DAMPING OF FLEXURAL VIBRATIONS IN TAPERED RODS OF POWER-LAW PROFILE: EXPERIMENTAL STUDIES

V Kralovic Department of Aeronautical and Automotive Engineering,
VV Krylov Loughborough University,
Loughborough, Leicestershire LE11 3TU, UK

1 INTRODUCTION

The present paper discusses the results of the experimental investigation of a damping system comprising a steel rod of a quadratic profile covered by a strip of absorbing layer located at its sharp tip, as shown in Figure 1. The combined effect of such a geometry and of an additional absorbing layer ensures that such a system represents an efficient way to reduce edge reflections of flexural waves in finite rods. The principle materialised by the above-mentioned combined action is known as the 'acoustic black hole effect'. This effect has been recently demonstrated experimentally for wedge-like structures of power-law profile[1], following its theoretical description[2-4]. As was described in Reference 5, this effect can take place also in rods of power-law profile, with possible applications to damping of impact-generated flexural vibrations in tennis racquets and golf clubs.

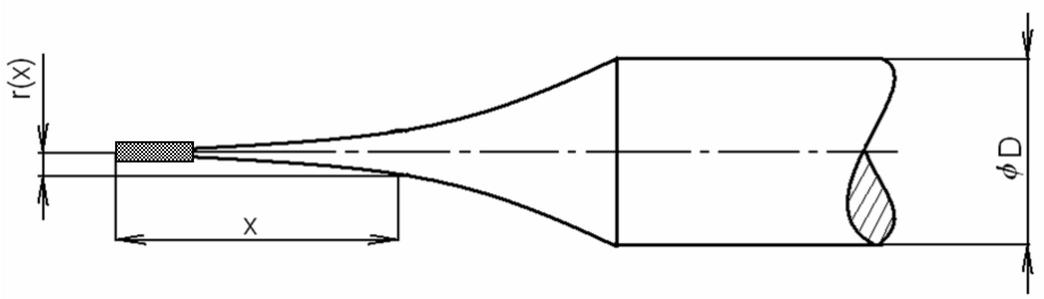

Figure 1. Geometry of a tapered cylindrical rod of power-law profile covered by an absorbing layer at its sharp end. Reduction of edge reflections in such a system represents an efficient method of suppression of resonant vibrations in finite rods.

It is well known that one of the efficient ways of suppression of resonant flexural vibrations of mechanical structures and their components (e. g. rods and plates) is to reduce reflections from free edges of such structures. In the quest of reduction of edge reflections, various methods have been suggested in the past. In particular, one of such methods introduces structures with graded impedance interfaces comprising finite plates of the same thickness but made of different materials, including plates made of highly damping materials placed at the edges[6]. In spite of the successful operation of such structures, their main disadvantage is the difficulty of manufacturing such graded impedance interfaces, which makes these structures not very suitable for practical applications.

To avoid complications associated with manufacturing structures comprising plates made of different materials, graded impedance interfaces can be materialised by means of changes in geometrical parameters of a structure. In particular, an efficient reduction of end reflections in plates can be achieved due to a gradual change in structural thickness of a plate from the value corresponding to the thickness of the basic plate to almost zero. Especially efficient reduction takes place if this change is made according to a power-law relationship between the local thickness of a plate $h(x)$ and the distance from the sharp end x: $h(x) = \varepsilon x^m$. If the exponent $m \geq 2$, then the sharp tips of such plates (wedges) represent the so-called acoustic black holes for flexural waves[1-5]. The physics of acoustic black holes can be understood using the geometrical acoustic theory of flexural wave propagation in structures of arbitrary geometry[7-9]. Ideally, such acoustic black holes provide zero reflections from tapered tips and thus reduce amplitudes of the associated resonant vibrations in plates of finite length. To make up for real manufactured plates of power-law profile characterised by the presence of imperfections and truncations, application of additional narrow strips of absorbing materials on the surfaces of the plates at their ends is paramount, which amplifies this effect very substantially and makes it practically viable. As was mentioned above, this method has been experimentally proved for plates (wedges) of power-law profile[1]. According to the theoretical calculations[2-4], the reflection coefficients in such wedges can be as low as 1–3%.

Damping of flexural vibrations in finite rods of circular cross-section with a profile described by a power-law relationship between the local radius $r(x)$ of the rod and the distance from the end x: $r(x) = \varepsilon x^m$ can be also described using the geometrical acoustic approach[7-9]. According to Reference 5, theoretical calculations of the reflection coefficients R_0 carried out for wedges of power-law profile[2-4] can be used to determine R_0 for rods as well. In the particular case, when the power-law exponent $m \geq 2$, the sharp tips of such rods represent the so-called one-dimensional acoustic black holes for flexural waves. Like in wedges, the group velocities of flexural waves approaching sharp edges of rods in this case gradually decrease to zero. Hence these waves ideally do not reflect back from the sharp ends. In reality, there are always edge truncations and other imperfections that cause significant part of the wave energy to reflect back. Like in the case of wedges, this problem can be solved by attaching narrow strips of absorbing materials at the sharp tips[2-4]. The combined effect of a specific geometry and of the added absorbing materials thus provides the maximum damping effect. In practical cases, only a part of the length of the bar is cut according to a power-law relationship (see Figure 1). As the sudden change in the profile results in some of the wave energy reflecting back, it is advisable to smoothen the transitional area to minimalise this effect.

It will be shown in this paper that the results of the measurements of mobilities in tapered rods are in qualitative agreement with the theory, thus demonstrating that a noticeable reduction of resonant vibrations can be achieved in a rod of power-law profile covered by strips of absorbing layers, in comparison with the uncovered rods of the same profile or with the free and covered uniform rods. This makes this unique method of damping very attractive for many practical applications.

2. MANUFACTURED RODS AND EXPERIMENTAL SETUP

In this section we describe experimental setup used for investigation of damping of flexural vibrations in finite rods of circular cross-section tapered at the ends according to a power-law relationship between the local radius of the rod and the distance from the end covered by an adhesive strips of absorbing layers of various materials and thickness.

Two rods of power-law profile were manufactured using a manual lathe. Since the production of rods of higher-order power-law profiles was too complicated by means of this technology, only rods of quadratic profile were manufactured. The length of the basic rod was 500 mm and its diameter was 12 mm. The values of the quadratic law parameter of the rod were $\varepsilon_1 = 10^{-3}$ mm^{-1} and $\varepsilon_2 = 3\times10^{-4}$ mm^{-1} (see Figure 2). Young`s modulus of the rod material (steel) was 2.1×10^{11} MPa, the mass density was 7850 kg/m^3, with the corresponding velocities of longitudinal and shear elastic waves in the material being equal to 6000 and 3200 m/s respectively.

Figure 2. Manufactured rods: a reference uniform rod (above) and two tapered rods of different quadratic profiles (below).

Measurements of point and cross mobilities (also known as driving point admittance) have been carried out in a wide frequency range (0–12500 Hz), of which only the most interesting part (0-6400 Hz) is displayed in Figures to follow. Measurements have been made for free rods of quadratic profile, for the same rods covered by adhesive strips of absorbing layers, and for a free and covered uniform reference rod.

The measurement set, shown in Figure 3, consisted of an electromagnetic shaker, accelerometer, force transducer, spectrum analyser, power amplifier and various supports. Experiments were carried out within the Noise and Vibration Laboratory of the Aeronautical and Automotive Engineering Department at Loughborough University.

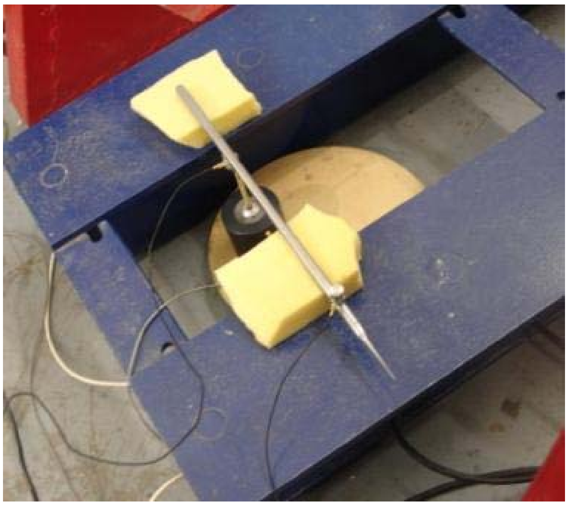

Figure 3. Experimental set-up

An electromagnetic shaker Ling Dynamic Systems 200 series provided the excitation input to the rod. It was attached to the bottom surface of the rod trough a force transducer Bruel & Kjaer type 8200. The force transducer was attached to the bottom surface of the rod via a super glue. The nearly free boundary conditions were provided by a set of foam supports placed on a massive steel frame to improve mechanical stability. A broad band accelerometer Bruel & Kjaer type 4371 was attached to the upper surface also via a super glue. A random signal was generated by the HP

3566 FFT analyser. The actual signal processing was then made using a computer with a Windows interface and installed software to process the signal from the analyzer. Other equipment included an ENDEVCO charge amplifier model 27218, the actual rods and several types of absorbing materials such as a ‚duct' tape (a 0.2 mm thick fabric-based tape with a soft and tacky pressure sensitive adhesive), a block of polystyrene, a block of foam with high density and a bitumen-like absorbing material with the thickness δ = 1.2 mm. Frequency response analysis was performed on the force transducer and accelerometer measurements. Of all absorbing materials used during experiments the best performance was achieved by a duct tape and a bitumen-like absorbing material. The obtained results are discussed in the following section.

3. EXPERIMENTAL RESULTS AND DISCUSSION

The measured point mobilities for the free reference rod and for the reference rod covered at the end by a strip of thick bitumen-like absorbing material are illustrated in Figure 4. As expected, the figure shows almost no effect of a partial coverage of adhesive damping strips attached at the end of the uniform reference rod on its frequency response.

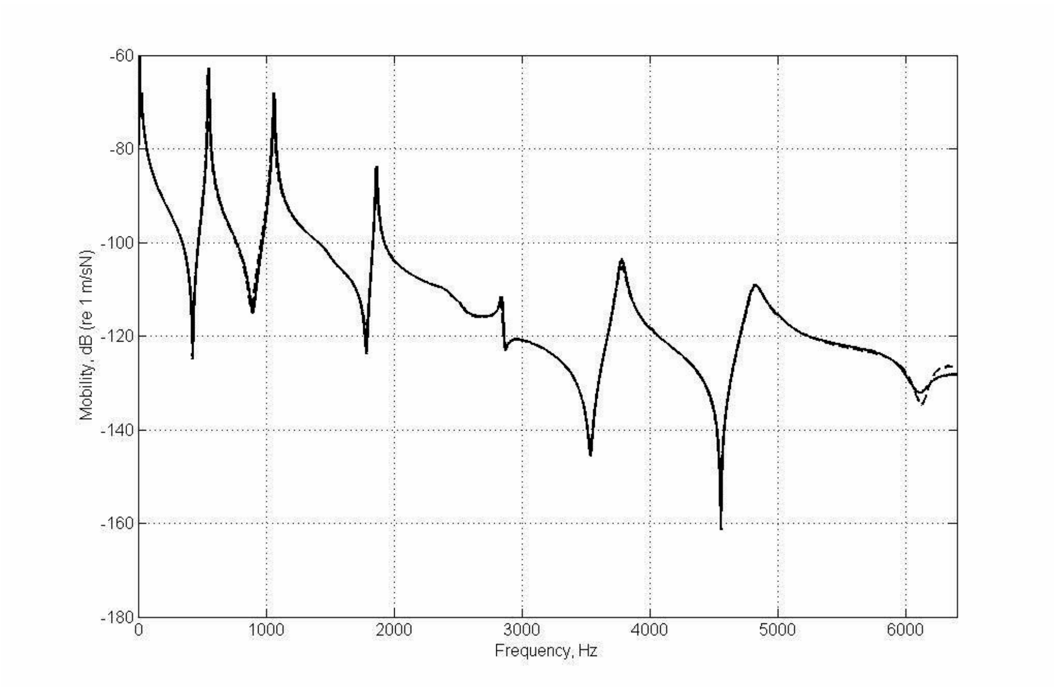

Figure 4. Measured point mobility for a free steel reference rod of constant diameter (solid curve) and for the same reference rod covered by a 20 mm long strip of bitumen-like absorbing layer with the thickness δ = 1.2 mm (dashed curve); the position of the shaker and accelerometer was at 100 mm from the end of the rod.

Experiments with non-uniform rods have been carried out for the two rods of different quadratic profiles described in the previous section. Figure 5 shows the measured point mobilities for an uncovered (free) rod of quadratic profile with $\varepsilon_1 = 10^{-3}$ mm^{-1} and for the same rod covered by a 12 mm long strip of bitumen-like absorbing layer. In comparison with the free quadratic rod, a noticeable suppression of resonant peaks, sometimes up to 15 dB, was achieved for a covered rod. This is to be attributed to the reduction of edge reflections of flexural waves from the sharp end of the finite rod due to the combined effect of a power-law profile and of a strip of thin absorbing material. The energy loss factor of the absorbing material is more efficient at higher frequencies. Hence, the combined effect is more noticeable at higher frequencies as well.

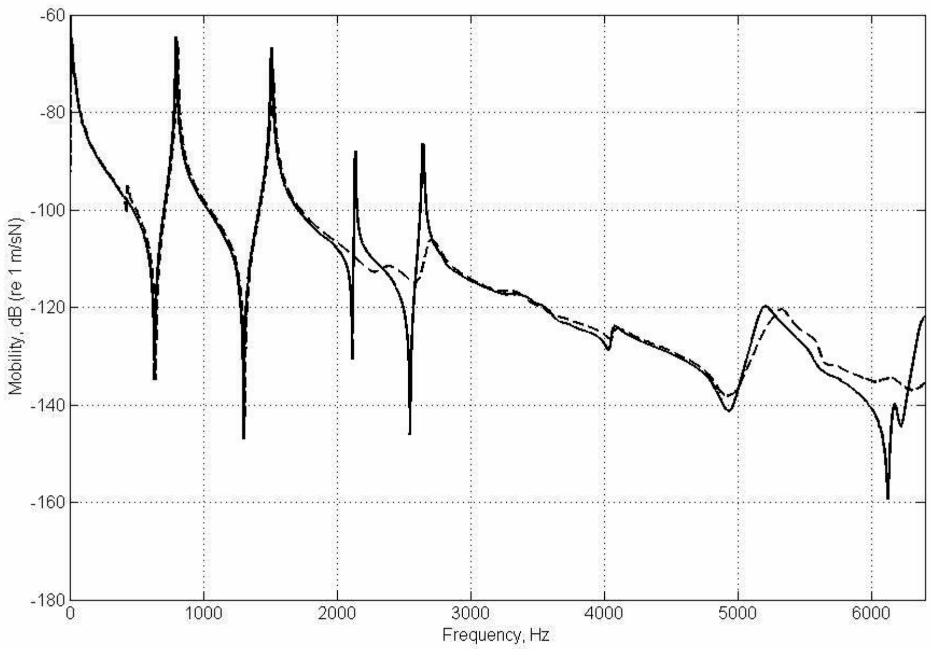

Figure 5. Measured point mobilities for a steel rod of quadratic profile with the quadratic parameter $\varepsilon_1 = 10^{-3}$ mm^{-1}: free rod (solid curve) and rod covered by a 12 mm long strip of bitumen-like absorbing layer with the thickness δ = 1.2 mm (dashed curve); the position of the shaker and accelerometer was at 120 mm from the uncut end.

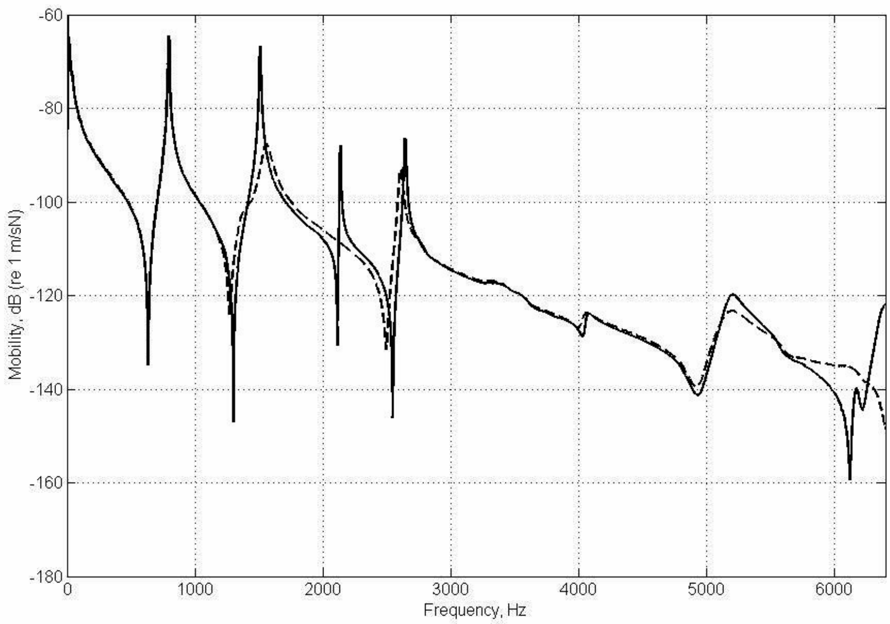

Figure 6. Measured point mobilities for a steel rod of quadratic profile with ε_1 = 10^{-3} mm^{-1}: free rod (solid curve) and rod with its tip inserted in a 5 mm long block of foam (dashed curve); the position of the shaker and accelerometer was at 120 mm from the uncut end.

This trend in reduction of resonant peaks can be further observed for various other absorbing layers. Figure 6, for example, shows measured point mobilities for the uncovered rod of quadratic profile (with the quadratic parameter $\varepsilon_1 = 10^{-3}$ mm^{-1}) and for the same rod with its tip inserted in a 5 mm long block of foam.

We recall that the above two pictures show the behaviour of a system consisting of a steal rod of power-law profile with the quadratic parameter $\varepsilon_1 = 10^{-3}$ mm^{-1} covered by various absorbing layers. As one can see, at the medium and higher frequencies the damping system benefits from such a treatment.

Even more encouraging results have been obtained for the second rod - with the quadratic parameter $\varepsilon_2 = 3\times10^{-4}$ mm^{-1}. Whereas in the previously discussed case the suppression of about 15 dB have been achieved, the behaviour of the 'sharper' rod showed suppressions of more than 20 dB when covered by absorbing layers. Figure 7 shows measured point mobilities of a free rod with the quadratic parameter $\varepsilon_2 = 3\times10^{-4}$ mm^{-1} and of the same rod covered by strips of duct tape.

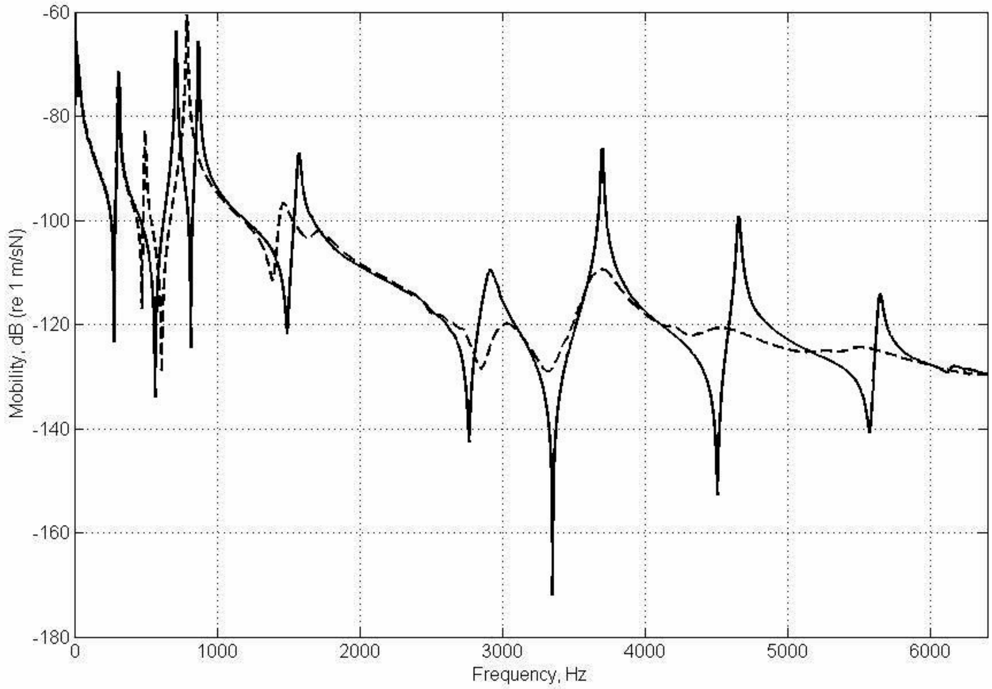

Figure 7. Measured point mobilities of a steel rod of quadratic profile with the quadratic parameter $\varepsilon_2 = 3\times10^{-4}$ mm^{-1}: free rod (solid curve) and rod covered by 3 layers of 15 mm long strips of duct tape (dashed curve); the position of the shaker and accelerometer was at 120 mm from the uncut end.

Point mobility measurements carried out for this rod showed a noticeable damping effect also with other attached damping layers. In Figure 8, point mobilities of a free quadratic rod with $\varepsilon_2 = 3\times10^{-4}$ mm^{-1} and of the same rod with its tip inserted in a 5 mm long foam block are displayed. Note a suppression of more than 20 dB in the higher frequency range.

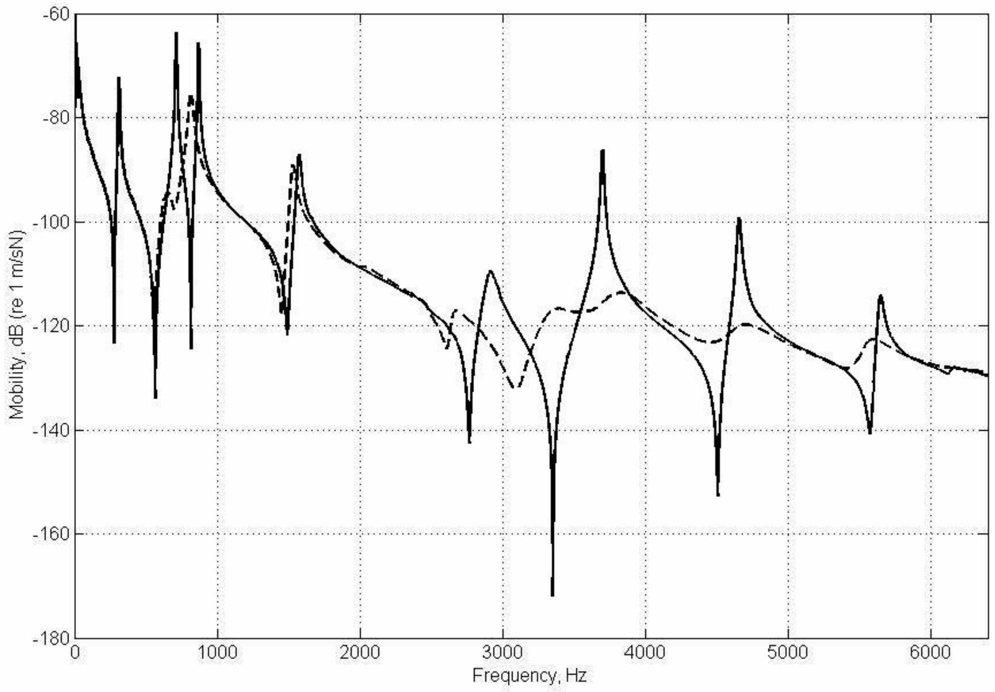

Figure 8. Comparison of point mobilities measured for a steel rod of quadratic profile (with $\varepsilon_2 = 3\times10^{-4}$ mm^{-1}): free rod (solid curve) and rod with its tip inserted in a 5 mm long block of foam (dashed curve); the position of the shaker and accelerometer was at 120 mm from the uncut end.

Experiments carried out for the two non-uniform rods of quadratic profile have both shown noticeable suppression of resonant peaks, especially in the medium and higher frequency range. During the measurements the effect of the quadratic parameter ε showed that with the reduction of its value the damping effect increases.

4. CONCLUSIONS

Experimental investigations of the dynamic behaviour of tapered rods of power-law profile covered by narrow strips of absorbing materials show a trend towards efficient damping of flexural vibrations in such structures. The advantage of applications of this unique damping technique is especially noticeable in the medium and higher frequency range.

Experiments carried out for free rods of quadratic profile and for the same rods covered by adhesive strips of absorbing layers demonstrated noticeable suppression of resonant vibrations. This can be attributed to a significant reduction of the reflection coefficients of flexural waves from the sharp tips of the rods. The results of these measurements also showed more significant damping obtained in the rod with the lower value of quadratic parameter.

Damping systems containing rods of power-law profile offer a variety of application possibilities. In particular, they can be applied for damping flexural vibrations in aerospace and civil engineering structures as well as in tennis racquets and golf clubs.

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