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## CHARACTERISTICS OF THERMOOPTICAL EXCITATION OF SOUND IN METALS

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The process of exciting sound in metals with absorption in them of intensity-modulated laser radiation is analyzed. Analytical expressions are given for the amplitudes of the surface and volumetric acoustical waves excited by the laser beam. The directivity diagrams of the volumetric waves are identified and their dependence on the beam width is investigated. The acquired results are compared with the experimental data.

Increased interest has recently been noted in literature in the problem of exciting acoustical waves in solids with absorption in them of intensity-modulated laser radiation [1-11]. At relatively low incident light powers such excitation is realized as a result of thermal expansion of the irradiated sector of the solid. For this reason, the corresponding excitation mechanism is commonly called a thermo-optical mechanism. Thanks to a number of intrinsic advantages - primarily the absence of mechanical contact with the solid medium and the possibility of wide adjustment of the frequency of the excited sound - this particular excitation method has been found to be quite promising for nondestructive testing equipment and equipment for experimental physical studies of solids.

The work analyzes certain characteristics of thermo-optical excitation of sound in metals - solids which are characterized by high light absorption and high heat conductivity. It is noted that such an examination has practical importance, since one most frequently deals with metals in the different technical applications of this particular method (see, for instance, the review [3]). In presenting the cited issues the authors will follow a technique proposed in works [6,7] which is based on the use of Green's dynamic tensor for an elastic semi-space with a free boundary [12].

Assume that an intensity-modulated, two-dimensional laser beam is perpendicularly incident to the surface of a metal (Fig. 1) which, for the sake of simpli-

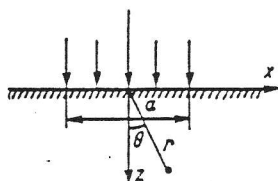


Fig. 1. The geometry of the problem of thermo-optical excitation of sound.

city, will be assumed to be isotropic with respect to its elastic properties. The components of the vector of shifts of the excited acoustical waves  $u_i$  must in this case satisfy the mechanical equation of movement

$$\rho \ddot{u}_i = \sigma_{ij,j}, \quad (1)$$

where  $\sigma_{ij}$  are the components of the tensor of stresses, while  $\rho$  is the density of the medium, and the linearized equation of state with consideration of the temperature effects [13,14]

$$\sigma_{ij} = 2\mu u_{ij} + [\lambda u_{kk} - \gamma K(T - T_0)] \delta_{ij}. \quad (2)$$

Here,  $u_{ij} = (1/2)(u_{i,j} + u_{j,i})$  are the components of the linearized tensor of deformation,  $\gamma$  is the thermal expansion coefficient,  $\lambda$  and  $\mu$  are the isothermal elastic Lamé constants,  $K = \lambda + 2\mu/3$  is the modulus of all-round compression, and  $T_0$  is the initial temperature. The linearized equation of thermal balance, in which the authors ignore the effect of viscosity, should be added to equations (1) and (2):

$$\rho c_v \dot{T} - \kappa T_{,zz} + \gamma K T_0 \dot{u}_{ii} = - \frac{\partial}{\partial z} [\beta I(x) f(t) \exp(-\alpha z)]. \quad (3)$$

In this equation  $c_v$  is the thermal capacity with a constant volume,  $\kappa$  is the heat conductivity coefficient,  $\alpha$  is the light absorption coefficient in the medium,  $\beta$  is the coefficient of light radiation passage in the solid medium,  $I(x)$  is the spatial distribution of the intensity of the incident radiation, and  $f(t)$  describes the intensity modulation law. Below, not limiting themselves to commutativity, the authors will assume the modulation law to be harmonic, i.e.,  $f(t) = 1 + m \cos \omega t$ , where  $m \leq 1$ . The field of excited acoustical waves, moreover, must satisfy the boundary conditions for the free surface of the solid:

$$\sigma_{ij} n_j = 0, \quad (4)$$

where  $n_j$  are the components of the vector of the unique perpendicular to the surface and the conditions of radiation to infinity. The temperature field at  $z = 0$  must satisfy a condition of continuity of the thermal stream of  $\partial T / \partial z = 0$ .

Analysis of the system of connected equations (1)-(3) in the general case is an extremely complex problem. One of the traditional simplifications, which allows its analytical solution, lies in ignoring the small dilation member in equation (3) [14] responsible for the so-called thermoelastic absorption of light. In this case equation (3) will not contain the variables  $u_i$  and may be solved independently of the other equations of the system. It is easy to show that this approximation is equal to ignoring the difference between the isothermal and adiabatic values of the elastic constants  $\lambda$  and  $\mu$ .

For further analysis, it is sufficient to examine solution of equation (3) in two maximal cases of the relation of the light penetration depth into the metal  $2\pi/\alpha$  and the length of the thermal wave  $\lambda_T = 2\pi/k_T = 2\pi/(\omega \rho c_v / 2\kappa)^{1/2}$ , or more precisely:  $2\pi/\alpha \gg \lambda_T$  and  $2\pi/\alpha \ll \lambda_T$ . The first case corresponds to high frequencies of the excited sound  $\omega$ , while the second corresponds to low frequencies. The quality  $2\pi/\alpha = \lambda_T$  for metals occurs at frequencies on the order of hundreds of megahertz. At  $2\pi/\alpha \gg \lambda_T$  the heat conductivity may be ignored in equation (3). Then, assuming for the sake of definiteness that  $m = 1$ , it is found that:

$$T = [\alpha \beta I(x) / \rho c_v] \exp(-\alpha z - i\omega t). \quad (5)$$

In the case of  $2\pi/\alpha \ll \lambda r$  equation (3) is also easily solved; in this case, with consideration of the boundary condition  $\partial T/\partial z = 0$ , it is found that:

$$T = [(1-i)k_T \beta I(x)/\rho c_V] \exp[-(1-i)k_T z - i\omega t]. \quad (6)$$

As is easy to see, expression (6) is quite similar to (5) and may be acquired from it through formal replacement of  $\alpha$  with  $(1-i)k_T$ . It is natural that the expressions for excited acoustical waves will differ just as little in each of these cases: in order to shift from a high frequency mode to a low frequency, it is sufficient to replace  $\alpha$  with  $(1-i)k_T$ . For this reason, only the case  $2\pi/\alpha \gg \lambda r$  is analyzed below.

Expressing the vibrational velocity of the particles of a medium  $\dot{u}_1$  through the scalar Lamé potentials  $\phi$  and  $\psi$  [14] through formulas  $\dot{u}_x = \partial\phi/\partial x - \partial\psi/\partial z$  and  $\dot{u}_z = \partial\phi/\partial z + \partial\psi/\partial x$ , from equations (1), (2), and (5), it is easy to find two equations for  $\phi$  and  $\psi$  (the factor  $\exp(-i\omega t)$  is dropped):

$$\Delta\phi + k_l^2\phi = [\rho/(\lambda+2\mu)]I(x)\exp(-\alpha z) = P(x, z), \quad (7)$$

$$\Delta\psi + k_t^2\psi = 0, \quad (8)$$

where  $k_{l,t} = \omega/c_{l,t}$  are the wave numbers of the longitudinal and transverse acoustical waves,  $c_l = [(\lambda+2\mu)/\rho]^{1/2}$  and  $c_t = (\mu/\rho)^{1/2}$  are their phase velocities, and  $p = \alpha\beta\gamma K/\rho c_V$ , while from relation (4) there are two boundary conditions

$$(\lambda+2\mu)\Delta\phi - 2\mu\left(\frac{\partial^2\phi}{\partial x^2} - \frac{\partial^2\psi}{\partial x\partial z}\right) = \rho I(x), \quad (9)$$

$$\mu\left[\Delta\psi + 2\left(\frac{\partial^2\phi}{\partial x\partial z} - \frac{\partial^2\psi}{\partial z^2}\right)\right] = 0. \quad (10)$$

Using equations (7) and (8), from conditions (9) and (10) one can shift to uniform boundary conditions of

$$k_l^2\phi + 2\left(\frac{\partial^2\phi}{\partial x^2} - \frac{\partial^2\psi}{\partial x\partial z}\right) = 0, \quad (11)$$

$$k_t^2\psi - 2\left(\frac{\partial^2\phi}{\partial x\partial z} - \frac{\partial^2\psi}{\partial z^2}\right) = 0, \quad (12)$$

which will be used below.

Thus, the examined problem about excitation of sound by laser radiation is reduced to solution of the boundary problem (7), (8), (11), and (12). Formal solution of this problem, written using the corresponding components of the Green tensor function [7,12], has the following appearance:

$$\begin{aligned} \phi(x, z) = & -\frac{1}{4\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{\infty} \left[ (e^{-\nu_l|z-z'|} - e^{-\nu_l(z+z')}) e^{ik(x-x')} \nu_l^{-1} - \right. \\ & \left. - 8 \frac{k^2 \nu_l}{F(k)} e^{-\nu_l(z+z') + ik(x-x')} \right] P(x', z') dx' dz' dk, \end{aligned} \quad (13)$$

$$\psi(x, z) = -\frac{i}{\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{\infty} \frac{k(2k^2 - k_l^2)}{F(k)} e^{-\nu_l z' - \nu_l(z+z') + ik(x-x')} P(x', z') dx' dz' dk. \quad (14)$$

Here  $\nu_{l,t} = (k^2 - k_{l,t}^2)^{1/2}$ ,  $F(k) = (2k^2 - k_l^2)^2 - 4k^2 \nu_l \nu_t$ , is the Rayleigh discriminator, integration in semiinfinite limits is performed in terms of the coordinate  $z'$ .

When acoustical field in the wave zone is the only concern, i.e., at  $k_{l,r} \gg 1$ , where  $r$  is the distance from the center of the irradiation field to the observation point, then integration in terms of  $k$  in (13) and (14) is performed by an asymptotic method of contour integration in the complex plane  $k$ . The poles of the subintegral expressions describe in this case the Rayleigh surface waves excited by the laser beam [7] (when writing the expressions (15) and (16), the imprecision in work [7] was corrected. This imprecision was the presence of excess components on the order of  $\alpha$ )

$$\varphi_R = - \frac{i p \Phi(k_R)}{F'(k_R)} \frac{4k_R^2 s}{(\lambda + 2\mu)(\alpha + q)} \exp(\pm i k_R x - qz), \quad (15)$$

$$\psi_R = - \frac{2k_R p \Phi(k_R)}{F'(k_R)} \frac{2k_R^2 - k_l^2}{(\lambda + 2\mu)(\alpha + q)} \exp(\pm i k_R x - sz), \quad (16)$$

where  $k_R$  is the wave number of the Rayleigh wave,  $F'(k_R)$  is the value of the arbitrary Rayleigh discriminator taken at the point  $k = k_R$ ,  $q = (k_R^2 - k_l^2)^{1/2}$ , and

$$s = (k_R^2 - k_l^2)^{1/2}, \quad \Phi(k_R) = \int_{-\infty}^{\infty} I(x) \exp(-i k_R x) dx,$$

while the saddle points are the fields of the longitudinal and transverse volumetric waves (the potentials  $\phi$  and  $\psi$ , respectively). In a polar system of coordinates of  $r$  and  $\theta$  the expressions for the amplitudes of the shifts of the harmonic longitudinal  $u_r$  and transverse  $u_\theta$  volumetric waves coupled with the potentials by the asymptotic relations  $u_r = -(1/c_l)\phi$  and  $u_\theta = (1/c_l)\psi$  assume the appearance:

$$u_r(r, \theta) = - \frac{p \Phi(k_l \sin \theta)}{(\lambda + 2\mu) \sqrt{2\pi k_l r c_l}} \left\{ \frac{k_l \cos \theta}{\alpha^2 + k_l^2 \cos^2 \theta} + \frac{i \cdot 2k_l^4 \sin \theta \sin 2\theta [(k_l/k_l)^2 - \sin^2 \theta]^{1/2}}{(\alpha - i k_l \cos \theta) F(k_l \sin \theta)} \right\} \exp[i(k_l r - \pi/4)], \quad (17)$$

$$u_\theta(r, \theta) = \frac{i p \Phi(k_l \sin \theta)}{(\lambda + 2\mu) 2 \sqrt{2\pi k_l r c_l}} \times \frac{k_l^4 \sin 4\theta}{[\alpha + k_l (\sin^2 \theta - k_l^2/k_l^2)^{1/2}] F(k_l \sin \theta)} \exp[i(k_l r - \pi/4)]. \quad (18)$$

The authors analyze formulas (17) and (18) for different relations of the characteristic width of the laser beam  $\alpha$  and wavelengths of the excited sound  $\lambda_{l,t} = 2\pi/k_{l,t}$ . In this case it is assumed that the inequalities  $\alpha \gg k_{l,t}$ ;  $k_T \gg k_{l,t}$  are almost always valid for metals. When it is recalled that  $\rho \sim \alpha$ ,  $k_T$ , it is easy to see that the expression for the excited transverse waves (18) in the main approximation is not a function of  $\alpha$ ,  $k_T$ , while in the formula for the longitudinal waves (17) only the first component which, is much less than the second at  $\theta \neq 0$ , is a function of  $\alpha$  and  $k_T$ . In the case of quite narrow laser beams this leads to the fact that the radiation of longitudinal waves perpendicular to the surface is negligibly small as compared with their radiation in other directions (Fig. 2a). According to the structure of expression (13), the cause of this is that the field of longitudinal waves observed in the depth of the solid made up of waves directly radiated by the heated field and the waves reflected from the free surface of the metal. The angular relation of the corresponding reflection coefficient is such that in a perpendicular direction there is mutual

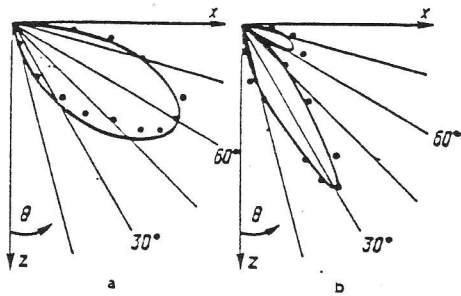


Fig. 2

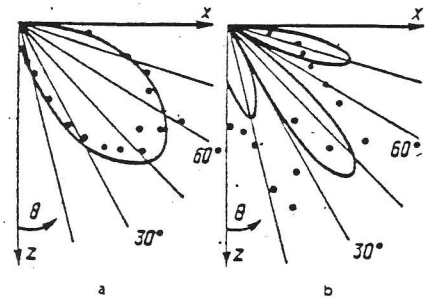


Fig. 3

Fig. 2. Normalized characteristics of the directivity of longitudinal (a) and transverse (b) acoustical waves excited by a narrow laser beam in aluminum ( $c_t = 6.40 \cdot 10^5$  cm/sec and  $c_t = 3.03 \cdot 10^5$  cm/sec): the curves indicate the theory and the points the experiment from [15].

Fig. 3. Normalized characteristics of the directivity of longitudinal acoustical waves excited by a laser beam of finite width  $a$  in aluminum at a frequency of 5 MHz: a)  $a = 1$  mm, b)  $a = 3$  mm; at this frequency  $k_T = 4.3 \cdot 10^3$  cm $^{-1}$ ,  $k_L = 4.9 \cdot 10$  cm $^{-1}$ , and  $k_t = 10.4 \cdot 10$  cm $^{-1}$ ; the curves are the theory and the points are from the experiment in [2].

compensation of these components of the radiation and this compensation is more complete the shallower the depth of localization of the thermal source created by light. Somehow this does not occur in the case of metals. For transverse waves (Fig. 2b) the absence of radiation in a perpendicular direction is caused by the symmetry of the problem and is not a function of the nature of the solid. The cited characteristics of radiation of sound by a narrow laser beam were experimentally observed for aluminum in work [15] (see Fig. 2). Similar patterns were also observed in the three dimensional case of incidence of narrow, axisymmetrical laser beam [11].

In the case of broad laser beams ( $k_L a \gg 1$ ) the first component in (17), despite its smallness, begins to play a substantive role, since the effect of the second member of the sum (17), and expressions (18) in this case is reduced thanks to the  $\Phi(k_L, \sin \theta)$  factors. In the limit at  $k_L a \rightarrow \infty$  this reflects the known fact that in the one-dimensional case in a solid, just as in a liquid, only longitudinal acoustical waves, which are propagated in a perpendicular direction to the surface, are excited [4, 6-8]. It is noted in this respect, that the first component in expression (17) describes that field into which the field (17) and (18) is shifted in the case of a maximal shift from a solid to a liquid, i.e., at  $\mu \rightarrow 0$ .

Figure 3 presents the experimental characteristics of the directivity of longitudinal waves excited by a laser beam of finite width  $a$  with even distribution of the intensity [2]. The figure also presents the corresponding theoretical relations calculated using formula (17). As is easy to see, there is quite good agreement between the theory and the experiment. The small divergence in the case of a broad beam (Fig. 3b) may be explained by the irregularity of the shape of the actual laser beam used in the experiments in work [2] and its cylindrical symmetry.

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