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(Submitted May 31, 1982)

Akust. Zh. 29, 790–798 (November–December 1983)

A general method is proposed for the analysis of sound radiation by cracks of arbitrary configuration propagating in bounded elastic solids. The method is based on the application of Huygens' principle and makes it possible to determine both the law governing the movement of the edges of a crack under the action of applied external stresses and the radiation from it.

PACS numbers: 46.30.Nz, 43.40.Le

It is well known that the formation and growth of cracks are accompanied by the radiation of acoustic waves into the solid containing the crack and into the surrounding space; the intensity of the radiation is sometimes so great as to be perceptible to the ear. The practical value of this effect, which is one of the manifestations of acoustic emission,[1,2] lies mainly in two aspects: its utility in physical experiments to study the dynamics of cracks and the feasibility of predicting processes of catastrophic damage to engineering structures by means of the effect. The latter aspect is particularly important insofar as the recording instrument in this case responds to dynamically active cracks, which present the greatest danger.

The existing theoretical studies of crack acoustics can be conditionally classified into two groups. The first group contains model-type studies, in which cracks of arbitrary configuration are described by means of equivalent three-dimensional multipole sources under various simplifying assumptions.[2–6] The second group includes studies characterized by the rigorous formulation of a boundary-value problem for cracks of simple geometry (e.g., semiinfinite linear cracks) and its subsequent solution with the recruitment of ponderous mathematical methods, in particular the Wiener–Hopf method.[7–11] The objective of the present study, which is more closely allied with the second group, is to develop a sufficiently general and, at the same time, rigorous approach based on Huygens' principle for analyzing the radiation of sound by cracks of arbitrary configuration (including finite cracks) and to discuss some specific results obtained by application of the method.

Let a growing crack L be situated in the interior of an elastically stressed solid bounded by a smooth surface. For definiteness we consider the surface to be plane (Fig. 1). The boundary conditions of zero normal stresses must be satisfied at the edges of the crack and on the free surface of the solid in this case. We adopt zero-valued initial conditions. It is more convenient to represent the basic problem of the crack in an elastically stressed medium as the superposition of two problems (this can be done in the linear formulation): 1) the problem of the stressed solid without the crack, which is of no interest in the given case; 2) the problem of a crack with nonzero normal stresses $n_i\sigma_{ij} = -n_j\sigma_{ij}^0$ applied to its edges in the absence of other stress sources.[12] Here the quantities σ_{ij}^0 represent the stresses of the first problem, calculated at the site of the crack (the net stresses, or the stresses acting on the edges of the crack in the basic problem, are equal to zero in this case, as required). We shall be concerned with the second problem.

The ensuing considerations are based on Huygens' principle for solid elastic media.[13] Applying it to the given problem (we treat the two-dimensional case), we choose a closed contour S running along the surface z = 0 around a cut denoting the possible path of crack propagation (this path is assumed to be known[12]) and closed by a semicircle of infinite radius. Then the corresponding mathematical representation of Huygens' principle for time-harmonic fields, being the analog of the first Helmholtz integral formula, takes the form

$$u_m(\mathbf{r}) = \int_S [n_j\sigma_{ij}G_{im}(\mathbf{r}, r') - n_jc_{ijkl}u_i(\mathbf{r}')G_{lm,k}(\mathbf{r}, \mathbf{r}')]\,dS, \qquad (1)$$

where the point $\mathbf{r}$ lies inside the contour S, u_i is the displacement vector, n_j is the outward unit normal to the contour, $G_{im}(\mathbf{r}, \mathbf{r}')$ is the dynamic Green's tensor, which satisfies the equation $c_{ijkl}G_{lm,ik} + \rho\omega^2 G_{im} = -\delta_{im}\delta(\mathbf{r}-\mathbf{r}')$ and the conditions at infinity, c_{ijkl} denotes the elastic constants, and $G_{lm,k} = \partial G_{lm}/\partial x_k$. The integral over the infinite semicircle vanishes because of the radiation conditions,[13] and so the integration in (1) is actually carried out only along the edges of the cut passing through the crack and along the boundary of the half-space. We note, in addition, that the first term in (1) vanishes in integration along the free surface owing to the boundary conditions on the surface z = 0. But if the boundary condition of zero stresses on the boundary is imposed on Green's function $G_{im}(\mathbf{r}, \mathbf{r}')$, i.e., if Green's tensor for the half-space $G_{im}^0(\mathbf{r}, \mathbf{r}')$ is introduced (see, e.g., Ref. 14), expression (1) is further simplified and reduced to an integral over the region of the crack. We recall that the quantities σ_{ij} on the surface of the crack are determined by the values of the external

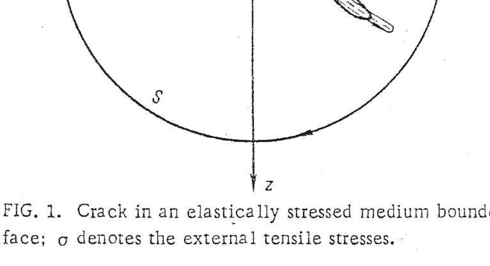

FIG. 1. Crack in an elastically stressed medium bounded by a smooth surface; σ denotes the external tensile stresses.

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stresses applied to the solid, i.e., are known. The values of u_i at the edges of the cut, describing the crack-growth dynamics, must be determined in some way from the known quantities σ_{ij}. The solution of this rather complicated problem in fracture mechanics has been the topic of an exhaustive literature (see Ref. 12), which we shall not discuss. We note, however, that u_i can also be determined, in particular, by solving the integral equation obtained directly from (1) as the observation point r tends to the surface of the crack. Once the values of u_i have been determined, the fields of all the waves excited by the crack, including surface waves, can be calculated by direct integration of (1).

Because of the extreme difficulty of determining the law of motion of the edges of the crack by means of the integral equation in the general case, it may be useful in practical calculations to let the role of u_i be taken by approximate solutions based on physically obvious assumptions (see below) or by solutions obtained for simpler (reference) problems.

To make the transition from expression (1), which is valid for time-harmonic external excitations, to the transient time-dependent problems typical of real growing cracks, it is necessary to investigate the following cases. Let the crack represent an initial break in continuity (cut), which opens out under the action of external stresses applied to the solid, without changing its length (such a crack is equivalent to the model of an "instantaneously propagating crack"). This type of situation is realized in practice when the stress intensity factors at the tips of the crack do not exceed their critical values at which the crack begins to grow.[12] In this case the field variables involved in (1) must be interpreted as the corresponding spectral densities $\sigma_{ij}(\mathbf{r}, \omega)$ and $u_i(\mathbf{r}, \omega)$. The final result can then be obtained by means of the inverse Fourier transform of $u_m(\mathbf{r}, \omega)$. A more complicated situation arises when the crack propagates at a finite rate, because now the stresses σ_{ij} on the edges of the cut depend in a nondegenerate way on the time and coordinates. To make use to Huygens' principle in this case it is necessary to take the Fourier transform of both sides of (1) with respect to the space coordinate measured along the cut and then, getting rid of the corresponding coordinate on the right-hand side of (1), to take the inverse Fourier transform with respect to it. Clearly, this can be done fairly simply in cases where the function $G_{im}(\mathbf{r}, \mathbf{r}')$ on the cut passing through the cut depends only on the difference between the corresponding coordinates, for example on the coordinates x and x', i.e., $G_{im}(x, x') = G_{im}(x-x')$. This is always true for a crack in an unbounded space or in situations where the crack is parallel to the free surface. Then the theorem of the convolution spectrum can be used to rewrite expression (1) in the form

$$u_m(\mathbf{r}, t)=\frac{1}{2\pi}\iint_{-\infty}^{\infty}\{[n_j^{(-)}\sigma_{ij}^{(-)}(\omega\ k)+n_j^{(+)}\sigma_{ij}^{(+)}(\omega, k)]G_{im}(\omega, k)$$
$$-c_{ijkl}[n_j^{(-)}u_i^{(-)}(\omega, k)+n_j^{(+)}u_i^{(+)}(\omega, k)]G_{lm,k}(\omega, k)\}e^{ikx-i\omega t}d\omega\, dk, \qquad (2)$$

where all the information about the law governing the motion of the tips of the crack is contained in the functions $\sigma_{ij}^{(-)}(\omega, k)$ and $\sigma_{ij}^{(+)}(\omega, k)$; the superscripts (−) and (+) refer to the lower and upper edges of the cut, respectively (see below for more details).

We have assumed up to now that boundary conditions are not imposed on the Green's function used in writing Huygens' principle or that the conditions of zero stresses on the free surface of the half-space are applied to it. However, in a number of situations, e.g., in the case of crack propagation in an unbounded medium, it is convenient to choose the Green's tensor in such a way that a definite type of boundary condition selected on the basis of convenience considerations will be satisfied on a surface passing between the edges of the crack. We illustrate the foregoing in the example of a linear normal-fracture crack propagating in an unbounded medium subjected to the action of tensile stresses $\sigma_{zz} = \sigma(t)$. According to the foregoing discussion, the action of the latter is equivalent to the action of stresses $-\sigma(t)$ applied to the edges of the crack (Fig. 2). Owing to the symmetry of the problem about the x axis in the given case, it is sufficient to investigate the field only in one half space, say the lower. We now have mixed boundary conditions on the surface z = 0 (Ref. 12):

$$\sigma_{zz}=-\sigma(t),\quad |x|\leqslant l,\quad \sigma_{zx}=0,\quad |x|\leqslant\infty,\quad u_z=0,\quad |x|>l \qquad (3)$$

with the initial conditions $u_x=u_z=\dot{u}_x=\dot{u}_z=0$. In order to apply Huygens' principle to the given problem we select the contour S as shown in Fig. 2. Then expression (1) acquires the form

$$u_m(x, z)=\int_{-\infty}^{\infty}[-\sigma_{zz}(x')G_{zm}(z, x-x')+c_{izkl}u_i(x')G_{lm,k}(z, x-x')]dx'. \qquad (4)$$

Various boundary conditions at z = 0 can be imposed on Green's function G_{im}, as mentioned. In particular, G_{im} can be the now-familiar Green's function G^0_{im} satisfying the condition of zero stress on the plane surface, which is the surface z = 0 in the given situation. Now the second term is omitted in the integral (4). With the remaining expression we can then determine both the law of motion of the edges of the crack and the radiation field [we note that the problem of determining the displacements of the edges of the crack in the general case reduces to solving a system of two integral equations in the given situation as a result of the mixed boundary conditions (3) at z = 0]. A different boundary condition at z = 0 can also be imposed on Green's function. Specifically, it can be required that it satisfy the relations

$$\sigma_{zx}=0,\quad u_z=0,\quad |x|<\infty. \qquad (5)$$

In this case, obviously, the first term is omitted in (4) [since the additional condition $u_x = 0$ for $|x| < \infty$ follows from (5)] and the radiation field can be calculated from the known values of $u_z|_{z=0} = u_z^0$, which can be determined in some way indirectly, for example experimentally or by solving the system of integral equations. It is clearly impossible to determine the displacements of the edges of the crack by means of (4) directly in this case, because choosing Green's function G_{im} from (4) eliminates the external forces σ acting on the surface of the crack. It may seem at first glance that the selection of Green's function satisfying the boundary conditions (5) would not offer any advantages in the calculations. However, this is not the

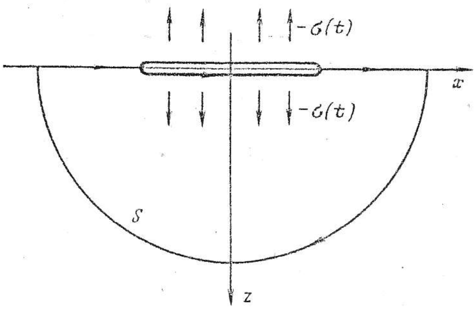

FIG. 2. Linear normal-fracture crack in an unbounded elastic medium.

case. The investigated Green's function, as expected, is far simpler (see below) than the function satisfying the free-surface conditions. The price that must be paid for this simplification is that the problem of determining the displacements u_z^0 of the edges of the crack in this case must be solved in isolation from the radiation problem. We note that the method of calculating the sound radiation from a crack on the basis of specification of the law governing the motion of its edges was evidently first proposed in Ref. 9 (see also Ref. 1), a large part of which is devoted to testing the efficiency of the method by numerical experiment. The formalism proposed above therefore enables us to explain the significance of the corresponding integral expressions in Ref. 9, which are derived as a direct result of solving the restated (with allowance for the specification of u_z^0) boundary-value problem.

After the foregoing general considerations, we now discuss the fundamental properties of the radiation from the investigated simple crack (see Fig. 2) in more detail, using the Green's function G_{lm} satisfying the boundary conditions (5) for the analysis. Bearing in mind the possibility of describing a propagating crack, we proceed from the Fourier-transformed expression (4), which is conveniently written as follows in the given situation:

$$u_m(x,z,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u_z^0(\omega,k)\,g_{zm}(z,\omega,k)\,e^{ikx-i\omega t}\,d\omega\,dk, \tag{6}$$

where the quantity $g_{zm}(z,\omega,k)=c_{zzkl}G_{lm,k}$ obviously describes the response of an elastic half-space with the boundary conditions (5) to a unit delta-type displacement of the surface points in the direction of the normal. The function g_{zm}, which has the significance of the renormalized Green's function, can be calculated from the known G_{lm} by passing to the limit $z' \to 0$. In the given situation, however, it is simpler to find it directly by solving the corresponding boundary-value problem. In the case of isotropic solids it is convenient to work with the Lamé potentials φ and ψ, which are related to the component u_x and u_z by the equations

$$u_x=\partial\varphi/\partial x-\partial\psi/\partial z,\quad u_z=\partial\varphi/\partial z+\partial\psi/\partial x, \tag{7}$$

and to transform from (6) to the expression

$$\Phi_n(x,z\ t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u_z^0(\omega,k)\,\Gamma_{zn}(z,\omega,k)\,e^{ikx-i\omega t}\,d\omega\,dk. \tag{8}$$

Here the index n takes the values 1 and 2, where $\Phi_1=\varphi$, $\Phi_2=\psi$, and Γ_{zn} denotes the Green's function for the potentials, which is related to g_{zm} by Eqs. (7). The potentials φ and ψ in this case must satisfy the wave equations

$$\begin{aligned}&\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial z^2}-\frac{1}{c_l^2}\frac{\partial^2\varphi}{\partial t^2}=0,\\&\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial z^2}-\frac{1}{c_t^2}\frac{\partial^2\psi}{\partial t^2}=0,\end{aligned} \tag{9}$$

where $c_l=[(\lambda+2\mu)/\rho]^{1/2}$ and $c_t=(\mu/\rho)^{1/2}$ are the longitudinal and transverse wave velocities, and also the boundary conditions at $z = 0$

$$\sigma_{zx}=0,\quad u_z=\delta(x-x')\,\delta(t-t'). \tag{10}$$

We note that the investigated two-dimensional problem is valid not only when the fields are independent of the third coordinate y (planar deformed state), but also in the case of thin plates (planar stressed state).[12,15] In the latter case relations (7) and Eqs. (9) remain valid, but now c_l and c_t must be interpreted as the limiting velocity of the lowest symmetrical Lamb mode $c_l=[(\lambda+2\mu)/\rho]^{1/2}\cdot(1-2\nu)^{1/2}/(1-\nu)$ and the velocity of the lowest SH-mode $c_t=(\mu/\rho)^{1/2}$, respectively, where ν is the Poisson ratio for the medium. The displacement u_y in this case is related to u_x and u_z by the equation $u_y=-[\nu d/2(1-\nu)]\,(u_{x,x}+u_{z,z})$, where d is the thickness of the plate. Applying the standard Fourier transform procedure to (9) and (10), we can readily show that the expressions for Γ_{z1} and Γ_{z2} have the form

$$\begin{aligned}&\Gamma_{z1}(z,\omega,k)=i\frac{2k^2-\dfrac{\omega^2}{c_t^2}}{\left(\dfrac{\omega^2}{c_l^2}-k^2\right)^{1/2}\dfrac{\omega^2}{c_t^2}}\exp i\left(\frac{\omega^2}{c_l^2}-k^2\right)^{1/2}z,\\&\Gamma_{z2}(z,\omega,k)=i\frac{2k}{\dfrac{\omega^2}{c_t^2}}\exp i\left(\frac{\omega^2}{c_t^2}-k^2\right)^{1/2}z.\end{aligned} \tag{11}$$

We now consider various regimes of growth of the investigated crack, which obviously are completely described by specification of the function u_z^0. In the special case of instantaneous opening of a crack with tip coordinates l and $-l$ under the action of uniform tensile stresses $\sigma(t)=\sigma h(t)$, where h(t) is the Heaviside step function, it is convenient to use the following simple approximation of the function $u_z^0(x,t)$ (Ref. 9):

$$u_z^0(x,t)=\begin{cases}u_z^0(t), & |x|\leqslant l,\\ 0, & |x|>l,\end{cases}$$

where $u_z^0(t)$ takes the values st for $0\leqslant t\leqslant 2l/c_l$ and $s2l/c_l$ for $t>2l/c_l$; $s=\sigma/\rho c_l$. The given approximation has a simple physical significance: The rate of opening of the crack is determined as the result of dividing the tensile stress σ by the wave resistance ρc_l of the medium to normal pressure. The crack opens up at this rate to the level $2sl/c_l$ corresponding to the steady state (Westergaard's solution). The ellipticity of the crack opening is disregarded in the given approximation, because it does not produce any fundamental changes in the radiation field. The spectral density of the variable u_z^0 is determined by means of the expression

$$u_z^0(\omega, k) = \frac{1}{2\pi} \int_0^\infty \int_{-l}^{l} u_z^0(t) e^{i\omega t - ikx} \, dt \, dx$$

and has the form

$$u_z^0(\omega, k) = \frac{2isl^2}{\pi\omega c_l} e^{i\frac{\omega}{c_l} l} \frac{\sin\frac{\omega}{c_l} l}{\frac{\omega}{c_l} l} \frac{\sin kl}{kl}. \quad (12)$$

Substituting (12) and (11) into (8) and using the steepest-descent method to compute the integrals with respect to k, we obtain expressions for the potentials φ and ψ in the far radiation field of the crack:

$$\varphi(R, \theta, t) = \frac{sl^2}{\pi^2 c_l} \int_{-\infty}^{\infty} \frac{e^{i\frac{\omega}{c_l} l}}{\omega} \frac{\sin\frac{\omega}{c_l} l}{\frac{\omega}{c_l} l} \frac{\sin\left(\frac{\omega}{c_l} l \sin\theta\right)}{\frac{\omega}{c_l} l \sin\theta}$$

$$\times \left(2\frac{c_t^2}{c_l^2}\sin^2\theta - 1\right)\left(-\frac{2\pi i}{\frac{\omega}{c_l} R}\right)^{1/2} e^{i\frac{\omega}{c_l} R - i\omega t} \, d\omega,$$

$$\psi(R, \theta, t) = \frac{sl^2}{\pi^2 c_l} \int_{-\infty}^{\infty} \frac{e^{i\frac{\omega}{c_l} l}}{\omega} \frac{\sin\frac{\omega}{c_l} l}{\frac{\omega}{c_l} l} \frac{\sin\left(\frac{\omega}{c_t} l \sin\theta\right)}{\frac{\omega}{c_t} l \sin\theta}$$

$$\times \sin 2\theta \left(-\frac{2\pi i}{\frac{\omega}{c_t} R}\right)^{1/2} e^{i\frac{\omega}{c_t} R - i\omega t} \, d\omega. \quad (13)$$

Here R and θ are the polar coordinates of the observation point: $x = R\sin\theta$ and $z = R\cos\theta$. An analysis of the integrands in (13) shows that the radiation directivity pattern of longitudinal waves (the potential φ) for the spectral component with frequency ω has a maximum at $\theta = 0°$ and the width of the major lobe is determined by the formula $\Delta\theta = 2 \arcsin(\pi c_l/\omega l)$. For example, for $2l = 1$ mm and a frequency of 3 MHz we have $\Delta\theta \approx 100°$. For $\omega l/\pi c_l \ll 1$ the directivity pattern no longer depends on ω or l and is determined by the factor $2(c_t^2/c_l^2)\sin^2\theta - 1$. The transverse-wave directivity pattern has a zero minimum at $\theta = 0°$, as is physically obvious, and for this reason it has two major lobes, the total width of which is determined by the expression $\Delta\theta = 2 \arcsin(\pi c_t/\omega l)$. To transform from the potentials φ and ψ to the displacements u_R and u_θ it is necessary to apply the relations $u_R \simeq \partial\varphi/\partial R$ and $u_\theta \simeq -\partial\psi/\partial R$, which are valid in the far zone. As a result, the expressions for u_R and u_θ differ from (13) only in the presence of $i\omega/c_l$ and $i\omega/c_t$ in the integrands. We now tender several remarks concerning the temporal waveform of the signal received in the far zone. Unfortunately, the integrals with respect to ω in (13) are too cumbersome to be evaluated by analytical methods. However, these integrals usually do not have to be computed along the entire frequency axis, because the acoustic-emission signals from cracks are recorded mainly by narrowband receivers, which cut a narrow spectrum of frequencies out of the received signal. But if we assume that the receiver is a wideband instrument, we can analyze the distortion of the original waveform of the signal $u_z^0(t)$ qualitatively. In particular, it is seen at once that the factor $[(\omega/c_{l,t})R]^{-1/2}$ describes the high-frequency attenuation of the signal as a result of cylindrical divergence, and the factors of the form $\sin[(\omega/c_{l,t})l\sin\theta]/(\omega/c_{l,t})\cdot l\sin\theta$ describe the additional spreading of the waveform in the lateral directions.

We next consider the case of a propagating crack whose tips move with constant velocities v_1 and v_2 in a time interval $0 \le t \le T$. The function $u_z^0(x, t)$ in this case can be approximated by the relation[9]

$$u_z^0(x, t) = \begin{cases} u_z^0(t), & v_1 t \le x \le v_1 t; \\ 0, & x < -v_1 t; \quad x > v_2 t; \end{cases}$$

$$u_z^0(t) = \begin{cases} s(v_1 + v_2)t/c_l, & 0 \le t \le T; \\ s(v_1 + v_2)T/c_l, & t > T; \end{cases}$$

its spectrum is given by the expression

$$u_z^0(\omega, k) = \frac{1}{2\pi} \int_0^\infty \int_{-v_1 t}^{v_2 t} u_z^0(t) e^{i\omega t - ikx} \, dt \, dx.$$

The integration of this expression in general form yields an extremely cumbersome result. It is therefore more practical to analyze a few important special cases. We first discuss the symmetrical, unlimited ($T \to \infty$) propagation of a crack with velocities $v_1 = v_2 = v$. In this case

$$u_z^0(\omega, k) = \frac{4s\omega}{i\pi c_l k^4 v^2}\left[1 - \left(\frac{\omega}{kv}\right)^2\right]^{-2} \quad (14)$$

Substituting expression (14) into (8) with allowance for (11) and applying the steepest-descent method, we obtain the following expressions for the potentials φ and ψ in the radiation far field:

$$\varphi(R, \theta, t) = -\frac{2sv^2}{\pi^2 c_l} \int_{-\infty}^{\infty} \frac{1}{\omega^3} \left(\frac{v^2}{c_l^2}\sin^2\theta - 1\right)^{-2}$$

$$\times \left(2\frac{c_t^2}{c_l^2}\sin^2\theta - 1\right)\left(-\frac{2\pi i}{\frac{\omega}{c_l} R}\right)^{1/2} e^{i\frac{\omega}{c_l} R - i\omega t} \, d\omega,$$

$$\psi(R, \theta, t) = -\frac{2sv^2}{\pi^2 c_l} \int_{-\infty}^{\infty} \frac{1}{\omega^3} \left(\frac{v^2}{c_t^2}\sin^2\theta - 1\right)^{-2}$$

$$\times \sin 2\theta \left(-\frac{2\pi i}{\frac{\omega}{c_t} R}\right)^{1/2} e^{i\frac{\omega}{c_t} R - i\omega t} \, d\omega. \quad (15)$$

It is inferred at once from (15) that the potentials φ and ψ are proportional to v^2 for small values of v and the directivity of the radiation is very weak. With an increase in v the directivity patterns become sharper in the normal direction. As in the case of instantaneous opening of a crack, longitudinal waves have the maximum intensity at $\theta = 0°$, and transverse waves are not radiated in this direction. An interesting feature typical of a propagating crack is the fact that the function $u_z^0(\omega, k)$ contains poles at $k = \pm\omega/v$, which must be taken into account formally in computing the integrals (8) for contour integrals on the complex plane of k. It is readily shown, however, that the contribution of the poles describes static fields localized near the tips of the propagating crack, since the crack propagation velocity v cannot be greater than the Rayleigh wave velocity (or the velocity of a wave that is analogous to a Rayleigh wave and that propagates along the edge of a thin plate).[12] If we make the formal assumption that the velocity v can be greater than c_l or c_t, the static fields will be converted into radiation analogous to Cerenkov radiation. Inasmuch as such a possibility is purely speculative, we shall not consider it in detail.

We now turn to the case of asymmetrical (unilateral) unlimited ($T \to \infty$) propagation of a crack with velocities $v_1 = 0$ and $v_2 = u$. In this case $u_z^0(\omega, k) = s[2(\omega/kv)-1] / 2\pi i k^3 v c_l (1-\omega/kv)^2 (\omega/kv)^2$ and, as is readily verified, the expressions for the potentials φ and φ acquire the form

$$\varphi(R, \theta, t) = -\frac{sv^2}{4\pi^2 c_l} \int_{-\infty}^{\infty} \frac{1}{\omega^3} \frac{\left(2 - \frac{v}{c_l}\sin\theta\right)}{\left(\frac{v}{c_l}\sin\theta - 1\right)^2} \left(2\frac{c_t^2}{c_l^2}\sin^2\theta - 1\right) \times \left(-\frac{2\pi i}{\frac{\omega}{c_l}R}\right)^{1/2} e^{i\frac{\omega}{c_l}R - i\omega t}\, d\omega,$$

$$\psi(R, \theta, t) = -\frac{sv^2}{4\pi^2 c_l} \int_{-\infty}^{\infty} \frac{1}{\omega^3} \frac{\left(2 - \frac{v}{c_t}\sin\theta\right)}{\left(\frac{v}{c_t}\sin\theta - 1\right)^2} \sin 2\theta \times \left(-\frac{2\pi i}{\frac{\omega}{c_t}R}\right)^{1/2} e^{i\frac{\omega}{c_t}R - i\omega t}\, d\omega.$$

An analysis of the derived expressions shows that the longitudinal-wave directivity patterns in the given case are asymmetrical about the z axis, deviating in the direction of movement of the crack.

Finally, we look briefly at the case of limited crack propagation. We assume here that $v_1 = v_2 = v$ and that the time T satisfies the condition $kvT \ll 1$, which implies that the size of the jump made by the crack is much smaller than the wavelength in the investigated part of the radiation spectrum. The constraints imposed on the law governing the motion of the crack are not of a fundamental nature and serve merely to simplify the calculations. It is readily verified that the expression for $u_z^0(\omega, k)$ takes the form

$$u_z^0(\omega, k) = \frac{i2sv^2}{\pi c_l}\left[(1-e^{i\omega T})\left(\frac{T^2}{\omega^2} - \frac{2}{\omega^2}\right) - \frac{i2T}{\omega^2}e^{i\omega T}\right]. \tag{16}$$

It follows from (16) that the quantity does not depend on k in the given approximation, and so the radiation directivity patterns are determined only by the properties of Green's function Γ_{zn} in expression (8). This fact is fully obvious insofar as the crack is equivalent to a point source for $kvT \ll 1$. It is also instructive to discuss the situation in which the crack experiences a symmetrical jump-type growth, not from zero, but from a certain initial state, which we characterize by the positions of the tips $x = \pm l_0$ (the second and subsequent stages of intermittent growth of the crack). It is readily verified that the function $u_z^0(\omega, k)$ for this case can be represented as follows for small jumps, i.e., for $kvT \ll 1$:

$$u_z^0(\omega, k) = \tilde{u}_z^0(\omega, k) + \tilde{\tilde{u}}_z^0(\omega, k) \tag{17}$$

Here $\tilde{u}_z^0(\omega, k)$ is a quantity analogous to the corresponding function for an instantaneously opening crack of length $2l_0$ [see expression (12)] and differing from it only in the replacement of the factor l^2 in (12) by $l_0 vT$ and of the quantity $\omega l/c_l$ by $\omega vT/c_l$. The function $\tilde{\tilde{u}}_z^0(\omega, k)$ represents nothing more than expression (16) multiplied by $\cos kl_0$. Thus, it follows from (17) that the radiation from the crack in the given situation can be represented by the superposition of the radiations from an instantaneously opening crack of length $2l_0$, but with a somewhat modified law governing the displacement of the edges [the quantity l in the temporal spectrum $\tilde{u}_z^0(\omega)$ is replaced by vT], and from a crack growing from zero and emitting radiation with a directivity pattern characterized by, in contrast with (16), the presence of the additional factor $\cos[(\omega l_0/c_{l,t})\sin\theta]$, which narrows the angular spectrum. This implies that the second and subsequent stages of intermittent growth of the crack are accompanied by a sharpening of the directivity pattern of the sound waves generated by it.

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Translated by J. S. Wood