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VERSION

AM (Accepted Manuscript)

LICENCE

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New Type of Vibration Dampers Utilising the Effect of Acoustic 'Black Holes'

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Short title: New type of vibration dampers

Summary

One of the well-known ways of damping resonant flexural vibrations of different engineering structures or their elements, e.g. finite plates or bars, is to reduce reflections of flexural waves from their free edges. In the present paper, a new efficient method of reducing edge reflections is described that utilises gradual change in thickness of a plate or a bar from the value corresponding to the thickness of the basic plate to almost zero. It is proposed to use specific power-law shapes of plates of variable thickness (wedges) that ideally provide zero reflection even for negligibly small material attenuation – the so-called 'acoustic black hole effect'. In particular, for powers $m \geq 2$ - in free wedges, and $m \geq 5/3$ – in immersed wedges, incident flexural waves become trapped near the edge and do not reflect back. Since, because of ever-present edge truncations in real manufactured wedges, the corresponding reflection coefficients are always far from zero, to make up for real wedges and make the systems more efficient it is proposed to deposit absorbing thin layers on wedge surfaces. It is shown that the deposition of thin damping layers on the wedge surfaces can dramatically reduce the reflection coefficients. Thus, the combination of a wedge with power-law profile and of thin damping layers can utilise the acoustic 'black hole' effect resulting in very effective damping systems for flexural vibrations.

PACS no 43.20.Dk, 43.40.At, 43.40.Tm

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1. Introduction

It is well known that one of the efficient ways of damping resonant flexural vibrations of different engineering structures or their elements, e.g. finite plates or bars, is to reduce reflections of flexural waves from their free edges. This can be achieved, for example, by introducing graded impedance interfaces along with placing damping material at the edges [1]. In addition to engineering applications, such structures, that simulate semi-infinite or infinite plates or bars, are of great interest for some physical experiments, e.g. on Anderson localisation of bending waves [2]. The main difficulty in this approach is to create suitable graded impedance interfaces. The authors of the paper [1] have suggested to attach to the end of the basic steel plate the graded impedance interface consisting of the combination of the finite aluminium plate, Lucite plate, and composite plate, all of them being of the same thickness as the basic steel plate. Using such a system the authors demonstrated experimentally that as much as 60-80 % of the energy could be damped for frequencies in the range of 2-10 kHz. This was much superior to the commonly used method of damping by embedding of edges of plates in sand, which results only in at most 30 % of the energy to be damped at frequencies above 2 kHz [3]. In spite of the encouraging results of reference [1], the technological difficulties of building and attaching several additional plates of different materials to the basic plate are restrictive for applying this method for practical purposes.

The alternative way of creating matching impedance interfaces is considered in the present paper. Instead of using combinations of finite plates made of different materials, it is proposed to utilise gradual change in thickness of a plate or a bar from the value corresponding to the thickness of the basic plate to almost zero. In other words, it is proposed to use elastic wedges as gradual impedance interfaces for flexural waves in plates and bars. The above-mentioned gradual change in thickness of a plate or a bar is to be made according

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to the special laws that ideally provide zero reflection even for negligibly small material attenuation – the so-called ‘acoustic black hole effect’. To make up for real wedges and to make the damping systems under consideration more efficient it is proposed to deposit thin absorbing layers on wedge surfaces near the edges.

To appreciate the ‘acoustic black hole effect’ for flexural waves one has to recall that when flexural elastic waves propagate towards edges of elastic plates of variable thickness gradually decreasing to zero, i.e. towards edges of thin elastic wedges of arbitrary shape, they slow down and their amplitudes grow. After reflection from the edge, with the module of reflection coefficient normally being equal to unity, the whole process repeats in the opposite direction [4-8]. Especially interesting phenomena may occur in the special case of wedge edges having cross sections described by a power law relationship between the local thickness h and the distance from the edge x: $h(x)= \varepsilon x^m$ (see Figure 1), where m is a positive rational number and ε is a constant [7-9]. In particular, for $m \geq 2$ - in free wedges, and for $m \geq 5/3$ – in immersed wedges, the flexural waves incident at an arbitrary angle upon a sharp edge can become trapped near the very edge and therefore never reflect back [8,9]. Thus, such structures materialise acoustic ‘black holes’, if to use the analogy with corresponding astrophysical objects. In the case of localised flexural waves propagating along edges of such wedges (these waves are also known as wedge acoustic waves) the phenomenon of acoustic ‘black holes’ implies that wedge acoustic wave velocities in such structures become equal to zero [7,8]. This reflects the fact that the incident wave energy becomes trapped near the edges and wedge acoustic waves simply do not propagate.

Note that ‘black hole’ effects are known not only for flexural waves in wedges of power-law profile. As has been demonstrated theoretically by several authors, the potential possibility of the effects of zero reflection can also exist for wave phenomena of different physical nature. In particular, these may occur for underwater sound propagation in a layer

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with sound velocity profile linearly decreasing to zero with increasing depth [10]. Similarly, the reflection may be absent for internal waves in a horizontally inhomogeneous stratified fluid [11,12] and for particle scattering in quantum mechanics [13]. For seismic interface waves propagating in soft marine sediments with power-law shear speed exponent equal to unity it is the wave velocity that may be equal to zero [14,15], exactly as in the case of the above-mentioned wedge acoustic waves in elastic wedges with power-law profile [7,8].

One must mention, however, that, whereas the conditions providing zero wave reflection can hardly be found in real ocean environment or for real atomic potentials, wedges of arbitrary power-law profile are relatively easy to manufacture. Thus, elastic solid wedges give the unique opportunity to materialise the above-mentioned zero-reflection effects normally associated with ‘black holes’ and to use them for practical purposes.

The unusual effect of power-low profile on flexural wave propagation in elastic wedges has attracted some initial attention in respect of their possible applications as vibration absorbers. Mironov [9] was the first to have pointed out that a flexural wave does not reflect from the edge of a quadratically-shaped wedge in vacuum ($m = 2$), so that even a negligibly small material attenuation may cause all the wave energy to be absorbed near the edge. Unfortunately, because of the deviations of manufactured wedges from the ideal power-law shapes, largely due to ever-present truncations of the wedge edges, real reflection coefficients in such homogeneous wedges are always far from zero [9]. Therefore, in practice such wedges can not be used as vibration absorbers.

As it will be demonstrated in the following sections, the situation can be radically improved by modifying the wedge surfaces. In particular, the deposition of thin damping layers on the surfaces of wedges having power-law profile can drastically reduce the reflection coefficients. Thus, the combination of a wedge with power-law profile and of thin damping layers can result in very effective damping systems for flexural vibrations.

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2. Theory

2.1. Flexural waves in free wedges of power-law profile

To explain the basic physics of the phenomenon let us consider the above-mentioned simplest case of a plane flexural wave propagating in the normal direction towards the edge of a free wedge described by a power-law relationship $h(x)= \varepsilon x^m$, where m is a positive rational number and ε is a constant (see Figure 1). Since flexural wave propagation in such wedges can be described in the geometrical acoustics approximation (see [4-9] for more detail), the integrated wave phase Φ resulting from the wave propagation from an arbitrary point x on the wedge medium plane to the wedge tip ($x = 0$) can be written in the form:

$$\Phi = \int_0^x k(x)dx \, . \qquad (1)$$

Here $k(x)$ is a local wavenumber of a flexural wave for a wedge in contact with vacuum: $k(x) =12^{1/4} k_p^{1/2}(\varepsilon x^m)^{-1/2}$, where $k_p = \omega/c_p$ is the wavenumber of a symmetrical plate wave, $c_p = 2c_t(1-c_t^2/c_l^2)^{1/2}$ is its phase velocity, and c_l and c_t are longitudinal and shear wave velocities in a wedge material, and $\omega =2\pi f$ is circular frequency. One can easily check that the integral in equation (1) diverges for $m \geq 2$. This means that the phase Φ becomes infinite under these circumstances and the wave never reaches the edge. Therefore, it never reflects back either, i.e. the wave becomes trapped, thus indicating that the above mentioned ideal wedges represent acoustic ‘black holes’ for incident flexural waves.

Real fabricated wedges, however, always have truncated edges. And this adversely affects their performance as potential vibration dampers. If for ideal wedges of power-law shape (with $m \geq 2$) it follows from equation (1) that even an infinitely small material attenuation, described by the imaginary part of $k(x)$, would be sufficient for the total wave energy to be absorbed, this is not so for truncated wedges. Indeed, for truncated wedges the lower

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integration limit in equation (1) must be changed from *0* to a certain value x_0 describing the length of truncation, thus resulting in the amplitude of the total reflection coefficient R_0 being expressed in the form [9]

$$R_0 = \exp(-2\int_{x_0}^{x} \operatorname{Im} k(x)dx) . \qquad (2)$$

According to equation (2), for typical values of attenuation in the wedge materials, even very small truncations x_0 result in R_0 becoming as large as 50-70 %.

2.2 Waves in wedges covered by thin absorbing layers

To improve the situation for real wedges (with truncations), let us now consider covering the wedge surfaces by thin damping layers (films) of thickness δ, e.g. by polymeric films. Note in this connection that the idea of applying absorbing layers for damping flexural vibrations of plates and beams is not new and has been successfully used since the 50-ies (see, e.g. [16-20]). The new aspect of this idea, which is discussed in the present paper, is the use of such absorbing layers in combination with the specific power-law geometry of a plate of variable thickness (a wedge) to achieve maximum damping.

Two types of wedge geometry will be considered: a symmetric wedge (Figure 2a) and a non-symmetric wedge bounded by a plain surface at one of the sides (Figure 2b). For each of these cases either two or only one of the sides can be covered by absorbing layers. Note that non-symmetric wedges are easier to manufacture as only one curved surface should be made. Non-symmetric wedges also have technological advantage in depositing absorbing layers: the latter can be deposited on a flat surface, which is much easier. From the point of view of theoretical description, there is no difference between symmetrical and non-symmetrical wedges as long as geometrical acoustics approximation is concerned and the wedge local

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thickness $h(x) = \varepsilon x^m$ is much less than the flexural wavelength. A possible application of a non-symmetric quadratic wedge-like damper covered by an absorbing film on one side is illustrated on Figure 3. The thick end of a damper is glued to the edge of a basic plate reflections from which are to be suppressed. To avoid reflections from the boundary between the plate and the wedge that may be caused by a sudden change in geometry the shape of the wedge is smoothened so that it gradually transforms into the plate with the thickness equal to the thickness of the basic plate.

Let us now discuss the effect of absorbing layers (films) covering wedge surfaces on flexural wave reflection. In what follows we consider only one possible film-induced attenuation mechanism – the one associated with in-plane deformations of the film under the impact of flexural waves. Such deformations occur on wedge surfaces as a result of the well known relationship between flexural displacements u_z and longitudinal displacements u_x in a plate: $u_x = -z(\partial^2 u_z/\partial x^2)$. Not specifying physical mechanism of the material damping in the film material, we assume for simplicity that it is linearly dependent on frequency, with non-dimensional constant ν being the energy loss factor, or simply the loss factor.

To analyse the effect of thin absorbing films on flexural wave propagation in a wedge in the framework of geometrical acoustics approximation one should consider first the effect of such films on flexural wave propagation in plates of constant thickness. The latter problem can be approached in different ways. For example, it can be solved using the non-classical boundary conditions taking into account the so-called ‘surface effects’ [21,22]. Alternatively, the energy perturbation method developed by Auld [23] can be used. However, the simplest way is to use the already known solutions for plates covered by damping layers of arbitrary thickness obtained by different authors with regard to description of damped vibrations in such sandwich plates [16-20].

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In particular, for a plate of constant thickness h covered by a visco-elastic layer of thickness δ on one of the surfaces the following expression for the additional loss factor ξ can be obtained (see, e.g. [17, 19]):

$$\xi = \frac{\nu}{1+(\alpha_2\beta_2(\alpha_2{}^2+12\alpha_{21}{}^2))^{-1}} . \qquad (3)$$

Here ν is the loss factor of the material of the visco-elastic layer, $\alpha_2 = \delta/h$, $\beta_2 = E_2/E_1$, and $\alpha_{21} = (1+\alpha_2)/2$, where E_1 and E_2 are respectively theYoung's moduli of the plate and of the visco-elastic layer. Note that formula (3) has been derived from the original more general expression of Oberst [16] by assuming that $\alpha_2\beta_2 = (\delta/h)(E_2/E_1) << 1$, which is almost always the case in practical applications. Note that equation (3) can be rewritten in the equivalent form [17] that is also used widely:

$$\xi = \frac{\nu\beta_2\alpha_2(3+6\alpha_2+4\alpha_2{}^2)}{1+\beta_2\alpha_2(3+6\alpha_2{}^2+4\alpha_2{}^2)} . \qquad (4)$$

Further simplification of the expression for the layer-induced additional loss factor ξ can be obtained in the case of very thin absorbing layers, $\alpha_2 = \delta/h << 1$. In this case the following linearised expression versus $\alpha_2 = \delta/h$ can be derived from equations (3) or (4):

$$\xi = 3\alpha_2\beta_2\nu . \qquad (5)$$

Using equation (5) and assuming that both surfaces of a wedge are covered by absorbing layers (see Figure 2a), one can arrive to the formula that takes into account the effects of both

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damping layers and of the wedge material and geometry on the imaginary part of a flexural wavenumber, Im $k(x)$:

$$\operatorname{Im} k(x) = \left[\frac{12^{1/4} k_p^{\ 1/2}}{h^{1/2}(x)}\right]\left[\frac{\eta}{4} + \frac{3}{2}\frac{\delta}{h(x)}\frac{E_2}{E_1}\nu\right]. \tag{6}$$

Here $h(x)$ is the local thickness of the wedge, and η is the loss factor of the wedge material. The additional flexural wave attenuation caused by the damping layers and described by the second term in (6) is proportional to the ratio of the film thickness δ to the plate's local thickness $h(x)$, and to the ratio of the Young's moduli, E_2/E_1, of the film and plate respectively.

In the case of a wedge with only one surface covered by a damping layer (Figure 2b) the additional damping is two times smaller and the expression for Im $k(x)$ has the form

$$\operatorname{Im} k(x) = \left[\frac{12^{1/4} k_p^{\ 1/2}}{h^{1/2}(x)}\right]\left[\frac{\eta}{4} + \frac{3}{4}\frac{\delta}{h(x)}\frac{E_2}{E_1}\nu\right]. \tag{7}$$

Let us consider a wedge of quadratic shape, i.e. $h(x) = \varepsilon x^2$, covered by damping layers on both surfaces (Figure 2a). Then, substituting equation (6) into (2) and performing the integration, one can obtain the following analytical expression for the resulting reflection coefficient R_0:

$$R_0 = exp(-2\mu_1 - 2\mu_2), \tag{8}$$

where

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$$\mu_1 = \frac{12^{1/4} k_p^{\ 1/2} \eta}{4\varepsilon^{1/2}} \ln\left(\frac{x}{x_0}\right), \tag{9}$$

$$\mu_2 = \frac{3 \cdot 12^{1/4} k_p^{\ 1/2} \nu \delta}{4\varepsilon^{3/2}} \frac{E_2}{E_1} \frac{1}{x_0^{\ 2}} \left(1 - \frac{x_0^{\ 2}}{x^2}\right). \tag{10}$$

As expected, in the absence of the damping film *(*$\delta = 0$ *or* $\nu = 0$, and hence $\mu_2 = 0$), the equations (8)-(10) reduce to the results obtained in [9] (where a typographical misprint has been noticed). If the damping film is present *(*$\delta \neq 0$ and $\nu \neq 0$), it brings the additional reduction of the reflection coefficient that depends on the film loss factor ν and on the other geometrical and physical parameters of the wedge and film.

In the case of a wedge of quadratic shape covered by damping layers on one surface only (Figure 2b) equations (8) and (9) remain unchanged, whereas equation (10) should be replaced by

$$\mu_2 = \frac{3 \cdot 12^{1/4} k_p^{\ 1/2} \nu \delta}{8\varepsilon^{3/2}} \frac{E_2}{E_1} \frac{1}{x_0^{\ 2}} \left(1 - \frac{x_0^{\ 2}}{x^2}\right). \tag{11}$$

In deriving equations (8)-(11) the effect of thin absorbing layers on flexural wave velocity has been neglected.

Note that geometrical acoustics approximation for the above-mentioned quadratic wedges ($m = 2$) is valid for all x provided that the following applicability condition is satisfied:

$$\frac{\omega}{c_t \varepsilon} >> 1, \tag{12}$$

where c_t is shear wave velocity in the wedge material. For the majority of practical situations this condition can be easily satisfied even at very low frequencies.

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2.3 Waves in wedges covered by absorbing layers of arbitrary thickness

The expressions (8)-(11) for the reflection coefficient of flexural waves in quadratic wedges covered by absorbing layers are valid if the thickness of layers is much smaller that the local thickness of the main wedge. Therefore, although being very useful and simple, these expressions can only be applicable for wedges covered by very thin absorbing layers (thin films). But even so, their applicability fails near wedge edges, where even very thin films may become comparable or even larger than wedge local thickness. Obviously, for equations (8)-(11) to be applicable for a wedge with truncation length x_0, the wedge local thickness at the point of truncation $h(x_0) = \varepsilon x_0^2$ must be much larger than the film thickness δ:

$$\varepsilon x_0^2 >> \delta . \tag{13}$$

In practice this means that the film thickness δ should be at least 3-4 times smaller than εx_0^2.

To extend the analysis to smaller values of wedge truncation and/or to thicker films one has to consider flexural wave propagation in wedges covered by damping layers of arbitrary thickness. As the wave phase behaviour near wedge edges is important for materialising the acoustic ‘black hole’ effect, the influence of the film thickness on flexural wave velocity dispersion in this case should be taken into account as well. Therefore, equations (3) and (4) (as well as the original formula derived by Oberst [16]), that only take into account the effect of an absorbing layer on the resulting loss factor, can no longer be used.

A more general analysis of the effect of damping layers on complex flexural rigidity of a sandwich plate with arbitrary parameters has been carried out in reference [17]. In what follows we consider a wedge with only one surface covered by an absorbing layer. Then,

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using the results of [17] for complex flexural rigidity of a sandwich plate in the case $\beta_2 = E_2/E_1 << 1$ and representing the surface mass density ρ_s of such plate as $\rho_s = \rho_w h + \rho_l \delta$, where ρ_w and ρ_l are the mass densities of the wedge material and of the absorbing layer respectively, one can derive the following expression for the complex wavenumber k of a flexural wave propagating in a plate covered by an absorbing layer of arbitrary thickness δ:

$$k = \frac{12^{1/4} k_p^{1/2}}{h^{1/2}} \left\{ \frac{1 + \tilde{\rho}\alpha_2}{\left[(1 - i\eta) + \beta_2 \alpha_2 (1 - i\nu)(3 + 6\alpha_2 + 4\alpha_2^2)\right]} \right\}^{1/4} . \tag{14}$$

Here $k_p = \omega/c_p$ is the wavenumber of a symmetrical plate wave for the main wedge material, $c_p = 2c_t(1-c_t^2/c_l^2)^{1/2}$ is the plate wave velocity, c_l and c_t are longitudinal and shear wave velocities in a wedge material (see also the discussion of equation (1)), and $\tilde{\rho} = \rho_l / \rho_w$. Other notations in equation (14) are the same as in the previous sections.

Using the fact that in the majority of practical situations $\eta << 1$ and $\nu << 1$, equation (14) can be simplified to the form

$$k = \frac{12^{1/4} k_p^{1/2} (1 + \tilde{\rho}\alpha_2)^{1/4}}{h^{1/2} [1 + \beta_2 \alpha_2 (3 + 6\alpha_2 + 4\alpha_2^2)]^{1/4}} \left\{ 1 + i \frac{1}{4} \frac{\eta + \nu \beta_2 \alpha_2 (3 + 6\alpha_2 + 4\alpha_2^2)}{[1 + \beta_2 \alpha_2 (3 + 6\alpha_2 + 4\alpha_2^2)]} \right\} . \tag{15}$$

From equation (15) it follows that the expression for $\operatorname{Im} k$ can be written as

$$\operatorname{Im} k = \frac{12^{1/4} k_p^{1/2} (1 + \tilde{\rho}\alpha_2)^{1/4} [\eta + \nu \beta_2 \alpha_2 (3 + 6\alpha_2 + 4\alpha_2^2)]}{4 h^{1/2} [1 + \beta_2 \alpha_2 (3 + 6\alpha_2 + 4\alpha_2^2)]^{5/4}} . \tag{16}$$

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As expected, in the limiting case of $\alpha_2 = \delta/h << 1$ equation (16) reduces to the earlier derived equation (7) for a wedge covered by a thin absorbing film on one of its surfaces.

Assuming that the wedge has a quadratic shape, i.e. $\alpha_2 = \alpha_2(x) = \delta/h(x) = \delta/\varepsilon x^2$ (see Fig.2b), and substituting equation (16) into equation (2), one can obtain the following expression for the resulting reflection coefficient R_0:

$$R_0 = \exp\left\{-\int_{x_0}^{x} \frac{12^{1/4} k_p^{1/2} (1+\tilde{\rho}\alpha_2(x))^{1/4} [\eta + \nu\beta_2\alpha_2(x)(3+6\alpha_2(x)+4\alpha_2(x)^2)]}{2h(x)^{1/2}[1+\beta_2\alpha_2(x)(3+6\alpha_2(x)+4\alpha_2(x)^2)]^{5/4}} dx\right\}. \qquad (17)$$

Unfortunately, equation (17) can not be simplified any further, and the integration in it should be carry out numerically.

3. Numerical examples

Let us first perform calculations according to the simplest formulae (8)-(11) and let us choose for illustration purposes the following values of the film parameters: $\nu = 0.25$ (i.e., consider the film as being highly absorbing), $E_2/E_1 = 0.3$ and $\delta = 15\ \mu m$. Let the parameters of the quadratically shaped wedge be: $\varepsilon = 0.05\ m^{-1}$, $\eta = 0.01$, $x_0 = 2\ cm$, $x = 50\ cm$ and $c_p = 3000\ m/s$. Then, e.g. for a wedge covered by absorbing films on both sides and at the frequency $f = 10\ kHz$ it follows from equations (8)-(10) that in the presence of the damping film $R_0 = 0.022$ (i.e. $2.2\ \%$), whereas in the absence of the damping film $R_0 = 0.542$ (i.e. $54.2\ \%$). Thus, in the presence of the damping film the value of the reflection coefficient is much smaller than for a wedge with the same value of truncation, but without a film. Obviously, it is both the specific geometrical properties of a quadratically-shaped wedge in respect of wave propagation and the effect of thin damping layers that result in such

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a significant reduction of the reflection coefficient. Note that almost all absorption of the incident wave energy takes place in the vicinity of the sharp edge of a wedge.

The effect of wedge truncation length x_0 on the reflection coefficient R_0 in the above example is shown on Figure 4 for the value of film material loss factor $\nu = 0.15$ and for the values of $\beta_2 = E_2/E_1$ equal to 0.1, 0.2 and 0.3. Calculations have been carried out according to equations (8)-(10). For comparison, the curve corresponding to the wedge not covered by absorbing films is shown on the same Figure as well. One can see that the behaviour of the reflection coefficient R_0 as a function of x_0 is strongly influenced by the parameter $\beta_2 = E_2/E_1$ describing relative stiffness of the absorbing film. The larger the film stiffness the higher values of truncation x_0 can be allowed to keep the reflection coefficient low. One should keep in mind, however, that, according to the applicability condition for equations (8)-(11) following from the limitation on the absorbing film thickness (see equation (13)), the results for R_0 obtained for very small values of truncation x_0 are not accurate. This will be discussed later in more detail, when numerical calculations according to the more precise equation (17) will be considered, instead of using simple analytical expressions (8)-(11)

The effect on the reflection coefficient R_0 of partial covering of a wedge by absorbing thin films is shown on Figure 5 for the values of the film material loss factor ν equal to 0.10, 0.15, 0.20 and 0.25 respectively. It is assumed that the films are partly covering both sides of the wedge with the total length $x = 50\ cm$, starting from its truncated edge and finishing at the symmetrically located points on the surfaces described by the coordinate x_1 of their projections on the axis x. The value of the wedge truncation length x_0 is $2\ cm$, and the value of relative film stiffness $\beta_2 = E_2/E_1$ is 0.2. Other parameters are the same as in Figure 4. Calculations have been carried out according to equations (8)-(10), where x in equation (10) has been replaced by x_1. One can see that the effect of the film length, characterised by

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the projection x_1, is noticeable only for small x_1. This reflects the fact that most of the wave energy loss occurs in the vicinity of the wedge edge. Naturally, the lower reflection coefficients are achieved for higher values of the film material loss factor.

Figure 6 illustrates the frequency dependence of the wedge reflection coefficient R_0 in the example considered for the wedge covered by absorbing thin films with the values of film material loss factor ν equal to 0.05, 0.10 and 0.15 respectively and for the uncovered wedge. Other parameters are the same as in Figure 5. As one can see, for all values of the film material loss factor the reflection coefficients decrease with the increase of frequency. Although for larger values of the film material loss factor (ν = 0.15) such increase is more rapid, the considered wedge-like structure seems to be more efficient as a damper at relatively high frequencies (higher than 5 kHz in the example considered).

Let us now turn to the calculations of the reflection coefficient R_0 according to the more precise formula (17), where the integration should now be performed numerically.

Figure 7 shows the reflection coefficient R_0 at frequency $f = 10\ kHz$ as a function of the wedge truncation length x_0 for the non-symmetric wedge covered by the absorbing film on one surface only (see Figure 2b). The parameters of the wedge and film are: $\varepsilon = 0.05$, $\delta = 10\ \mu m$, $\nu = 0.2$, $\eta = 0.01$, $x_0 = 2\ cm$ and $E_2/E_1 = 0.3$. A dotted curve corresponds to the reflection coefficient calculated according to the simple analytical expressions (8), (9) and (11). A solid curve corresponds to the reflection coefficient calculated according to the more precise formula (17). For comparison, the behaviour of the reflection coefficient for an uncovered wedge is shown on Figure 7 as well. It is clearly seen that the curves calculated according to the simplified equations and to the more precise formula (17) almost coincide with each other everywhere except very small values of x_0, where the approximation of thin film becomes invalid. As expected, the values of R_0 calculated for very small x_0 according to equation (17) are always larger than zero since even for the ideal case of the absence of

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wedge truncation ($x_0 = 0$) the presence of the film implies that as $x \to 0$ the wedge covered by a film reduces gradually to a film of constant thickness only, and the reflection coefficient is determined entirely by an absorbing film. Nevertheless, the curve calculated according to the simplified analytical formulae (equations (8), (9) and (11)) represents a very good approximation of the reflection coefficient for realistic values of wedge truncation x_0.

The reflection coefficient for a wedge covered by absorbing films of increasing thickness is shown on Figure 8 as a function of x_0. The values of the film thickness δ are: 10, 100 and 200 μm. Calculations have been carried out using equation (17). Other parameters of the wedge and film are the same as on Figure 7. As expected, thicker films result in lower values of the reflection coefficient R_0 in a wider range of wedge truncation length x_0.

Finally, Figure 9 shows the effect of relative film stiffness $\beta_2 = E_2/E_1$ on the behaviour of the reflection coefficient R_0 for a wedge covered by a thick absorbing film ($\delta = 200\ \mu m$). The values of $\beta_2 = E_2/E_1$ are 0.3, 0.03, and 0.003. All other parameters are the same as on Figure 7. Calculations have been carried out according to equation (17). It can be seen that in the considered case of thick absorbing films even a very 'soft' film results in significant reduction of the reflection coefficient for small values of the truncation length x_0.

4. Conclusions

Some theoretical results have been reported on the possible practical utilisation of the acoustic 'black hole' effect for flexural waves propagating in quadratically-shaped elastic wedges covered by thin absorbing layers. It has been demonstrated that the presence of thin absorbing layers on the surfaces of quadratically-shaped elastic wedges can result in very low reflection coefficients of flexural waves from their edges even in the presence of edge truncations. As a result of this, such wedges can be used as efficient devices providing very low reflection coefficients from edges of structural elements to be damped, e.g. finite plates

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or bars. This in turn would result in very efficient damping of basic structures' resonant vibrations. The additional advantage of using the proposed wedge-like vibration dampers over traditional types of vibration absorbers lays in the fact that wedge-like dampers are compact and relatively easy to manufacture. They can be produced as separate parts to be attached to the structure to be damped, or they can be integrated into such a structure on a design stage as its inseparable components.

In spite of the promising theoretical results described in this paper, further theoretical and experimental investigations are needed to validate the principle and to explore the most efficient ways of developing the proposed wedge-like dampers of flexural vibrations.

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SEPARATE LIST OF FIGURE CAPTIONS

Figure 1. Truncated elastic wedge of power-law profile.

Figure 2. Truncated quadratic wedges covered by thin damping layers: *a*) Symmetric wedge covered on both sides; *b*) Non-symmetric wedge that is covered on one side only.

Figure 3. Application of a non-symmetric quadratic wedge-like damper covered by an absorbing film on one side (*a*) for suppression of flexural wave reflections from one of the edges of an elastic plate (*b*).

Figure 4. Effect of wedge truncation length x_0 (in *m*) on the reflection coefficient R_0: solid curve corresponds to an uncovered wedge, dashed, dotted and dash-dotted curves correspond to a wedge covered by absorbing films with the values of relative film stiffness E_2/E_1 equal to 0.1, 0.2 and 0.3.

Figure 5. Effect of partial covering of a wedge by absorbing thin films on the reflection coefficient R_0: solid, dashed, dotted and dash-dotted curves correspond to the values of the film material loss factor ν equal to 0.10, 0.15, 0.20 and 0.25 respectively; the value of the wedge truncation length x_0 is $2\ cm$, and the value of relative film stiffness $\beta_2 = E_2/E_1$ is 0.2. Other parameters are the same as in Figure 4.

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Figure 6. Frequency dependence of the reflection coefficient R_0: solid curve corresponds to the uncovered wedge, dashed, dotted, and dash-dotted curves correspond to wedges covered by thin absorbing films with the values of film material loss factor ν equal to 0.05, 0.10 and 0.15 respectively. Other parameters are the same as in Figure 5.

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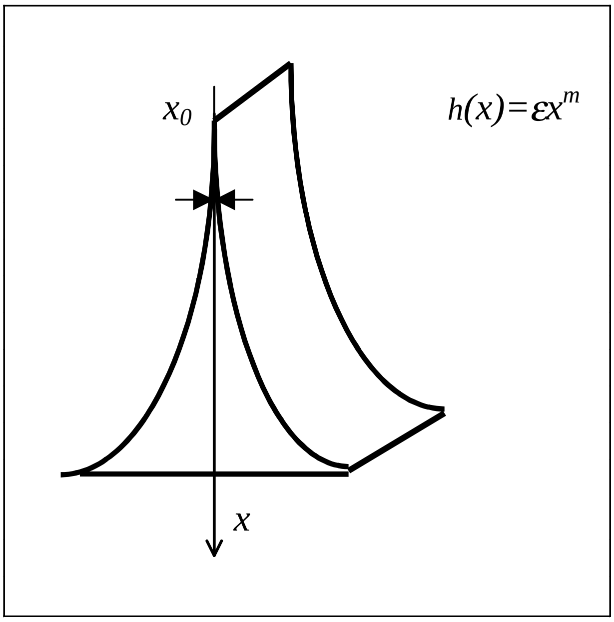
FIGURES



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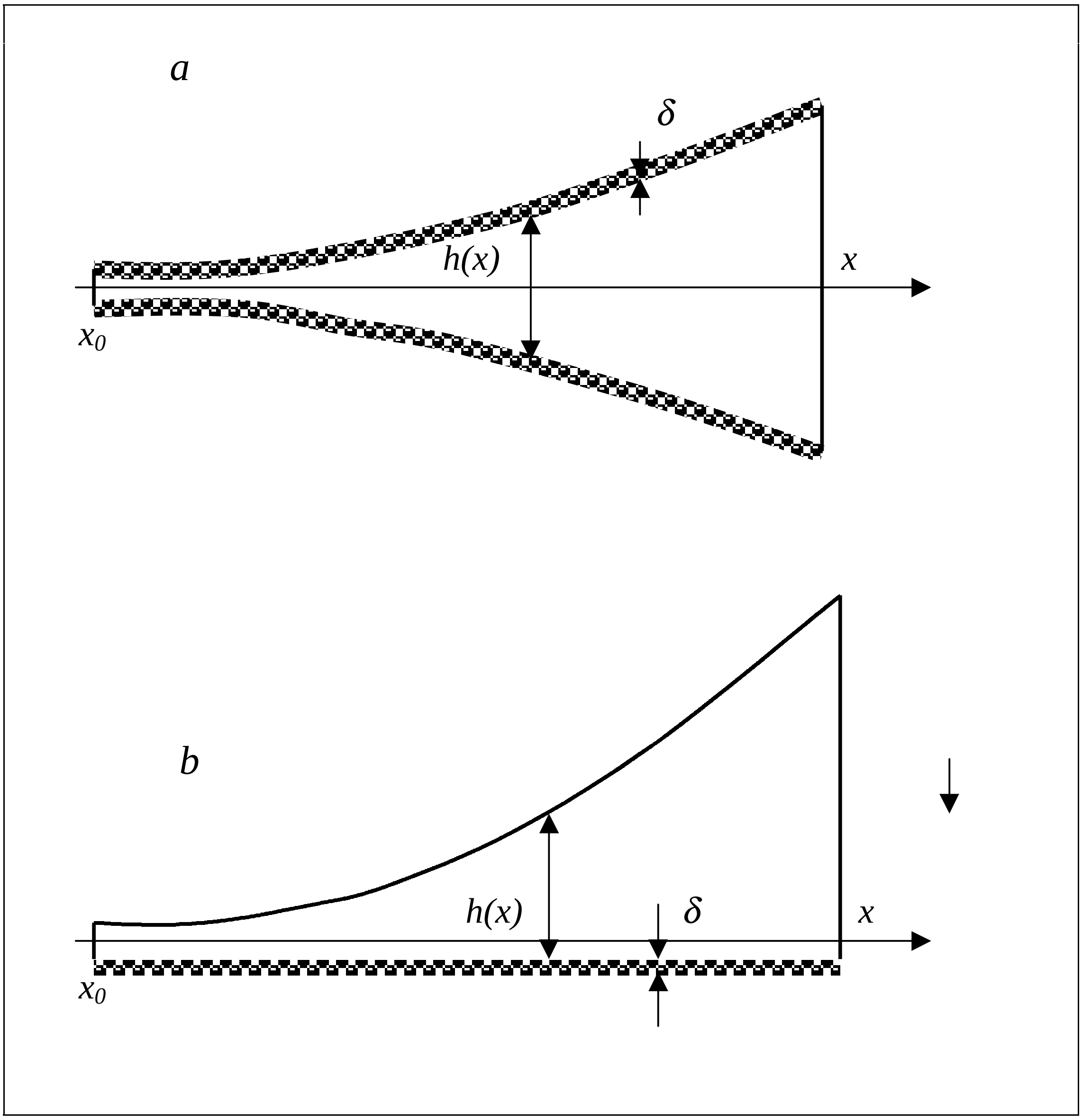

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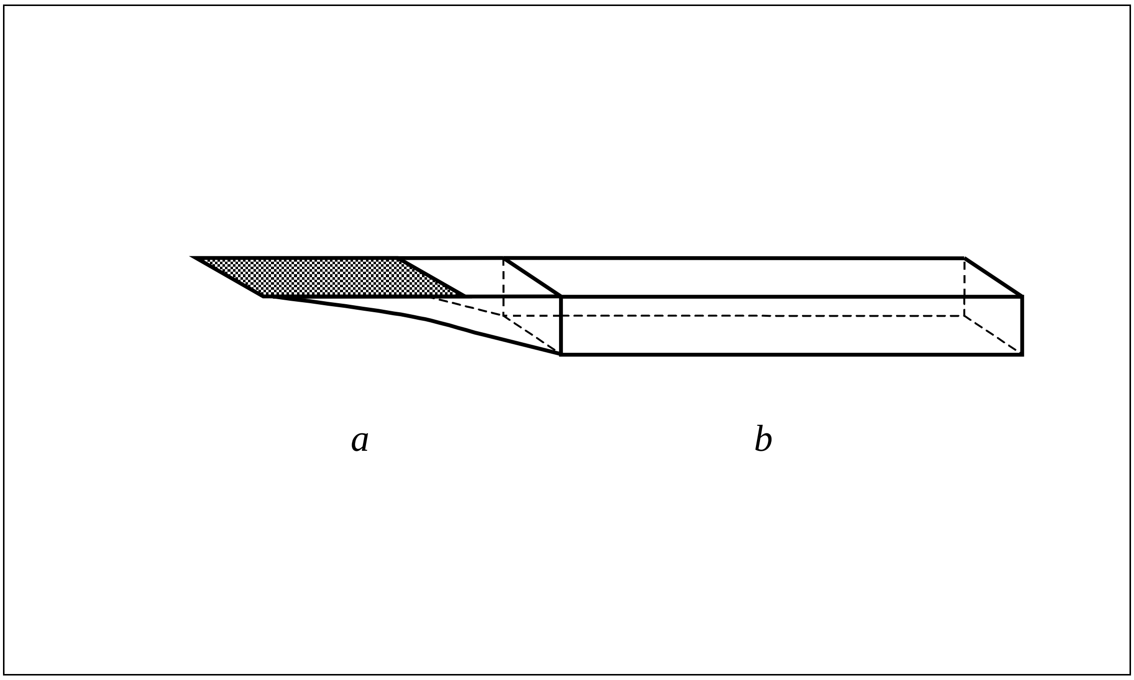

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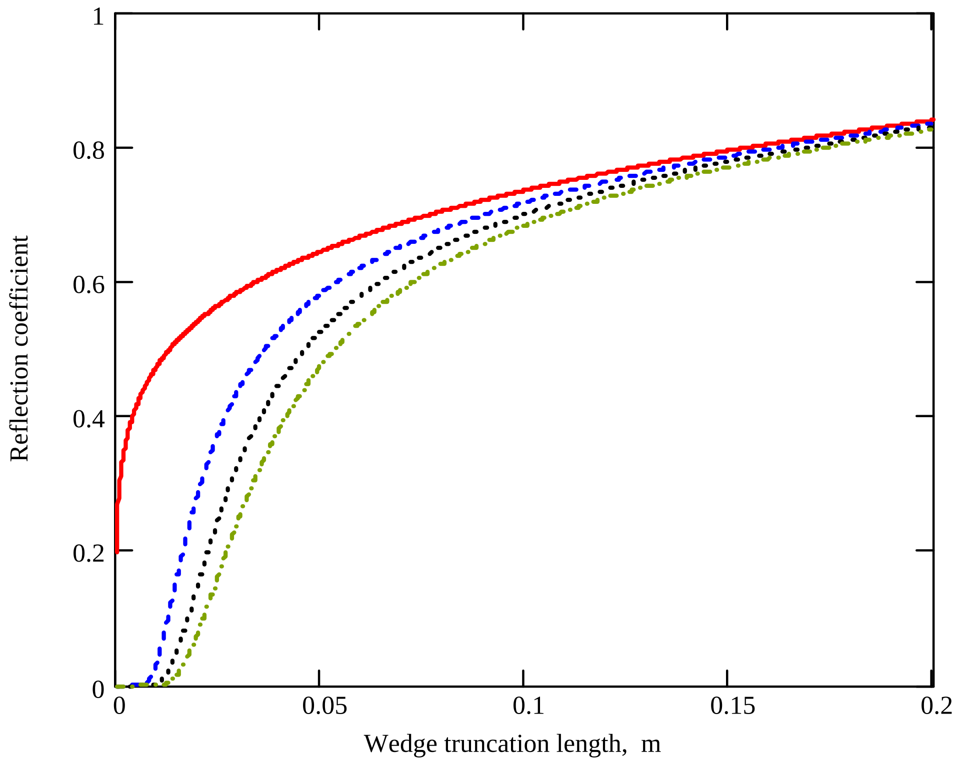

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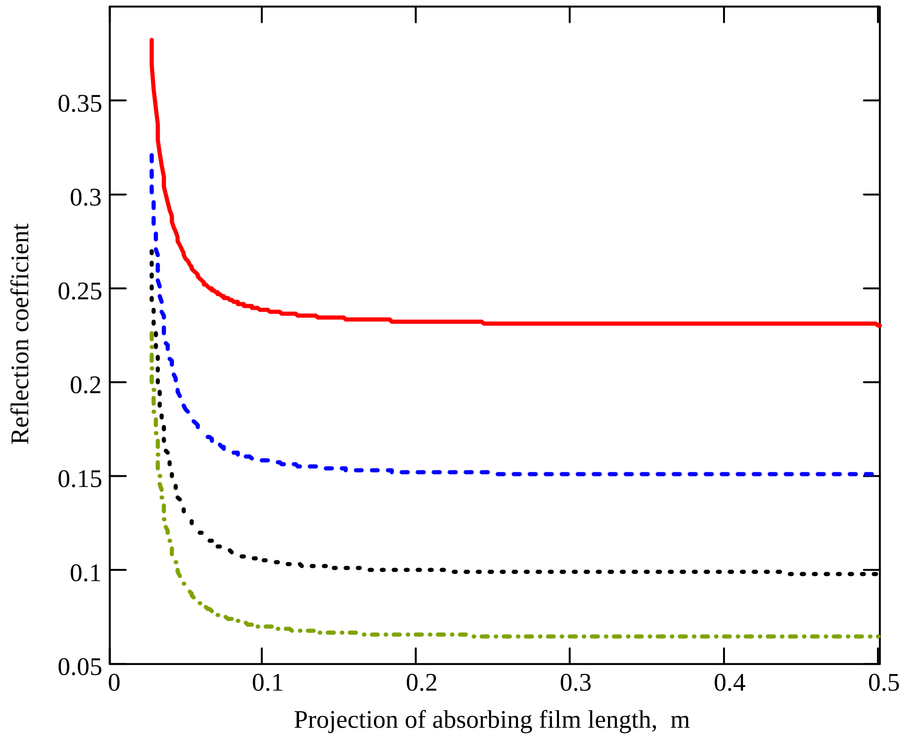

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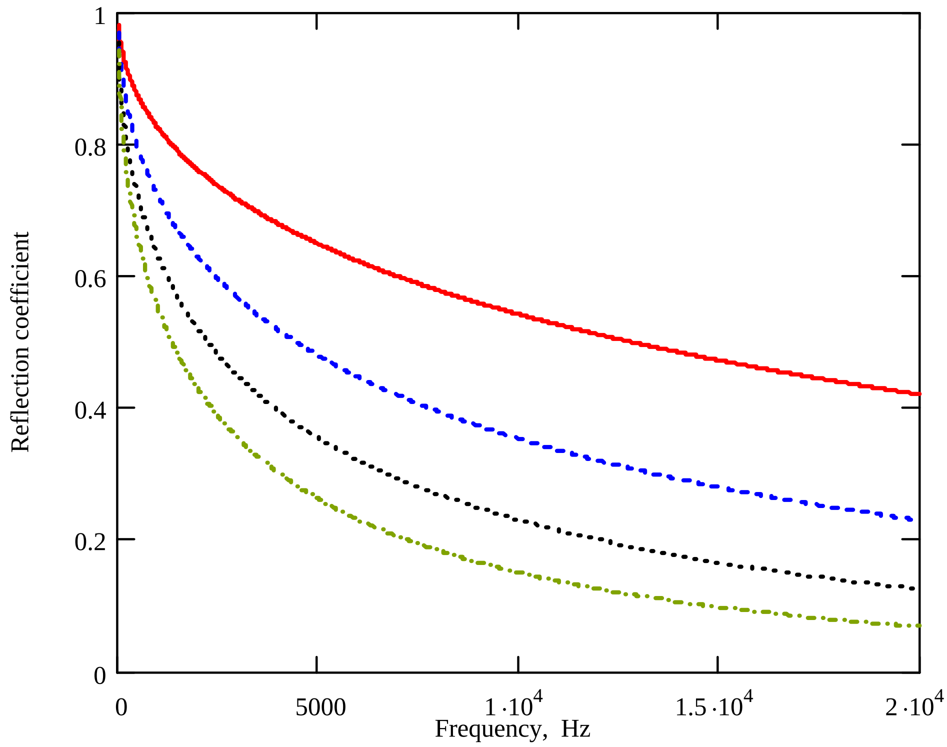

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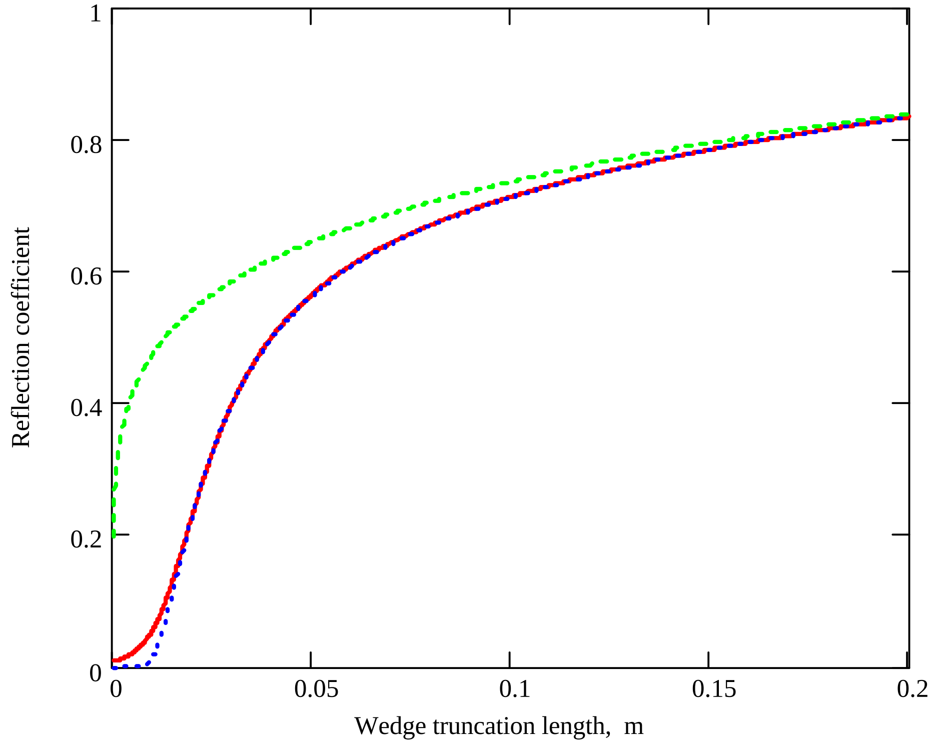

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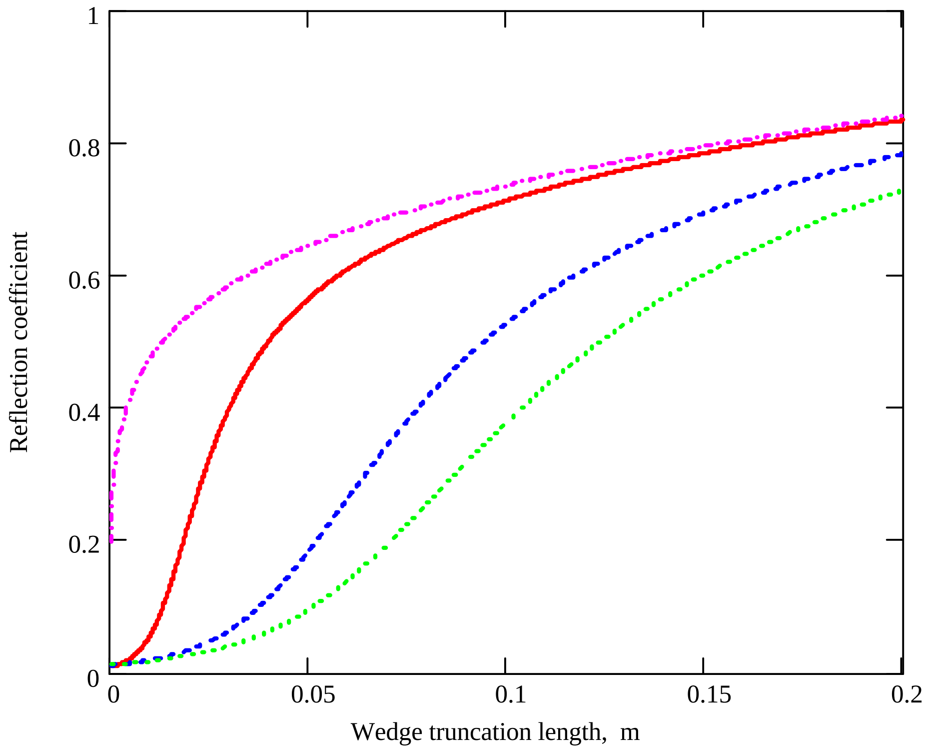

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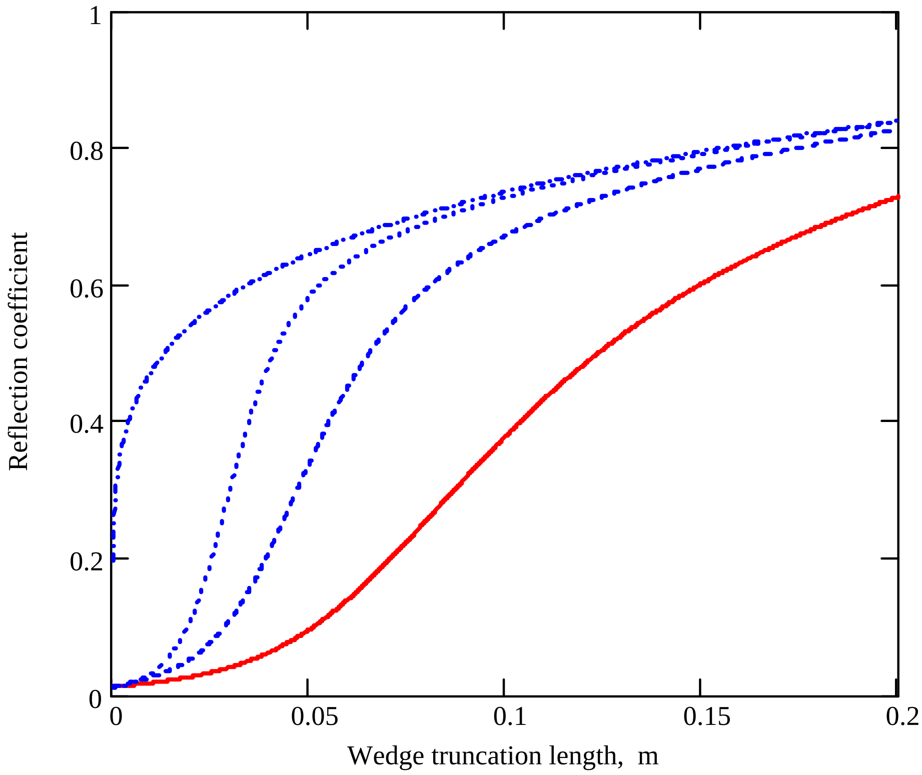

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