A perturbation solution for long wavelength thermoacoustic propagation in dispersions

A Perturbation Solution for Long Wavelength Thermo-Acoustic Propagation In Dispersions [*]

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Abstract

In thermoacoustic scattering the scattered field consists of a propagating acoustic wave together with a non-progagational "thermal" wave of much shorter wavelength. Although the scattered field may be obtained from a Rayleigh expansion, in cases where the particle radius is small compared with the acoustic wave length, these solutions are ill-conditioned. For this reason asymptotic or perturbation solutions are sought. In many situations the radius of the scatter is comparable to the length of the thermal wave. By exploiting the non-propagational character of the thermal field we obtain an asymptotic solution for long acoustic waves that is valid over a wide range of thermal wavelengths, on both sides of the thermal resonance condition. We show that this solution gives excellent agreement with both the full solution of the coupled Helmholtz equations and experimental measurements. This treatment provides a bridge between perturbation theory approximations in the long wavelength limit and high frequency solutions to the thermal field employing the geometric theory of diffraction.

Key words:
PACS: 43.35.B, 82.70.K, 34.50.B, Helmholtz equation, ultrasound spectroscopy, low frequency limit

1 Introduction

Ultrasound spectroscopy is an increasingly popular technique for characterising the physical properties of soft solids. It is a non-invasive technique that can address the extensive range of particle sizes encountered in soft solids and can be used with optically opaque materials. This technique has now been adopted in many manufacturing processes encountered in the food, chemical and pharmaceutical industries, for example.

The theory of ultrasound propagation in dispersions was first formulated and solved by Lord Rayleigh in 1897 [1]. Subsequently Epstein and Carhart [2] applied the theory to water fogs and Allegra and Hawley [3] derived an approximation for solid particles, providing further experimental verification. The transmission problem is characterised by a system of two coupled Helmholtz equations which correspond to two distinct scattered fields; an acoustic reflection of the incident acoustic radiation of wavenumber K; and a much shorter attenuated "thermal" wave of wavenumber L. The wavelength of the thermal wave is the distance heat can diffuse within one period of oscillation and so is proportional to the inverse square of the frequency, whereas the acoustic wavelength is inversely proportional to the frequency. Consequently the ratio of the two wavelengths varies with the square root of frequency. A full description may be found in McClements and Povey [4], and Povey [5] who additionally utilized a result of Lloyd and Berry [6] to account for the effects of multiple scattering. For a rigorous development of the Lloyd-Berry theory see Linton and Martin [7]. These theoretical descriptions have been developed to describe accurately acoustic propagation in practical cases such as those of food and similar emulsion, up to about 40% v/v of the dispersed phase, although deficiencies [8] in the multiple scattering description prevent the achievement of accurate results at higher concentrations.

For a single spherical scatterer an exact solution of the system of coupled equations can be obtained as an infinite sum of spherical Hankel and Bessel functions. It has been shown that this solution in the form of Rayleigh expansions is ill-conditioned [8], giving rise to numerical problems in evaluating the far field radiation pattern, particularly when in the long wavelength limit $Ka \ll 1$, where a is the particle radius. Since this is the case for many applications of ultrasound, there is a need for a more robust solution in the low frequency limit.

Although Ka is small in many applications, the length of the thermal wave can be small or large compared with the particle size, depending upon the frequency. For low frequencies and very small particles, where the angular frequency $\omega \ll \frac{\sigma}{a^2}$ (where σ is the thermal diffusivity), both $Ka \ll 1$ and $La \ll 1$. However, it is also possible to find ranges of particle size and frequency for

which $Ka \ll 1$ and $La \gg 1$. We have examined both of these limits in previous papers [8], [10]. For the case where both Ka and La are small we developed an asymptotic technique for low frequencies introduced by Kleinman [12] to obtain a low frequency potential scattering theory (LFPST) [8], [9]. In the other limit where Ka is small but where La is large, we developed a hybrid method by combining the low frequency potential solution for the acoustic wave with the geometric theory of diffraction for the short wave length thermal wave [10]. In a significant departure from the above methods discussed in [11] all wave modes are expanded as a series in Ka, leaving dependence on La implicit in the coefficients. This avoids assumptions on the size of La.

In this paper we consider the intermediate case of $|La|$ of order unity. This is the regime of maximum attenuation per unit wavelength and so is of most practical interest and links together our two previous asymptotic limits. Although this situation is covered in [11] it has its own independent interest, as we show that in this regime the two sets of boundary conditions decouple, greatly simplifying the analysis. In the next section we outline the equations governing thermoacoustic scattering and set up the perturbation solution. In section three we use this approach to find an analytic solution for scattering by an isolated sphere up to second order. Finally in section four we discuss the form of the far field radiation pattern and compare our results with experimental measurements and the full solution of the thermoacoustic scattering problem for a single sphere.

2   Thermoacoustic Scattering

We now introduce the equations and boundary conditions for thermoacoustic scattering in emulsions keeping our variables consistent with those of [8] and [11]. We consider the scattering of a plane sound wave of frequency ω by a droplet with boundary B that has contrasting thermal and compressive properties. For simplicity we will neglect the effects of viscosity as these are of secondary importance in oil-water emulsions, though these can be incorporated in the formulation [2]. We define the domains D_1 and D_2 to be regions outside and inside the droplet respectively and $\mathbf{n}$ to be outward normal on B.

By assuming an harmonic time dependence we introduce the velocity potential Φ which is the sum of a compressional wave potential φ and thermal wave potential ψ

$$\Phi = e^{-i\omega t} \left(\varphi + \psi + \varphi_0 \right) \ ,$$

$\varphi_0 = e^{iKz}$ is the incoming wave and φ and ψ satisfy separate Helmholtz equations

$$\left(\nabla^2 + K^2 \right) \varphi = 0 \quad \text{and} \quad \left(\nabla^2 + L^2 \right) \psi = 0 \quad \text{in} \ \ D_1. \tag{1}$$

Here K is the acoustic wavenumber and $L = \left(\frac{\omega}{2\sigma}\right)^{1/2}(1+i)$. The corresponding thermal fluctuations are given by $T = e^{-i\omega t}\left(\Gamma_c\varphi + \Gamma_t\psi + \Gamma_c\varphi_0\right)$ where defintions of Γ_c and Γ_t are given in [4].

Equivalent equations hold inside the droplet, so that

$$\Phi = e^{-i\omega t}\left(\varphi' + \psi'\right),$$

with

$$\left(\nabla^2 + K'^2\right)\varphi' = 0 \quad \text{and} \quad \left(\nabla^2 + L'^2\right)\psi' = 0 \quad \text{in } D_2. \tag{2}$$

We use primes to denote quantities in the droplet phase.

The external scattered fields φ and ψ must satisfy the Sommerfeld radiation condition at infinity [13], [14],

$$\lim_{r\to\infty} r\left(\frac{\partial\varphi}{\partial r} - iK\varphi\right) = 0, \quad \lim_{r\to\infty} r\left(\frac{\partial\psi}{\partial r} - iL\psi\right) = 0, \tag{3}$$

while at the droplet boundary the normal velocity, pressure, temperature and heat flux must be continuous, giving the following four boundary conditions on the boundary B

a) normal velocity

$$\frac{\partial}{\partial n}\left(e^{iKz} + \varphi + \psi\right) = \frac{\partial}{\partial n}\left(\varphi' + \psi'\right), \tag{4}$$

b) pressure-

$$e^{iKz} + \varphi + \psi = \hat{\rho}\left(\varphi' + \psi'\right), \quad \hat{\rho} = \rho'/\rho \tag{5}$$

c) temperature-

$$\Gamma_c\left(e^{iKz} + \varphi\right) + \Gamma_t\psi = \Gamma'_c\varphi' + \Gamma'_t\psi', \tag{6}$$

d) heat flux-

$$\Gamma_c\frac{\partial}{\partial n}\left(e^{iKz} + \varphi\right) + \Gamma_t\frac{\partial}{\partial n}\psi = \hat{\tau}\left(\Gamma'_c\frac{\partial}{\partial n}\varphi' + \Gamma'_t\frac{\partial}{\partial n}\psi'\right), \quad \hat{\tau} = \tau'/\tau. \tag{7}$$

Here $\hat{\rho}$ and $\hat{\tau}$ are the repectively the ratios of density and thermal conductivity between the dispersed and continuous phases.

Rather than solving this pair of coupled Helmholtz equations we shall seek a perturbation solution for the limit when the acoustic wavelength is large compared to the size of the droplet. In the limit $Ka \ll 1$, $|La| \sim 1$, the ratios of the thermal coefficients

$$\frac{\Gamma_c}{\Gamma_t} = -K^2 a^2 G_c \qquad \frac{\Gamma'_c}{\Gamma'_t} = -K^2 a^2 G'_c, \tag{8}$$

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where a is a typical dimension of the droplet with G_c and G'_c order one quantities. With this change of parameters, the thermal boundary conditions (equations (6) and (7)) become

$$-K^2 a^2 G_c \left(e^{iKz} + \varphi \right) + \psi = \hat{\Gamma} \left(-K^2 a^2 G'_c \varphi' + \psi' \right), \quad \hat{\Gamma} = \Gamma'_t / \Gamma_t, \tag{9}$$

$$-K^2 a^2 G_c \frac{\partial}{\partial n} \left(e^{iKz} + \varphi \right) + \frac{\partial}{\partial n} \psi = \hat{\tau} \hat{\Gamma} \left(-K^2 a^2 G'_c \frac{\partial}{\partial n} \varphi' + \frac{\partial}{\partial n} \psi' \right). \tag{10}$$

Thus we see that the acoustic wave only contributes to the temperature perturbation at second order in Ka. Consequently the thermal field will be of order $K^2 a^2$, and so will only affect the scattered potential at this order.

3 Low Frequency Thermoacoustic Scattering by an Isolated Spherical Droplet

Radiating solutions of the Helmholtz equation satisfying the radiation condition have the asymptotic form [13], [14], [16],

$$\varphi = \frac{e^{iKr}}{r} \sum_{n=0}^{\infty} \frac{f_n(\theta, \Omega)}{r^n}, \tag{11}$$

where f_n are differentiable functions of K and the polar angles. Consequently, φ is not regular at infinity as $r^2 \frac{\partial \varphi}{\partial r}$ is unbounded as $r \to \infty$. However, following Kleinman [12] we can transform the Helmholtz equation into a regular problem by introducing the function

$$\tilde{\varphi} = e^{-iK(r-a)} \varphi, \tag{12}$$

so that φ is a regular function satisfying

$$\nabla^2 \tilde{\varphi} = -\frac{2iK}{r} \frac{\partial}{\partial r} (r\tilde{\varphi}) \tag{13}$$

with

$$\lim_{r \to \infty} r^2 \frac{\partial \varphi}{\partial r} \to 0 \quad \text{as} \quad r \to \infty.$$

This transformation allows us to seek a regular perturbation solution for $\tilde{\varphi}$, ψ, φ' and ψ' as power series in iKa,

$$(\tilde{\varphi}, \varphi', \psi, \psi') = \sum_{n=0}^{\infty} (iKa)^n \left(\tilde{\varphi}_n, \varphi'_n, \psi_n, \psi'_n \right) \tag{14}$$

(Without the transformation it is necessary to match a solution for $r \sim a$ to an outer solution for $r \sim 1/K$ in order to enforce the radiation condition). For $|Ka| < \ln(2)$ these series converge (see [12]), and in practice are rapidly

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convergent. The error is of size $|Ka|^{m+1}$ if the series is truncated at order m, and so we are guaranteed to obtain an accurate estimation of the potential with only a few terms. Substitute the expansions (14) into equations (1) and (2) we obtain, for $n \geq 0$, the equivalent system:

$$\nabla^2 \tilde{\varphi}_n = -\frac{2}{ar}\frac{\partial}{\partial r}\left(r\tilde{\varphi}_{n-1}\right), \qquad \text{for } r > a, \tag{15}$$

together with

$$\nabla^2 \varphi'_n = \frac{\hat{c}}{a^2}\varphi'_{n-2}, \qquad \text{for } r < a. \tag{16}$$

where $\hat{c} = K'^2/K^2$. In equations (15) and (16) we understand that $\tilde{\varphi}_{-1} = \varphi'_{-1} = \varphi'_{-2} = 0$. Thus the solution for the acoustic wave is reduced to a series of potential problems. We are still left with Helmholtz equations for the thermal wave at each order $n \geq 2$,

$$\left(\nabla^2 + L^2\right)\psi_n = 0 \quad \text{for } r > a, \tag{17}$$

and

$$\left(\nabla^2 + L'^2\right)\psi'_n = 0 \quad \text{for } r < a. \tag{18}$$

However, because iLa has a negative real part of order unity, the solutions decay exponentially away from the boundary and so regularity issues at infinity are avoided.

The boundary conditions at $r = a$ can be grouped into two pairs. For $m \geq 2$, the thermal boundary conditions at order $|Ka|^n$, are

$$\psi_n + G_c\left(\frac{1}{(n-2)!}\cos^{n-2}\theta + \tilde{\varphi}_{n-2}\right) = \hat{\Gamma}\left(\psi_n + G'_c\varphi'_{n-2}\right) \tag{19}$$

$$a\frac{\partial \psi_n}{\partial r} + G_c\left(\frac{n-2}{(n-2)!}\cos^{n-2}\theta + a\frac{\partial \tilde{\varphi}_{n-2}}{\partial r} + \tilde{\varphi}_{n-3}\right) = \hat{\Gamma}\hat{\tau}a\left(\frac{\partial \psi'_n}{\partial r} + G'_c\frac{\partial \varphi'_{n-2}}{\partial r}\right), \tag{20}$$

which provide boundary conditions for the solution of equations (17) and (18) for the thermal field. Once ψ_n and ψ'_n are known, the solution for the acoustic field at order n may be found from the velocity and pressure boundary conditions

$$\frac{1}{n!}\cos^n\theta + \tilde{\varphi}_n + \psi_n = \hat{\rho}\left(\varphi'_n + \psi_n\right) \tag{21}$$

$$\frac{n}{n!}\cos^n\theta + a\frac{\partial \tilde{\varphi}_n}{\partial r} + \tilde{\varphi}_{n-1} + a\frac{\partial \psi_n}{\partial r} = a\frac{\partial \varphi'_n}{\partial r} + a\frac{\partial \psi'_n}{\partial r} \tag{22}$$

Thus, we see that at each order the boundary conditions decouple allowing the thermal and acoustic waves to be calculated sequentially, in the order $\tilde{\varphi}_0$, $\tilde{\varphi}_1$, ψ_2, $\tilde{\varphi}_2$, ψ_3, $\tilde{\varphi}_3$.... From the decomposition of the incoming plane wave into spherical harmonics, we can deduce that ψ_n and ψ'_n only contain the first $n-2$ spherical harmonics so that ψ_n and ψ'_n can be written as finite sums of the form

$$\psi_n = \sum_{m=0}^{n-2} B_{nm} h_m \left(Lr \right) P_m(\cos\theta), \tag{23}$$

$$\psi'_n = \sum_{m=0}^{n-2} B'_{nm} j_m \left(L'r \right) P_m(\cos\theta). \tag{24}$$

Similarly $\tilde{\varphi}_n$ and φ_n will only contain the first n spherical harmonics and so may be written as

$$\tilde{\varphi}_n = \sum_{m=0}^{n} A_{nm} \frac{a^{m+1}}{r^{m+1}} P_m(\cos\theta) + I_n(r,\theta) \tag{25}$$

$$\varphi'_n = \sum_{m=0}^{n} A'_{nm} \frac{r^m}{a^m} P_m(\cos\theta) + I'_n(r,\theta). \tag{26}$$

where I_n and I'_n are particular integrals of the inhomogeneous equations.

At order $n = 0$, the solution is $\tilde{\varphi}_0 = 0$ and $\varphi'_0 = 1/\hat{\rho}$. At this order the incident field is uniform, and so there is no reflected field outside the particle, merely an increase in the pressure within the droplet to balance the external field.

The leading contribution to the reflected field occurs at order $|Ka|$ in the form of a dipole field

$$\tilde{\varphi}_1 = A_{11} \frac{a^2}{r^2} \cos\theta, \qquad \varphi'_1 = A'_{11} \frac{r}{a} \cos\theta. \tag{27}$$

where the coefficients A_{11} and A'_{11} are given by

$$A_{11} = \frac{\hat{\rho} - 1}{1 + 2\hat{\rho}}, \qquad A'_{11} = \frac{3}{(1 + 2\hat{\rho})}.$$

However, $\tilde{\varphi}_1$ decays quadratically as $r \to \infty$ and does not contribute to the far field. In order to compute $\tilde{\varphi}_2$ we must find the leading order contribution to the thermal field. Thus at $n = 2$ we only have the zeroth harmonic appearing in ψ_2, giving

$$\psi_2 = \frac{\hat{\tau}(\hat{\Gamma} G'_c - \hat{\rho} G_c)(L'a - \tan L'a)}{\hat{\rho}[\hat{\tau} L'a + (1 - \hat{\tau} - iL'a) \tan L'a]} \frac{a}{r} e^{iL(r-a)}. \tag{28}$$

Substituting this value for ψ_2 into the boundary conditions (21) we find the solution for $\tilde{\varphi}_2$ in the form

$$\tilde{\varphi}_2 = A_{22} \frac{a^3}{r^3} P_2(\cos\theta) + \left(A^a_{20} + A^t_{20} \right) \frac{a}{r} - A_{11} \frac{a(r-a)}{r^2} \cos\theta. \tag{29}$$

where

$$A_{22} = \frac{2(\hat{\rho} - 1)}{3(2 + 3\hat{\rho})},$$

$$A_{20}^a = \frac{\hat{\rho} - \hat{c}}{3\hat{\rho}},$$

$$A_{20}^t = \frac{(\hat{\Gamma}\hat{\tau} - 1)(\hat{\Gamma}G_c' - \hat{\rho}G_c)(L'a - \tan L'a)(iLa - 1)}{\hat{\Gamma}\hat{\rho}[\hat{\tau}L'a + (1 - \hat{\tau} - iL'a)\tan L'a]}.$$

Here we have divided the monopole reflection term into the acoustic reflection A_{20}^a plus an additional term, A_{20}^t due to thermal effects.

Far from the droplet the reflected acoustic wave has the form

$$\varphi \sim \frac{e^{iKr}}{r} f(\theta)$$

and so for small $|Ka|$ the far field radiation pattern is given by

$$f(\theta) = K^2 a^3 \left(A_{11} \cos\theta - A_{20}^a - A_{20}^t \right) + O(|Ka|^3) \text{ as } r \to \infty. \qquad (30)$$

ψ is not present in the far field due to the non-propagational character of the thermal field.

The advantage of the perturbation method outlined here is that it can easily be extended to higher order in Ka and can also be applied to multiple particles or non-spherical particles when applied with the general potential theory approach developed by Kleinman [12].

The far field radiation pattern may be divided into three terms. The terms with coefficients A_{11} and A_{20}^a come from acoustic scattering. These coefficients depend only on the ratios of density and sound of the two phases, and are independent of $|La|$ and the thermal properties of the two phases. Thermal effects are confined to the term A_{20}^t. Provided that the acoustic attenuation in the pure phases is small, the imaginary parts of the acoustic terms A_{20}^a and A_{11} will be small, and so the imaginary part of f will be dominated by the thermal term A_{20}^t.

4 Thermoacoustic Attenuation in an Oil-in-Water Emulsion

We can now use the results of this calculation to predict the attenuation of ultrasound in an oil-in-water emulsion. This is compared with experimental measurements by Hermann et al [17] on a 5% silicone oil-in-water emulsion and full numerical solutions of equations (1) to (7)). In order to account for the effects of multiple scattering, we use the theory of Lloyd and Berry [6],

which gives the effective wave number $\tilde{K}$ for a dispersion of droplets at volume fraction $\phi = 4\pi a^3 n_0/3$ as

$$
\begin{aligned}
\left(\frac{\tilde{K}}{K}\right)^2 &= 1 + \frac{3\phi f(0)}{K^2 a^3} \\
&+ \frac{9\phi^2}{4K^4 a^6}\left\{f^2(\pi) - f^2(0) - \int_0^\pi d\theta \frac{1}{\sin(\theta/2)}\{\frac{d}{d\theta}f^2(\theta)\}\right\}.
\end{aligned}
\tag{31}
$$

This theory assumes that multiple scattering may be accounted for by a linear superposition of the reflected acoustic fields, and so requires that there is no overlap between the thermal fields of different particles. Consequently this approximation is expected to break down for $\phi > |La|^3$ when overlapping of the thermal fields becomes significant.

Substituting the expression for f given in equation (30) we find that

$$
\left(\frac{\tilde{K}}{K}\right)^2 = 1 + 3\phi(A_{11} - A_{20}^a - A_{20}^t) + 3\phi^2 A_{11}(2A_{11} - 3A_{20}^a - 3A_{20}^t)
$$

The dispersion wave number $\tilde{K}$ may be written as $\tilde{K} = \omega/\tilde{v} + i\alpha_{tot}$, where $\tilde{v}$ is the effective sound speed in the dispersion and α_{tot} is the total attenuation resulting from the attenuation in the two separate phases and the scattering at the droplet surface. In order to separate the attenuation due to scattering, it is necessary to subtract the contributions from the phases $\alpha = \{\alpha_{tot} - \alpha_c(1 - \phi) - \alpha_p\phi\}$ where α_c, α_p denote the attenuation coefficients of the continuous phase and the particles.

Measurements of the attenuation were made over a range of frequencies from 0.5 to $10 MHz$, and for droplet sizes ranging from 230 to $760nm$ in the Silicone oil emulsion. For these parameter values the values of Ka ranged from 10^{-4} up to 10^{-2} so that the all the data satisfy $|Ka| \ll 1$. The calculations of the full thermoacoustic scattering problem were performed using Maple 6 with 24 point arithmetic [8] to ensure accuracy.

The form of the second-order solution for the far field radiation pattern suggest that the attenuation from different frequencies and droplet sizes may be superimposed to form a master curve by plotting attenuation per unit wavelength $\alpha\lambda$ against La. In figure 1 it can be seen that the experimental data does indeed follow a single master curve, and are in remarkably good agreement with the two sets of theoretical calculations. The two theoretical solutions are virtually identical and indicating that for these conditions the higher order terms in $|Ka|$ indeed small.

All the curves show a maximum in the attenuation at a value of $|La|$ of ap-

proximately 2.5. For small values of $|La|$ the attenuation scales as $|La|^2$. From equation (28) it can be seen that for $|Ka| \ll |La| \ll 1$ the thermal field outside the particle is of order $(Ka)^2|La|^2$. At very low frequencies, or for very small droplets, the distance heat can diffuse within one period of oscillation is much larger than the droplet radius. Consequently heat produced within the droplet due to the differences in thermal expansivity of the two phases diffuses over a large volume of fluid, of radius $1/L$. Thus although the thermal field has a large extent, its magnitude is very small. Since the temperature perturbation within thermal field falls off like $1/r$ the thermal gradient at the surface of the droplet will be of order $|La|^2$. A more complete analysis for the regime where $|Ka|, |La| \ll 1$ is given in our earlier paper on the low frequency potential scattering approximation.

By varying the particle size and frequency it is also possible to obtain a situation in which Ka is small, but $|La|$ is large. The attenuation is also small in this limit and now scales as $|La|^{-1}$. In this regime the diffusion length is much shorter than radius of the droplet and so only a fraction $|L'a|^{-2}$ of the heat generated within the droplet is able to diffuse out. This is contained with a boundary of length $1/L$ outside of the droplet and giving a thermal gradient of order $1/|La|$. As the thermal wavelength is now much smaller than the droplet radius the solution of the thermal wave is analogous to a high frequency radiation problem, and so may be analysed using the geometric theory diffraction [10].

The maximum attenuation is found when $|La|$ is of order one. This corresponds to the case when the diffusion length is equal to radius of the droplet. The frequency of the ultrasound is low enough to allow time for all the heat to diffuse out of the particle, but to keep it confined within a shell of thickness a around the droplet.

5 Conclusions

The perturbation solution presented in this paper provides an accurate solution for the scattered fields in long wavelength acousic limit for arbitrary thermal wavelengths. It has the potential to make an important practical contribution to ulrasound spectroscopy by converting an ill-conditioned problem amenable to inversion and the determination of particle size distribution from attenuation and velocity spectra. By simplifying the field structure it allows one to probe the region of parameter space in which the attenuation per wavelength is maximised.

Acknowledgments

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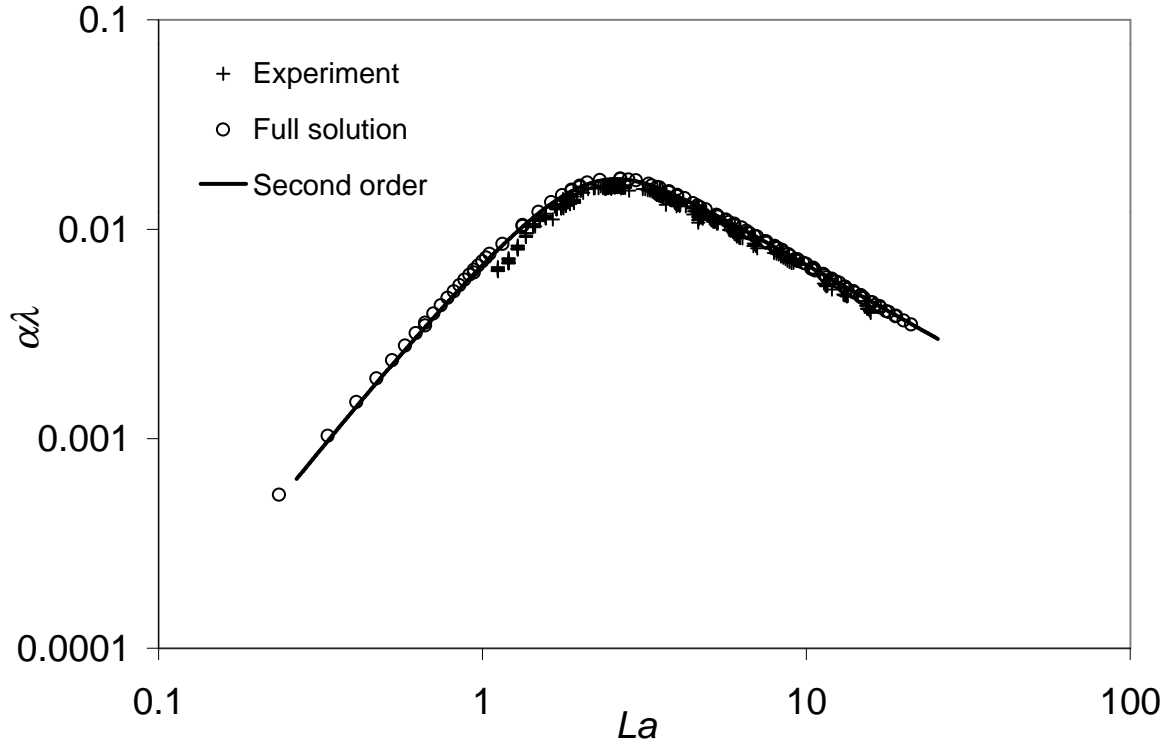
Fig. 1. *Plot of $\alpha\lambda$, the attenuation per wavelength against La, the product of thermal wave number and droplet radius for 5% silicone oil emulsion for four different droplet radii, $a = 2.3$—7.6×10^{-7} m. The continous line shows the result for the asymptotic solution taken to order $K^2 a^2$; the circles show the results from the full solution of equations (1) to (7) ; and the plus symbols denote experimental data of reference [17].*

	Silicone oil	Aqueous phase
Sound velocity $\upsilon(m\ s^{-1})$	1004	1482
Density $\rho(kg\ m^{-3})$	975	998.2
Thermal expansivity $\beta(K^{-1})$	9.4×10^{-4}	2.13×10^{-4}
Specific heat $C_p(J\ kg^{-1}K^{-1})$	1460	4182
Thermal conductivity $\tau(W\ m^{-1}K^{-1})$	0.15	0.591

Table 1
Values of physical parameters for silicone oil in water emulsions.