Recent progress in the theory of localised elastic waves in immersed wedge-like structures

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Recent progress in the theory of localised elastic waves in immersed wedge-like structures

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Abstract

Recent analytical results in the theory of antisymmetric localised elastic modes propagating along edges of immersed wedge-like structures of arbitrary shape and curvature are briefly described with the emphasis on methodological aspects of using geometrical-acoustics approach for developing the theory. The velocities of localised modes are calculated for wedges of linear geometry, wedges with a cross section described by a power law, and curved cylindrical wedges. It is shown that deviations of a wedge shape and curvature from linear geometry result in frequency dispersion of wedge modes. The comparison is given with the experimental investigations and numerical calculations of wedge waves carried out by independent researchers.

1. Introduction

Antisymmetric localised elastic waves propagating along tips of solid wedges in contact with vacuum have been predicted in 1972 using numerical calculations (Lagasse [1] and Maradudin et al. [2]). It has been shown in [1,2] that such waves, now often called wedge acoustic waves, are characterised by low propagation velocity (generally much lower than that of Rayleigh waves), and their elastic energy is concentrated in the area of about one wavelength near the wedge tips. Since 1972, wedge acoustic waves have been investigated in a number of papers with regard to their possible applications to signal processing and to non-destructive testing of special engineering constructions (see, e.g., [3-8] and references there).

The first experimental work studying wedge wave interaction with liquids has been published by Chamuel [9] in 1993. In the year 1994, the existence of localised flexural elastic waves on the edges of wedge-like immersed structures has been described theoretically by the present author [10]. These advances were followed by the experimental investigations of wedge waves in immersed structures which considered samples made of different materials and having different values of wedge apex angle [11,12]. Recently, finite element calculations have been carried out [13] for Plexiglas and brass wedges with the values of apex angle in the range from 20 to 90 degrees and an analytical theory based on geometrical-acoustics approach has been developed for the same range of wedge apex angle [14]. In the paper [15] dealing with finite element calculations of the velocities and amplitudes of wedge waves, among other results, calculations have been carried out of the velocities of waves propagating along the edge of a cylindrical wedge-like structure bounded by a circular cylinder and a conical cavity.

The above mentioned theoretical calculations of wedge elastic waves in immersed solid structures and their experimental investigations are important for the explanation of many as yet poorly understood phenomena in related fields of structural dynamics, physics and environmental acoustics and may result in many useful practical applications. In particular, it is expected that these waves play an important role in the dynamics of wedge-shaped civil engineering off-shore structures (such as piers, dams, wave-breakers, etc.), and in the formation of vibration patterns and resonance frequencies of propellers, turbine blades, disks, cutting tools and airfoils. They may be responsible for specific mechanisms of helicopter noise, wind turbine noise and ship propeller noise. Promising mechanical engineering applications of wedge elastic waves include non-destructive evaluation of specific engineering constructions (with edges), measurements of cutting edge sharpness, environmentally friendly water pumps and domestic ventilators utilising wave-generated flows. Another possible application earlier suggested by the present author [10] may be the use of wedge waves for in-water propulsion of ships and submarines, the main principle of which being similar to that used in nature by fish of the ray family.

In the present paper we give a brief overview of the recent results in the theory of antisymmetric localised elastic modes propagating along edges of immersed wedge-like structures of arbitrary shape and curvature. All these results have been obtained using the geometrical-acoustics approach. The particular cases to be considered are wedges of linear geometry, wedges with a cross section described by a power law, and curved cylindrical wedges.

2. Geometrical-acoustics approach

The approximate analytical theory of localised elastic waves in immersed solid wedges is based on the geometrical acoustics approach considering a slender wedge as a plate with a local variable thickness $d(x)$, where x is the distance from the wedge tip measured in the middle plane. In the case of "linear" wedge $d(x) = x\Theta$, where Θ is the wedge apex angle.

Applying the well known procedure for geometrical acoustics calculations of waveguide modes, one can derive the following Bohr - Sommerfeld type equation for the velocities c of the localised wedge modes propagating in y-direction (along the tip) [6-8]

$$\int_0^{x_t} \left[k^2(x) - \beta^2\right]^{1/2} dx = \pi n . \qquad (2.1)$$

Here $k(x)$ is a local wavenumber of a quasi-plane plate flexural wave (as a function of the distance x from the wedge tip), $\beta = \omega/c$ is yet unknown wavenumber of a localised wedge mode, $n = 1,2\ 3, ...$ is the mode number, and x_t is the so called ray turning point being determined from the equation $k^2(x) - \beta^2 = 0$.

In the case of linear wedge in vacuum $k(x) = 12^{1/4}\, k_p^{1/2}(\theta x)^{-1/2}$, and $x_t = 2\sqrt{3}\, k_p/\Theta\beta^2$, where $k_p = \omega/c_p$, so that the solution of Eq. (2.1) yields the extremely simple analytical expression for wedge wave velocities [6-8]:

$$c = c_p n \Theta/\sqrt{3}. \qquad (2.2)$$

The expression (2.2) agrees well with the other theoretical calculations [1-5] and with the experimental results [3]. Note that, although the geometrical acoustics approach is not valid for the lowest order wedge mode ($n = 1$) [7], in practice it provides quite accurate results for wedge wave velocities in this case as well.

3. Effect of liquid loading

To calculate the velocities of wedge modes in a wedge embedded in liquid one has to make use of the expression for a plate wave local wavenumber $k(x)$ which takes into account the effect of liquid loading [10,14]. The starting point to derive $k(x)$ for this case is the well known dispersion equation for the lowest order flexural mode in an immersed plate. For shortness, we restrict ourselves in this paper only with the case $\rho_f/\rho_s \approx 1$ typical for light solid materials in water and limit our analysis by a subsonic regime of wave propagation ($k > \omega/c_f$), where ρ_s and ρ_f are respectively the mass densities of solid and liquid, and c_f is the velocity of sound in liquid. For the sake of simplicity, we impose even a more severe restriction on wave velocities considering very slow propagating plate flexural modes ($k >> \omega/c_f$) and using the approximation of incompressible liquid. Then, for $kd << 1$ typical for thin plates, the wavenumber $k(x)$ in the case of linear wedge $d = d(x) = x\Theta$ has the form

$$k(x) = \left[\sqrt{6}\,\frac{c_l}{c_t}\,\frac{1}{\sqrt{c_l^2 - c_t^2}}\,\frac{\sqrt{\rho_f}}{\sqrt{\rho_s}}\,\frac{\omega}{(x\theta)^{3/2}}\right]^{2/5}, \qquad (3.1)$$

where c_l and c_t are longitudinal and shear velocities in wedge material. Substituting (3.1) into (2.1) and performing some simple transformations, one can derive the following analytical expression for wedge wave velocities c [14]:

$$c = c_t A^{-5/2} D^{-3/2} (\pi n)^{3/2} \Theta^{3/2}, \qquad (3.2)$$

where $A = 6^{1/5}(\rho_f/\rho_s)^{1/5}(1- c_t^2/c_l^2)^{-1/5} = 6^{1/5}(\rho_f/\rho_s)^{1/5}\ [2(1-\sigma)]^{1/5}$ is a nondimensional parameter which depends on the relation between the mass densities ρ_f/ρ_s and on the Poisson ratio σ, and $D = \int_0^1 (x^{-6/5} - 1)^{1/2} dx = 2.102$. Comparison of Eqs. (3.2) and (2.2) shows that the effect of liquid loading results in significant decrease of wedge wave velocities in comparison with their values in vacuum. It has been demonstrated [14] that Eq. (3.2) provides a very good agreement with the corresponding experimental data of Chamuel [11].

Although the above described geometrical acoustics theory has been developed for slender elastic wedges, it can be used successfully for description of the effect of liquid loading also for wedges with large values of wedge apex angle. In this case one should apply it for calculating relative values of wedge wave velocity, in comparison with those for wedges in vacuum. The results of such calculations [14] show a remarkably good agreement with the experimental data of de Billy [12] presented as the ratio c_{wat}/c_{vac} between the velocities of the first order localised modes in immersed Plexiglas wedges and in the same wedges in vacuum.

4. Waves in wedges of non-linear shape

Here we briefly describe the generalisation of the theory by introducing a power law relationship between the local thickness d and the distance from the tip x: $d = \varepsilon x^m$, where m is any positive rational number. Then, for immersed wedges, the Eq. (3.1) corresponding to a subsonic regime of wave propagation should be replaced by the expression

$$k(x) = \left[\sqrt{6} \frac{c_l}{c_t} \frac{1}{\sqrt{c_l^2 - c_t^2}} \frac{\sqrt{\rho_f}}{\sqrt{\rho_s}} \frac{\omega}{(\varepsilon x^m)^{3/2}} \right]^{2/5} . \qquad (4.1)$$

Substituting Eq. (4.1) into (2.1) and performing simple manipulations, one can easily obtain the general relationship for wedge wave velocities of localised elastic modes propagating in non-linear immersed wedges:

$$c = \frac{\omega^{1+2/(3m-5)} B^{2/(3m-5)} G_m^{\;3m/(3m-5)}}{(2\pi m \varepsilon^{1/m})^{3m/(3m-5)}} . \qquad (4.2)$$

Here $B = \sqrt{6} \frac{c_l}{c_t} \frac{1}{\sqrt{c_l^2 - c_t^2}} \frac{\sqrt{\rho_f}}{\sqrt{\rho_s}}$ and $G_m = (5/3m) \int_{-\pi/2}^{\pi/2} \tan\varphi \sin\varphi \cos^{(5-3m)/3m} \varphi d\varphi$.

It is clearly seen from (4.2) that deviation of a wedge shape from linear geometry ($m = 1$ and $\varepsilon = \Theta$) results in dispersion of wedge wave velocities. For linear wedges, as expected, the velocity c is independent of ω and reduces to the earlier derived expression (3.2). The case of $m = 5/3$ requires a more careful investigation since both the nominator and the denominator in Eq. (4.2) have singularities for this value of m. It is interesting to notice that the effect of liquid loading eliminates anomalous behaviour of wedge wave velocities for quadratic wedges ($m = 2$) in contact with vacuum. As a separate analysis shows [16], in the latter case the velocities of all wedge modes are equal to zero, unless there is a truncation on the wedge tip.

5. Waves in cylindrical wedge-like structures

In this section we demonstrate that wedge wave propagation in cylindrical and cognate wedge-like structures can be considered in a rather simple way using the earlier developed approximate analytical theory of localised elastic modes in a wedge curved in its plane [7].

To calculate the velocities of wedge waves in a curved wedge one has to consider two possible types of curved wedges: wedges curved in their own plane (in-plane curvature) and wedges curved perpendicular to their own plane (anti-plane curvature). In both cases we assume that the radius of curvature is large enough in comparison with characteristic wavelengths.

Let us first consider the case of in-plane curvature and assume for certainty that the radius of curvature is positive (a convex edge) and has a value r_0. Then, using the description of basic geometrical acoustics relations in cylindrical co-ordinates [17] in which the edge of a curved wedge is described by the equation $r = r_0$, one can rewrite the governing equation (2.1) as

$$-\int_{r_0}^{r_t}\left[k^2(r-r_0)-\beta^2(r_0^2/r^2)\right]^{1/2}dr=\pi n, \tag{5.1}$$

where r_t is the co-ordinate of a ray turning point. Considering wedges in vacuum and assuming that the radius of curvature r_0 is large enough ($r_0 >> |r_0 - r_t|$), one can derive the following approximate expression for phase velocities of wedge waves propagating along a convex curved edge:

$$c=\frac{c_p\Theta n}{\sqrt{3}}\left(1+\frac{\sqrt{3}}{2}\frac{n^2\Theta}{k_p r_0}\right). \tag{5.2}$$

According to Eq.(5.2), wedge waves in curved wedges are dispersive. For an edge with a negative curvature (a concave edge), Eq. (5.2) remains valid if we replace r_0 by $-r_0$.

Let us now consider the case of wedge with anti-plane curvature of radius r_{0A}.. Obviously, because of the symmetry considerations, wedge wave velocities in such a wedge should not depend on the sign of anti-plane curvature, i.e., they should not change after replacing r_{0A} by $-r_{0A}$.. Therefore, one can conclude that, in the first approximation versus r_{0A}^{-1}, anti-plane curvature does not bring changes in wedge wave velocities.

Keeping all these in mind, we can now easily calculate the velocities of wedge modes propagating along a cylindrical wedge formed by the intersection of a cylinder of radius R with a conical cavity characterised by the rotation angle θ [15]. The resulting structure represents a wedge with the apex angle θ having both in-plane and anti-plane curvatures. According to the discussion above, the only geometrical parameter we need is radius of in-plane curvature r_0.. It is easy to show that for the geometry considered $r_0 = R/\sin(\theta/2)$. Using this value of r_0 in Eq. (5.2) for the computation of velocities of two lowest-order wedge modes gives an excellent agreement with the existing finite element calculations [15].

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