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PUBLISHER

Chinese Society for Composite Materials

VERSION

AM (Accepted Manuscript)

PUBLISHER STATEMENT

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Harvey, Christopher M., Matthew R. Eplett, Zhaopeng Yang, and Simon Wang. 2017. "The Mechanics of Interface Fracture in Layered Composite Materials: (3) Experimental Assessments - Macroscopic Interface Fractures". figshare. <https://hdl.handle.net/2134/25526>.

THE MECHANICS OF INTERFACE FRACTURE IN LAYERED COMPOSITE MATERIALS: (3) EXPERIMENTAL ASSESSMENTS – MACROSCOPIC INTERFACE FRACTURES

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Keywords: Composite materials, Energy release rates, Experimental assessments, Fracture toughness, Mixed mode partition theory

ABSTRACT

Several valuable and popular mixed mode partition theories are assessed against experimental results for the prediction of delamination toughness of fiber-reinforced laminated composites. It is shown that Wang and Harvey's classical partition theory [1-6] gives the most accurate predictions; Davidson et al.'s non-singular field partition theory [7, 8] also gives accurate predictions; but the Wang and Harvey's shear deformable partition theory [1-6] and the singular-field partition theory [7-10] gives poor predictions. It is concluded that the Wang and Harvey's classical partition theory [1-6] governs macroscopic interface fractures.

1 INTRODUCTION

Partition of mixed-mode fractures is one of the fundamental research topics in the mechanics of interface fractures in layered composite materials. There has been a great deal of confusion concerning it due to the many complexities arising from the involvement of factors such as interface properties, crack extension size, material size, and analytical derivation methodologies, to name only a few. Based on a powerful orthogonal pure mode partition theory (OPPT) [1-6] Wang and Harvey and their colleagues have carried out a systematic development of partition theories based on classical and shear-deformable beam and plate/shell theories, and 2D elasticity theory for mixed-mode interface fractures including brittle, cohesive, homogeneous and bi-material interfaces under general loading conditions. Multi-scale interface fractures have been considered including delamination in macroscopic fiber-reinforced laminated composites, spallation of macroscopic/microscopic thermal barrier coatings in aero engine turbine blades, spallation of microscopic α -Al₂O₃ films grown by oxidation, telephone cord buckling of microscopic thin films driven by pocket energy concentration, and adhesion toughness of multilayer graphene membranes. Excellent predictions have been observed in comparison with experimental tests. Most of the previous confusions have been cleared. Here, some comparisons are presented for prediction of interface delamination toughness in macroscopic fiber-reinforced laminated composites.

2 SEVERAL IMPORTANT PARTITION THEORIES

Fig. 1 (a) shows a layered composite double cantilever beam (DCB) with its associated geometry, two tip bending moments, and two tip axial forces. The partition is based on the bending moments and axial forces acting at the crack tip B, which are shown in Fig. 1 (b).

2.1 The Williams partition theory

Williams was one of the first researchers to attempt to partition a mixed mode [11]. His theory was developed for either isotropic materials or unidirectional composite materials.

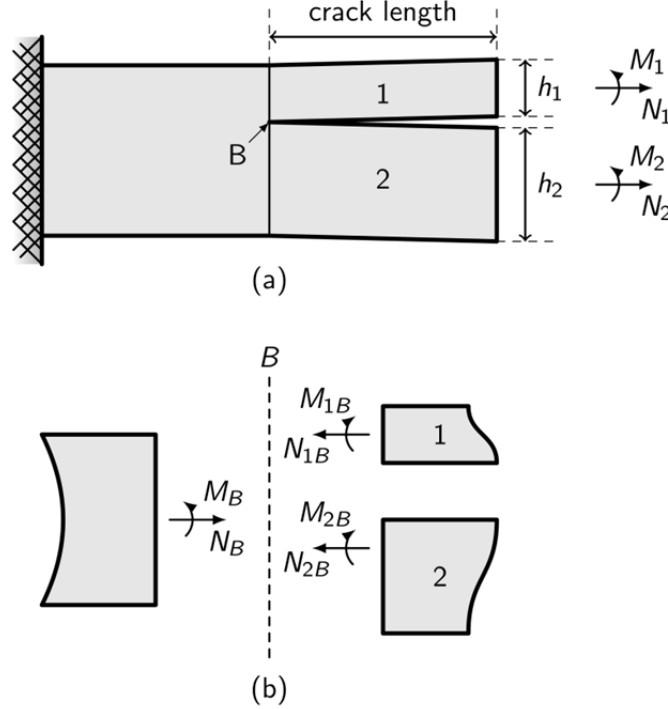


Figure 1: A layered composite double cantilever beam. (a) General description. (b) Crack tip forces.

Williams partition, denoted by G_{IW} and G_{IIV} , is now reproduced here. Again, for consistency, the notation has been changed where appropriate to match the conventions in this paper.

$$G_{IW} = \frac{6(M_{2B} - M_{1B}\gamma^3)^2}{b^2 h_1^3 E \gamma^3 (1 + \gamma^3)} \quad (1)$$

$$G_{IIV} = \frac{18\gamma(M_{1B} + M_{2B})^2}{b^2 E h_1^3 (1 + \gamma)^3 (1 - \gamma + \gamma^2)} + \frac{(1 - \gamma)^2 (N_{2B} - \gamma N_{1B})^2}{2b^2 h_1 E \gamma^3 (1 + \gamma)} \quad (2)$$

2.2 The Suo-Hutchinson partition theory

Suo and Hutchinson [9, 10] considered a crack in a semi-infinite strip of orthotropic material and derived expressions for the mixed-mode intensity factors based on 2D elasticity theory, which are analytical except for one parameter, which is determined numerically. This partition is now reproduced here. For consistency, the notation has been changed where appropriate to match the conventions used elsewhere in this paper. This partition theory assumes that a square-root singular field exists, so the partition is expressed in terms of stress intensity factors. The mode I and II stress intensity factors K_{ISH} and K_{IISH} are

$$K_{ISH} = \frac{N}{\sqrt{2h_1 U}} \cos(\omega) + \frac{M}{\sqrt{2h_1^3 V}} \sin(\omega + \varepsilon) \quad (3)$$

$$K_{IISH} = \frac{N}{\sqrt{2h_1 U}} \sin(\omega) - \frac{M}{\sqrt{2h_1^3 V}} \cos(\omega + \varepsilon) \quad (4)$$

where M and N are linear combinations of the applied loads:

$$bN = -N_1 + C_1(N_1 + N_2) - C_2 \left[M_1 + M_2 + \frac{h_1}{2}(N_2 - \gamma N_1) \right] / h_1 \quad (5)$$

$$bM = M_1 - C_3 \left(M_1 + M_2 + \frac{h_1}{2}(N_2 - \gamma N_1) \right) \quad (6)$$

$$C_1 = \frac{1}{1+\gamma} \quad \text{and} \quad C_2 = \frac{6\gamma}{(1+\gamma)^3} \quad \text{and} \quad C_3 = \frac{1}{(1+\gamma)^3} \quad (7)$$

The geometric factors U , V and ε are functions of γ :

$$U = \frac{\gamma^3}{3 + 6\gamma + 4\gamma^2 + \gamma^3}, \quad V = \frac{\gamma^3}{12(1 + \gamma^3)}, \quad \frac{\sin \varepsilon}{\sqrt{UV}} = \frac{6(1 + \gamma)}{\gamma^3} \quad (8)$$

The quantity ω is determined from the following approximate formula:

$$\omega = 52.1^\circ - 3^\circ/\gamma \quad (9)$$

For comparison with the energy release rates from the beam theories, the relationship between energy release rate and stress intensity factor for plane stress is

$$K^2 = EG \quad (10)$$

For plane strain E may simply be replaced by $E/(1-\nu^2)$. The Suo-Hutchinson partition theory is usually called singular field partition theory.

2.3 Davidson et al.'s partition theory

Davidson et al.'s partition theory [7, 8], which is based on 2D elasticity, is given by the following formula:

$$\frac{G_{II}}{G} = \frac{[N_c \sqrt{c_1} \cos \Omega + M_c \sqrt{c_2} \sin(\Omega + \Gamma)]^2}{c_1 N_c^2 + c_2 M_c^2 + 2\sqrt{c_1 c_2} N_c M_c \sin \Gamma} \quad (11)$$

where N_c and M_c are the concentrated crack tip force and moment respectively. Details of all the quantities in Eq. (11) can be found in Refs. [7, 8] and are not copied here; however, giving the details of Ω , which is called the 'mode mix parameter', is worthwhile.

$$\Omega = \begin{cases} -24 & \eta < -0.468 \\ 60.409\eta - 41.738\eta^3 & \text{if } -0.468 < \eta < 0.468 \\ 24 & \eta > 0.468 \end{cases} \quad (12)$$

Note that η in Eq. (12) is given by $\eta = \log_{10}(\gamma)$. The mode mix parameter Ω is determined with the aid of experimental data. Davidson et al.'s partition theory is usually called non-singular field partition theory.

2.4 The Wang-Harvey partition theories

For generally layered composite materials, in the Wang-Harvey Euler beam or classical partition theory [1-6], the mode I and II components of the total energy release rate, denoted by G_{IE} and G_{IIE} respectively, are

$$G_{IE} = c_{IE} \left(M_{1B} - \frac{M_{2B}}{\beta_1} - \frac{N_{1B}}{\beta_2} - \frac{N_{2B}}{\beta_3} \right) \left(M_{1B} - \frac{M_{2B}}{\beta'_1} - \frac{N_{1B}}{\beta'_2} - \frac{N_{2B}}{\beta'_3} \right) \quad (13)$$

$$G_{IIE} = c_{IIE} \left(M_{1B} - \frac{M_{2B}}{\theta_1} - \frac{N_{1B}}{\theta_2} - \frac{N_{2B}}{\theta_3} \right) \left(M_{1B} - \frac{M_{2B}}{\theta'_1} - \frac{N_{1B}}{\theta'_2} - \frac{N_{2B}}{\theta'_3} \right) \quad (14)$$

where c_{IE} and c_{IIE} are

$$c_{IE} = G_{\theta_i} \left[\left(1 - \frac{\theta_1}{\beta_1} \right) \left(1 - \frac{\theta_1}{\beta'_1} \right) \right]^{-1} \quad (15)$$

$$c_{IIE} = G_{\beta_i} \left[\left(1 - \frac{\beta_1}{\theta_1} \right) \left(1 - \frac{\beta_1}{\theta'_1} \right) \right]^{-1} \quad (16)$$

(θ_i, β_i) and (θ'_i, β'_i) represent the two sets of orthogonal pure modes where the range of i is from 1 to 3, details of which can be found in the studies [1-6]. The partitions are easily reduced for isotropic materials [1-6]. A thickness ratio $\gamma = h_2/h_1$ is now introduced. The present classical partitions for isotropic materials reduce to

$$G_{IE} = c_{IE} \left(M_{1B} - \frac{M_{2B}}{\beta_1} - \frac{N_{1Be}}{\beta_2} \right) \left(M_{1B} - \frac{M_{2B}}{\beta'_1} \right) \quad (17)$$

$$G_{IIE} = c_{IIE} \left(M_{1B} - \frac{M_{2B}}{\theta_1} - \frac{N_{1Be}}{\theta_2} \right) \left(M_{1B} - \frac{M_{2B}}{\theta'_1} - \frac{N_{1Be}}{\theta'_2} \right) \quad (18)$$

where c_{IE} and c_{IIE} are still given by Eqs. (15) and (16) and

$$N_{1Be} = N_{1B} - N_{2B}/\gamma \quad (19)$$

The two sets of orthogonal pure modes (θ_i, β_i) and (θ'_i, β'_i) are given as

$$\theta_1 = -\gamma^2 \quad (20)$$

$$\theta_2 = -\frac{6}{h_1} \quad (21)$$

$$\beta_1 = \frac{\gamma^2(3+\gamma)}{1+3\gamma} \quad (22)$$

$$\begin{aligned} \beta_2 &= \frac{2(3+\gamma)}{h_1(\gamma-1)} \quad \text{for } \gamma \neq 1 \\ &= 1 \quad \text{for } \gamma = 1 \end{aligned} \quad (23)$$

$$\theta'_1 = -1 \quad (24)$$

$$\theta'_2 = -\frac{6(1+\gamma)}{h_1(1+\gamma^3)} \quad (25)$$

$$\beta'_1 = \gamma^3 \quad (26)$$

The isotropic G_{θ_i} and G_{β_i} for use in Eqs. (15) and (16) are

$$G_{\theta_1} = \frac{24\gamma}{Eb^2h_1^3(1+\gamma)} \quad (27)$$

$$G_{\beta_1} = \frac{72\gamma(1+\gamma)}{b^2Eh_1^3(1+3\gamma)^2} \quad (28)$$

According to the Wang-Harvey Timoshenko beam or shear deformable partition theory [1-6], the mode I and II components of the energy release rates denoted by G_{IT} and G_{IIT} respectively are for generally layered composite materials,

$$G_{IT} = c_{IT} \left(M_{1B} - \frac{M_{2B}}{\beta_1} - \frac{N_{1B}}{\beta_2} - \frac{N_{2B}}{\beta_3} \right)^2 \quad (29)$$

$$G_{IIT} = c_{IIT} \left(M_{1B} - \frac{M_{2B}}{\theta_1} - \frac{N_{1B}}{\theta_2} - \frac{N_{2B}}{\theta_3} \right)^2 \quad (30)$$

where

$$c_{IT} = G_{\theta_1} \left(1 - \frac{\theta_1}{\beta_1} \right)^{-2} \quad (31)$$

$$c_{IIT} = G_{\beta_1} \left(1 - \frac{\beta_1}{\theta_1} \right)^{-2} \quad (32)$$

The partitions for isotropic beams reduce to

$$G_{IT} = c_{IT} \left(M_{1B} - \frac{M_{2B}}{\beta_1} - \frac{N_{1Be}}{\beta_2} \right)^2 \quad (33)$$

$$G_{IIT} = c_{IIT} \left(M_{1B} - \frac{M_{2B}}{\theta_1} - \frac{N_{1Be}}{\theta_2} \right)^2 \quad (34)$$

where c_{IT} and c_{IIT} are still given by Eqs. (31) and (32).

Finally, the Wang-Harvey averaged partition theory is the average of the Wang-Harvey Euler beam and Timoshenko beam partitions. The mode I and II components of the energy release rate from the averaged partition theory are denoted by G_{IA} and G_{IIA} respectively. They are

$$G_{IA} = (G_{IE} + G_{IT})/2 \quad (35)$$

$$G_{IIA} = (G_{IIE} + G_{IIT})/2 \quad (36)$$

3 EXPERIMENTAL ASSESSMENTS

The first set of experimental results is from the work [12]. The DCB was made from laminated unidirectional composite materials and the upper beam was loaded by a bending moment, i.e. $M_{1B} \neq 0, M_{2B} = N_{1B} = N_{2B} = 0$. The ERR partitions G_I and G_{II} at failure are shown in Fig. 2 from various partition theories presented in Section 2. The Wang-Harvey Euler beam or classical plate/shell partition theory is the closest to the linear failure locus which is expected to be the accurate failure criterion for the DCB materials of low toughness in consideration. The Wang-Harvey Timoshenko beam or shear deformable plate/shell partition theory is the furthest to the linear failure locus. The Wang-

Harvey averaged partition theory is very close to the Suo-Hutchinson theory, but both of which are far away from the linear failure locus. In contrast, the Williams partition theory is closer.

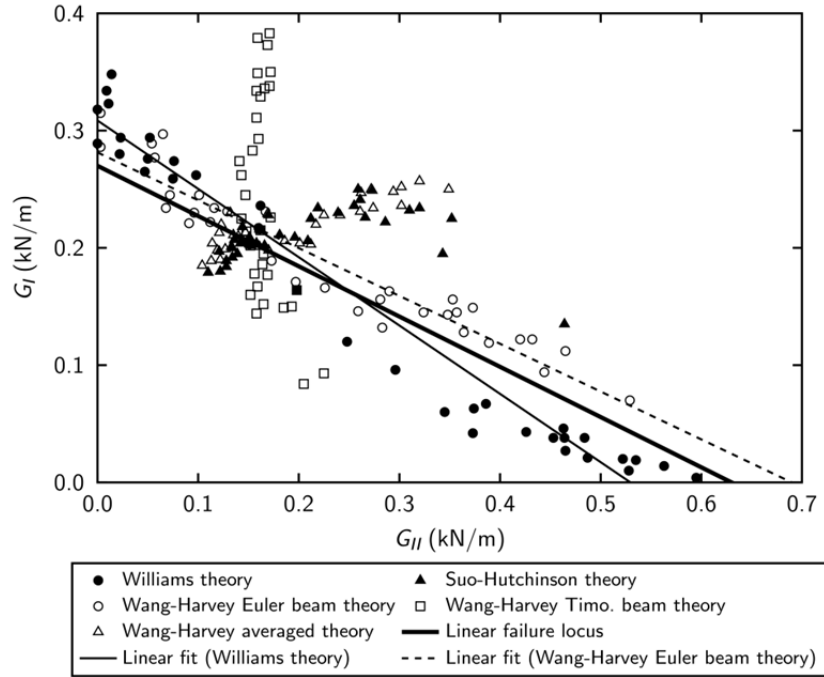


Figure 2: A comparison of partitions from various partition theories and the linear failure locus for epoxy-matrix/carbon-fibre composite specimens.

As far as the authors' knowledge is concerned, the work in Refs. [7, 8] represents some of the most comprehensive and thorough experimental test data available for the study of interfacial delamination toughness in generally laminated composite beams. Davidson et al.'s non-singular-field partition theory [7, 8] was identified as the best performer in predicting interface fracture toughness. The present Fig. 2 identifies the Wang-Harvey Euler beam or classical plate/shell partition theory is the best performer. In the following assessments, the experimental test data [7, 8] are used to assess the two best performers. In addition, the singular-field partition from finite element method simulation based on 2D elasticity is also assessed.

Unidirectional (UD) specimens made from C12K/R6376 material with midplane and offset delaminations are considered first. The results are shown in Fig. 3. As expected, all three partition approaches give largely identical partition results for midplane delaminations. By using these results, a failure locus is experimentally determined in terms of the total critical ERR G_c and the partition G_{II}/G and this is shown in Fig. 3 as the solid piecewise straight line. The error bars show plus/minus one standard deviation from each data point based on Davidson et al.'s testing of at least five specimens for each test [7, 8]. Up to plus/minus one standard deviation of the failure locus is also shown by the shadowed area. The different partition theories are assessed against this failure locus for offset delamination. It is seen that Wang and Harvey's Euler beam partition theory and Davidson et al.'s non-singular-field partition theory again give largely identical partition results and agree very well with the failure locus; however, the singular-field partition results are generally not in good agreement with this failure locus. It is surprising to see the excellent—almost identical—agreement between Wang and Harvey's Euler beam partition theory and Davidson et al.'s non-singular-field partition theory, because the former is derived completely analytically, and the latter is derived with the aid of experimental work. In order to investigate this observation further, Fig. 4 shows the difference between the partitions G_{II}/G from both partition theories over a range of bending moment ratios, M_{2B}/M_{1B} , and thickness ratios, $\log_{10}(1/\gamma)$. Within the range

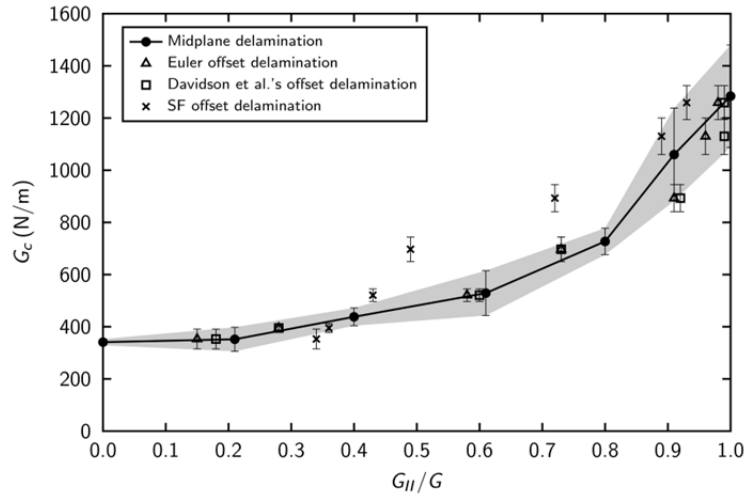


Figure 3: Fracture toughness of midplane and offset delaminations in unidirectional laminates made from C12K/R6376 [7].

$1/3 < \gamma < 3$, or with reference to Eq. (12), the range $-0.468 < \eta < 0.468$, the two approaches are approximately identical, which is strong support for the theoretical basis behind Wang and Harvey's Euler beam partition theory. Cross data markers for each UD specimen test point ($M_{2B}/M_{1B}, \gamma$) tested in Ref. [7] are also overlaid onto Fig. 4. It is interesting to note that every test point lies in the region where there is excellent agreement between the two partition theories. This begs the question, outside of the region $1/3 < \gamma < 3$, which theory is better? Although this is not conclusive, the data presented in Ref. [13] shows that Wang and Harvey's Euler beam partition theory agrees well with the experimental measurements when $\gamma < 1/3$ and much better than Davidson et al.'s non-singular-field partition theory.

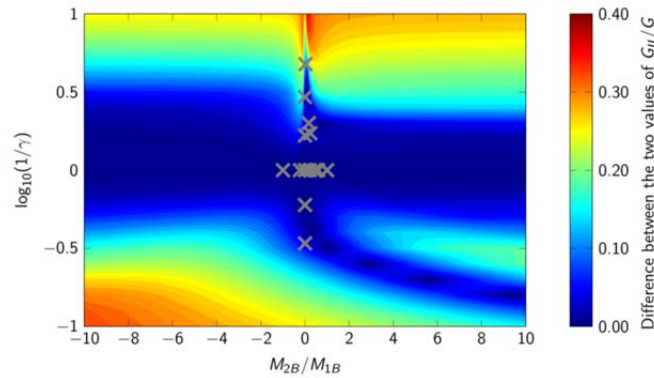


Figure 4: Difference between G_{II}/G from Wang and Harvey's Euler beam partition theory and Davidson et al.'s partition theory with overlaid test points for unidirectional beams [7].

Multidirectional (MD) specimens [8] which are made from T800H/3900-2 graphite epoxy material are considered next. The partition results are given in Fig. 5. The straight line in Fig. 5 is the failure locus obtained from UD midplane delamination tests. As the test results fall almost exactly on the line, they are not plotted on the figure for clarity. It is impressive to see that partition results from Wang and Harvey's Euler beam partition theory for the MD specimens fall almost exactly on the line except for one specimen. This test however has a large standard deviation for its fracture toughness measurements. Reference [8] says that there may have been some errors in the testing of this specimen. It is also noted that both Davidson et al.'s non-singular-field partition theory [7, 8] and the singular field approach have better agreement with the failure locus than they do for the UD specimens in Fig. 3.

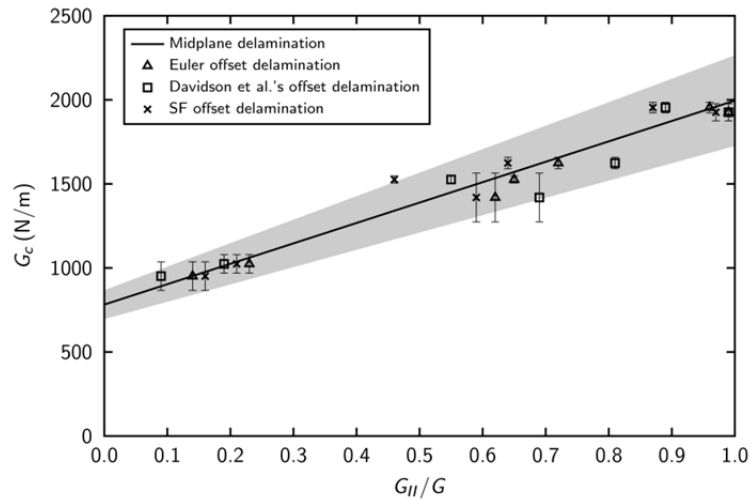


Figure 5: Fracture toughness of midplane and offset delaminations in multidirectional laminates T800H/3900-2 [8].

4 CONCLUSIONS

By using some of the most comprehensive and thorough experimental test data to be found in the literature, several valuable and popular partition theories are assessed. Wang and Harvey's Euler beam or classical plate/shell partition theory [1-6] has excellent agreement with experimental test results and gives very accurate predictions of macroscopic interfacial fracture toughness of laminated composite beams with arbitrary layups, various thickness ratios and various loading conditions. It is a very valuable theory for academic research of fracture and fatigue of advanced materials. Furthermore, it can play a very valuable role in the design of engineering structures made of layered materials. Davidson et al.'s non-singular-field partition theory [7, 8] has excellent agreement with experimental test results and with Wang and Harvey's Euler beam or classical plate/shell partition theory [1-6] (inside the range $1/3 < \gamma < 3$) for UD laminated composite materials. Its accuracy is still very good for MD laminated composite beams; however, it has been observed and argued that overall Wang and Harvey's Euler beam or classical plate/shell partition theory [1-6] offers improved accuracy. In general, the singular-field approach based on 2D elasticity and the finite element method give poor predictions.

REFERENCES

- [1] S. Wang S and C.M. Harvey, A theory of one-dimensional fracture. *Composite Structures*, **94**, 2012, pp.758–767 (doi: 10.1016/j.compstruct.2011.09.011). Also a plenary lecture at the 16th International Conference on Composite Structures (ICCS16), 28–30 June 2011, Porto, Portugal.
- [2] S. Wang and C.M. Harvey, Mixed mode partition theories for one dimensional fracture, *Engineering Fracture Mechanics*, **79**, 2012, pp. 329-352 (doi: 10.1016/j.engfracmech.2011.11.013).
- [3] C.M. Harvey and S. Wang, Mixed mode partition theories for one dimensional delamination in laminated composite beams, *Engineering Fracture Mechanics*, **96**, 2012, pp. 737-759 (doi: 10.1016/j.engfracmech.2012.10.001).
- [4] C.M. Harvey and S. Wang, Experimental assessment of mixed-mode partition theories, *Composite Structures*, **94**, 2012, pp. 2057-2067 (doi: 10.1016/j.compstruct.2012.02.007).
- [5] C.M. Harvey, M.R. Eplett and S. Wang, Experimental assessment of mixed-mode partition theories for generally laminated composite beams, *Composite Structures*, **124**, 2015, pp. 10-18 (doi: 10.1016/j.compstruct.2014.12.064).
- [6] S. Wang, C.M. Harvey and J.D. Wood, The Mechanics of Interface in Layered Composite Materials: (1) Brittle Homogeneous and Bi-material Interfaces, *The 21st International Conference on Composite Materials*, Xi'an, 20-25th August 2017.

- [7] B.D. Davidson, S.J. Gharibian and L. Yu, Evaluation of energy release rate-based approaches for predicting delamination growth in laminated composites, *International Journal of Fracture*, **105**, 2000, pp. 343-365 (doi: 10.1023/A:1007647226760).
 - [8] B.D. Davidson, R.D. Bialaszewski and S.S. Sainath, A non-classical, energy release rate based approach for predicting delamination growth in graphite reinforced laminated polymeric composites, *Composite Science and Technology*, **66**, 2006, pp.1479-1496 (doi: 10.1016/j.compscitech.2004.10.031).
 - [9] Z. Suo and J.W. Hutchinson, Interface crack between two elastic layers, *International Journal of Fracture*, **43**, 1990, pp.1-18 (doi:10.1007/BF00018123).
 - [10] J.W. Hutchinson and Z. Suo, Mixed mode cracking in layered materials, *Advances in Applied Mechanics*, **29**, 1992, pp.63-191 (doi:10.1016/S0065-2156(08)70164-9).
 - [11] J.G. Williams, On the calculation of energy release rates for cracked laminates, *International Journal of Fracture*, **36**, 1988, pp.101-119 (doi:10.1007/BF00017790).
 - [12] M. Charalambides, A.J. Kinloch, W. Wang, J.G. Williams, On the analysis of mixed-mode failure, *International Journal of Fracture*, **54**, 1992, pp. 269-291 (doi:10.1007/BF00035361).
- M. Conroy, B.F. Sørensen, A. Ivankovic, Combined numerical and experimental investigation of mode-mixity in beam like geometries. In: *Proceedings of the 37th annual meeting of the Adhesion Society*, San Diego CA, February 2014.