Experimental assessment of mixed-mode partition theories for fracture toughness in laminated composite beams

Experimental assessment of mixed-mode partition theories for generally laminated composite beams

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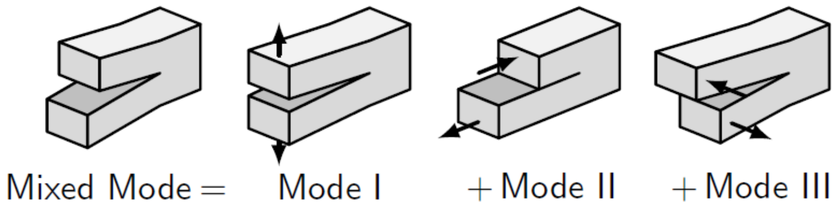
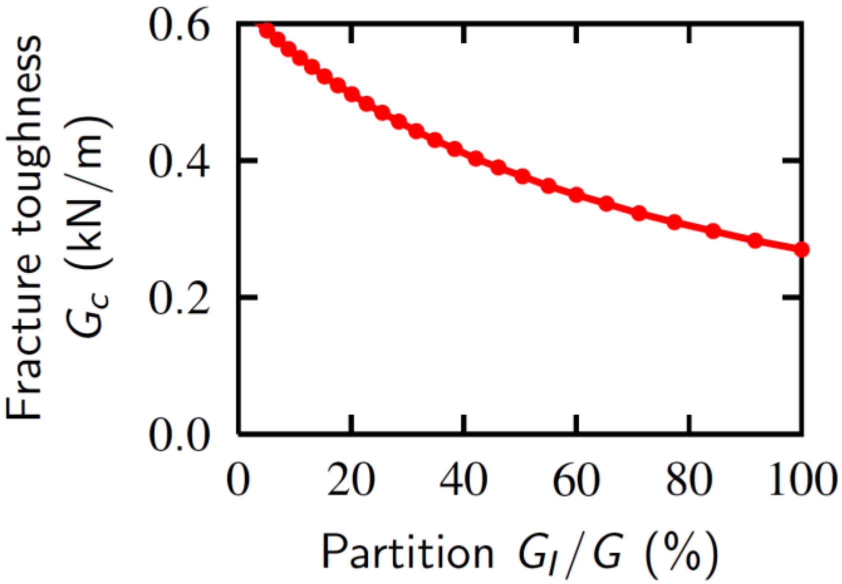
Introduction

- Fracture toughness depends on the fracture mode partition.
- Predicting fracture toughness requires knowledge of the partition of a mixed-mode fracture.
- It is therefore essential to have a correct analytical partition theory in order to predict the fracture toughness.





Introduction

- Previous work[1] by the authors shows that Loughborough University's Euler beam (EB) partition theory performs very well when predicting the mode-dependent fracture toughness.
- Davidson et al.'s (Syracuse, NY) non-singular field (NSF) partition theory[2] is developed based on experimental fracture toughness measurements. It also works very well.
- Therefore, it is reasonable to speculate that Davidson et al.'s NSF partition theory approaches to Loughborough's EB partition theory.

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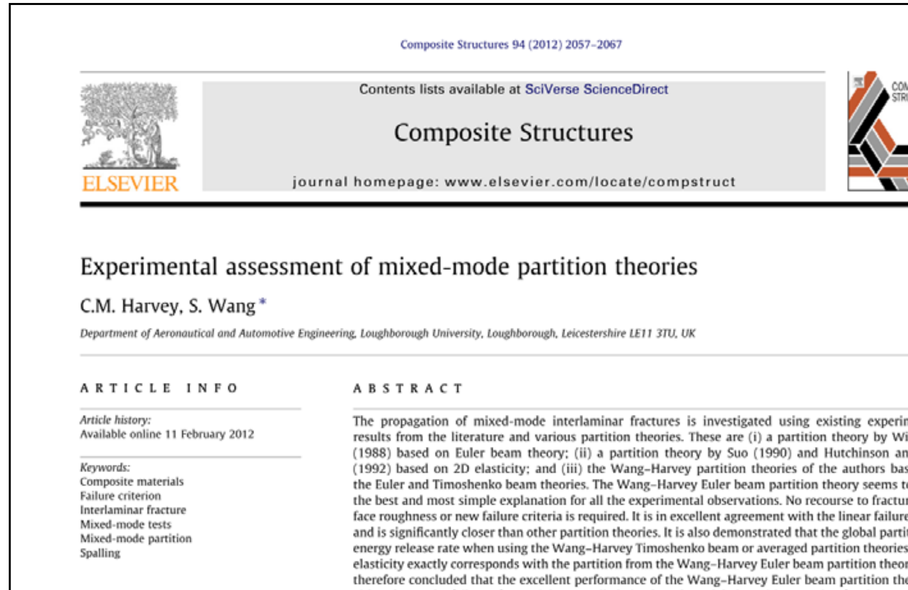
[1] Harvey, Wang (2012), Compos Struct 94.
[2] Davidson et al. (2000), Int J Fract 105.

Aims

- To assess the EB[1] and NSF[2] theories thoroughly using Davidson's et al.'s experimental fracture toughness data[2].
- To explore the connections between the two theories[1,2].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Experimental assessment of mixed-mode partition theories

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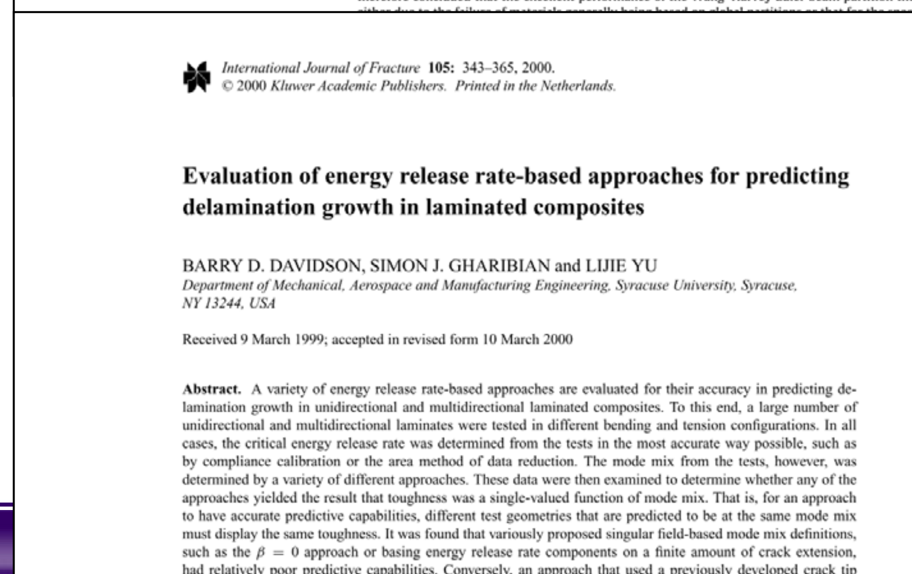
ABSTRACT

The propagation of mixed-mode interlaminar fractures is investigated using existing experin results from the literature and various partition theories. These are (i) a partition theory by Wi (1988) based on Euler beam theory; (ii) a partition theory by Suo (1990) and Hutchinson an (1992) based on 2D elasticity; and (iii) the Wang–Harvey partition theories of the authors bas the Euler and Timoshenko beam theories. The Wang–Harvey Euler beam partition theory seems t the best and most simple explanation for all the experimental observations. No recourse to fractu face roughness or new failure criteria is required. It is in excellent agreement with the linear failure and is significantly closer than other partition theories. It is also demonstrated that the global parti energy release rate when using the Wang–Harvey Timoshenko beam or averaged partition theories elasticity exactly corresponds with the partition from the Wang–Harvey Euler beam partition theor therefore concluded that the excellent performance of the Wang–Harvey Euler beam partition the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Evaluation of energy release rate-based approaches for predicting delamination growth in laminated composites

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Abstract. A variety of energy release rate-based approaches are evaluated for their accuracy in predicting delamination growth in unidirectional and multidirectional laminated composites. To this end, a large number of unidirectional and multidirectional laminates were tested in different bending and tension configurations. In all cases, the critical energy release rate was determined from the tests in the most accurate way possible, such as by compliance calibration or the area method of data reduction. The mode mix from the tests, however, was determined by a variety of different approaches. These data were then examined to determine whether any of the approaches yielded the result that toughness was a single-valued function of mode mix. That is, for an approach to have accurate predictive capabilities, different test geometries that are predicted to be at the same mode mix must display the same toughness. It was found that variously proposed singular field-based mode mix definitions, such as the $\beta = 0$ approach or basing energy release rate components on a finite amount of crack extension,

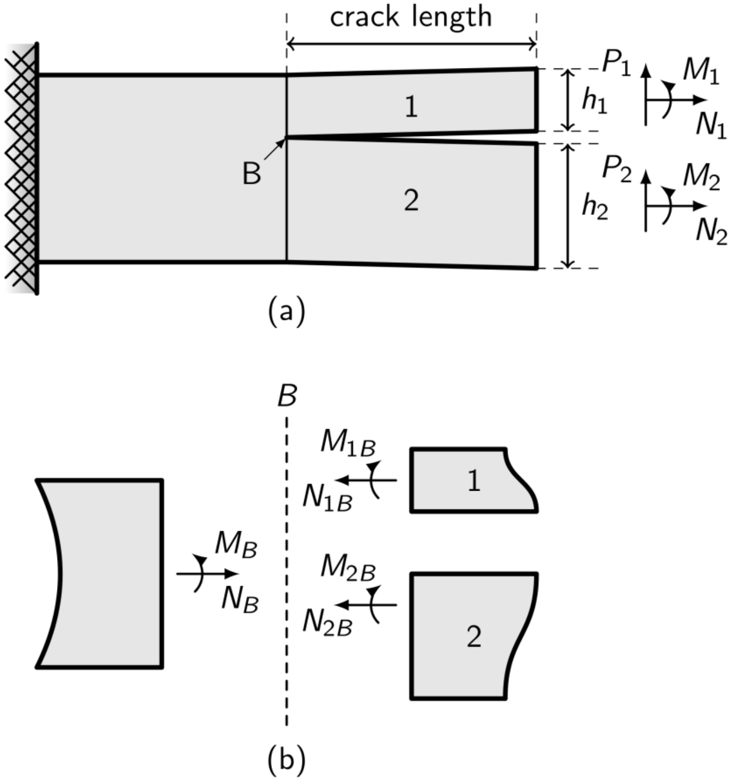
[1] Harvey, Wang (2012), Compos Struct 94.
[2] Davidson et al. (2000), Int J Fract 105.

Partition theories

- Loughborough University's EB partition theory (completely analytical):

$$G_{IE} = c_{IE}\left(M_{1B} - \frac{M_{2B}}{\beta_1} - \frac{N_{1B}}{\beta_2} - \frac{N_{2B}}{\beta_3}\right)\left(M_{1B} - \frac{M_{2B}}{\beta_1'} - \frac{N_{1B}}{\beta_2'} - \frac{N_{2B}}{\beta_3'}\right)$$

$$G_{IIE} = c_{IIE}\left(M_{1B} - \frac{M_{2B}}{\theta_1} - \frac{N_{1B}}{\theta_2} - \frac{N_{2B}}{\theta_3}\right)\left(M_{1B} - \frac{M_{2B}}{\theta_1'} - \frac{N_{1B}}{\theta_2'} - \frac{N_{2B}}{\theta_3'}\right)$$



- Davidson et al.'s NSF partition theory:

$$\frac{G_{II}}{G} = \frac{[N_c\sqrt{c_1}\cos\Omega + M_c\sqrt{c_2}\sin(\Omega+\Gamma)]^2}{c_1N_c^2 + c_2M_c^2 + 2\sqrt{c_1c_2}N_cM_c\sin\Gamma}$$

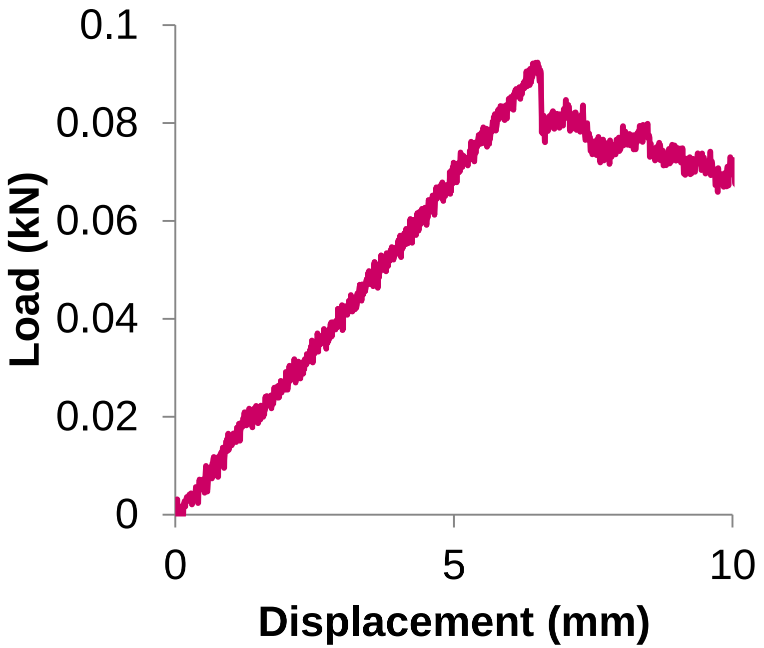
  - The mode mix parameter Ω is determined with the aid of experimental data.

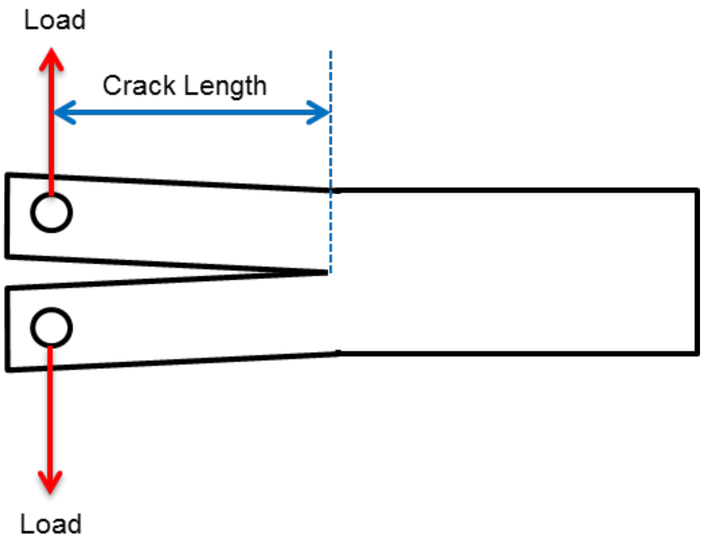
$$\Omega = \begin{cases} -24 & \log(h_2/h_1) < -0.468 \\ 60.409\eta - 41.738\eta^3 \quad \text{if} & -0.468 < \log(h_2/h_1) < 0.468 \\ 24 & \log(h_2/h_1) > 0.468 \end{cases}$$

Davidson et al.'s fracture testing methods

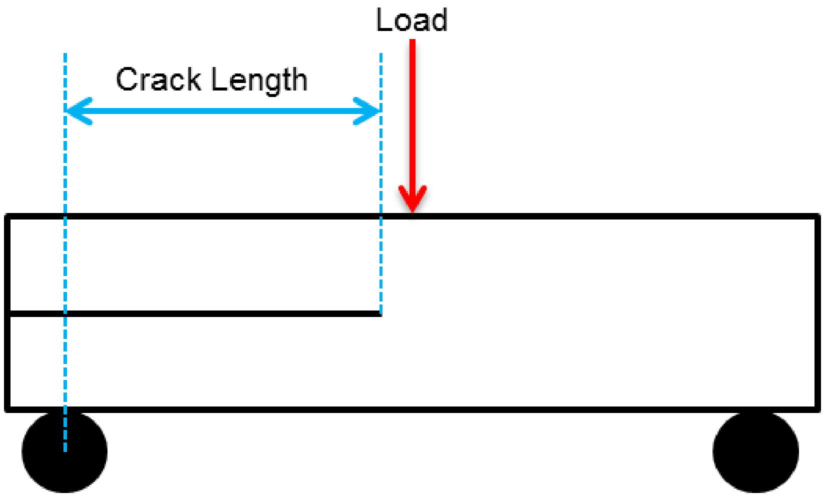
Double cantilever beam (DCB) test

- Gives the pure mode I fracture toughness G_{Ic}





Davidson et al.'s fracture testing methods

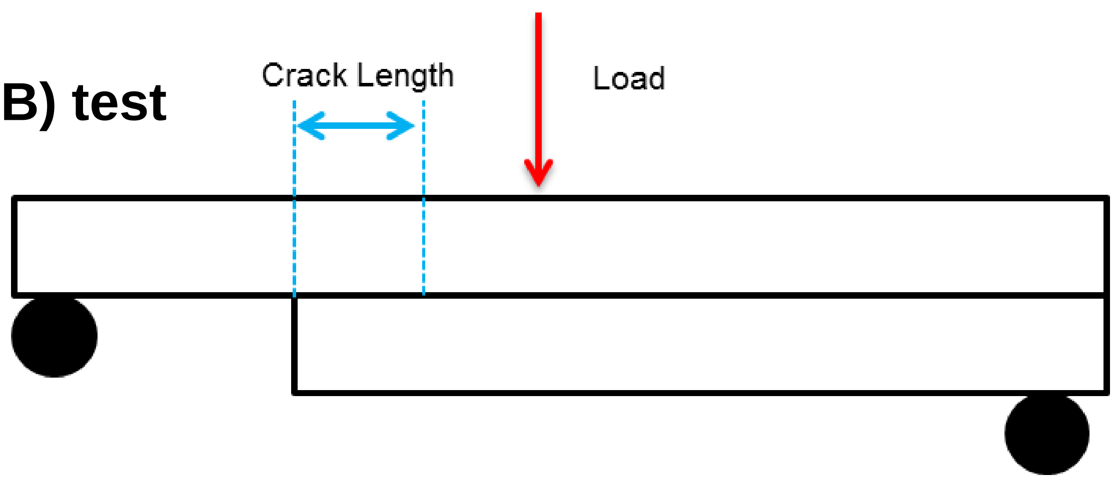


End-notched flexure (ENF) test

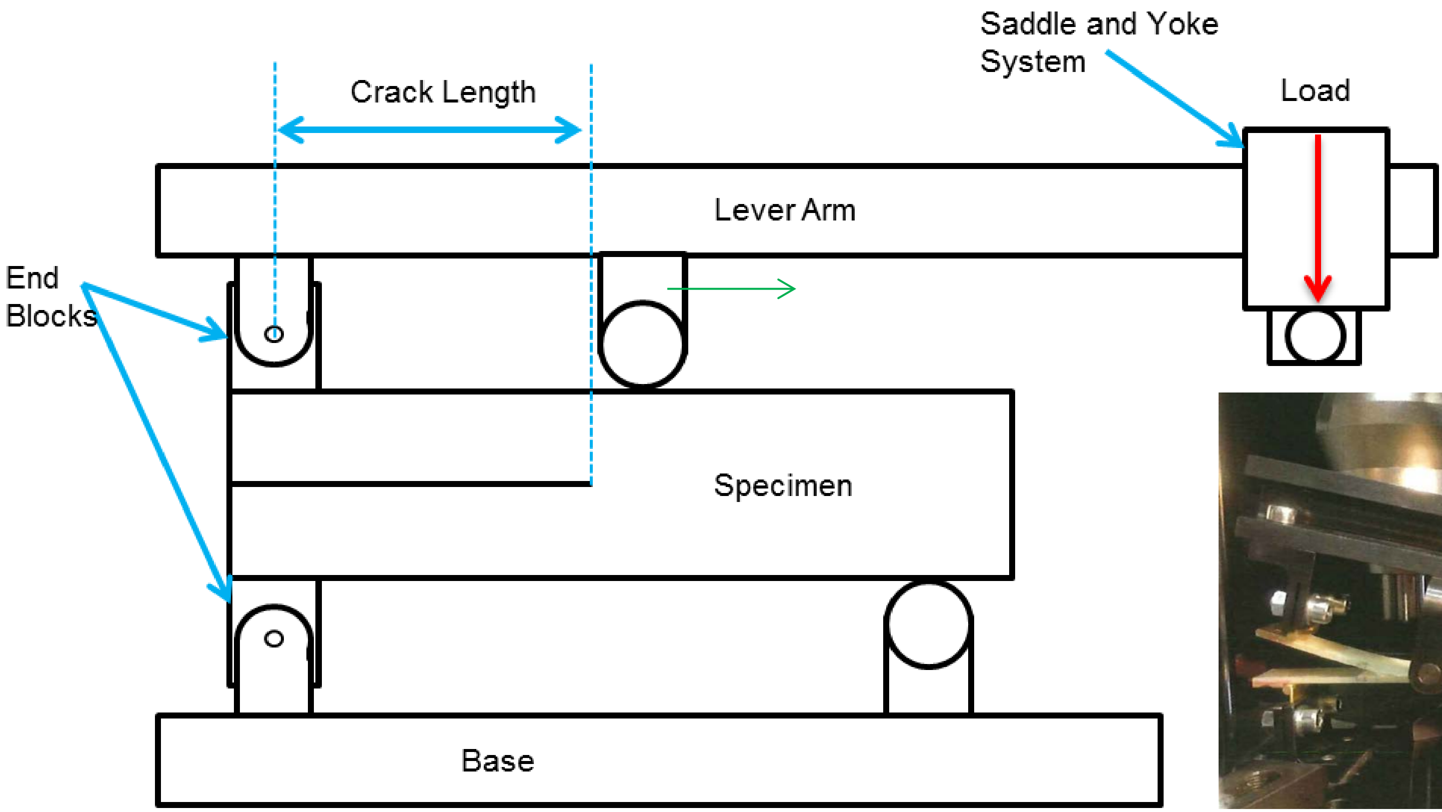
- Gives the pure mode II fracture toughness G_{IIc}

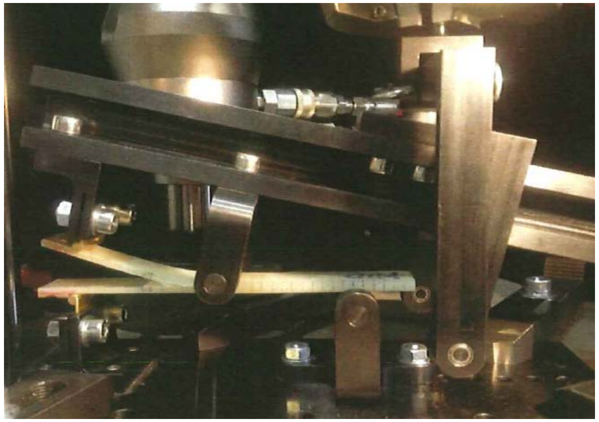
Single leg bending (SLB) test

- Mixed mode



Davidson et al.'s fracture testing methods

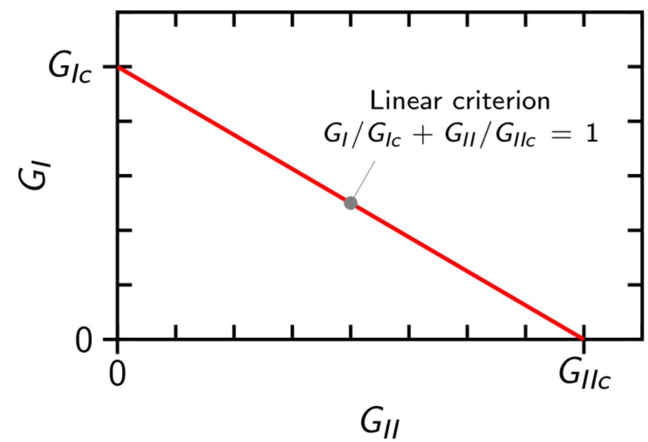




Mixed mode bending (MMB) test

Methodology

- **Objective** was to compare the fracture toughness obtained at each partition G_{II}/G against the failure locus.
- However, the failure locus is not readily available:
  - In early work, a linear failure locus was assumed.
  - In Davidson et al.'s work, unidirectional (UD) midplane delamination testing work was used to obtain the failure locus.



  - This is possible because all existing partition theories agree for UD midplane delamination.
- Therefore, in Davidson et al.'s work and in our work, the UD midplane failure locus is used to assess the accuracy of a partition theory for offset delaminations and general layups.

Loughborough University

Test specimens

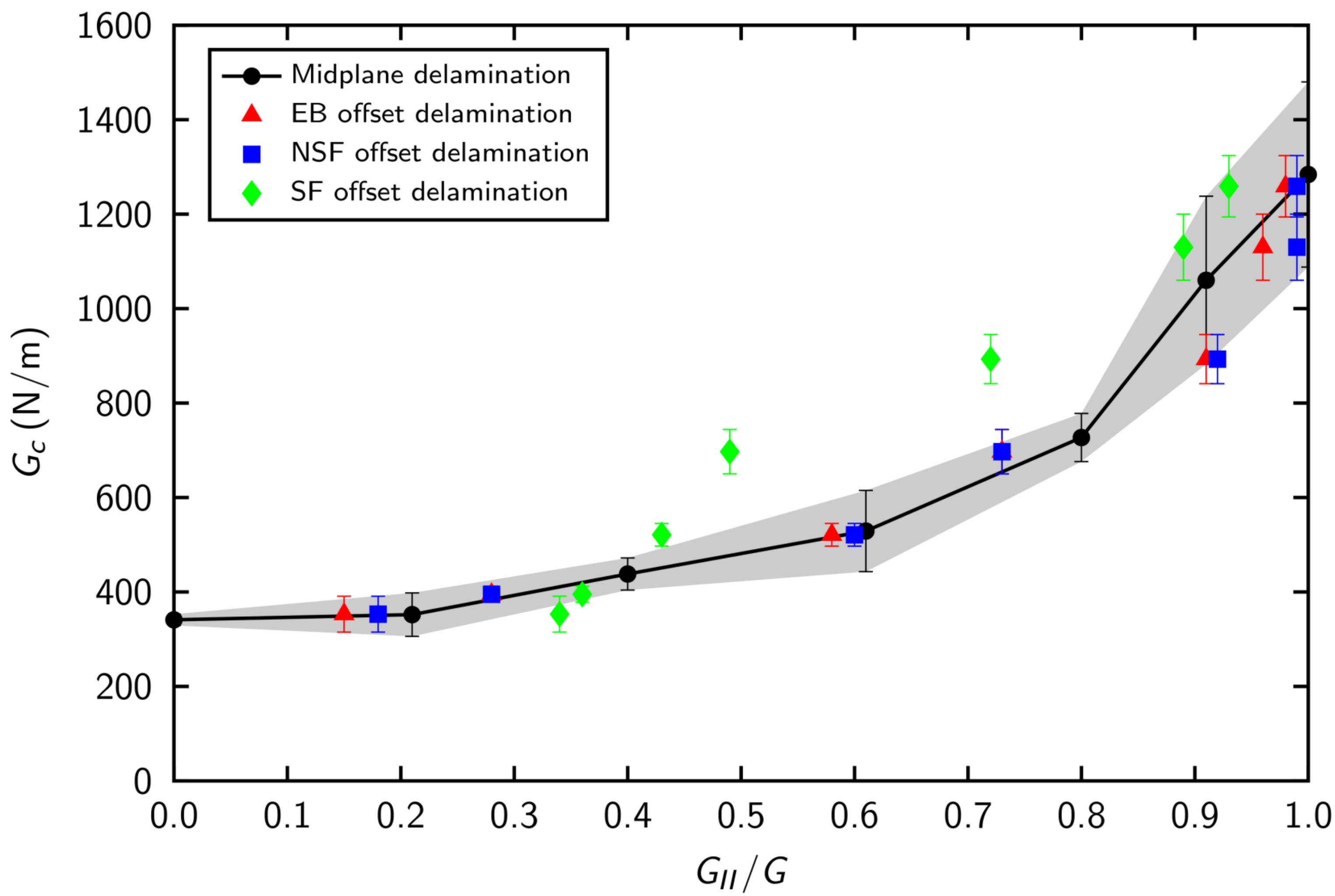
- Davidson et al. (2000), Int J Fract 105.
  - C12K/R6376 graphite/epoxy (relatively low toughness)
- Davidson et al. (2006), Compos Sci Tech 66.
  - T800H/3900-2 graphite/epoxy (relatively high toughness)

- 3 specimen types and DCB, ENF, MMB, SSLB, UENF tests
  1. UD 0 / 0 interface
  2. Constrained UD 0 / 0 interface (‘d’ = delamination location)
     - $[0/10/-15/0_{10}/-15/10/0/d]_s$ Layup A
     - $[(0/\pm15/0)_3/d/(0/\pm15/0)/(0/\mp15/0)_4)$ Layup B
     - $[(0/\pm15/0)_4/(0/\mp15/0)/d/(0\mp15/0)_3)$ Layup C
  3. Multi-Directional (MD) interfaces
     - 5A $[(0\mp45/90)_s/d/(45/90/0/-45_2/0/90/45)_s/(0\mp45/90)_s]$ Layup D
     - 12A $[(\pm45/0_2/\mp45/\pm45/0_2/\mp45)_s/d/(0\pm45/\mp45/0)_s]$ Layup E
     - 19A $[(\mp45/0_8/\pm45)_s/d/(\mp45/0_8/\pm45)]$ Layup F

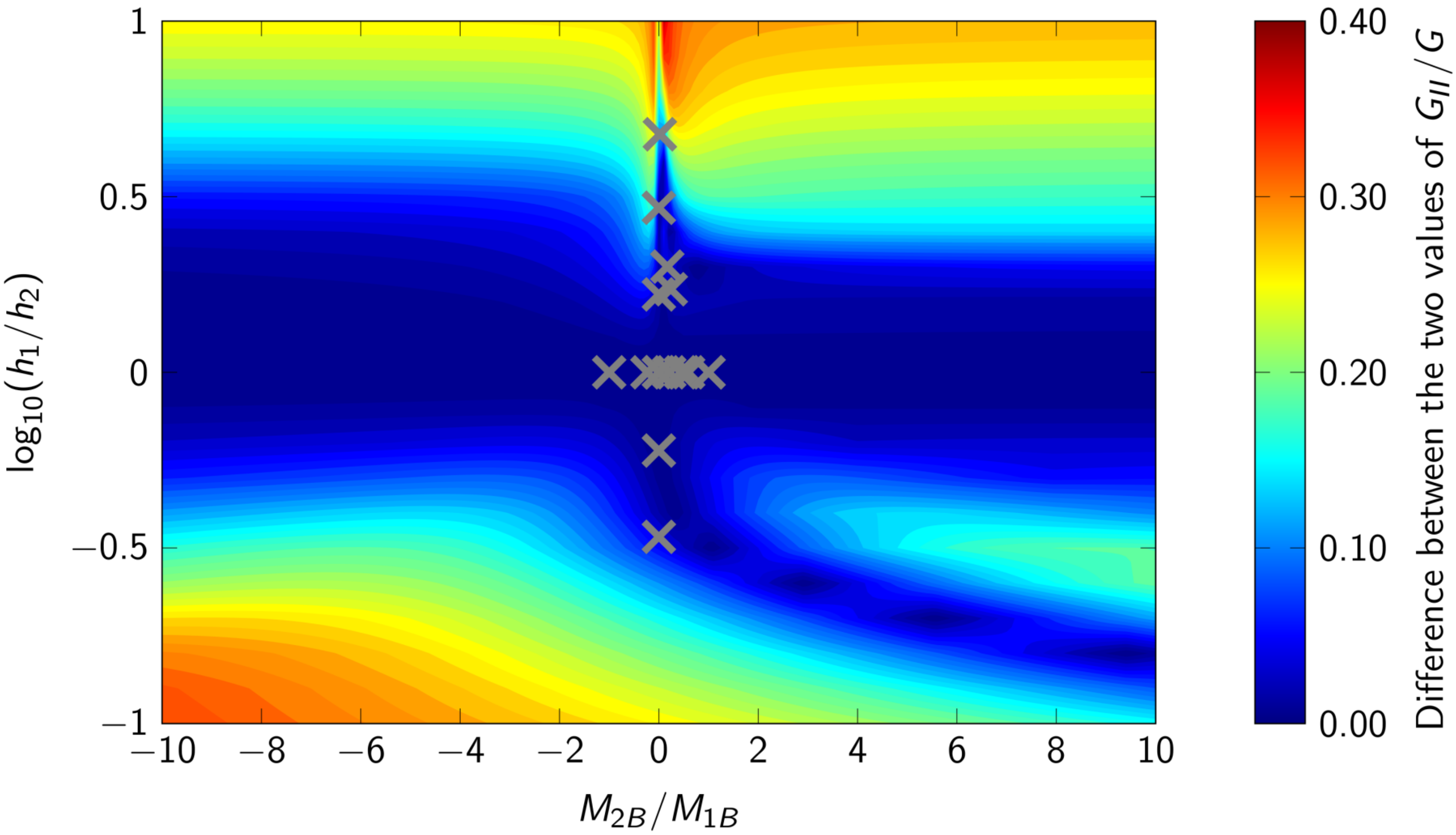
UD 0/0 Interface

Data from Davidson et al. (2000)

C12K/R6376 graphite/epoxy (relatively low toughness)



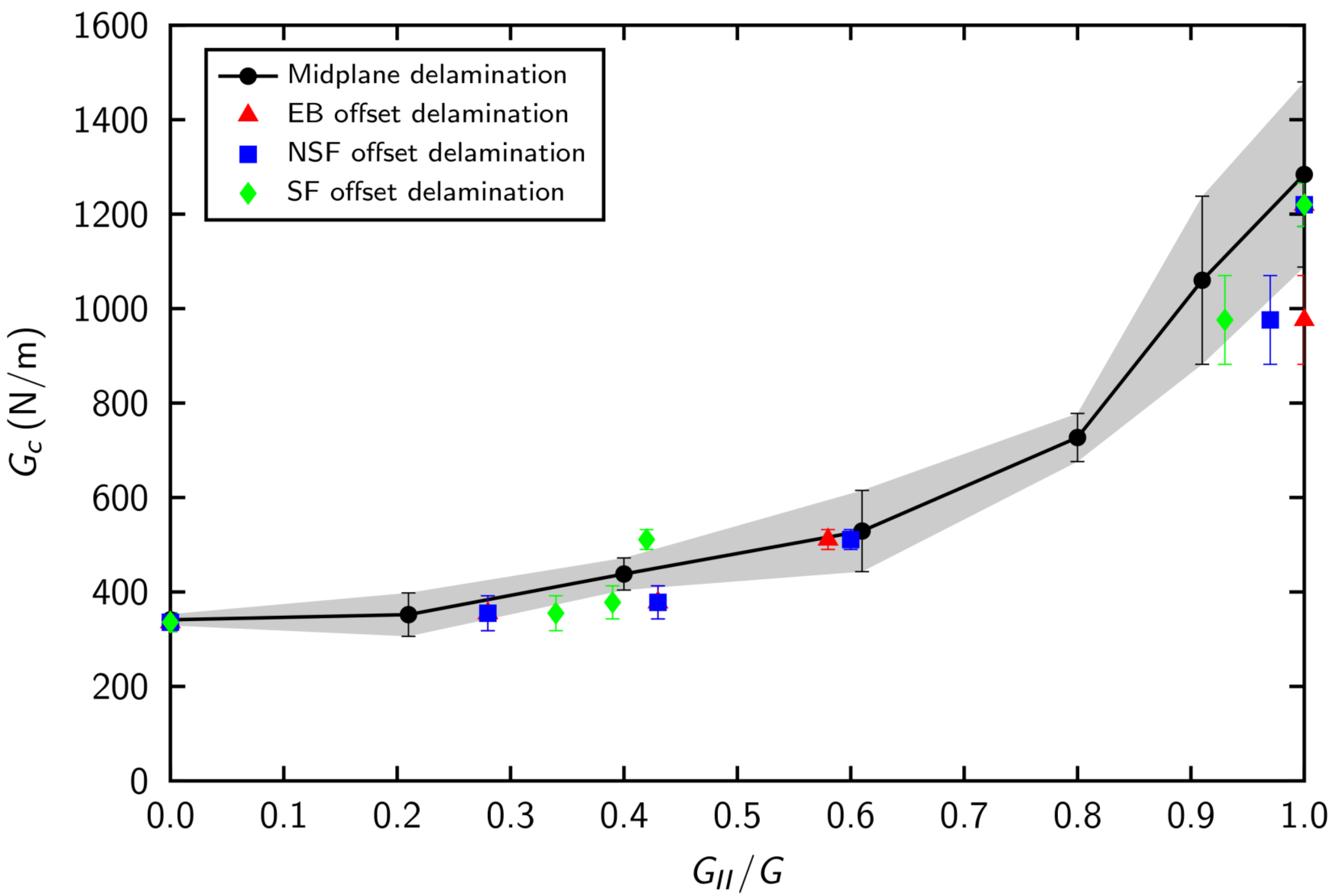
Difference between EB and NSF partitions



Constrained UD 0 / 0 interface

Data from Davidson et al. (2000)

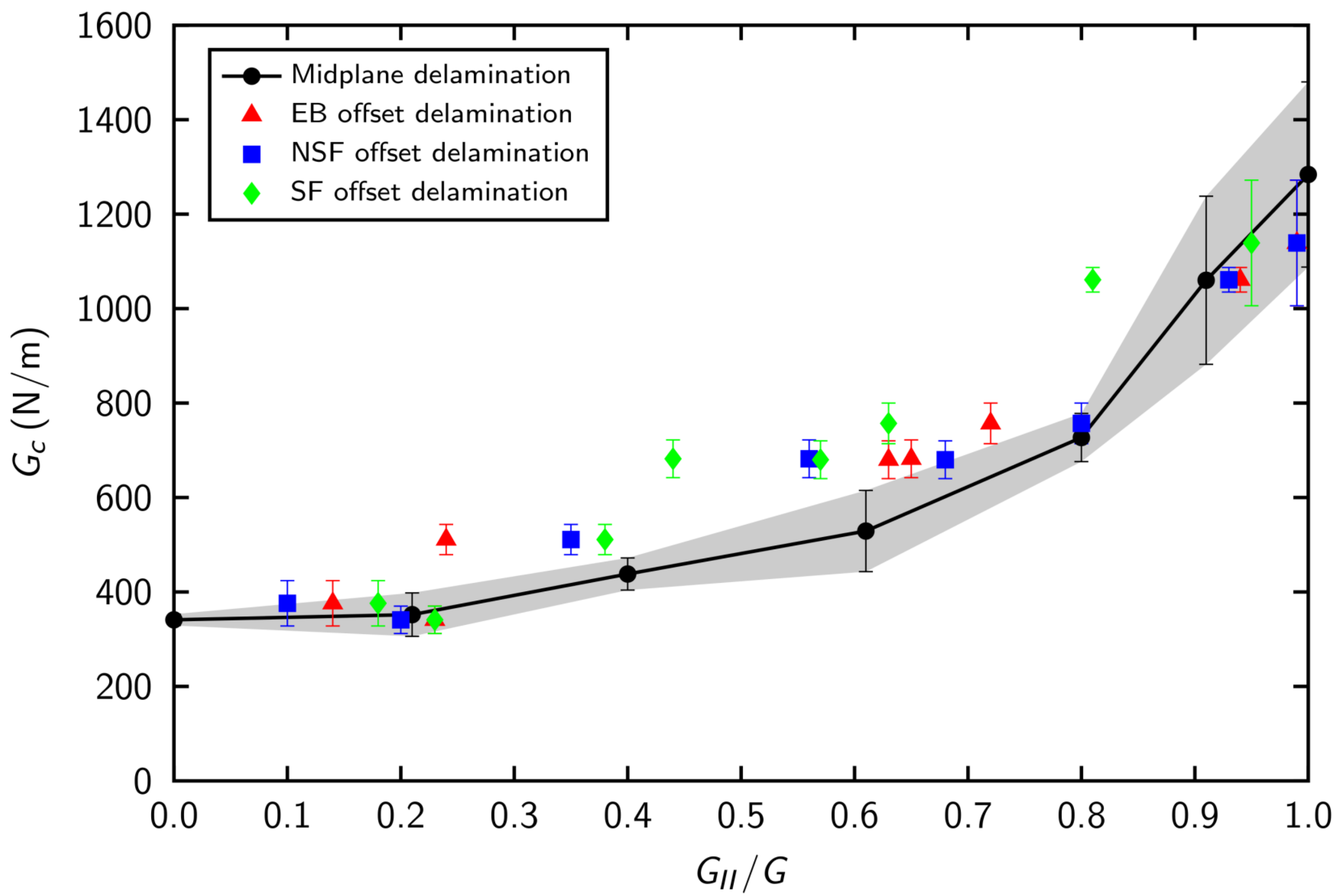
C12K/R6376 graphite/epoxy (relatively low toughness)



Multi-Directional (MD) interfaces – First set

Data from Davidson et al. (2000)

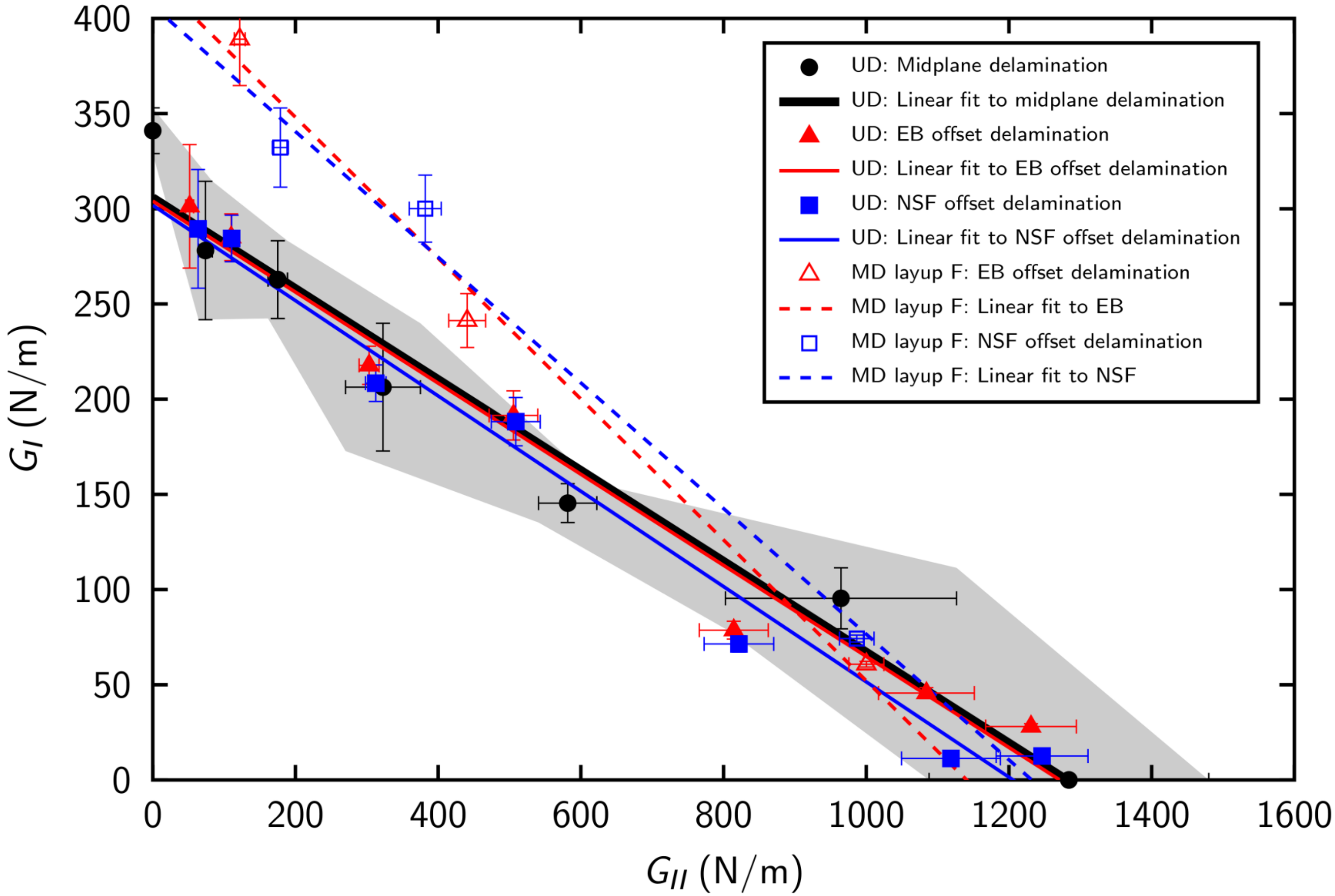
C12K/R6376 graphite/epoxy (relatively low toughness)



Multi-Directional (MD) interfaces – First set

Data from Davidson et al. (2000)

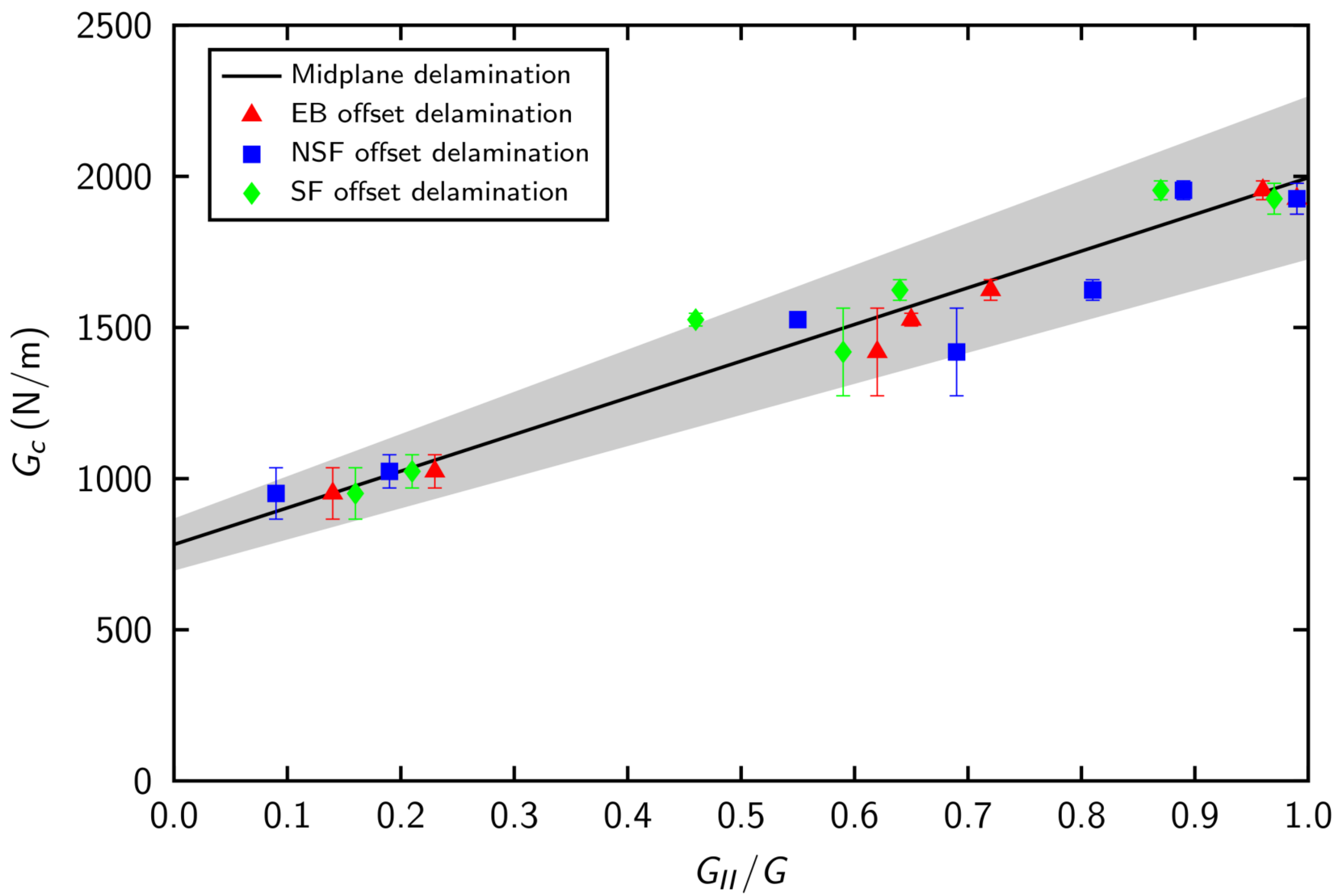
C12K/R6376 graphite/epoxy (relatively low toughness)



Multi-Directional (MD) interfaces – First set

Data from Davidson et al. (2006)

T800H/3900-2 graphite/epoxy (relatively high toughness) : (Layups D – F together)



Conclusions

- SF does not have great agreement with the fracture toughness data
  - Care must be taken when using the 2D finite element method. ERR partitions will be based on the SF and this gives poor predictions offracture toughness.

- NSF theory – based on experimental data – works very well in predicting mixed-mode fracture toughness.
- EB theory – completely analytical – works very well in predicting mixed-mode fracture toughness.

Thank you for listening

Questions?

Appendix

Table 1: Unidirectional material properties

	C12K/R6376 graphite/epoxy	T800H/3900-2 graphite/epoxy
E_{11} (GPa)	146.86	154.72
E_{22} , E_{33} (GPa)	10.62	7.58
μ_{12} , μ_{13} (GPa)	5.45	4.27
μ_{23} (GPa)	3.99	2.88
ν_{12} , ν_{13} , ν_{23}	0.33	0.32
E_{1f} (GPa)	114.15	143.13

Data from Davidson et al. (2000) for C12K/R6376 graphite/epoxy

Data from Davidson et al. (2006) for T800H/3900-2 graphite/epoxy

Appendix

Table 2: Fracture toughness of midplane and offset delaminations in unidirectional laminates made from C12K/R6376.

Test	n_1/n_2	$\gamma = h_2/h_1$ **	M_2/M_1	Calculated partition, G_{II}/G SF	Davidson et al.	Euler	G_c (N/m)	$\pm\sigma$ (N/m) error
DCB	16/16	1.00	-1.00	0.00	0.00	0.00	341	12
SSLB	16/16	1.00	0.00	0.40	0.43	0.43	438	34
ENF	16/16	1.00	1.00	1.00	1.00	1.00	1284	196
MMB*	12/12	1.00	-0.23	0.21	0.23	0.23	352	46
MMB*	12/12	1.00	0.01	0.40	0.44	0.44	438	34
MMB*	12/12	1.00	0.21	0.61	0.64	0.64	529	86
MMB*	12/12	1.00	0.44	0.80	0.83	0.83	727	51
MMB*	12/12	1.00	0.59	0.91	0.92	0.92	1060	178
MMB*	12/12	1.00	1.00	1.00	1.00	1.00	1284	196
USLB	8/24	2.94	0.00	0.34	0.18	0.15	353	38
USLB	12/20	1.67	0.00	0.36	0.28	0.28	395	17
USLB	20/12	0.60	0.00	0.43	0.60	0.58	521	24
USLB	24/8	0.34	0.00	0.49	0.73	0.73	697	47
UENF	25/5	0.21	0.02 (0.004)	0.72	0.92	0.91	893	52
UENF	20/10	0.50	0.17 (0.10)	0.89	0.99	0.96	1130	70
UENF	20/12	0.58	0.24 (0.18)	0.93	0.99	0.98	1259	65

* 24-ply UD MMB laminates, ply thickness $t_p = 0.155$ mm (for all other UD laminates, ply thickness $t_p = 0.146$ mm)

** These thickness ratios refer to the actual average thickness ratio as measured from the test specimens

Data from Davidson et al. (2000)

Appendix

Table 3: Fracture toughness of midplane and offset delaminations in constrained unidirectional laminates made from C12K/R6376.

Test	n_1/n_2	$\gamma = h_2/h_1$	Stacking sequence	M_2/M_1	Calculated partition, G_{II}/G SF	Davidson et al.	Euler	G_c (N/m)	$\pm\sigma$ (N/m) error
DCB	16/16	1.00	A	-1.00	0.00	0.00	0.00	336	20
SSLB	16/16	1.00	A	0.00	0.39	0.43	0.43	378	35
ENF	16/16	1.00	A	1.00	1.00	1.00	1.00	1220	46
USLB	12/20	1.67	B	0.00	0.34	0.28	0.28	355	37
USLB	20/12	0.60	C	0.00	0.42	0.60	0.58	511	21
UENF	20/12	0.60	C	0.22 (0.22)	0.93	0.97	1.00	976	94

Stacking sequence (ply thickness $t_p = 0.159$ mm):

A : $\left[0/10/-15/0_{10}/-15/10/0/\mathrm{d}\right]_s$

B : $\left[(0/\pm 15/0)_3/\mathrm{d}/(0/\mp 15/0)/(0/\mp 15/0)_4\right]$

C : $\left[(0/\pm 15/0)_4/\mathrm{d}/(0/\mp 15/0)/(0/\mp 15/0)_3\right]$

Data from Davidson et al. (2000)

Appendix

Table 4: Fracture toughness of midplane and offset delaminations in multidirectional laminates made from C12K/R6376

Test	n_1/n_2	$\gamma = h_2/h_1$	Stacking sequence	M_2/M_1	Calculated partition, G_{II}/G			G_c (N/m)	$\pm\sigma$ (N/m) error
					SF	Davidson et al.	Euler		
USLB	8/24	3.00	D	0.00	0.18	0.10	0.14	376	48
USLB	24/8	0.33	D	0.00	0.63	0.80	0.72	757	43
USLB	12/24	2.00	E	0.00	0.23	0.20	0.23	341	29
USLB	24/12	0.50	E	0.00	0.57	0.68	0.63	680	40
UENF	24/12	0.50	E	0.17 (0.14)	0.95	0.99	0.99	1139	133
USLB	12/24	2.00	F	0.00	0.38	0.35	0.24	511	32
USLB	24/12	0.50	F	0.00	0.44	0.56	0.65	682	40
UENF	24/12	0.50	F	0.11 (0.06)	0.81	0.93	0.94	1061	26

Stacking sequence (ply thickness $t_p = 0.152\,\text{mm}$):

D : $\left[(0/\mp 45/90)_s / \mathrm{d} / (45/90/0/-45_2/0/90/45)_s / (0/\mp 45/90)_s\right]$

E : $\left[(\pm 45/0_2/\mp 45/\pm 45/0_2/\mp 45)_s / \mathrm{d} / (0/\pm 45/\mp 45/0)_s\right]$

F : $\left[(\mp 45/0_8/\pm 45)_s / \mathrm{d} / (\pm 45/0_8/\mp 45)\right]$

Data from Davidson et al. (2000)

Appendix

Table 5: Fracture toughness of offset delaminations under the loading case

Test	n_1/n_2	$\gamma = h_2/h_1$	Stacking sequence	Calculated partition, G_{II}/G			G_c (N/m)	$\pm\sigma$ (N/m) error
				SF	Davidson et al.	Euler		
USLB	8/24	3.00	D	0.18 (0.16)	0.10 (0.09)	0.14 (0.14)	376 (951)	48 (85)
USLB	8/24	2.94	UD	0.34	0.18	0.15	353	38
USLB	12/24	2.00	E	0.23 (0.21)	0.20 (0.19)	0.23 (0.23)	341 (1024)	29 (55)
USLB	12/24	2.00	F	0.38	0.35	0.24	511	32
USLB	12/20	1.67	UD	0.36	0.28	0.28	395	17
USLB	12/20	1.67	B	0.34	0.28	0.28	355	37
USLB	20/12	0.60	UD	0.43	0.60	0.58	521	24
USLB	20/12	0.60	C	0.42	0.60	0.58	511	21
USLB	24/12	0.50	E	0.57 (0.59)	0.68 (0.69)	0.63 (0.62)	680 (1419)	40 (145)
USLB	24/12	0.50	F	0.44 (0.46)	0.56 (0.55)	0.65 (0.65)	682 (1526)	40 (21)
USLB	24/8	0.34	UD	0.49 (0.49)	0.73 (0.73)	0.73 (0.73)	697 (1807)	47 (91)
USLB	24/8	0.33	D	0.63 (0.64)	0.80 (0.81)	0.72 (0.72)	757 (1624)	43 (34)
USLB*	18/6	0.33	UD	(0.48)	(0.73)	(0.73)	(1682)	(166)

* For this specimen only, ply thickness $t_p = 0.182\,\text{mm}$, otherwise see Tables 2, 3, 4 and 6.

Data from Davidson et al. (2000)

Appendix

Table 6: Fracture toughness of midplane and offset delaminations in multidirectional laminates T800H/3900-2.

Test	n_1/n_2	$\gamma = h_2/h_1$	Stacking sequence	M_2/M_1	Calculated partition, G_{II}/G: SF	Davidson et al.	Euler	G_c (N/m)	$\pm\sigma$ (N/m) error
USLB	8/24	3.00	D	0.00	0.16	0.09	0.14	951	85
USLB	12/24	2.00	E	0.00	0.21	0.19	0.23	1024	55
USLB	24/12	0.50	F	0.00	0.46	0.55	0.65	1526	21
USLB	24/12	0.50	E	0.00	0.59	0.69	0.62	1419	145
USLB	24/8	0.33	D	0.00	0.64	0.81	0.72	1624	34
UENF	24/12	0.50	F	0.10 (0.07)	0.87	0.89	0.96	1954	31
UENF	24/12	0.50	E	0.17 (0.15)	0.97	0.99	0.99	1926	51

Stacking sequence (ply thickness $t_p = 0.179$ mm):

D : $[(0/\mp 45/90)_s / d / (45/90/0/-45_2/0/90/45)_s / (0/\mp 45/90)_s]$

E : $[(\pm 45/0_2/\mp 45/\pm 45/0_2/\mp 45)_s / d / (0/\pm 45/\mp 45/0)_s]$

F : $[(\mp 45/0_8/\pm 45)_s / d / (\pm 45/0_8/\mp 45)]$

Data from Davidson et al. (2006)