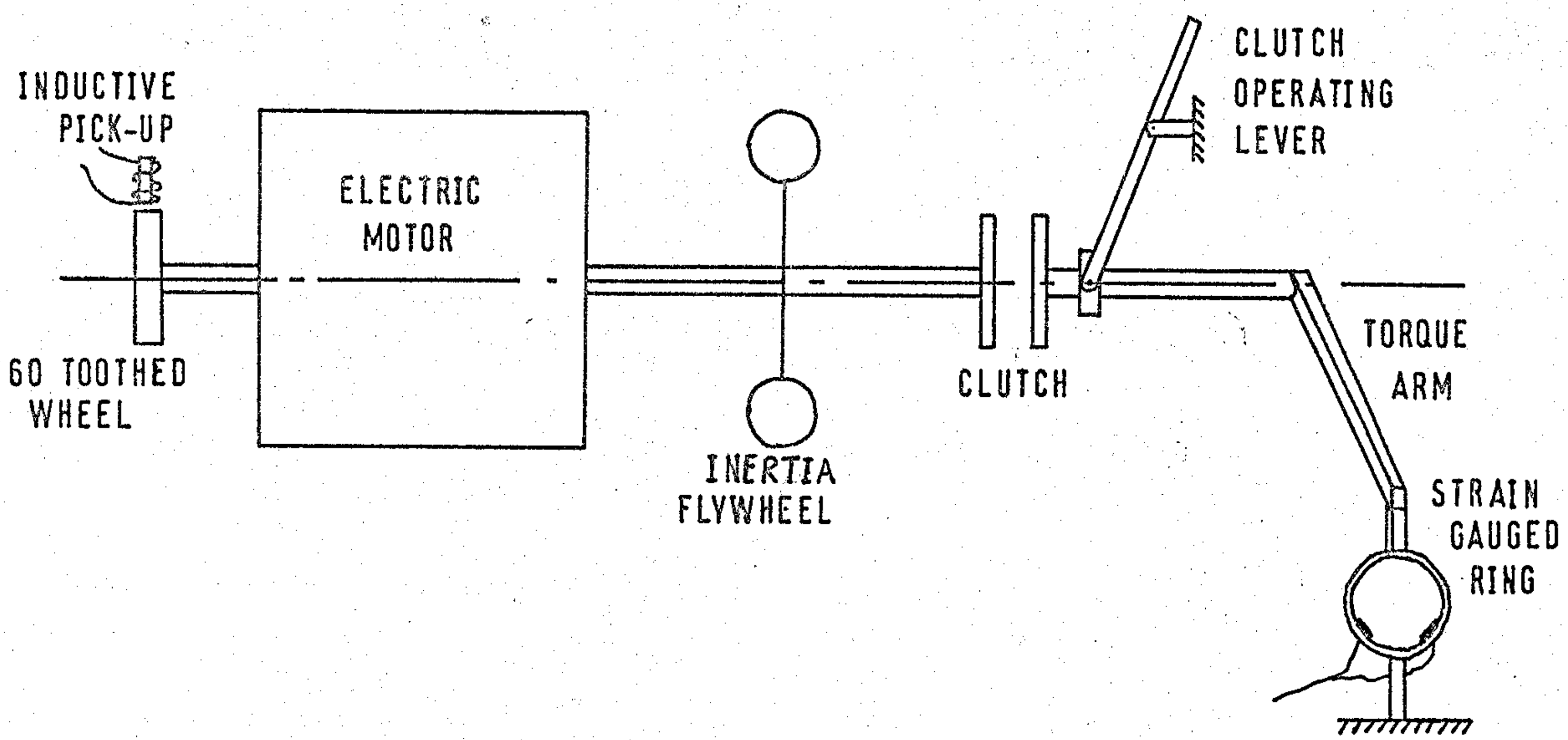
SEX	
MALE	71%
FEMALE	29%
NUMBER OF MILES DRIVEN PER WEEK	
0 - 50	24%
50 - 100	22%
100 - 200	20%
OVER 200	34%
MARRIED	60%
UNMARRIED	40%
EMPLOYMENT	
TRANSPORT	5%
MANUAL	24%
CLERICAL	24%
PROFESSIONAL	47%

TABLE NUMBER 4.1

ARITHMETIC MEAN OF ALL VEHICLE TEST RESULTS

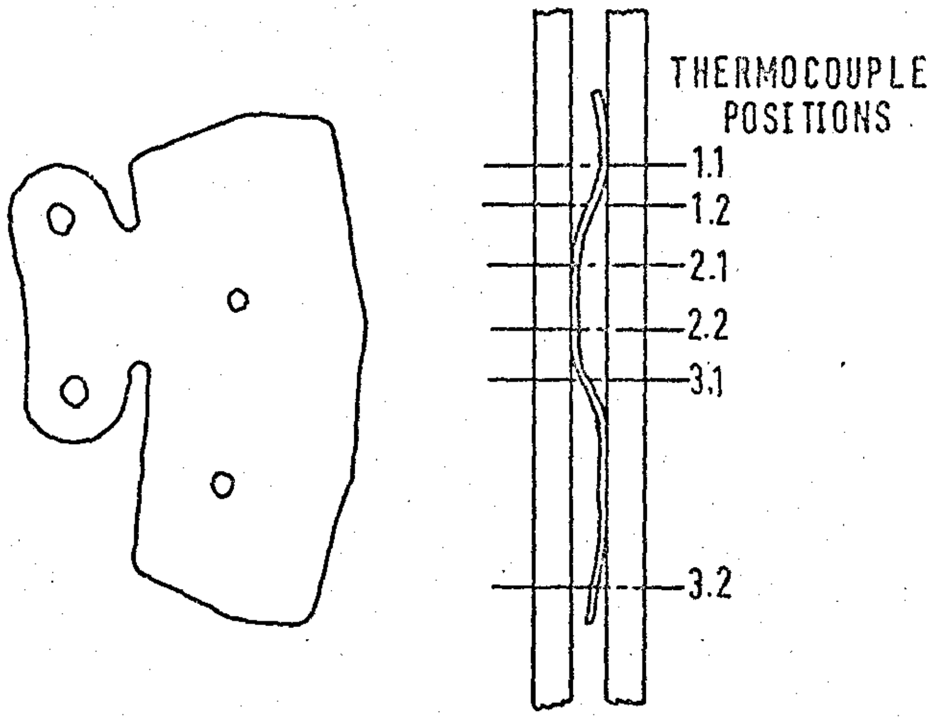
Test Conditions	Gradient	Heat Generated kJ	Propellor Shaft Acceleration (RAD/S^2)	Maximum Engine Speed (RAD/S)	Minimum Engine Speed (RAD/S)	Average Engine Speed (RAD/S)	Total Slip Time (S)	Slip Time Prior to Start of Propellor Shaft Rotation (S)	Clamp Load % of Maximum
LEVEL ROAD	LEVEL	9.98	84.0	196.0	112.0	159.0	2.8	0.5	30
QUICK START	LEVEL	17.87	117.0	252.0	124.0	211.0	2.4	0.4	59
MEDIUM GRAD	1 : 17	17.95	82.0	211.0	112.0	169.0	3.4	1.2	48
SLIGHT GRAD	1 : 28	14.42	86.0	200.0	107.0	160.0	3.1	1.0	43
SEVERE GRAD	1 : 5	34.40	45.0	248.0	101.0	179.0	4.5	1.1	63

TABLE NUMBER 4.2



TEST RIG FOR THE DETERMINATION OF VARIATION IN COEFFICIENT OF FRICTION

FIGURE NO. 4.1



A BORG-LYTE SEGMENT

FIGURE NO. 4.2

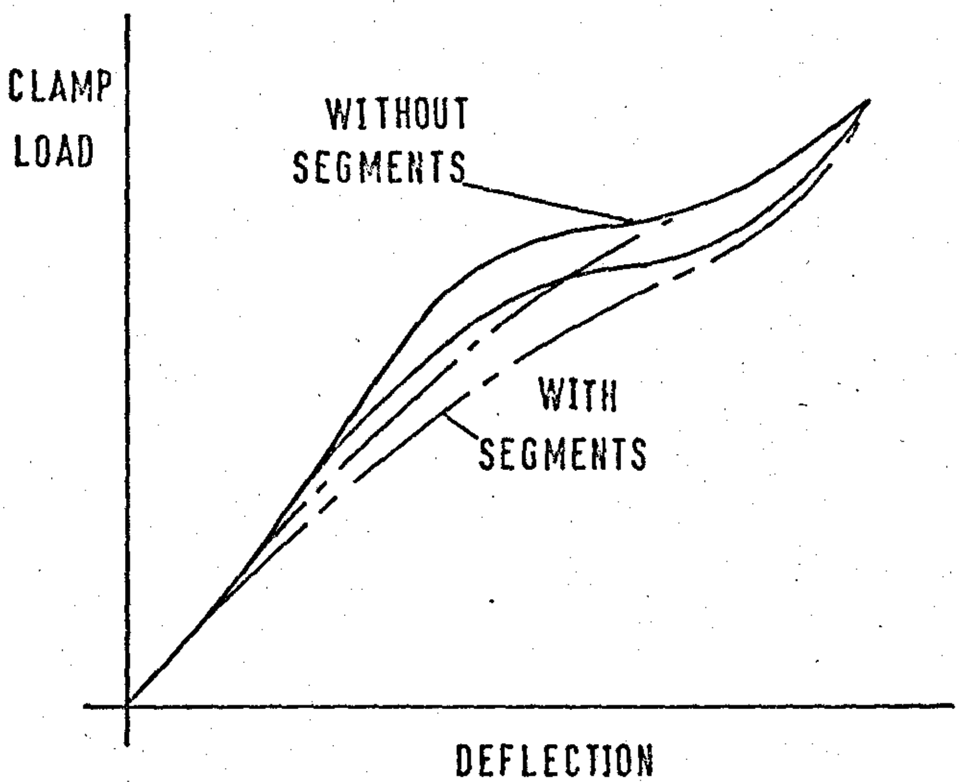


FIGURE NO. 4.3

LOUGHBOROUGH UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF TRANSPORT TECHNOLOGY

VEHICLE TEST

Test Number: ______________

Date of Test: ______________

1. Sex: Male ☐ Female ☐

2. Average number of miles driven per week:

0 - 50	50 - 100	100 - 200	200 - 500	500 - OVER
☐	☐	☐	☐	☐

3. Weight: st. lbs.

4. Height: ft. ins.

5. What type of vehicle do you normally drive?

____

6. Marital status at present time:

Married ☐ Unmarried ☐

7. Employment:

Transport ☐

Light manual ☐

Heavy manual ☐

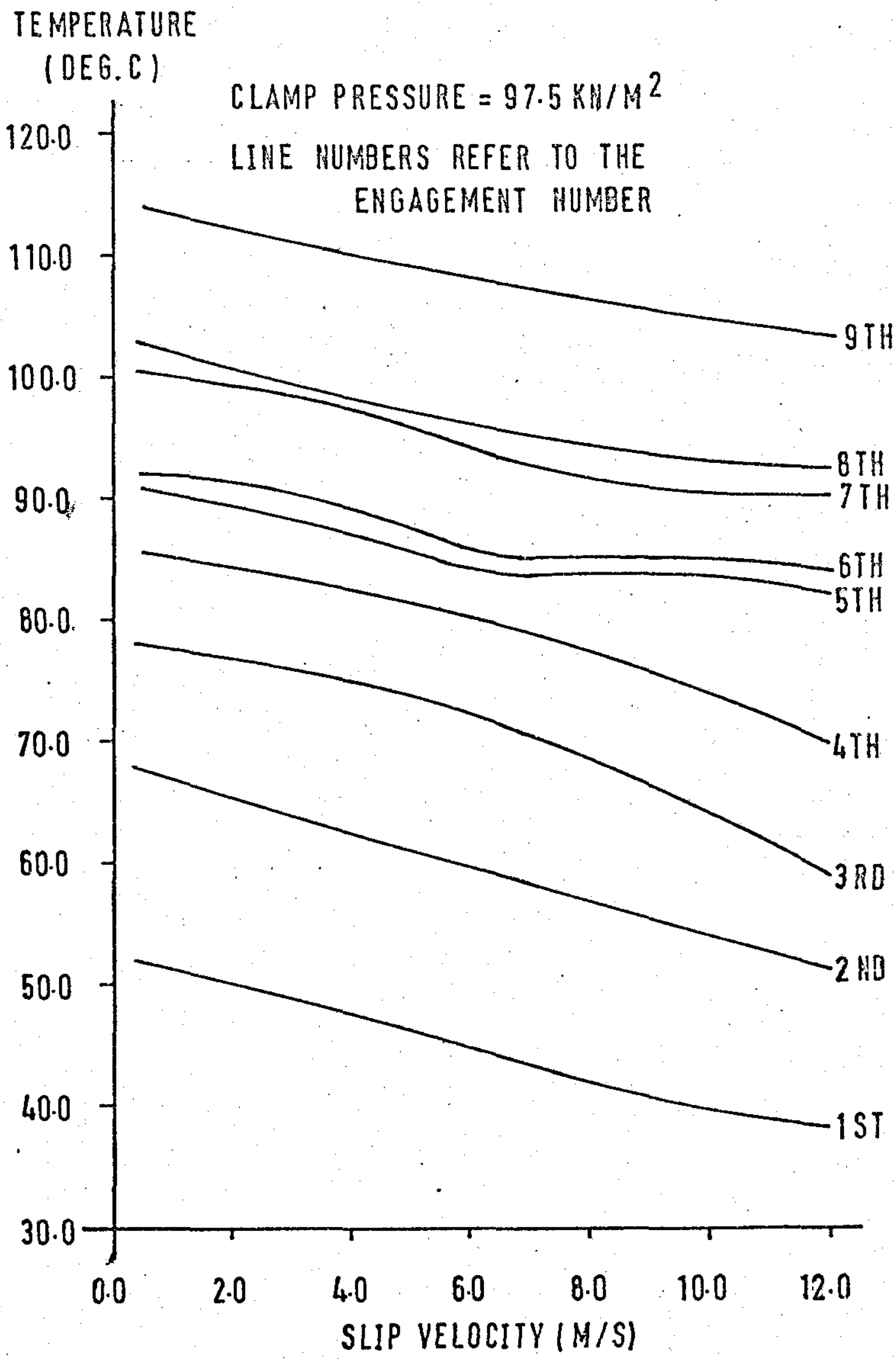
Clerical ☐

Professional ☐

8. Job description:

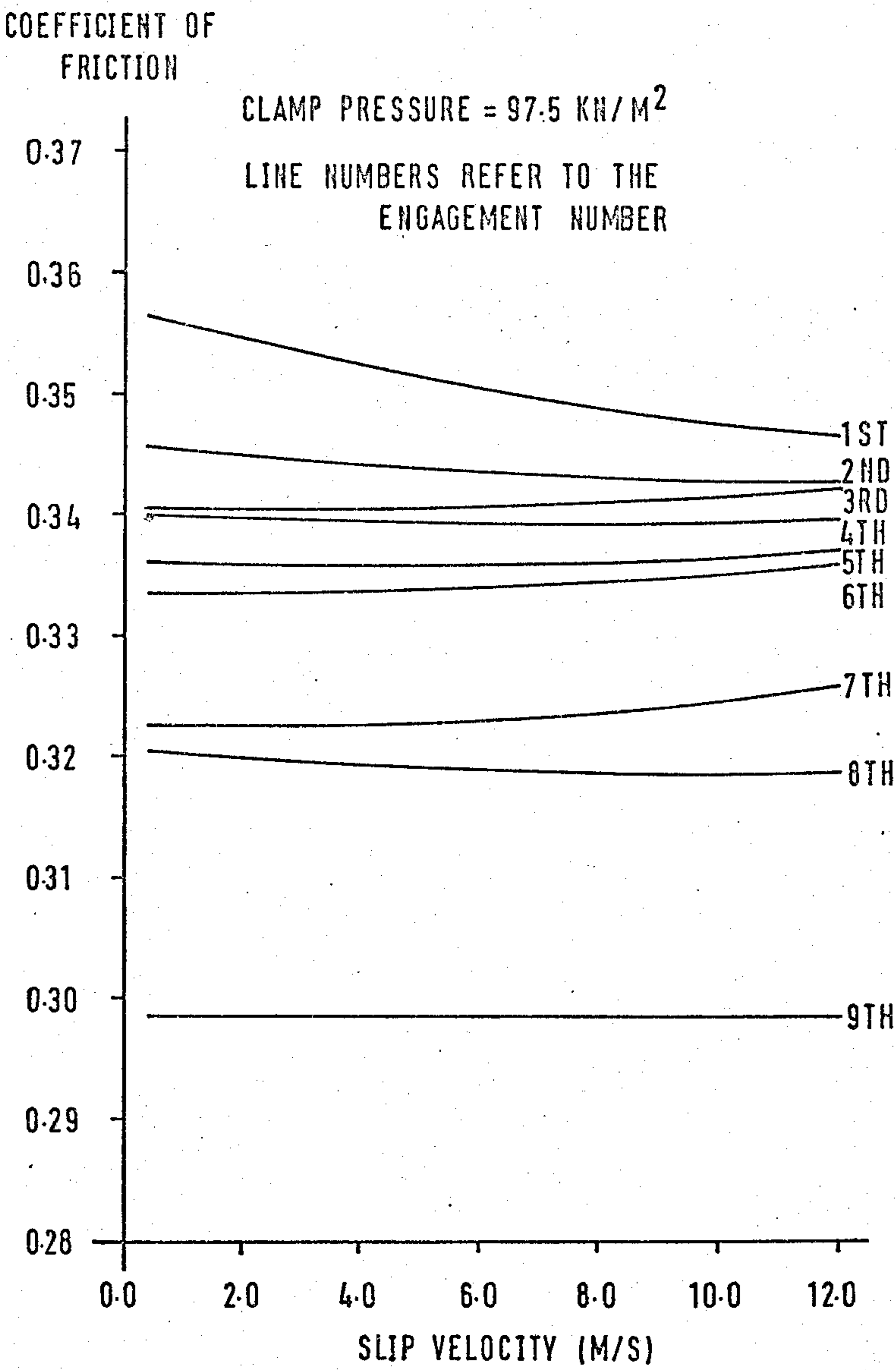
FIGURE NO. 4.4

SURFACE TEMPERATURE VARIATION WITH SLIP VELOCITY



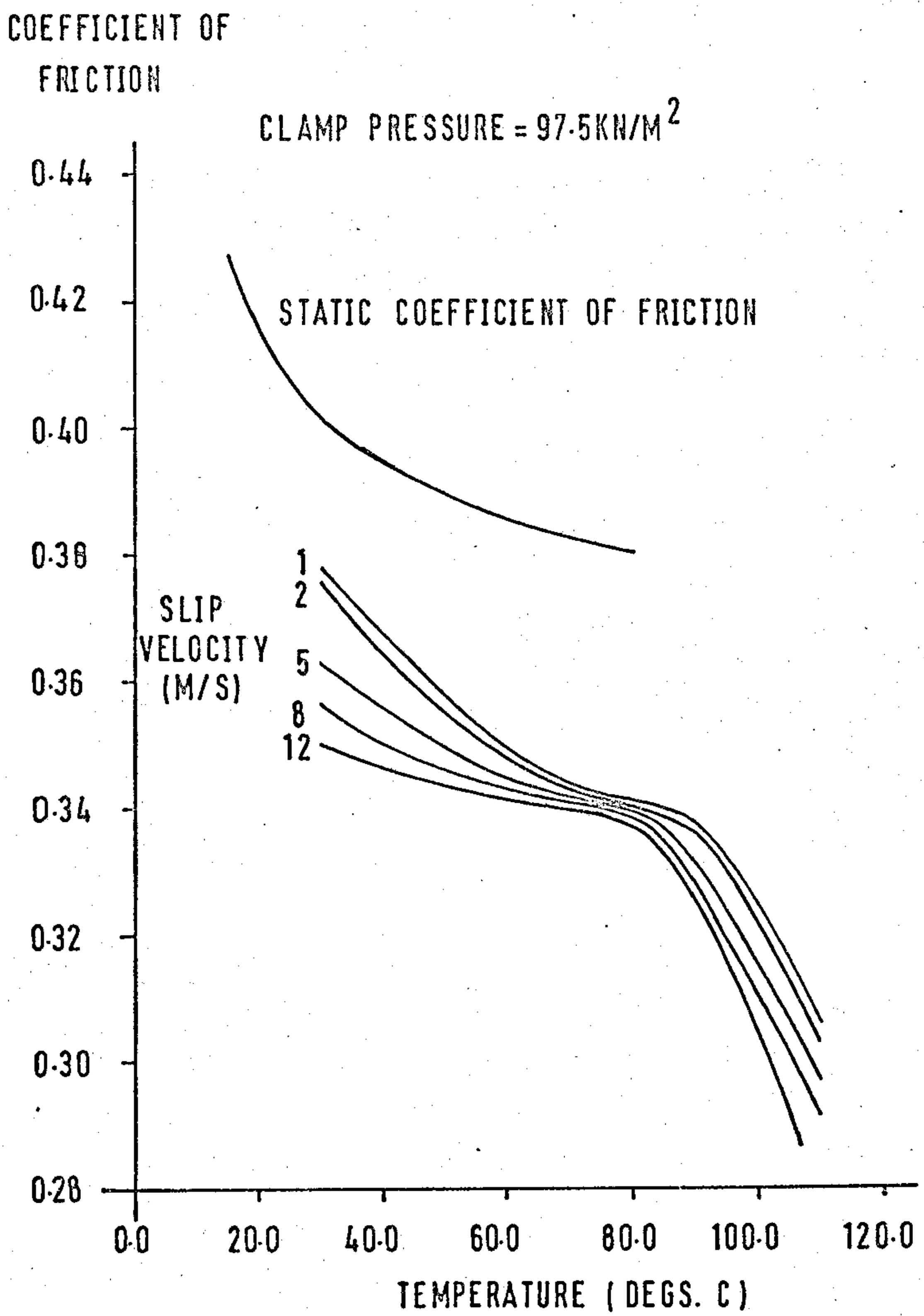
GRAPH NO. 4.1

COEFFICIENT OF FRICTION VARIATION WITH SLIP VELOCITY



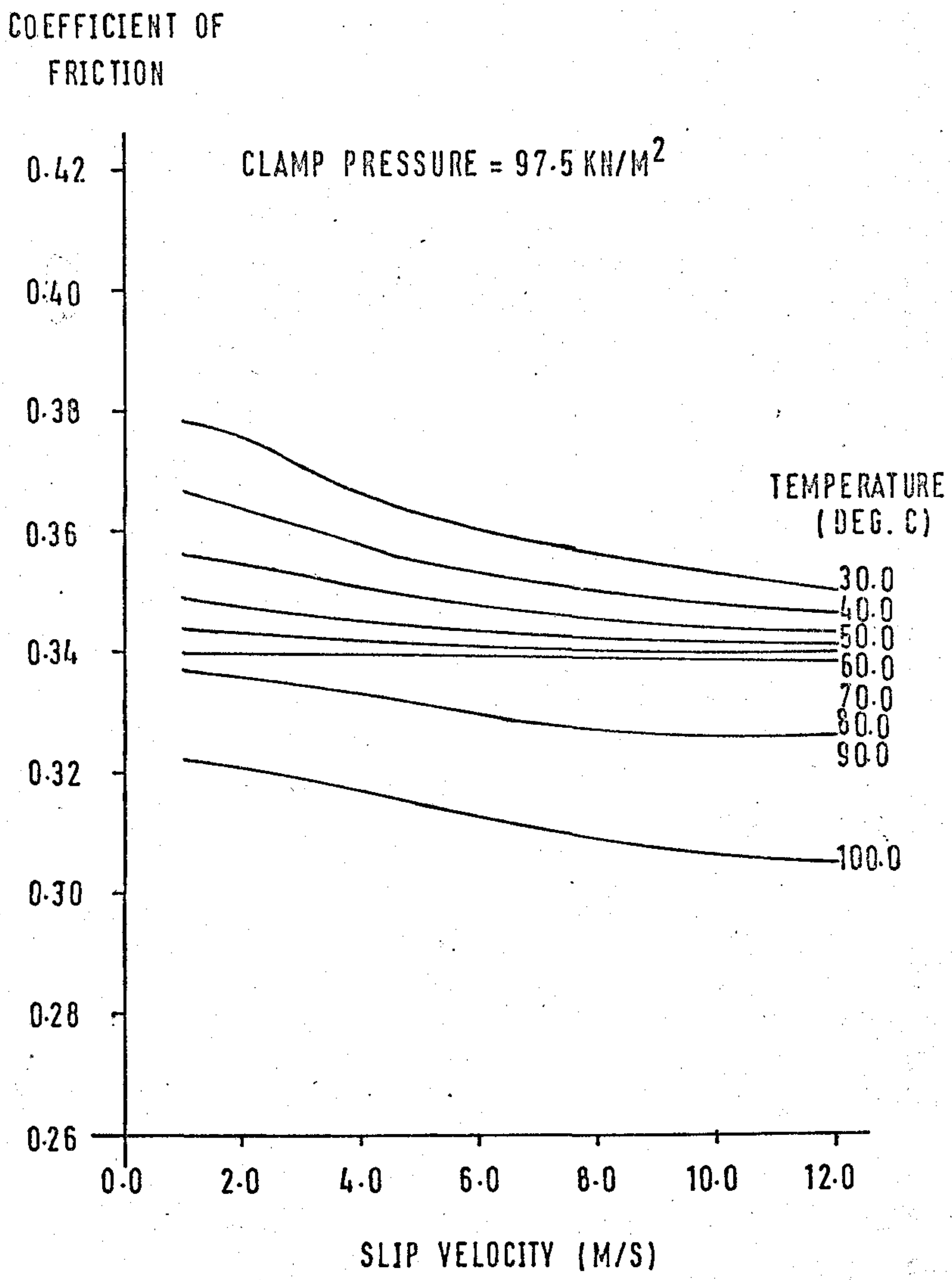
GRAPH NO. 4.2

TEMPERATURE VS COEFFICIENT OF FRICTION



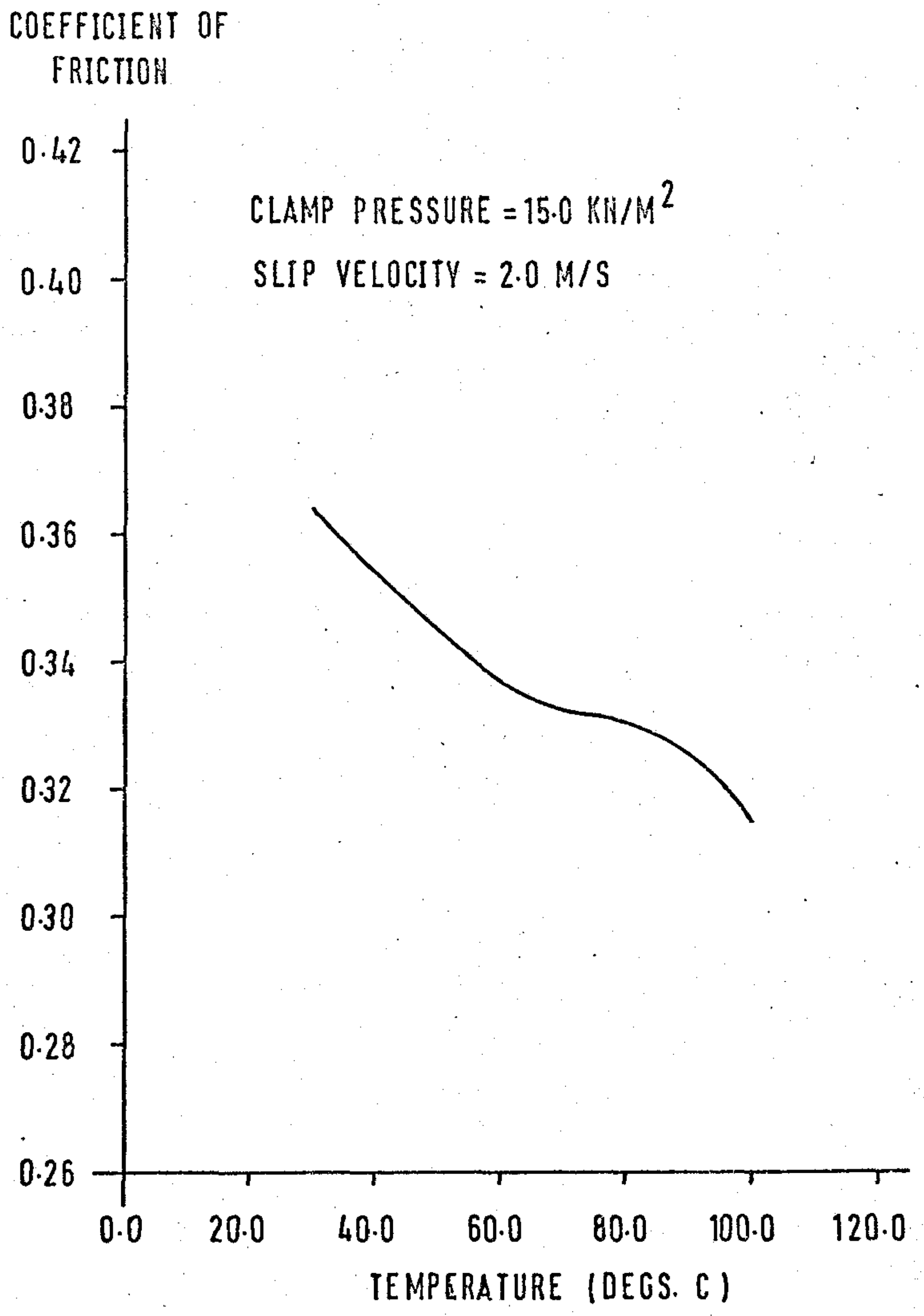
GRAPH NO. 4.3

SLIP VELOCITY VS COEFFICIENT OF FRICTION



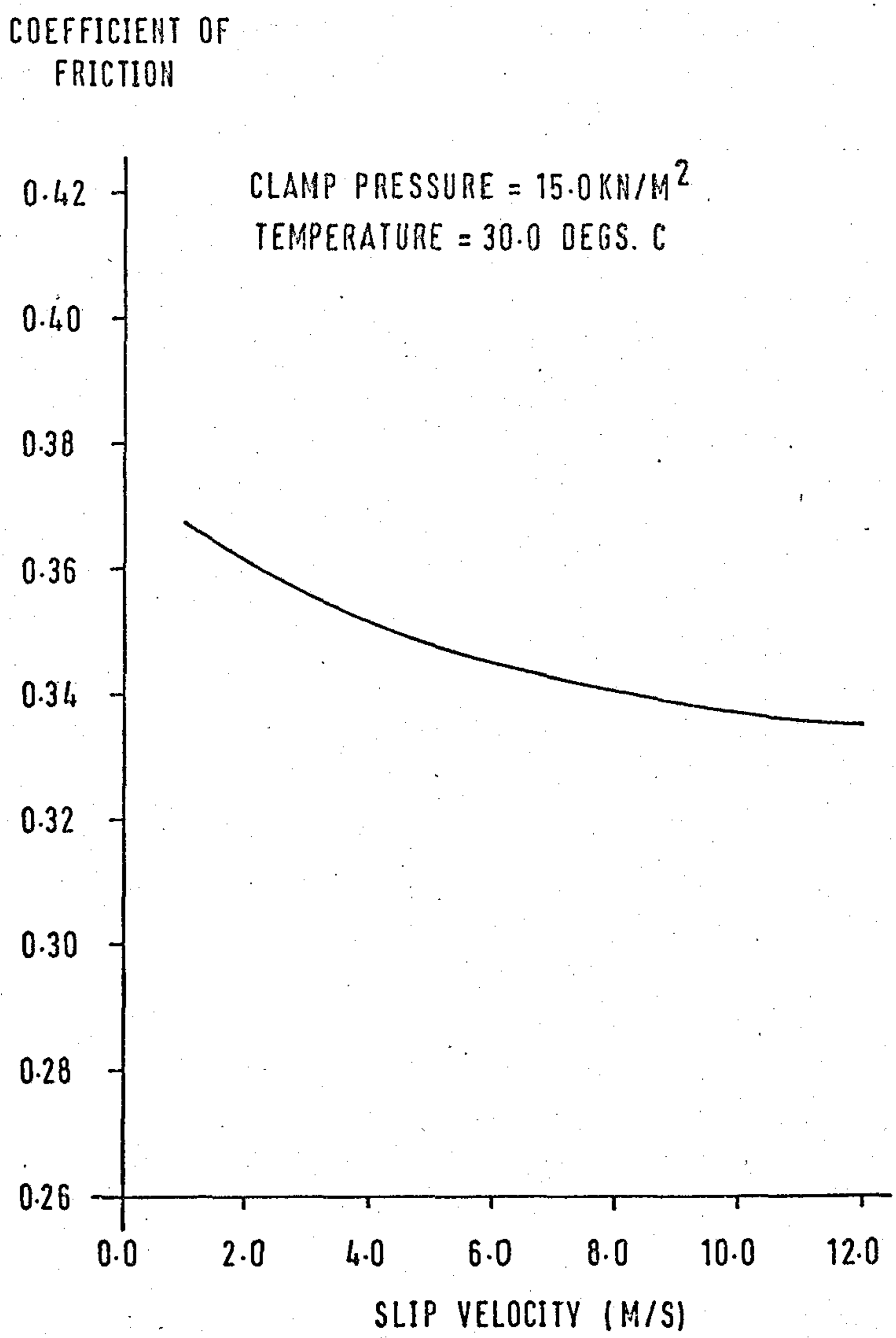
GRAPH NO. 4.4

TEMPERATURE VS COEFFICIENT OF FRICTION



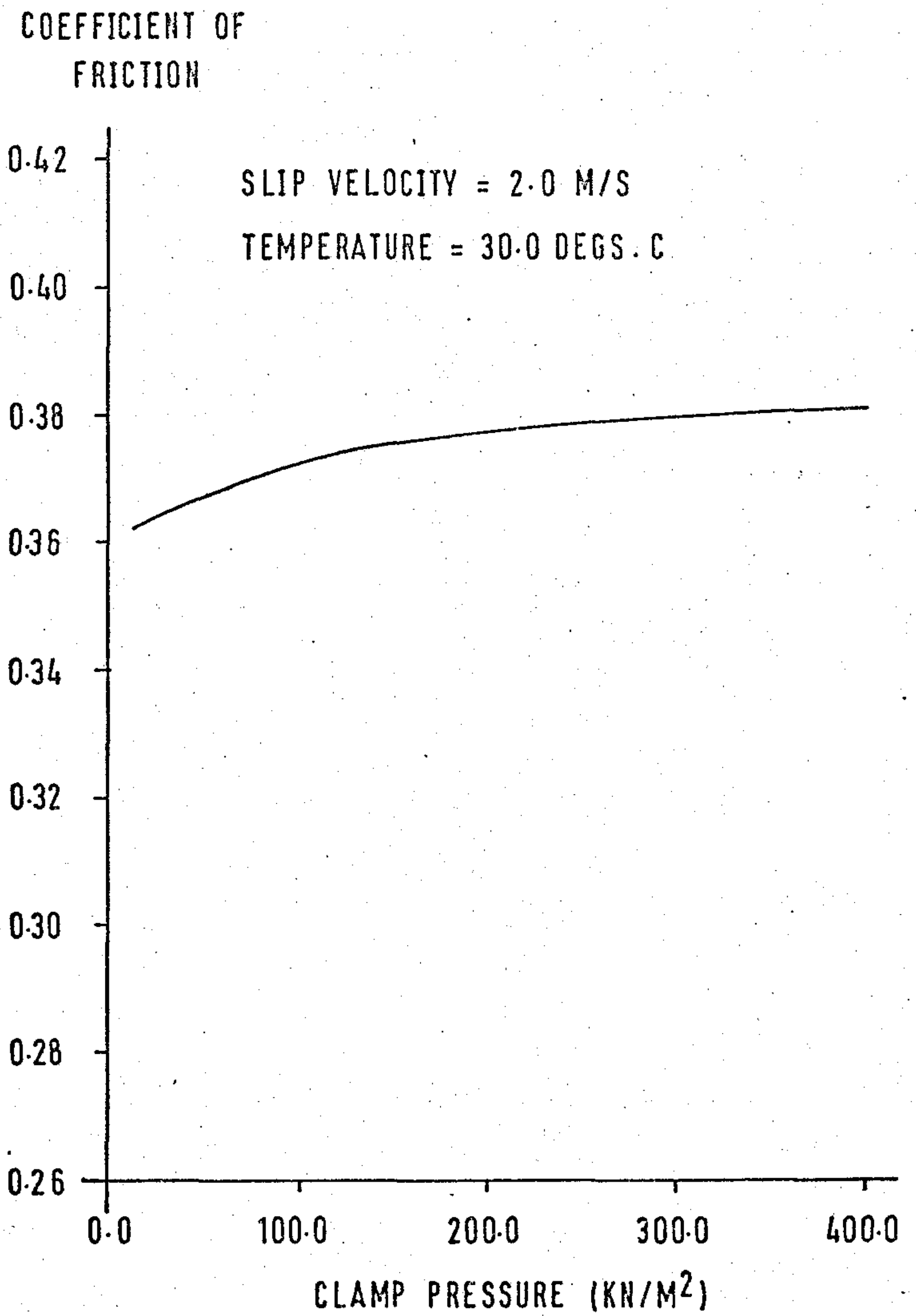
GRAPH NO.4.5

SLIP VELOCITY VS COEFFICIENT OF FRICTION

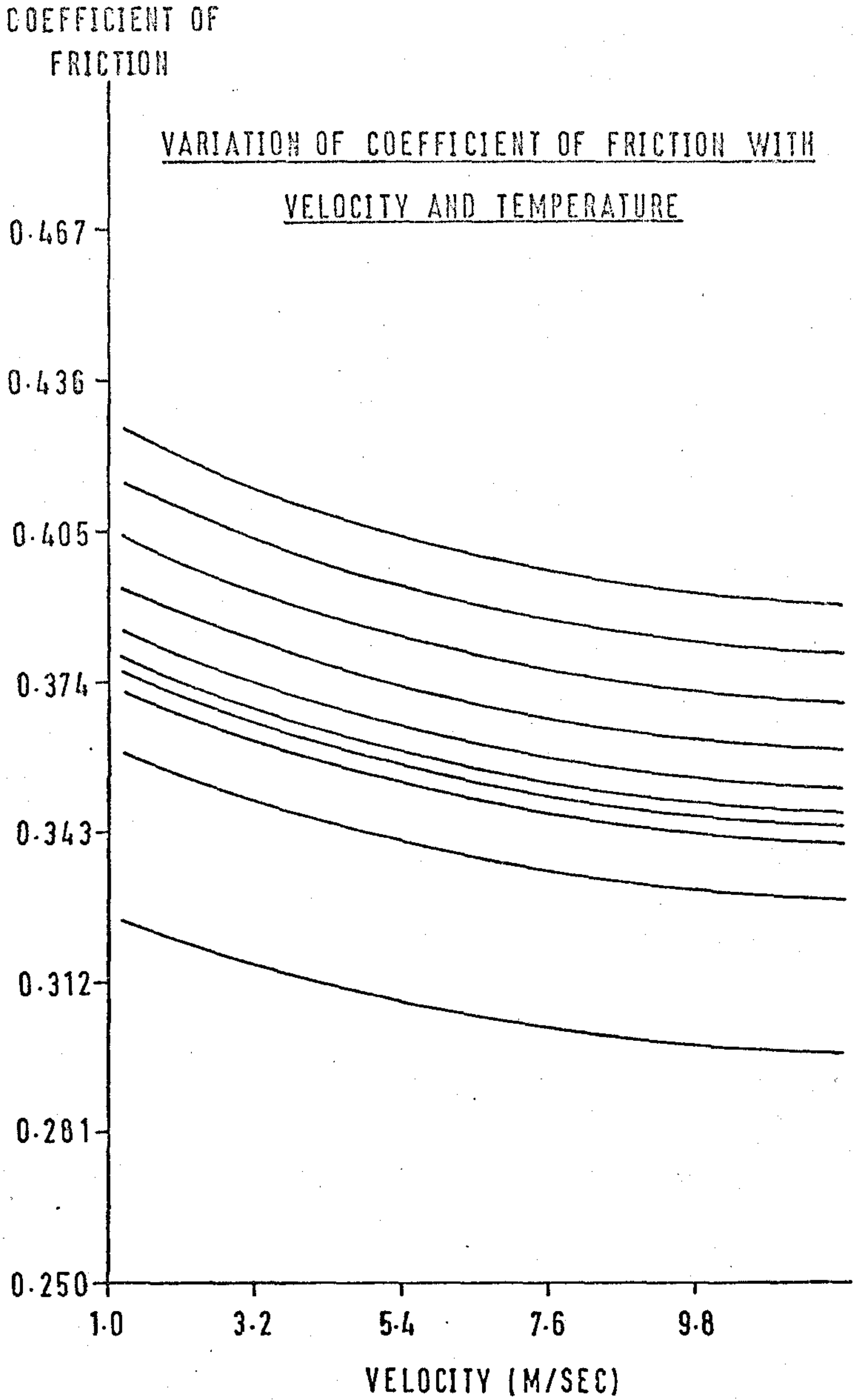


GRAPH NO. 4.6

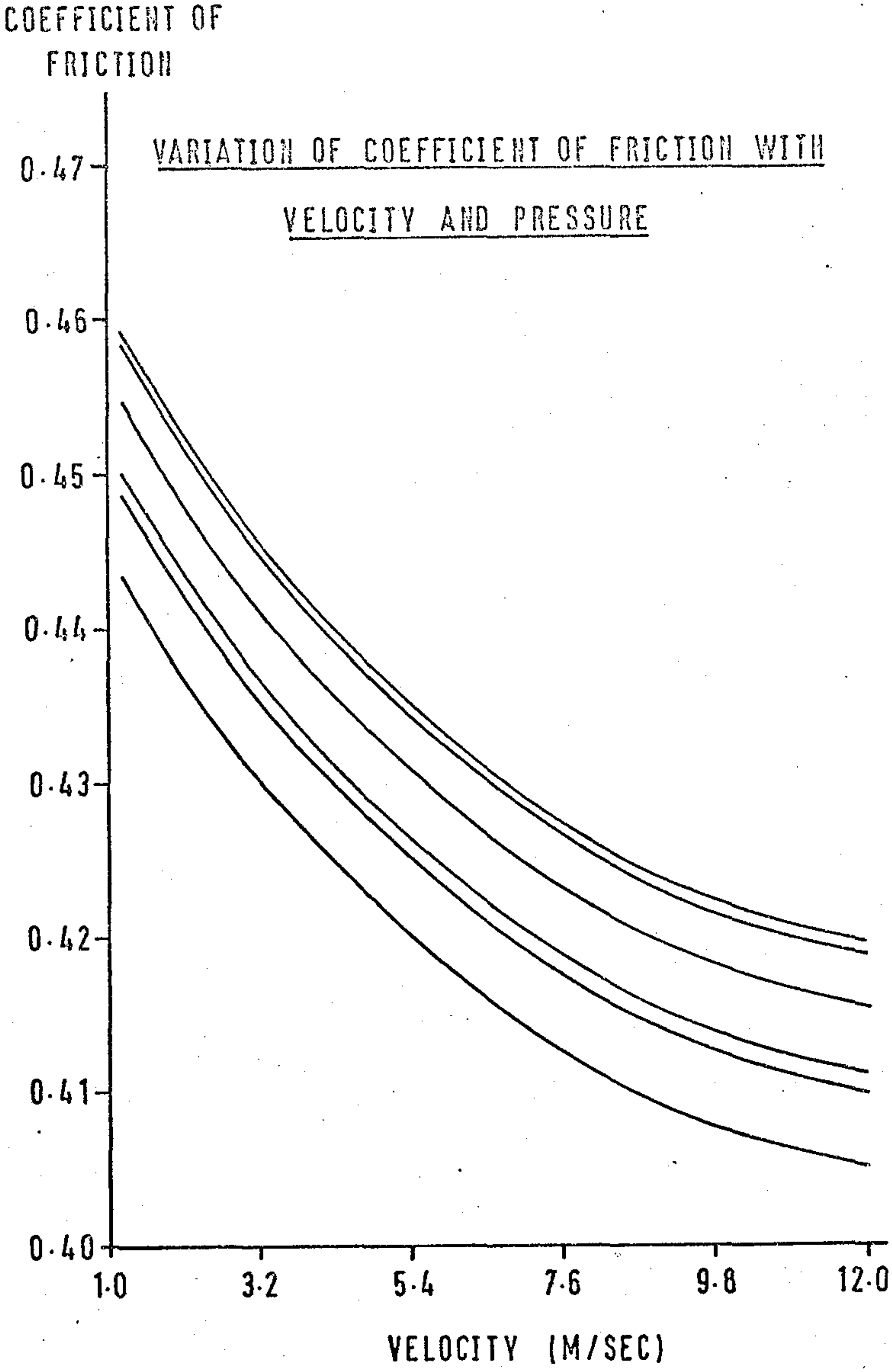
CLAMP PRESSURE VS COEFFICIENT OF FRICTION



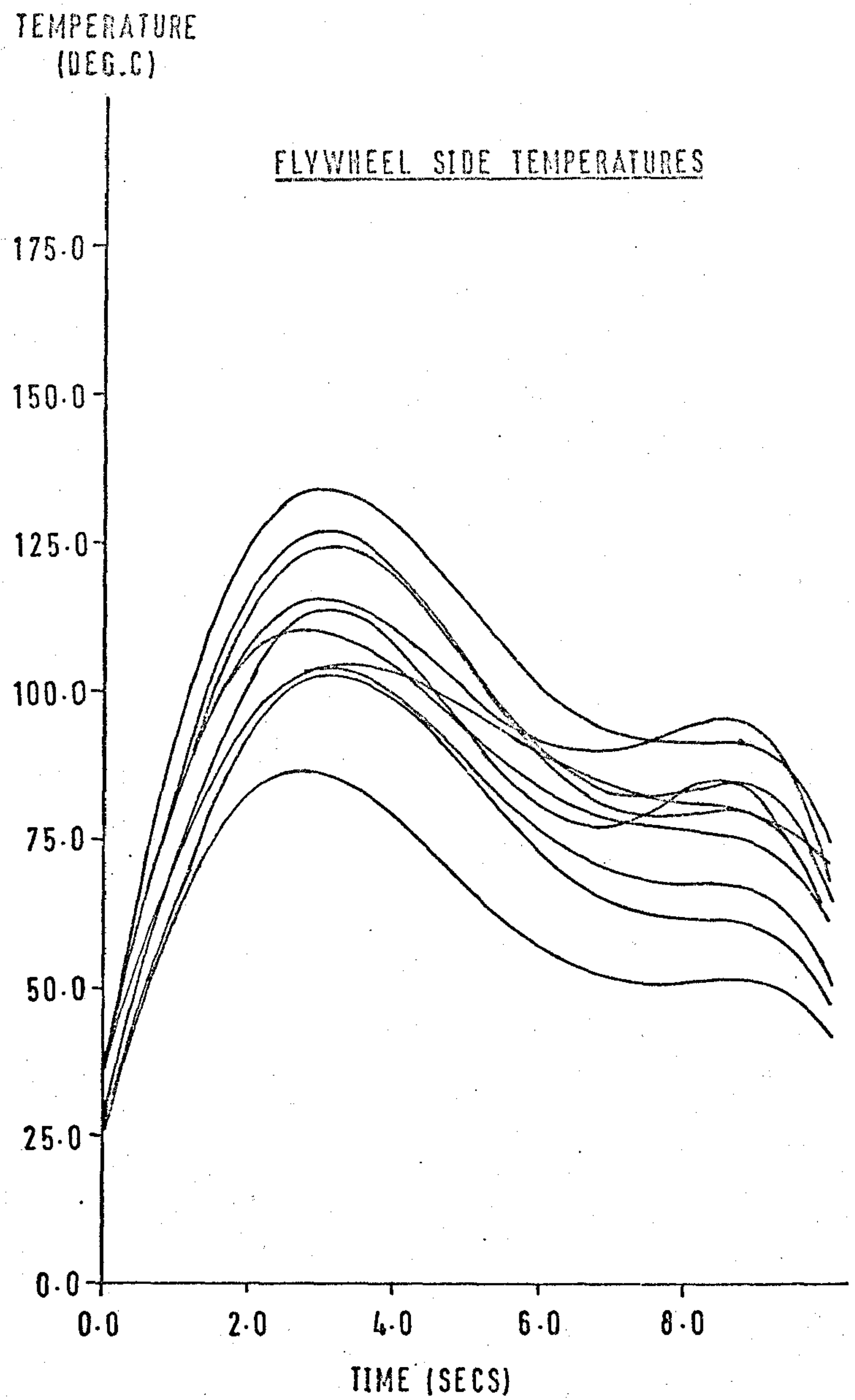
GRAPH NO. 4.7


COEFFICIENT OF FRICTION
VARIATION OF COEFFICIENT OF FRICTION WITH VELOCITY AND TEMPERATURE
0·467
0·436
0·405
0·374
0·343
0·312
0·281
0·250
1·0
3·2
5·4
7·6
9·8
VELOCITY (M/SEC)

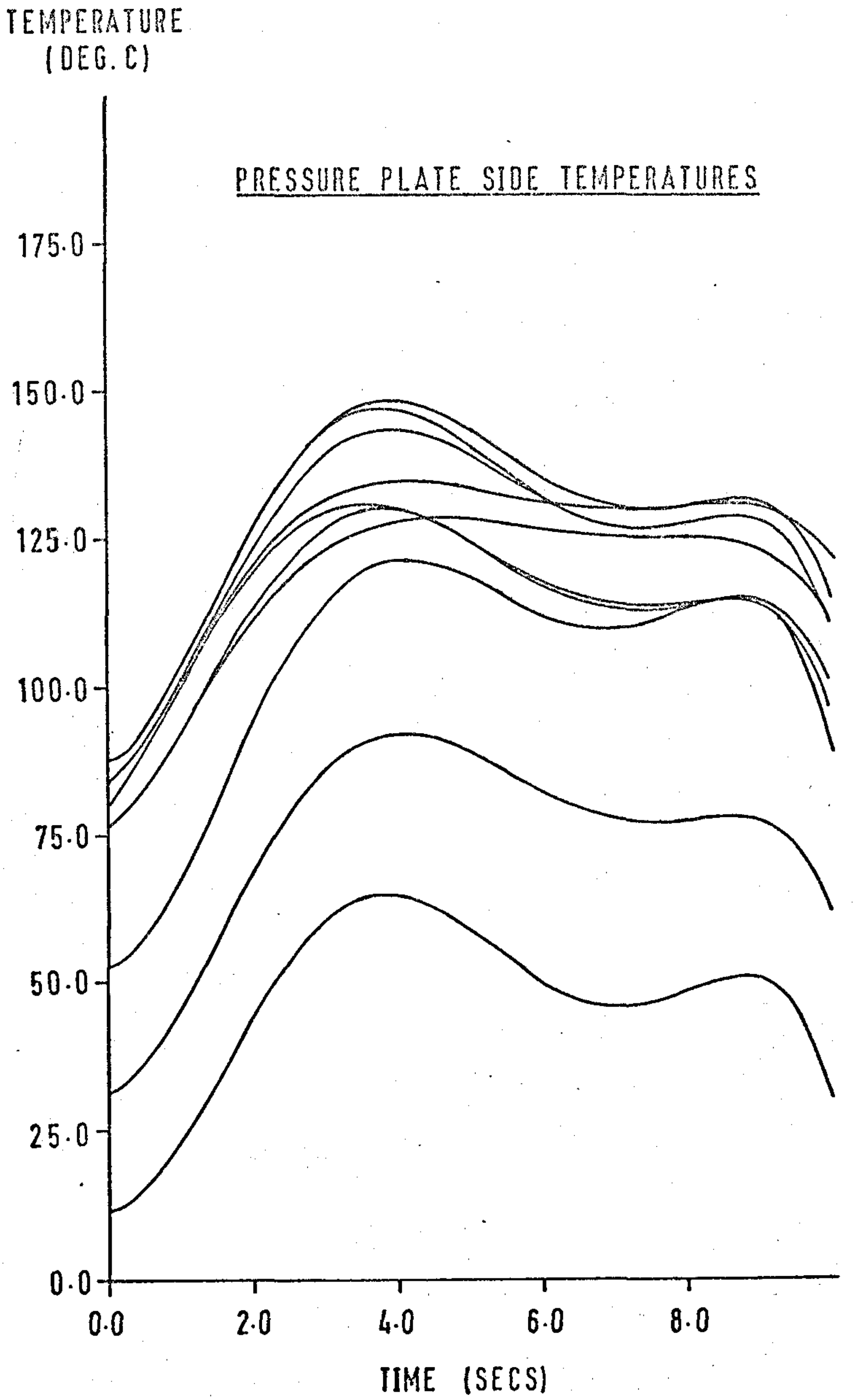
GRAPH NO. 4.8


COEFFICIENT OF FRICTION
VARIATION OF COEFFICIENT OF FRICTION WITH VELOCITY AND PRESSURE
0.47
0.46
0.45
0.44
0.43
0.42
0.41
0.40
1.0
3.2
5.4
7.6
9.8
12.0
VELOCITY (M/SEC)

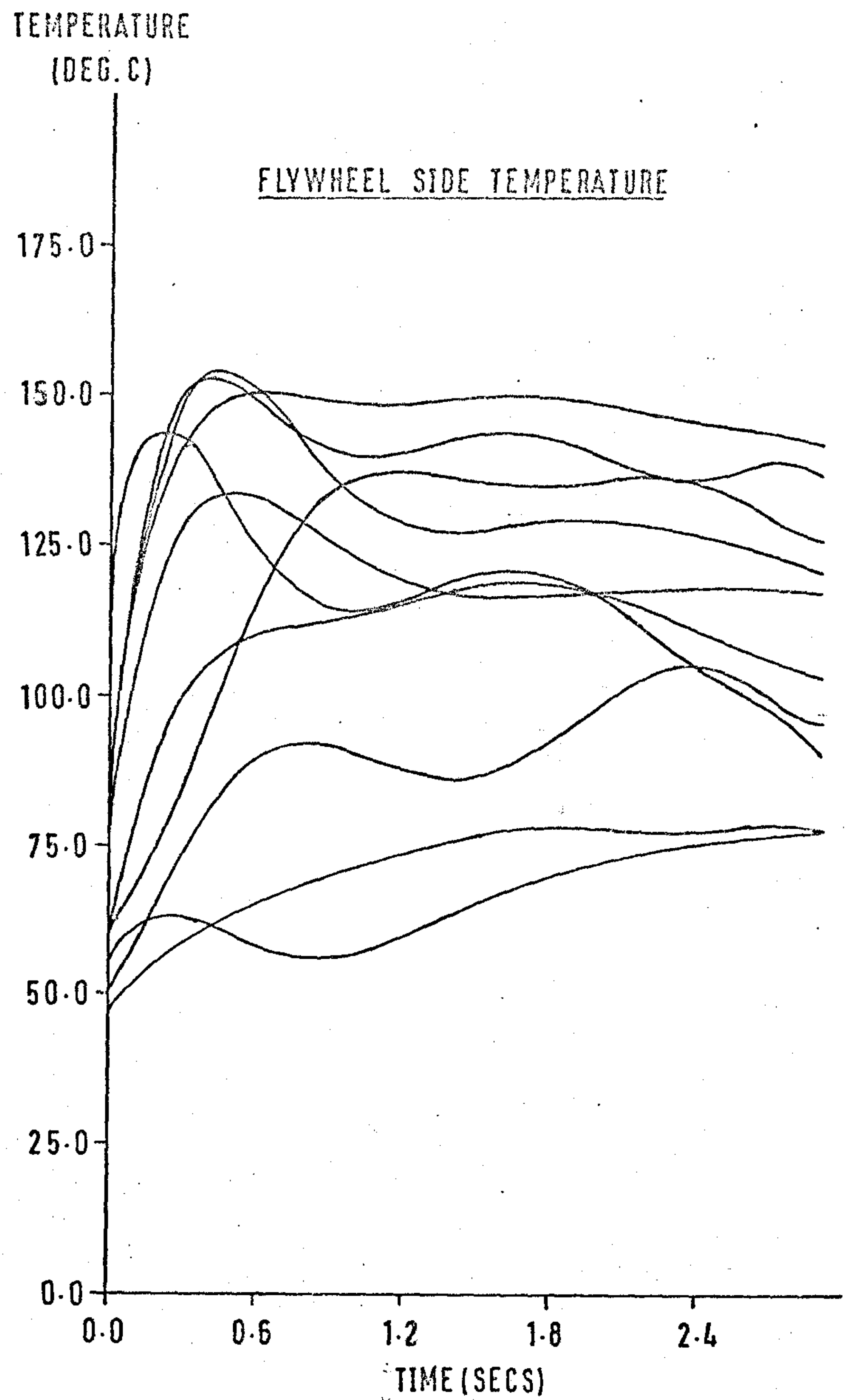
GRAPH NO. 4.9


TEMPERATURE
(DEG.C)
FLYWHEEL SIDE TEMPERATURES
175.0
150.0
125.0
100.0
75.0
50.0
25.0
0.0
0.0
2.0
4.0
6.0
8.0
TIME (SECS)

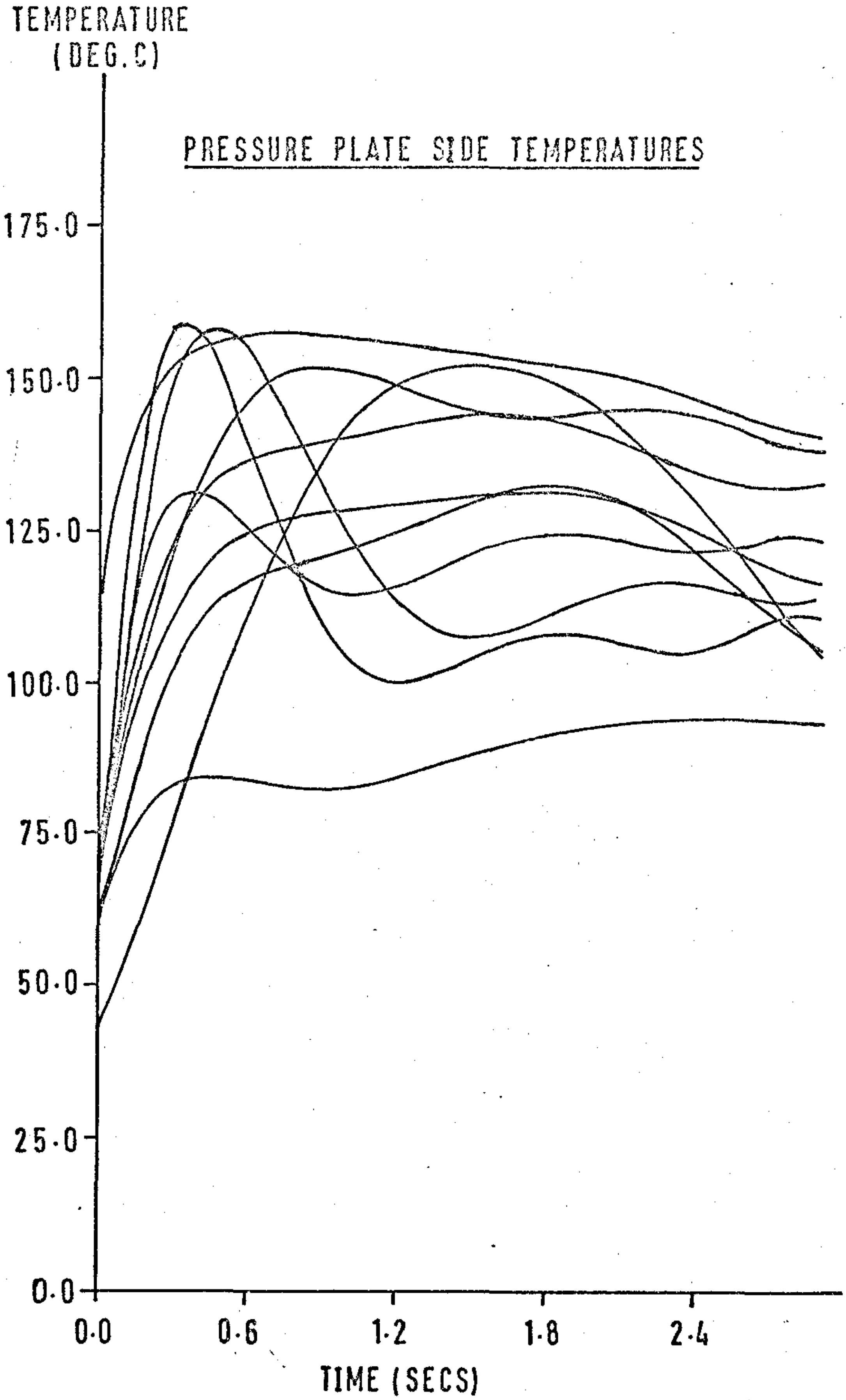
GRAPH NO. 4.10


TEMPERATURE
(DEG. C)
PRESSURE PLATE SIDE TEMPERATURES
175.0
150.0
125.0
100.0
75.0
50.0
25.0
0.0
0.0
2.0
4.0
6.0
8.0
TIME (SECS)

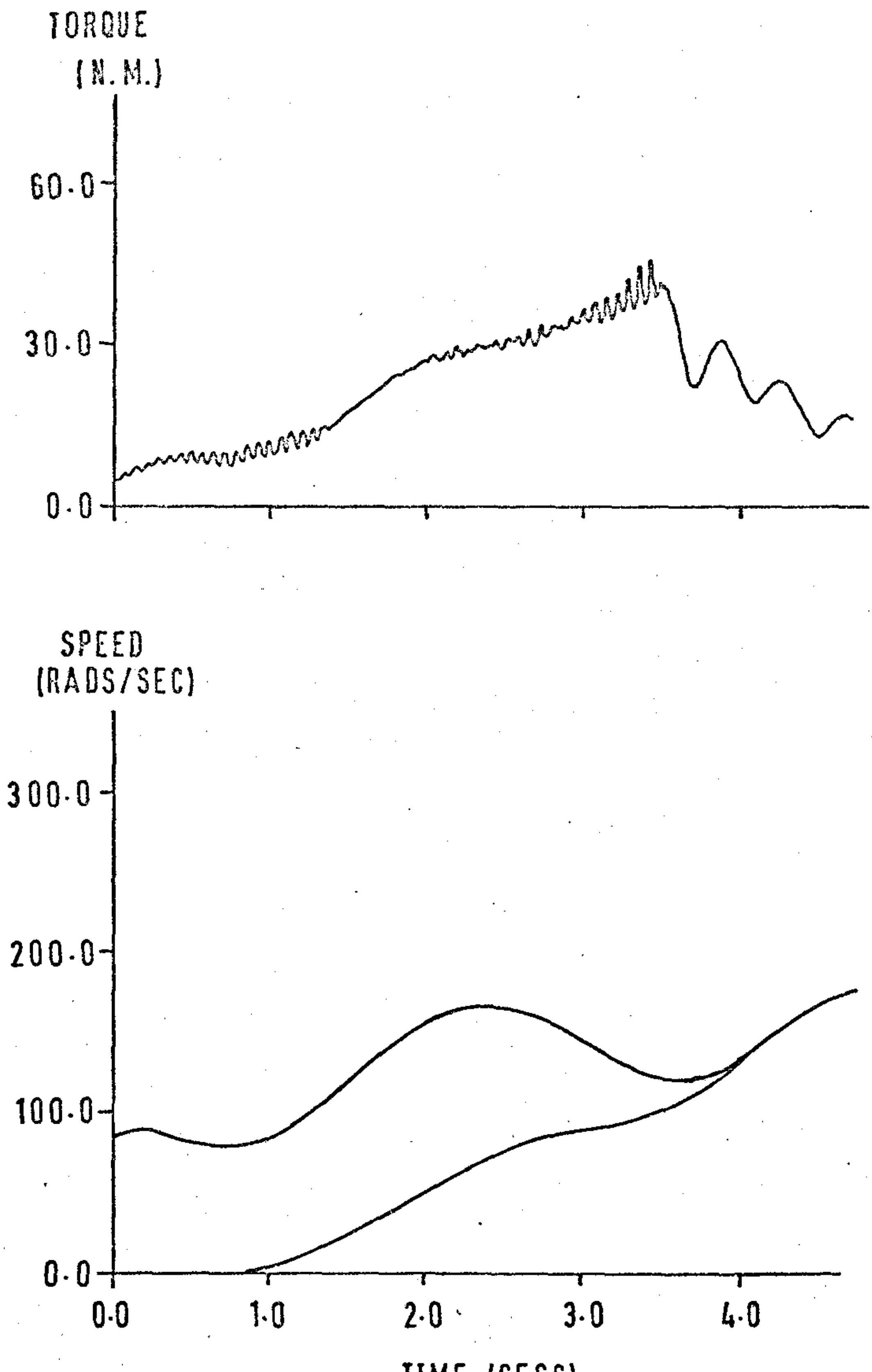
GRAPH NO. 4.11


TEMPERATURE
(DEG. C)
FLYWHEEL SIDE TEMPERATURE
175·0
150·0
125·0
100·0
75·0
50·0
25·0
0·0
0·0
0·6
1·2
1·8
2·4
TIME (SECS)

GRAPH NO. 4.12

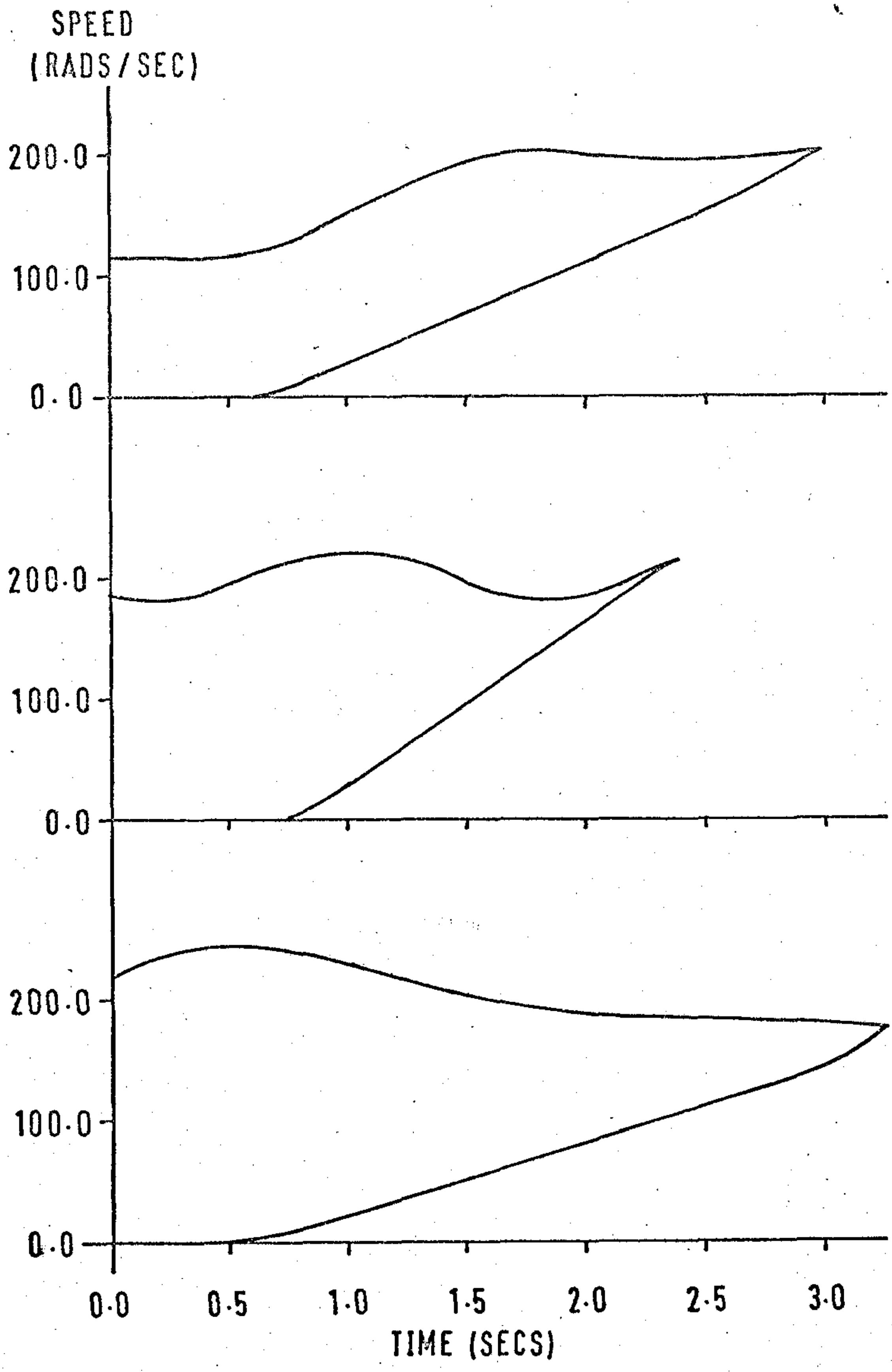

TEMPERATURE
(DEG.C)
PRESSURE PLATE SIDE TEMPERATURES
175·0
150·0
125·0
100·0
75·0
50·0
25·0
0·0
0·0
0·6
1·2
1·8
2·4
TIME (SECS)
GRAPH NO. 4.13

TEST VEHICLE LEVEL ROAD ENGAGEMENT



GRAPH NO. 4.14

TYPICAL ENGAGEMENTS



GRAPH NO. 4.15

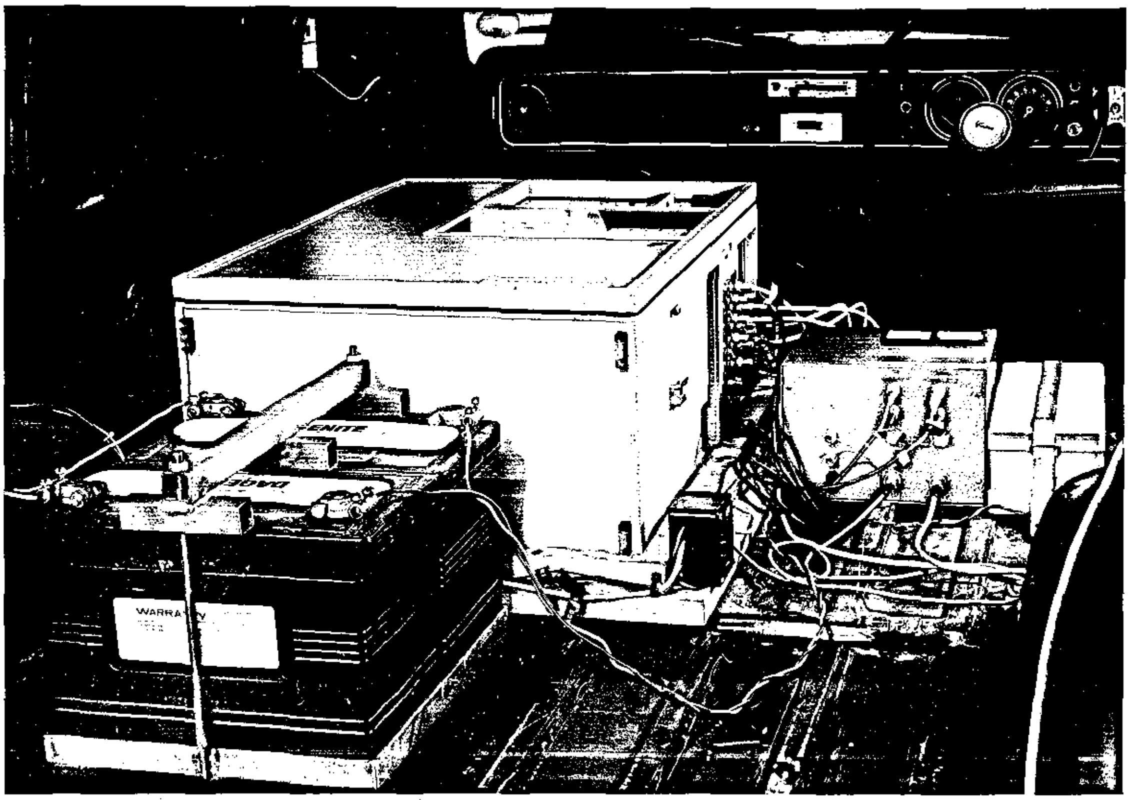


PLATE NO 4·1

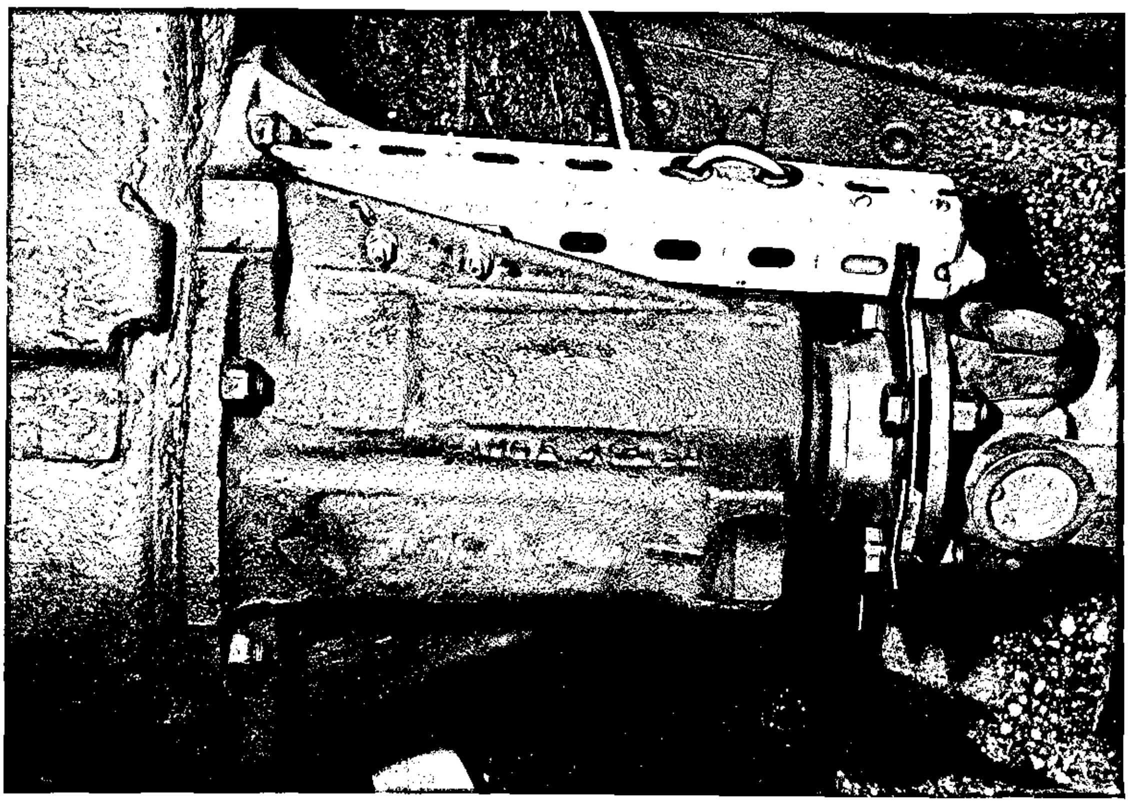


PLATE NO 4·2

PLATE NO 4•3

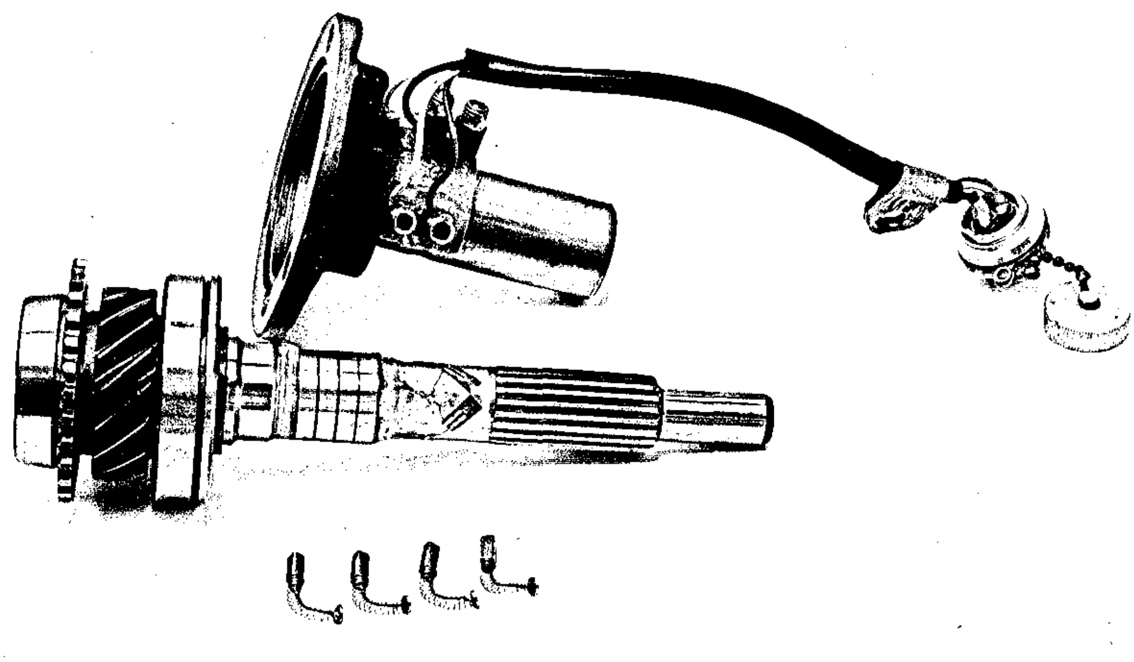


PLATE NO 4•4

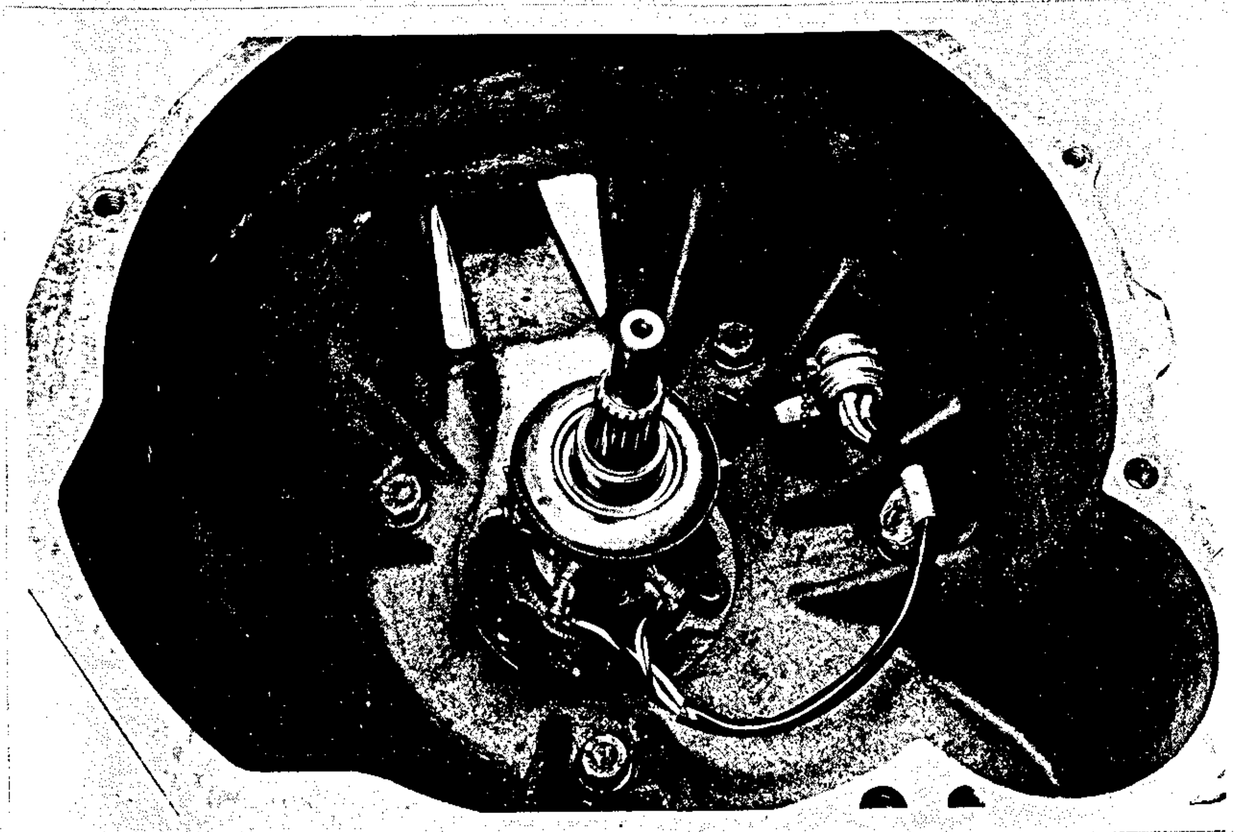


PLATE NO 4•5

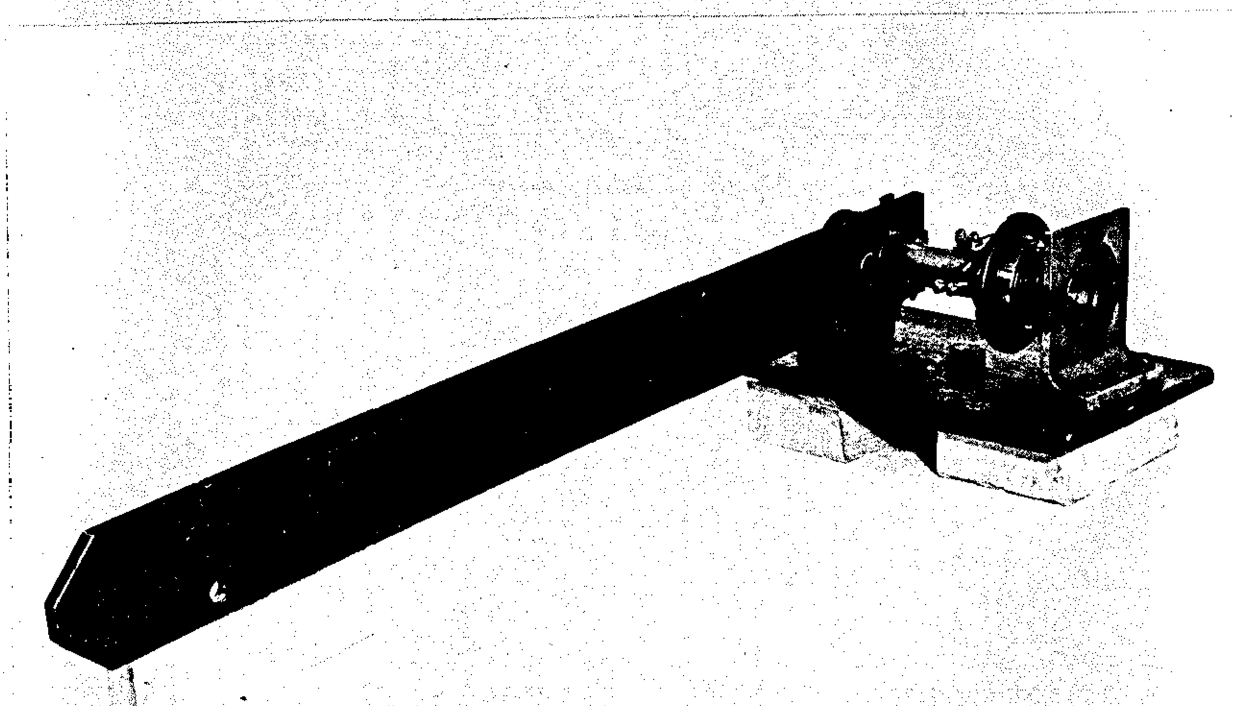


PLATE NO 4•6

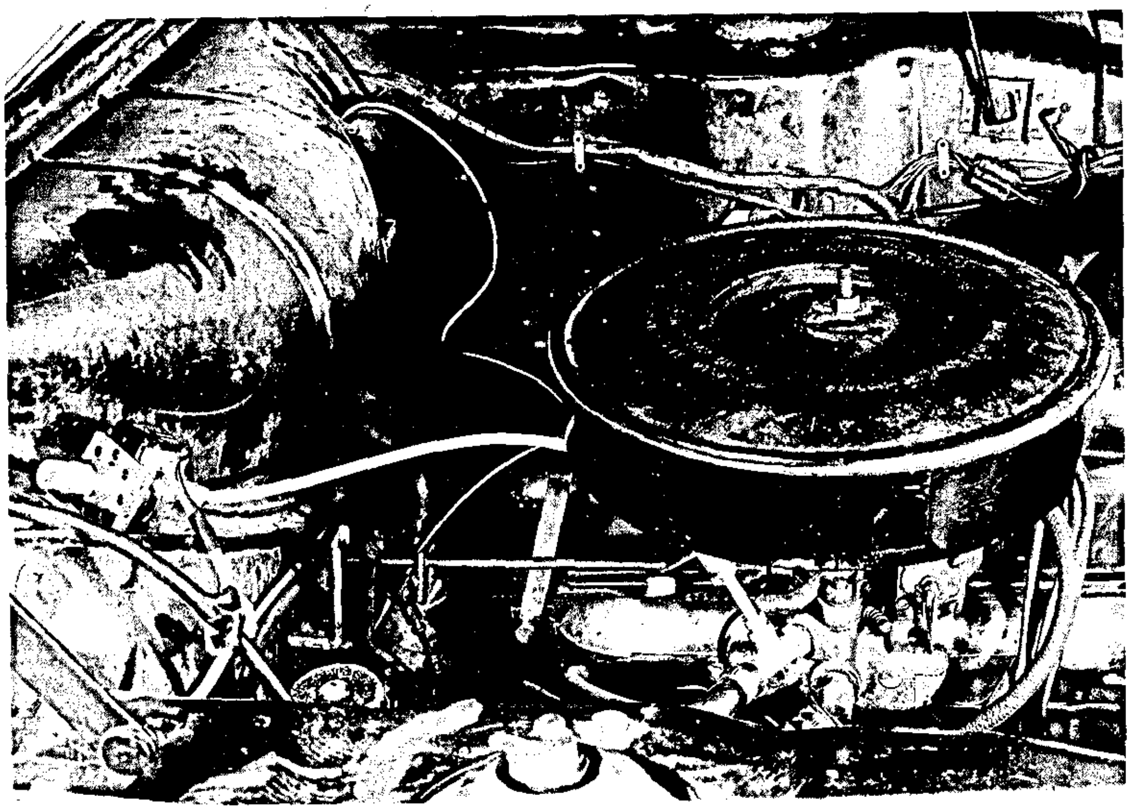


PLATE NO 4·7

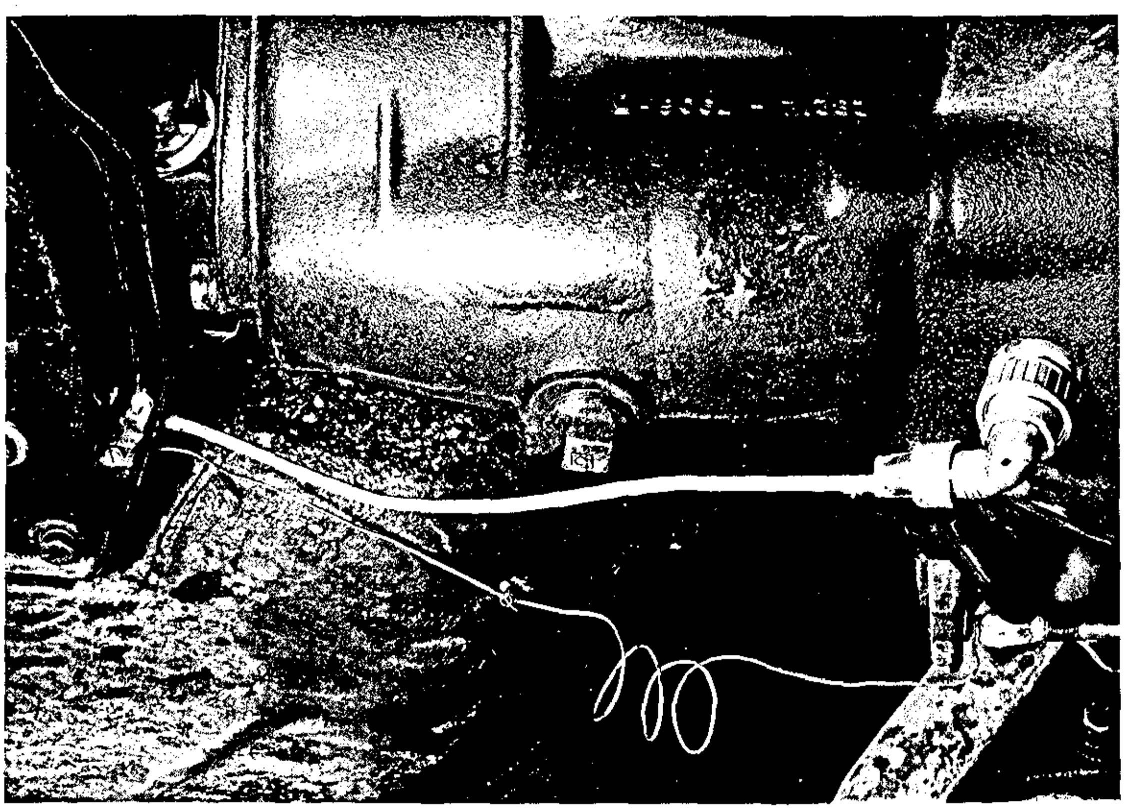


PLATE NO 4·8

CHAPTER 5

VEHICLE SIMULATION TEST RIG

At this point in the research a study of the purely theoretical approach had been made and engagements under various different physical conditions with a wide variety of drivers observed. What was then needed was a method whereby the vehicle-driver combination could be simulated without the complexity of a vehicle. This suggested a test rig with automatic control of clutch engagement, throttle and data acquisition. As with the test rig used in chapter 4 the data acquisition required a rapid sampling rate, but also it was necessary to control the rate of application of the clutch according to the conditions being simulated.

In order to accomplish this a mini computer with sixteen analogue to digital input channels and two digital to analogue output channels was employed, Plate No. 5.2. Using this equipment meant that the test rig could be operated by someone not skilled in the ways of either the mini computer or the test rig. The program, shown in Appendix II, was written to control the test rig and was of a conversational type, i.e. once loaded into the computer it worked on a question and answer basis with the operator.

Test Rig

If a vehicle is considered on the point of moving away from rest on a gradient the forces to be overcome by the torque transmitted through the clutch are:

1. The inertia of the vehicle to be accelerated
2. Rolling resistance of the vehicle
3. Component of the mass of the vehicle acting down the slope.

Each of these forces had to be simulated by the test rig, therefore taking each in turn, the inertia to be accelerated was simulated by replacing the mass of the vehicle plus the inertia of the rotating parts of the drive-line downstream of the engine, by a flywheel of equivalent inertia.

The component of the vehicle mass acting down the slope and the rolling resistance was taken care of by bolting a disc brake assembly to the end of the clutch output shaft, thus enabling a constant torque to be applied to the shaft. Plate No. 5.3 shows the mounting position of the disc brake assembly as well as its method of application. The brake pedal was added to provide an emergency stop mechanism.

The test rig, Plate No. 5.1, comprised of an engine driving a clutch assembly, mounted remote from the engine to allow for ease of access, with the output shaft of the clutch driving through a dynamometer to the inertia flywheel and disc brake. The reason for the dynamometer was that its power absorbtion by drag was proportional to speed to the power 2.8 ($\omega^{2.8}$) and the aerodynamic drag of a vehicle results in a power loss proportional to speed cubed (v^3). The clutch was engaged and disengaged using an electro-hydraulic ram, Plate No. 5.4, which was controlled either manually or by the computer through the control box shown below the ram mounting frame in the plate.

Plate No. 5.5 shows the system used for manipulating the throttle, the diaphragm actuator, being previously used on a distributor automatic timing advance and retard mechanism, was connected into the throttle linkage so that when the engine came under load during an engagement the increased inlet manifold depression opened the throttle. By varying the internal springs of the device the correct response was obtained to prevent the engine stalling during an engagement.

In addition to the basic hardware of the test rig instrumentation was needed in order to monitor the parameters relevant to the engagement.

Instrumentation

The computer was used to log and store eight 0 to ±1 volt input channels at time intervals decided by the operator, the parameters monitored were:

1. A calibration signal of 0.492 volts.
2. Engine speed.
3. Brake dynamometer speed.
4. Clutch torque.
5. Clamp load.
6. Flywheel surface temperature.
7. Pressure plate surface temperature.
8. Ram displacement.

The calibration signal was used as a check on the accuracy of the computers analogue to digital convertor and was also used to trigger the commencement of a cycle. That is when the calibration signal was switched to channel number 1 the computer sensed this and commenced the engagement.

The engine speed and brake dynamometer speed were measured using sixty toothed wheels, see right hand side of Plate No. 5.5, and inductive pickups. The signals passing through a frequency to voltage circuit to be displayed on the test rig console on meters and also passing to the computer.

Clutch torque was measured by strain gauging the first motion shaft of the clutch assembly under test, a full strain gauge bridge arrangement being used to minimise errors from shaft bending. As the shaft was rotating the difficulty came in recording the strain signal, to do this a telemetry system was developed, and can be seen attached to the shaft left hand side of Plate No. 5.5. The signals from the strain gauges were D.C. amplified and then converted into 10 μ sec. pulses the frequency of which was proportional to shaft strain. These pulses were fed into an inductive coil rotating with the shaft; a stationary inductive pick-up was positioned within 0.050 inches of this rotating coil thus collecting the strain signal. This signal was then passed through a frequency to voltage converter the resulting analogue signal going to the computer and a display meter graduated in μ-strain.

As on the test vehicle clamp load could not be measured directly and so a link which can be seen in Plate No. 5.4 at the end of the hydraulic ram was strain gauged and a calibration carried out between a load cell clamped between the clutch plates and the rod strain.

Thermocouples were inserted into the pressure plate and flywheel at the surface in order to measure the temperatures and as these components rotated during an engagement, the system of telemetry used on the test rig in chapter 4 had to be used in order to record the signals. Plate No. 5.6 shows mounting position of this equipment on the test rig.

The final parameter to be measured was the displacement of the hydraulic ram which was used as a feedback for the control of the ram and was also monitored by the computer. Plate No. 5.7 shows the linear displacement transducer, which was used for the measurement of ram displacement, fixed in position at the back side of the ram.

The operation of the test rig, the connection of signal leads and computer program loading instructions are outlined in Appendix II along with a program listing.

Program Theory

From the experience gained with the test vehicle the following description is considered to be representative of how a driver engages a clutch. The clutch pedal is released quickly until the engine speed faulters or the vehicle shudders, whereupon the rate of release of the pedal is reduced i.e. during the period when the vehicle starts to move. The clutch pedal is fully released when the slip speed approaches zero. During the engagement the driver applies more throttle dependant upon the behaviour of the engine i.e. if the engine decelerates rapidly the throttle is opened and vica versa.

In order to simulate exactly this method of control much more sophisticated equipment than that available would be required and so the above description was simplified to the flow chart, Figure No. 5.1; modulation of the throttle being taken care of as previously described. The program written to perform the operation of the test rig was also required to sample the eight data input channels and process the signals ready for output by the required peripheral device between engagements.

The time interval of sampling was set by the operator, dependant upon the type of conditions being simulated, even so the intervals had to be multiples of 0.1 secs. as this allowed the program to be simplified somewhat. Also, this timing constraint allowed the logic outlined in Figure No. 5.1 to be carried out at 0.01 secs. intervals which enabled effective control of the ram engagement. The time required to sample the eight input channels was less than 0.01 secs. therefore the sampling could be carried out without affecting the control section of the program. The maximum number of samples allowed by the program was 100, if this number was reached before the engagement was complete the program printed an error message, before outputing the results. This constraint on the number of samples was due to lack of core storage within the computer and the unavailability of a mass storage device.

At synchronisation of the two clutch members the program automatically stopped the engagement and disengaged the clutch, preventing over heating of the disc brake when simulating gradient engagements.

Results

The data from each series of tests was output by the computer on punched paper tape for processing by a larger machine, which drew graphs similar to that shown, Graph No. 5.1. This graph is similar to the bottom graph shown in a previous chapter, Graph No. 4.15, which was drawn from the test vehicle results. The engine speed was allowed to fall off as the clutch engaged therefore the clutch did less work during the engagement.

The torque signal shown can be seen to increase fairly linearly with time during the engagement. This signal is the average torque and not the true signal, the reason for this is that the torque signal fluctuated at a frequency of approximately 13 HZ, as did the test vehicle signals, but as the sampling frequency of the computer was only 10 HZ, a phenomenon known as aliasing was encountered. This is illustrated in Figure No. 5.2; if the top line is the true signal and the chain dotted lines indicate sample intervals the bottom line will be the signal "seen" by the computer. In order to show clearly the 13 HZ frequency and assuming that three points are needed in order to define one full cycle of a wave form, a sampling frequency of 39 HZ would be required. This sampling frequency would need a time interval between samples of approximately 26 milliseconds which was not possible if all eight channels were to be sampled and the ram controlled. The torque signal was smoothed by statistically curve fitting when the results were processed on the larger machine.

Table No. 5.1, gives details of a number of engagements carried out under different conditions each set of figures is an average of at least six engagements. Due to the newness of the dynamometer used on the test bed, the level road engagements are not strictly level road. This was because the drag torque due to the bearings and seals within the dynamometer was higher than the level road drag torque of the test vehicle. This charactersitic will diminish as the rig is used more often but even so the figures shown in the first row of the table agree well with those obtained during vehicle testing. The temperatures measured during these tests indicate that a normal start from rest on a level road raised the surface temperature in the region of ten degrees which would cause a bulk temperature rise of around one or two degrees.

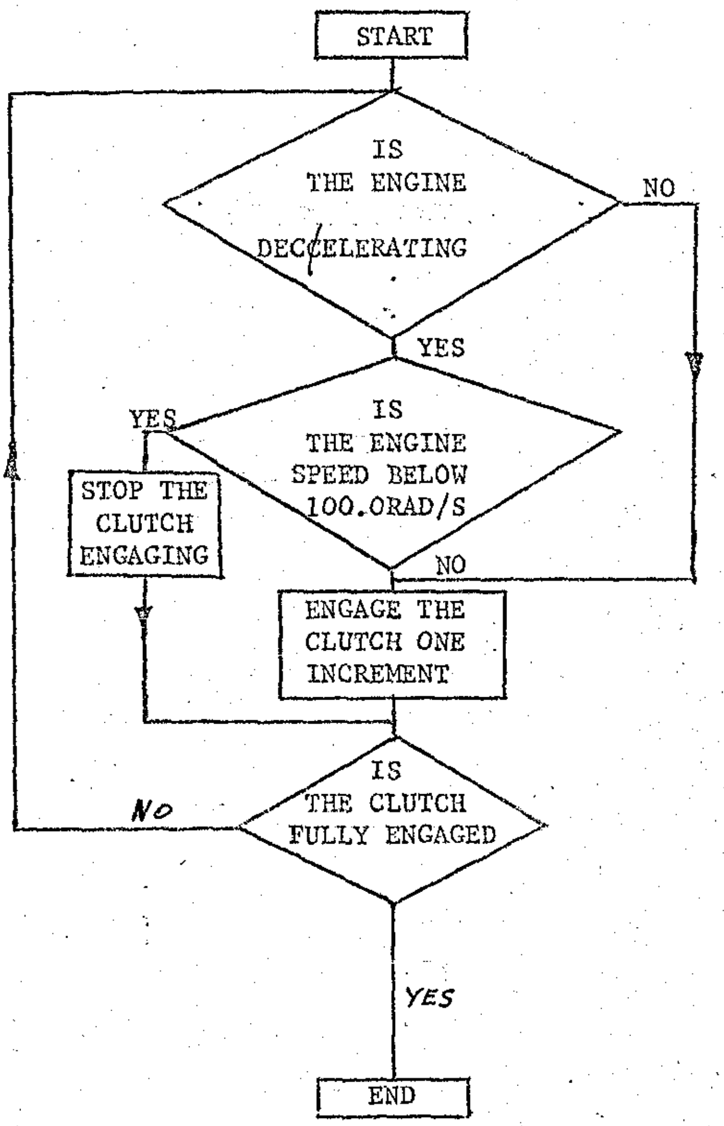
The next stage was to investigate if gradients could be simulated effectively by the test rig, a load was applied by means of the disc brake assembly and the next three rows of figures show the affect of carrying this out. The slip times increased, the heat generated rose and so did the measured surface temperatures, following a similar pattern to the test vehicle results. The choice of gradient was difficult and the figures shown are approximate values calculated from the measured torque required to cause the output shaft to rotate.

Finally the effect of engine speed was investigated by raising the initial engine speed before the clutch started to engage. The final two rows in Table No. 5.1, show the results obtained, the slip time increased slightly as did the output shaft acceleration but the major increase came in the heat generated.

These results indicate the need to keep engine speed down during the engagement period in order to reduce clutch temperatures but that more thermal damage occurs when engaging the clutch whilst on a gradient. This test rig gives results similar to those of the test vehicle and provides a simple-to-use test facility. However, the theoretical model predicts short slip times which indicates a necessity for a closer look at the theory now that the test rig has been used to simulate a driver-vehicle combination with some degree of success.

ENGAGEMENT CONDITIONS	MAXIMUM ENGINE SPEED RADS/SEC	MINIMUM ENGINE SPEED RADS/SEC	AVERAGE ENGINE SPEED RADS/SEC	SLIP TIME SECS	HEAT GENERATED KJ	OUTPUT SHAFT ACCELERATION RAD/S/S	AVERAGE TEMPERATURE RISE DURING THE ENGAGEMENT DEGS.C	
							PRESSURE PLATE	FLYWHEEL
LEVEL ROAD	230	122	190	2.1	10.6	80.0	12	10
1 : 10	251	84	190	2.5	17.1	62.0	20	19
1 : 7	285	123	245	3.6	30.4	69.0	29	25
1 : 3	302	131	238	5.2	50.7	65.0	44	37
LEVEL ROAD INITIAL SPEED 250.0 RADS/SEC	272	153	224	2.4	15.0	82.0	14	13
LEVEL ROAD INITIAL SPEED 300.0 RADS/SEC	312	181	260	2.4	18.2	89.0	16	21

TABLE NO. 5.1



FLOW DIAGRAM OF CLUTCH ENGAGEMENT

FIGURE NO. 5.1

THE AFFECT OF ALIASING ON AN INPUT SIGNAL

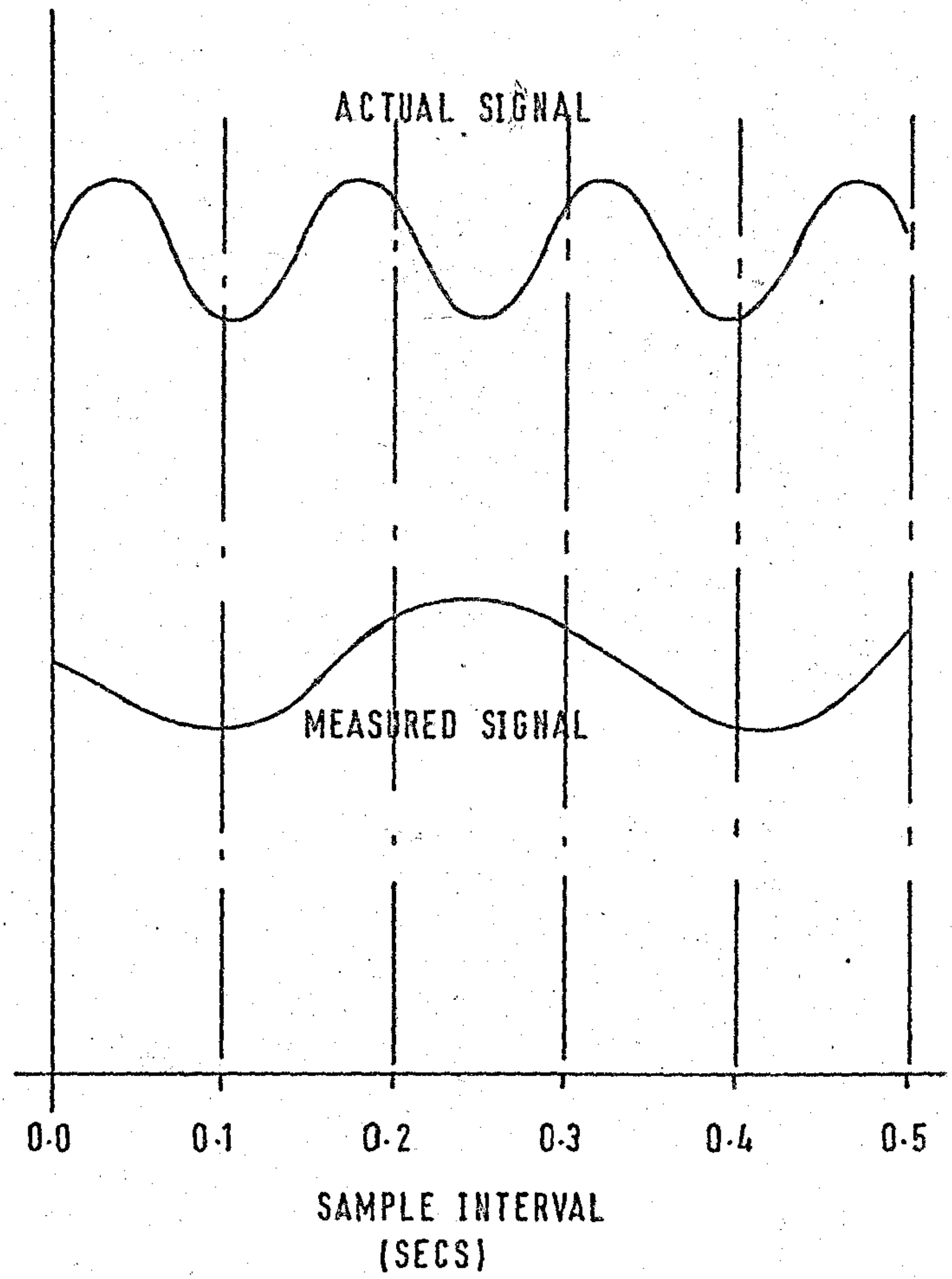
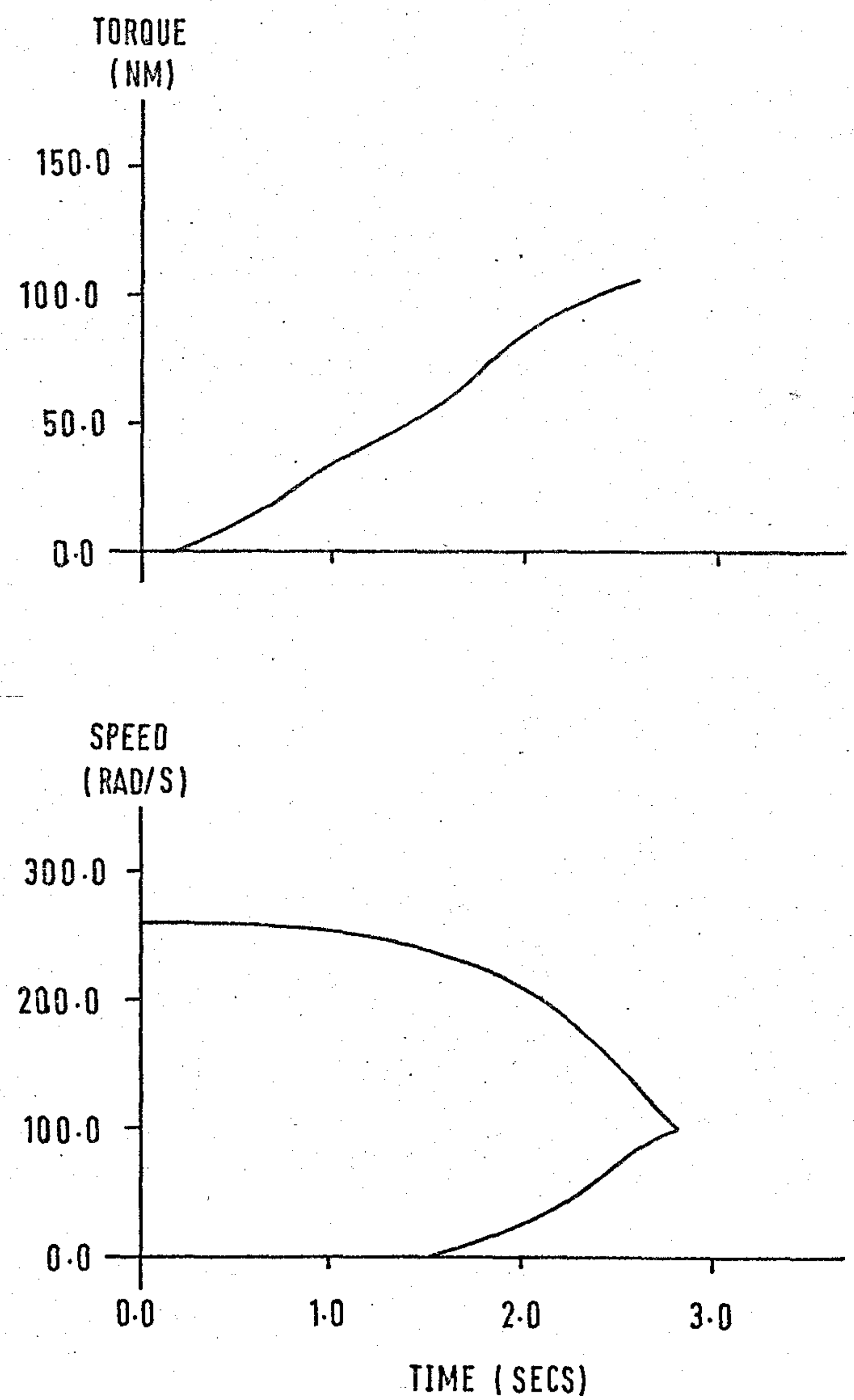


FIGURE NO. 5.2

A TYPICAL TEST RIG ENGAGEMENT



GRAPH NO. 5.1

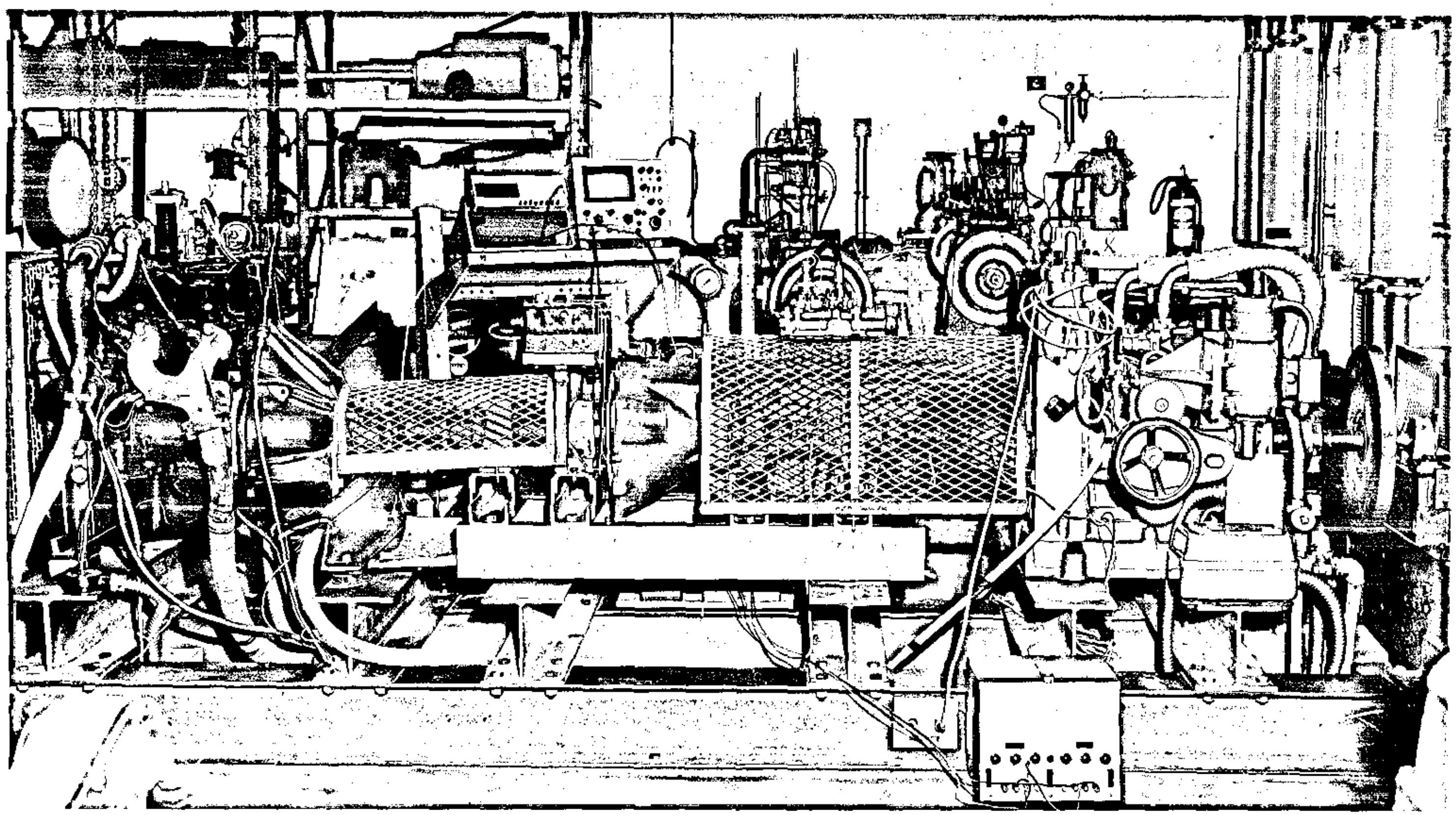


PLATE NO 5·1

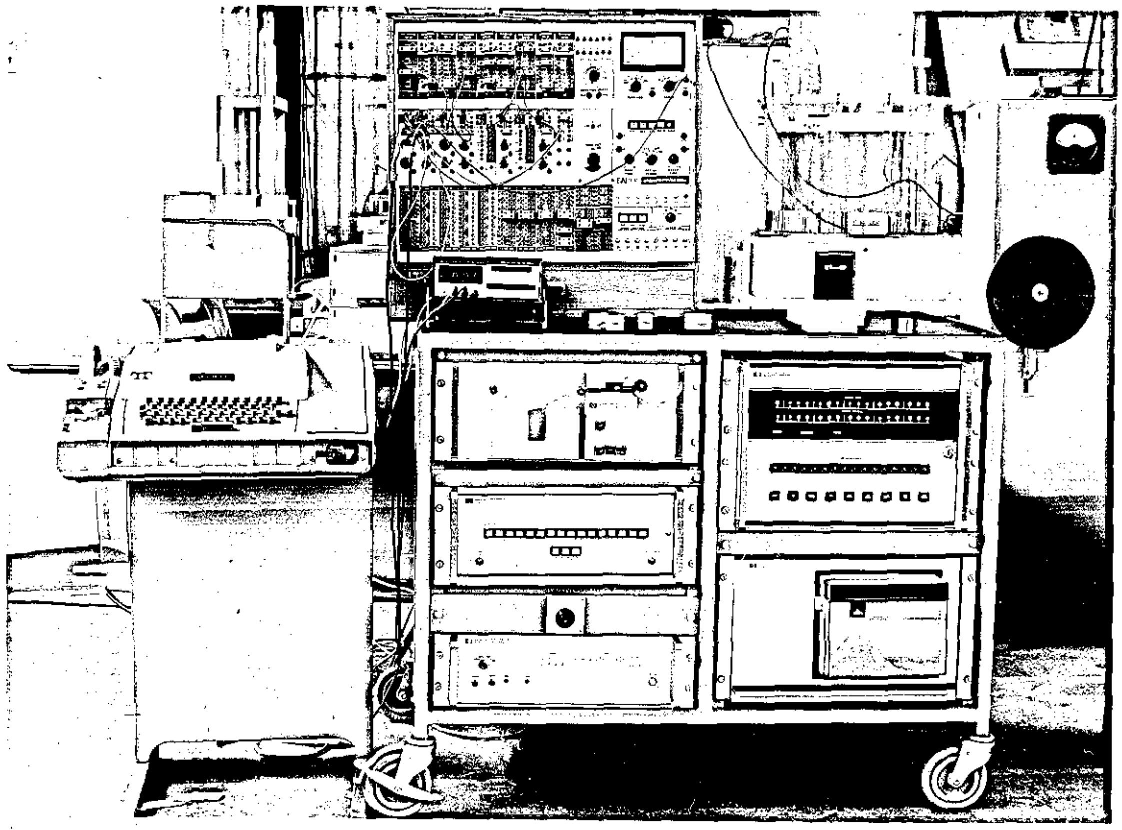


PLATE NO 5·2

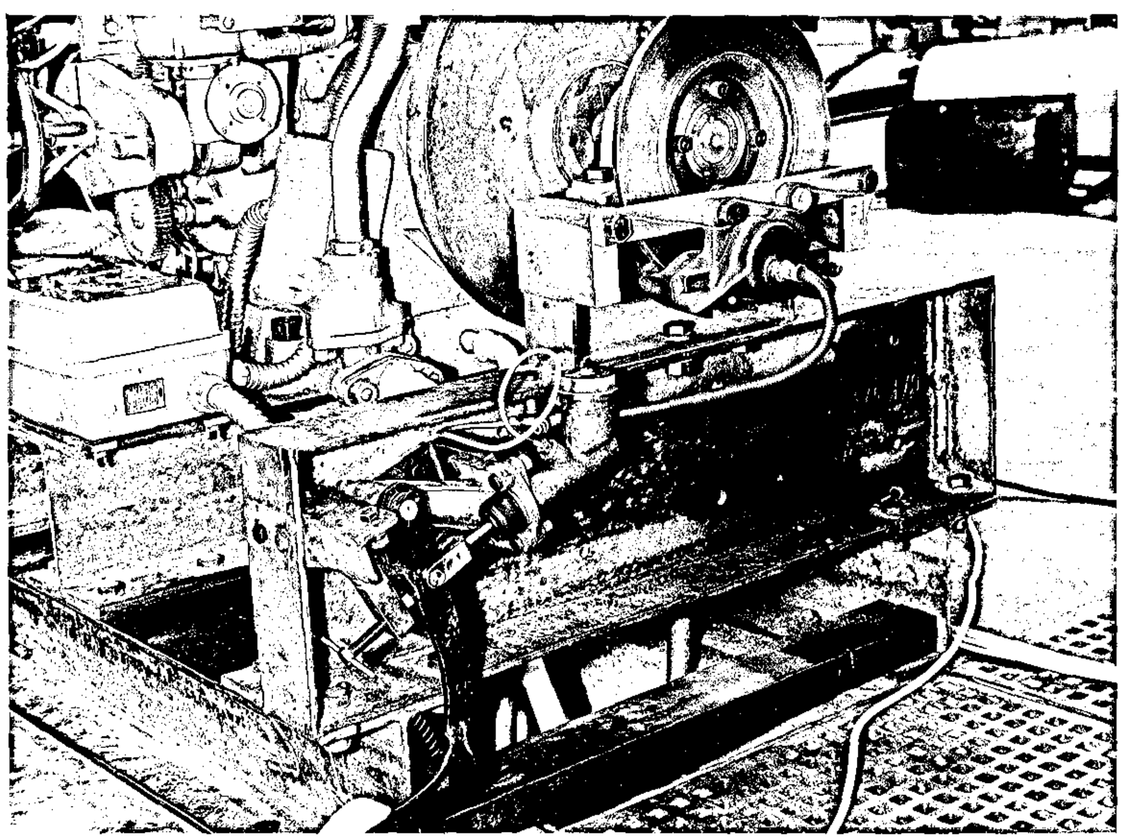


PLATE NO 5·3

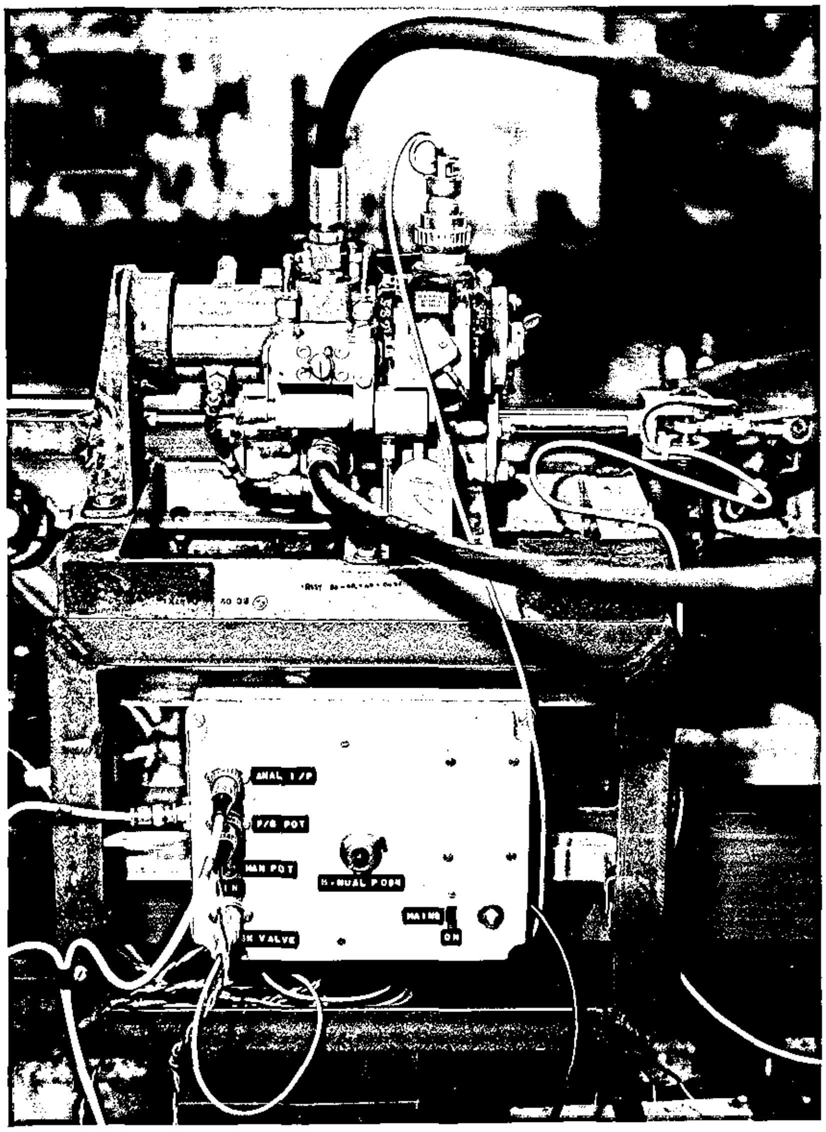


PLATE NO 5·4

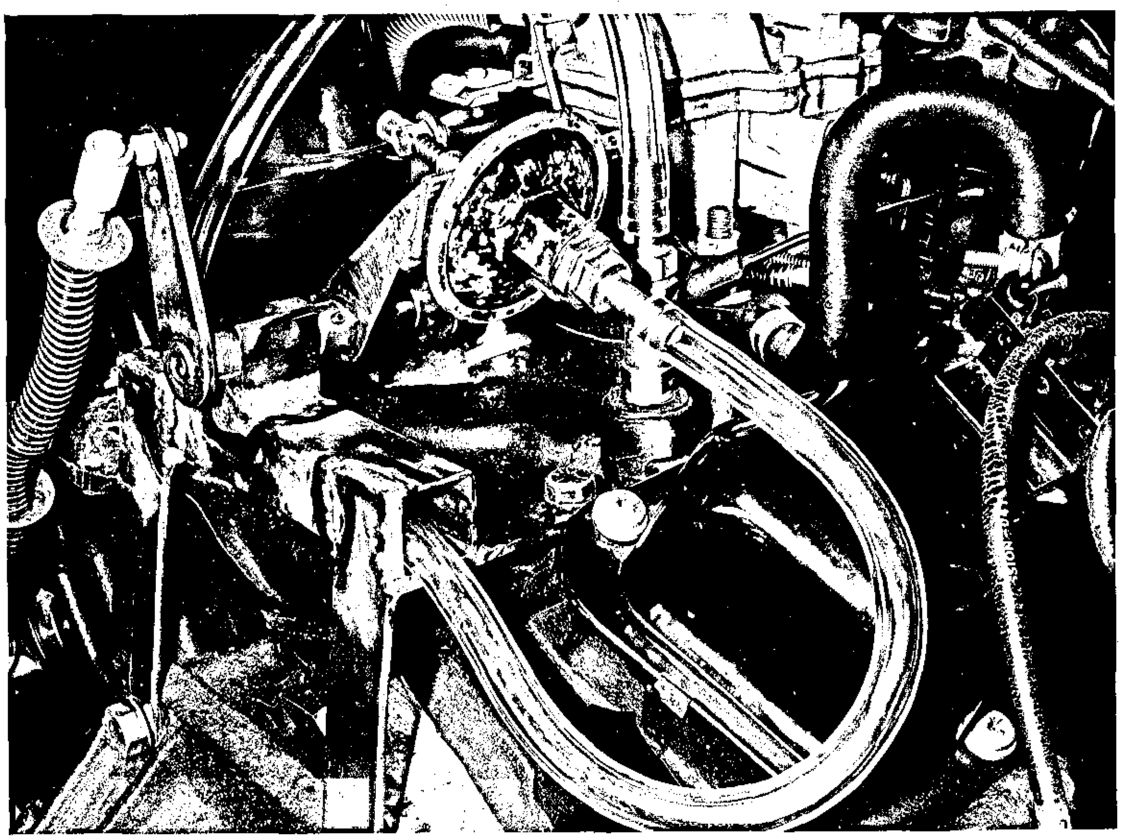


 PLATE NO 5·5

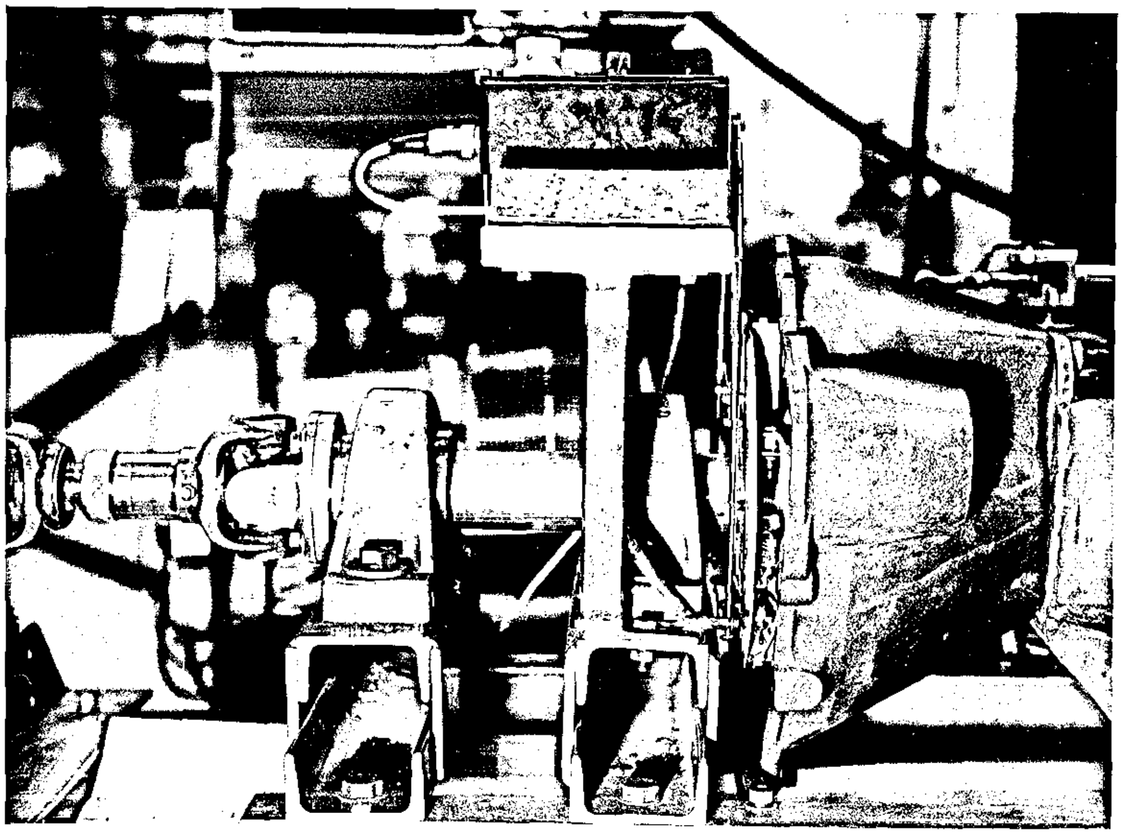


PLATE NO 5·6

PLATE NO 5·7

CHAPTER 6

REVIEW OF MATHEMATICAL MODEL

Introduction

The digital computer simulation of a clutch engagement has been shown to predict slip times and heat generated which while agreeing closely with theoretical work carried out by other researchers working in this field, does not agree with the values obtained from the test vehicle or test rig results. Also the predicted temperatures were far higher than those measured on the rig tests and although some of this discrepancy (less than 10%) could be attributed to the method of measurement of the surface temperatures there is also a large quantity of heat flowing radially across the surface of the clutch, thus presenting a two dimensional heat conduction problem. If this heat flow radially were to be included in the model the complexity of the relevant equations would increase from n to n^2 where n is the number of mesh points considered within the material. Therefore it was felt that rather than push the model into a more complex form, for what would be a minimal gain it would be better to keep the simplicity and accept the limitations, a more complete discussion will be given later.

Regarding the slip times and heat generated however it was felt that large improvements could be made by incorporating into the theory what were considered to be two major stumbling blocks:

1. The constraint of a constant engine torque
2. The human element in clutch manipulation

Engine Characteristics

Before test work began with the vehicle the engine was removed and fitted onto an engine test bed, upon which a full performance test was carried out. Therefore it was possible to interpolate, from the graphs drawn; the engine power being delivered, given a value for engine speed and manifold depression, see Graph No. 6.1. This graph was approximated by a number of polynomials and included into the digital simulation, by adding the solution of one more differential equation to the problem.

$$\frac{\partial \Delta P}{\partial t} = f(t)$$

The function f(t) being the response of the engine to the throttle, which was estimated from the vehicle test results.

Thus allowing the model to respond as the vehicle would to whatever inputs were fed via the clutch pedal. This only left the nature of these inputs to be considered and from the vehicle test results they appeared to be of a linear ramp form similar to Figure No. 6.1.

Operator Characteristics

The initial steep ramp is during the period where the clamp load is raised from zero to the value at which the propeller shaft commenced to rotate and is dependant upon the engagement conditions. The secondary ramp is where the vehicle commences to move and continues until the clutch plates are synchronized. The final ramp generates no heat as the two plates are rotating at the same speed and are not therefore included in the model.

For the initial ramp the model shown in Figure No. 6.2 was considered where the output side of the clutch is stationary and the engine speed constant. Thus giving the following equations:

$$\frac{\partial P}{\partial t} = f(\text{gradient}, t)$$

$$\frac{\partial \omega_e}{\partial t} = \frac{T_e - T_c}{I_e} = 0 \text{ as } T_e = T_c .$$

$$\frac{\partial Q}{\partial t} = T_c * \omega_e$$

The integration of the above differential equations yielding the heat generated during the period before the propeller shaft started to rotate and also giving the initial conditions for the integration process governing the synchronizing time of the clutch.

During the period of time when the clutch plates are synchronizing the model in use was the same as outlined in chapter 3, with the inclusion of the engine characteristics. But the clutch was engaged at a rate dependent upon how the engine performed i.e. if the engine accelerated, the rate of clutch engagement was increased and vice versa. Also built into the program was a safeguard so that if the engine speed fell below 100.0 rads/s the clutch application ceased until the engine speed again rose above 100.0 rads/s. The values for the rate of application being deduced from the practical tests.

The model, after the inclusion of these modifications gave a more representative simulation of an actual engagement than the previous purely theoretical models and allowed for detailed studies of parametric variations or repeated engagements to be made easily. Appendix III, contains the printout of the two programs written using the two different types of temperature solution and also gives a detailed description of how to run the programs and what the various switching options are.

Results

The results of a digital simulation can be seen tabulated in Appendix III, but Graph No. 6.2 depicts a typical engagement and is in a similar form to the results obtained from both the test vehicle and the test rig. For comparison, values of results from several runs have been tabulated, Table No. 6.1, contains level road engagements only. The first row being the results from a normal engagement, with an initial engine speed of approximately 200.0 rads/s which was allowed to decay during the engagement. These results are similar to those obtained from the test vehicle although the slip times are shorter. In chapter 4 it was mentioned that the clamp load was applied until the clutch was transmitting enough torque to move the vehicle away from rest, the length of this period varying with the gradient and the driver. In the digital simulation this time period was controlled by applying the clamp load at the same rate, regardless of gradient, until the clamp load required to prevent the vehicle rolling backwards was reached.

The next two rows of Table No. 6.1 show the affect of varying the response of the engine i.e. how much throttle was used during the engagement period. These results indicate that for a fast engagement it is better to use minimal throttle and this will also give lower heat generation. This may at first appear contrary to what actually happens but in practice a fast clutch engagement is usually, hand in hand with a fast "getaway", which will mean high engine speeds and high propeller shaft accelerations. In the case shown the vehicle was merely required to move away smoothly which can be carried out with low engine speeds, but at the sacrifice of acceleration.

Next in the table the effect of initial engine speed upon the engagement is presented, with all other parameters remaining constant. In this case as engine speed rises, the heat generation and slip time increases, this is because the clutch was required to transmit the same torque in order to move the vehicle from rest, but at the higher engine speed more work must be done. The final row in Table No. 6.1 shows the effect of increasing the engagement speed which brings down the slip time and heat generated but increases the propeller shaft acceleration. All these parameters are varied by the driver, during the engagement period, in order to obtain the desired engagement.

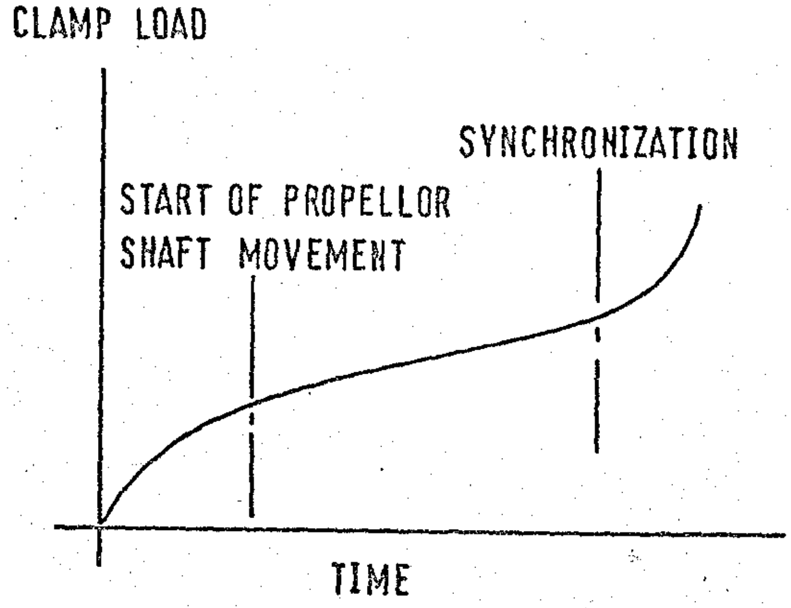
Table No. 6.2 shows the effect of gradient variation i.e. longer slip times and higher heat generation but the slip times do not compare with those obtained during the vehicle tests. The cause of this discrepancy is the human element, only a very simple model of the vehicle has been considered. The digital model could, by varying engine response, engagement speed and initial condition be made to give results similar to those measured. Doing this for a particular vehicle would then enable parametric studies to be carried out and the effect of the variations for that particular vehicle noted.

ENGAGEMENT CONDITIONS	MAXIMUM ENGINE SPEED RAD/S	MINIMUM ENGINE SPEED RAD/S	AVERAGE ENGINE SPEED RAD/S	SLIP TIME S	HEAT GENERATED KJ	OUTPUT SHAFT ACCELERATION RAD/S/S	AVERAGE TEMPERATURE RISE DURING THE ENGAGEMENT DEGS. C	
							PRESSURE PLATE	METAL MEMBER
LEVEL ROAD STANDARD	200.0	149.0	183.0	2.15	9.50	77.0	44.0	40.0
LEVEL ROAD FASTER ENGINE RESPONSE	200.0	167.0	188.0	2.31	10.43	80.0	47.0	43.0
LEVEL ROAD SLOWER ENGINE RESPONSE	200.0	146.0	183.0	2.0	9.10	81.0	43.0	39.0
LEVEL ROAD INITIAL ENGINE SPEED. 250.0	250.0	185.0	224.0	2.4	13.91	84.0	59.0	56.0
LEVEL ROAD INITIAL ENGINE SPEED.300.0	300.0	212.0	278.0	2.6	19.22	89.0	82.0	76.0
INCREASED ENGAGEMENT SPEED	200.0	130.0	188.0	1.6	8.05	93.0	45.0	40.0

TABLE NO. 6.1

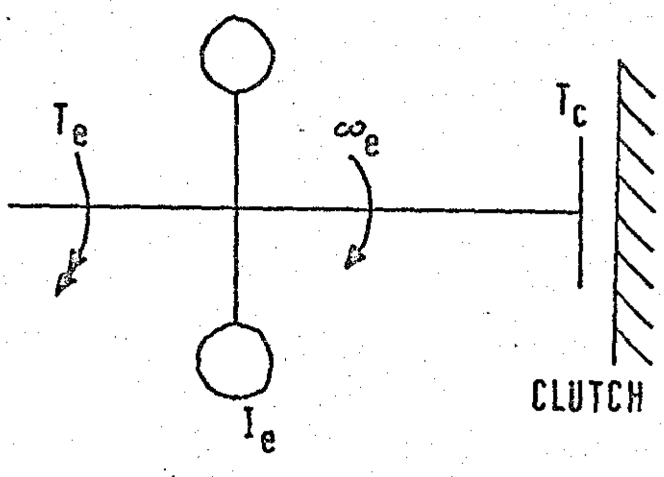
ENGAGEMENT CONDITIONS	MAXIMUM ENGINE SPEED RADS/S	MINIMUM ENGINE SPEED RADS/S	AVERAGE ENGINE SPEED RADS/S	SLIP TIME S	HEAT GENERATED KJ	OUTPUT SHAFT ACCELERATION	AVERAGE TEMPERATURE RISE DURING THE ENGAGEMENT DEG. C.	
							PRESSURE PLATE	METAL MEMBER
GRADIENT 1:5	200.0	135.0	182.0	2.63	26.02	84.0	113.0	108.0
GRADIENT 1:10	200.0	144.0	182.0	2.47	17.15	76	73.0	68.0
GRADIENT 1:20	200.0	150.0	183.0	2.33	13.23	75.0	57.0	53.0

TABLE NO. 6.2



DRIVER APPLICATION OF CLAMP LOAD

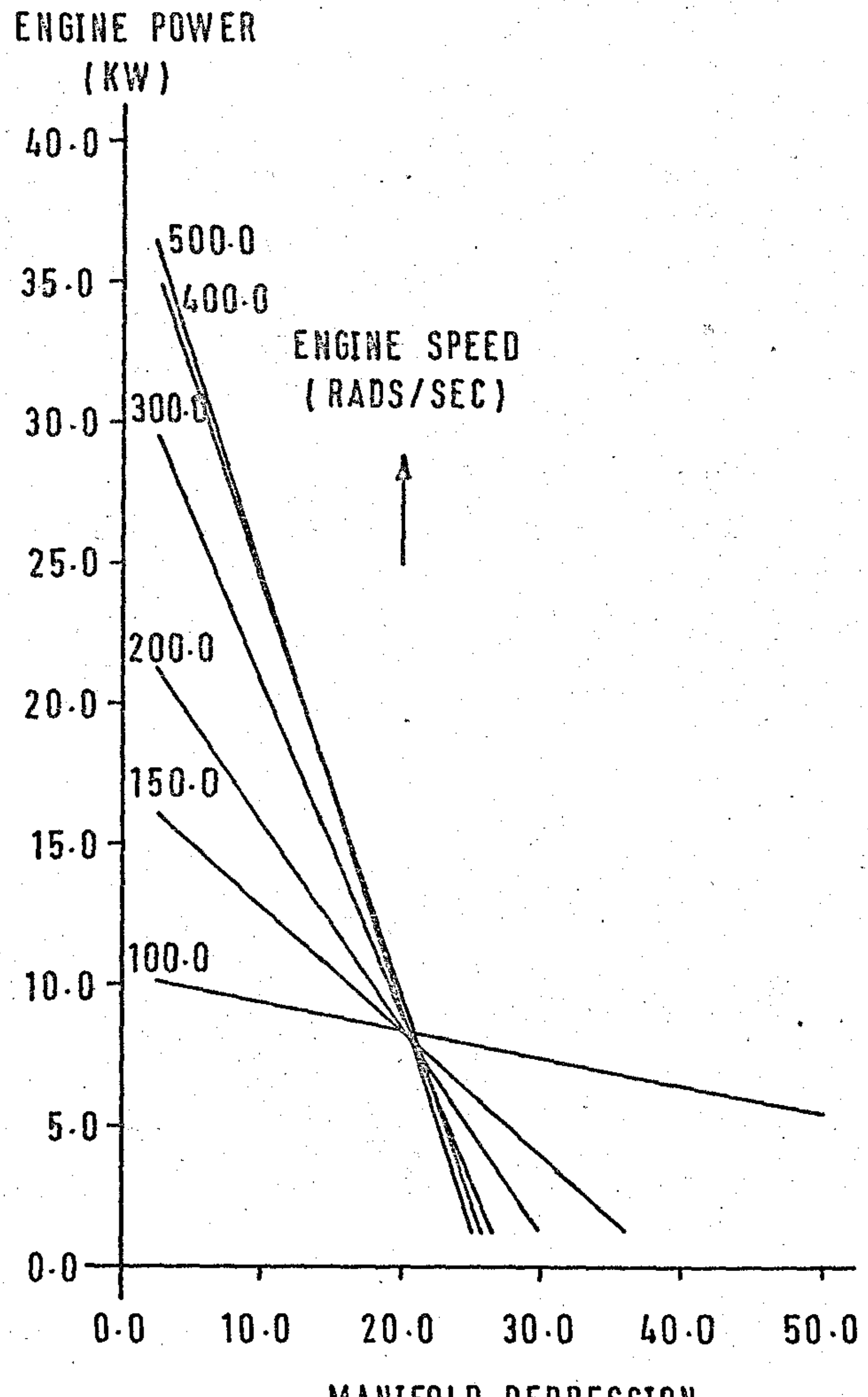
FIGURE NO. 6.1



MATHEMATICAL MODEL OF CLUTCH SLIP
WHILST VEHICLE IS STATIONARY

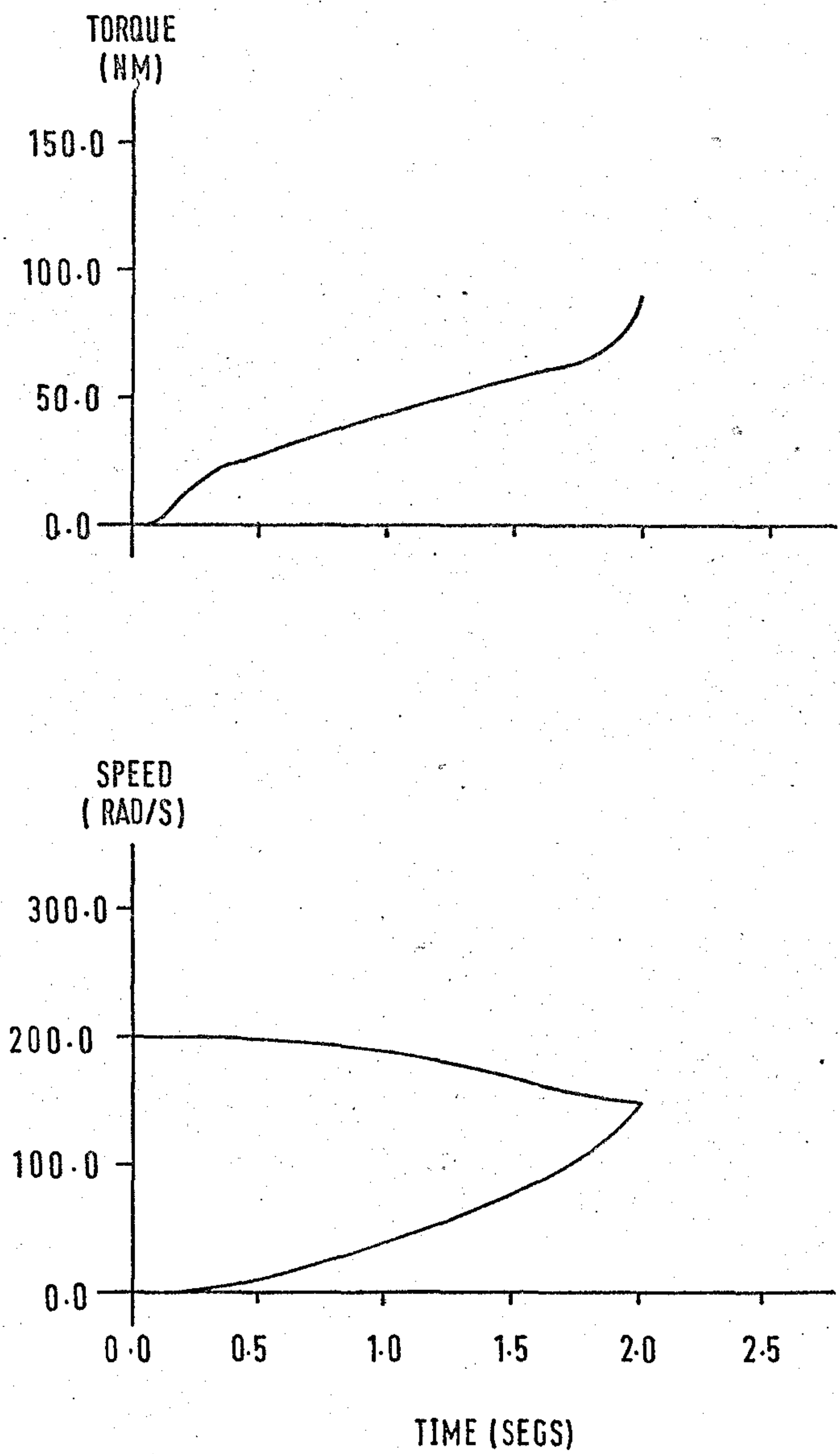
FIGURE NO. 6.2

TEST VEHICLE ENGINE PERFORMANCE



GRAPH NO. 6.1

A TYPICAL THEORETICAL ENGAGEMENT



GRAPH NO. 6.2

CHAPTER 7

DISCUSSION

The discussion follows a similar pattern to the rest of this thesis, in that each topic will be dealt with in turn and at the end, the research as a whole will be reviewed.

Initially a mathematical model was produced which was constrained by the same assumptions as were previous theoretical models and it was found that the two different approaches i.e. the analytical and digital, yielded the same results. Now, as one of the more obviously erring assumptions was that the clamp load was applied in zero time, this was removed from the digital model and replaced by assuming the clamp load to increase linearly throughout the engagement period until full clamp load had been applied. This modification was shown to increase the slip time, heat generated and ultimately the surface and bulk temperature of the assembly. The reason why this happened was that the model was still constrained by another assumption; that of constant engine torque being delivered by the engine throughout the engaging period which again is obviously in error. In addition, whilst looking at the problem it was decided that the coefficient of friction variation was not as simple as had previously been assumed.

These points led to two separate investigations being instigated:

1. The variation of coefficient of friction during the engagement.
2. The manner of clutch manipulation by the driver during the engagement.

Investigation into coefficient of friction and surface temperature

For the first investigation an existing test rig was utilized, and a clutch assembly fixed therein. Numerous thermocouples were inserted into the friction materials and also at the friction surface of the pressure plate and the flywheel. From the results of many repeated engagements it was hoped to build up a pattern of temperature variation across the cushion segments in addition to defining the coefficient of friction variation with clamp pressure, surface temperature and slip velocity. To reduce errors in measurement ,calibrations were carried out through all the signal conditioning equipment and also through the recording system. Before each test spot checks were made to confirm that the equipment, signal conditioning and recording was still functioning correctly. This procedure was followed throughout all test work carried out during this research.

For the friction material a sample of the results was presented in chapter 4 and showed some scatter and inconsistancy. As inaccuracies in the equipment and measuring system had been cut to a minimum by the "through" calibration and spot checks, the answer must be sought elsewhere. Firstly, after discussion with a friction material manufacturer, pressure plates were examined which had surface patches where the metal appeared to have locally melted and had flowed in the direction of slip, this phenomenon was caused by what is known as heat spotting. It can occur in clutches under light or heavy duties and its choice of position appears to be arbitrary. Secondly, the friction material used was an asbestos yarn, bound with zinc wire coiled and set in a resin, so if one of the thermocouples was in the near vicinity to the zinc, it would receive the heat from the interface at a much faster rate than a thermocouple positioned farther away. These two explanations are suggested as the cause of the varied results obtained, during the measurement of the friction material temperatures.

However the temperatures measured by the thermcouples inserted at the interface surface of the pressure plate and flywheel gave repeatable results. In order to check the validity of the mathematical model, the data relating to the test rig was inserted into the program but the resulting temperatures were much higher than those measured i.e. maximum surface temperature of around 85^{o}C against 180^{o}C predicted. Bulk temperature of 50^{o}C against 130^{o}C predicted. The bulk temperature of the assembly was measured after 10 seconds had elapsed, since the end of the engagement but it is realised that for a true bulk temperature a settling time in the region of 10 mins. should be allowed. During a cooling curve carried out on the assembly the theoretical results drew closer to the measured as time increased.

A first reaction was to suspect the thermocouples but on checking the response of these by applying a large quantity of heat to the metal surface and monitoring the rise time of the embedded thermocouples to that of one sandwiched on the surface, there was no detectable difference. Some of the measured against predicted difference of the clutch temperatures can be attributed to experimental error, in that the program simulates the ideal case whereas on the rig uneven clamping and excessive wear particles cause inefficient thermal contact at the mating surface. Both conditions tend to lower the measured temperatures by allowing heat to flow radially outwards. Even so the discrepancies were large enough that the assumptions surrounding the theoretical model became suspect and required examination.

The major assumption was that of a one dimensional heat conduction model, Figure No. 7.1 shows a scaled diagram of the flywheel and of the assumed model, with the mass of metal of each printed at the side. As the metal members of the clutch absorb approximately 98% of the total heat generated the friction materials may be neglected for this argument. If the annulus of Figure No. 7.1 is assumed to have the same mass as the actual flywheel, the transient temperatures predicted are reduced to nearer the measured values. This suggests a two dimensional model would give more accurate results, and so in order to substantiate this an extra thermocouple was inserted radially out from one of the others in order to monitor the heat flow in that direction. The results showed that heat was flowing outwards from the rubbing surface very rapidly. From the above evidence it was concluded that the temperatures measured were accurate but the one dimensional model is not a true description of the thermal system. As has already been pointed out the complexity and execution time would rise from n to n^2 where n is the number of mesh points under consideration if the change from a one dimensional to a two dimensional model was made. One of the aims of this research was to produce a usable tool and so the decision was made to utilise the one dimensional model accepting the over prediction, but noting that if the model is used only for parametric studies the absolute values can be ignored. In order to correctly test the validity of this assumption it would be necessary to program the two dimensional model, which could be performed at a later date as an academic exercise. However, the assumption would appear correct from the results obtained earlier when the one dimensional model was constrained to have the same mass as the whole flywheel.

The measurement of coefficient of friction and its variation with surface temperature, clamp pressure and slip velocity has been presented as a series of graphs which define this variation. The methods used to obtain these results were tedious but could be automated using the on-line computer. As the technique used to describe the variation was of an empirical nature the results should be understood to apply only for the type of clutch i.e. size, friction material used and the past history of usage. The clutch used for the test was new and the lining was gently but fully bedded before the test. This procedure was employed for all the test work. The effect of making three different assumptions in so far as the simulation is concerned, with regards to coefficient of friction variation, can be seen below:

1. Assuming kept constant. At average level between the static and dynamic coefficient of friction, gives slip time 0.5 secs. and heat generated 6.02 KJ.
2. Assuming variation with slip velocity only. Gives slip time 0.56 secs. and heat generated 6.31 KJ.
3. Assuming variation with slip velocity, surface temperature and clamp pressure. Gives slip time 0.53 secs. and heat generated 6.09 KJ.

These figures were obtained from the digital program and show that the above assumptions for coefficient of friction variation have little affect on the heat generated and in addition to this as large a variation can be obtained by an error of 10% in the vehicle mass.

Vehicle Test Work

As with all the instrumentation mentioned previously, that of the test vehicle was calibrated through the whole of the signal conditioning equipment as well as the recording system, in order to minimise experimental error. Trouble was experienced when measuring the dynamic torque in the first motion shaft, because the signal was extracted from the rotating shaft using a slip ring and brush assembly. If the slip rings became contaminated, which happened frequently even though care had been taken to seal all the sources of contamination, the signal to noise ratio became bad enough to render the results meaningless and so a close watch had to be kept and the assembly dismantled at regular intervals for cleaning purposes. As a secondary torque measuring system a correlation was obtained between inlet manifold depression, engine speed and torque delivered, but this correlation assumed steady state conditions which, for the engagement period, was a reasonable assumption. A comparison of the total heat generated and the maximum torque transmitted during the take-up period show agreement within 5% for the two methods used.

The method for measuring clamp load was strain gauging of a rod in the clutch linkage and was susceptable to inaccuracies caused by hysteresis in the clutch actuating linkage, but if the clamp load was continually increased throughout the engagement these inaccuracies had negligible affect. The signals from the linear displacement transducer monitoring the clutch pedal movement showed that in general the pedal was released continually throughout the engagement and that the only point of modulation occurred when the initial "bite" of the plates was felt.

The results were split into three different patterns of engagements, these patterns forming general groups under which the majority of the engagements could be classified although there was some overlap between the three groups. It was observed, from the results what operating parameters affected the engagement characteristics but when a correlation was attempted using statistical techniques, no definite conclusions could be drawn. In order to arrive at definite correlations much more test work would be required, coupled with detailed questionnaires to define the type of driver. Also the instrumentation and processing of results would need improving but this was not considered economic or indeed worthwhile for this program of research. These results show how the clutch was manipulated by the drivers and gave indications of which characteristics affect the heat generated and hence the temperature rise within the clutch. Also, these results formed a good basis for comparison with the rig test work and theoretical work that followed.

Vehicle Simulation Test Rig

As mentioned in the chapter describing this test rig, it was at the time of testing not possible to carry out true level road tests because of the drag in the dynamometer seals, which effectively caused the simulation of approximately 1 : 30 gradient but this affect will disappear with use. The results obtained compare well with those from the test vehicle but it was difficult to choose exactly the type of gradient required to be simulated because of the insensitivity of the disc brake assembly. To combat this, if the disc pads were replaced with ones made from a material having a lower coefficient of friction, the adjustment of drag torque would be more sensitive.

The basic hardware available within the computer, for the timing of the interrupt intervals only allowed intervals in orders of ten from 0.0001 to 1000.0 seconds and also the software for driving the teleprinter did not allow interruption whilst it was in operation, without an error condition being set. This meant that a 100 second interval was the only one which would allow the printout of results between engagements, thus limiting the repeatability of engagements. This can be overcome but requires more complex programming, at a systems level, which was not felt necessary at this stage. The program was capable of carrying out multiple engagements automatically but for each engagement the conditions had to remain the same, even so this test rig is capable of testing clutches under similar conditions to those encountered in service. To enable more realistic conditions to be simulated the test rig would require modifying to allow automatic application of drag torque and the setting of engine speeds by computer. All this could be accomplished reasonably simply from both the mechanical and programming side and the result would be a test rig capable of being programmed to simulate a given test route or set of circumstances.

Theoretical Model

By inserting the engine performance characteristics in to the program and also applying the clutch dependant on the behaviour of the engine, slip times and heat generation of more sensible levels were obtained which agree favourably with the results from both the test rig and the test vehicle. The problem comes when trying to simulate the driver, the model as programmed will engage the clutch in a manner not unlike some of the practical tests but if the conditions are changed the model does not necessarily follow the same pattern as the practical tests, even though the heat generated agrees, as can be seen from the results. This means that although the affect of changing parameters within the vehicle may be observed, in relation to the heat generation, it will not be possible to see the affect of this parametric variation on the pattern of engagement. However given the results of a test on a particular driver, he could be simulated with some degree of accuracy but as the model stands it is not possible to simulate say driver "A" without prior knowledge of his driving technique. To do this would require detailed studies of driver behaviour to provide a correlation between his temperament and driving technique.

The difference in temperatures between those predicted and those measured has already been discussed, suffice to say that in respect to the simulation of actual engagements the predicted peak temperatures were found to be approximately double the measured. Care will have to be taken when using this model for predictions of magnitudes but the model will be invaluable for comparing situations.

To summarise this program of research, a theoretical model has been produced giving a usable design tool bearing in mind the limitations mentioned above. A method for defining the variation of coefficient of friction with pressure, temperature and velocity has been tried and proved to work but its affect when incorporated into the mathematical model was found to be small. A computer controlled test rig has been designed and the results were in agreement with tests carried out on subjects in a specially instrumented vehicle.

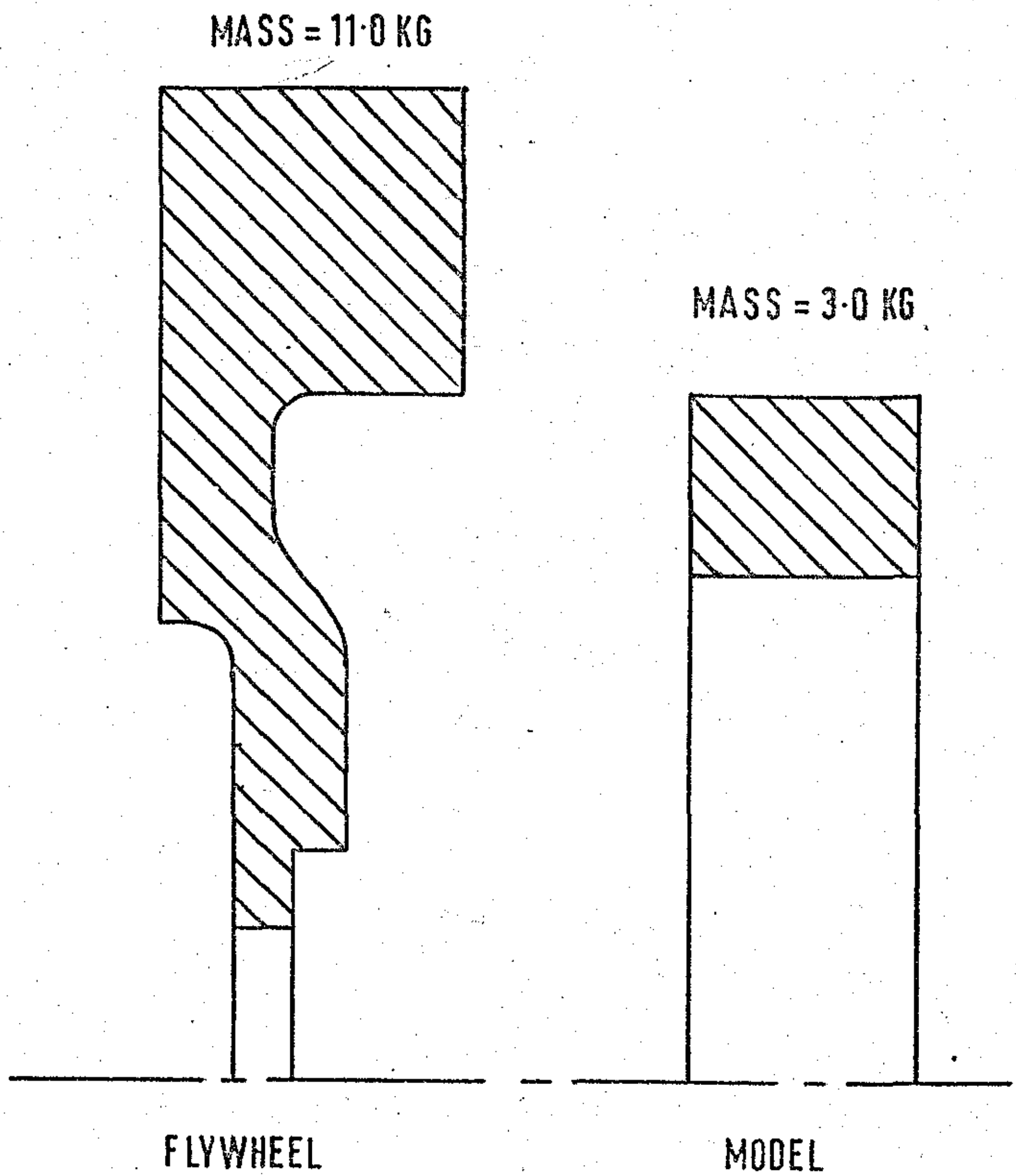


FIGURE NO. 7.1

CHAPTER 8

CONCLUSIONS

1. A mathematical model has been produced which will enable parametric studies of a clutch to be made. The model can simulate the driver/vehicle combination and is not confined to the dynamics of the clutch as was the case in earlier publications.

2. A computer controlled test rig has been developed which will allow clutches to be tested under "in service" conditions and excludes all the tedious data reduction.

3. Both the digital simulation and the test rig simulation provide an ideal basis for a similar study aimed at the problem of clutch judder.

4. Assuming the coefficient of friction to be constant instead of allowing it to vary with pressure, temperature and slip speed had a similar effect to making a small error in the vehicle mass. A method of defining the variation of coefficient of friction was tried but was not found to be satisfactory for general use because of the lengthy reduction of results involved.

5. The most notable difference between the operating characteristics of the different drivers was the way in which they modulated the engine throttle. A driver who attempted to keep the engine speed high during an engagement, by progressively opening the engine throttle whilst engaging the clutch, generated a significantly larger amount of heat than a driver who allowed the engine speed to remain low throughout the engagement.

6. A prolonged slip time caused by gradient starts or bad driving technique, can generate at least as much heat during an engagement as would a so called racing start where high engine speed is maintained throughout the slip period.

CHAPTER 9

FURTHER WORK

During the test work carried out in this research it was noticed that the torque signal near the end of the synchronization period began to oscillate at a frequency of approximately 13 HZ. This phenomenon was observed during both the vehicle tests and the rig tests even though, on the test rig, the driveline was very much stiffer and the only flexible part was the cushion springs of the centre plate. On the test vehicle there was little physical sensation of vibration, but under certain conditions a transmission vibration could be felt. It is thought that these torque fluctuations are the exciting force behind what is more commonly known as clutch judder. This is a condition where the whole of the driveline vibrates severely and does inhibit clutch take-up as well as causing driveline damage.

The engine test rig provides a good starting point for a program of research into clutch induced vibrations, in that relevant parameters can be measured and the control and data acquisition can be carried out automatically using the computer. The program for sampling the input channels would need a minor modification to allow a sampling rate fast enough to define frequency up to 20 HZ. In addition, the affect of varying driveline stiffnesses could easily be seen by inserting rubber couplings of different torsional stiffness into the existing driveline. In time the test rig could be extended to enable the engine to drive the dynamometer through a gearbox and axle each mounted to allow "in-service" movements.

Again testing on vehicles prone to judder would be necessary in order to try and isolate the source and to define operating factors which affect it. There would be a need for a mathematical simulation, which could possibly be built around the existing model but perhaps the use of one of the new sophisticated analogue simulation languages may simplify the solution of the numerous differential equations governing the torsional oscillation in a system as complex as the driveline. Building the new model around the existing one, which could be supplied with all the data required to simulate the test rig, would provide a method of cross checking results as the research developed.

Although many hours of research have been spent by companies into trying to solve the problem of judder there is still a void and much disagreement between rivals as to the primary causes. So a program of research into this phenomenon, along the suggested lines, would not only be timely but attract wide interest throughout the industry.

The vehicle tests carried out during this program of research into clutch take-up has allowed a picture to be drawn describing the characteristics of a driver and has shown the type of engagement that is liable to cause the most damage. But this only covers starting from rest and use of the clutch pedal, what is needed is the development of a fully instrumented vehicle upon which measurement of the following parameters is possible:

1. Steering angle
2. Clutch usage
3. Brake usage
4. Fuel consumption
5. Driveline torque

From such data obtained for many drivers round a varied test route which included town work as well as country driving, it should be possible to correlate control usage to economy of operation. Also engine usage i.e. power levels used, would provide useful information for the design of vehicle motive power units and transmissions for future use in towns and the design of invalid vehicles. The study would require a joint ergonomic and engineering team in order to cover all aspects of the topic completely.

Another subject which requires study is the mechanism of friction in relation to friction materials but as already stated earlier in this work, a theoretical study of such a non-homogeneous material would be both complex and fraught with disaster from a practical point of view. Although an empirical study of small samples and the correlation of these results to in service components would prove a boon to the industry.

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APPENDIX I

SOLUTION OF THE ONE DIMENSIONAL UNSTEADY STATE HEAT-CONDUCTION EQUATION

The equation to be solved is:

$$\rho.Cp.\frac{\partial\theta}{\partial t} = k.\frac{\partial^2\theta}{\partial x^2} \quad ....(A1.1)$$

1. Boundary Conditions

1. At the geometrical centre of the centre plate, the heat flow across the boundary is assumed to be zero i.e. the heat inflow from both directions is the same.

$$\text{Therefore } Q_o = -k.A.\frac{\partial\theta}{\partial x} = 0$$

$$\text{or} \quad \frac{\partial\theta}{\partial x} = 0 \quad ....(A1.2)$$

2. At the back face of the rotating metal member i.e. pressure plate or flywheel, heat is lost to the surrounding air by forced convection.

$$Q_f = +h_f.A.(\theta_w - \theta_{f1}) \quad ....(A1.3)$$

3. At the surface of the friction material heat is generated due to friction and also heat is flowing back across the resistance from the metals surface.

Heat flow to friction material surface

$$\lambda . Q_g + h_I . \frac{\partial \theta}{\partial x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (A1.4)$$

Similarly at the metals surface

$$\text{Heat inflow} = (1 - \lambda)\, Q_g + h_I . \frac{\partial \theta}{\partial x} \quad \ldots\ldots\ldots\ldots\ldots (A1.5)$$

These are the boundary equations which enable equation (A1.1) to be solved.

2. Implicit Method

Using the following finite difference approximation to derivatives

$$\frac{\partial \theta}{\partial t} = \frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t}$$

and
$$\frac{\partial^2 \theta}{\partial x^2} = \frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{\Delta x^2}$$

equation (A1.1) can be represented by the following equation

$$\frac{\rho . Cp.}{\Delta t}(\theta_{i,j+1} - \theta_{i,j}) =$$

$$\frac{k}{2\Delta x^2}(\theta_{i+1,j+1} - 2\theta_{i,j+1} + \theta_{i-1,j+1} + \theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j})$$

Note that the derivative $\frac{\partial^2\theta}{\partial x^2}$ has been replaced by its finite-difference representation on two time rows (j +1) and (j).

This equation reduces to:

$$\underline{\underline{-F_o.\theta_{i+1,j+1} + 2(1 + F_o).\theta_{i,j+1} - F_o.\theta_{i-1,j+1}}}$$

$$\underline{\underline{= F_o.\theta_{i+1,j} + 2(1 - F_o).\theta_{i,j} + F_o.\theta_{i-1,j}}} \quad \ldots\ldots\ldots\ldots\ldots (A1.6)$$

But this equation is only applicable to stations inboard of the boundaries, what happens at the boundaries is given by equations (A1.2) to (A1.5) and will be used to derive modifications to equation (A1.6) which are applicable at the boundaries.

Geometrical centre of centre plate

Boundary Equation (A1.2)

$$\left.\frac{\partial\theta}{\partial x}\right|_{t=j} = 0 = \frac{\theta_{i+1,j} - \theta_{i-1,j}}{2.\Delta x}$$

Using a central difference approximation.

Simplifying gives

$$\theta_{i+1,j} = \theta_{i-1,j}$$

and similarly ....(A1.7)

$$\theta_{i+1,j+1} = \theta_{i-1,j+1}$$

A pictorial description of the finite difference approximation to the model is shown in (figure A1.1).

If equation (A1.6) is applied at station (i = 1) temperatures at a fictitious mesh point outside the material are required i.e. $\theta_{o,j}$ and $\theta_{o,j+1}$ but by using the boundary equations (A1.7) these two temperatures can be eliminated.

Equation (A1.6) becomes:

$$-F_o.\theta_{i+1,j+1} + 2(1 + F_o).\theta_{i,j+1} - F_o.\theta_{i+1,j+1} =$$

$$F_o.\theta_{i+1,j} + 2(1 - F_o).\theta_{i,j} + F_o.\theta_{i+1,j}$$

or $$-F_o.\theta_{i+1,j+1} + (1+F_o).\theta_{i,j+1} = F_o.\theta_{i+1,j} + (1-F_o).\theta_{i,j} \quad ....(A1.8)$$

Which is the finite difference equation applying at station (i=1).

Back face of metal member

Boundary equation (A1.3)

$$Q_f = +h_f.A.(\theta_w - \theta_{f1})$$

In order to take account of this forced convection term at the back side of the metal, the heat flowing across the metal surface in the direction normal to the surface is $= -k.A.\frac{\partial\theta}{\partial x} = Q_f$, thus equation (A1.3) becomes

$$Q_f = -k.A.\frac{\partial\theta}{\partial x} = h_f.A.(\theta_w - \theta_{f1})$$

Which when the derivative is replaced by a central finite difference approximation becomes:

$$\frac{h_f.A.}{2}(\theta_{i,j+1} + \theta_{i,j} - 2\theta_{f1}) =$$

$$-\frac{k.A}{4\Delta x}(\theta_{i+1,j+1} - \theta_{i-1,j+1} + \theta_{i+1,j} - \theta_{i-1,j}) \quad \text{..........(A1.9)}$$

applying equation (A1.6) to the boundary at (i = n1+n2+2) gives again a requirement for temperatures at a fictitious mesh point, (n1+n2+3) but these temperatures also appear in equation (A1.9) which can be rearranged to give:

$$\theta_{i+1,j+1} = \theta_{i-1,j+1} - 2.B_o.\theta_{i,j+1} -$$

$$\theta_{i+1,j} - 2.B_o.\theta_{i,j} + \theta_{i-1,j} + 4.\theta_{f}.B_o \quad \text{...............(A1.10)}$$

and $$B_o = \frac{h_f.\Delta x}{k}$$

If equation (A1.10) is substituted into equation (A1.6) the temperatures at the fictitious mesh point cancel:

$$- F_o(\theta_{i-1,j+1} - 2B_o.\theta_{i,j+1} - \theta_{i+1,j} -$$

$$2B_o.\theta_{i,j} + \theta_{i-1,j} + 4B_o.\theta_{f1}) +$$

$$2(1 + F_o).\theta_{i,j+1} - F_o.\theta_{i-1,j+1} =$$

$$F_o.\theta_{i+1,j} + 2(1 - F_o).\theta_{i,j} + F_o.\theta_{i-1,j}$$

which after simplification becomes:

$$\underline{\underline{-2F_o.\theta_{i-1,j+1} + (2(1 + F_o) + 2F_o.B_o).\theta_{i,j+1} =}}$$

$$\underline{\underline{(2(1 - F_o) - 2F_o.B_o).\theta_{i,j} + 2F_o.\theta_{i-1,j} + 4F_o.\theta_{f1}}} \quad \text{(A1.11)}$$

and applies at station (i = n1+n2+2)

Boundary equation (A1.4)

$$Q_1 = \lambda . Q_g + h_I . A . \frac{\partial \theta}{\partial x}$$

where $\lambda = \frac{k_1}{k_2}\sqrt{\frac{\alpha_2}{\alpha_1}}$ and $\alpha 1,2 = \frac{k_{1,2}}{\rho_{1,2} . Cp_{1,2}}$

or $Q_1 = -k.A.\frac{\partial \theta'}{\partial x} = -(\lambda . Q_g + h_I . A . \frac{\partial \theta}{\partial x})$

$\frac{\partial \theta'}{\partial x}$ is fictitious temperature gradient at the boundary.

Using finite difference approximation the above equation becomes:-

$$-\frac{k.A.}{4\Delta x}(\theta'_{i+1,j+1} - \theta_{i-1,j+1} + \theta'_{i+1,j} - \theta_{i-1,j}) =$$

$$-(\frac{\lambda .}{2}(Q_{g,j+1} + Q_{g,j}) + \frac{h_I . A}{4}(\theta_{i+1,j+1} -$$

$$\theta_{i-1,j+1} + \theta_{i+1,j} - \theta_{i-1,j})$$

rearranging

$$\theta'_{i+1,j+1} = \theta_{i-1,j+1} - \theta'_{i+1,j} + \theta_{i-1,j}$$

$$+ \frac{2.\Delta x.\lambda}{k.A} (Q_{g,j+1} + Q_{g,j}) + \frac{h_I.\Delta x}{k}$$

$$(\theta_{i+1,j+1} - \theta_{i-1,j+1} + \theta_{i+1,j} - \theta_{i-1,j})$$

simplifying

$$\theta_{i+1,j+1} = -\theta'_{i+1,j} + (1 - B_I).\theta_{i-1,j} +$$

$$B_I.\theta_{i+1,j} + 2B_{Q1} (Q_{g.j+1} + Q_{g,j}) \quad \text{..................(A1.12)}$$

where $\quad B_I = \dfrac{h_I.\Delta x}{k} \qquad B_{Q1} = \dfrac{\Delta x.\lambda}{k.A}$

The temperatures at the fictitious mesh point (i + 1) can now be eliminated by substitution into equation (A1.6), equation (A1.12).

$$-F_o.(-\theta'_{i+1,j} + (1 - B_I).\theta_{i-1,j+1} + B_I.\theta_{i+1,j+1} +$$

$$(1 - B_I).\theta_{i-1,j} + B_I.\theta_{i+1,j} + 2B_{Q1}. (Q_{g,j+1} + Q_{g,j})) +$$

$$2(1 + F_o).\theta_{i,j+1} - F_o.\theta_{i-1,j+1} = F_o.\theta'_{i+1,j} +$$

$$2 (1 - F_o).\theta_{i,j} + F_o.\theta_{i-1,j}$$

simplifying, gives

$$\underline{\underline{F_o.(2 - B_I).\theta_{i-1,j+1} + 2(1 + F_o).\theta_{i,j+1} - F_o.B_I.\theta_{i+1,j+1}}} =$$

$$\underline{\underline{+ F_o.(2 - B_I).\theta_{i-1,j} + 2(1 - F_o).\theta_{i,j} + F_o.B_I.\theta_{i+1,j}}} +$$

$$\underline{\underline{F_o.B_{Q1}.(Q_{g,j+1} + Q_{g,j})}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (A1.13)$$

which applies at station (nl + 1)

<u>Boundary equation</u> (A1.5)

$$Q_2 = (1 - \lambda).Q_g - h_I.A.\frac{\partial\theta}{\partial x} = -k.A\frac{\partial\theta'}{\partial x}$$

which becomes in finite difference form

$$\frac{(1-\lambda)}{2}.(Q_{g,j+1} + Q_{g,j}) - \frac{h_I.A}{4,\Delta x}(\theta_{i+1,j+1} - \theta_{i-1,j+1} + \theta_{i+1,j} - \theta_{i-1,j})$$

$$= -\frac{k.A}{4.\Delta x}(\theta_{i+1,j+1} - \theta'_{i-1,j+1} + \theta_{i+1,j} - \theta'_{i-1,j})$$

rearranging

$$\theta'_{i-1,j+1} = \theta_{i+1,j+1} + \theta_{i+1,j} - \theta'_{i-1,j} - B_I.(\theta_{i+1,j+1}$$

$$-\theta_{i-1,j+1} + \theta_{i+1,j} - \theta_{i-1,j}) + 2.B_{Q2}.\ (Q_{g,j+1} - Q_{g.j})$$

where

$$B_{Q2} = \frac{(1-\lambda).\Delta x}{k.A}$$

and simplifying

$$\theta'_{i-1,j+1} = (1 - B_I).\theta_{i+1,j+1} + B_I.\theta_{i-1,j+1} + (1 - B_I).\theta_{i+1,j}$$

$$+ B_I.\theta_{i-1,j} - \theta'_{i-1,j} + 2.B_{Q2}.(Q_{g,j+1} + Q_{g,j}) \quad \ldots\ldots\ldots\ldots(A1.14)$$

Substituting (A1.14) into (A1.6) to eliminate the temperatures at the fictitious mesh point gives:

$$-F_o.\theta_{i+1,j+1} + 2(1 - F_o).\theta_{i,j} - F_o.(\ (1 - B_I).\theta_{i+1,j+1}$$

$$+ B_I.\theta_{i-1,j+1} + (1 - B_I).\theta_{i+1,j} + B_I.\theta_{i-1,j} - \theta'_{i-1,j} +$$

$$2.B_{Q2}.(Q_{g,j+1} + Q_{g,j})\ ) = F_o.\theta_{i+1,j}$$

$$+ 2(1 - F_o).\theta_{i,j} + F_o.\theta'_{i-1,j}$$

simplifying gives

$$-F_o.(2 - B_I).\theta_{i+1,j+1} + 2.(1 + F_o).\theta_{i,j+1} - F_o.B_I.\theta_{i-1,j+1}$$

$$= F_o.(= - B_I).\theta_{i+1,j} + 2.(1 - F_o).\theta_{i,j} + B_I.F_o.\theta_{i-1,j}$$

$$+ 2.F_o.B_{Q2}.(Q_{g,j+1} + Q_{g,j}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (A1.15)$$

Equations (A1.6), (A1.8), (A1.11), (A1.13) and (A1.15) form a set of simultaneous linear equations the solution of which yield the temperature distribution at the next time step.

Solution

Although theoretically, stability with the implicit technique is not a problem, however unless F_o is chosen carefully the results may be erroneous. Therefore when choosing the time step length and the mesh size the solution should be constrained to keep F_o low enough to maintain an accurate solution. In the digital computer program written the value of F_o is kept close to 0.1 and the mesh size is varied automatically to ensure this condition is always met.

The set of equations resulting from the expansion of equation (A1.6) at all the mesh points inside the boundary and of application of equation (A1.8), (A1.11), (A1.13) and (A1.15) at the boundaries are more easily manageable if written in matrix notation:

$$\bar{A}.\hat{\theta}_{j+1} = \bar{B}.\hat{\theta}_j + \bar{C} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (A1.16)$$

where matrix $\bar{A}$ is:

$$\begin{bmatrix}
2(1+F_{o1}) & -2F_{o1} & & & & & & \\
-F_{o1} & 2(1+F_{o1}) & -F_{o1} & & & & & \\
& \ddots & \ddots & \ddots & & & & \\
& -F_{o1} & 2(1+F_{o1}) & -F_{o1} & & & & \\
& & +F_{o1}(2-B_{I1}) & 2(1+F_{o1}) & -F_{o1}.B_{I1} & & & \\
& & & -F_{o2}.B_{I2} & 2(1+F_{o2}) & -F_{o2} & & \\
& & & & \ddots & \ddots & \ddots & \\
& & & & -F_{o2} & 2(1+F_{o2}) & -F_{o2} \\
& & & & & -2F_{o2} & 2(1+F_{o2})+F_o B_{o2}
\end{bmatrix}$$

Matrix $\bar{B}$ is:

$$
\begin{bmatrix}
2(1-F_{o1}) & 2F_{o1} & & & & & & & \\
F_{o1} & 2(1-F_o) & F_{o1} & & & & & & \\
\ddots & \ddots & \ddots & \ddots & & & & & \\
 & F_{o1} & 2(1-F_{o1}) & F_{o1} & & & & & \\
 & & F_{o1}(2-B_{I1}) & 2(1-F_{o1}) & F_{o1}B_{I1} & & & & \\
 & & \ddots & \ddots & \ddots & \ddots & & & \\
 & & & F_{o2}B_{I2} & 2(1-F_{o2}) & F_{o2}(2-B_{I2}) & & & \\
 & & & & F_{o2} & 2(1-F_{o2}) & F_{o2} & & \\
 & & & & \ddots & \ddots & \ddots & \ddots & \\
 & & & & & & F_{o2} & 2(1-F_{o2}) & F_{o2} \\
 & & & & & & & 2F_{o2} & 2(1-F_{o2})-F_oB_O
\end{bmatrix}
$$

$$\bar{\theta}_{j+1} = \begin{bmatrix} \theta_{i,j+1} \\ \theta_{2,j+1} \\ \vdots \\ \theta_{n1+n2+1,j+1} \\ \theta_{n1+n2+2,j+1} \end{bmatrix} \qquad \bar{\theta}_j = \begin{bmatrix} \theta_{1,j} \\ \theta_{2,j} \\ \vdots \\ \theta_{n1+n2+1,j} \\ \theta_{n1+n2+2,j} \end{bmatrix}$$

and $$\bar{C} = \begin{bmatrix} o \\ o \\ \vdots \\ o \\ F_{o1}B_{I'}(Q_{g,j+1} - Q_{g,j}) \\ F_{o2}B_{I2}\ (Q_{g,j+1} - Q_{g,j}) \\ o \\ \vdots \\ o \\ 2B_o\Theta_f \end{bmatrix}$$

But $\bar{B}.\bar{\theta}_j + \bar{C}$ is known at any given time interval thus equation (A1.16) becomes:

$$\bar{A}.\bar{\theta}_{j+1} = \bar{D} \quad \text{.............................(A1.17)}$$

where $$\bar{D} = \bar{B}.\bar{\theta}_j + \bar{C}$$

Further, matrix $\bar{A}$ is tridiagonal in form which much simplifies the solution for the temperature matrix $\bar{\theta}_{j+1}$, providing the leading diagonal is dominant. As F_o is constrained by the program to be approximately equal to 0.1 it can be seen that providing B_I and B_o are of order 1.0 or less (which is the case) the leading diagonal is of order 2.0 and the other two diagonals of order 0.1. The algorithm suggested by Alberg, Wilson and Walsh (33) enables the solution of the matrix equation (A1.17) more efficiently and without significant loss in accuracy.

Consider the matrix equation

$$\begin{bmatrix} b_1 & c_1 & & & & \\ a_2 & b_2 & c_2 & & & \\ & a_3 & b_3 & c_3 & & \\ & & \ddots & \ddots & \ddots & \\ & & & a_{n-1} & b_{n-1} & c_{n-1} \\ & & & & a_n & b_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_{n-1} \\ d_n \end{bmatrix}$$

form the following parameters, for $k = 1,2,\ldots\ldots n$

$$pk = a_k\, q_{k-1} + b_k \quad (q_o = 0)$$

$$q_k = -C_k/p_k$$

$$U_k = (d_k - a_k.U_{n-1})/\,P_k \quad (U_o = 0)$$

which yields the equivalent set of equations:

$$x_k = q_k x_{k+1} + U_k \quad \text{for} \quad k = 1,2, \ldots\ldots, n-1$$

$$x_n = U_n$$

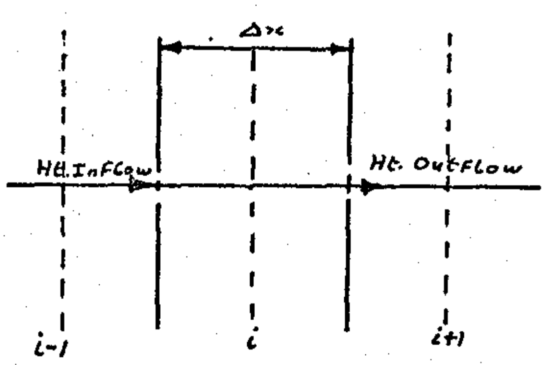
The required solution is then obtained from back substitution

3. <u>Explicit Method</u>

In deriving the finite difference equations governing the flow of heat through the system, the following general equation is used at all the mesh points.

Heat inflow - Heat outflow = Heat Stored ....(A1.18)

General equation at any point inside the boundary



$$\text{Ht. Stored} = \frac{\rho.Cp.V\ (\theta_{i,j+1} - \theta_{i,j})}{\Delta t} \quad \ldots\ldots\ldots\ldots\ldots\ (A1.19)$$

$$\text{Ht. Inflow} = \frac{-k.A\ (\theta_{i,j} - \theta_{i-1,j})}{\Delta x} \quad \ldots\ldots\ldots\ldots\ldots\ (A1.20)$$

$$\text{Ht. Outflow} = \frac{k.A\ (\theta_{i,j} - \theta_{i+1,j})}{\Delta x} \quad \ldots\ldots\ldots\ldots\ldots\ (A1.21)$$

Insertion of equations (A1.19), (A1.20) and (A1.21) into equation (A1.18) reveals.

$$\frac{\rho.Cp.V(\theta_{i,j+1} - \theta_{i,j})}{\Delta t} =$$

$$-\frac{k.A}{\Delta k}(\theta_{i,j} - \theta_{i-1,j} + \theta_{i,j} - \theta_{i+1,j})$$

which after simplification and substitution of $F_o = \frac{k.\Delta t}{\rho.Cp.\Delta x^2}$

and $V = A.\Delta x$ gives

$$\theta_{i,j+1} - \theta_{i,j} = -F_o.(2\theta_{i,j} - \theta_{i-1,j} - \theta_{i+1,j})$$

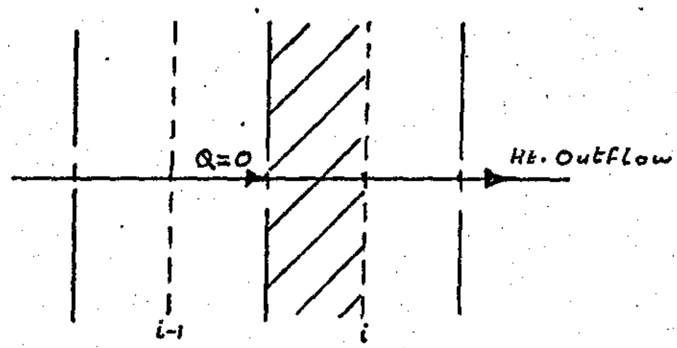
or

$$\underline{\underline{\theta_{i,j+1} = (1 - 2F_o).\theta_{i,j} + F_o.(\theta_{i-1,j} + \theta_{i+1,j})}} \quad \ldots\ldots\ldots\ (A1.22)$$

but for a stable solution $(1 - 2F_o) < 0$ or $F_o < \frac{1}{2}$

At the geometrical centre of the plates boundary equation (A1.2) applies

$$\frac{\partial\theta}{\partial x} = 0 \quad \text{or there is no heat flow}$$



$$\text{Heat inflow} = 0$$

$$\text{Heat outflow} = \frac{k.A.}{\Delta t}(\theta_{i,j} - \theta_{i+1,j}) \quad \ldots\ldots\ldots\ldots\ldots(A1.23)$$

$$\text{Heat stored} = \frac{\rho.Cp.V/2\,(\theta_{i,j+1} - \theta_{i,j})}{\Delta t} \quad \ldots\ldots\ldots\ldots\ldots(A1.24)$$

Inserting (A1.23) and (A1.24) into (A1.18) gives:

$$\frac{\rho.Cp.V/2}{\Delta t}(\theta_{i,j+1} - \theta_{i,j}) = -\frac{k.A}{\Delta t}(\theta_{i,j} - \theta_{i+1,j})$$

or simplifying

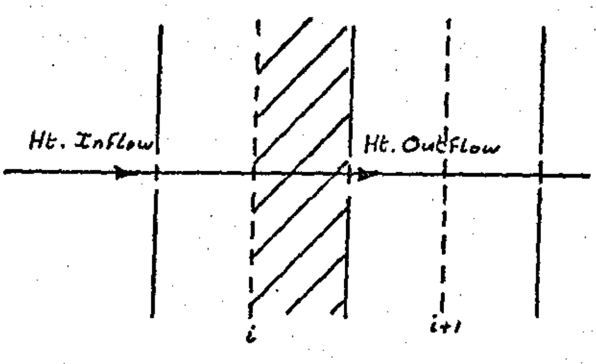
$$\theta_{i,j+1} - \theta_{i,j} = -2F_o . (\theta_{i,j} - \theta_{i+1,j})$$

$$\theta_{i,j+1} = (1 - 2F_o).\theta_{i,j} + 2F_o.\theta_{i+1,j} \qquad \text{..................(A1.25)}$$

for stable solution $F_o < 2$

At the back face of the rotating metal member boundary equation (A1.3) applies.

$Q = h_f.A. (\theta_w - \theta_{f1})$ or forced convective heat transfer.



$$\text{Heat inflow} = \frac{-k.A}{\Delta x} (\theta_{i,j} - \theta_{i-1,j}) \qquad \text{..................(A1.26)}$$

$$\text{Heat outflow} = h_f.A.(\theta_{i,j} - \theta_{f1}) \qquad \text{..................(A1.27)}$$

$$\text{Heat Stored} = \frac{\rho.Cp.V/2}{\Delta t} . (\theta_{i,j+1} - \theta_{i,j}) \quad \ldots\ldots\ldots\ldots\ldots (A1.28)$$

Substituting (A1.26), (A1.27) and (A1.28) in (A1.18) gives:

$$\frac{\rho.Cp.V/2}{\Delta t} . (\theta_{i,j+1} - \theta_{i,j}) = - \frac{k.A.}{\Delta x} . (\theta_{i,j} - \theta_{i-1,j})$$

$$- h_f.A. \ (\theta_{i,j} - \theta_{f1})$$

which after simplification becomes:

$$\theta_{i,j+1} - \theta_{i,j} = -F_{o2} \ (\theta_{i,j} - \theta_{i-1,j}) - F_o.B_{o2}.(\theta_{i,j} - \theta_{f1})$$

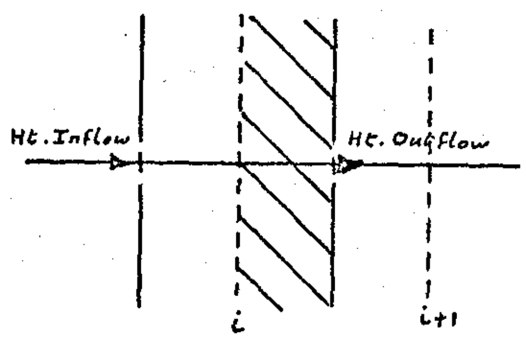
where $$B_o = \frac{hf.\Delta x}{k}$$

$$\underline{\underline{\theta_{i,j+1} = (1 - 2F_o.(1 + B_o))\theta_{i,j} + 2F_o.\theta_{i-1,j} + 2F_o.B_o.\theta_{f1}}} \quad \ldots\ldots (A1.29)$$

for stable solution $F_o(1 + B_o) < \frac{1}{2}$

At the friction interface boundary equations (A1.4) and (A1.5) apply.

Friction material



Heat outflow = Boundary equation (A1.4)

$$= -\lambda.Q_g - \frac{h_I}{\Delta x}.(\theta_{i,j} - \theta_{i+1,j}) \quad \text{...............(A1.30)}$$

$$\text{Heat inflow} = \frac{-k.A.}{\Delta x}.(\theta_{i,j} - \theta_{i-1,j}) \quad \text{..............(A1.31)}$$

$$\text{Heat Stored} = \frac{\rho.Cp.V/2}{\Delta t}.(\theta_{i,j+1} - \theta_{i,j}) \quad \text{.............(A1.32)}$$

Substitution of (A1.30), (A1.31) and (A1.32) into (A1.18) gives:

$$\frac{\rho.Cp.V/2}{\Delta t}.(\theta_{i,j+1} - \theta_{i,j}) = -\frac{k.A}{\Delta x}.(\theta_{i,j} - \theta_{i-1,j}) + \lambda Q_g$$

$$+ \frac{k_I}{\Delta x}.(\theta_{i,j} - \theta_{i+1,j})$$

or simplifying

$$\theta_{i,j+1} - \theta_{i,j} = -F_o.(\theta_{i,j} - \theta_{i-1,j})$$

$$+ 2F_o.B_{Q1}.Q_g - 2F_o.B_I.(\theta_{i,j} - \theta_{i+1,j})$$

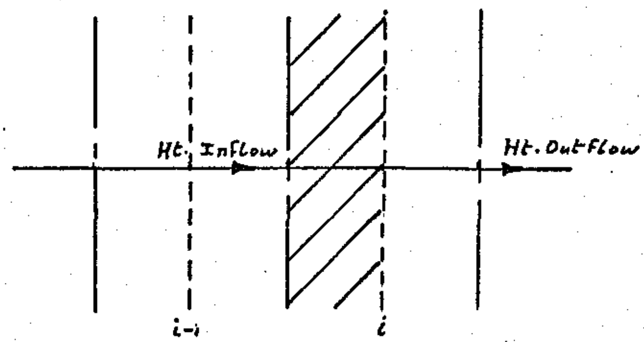
where $B_{Q1} = \dfrac{\lambda.\Delta x}{k.A}$ and $B_I = \dfrac{h_I.\Delta x}{k.A}$

$$\theta_{i,j+1} =$$

$$\frac{(1 - 2F_o(1 + B_I)).\theta_{i,j} + 2F_o.(\theta_{i-1,j} + B_I.\theta_{i+1,j}) + 2F_o.B_{Q1}.Q_g}{} \quad \ldots.(A1.33)$$

for a stable solution $F_o(1 - B_2) < \frac{1}{2}$

Metal Member



$$\text{Heat inflow} = (1 - \lambda).Q_g - \frac{h_I}{\Delta x}.(\theta_{i,j} - \theta_{i-1,j}) \quad \ldots\ldots\ldots(A1.34)$$

$$\text{Heat outflow} = + \frac{k.A}{\Delta x}.(\theta_{i,j} - \theta_{i+1,j}) \quad (A1.35)$$

$$\text{Heat Stored} = \frac{\rho.Cp.V/2}{\Delta t}.(\theta_{i,j+1} - \theta_{i,j}) \quad \ldots\ldots\ldots\ldots(A1.36)$$

Substitution of (A1.34), (A1.35) and (A1.36) in (A1.18) gives:

$$\frac{\rho.Cp.V/2}{\Delta t}.(\theta_{i,j+1} - \theta_{i,j}) =$$

$$(1 - \lambda).Q_g - \frac{h_I}{\Delta x}.(\theta_{i,j} - \theta_{i-1,j}) - \frac{k.A.}{\Delta x}.(\theta_{i,j} - \theta_{i+1,j})$$

or simplifying

$$\theta_{i,j+1} - \theta_{i,j} = B_{Q2}.F_o.Q_g -$$

$$2B_I.F_o.(\theta_{i,j} - \theta_{i-1,j}) - 2F_o.(\theta_{i,j} - \theta_{i+1,j})$$

where $$B_{Q2} = \frac{(1 - \lambda).\Delta x}{k.A}$$

$$\underline{\underline{\theta_{i,j+1} =}}$$

$$\underline{\underline{(1 - 2F_o.(1 + B_I)).\theta_{i,j} + 2F_o.(\theta_{i+1,j} + B_I.\theta_{i-1,j}) + 2B_{Q2}.F_o.Q_g}} \quad \ldots.(A1.37)$$

for a stable solution $F_o(1 + B_I) < \frac{1}{2}$

Equations (A1.25), (A1.29), (A1.33) and (A1.37) describe the new temperatures at the boundaries and equation (A1.22) describes inboard temperatures. Using this method each temperature could be solved separately i.e. it is not necessary to form a block of simultaneous linear equations as with the implicit technique, although it is convenient to represent them in matrix form:

$$\bar{\theta}_{j+1} = \bar{\theta}_j . \bar{B} + \bar{C} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(A1.38)}$$

where $\bar{\theta}_{j+1} = \{\theta_{1,j+1}\theta_{2,j+1}, \ldots\ldots\ldots, \theta_{n1+n2+2,j+1}\}$

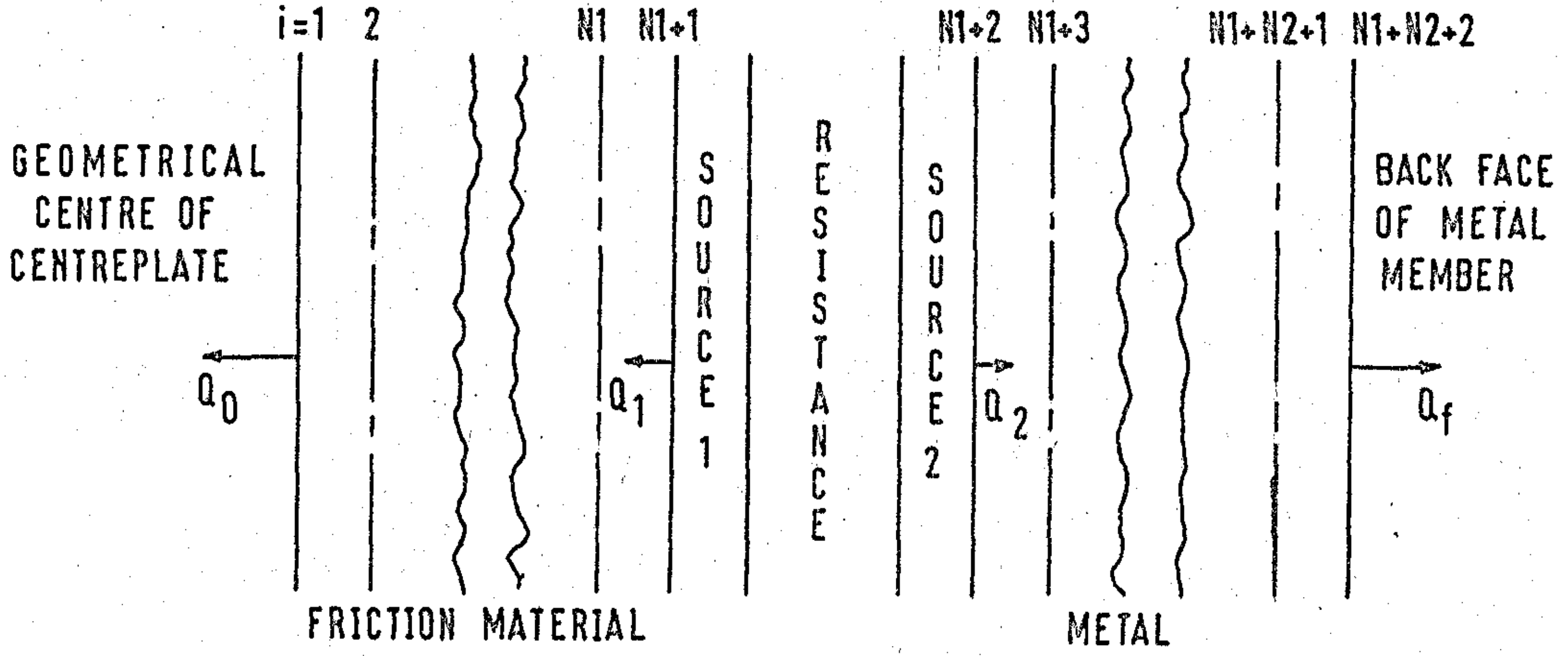
$\bar{\theta}_j = \{\theta_{1,j}, \theta_{2,j}, \ldots\ldots\ldots, \theta_{n1+n2+2,j}\}$

$\bar{C} = \{0,0, \ldots\ldots\ldots, 2B_{Q1}.F_{o1}.Q_g, 2B_{Q2}.F_{o2}.Q_g, \ldots\ldots\ldots\ 2F_{o2}.B_O.\theta_{f1}\}$

and $\bar{B}$ =

$$\begin{bmatrix}
(1-2F_o) & 2F_o & & & & \\
F_o & (1-2F_o) & F_o & & & \\
\vdots & \vdots & \ddots & & & \\
2F_o & (1-2F_o(1-B_I)) & -2F_oB_I & & & \\
 & 2F_oB_I & (1-2F_o(1+B_I)) & 2F_o & & \\
 & & \ddots & \ddots & \ddots & \\
 & & F_o & (1-2F_o) & F_o & \\
 & & & 2F_o & (1-2F_o(1+B_o)) &
\end{bmatrix}$$

The solution of (A1.38) at each time interval yields the required solution. In order to reduce errors and instabilities in the mathematics the computer program written to solve (A1.38) kept $F_o \approx 0.1$ by choosing n1 and n2 appropriately.



THERMAL MODEL OF RUBBING MEMBERS

FIGURE NO. A1.1

APPENDIX II

OPERATING INSTRUCTIONS FOR THE VEHICLE
SIMULATION TEST RIG WITH PROGRAM LISTING

This appendix deals with the operation of the vehicle simulation test rig and the mini-computer system used for data acquisition and control.

The Computer

The program written samples the first eight analogue input channels, these channels will allow voltage levels of 0 to $\pm$ 1.0 volts but if a signal level above this is fed in it will be truncated, up to a level of 2.5 volts which if exceeded will result in damage to the input buffers.

Channel number one should be fed with a D.C. signal which can be switched on and off, this will be used to commence the test series.

Channel numbers two and three are used to record the engine speed and the propeller shaft speed respectively. All other channels may be connected arbitarily because all the calibration coefficients are entered before each test. The calibration coefficients are the values of the linear relationship describing the voltage variation of each signal.

Procedure

(1) The computer system should be switched on for at least one hour to allow the analogue to digital convertor to stabalise, this can be checked by using the zero calibration switch on the hardware itself.

(2) The system tape, produced from the listings shown, in accordance with the computer manual, should be loaded into the computer by placing the system tape into the tape reader and readying that reader, then pressing the buttons PRESET and LOAD located on the computer panel. The tape will now be read in and the computer should halt with the message

102077 (IN BINARY) lighted on the display register located on the computer front panel above the switches. This message indicates a successful load, if any other message is displayed reference to the computer operators manual is required for diagnosis of the error.

(3) The calibration coefficients should have been punched on the paper tape in free format each channel taking a full line and being in the following order:

C1(1) C1(2) (CR) (LF)
. . . .
. . . .
. . . .
C8(1) C8(2) (CR) (LF)

where

True value = C1(1) + CI(2) * voltage level input

and

(CR) - Carriage return character

(LF) - Line feed character

This tape is loaded into the photo reader and that reader readied.

(4) To commence the input of data switch number one on the switch register (located on the front panel) should be lighted and then the following switches pressed:

LOAD ADDRESS

PRESET

RUN

(5) The computer will print the message TEST NUMBER and wait for input data, the data should be input on the teletype followed by (CR) and (LF).

(6) The computer then asks for the date which should be input in numerical form each number separated by a comma.

(7) The calibration coefficients are then read in and printed on the teletype under their channel headings.

(8) The next parameter required is the number of engagements to be carried out, the engagements are made automatically at set intervals and results output inbetween.

(9) The time interval between the samples i.e. the reciprocal of the sampling frequency is now required and should be an exact multiple of 0.1 seconds, because this is the smallest interval allowed.

(10) The time interval between engagements should be entered, the minimum value of which is 100 seconds, in order to allow for the output of results before the next engagement is commenced, the total interval required should be entered in seconds.

(11) Then the option is given for the output of results by either the paper tape punch (enter 1) or the teleprinter (enter 2). If output by the teleprinter is required the time interval between engagements must be greatly increased to enable printout between engagements.

(12) The computer now tells the operator to start the engine and pauses, the engine should be allowed to warm up whilst the equipment is checked for malfunction.

(13) The RUN button is then pressed and the computer asks for the required drag torque to be applied by the disc brake assembly, after which the RUN button is pressed to disengage the clutch. This prevents overheating of the disc brake assembly.

(14) The computer waits in a loop, sampling channel number one until a voltage of more than 0.2 volts is applied when the test series will commence.

After the final engagement has been carried out and the results output the computer halts, if another series of tests is required return to stage (4) and recommence data input.

The following pages contain, first a sample of the data input printout and then a listing of the program used.

```
TEST NO.
1
DATE.
13,9,74
CHANNEL CALIBRATION COEFFICIENTS Y=A0+A1*X+A2*X*X+..
CHANNEL NO. 1
  .000000E+00 1.000000E+00
CHANNEL NO. 2
  .000000E+00  .512000E+03
CHANNEL NO. 3
  .000000E+00  .512000E+03
CHANNEL NO. 4
 -.800000E+01 -.162500E+03
CHANNEL NO. 5
  .000000E+00 1.000000E+00
CHANNEL NO. 6
 -.590000E+02 -.397000E+03
CHANNEL NO. 7
 -.104000E+03 -.524000E+03
CHANNEL NO. 8
  .000000E+00 1.000000E+00
ENTER NO. OF ENGAGEMENTS TO BE CARRIED OUT
6
ENTER TIME INTERVAL BETWEEN SAMPLES
0.1
ENTER THE TIME BETWEEN ENGAGEMENTS
AFTER THE ENGAGEMENT 100 SECS
ELAPSE TO ALLOW OUTPUT OF RESULTS
TOTAL TIME INTERVAL SHOULD BE ENTERED IN SECS
120.0
TYPE 1 FOR PUNCH TAPE OUTPUT OR 2 FOR TELETYPE OUTPUT
2
START THE ENGINE
PAUSE
APPLY REQD. DRAG TORQUE
 PRESS RUN TO DISENGAGE RAM
 PROGRAM WAITS FOR TRIGGER SIGNAL TO COMMENCE TEST
PAUSE
```

```
FTN,B,L
      PROGRAM CENG
      DIMENSION CF(8,2),X(8)
      COMMON IBUF(8,100)
      WRITE(2,200)
C...THIS PROGRAM ATTEMPTS TO KEEP THE ENGINE SPEED FAIRL
C   CONSTANT THROUGHOUT THE ENGAGEMENT
      READ(1,*)NT
      WRITE(2,201)
      READ(1,*)N1,N2,N3
      WRITE(2,204)
      DO 1 I=1,8
      WRITE(2,202)I
      READ(5,*)(CF(I,J),J=1,2)
      WRITE(2,203)(CF(I,J),J=1,2)
    1 CONTINUE
      WRITE(2,205)
      READ(1,*)NENG
      WRITE(2,206)
      READ(1,*)STIM
      WRITE(2,207)
      READ(1,*)BT
      WRITE(2,208)
      READ(1,*)NT
      WRITE(2,209)
      PAUSE
      WRITE(2,210)
      PAUSE
      CALL DISEG
      IOVER=0
      INDIC=0
      NUMS=100
      NREPT=1
      IREPT=1
      NBT=IFIX(BT-100.0)
      NST=IFIX(STIM/0.1)
   10 CALL INIT(IOVER,IREPT,NST,NUMS,NBT,INDIC)
      NREPT=NREPT+1
      I=0
      IOVER=500
   12 CONTINUE
      IF(IOVER-200)11,11,12
   11 CONTINUE
      IF(IOVER-100)13,14,13
   14 WRITE(2,213)
  213 FORMAT(37HCLUTCH NOT ENGAGED AT END OF SAMPLING,/,
```

```
     124HTRY LARGER TIME INTERVAL)
   13 CONTINUE
      IF(NT-1)2,2,3
C...PUNCH TAPE OUTPUT
    2 WRITE(4,211)INDIC
    4 I=I+1
      DO 5 J=1,8
      XVAL=FLOAT(IAND(177700B,IBUF(J,I)))/32768.0*CF(J,2)+CF(J,1)
    5 X(J)=XVAL
      WRITE(4,212)(X(J),J=1,8)
      IF(I-INDIC)4,6,6
    3 I=I+1
      DO 7 J=1,8
      XVAL=FLOAT(IAND(177700B,IBUF(J,I)))/32768.0*CF(J,2)+CF(J,1)
    7 X(J)=XVAL
      WRITE(2,212)(X(J),J=1,8)
      IF(I-INDIC)3,6,6
    6 IF(NREPT-NENG)8,8,9
    8 IF(NREPT-IREPT)10,10,6
    9 CONTINUE
      STOP
  200 FORMAT(8HTEST NO.)
  201 FORMAT(4HDATE)
  202 FORMAT(11HCHANNEL NO.,I2)
  203 FORMAT(4E13.6,/)
  204 FORMAT(42HCHANNEL CALIBRATION COEFFICIENTS Y=A0+A1*X,
     110H+A2*X*X+...)
  205 FORMAT(42HENTER NO. OF ENGAGEMENTS TO BE CARRIED OUT)
  206 FORMAT(35HENTER TIME INTERVAL BETWEEN SAMPLES )
  207 FORMAT(34HENTER THE TIME BETWEEN ENGAGEMENTS,/,
     129HAFTER THE ENGAGEMENT 100 SECS,/,
     233HELAPSE TO ALLOW OUTPUT OF RESULTS,/,
     345HTOTAL TIME INTERVAL SHOULD BE ENTERED IN SECS)
  208 FORMAT(42HTYPE 1 FOR PUNCH TAPE OUTPUT OR 2 FOR TELE
     1,11HTYPE OUTPUT)
  209 FORMAT(16HSTART THE ENGINE)
  210 FORMAT(23HAPPLY REQD. DRAG TORQUE,/,1X,9HPRESS RUN
     1,17H TO DISENGAGE RAM,/,1X,25HPROGRAM WAITS FOR TRIGGER
     2,24H SIGNAL TO COMMENCE TEST)
  211 FORMAT(I4)
  212 FORMAT(1X,F5.3,2X,3(F5.1,2X),F5.3,2(2X,F5.1),2X,F5.3)
      END
      END$
```

```
ASMB,R,L,B,T
      NAM INIT
      ENT INIT,CONT
      EXT .ENTR,CONTR
      COM IBUF(800)
OVER  BSS 1
IREPT BSS 1
NST   BSS 1
NUMS  BSS 1
NBT   BSS 1
INDIC BSS 1
INIT  NOP
      JSB .ENTR
      DEF OVER
      LDA =B377
      STA RAMV
      LDA =B6
      STA TBG
*SET INTERRUPT CONTROL
      LDA INSTR
      STA 10B
      LDA ACONT
      STA 25B
      LDA =D500
      STA OVER,I
*NEGATE COUNTER FOR TIME BETWEEN SAMPLES
      LDA NBT,I
      CMA
      STA NNBT
      STA NNNBT
*NEGATE TOTAL NUMBER OF SAMPLES
      LDA NUMS,I
      CMA,INA
      STA NNUMS
*NEGATE SAMPLE TIME COUNTER
      LDA NST,I
      CMA,INA
      STA NNST
      STA NNNST
*INITIATE COUNTER OF NUMBER OF SAMPLES
      CLA
      STA INDIC,I
```

```
*DEFINE ADDRESS OF ARRAY STOREAGE
       LDA ADD
       STA IADD
*CHECK IF THIS IS THE FIRST ENGAGEMENT
       LDA IREPT,I
       CMA,INA
       ADA =B1
       SZA
       JMP GO
*WAIT FOR TRIGGER
       LDA =D-10
       STA NNCON
       CLA
       OTA 11B
RAND   STC 11B,C
       SFS 11B
       JMP *-1
       LIA 11B
       AND =B177700
       ADA =B-14600
       SSA
       JMP RAND
       CLC 11B
*SET INTERRUPT TIME INTERVAL AT 0.1 SEC
GO     LDA =B2
       OTA 10B
       STF 0
       STC 10B,C
       JMP INIT,I
*
*CONTINUETOR SECTION
*
CONT   NOP
       STC 10B,C
*SAVE REGISTERS
       STA SAVA
       STB SAVB
       ERA,ALS
       SOC
       INA
       STA SAVEO
```

```
*IS THIS ENGAGEMENT OVER
       LDA OVER,I
       CMA,INA
       ADA =B377
       SSA,RSS
       JMP SEND
       ISZ NNCON
       JMP *+2
       JMP *+6
       LDB RAMV
       JSB CONTR
       STB OVER,I
       STA RAMV
       JMP END
       LDA =D-10
       STA NNCON
*CHECK IF CORRECT NUMBER OF SAMPLES TAKEN
       ISZ NNUMS
       JMP *+2
       JMP DISN1
*INCREMENT SAMPLE INTERVAL COUNTER
       ISZ NNNST
       JMP END
       LDA NNST
       STA NNNST
*SAMPLE REQD. CHANNELS
       LDA =D-8
       STA NC
       CLA
       OTA 11B
       LDA =B40000
       OTA 11B
SAMP   STC 11B,C
       SFS 11B
       JMP *-1
       LIA 11B
       STA IADD,I
       ISZ IADD
       ISZ NC
       JMP SAMP
       CLC 11B
*INCREMENT SAMPLE COUNTER
       ISZ INDIC,I
       JMP END
```

```
*RESTART T.B.G. WITH 10 SEC INTERVAL
*FOR WAITING PERIOD BETWEEN ENGAGEMENTS
SEND  NOP
      LDA =D200
      STA OVER,I
      JMP DISEN
BEND  CLC 10B
      LDA TBG
      OTA 10B
      STF 0
      STC 10B,C
      LDA =B4
      STA TBG
      ISZ NNNBT
      JMP END
*START NEXT ENGAGEMENT
      ISZ IREPT,I.
      JMP END
*DISENGAGE RAM
DISN1 LDA =B377
      STA OVER,I
DISEN LDA =B377
      OTA 13B
      CLC 13B
      JMP BEND
*RESET REGISTERS
END   LDA SAVEO
      CLO
      SLA,ELA
      STF 1
      LDA SAVA
      LDB SAVB
      JMP CONT,I
*CONSTANTS SECTION
RAMV  BSS 1
TBG   BSS 1
NNCON BSS 1
NNBT  BSS 1
NNNBT BSS 1
NNUMS BSS 1
NNST  BSS 1
NNNST BSS 1
SAVA  OCT 0
SAVB  OCT 0
SAVEO OCT 0
NC    OCT 0
IADD  BSS 1
ADD   DEF IBUF
INSTR OCT 114025
ACONT DEF CONT
      END
```

```
ASMB,R,L,B,T
      NAM CONTR
      ENT CONTR
CONTR NOP
*THIS PROGRAM CONTROLS THE RAM ENGAGEMENT IN AN
*ATTEMPT TO KEEP THE ENGINE SPEED ABOVE 1000REVS/MIN
*SAMPLE CHANNEL 2 AND 3 , MASK OUT REQD. BITS AND
*STORE IN WE AND WV
      STB RAMV
      LDA =B1
      OTA 11B
      STC 11B,C
      SFS 11B
      JMP *-1
      LIA 11B
      AND =B177700
      STA WE
*
      LDA =B2
      OTA 11B
      STC 11B,C
      SFS 11B
      JMP *-1
      LIA 11B
      AND =B177700
      STA WV
      CLC 11B
*CHECK IF SPEEDS SYNCHRONISED
      LDA WE
      CMA,INA
      ADA WV
      ADA =B1200
      SSA,RSS
*CHECK IF RAM FULLY ENGAGED
      JMP FENG
      LDA RAMV
      SZA,RSS
      JMP FENG
      LDA WE
      CMA,INA
      ADA =B15400
      SSA,RSS
      JMP INC
```

```
*IS ENGINE ACCELERATING
        LDA WE
        CMA,INA
        ADA WED
        LDB WE
        STB WED
        LDB =D500
        SSA,RSS
        JMP CONT
*ENGAGE RAM
        LDA RAMV
        ADA CODE
        STA RAMV
        JMP CONT
*DISENGAGE RAM
INC     LDA RAMV
        ADA CODE1
        STA RAMV
        LDB =D500
        JMP CONT
*DISEOF CONTROL
FENG    CLA
        STA RAMV
        LDB =B377
CONT    LDA RAMV
        OTA 13B
        CLC 13B
        JMP CONTR,I
* CONSTANTS SECTION
WED     BSS 1
WE      BSS 1
WV      BSS 1
RAMV    BSS 1
CODE    OCT -2
CODE1   OCT 2
        END
```

```
ASMB,R,L,B,T
      NAM DISEG
      ENT DISEG
DISEG NOP
*THIS ROUTINE DISENGAGES THE RAM
      LDA =B377
      OTA 13B
      CLC 13B
      JMP DISEG,I
      END
```

APPENDIX III

DIGITAL SIMULATION OF A CLUTCH ENGAGEMENT

The data required for running the program and the format in which it must be punched is outlined below, along with the various options and how these options may be called. The program will simulate either a test rig similar to the one used in chapter 4 or a vehicle.

Program Data

All data is in free format i.e. all the numbers on each card must be separated by at least one space and if there is not enough room on one card continue to the next. In the case of a test rig being simulated all the parameters not relevant should be given the value one unless otherwise stated below.

Card No. 1

This is a title card which allows a heading to preceed each printout of results, the program reads 80 spaces i.e. 1 card and prints the same. The first column should be left blank as this column is used to control paper feed on the line printer.

Card No. 2

This card allows the option of the engine torque being dependant upon engine speed and manifold depression or dependant on engine speed only according to whether a 1 or 2 is punched on the card.

OPTION No. 1

Card No. 3

A number giving the order of the polynomials describing the engine performance characteristics, as discussed in chapter 6.

Card No. 4

The polynomials defining the graph, as shown in chapter 6 Graph No. 6.1 punched in the form:

PC(1),PM(1),PC(2),PM(2),...

where PC(1) describes the intersect variation of each straight line and PM(1) the variation in slope of each line with respect to the engine speed. The maximum polynomial allowable is a fifth order.

OPTION No. 2

Card No. 3

A number giving the order of polynomial describing the variation in engine torque with engine speed.

Card No. 4

The polynomial coefficients, in the order:

P(1),P(2),P(3),....

up to a seventh order polynomial may be entered or a constant torque may be assumed by setting card no. 3 to one and entering the single value on card no. 4.

Card No. 5

Read the total inertia of all four road wheels and the drive-line downstream of the clutch (referred to the rear wheels) and the engine inertia.

Card No. 6

Read the gear ratio, the drive axle ratio and the driving wheels rolling radius.

Card No. 7

Read the mass of the vehicle, the aerodynamic drag coefficient, the rolling drag coefficient and the projected frontal area of the vehicle.

Card No. 8

Read the outside and inside radius of the friction material.

Card No. 9

Read the ambient pressure (N/m^2), the ambient temperature (Deg.C) the component of the wind velocity normal to the front of the vehicle and the gradient (sine of the angle)

Card No. 10

This card allows the choice of how the coefficient of friction is assumed to vary, one with slip velocity only and two with slip velocity, clamp pressure and surface temperature.

OPTION No. 1

Card No. 11

Read the static coefficient of friction and the coefficient of friction at high slip velocities. If a constant coefficient of friction is required set the static coefficient of friction to zero and enter the required value next.

OPTION No. 2

Card No. 11

Read the orders of the polynomials used to describe the variation of coefficient of friction with clamp pressure, surface temperature and slip velocity, respectively. The next three or more cards contain the coefficients in the same order i.e.

Pressure Variation	P(I) I = 1, NP
Temperature Variation	T(I) I = 1, NT
Velocity Variation	V(I) I = 1, NV

Cards No. 12 and 13

Read the thermal conductivity the specific heat, the density and the material thickness, for the metal member and the friction material respectively.

Card No. 14

Read the heat transfer coefficient across the interface resistance and the forced convection heat transfer coefficient.

Card No. 15

This is an option card which allows the choice of how the clutch is to be engaged. The options allowed are:

1. Linear ramp
2. Instantaneous application
3. Polynomial application
4. The clutch is engaged as the driver would, this option is only to be used when the full engine performance characteristics are being used.

Card No. 16

This card controls the type of simulation required, if set to 1 a vehicle will be simulated and if set to 2 a test rig will be simulated.

OPTION No. 1

Card No. 17

Read the initial clamp load, the final clamp load and the time over which the clutch is to be engaged.

OPTION No. 2

Card No. 17

Read the maximum clamp load only.

OPTION No. 3

Card No. 17

Read the order of the polynomial describing the required shape of clamp load application.

Card No. 18

Read the coefficients of the above polynomial in the order:

PP(I) I=1, NPP

OPTION No. 4

Card No. 17

Read the maximum allowable clamp load and on a separate card the speed of engagement of the clutch and the response of the engine.

Card No. 18

Read the initial conditions for the engine speed, propeller shaft speed and the heat generated.

Card No. 19

Read an integer, which is equal to the time interval required during the printout of the dynamic results divided by 0.01 secs.

Card No. 20

Read the stability factor, which is a number in the range 0.0 to 0.5 which controls the stability of the temperature solution and should usually be set to 0.1. Read also the time interval of the printout for the temperatures during the engagement, the total time for which cooling is assumed and the printout time interval during cooling.

Card No. 21

This is a switching card which allows the following options dependant upon the number punched in columns 1 and 2 of the card.

Number	Program action
1	Re-read torque characteristics Cards No. 2,3 and 4
2	Re-read engine and wheel inertias Card No. 5.
3	Re-read gear and axle ratios and rolling radius Card No. 6.
4	Re-read vehicle mass, drag coefficients and projected frontal area Card No. 7.
5	Re-read friction material inside and outside radii Card No. 8

6	Re-read ambient pressure and temperature, wind speed and gradient, Card No. 9.
7	Re-read details of coefficient of friction variation, Cards No. 10 and 11
8	Re-read thermal characteristics of the clutch components, Cards No. 12 and 13.
9	Re-read heat transfer coefficient across the interface and the forced convection heat transfer coefficient, Card No. 14.
10	Re-read details of the type of engagement to be carried out, Cards No. 15, 16 and 17
14	Repeat the engagement already carried out.
15	Read in a complete new set of data.

New boundary conditions must be read in after each of the above options and also new timing controls and a title card. If no repeat is required the control card should be left blank, thus terminating the run.

Program Listing

The main body of both programs are the same i.e. the read data section, the integration of the dynamic equations and the printout section, therefore only one full program is listed here and the relevant inter-changeable subroutines of the other technique.

COMPLETE DIGITAL SIMULATION PROGRAM

IMPLICIT TEMPERATURE SOLUTION

```
      MASTER CLUTCH
      REAL NSPEED
      REAL MASS,MUS,MUIN,MU,KF,KS
      EXTERNAL DERIV
      EXTERNAL PRELUDE
      DIMENSION TAT(30,30),TXT(30),TCT(30),TDT(30),TBT(30,30)
      DIMENSION G(5),Y(5),F(5),YP(5),D(5,15),DP(5,15),PPP(7)
      COMMON/TEMPER/KS,KF,RHOS,RHOF,CPS,CPF,PPT,FMT,U,FCH,TAMB
      COMMON/ENGINE/PM(5),PC(5),IMC
      COMMON/COEFF/CT(10),CV(10),CP(10),NV,NT,NP
      COMMON/INTEG/RR,DAR,GR,VW,CD,AD,AP,RHO,GRAD,MASS,IRTYPE,DPP,DPP
     1),NPP,RAMTIM,EI,A,V,MU,Z,TE,TD,TC,TOR(7),NTOR,IMP,TEMP,R1,R2,MU
     2MIN,IDERI,AIE,PSA,ITOR,PMAX,PRERAMP,NSPEED,ENGPER
C...READ DATA DEPENDANT ON SWITCHING INTEGER "NREPET" WHICH ON ENTRY
C   SET TO "15" CAUSING THE PROGRAM TO READ IN ALL THE RELEVENT DATA
C   THIS PARAMETER IS RESET TO 15 AFTER EXECUTION THE PROGRAM READS I
C   A FRESH SET OF DATA
C
C
C   NREPET              PROGRAM ACTION
C     1                 REREAD ENGINE TORQUE CHARACTERISTICS
C
C     2                 REREAD ENGINE AND WHEEL INERTIAS
C
C     3                 REREAD GEAR AND AXLE RATIOS AND ROLLING RADIUS
C
C     4                 REREAD VEHICLE MASS , DRAG COEFFICIENTS AND PROJE
C                       FRONTAL AREA OF VEHICLE
C
C     5                 REREAD CLUTCH PLATE INSIDE AND OUTSIDE RADII
C
C     6                 REREAD AMBIENT PRESSURE AND TEMPERATURE , WIND SP
C                       AND GRADIENT
C
C     7                 REREAD DETAILS OF COEFFICIENT OF FRICTION VARIATI
C
C     8                 REREAD THERMAL PROPERTIES AND THICKNESS OF METAL
C                       OF FRICTION MATERIAL
C
C     9                 REREAD HEAT TRANSFER COEFFICIENT ACROSS INTERFACE
C                       THE FORCED CONVECTION COEFFICIENT
C
C    10                 REREAD DETAILS OF THE TYPE OF ENGAGEMENT REQUIRED
C
C    14                 REREAD BOUNDARY CONDITIONS AND REPEAT SAME ENGAGEM
C                       NEW BOUNDARY CONDITIONS MUST BE SUPPLIED ON EACH
      NREPET=15
    6 READ(1,201)
      WRITE(2,222)
  222 FORMAT(//)
```

```
      WRITE(2,201)
C...READ A TITLE CONTAINED IN 80 SPACES ON A CARD THE FIRST COLUMN LE
      GO TO (0,10,11,12,13,14,15,18,19,20,0,0,0,9,0),NREPET
      READ(1,202)ITOR
C...  ITOR = 2 ENGINE TORQUE IS ASSUMED TO BE A FUNCTION OF ENGINE SP
C
C...  ITOR = 1 ENGINE TORQUE IS A FUNCTION OF ENGINE SPEED AND MANIFO
C
C...TOR(I) CONTAINS THE COEFFICIENTS OF THE ENGINE TORQUE CURVE POLYN
C...NTOR IS THE NUMBER OF COEFFICIENTS
C...PC(I) AND PH(I) CONTIAIN THE COEFFICIENTS OF THE POLYNOMIALS DESC
C      THE VARIATION OF TURQUE WITH MANIFOLD DEPRESSION AND ENGINE SP
C... IMC CONTAINS THE NUMBER OF COEFFICIENTS
      IF(ITOR.EQ.2)GO TO 7
      READ(1,202)IMC
      READ(1,203)(PC(I),PH(I),I=1,IMC)
      WRITE(2,103)(PC(I),PH(I),I=1,IMC)
      GO TO 8
    7 READ(1,202)NTOR
      READ(1,203)(TOR(I),I=1,NTOR)
      WRITE(2,104)(TOR(I),I=1,NTOR)
    8 IF(NREPET.NE.15)GO TO 9
   10 READ(1,203)AIW,AIE
      WRITE(2,222)
C...AIW IS THE INERTIA OF THE FOUR ROAD WHEELS
C...AIE IS THE ENGINE INERTIA
      WRITE(2,105)AIW,AIE
      IF(NREPET.NE.15)GO TO 9
   11 READ(1,203)GR,DAR,RR
      WRITE(2,222)
C...GR IS THE FIRST GEAR RATIO
C...DAR IS THE DRIVE AXLE RATIO
C...RR IS THE ROLLING RADIUS
      WRITE(2,106)GR,DAR,RR
      IF(NREPET.NE.15)GO TO 9
   12 READ(1,203)MASS,AD,CD,AP
      WRITE(2,222)
      WRITE(2,107)MASS,AD,CD,AP
C...MASS IS THE TOTAL MASS OF THE VEHICLE
C... AD AND CD ARE DRAG COEFFICIENTS
C... AP IS THE VEHICLE PROJECTED FRONTAL AREA
      IF(NREPET.NE.15)GO TO 9
C...R1 IS THE CLUTCH OUTSIDE RADIUS
C...R2 IS THE CLUTCH INSIDE RADIUS
   13 READ(1,203)R1,R2
      WRITE(2,222)
      WRITE(2,108)R1,R2
      IF(NREPET.NE.15)GO TO 9
   14 READ(1,203)PA,TA,VW,GRAD
      WRITE(2,222)
```

```
C... PA IS THE AMBIENT PRESSURE
C...TA IS THE AMBEINT TEMPERATURE
C...VU IS THE WIND SPEED
C...GRAD IS THE GRADIENT SINE OF THE ANGLE OF THE SLOPE
      WRITE(2,109)PA,TA,VU,GRAD
      IF(NREPET.NE.15)GO TO 9
      WRITE(2,222)
   15 READ(1,202)IMU
C...IMU=1 COEFFICIENT OF FRICTION VARIES WITH SLIP SPEED ONLY
C...IMU=2 COEFFICIENT OF FRICTION VARIES WITH SLIP SPEED,TEMPERATURE
C    CLAMP PRESSURE
      IF(IMU-1)0,0,16
C...COEFFICIENT OF FRICTION IS ASSUMED ONLY TO VARY WITH SLIP SPEED
      READ(1,203)MUS,MUIN
C...MUS IS THE STATIC COEFFICIENT OF FRICTION
C...MUIN IS THE COEFFICIENT OF FRICTION AT HIGH SLIDING VELOCITY
      WRITE(2,110)MUS,MUIN
      GO TO 17
   16 READ(1,202)NP,NT,NV
C...NP IS THE NUMBER OF COEFFICIENTS IN POLYNOMIAL CP  DESCRIBING MU
C    VARIATION WITH PRESSURE
C...NT IS THE NUMBER OF COEFFICIENTS IN POLYNOMIAL CT DESCRIBING MU
C    VARIATION WITH TEMPERATURE
C...NV IS THE NUMBER OF COEFFICIENTS IN POLYNOMIAL CT DESCRIBING MU
C...VARIATION WITH VELOCITY
C...COEFFICIENT OF FRICTION VARIES WITH VELOCITY , TEMPERATURE AND PR
      READ(1,203)(CP(I),I=1,NP)
      READ(1,203)(CT(I),I=1,NT)
      READ(1,203)(CV(I),I=1,NV)
      WRITE(2,118)(CP(I),I=1,NP)
      WRITE(2,119)(CT(I),I=1,NT)
      WRITE(2,120)(CV(I),I=1,NV)
   17 IF(NREPET.NE.15)GO TO 9
   18 READ(1,203)KS,CPS,RHOS,PPT
      READ(1,203)KF,CPF,RHOF,FMT
      WRITE(2,222)
C...K IS THE THERMAL CONDUCTIVITY
C...CP IS THE SPECIFIC HEAT
C...RHO IS THE DENSITY
C...S REFERS TO THE METAL MEMBER AND F TO THE FRICTION MATERIAL
C...PPT IS THE THICKNESS OF THE METAL MEMBER
C...FMT IS THE THICKNESS OF THE FRICTION MATERIAL
      WRITE(2,111)KS,CPS,RHOS,PPT,KF,CPF,RHOF,FMT
      IF(NREPET.NE.15)GO TO 9
   19 READ(1,203)U,FCH
      WRITE(2,222)
C... U IS THE HEAT TRANSFER COEFFICIENT ACROSS THE FRICTION INTERFACE
C...FCH IS THE FORCED CONVECTION COEFFICIENT
      WRITE(2,112)U,FCH
      IF(NREPET.NE.15)GO TO 9
```

```
   20 READ(1,202)IRTYPE
      WRITE(2,222)
  121 FORMAT(10X,21HMAXIMUM CLAMP LOAD IS,G13.6)
C... IDERI EQUALS 1 VEHICLE IS SIMULATED
C... IDERI EQUALS 1 TEST RIG SIMULATED
      READ(1,202)IDERI
      IF(IRTYPE.EQ.4)GO TO 25
C...IRTYPE = 1 LINEAR RAMP ENGAGEMENT
C
C...IRTYPE = 2 INSTANTANEOUS ENGAGEMENT
C
C...IRTYPE = 3 POLYNOMIAL ENGAGEMENT
C
C...IF IRTYPE = 4 P/S ACCELERATION KEPT CONSTANT PSAMAX
      IF(IRTYPE-2)0,21,22
      READ(1,203)AM,AC,RAMTIM
      DPP=(AM-AC)/RAMTIM
      WRITE(2,113)AM,AC,RAMTIM
      GO TO 23
   21 READ(1,203)AC
      WRITE(2,114)AC
      GO TO 23
   22 READ(1,202)NPP
      READ(1,203)RAMTIM
      READ(1,203)(PPP(I),I=1,NPP)
      WRITE(2,115)(PPP(I),I=1,NPP)
      AC=PPP(1)
      DO 24 I=2,NPP
   24 DPPP(I)=PPP(I)/RAMTIM/(I-1)**2
      GO TO 23
   25 CONTINUE
      READ(1,203)PMAX
      WRITE(2,121)PMAX
      READ(1,203)NSPEED,ENGPER
      WRITE(2,122)NSPEED,ENGPER
  122 FORMAT(10X,21HRATE OF ENGAGEMENT = ,G13.6,/,10X,18HENGINE RESPON
     1 = ,G13.6)
      MU=0.34
      TO=RR*MASS*9.807*(AD+GRAD)/DAR/GR/0.96
      Y(3)=TO/MU/(R1+R2)*1.5
      IF(Y(3).LT.300.0)Y(3)=300.0
      PRERAMP=1500.0
      PRETIME=Y(3)/1500.0
      Y3=Y(3)
      Y(3)=0.0
   23 CONTINUE
    9 CONTINUE
      WRITE(2,222)
      READ(1,203)Y(1),Y(2),Y(4),Y(5)
      IF(NREPET.NE.14)T=0.0
```

```
      WRITE(2,222)
      READ(1,202)IPRINT
      READ(1,203)FSTAB,DTPRINT,TOTIM,TINC
C
C...IPRINT = THE REQUIRED TIME INTERVAL IN PRINTOUT DIVIDED BY 0.01
C
      TPRINT=FLOAT(IPRINT)*0.01
      Y(3)=AC
      WRITE(2,116)(I,Y(I),I=1,5)
      WRITE(2,117)TPRINT
      EI=(MASS*RR*RR+AIW)/DAR**2/GR**2
      A=3.142*(R1**2-R2**2)
      RHO=PA/0.287/(TA+273.0)
      TE=0.0
      TC=0.0
      PSA=0.0
      TD=0.0
      HO=0.001
      DT=0.01
      CALL STABLE(NF,NS,FSTAB,ND,DT)
      WRITE(2,222)
      WRITE(2,100)
      IF(IRTYPE.NE.4)GO TO 30
      N1=5
      N=5
      IFAIL=0
      ITEST=1
      H=0.0005
      ITIME=IFIX(T/60.0)
      TIME=T-60.0*FLOAT(ITIME)
      WRITE(2,101)ITIME,TIME,Y(1),Y(2),Z,Y(3),Y(4),Y(5),MU,V,TE,TC,TD,
     1A
      G(1)=0.01
      G(2)=0.01
      G(3)=0.01
      G(4)=0.01
      G(5)=0.01
      NP=10
      IP=IPRINT
   26 CALL D02AHF(T,Y,G,ITEST,N,HO,H,PRELUDE,F,YP,D,DP,N1,IQ,IFAIL)
      NP=NP+1
      IF(NP.GE.10)GO TO 31
      IF(Y(3).LT.Y3)GO TO 26
      PRERAMP=0.0
      GO TO 26
   31 CONTINUE
      NP=0
      DQ=Y(5)*500.0/DT
      CALL TDIST(T,DQ,DT,DTPRINT,IENTRY,R1,R2,FSTAB,TOTIM,TINC,ND,TAT,
     1T,TCT,TBT,TDT,NF,NS)
```

```
      IP=IP+1
      IF(IPRINT-IP)0,0,29
      IP=0
      ITIME=IFIX(T/60.0)
      TIME=T-60.0*FLOAT(ITIME)
      WRITE(2,101)ITIME,TIME,Y(1),Y(2),Z,Y(3),Y(4),Y(5),MU,V,TE,TC,TD
     1A
   29 CONTINUE
      IF(Y(3).LT.Y3)GO TO 26
   30 CONTINUE
C
C...INTEGRATE DYNAMIC EQUATIONS
C
      IP=0
      IFAIL=0
      IETEST=1
      G(1)=0.001
      G(2)=0.001
      G(3)=0.001
      G(4)=0.001
      G(5)=0.001
      G(1)=0.01
      G(2)=0.01
      G(3)=0.01
      G(4)=0.01
      G(5)=0.01
      TEMP=TA
      N=5
      H0=0.01
      H=0.0005
      N1=5
      IENTRY=0
      CALL TDIST(T,DQ,DT,DTPRINT,IENTRY,R1,R2,FSTAB,TOTIM,TINC,ND,TAT,
     1T,TCT,TBT,TDT,NF,NS)
      WRITE(2,101)ITIME,TIME,Y(1),Y(2),Z,Y(3),Y(4),Y(5),MU,V,TE,TC,TD,
     1A
    4 CALL D02AHF(T,Y,G,IETEST,N,H0,H,DERIV,F,YP,D,DP,N1,IQ,IFAIL)
      DQ=Y(5)*500.0/DT
      CALL TDIST(T,DQ,DT,DTPRINT,IENTRY,R1,R2,FSTAB,TOTIM,TINC,ND,TAT,
     1T,TCT,TBT,TDT,NF,NS)
      TEMP=(TXT(NF+1)+TXT(NF+2))/2.0
      TEMP=TEMP+TA
      IP=IP+1
      IF(IPRINT-IP)0,0,2
      IP=0
      ITIME=IFIX(T/60.0)
      TIME=T-60.0*FLOAT(ITIME)
      WRITE(2,101)ITIME,TIME,Y(1),Y(2),Z,Y(3),Y(4),Y(5),MU,V,TE,TC,TD,
     1A
    2 IF(Y(1)-Y(2)-0.1)3,3,4
```

```
    3 WRITE(2,102)
      WRITE(2,101)ITIME,TIME,Y(1),Y(2),Z,Y(3),Y(4),Y(5),MU,V,TE,TC,TD
     1A
      IENTRY=-10
      CALL TDIST(T,DQ,DT,DTPRINT,IENTRY,R1,R2,FSTAB,TOTIM,TINC,ND,TAT
     1T,TCT,TBT,TDT,NF,NS)
      DT=0.05
      IENTRY=-99
      CALL TDIST(T,DQ,DT,DTPRINT,IENTRY,R1,R2,FSTAB,TOTIM,TINC,ND,TAT
     1T,TCT,TBT,TDT,NF,NS)
      READ(1,200)NREPET
      IF(NREPET)5,5,6
    5 CONTINUE
      STOP
  201 FORMAT(80H
     1                                                   )
  202 FORMAT(4I0)
  203 FORMAT(10F0.0)
  103 FORMAT(10X,46HROUTINE VARIES THE ENGINE TORQUE ATTEMPTING TO,/,1
     1,56HKEEP THE ENGINE SPEED CONSTANT THROUGHOUT THE ENGAGEMENT,/,1
     2,63HTHE ENGINE PERFORMANCE IS DESCRIBED BY THE FOLLOWIN POLYNOM
     3S,/,10X,2HPC,10X,2HPM,/,5(2X,G15.6,5X,G15.6,/))
  104 FORMAT(10X,63HROUTINE VARIES TORQUE ACCORDING TO SUPPLIED ENGINE
     1ORQUE CURVE,/,10X,29HPOLYNOMIALS COEFFICIENTS ARE:,/,7(40X,G15.
     2))
  105 FORMAT(10X,42HINERTIA OF FOUR ROAD WHEELS PLUS DRIVELINE,G13.6,
     1G M**2,/,10X,17HINERTIA OF ENGINE,G13.6,7HKG M**2)
  106 FORMAT(10X,14HIST GEAR RATIO,G13.6,/,10X,16HDRIVE AXLE RATIO,G1
     1,/,10X,19HTYRE ROLLING RADIUS,G13.6,1HM)
  107 FORMAT(10X,12HVEHICLE MASS,G13.6,2HKG,/,10X,17HDRAG COEFFICIENT
     1,15X,2HAD,G13.6,/,15X,2HCD,G13.6,/,10X,22HPROJECTED FRONTAL ARE
     213.6,4HM**2)
  108 FORMAT(10X,21HOUTSIDE CLUTCH RADIUS,G13.6,1HM,/,10X,20HINSIDE C
     1CH RADIUS,G13.6,1HM)
  109 FORMAT(10X,34HAMBIENT CONDITIONS AT TIME OF TEST,/,15X,8HPRESSU
     1G13.6,6HN/M**2,/,15X,11HTEMPERATURE,G13.6,5HDEG C,/,15X,10HWIND
     2EED,G13.6,5HM/SEC,/,15X,8HGRADIENT,G13.6)
  110 FORMAT(10X,63HCOEFFICIENT OF FRICTION IS ASSUMED TO VARY ONLY W
     1 SLIP SPEED,/,15X,18HSTATIC COEFFICIENT,G13.6,/,15X,19HDYNAMIC
     2FFICIENT,G13.6)
  118 FORMAT(10X,71HCOEFFICIENT OF FRICTION VARIES WITH VELOCITY,, TE
     1RATURE AND PRESSURE,/,10X,21HPRESSURE COEFFICIENTS,/,(10X,5G15.
  119 FORMAT(10X,24HTEMPERATURE COEFFICIENTS,/,(10X,5G15.6))
  120 FORMAT(10X,21HVELOCITY COEFFICIENTS,/,(10X,5G15.6))
  111 FORMAT(10X,43HTHERMAL PROPERTIES OF THE CLUTCH COMPONENTS,//,14
     1HTHERMAL,8X,8HSPECIFIC,21X,8HMATERIAL,/,12X,12HCONDUCTIVITY,5X,
     2RAVITY,8X,7HDENSITY,7X,9HTHICKNESS,/,13X,9HW/M/DEG K,6X,10HJ/KG
     3G K,6X,7HKG/M**3,11X,1HM,/,2(10X,4G15.6,/))
  112 FORMAT(10X,45HCOEFFICIENT OF HEAT TRANSFER ACROSS INTERFACE,G13
     112HW/M**2/DEG K,/,10X,43HFORCED CONVECTION HEAT TRANSFER COEFFI
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   2HT,G13.6,12HW/M**2/DEG K)
113 FORMAT(10X,43HCLAMP LOAD APPLIED ACCORDING TO LINEAR RAMP,/,20X
   1HCLAMP LOAD = ,G13.6,9H* TIME + ,G13.6,/,15X,33HTIME OF APPLICA
   2N OF CLAMP LOAD,G13.6,4HSECS)
114 FORMAT(10X,34HCLAMP LOAD APPLIED INSTANTANEOUSLY,/,15X,18HMAXIM
   1CLAMP LOAD,G13.6,1HN)
115 FORMAT(10X,44HCLAMP LOAD APPLIED ACCORDING TO A POLYNOMIAL,/,10
   13HPOLYNOMIAL COEFFICIENTS,/,3(10X,4G15.6,/))
116 FORMAT(10X,18HINITIAL CONDITIONS,//,5(5X,2HY(,I1,1H),G13.6))
117 FORMAT(10X,23HPRINT OUT TIME INTERVAL,G13.6,/)
100 FORMAT(3X,4HTIME,4X,7HENG.SP.,2X,7HP/S SP.,1X,8HSLIP SP.,2X,7HC
   1OAD,1X,8HMAN.DEP.,2X,6HHT.GEN,2X,8HCO.OF FR,2X,7HVEH.SP.,1X,8HE
   2TOR,,2X,7HCL.TOR.,1X,8HDRAG TOR,1X,8HP/S ACC.,/,2X,7HMIN-SEC,2X
   3RAD/SEC,2X,7HRAD/SEC,2X,7HRAD/SEC,3X,4HK.N.,5X,5HMM.HG,4X,6HJOU
   4,14X,5HM/SEC,4X,4HN.M.,5X,4HN.M.,5X,4HN.M.,3X,10HRAD/SEC**2,//)
101 FORMAT(1X,I2,1H-,F6.3,1X,3(F6.1,3X),1X,F5.0,4X,F5.2,4X,F5.2,4X,
   13,4X,F5.1,2X,3(2X,F5.1,2X),2X,F5.1)
102 FORMAT(10X,32HCONDITIONS AT TIME OF ENGAGEMENT)
200 FORMAT(I2)
    END
    SUBROUTINE PRELUDE(DY,Y,T)
    REAL MU,MUIN,MUS,MASS
    DIMENSION DY(5),Y(5)
    COMMON/INTEG/RR,DAR,GR,VW,CD,AD,AP,RHO,GRAD,MASS,IRTYPE,DPP,OPP(
   1),NPP,RAMTIM,EI,A,V,MU,Z,TE,TD,TC,TOR(7),NTOR,IMU,TEMP,R1,R2,MUS
   2MIN,IDERI,AIE,PSA,ITOR,PMAX,PRERAMP,NSPEED,ENGPER
    DY(3)=PRERAMP
    TC=MU*(R1+R2)*Y(3)
    DP=AMANDE(Y(1),TE)
    IF(DP)0,2,2
    DP=0.0
    TE=ENGTOR(Y(1),DP)
    GO TO 1
  2 TE=TC
  1 CONTINUE
    Y(4)=DP
    DY(5)=Y(1)*TC/1000.0
    DY(1)=(TE-TC)/AIE
    DY(2)=0.0
    DY(4)=0.0
    RETURN
    END
    FUNCTION AMANDE(XWE,TE)
    COMMON/ENGINE/PM(5),PC(5),IMC
    XM=PM(IMC)
    XC=PC(IMC)
    DO 1 J=1,IMC-1
    XM=XM*XWE+PM(IMC-J)
  1 XC=XC*XWE+PC(IMC-J)
    XDP=(TE/1000.0*XWE-XC)/XM
```

```
      AMANDE=XDP
      RETURN
      END
      SUBROUTINE TDIST(T,Q,DT,DTPRINT,IENTRY,R1,R2,FSTAB,TOTTIM,TINC
     1A,X,C,B,D,NF,NS)
C
C...THIS ROUTINE SOLVES THE UNSTEADY HEAT CONDUCTION EQUATION AND ST
C   THE RESULTANT TEMPERATURE DISTRIBUTION ON DISK , AT THE END OF A
C   ENGAGEMENT THE DISTRIBUTION IS OUTPUT IN A SHORTEND FORM
C
C...R1 AND R2 ARE THE CLUTCH INSIDE AND OUTSIDE RADIUS RESPECTIVELY
C
C...T IS THE TIME FROM THE BEGINING OF ENGAGEMENT
C
C...Q IS THE TOTAL HEAT GENERATED UP TO THIS TIME
C
C...DT IS THE TIME INTERVAL OVER WHICH INTEGRATION TAKES PLACE
C
C...DTPRINO IS THE TIME INTERVAL OF THE PRINT-OUT
C
C...IENTRY = 0 ON FIRST ENTRY
C...IENTRY = +VE ON SUBSEQUENT ENTRIES DURING THE ENGAGEMENT
C...IENTRY = -10 ON LAST ENTRY OF ENG175M5NT
C...IENTRY = -09 ON ANY SUBEQUENT ENTRY AND ASSEMBLY IS ASSUMED TO CO
C
C...FSTAB IS THE APPROXIMATE VALUE OF THE FOURIER NO. TO BE USED
C
C...TOTTIM IS THE TIME OVER WHICH COOCRE7,9S 1SSUMED
C
C...TINC IS THE INTEVAL OF PRINT-OUT
C
      REAL KS,KF
      DIMENSION A(ND,ND),X(ND),C(ND),B(ND,ND),D(ND),Z(1)
      COMMON/TEMPER/KS,KF,RHOS.RHOF,CPS,CPF,PPT,FMT,U,FCH,TAMB
      IF(IENTRY)2,0,1
      IENTRY=10
    8 CONTINUE
      QJ=0.0
      QOLD=0 0
      AF=(R1**2-R2**2)*3.142
      DXS=PPT/FLOAT(NS-1)
      DXF=FMT/FLOAT(NF-1)
      FOS=DT*KS/(RHOS*CPS*DXS**2)
      FOF=DT*KF/(RHOF*CPF*DXF**2)
      AL=KF/KS*SQRT(KS*RHOF*CPF/KF/RHOS/CPS)
      BO=FCH*DXS/KS
      BQF=AL*DXF/KF/AF
      BQS=(1-AL)*DXS/KS/AF
      BIS=U*DXS/KS
      BIF=U*DXF/KF
```

```
      ICH=0
      IFAIL=0
      CALL F01CAF(A,ND,ND,IFAIL)
      CALL F01CAF(B,ND,ND,IFAIL)
      CALL F01CAF(C,ND,1,IFAIL)
      CALL F01CAF(D,ND,1,IFAIL)
      NF1=NF+3
      CALL SETA(A,FOS,ND,NS,NF1)
      CALL SETA(A,FOF,ND,NF,2)
      CALL SETB(B,FOS,ND,NS,NF1)
      CALL SETB(B,FOF,ND,NF,2)
      A(1,1)=(1.0+FOF)*2.0
      A(1,2)=-FOF*2.0
      A(NF+1,NF)=-FOF*(2.0-BIF)
      A(NF+1,NF+1)=2.0*(1.0+FOF)
      A(NF+1,NF+2)=-FOF*BIF
      A(NF+2,NF+1)=-FOS*BIS
      A(NF+2,NF+2)=2.0*(1.0+FOS)
      A(NF+2,NF+3)=-FOS*(2.0-BIS)
      A(NF+NS+2,NF+NS+2)=2.0*(1.0+FOS)+2.0*FOS*BOS
      A(NF+NS+2,NF+NS+1)=-2.0*FOS
      B(1,1)=2.0*(1.0-FOF)
      B(1,2)=2.0*FOF
      B(NF+1,NF)=FOF*(2.0-BIF)
      B(NF+1,NF+1)=2.0*(1.0-FOF)
      B(NF+1,NF+2)=FOF*BIF
      B(NF+2,NF+1)=FOS*BIS
      B(NF+2,NF+3)=FOS*(2.0-BIS)
      B(NF+2,NF+2)=2.0*(1.0-FOS)
      B(NF+NS+2,NF+NS+1)=2.0*FOS
      B(NF+NS+2,NF+NS+2)=2.0*(1.0-FOS)-2.0*FOS*BOS
      IF(IENTRY.EQ.-99)GO TO 3
    1 QJ=QJ1
      QJ1=Q-QOLD
      QOLD=Q
      C(NF+1)=BQF*FOF*(QJ1+QJ)*2.0
      C(NF+2)=BQS*FOS*(QJ1+QJ)*2.0
      C(NF+NS+2)=4.0*BOS*FOS*TAMB
C
C...MULTIPLY MATRIX B BY MATRIX X
C
      IFAIL=0
      CALL F01CKF(D,B,X,ND,1,ND,Z,1,1,IFAIL)
C
C...ADD MATRIX C TO MATRIX D
C
      CALL F01CGF(D,ND,1,1,1,C,ND,1,1,ND,1,1,IFAIL)
C
C...SOLVE TRIDIAGONAL MATRIX
C
```

```
      CALL TRIDIAG(A,X,D,ND)
      ICH=ICH+1
      WRITE(4)T,(X(I),I=1,ND)
      RETURN
    2 IF(IENTRY.EQ.-99)GO TO 8
      WRITE(2,101)FOS,BIS,BQS,DXS,NS,FOF,BIF,BQF,DXF,NF
  101 FORMAT(//,22X,11HFOURIER NO.,8X,3HBI ,12X,3HBQ ,12X,3HDX ,5X,16
     1.OF MESH PTS. ,/,10X,10HMETAL MEMB,4G15.8,6X,I3,/,8X,12HFRICTIO
     2AT,4G15.8,6X,I3,//)
      REWIND 4
      WRITE(2,102)
  102 FORMAT(//,10X,42HTEMPERATURE DISTRIBUTION DURING ENGAGEMENT,//)
      NITS=NS/2
      NITF=NF/2
      WRITE(2,103)(I,I=1,NF+1,NITF),(I,I=NF+2,NS+NF+2,NITS)
  103 FORMAT(13X,4HTIME,10X,17HFRICTION MATERIAL,25X,12HMETAL MEMBER,
     11X,3(4X,I2,4X),10X,3(4X,I2,4X))
      IPRIN=IFIX(DTPRINT/DT)
      IDP=IPRIN
      ICH=ICH-1
    5 READ(4)T,(X(I),I=1,ND)
      IF(IDP.LT.IPRIN)GO TO 4
      IDP=0
      WRITE(2,104)T,(X(I),I=1,NF+1,NITF),(X(I),I=NF+2,NS+NF+2,NITS)
    4 IDP=IDP+1
      IDPP=IDPP+1
      IF(IDPP.LE.ICH)GO TO 5
  104 FORMAT(13X,F5.3,2X,3(2X,F6.1,2X),3X,4H****,3X,3(2X,F6.1,2X))
      RETURN
    3 WRITE(2,108)
  108 FORMAT(//,10X,27HTHE ASSEMBLY IS NOW COOLING,//)
      C(NF+2)=0.0
      C(NF+1)=0.0
      TIME=FLOAT(IFIX(T/TINC))*TINC
      WRITE(2,107)(I,I=1,NF+1,NITF),(I,I=NF+2,NS+NF+2,NITS)
    7 TIME=TIME+TINC
    6 T=T+DT
      IFAIL=0
      CALL F01CKF(D,B,X,ND,1,ND,Z,1,1,IFAIL)
      CALL F01CGF(D,ND,1,1,1,C,ND,1,1,ND,1,1,IFAIL)
      CALL TRIDIAG(A,X,D,ND)
      IF(T.LT.TIME)GO TO 6
      ITM=IFIX(TIME/60.0)
      ITS=IFIX(TIME)-ITM*60
      WRITE(2,106)ITM,ITS,X(1),X(NITF),X(NF+1),X(NF+2),X(NF+2+NITS),X
     1+2+NS)
  106 FORMAT(10X,2X,I4,4X,I2,4X,3(2X,F6.1,2X),3X,4H****,3X,3(2X,F6.1,
     1)
  107 FORMAT(16X,4HTIME,30X,12HTEMPERATURES,/,12X,4HMINS,3X,4HSECS,4X
     1(4X,I2,4X),10X,3(4X,I2,4X))
```

```
      IF(TIME.LT.TOTTIM)GO TO 7
      RETURN
      END
      SUBROUTINE SETA(A,FO,ND,N,M)
      DIMENSION A(ND,ND)
      Q=2.0*(1.0+FO)
      DO 1 I=M,N+M-2
      A(I,I-1)=-FO
      A(I,I)=Q
    1 A(I,I+1)=-FO
      RETURN
      END
      SUBROUTINE STABLE(NF,NS,FSTAB,ND,DT)
      REAL KS,KF
      COMMON/TEMPER/KS,KF,RHOS,RHOF,CPS,CPF,PPT,FMT,U,FCH,TAMB
      DXF=SQRT(DT*KF/FSTAB/RHOF/CPF)
      NF=IFIX(FMT/DXF)+1
      IF(NF.GE.5)GO TO10
      NF=5
      DXF=FMT/FLOAT(NF-1)
      FSTAB=DT*KF/DXF**2/RHOF/CPF
      WRITE(2,105)
  105 FORMAT(10X,43H*** STABILITY FACTOR HAS BEEN CHANGED ****)
   10 DXS=SQRT(DT*KS/FSTAB/RHOS/CPS)
      NS=IFIX(PPT/DXS)+1
      ND=NF+NS+2
      WRITE(2,100)FSTAB
  100 FORMAT(10X,49HTHE ROUTINE HAS CHOSEN THE MESH SIZE REQUIRED TO
     110X,48HKEEP THE STABILITY FACTOR APPROXIMATELY EQUAL TO,G13.6)
      RETURN
      END
      SUBROUTINE SETB(B,FO,ND,N,M)
      DIMENSION B(ND,ND)
      Q=2.0*(1.0-FO)
      DO 1 I=M,N+M-2
      B(I,I-1)=+FO
      B(I,I)=Q
    1 B(I,I+1)=+FO
      RETURN
      END
      SUBROUTINE TRIDIAG(A,X,B,N)
      DIMENSION X(N),A(N,N),P(30),Q(30),B(N),U(30)
C
C...THIS SUBROUTINE SOLVES A TRIDIAGONAL MATRIX
C
      P(1)=A(1,1)
      U(1)=B(1)/P(1)
      Q(1)=-A(1,2)/P(1)
      DO 1 I=2,N-1
      P(I)=A(I,I-1)*Q(I-1)+A(I,I)
```

```
      Q(I)=-A(I,I+1)/P(I)
    1 U(I)=(B(I)-A(I,I-1)*U(I-1))/P(I)
      P(N)=A(N,N-1)*Q(N-1)+A(N,N)
      U(N)=(B(N)-A(N,N-1)*U(N-1))/P(N)
      X(N)=U(N)
      DO 2 I=1,N-1
      IN=N-I
    2 X(IN)=Q(IN)*X(IN+1)+U(IN)
      RETURN
      END
      FUNCTION COOFF(T,P,V)
      COMMON/COEFF/CT(10),CV(10),CP(10),NV,NT,NP
C
C...THIS FUNCTION CALCULATES THE VALUE OF THE DYNAMIC COEFFICIENT OF
C
      XP=CP(NP)
      DO 1 I=1,NP-1
    1 XP=XP*P+CP(NP-I)
      XT=CT(NT)
      DO 2 I=1,NT-1
    2 XT=XT*T+CT(NT-I)
      XV=CV(NV)
      DO 3 I=1,NV-1
    3 XV=XV*V+CV(NV-I)
      COOEF=0.412/0.0475*XP*XT*XV
      RETURN
      END
      FUNCTION ENGTOR(XWE,XDP)
      COMMON/ENGINE/PM(5),PC(5),IMC
      XM=PM(IMC)
      XC=PC(IMC)
C
C...THIS FUNCTION CALCULATES THE ENGINE TORQUE FROM MANIFOLD DEPRESSI
C...AND ENGINE SPEED
C
      DO 1 J=1,IMC-1
      XM=XM*XWE+PM(IMC-J)
    1 XC=XC*XWE+PC(IMC-J)
      TE=(XM*XDP+XC)*1000.0/XWE
      RETURN
      END
      SUBROUTINE DERIV(DY,Y,T)
      REAL NSPEED
      REAL MU,MUTH,MUS,MASS
      DIMENSION DY(5),Y(5)
      COMMON/INTEG/RR,DAR,GR,VW,CD,AD,AP,RHO,GRAD,MASS,IRTYPE,DPP,DPP(
     1),NPP,RAMTIM,EI,A,V,MU,Z,TE,TD,TC,TOR(7),NTOR,IMU,TEMP,R1,R2,MUS
     2MIN,IDERI,AIE,PSA,ITOR,PMAX,PRERAMP,NSPEED,ENGPER
C
C...THIS SUBROUTINE CALCULATES THE DERIVITIVES OF THE FOLLOWING PARAME
```

```
C
C    DY(1)                 ENGINE SPEED  WE
C
C    DY(2)                 P/S SPEED  WV
C
C    DY(3)                 CLAMP LOAD  P
C
C    DY(4)                 MANIFOLD DEPRESSION  DPP
C
C    DY(5)                 HEAT GENERATED  Q
C
C
C...THREE TYPES OF ENGAGEMENT ARE POSSIBLE
C
C    ITYPE = 1 IS A LINEAR RAMP  P=M*T+C
C
C    ITYPE = 2 IS AN INSTANTANEOUS ENGAGEMENT T=0
C
C    ITYPE = 3 IS A POLYNOMIAL ENGAGEMENT P=C+M*T+N*T**2+...
C
C
C...ENGINE TORQUE IS CHOOSEN TO GIVE CONSTANT ENGINE SPEED THROUGHOU
C    THE ENGAGEMENT , IF THIS IS POSSIBLE
C
      A1=1.0
      IF(Y(2).GT.Y(1))A1=-1.0
      Z=Y(1)-Y(2)
      IF(IMU-1)0,0,18
      IF(MUS.EQ.0.0)GO TO 30
      MU=MUIN+(MUS-MUIN)*EXP(-1.88*Z*(R1+R2)/2.0)
      GO TO 19
   30 MU=MUIN
      GO TO 19
   18 VC=(R1+R2)/2.0*Z
      PRESS=Y(3)/A/1000.0
      MU=COOFF(TEMP,PRESS,VC)
   19 CONTINUE
      V=Y(2)/GR/DAR*RR
      ETA=(0.96-0.000707*V-0.000020*V**2)*(0.977-0.001765*V)
      FD=MASS*9.807*(AD+GRAD)+RHO/2.0*AP*CD*(V+VW)*ABS(V+VW)
      TD=FD*RR/DAR/GR/ETA
      IF(IRTYPE.NE.1)GO TO 1
C...LINEAR RAMP
      IF(T.GE.RAMTIM)GO TO 2
      DY(3)=DPP
      GO TO 3
    2 DY(3)=0.0
      GO TO 3
    1 IF(IRTYPE-2)4,2,4
C...INSTANTANEOUS ENGAGEMENT
```

```
    4 IF(IRTYPE.NE.3)GO TO 3
      IF(T.GE.RAMTIM)GO TO 2
C...POLYNOMIAL ENGAGEMENT
      XA=XA*T+DPPP(NPP)
      DO 7 IPP=1,NPP-1
    7 XA=XA*T+DPPP(NPP-IPP)
      DY(3)=XA
      GO TO 3
    3 CONTINUE
C
C...CALCULATE ENGINE TORQUE
C
C...IF ITOR = 1 ENGINE TORQUE DEPENDS UPON MANIFOLD DEPRESSION AND E
C
C...IF ITOR = 2 ENGINE TORQUE DEPENDS ONLY UPON ENGINE SPEED
C
      IF(ITOR.NE.1)GO TO 8
      IF(Y(3).GE.PMAX)GO TO 12
      IF(DY(1))9,10,11
   10 DY(4)=0.0
      GO TO 12
    9 DY(4)=ENGPER
      IF(DY(2)-40.0)0,0,23
      DY(4)=-30.0
   23 CONTINUE
      IF(Y(4).GT.0.0)GO TO 12
      DY(4)=0.0
      Y(4)=0.0
      GO TO 12
   11 DY(4)=20.0
      IF(Y(4).LT.30.0)GO TO 12
      DY(4)=0.0
      Y(4)=30.0
   12 TE=ENGTOR(Y(1),Y(4))
      GO TO 13
    8 CONTINUE
      DY(4)=0.0
      TE=TOR(NTOR)
      IF(NTOR.EQ.1)GO TO 13
      DO 14 IT=1,NTOR-1
   14 TE=TE*Y(1)+TOR(NTOR-IT)
   13 CONTINUE
      TC=A1*MU*Y(3)*(R1+R2)
C...SET-UP DERIVITIVES
C
C...IDERI =1 SIMULATES A VEHICLE
C
C...IDERI =2 SIMULATES A TEST RIG WITH ONE STATIONARY MEMBER
      IF(IDERI.NE.1)GO TO 15
      DY(1)=(TE-TC)/AIE
```

```
      IF(IRTYPE.NE.4)GO TO 21
      IF(DY(1))0,22,22
      IF(Y(1)-100.0)0,0,20
      DY(3)=100.0
      GO TO 21
   20 DY(3)=NSPEED
      GO TO 21
   22 DY(3)=1500.0
      IF(Y(3)-PMAX)21,21,0
      DY(3)=0.0
      Y(3)=PMAX
   21 CONTINUE
      DY(2)=(TC-TD)/EI
      DY(5)=Z*TC/1000.0
      PSA=DY(2)
      IF(V-0.01)0,0,15
      IF(DY(2))0,0,15
      GGL=TD/GR/MU/(R1+R2)
      DY(3)=TD/EI*1.01
      WRITE(2,17)Y(3),GGL
      Y(3)=GGL
   17 FORMAT(///,10X,'**** CLUTCH FORCE ALTERED FROM',F12.2,'TO',F12.
     1****',///)
   15 IF(IDERI.NE.2)GO TO 16
      DY(1)=0.0
      DY(2)=(TC-TD)/EI
      DY(3)=0.0
      DY(4)=0.0
      DY(5)=Z*TC/1000.0
   16 CONTINUE
      RETURN
      END
      FINISH
```

INTERCHANGEABLE SUBROUTINES FOR

EXPLICIT TEMPERATURE SOLUTION

```
      SUBROUTINE TDIST(T,Q,DT,DTPRINT,IENTRY,R1,R2,FSTAB,TOTTIM,TINC,
     1A,X,C,D,NF,NS)
C
C...R1 AND R2 ARE THE CLUTCH INSIDE AND OUTSIDE RADIUS RESPECTIVELY
C
C...T IS THE TIME FROM THE BEGINING OF ENGAGEMENT
C
C...Q IS THE TOTAL HEAT GENERATED UP TO THIS TIME
C
C...DT IS THE TIME INTERVAL OVER WHICH INTEGRATION TAKES PLACE
C
C...DTPRINT IS THE TIME INTERVAL OF THE PRINT-OUT
C
C...IENTRY =0 ON FIRST ENTRY
C...IENTRY = +VE ON SUBSEQUENT ENTRIES DURING THE ENGAGEMENT
C...IENTRY = -10 ON LAST ENTRY OF ENGAGEMENT
C...IENTRY = -99 ON ANY SUBSEQUENT ENTRY AND ASSEMBLY IS ASSUMED TO C
C
C...FSTAB IS THE APPROXIMATE VALUE OF THE FOURIER NO. TO BE USED
C
C...TOTTIM IS THE TIME OVER WHICH COOLING IS ASSUMED
C
C...TINC IS THE INTERVAL OF PRINT-OUT
C
      REAL KS,KF
      DIMENSION A(ND,ND),X(ND),C(ND),D(ND)
      COMMON/TEMPER/KS,KF,RHOS,RHOF,CPS,CPF,PPT,FMT,U,FCH,TAMB
      IF(IENTRY)2,0,1
      IENTRY=10
    8 CONTINUE
      QJ=0.0
      QOLD=0.0
      AF=(R1**2-R2**2)*3.142
      DXF=FMT/FLOAT(NF-1)
      DXS=PPT/FLOAT(NS-1)
      FOS=DT*KS/(RHOS*CPS*DXS**2)
      FOF=DT*KF/(RHOF*CPF*DXF**2)
      AL=KF/KS*SQRT(KS*RHOF*CPF/KF/RHOS/CPS)
      BO=FCH*DXS/KS
      BQS=(1.0-AL)*DXS/KS/AF
      BQF=AL*DXF/KF/AF
      BIS=U*DXS/KS
      BIF=U*DXF/KF
      ICH=0
      IFAIL=0
      CALL F01CAF(A,ND,ND,IFAIL)
      CALL F01CAF(C,ND,1,IFAIL)
      CALL F01CAF(D,ND,1,IFAIL)
      NF1=NF+3
      CALL SETA(A,FOS,ND,NS,NF1)
      CALL SETA(A,FOF,ND,NF,2)
      A(1,1)=(1.0-2.0*FOF)
```

```
      A(1,2)=2.0*FOF
      A(NF+1,NF)=2.0*FOF
      A(NF+1,NF+1)=(1.0-2.0*FOF*(1.0+BIF))
      A(NF+1,NF+2)=2.0*FOF*BIF
      A(NF+2,NF+1)=2.0*FOS*BIS
      A(NF+2,NF+2)=(1.0-2.0*FOS*(1.0+BIS))
      A(NF+2,NF+3)=2.0*FOS
      A(NF+NS+2,NF+NS+1)=2.0*FOS
      A(NF+NS+2,NF+NS+2)=(1.0-2.0*FOS*(1.0+BOS))
      IF(IENTRY.EQ.-99)GO TO 3
    1 QJ=Q-QOLD
      QOLD=Q
      C(NF+1)=2.0*FOF*BQF*QJ
      C(NF+2)=2.0*FOS*BQS*QJ
      C(NF+NS+2)=2.0*FOS*BO*TAMB
C...MULTIPLY MATRIX B BY MATRIX X RESULT IN X
      IFAIL=0
      CALL F01CKF(X,A,X,ND,1,ND,D,ND,3,IFAIL)
C...ADD MATRIX C TO MATRIX X RESULT IN X
      CALL F01CGF(X,ND,1,1,1,C,ND,1,1,ND,1,1,IFAIL)
      ICH=ICH+1
      WRITE(4)T,(X(I),I=1,ND)
      RETURN
    2 IF(IENTRY.EQ.-99)GO TO 8
      WRITE(2,101)FOS,BIS,BQS,DXS,NS,FOF,BIF,BQF,DXF,NF
  101 FORMAT(//,22X,11HFOURIER NO.,8X,3HBI ,12X,3HBQ ,12X,3HDX ,5X,16H
     1 OF MESH PTS. ,/,10X,10HMETAL MEMB,4G15.8,6X,I3,/,8X,12HFRICTION
     2AT,4G15.8,6X,I3,//)
      REWIND 4
      WRITE(2,102)
  102 FORMAT(//,10X,42HTEMPERATURE DISTRIBUTION DURING ENGAGEMENT,//)
      NITS=NS/2
      NITF=NF/2
      WRITE(2,103)(I,I=1,NF+1,NITF),(I,I=NF+2,NS+NF+2,NITS)
  103 FORMAT(13X,4HTIME,10X,17HFRICTION MATERIAL,25X,12HMETAL MEMBER,/
     11X,3(4X,I2,4X),10X,3(4X,I2,4X))
      IPRIN=IFIX(DTPRINT/DT)
      IDP=IPRIN
      ICH=ICH-1
    5 READ(4)T,(X(I),I=1,ND)
      IF(IDP.LT.IPRIN)GO TO 4
      IDP=0
      WRITE(2,104)T,(X(I),I=1,NF+1,NITF),(X(I),I=NF+2,NS+NF+2,NITS)
    4 IDP=IDP+1
      IDPP=IDPP+1
      IF(IDPP.LE.ICH)GO TO 5
  104 FORMAT(13X,F5.3,2X,3(2X,F6.1,2X),3X,4H****,3X,3(2X,F6.1,2X))
      RETURN
    3 WRITE(2,108)
  108 FORMAT(//,10X,27HTHE ASSEMBLY IS NOW COOLING,//)
```

```
      C(NF+2)=0.0
      TIME =FLOAT(IFIX(T/TINC))*TINC
      C(NF+1)=0.0
      WRITE(2,107)(I,I=1,NF+1,NITF),(I,I=NF+2,NS+NF+2,NITS)
    7 TIME=TIME+TINC
    6 T=T+DT
      IFAIL=0
      CALL F01CKF(X,A,X,ND,1,ND,D,ND,3,IFAIL)
      CALL F01CGF(X,ND,1,1,1,C,ND,1,1,ND,1,1,IFAIL)
      IF(T.LT.TIME)GO TO 6
      ITM=IFIX(TIME/60.0)
      ITS=IFIX(TIME)-ITM*60
      WRITE(2,106)ITM,ITS,X(1),X(NITF),X(NF+1),X(NF+2),X(NF+2+NITS),X(
     1+2+NS)
  106 FORMAT(12X,I4,4X,I2,4X,3(2X,F6.1,2X),3X,4H****,3X,3(2X,F6.1,2X))
  107 FORMAT(16X,4HTIME,30X,12HTEMPERATURES,/,12X,4HMINS,3X,4HSECS.4X,
     14X,I2,4X),10X,3(4X,I2,4X))
      IF(TIME.LT.TOTTIM)GO TO 7
      RETURN
      END
      SUBROUTINE STABLE(NF,NS,FSTAB,ND,DT)
      REAL KS,KF
      COMMON/TEMPER/KS,KF,RHOS,RHOF,CPS,CPF,PPT,FMT,U,FCH,TAMB
      DXF=SQRT(DT*KF/FSTAB/RHOF/CPF)
      NF=IFIX(FMT/DXF)+1
      IF(NF.GE.5)GO TO 10
      NF=5
      DXF=FMT/FLOAT(NF-1)
      FSTAB=DT*KF/DXF**2/RHOF/CPF
      WRITE(2,105)
  105 FORMAT(10X,43H*** STABILITY FACTOR HAS BEEN CHANGED ****)
   10 DXS=SQRT(DT*KS/FSTAB/CPS/RHOS)
      NS=IFIX(PPT/DXS)+1
      ND=NF+NS+2
      WRITE(2,100)FSTAB
  100 FORMAT(10X,49HTHE ROUTINE HAS CHOSEN THE MESH SIZE REQUIRED TO ,
     110X,48HKEEP THE STABILITY FACTOR APPROXIMATELY EQUAL TO,G13.6)
      RETURN
      END
      SUBROUTINE SETA(A,FO,ND,N,M)
      DIMENSION A(ND,ND)
      Q=1.0-2.0*FO
      DO 1 I=M,N+M-2
      A(I,I)=Q
      A(I,I-1)=FO
    1 A(I,I+1)=FO
      RETURN
      END
```

SAMPLE SET OF RESULTS,

SIMULATION OF A 1 : 10 GRADIENT TAKE-UP.

MAXIMUM CLAMP LOAD IS 4200.00

RATE OF ENGAGEMENT = 500.000
ENGINE RESPONSE = -2.50000

INITIAL CONDITIONS

Y(1) 200.000 Y(2) 0.000000E 00 Y(3) 0.000000E 00 Y(4) 30.0000
PRINT OUT TIME INTERVAL 0.100000

THE ROUTINE HAS CHOSEN THE MESH SIZE REQUIRED TO
KEEP THE STABILITY FACTOR APPROXIMATELY EQUAL TO 0.100000

	FOURIER NO.	BI	BQ	DX	NO OF MESH PTS.
METAL MEMB	0.92459421E-01	0.44444444E-02	0.34097608E-02	0.20000000E-02	6
FRICTION MAT	0.97293814E-01	0.44150110E-01	0.36856558E-02	0.33333333E-03	10

OUTSIDE CLUTCH RADIUS 0.905000E-01M
INSIDE CLUTCH RADIUS 0.667000E-01M

AMBIENT CONDITIONS AT TIME OF TEST
PRESSURE 101.500 N/M**2
TEMPERATURE 15.0000 DEG C
WIND SPEED 0.000000E 00M/SEC
GRADIENT 0.996000E-01

COEFFICIENT OF FRICTION IS ASSUMED TO VARY ONLY WITH SLIP SPEED
STATIC COEFFICIENT 0.450000
DYNAMIC COEFFICIENT 0.340000

THERMAL PROPERTIES OF THE CLUTCH COMPONENTS

THERMAL CONDUCTIVITY W/M/DEG K	SPECIFIC GRAVITY J/KG/DEG K	DENSITY KG/M**3	MATERIAL THICKNESS M
45.0000	155.000	7850.00	0.100000E-01
0.755000	388.000	1800.00	0.300000E-02

COEFFICIENT OF HEAT TRANSFER ACROSS INTERFACE 100.000 W/M**2/DEG K
FORCED CONVECTION HEAT TRANSFER COEFFICIENT 43.0000 W/M**2/DEG K

ROUTINE VARIES THE ENGINE TORQUE ATTEMPTING TO
KEEP THE ENGINE SPEED CONSTANT THROUGHOUT THE ENGAGEMENT
THE ENGINE PERFORMANCE IS DESCRIBED BY THE FOLLOWIN POLYNOMIALS.

PC	PM
-5.42000	0.817900
0.163908	-0.863235E-02
-0.118515E-03	0.577556E-05
-0.580300E-07	0.101552E-08

INERTIA OF FOUR ROAD WHEELS PLUS DRIVELINE 3.30000 KG M**2
INERTIA OF ENGINE 0.250000 KG M**2

IST GEAR RATIO 3.54300
DRIVE AXLE RATIO 3.90000
TYRE ROLLING RADIUS 0.286000 M

VEHICLE MASS 1210.00 KG
DRAG COEFFICIENTS
AD 0.180000E-01
CD 0.557000
PROJECTED FRONTAL AREA 0.930000 M**2

TIME SEC	ENG.SP. RAD/SEC	P/S SP. RAD/SEC	SLIP SP. RAD/SEC	CL.LOAD K.N.	MAN.DEP. MM.HG	HT.GEN KJOULES	CO.OF FR	VEH.SP. M/SEC	ENG.TOR. N.M.	CL.TOR. N.M.	DRAG TOR N.M.	P/S AC RAD
0.000	200.0	0.0	0.0	0.	30.00	0.00	0.340	0.0	0.0	0.0	0.0	0
0.001	200.0	0.0	0.0	1.	33.07	0.00	0.340	0.0	0.1	0.1	0.0	0
0.101	200.0	0.0	0.0	152.	30.68	0.08	0.340	0.0	8.1	8.1	0.0	0
0.201	200.0	0.0	0.0	302.	28.28	0.32	0.340	0.0	16.1	16.1	0.0	0
0.301	200.0	0.0	0.0	451.	25.89	0.73	0.340	0.0	24.1	24.1	0.0	0
0.401	200.0	0.0	0.0	602.	23.49	1.29	0.340	0.0	32.1	32.1	0.0	0
0.501	200.0	0.0	0.0	752.	21.10	2.01	0.340	0.0	40.2	40.2	0.0	0
0.671	200.0	3.1	196.9	927.	18.29	3.55	0.340	0.1	49.6	49.6	30.8	35
0.771	199.9	7.0	192.9	997.	17.41	4.56	0.340	0.1	52.5	53.3	30.8	42
0.871	199.3	11.4	187.9	1047.	17.16	5.60	0.340	0.2	53.5	56.0	30.8	46
0.971	198.0	16.4	181.6	1097.	16.91	6.66	0.340	0.3	54.7	58.6	30.8	51
1.071	196.2	21.8	174.3	1147.	16.66	7.73	0.340	0.5	56.0	61.3	30.8	56
1.171	193.8	27.7	166.0	1197.	16.41	8.79	0.340	0.6	57.4	64.0	30.8	61
1.271	190.9	34.2	156.7	1247.	16.16	9.85	0.340	0.7	58.9	66.6	30.9	66
1.371	187.6	41.1	146.5	1297.	15.91	10.88	0.340	0.9	60.6	69.3	30.9	71
1.471	183.9	48.5	135.4	1347.	15.66	11.88	0.340	1.0	62.3	72.0	30.9	76
1.571	179.9	56.5	123.4	1397.	15.41	12.82	0.340	1.2	64.1	74.7	30.9	81
1.671	175.5	64.9	110.7	1447.	15.16	13.71	0.340	1.3	66.1	77.3	30.9	86
1.771	170.9	73.8	97.1	1497.	14.91	14.53	0.340	1.5	68.1	80.0	30.9	91
1.871	166.0	83.2	82.8	1547.	14.66	15.26	0.340	1.7	70.1	82.7	31.0	96
1.971	160.9	93.1	67.8	1597.	14.41	15.90	0.340	1.9	72.3	85.4	31.0	101
2.071	155.6	103.5	52.1	1647.	14.16	16.42	0.340	2.1	74.5	88.0	31.0	106
2.171	150.1	114.4	35.7	1697.	13.91	16.81	0.341	2.4	76.8	90.8	31.0	111
2.271	144.2	125.9	18.2	1747.	13.66	17.06	0.347	2.6	79.3	95.4	31.1	120
2.371	138.3	137.9	0.4	1802.	13.49	17.14	0.444	2.9	81.6	125.8	31.1	176
2.471	144.3	144.3	0.0	1876.	14.08	17.15	0.450	3.0	78.3	132.6	31.1	189

CONDITIONS AT TIME OF ENGAGEMENT

2.471	144.3	144.3	0.0	1876.	14.08	17.15	0.450	3.0	78.3	132.6	31.1	189

TEMPERATURE DISTRIBUTION DURING ENGAGEMENT

TIME	FRICTION MATERIAL				METAL MEMBER		
1	6	11		12	15	18	
0.001	0.0	0.0	0.0	****	0.0	0.0	0.0
0.101	0.0	0.0	1.9	****	1.7	0.0	0.0
0.201	0.0	0.0	5.8	****	5.2	0.1	0.0
0.301	0.0	0.1	10.8	****	9.8	0.6	0.0
0.401	0.0	0.2	16.9	****	15.2	1.3	0.1
0.501	0.0	0.5	23.7	****	21.5	2.5	0.3
0.601	0.0	1.1	31.0	****	28.2	4.1	0.7
0.701	0.1	1.8	37.7	****	34.2	6.1	1.4
0.801	0.1	2.8	43.8	****	39.8	8.5	2.3
0.901	0.2	4.0	49.2	****	44.7	11.0	3.5
1.001	0.4	5.4	54.0	****	49.2	13.7	4.9
1.101	0.6	6.9	58.4	****	53.3	16.4	6.7
1.201	1.0	8.5	62.3	****	56.9	19.2	8.6
1.301	1.4	10.2	65.6	****	60.1	22.0	10.7
1.401	1.9	12.0	68.4	****	62.9	24.7	13.0
1.501	2.6	13.8	70.6	****	65.1	27.5	15.4
1.601	3.3	15.6	72.1	****	66.7	30.1	17.8
1.701	4.1	17.4	73.0	****	67.8	32.6	20.4
1.801	5.0	19.1	73.1	****	68.3	35.0	22.9
1.901	6.0	20.8	72.4	****	68.0	37.2	25.4
2.001	7.0	22.4	70.9	****	67.1	39.2	27.8
2.101	8.2	23.9	68.5	****	65.4	41.0	30.2
2.201	9.3	25.3	65.2	****	62.9	42.5	32.5
2.301	10.5	26.5	60.9	****	59.5	43.6	34.6
2.401	11.7	27.5	55.6	****	55.2	44.4	36.5

THE ASSEMBLY IS NOW COOLING

TIME		TEMPERATURES						
MINS	SECS	1	6	11		12	15	18
0	30	38.7	38.8	39.6	****	43.7	43.7	43.8
1	0	41.9	41.9	42.1	****	43.2	43.3	43.3