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The coherent shear wave in suspensions

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Abstract. We consider a disordered suspension of spherical silica particles in water. For a particle size of a few hundred nanometres and concentration (volume fraction) around 0.15 to 0.2, experiments conducted in the MHz range have shown that the non ideal nature of water must be taken into account for the "longitudinal" coherent wave attenuation to be understood, because of wave conversions, from longitudinal to shear and then back to longitudinal, occurring at each pair of scattering events. We are interested here in the properties of the "shear" coherent wave that are given by the expansion of its squared wavenumber, around that in the absence of particles, in powers of the concentration. At 1 MHz and a particle radius of 0.05 μm, we show that convergence of the modal series involved in that expansion may be reached after three terms: we use ten terms subsequently. We study the evolution of both the effective shear velocity and attenuation with concentration, as well as that of the effective shear viscosity deduced therefrom.

1. Introduction

We study the propagation of the "shear" coherent wave in a disordered suspension of silica spheres in water. Forrester [1] et al. have shown that for particle radii smaller than a few micrometres the attenuation of the "longitudinal" coherent wave in such a suspension could be well predicted by a multiple scattering model, as long as the shear-mode effects, due to water viscosity, were taken into account, as in Ref. [2]. Including those effects leads to the introduction of another coherent wave, called the "shear" coherent wave (with quotes omitted in the following), because its wavenumber ξ_S reduces to that, k_S, of the shear wave in (viscous) water in the absence of particles. It is in the properties of this shear coherent wave that we are interested here.

The physical properties of both water and silica are the same as in Ref. [1], and the numerical study is conducted at one frequency only, $f = 1$ MHz. All particles have the same radius $a = 0.05$ μm, so that letting k_C denote the compressional wavenumber in water in the absence of particles, while low frequency approximations may be used when dealing with compression waves, one must be careful with shear waves, as $k_C a \simeq 2\times 10^{-4}$, and $k_S a \simeq 0.1\ 1+i$.

The properties of the shear coherent wave are deduced from the calculation of its wavenumber ξ_S, as a function of the concentration (volume fraction) c of solid, using the multiple scattering model of Ref. [2], and section 2 is dedicated to numerical issues that need to be taken care of in doing this. The

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effective shear velocity and attenuation are studied as a function of the concentration in section 3, and preliminary work on the definition of a prospective effective shear viscosity is conducted in section 4.

2. The effective shear wavenumber

2.1. Effective shear wavenumber expansion

After Eqs. (16,29-32) of Ref. [2], the effective "shear" wavenumber ξ_S may be expanded around k_S in powers of the concentration ($c = 4\pi n_0 a^3/3$),

$$\frac{\xi_{1S}^2}{k_S^2} = 1 + \frac{c}{k_S a}^3 \delta_1^{SS} \tag{1}$$

$$\frac{\xi_S^2}{k_S^2} \simeq \frac{\xi_{1S}^2}{k_S^2} + \frac{c^2}{k_S a}^6 \delta_2^{SS} + \delta_2^{SC} + O\ c^3 \tag{2}$$

The first order term (in concentration), δ_1^{SS}, involves a modal series of the form $\sum_{n=0}^{+\infty} f_n T_n^{SS}$, while δ_2^{SS} and δ_2^{SC} involve respectively series of the form $\sum_{n=0}^{+\infty}\sum_{m=0}^{+\infty} g_{nm} T_n^{SS} T_m^{SS}$ and $\sum_{n=0}^{+\infty}\sum_{m=0}^{+\infty} h_{nm} T_n^{SC} T_m^{CS}$, with f_n, g_{nm}, h_{nm} more or less complicated functions of n and m, and the T_n^{pq} (p,q=S,C) the modal scattering coefficients of a single particle for an incident compressional (p=C) or shear (p=S) wave into either a compressional (q=C) or a shear (q=S) one. They are defined in Ref. [2] as associated with the Debye potentials of either compressional or shear waves, with no dependence with the azimuthal angle (see Eqs. 5,6 in Ref. [2]) . Their decay with the increase of mode number n is shown in Figure 1.

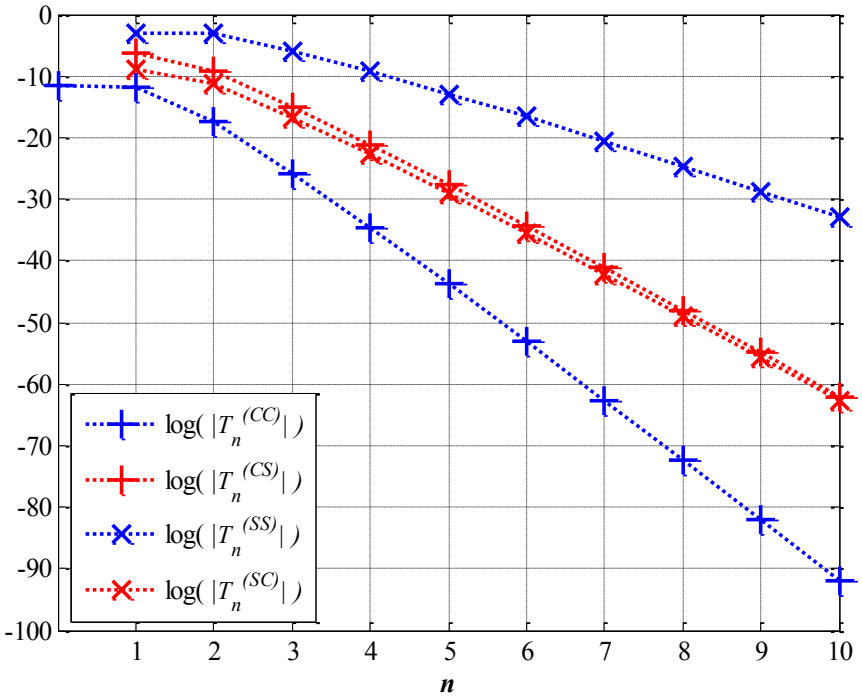


Figure 1. The modal scattering coefficients (amplitudes) as functions of mode number.

The scattering coefficient of a compressional wave into a compressional one is the fastest to decrease, that of a shear wave into a shear one the slowest. The first one is also the lowest in magnitude, while the second one the largest. However, all coefficients are smaller than 10^{-10} as soon as n is larger than 4, and we can suppose that all the series involved in Eqs. (1,2) should have achieved convergence after summing up 5 terms at most.

Convergence of the $\delta_{2N}^{SS} = \sum_{n=0}^{N} \sum_{m=0}^{N} g_{nm} T_n^{SS} T_m^{SS}$ series may be studied through the plot of its real and imaginary parts versus N, as done in Figure 2, where all the δ terms are plotted against N, showing that N=2 should be sufficient for all series to reach convergence. While this information could be useful to carry out analytic approximations of Eqs. (1,2), a value of N=10 has been used nevertheless to obtain all the subsequent results.

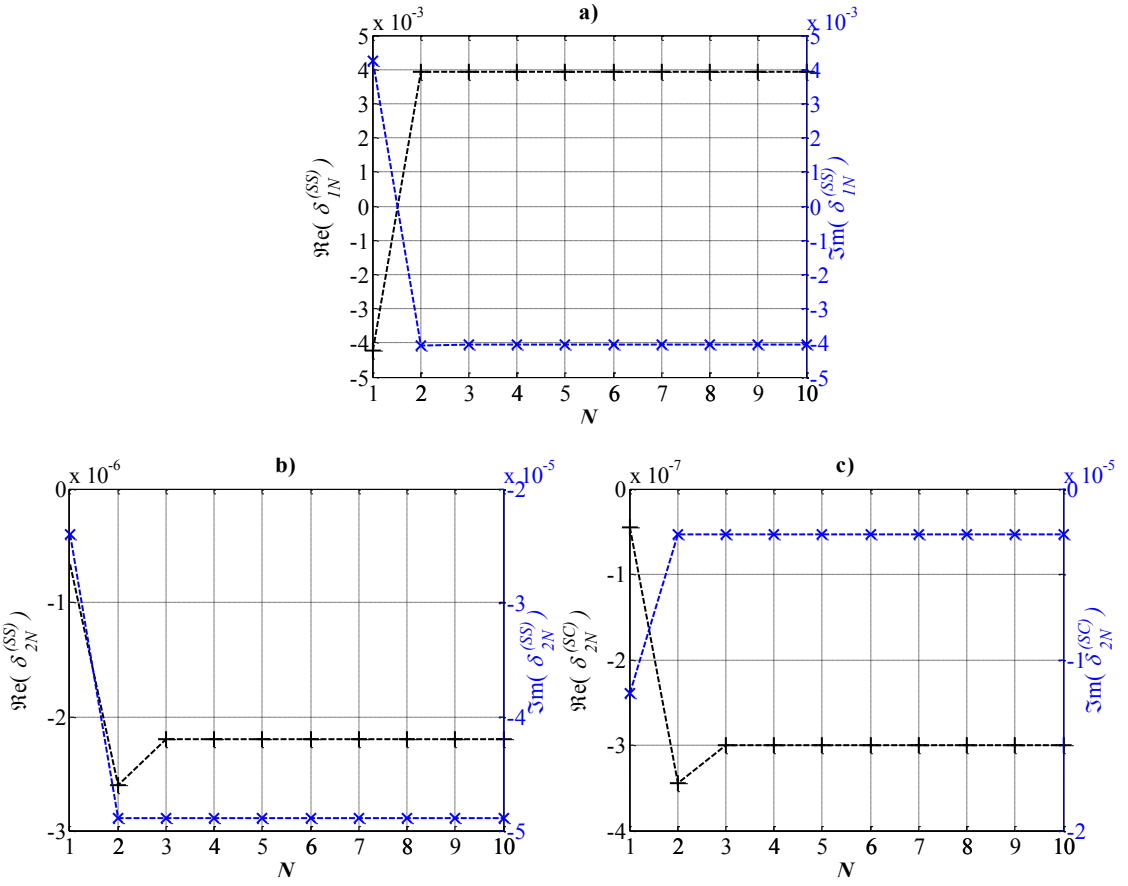


Figure 2. Convergence of the first and second order terms of the effective wavenumber expansion Eqs. (1,2). N is the order at which all modal series are truncated.

Eq. (2) corresponds to the expansion of $\frac{\xi_S^2}{k_S^2}$ in powers of the concentration c, up to order 2. This expansion was carried out [2] under the assumption of a small enough concentration for all successive orders to be decreasing in magnitude. However, each term of a given order r in Eq. (2) is multiplied,

not only by the small quantity c^r, as one would have expected, but by a larger one, $\frac{c^r}{k_S a^{3r}}$, so that, while the delta terms relating to the terms of order 2 in concentration (Figure 2) are two orders of magnitude smaller than the delta term relating to the order 1 term in concentration, Figure 3 shows that the whole term of order 2 in concentration in Eq. (2) is no longer smaller than the whole term of order 1 in concentration as soon as the concentration gets higher than about 0.25. This shows clearly the limit of the model, and the subsequent analysis considers only concentration up to 0.2.

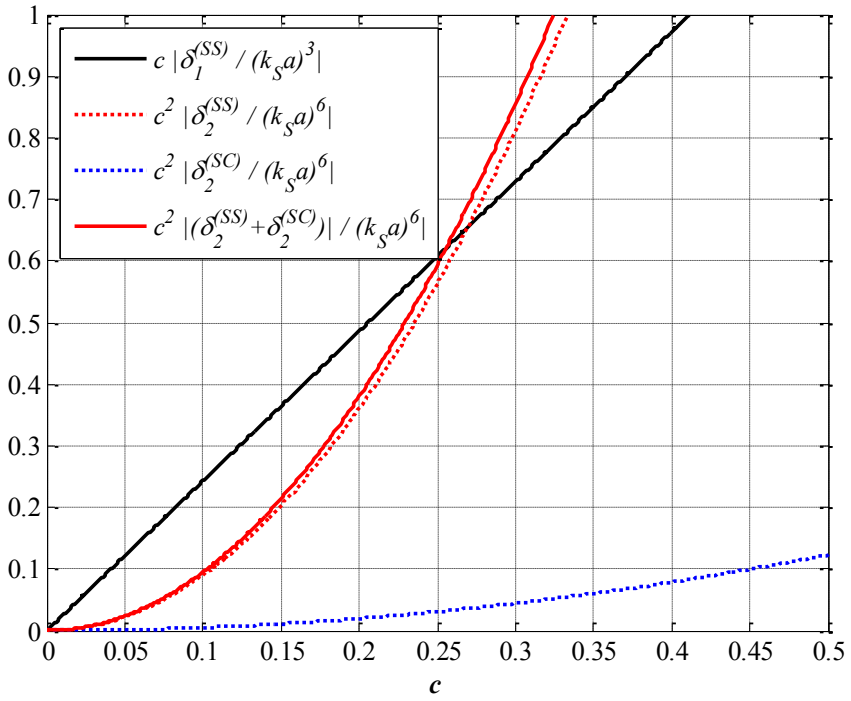


Figure 3. Magnitude of the terms in the effective shear wavenumber expansion in concentration.

2.2. Properties of the effective shear wave

The effect of the concentration of scatterers on the velocity and the attenuation of the effective shear wave is shown in Figure 4. Not surprisingly, the introduction of solid scatterers in water increases the shear velocity and decreases the shear wave attenuation, compared to the situation in the absence of scatterers. The deviation from the corresponding values in the absence of scatterers, however, is less than 10 %. As the effective wavenumber expansion in concentration is limited to order 2 in concentration, both the velocity and attenuation curves exhibit a quadratic behavior, but the concentration c_{crit} at which the velocity reaches a maximum is slightly lower than that at which the attenuation is lowest. As the quadratic behavior of the curves is the most obvious for concentration values around c_{crit}, it might be an indication that, for this particular frequency under study, order 3 in concentration can no longer be neglected for $c \geq c_{crit}$.

Figure 5 shows that the real and imaginary parts of the effective shear wavenumber are practically equal, just like those in the absence of scatterers, and the next section discusses the possibility of defining an effective shear viscosity by analogy with a pure viscous fluid.

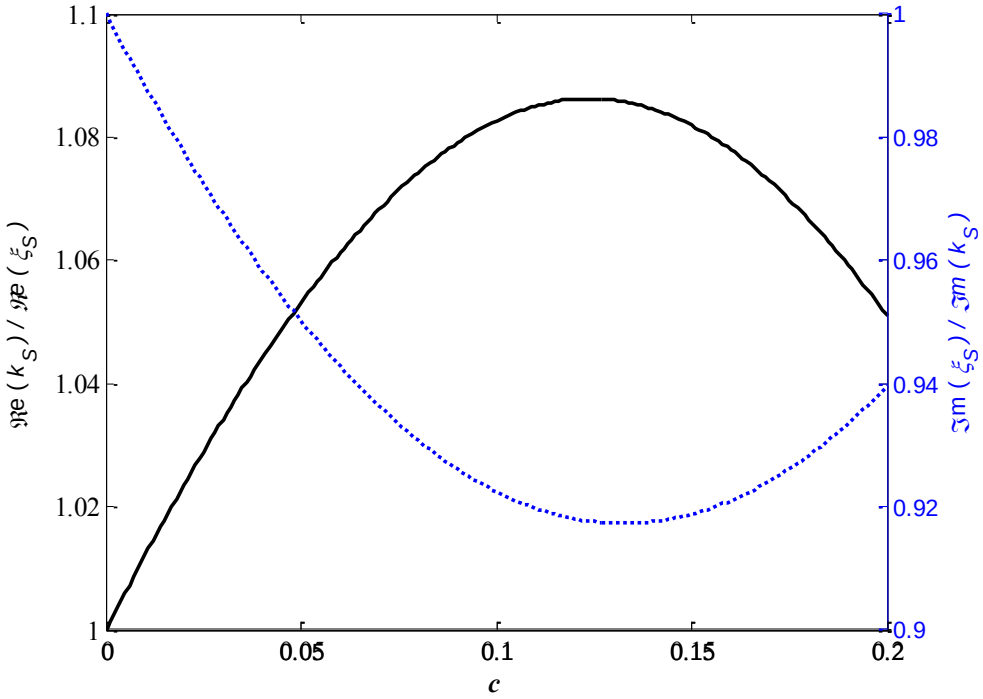


Figure 4. Ratios of the effective shear wave velocity (solid line, left vertical axis) and attenuation (dotted line, right vertical axis) to those of the shear wave in the absence of scatterers, versus concentration.

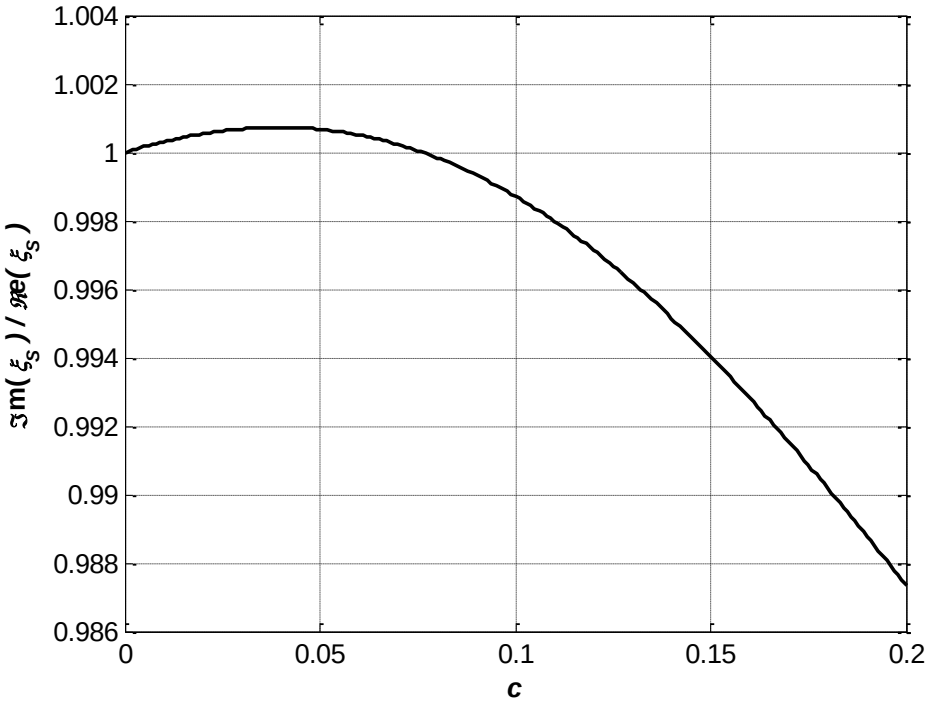


Figure 5. The effective shear wavenumber: ratio of its imaginary part to its real part versus concentration.

3. Towards the definition of an effective shear viscosity

In a viscous fluid such as water, and supposing the exp(-$i\omega t$) time dependence,

$$k_S^2 = \frac{\rho}{\mu}\omega^2, \qquad \mu = -i\omega\eta, \tag{3}$$

so that the shear wavenumber has equal real and imaginary parts. The fact that the effective shear wavenumber calculated in the preceding section almost satisfies this condition suggests that one could define an effective medium, at least from the shear wave point of view, with a real effective mass density ρ_{eff} and a (practically) real effective shear viscosity η_{eff} obeying

$$\frac{\eta}{\eta_{\text{eff}}} = \frac{\xi_S^2}{k_S^2}\frac{\rho}{\rho_{\text{eff}}}. \tag{4}$$

There have been many published works regarding the effective density of a random configuration of scatterers in a host matrix, with nearly as many formulas for that density. All agree on the complex nature of the effective mass density, as well as on its real static limit, which is given [4,5] by a simple volume average in the case of an elastic matrix,

$$\rho_{\text{eff}} = (1-c)\rho_{\text{water}} + c\rho_{\text{Silica}}\,, \tag{5}$$

and by a different one [4] in the case of an inviscid fluid matrix.

Taking into account the shear viscosity of water through Eq. (3) is equivalent to considering water as a solid with a purely imaginary second (shear) Lamé coefficient μ, and thus adopting Eq. (5) with Eqs. (2, 4) allows an effective viscosity, η_{eff}, to be obtained. This effective viscosity η_{eff}, which is almost real, is plotted in Figure 6 (black curve), and has an imaginary part of less than 2 % of the viscosity of pure water η. The other curves in Figure 6 correspond to a few hydrodynamic models of viscosity for hard sphere suspensions in water that are described in Ref. [5]. While all hydrodynamic models provide quite different results, they all show a monotonic increase of the effective viscosity with concentration, which is not the case in our calculation (black curve). This dramatically different behavior may be explained from the difference in the assumptions of the hydrodynamic models and the one used here. All of them correspond to static expressions of the effective mass density and viscosity, whereas our calculation, while using a static approximation for the effective density (Eq. (5)), has been obtained from a multiple scattering model at intermediate frequency $k_C a \simeq 2\times10^{-4}$, and $k_S a \simeq 0.1\ 1+i$. It is quite clear that one should use here a frequency dependent approximation of the effective mass density, but this is a topic of further investigation. A comparison of our model calculations of the effective compressional and shear wavenumbers with those obtained from various self consistent models such as [6,7] is also an area for future work.

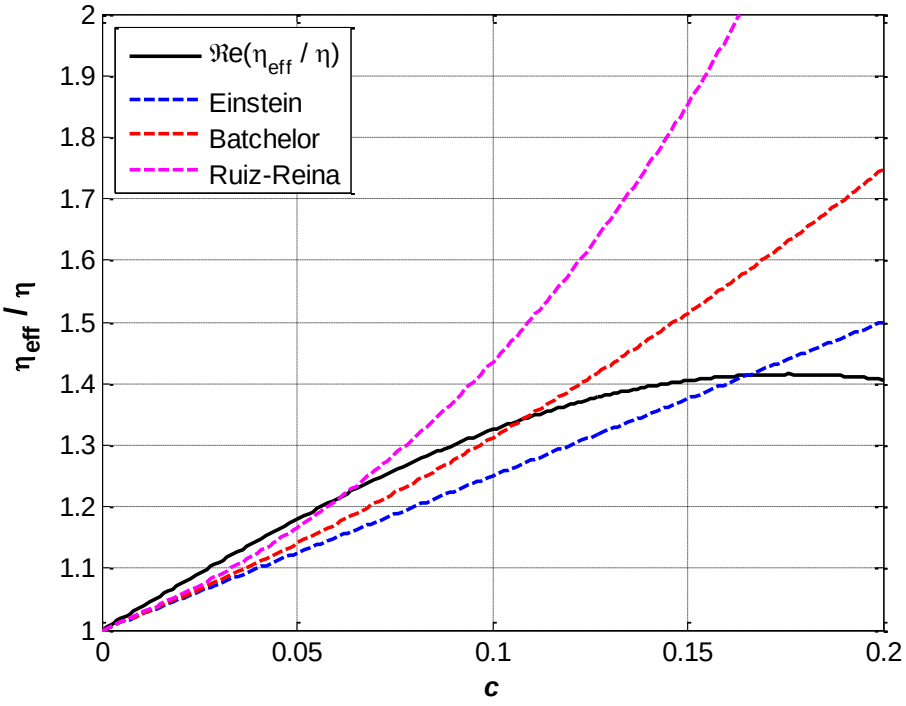


Figure 6. The effective shear viscosity as a function of concentration. Hydrodynamic models follow Ref. [5].

4. Conclusion

This work is a preliminary study of the properties of the shear coherent wave in suspensions of spherical particles. It highlights the truncation of the series in partial wave orders as well as the problem with convergence of the concentration series. Although an effective viscosity has been determined, its validity requires use of a frequency dependent effective mass density. Further work will include the introduction of self consistent models such as in Refs. [6, 7] and their relation with the model used here. The relation between hydrodynamic models and multiple scattering effective theories should be also investigated further, as begun by the authors of Ref. [8].

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