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Dynamic interfacial fracture of a thin-layered structure

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Abstract

To calculate the dynamic energy release rate of a crack is important for understanding a structure's fracture behavior under transient or varying loads, such as impact and cyclic loads, when the inertial effect can be significant. In this work, a method is proposed to derive an analytic expression for the dynamic energy release rate of a stationary crack under general applied displacement. An asymmetric double cantilever beam with one very thin layer is considered as a special case, with vibration superimposed onto a constant displacement rate applied to the free end. The resulting expression for dynamic energy release rate is verified using the finite-element method (FEM) in conjunction with the virtual crack closure technique. The mode-mixity of the dynamic energy release rate is also calculated. The predicted total dynamic energy release rate and its components, G_I and G_{II}, are both in close agreement with results from FEM simulations.

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Keywords: dynamic energy release rate, inertial effect, modal analysis, dynamic energy release rate partition

1. Introduction

Dynamic fracture has been an active research topic for several decades. During this time, various physical quantities have been derived as direct counterparts to those from quasi-static fracture, such as dynamic stress intensity factor and dynamic energy release rate. It is important to be able to calculate these quantities to understand the fracture behavior of a structure under transient or varying loads. In some engineering applications, loading can

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be considered as applied displacement (e.g. the response of a vehicle suspension system; the earthquake response of a structure; the drilling of a plate at a prescribed feed rate); and in others as applied load (e.g. buildings or off-shore structures subject to wind or wave loading). In this work it is convenient to consider applied displacement.

The dynamic effects on a structure result from its inertia and the material's strain-rate sensitivity. In this study, only the inertial effect is investigated. Conventionally, dynamic fracture is studied with either stress-based approaches or energy-based approaches. Stress-based approaches include the transmission of sudden load through stress wave propagation and superposition of stresses near the crack tip (Green's method), and the Laplace transform technique (Wiener-Hopf method) to solve initial-boundary value problems (Freund, 1990). Energy-based approaches include applying the 'crack tip energy flux integral'. For an engineering structure, however, there appear to be few analytic solutions for dynamic energy release rate or dynamic stress intensity factor, although some approximation methods have been used, such as the 'kinetic energy distribution' method (Blackman et al., 1996) and the 'displacement rate' method (Smiley and Pipes, 1987), but these only considered constant loading rates and assumed the dynamic deflection the same as the static deflection.

In this work, an analytic expression for the dynamic energy release rate of a stationary crack is derived for general applied displacement. An asymmetric double cantilever beam with one very thin layer (Fig. 1a) is considered as a special case, with vibration superimposed onto a constant displacement rate applied to the free end.

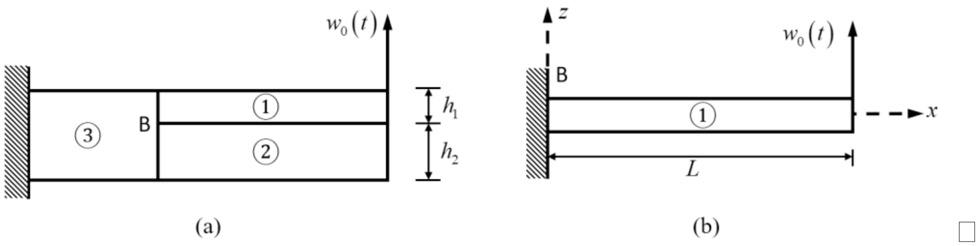

Fig. 1. (a) an asymmetric double cantilever beam; (b) effective boundary condition on beam section ①

2. Theory

2.1. Assumptions

For the beam structure shown in Fig. 1a, the energy release rate at the crack tip at point B is given by Eq. (1), where M_{1B}, M_{2B}, M_B are bending moments of beam section ①,②,③ at crack tip, respectively; and I_1, I_2, I are corresponding secondary moments of area. Note that any undefined nomenclature has its conventional meaning.

$$G = \frac{1}{2bE}\left(\frac{M_{1B}^2}{I_1} + \frac{M_{2B}^2}{I_2} - \frac{M_B^2}{I}\right). \tag{1}$$

The beam thickness is assumed to be thin (i.e. Euler-Bernoulli beam theory can be applied) and axial forces insignificant. For a thin-layered structure, where $h_2 >> h_1$, beam sections ② and ③ can be treated as rigid compared to beam section ① (i.e. $M_B/I_B \approx 0$ and $M_{2B}/I_{2B} \approx 0$). The total energy release rate for this thin beam configuration therefore becomes

$$G = \frac{1}{2bE}\left(\frac{M_{1B}^2}{I_1}\right). \tag{2}$$

Consequently, the boundary condition of beam section ① at the crack tip B is effectively as shown in Fig. 1b.

2.2. Beam's transverse deflection under general displacement excitation

If the general applied displacement $w_0(t)$ can be expanded into the summation of several functions

$$w_0(t)=\sum_{j=1}^{n} w_{0j}(t). \tag{3}$$

with each having a finite number of linearly-independent derivatives (Grant, 1983), or of periodic functions, then shifting functions can be introduced to derive the transverse deflection of the beam by forcing homogenous conditions. Under the above conditions, the transverse deflection of the beam is of the form

$$w(x,t)=w_{\mathrm{fv}}(x,t)+\sum_{j=1}^{n} F_j(x) w_{0j}(t). \tag{4}$$

where $w_{\mathrm{fv}}(x,t)$ is the free vibration of the beam and $F_j(x)$ are the shape functions.

To demonstrate the process and facilitate the derivation, the applied displacement is taken here as the sum of one linear term and one harmonic term, that is, $w_0(t)=vt+H\sin(\theta^2 t)$.

The governing equation for vibration of an Euler-Bernoulli beam (Rao, 2007) is

$$EIw^{(4)}(x,t)+\rho A\ddot{w}(x,t)=0. \tag{5}$$

The transverse deflection is thus assumed to be of the form as

$$w(x,t)=w_{\mathrm{fv}}(x,t)+F_1(x)vt+F_2(x)H\sin(\theta^2 t). \tag{6}$$

By combining Eqs. (5) and (6) and forcing homogenous conditions, the following differential equations are obtained:

$$EIw_{\mathrm{fv}}^{(4)}(x,t)+\rho A\ddot{w}_{\mathrm{fv}}(x,t)=0, \quad F_1^{(4)}(x)=0, \quad EIF_2^{(4)}(x)-\rho A\theta^4 F_2(x)=0. \tag{7}$$

The boundary conditions for the first of Eqs. (7) are $w_{\mathrm{fv}}(0,t)=0$, $w_{\mathrm{fv}}^{(1)}(0,t)=0$, $w_{\mathrm{fv}}(L,t)=0$, $w_{\mathrm{fv}}^{(2)}(L,t)=0$. Note that $w_{\mathrm{fv}}(x,t)$ is the free vibration of a fixed-pinned beam.

The solution for the free vibration of an Euler-Bernoulli beam can be found by using the separation method (Rao, 2007; Blevins, 1979) as

$$w_{\mathrm{fv}}(x,t)=\sum_{i=1}^{\infty} W_i(x)\left(A_i\cos\omega_i t+\frac{B_i}{\omega_i}\sin\omega_i t\right). \tag{8}$$

where $W_i(x)$ is the ith normal mode given in

$$W_i(x)=\sqrt{\frac{1}{\rho AL}}\left[\cos\frac{\lambda_i}{L}x-\cosh\frac{\lambda_i}{L}x-\sigma_i\left(\sin\frac{\lambda_i}{L}x-\sinh\frac{\lambda_i}{L}x\right)\right]. \tag{9}$$

and A_i and B_i are determined by initial conditions. In Eq. (9), σ_i is given in Eq. (10), and λ_i is a dimensionless parameter determined solely from the boundary conditions.

$$\sigma_i = \frac{\cos\lambda_i - \cosh\lambda_i}{\sin\lambda_i - \sinh\lambda_i}. \tag{10}$$

The solution for the shifting function $F_1(x)$ is

$$F_1(x) = -\frac{1}{2L^3}x^3 + \frac{3}{2L^2}x^2. \tag{11}$$

and the solution for $F_2(x)$ is

$$F_2(x) = C_1\sin kx + C_2\cos kx + C_3\sinh kx + C_4\cosh kx. \tag{12}$$

where $k = \theta\cdot\sqrt[4]{\rho A/EI}$ and the coefficients C_1, C_2, C_3 and C_4 are determined from

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ \sin kL & \cos kL & \sinh kL & \cosh kL \\ -\sin kL & -\cos kL & \sinh kL & \cosh kL \end{bmatrix}\begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{Bmatrix}. \tag{13}$$

2.3. Dynamic energy release rate

Substituting Eqs. (8), (11) and (12) into Eq. (6), the deflection of beam under the supposed displacement is derived. Now the bending moment at crack tip B is obtained from

$$M_{1\mathrm{B}} = EIw^{(2)}(0,t) \tag{14}$$

Finally, by expanding Eq. (14) and combining with Eq. (2), the total dynamic energy release rate at the crack tip B is obtained in Eq. (15), and K_{i1}, K_{i2}, K_{i3} and K_{i4} are given in Eq. (16).

$$G = \frac{EI}{2b}\left\{ \begin{array}{l} \sqrt{\dfrac{\rho A}{EI}}\displaystyle\sum_{i=1}^{\infty}\left[\frac{2v}{\lambda_i}\left((-1)^i\sqrt{\sigma_i^2+1} - \sqrt{\sigma_i^2-1}\right) - \frac{2}{L}H\theta^2 C_j K_{ij}\right]\sin(\omega_i t) \\ + \dfrac{3}{L^2}vt + k^2(C_4 - C_2)H\sin(\theta^2 t) \end{array} \right\}^2. \tag{15}$$

$$\begin{Bmatrix} K_{i1} \\ K_{i2} \\ K_{i3} \\ K_{i4} \end{Bmatrix} = \frac{1}{k^4 - \beta_i^4}\begin{bmatrix} \sin kL & -\sin kL & -1 & 0 \\ \cos kL & -\cos kL & 0 & -1 \\ -\sinh kL & -\sinh kL & -1 & 0 \\ -\cosh kL & -\cosh kL & 0 & -1 \end{bmatrix}\begin{Bmatrix} k^2\phi_i^{(1)}(L) \\ \phi_i^{(3)}(L) \\ 2k\beta_i^2 \\ 2\sigma_i\beta_i^2 \end{Bmatrix}. \tag{16}$$

and $\phi_i(x)$ is

$$\phi_i(x) = \cos\frac{\lambda_i}{L}x - \cosh\frac{\lambda_i}{L}x - \sigma_i\left(\sin\frac{\lambda_i}{L}x - \sinh\frac{\lambda_i}{L}x\right). \tag{17}$$

Note that the dynamic energy release rate in Eq. (15) is the total energy release rate. For the majority of engineering applications where the fracture toughness is mode-mixity-dependent, the total dynamic energy release has to be partitioned into its components, G_{I} and G_{II}. Wood et al. (2017), building on Harvey and Wang (2012), provided a partition theory for a thin layer on a thick substrate, which can be applied in this study.

3. Numerical verification

To verify the analytical expression for dynamic energy release rate derived above, the asymmetric double cantilever beam shown in Fig. 2 is considered, with vibration superimposed onto a constant displacement rate applied to the free end.

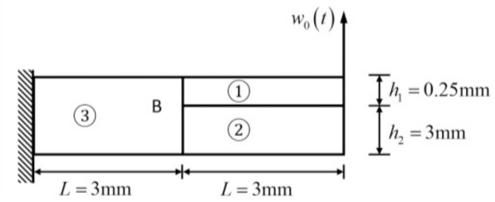

Fig. 2. Geometry for numerical verification.

An isotropic elastic material is assumed with a Young's modulus of 50 GPa and a Poisson's ratio of 0.3. For the applied displacement, the constant loading rate is 10 mm s^{-1} together with a vibration of amplitude of 1 μm and an angular frequency of 160 000 rad s^{-1}.

The finite-element method is used here in conjunction with the virtual crack closure technique to determine the dynamic energy release numerically. For a plane-stress problem, the comparison between numerical and analytical methods for total dynamic energy release rate is shown in Fig. 3.

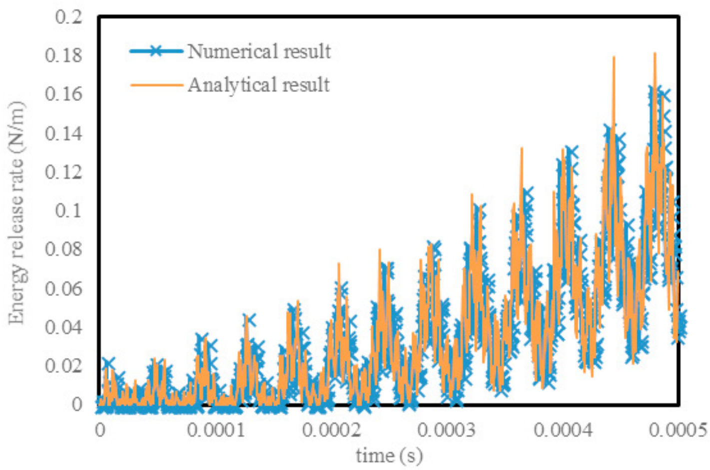

Fig. 3. Comparison of results for total dynamic energy release rate.

Wood et al.'s (2017) partition theory for a thin layer on a thick substrate gives $G_{\mathrm{I}}/G = 0.6227$ (or equivalently, $G_{\mathrm{II}}/G = 0.3773$). These values can be applied directly to partition the analytical total dynamic energy release rate; the comparisons of its components, G_{I} and G_{II}, are shown in Fig. 4.

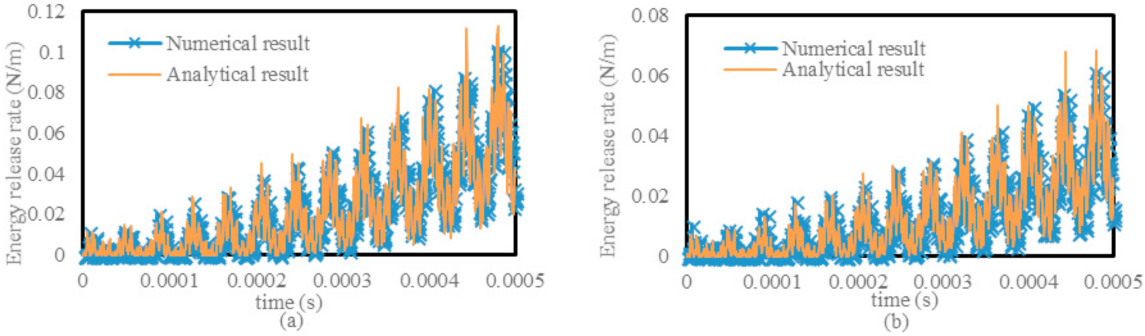

Fig. 4. Comparison of results for components of dynamic energy release rate: (a) Mode I, G_{I} ; (b) Mode II, G_{II} .

The analytical results for both the total dynamic energy release rate and its components are in very good agreement with the results from the numerical simulation. The analytical solution captures not only the amplitude of dynamic energy release rate but also the frequency of its variation.

It should be noted that the analytical solution for the present problem of an asymmetric double cantilever with one very thin layer can be directly applied to a symmetric double cantilever beam with equal and opposite applied displacements. This is because the boundary condition in Fig. 1b applies in both cases. The dynamic energy release rate is double that in Eq.(15), and its partition is $G_{\mathrm{I}}/G=1$ (or equivalently, $G_{\mathrm{II}}/G=0$).

4. Conclusion

A method has been proposed to derive an analytic expression for the dynamic energy release rate of a stationary crack under general applied displacement. When the method is applied to an asymmetric double cantilever beam with one very thin layer, and with vibration superimposed onto a constant displacement rate acting at the free end, very good agreement is obtained with results from FEM simulation. Furthermore, the analytical components of dynamic energy release rate, G_{I} and G_{II} , calculated by using Wood et al.'s (2017) partition theory for a thin layer on a thick substrate, are also in very close agreement with the FEM simulation results.

This work is foundational for the consideration of crack propagation under transient or varying loads, and with arbitrary through-thickness crack locations. These extensions are under active development by the authors and will provide classical solutions that are relevant to numerous modern and relevant engineering problems.

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