Simplified analytical models for prediction of vehicle interior noise

Simplified analytical models for prediction of vehicle interior noise

Victor V. Krylov

Department of Aeronautical and Automotive Engineering, Loughborough University,
Loughborough, Leicestershire LE11 3TU, UK

Abstract

The present paper describes some preliminary results in the developing of simplified analytical models that could be used as effective engineering tools for prediction and mitigation of vehicle interior noise. The structural simplification is based on understanding the physics of both generation of predominant modes of structural vibrations at particular frequencies and radiation of sound by vibrating surfaces into the vehicle interior. It is expected that the proposed approach will lead to the development of effective analytical tools that could be suitable for use on a design stage.

1. Introduction

The need for developing simplified models for prediction of vehicle interior noise arises from the fact that in the low and medium frequency ranges (10-600 Hz) the main sources of vehicle interior noise are structure-borne. This means that noise is generated mainly by vibrations of vehicle structural components excited either by internal sources (engine and other rotating components) or by external sources (road surface irregularities and pressure fluctuations in the surrounding air boundary layer). To predict the noise level in the vehicle interior one should be able to calculate the amplitudes and the very complex modal shapes of structural vibrations at all internal surfaces, including frame, doors and windows, dashboard, etc. After that, their excitation by internal or external forces should be considered, as well as their coupling with acoustic modes in a vehicle compartment.

There are different approaches to solving this problem. The most commonly used approach is based on the applying finite element analysis to the calculation of structural and acoustic modes (see. e.g. [1]). The main disadvantage of this approach is that it requires a lot of computation efforts and is reliable in predicting structural modes only at very low frequencies. To improve the situation at higher frequencies a combination technique is being used that applies experimental measurements of structural vibrations for subsequent calculation of the acoustic response using finite element technique [2,3]. Some other approaches (see, e.g. [4]) attempt to evaluate contributions of particular vehicle panels as noise sources and apply different experimental and modelling techniques, including equivalents of the transfer path analysis.

Neither of the existing techniques can be considered satisfactory, and further research is needed to improve understanding the problem and to develop reliable methods that could be used in practice, especially on a vehicle design stage. It is always desirable to know which parts of the vehicle structure contribute the most into the compartment interior noise at particular frequencies. And it is those parts of the structure that should be mitigated in the first place - to make the overall reduction of the noise most noticeable.

The proposed in this paper analytical approach to prediction of vehicle interior noise is based on a maximum possible simplification of a vehicle structure and of the acoustic interior. The structural simplification is based on understanding the physics of the problems of both generation of predominant modes of structural vibrations by particular sources at particular frequencies and radiation of sound by the excited structural vibrations into the vehicle interior. The results are expressed in terms of relatively simple analytical formulas that give the value of the internal sound pressure as a function of road irregularity, vehicle speed, properties of suspensions, resonance frequencies and modal shapes of structural and acoustic modes and their coupling to each other. Analytical results are illustrated by example calculations of resonance frequencies of acoustic and structural modes and of their coupling coefficients. It is shown that relatively few acoustic and structural modes are coupled effectively, so that only well-coupled (influential) structural modes need to be mitigated.

The present paper aims to illustrate the principle of the proposed analytical approach, rather than to obtain any precise results. To make the approach suitable for practical use, further developments will be needed to model a real vehicle structure as fully as possible. It is expected that on this way the proposed approach will lead to the development of effective analytical tools for identification of main contributors to vehicle interior noise and will assist in its more reliable prediction and mitigation.

2. General Approach to the Problem

It is convenient to describe the generated acoustic field using the potential φ related to air particle velocity $\boldsymbol{v}$ and acoustic pressure p' as $\boldsymbol{v} = grad\varphi$ and $p' = -\rho_0\ \partial\varphi/\partial t$. Any of these variables should satisfy the corresponding acoustic wave equation inside the vehicle interior V and the boundary conditions on the internal walls S (Figure 1).

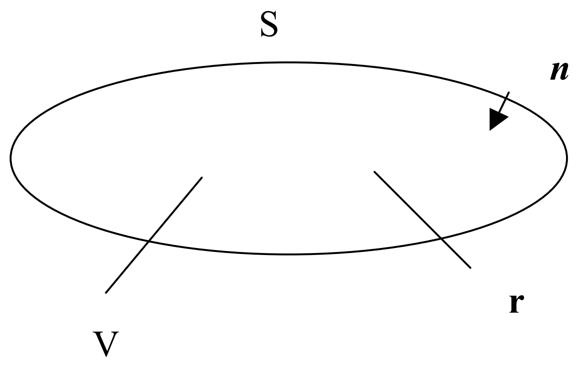

Figure 1. On the description of the acoustic fields in an enclosed volume

As it is well known (see, e.g. [5]), a time-harmonic acoustic field $\varphi(\mathbf{r})$ (the factor $exp(-i\omega t)$ is assumed) inside any closed volume V surrounded by the surface area S can be represented using the mathematical formulation of Huygens' principle (also called Helmholtz theorem):

$$\int_S \left(G\frac{\partial\varphi}{\partial \mathbf{n}} - \varphi\frac{\partial G}{\partial \mathbf{n}} \right) dS + \int_V G f dV = \begin{cases} \varphi(r), & \vec{r} \in V \\ 0, & \vec{r} \notin V \end{cases} . \tag{1}$$

Here $G(\mathbf{r},\mathbf{r}')$ is the acoustic Green's function satisfying the inhomogeneous Helmholtz equation

$$\Delta\varphi + k^2\varphi = \delta(\mathbf{r} - \mathbf{r}') \tag{2}$$

and certain radiation or boundary conditions, where $k = \omega/c$ is the acoustic wavenumber, c is sound velocity, $\delta(\mathbf{r} - \mathbf{r}') = \delta(x-x')\delta(y-y')\delta(z-z')$ is a 3-d Dirac's delta-function, $\boldsymbol{n}$ is a unit vector of inward normal to the surface, and $f(\mathbf{r}')$ is a distribution of internal sources.

If to choose the Green's function in such a way that it satisfy the Neumann boundary condition on the surface S, i.e. $\partial G/\partial\boldsymbol{n} = 0$, and assume that there are no internal sources within V, then it follows from (1) that

$$\varphi(r)=\int_S G(\mathbf{r},\mathbf{r}')v_n(\mathbf{r}')d\mathbf{r}' . \qquad (3)$$

Here we have taken into account that, according to the boundary conditions imposed upon the acoustic field on S, the normal components of acoustic particle velocity should be equal to the normal components of structural movement on the boundary: $\partial\varphi/\partial n = v_n$.

3. Acoustic Modes of Vehicle Interior

We recall that the Green's function for the enclosed volume V can be expressed as the sum of acoustic modes characterised by their resonance frequencies ω_m, attenuation δ_m (so that $k_m = (\omega_m + i\delta_m)/c$) and modal shapes $\Phi_m(\mathbf{r})$ (see, e.g. [6]):

$$G(\mathbf{r},\mathbf{r}')=\sum_{m=0}^{\infty} a_m \frac{c^2}{V}\frac{\Phi_m(\mathbf{r})\Phi_m(\mathbf{r}')}{(\omega_m{}^2-\omega^2-2i\delta_m\omega)} . \qquad (4)$$

Here a_m are constants depending on shape of the enclosure and mode type, and the summation over m means summation over the total number of acoustic modes (in practice this often means triple summation).

Substitution of (4) into (3) results in the expression

$$\varphi(\mathbf{r})=\sum_{m=0}^{\infty} a_m \frac{c^2}{V}\int_S \frac{\Phi_m(\mathbf{r})\Phi_m(\mathbf{r}')}{(\omega_m{}^2-\omega^2-2i\delta_m\omega)} v_n(\mathbf{r}')d\mathbf{r}' \qquad (5)$$

which describes the relationship between generated acoustic modes of the enclosure and the total distribution of normal vibration velocities over the boundary S at a given frequency ω. Obviously, only those acoustic modes will be generated which resonance frequencies ω_m are relatively close to ω.

Finding the Green's function (4) for each particular form of a closed volume and boundary conditions is a rather cumbersome task that can be solved analytically for only limited geometrical configurations. The most simple is a rectangular domain characterised by the length L_x, width L_y, and height L_z, so that $V = L_xL_yL_z$. In this case the well known expressions for the resonance frequencies $\omega_m/2\pi = f_m = f_{ijk}$ and for the mode shapes $\Phi_m(r, r') = \Phi_{ijk}(r, r')$ have the following forms respectively:

$$\mathrm{f}_{\mathrm{ijk}} = \frac{c}{2}\left[\left(\frac{i}{L_x}\right)^2+\left(\frac{j}{L_y}\right)^2+\left(\frac{k}{L_z}\right)^2\right]^{1/2} , \qquad (6)$$

$$\Phi_{ijk}(r,r') = \cos\left(\frac{i\pi x}{L_x}\right)\cos\left(\frac{j\pi y}{L_y}\right)\cos\left(\frac{k\pi z}{L_z}\right), \qquad (7)$$

where $i, j, k = 0, 1, 2, \ldots$ Coefficients a_m are 2 – for axial modes (two of the indexes i,j,k are zero), 4 – for tangential modes (one of the indexes i,j,k is zero), and 8 – for oblique modes (all indexes i,j,k are different from zero).

So far we considered the boundary particle velocities v_n as given values. However, they in turn represent the reaction of the vehicle structure on the internal or external forces that will be discussed in the following sections.

4. Dynamic Forces Applied from Road to a Vehicle

To calculate analytically the response of a vehicle structure on external or internal forces one should specify the forces and consider a vehicle model structure of certain degree of complexity. The first very obvious simplification is that we neglect the effect of air loading on structural response. Secondly, we simplify the actual vehicle structure by the simplest forms, so that analytical description was at all possible.

The problem of describing vibration modes of vehicle structures is extremely difficult, and there can be different approaches to model them analytically using combinations of plates, beams, shells, added masses, etc. To illustrate the principle, we consider in this paper one of the simplest possible models of vehicle structure. Namely, we assume that the vehicle is made up of a single curved thin plate that is simply supported in front of the vehicle compartment and along the side walls (see Figure 2). We also assume that the smallest radius of the plate curvature is large in comparison with the flexural wavelengths of interest, so that the shell-type behaviour due to the curvature shall not be accounted for. The two side walls are considered as absolutely rigid vertical plates (not shown on Figure 2).

Considering the effects of external forces, we will assume that the above structure is affected by four vertical forces acting on its bottom and modelling the impact of vehicle suspensions reacting on road irregularities (only two such forces, $\boldsymbol{F_f}$ and $\boldsymbol{F_r}$, associated with any of the front and rear suspensions are shown on Figure 2).

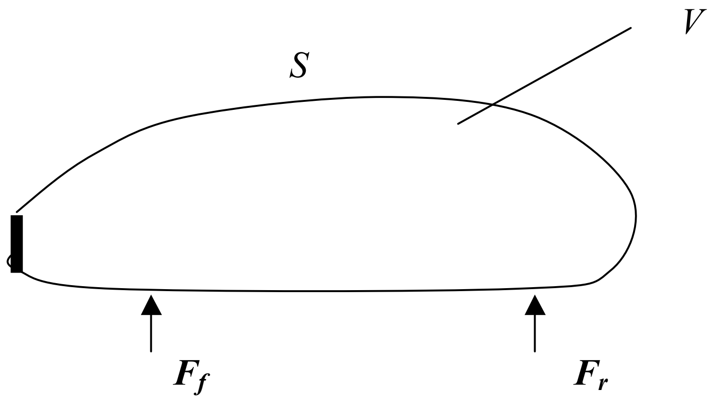

Figure 2. Simple model of a vehicle structure as a curved finite plate

The amplitudes of the forces F_f and F_r can be easily calculated using a quarter-vehicle model (Figure 3) taking into account only axle hope resonances, i.e. considering the main body of a vehicle as immobile in a vertical direction [7]. Let us also assume that the road surface irregularities (roughness) are characterised by the function $z_1 = g(x)$. Normally this function is considered as a random one. Here, however, for simplicity we consider it as a periodic corrugation of height h and space periodicity d: $g(x) = h\cos(2\pi x/d)$. Then, if a vehicle moves at speed v, the tyre contact displacement z_1 can be described as periodic function of time t: $z_1(t) = h\cos(\omega t)$, where $\omega = 2\pi v/d$.

The differential equation describing vertical displacement z_2 of each wheel versus its static position has the form

$$m\frac{\partial^2 z_2}{\partial t^2} + Q\frac{\partial z_2}{\partial t} + Kz_2 = K_1 z_1(t), \qquad (8)$$

where $K = K_1 + K_2$ is a combined stiffness of a tyre and a suspension, m is a mass of a wheel, and Q is a total damping coefficient. Then, the force exerted on the vehicle from a suspension, e.g. from one of the front suspensions, can be written in the form

$$F_f(t) = K_2 z_2(t) \qquad (9)$$

The solution of equation (8) in the Fourier domain has the form

$$z_2(\omega) = \frac{\omega_1^{\,2}}{\omega_0^{\,2} - \omega^2 - 2i\alpha\omega} z_1(\omega) = T(\omega) z_1(\omega), \qquad (10)$$

where $\omega_0 = (K/m)^{1/2}$ is the wheel hope resonance frequency, $\omega_1 = (K_1/m)^{1/2}$ is the tyre 'jumping' resonance frequency, $\alpha = Q/2m$ is a normalised damping coefficient, and $z_1(\omega)$ is the Fourier component at frequency ω corresponding to the road corrugation profile. We recall that in the considered case of periodic corrugation $z_1(t) = h\cos(\omega t)$, where $\omega = 2\pi v/d$. Hence, $z_1(\omega) = h$.

For the rear axle, the vertical displacement of a tyre contact area can be described by the retarded function $z_1(t-L/v)$, where L is a wheel base. Therefore, the displacement $z_2(\omega)$ in this case will be described by the same formula (10) in which $z_1(\omega)$ should be replaced by $z_1(\omega)\exp(i\omega L/v)$. Typical values of wheel hope resonance frequencies $f_0 = \omega_0/2\pi$ are 8 – 12 Hz, whereas for typical vehicle speeds of 100 km/h and surface corrugations periodicity d of 20 cm the input frequency $f = \omega/2\pi = v/d$ is around 140 Hz. This means that the forces resulting from the road surface corrugation are highly attenuated by suspensions. Nevertheless, it is these forces that give the largest contribution into interior noise of moving vehicles.

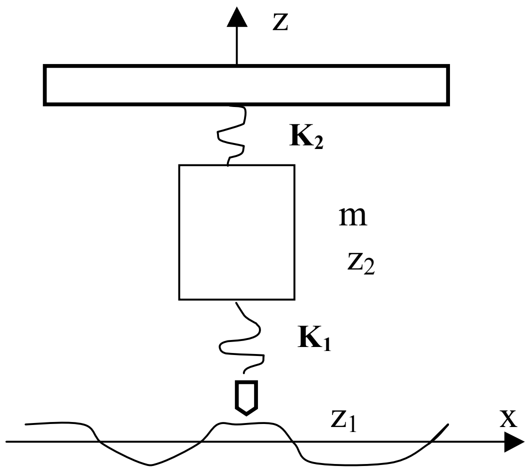

Figure 3. On the calculation of vehicle suspension forces

5. Calculation of Vehicle Structural Response

Let us now consider the response of a simplified structural model of a vehicle on the suspension forces described in the previous section.

Referring to the simple model of a vehicle structure made up of a single curved plate of width L_y and total length L_l which is simply supported in the front of the compartment and along the side walls (see Figure 2), we can use the corresponding expression for its Green's function $G(\rho,\rho')$ that should satisfy the equation of plate flexural vibrations

$$D\left(\frac{\partial^4 w}{\partial l^4}+2\frac{\partial^4 w}{\partial l^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)-\rho_{0s}h_s\omega^2 w=\delta(\rho-\rho') \tag{11}$$

and the boundary conditions for simply supported edges. Here w is flexural displacement of a plate surface, $D = Eh_s^3/12(1-\sigma^2)$ is the plate's bending stiffness, E, ρ_{0s} and h_s are its Young's modulus, mass density and thickness respectively, $\delta(\rho-\rho') = \delta(l-l')\delta(y-y')$ is a 2-D Dirac's delta-function.

The modal shapes of such plate and the corresponding resonance frequencies $\omega_p/2\pi = f_p = f_{st}$ can be written as follows (see, e.g. [5]):

$$\Psi_{st}(l,y)=\sin\left(\frac{s\pi l}{L_l}\right)\sin\left(\frac{t\pi y}{L_y}\right), \tag{12}$$

$$f_{st}=\frac{\pi}{2}\left(\frac{D}{\rho_{0s}h_s}\right)^{1/2}\left[\left(\frac{s}{L_l}\right)^2+\left(\frac{t}{L_y}\right)^2\right], \tag{13}$$

where, s,t = 1,2,3… Consequently, for the Green's function of such a plate, $G_s(\rho,\rho')$, one can have

$$G_s(\rho,\rho')=\frac{4}{M}\sum_{p=1}^{\infty}\frac{\Psi_p(\rho)\Psi_p(\rho')}{(\omega_p{}^2-\omega^2-2i\delta_p\omega)}, \tag{14}$$

where $M = \rho_{0s}h_sL_lL_y$ is the total mass of the plate.

Since we need to describe normal particle velocities of the structure v_n, rather than flexural displacements w, we should use the Green's function for velocity (often called mobility function) which is obtained by multiplication of $G_s(\rho,\rho')$ by $-i\omega$.

For arbitrary normal stresses $F(\rho')$ applied to the structure the resulting distribution of normal particle velocity v_n can be expressed in the form

$$v_n(\rho) = -i\omega \int_S G_s(\rho, \rho') F(\rho') d\rho' \ . \qquad (15)$$

In particular, in the case of four vertical concentrated forces applied from the front (left and right) and rear (left and right) suspensions:

$$F(\rho') = F_{fl}\delta(l'-l_{fl})\delta(y'-y_{fl}) + F_{fr}\delta(l'-l_{fr})\delta(y'-y_{fr}) + F_{rl}\delta(l'-l_{rl})\delta(y'-y_{rl}) + F_{rr}\delta(l'-l_{rr})\delta(y'-y_{rr}). \qquad (16)$$

For simplicity, let us consider initially the effect of the front left suspension only. Then, using formula (9), we can rewrite $F(\rho')$ in the form

$$F(\rho') = K_2 z_2(\omega)\delta(\rho'-\rho_{fl}). \qquad (17)$$

Substituting (17) into (15) and using the properties of delta-function, we obtain

$$v_n(\rho) = -i\omega K_2 z_2(\omega) G_s(\rho, \rho_{fl}) \ . \qquad (18)$$

6. Calculation of Acoustic Pressure in the Vehicle Interior

Substituting (18) and (14) into (5) and using (9) and the relationship $p' = -\rho_0\ \partial\varphi/\partial t. = \ = i\omega\rho_0\varphi$, one can obtain the final formula for the acoustic pressure generated within the vehicle compartment:

$$p'(\mathbf{r}) = \frac{4c^2 \rho_0 K_2 T(\omega) z_1(\omega)}{\omega^2 \rho_{0s} h_s V} \times \sum_{m=0}^{\infty}\sum_{p=1}^{\infty} a_m F_{mp}(\omega) S_{mp} \Phi_m(\mathbf{r}) \Psi_p(\rho_{fl}) \qquad (19)$$

where

$$F_{mp}(\omega) = \frac{\omega^4}{(\omega_m{}^2 - \omega^2 - 2i\delta_m\omega)(\omega_p{}^2 - \omega^2 - 2i\delta_p\omega)} \qquad (20)$$

and

$$S_{mp} = \frac{1}{L_l L_y} \int_S \Phi_m(\mathbf{r}') \Psi_p(\rho') d\mathbf{r}' \ . \qquad (21)$$

The function $F_{mp}(\omega)$ defined by the non-dimensional expression (20) can be called the frequency overlap function of the acoustical and structural modes characterised by the overall indexes m and p.

Similarly, the non-dimensional factor S_{mp} defined by the expression (21) can be considered as the corresponding coefficient of spatial coupling between acoustical and structural modes. It is the product $F_{mp}(\omega)S_{mp}$ that determines the amplitudes of the resulting acoustic pressure inside the vehicle compartment.

The expression (19) describes the acoustic pressure generated by the road irregularity $z_1(\omega)$ via the force of the amplitude $K_2T(\omega)z_1(\omega)$ exerted from the front left vehicle suspension. To add contributions of the remaining three suspensions, one should take summation of three more expressions (19) in which the term $\Psi_p(\rho_{fl})$ should be replaced by $\Psi_p(\rho_{fr})$, $\Psi_p(\rho_{rl})$ and $\Psi_p(\rho_{rr})$ respectively. In real situations, all these forces should be described statistically, using stochastic functions $z_1(\omega)$ associated with road surface roughness. However, we will not discuss these aspects in this paper.

One can see from the expression (19) that the resulting acoustic pressure is formed as a summation over products of all structural and acoustic modes. However, because of the double filtration – over time and space described by the products $F_{mp}(\omega)S_{mp}$ – only relatively few of the structural and acoustic modes interact effectively and give noticeable contributions.

First of all, it is clear that, because of the time filtration, only those acoustic and structural modes should be taken into account which resonance frequencies, ω_m and ω_p respectively, are close enough to each other, so that the functions $(\omega_m^2 - \omega^2 - 2i\delta_m\omega)^{-1}$ and $(\omega_p^2 - \omega^2 - 2i\delta_p\omega)^{-1}$ in (20) overlap effectively.

In addition to this, due to the spatial filtration described by the mode coupling coefficients S_{mp} in (21), only those overlapping acoustical and structural modes should be taken into account for which the values of S_{mp} are big enough.

7. Example Calculation and Discussion

For the purpose of numerical illustration, we consider a medium-size car. For calculation of acoustic modes we approximate its compartment as a parallelepiped characterised by the length L_x =

$2.2m$, width $L_y = 1.2m$ and height $L_z = 1.0m$. For calculation of structural modes we consider the same compartment as being enveloped by a smoothly curved thin plate (see Figure 2) with the width equal to L_y and with the total length $L_l = 2L_x + 2L_z$. For illustration purposes we assume that the above-mentioned curved plate is made of steel with $\rho = 7700\ kg/m^3$, $h_s = 0.015m$, $E = 1.95\ 10^{11}\ N/m^2$ and $\sigma = 0.31$. Let us also assume that $\delta_m/\omega_m = \delta_p/\omega_p = 5\%$.

The results of calculation of the resonance frequencies of all acoustic modes in the frequency range from 0 to 230 Hz, starting from the lowest order $(i,j,k) = (1,0,0)$, and of some structural modes, starting from $(s,t) = (10,1)$, are shown in Table 1.

Acoustical modes		Structural modes	
Mode indexes, (i, j, k)	Resonance frequencies, F_{ijk} (Hz)	Mode indexes, (s, t)	Resonance frequencies, f_{st} (Hz)
(1,0,0)	77	(10,1)	113
(0,1,0)	142	(11,1)	131
(2,0,0)	154	(12,1)	152
(1,1,0)	161	(13,1)	174
(0,0,1)	170	(10,2)	188
(1,0,1)	187	(11,2)	206
(0,1,1)	221	(12,2)	227

Table 1. Calculated resonance frequencies of acoustical and structural modes

Let us consider the two pairs of acoustic and structural modes with very close resonance frequencies: the ones characterised by the frequencies 154 Hz and 152 Hz respectively, and the ones characterised by the frequencies 170 Hz and 174 Hz respectively. Since the relative differences between these two pairs of resonance frequencies are smaller than $\delta_m/\omega_m = \delta_p/\omega_p = 5\%$, the chosen acoustical and structural modes overlap very effectively, so that the corresponding overlap functions have large maxima between the acoustical and structural resonance frequencies.

To calculate the coefficients of spatial coupling S_{mp} for these two pairs of modes, we introduce the surface co-ordinate l along the plate length and perform the integration along its surface according to (21). As a result, one can obtain the corresponding values of S_{mp}: 0.0336 for the first pair, and 0.0189 – for the second.

Substituting these values of S_{mp} into formula (19), one can calculate the resulting acoustic pressure as function of frequency or vehicle speed. Obviously, the frequency response of the described simple model associated with these two pairs of modes will have predominant maxima centred at the two frequencies: 153 Hz and 172 Hz. Contributions of other 'close' pairs of modes from Table 1 can be evaluated in a similar way. However, one can expect that they will be smaller.

To suppress the interior noise at the dominant frequencies one should suppress the corresponding structural modes. This can be done, for example, by attaching passive vibration absorbers to the structure on the locations where spatial distributions of structural vibrations according to the mode shapes have maxima. On a practical level this can be interpreted as minimising the effect of the particular panel or part of the structure that predominantly influence the interior noise. In reality, however, one should remember that it is not a part of the structure which contribution is being minimised. It is the whole structural mode that is being suppressed, with the resulting reduction of vehicle interior noise.

Another possible way of reducing vehicle interior noise that can be considered from analysing formula (19) is influencing the structural resonance frequencies so that to move them from the acoustic resonance frequencies as far away as possible. However, this way seems to be difficult and possibly counterproductive, because after moving the resonance frequencies of the chosen structural modes away from the acoustic ones one can disturb the resonance frequencies of other structural modes, so that they could approach the acoustic ones and make new effectively interacting pairs.

Although in this paper we have considered the simplest model of a vehicle, it is expected that the basic physical mechanisms of generating vehicle interior noise following from this model will remain the same for more complex and more precise vehicle structural models. Introduction of such more complicated but still manageable analytical models will lead to the development of effective analytical tools for identification of main contributors to vehicle interior noise and will assist in its more reliable prediction and mitigation.

References

[1] D.J. Nefske, J.A. Wolf Jr. and L.J. Howell, *Structural-acoustic finite element analysis of the automobile passenger compartment: a review of current practice*, Journal of Sound and Vibration, Vol. 80, Academic Press (1982), pp. 247-266.

[2] S.H. Kim, J.M.Lee and M.H.Sung, *Structural-acoustic modal coupling analysis and application to noise reduction in a vehicle passenger compartment*, Journal of Sound and Vibration, Vol. 225, No. 5, Academic Press (1999), pp. 989-999.

[3] T.C. Lim, *Automotive panel noise contribution modelling based on finite element and measured structural-acoustic spectra*, Applied Acoustics, Vol. 60, Elsevier (2000), pp. 505-519.

[4] M. Vanolo, D. Storer and H. van der Auweraer, *Associating the vibrations of car body panels to the internal acoustic response*, Proc. ISMA-25, Leuven, Belgium, Sept.13-15, 2000

[5] M.C. Junger and D. Feit, *Sound, Structures and their Interaction,* MIT Press, Cambridge MA, 1972.

[6] P. Filippi, D. Habault, J.P. Lefevre and A. Bergassoli, *Acoustics: Basic Physics, Theory and Methods*, Academic Press, 1999.

[7] V.V. Krylov, *Generation of ground elastic waves by road vehicles*, Journal of Computational Acoustics, Vol. 9, No. 3, World Scientific (2001), pp. 919-933.