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A STUDY OF INDIVIDUALISED
SYSTEMS FOR MATHEMATICS INSTRUCTION
AT THE POST-SECONDARY LEVELS

by

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A Doctoral Thesis

Submitted in partial fulfilment of the
requirements for the award of Doctor of
Philosophy of the Loughborough University
of Technology. June 1977.

Supervisor: Professor A.C. BAJPAI,
Department of Engineering Mathematics,
and Director of CAMET.

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A STUDY OF INDIVIDUALISED SYSTEMS FOR MATHEMATICS INSTRUCTION
AT THE POST-SECONDARY LEVELS

- Alexander J. Romiszowski -

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SECTION A

THE "WHAT, WHEN, WHO AND WHY" OF INDIVIDUALISATION

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CHAPTER 1

BACKGROUND AND JUSTIFICATION FOR THE AREA OF STUDY

- 1.1 Early sources of the author's interest in individualised mathematics teaching and the problem of the mature student.
- 1.2 Work in remedial mathematics instruction at the Middlesex Polytechnic and elsewhere.
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CHAPTER 1

BACKGROUND AND JUSTIFICATION FOR THE AREA OF STUDY

1.1 EARLY SOURCES OF THE AUTHOR'S INTEREST IN INDIVIDUALISED MATHEMATICS TEACHING AND IN THE PROBLEMS OF THE MATURE STUDENT

The author's interest in the process of mathematics teaching was first aroused during his secondary schooldays. The school he was attending - a small country Grammar School in the Cotswolds - was much more renowned for its sporting record than for academic excellence. Particularly in Mathematics, standards were not high. Typically, only about half of the "A" stream were even entered for the Ordinary Level G.C.E. examination in Mathematics and typically about half of those entered gained passes.

Two years before the author's "O level ordeal" a new teacher took over the fourth and fifth form mathematics teaching. The next year, all the "A" stream were entered for "O" level mathematics and all passed their G.C.E. examinations. It was about this time that the author resolved to be a teacher of mathematics. He may also have been influenced by the fact that the new teacher was his mother.

However, life held many surprises, and the author found himself first studying engineering, followed this by a Diploma in Education, but only taught for a year or so before being "swallowed by industry". He spent two years in the motor car industry, with the original intention of

"gaining industrial experience", but soon was again diverted into education and training. Much experience was gained designing training and re-training schemes at all levels, from apprentices, foremen, shop stewards, to middle and top management.

The period of teaching had been spent partly in a grammar school, but mainly in a private "cramming college" preparing mature students for a second, third (or further) attempt at G.C.E. mathematics and science examinations. Now, again the author was often faced with groups of trainees who had "tried and failed" before or who had to retrain in later life in order to keep down a job. It was obviously necessary to treat such mature, non-academic students differently. But typically, the courses organised for them were no different in structure or style from those they had experienced (and hated?) at school.

The author began investigating alternative course structures, better suited to the adult in industry and as this was 1963-65, inevitably experimented with the use of programmed instruction. An individual-progress system of apprentice training was devised, based on programmed booklets for the theory content of a one-year course and individual instruction, from instructors, on the practical aspects. This course was highly successful, and became the model for apprentice training in many engineering firms. The programmes were published by International Tutor Machines and by Heinemann's. They were adopted by the Engineering Industry Training Board as part of their recommended training materials (Romiszowski, 1965, 1966).

Similar individualised training schemes were devised for other groups of trainees, including foremen (Principles of supervision, work study, etc.), accounts clerks (how to use a slide rule, tables, etc.), and middle management (statistics for business, quality control, etc.). It was found that the programmed booklet approach only broke down if the trainees had poor reading skills. Otherwise, the introduction of individualised, self-paced, learning (though not to the total exclusion of group work) improved learning efficiency, attitude to the course and to the job. The mature trainee in particular welcomed such aspects as the ability to make his mistakes in private, not observed by his colleagues or superiors, or the ability to take the study material home, spend as long as he needed to learn, and indeed keep the material permanently for later reference and revision. Such aspects tended to reduce the trainees' anxiety at being "back in school".

1.2 WORK WITH REMEDIAL MATHEMATICS INSTRUCTION

In 1965 the author joined the Enfield College of Technology to work in the "Programmed Instruction Centre", specialising in the development of such individualised, programmed courses both for the college and for outside industrial clients. He spent part of 1967 in the U.S.A. on sabbatical, studying other systems of individualised learning including computer assisted instruction. On his return the opportunity arose to combine his interests in individualised systems of instruction with his earlier interests in mathematics teaching. A computer-managed,

individualised, remedial mathematics course was devised for the first year social science and business studies undergraduates. The course components were, firstly, a set of diagnostic tests covering the mathematics skills considered to be pre-requisite to the statistics content of the degree courses, secondly self-instructional materials mainly in the form of programmes (texts and teaching machines, thirdly tutorial back-up. The role of the computer was to perform a "first line" diagnosis of mathematics difficulties (thus reducing the number of students that need to be interviewed directly by the tutor) and a first "prescription" of the self-instructional units that a given student should study. The system, as it was first conceived, is described in a paper (Hamer and Romiszowski, 1970) which may be found in the appendix to this study. It was designed and pre-tested in 1968 and, from 1969 to date, has been coping quite well with the remedial mathematics needs of the students at Enfield. Since the Enfield College became part of the new Middlesex Polytechnic, the use of the system has spread to other groups of students and to courses with differing mathematical needs. Outside interest in the scheme was also generated. Several other Polytechnics and some Universities have adopted the system (or at least the basic idea). The service forms an integral part of several Course submissions to the Council for National Academic Awards (CNAA) from Middlesex Polytechnic. We have several times been involved in projects with local schools, to help with the teaching

of mathematics during periods of teacher shortage. Recently we have been involved as consultants to the Open University in the preparation of remedial mathematics units for some Open University courses. One course unit, a "Developmental and Diagnostic Mathematics" booklet, based on the Enfield system, is currently in use (Open University, 1977) and there are plans to produce more such units for the Open University in the future.

These activities have resulted in a growing interest and involvement on the part of the author with the problems of mathematics teaching in general and with the use of educational technology for mathematics teaching in particular.

Various alternative schemes and systems were tried at Enfield, investigating among other aspects the role of the tutor in such individualised systems. Three different schemes for running the course were tried utilizing various levels of tutor support to the individualised materials. Investigations were made of systems not utilizing programmed materials. Tests were made of the "Keller Plan". Some of these investigations are described in the following chapters and also in a paper included in the appendix (Romiszowski, Bajpai and Lewis 1976).

The author's growing interest in "individualised mathematics for adults" led to involvement with other similar projects. For some years close co-operation was maintained with the late Kenneth Gray, then organising the "Surrey Autotutor Project" and interesting team-teaching experiments

at the France Hill School in Surrey (Surrey County Council, Education Department, 1970) and with Charles Blake, then responsible for Educational Technology in the Wiltshire Local Education Authority and deeply involved with the development of learning materials for elementary and secondary level mathematics (the Wiltshire programmes, published from 1967 to 1970).

Farther afield, but back in the area of adult education, the author was between 1969 and 1973 involved with projects of the Council of Europe concerned with "Permanent" or "Continuing" adult education. He investigated the potential in general of self-instructional techniques for adult education (Romiszowski and Biran, 1970) and more specifically was involved in a multi-national project to develop some experimental materials in the field of mathematics for adults in business. This project, though short-lived, produced some experimental materials in self-instructional format on the topics of Vectors and Matrices.

The original versions of these materials were written at the Centre for the Advancement of Mathematical Education in Technology (CAMET) at the University of Loughborough under the direction of Professor A.C. Bajpai. Alternative versions of the materials, utilizing a different method of organisation of the material in the book (information mapping) were prepared by the author at Enfield and a small pilot experiment comparing the two versions was performed (Romiszowski and Ellis, 1973). Plans to evaluate and compare yet other

alternatives did not materialise, as the alternative versions utilizing television and other media, which were the responsibility of a French institute, were never completed.

However, the author's interest in alternative ways of organising self-instructional material and in alternative systems for the implementation of individualised instruction continued.

1.3 WORK IN BRAZIL

During 1974 to 1975, the author had the opportunity to extend his work in this field to encompass the special problems of the developing countries. He worked as a member of an international team in a United Nations-sponsored project in Brazil. The aims of the project were to re-design the technical education systems of the country, including technical teacher training and even the secondary school curriculum. The reason for this latter was a decision taken by the Brazilian Ministry of Education and Culture that all children should, during their secondary schooling, learn a directly employable skill (trade or profession) in addition to traditional academic subjects. The reasons behind this move were to promote the development of the various skills required by the rapid industrialisation and urbanisation of the country, and also to stem the traditional drift of all academically successful students into the "traditional" universities and the "traditional" professions of law and medicine. The problem of over-production of trained personnel in certain popular top-level professions at the

expense of other professions and the total neglect of the lower (technician and craftsman) levels, is common to many developing countries.

Among a variety of tasks tackled during 1974/75, (which included the design of teacher training on an in-service basis, task analysis of key professions in order to develop appropriate school curricula and the training of teams of teachers to write textbooks, self-study materials, produce audio-visual media etc.), the author had several opportunities to implement and evaluate individual self-instructional systems. Two of these were directly concerned with the teaching of mathematics.

One, the IRDEB project, concerned the production of a correspondence course at the secondary school level destined for adults who had not had the opportunity to attend school, or had not completed their schooling for various reasons. This problem is quite serious both at elementary and secondary level. In the state of Bahia for example there are currently about 67,000 persons over 14 years old, who have not completed the first four years of elementary education - this in a total population of about 4 million. For some years now the problem at the elementary level has been tackled by evening classes and, since 1973 also by a radio-based distance education system run somewhat on the lines of our Open University, but with regular group meetings in the village hall every day, under the supervision of trained group leaders.

The author's task was to help extend this system to the upper secondary level. Due to limited resources, it was impossible to utilise the same model as was used at the primary level. There was neither money nor personnel available to act as group organisers. There were no funds for radio time. Thus the "Instituto de Radiodifusao da Bahia (IRDEB) which is the institution responsible for the project, adopted a correspondence course model. With limited tutorial support, the role of the primary instructional medium (in this case print) becomes much more critical. The author was responsible for the training of the original team of teachers who are currently producing and testing the science and mathematics sections of the correspondence courses. These courses have started operation on an experimental basis, in 1977, so it is still rather early to be able to judge the success of the project. Some evaluation of the mathematics materials has been completed. This has revealed a series of problems due partly to the lack of human support to the (academically poor quality) adult students, and partly due to the rather condensed nature of the printed materials, (necessarily due to economic restrictions on the quantity of material the project can print and send through the post). The author hopes to continue his association with this project in the future.

However, it is the second project, the BASG-M. project, which is the subject of the research in this study. This project resulted from an analysis of the state of mathematical education in the secondary school system.

This analysis considered the curriculum required for the new "semi-vocational" direction that the secondary education system is adopting and also the state of current teaching in the schools. One aspect which came to light was that many students enter the upper-secondary level (age 15) with very large gaps in their mathematical education. Clearly, much of the problem lies lower down in the schools. The author's experiences of visiting and working with schools throughout Brazil indicates that the supply of mathematics teachers (both quantity and quality) is well below requirements. Several "crash" systems for the training of teachers are in operation, many utilizing correspondence, "Open University" or in-service models. These have been described by the author elsewhere (Romiszowski, 1974a). However, the problems and shortcomings in the existing educational systems are such (and the growth rate in demand so rapid) that there is no way in which the expansion of "traditional" educational provision could cope. The Federal Ministry of Education has several times stated that only by the massive and systematic use of new educational technologies can one hope to close the ever widening gap between educational supply and demand.

Several large projects are in operation, attempting various approaches to the solution. These approaches include mass-media based courses (television, radio and the press), correspondence, the use of rapidly trained "para-educational" staff to administer media-based courses, systems which do without teachers altogether, in-service training of local volunteers as teachers, a "multiplier" or "chain reaction" scheme whereby every graduate from a teacher training course

not only teaches, but also runs other teacher training courses, etc. Some of these schemes and systems have been analysed by the author elsewhere (Romiszowski, 1974a; Romiszowski 1975; Romiszowski and Pastor 1977).

The BASG-M Project is one such scheme. It is intended more as "back-up" to the teacher in the classroom. The name is self-explanatory; Bases for Access to the Second "Grau" (level). The Brazilian educational system is now divided into two "levels"; the first level is eight years of "compulsory" schooling (between the ages of 7 and 15) and the second level is three further years of non-compulsory attendance.

Attendance in the "2^o Grau" level is not very high. In some of the poorest states it was as low as 2% of the relevant age group. Recent efforts have improved this considerably, so that now about 20% - 40% is a more typical figure.

However, school attendance is one thing. Getting taught is another. It is recognised that the standard of teaching is not always very high in the "1^o Grau", thus students enter the "2^o Grau" often with very incomplete mastery of the lower school curriculum. This is particularly serious in such subjects as mathematics, where gaps in previous learning inhibit (indeed often render impossible) progress later on.

The BASG-M project's aims are to produce self-study materials which the teacher in the upper level may prescribe individually to students, in order to "fill the gaps which they bring from the lower level".

Materials in Mathematics and in Portuguese are being prepared. The author was involved in the project dealing with mathematics (BASG-M). In 1974 he trained the team of teachers responsible for the preparation of self-study materials to cover the entire lower school mathematics curriculum. During 1975 he gave assistance to the team of writers, as part of his contract with the United Nations. When his United Nations contract terminated, the State Secretariat of Education of the State of Bahia (where the BASG-M project is based) requested further help. The author negotiated that his contract with the Middlesex Polytechnic should become a part-time contract, thus enabling him to spend up to half the year in Brazil. He has continued to consult to the BASG-M project. During August - December 1976, the first field trials of the materials so far produced were carried out. It was during this period that the author carried out the research which is reported in the last section of this study.

The background to this research is fully described in Chapter 10. We shall mention here only the gravity of the situation encountered in the schools which took part in the field testing.

Firstly, pre-testing of the students on their knowledge of the "first level" curriculum, revealed that the predicted "large number of weak students" was indeed very large - nearly 100% of the students in the Second level. Of 1200 students tested, not one reached an 80% level on all the module tests prepared for BASG-M and, more seriously, nearly half the group failed to reach the 80% criterion on

even one of the seven tests administered. Indeed, the mean scores on the tests hovered around 20% to 30%. As these were tests of basic arithmetic and concepts of sets - all material supposedly covered in previous years, and all basic to further progress, it appeared that the task for the teachers in the second level would be to re-teach all the earlier work, rather than to fill some gaps.

But secondly, there was a shortage of teachers in the second level. In the college used for this study (a large school with a 16-form entry of between 40 - 45 students in each form) one third of the students who had entered the second level in January 1976, had, by August when our study began, not received a single lesson of mathematics due to teacher shortages. Indeed the groups we received for our experiments, were exactly these teacher-less groups. It was hoped that our activities would help alleviate this chronic lack of mathematics teachers. This was indeed the case, as all the personnel involved in preparing the materials - the writers, the mathematics teachers who were consulting on content and the author - adopted the roles of teacher, tutor or monitor as the experimental design required. In one way or another, therefore, the classes we were allocated for our experiments received an above average amount of teacher attention during the months of the experiment.

Therefore, we conclude, that the experiments described later were conducted in a school environment which would be

classified as extremely deprived by U.K. standards (at least in so far as Mathematics Education is concerned). However, we have reasons to believe that the conditions were by no means exceptional for Brazil. Indeed if, as seems to be the case, the level of the local educational system is related to the relative wealth or poverty of the particular state, we would place Bahia about half-way up the list. Whereas the rich, industrial, Rio de Janeiro - Sao Paulo belt is relatively well equipped educationally (differing little perhaps from a highly urban overcrowded area in Britain - say Wolverhampton/Birmingham) the poorest states in the North-East (Maranhão, Rio Grande do Norte) and the vast, nearly uninhabited areas of the Amazon Basin (Mato Grosso, Goiás, Amazonas) have much poorer educational facilities than the State of Bahia. After all, the college the author is describing is one of the largest state schools in the State capital, which has a population of 2 million, a growing industrial belt and an oilfield.

1.4 IDENTIFICATION OF THE PROBLEM AREAS

From the viewpoint of the "instructional designer" or "curriculum developer" the situation described above is in marked contrast to the situation which exists in the United Kingdom, or Europe, or the U.S.A. In these countries the innovator is generally faced with the problem of modifying existing entrenched systems. In a situation such as the one described in Brazil, often there is no existing system (or at best a sketchy and not too entrenched one). One is starting with a "tabula rasa". One may design, for example, teacher-less systems concerning

oneself solely with their educational efficiency, and not with the problems of redundant or dissatisfied teachers. Also, as the base-line is so poorly developed, almost any systematically designed educational effort is bound to give significant improvements. It would be easy to concentrate solely on the situation in Brazil and concentrate on the development and testing of instructional systems designed to overcome the types of problems outlined earlier.

However, the author's interest in the use of individualised instruction is not restricted to the problems of developing countries. He believes that individualised self-instruction will play a growing role in the educational systems of the developed world. He predicts that programmed instruction "discovered" in the 1950's and almost forgotten by the 1970's will be rediscovered in the 1980's (not the same types of products as the early programmes, of course, but the process, as a guiding philosophy of teacher training). Particularly in the field of mathematics education, he believes that automated methods, making less use of the teacher, will become the rule. Current research (described later) suggests that by the turn of the century machines may be more efficient at teaching mathematics (particularly the higher level skills such as problem solving) than are most of the teachers currently working in schools for the low salaries we pay. It may well be an economic fact of life that school systems cannot afford good mathematics teachers. The logical ("technological") solution would be to seek efficient teacher-substitutes, or at any rate teacher-aids (devices which can turn a mediocre teacher

into part of an efficient teaching system). Finally, the author's experience both in the United Kingdom and Brazil does not lead him to believe that the quality of teachers of mathematics is any different. The differences between the two countries is one of quantity - teachers are just that much scarcer in Brazil. But the situation in the U.K. is not all that rosy either. After all, Shirley Williams, whilst cutting general teacher training by half, is attempting to expand the production of trained teachers of mathematics.

The author is therefore led to believe that countries like Brazil, with currently more acute problems of supply (but with smaller problems of entrenched attitudes of a traditional teaching profession), with adequate financial resources for educational development (springing from the new industrial revolution) and with an attitude to "get on in a hurry", may well spearhead the practical application of novel solutions to educational problems which may in later years be the models for the more traditional "developed" world to follow.

It is in this light therefore that the author has approached his study of individualised mathematics instruction.

(a) Firstly, individualisation needs to be defined.

What do we mean by individualised instruction?

This is necessary as there are nearly as many meanings given to this term as there are authors who use it. The author is of the opinion that there is no adequate short definition, but that one needs to define the characteristics of a given course that

adapt to the individual learner's needs, the frequency of such adaptations, and how decisions to adapt the course are arrived at. This is the "what, when and who" of individualisation.

(b) Secondly, the rationale for adopting individualised approaches to teaching and learning should be clarified. Again there are several sources for a rationale, economical, humanist, educational and psychological. Examining the psychological viewpoints, one discovers that behaviourists and the cognitive school, whilst both supporting "individualisation" do so for different theoretical reasons and thus advocate quite different methods of achieving it. This is the "why" of individualisation.

(c) Thirdly, therefore, one should attempt to clarify the "how" of individualisation. Not only are there different general theoretical viewpoints on how one should go about this process. The detailed practical "approaches", "schemes", "plans" or "systems" have detailed differences, and even if they don't they adopt such important-sounding names as to give the impression that the roads to individualisation are legion. Some simple classifying or analysis scheme is badly needed in order to allow the teacher to "see the wood for the trees".

(d) Finally, in addition to these general considerations which need clarification, the author would add a list of specific considerations which have typically been the source of problems for the designers of individualised systems of instruction. Some of these are connected with the individual components that make up any individualised system of instruction, namely:-

- The students; how do student attitudes skills, age or individual differences affect the way they learn; how should one take these into account when designing the system?
- The teachers/tutors/monitors; how can one best make use of the human elements in the system; how can we ensure that these elements will perform, will be enthusiastic, satisfied, motivated?
- The instructional materials; how can we ensure that they are effective; what is the best way of preparing them, what is the most effective format for presentation?

Others are connected with the processes that occur in the system, for example:-

- communication; how can we best ensure effective communication between the learner and the instructional materials, tutors, etc.; what media should we be employing?

- feedback; what information do we need in order to control the system; to improve or re-design the system; how can we collect it; how will we handle it?

Still others are connected with the implementation and management of innovative systems. These are more the preserve of the administrator. One could go on adding to this list of considerations. However, the ones that interest the author most have all been mentioned already. We shall proceed now to identifying which of these many considerations are the subject of the present study.

1.5 AIMS AND ORGANISATION OF THE PRESENT STUDY

The aims of this study are three-fold;

- (a) Firstly, to clarify the general considerations listed above - the "what, when, who and why" of individualised instruction. This is treated from the viewpoint of mathematical education, but to give a balanced view must necessarily consider the broader, more general, theoretical positions as well. In order to limit the scope of the study, the author has considered one aspect of another general question; "for whom" should one design individualised instruction. This question has been restricted to consideration of the adult, mature student who is a non-specialist, but who needs specific mathematical knowledge or skills in order to study another subject, or keep down a job, or simply to pass an examination he should have passed earlier in life.

- (b) Secondly, to review and attempt to classify the variety of practical systems for the individualisation of mathematics teaching. This aim is strictly limited to systems which are commonly used for mathematics teaching. The author has tended to cover mathematics teaching at all levels, in order to construct as complete a picture of the field as possible. However, in considering specific applications and research, he has placed stress generally on such systems as are applicable to the needs of the mature learner.
- (c) Thirdly, to select some of the specific considerations listed above for further more detailed study by a programme of research. The author has selected the consideration of the components of the system, in particular some aspects of the role of the teachers and of the instructional materials. He has been led to select these as there is less research on these factors than there is on, for example, individual differences or the selection of media. Also, these two areas are highlighted as of critical importance by the review of current practices (aim "b"). Finally, these two areas are also connected to key factors in the author's system for classification and analysis of individualised systems (aim "a").

The Organisation of the study follows the statement of aims set out above. There are three major sections:

Section A The "what, when, who, and why" of individualisation.

Section B Review of Current Practices and Research.

Section C The author's research.

Sections A and B are the result of the author's practical experiences with individualised systems of instruction, coupled to a great deal of reading and library searches performed specifically for this study. In this the author has been fortunate to have gained access to much literature on both sides of the Atlantic, help and advice from other researchers in the field, and to have had the opportunity to attend a number of specialist conferences related to his area of study. He spent two periods of study in North America - two weeks in April 1976 in the U.S.A. and six weeks in July/August 1976 in Canada. During these visits he had access to the libraries of several North American universities and also to the microfilm libraries such as the ERIC collections and University Microfilms. In the United Kingdom he has had access to the libraries at his own Middlesex Polytechnic, at Loughborough University and at the Institute of Education of London University. During the period of his study he has attended specialist conferences devoted to educational technology in England, the U.S.A., Poland, Czechoslovakia, Chile and Brazil. All of these have served as inputs to his information store on the use of individualised instruction. Of particular usefulness, in a broader sense were two specialist international congresses, namely the

Third International Congress on Mathematical Education in Karlsruhe, Germany, in August 1976 and the Latin-American and Caribbean Congress on New Forms of Post-Secondary Education, held in Caracas, Venezuela, in September 1976.

Section C contains summaries of some of the author's earlier work on individualised systems and some pilot studies, from which he develops the rationale for the main research study, carried out in Brazil between August and December 1976. This study is reported in detail.

Examples of some of the materials used in the researches, together with bibliographies, raw experimental data, etc., are included in the appendices.

CHAPTER 2

THE INDIVIDUALISATION OF INSTRUCTION

2.1 Introduction - The many meanings of "individualisation".

2.2 Individualised Learning or Instruction?

Why the author favours the use of the term "instruction".

2.3 Approaches to the Individualisation of Instruction

A rapid historical overview of individualised instruction - 1880 to the present.

2.4 Sorting out the approaches.

Past attempts to classify the various types of individualised instruction systems.

2.5 Key factors in describing individualisation schemes.

The "what", "when", "who", and "how" of individualisation.

2.6 A proposed form for the description of individualised schemes.

The analysis form developed by the author as an aid to the description of existing systems and to the design of proposed systems of individualised instruction.

2.7 Towards a Taxonomy of Individualised Instruction.

Do we have a comprehensive taxonomy? Do we need one?

CHAPTER 2

THE INDIVIDUALISATION OF INSTRUCTION

"To many educators, the "one tutor - one student" relationship - the "Socratic dialogue" - represents the ideal for higher education. They bemoan that the tutorial system, as practised in Oxford or Cambridge, cannot be economically applied in the less well endowed universities. But as a product of the Oxbridge system, I do not bemoan this.

If the tutor is indeed Socrates, and if the students are highly motivated, articulate and informed, then there may be no better system.

But if (as was the case in my time) the students are less than perfect, or the tutors untrained in the skills of tuition (and generally much more interested in their research than in the students) then I can think of no more disastrous system."

[Quote from speech by Sir Walter Perry, Vice Chancellor of the Open University, delivered at the International Conference on "New Methods of Post-Secondary Education", Caracas, Venezuela, September 1976.]

2.1 Introduction

The above mentioned conference focussed on two main themes:- education at a distance (e.g. the Open University) and individualisation (e.g. the Keller Plan).

The quotation is from the opening speech of a panel discussion under the title "Individualisation and education at a distance" - two strange bedfellows one may think. And yet, the discussion suggested that in some respects, some Open-University-type courses are more individualised than many "traditional" school or college-based courses.

Other speakers disagreed. It soon became obvious that there were as many separate concepts of "individualisation" as there were members of the discussion panel.

This chapter is devoted to identifying the main characteristics of "individualisation", investigating attempts to construct a taxonomy for classifying and comparing the many practical schemes or plans for the individualisation of instruction, and suggesting a method for describing in a standardised way exactly how a particular system is in fact "individualised". In later chapters this method will be applied to the analysis of existing schemes for the individualisation of mathematics teaching.

2.2. Individualised Learning or Instruction?

It has often been said that all learning is individualised. After all the learner learns - nobody can do it for him.

Teachers can help learners to learn, and this they can do in a variety of ways. Some of these ways take the individual

learner into consideration more than do others. In this sense they are more individualised.

Teachers may help learners to learn in ways other than by instruction. They may exercise guidance or counselling functions, or remove obstacles to efficient learning (such as cramped or noisy study conditions), or provide extra resources (by lending books) and so on. However, our concern here will be restricted to the processes of communicating information to the learner, stimulating relevant learning activities, evaluating the result of those activities and taking remedial action if necessary. This we shall term instruction. We shall therefore be concerned with the individualisation of instruction.

2.3 Approaches to the Individualisation of Instruction

Of course instruction (some of it) has always been individualised - right back to the one-family unit of the stone age, through the age of Socrates and Aristotle, to the "Oxbridge" type of university system (see quote above). It is only in the more recent history of mankind that the major part of instruction has ceased to be individualised (or "small-group") and has become "large-group-centred". This has perhaps been one of the "penalties" of rampant civilisation - or at any rate it appears to be perceived as such, as attempts to find new ways of individualising instruction recur in every generation. Rousseau's "Emile"

is still sometimes held up quite seriously as a model for modern education. Quintillian is reputed to have said that "if only books could be so written that one page would only reveal itself to the reader once the previous page was quite understood, our educational problems would be over" and MacPherson (1972) quotes the following gem.

"To accompany the Studebaker Economy Practice Exercises in Arithmetic (a set of programmed materials, complete with cardboard frames, immediate reinforcement and the like) published in 1916, the publishers stated,

With the increasing scientific studies in education, the traditional "average" child has passed out of existence and the complexity of the teaching problem has become much more apparent.

Consequently educators have ceased their attempts to apply "average" methods of instruction. It is now fully realized that if the processes of education are to yield adequate return for the effort, time and money spent, the extreme and numerous individual differences always present in every class must be recognized and treated.

Since one grows educationally only when his particular needs are met, and since the customary mass instruction cannot possibly furnish the variety of methods and materials necessary to meet the

varying abilities represented in any group of individuals, it is imperative that a form of instruction be used that will, to the largest degree, permit each child to work according to his needs. It could have been written in 1972. In fact, it probably will be. "

Thus one observes that the hankering after a return to the (economically unfeasible) one learner - one tutor relationship, has led to speculations and designs for "tutor-substitutes". And long before the self-instruction boom" ushered in by Pressey (1926) and Skinner (1961a) (and apparently the Studebaker Exercises), attempts of various types had begun to organise education in ways which would optimise the amount of consideration given to the individual learner and his individual problems.

The development of individualized instructional programmes began in the later decades of the nineteenth century (Harris, 1960). Although the main current of educational practice has continued, fixed in its course, an increasing number of programmes that make schooling more adaptable to differences among students have been proposed and developed (De Haan and Doll, 1964, Shane 1962). Arguments for breaking down uniformity of instruction gained support with the appearance of instruments for measuring human abilities shortly after the turn of the century. It has become clear that students differ not only in intelligence but in creativity (Wallach and Kogan, 1965) and in various elements of intellect (Guildford, 1967). It also became clear that great differences between competence and

performance are possible, and that inequalities in intellect, physical ability, and social behaviour, marked in childhood, "increase as students move through the grades" (Thomas and Thomas, 1965).

Some of the early proposals for changing the traditional system were referred to as individualized programmes, others exhibit the hallmarks of individualization even though they do not bear the title, and to these may be added still others, not necessarily associated with the school but obviously individualised: tutorials, correspondence courses, and informal programmes of independent study.

Together such programmes constitute a diverse family. They are based on different interpretations of individualisation. They are inspired by different philosophies and theories, influenced by different technologies, in fact, the term individualised instruction is used to describe such a varied assortment of curricula that it is no longer a useful, restrictive category of instructional methods.

It has become necessary to qualify the term by defining exactly what about a course is individualised and how it is individualised. This we shall attempt to do in a moment. But to illustrate the diversity, let us consider just some of the well known plans which have appeared in the last 100 years or so, both in the U.S.A. and in Britain.

In the U.S.A. by 1888 Preston Search had initiated the Pueblo Plan, a laboratory scheme permitting a student to pace his own coverage of the course (Search 1894). Parkhurst's Dalton Laboratory Plan (1922) and Carleton Washburne's Winnetka Plan (1963) presented self-instructional units that each student worked through as fast as he could.

In Britain, elements of individualisation were introduced with the monitorial system (now being revived as part of the Keller Plan). The "free choice" approach at Summerhill (Neil 1960) is a completely different aspect of the concept. And in the Leicestershire Plan (Featherstone 1967) the "integrated day" concept presented yet another approach to adapt the school to the individual.

With the boom of programmed instruction in the early 1960's we were treated to yet another interpretation: individualisation as self-paced individual study of prescribed material (usually common to all the students in a group). Briefly, the "self-pacing" element caught the imagination of educators and linear programmes were seen to be the new road to individualisation.

However, soon people remembered that there were other factors involved in learning than the pace of reading. Branching programmes (Crowder 1963) appeared on the scene, soon to be followed by even more complex "adaptive" teaching machines such as the SAKI keyboard instructor (Pask 1960) and the Edison Responsive Environment, or "talking typewriter" (a multi-media reading machine which adapts to the individual students learning pattern in complex ways - O.K. Moore 1962).

This trend has continued with the development of computer-based learning systems, commencing as rather sophisticated branching teaching machines (e.g. PLATO I) moving to programmes of "drill-and-practice" in routine skills (Suppes 1968) and finally computer-based instructional

systems which simulate real-life problems in business technology or society. We are on the verge of developing viable systems of "conversational" programming which would enable the simulation of the one-to-one tutorial situation in all its essential aspects - "the general purpose adaptive teaching machine".

Yet another approach to individualisation has been the growth in independent study:- project-based work, "Quest" programmes, "student-directed learning", "resource-based learning" and so on. Such independent study may be carried out by the individual alone, or it may be designed for small-group work, (yet it is still termed individualisation, as it breaks down the larger group and ensures that each member of the small group has a task to perform).

Yet other types of group situations have been developed with the individual in mind. In sensitivity training, T-groups are used so that individuals can learn to react appropriately to other individuals. In more specific training situations, role-playing and other simulation techniques are employed. Games are now commonly employed as instructional techniques - not only in elementary schools, but right up through the school system, into university, business and the professions. A well structured instructional game ensures that each individual engages in learning-directed activities, whilst involved in a group situation.

And finally, talking of groups (and group size) the simple expedient of grouping (according to ability, interest or what have you), although currently passing through a

phase of unpopularity, is seen by some as a powerful and indeed necessary way of individualising instruction. To quote MacPherson (1972) again:-

"Someday, someone will explain why we in education were captured by psychology rather than sociology. Perhaps the psychologists simply got here first. Whatever the reason, we were. In all sorts of subtle ways, we think about learning as though it were an individual phenomenon.

In African Genesis, Robert Ardrey relates an instructive incident. While carrying a small black object, a friend of Konrad Lorenz was seen by a crow, who immediately set up a characteristic cry. From that time onwards, no matter where he went, the man was feared by local crows. Crows unborn at the time of the incident learned to fear him. The question is: how would we find out about all this by studying single crows? To drop the metaphor, is it possible that there are important properties of human societies which we learn by studying groups? In fact, is it possible that we are, by some instinct, herd animals and that some learning is, to begin with, most naturally a group phenomenon? "

2.4 Sorting out the approaches

With so many varieties of approaches to the individualisation of instruction it is necessary to develop some way of clarifying and describing them.

One may adopt a variety of parameters. Maurice Gibbons, in his book "Individualised Instruction" A descriptive Analysis (1971) adopts a classification of strategies for individualisation into strategies which are employed in the group situation and strategies which impinge on the individual. He then sub-divides each into "Active - Responsive - Permissive in function of who directs the activity (who makes the decisions).

His final sub-division is concerned with the teacher-student interaction (in group situations, whether this is with the whole group or with sub-groups, and in the individual learning situations, whether the teaching is "direct" from a human teacher, or is indirect, (i.e. mediated), Gibbons' classification is presented in figures 2.1 and 2.2.

The classification says nothing about what aspects of the course are individualised (objectives, methods etc.) In order to show this, Gibbons develops a rather complex profile-form. Examples of the profiles of several well known individualisation schemes are shown in figure 2.3.

These profiles may serve as a basis for comparison between systems but it is difficult to see exactly what conclusions we should draw from them. Also the original "family tree" classification is somewhat rigid. Many instructional systems are partly individual-study, partly group-study indeed MacPherson (1972) suggests that any well-designed system probably would be a mixture. Some decisions regarding options in the course will be taken by the learner - others must be taken by the teacher (or even

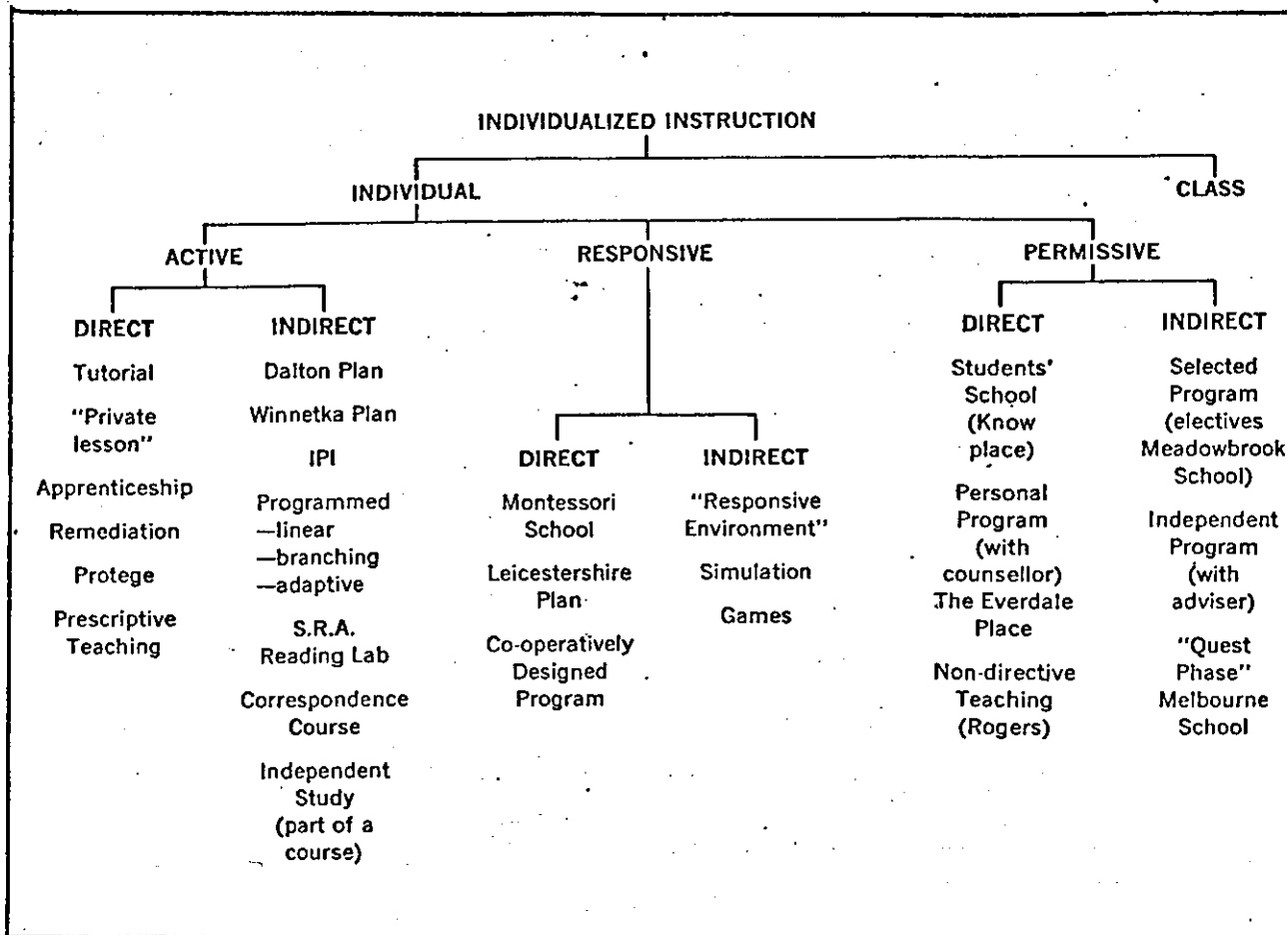


Figure 2.1 An Elementary Classification of Individualized Instructional Programmes (adapted from GIBBONS (1971))

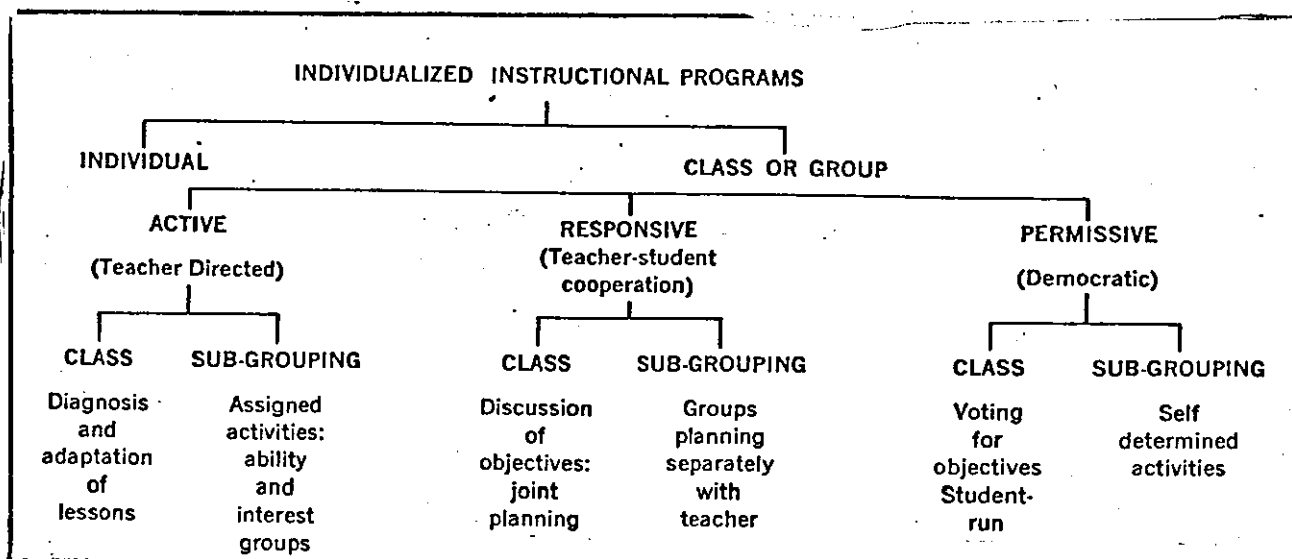


Figure 2.2 A Rough Classification of Individualization through Group Practices (adapted from GIBBONS (1971))

Figure 2.3 Examples of "Profiles" of Certain Individualisation Schemes, proposed by GIBBONS (1971)

1. PERCENTAGE OF STUDENT BODY				
2. PERCENTAGE OF SCHOOL TIME				
3. ATTENDANCE	Optional	School Not Class	Class Not Sub-Group	Mandatory
4. MATERIALS FOR STUDY	Individual Choice	Individual Prescribed	Sub-Group Prescribed Or Discussed	Class/Grade Prescribed
5. METHOD OF STUDYING MATERIALS	Individual Choice	Individual Prescribed	Sub-Group Prescribed Or Discussed	Class/Grade Prescribed
6. PACE OF STUDY	Individual Choice	Individual Prescribed	Sub-Group Prescribed Or Discussed	Class/Grade Prescribed
7. ACTIVITY	Individual Choice	Individual Prescribed	Sub-Group Prescribed Or Discussed	Class/Grade Prescribed
8. DECISION-MAKING	Student (Permissive)	Student and Teacher (Responsive)	Teacher (Active)	Administrative Authority
9. TEACHING FOCUS	Values	Processes	Skill Concepts	Content
10. TEACHING FUNCTION	Teacher Available	Teacher Guides	Teacher Presents	Teacher Directs
11. TEACHING METHOD	Unspecified Discovery (Permissive)	Guided Discovery (Problem Solving)	Explanation and Discussion	Drill Exercise Repetition
12. ENVIRONMENT	Community	School	Classroom or Resource Area	Desk
13. TIME STRUCTURE	Non-Structured	Fluid	Structured Non-Structured	Structured
14. EVALUATION	Student Self-Evaluation	Broad Assessment	Quantity Of Work	Exam-Class Rank
15. PURPOSES OF PROGRAM	Continuous Development To Maturity	Adjustment	Understanding	Efficient Mastery

THE DALTON PLAN

LINEAR, PROGRAMMED INSTRUCTION

1. PERCENTAGE OF STUDENT BODY				?
2. PERCENTAGE OF SCHOOL TIME				?
3. ATTENDANCE	Optional	School Not Class	Class Not Sub-Group	Mandatory
4. MATERIALS FOR STUDY	Individual Choice	Individual Prescribed	Sub-Group Prescribed Or Discussed	Class/Grade Prescribed
5. METHOD OF STUDYING MATERIALS	Individual Choice	Individual Prescribed	Sub-Group Prescribed Or Discussed	Class/Grade Prescribed
6. PACE OF STUDY	Individual Choice	Individual Prescribed	Sub-Group Prescribed Or Discussed	Class/Grade Prescribed
7. ACTIVITY	Individual Choice	Individual Prescribed	Sub-Group Prescribed Or Discussed	Class/Grade Prescribed
8. DECISION-MAKING	Student (Permissive)	Student and Teacher (Responsive)	Teacher (Active)	Administrative Authority
9. TEACHING FOCUS	Values	Processes	Skill Concepts	Content
10. TEACHING FUNCTION	Teacher Available	Teacher Guides	Teacher Presents	Teacher Directs
11. TEACHING METHOD	Unspecified Discovery (Permissive)	Guided Discovery (Problem Solving)	Explanation and Discussion	Drill Exercise Repetition
12. ENVIRONMENT	Community	School	Classroom or Resource Area	Desk
13. TIME STRUCTURE	Non-Structured	Fluid	Structured Non-Structured	Structured
14. EVALUATION	Student Self-Evaluation	Broad Assessment	Quantity Of Work	Exam-Class Rank
15. PURPOSES OF PROGRAM	Continuous Development To Maturity	Adjustment	Understanding	Efficient Mastery

Figure 2.4 More "Profiles" from GIBBONS (1971)

1. PERCENTAGE OF STUDENT BODY				
2. PERCENTAGE OF SCHOOL TIME				
3. ATTENDANCE	Optional	School Not Class	Class Not Sub-Group	Mandatory
4. MATERIALS FOR STUDY	Individual Choice	Individual Prescribed	Sub-Group Prescribed Or Discussed	Class/Grade Prescribed
5. METHOD OF STUDYING MATERIALS	Individual Choice	Individual Prescribed	Sub-Group Prescribed Or Discussed	Class/Grade Prescribed
6. PACE OF STUDY	Individual Choice	Individual Prescribed	Sub-Group Prescribed Or Discussed	Class/Grade Prescribed
7. ACTIVITY	Individual Choice	Individual Prescribed	Sub-Group Prescribed Or Discussed	Class/Grade Prescribed
8. DECISION-MAKING	Student (Permissive)	Student and Teacher (Responsive)	Teacher (Active)	Administrative Authority
9. TEACHING FOCUS	Values	Processes	Skill Concepts	Content
10. TEACHING FUNCTION	Teacher Available	Teacher Guides	Teacher Presents	Teacher Directs
11. TEACHING METHOD	Unspecified Discovery (Permissive)	Guided Discovery (Problem Solving)	Explanation and Discussion	Drill Exercise Repetition
12. ENVIRONMENT	Community	School	Classroom or Resource Area	Desk
13. TIME STRUCTURE	Non-Structured	Fluid	Structured Non-Structured	Structured
14. EVALUATION	Student Self-Evaluation	Broad Assessment	Quantity Of Work	Exam-Class Rank
15. PURPOSES OF PROGRAM	Continuous Development To Maturity	Adjustment	Understanding	Efficient Mastery

SUMMERHILL "FREE SCHOOL"

LEICESTERSHIRE PLAN

1. PERCENTAGE OF STUDENT BODY				
2. PERCENTAGE OF SCHOOL TIME				
3. ATTENDANCE	Optional	School Not Class	Class Not Sub-Group	Mandatory
4. MATERIALS FOR STUDY	Individual Choice	Individual Prescribed	Sub-Group Prescribed Or Discussed	Class/Grade Prescribed
5. METHOD OF STUDYING MATERIALS	Individual Choice	Individual Prescribed	Sub-Group Prescribed Or Discussed	Class/Grade Prescribed
6. PACE OF STUDY	Individual Choice	Individual Prescribed	Sub-Group Prescribed Or Discussed	Class/Grade Prescribed
7. ACTIVITY	Individual Choice	Individual Prescribed	Sub-Group Prescribed Or Discussed	Class/Grade Prescribed
8. DECISION-MAKING	Student (Permissive)	Student and Teacher (Responsive)	Teacher (Active)	Administrative Authority
9. TEACHING FOCUS	Values	Processes	Skill Concepts	Content
10. TEACHING FUNCTION	Teacher Available	Teacher Guides	Teacher Presents	Teacher Directs
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12. ENVIRONMENT	Community	School	Classroom or Resource Area	Desk
13. TIME STRUCTURE	Non-Structured	Fluid	Structured Non-Structured	Structured
14. EVALUATION	Student Self-Evaluation	Broad Assessment	Quantity Of Work	Exam-Class Rank
15. PURPOSES OF PROGRAM	Continuous Development To Maturity	Adjustment	Understanding	Efficient Mastery

the system itself). Some activities will involve teacher-student contact - others will be mediated by "packaged" presentations (both self-instructional and group).

We ought perhaps to focus more on "who makes what decisions, and when". A simple classification is suggested by Edling (1970) which can be summarised in the following diagram.

Figure 2.5 Edling's Model for the Classification of Individualised Systems

METHODS AND MEDIA	OBJECTIVES	
	SCHOOL DETERMINED	LEARNER SELECTED
SYSTEM DETER- MINED	"Individually diagnosed and prescribed" (IPI) (PLAN) (Some CAL)	"Personalised" (many CAI systems)
LEARNER SELECTED	"Self-directed" (Learning resource centres) (Some multi-media kits)	"Independent study (Project QUEST)

Chart adapted from Edling (1970)

This classification, though useful, makes no direct mention of learner groupings or learning pace (often considered key factors in individualisation), although all the examples quoted by Edling happen to involve self-instruction or small-groups under self-paced learning conditions, as if this was an essential ingredient.

However, it is by no means clear whether a CAI course which allows the learner to select his objectives, is any more or less "individualised" than the system at Summerhill (1960) which also allows the student to select the lessons he attends. If there is a difference then surely it lies in what happens within the lesson. How does the lesson adapt to the learner? Which experience is more individualised, a linear programme in which all students read the same material (albeit at their own pace) or a traditional lesson where all the students have been selected by some diagnostic procedure so that the teacher has an inventory of their learning problems?

We might add a "third dimension" to Edling's model, by constructing a hierarchy of potential individualisation among media, based on the degree to which a medium can adapt a presentation automatically to the needs of the individual learner. This is the degree to which a given instructional system is a self-regulating system. We may measure this by the frequency of decisions taken by the system.

In the majority of current CAI programmes now in use, one sees only a limited level of self-regulation or adaptiveness. In highly specific subjects (tracking skills or Suppes' "drill and practice" mathematics courses for example) the adaptive machine may have the edge on the

human teacher, but in most academic "open-ended" disciplines the personal tutor, backed up by efficient diagnostic procedures is still hard to beat. The more open-ended the subject and the more inquisitive and sophisticated the student, the greater the need for adaptive capability. Hence the relative lack of acceptance of "traditional" programmed instruction at university level.

A fourth dimension to consider is the level within a course at which individualisation takes place. Is it:

- (a) at the course level - do students simply exercise their option to take or not to take a given course? Do they do this, incidentally, on the basis of a systematic consideration of the course objectives or simply by their individual whims?
- (b) at the course unit level - course options are planned in the light of the overall objectives of the course, the inter-dependencies of the units and the resources available. Neglecting the constraint of resources, reasonable course options are few in most courses with tightly defined, specific objectives, and many when objectives are ill-defined.
- (c) at the lesson level - lessons usually form a sequence, one building on the other. If this is so, is it reasonable at this level to talk of learner-selected objectives? Do we mean simply that learning rate, sequence and perhaps to some extent the methods and media, are individualised? Or do we mean that the learner can select some (e.g. enrichment) objectives, over and above the common core of essential objectives?

(d) at the individual, detailed objective level - the options would appear much the same as at the lesson level (i.e. learning rate, sequence, methods/media) but on a more "micro" scale, and therefore individual choice is exercised more often.

One may consider these "levels" of individualisation as another way of expressing the frequency with which decisions are made - in terms of four broad frequency bands.

This concept of "frequency" of decision making is quite useful, indeed Tosti and Harmon (1972) in attempting to develop a taxonomy for decision making in individualised instruction, state:-

"In the principles of instructional management, a distinction between individualised and non-individualised instruction can be made. This distinction should not be made on the basis of whether a hundred students are experiencing the same learning activity at the same time, since it is possible that every one of them should be engaged in this activity at this time. Nor should the distinction be made on the basis of whether or not the instructional system allows a student to progress at his own pace, as a book can. Instead, the degree of individualisation must be defined in terms of instructional management. This means that:

Individualised instruction is a function of the frequency with which the decision to change the instructional presentation is made as a result of the assessment of an individual student's achievements, needs or aspirations.

The presence of large groups in an instructional setting tends to inhibit the individualisation process due to the logistics of data collection, processing, assigning and so forth. Still, limited individualised instruction occurs in almost every classroom. When an instructor stops to modify his presentation as a function of a student's query, that student (but not the others who are in the room) is receiving a form of individualised instruction.

They suggest the following taxonomy of instructional management strategies, related to the individualisation of instruction:-

1. Aspiration Management. Purpose: To select those objectives required to meet a given student's aspirations or interest.
2. Achievement Management. Purpose: To ensure that the student has mastered the objective specified.
3. Prescriptive Management. Purpose: To ensure that a given student receives the materials appropriate to his individual characteristics to best meet the objectives.
4. Motivation Management. Purpose: To ensure alert and continual student interaction with the educational stimuli in order to increase

individual learning rates and performance levels.

5. Enrichment Management. Purpose: To provide for access to additional information relevant to the objectives but not necessary for their attainment.
6. Maintenance Management. Purpose: To ensure long-term maintenance of the student's continuing ability to perform at a pre-specified criterion level.
7. Support Management. Purpose: To ensure that such data be collected as necessary to keep the instructional system operating effectively and to provide individuals outside the system with information they require to evaluate and revise the existing instructional system.

This taxonomy is a management decision-making tool. As such it appears to have much promise. However, our current intention is not to manage a particular instructional system, but to conceptualise and describe the factors in a course which are susceptible to individualisation, and to identify how and when decisions can be taken, [in order to later make decisions concerning specific instructional systems - decisions on what should be individualised, who should direct the individual choice and with what frequency should these decisions be made.]

2.5 Key factors in describing individualisation schemes

We have identified four relevant questions that one may ask of any instructional scheme that purports to be "individualised".

- a. What is to be individualised
 - b. When (with what frequency) will the course adapt to the individual
 - c. Who decides
 - d. How does the system adapt to the individual
- a. What may be individualised?

The majority (though not all) of individualisation schemes allow the learner to work on his own, at his own pace, for at least part of his study time. However, many other characteristics of a course can be (and in some schemes are) individualised. Comprehensive lists of the characteristics that might be individualised have been prepared by Davies (1971). Gibbons (1971) prepared a classification of individualised instruction schemes and a list of factors in which they differ.

Some of the more obvious (and more important) characteristics which may be individualised, include:-

Pace of study. Students may be constrained to learn at a pre-determined pace (as when listening to a lecture or viewing a TV programme) or they may be allowed to work at varying paces (as in programmed instruction, independent study, small group discussions).

Materials or Media. Students may be allowed to choose (or be assigned to on the basis of past performance) alternative versions of a lesson in different media, or alternative lessons leading towards the same objectives.

Methods of Study. Students may receive alternative lessons differing in the instructional "strategy" adopted (e.g. expository or discovery) and/or in the detailed "tactics" of instruction (e.g. choice of examples, number of problems to be worked, amount of hints given to the learner, sequence of topics, etc.)

Content of Study. Students may receive alternative lesson content - either as a means of tailoring a course to the individual's own objectives (the liberal view of education) or as a means of selecting material familiar or interesting to the individual to be the vehicle for the attainment of broader educational objectives (the "process" view of education).

Objectives of Study. Course objectives may be varied - either for liberal reasons, or in order to adapt courses to the different aptitudes of individuals or the different needs of the organisation (the "product" view of education).

This list can be extended or compressed. The profiles of Gibbons (1971) shown in figures 2.3 and 2.4 are based on a much extended list of course characteristics. The classification of Edling (1970) shown in figure 2.5 considers only two groups of characteristics.

Figure 2.6 shown here, represents the definition of individualisation used in the Swedish IMU project (discussed in later chapters). This considers five main variables of individualisation (Larsson 1973). The dots on the arrows represent (by their position) the planned degree of individualisation in the IMU project.

Fig. 2.6

Total individualization		No individualization
1. The work of each pupil is directed at achieving a goal that is adapted to his particular ability and interests. Thus each pupil follows his own course, which develops gradually as the teacher plans for each pupil individually. In setting up goals and planning the teacher and pupil confer.	Goals Course	The same for all pupils. The teacher plans for the class as a whole. The pupils do not participate in planning or setting up goals.
2. Instructions are given to one pupil at a time. The instructions are shaped by what the teacher knows about the pupil's ability and modified according to the pupil's questions and answers.	Instruc-tions	All pupils are given the same instructions. The instructions are not influenced by the pupils' reactions. "Lecturing." Singletrack study material.
3. The number of tasks and their degree of difficulty varies from pupil to pupil. The teacher chooses tasks for one pupil at a time.	Tasks	All pupils work with the same tasks. The teacher chooses tasks for the whole class.
4. Each pupil works at his own rate and with methods and material that suit his ability. The pupil "steers" his work himself. The teacher functions as a tutor.	Methods and Rate of Work	The pupils are "steered" by the teacher and all work at the same rate and in the same way.
5. The teacher evaluates the work of each pupil on the basis of the pupil's ability. This means that the pupil's achievements are compared with the goals on which the teacher and the pupil have agreed. The marks refer to the goals. The pupil is given a chance to evaluate his work himself. There is no comparison between pupils.	Evaluation Marking	The teacher evaluates the work of the pupil by comparing his achievements with those of the other pupils. Then the achievements of the pupils are ranked and they are given marks according to certain norms. The marks refer to the norms. No consideration is taken to the prerequisites of the pupils in evaluation.

b. When does individualisation take place?

As Gibbons (1971) points out, the term "... individualised instruction" suggests a distinction to "non-individualised instruction", but it is impossible to un-individualise instruction. Every programme is unavoidably individual to some degree by the perception each person has of it and the response he makes to it."

Thus individualisation is a relative thing - a matter of degree. Gibbons attempts to construct a scale by which the degree of individualisation of a scheme with respect to certain factors may be described - a profile of individualisation. Larsson (1973) offers a simpler profile.

Tosti and Harman (1972) define "individualised instruction" as:- "a function of the frequency with which the decision to change the instructional presentation is made".

The present author has suggested (Romiszowski, 1976a) a scale of "frequency bands" depending on whether individualisation decisions are taken

- for a course
- for each unit within a course
- for each objective within a unit
- for each learning step taken to achieve the objective.

This scale is elaborated later in this chapter, into a classification scheme for individualised instruction which combines the "what" "when" and "who decides" factors.

c. Who decides?

The decisions to individualise may be taken by

1. The Student himself, when he chooses a particular course option, or a particular text book.
2. The teacher, who may prescribe individual objectives or media or extra content.
3. The System itself which may have built into it a diagnostic device which automatically adapts the presentation to the individual student. This is the case with the "branching" style of programmed learning (Crowder, 1963) and with many computer-based learning systems.
4. Any combination of Student, Teacher and System
This is the most common situation, involving a joint decision between Student and Teacher (e.g. in IPI), between Student and System (e.g. in Resource-Based Learning), or between all three (e.g. in Project "PLAN").

d. How does the system adapt to the individual?

Chapters 4 and 6 of the present study analyse some of the systems for individualising instruction which have been applied to the teaching of mathematics.

- These include - programmed instruction-based systems (IMU)
- worksheet/workbook-based systems (FIFE etc.)
 - activity packs or learning packages (L.A.P's)
 - individually prescribed systems (IPI) (PLAN)
 - computer-based systems (STANFORD) (PLATO)

- laboratory-based systems
- A/V media-based systems
- audio-tutorial systems (POSTELTHWAITE)
- PSI/Keller Plan - (materials + tutors system)
- By appointment/Clinic remedial systems

These systems are related to the classification of individualisation to be developed in this chapter, and to the theoretical viewpoints on mathematics instruction presented in Chapter 3.

Two factors are identified as of importance:-

1. the style and type of instructional materials employed in the system, and the way in which the learners select and have access to the materials,
2. the role of the tutor or instructor (if any) in the system, both as a medium of instruction and as a medium for management and control.

It is apparent that there are innumerable different ways of going about the "HOW" question - in reality, a reply to this question generally requires a detailed description of the system in question. However, the "WHAT", "WHEN" and "WHO" questions may be used to compare the general structure and philosophy of different schemes.

2.6 A proposed form for the description of individualised schemes

We have found it useful to use the form shown below (fig.2.7) in order to analyse and compare existing systems of individualised instruction. It has also proved to be a useful planning tool when deciding which aspects of a

INDIVIDUALISED SYSTEM

"WHO" MAKES THE DECISIONS

		"WHAT" COURSE PARAMETERS INDIVIDUALISED				
		TIME	MATERIALS + MEDIA	SEQUENCE + METHODS	CONTENT	OBJECTIVES
"WHEN" FREQUENCY OF DECISION TO INDIVID- UALISE	1. TOTAL COURSE	Student	/	/	/	System
	2. EVERY UNIT	Tutor	Tutor + Student		/	/
	3. EVERY LESSON	Student (System)	/	System	/	/
	4. EVERY LEARNING STEP	Student (System)	/	System	(System)	/

(A) Example of use of analysis form to describe the individualised remedial mathematics course at Middlesex Polytechnic. (Lower half completed for a "branching P.I." unit)

	3. EVERY LESSON	Student	/	/	/	/
	4. EVERY LEARNING STEP	Student	/	/	/	/

(B) Example of lower half, completed for a Linear P.I. unit

	3. EVERY LESSON	Student + Tutor	(Tutor)	Tutor	(Tutor)	/
	4. EVERY LEARNING STEP	Student + Tutor	(Tutor)	Tutor	(Tutor)	(Tutor)

(C) Example of lower half completed for an individual tutorial.

(NOTE: Brackets are used to indicate occasional rather than systematic and regular decisions.)

course ought to be individualised and which ought not to be. We shall apply this form for this purpose a little later when considering what ought to be individualised in mathematics education.

The example shown in figure 2.7 is an analysis of the individualised remedial mathematics course offered at the Middlesex Polytechnic described by the author in papers to be found in the annexes (Hamer & Romiszowski (1969); Romiszowski Bajpai and Lewis, 1976).

A few words of explanation (the answers to the "HOW" question) will help to explain the chart.

- 1) At the "course" level, course objectives are adapted by the "system" to the individual's needs - a diagnostic test (marked automatically) prescribes the units (these are 22 in all) which need to be studied. Theoretically, there is no overall time limit for the student to complete the prescribed units. He may plan his own timetable.
- 2) At the "unit" level there is no choice of objectives, the objectives are the same for any student studying the unit. However, generally, there is a choice of methods and media available to achieve the objectives. The choice is made by the tutor and student jointly (typically the choice would be between a branching P.I. unit, a teaching machine a printed assignment to take home or an individual tutorial). However, mastery of the unit's

objectives is always monitored by the tutor, who will require the student to repeat a unit, use an alternative presentation, etc. if required in order to achieve mastery. Thus the time spent on a unit is effectively adjusted to the individual student's needs by the tutor.

- 3) + 4) At the "lesson" and "step" levels, the analysis would be different for alternative sets of materials. The bulk of the materials used are branching programmes in teaching machines. The analysis (as shown for these materials) indicates standard lesson objectives + media, but the content (example: type and number of problems), and the methods (alternative explanations) are varied automatically by the system to suit the individual learner's error pattern. The student of course controls his learning rate, but is occasionally forced by the system to repeat sections or to attempt extra problems, with the consequent time penalty.

In the case of a different medium of instruction, the analysis at the learning step level may be quite different. For example, a linear (Skinnerian) learning programme individualises only the learning rate. Methods and content stay the same for all students. On the other hand, when the tutor gives an individual tutorial all the factors (except the overall objectives in this case) may be adapted to the individual student. The extent to which this happens

in practice depends on the skill and creative adaptability of the tutor. At the least creative extreme, he is no more (perhaps less) adaptive than a simple branching programme.

These examples illustrate the sensitivity of the chart suggested, as a form for describing a given individualised scheme. Other classifications (e.g. Edling's shown in figure 2.5) deal only in gross generalisation, not defining the frequency of decision. The profile charts suggested by Gibbons (figures 2.3 and 2.4) analyse the system in perhaps too great an amount of detail, thus becoming rather complex and difficult to understand.

The suggested chart is simpler to understand and use, both for analysis of existing schemes or the design of new ones. We shall use it for both these purposes in the later chapters.

Contrasting the author's description system with that of Larsson's (see figure 2.6) there are certain points of interest.

The first two categories of Larsson's ("Goals Course" and "Instructions") equate approximately to the author's "Objectives" and "Content".

Secondly, Larsson's factor "Tasks" is somewhat unclear. As he says: "the teacher chooses tasks for one student at a time -", it suggests that "tasks" are a part of "content" in the author's definition. However, the "number and degree of difficulty" of the tasks suggests that this factor, in the author's classification is part of the "Methods" of instruction - the detailed instructional tactics to

achieve a particular objective. In the author's chart, individualised content suggests the sort of situation advocated by Bruner (1966) that process-type objectives (e.g. learning to think, learning to learn) may be achieved by means of various routes, utilising various subject matter. This is the philosophy of the "free-discovery" approach (and of the mathematics laboratory to a certain extent).

Thirdly, Larsson combines "methods" and "rate" as one characteristic, whilst the author separates them. There are two very good reasons for this: (1) many individualised systems (e.g. linear programmed instruction) individualise the rate of learning but in no way individualise the methods, and (2) the factor "methods" is a multi-faceted factor, as indicated above, so that there may be a case for sub-dividing it further (e.g. sequence, strategies, and tactics) rather than combining with other factors. There may be a case for sub-dividing the author's chart further here.

Fourthly, Larsson includes the factor "Evaluation/Marking". The author sees no need for this due to the implied definition of the factor "objectives". Objectives in this context are "performance" or "behavioural" objectives (if they were otherwise, e.g. topic titles, they would classify as "content"). A characteristic of a well stated performance objective (Mager 1962) is that it defines the conditions under which and the standard to which the defined programme should be demonstrated to show that learning is

complete. Therefore stating the objective defines the evaluation instrument and the marking scheme. Conversely, varying this implies varying the objective. Thus a course which is individualised with respect to its objectives, is individualised to the same degree with respect to evaluation procedures and standards. The only extra bit of individualisation which would creep in at this stage would be the introduction of "alternative, equivalent, tests" for a given objective and allowing the student to choose. However, as they are equivalent (with respect to the objective) the author does not feel that this type of procedure warrants a separate classification. Rather it is alternative content, which happens to be a test.

Fifthly, Larsson's technique of showing the "degree of individualisation" by marking a point on the "total individualisation— no individualisation" continuum is visually attractive and certainly may indicate where the emphasis of individualisation lies (which factors are very individualised and which not so much). However, it is not a very quantitative measure. The author's four "levels of individualisation", whilst not being totally quantitative, do indicate in broad terms the frequency with which individualisation decisions are made. At the first (course) LEVEL? Decisions are made concerning "which units". (Once only or at a few key points during the course, e.g. between units). At the "unit" level, decisions are made concerning "which lesson" (i.e. daily). At the "lesson" level decisions are made concerning that lesson's structure, content, etc., (i.e. hourly). At the "learning

step" level decisions on an individual-student basis are made on a more or less continuous basis.

Lastly, Larsson's model does not indicate precisely who makes the individualisation decisions. He seems to suggest that at the "total individualisation" extreme, it is the teacher who makes most of the decisions regarding instructions and tasks, they collaborate on objective selection and evaluation, and students select methods and rate. This may well be true at the "course" or "unit" level, but as the IMU units are entirely in the form of programmed instruction, once the student is within a given unit (i.e. at the "lesson" or "step" level) the teacher's and the student's decision-making freedom is much curtailed, yet individualisation continues, having been built into the "system" (programmed text) by the programmer.

2.7 Towards a Taxonomy of Individualised Instruction?

The above discussion and the form suggested by the author are not presented as a complete system of classification or description of techniques for the individualisation of instruction. The construction of a useful taxonomy for individualised instruction is a task which has interested a number of researchers, including Gibson (1971) and Tosti and Harmon (1972). The systems of classification they have suggested, which were discussed in the earlier part of this chapter, are incomplete. Gibson's is comprehensive in attempting to

include all the varieties of individualised instruction known in 1971, but not very specific in distinguishing all the factors which may differ between them. His "family tree" classification (figures 2.1 and 2.2) throw such different systems as the Dalton Plan, IPI and a linear programmed text into the same group. His profiles may produce a very similar trace for a one-hour study assignment and for a system which involves the whole school life of a student (see the profiles of linear programmed instruction and the Summerhill free school in figures 2.3 and 2.4).

Tosti and Harmon (1972) by no means present a complete taxonomy. They commence by stating:

"The quality of an educational system depends upon:

1. The quality of presentation and display
2. The nature and extent of student participation
3. The accuracy of decisions regarding the assignment of the most appropriate learning activities for a student considering his needs, abilities, aspirations and motivations.

To date most innovations in education have been primarily concerned with the first function. Films, educational television, typical audio-visual devices and even team teaching are examples of this. "

They take the opposite approach and therefore concentrate on the third factor. The types of decision making they suggest as a "taxonomy" were discussed earlier in this chapter. However they do conclude the paper with the following statement:

"We have not detailed the strategies that would be included in the taxonomy, but have only highlighted a few of the more common ones. In describing the instructional management devices for media that may be employed, we have treated them as isolated devices. But each of these instructional management media (the teacher, the computer, the para-professional aide or the peer group member and the student himself has some advantages and limitations. To the designer of instructional systems, the task will be to develop a total system which will incorporate the best instructional management and media mix. We expect the system of the future to be a combination of teacher, computer, proctor and student. The development of such a mix will be based on a general technology of instructional management which will result from continued research into strategies. "

Indeed, a full taxonomy of individualisation would be much more complex. As Robert B. Davis pointed out in his editorial to a special issue of Educational Technology, devoted to the question of the individualisation of mathematics instruction (Davis 1972).

"If a taxonomy of individualization methods, ever does develop, it will need to provide for at least the following kinds of variation, which already play a role in presently existing approaches:

Decisions: What kinds of decision points appear as an individual student progresses through the program? How often? Who makes the decision? Under what sort of constraints?

What external or reality pressures are brought to bear on the student? Does he receive "reinforcers" such as tokens (or actual cash payments)? Is the experience of success itself a major reinforcer? How about competition with other students? Persuasion? Sanctions? Adult-devised "temptations" (as in the TV program Sesame Street, which strives to be something children will want to watch)?

What levels of technology are involved? Merely paper-and-pencil (as in many LAPs)? Inexpensive or free materials (such as pebbles and bottle caps)? "Medium-level" technology, such as desk calculators, slide rules, audio cartridges, etc.? Sophisticated technology, such as time-shared computers, CAI, videotape, etc.?

How is "time-sharing" or "space-sharing" used - e.g., is part of the day allocated to one approach, and part to another; is some space in the room or building used in one mode, while other space is used differently?

To this list might be added:

"What assumptions are made about the nature of cognition?

What assumptions are made about the nature of motivation?

What priorities rank highest in the design and operation of the system? "

The ERIC Information Analysis Centre for Science Mathematics and Environmental Education has recently published a preliminary taxonomy of instructional strategies, individualised or otherwise (Merrill and Wood, 1974). The purpose of this paper is to develop a taxonomic vocabulary and a model for portraying instructional strategies. Instructional strategies are defined as sequences of two or more instructional displays. To describe individual displays, eight variables are identified: content type, content mode, content representation, prompting, response conditions, response mode, response representations and feedback.

However, even this paper, nearly a hundred pages long, is really only an outline for a taxonomy, rather than a completely elaborated classification system.

Otherwise, however, little new work has appeared in the literature which would contribute towards the construction of such a comprehensive taxonomy. Perhaps interest has waned. Perhaps there is no real practical need for it. There are some writers (e.g. Gilbert, 1962) who maintain that the effort which went into the production of Bloom's (1956) and Krathwohl's (1964) taxonomies of educational objectives has not paid off;

that the pseudo-scientific jargon which has been created by the labelling of objective-types has led to confusion in the stating of objectives rather than clarity. Perhaps the design of individualised instructional systems (as is the case with statement of clear objectives) can "better be achieved by the commonsense application of a few basic principles", than by the development of a comprehensive classification system.

The author offers his descriptive form in this context. It is suggested as a practical aid to description design or analysis. If each of the blocks which indicates individualisation decisions is described further, in a sentence or paragraph, to indicate HOW the individualisation takes place, (as was done in this chapter for the example of the remedial mathematics course at the Middlesex Polytechnic) then the form may also be used as a basis for description. If a given course is being analysed or designed, completion of the form may act as an organizer of one's thoughts, as an aid to the "common sense application of basic principles."

CHAPTER 3

MATHEMATICS LEARNING AND TEACHING AND ITS RELATIONSHIP TO THE INDIVIDUALISATION OF INSTRUCTION

3.1 Introduction

The aims and scope of the review.

3.2 Major Theoretical Viewpoints

3.2.1. The Behaviourist position as exemplified by Skinner,

(and practical techniques for applying it to instruction, developed by Gilbert, Glaser, Evans and others).

3.2.2. The Neo-Behaviourist Viewpoint as exemplified by R. Gagné.

(The eight categories of learning and the conditions necessary for each to take place.)

3.2.3. The Cognitive/Developmental Viewpoint as exemplified by Piaget and Bruner.

(The concrete, iconic and symbolic stages in learning, and practical procedures designed to follow these stages, as developed by Bruner and Dienes.)

3.2.4. The Subject-Matter Viewpoint as exemplified by Ausubel.

("Meaningful reception learning", in contrast to discovery learning.)

3.2.5. The Cybernetic Viewpoint as exemplified by Landa

(Algorithmic and Heuristic problem-solving processes. The need to identify the exact mental operations involved in problems before attempting to teach them. Matching instructional process to thinking process.)

3.3 Comparisons and Contrasts Between the Viewpoints

3.3.1. Objectives of Mathematics Education

Inputs (content), outputs (skills), or processes (thinking) ?

The extreme positions adopted by Ausubel, Skinner and Bruner.

The unifying positions of Gagné and Landa.

The author's viewpoint - the systems approach.

3.3.2. Methods of Mathematics Instruction

The expository/discovery controversy.

The varieties of discovery-learning as indicated by Biggs and Landa.

The author's suggested continuum of discovery - expository methods.

3.3.3. The Individualisation of Mathematics Instruction

Course characteristics which may reasonably be individualised in mathematics courses. The rationale for individualisation.

3.3.3.1. - The reception learning viewpoint.

3.3.3.2. - The discovery learning viewpoint.

General Models for the implementation of individualised schemes.

3.3.3.3. - The Mastery Learning model - a neo-behaviourist model, favouring reception learning.

3.3.3.4. - The Cyclic Learning model - a cognitive/developmental model favouring discovery learning.

Other viewpoints.

3.3.3.5. The Cybernetics viewpoint.

Machine systems and cybernetic control,
as visualised by Landa and as realised
by Pask.

3.4 Conclusion - Other Considerations

3.4.1. The Prescriptive/Democratic/Cybernetic control
Controversy.

What indeed is the role of the teacher in an
individualised mathematics system? How should
he control the learning process? Could this
function be effectively automated?

3.4.2. The Humanist and Cultural Positions

The humanistic arguments for individualisation.
The arguments for the retention of the teacher.
The potential de-humanising effect of automation.

3.4.3. The Author's View

Contrasting the theory with reality - the true
state of the mathematics teaching profession.
The crying need for technological aids and systems.
The special need of developing countries.
The author's past work and current research
plans related to this need.

CHAPTER 3

MATHEMATICS LEARNING AND TEACHING AND ITS

RELATIONSHIP TO THE INDIVIDUALISATION OF INSTRUCTION

3.1 INTRODUCTION

In order to give a framework to this study of individualised instruction of mathematics, this chapter presents a summary of some current thinking and practice concerning the learning and teaching of mathematics.

Firstly, major current theoretical viewpoints with regard to learning and instruction in general are reviewed.

Secondly, current models for mathematics instruction, arising out of the theories are discussed. This discussion focuses in particular on the variety of viewpoints the models express concerning the objectives of teaching mathematics, the instructional strategy to adopt, and the need for individualisation of the learning-teaching process.

Current practice in the implementation of these models - at the elementary, secondary, and post-secondary (higher and further education) levels will be reviewed in the following chapters,

Teaching, or "instruction" (this term is preferred in the present context) has as its purpose the promotion of learning in individuals. Therefore theories of instruction are necessarily based on theories of learning. The history of learning theory has been eventful and colourful, marked by a series of "feuds" between partisan groups fighting under different names in different epochs.

We have had "Associationists" fighting the "Humanists" the "Connectionists" against the "Gestalt" school, the "Behaviourists" at loggerheads with the "Cognitive" psychologists. Despite these changes in name, however, the battles fought in each epoch have been remarkably similar. Each generation of psychologists has fought skirmishes on, for example, the "Nature/Nurture" controversy, the "Rote Learning/Meaningful learning" question, and the "Aims/Means" argument.

Today there is a growing realisation that such questions are not resolvable one way or the other, but that there is an element of validity in all the positions:-

- that both Nature (heredity) and Nurture (experience) play important parts in the learning process; that some things (e.g. bare facts) are best learnt by rote while other things (e.g. concepts) are better learnt in some meaningful context; that one may have a variety of different aims (objectives) for teaching a topic and that these may require a variety of different instructional methods.

However, the old battle-cries are still with us - often in modern up-dated jargon.

Currently, "fashionable" controversies include .

- "discovery" versus "exposition" in teaching,
- "products" versus "processes" of learning,
- "learning environments" versus "knowledge structures" as the keys to the control of instruction.

Some emerging controversies include

"algorithmic/heuristic" problem-solving strategies,
"deductive/conversational" programming of instruction-

and so on.

On analysis, however, many of these "new" controversies are found to bear remarkable resemblances to the older ones. Indeed it is interesting to note that most of the current catch-phrases in mathematical education have been around for a long time. Young (1906) when writing on the teaching of mathematics, makes a very useful distinction between "methods" and "modes" of instruction (current equivalent terminology might be "strategies" and "tactics").

Under the "methods" title he lists, among others, the "discovery" or "heuristic" method, the "laboratory" method, and the "expository" method. Among his "modes" one finds the "lecture", the "Socratic dialogue", inductive and deductive presentations, and so on. It is true that Young's usage of these terms sometimes differs slightly from current usage, but one is left wondering just how many "new" ideas really are all that new.

Indeed one should add that current usage of many such terms is so un-standardised that it is not easy to see whether any really important changes in the basic premises underlying learning and instructional theories have taken place since the turn of the century.

An analysis of some major current theoretical viewpoints on learning and instruction, considering both their characteristics and their origins, will help to clarify the position.

3.2 MAJOR THEORETICAL VIEWPOINTS

3.2.1 THE BEHAVIOURIST POSITION AS EXEMPLIFIED BY SKINNER

This viewpoint, exemplified by the position of Skinner (1961b) is based on a definition of learning as an observable change in behaviour (not caused by physical maturation or

growth). The structure of internal thinking and learning processes is considered irrelevant to the process of instruction, which is seen as the structuring of the environment in such a way as to maximise the probability of the desired new behaviour being learnt. Desired behaviours are taught by a series of successive approximations, commencing from an already established behaviour and working towards the desired behaviour - the process is based on the principle of reinforcement expounded by Skinner - a somewhat more precise re-statement of Thorndike's (1927) "Law of Effect".

The law of effect states the observed phenomenon that behaviour which produces desirable or pleasant effects tends to be repeated (the corollary being that behaviour resulting in unpleasant effects tends not to be repeated).

Skinner defined reinforcement of behaviour as the supplying of a "reinforcer" in order to increase the probability of given behaviour being exhibited (e.g. food for a hungry dog, as a "reward" for "begging"). A reinforcer is defined as "any object or event which is found to reinforce" - a delightfully circular argument. Instruction is equated to the conditioning of desired behaviour - termed "operant conditioning". An operant is a unit of behaviour (a response), together with the environmental condition which triggers it off (the stimulus).

The instructor or trainer,

- (1) arranges the stimulus,
- (2) observes the learner's response,
- (3) reinforces desired responses and withholds reinforcement if responses are not desired.

(Sometimes "punishment" of undesired responses

is employed, but it is not favoured.)

Complex behaviours are "shaped" by first reinforcing any already learnt behaviour which approximates to the desired, and then in gradual stages only reinforcing successively closer approximation. Using the dog example, the stages might be: sitting when food is shown, sitting on haunches, but raising a paw - sitting but raising both front paws off the ground momentarily - progressively longer periods of "balancing" on haunches. In this example, the food is the reinforcer (the trainer arranges that this is so by not feeding the dog some time before training) and also the stimulus for the "begging" response, but generally stimulus and reinforcer may be quite separate objects or events. They always have one characteristic in common however - they are external to the learner. The trainer manipulates and controls the learner's environment, in order to control and shape the learner's behaviour.

Thus Skinner's "theory of instruction" requires no theory of learning concerning the internal learning processes. Using systems terminology, it is an input-output learning theory, treating the learner as a "black box".

In passing, one should distinguish Skinner's "operant conditioning" from the "classical" conditioning described by Pavlov. . . . Pavlov's dogs learnt to exhibit a well established and "natural" behaviour (salivating) in a new stimulus situation (ringing bell) by the expedient of fusing the new stimulus of the bell with the "natural" stimulus of the sight of food. Skinner's dog, in the example above, is learning an entirely new behaviour to a familiar stimulus.

Classical conditioning serves as a paradigm for relatively little human learning, mainly very early infant learning of reflex responses and in later life the inadvertent learning of fears, attitudes and habits. It is consequently of limited relevance to the teaching of mathematics or any academic discipline. Operant conditioning, on the other hand, has been applied to the learning of subject matters, through the techniques of programmed instruction. This involved a certain amount of intellectual gymnastics (in order to equate the learning of verbal information in small units to the "successive approximations" to desired behaviour used as steps in the animal training laboratory) and sweeping generalisations (e.g. that success in intellectual tasks is reinforcing for all human learners). However, early developments in programmed instruction showed great promise and nowhere more than in the field of mathematics instruction. Perhaps as a result of needs, but partly also because it was felt that mathematics was particularly suitable for programming (and possibly particularly easy to programme) the early 1960's saw a boom in the production of programmed instruction for school mathematics, reaching a peak in the U.S.A. in about 1965 (Hendershot 1966) and in the U.K. in 1967 (Basu, Cavanagh & Jones, 1967). For a brief period the "Key to the individualisation of instruction" seemed to have been discovered. However, as De Vault and Kriewall (1970) put it,

"After a brief flurry, stirred up by the belief that a solution had been found, hopes waned considerably

while criticism of the process grew. Not only was it hard to find convincing evidence that the quality of the learning experience had been enhanced by the use of these materials, but also there was a growing conviction that indeed the experience provided by such materials was the very opposite of individualization, at least insofar as the purpose was concerned. Every learner travelled the same path to the same end, varying only in the time at which he arrived at the prescribed objective."

Although school use of programmed instruction failed to grow, indeed declined, development of the technique continued. Alternatives to the linear format included the "branching" programmes pioneered by Crowder (1960) and the "adjunct" programmes (based on self-marked multiple-choice testing) which actually pre-dated Skinner's work (Pressey, 1926) enjoyed a brief vogue. The more important long-term developments occurred in the field of programme-planning techniques. Mager (1962) refined a methodology for stating objectives in behavioural terms which has had impact well outside the strict confines of programmed instruction. Gilbert (1962) developed a methodology for the analysis of behaviour and design of training which, although not generally accepted, still forms the basis of a highly successful approach to instructional problems, particularly in the industrial training field. His methodology, which he termed "Mathetics" (note: not

mathematics, though it has similar Greek roots*) classified behaviour, as composed of combinations of three basic structures - chains, multiple discriminations and generalisations. He developed a basic instructional model, involving 3 stages to any exercise (he termed these "demonstrate", "prompt" and "release"), subsidiary tactics for each of the three basic behavioural structures, and powerful rules for deciding what theoretical content to include, how to select examples, how to sequence instruction, etc.

The "mathetics" methodology, in its original form did not live up to Gilbert's claims, of being the technology of education. It proved well adapted to "training" situations where the final outcomes of instruction are observable behaviours, but somewhat difficult to use in "academic" disciplines, where "subject matter" rather than "job performance" was the starting point for instructional design. In this domain the "Rule and Example" (RULEG) programming technique developed by Evans et al (1962) was more popular. Gilbert has more recently attempted to extend the power of mathetics to the academic disciplines, with some success (Gilbert 1967 and 1969).

However, the long term importance of mathetics, and of the other techniques associated with the beginnings of programmed instruction lie not so much in their elegance (or lack of it) as in the catalytic effect they have had

* Greek: MATHEISIS - The process of learning
MATHETIC - pertaining to learning
MATHEMATIC - Something learned; a science

1816 Usage MATHETIC EXERCISES - Exercises by which progress is made and proficiency obtained.
(Source: Oxford English Dictionary)

on the development of theories of instruction which are capable of being tested out and verified. The programmed instruction movement was the first instructional methodology to "lay itself open" to experimental verification at every stage. Empirical testing and revision were key points in the process of programme development and have since become generally accepted as key elements in any process of instructional design. In particular, the stating of instructional outcomes as specific observable behaviours has become a common practice even among instructors who would reject all other aspects of the behaviourist model of learning.

Finally, many of the behaviourist concepts have become fused into more complex, more "complete" theories of instruction. A good example of such an eclectic theory is the one of Robert Gagne.

3.2.2. THE NEO-BEHAVIOURIST VIEWPOINT AS EXEMPLIFIED BY R. GAGNÉ

Gagné has produced a series of books and dissertations over the last decade, expounding his views on learning and instruction. (Gagné, 1965, 1970, 1974). A study of these reveals a gradual, but constant, change and evolution in his viewpoint. Certain characteristics have remained throughout and these distinguish him from the strict behaviourists on two points:

- (a) he admits a large variety of different types of learning (called variously, learning categories, learning outcomes, or, recently, intellectual skills). Each type of learning is associated with characteristic strategies of instruction.

(b) he admits to some interest in the functioning of the internal mental processes which govern learning.

In his early work, Robert Gagné (1965) suggests a hierarchical list of eight categories of learning. The list is hierarchical in the sense that it proceeds from very simple conditioning-type learning, up to complex learning, such as is involved in problem solving. The list is also hierarchical in the sense that lower levels of learning are prerequisite to higher levels. Figure 3.1 presents the eight categories and illustrates their hierarchical structure. A few words about these categories may help to relate Gagné's work to our earlier discussion.

Gagné distinguishes between 8 different types of learning, as follows:

1. Signal learning

This may be equated with the Pavlovian conditioned response. The subject learns that a given event is the signal for another event, as the dinner bell was the signal for Pavlov's dogs' dinner. So he responds to the signal as he would to the event, i.e. instead of salivating when dinner arrived the dogs did so at the signal. Similarly, a child may learn that its mother's frown is the signal of pain to come, so it responds to the frown as it would to the pain itself.

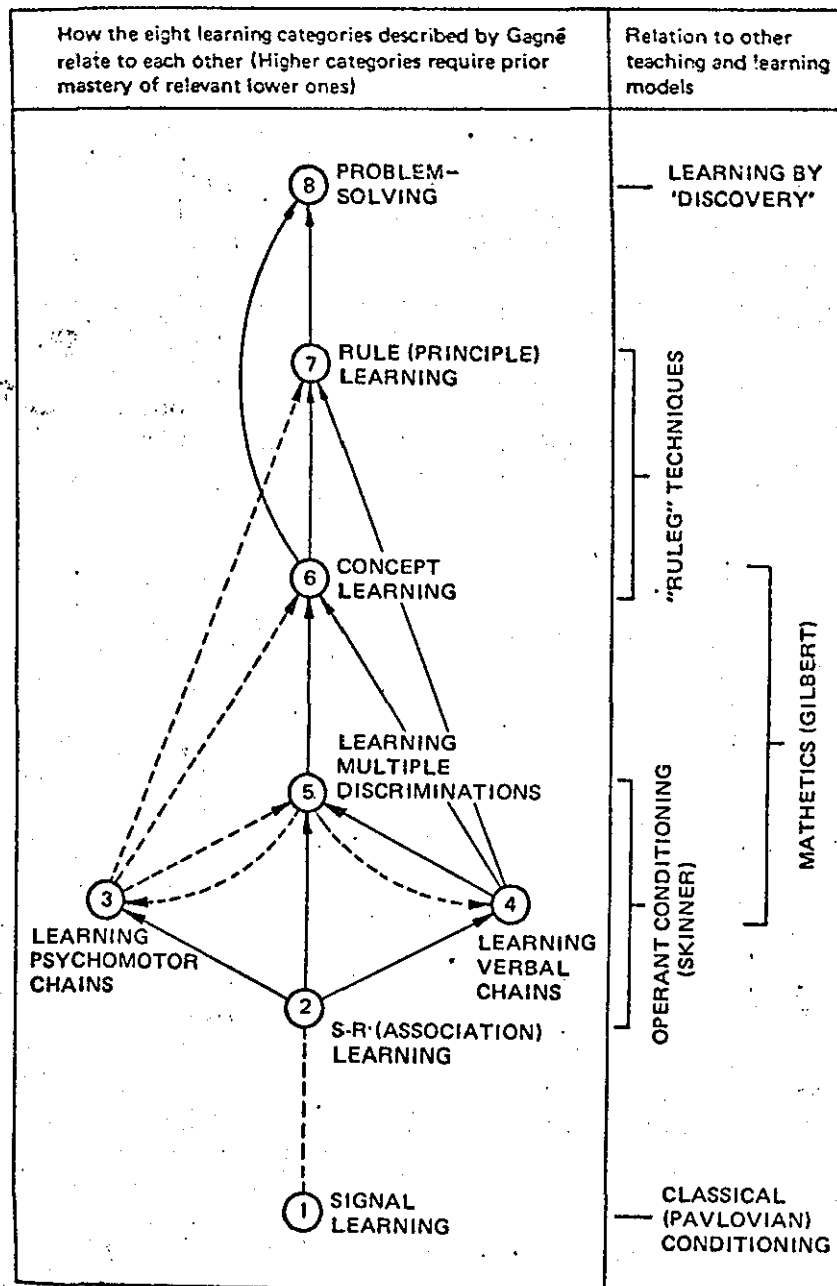
It is characteristic of this type of learning that the stimulus and the response must be closely associated in time - the stimulus of course precedes the response, and it will not produce the desired learning if it takes place too many seconds before the response.

Figure 3.1.

Diagram illustrating the hierarchical nature of Gagné's Categories of Learning.

The arrows indicate that certain learning categories are always (solid arrow) or occasionally (dotted arrow) prerequisite to the learning of other categories. The diagram also shows (on the right) the eclectic nature of Gagné's hierarchy, by indicating how it overlaps with the areas of work of other authors discussed.

(From Romiszowski, 1974 - "The Selection and Use of Instructional Media: A Systems Approach.")



2. Stimulus-Response learning. (S-R).

This is differentiated from signal learning in that the response is not a generalised emotional one, but a very precise act. Gagné gives the following characteristics of this type of learning:

(a) The learning is typically gradual: some repetition of the association between the stimulus and the response is usually necessary.

(b) The response becomes more precise as the repetitions take place (this is what Skinner calls 'shaping').

(c) The controlling stimulus becomes more precise.

(d) There is reward, or reinforcement, for exhibiting the required response and there is no reward when the behaviour is incorrect.

3. Chaining.

Chaining is the type of learning we have already described when discussing Gilbert's work. Characteristics of this type of learning are:

(a) The individual links in the chain must be established first.

(b) Again time is a factor - the events in the chain must occur close together in time.

(c) If both the other two conditions are satisfied, learning a chain is not a gradual process, it occurs on a single occasion. In practice the occasion may have

to be repeated, because the individual links may not be well enough established.

4. Verbal chaining.

Gagné says 'verbal association might well be classified as only a subvariety of chaining ... But because these chains are verbal and because they explain the remarkable versatility of human processes, verbal association has some unique characteristics.' He gives the example of a man learning the French for 'match' (alumette) in the following way: the word 'match' acts as a stimulus for the mental picture of a match. A match 'illuminates'. The syllable 'lum' occurs in illuminate as it does in alumette, so a chain is established.

The conditions for effective learning of verbal chains, according to Gagné, are:

- (a) Each link must be established previously - the link in the individual's mind between the word 'match' and the object match must be clear.
- (b) 'Response differentiation' must have taken place; i.e. the individual must know how to say 'alumette' well enough so that the key syllable 'lum' means something to him and can be used as a link with the word 'illuminate'.
- (c) A 'coding connection' (Gilbert uses the term mediator) must be established. The mental picture of a flaming match and the word 'illuminate' must be associated. 'Clearly, people will tend to use different 'coding connections', depending on their previous

history; highly verbal people will have more codes available than less verbal people.

5. Discrimination learning.

This is the same category as Gilbert's multiple discriminations.

The conditions of this type of learning are as follows:

- (a) Necessary Ss ——— Rs must already be established.
- (b) Interference from conflicting stimuli must be reduced to a minimum. That is to say, distinctions must be emphasised; interference is anything which might add to confusion of the stimuli and uncertainty about which of several responses is required.

6. Concept learning.

This may be compared to the 'generalisation' of mathematics. In this form of learning a stimulus is classified in terms of its abstract properties, as shape, position, number etc. A child learns that a green block A is called a 'cube'. It is then told that block B, which is twice the size of A and red in colour, is also a cube. Concept learning is the type of learning which enables him to identify a cube on the basis of an internalised representation (an idea) which is independent of the dissimilarities of the two objects - in the Platonic sense, an 'idea of cubeness'. Gagné draws a distinction between a 'concrete concept', which depends on the observable properties of objects, and a 'defined concept', which identifies a class of objects whose common properties are not determinable by observation, but are a matter of

verbal definition. (For example, a broom and a screwdriver are both 'tools', but cannot be observed to have much else in common.) Conditions for this type of learning are as follows:

- (a) Necessary S — R's must be established.
- (b) A variety of stimulus situations must be presented, so that the conceptual property common to all of them can be discriminated.
- (c) The learning of a new concept may be gradual, because of the need for a variety of stimulus situations.

7. Rule learning.

In a formal sense, a rule is a chain of two or more concepts. The simplest type of rule may be 'If A, then B' - e.g. 'If two angles in a triangle are equal, then the sides opposite the angles are equal. This may be distinguished from a 'simple verbal fact to be memorised' in that if the rule is correctly learnt, then the learner will be able to apply it in all relevant situations, and he may not necessarily be able to state the rule in words. The conditions for this type of learning are as follows:

- (a) The concepts to be linked must be clearly established
 - the learner must know what a 'feminine noun' is and what the 'feminine article' is.
- (b) A simple process of chaining can then take place.
- (c) The learning of a rule can take place on a single occasion.

8. Problem-solving.

Once a human being has acquired some rules he can combine these rules into a great variety of higher order rules. In doing this he can use what he already knows to solve problems which are new to him, (though they may or may not be so to other people). This 'problem-solving' takes place at all levels, from Joe Bloggs working out how to change a tyre without getting his clothes dirty, to Einstein producing the theory of relativity. The conditions of this type of learning are as follows:

- (a) The learner must be able to identify the essential features of the response that will be the solution before he arrives at the solution.
- (b) Relevant rules are used and recalled.
- (c) The recalled rules are combined so that a new rule emerges. (Gagné admits little is known about the nature of this 'combining event').
- (d) Though the overall process of solving a problem may take a very long time, Gagné thinks that the solution is actually arrived at in a 'flash of insight'.

Thus we see that Gagné's model embraces the models of Skinner and of Gilbert, which we presented earlier. It also includes the very primitive type of Pavlovian conditioning which is only of marginal concern to teachers above the kindergarten level. Where it differs is that it extends these models to define categories of higher order learning. One point it makes, is to stress just how much success at

higher orders depends on adequate mastery of lower order learning. Another important point that Gagné (1970) makes concerning rule-learning and problem-solving is that the outcomes of these two types of learning are essentially the same - the difference lies in the processes by which the learning took place. A higher-order rule, formed from two or more subsidiary rules may be learnt as a "type 7" task, if the teacher recalls the relevant rules and demonstrates how they combine to give a more powerful rule. This he would do by definition and example - the "expository" approach.

Alternatively, the teacher might commence by presenting an example as a new problem to be solved. The learner must recall the relevant simpler rules and "discover" the higher order rule necessary to solve the problem, although the teacher would normally prompt or guide this process along by his use of hints and leading questions - the "guided discovery" approach.

Thus, for higher order intellectual learning, Gagné presents two alternative strategies - the expository (from rule to examples) strategy, which he favours on the grounds of consuming less learning time in general, and the "guided discovery" (from examples to rule) strategy, which he favours when long term recall and/or transfer to other similar learning tasks is required.

In a recent book (Gagné 1974) he has further extended his model to include yet more types of learning and approach yet closer to the cognitive school, which we now turn to discuss.

3.2.3 THE COGNITIVE/DEVELOPMENTAL VIEWPOINT AS EXEMPLIFIED BY PIAGET AND BRUNER.

The influence of Jerome Bruner on the teaching of mathematics, particularly elementary school mathematics in the U.S.A. has been immense. He is probably the foremost living proponent of the discovery approach in mathematical education. However, he is not by any means the inventor of the discovery approach. As already noted (Young 1906) this concept was well known in mathematics education at the end of the last century. Bruner's (1966) approach to discovery learning is characterised by 3 stages, which he calls enactive, iconic and symbolic. These stages are firmly based on the developmental psychology of Jean Piaget. Piaget has been probably the most prolific living researcher in developmental psychology. His interests have centred on the study and definition of the stages of cognitive development of the child. We shall not reiterate here the Piagetian stages of cognitive development. This is available in many other works (1952, 1958, 1963, 1968). We shall concentrate on the characteristics of Piaget's view of the growth of intelligence as they may relate to the process of instruction.

Piaget views the development of intelligence as part of the more general process of biological development. Gallagher (1964) has suggested five major themes running through Piaget's work.

1. Continuous and progressive changes take place in the structures of behaviour and thought in the developing child.

2. Successive structures make their appearance in a fixed order.
3. The nature of accommodation (adaptive change to outer circumstances) suggests that the rate of development is, to a considerable degree, a function of the child's encounters with his environment.
4. Thought processes are conceived to originate through a process of internalizing actions. Intelligence increases as thought processes are loosened from their basis in perception and action and thereby become reversible, transitive, associative, and so on.
5. A close relationship exists between thought processes and properties of formal logic.

For Piaget the child is a developing organism passing through biologically determined cognitive stages. These stages are more or less age-related, although wide variations in cultures or environments will yield differences in individual rates of development. One might view the process of cognitive growth as a drama. The script or scenario describing the drama's plot and characters is given by the biological component. The role of the director - that of determining the onset and pace of the episodes - is a function of the environment.

Although development is a continuous process of structural change, it is still possible to characterise certain growth periods by the formal logical structures most useful for describing the child's cognitive functioning during that time span. These growth periods, when a temporary stability of cognitive functioning is achieved, define for Piaget the major stages of intellectual growth.

There is one other principle which is extremely important for an understanding of Piaget's system and its impact on education. This is the principle of autoregulation or equilibration. Piaget sees the development of intelligence as a sequence of successive disequilibria followed by adaptations leading to new states of equilibrium. The imbalance can occur because of a change occurring naturally as the organism matures. It can also occur in reaction to an input from the environment. Since disequilibrium is uncomfortable, the child must accommodate to new situations through active modification of his present cognitive structure.

Piaget observes that only in man can intelligence develop to the point where the domain of ideas and symbols can serve as the "environmental" source of disequilibrium. That is, we can construct intellectual universes, for example, "transintuitive spaces, which can stimulate our own cognitive growth as surely as the confrontation by a baby with the problem of reaching his pacifier can lead to new insights of equilibria on his part." (Shulman 1970)

Piaget has written little specifically directed at problems of education. He has repeatedly disavowed any expertise in the pedagogical domain. Yet, either directly or through such interpreters as Bruner, his influence has been strongly felt.

Piaget's emphasis upon action as a prerequisite to the internalization of cognitive operations has stimulated the focus upon direct manipulation of mathematically relevant materials in the early grades. His description of cognitive

development occurring through auto-regulation has reinforced tendencies to emphasize pupil-initiated, problem-solving activities as a major vehicle of mathematics instruction.

Much of the work of such practical innovators as Z.P.Dienes (1960, 1964), and such theoreticians as Bruner is directly based on Piaget.

The general learning process described by Bruner (1966) occurs in the following manner. First, the child finds in his manipulation of the materials regularities that correspond with intuitive regularities he has already come to understand. Notice that what the child does, according to Bruner, is to find some sort of match between what he is doing in the outside world and some models or templates that he has already grasped intellectually. For Bruner, it is rarely something outside the learner that is discovered. Instead, the discovery involves an internal reorganization of previously known ideas in order to establish a better fit between those ideas and the regularities of an encounter to which the learner has had to accommodate.

Bruner almost always begins with a focus on the production and manipulation of materials. He describes the child as moving through three levels of representation as he learns. The first level is the enactive level, where the child manipulates materials directly. He then progresses to the iconic level, where he deals with mental images of objects but does not manipulate them directly. Finally he moves to the symbolic level, where he is strictly manipulating symbols and no longer mental images of objects.

This sequence is based on Bruner's interpretation of the developmental theory of Jean Piaget. The combination of these concepts of manipulation of actual materials as part of a developmental model and the Socratic notion of learning as internal reorganization into a learning-by-discovery approach is the unique contribution of Bruner.

Bruner's position is in strong opposition to both Skinner's and Gagne's (1970) in that he rates the internal thought processes as of paramount importance, and the final outputs, or products, of secondary and much lower importance. The behaviourist viewpoint disregards internal processes altogether, using observable behaviours as the only measures by which to assess instruction. Gagne would allow both, though rating specific skills higher than generalised mental capabilities. However, in his recent writings (1974, 1975) Gagne has approached much closer to the cognitive position adopted by Bruner.

3.2.4 THE SUBJECT-MATTER VIEWPOINT AS EXEMPLIFIED BY AUSUBEL.

David Ausubel (1968) has been a powerful (though perhaps a waning) influence on instructional thinking. He stands in opposition to the "discovery" movement, claiming that much of the apparent superiority of "discovery" over "exposition" is due to research generally comparing discovery techniques with "rote-learning" approaches. Ausubel argues that much instruction, particularly at higher levels of education, is (and has always been) successfully performed by the process of exposition leading to "meaningful reception learning". He states:-

"In reception learning (rote or meaningful) the entire content of what is to be learned is presented to the learner in final form. The learning task does not involve any independent discovery on his part. He is required only to internalise or incorporate the material ... that is presented to him so that it is available or reproducible at some future date. In the case of meaningful reception learning, the potentially meaningful task or material is comprehended or made meaningful in the process of internalisation. In the case of rote reception learning, the learning task either is not potentially meaningful or is not made meaningful in the process of internalisation.

The essential feature of discovery learning ... is that the principal content of what is to be learned is not given but must be discovered by the learner before he can incorporate it meaningfully into his cognitive structure. The distinctive and prior learning task, in other words, is to discover something ... The first phase of discovery learning involves a process quite different from that of reception learning. The learner must reorganize or transform the integrated combination in such a way as to generate a desired end-product or discover a missing means-end relationship. After discovery learning itself is completed, the discovered content is made meaningful in much the same way that presented content is made meaningful in reception learning.

It is evident, therefore, that reception and discovery learning are two quite different kinds of processes, and ... that most classroom instruction is organized along the lines of reception learning. Verbal reception learning is not necessarily rote in character. Much ideational material (concepts, generalisations) can be internalised and retained meaningfully without prior problem-solving experience, and at no stage of development does the learner have to discover principles independently in order to be able to understand and use them meaningfully. "

(Ausubel 1968)

3.2.5 THE CYBERNETIC VIEWPOINT AS EXEMPLIFIED BY LANDA

In contrast to all the theoreticians mentioned so far, who are American, Leo Nakhmanovich Landa is Russian. His work has been based much more on the European psychological tradition, oriented (as in the Gestalt school) much more towards the study of internal thought processes than has been the case in the U.S.A. It extends over a considerable period of years, parallel to the years during which most of the previously mentioned American work was published. Already in 1955 his thesis on the "Formation in Students of General Methods of Reasoning" was exploring algorithmic and heuristic methods of instruction, the construction of mathematical models of the learning and teaching process and the automatisisation and programming of instruction in problem-solving; topics which became important areas of research in the U.S.A. only a decade or so later. His works

have only recently become available in the English language, with the publication in 1974 in the United States of "Algorithmization in Learning and Instruction", first published in Russian in 1966. Much of his more recent work, published in the Soviet Union as separate papers, has appeared in the U.S.A. in 1976 as a second book "Instructional Regulation and Control - Cybernetics, algorithmization, and heuristics in education".

These works show that although working in parallel with, and geographically isolated from, North American thinkers, he was not uninformed of developments in the West. However, his different psychological background and his training in mathematics and cybernetics have combined to form a view of learning and instruction which often resolves apparent differences between opposing psychological "camps" and which is particularly understandable to mathematicians.

Landa's first book (Landa, 1974) summarises two decades of research on the learning teaching processes. Most of this work was concerned with the intellectual processes involved in the learning of Mathematics and Russian Grammar. Landa describes the objectives of this work as follows:-

"The research was directed at the solution of these problems as applied to the instruction of geometry.

During the research, an attempt was made to determine the operations which "make up" the process of thinking through a geometrical problem (by proof) and the process of searching for its solution. The system of these operations was

formed as a specific system of rules and as a sufficiently general procedural prescription which shows what must be done with the conditions of the problem in the process of its solution, and how to think as one searches for the proof. The reduction of the process of searching for proof into separate, sufficiently elementary operations and instructing students in them showed that, in a relatively short time, it was possible to raise sharply the efficiency of instruction and to teach students how to solve problems of a kind they had not been able to solve up until then.

After the completion in 1955 of research to discover sufficiently general procedures (methods) of thought for the solution of geometrical problems, and experiments in teaching them to students, the question arose as to whether these methods were specific only to mathematics. If not, then would it not be possible to establish the common features of methods of thought, for the solution of problems from different subject-matter fields?

In order to answer the question of whether the methods were specific only to mathematics, it was necessary to investigate methods of solution for some non-mathematical problems. Grammar problems were chosen as the object of investigation. The process of solving grammar problems was analyzed in exactly the same way as the process of solving geometrical problems. It turned out that the procedures for solving both kinds of problems

had much in common and that one may also formulate sufficiently general procedures for the solution of grammar problems quite as for the solution of geometry problems. But an essential difference between these procedures was also brought to light. The procedures in the search for proof in the solution of geometry problems were precisely methods of search, i.e., they had a heuristic character; and, therefore, the way to accomplish them depended essentially on the type of problem, its complexity, etc. These methods do not completely and univocally determine all of the students' actions as they search for the solution, and therefore, they do not guarantee - as applied to each specific problem - its inevitable solution. For grammar problems, we succeeded in designing procedures which completely and univocally determine the students' actions, and when they are applied correctly, guarantee the inevitable solution of real problems encountered in practical work in school. In other words, these methods have an essentially algorithmic character. "

Subsequent work was concerned with investigations on how to establish problem-solving algorithms, how to identify intellectual operations which are not capable of algorithmization, how to teach by the use of algorithms and how to teach students to develop their own algorithms for the solution of new problems.

Landa has therefore been little concerned with the "lower level" type of learning, such as stimulus-response or chaining. He has been primarily interested in problem-solving activity (as in the case of geometrical proofs) and rule-following activity (as in the rules of grammar). These two types of learning seem to correspond to Gagné's higher-order categories. However, Gagné discusses these somewhat superficially and with examples drawn almost exclusively from the early elementary grades. It will therefore be interesting for our purposes to study Landa's position, based on more extensive research with older students.

In connection with rules and rule-learning Landa states:

"The application of rules, particularly the recognition of situations where the rule is applicable, is achieved by means of special operations. Just as it is impossible to solve a manufacturing problem (for example, to make something) without carrying out specific component manipulations (operations), it is also impossible to solve an intellectual problem (a grammatical, mathematical problem or one pertaining to physics, etc.) without carrying out specific intellectual operations. The execution of a specific aggregate of intellectual operations to solve a problem is an objective necessity. But if this is so, then it is incorrect to think that some problems may be solved without executing the operations.

The opinions that something is "obvious at once", that something "is immediately grasped" without any operations, "comes to mind of itself", etc., are illusions engendered by the fact that there is no awareness of many operations because of their automatiséation.

The opinion that one may solve problems "without operations" is untenable. An objective analysis of the structure of knowledge, in particular, grammatical knowledge, shows that intellectual processes which seem at first glance to be simple and not to require any special operations, are actually very complex and break down into a considerable number of operations. If this were not so, if it did not just seem simple to identify, for example, the subject, then all the students would identify this part of the sentence without mistakes. This is not so in reality. "

Landa immediately draws our attention to the "recognition of situations when and where the rule is applicable". Rule-learning is incomplete until the learner can apply the rule correctly and on appropriate occasions. This latter aspect is not stressed by Gagné. For Gagné, the student has learnt the rules for simplifying fractions when he can respond correctly to the request "simplify this fraction in order to..." For Landa a rule has been learnt only once the student can respond to the request "show me what you should do to this fraction in order to ...". To use Gagné-type terminology, Landa considers that rule

mastery is made up of two elements (1) Discrimination of appropriate and inappropriate occasions for application of the rule and (2) Correct application of the rule. Landa refers to the first of these as the "logical thinking" component. He has the following to say concerning logical thinking:

"In pedagogy, there exist two ways to form habits of logical thinking. The first way is the unconscious mastery of logical methods by the students. This takes place during the process in which specific material from the textbooks is learned and during the solution of problems by practice. The second way is that of conscious mastery of those methods when the teacher specially draws the students' attention to those logical means with whose help the solutions of problems are achieved.

Progressive-minded pedagogues of the past always attributed great importance to the special instruction of students in methods of logical thinking. The majority of contemporary pedagogues also admit the necessity of such instruction. Then why is the first way still the most prevalent one in practice up to the present day?

The level of development in psychology and logic, even several decades ago, was such that the creation of a scientifically based general theory for the instruction in systems of intellectual operations, in particular, algorithms of intellectual activity, was hardly possible. At the present time, thanks to new ideas which have appeared in psychology,

logic, and cybernetics, not only is it possible to raise the question about the creation of such a theory, but it is indispensable.

Landa's theory of instruction is based on:-

- (1) Firstly, analysis of the topic, in order to identify the thought processes (the operations) necessary to master it. This is the stage of attempting to define the algorithms that should be used for problem-solving.
- (2) Secondly, analysis of the learners, their existing thought processes (the algorithms they have already mastered) and their psychological characteristics (individual differences etc.) in order to devise a "teaching algorithm" (a set of operations that should be carried out during instruction in order to ensure efficient learning.

He admits that it is not always possible to "algorithmicise" a process. Indeed a true algorithm must satisfy three conditions which he defines:-

" Specificity indicates the fact that all actions of the user of an algorithm (a person or a machine) are unambiguously determined by instructions (rules), and that these instructions are identically (or uniformly) understandable and understood by all users, since they are addressed to sufficiently elementary operations to be performed in the same way by all users of the algorithm; having started from the same initial conditions and proceeding in accordance with the instructions, all users will arrive at a single, identical result.

Generality means applicability of an algorithm to an entire (often infinite) set of problems belonging to a particular class, rather than to a single problem, or, in other words, that any objects from some class of objects, and not just certain particular ones, may constitute the initial data of a problem (for example, numbers, logical expressions, etc.)

Resultivity indicates that the algorithm is always directed toward achieving the sought-after result, which the user, once he (or it) possesses the appropriate initial data, always achieves. We note, however, that resultivity does not mean that the goal is achieved with any type of initial data. The initial data may be deficient such that the algorithm may terminate without result or continue indefinitely without arriving at the desired result."

However, he maintains that only when one has analysed a particular thought process to the extent that one can state to what extent these three characteristics are met, is one ready to consider the instructional process that should be adopted in order to teach it. In his second book Landa (1976) examines the problem of matching instructional process to the type of thinking process. We shall not go into detail on this here. Just by way of illustration we reproduce one of a series of charts which define the type of thought process, or prescription involved. This chart analyses the variety of degrees in which a given prescription may satisfy the conditions of

specificity and resultivity.

	Algorithm Prescrip.	Semi- Algorithm. Prescrip.	Semi- Heurist. Prescrip.	Heuristic Prescription
All people solve	+	+	+	-
All solve correctly	+	+	-	-
All solve identically	+	-	-	-

Where such an analysis will yield detailed instructional tactics relevant to specific learning problems, Landa is quite clear about the general, overall strategies that one should follow in teaching problem-solving.

"The instruction in algorithms, like the instruction in methods of non-algorithmic character, may be achieved in different ways. One way is to present the algorithms in a ready-made form so that the students have only to learn them, and then to reinforce these algorithms during exercises. With another method, algorithms are not handed to the students in prepared form, but the students discover them themselves. The instruction is arranged in such a way that the students independently find the necessary and, at the same time, efficient systems of operations. It is important, when this is done, that the assimilation of these systems of operations is achieved not by rote-learning, but as a result of properly designed exercises. It is clear that, in specific cases, the first form of instruction may turn out to be expedient.

But, in our opinion, the second method should be the important, basic, and leading one in instruction. It is this method which we used in experimental instruction. We did not impose algorithms on the students. We did not provide them in advance or present them in ready-made form. The students did not have to specially learn them. The systems of operations forming the basis of the algorithms were gradually assembled by the students in the process of independent, active, practical, linguistic actions. The verbal formulation of the algorithms or of their separate elements and the representation of their diagrams was only the result, the sum of the formation of separate operations. "

We note here a marked similarity of viewpoint between Landa and Gagné. Gagné's "rule-learning" appears equivalent to Landa's "learning of ready-made algorithms". Gagné's "problem-solving" is equivalent to the "discovery of the algorithm" in Landa's system. The one point of difference between the two theoreticians appears to be the relative importance of these two types of learning. Gagné favours rule-learning as being faster, the problem-solving approach paying off only when longer-term retention or transfer of skill are important factors. Landa seems to favour the discovery approach, perhaps because as a cybernetician his preoccupation would naturally be focussed on generalisation and transfer of skills between disciplines. It is doubtful if Landa would consider any skill of no transfer-value as worth learning.

In this last point, Landa's viewpoint is very close to Bruner's. However, one should stress that the discovery approach suggested by Landa is far removed from the "free discovery" advocated by Bruner. Gagné's "guided discovery" approach is already much more rigorous than Bruner's. For Gagné any discovered exercise has specific pre-set objectives and the teacher has only been successful if he has guided the learner to achieve them. But he only gives general guidelines on how a teacher may attempt to do this. Landa would go much further. We can read a criticism of the position of Gagné (and also of Bloom's taxonomy of objectives - Bloom et al. 1958) into the following paragraph (Landa 1976).

"We note that different types of problems require the ability to perform different cognitive operations. This situation is analogous to what takes place with physical operations during various forms of physical activity; driving a car requires knowledge of one kind of operation, working a lathe requires knowledge of others. Very often in psychology and logic the characterisation of cognitive activity is limited to indications of operations such as analysis, synthesis, generalisation, discrimination, and a few others. This characterisation is correct, but not sufficient. The specific operations in which analysis and synthesis appear are extremely diverse (there are hundreds, if not thousands, of them), and it is possible to know some analytical operations and not to know others. The

task facing psychology, especially educational psychology, requires it not to be limited by simple characterizations of cognitive activity such as that involving analysis and synthesis, but rather to discover the specific cognitive actions (operations) in which analysis and synthesis manifest themselves and which are required for the solution of a given class of problems."

3.3 COMPARISONS AND CONTRASTS BETWEEN THE VIEWPOINTS

3.3.1 OBJECTIVES OF MATHEMATICS EDUCATION

The viewpoints discussed above stand in sharp contrast as regards the general objectives of mathematics instruction.

Skinner, in his work on the programming of arithmetic and mathematics, considered only discrete, observable, definable behaviours. The objectives of most linear programmed materials have therefore been concerned with the operations of mathematics; calculations and drills. Problem-solving was largely neglected, apart from the solution of "stock problem types" - in other words, the programmes taught a specific solution algorithm for a specific type of familiar problem.

There were, of course, exceptions, notably the "Productive Thinking" programmes developed by Covington and Crutchfield at Berkley (1965) which successfully taught certain strategies of creative thinking, and several attempts to write "conversational" problem-solving programmed courses, adopting one of the many varieties of "branching" programmed instruction, or specially developed programming techniques (e.g. Structural Communication). Indeed,

there is no reason why programmed materials cannot be used to teach certain types of problem-solving, but they must generally, in so doing, depart from the rigid Skinnerian model of programme writing.

In direct contrast, for Bruner the emphasis is upon the kinds of processes learned by the student (rather than the specific subject-matter "products" he may acquire). One paragraph from "Toward a Theory of Instruction" communicates the essence of educational objectives for Bruner.

"Finally a theory of instruction seeks to take account of the fact that a curriculum reflects not only the nature of knowledge itself (the specific capabilities) but also the nature of the knower and of the knowledge-getting process. It is the enterprise par excellence where the line between the subject matter and the method grows necessarily indistinct. A body of knowledge, enshrined in a university faculty and embodied in a series of authoritative volumes, is the result of much prior intellectual activity. To instruct someone in these disciplines is not a matter of getting him to commit results to mind. Rather, it is to teach him to participate in the process that makes possible the establishment of knowledge. We teach a subject not to produce little living libraries on that subject, but rather to get a student to think mathematically for himself, to consider matters as a historian does, to take part in the process of knowledge-getting. Knowing is a process, not a product." (Bruner 1966)

Gagné has come out in substantial agreement with Bruner on the priority of processes over products as the objectives of instruction. His emphasis, however, is not on teaching general strategies or heuristics of discovery; he is much more concerned with the teaching of the rules or intellectual skills that are relevant to particular instructional domains.

"Obviously, strategies are important for problem-solving, regardless of the content of the problem. The suggestion from some writings is that they are of overriding importance as a goal of education. After all, should not formal instruction in the school have the aim of teaching the student "how to think"? If strategies were deliberately taught, would not this produce people who could then bring to bear superior problem-solving capabilities to any new situation? Although no one would disagree with the aims expressed, it is exceedingly doubtful that they can be brought about solely by teaching students "strategies" or "styles" of thinking. Even if these can be taught (and it is likely that they can), they do not provide the individual with the basic firmament of thought, which is a set of externally-orientated intellectual skills. Strategies, after all, are rules which govern the individual's approach to listening, reading, storing information, retrieving information, or solving problems. If it is a mathematical problem the individual is engaged in solving, he may have acquired a strategy of applying relevant subordinate rules in a certain order - but he must also have

available the mathematical rules themselves.

If it is a problem in genetic inheritance, he may have learned a way of guessing at probabilities before actually working them out - but he must also bring to bear the substantive rules pertaining to dominant and recessive characteristics. Knowing strategies, then, is not all that is required for thinking; it is not even a substantial part of what is needed. To be an effective problem-solver, the individual must somehow have acquired masses of organized intellectual skills." (Gagné and Briggs, 1974),

For Gagné, the objectives of instruction are intellectual skills or capabilities that can be specified in operational terms, can be task-analysed, and then can be taught. Gagné would subscribe to the position that psychology has been successful in suggesting ways of teaching only when objectives have been made operationally clear. When objectives are not clearly stated, the psychologist can be of little assistance. Objectives clearly stated in behavioural terms are the cornerstones of Gagné's position.

Ausubel strongly rejects the notion that any kind of process, be it strategy or skill, should hold priority among the objectives of education. He remains a militant advocate of the importance of mastering well-organized bodies of subject-matter knowledge as the most important goal of education.

"... As far as the formal education of the individual is concerned, the educational agency largely transmits ready-made concepts, classifications, and propositions. In any case, discovery methods of teaching hardly constitute an efficient primary means of transmitting the content of an academic discipline.

It may be argued with much justification, of course, that the school is also concerned with developing the student's ability to use acquired knowledge in solving particular problems, that is, with his ability to think systematically, independently, and critically in various fields of inquiry. But this function of the school, although constituting a legitimate objective of education in its own right, is less central than its related transmission-of-knowledge function in terms of the amount of time that can be reasonably allotted to it, in terms of the objectives of education in a democratic society, and in terms of what can be reasonably expected from most students ...," (Ausubel, 1968)

We may thus observe that, while Gagne and Ausubel tend to agree that exposition is a more generally useful form of instruction than discovery, they disagree regarding the appropriate objectives of instruction.

Ausubel is indeed closer to Skinner in putting the products of learning mathematics before the processes involved, although he defines them in terms of subject content (inputs), whilst Skinner would refer to specific operations (outputs). Gagne and Bruner agree that processes

are more important than products, but differ again in the way they define and measure the processes.

The position of Landa with respect to the objectives of instruction is stated in characteristically cybernetic terminology, but embraces all the viewpoints discussed so far, managing to mould them into a coherent whole.

Concerning objectives, Landa (1976) states:

"One of the major shortcomings of instructional programmes is the fact that their objectives are often formulated in an extremely general and indefinite way, with the result that they cannot fulfill their basic function: the direction and regulation of instructional activity.

As a rule, they amount to a mere enumeration of the material being studied. If the material in question consists of skills which are to be developed during the instructional process, then these skills are usually merely named, without reference to their composition or structure.

Thus, for example, in one handbook we read: "the student must be taught alertness, intuition, the ability to orient himself to the material, and the ability to grasp connections between facts". (Nemytov, 1947)

This statement could be paraphrased from the writings of Skinner or other behaviourists. However, Landa goes on:

"These are, of course, legitimate objectives. But is it really possible to develop these qualities, not knowing their make-up, the "elements" of which they are composed, or what happens inside a person's head when he displays "intuition", "alertness", or rapid orientation? It is precisely these questions which are not answered, as a rule, in textbooks and manuals. The teacher is asked to shape processes about which he or she is given no information and the substance of which is unknown to him (her). It is clear that this sort of description of educational objectives gives the teacher very little to work with and cannot significantly influence instructional activity. "

Obviously unlike Skinner or Mager (1962) Landa is not concerned with merely re-phrasing "alertness" in the form of observable measurable behaviours which can be used as "indicators of alertness (Mager 1972). He is concerned with the internal processes implied by "alertness" in "what happens inside a person's head". Thus Landa now appears to be taking a cognitive viewpoint, as might Bruner. But there is a sharp difference in how Bruner and Landa would set about defining what goes on in a person's head.

Bruner's approach, based on Piaget's work, uses the idea of conceptual schemata. The learner learns new concepts by assimilating them into the schema (or structure) of existing previously learnt concepts. If this proves difficult he may accommodate (or re-structure) his existing

schema in order to assimilate the new concept. How this happens in specific instances is not specified in detail. It is implied that one learner's schema may be quite different from another's. Thus Bruner's schema is not quite the same as the "logical structure" of a topic referred to by Ausubel. But Landa combines these two concepts:-

"How are we to specify the structure of the processes which are to be given to the teacher as instructional objectives? How can we establish the components which constitute the process which is to be taught to the student, and the interrelationships between these components? It can be done by the same method used in other sciences, and especially cybernetics: the construction of models.

Let us assume that a particular skill, for example, the ability to prove that a given object is a member of a given class of objects is to be taught.

In order to properly map out a programme of instruction, it is first necessary to determine the components of this skill; and to do this, it is necessary to analyze the "proving process" into elementary operations and to determine their structure. On the basis of formal considerations, observations, and, when necessary, experiments, an hypothesis is formed as to what proving membership of an object in a particular class means i.e., the sequence of operations which must be

followed in order to carry out the proof is determined. The discovery of these operations and their structure constitutes the construction of a model of the process in question, "

A correct model of a thought process should appear precisely as that programme of thinking activity which the learner should assimilate, which must be made the basis of the instructional programme and which is to be furnished to the teacher as a precisely defined objective.

But designing and teaching students the programmes of learning and thinking activity is only one of the tasks facing educational processes. Instruction cannot and should not be equated solely to teaching students programmes of activities given from outside. One of the most important tasks is to teach students to discover and design programmes of efficient activity independently, i.e., to develop in them the ability of self-programming.

To design a programme independently, however, it is necessary to know how to do this, i.e. to possess a programme for designing a programme, or the programme of the second order. There may and must exist programmes also of still higher orders which represent, each, programmes of higher and higher self-organization and self-control. "

The author's viewpoint concerning the objectives of mathematics education are very much in line with Landa's

Even a cursory analysis of the realities of mathematics in school and society, shows that one cannot really divorce the learning inputs (the subject and its topics), the products (specific skills) and the processes (learning to learn, to think, to solve problems, etc.). Of course one wishes to teach for transfer, but one also ought to teach certain basic skills useful in life and certain defined content which is job-related or life-related. Hence, extreme positions with regard to the objectives of mathematics instruction, such as adopted by Ausubel (content or "input"), Bruner (thinking or "process") and Skinner (skills or "products") have not on the whole been helpful. Although the respective authors may have only been stressing one of these components in order to "redress the balance" as it were between the direction of mathematics teaching as they saw it, and the direction that they felt it should take, their well-publicised viewpoints have resulted in the formation of "ideological camps" among educators. Hence, the popular success of the discovery movement in mathematics teaching (largely due to Bruner, Dienes and the Piagetian school), in seeking to redress the balance in a curriculum too much oriented towards rote drill and practice of specific techniques, has swung the scales too far in the opposite direction. Higher order cognitive skills are taught (more or less efficiently) to the near exclusion of basic applicable skills. Teachers at the higher level complain that the students reach them

with inadequate mathematical training and often reverse the process, reverting to a highly expository, highly "drill-and-practice" approach in order to "make up for lost ground".

In the author's opinion such extreme approaches are generally harmful and could be avoided by a "total systems" viewpoint in the planning of curricula and courses. Insofar as objectives are concerned, it would be useful to consider the given course as a system. In order to define the system fully one should define as completely as possible the system inputs, products and internal processes. Among the inputs is the essential mathematical content, which is derived from an expert's analysis of what is necessary and relevant for the given student group in the light of their aspirations, jobs or positions in society. Among the outputs are included the skills that the students will gain. Two types of skills must be considered; the directly measurable job-related or life-related skills that Gagné refers to - being able to multiply, transpose equations, solve "stock" problems, etc. - and the generalisable skills that Bruner emphasises - being able to see the problem, "go beyond the information given", formulate a mathematical model of a real life problem, etc. Both types of skill should be given equal emphasis. Both should be actively evaluated. Traditionally, much testing of directly applicable skills has been done; much less attention has been given to the "transfer" skills.

Finally, one should be defining the processes of learning in much more detail than has been customary. One should be matching learning processes to learning outcomes in a much more systematic manner. Most especially, one should be

learning from one's efforts by planned, systematic use of feedback from the evaluation stages. In order to be systematic in this way one needs models (mathematical models?) of the teaching learning process, capable of being continually put to the test.

This is the essence of Landa's theoretical approach. It is also behind Gagne's intuitive psychological approach, but Landa's suggestions are much more rigorous and precise. In the author's opinion, it is exactly this type of systematic rigour which is required.

Mathematics teaching has been for too long entrusted to loosely-knit groups of individuals. Such an approach has shown itself ineffective. Lessons of successes and failures are not disseminated. Poor quality teaching of the "new mathematics" has been allowed to go on for so long that a crisis situation has developed, which threatens to discredit (and possibly abolish) some modern mathematics curricula, although the fault lied not in "what you do but the way that you do it". On the other hand, succesful techniques developed by some teachers, spread very slowly, if at all. How is it that Polya's excellent book, "How To Solve It", lay fallow from 1945 to 1971 in two small-run hardback printings, and has only really "caught on" since being adopted by the Open University (it now sells in paperback more copies per annum than it sold in its first quarter century of existence).

The author feels strongly that this sort of situation could be eliminated just as a similar amateurism was eliminated from the NASA space projects when confronted with the reality of Sputnik - and by the same methods.

These methods are systems thinking applied to the planning process, systematic control techniques (e.g. PERT) the use of modern aids (the computer) for the documentation and evaluation of feedback on a previously untried scale, and full teamwork between all personnel concerned, controlled by common objectives (Management by objectives).

Is there any reason why any of these techniques could not be applied to the teaching-learning process - apart from human nature and resistance to change?

The first step in system design is to define the desired system outputs - the objectives. In mathematics this is easier than in many other subject areas. At least most mathematical objectives are directly measurable, including such at first glance "woolly" objectives as "seeing the problem in mathematical terms". The only real difficulty is to obtain agreement among mathematicians as to which objectives are worth pursuing. There are often arguments for and against the inclusion of some specific mathematical topic in a given curriculum. However, if these arguments are not easily resolvable it is because either

- (a) it is a fringe topic, not essential to the main purpose of the course, though perhaps desirable (if we have the time) or
- (b) the mathematicians concerned have different views concerning the main purpose of the course. In that case, one should resolve the objectives at that level first.

3.3.2 METHODS OF MATHEMATICS INSTRUCTION

In the discussion so far, we have already touched several times on the general instructional strategies favoured by the various psychologists.

We have noticed a polarisation between the discovery-learning camp and the supporters of reception learning. The extreme supporter of the reception learning mode is Ausubel, but he is not on his own. Hess (1968) considered that not all students benefitted from discovery methods and that not all teachers were skilled at employing them. Newton (1968) notes that "dishonesty of inquiry teaching" stating that much of what passes for "discovery" is in fact "guidance" and the rest is "anarchy". The research is not very conclusive. Even proponents of the discovery approach such as Joseph Scandura have on occasions encountered unfavourable evidence, for example, that failure on a discovery task may inhibit later learning (Scandura 1976). Certainly, expository teaching is still extremely common, particularly at the higher levels of mathematics education. The discovery approach as a "blanket" strategy is more accepted during the early school years.

One problem, pointed out by all, is that teaching through discovery is more time-consuming than teaching through exposition. Thus the discovery approach can best be defended when some benefits will accrue from the extra time spent. These benefits, according to Gagné, are better long-term recall (in the absence of practice) and better transfer of skill to other similar problem types. This would seem to restrict discovery-learning to the higher-order types of learning activity - learning principles or rules and learning how to solve problems.

We have noted also that whereas Ausubel and Bruner represent extreme viewpoints for or against discovery methods, most other writers adopt intermediary positions, accepting that both strategies may be used to good advantage and suggesting criteria for selection between them. We have also noted that discovery-learning as defined by Bruner is quite different from the guided discovery approach of Gagné and the Landa's approach is different again.

A useful classification of discovery methods was suggested by Biggs (1972)

1. Impromptu discovery
2. Free exploratory discovery
3. Guided discovery
4. Directed discovery
5. Programmed learning.

It is interesting that she should include programmed learning in a list of "discovery" methods. Many writers would classify programmed learning as the opposite extreme - the fully expositive technique for receptive learning. Belbin (1964) for example has even performed research comparing "programmed learning and discovery learning" for various tasks.

As we shall shortly be considering techniques for the individualisation of instruction and as programmed learning shall be one of these, it will be useful to consider briefly this apparent confusion of terms. It is true that most linear programmed learning materials have been based on the RULEG (Rule — Example) model for programme writing (Evans, Homme and Glaser, 1962). This model is a classically expository

approach to instruction. However, discovery-type programmes have been produced, generally employing some style of branching. Even linear programmes can follow a rather restricted form of discovery model - the Eg-RUL model, in which the learner has to induce the rule from given examples.

Landa (1976) speaks of:

- linear programmes - both RUL-EG and EG-RUL
- intrinsic programmes - such as Crowder's (1960) branching text
- extrinsic programmes - which react not to one response of the learner but to his response pattern over a period of time. (Project PLAN is an example which is discussed later.)
- adaptive programmes - which learn from the student's response pattern and adapt the programme of instruction "on line". (An example was the SAKI keyboard instructor of the 1960's.)
- structure-diagnostic programmes - which reacts not only to the responses the student makes (whether they are right or wrong and why) but also diagnoses the underlying psychological reasons for each mistake.

Landa describes a course in Russian grammar which has the above structure-diagnostic capabilities. Whenever a student makes a mistake, the programme enters into a diagnostic procedure to "discover the psychological reasons for the mistake" and may therefore offer different remedial actions to different students making the same mistake.

This programme can be presented as a text, but is somewhat unwieldy and is better presented by computers.

If we take programmed instruction in this very general sense, to include all such variations, then indeed it is obvious that we may have programmed discovery learning. Indeed, Landa's position, as outlined earlier, is that for successful instruction, even in mathematical problem solving, the instructional process must be programmed.

Modifying Biggs' (1972) classification somewhat we might equate the viewpoints of our theoreticians as follows:-

Impromptu discovery	Unplanned learning. Occurs in every instructional situation. Sometimes useful sometimes not.
Free exploratory discovery	Bruner's position. Broad learning objectives are fixed; otherwise the learner is free to choose sub-goals, methods, etc.
Guided Discovery	Gagne's model. Objectives for each learning step are fixed. The learner is free to explore methods, but with guidance and help at every stage.
Adaptive/diagnostic programmed discovery	Cybernetic approaches such as "Dialogue CAI", Pask's conversational programming, and Landa's structure-diagnostic programmes.
Linear/Intrinsic programmed discovery	Rigidly directed. (EG-RUL) (INTRINSIC PROGRAMMES)

To complete the picture, we may attempt a similar classification of reception learning.

Meaningful reception learning (a) Inductive reasoning (From particular to general)	This is really EG-RUL, but the learner receives the argument; he does not have to discover the rule. This is the way that most mathematical discovery and problem solving takes place. It would seem to the author reasonable to use this approach, even when "talking-through" a problem solution in an expository manner. Can be programmed.
Meaningful reception learning (b) Deductive reasoning (as favoured by Ausubel)	As understanding of concepts is shown by the ability to apply them to examples, The RUL — EG model (student receives rule and demonstrates understanding by application to suitable range of examples) is an appropriate model here. Can be programmed.
Rote reception learning (Drill-and-Practice)	Learning of facts, statements and operations without understanding the concepts involved. Memorisation.
Impromptu reception learning	Facts and observations, originally unplanned, supplied by the teacher, other resources or other students.

The two tables, taken together may be considered as one continuum of sorts. Indeed the division between programmed discovery and inductive reception learning is very blurred. A teacher, talking through a piece of inductive reasoning is posing and answering questions, groping for rules or theorems which may help in the solution. As soon as he commences to throw those questions open to the students, he is in the rigidly directed discovery mode. Indeed a very

common, if not particularly good, classroom tactic is to pose a question; if no answer is forthcoming to re-phrase or prompt the answer; and if still no response from the class, to answer it oneself, explain why that's the answer and finally get the class to recite it in chorus. Here we have "descended" in our hierarchy of modes, from directed discovery, through inductive reception learning to rote drills (a measure of the teacher's failure, perhaps).

To use programmed learning as an example:-

A sequence asking the student to induce a rule from a series of examples, is "discovery", unless the frames are so prompted that it is obvious what the required answer shall be, or, the student is given a set of multiple-choice answers where the incorrect alternatives are so obviously incorrect that he cannot help but choose the right one, whether he fully understands the reasons for his choice or not. This is a very common occurrence in programmed texts which use a lot of prompted frames. In effect the student's activity is little different from reading an expository text, written in an inductive style.

A (real life) example:

Look at the table of "coefficients of expansion of some metals " on page 6. Notice that the coefficient of expansion is positive in all cases.

We may say therefore that in general metals _____ (expand/contract) when heated.

The sequence goes from example to rule. It is inductive. The student selects the answer "expand". Can we really say he has "discovered" the rule?

Contrast this with

Have a look at the table on page 6.
See if you can state a general rule about what happens to metals when they are heated.

This one is already pretty strongly directed "discovery".

3.3.3 THE INDIVIDUALISATION OF MATHEMATICS INSTRUCTION

The various theoretical viewpoints vary also in their positions regarding the individualisation of the instructional process.

3.3.3.1 The reception-learning viewpoint

David Ausubel, in agreement on this point with Robert Gagné, sees the key to efficient instruction in careful sequencing and in ensuring that all necessary pre-requisite learning had been satisfactorily completed. "The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly". (Ausubel, 1968). The difference between Ausubel and Gagné on this point is restricted to how "what the learner already knows" is defined - Gagné would construct a learning hierarchy in terms of his eight

categories of learning. Ausubel would define the content - the knowledge structure that is to be, or has been, learnt. This difference makes little difference from the point of view of individualisation. As it is to be expected that different students will have different levels of mastery of the pre-requisites, one should plan differentiated starting points and individualised remedial sequences. As in general the students will progress at a constant pace through the material to be learnt, the problem of differentiation is a constant one, occurring at the "lesson" level and even the "learning step" level (reference to the four levels of individualisation defined in Chapter 2).

How to achieve this differentiation is another matter. In a large-class situation, the expedient of breaking the large group into small groups still leaves a problem for the proponent of the expository approach. One teacher may only give one exposition at one time. This is not a problem to the proponent of discovery learning, as he can set a common "project" and then monitor and guide individual progress. But for the expository approach, individualisation implies, of necessity, the "packaging" of the expository presentation in some reproducible medium suitable for individual or small-group use.

Thus individualisation implies "modular packages" of some form - printed, programmed, audiovisual, etc., in order to make the information to be transmitted available to each receiver (student) when he needs it and for as long as he needs it.

Concerning the objectives and content of a course, both Ausubel and Gagné would restrict the student's free choice, Ausubel probably more severely than Gagné.

Whether there would be any free choice at "course" level is a matter of educational philosophy and politics rather than psychology. Clearly it is common at the higher levels of education to select certain course options and to omit others. Whether this is a desirable characteristic of, say, elementary education is a moot point. Certainly to place total responsibility for such decisions on the student (as for example at Summerhill - Neil 1960) is not supported by many educators as a procedure for the elementary level. It is quite another matter to allow a limited student choice at the "course unit" level. Particularly in Mathematics this will be limited by the structure of the subject itself. Exactly because mathematics is a highly structured, sequential subject, it is more difficult to let the student take the initiative in deciding which topics of a course he would study. However, a "common core" curriculum of essential integrated topics, together with a large number of extra electives to choose from is a feasible and currently oft-practised course structure at all levels of schooling and in higher education. Now let us consider the lower levels of analysis. A student has elected to study a particular course unit. It is highly probable that the content of this course unit will be highly structured, that, in order to achieve the final objectives of the unit the student must master all the sub-objectives to a satisfactory standard and (as these are inter-related hierarchically) in a particular pre-

determined sequence (there may sometimes be alternative equivalent pathways through the material, but not total freedom). Thus the viewpoints which see learning as "climbing up a hierarchy of inter-dependent learning tasks" do not generally allow for many of the characteristics of a course (as listed in Chapter 2) to be individualised

- (1) Learning pace; certainly may be individualised.
- (2) Individual or small-group learning materials; usually.
- (3) Alternative sets of materials employing different media, different methods or different levels of difficulty in order to achieve the same objectives; occasionally (this is limited more by economic and practical constraints than by theoretical viewpoint)
- (4) Alternative sequences for the study of the lessons or units; rarely (only when the structure of the course allows this logically)
- (5) Varied objectives or standards of assessment, adjusted to the individual needs of the learner; hardly ever (and then only at the discretion of the teacher, not the student):

The above statements are of course generalisations. They describe adequately the typical expository, reception-learning viewpoint on individualisation. Most published programmed instruction, most multi-media systems of instruction, the Keller Plan, IPI, and many other well known systems of individualised instruction fall more or less into this category (particularly individualised mathematics schemes). Exceptions are the mathematics laboratory,

project PLAN (to a certain extent), the Swedish IMU (to a certain extent) and several of the British schemes (e.g. the Fife Mathematics project).

3.3.3.2 The "discovery learning" viewpoint

These latter schemes are based (totally or partially) on the theoretical viewpoints of Piaget, Bruner, Dienes and the cognitive school.

As mentioned earlier, Gagné seems to be undergoing some sort of a conversion. In his earlier works (e.g. Gagné 1968) he very usefully constructed a bridge between the behaviourist and cognitive camps, using the best aspects of both positions to form his theory of instruction. Now in his most recent book (Gagné 1974) he seems to have crossed into the humanist camp as well. This is what he says on individualisation:

"Experience with programmed instruction and other modes of delivery suggests that individualized instruction is often not only more effective and more efficient than group instruction, but is also more responsive to the needs of the learner. It therefore may also be characterized as more human than group methods, 'because it: (1) allows realistic goals to be set for each learner; (2) provides various materials or resources for a given goal, thus adapting to individual competencies and backgrounds; (3) provides privacy when difficulty is encountered; (4) permits the learner to work at his own rate; and (5) provides consistent individual feedback rather than hit-or-miss or inappropriate feedback. "

It is the first of these four reasons which appears to be at odds with Gagné's previous position. If learning (particular of mathematics) is such a hierarchical procedure as Gagné outlined earlier, and if the overall objectives of teaching a given mathematics course are fixed more by practical and political factors, how do we set "realistic goals" for each learner?

This is not a problem for the cognitive school who would no doubt accept all four of Gagné's reasons as the tenets of their faith in individualisation, (and in particular the first). This might be re-phrased in terms of the individual student's "readiness" (both from a developmental and a "prior-skills" point of view) and also his willingness (from a motivational point of view) to participate in a given learning activity. The Piagetian concept of readiness is much to the fore in the cognitive position vis-a-vis individualisation.

Thus the cognitive position can be summarised as:

- (1) Learning pace; certainly may be individualised
- (2) Individual or small group learning materials; usually (with more accent on group project work than in the reception-learning camp).
- (3) Alternative sets of materials for the same objective; usually. (the learner may exert his preferences here much more than in the expository approach, where the alternatives are generally prescribed as a result of a diagnostic process).
- (4) Alternative sequences for the study of the lessons or the units; usually whenever it is not completely ruled out by the logic of the topic).

(5) Varied objectives or standards of assessment, adjusted to the individual needs of the learner; usually (the aim is individual development, not mastery of specific content)

The potential problem that arises for the cognitive position in the case of mathematics is as follows: As mathematics is a highly structured interdependent body of knowledge, even if the overall objectives of a given course are not to be exactly the same for each individual, how can we ensure that the cognitive schema formed by a given individual is coherent if he has total autonomy over course content, sequence, standards etc.?

Well, part of the answer is that in practice he does not. The teacher's role is to influence the student's choice, so that the goals he sets himself are "realistic". The other part of the answer lies in the commonly-used cyclical instructional model.

Returning at this point to the question concerning Gagné's position on "realistic goals". Gagné is not very clear on this, but perhaps he does not mean "realistic" in the sense of individual student preferences, but simply in terms of the time the student will spend to reach goal X or, alternatively, how far towards goal X will he progress, by next Monday. This is a slightly different use of the term "realistic goal" than that used by the cognitive school, and quite compatible with Gagne's earlier position.

This connotation does not suggest variable objectives but rather variable short-term targets in order to eventually achieve the pre-set objectives. This concept of realistic goals lies at the back of the "performance contracting" model of

course management, and this model, in its most refined form has become known as the "Mastery-Learning Model". We shall conclude this comparison of theoretical viewpoints by contrasting this model with the other popular models of today, the "Cyclic Learning Model" of the cognitive school.

3.3.3.3 Mastery Learning

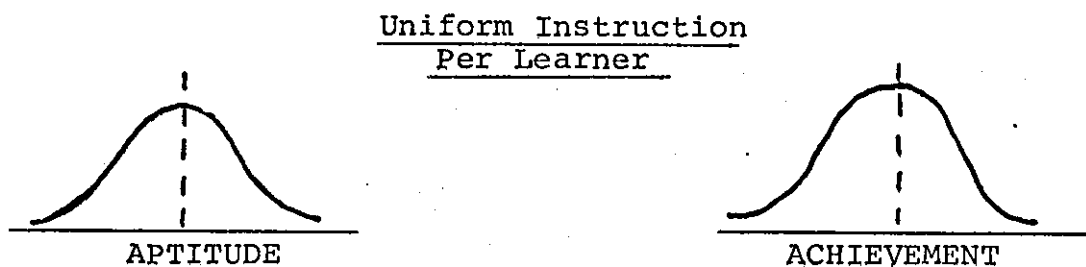
This model has been suggested and developed principally by Bloom (1968) and Carroll (1963). Its major point of difference from traditional learning models is that it does not accept differentiated achievement among students as a necessary consequence of different "aptitudes".

Carroll defined aptitudes as measuring the amount of time required to learn a task to a given criterion level under ideal instructional conditions. In its simplest form, his model proposed that if each student was allowed the time he needed to learn to some level and he spent the required learning time, then he could be expected to attain the level. However, if the student was not allowed enough time, then the degree to which he could be expected to learn was a function of the ratio of the time actually spent in learning to the time needed:

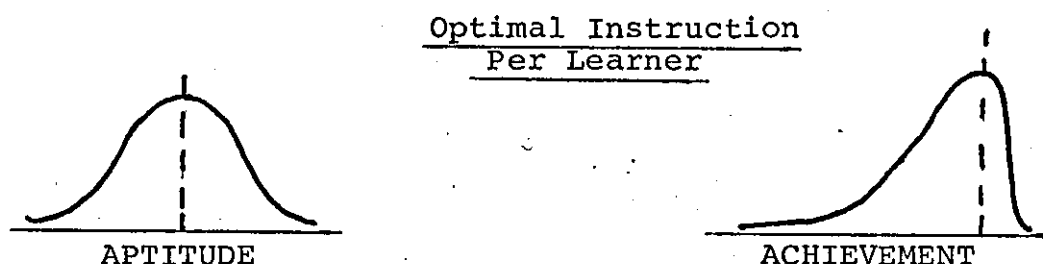
$$\text{Degree of Learning} = f \left(\frac{\text{time actually spent}}{\text{time needed}} \right)$$

Bloom transformed this idea into a practical set of procedures for mastery learning.

Bloom argued that if students were normally distributed with respect to aptitude for a subject and if they were provided uniform instruction in terms of quality and learning time, then achievement at the subject's completion would be normally distributed. Further the relationship between aptitude and achievement would be high. This situation can be represented as follows:



However, if students were normally distributed on aptitude but each learner received optimal quality of instruction and the learning time he required, then a majority of students could be expected to attain mastery. There would be little or no relationship between aptitude and achievement. This situation can be represented as follows:



The mastery learning strategy Bloom proposed to implement these ideas was designed for use in the classroom where the time allowed for learning is relatively fixed. Mastery was defined in terms of a specific set of major

objectives (content and cognitive behaviours) the student was expected to exhibit by a subject's completion. The subject was then broken into a number of smaller learning units (e.g. two weeks' instruction) and the unit objectives were defined whose mastery was essential for mastery of the major objectives. The instructor taught each unit using typical group-based methods, but supplemented this instruction with simple feed-back correction procedures to ensure that each student's unit instruction was of optimal quality. The feedback devices were brief, diagnostic (formative) tests administered at the units' completion. Each test covered all of a particular unit's objectives and thus indicated what each student had or had not learned from the unit's group-based instruction. Supplementary instructional correctives were then applied to help the student overcome his unit learning problems before the group instruction continued. (Block 1971)

The following correctives were used: small-group study sessions, individualized tutoring, alternative learning materials (additional textbooks, workbooks, programmed instruction, audio-visual methods, and academic games), and reteaching. The small-group sessions and the individualised tutoring, for example, added an important personal-social component to each student's learning not typically found in large-group instruction. The workbooks and programmed instruction provided the student with the drill he may have required.

Thus, as originally propounded by Bloom, and applied by various researchers, the Mastery Model was a method of implementing individualisation within the large-group school or college classroom. Since then however, the basic concept has been adopted by various other persons to fully individualised learning schemes. A system of instructional design termed "competency-based education" has become popular in the U.S.A. It is defined as:-

COMPETENCY BASED EDUCATION	=	MASTERY LEARNING	+	MODULAR INDIVIDUALISED INSTRUCTION
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The Keller-Plan (or Personalised System of Instruction) (Keller 1968) consciously embraces the principle of mastery learning. Some of these plans and systems are discussed in detail in later chapters.

The research so far on the application of the mastery-learning model has been very encouraging. A book of papers entitled *Mastery Learning* (Block 1971) lists much of the research including the following studies in the teaching of mathematics:

Collins (1969) applied the mastery model to teaching freshmen mathematics at Purdue University.

The research involved two modern algebra courses for liberal arts majors (n = 50 approximately) and two calculus courses for engineering and science majors (n = 40 approximately).

In the modern algebra classes, 75% of the mastery compared to only 30% of the non-mastery students achieved the mastery criterion of an A or B grade. The calculus classes' results were similar: 65% of the mastery compared to 40% of the non-mastery students achieved the criterion.

In both the modern algebra and the calculus courses, D and F grades were for all practical purposes eliminated for mastery students.

In another study (Collins 1970) he reports similarly promising results when teaching modern mathematics in the secondary school level.

Kersh (1970) investigated the use of the strategy in the elementary school, on fifth grade arithmetic.

The results indicated that on the same achievement test and using the same mastery standard, there were significant increases in the proportion of experimental students (mastery class) attaining mastery compared to the proportion of the teacher's students from the previous year (control class) attaining mastery. These increases ranged for one advantaged class from 19% mastery in the 1966 control class to 75% mastery in the same teacher's 1967 mastery learning class. Moreover, a disadvantaged class increased from 0% attaining mastery in 1966 to 20% attaining it in the 1967 mastery learning class. Note, in these examples, that the disadvantaged mastery class performed as well as an advantaged control class. Perhaps the strategy might be helpful in at least partially overcoming the cumulative deficit in learning manifested by socio-economically disadvantaged students.

Just to show once more that there is nothing really new under the sun, Block quotes a research by R.B. Thompson (1941) a full twenty years before the Mastery Model was developed by Bloom. The description of the instructional

process adopted with the experimental group, fits all the important criteria of the mastery model.

The results were also most convincing.

"The results of the studies indicate consistent gains in arithmetic achievement for the experimental groups over various periods of time. In one study, in a ten-week period the experimental group gained 1.41 years in arithmetic achievement as measured by standardized tests, while the control group gained just .40 year.

Thompson concludes that the use of diagnostic examinations and remediation to individualize instruction is one very effective way to teach mathematics. He claims the method was effective because: (1) no pupil wasted time working on topics he had previously mastered; (2) the student did not have to wait for his whole class; and (3) no student left any particular topic until he had thoroughly mastered it. "

3.3.3.4 Cyclic Learning

The mastery learning model insists that all students should follow much the same course, towards the same objectives, receiving remedial instruction on each module or unit, until they can demonstrate total mastery, and only then being allowed to move on to study the next unit. Much of this is opposed to the developmental school of thought

We have earlier posed the question: "how to ensure the formation of a coherent schema" without this sort of pre-planned sequence and tests of mastery. The answer to this is partly "the personal influence of the teacher" (we shall come back to this point) and partly the cyclic learning process which is commonly adopted in the "free exploratory discovery" model of learning.

Bruner in describing the process of mathematics learning identified three stages in the learning of a new mathematical concept - the enactive, iconic and symbolic stages. Optimum learning should pass through these three stages. These stages are identifiable in most of the practical procedures for working in the "mathematics laboratory" mode, notably in the work of Dienes (1960) although he later sub-divided the three stages to give six in all (Dienes 1970). They are also discernible in the theoretical work (and in the structure of his practical textbooks) of Richard Skemp (1971) both at the level of simple concept formation with young children and at the more advanced levels in secondary school when the enactive stage may take the form of "playing" with previously learnt concepts and rules. Skemp's very useful construct of "intuitive" and "reflective" intelligence enables one to visualise the "playing" with previously learnt abstract ideas in the same terms as the play of the young child learning to discriminate shapes. Another proponent of the cyclic process at the reflective, abstract level is Polya (1963) who describes an exploratory phase, followed by a formalizing phase and leading finally to the

assimilation of new ideas.

Servais and Varga (1971) paraphrase the description of Dienes (1960) and Polya (1963) in order to emphasise the similarities:-

Dienes	Polya
The preliminary or play stage corresponds to rather undirected, seemingly purposeless activity usually described as play. In order to make play possible, freedom to experiment is necessary.	A first, exploratory phase which is close to action and perception and moves on an intuitive, heuristic level.
The second stage is more directed and purposeful. At this stage a certain degree of structured activity is desirable.	A second, formalizing phase ascends to a more conceptual level, introducing terminology, definitions and proofs.
The next stage really has two aspects: one is having a look at what has been done and seeing how it is really put together (logical analysis); the other is making use of what we have done (practice). In either case this stage completes the cycle, the concept is now safely anchored with the rest of experience and can be used as a new toy with which to play new games.	The phase of assimilation comes last: there should be an attempt to perceive the 'inner grounds' of things; the material learnt should be mentally digested, absorbed into the system of knowledge, into the whole mental outlook of the learner. This phase paves the way to applications on one hand, to higher generalizations on the other.

That Polya and Dienes should be in such close agreement is interesting, as both have extensive experience of mathematics teaching, but of different kinds. Dienes has worked mainly with young children on concept formation, Polya has worked mainly at secondary school and university levels, on problem-solving. The background of Dienes is strongly influenced by Piagetian psychology. Polya based his original studies of problem solving on mathematical premises, particularly Work of Descartes and the studies of heuristic method in the 19th century.

However, the value of the cyclic nature of the process of mathematics learning suggested by such different authors, lies in the fact that a given student will return to previously learnt concepts with regularity and each time he returns he will have the opportunity to extend his knowledge. Thus, it can be argued, there is no need to have a rigid system of pre-determined modules of study, each with its own behaviourally stated objectives, which all learners must master in order to gain the "right" to proceed. We have instead the concept of a cyclic process, passing through the enactive or 'play' stage (when certain regularities and rules are discovered) to the iconic or "representational" stage (when the rules and relationships are represented in some, perhaps graphical or diagrammatical form) and finally to the symbolic stage (at which a mathematical language is used - perhaps even invented by the learners - to describe the relationships which have been discovered). It is natural, that at this stage the learner would wish to "play" (in a physical or reflective

sense) with something where his new knowledge can be practised, so he will naturally tend to select new activities related with the previous ones. If, from the point of view of his general education in mathematics, he has not gone as deeply as he should into a particular concept area, this will not matter in the long run, as further activities and further learning cycles will eventually throw up the need for further study in depth. The learner will return to further study of previously learnt topics as he needs to in order to progress along his chosen path.

These two models for the individualisation of mathematics instruction spring from two opposed viewpoints, concerning the goals and methods of the instructional process. Both are applied in practice. Both work - better for some than for others. Both have their critics. In the next chapter, we shall examine several practical models for the individualisation of mathematics and shall observe that in practice not all models are totally mastery or totally free-discovery "cyclic". There are many "hybrid" models and some of these are among the most successful.

3.3.3.5 The cybernetic viewpoint

Cybernetics, as the "study of regulation and control in complex systems both living and man-made" is concerned with the discovery of general rules which govern the functioning of any system.

In the field of learning, for example, one area of interest is to simulate the human learning and thinking process by machines. In educational cybernetics in particular, the concentration of the research is to discover mathematically-expressable relationships between instructional processes and learning effects.

Not surprisingly, therefore, the concentration of effort by cybernetically-oriented researchers has been in automated instructional systems - teaching machines and in particular computer assisted instruction. The attraction of computers is three-fold. Firstly one can programme a computer to execute a particular instructional strategy faithfully. One can simulate (more or less perfectly) certain learner-tutor interactions and study them in much greater detail than is possible in the real-life situation. Secondly, the data collection, storage and analysis capabilities of the computer make it an ideal base for research. Thirdly, many cyberneticians would assert that the complexity of the teaching-learning process is such that only with the help of the data-processing capabilities of a computer can we hope to improve the teaching-learning process from its present primitive "neolithic" state of development. Only by matching the variety of response of the tutor to the variety of response possible from the learners and all other influencing factors, can one ensure effective control of the educational process. "Control" in the cybernetic sense of course, as Landa (1976) points out:

"The word "control" as applied to education has for many an odious connotation. It is often thought that to control, for example, the development of the student's personality or thinking means to exercise a dictatorial influence over him and to regulate all of his behavior and cognitive activity.

It is, of course, possible to exert control of this sort. But the concept of control in its current scientific sense does not at all correspond to such a rigid approach. Control is understood in modern science as any influence exerted by one system on another (e.g. by one machine on another, by a human being on a machine, a human being on another human being, etc.) in order to achieve a specific goal.

All forms of education or instruction presuppose a specific goal: to transmit to the student a particular body of knowledge, or to develop in him specific skills, habits, abilities, motives, character traits, etc.

The means for attaining the goal (the means of instructional control) may be extremely varied--ranging from dictatorial to the most liberal forms, depending on the goals of the educational or child rearing process, the educational philosophical conceptions of the teacher or parent, and his or her educational abilities.

The most important objective of control in the area of education and upbringing, i.e., of educational control, is the development in the student of the capacity for self-control, i.e. the capacity for independent regulation of his own mental processes and behaviour.

The capacity for self-control and self-regulation is not, however, an inborn characteristic; its development too must be controlled; so that it, too, must be taught. It is evident, moreover, that this is more difficult to teach than to control anything else, especially where the goal is to develop the processes of self-regulation which underly creative activity. The problem of developing the qualities of independence, of creativity, and of freedom (in the psychological sense) not only does not obviate the necessity for control in the course of the educational and upbringing process, but indeed requires control of a particularly skillful and complete sort.

A characteristic feature of contemporary education, in spite of all its achievements, is the fact that it still proceeds as a poorly controlled process. Much necessary knowledge and many necessary skills, abilities, character traits, etc., are not developed in the student, or developed incompletely, slowly, or at the cost of excessive expenditure of time, energy and financial resources. "

Surely this is not the statement of a mechanistically oriented scientist. The humanist and cognitive schools would have difficulty in disagreeing with the sentiments expressed even if they might question the conclusions.

Is the cybernetic approach a reality? To what extent can one analyse and simulate the student-tutor interaction? There is no room to go into great detail on this point, but it will suffice to quote the work of one British cybernetician working in the educational field - Gordon Pask.

In the late 1950's Pask (1960) developed the highly successful SAKI adaptive teaching machine for keyboard skills. The original SAKI machines were used for training punch-card operators. Typist training machines were later developed on the same basic principles. The machine presented data to be typed. The trainee would type a particular character (which was not marked, so that he had to use a touch-typing skill). His error pattern was used to control the programme (Lewis and Pask 1965).

"Each illuminated number poses a problem to which the trainee must respond by pressing, within a short time allowance, the appropriate key. To help him find this key without looking at the keyboard, a further display of "cue information" lights is provided immediately below the exercise card. This display duplicates the spatial layout of the keyboard being used, and a light appears in the appropriate position to tell the trainee where the correct key is located. If he presses this key, the problem light and cue

light extinguish, and the next pair are illuminated immediately. During the early stages of learning the figures move rather slowly, at approximately three-second intervals. But as the trainee gains in speed and accuracy, the waiting times become shorter and the cue lights diminish in intensity. This is done differentially, so that the machine continues to wait longer and to give cue information on those keys that persist in giving difficulty. Furthermore, it will slow down and restore cue information for any key on which the trainee suffers a sudden relapse.

Whereas the simpler "branching" machines take corrective action on the basis of just one response, Saki adjusts its program in accordance with an integrated performance measure (based on speed and accuracy) secured over a whole series of responses. The result is a continuous adjustment of difficulty and time pressure which keeps the trainee working always near the limit of his ability. "

The results with this machine were most impressive, learning taking place in a fraction of the normally expected time. Similar machines were built for perceptual coding skills, maintenance skills of electronic equipment, and some simple concept formation skills. These all seem rather low-level learning tasks. Can the same techniques

be applied to, say, mathematics? The answer is that of course they can. For example, the Drill-and-Practice computer-based programmes at Stanford University (Suppes 1968) have certain similarities to the SAKI machine in the logic that is used to differentially branch the learners to material at an appropriate level of difficulty. As it happens, the logic of the SAKI machine is somewhat more complex than what is necessary at Stanford.

However, drill-and-practice is still a fairly low-level learning activity as compared to say the formation of mathematical concepts or problem-solving.

By the 1970's Pask was working on concept formation. His well known "Serialist/Holist" experiments investigated learning styles and strategies in concept formation. Observation of subjects when they were given freedom to choose their mode of study identified two distinct groups - the serialists, who followed the linear progression of the logic of the topic under study, as it was given to them in a "course-map", and the Holists, who did not follow a step by step sequence but tended to form global hypotheses concerning the problem under study, and then test it out by selecting information out-of-order.

The subject matter of the experiments were the taxonomies of two imaginary families of Martian fauna the Clobbits and the Glandlemullers. (Pask and Scott, 1972).

"Having observed the different orders in which serialists and holists worked on the Clobbit exercise, the experimenters could construct linear instructional programmes which matched

these strategies. This they did for a second taxonomy (the Gandlemullers), constructing two linear programmes which each provided a complete description of the taxonomy but differed in the concepts they instructed and their structure.

Since subjects had already been classified as either serialists or holists on the basis of the Clobbit experiment the Gandlemuller programmes could be used in either a matched or a mismatched fashion. Matched groups were serialists given the serialist programme and holists given the holist programme, and the mismatched sample also contained two learner/programme groups, holist/serialist and serialist/holist. In all cases students had to complete each frame of their programme successfully before moving to the next and were required to repeat the entire programme until they achieved an error-free run. A 30-item test designed to determine both factual knowledge and ability to generalize was then administered. The test results were unequivocal. All members of matched groups scored between 28-30 whereas scores for mismatched students fell between 7-21 with an average of 14. Such a difference hardly needs a statistician to assess its significance. "

Armed with information from series of such experiments, Pask and his collaborators have now constructed systems, both mechanical and "paper and pencil" which simulate the tutorial function, which learn about the individual student's learning style, and strategies, which use what they learn to adapt the instructional process to the learning style of the individual and which allow the student to take-over a greater proportion of the decisions concerning the content and structure of his course (Daniel 1966). In short, these experiments, and others like them, are beginning to achieve what the cognitive/humanist camps have been preaching as the aim of education for longer than we recall, but have never managed to implement due to lack of "effective control".

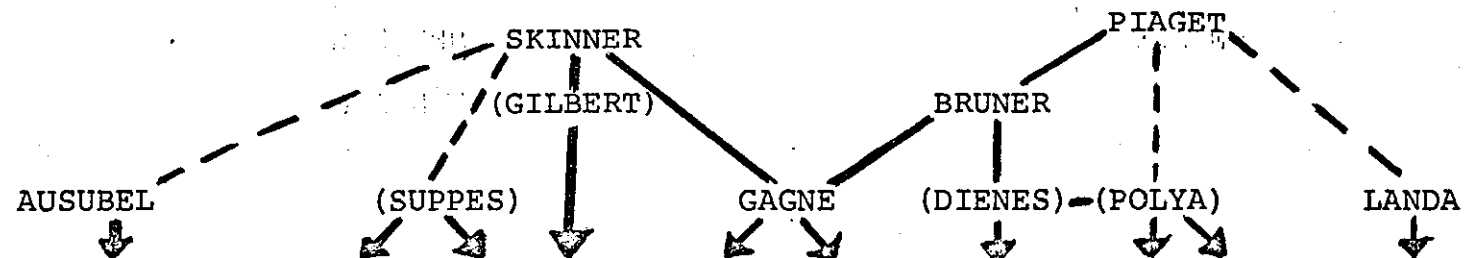
3.3.4 SUMMARY - THE AUTHOR'S ANALYSIS

It may be helpful to summarise, in diagrammatic form, the main similarities and differences between the various psychological viewpoints discussed in this chapter. In the chart shown overleaf the author has attempted to emphasise how some earlier viewpoints have concerned themselves more with one aspect (e.g. inputs) of education and how the total-systems viewpoint is beginning to act as a unifying influence.

At the end of the chart the author has placed some existing individualised systems for mathematics instruction. Most of these are discussed at length in later chapters.

SUMMARY CHART

THE WRITERS MENTIONED



KEY EMPHASIS

INPUTS TO LEARNING

(Subject Matter)

PRODUCTS OF LEARNING

(Specific
behaviours)

PROCESSES OF LEARNING

(Concept formation,
thinking, learning to learn,
etc.)

TOTAL LEARNING SYSTEM

(Inputs, processes
and outputs are
considered
equally.)

KEY CONCEPTS CONCERNING LEARNING

RECEPTION- LEARNING

(Rote v.
meaningful)

SHAPING BEHAVIOUR

- Reinforcers,
- Behavioural objectives
- types
- Learning tasks
- types
- Conditions for learning
- internal/external

DEVELOPING COGNITIVE SCHEMA

- Assimilation
- Accommodation
- Stages of learner's development
- Readiness for learning.

REGULATION AND CONTROL OF LEARNING

Cybernetic control
Self-regulation
Algorithmic and
heuristic learning
processes.

KEY
SUGGESTIONS
CONCERNING
INSTRUCTION

Logical Sequence of a subject.
General - Specific.
(Use of Advance organisers)
Match presentation to subject.

Hierarchies of pre-requisites.
- Simple - Complex
- Specific - General
(Use of mediators)
Match presentation to learning type

Conceptual networks or schema.
Student-selected sequence.
Concrete - Iconic - Symbolic.
(Use of apparatus/lab.)
Student selects presentation.

Knowledge structures
Adaptive/diagnostic sequence.
Can vary in all respects of sequence or presentation on an individual basis
(student/system interaction)
ADAPTIVE-LEARNING MODEL

MASTERY-
LEARNING MODEL

CYCLIC-LEARNING MODEL

TYPICAL POSITION
CONCERNING
INDIVIDUALISATION

Objectives
Content
Sequence

Methods

Materials)
Media)

Time

—
(Remedial options)
—

—
(Occasionally)

(Learning rate)

—
(Remedial options)
(Alternative paths)

(Matched to learning type)
(Usually options for individual styles)
(Rate and occasion)

(To student's needs)
(To existing schema)
(Student choice)

(To student needs)
(Student may choose)

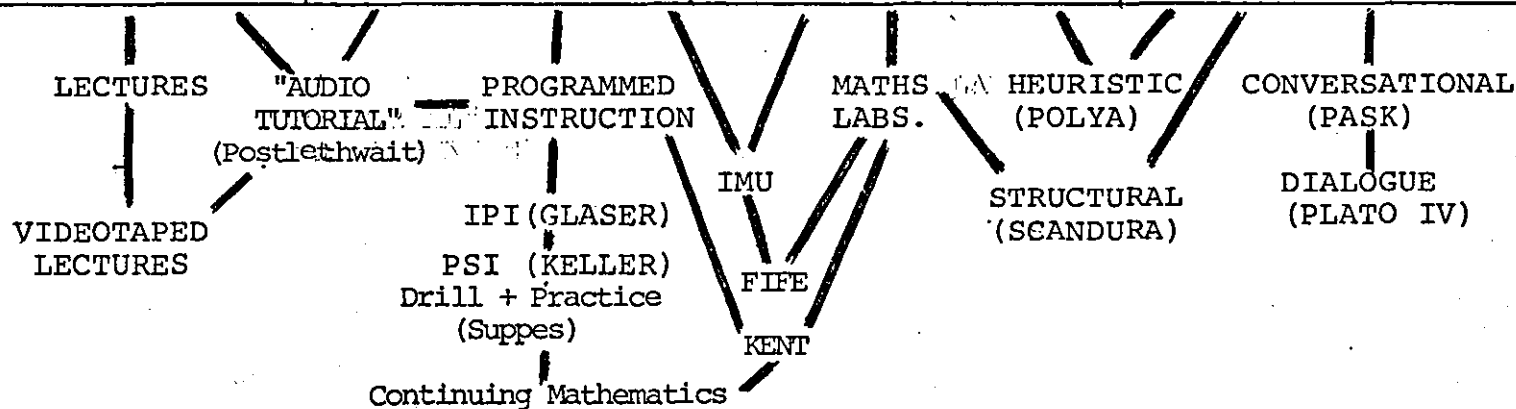
(Rate and occasion)

(Diagnosis)
(Adaptation)
(Preferred learning style)
(Adaptation)

(Options if found necessary)

(Rate and occasion)

RELATIONSHIP TO
SOME SCHEMES
OF MATHEMATICS
INSTRUCTION



3.4. CONCLUSION - OTHER CONSIDERATIONS

3.4.1. The prescriptive/democratic/cybernetic controversy

Apart from the differences discussed so far, many of the various viewpoints on the teaching of mathematics may be classified as being either highly "prescriptive" or highly "democratic". The "prescriptive" approach (characterised by both Ausubel and Gagné) supports individualisation on the basis of a comparison between the individual student's profile and some ideal model. Such a comparative diagnosis leads to an individual "prescription" of learning activities for the student.

The "democratic" or "student-centred" approach (favoured by the Discovery-learning school) supports individualisation for the "student's own sake" - to adapt the course to his needs, to give him responsibility, to develop him as an individual.

There is a certain amount of "partisan warfare" between these two points of view, on philosophical grounds. For example, Easley and Witz (1972) criticize the prescriptive approach strongly:-

"Some of the research in the Piagetian tradition, and many of our own experiences, raise serious questions about individualized programs that operate in this way. The basic issue is whether we should take seriously the organization which a child brings to a learning experience, and the natural

dynamics of such organization, or view the child primarily in relation to the desired learning (Brown et al., 1966). In our own work with Piagetian interviews (Witz, 1971), we are constantly struck with the phenomenon that children often have systems of ideas, or operate within presuppositional and functional contexts, which are very different from ours, which pervade much of their perception of their instructional experiences, and which are rarely identifiable by an adult in personal contact with the child at the time (much less by tests!)

On the other hand the supporters of the prescriptive approach argue that there are sharp limits, particularly in the study of mathematics, to what should reasonably be left to the student's choice. A student, by definition, is not an expert in mathematics nor in the best way to study it. A teacher, with a certain amount of experience of various students, may make a better judgment of a student's needs, but better still is the combined experience of many teachers, built into a diagnostic/prescriptive routine. Certainly, the observation of Easley and Witz that individual differences in learning style and conceptual organisation between learners are rarely identified by an adult in personal contact do not necessarily imply that removing the adult

from the decision making process and substituting him by the student will improve matters. Whoever decides among learning alternatives, be it teacher or student, requires aid. The totally open free-learning, unguided discovery, approach is seen by the prescriptive school as bordering on anarchy. As Professor B.F. Skinner put it during a lecture at University College London in 1967, "the dichotomy between education and training is false ... when you know the required outcome and can specify the means which lead to it, you are training ... when you have no idea of what you wish to achieve and only a hazy idea of the methods you plan to employ then you are in the business of education ... the ultimate cop-out is to delegate it all to the student!"

The bridge between these opposing camps is being constructed by the cyberneticians. Work such as that of Pask, described earlier, suggests that machine-based systems can be constructed which can learn from the learner, can adapt to the learner's "strategy for learning" and can re-design the presentations (on a conversational/tutorial pattern) in ways superior to those achieved by human tutors. This was obviously the case with machines such as SAKI (described earlier) for complex high-speed sensori'motor skills. Recent work such as the "CASTE" system (and also the PLATO IV system in the USA) are attempting to achieve similar results in the conceptual and problem-solving area. Pask (1972) in an article setting out his views on the state and prospects of computer assisted instruction, states:-

"First, the CAI system must learn because there are salient individual differences between students and changes in problem solving method that occur in the course of learning. Fortunately, students fall into theoretically predictable (and empirically verified) categories on a given occasion, i.e. any student who is free to do so adopts one class of learning strategy in respect of a certain subject-matter; moreover, there is a general individual disposition to adopt one or other class at the outset.

Next, the CAI system must learn in order to interact with the student on his own terms. There is a current dispute about how much liberty a student should be allowed and the amalgamated result indicates "not too much, but not too much restriction either". The fact is that if a student is given liberty to explore, he (any pretrained student, or anyone with the requisite foresight and self-awareness to begin with) can learn fast. But if he is given liberty, then the CAI system must match its strategy to his style and it can only do this if its data structure images his concept of the material."

We have here a strong argument for computer-assisted instruction, on the grounds that it can do a better job than can teachers. Pask continues this theme as follows:

"Most people in the educational profession (teachers, psychologists, university professors, curriculum designers, ETV producers, graduate students, and CAI merchants) have a sense of vocation. Many of these practitioners owe their vocations to having at least glimpsed some moment of excellence; the power of evolving symbolism, the sheer joy of comprehension. The phenomena in question are varied. A child suddenly learns to learn, and you account for it by some sort of neurophysiological change; but you know that the explanation is phoney and really you saw a miracle. A culture is engendered by a project; sometimes by an idea; or an adult who seems to have died in his twenties comes alive again. A design class in California, where the students were ignorant of electronics, gets, uses and innovates with, laser technology; all in a week. Von Foerster has over 100 well-documented examples, Illich more than 1,000. Papert has legions of instances, my own students have many.

In general life, the phenomena dubbed "moments of excellence" are rare; a circumstance that is only in part attributable to the dissonance established

by conventional training in scientific techniques. The chief reason for their rarity can be uncovered and represented formally; I shall gloss a lengthy argument by saying the world of learning and knowledge does not contain enough situations that count in a valid, non-trivial, very profound sense as "conversations".

That, I think, is CAI's important role; to foster conversation which is coupled to a corpus of wisdom (some of it encoded no doubt, but some of it not) and thus to increase the frequency with which moments of excellence occur.

If I did not believe that CAI could do that (it is a belief, though a reasonably founded one) I should not be in the field. "

This strongly-stated point of view which would raise the hackles of many a teacher, may lead us into a brief consideration of society's viewpoint with respect to individualisation in general and individualised mathematics instruction in particular.

The views expressed so far have all been concerned with alternative concepts of "improved learning"

- the task analysis-oriented "prescriptive" view, as expressed in the "mastery-learning" model,
- the individual-development-oriented "democratic" view, as expressed in the "free discovery learning" model,

- the "moments of excellence" creativity-oriented view, as expressed in Pask's computer-based "conversational" model.

3.4.2 The Humanist and Cultural positions

Two other viewpoints in the literature, supporting individualisation, are the "humanist" viewpoint, as expressed by Gagné and Briggs (1974) and by O'Daffer (1976) and the "cultural push" viewpoint expressed by Davis (1972).

The humanist viewpoint supports individualisation as "the only way to treat humans". Humans are all different and should be treated as such. O'Daffer therefore argues that individualisation is not one or other methodology, one or other psychological position, but "the basic issue in any individualised programme is not which modes of instruction should we use but rather at which time, with what types of content shall we use which mode with which students". It appears that this viewpoint is not necessarily at odds with any of the three approaches outlined above.

The "cultural push" viewpoint (Davis (1972) views individualisation as a step toward building a more vital and viable subculture within the school (and, where possible, within the community) that values mathematics and immerses itself in mathematics. From this point of view, the student working by himself is important mainly as he can report his discoveries and accomplishments back to his friends; and small-group collaboration of congenial and devoted students is even more important.

This last is in a sense a sociological argument for individualisation, interesting if only because it is from the sociology camp that most of the strongest opposition to individualisation has come. The objections of Eric MacPherson (1972) have already been described in Chapter 2. His objections are based on the "social animal" nature of man. Jackson (1968) has criticised the individualisation on many counts but in particular he regrets any loss in student teacher contact:

"When the teacher is in constant attendance he is available not only to call attention to errors and to affirm correct responses, but also to beam with pleasure and to frown with disappointment. "

and then again:

"... although a computer can store almost countless pieces of information about a student, it cannot know him as one person knows another.

Only humans care about humans, machines never do. "

And finally, Nichols (1972) (in direct disagreement with Gagné, Briggs, O'Daffer and others we have mentioned) when discussing programmed instruction, computer assisted instruction and the IPI system, states:

" Completely individualized systems of instruction are based on the differences between individuals. Perhaps we should

attempt to find out in what ways individuals are alike, rather than different, and capitalize on that to bring them together. Isn't this what the world needs today?

All three of the systems I mentioned earlier are basically dehumanizing. They successfully eliminate the direct interaction of the mature and the disciplined mind with that of a novice attempting to master basic skills and concepts of a given discipline. Because of this, these systems, while they succeed in teaching students some basic skills, may impoverish them intellectually and socially. "

There is indeed some evidence that some primitive self-study systems are incapable of teaching higher-level intellectual skills and may even inhibit them - this will be discussed in later chapters. With respect to mathematics teaching in particular, the sort of problem-solving approach as outlined by Polya (1945) in his book "How to Solve It", based as it is on "Heuristics" rather than prescriptive algorithms appears particularly difficult to automate. And perhaps too much emphasis on computational drills may indeed inhibit problem-formulation and problem solving.

3.4.3 The Author's View

However, this need not be so. It is becoming possible to computerise the heuristic, conversational tutorial. Excessive drill-and-practice routines, to the exclusion

of problem-solving activity should be avoided, but one need not invest in computer-assisted-instruction to observe instances of courses concentrating on drills. Many teachers administer mathematics courses which are as operations-oriented, as rote-learning-oriented, as the most outdated of linear programmed instruction - only the teachers are not so efficient at reaching their limited goals. They do this because they know no better, because they have learnt their mathematics in that way and because they have not the mathematical skills necessary to enter into a "conversational tutorial" mode of instruction with their students (even if class members allowed them to attempt it.)

It is here that the author sees the greatest justification for the study and development of teacher-support-systems (perhaps even teacher-replacement-systems) for large parts of mathematics instruction. We have no alternative. The traditional approach to teaching of mathematics has been singularly unsuccessful, more so as mass educational opportunities expanded. This inevitably may be traced in a large part to the teacher force - both quantity and quality.

It is not the function of this study to investigate or comment upon the quality of mathematics teachers in any great depth. Suffice it to observe that quality and quantity are inextricably linked (by the laws of supply and demand) to the value that society appears to put on mathematics. As long as priorities (as expressed by salary levels) remain as they are, educational systems

will continue to be short of highly qualified teachers, particularly in certain key areas such as mathematics. Land (1970) put his plea for technological aids in the teaching of mathematics thus:-

"I would like to think that every potential mathematician would have the opportunity of learning his mathematics from a properly qualified teacher. Today, however, we have the situation in which there are 5603 mathematics graduates teaching in 5729 maintained secondary schools to 2,832,581 pupils. This is less than one graduate per secondary school and one to 500 pupils. With the usual allocation of time to mathematics, this means about 70% of pupils are not being taught by mathematics graduates. With no longer selection for secondary education and streaming becoming socially unacceptable, the chance of a potential mathematician being taught by a mathematics graduate could be as low as about one in three. These figures are from the Department of Education and Science 'Statistics of Education', published in 1969. For Establishments of Further Education, the same source gives 1379 mathematics graduates teaching in 738 major establishments of further education to a total of 1,774,000 students. There is no information as to

how many of these are required to do any mathematics, but the ratio is one mathematics graduate to about 1,300 students. These figures leave us in no doubt about the need for help from educational technology. "

The position has not improved since 1969, despite the current general "glut" of teachers in Great Britain. It is significant that the Secretary for Education, Shirley Williams, whilst cutting teacher training in general by nearly half, is encouraging Colleges of Education to mount one-year "crash courses" to retrain teachers of other disciplines as mathematics teachers. Apparently insufficient trainees are coming forward.

The question arises whether it would not be more cost-effective to devote the resources to be spent on these crash-courses to the development of "multipliable" teacher-substitutes, perhaps in the form of part mass-media, part individualised packages. The answer is that we do not really know. Despite the considerable amount of research on media-based system for mathematics instruction, we as yet cannot categorically answer questions related to their effectiveness, efficiency, cost effectiveness and particularly cost-benefit (a question which implies the quantification of any transfer-inhibiting or other detrimental factors). Hence there is a need for further research, perhaps research of a more extensive and different type from what has been practised to date.

The author has recently been working for some years in Brazil. There the answers are perhaps clearer. The Federal Ministry of Education has repeatedly stated that there is no way whatever, in which the country's current educational needs (at any level and particularly in all technological and scientific subjects) can be met by "traditional" approaches to teaching.

The alternatives being tried include the mass media (two states, each bigger in area than the U.K. have primary and secondary educational systems entirely based on radio, television and print - no teachers as we know them), use of para-educational staff (the Keller plan, using proctors/monitors was after all conceived in Brazil), programmed correspondence courses (much in-service teacher training - 60% of practising teachers are untrained, and much adult education) and the chain-reaction "multiplier" approach (in which every course graduate leaves the course with a kit to enable him to repeat the course in his locality - every trainee a trainer - much used in technical teacher training).

Most of these very large and ambitious projects are working reasonably well - not always better than equivalent traditional solutions, but not always worse. That they are working at all, given the low-budget, rapid, amateurish way in which most of them have been set up, is incredible. That they may be improved further by better design of the basic system of control and of the instructional materials used is beyond doubt. The author has already

performed some studies which show this (Romiszowski 1975, 1977).. That there is need for further basic research in order to facilitate the design and improvement of such systems is indisputable. That such systems are of paramount importance to developing countries is obvious. That they are also of growing importance in the developed world (witness the spread of the Keller Plan in the U.S.A. or of the Open University concept throughout the world) is true. That they will be of particular importance in the post-secondary levels of education especially in the training and re-training of adults and in continuing (or recurrent) education, is likely, (as shown by a study which the author performed for the Council of Europe - Romiszowski & Biran, 1970).

It is for the abovementioned set of reasons that the author has concentrated in this study on:-

- Individualised instruction as it relates to the adult learner of mathematics, particularly the non-specialist being trained or re-trained in a mathematically-based skill, or the adult in a continuing education programme. This is the theme of the next chapter.
- Individualised systems of instruction which are principally media-based (as opposed to teacher-based). This does not mean that the teacher is excluded, but that a substantial part of the instructional tasks are delegated to other media. Such systems, applied to mathematics instruction at any level are reviewed in Chapters 5 to 9.

- Research which is based on currently available, reasonably cheap and "multipliable" instructional media, such as may readily be adopted by any country regardless of its state of technological development.
- Research which is based in a developing country (Brazil) in situations in which the "traditional" systems of education are non-existent or inefficient.

CHAPTER 4

THE ADULT LEARNER OF MATHEMATICS

4.1 Introduction

The meaning of "adult learner" in the present context. The variety of aims, pre-requisites, attitudes and problems which may be encountered among adult learners of mathematics.

4.2 Aims of the Adult Learner

Why teach mathematics to adults.

The "Marthas and Marys" of adult education.

Four categories of adult learners and their aims.

The aims of teaching mathematics to non-specialists.

4.3 Knowledges and Skills that the Learner brings to the Course

The variety of pre-knowledge that one might expect in any adult non-specialist group.

Examples from the author's work with remedial mathematics in Brazil and the U.K.

4.4 Attitudes towards the learning of mathematics.

The need to see practical relevance. The effects of previous successes or failures. Anxiety.

Strong arguments for individualised self-instruction.

The author's experiences.

4.5 The Learning Problems of the Adult

The scarcity of research on adult learners. Some surveys of adults self-assessment of their learning problems. How to study; what to study; Anxiety; motivation; habits; individual differences.

Arguments for individualisation.

4.6 Conclusion - Adult Learner and Individualisation

The author's summary of factors which make individualised instruction particularly suitable for the adult non-specialist learner of mathematics, and his suggestions as to the characteristics of appropriate individualised systems.

CHAPTER 4.

THE ADULT LEARNER OF MATHEMATICS

"The current state of the literature in the field is so poor. While a great deal has been written on the subject of Adult and Continuing Education from the point of view of the design of distance teaching systems little if anything appears to have been written on the teaching of particular subjects."

R.M. Pengelly, Great Britain. Opening of invited paper on "Adult and continuing education in mathematics" at the Third International Conference on Mathematics Education, Karlsruhe, August 1976.

4.1 INTRODUCTION

The above quotation emphasises the need for research particularly related to the learning of mathematics at the post-secondary level. By the post-secondary level we do not mean necessarily formal higher education as, for example, in a university mathematics course, or a science course with related mathematics content, but would include any mathematics education occurring at an age which normally would be considered post-secondary. Thus, apart from university courses, and further education courses of a formal nature, we would include in this category mathematics being learnt by people already in

an occupation in order to gain further job skills, promotion prospects etc. and also mathematics being learnt by adults as part of an adult education course aimed at giving a general education to people who did not obtain one when younger, or who would like a second chance at improving their educational standard.

Obviously the content of mathematics courses for these various groups of clientele would be extremely different and possibly the learning problems of these groups as well. For example, at one extreme the post-secondary school student in a university mathematics course, although he is studying a very high level of mathematics, he would be bringing to this study a collection of previous knowledge and skills which he has learnt in his school days efficiently, and probably with a modicum of enjoyment. At the other end of the extreme, the foreman in industry who has to learn some statistics for quality control, or the adult in an adult education programme may either have never attempted to learn the requisite mathematics, or have certainly not practised any mathematics of that type for a considerable number of years. And quite possibly when he was last exposed to mathematics learning, experienced failure and disappointment.

We may, therefore, expect differences among these groups of clientele in terms of their aims in learning mathematics, in terms of the knowledge and skills which they bring to this task, in terms also of the attitudes

which they bring to the task, and finally, in terms of the learning problems which they bring.

4.2. Aims of the Adult Learner

In his article entitled "Why Teach Mathematics to Most Children?" Geoffrey Matthews (Matthews 1976) identifies two broad aims, one for the under-thirteens and one for the upper secondary school children. For the under-thirteens he sees the aim of the teacher to be to provide appropriate experiences so that each child may develop his mathematical skills and knowledge to the utmost of his ability. He labels this a birthright of all children, that they should have the chance to show how far they can develop, the criterion for success being ability rather than ancestry.

For the over-thirteens, he sees the main function of mathematics teaching as concerned with imparting skills concerned with problem-solving and problem-construction. Thus a general objective at this level might be to teach in such a way as to foster transfer, generalisation and creativity. Our aim in this section will be to consider whether there is any such general aim, or perhaps a group of aims which could be defined for the teaching of mathematics at the post-secondary levels.

Firstly, one must observe an important difference between the sort of problem discussed by Prof. Matthews in his article, and the problem of the adult student. Primary and secondary education is compulsory, and we therefore have a problem to decide what is the most useful mix of disciplines and educational experiences with which to fill these years of schooling. However, post-secondary education

is not directly compulsory, although sometimes, due to the pressures of life, it does seem to become so for some people. Thus, in general, we may assume that adults, be they in university courses, further education or supplementary adult education programmes, are participating because they have either a need to acquire certain knowledge and skills, or alternatively, have a wish to improve their general education. This is put nicely by Elton (1975) as 'the Martha's and Mary's of education'. The Martha's he describes as follows:

"They are highly motivated, they need security in their study and basically one gets so often the question from them, either straight or disguised, conscious or sub-conscious, 'what will I be examined on?'. This group is excellently served by what I might call the traditional Ed Tech approach: the specification of aims and objectives, the provisions of materials which are designed to satisfy those aims, the provision of tests and self-tests to reassure the student that he is in fact satisfying them. Clearly the Open University has done a magnificent job in that line. "

Elton is obviously talking here of university education, but in general his comments might apply to any group of adult learners. The Mary's, the second group, he defines as the 'love of education' group.

"They would also like a degree because a degree is an accepted status symbol, a degree is a sign of achievement. This century is the century of the paper qualification man and woman, and nobody is really exempt from this. But their love of education conflicts with a directiveness of most degree courses and a question now arises whether we could not adapt our degree courses so that there is not a conflict in aims between the love of education and the desire for a degree of these students. "

The second group, although probably quite numerous in university courses, is less frequently met in vocationally oriented courses or, with respect particularly to mathematics in service mathematics courses which form parts of larger courses. With respect to service courses in particular, McKeen, Newman and Purtle (1974) observed when teaching service mathematics sequences, for social science and biology science units that

"To begin with, few of the students taking the sequence really like mathematics; they were taking the sequence only because of a college or departmental requirement. Because the student attitude is relatively negative and mathematical aptitudes largely minimal, most faculty members avoid teaching the sequence like the plague. When "trapped" into teaching such a course the presentation was often sterile.

The result was to be expected - the students actively avoided as much work as possible. Performance levels were marginal. "

The author's experience with remedial mathematics tuition for social science degree students was not quite as negative as the one described above, but certainly the aims of the students were very precisely delimited by the needs of their main course. (Hamer & Romiszowski, 1969). Experience in running foremanship and supervision courses involving quality control and statistics supported this finding at the vocational level of mathematics for industry, (Romiszowski, 1968). At the general level of mathematics courses as revision or refresher courses in adult education programmes, one notices the same two groups re-appearing, as were identified by Elton at the higher education full-time level. Some adults participate in general education of this nature for the love of learning, but some, (in the author's experience in Brazil the majority), are interested primarily in the paper qualification which will open doors to promotion and job prospects and are therefore interested primarily in 'what will I be examined on?'.

We shall concentrate here to consider the last three of the groups of clientele that have been outlined above, and shall largely ignore the post-secondary specialist in mathematics. This is partly because his learning problems are minimal as compared to the other three groups and our research has been concerned primarily with the second and

fourth groups. We are therefore interested in this chapter mainly with the aims and methods of teaching mathematics to non-mathematicians. In an article discussing this particular theme, Elton (1971) considers the aims that have been set up for such courses. On the one hand he considers the position of the Association of Teachers in training colleges, in departments of education who have stated the aims as

- 1) to establish healthy attitudes towards the subject
- 2) to foster an understanding of the nature and significance of mathematics, and
- 3) to give pupils the opportunity of discovering simple mathematical truths through their own experience.

He contrasts this position with the aims as stated in a degree course in business studies, namely,

- 1) to give an understanding of mathematical concepts and techniques relevant to business studies, including data processing, and
- 2) to give such technical competence as the study of statistics requires.

Elton characterises the first position as being stated entirely from the point of view of the interests of mathematicians and teachers of mathematics. The second, on the other hand, he considers as erring in the opposite direction in that it reduces mathematics purely to a "tool subject" and not a very versatile tool at that.

Elton's argument continues that not only should the aims of a mathematics course for non-specialists include both aspects, but how much of each aspect would need to be defined for each case separately. He is arguing, in effect, that it is not possible to establish a general aim for the teaching of mathematics to non-specialists, but that every case has to be considered on its own merits.

The author would tend to agree in principle with this position as is exemplified by the case described in the following paragraph:

4.3 Knowledges and Skills that the learner brings to the Course

The author has recently been concerned with adult education courses in Brazil, which aim to give a general education up to equivalent of British 'O' level or 'A' level to adults who never went to school or, alternatively, failed to obtain a qualification whilst at school. The author's particular area of interest with these courses was in the teaching of course production teams, in the area of mathematics and science. One practical constraint of these courses was that of necessity they were fairly condensed, completing in one or two years the equivalent of a full school curriculum. Clearly, the learning problems of adults who had never attended school, or not completed their school past the first or second grade would be quite different from those who had gone right through the school system and had failed to reach the appropriate standards to get a school leaving certificate. One group would be revising and remembering material which they had learnt, albeit badly, whereas the other group would be

starting completely afresh. Given these constraints, to what extent should one adhere to the normal school curriculum? As it is impossible in the time to complete all the topics to the same standards, where should one make cuts in the adult curriculum? In basic skills and operations, or in conceptual understanding? This matter was confounded even more by a situation that the adults ~~whose~~ are at the present moment participating in courses of this nature, are students who even if they went through the primary school system will now be revising ~~the~~ course which is based on a modern mathematics approach, whereas at the time of their schooling they would have studied a course based on a very much more traditional approach to mathematics. Should one insist with these adults that they should re-learn all the mathematics that they have studied, or should one concentrate on developing and refining the few skills that they have brought with them from their schooling?

A second example is drawn from the author's experience in operating the remedial mathematics "clinic" at the Middlesex Polytechnic (Hamer & Romiszowski 1970; Romiszowski et al 1976). Approximately 300 first-year social science undergraduates are diagnostically tested each October, in order to establish whether they have any remedial mathematics needs. Only about 100 generally demonstrate satisfactory competence in basic pre-requisite skills (limited to arithmetic, some algebra and graphical work); the rest demonstrate needs ranging from a quick refresher on simultaneous equations, to systematic revision starting with fractions and decimals.

These two examples emphasise the need for flexible adjustments in the aims, objectives and content of mathematics courses for adult non-specialists - a need probably greater at this level than at the primary and secondary levels.

4.4 Attitudes towards the learning of Mathematics

The attitudes of adults towards the learning of mathematics may be expected to vary considerably. The two factors already discussed - aims and previously acquired skills - have a strong influence on the formation of individual attitudes.

The influence of the learner's aims is tied to the adult learner's need to see relevance in his learning activities. This demand for clear relevance becomes very much stronger with age (Belbin & Belbin 1972, Tough 1967). Whereas younger students will participate in many learning activities simply because they are asked to, or told to, the older student will question the relevance of these activities to the overall objectives of the course and to his own aims in attending the course. His reaction to apparent irrelevance may range from a loss of interest and motivation (switching off) to irregular attendance with withdrawal from the course (dropping-out). Chronic irrelevance may lead to active revolt, (as occurred in May 1968 in Paris among the university students[.

The traditional methods of teaching mathematics have often ignored the need to establish relevance. This is now realised at the school level and has led to such

curriculum projects as "Mathematics Applicable" (Bentley, 1976; Malvern 1976; Ormell 1976) produced at Reading University for use in the sixth form, and the "Continuing Mathematics" project (Schools Council, 1975) which is also very much "practical-applications-oriented". However, the post-secondary level has largely been ignored. It is still quite difficult, for example, to locate published textbooks on calculus which avoid kinematics and science applications, substituting in their place valid examples from the social sciences. Furthermore, in the author's experience at the Middlesex Polytechnic, social science undergraduates specialising in sociology do not understand, are not interested and consequently do not learn from calculus problems drawn from economics, and economists do not like sociology-based examples. Once again we see that the need for adaptability of course content to the individual's needs (as he sees them) is particularly important at the post-secondary level.

The influence of the pupil's previously acquired mathematics skills on his attitude is a function of not only which skills he has (or has not) learnt in the past, but most importantly of how he learnt them - what were the teaching methods; what difficulties did he have, how were they handled. All too often, at the post-secondary level, the non-specialist learner of mathematics is an unwilling learner, having experienced failure and frustration in the past, perhaps having developed "blocks" against the learning of mathematics. If not an unwilling learner, he will be at any rate an anxious learner. He may also be a "shy" learner - knowing

his weaknesses, prepared to try again, but unwilling to "show his ignorance" to his colleagues - other students, or even the teacher. This is a common source of "lack of participation" in adult learning (Cooper-Marsh 1970). It was also one of the "classic" arguments used to sell programmed instruction and teaching machines to industry, particularly when the programmes were destined for the supervision or management levels. Certainly this was supported by the comments of users of individualised programmes of instruction in a survey carried out by the author (Romiszowski 1967) and would still stand as a further favourable factor for the individualisation of a substantial part of mathematics teaching to adults.

Anxiety is a general characteristic of the adult learner, showing itself in various ways. One of them is the adult's greater preoccupation with being correct. "All older people prefer accuracy to speed" (Belbin & Belbin 1972) .. "if a trainee is allowed - and in fact, encouraged by the design of the training programme - to detect and correct his own errors in the early stages, he will be freed from disillusionment and despair when his errors are later detected by those who supervise him." Yet again a suggestion that the individualised, self-correcting, self-instructional approach should be of particular benefit to adult learners in overcoming their anxieties and allowing them to "place accuracy before speed". There are of course limits to the practicability

of this maxim. The author well remembers an early "failure" with a linear programmed text he was developing to teach graphical methods (break-even charts, multiple activity charts, machine-loading charts) to foremen in the motor car industry. The first draft was pre-tested on a group of apprentices in training, familiar with programmed instruction, (due to the non-availability of foremen). After revision, the text was applied on the next foremanship course. The first chapter, completed by all the apprentices in under half an hour, was not completed by any foremen in less than 2 hours. This phenomenon disappeared once the foremen learnt that they and only they would be checking their answers to the frames of the programme.

4.5 The learning problems of the adult

It is surprising how little work has been published concerning the learning problems of the adult. The considerable body of literature on adult education seems to concentrate on the "why, when, who and what" aspects of the subject. The "how" aspect, when discussed at all, has generally been treated from the organisational and curriculum-content points of view rather than from the learning problems/teaching methods approach. There is a body of research on adult learning but much of it relates to academic studies of rote-memorisation of nonsense syllables, short-term memory and the like. A good selection of such studies and extensive bibliographies

may be found in Talland (1968). Studies more directly related to the teaching learning process in real classrooms have generally been of the anecdotal or questionnaire-based surveys of student and teacher activities or opinions (Tough 1967; Cooper-Marsh, 1970). The realisation that this is an important field has grown in universities and polytechnics, with the expansion of higher education provision to less able and less well prepared sections of society. The Middlesex Polytechnic, as indeed most of the Polytechnics, The Open University and several of the "traditional" universities are currently developing learning-to-learn schemes and study-skills packages for the use of their own students, but as yet little hard data has emerged. Reports such as that of Cooper-Marsh assemble questionnaire and observation-based evidence to support such hypotheses as:-

1. the development of the student as a person is the most important aim in adult education
2. adult students engage in adult education primarily to learn
3. active student participation in the teaching and learning process are advantageous for individual development and active learning.
4. socialisation of the adult in his various roles as e.g. worker, citizen, spouse, parent, homemaker, friend, voluntary group member, and retired person is an integral function of adult education.

5. active student responsibility in the educating process, including power to influence the form and content of his own education and a share in evaluating his own progress, is a necessary part of the socialisation of the adult.

Tough (1967) specifically investigated independent learning of topics which the adult student himself had attempted to learn without direct, organised teacher help (he calls these "Adult Self-Teaching Projects") and argues that much of adult education is made up of such projects (some teacher-set, others set by the adult learner himself). Of course the students may and do need assistance from teachers, libraries, friends, or any other imaginable resource. Tough identifies 12 tasks which may form part of a self-teaching project and presents information on which tasks were found to be more difficulty, and which needed most assistance. The table below summarises some of the findings for a group of 40 adult college graduates, interviewed by Tough (1967).

Task	No. who Experienced some Difficulty	No. who Found Task to be One of Two Most Difficult	No. who sought a large amount of assistance	No. who would have asked for more assistance
Deciding activities	17	17	22	18
Dealing with difficult parts	15	16	18	14
Estimating level	11	8	22	9
Deciding about time	13	7	10	2
Dealing with dislike of activities	10	7	2	3
Obtaining resources	11	6	28	9
Dealing with lack of desire	7	5	5	1
Deciding whether to continue	3	4	3	3
Deciding about place	6	3	6	5
Dealing with doubts about success	10	3	8	7
Choosing the goal	17	2	19	11
Deciding about money	5	2	4	2

It would seem that these students had most problems associated with deciding how to study (learning activities, overcoming difficult parts, estimating the right level of difficulty) and next down the list come what to study (choosing goals). Problems of finance, motivation and how to self-evaluate progress were less troublesome for most students. Tough summarises his report by confirming that adults in the self-teaching situation do indeed take over

many of the tasks that a teacher generally performs, but find that certain tasks (listed above) are more difficult and require the assistance of other people or resources to a considerable degree. He does not consider how to organise the support systems required by the "self-teacher". It seems to the author however, that many of Tough's findings are relevant to the "prescriptive/student-directed controversy as it applies to adult learning.

Perhaps the most thorough and most systematic British research on the learning problems of adults is the work performed by the Industrial Training Research Unit of University College, London. This has been reported by Belbin and Belbin (1972) in summary, and in innumerable research papers. The work is concerned with the training and re-training of adults in industry, particularly at the operative, clerical and supervisory levels. However, some of the findings would appear to have general relevance. In particular the work is useful as it suggests practical actions to be taken to overcome the problems. Some of the problems and solutions discussed by Belbin & Belbin are

- (a) Anxiety - already discussed above. Suggested solutions include reducing tension by such measures as: social support (an argument for group-instruction); an acceptable instructor (or instructional medium); a secure future (by continuous assessment schemes and the preclusion of failures - apparently an argument for the mastery-learning model).

- (b) Loss of interest or motivation. This may spring from the adult being treated as a child (a loss of status in the eyes of his peers), from lack of obvious relevance to his needs, or from lack of success (discussed above). One should combat this by creating an adult atmosphere, using methods and materials designed for adults, explaining relevance, demonstrating relevance, avoiding the probability of student errors, but when errors occur, supplying immediate explanations of the error (feedback), preferably not in front of other trainees.
- (c) Different study habits. These include a preference for longer study sessions than younger students, a tendency to continue past the end of a session if there is an unresolved problem, a higher level of pre-occupation with correct performance, a preference for "whole" rather than "part" methods of instruction, often a slower rate of learning rate, particularly in the early stages of a new unfamiliar learning task. All these factors should be taken into consideration during the design of training schedules for adults. In general, they imply a more flexible, more student-directed approach than is commonly employed with younger age groups.
- (d) Finally, individual differences. These tend to become accentuated with increased age. Hence greater importance is attached to individualised

instructional methods, adaptive to the needs of the trainee. As an example Belbin quotes the results of an investigation of how instructors' approach to instruction influences final results on an adult training course. The study interviewed and observed 116 instructors. The results below speak for themselves.

	Statements mainly;	
	Trainee-oriented	Instructor-oriented
Record of 116 instructors:		
5 with best record	4	1
4 with worst record	0	4

In summary, therefore, many of the findings of the ITRU, although related to industrial training, may well be relevant for adult training in general - certainly the adult on a vocationally oriented "service mathematics" course, or an adult in a continuous education programme. Many of these suggestions indicate yet once more that "individualised" (private, self-paced, self-correcting, practical-application-based, adaptive to the learner's learning strategies) programmes of instruction can play an important role in adult courses, though not to the exclusion of group work and teacher-led work.

4.6 Conclusion - the adult learner and individualisation

It is noted that the mature learner:-

1. Probably has much clearer long-term objectives for the learning of specific mathematics content.
2. Requires (more than the young learner) to see the relevance of what he is studying to his long-term objectives.
3. Often has already studied mathematics and has encountered problems. He may have "blocks" or exhibit "avoidance behaviour" with respect to mathematics.
4. Has well-developed (good or bad) study habits, which will be difficult to change.
5. Has developed "status" in his peer group, which may cause learning problems due to fears of being "shown up".
6. Is generally in a hurry to learn what is required, in the minimum of time.
7. Prefers to apply new techniques directly to his own problem, rather than to hypothetical "general" problems.
8. Is more capable of evaluating his own progress, given the required criteria; than the young learner.

This suggests that individualised techniques should be of great value in the teaching of mathematics to (particularly) adults who are not primarily planning to be mathematicians, but require a certain body of skills for a certain precise reason.

A good system would probably:-

- (a) rely heavily on self-study from pre-prepared materials, developed to specific pre-determined objectives.
- (b) these materials should be developed specifically for the application, using examples relevant to the practical situation of the learner.
- (c) the materials should be produced in styles which are already familiar to the learner.
- (d) if possible the materials should not force a particular study style on the learner, but should be capable of being used in a variety of ways.
- (e) the materials should have ample self-tests for the purpose of self-assessment/evaluation.
- (f) these tests should, when possible, be realistic problems, rather than abstract, or taken from an unfamiliar application.
- (g) some human tuition will usually be required and certainly diagnostic/counselling interviews would be required in the case of any mathematics "blocks" existing from prior schooling.

In summary, a materials/tutor system, in which the materials are flexible and very carefully prepared and the tutor is well trained and well briefed in their administration.

SECTION B.

REVIEW OF EXISTING SYSTEMS FOR THE INDIVIDUALISATION
OF MATHEMATICS INSTRUCTION

CHAPTER 5 DEFINING THE SCOPE AND PURPOSE
 OF THE REVIEW

CHAPTER 6 PRINT-BASED INSTRUCTIONAL SYSTEMS

CHAPTER 7 MULTI-SENSORY INSTRUCTIONAL SYSTEMS

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CHAPTER 9 SOME PROBLEMS IDENTIFIED

SECTION B

REVIEW OF EXISTING SYSTEMS FOR THE INDIVIDUALISATION OF MATHEMATICS INSTRUCTION

CHAPTER 5

DEFINING THE SCOPE AND PURPOSE OF THE REVIEW

5.1 The Levels of Individualisation

The "course", "course-unit", "lesson" and "learning step" levels.

Individualisation techniques for the "course" and "course unit" levels.

Techniques for the "lesson" and "step" levels.

The role of media.

The author's decision to focus on these lower levels.

5.2 The Media of Instruction

The teacher-teaching or the learner-learning?

Teacher-based and media-based systems of instruction.

Resource-based learning.

5.3 Organisation of the review

Print-based systems; Multi-sensory systems;

Computer-based systems.

SECTION B

REVIEW OF EXISTING SYSTEMS FOR THE INDIVIDUALISATION OF MATHEMATICS INSTRUCTION

CHAPTER 5. DEFINING THE SCOPE AND PURPOSE OF THE REVIEW

As we have already seen, from Chapter 2, there is a very large and ever growing variety of individualised systems in use. There are various ways of classifying them.

Gibson's (1971) major first sub-division, into those based on individual, self-study (or small-group) learning models on the one hand, and group-dynamics learning models on the other, could be one way of restricting the scope of a review of this nature. However, a glance at figures 2.1 and 2.2 in Chapter 2 will show that each of these two sub-categories still contains a bewilderingly varied collection of systems.

The author prefers to use two other parameters to define the boundaries of this study. These two factors are:

- (1) The 4 levels at which individualisation takes place,
- (2) The principal medium or agent of instruction.

5.1 The Levels of Individualisation

As already discussed in Chapter 2, virtually any course may be individualised as a whole - the student may choose to take it or leave it. It is only at the course-unit level that individualisation really begins to have meaning. However, at this level, individualisation decisions are restricted to a choice of units that will make up the course. These units may vary in objectives, content, instructional

methods or delivery media (or any combination of these). The student may select these himself, may be guided by a teacher, or may have his selection options restricted by some master plan, be it a compulsory common-core curriculum or a decision algorithm built into a computer-managed-instructional system such as project PLAN (Flanagan 1968).

A justification for individualising at the course unit level, in mathematics, is given by Kaufman and Steiner (1970).

"For the development of curricula the didactics of mathematics has to prepare a store of didactically structured units of mathematics, organized on different levels of abstraction and in different sequences. Then the adaptation of a piece of mathematical content to a teaching situation will mainly consist in a selection of suitable units out of the store.

Even if such a store were available a major problem still existing would be the availability of qualified teachers capable of making judicious selections from this store. The problem of availability of qualified teachers is world wide. "

This paragraph is from an introduction of a paper on the Comprehensive School Mathematics Project (CSMP). This project has developed a curriculum which is individualised at 3 levels; course unit, lesson, and learning step.

"A feasible solution to the aforementioned problems seems to be the total individualization of the mathematics curriculum. This, of course, is based on the assumption that besides the teacher there are many other media by which mathematics can be learned such as well-arranged books, carefully selected problem sets, programmed materials, worksheets, models, curriculum-bound games, films, slides, video and audio tapes, computers, etc. "

We shall consider the "lesson" and "step" levels later. At the "course unit" level, individualisation is organised by means of a curriculum matrix.

"The curriculum matrix is a 3-dimensional array in which the full pre-college mathematics curriculum for all students is organized. The three dimensions are content, levels of sophistication and individual student characteristics. The actual instructional materials which fill such an array are called "activity packages". An activity package is a multi-media unit of instruction prepared for a particular type of student on some piece of mathematical content at a specific level of sophistication. To each package are assigned well stated prerequisites and goals, which are measured by a pre-test and a post-test. The package itself might make use of various media such as books, programmed instruction, video and audio tapes, games, teachers, computer assisted instruction. etc. "

The curriculum matrix acts as a guide to learners and students in the selection of modules (activity packages) suited to the learner's current needs, interests, and state of readiness. This procedure is one example of modular scheduling described in detail by King (1972) and Petrequin (1968). Petrequin's book is the classic text on modular scheduling or "modular-flexible programming", being a collection of papers on how to individualise at the course-unit level in the large-group situation, in small group learning, in independent study projects and in resource-based learning.

Another set of techniques for individualisation at the course unit level, now suffering from unpopularity on social-justice grounds, are various forms of streaming and grouping into ability groups. These are well known and do not need much discussion here. To complete the list of possibilities, we may quote some that are listed in an article by Stephen Willoughby in the special issue of the Mathematics Teacher, devoted to the subject of individualisation (Willoughby 1976).

- "1. Some students have simply been eliminated from the schools or the mathematics classes by the school authorities.
2. Various forms of homogeneous grouping or tracking have been tried.
3. Grouping within classes has been tried by many teachers. Some attempts have

included grouping together students of similar need and achievement; others have involved placing students with different needs and abilities together in the hope that they would help each other.

4. Students (and their parents) have been allowed to choose which courses they would study, especially in the upper grades.
5. Some students have been used as teachers' helpers to try to improve the learning of other students. Sometimes the helpers are the same age as those to be helped, and sometimes they are older.
6. Differentiated assignments have been tried, usually consisting of special remedial work or enrichment.
7. Occasionally parents have been asked to provide special help for certain students, either directly or through a paid tutor.
8. Teachers often provide special help or inspiration to specific students outside the regular class period as well as within the class.
9. Various forms of team teaching have been tried to help meet the needs of individual pupils.
10. Flexible scheduling on a school-wide basis has also been tried in an attempt to meet the needs of individuals more efficiently.

11. Nongraded schools have been created to allow for progress through ability, achievement, and so on, rather than simply through the process of aging.
12. Continuous-progress plans (similar to item 11) have also been used.
13. "Free schools" based on the Summerhill School plan have been tried recently.

The reader will agree that the items in this list are all organisational solutions which affect grouping, scheduling or grading systems and may have little influence on the instructional methods employed. Indeed all these strategies could be applied at the "course unit" level without any attempt at individualisation at the "lesson" or "learning step" levels,

Considering the points made in the concluding paragraph of Chapter 3, concerning the reasons for individualising mathematics instruction, it is obvious that we are more interested in systems which permit individualisation at these lower levels. We are particularly interested in studying systems which modify the way that students learn (not just what they learn or when they learn) and systems that supplement or extend or even partly replace the human teacher (we come back to this aspect later in the chapter).

Of the list of 13 techniques above, perhaps the only one which of necessity modifies the way that teachers teach (and therefore the way that learners learn) is the team-

teaching approach. This approach generally forms part of a more complex system of individualisation. The teachers released from large-group lecturing perform small-group or individual tutorial functions or manage resource-based learning systems. The technique has a history going back to the 1950's (National Association of Secondary School Principals 1957) and is backed up by a small but favourable body of research (Sigmund 1973; Shaplin and Olds, 1964). A very full account may be found in the book by David Warwick (1971). This technique shall be discussed as part of other systems which make use of it, rather than in its own right. The rest of our list of 13 techniques shall not be discussed in detail.

Let us now turn to the remainder of the list of techniques (Willoughby 1976):

- "14. Computer-assisted instruction in one form or another has been advocated and installed in many places in the recent past.
15. Programmed instruction with and without machines has been advocated and employed, often with some initial success, but never with any truly wide popularity.
16. All sorts of other methods based on advances in technology have been explored. These have included individual and small-group use of movies, video tapes, audio tapes (often combined with filmstrips or slides), teaching machines.

17. Partial or complete independent study,
in which children read appropriate material
to learn certain topics, has been fairly
popular lately. Usually some collection of
learning goals is specified, and students
progress by reading standard texts and
participating in related activities.
18. A system of contracts in which children agree
to complete a certain amount of work in a
certain time has been tried in many schools.
19. Performance-based curricula, in which the
goal is to have the students perform certain
activities at a certain level of competency.
20. Mathematics laboratories and resource or
materials centers for mathematics have been
set up as one means to help individualize
instruction. "

The remainder of our list includes techniques which imply that individualisation proceeds to the lower "lesson" and possibly "learning step" levels. Most of them also imply the use of media to take over a substantial part of the instructional task from the teacher. In view of the comments in Chapter 3 regarding the world shortage of mathematics teachers and the impossibility of supplying demand for mathematics instruction by "traditional" means, we are particularly interested in this category of techniques.

Thus the first criterion for selecting a particular system for study has been that it takes the process of individualisation to at least the "lesson" level of our 4-level classification (Chapter 2). In other words,

systems in which decisions regarding individual students are taken frequently.

5.2 The media of instruction

The second criterion for inclusion was that the principal medium of instruction used by the system was not the teacher. This does not mean that the teacher has been eliminated from the system, nor does it necessarily mean that his importance has been diminished. His role has been changed, though.

In his excellent book "Resources for Learning", L.C. Taylor (1971) says:-

"We can conceive of a number of different systems of learning being used in secondary schools. Foremost among them is the teacher-based system familiar to us all in the classroom. Other systems already tried here or abroad are these; book based, book-and-boy based, assignment based, programmed learning based, correspondence based, radio and television based, computer based. At first sight, it seems a daunting complexity, but we may find the appearance of conflict between these systems misleading. The passionate exclusiveness of rivals may be a measure of a basic similarity, just as snobbery is most intense where differences between incomes are small. What essential elements have these systems of learning in common which distinguish them all from conventional teaching?

We can start by noting that whatever system is used in a school the teacher is assumed to be present. The most extreme exponents of programmed learning threatened to reduce the teacher to a cipher, but with regard to schools this was 1984 stuff and scarcely conceivable. Teachers are plainly of critical importance in caring for and about children; the inspiration, encouragement, control, guidance they provide matters profoundly. Further, no one but the teacher on the spot can perceive and supply the particular needs of a particular child at a particular moment. Neither the teacher's pastoral nor his tutorial functions can be replaced. When we talk about alternative systems of learning, then, we are asking only where, principally, the burden of instruction should rest. "

He then goes on to make the point that the essential differences between teacher-based and media-based instruction is that the accent moves from the "teacher-teaching" to the "learner-learning". Learning occurs at various times but always through learner activity.

"For this sense of activity to be possible, the things I am learning from must be there when I need them. They must have some measure of permanence. Whether contrived or natural they must provide a collection of physically existing, relatively stable objects from which I can learn. On this

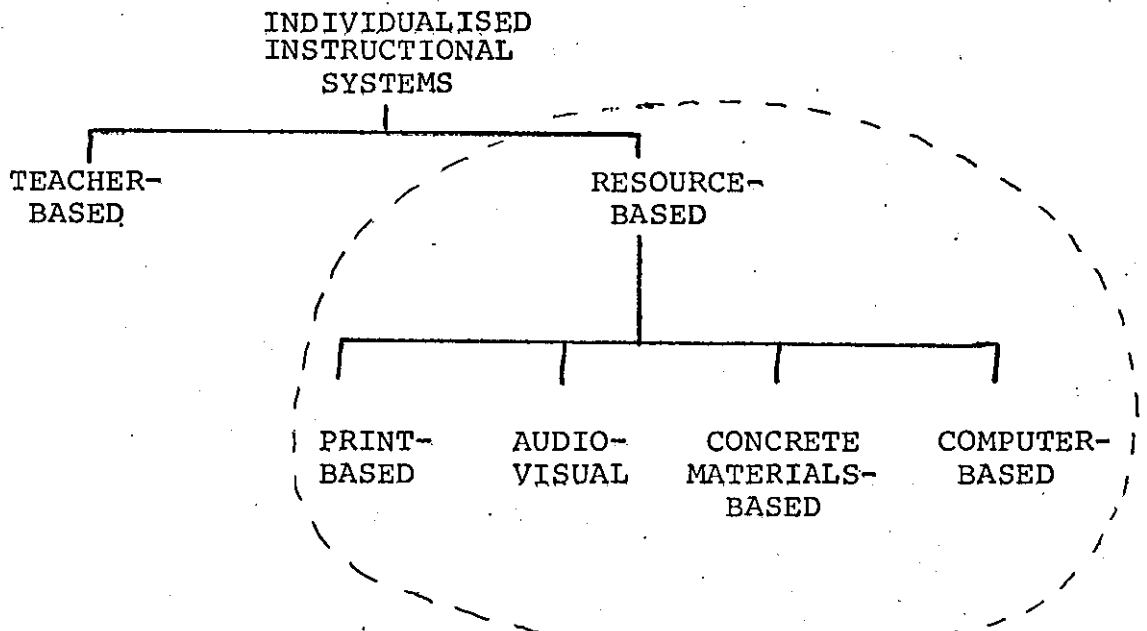
essential common ground, improbably and doubtless to their mutual dismay, would stand Rousseau and Charlotte Mason, Montessori and Skinner. "

One may add Piaget, Bruner, Gagné and Landa to this list, each one justifying inclusion on slightly different grounds, disagreeing perhaps on the exact nature of the materials to be made available, but agreeing on the basic premise that much of learning occurs during contact with resources other than the teacher. Of course the teacher in the "traditional" teaching situation (whatever that is) supplements his presentation with visual aids, refers the learners to textbooks and sets reading assignments and so on. However, he remains the principal medium of instruction, the principal learning resource at the learners' disposal (perhaps that is what we mean by "traditional"). The systems which will interest us in this study however, are the "non-traditional", the systems where the bulk of basic instruction is taken over by resources other than the teacher. Such systems, as Taylor points out,

"are made from the same cloth frayed out into threads or, in the case of the computer, overlaid with the dazzle of electronic millinery. For brevity we can group all these systems together and, since they rely on collections of learning situations and materials, call them 'resource based' or, alternatively, 'package based'. "

5.3 Organisation of the review

However, there are so many different resources now used for instruction and so many different resource-based learning systems in use - even for mathematics instruction - that it will be well to subdivide them. The classification we shall use is shown in the following diagram.



The dotted circle encloses the area on which we shall concentrate this review.

The print-based category includes any system which teaches principally (but not exclusively) by means of printed materials (including pictures and diagrams). It includes programmed learning materials (verbal), reading assignments as in the P.S.I. system, worksheets or workbooks, and of course the old friend the textbook.

These are discussed in Chapter 6.

The computer-based category includes any system which makes extensive use of the computer, either to administer instruction (CAI), or to manage the instructional process (CMI). It should be noted that both CAI and CMI systems make extensive use of the printed word. In the case of some CMI systems, nearly all basic instruction is by book or worksheet, so really they would classify quite well under the previous category. However, it is convenient to consider the category separately in order to study together all aspects of the contribution which computers make to the process of individualisation. This category is discussed in Chapter 8.

The other two categories (audio-visual and concrete materials) will be considered together in Chapter 7. This is partly because there are fewer examples of system types in this category, so it is convenient to combine them into one chapter. Another reason is that many of the systems which use audio-visual learning materials combine them with practical exercises, using concrete materials or "manipulatives". Examples of such are the LAP's from the U.S.A. and the Fife mathematics project in the U.K. Many of them also use printed materials, books, worksheet, workcards, but not as the exclusive or main learning resource. We have therefore termed the combined categories "multi-sensory learning systems".

CHAPTER 6

PRINT-BASED INSTRUCTIONAL SYSTEMS

6.1 Programmed Instruction-Based Systems

6.1.1 Characteristics of Programmed Instruction

Early characteristics. Later research.

The two possible meanings of programmed instruction.

Recent developments in programming styles and principles.

6.1.2 Some Examples

6.1.2.1. The Kent Mathematics Project

6.1.2.2. The Birmingham Scheme.

6.1.2.3. The Continuing Mathematics Project.

6.1.2.4. Programmed Learning at Loughborough University.

6.1.2.5. The I.M.U Project in Sweden.

6.1.2.6. Use of I.M.U. in the U.K.

6.2 Systems not using Programmed Instruction Materials

6.2.1 Programmed Instruction as Product or Process

6.2.2 Individually Prescribed Instruction (I.P.I.)

6.2.2.1. General characteristics

6.2.2.2. Use in Mathematics.

6.2.2.3. Research on I.P.I.

6.2.3 Personalised System of Instruction (.P.S.I.)

6.2.3.1. Characteristics

6.2.3.2. Applications

6.2.3.3. Research

CHAPTER 6

PRINT-BASED INSTRUCTIONAL SYSTEMS

6.1. PROGRAMMED INSTRUCTION-BASED SYSTEMS

6.1.1 CHARACTERISTICS OF PROGRAMMED INSTRUCTION

In the early, heady, days when programmed instruction was still a new-born revolutionary its generally-acknowledged father predicted a rosy a future:

"There is a simple job to be done. The task can be stated in concrete terms. The necessary techniques are known. Nothing stands in the way but cultural inertia."

(Skinner, 1954).

Looking back over twenty-odd years, it would seem that the job was not that simple, the tasks not all equally simple to state, the techniques not all as powerful as predicted - or is it just that cultural inertia is the proverbial "immovable object"?

On the one hand, programmed instruction has not fulfilled its early promise. On the other hand, the influence of the programmed instruction movement in education has gone much further and much deeper than many people care to admit.

Programmed instruction was the first teaching movement to insist on clearly stated objectives (Mager, 1962), Now this is the rule rather than the exception.

Programmed instruction was the first teaching movement to lay itself open to evaluation at every stage - now we have accountability in education.

Of course not all recent developments in educational thinking can be ascribed to programmed instruction. Rather, the necessities of the age fostered its growth and it in turn helped develop techniques to satisfy these necessities. Necessity is the mother of invention - that is why Pressey's (1926) excellent work on self-instruction remained dormant for 30 years. That is why the principles already known to Quintillian and Rousseau and proved scientifically by Thorndike (1927) were not applied earlier. That is probably why early attempts to install individualised systems of instruction at Winneka and elsewhere failed, (Washburne & Marland, 1963).

Now necessity has caught up with the invention, indeed has outstripped the early versions of programmed instruction as it was launched in the 1950's and 1960's. We have new forms of programmes and new forms of programming. Indeed the very term "programmed instruction" has grown to have two quite distinct meanings. To quote Biran (1974)

"In considering the limits of applicability of programmed instruction one must bear in mind the different levels on which this term is used. At the more general level of 'a systematic strategy of teaching, aided by technological aids and psychological tactics', programmed instruction merges into modern teaching methodology, and a separate discussion of its limitations becomes meaningless. At the most restricted level of 'structured, individual self-instruction',

programmed instruction is a teaching technique in competition with others, such as independent learning in small groups, teacher-led instruction, project work, etc. Each of these has advantages, disadvantages and pre-requisites on which a decision about the usefulness of a particular technique in particular circumstances can be based.

These two definitions - the "process" and the "product" definitions highlight the difficulty of defining the characteristics of "programmed-instruction-based learning systems". Do we include only systems which present the learner with self-instructional, structured, individual learning tasks (usually programmed texts), or do we include all systems which have been developed according to the principles of programmed instruction?

One could argue that in the latter case, all individualised learning systems (excluding perhaps those based on small-group-dynamics) should be included. The concept of programmed learning, as held by most "practitioners" has changed much over the years. Leith (1969) summarised research on the "traditional" characteristics of programmed instruction, showing that most of them are either inessential or downright unnecessary. A more recent analysis has been made by Hartley (1974). The extract which follows is an analysis of the trends prepared by the present author and originally published in his book on "The Selection and Use of Instructional Media" (Romiszowski 1974).

Much of the recent research has been summarized by

George Leith in a pamphlet entitled "Second Thoughts on Programmed Learning" (1969).

1. Small steps. Over-use of small steps leads to boredom on the part of the students, more often than not. Current practice favours a step size as large as the student can manage. This is generally established by producing first drafts of programmes which are almost sure to be too difficult for the student, and then simplifying those sections which give students difficulty during validation.
2. Error-free learning. Skinner suggested the maximum permissible error rate on the frames of a linear programme to be in the region of 5%. Subsequent studies have suggested that this figure is not very critical. Current programming practice pays much less attention to error rates within all the frames of a programme, and concentrates instead on analysis of student performance on post-tests and on key criterion frames within the programme.
3. Overt responding. Again, studies have shown that in most instances, students who do not write down responses to a programme's frames, but simply think them out, do just as well on final post-tests and generally take less time in studying the programme. Leith carried out a series of experiments on this factor, which seem to indicate that less mature students

benefit more from responding overtly, and that the benefit is related to the type of subject being studied - e.g. learning to spell is improved if students write their responses, learning concepts is not.

4. Self-paced learning. This was held to be one of the main reasons for the success of early programmes - 'the learner can proceed at his own pace.- The pace he chooses, is often much slower than the pace he could proceed at if he were given some indication of what was expected of him. Today, programmes are sometimes designed with built-in pacing (a characteristic of some tape-slide programmes).
5. Individual learning. Allied to the above point, if self-paced learning is abandoned, one can also abandon individual presentations and revert to group instruction (with a consequent saving of cost on hardware). Experiments have shown no loss of effectiveness for some programmes when used in the group situation, though this depends largely on the subject and the complexity of student responses demand. However, even in complex, structured subjects such as mathematics, benefits have sometimes been gained from allowing students to work together in pairs or in small groups.
6. Programming styles. As already mentioned, programmers now tend to use both linear and branching sequences in the same programme, depending on the learning task they are tackling. There are

also other programming styles in use. Some of these have developed their own jargon and sets of techniques. Examples of some notable current techniques (by no means an exclusive list) are:

- (a) Mathetics - the creation of Tom Gilbert (1962) mentioned in some detail in Chapter 3 of this work)
- (b) Information mapping - the creation of Robert E. Horn (1969), this attempts to obey all the established principles of good communication, transforming them into a complex set of rules for the layout of information in a book. Features include the titling, sub-titling and cross referencing of all blocks of information in an attempt to make the purpose of the information clear and its retrieval easy.
- (c) Structural communication - the creation of Anthony Hodgson (1974). This is a technique based on quite a different approach to learning than the previously described (mainly behavioural models. It does not pre-suppose a correct (or even a limited range of correct) answers. Rather, it is designed to enable the student to respond to a block of information presented to him, in as open-ended a manner as is possible in a self-instructional exercise. The student composes a response from a set of 'building blocks' and the structure of this

response is used to control the subsequent 'discussion' between author and student. Although textbook presentations in this format exist, it is better suited as a technique for computer-assisted instruction.

7. Presentation styles. As already mentioned, 'traditional' teaching machines presenting only verbal information (plus occasional diagrams) are on the decline. There is also a decline in the production of 'traditional' programmed texts. However, audio-visual programming is on the increase, and one can also find examples of programmed film sequences. Group presentations are presented by some teachers, frame by frame on an overhead projector.

The most significant development is the concept of the instructional 'package'. This is an assembly of instructional materials designed around clearly defined objectives, complete with tests and other necessary controls, teacher notes, etc., and utilising the most appropriate instructional media.

8. Implementation techniques. The idea that programmed instruction might replace the teacher, is as dead as the dodo. True, a good programme may be as effective as most teachers in achieving the limited objectives it sets out to achieve. However, there are whole classes of educational objectives which are not suitable for programmed instruction (at any

rate, for 'traditional' programmed texts or teaching machines).

9. The Teacher as manager. It is this role of the teacher which is now being highlighted by developments in programmed learning and in the wider concept of a systems approach. A recent booklet with the above title has recently been published by the National Council for Educational Technology (Taylor 1970), outlining the current trends, the probable changes in the role of the teacher and the necessary changes in both the structure of schools and the structure of teacher training.

10. From programmed learning to a technology of education. Programmed learning has grown, but in its growth it has shed many of the characteristics by which it was earlier characterised. 'Traditional' programmed instruction still flourishes, though the great revolution forecast by Pressey, Skinner, Crowder and others has not taken place. Or has it?

This changing nature of programmed instruction is well illustrated by Fig.6.1, taken from Hartley (1974).

For our purposes, we shall attempt to combine some aspects of both the "product" and the "process" definitions. A programmed-instruction-based learning system shall be one which (1) has an overall structure which incorporates most of the principles listed in the 1968/1974 columns of Hartley's table and (2) employs self-instructional programmed materials (not necessarily texts) as the major (not necessarily only) instructional medium.

The changing nature of programmed instruction (each cell indicates areas of research)

1954	1961	1968	1974
	behavioural objectives task analysis	task analysis behavioural objectives subject analysis	systematic analyses and constraints levels of objectives techniques of structuring material
small steps overt responses reinforcement self-pacing	small steps logical sequencing active responding immediate feedback self-pacing	task structures and related teaching strategies presentation as a communication problem	task structures and related teaching methods, strategies and tactics presentation as a communication problem; problems of learning from text, diagrams, etc.; appropriate instructional decisions individual differences and methods of learning
revision	evaluation (i) revision (ii) comparison studies	evaluation : revision	evaluation of (i) objectives and methods (ii) techniques: retrospective and concurrent decision making evaluation as illumination
			installation and implementation inservice teacher training

FIGURE 6.1 (From Hartley, 1974)

6.1.2 SOME EXAMPLES

6.1.2.1(a) The Kent Mathematics Project

This started as a small one-school project at the Ridgeway School (Banks, 1969).

A course of about 20 to 25 hours' work was first organised to start in November 1966, and tried out on two groups, 3 U1 and 2 U1 (3rd and 2nd year A students - Secondary Modern), working for two of their three one-hourly periods a week. The course was organised so that the students selected their own path through the material conditional only upon the availability of equipment and the sequencing of some of the topics. At the start, the course used programmes, written and taped, worksheets for structural work and exercises on the desk calculator, post-programme exercises, practice exercises on a frequency distribution, work on the spirograph and discovery experiments on the number of squares on a chess-board and the number of cubes in a litre cube, and the relationship between the diagnosis and sides of polygons.

The students were supplied with a sheet of general instructions and a Record of Work sheet and the course organised into tasks which were identified by a cross-reference of a letter and number on the master matrix displayed on the Wall-Board.

Each student was supplied with a blank form of the matrix, to be completed as each task was finished. This was used as a Record of Work sheet and kept in the student's folder.

The task allocated to each cell in the master matrix was designed to take from between 40 and 75 minutes to complete, and the students consulted the matrix to select their next task. When the task was completed, the student's Record of Work sheet was written up by the student with the title, date and time taken and then submitted to the teacher for checking, inserting initials and ticking off on the Progress Chart.

Each programme carried a test and each task was not finalised by the teacher until it had been completed to his satisfaction. There were no queues and students never had to wait longer than a few minutes for any help and the teacher was able to give individual attention to the students all the time.

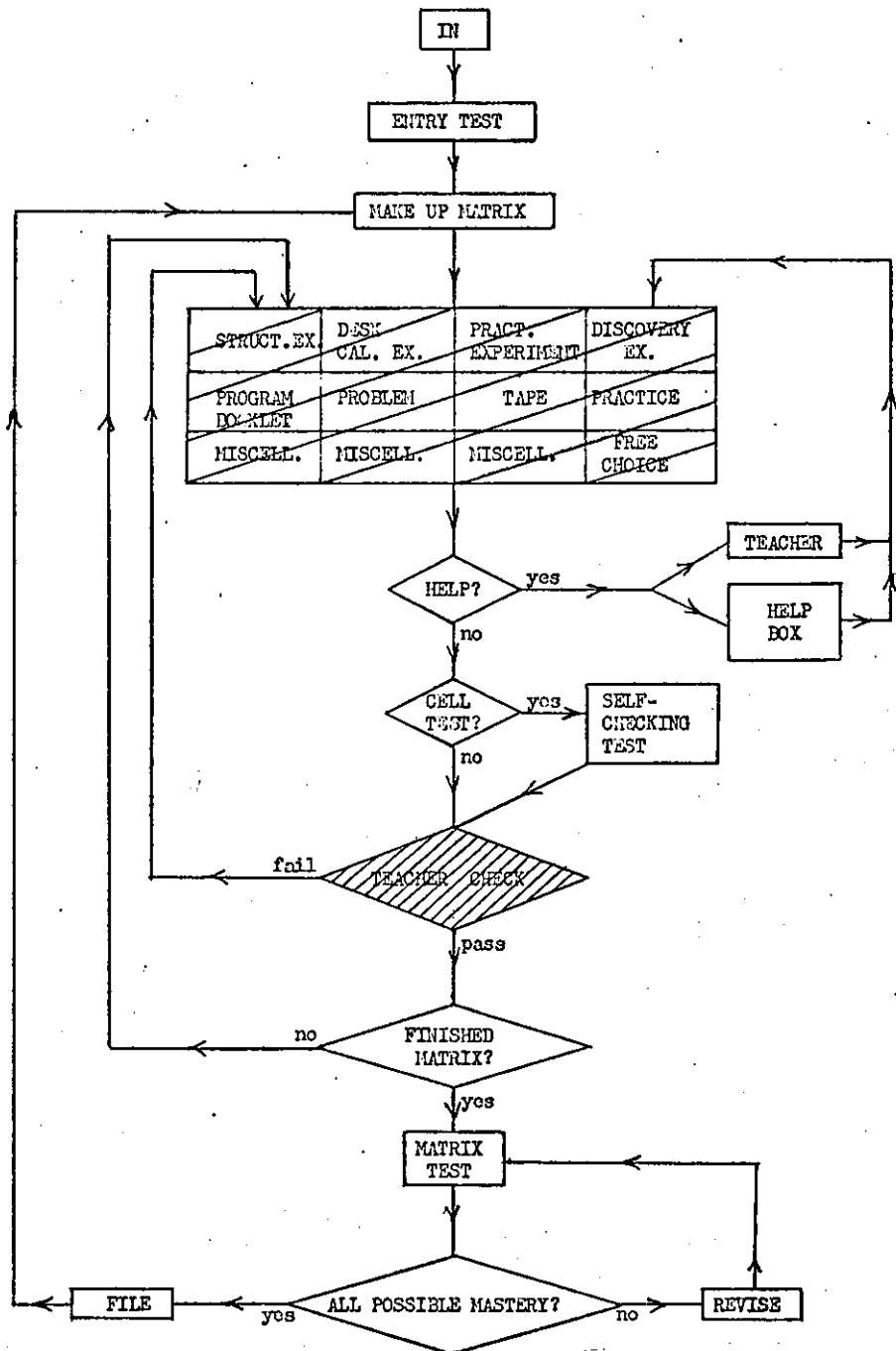
Figure 6.2, overleaf, illustrates the method of operation of the system.

By 1970, fifteen schools in Kent were participating in the project, and also two London Comprehensive Schools, involving in all about 1750 schoolchildren.

"The two experimental schools in London, the Gilliatt Girls' Comprehensive, Fulham, and the Ladbroke Girls' Comprehensive, Kensington, successfully started their First Year students at the end of October, 1969. These two schools entered the scheme at the instigation of Professor Geoffrey Matthews, who developed the work through the Department of Mathematical Education, Chelsea College of Science and Technology.

FIGURE 6.2 (From Banks, 1970)

WEST KENT EXPERIMENTAL MATHEMATICS COURSE



The problems expected from language difficulties because the London schools have a high percentage of immigrant children have not been as serious as was expected, probably because there are tapes for poor readers and much of the material has been designed with a minimum of verbiage. There is also a high motivation factor with the children to master the language they encounter, and the girls in these two schools have worked well enough for Professor Matthews to describe the scheme as a "disaster package". "

(Banks, 1970).

In time the system spread to other schools, was adopted by the L.E.A. and became the "Kent Mathematics Project". (Mathematical Association, 1976). It originated the I.L.E.A. S.M.I.L.E. Project and it is believed that some schools in Birmingham are also developing the scheme from an earlier version. The Project offers a personal course in Mathematics for each student between 9 and 16 years, and uses a material-bank of worksheets, programmed booklets and tapes, organised into mathematics levels and areas.

When completed, the material-bank will contain courses in all 'O' level, C.S.E. and non-examinable topics, systematised so that the teacher can select material according to the ability and special interest of each student. Much of the work is practical, especially in lower levels, and the Project aims to select the best approach of all present-day learning approaches from entirely

open-ended activities to rote-learning, according to the concept being developed. Material is organised into a hierarchical structure of concept-building, and students receive from the teacher a bundle of up to 11 tasks at a time. When tested, results provide the teacher with relevant information about selecting the next bundle to suit the requirements of each individual.

The teacher's role is quite critical in the system. He is not a mere "selector of bundles". Banks explains the teacher's role as follows (Banks, 1971):

Professor Matthews wrote that the scheme is virtually teacher-proof but this is only true to a point. Certainly, as Head of Mathematics in my school, I am happier that all the children are obtaining a Mathematics course that I approve of, and the non-specialist teachers certainly feel that they are painlessly bringing themselves up-to-date in the subject as they help the children. It is, however, at the point in the scheme when the children need help, or receive suggestions for extending their work, or when they need their next matrix made up, that the course becomes different according to the teacher, and some teachers are extracting the best from the scheme, some are not. Nevertheless, it does seem that even if a teacher cannot derive the maximum from the material, the children do better than if the teacher were teaching them in the traditional way, and I suppose it is in these terms that Professor Matthews sees it as a "Disaster Kit".

The Kent Mathematics Project is an exercise in educational technology and uses a totally self-evaluating system. All aspects of materials and classroom organisations are thus validated during development stages.

The evaluation and student assessment procedures are very thoroughly worked out (Banks, 1972).

6.1.2(2). The Birmingham Scheme

The starting point of this project was the identification, by the City of Birmingham Education Authority, of a potentially serious shortage of specialist mathematics teachers, especially in the non-selective secondary schools. The mathematics inspectorate realized that such a shortage would mean that much mathematics teaching would need to be done by non-specialists and decided to provide as much help for these as was possible.

The St. Albans Programmed Learning Centre was initially approached in the Spring of 1970 to advise on the provision of suitable learning programmes for issue to schools.

This centre, rather than simply suggest some existing materials, drew up proposals for a project to produce a special-purpose system of individualised mathematics (Davies and Needham, 1972).

- "1. Groups of maths teachers to be formed, to try to draw up agreed schemes of work in the form of syllabi, and to examine existing programmed material for suitability of content and approach.

2. Selected maths teachers to be given a minimum training in the design and production of teachers' guide notes, test construction and production of student work material matched to learning objectives.
3. A small production facility to be provided, this to be extended if the scheme proved viable; either one full-time or two part-time schools liaison officers to be provided to oversee the development of the scheme, with provision for a full-time coordinator.
4. Material to be provided in the form of strongly teacher-supportive 'tests' each containing three or four units of work material ('packages' to cover approximately six weeks' work on various aspects of mathematics."

Several working parties were set up to implement the proposals. A series of self-instructional packages was produced in 1970/71 (Mathematical Association, 1976).

Each package contains a teacher's guide giving the minimum objectives of the package, suggested lesson plans, notes about the pupils' worksheets and any relevant background information as well as a copy of each worksheet and answers. The boxes contain enough work material for a class of thirty pupils. Also included are suggestions for further work for quicker pupils. The first packages were delivered to schools in January, 1971.

The Scheme has now a Management Committee and a small permanent staff but volunteer teachers are still closely involved. There are sufficient packages to cover the first

two years of a secondary course and a start has been made on producing material for the third year.

6.1.2.3. The Continuing Mathematics Project

The Continuing Mathematics Project is unlike the Kent or Birmingham schemes in that it is not based in certain schools but is a national project, sponsored by the Council for Educational Technology, the Department of Education and Science, the Schools Council and many other bodies. It is a new project, not yet fully implemented. Its purpose is as follows (Schools Council, 1975);

"It is now generally accepted that students entering tertiary education, over a wide range of disciplines, require some familiarity with mathematics and its uses in their chosen fields. However, there are insufficient mathematics teachers in the schools to satisfy the demand from both 'A' level mathematics specialists and from non-specialists who require mathematics as a background to their own subjects.

It is the purpose of the Continuing Mathematics Project to point the way to some alleviation of this problem. By devising material which a Sixth Form student can use independently of a specialist mathematics teacher, it is hoped that any student needing some understanding of mathematics for his main subject, be it Biology, Economics, Geography, can at least start on the road that he will need to follow when he takes his subject further after school. By working independently the student will not impose further strain on the small number of mathematics specialist available."

The materials mentioned have been developed at Sussex University and are currently being prepared for printing and distribution to schools. Prior to this, all the materials were, of course, tested and revised as necessary.

While the views of 'experts' in mathematics and educational technology are sought where possible, the main evaluation activity has involved the help of teachers, lecturers and students in trial institutions.

Following revision after developmental testing, units have been tested in schools, colleges and universities in Britain. Over 50 institutions have taken part since 1972. Students and tutors have provided valuable information by means of comment sheets, questionnaires and discussions with Team members.

In practice, the materials will be used for independent study with the minimum of teacher involvement. The project makes a careful distinction between independent study and individualised study.

"Individualised and independent study are often confused. In individualised study the student has the support of a teacher familiar with the material the student is studying. The learning proceeds at the pace of the individual student, but is facilitated by the helpful intervention of the teacher.

In independent study, as the Project understands it, the student is largely responsible for his own learning. A tutor will be present to provide general guidance through the course of material, but will not be there to help him with specific

points of learning. The Project's independent learning is so devised that it is not necessary for the teacher to be a Mathematics specialist."

The unit structure of the material and the definition of three broad types or categories of units, allow flexibility of use with a wide range of students having differing needs and interests. Students might use single units to fill in a gap in their knowledge or might use a sequence of related units to meet some specified need of their specialist subjects.

Usually, the tutor's role will be that of adviser rather than teacher. At the beginning of a course the tutor will advise the student on the units which best fit his needs and interests. He will also suggest a route through the materials and help the student establish a timetable aimed at ensuring completion of the course in a reasonable time.

6.1.2.4. Programmed-Learning at Loughborough

The Centre for the Advancement of Mathematical Education in Technology has for some years been producing and encouraging the use of a series of programmed mathematics text books for Engineers. Titles include;

Ordinary Differential Equations (Bajpai et al 1970)

Fourier Series and Partial Differential Equations (Calus et al 1970b)

Algol and Fortran (Bajpai et al 1972a), Mathematics for Engineers and Scientists, Vols. I and II (Bajpai et al 1972b).

These texts are used by many universities and technical colleges, in the U.K. and overseas.

At Loughborough, programmed texts in mathematics have been used with applied science and engineering students in two categories of situation:

1. remedial work with individual students;
2. normal coverage of parts of the first and second year undergraduate courses.

In category (1) would be included the case where a student has missed a section of the course, because of illness or other reasons, or where a postgraduate student finds his undergraduate course has not given him the particular mathematical background which he requires. Although the number of students in this category is relatively small, it must be remembered that to each such student his problem is large and a programmed text offers a solution to which there is no comparable alternative. In this type of situation a programmed text is obtained on loan through personal contact between student and lecturer.

The use of programmed texts as a normal part of courses occurs in a variety of ways. Bajpai and Calus (1970c) describe the use as follows:

"In undergraduate courses our programmes have always been used in conjunction with lectures and tutorials. Although the texts have been written so that they are complete in themselves, and can therefore be used in the remedial situation already described, we do not aim at replacing lectures entirely by programmed instruction. Certain aspects of a mathematical topic are better dealt with in a lecture, other aspects lend themselves particularly well to being programmed. The lecturer can inspire, excite and motivate where the printed word will not, especially if he makes use of visual aids. For instance, analogue and

digital computer demonstrations can be used in lectures to introduce the concept of a differential equation and its solution, as described in an article in the new International Journal of Mathematical Education in Science & Technology. On the other hand, when the task is to go through the various forms of the trial solution for finding the particular integral of a second order differential equation, or to obtain the Laplace transforms of some standard functions, the programmed text has its advantages. Group tutorials give students the opportunity to discuss any difficulties which may have arisen in learning from the texts. Some experiences with undergraduate students (Hogg, 1967; Unwin & Spencer, 1967) have indicated that use of programmed texts under supervision is both unnecessary and undesirable. Such students usually prefer to be on their own when they are working, without someone looking over their shoulder. In any case, we would consider it wasteful of staff time to have a member of staff in attendance while university students work through programmes, though we can appreciate that in other situations it may be necessary. Furthermore, Loughborough students are often given a programmed text as a vacation task."

Another way that has been employed is to use the programme, in sections as the study units of a PSI Keller-Plan-Type of course. This application is fully described in a paper which is appended to this study (Romiszowski, Bajpai & Lewis, 1976)

6.1.2.5 The I.M.U. Project in Sweden

The I.M.U. project is perhaps the best documented and most researched system of individualised mathematics instruction in Europe - certainly at the secondary level. The project commenced in 1964 on a pilot basis and was extended to many schools during 1968-71 (Larsson 1973).

In the autumn of 1963 a study commissioned by the National Board of Education was set up with the purpose of comparing the effects of completely individualized teaching and conventional teaching in mathematics in grades 7 and 8. The results of the first year of the experiment were such that it was considered worthwhile to follow up the work. Therefore in the autumn of 1964 the National Board of Education started the I.M.U. project. The original study was incorporated into the project in the form of a preliminary study.

When the project started, the aims were formulated thus:-

- to construct and test self-instructional study material in mathematics
- to test suitable teaching methods and schemes for the use of this material
- to try out different ways of grouping the pupils and making use of the teachers
- to measure by means of the material constructed the effects of individualised teaching.

The experiments that have been carried out in the middle level of the comprehensive school and the upper secondary school cover to a varying extent the factors mentioned above.

Various versions of the materials used in the project were prepared between 1964 and 1968. The third version (referred to as the I.M.U. Upper Level, version 3) was extensively tested in a series of thorough studies summarised by Larsson. Materials revision has continued and today most materials are in a fifth version. The materials are programmed texts, and printed tests. No manipulative materials were prepared, although, due to the great flexibility and high level of individualisation of the scheme, there is nothing to prevent individual teachers from introducing such additional materials if they so desire.

Larsson describes the system as follows:

" The principle behind the model for I.M.U. Upper Level is that there should be no grade differentiation and no division into general and special courses. Instead the material is built up of nine units, which together cover the upper level course in mathematics. Starting with a common curriculum for all pupils, the subject material is then structured according to the degree of difficulty within each module. Fig.6.3 outlines the principles on which a module is based. "

FIGURE 6.3 Diagram of the structure of a typical module of the I.M.U. programmed mathematics materials.
(Adapted from Larsson 1973)

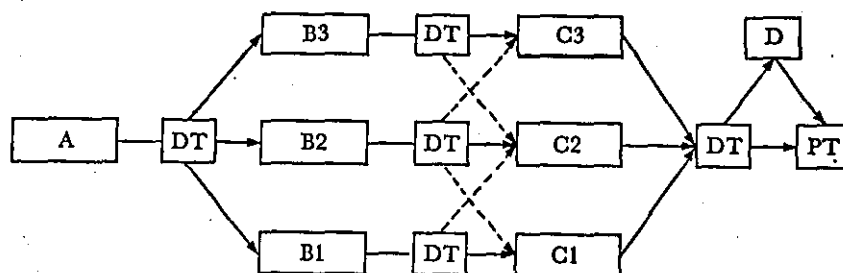


FIGURE 6.4 Summary of the ten research studies into the I.M.U. materials reported by Larsson (1973).
(Continued on next page.)

Study	Category	Number of indiv.	Measur. occasion	Instrument/technique
1. Main investigation	pupils Random sample from the population of IMU pupils of approx. 12,000	1008	August '68	standard tests in mathematics and Swedish marks in mathematics and Swedish attitude schedule
			December '68	marks in mathematics questionnaire
			May '69	achievement tests in mathematics marks in mathematics questionnaire attitude schedule
			December '69	marks in mathematics
			March—April '70	achievement tests in mathematics standard tests in mathematics and Swedish marks in mathematics questionnaire attitude schedule
	teachers All teachers who taught with IMU in the experimental classes during the experimental period	294	December '68	questionnaire
		295	May '69	questionnaire
		293	March—April '70	questionnaire
	assistants All assistants who worked with IMU in the experimental classes	67	December '68	questionnaire
		66	May '69	questionnaire
		68	March—April '70	questionnaire

FIGURE 6.4 (continued)

Study	Category	Number of indiv.	Measur. occasion	Instrument/technique
2. Goal testing study	pupils All IMU pupils who studied mathematics in grade 9	8600	May '71	goal tests in mathematics
3. Material study	pupils Random sample from the population of IMU pupils of approx. 12,000	374	Continuously during experimental period 1968—1971	diagnostic tests prognostic tests questionnaires
4. Job analysis study I	school principals teachers assistants	32 40 14	Spring term '69 and Autumn term '69	personal interviews
Job analysis study II	teachers assistants All teachers and assistants who worked with IMU during the school year 1970/71 in grades 8 and 9	358 82	May '71 May '71	questionnaire questionnaire
5. Parent study	parents Parents of the pupils who participated in the material study	364	May—September '71	questionnaire
6. Observation study	teachers assistants pupils All teachers and assistants and a sample of the pupils working at 5 experimental schools and at 3 control schools	19 5 468	school year 1970/71, five observation periods	observation schedule
7. Study of anxious pupils I	pupils Random sample of IMU pupils and a group of control pupils	649 (IMU) 839 (control)	September '68 March—April '70	intelligence test marks in mathematics school motivation test anxiety scale school motivation test anxiety scale
Study of anxious pupils II	pupils	96	October '70	achievement test in mathematics marks in mathematics school motivation test anxiety scale attitude test
8. Study of different ways of presenting mathematical material	pupils Seven small part-studies of pupils not normally working with IMU	449	Spring term '68 to spring term '71 (Seven occasions)	intelligence tests achievement tests
9. Work of project consultants	principals teachers assistants		Continuously during the school years 1969/70 and 1970/71	personal interviews visiting diaries
10. Study of single pupils	pupils teachers	80 154 60 76	Spring term '70 Spring term '71 Spring term '70 Spring term '71	questionnaire questionnaire questionnaire questionnaire

Each module comprises four parts, components, the first three of which belong to the basic course. They are called components A, B and C. Component A is common for all pupils. The B and C components are divided into levels of difficulty, hereafter called booklets. For the B component there are 2 or 3 booklets, called B1, B2, B3 or B1, B2 - 3. The C component comprises 3 booklets, C1, C2 and C3. The degree of difficulty is easiest in the B1 and C1 booklets, while B2 -3 or B3 and C3 are the most difficult. The different booklets cover roughly the same material but the way in which the instructions are presented and the number of extra tasks vary. The D component is not part of the basic course. It exists only on one level and comprises both revision tasks and certain tasks of a more independent nature. Each component except the D component is completed with a diagnostic test (DT).

The material is individualised in both the rate of work and the "degree of penetration". As a result pupils within one grade can reach different points in the material. The intended "normal rate of study" is three modules per year.

In principle the pupils are free to choose which booklet they like. The idea is that the pupils should together with their teacher go through what they have achieved earlier and on the basis of this and other experiences choose a suitable level. It is possible and permissible to change level both within and between modules. The constructors of the material have indicated certain figures for guiding the spread between the different levels in a

component, but neither pupils nor teachers are obliged to follow these figures.

A very comprehensive programme of research was executed during the years 1968-71. (See Figure 6.4.)

Upwards of 8,000 students took part in the studies, nearly 500 teachers and other staff and nearly 400 parents. The results of the 17 reports (mainly in Swedish) are summarised by Larsson (1973).

This project, though highly individualised, is not individualised in all aspects. A model for the degree of individualisation was drawn up in the plan of the project (reproduced in Figure 2.6 and the desired degree of individualisation was carefully defined for each parameter in the model (Larsson 1973). (See Chapter 2).

The method manual states the desired degree of individualisation with regard to goals and course:

"the fact that the pupil participates in setting the goals and planning the work and thereby not only sees the goals clearly but feels them to be his own personal goals should lead to better motivation and more purposeful work. The reason why we are not attempting to achieve total individualisation in the planning of goals and syllabus is that the demand made in the curriculum for individualisation is balanced by the demands for an extended common frame of reference. "

The actual degree of individualisation used is very close to the desired level. The number of routes through I.M.U.'s nine modules are legion. In principle it is possible for the pupils to choose their level not only between components but also within them, so that some sections are taken from a lower level, others from a higher one. In practice, however, individualisation has not been pressed so far. According to the data of the material study, the pupils work right through a level that they have chosen (or been advised to choose). Thus in that respect I.M.U. admits a greater degree of individualisation than has been utilized. Individualisation is not only made possible by the choice of different levels and the different goals that exist for them, however, since within the material sections can be shortened or expanded through the intervention of the teacher and this is in fact what has happened. The pupils have also to a great extent been given the opportunity of controlling their choice of level themselves, as can be seen in Fig.6.5. The table shows the pupils' answers to the question: "What happened when you got the booklet you are working on now?"

FIGURE 6.5 Pupils' choice of booklet, spring 1970, grade 8.

Alternatives	no.	%
I chose it myself	401	39.8
I talked to the teacher and then chose it myself	229	22.7
I talked to the teacher and then he/she chose it for me	62	6.2
The teacher chose it for me	222	22.0
Other alternatives	94	9.3
Total	1008	

63 per cent of the pupils say that they chose the booklet themselves, 23 per cent after having spoken to the teacher, 28 per cent say that the teacher chose the booklet, 6 per cent after having spoken to the pupil, and the remaining 9 per cent gave other alternatives which mainly consist of the answer "was to have Booklet A", which makes the question of who chose insignificant.

The desired degree of individualisation with regard to instructions is:

"if the same instructions are to be given to several pupils, the teacher should make use of group teaching. Group activities can also be valuable from the point of view of motivation. Thus the aim should not be to achieve total individualisation here, either."

The actual degree of individualisation can be said to be greater than the desired degree. Data from both teachers and pupils show that group teaching and group activities occur less frequently than was intended, not least as a means of creating motivation.

On the subject of the distribution of tasks, the method manual gives total individualisation as the aim.

The actual degree of individualisation cannot be said to be total. Since the pupils choose certain levels within the components, either themselves or in consultation with a teacher/teacher assistant or is directed by a teacher, the tasks will vary both in number and degree of difficulty. The difficulty of the material is not so highly individualised, however, as the rate of working.

Concerning methods and rate of work, the method manual states as follows:-

"the pupils is quite clear as to the goal and if he is equipped with aids that enable him to evaluate his achievements continuously, it should be possible to train him to direct his studies himself. Teaching him this is in itself a goal, but the self-direction should also lead to more effective work.

Who knows best which is the optimal rate of work, how much repetition is necessary and so on? Often it is probably the pupil himself."

The actual degree of individualisation probably approaches the total level for certain groups of pupils, at least as far as rate of work is concerned. For quick pupils the individualisation of the rate of work has been as good as total, but for the slower pupils, however, the freedom to choose methods and rate of work themselves has had to be limited in order to prevent the spread of the material becoming too great, that is to say allowing too many pupils to lag behind in the material.

Finally evaluation and marking have been discussed in the method manual:

"School marks have two functions:

- a) to stimulate the pupil during his education and facilitate his and the teacher's planning of his future studies. These purposes are best served if the marks are related to the goals. Goal-related tests and mark-setting based on such tests mean that the pupil is always competing with an opponent of equal merit (himself). This should stimulate all pupils into making greater achievements in their work. Tests related to norms are of little help in planning and are probably in the long run only stimulating for the best pupils.

b) to serve after the completion of the education as an instrument of selection. As far as the comprehensive school is concerned, marks need only be given for this purpose at the end of grade 9. Even as means of selection, a full description of the pupil's achievements in his school work, based on goal-related tests, should be preferable to the norm-related marks we use now. These provide only an approximate ranking of the pupil's achievements."

The actual degree of individualisation with regard to evaluation and marking has been at the end "no individualisation". Only norm-related marks have been used. Setting of marks are made in reference to course choice although no division is made of common and special course in the teaching situation.

From the above descriptions we can see that the I.M.U. project, despite the use of "prescriptive" learning materials (i.e. programmed texts) was never designed to be a totally prescriptive "Mastery-Learning" system (in the sense that all students are expected to master all objectives and progress is controlled by mastery). It had however planned to incorporate one aspect of the mastery learning model (i.e. criterion-referenced evaluation procedures) but in practice had not done so.

6.1.2.6 Use of I.M.U. in the U.K.

The I.M.U. materials have been translated into English, and several schools have experimented with their use under the auspices of the Nuffield Resources for Learning Project. Reported results (somewhat subjective) are generally favourable. (Williams 1973; Senior, 1972).

In some schools the students liked the materials very much and wished to continue with the materials into the second year.

The teachers were generally favourably inclined but would have preferred a textbook and/or workbook format to the programmed text format, in order to facilitate ancillary classroom work.

Both students and teachers favoured a greater proportion of classroom group work than the system had forseen (this was also true in the Swedish experience). It was difficult, without extra teacher-led work, to complete the modules in the pre-planned time.

6.2 SYSTEMS NOT USING PROGRAMMED INSTRUCTION MATERIALS

6.2.1 Programmed Instruction as Product or Process

The systems to be described in the remaining part of this chapter are print-based instructional systems - that is, the principal medium of instruction is the printed word, in the form of books, workcards, etc. However, these systems do not use strictly "programmed" materials, in the sense of linear or branching programmed texts.

However, the systems all follow certain, if not all, the principles of programming. The materials generally have built-in self-tests. They have generally been tested and revised empirically prior to large scale implementation. They generally follow the mastery model, for teaching to a pre-determined criterion.

Indeed the two major systems which we shall now examine - I.P.I. and P.S.I. - have both evolved out of the programmed instruction movement. They are adaptations of the programmed instruction model to the particular needs of young school children (in the case of I.P.I.) and adult undergraduates (in the case of P.S.I.).

Although they do not (as a rule) use programmed instruction materials, both these systems have incorporated in their philosophy many of the principles of programmed instruction. Thus, if one were to adopt the "wider" definition of programmed instruction, given by Biran (1974), we could well include these systems within the previous class, as products of the same general programming process. In the words of Susan Markle (1970) "programmed instruction is a process" capable of producing a variety of products.

However, we are adopting here Biran's "narrower" definition of programmed instruction as "individual self-instruction from (mainly) printed materials, demanding frequent student responses". This is a "product" definition as opposed to Markle's "process" definition.

The examples given earlier in the chapter were of projects or schemes all based on extensive use of programmed instruction materials - all examples (drawn from mathematical education) of the general "programmed instruction system". The two systems which follow, I.P.I. and P.S.I., qualify, in the author's opinion, to be considered as "general" systems. Despite the similarity of origins, they are implemented in quite different ways. They have been applied to a wide variety of subject areas (including mathematics) and at various educational levels. They have their own literature and research. The I.P.I. system has perhaps the widest application of any system for the individualisation of mathematics instruction (though mainly in the U.S.A. and at elementary level). The P.S.I. system is possibly the "fastest growing" system of individualisation (particularly at post-secondary levels).

We shall therefore examine these two systems in greater detail in the remainder of this chapter.

6.2.2. Individually Prescribed Instruction (I.P.I.)

6.2.2.1. General Characteristics

The I.P.I. system was devised at the Learning Research and Development Center (LRDC) University of Pittsburgh, commencing in 1963 (Scanlon, 1970). Since then it has been adopted by a progressively growing number of school boards (Scanlon, 1969; Papiernick 1970; Research for Better Schools, Inc. 1966, 1968 and 1969). It is applied to most subjects of the American elementary school curriculum, including mathematics.

The LRDC, in conjunction with Research for Better Schools, has produced and disseminated a vast amount of learning materials, teacher guides, descriptive articles and teacher reports (Research for Better Schools, Inc. - "Degree of implementation" reports - every semester since fall 1968; Lindvall and Bolvin 1968 - teacher preparation; Lindvall and Cox 1969 - plan for evaluation; Wine and Fisher 1969 - Audiovisual materials for mathematics; Lindvall 1968 - planning objectives; Lindvall and Nitko 1969 - criterion referenced testing in I.P.I; and many others.)

A unique feature of I.P.I. is its requirement that each pupil's work be guided by a written Prescription prepared to meet his individual needs and interests. The Prescription is a link between pupil and teacher. The teacher communicates to the pupil the choices made to achieve an objective. Information about the pupil and his progress is communicated to the teacher. The data to be used for Prescription writing should include:

(1) general ability level in the given subject, (2) the degree of mastery or lack of mastery in each skill in the particular learning unit to which the pupil is assigned, (3) information on progress in previous units directly related to the skills in the current unit, (4) detailed information on pupil progress, including test results, in his current unit, and (5) general learning characteristics of the pupil as they relate to specific assignments.

The pupils proceed through the prescribed curriculum with a minimum of teacher direction and instruction. The teacher gives assistance only when requested. This frees the teacher for instructional decision making, tutoring and evaluation of student progress. The marking of tests and the tabulation of student data are done by teacher aides, or by the pupils themselves.

Inherent in the I.P.I. design is its capability for progressive adaptation. In this respect the availability accuracy and format of the Prescriptions are crucial. With the information they contain the teachers can organise the classes for small/large group instruction or for individualised tutoring, and can strengthen the instructional procedures, as required, in relation to each individual.

6.2.2.2. Use in Mathematics

In I.P.I. Mathematics, the total curriculum is broken down into ten content areas; Numeration/Place Value, Addition/Subtraction, Multiplication, Division, Fractions, Money, Time, Systems of Measurement, Geometry and Applications. Each content area is broken down into levels of difficulty. When put into a matrix format, each cell represents a unit.

This matrix is presented in Figure 6.6 (Hosticka 1972).

Figure 6.6. Diagram of Units.

	A	B	C	D	E	F	G
Numeration/ Place Value	X	X	X	X	X	X	X
Addition/ Subtraction	X	X	X	X	X	X	X
Multiplication		X	X	X	X	X	X
Division		X	X	X	X	X	X
Fractions	X	X	X	X	X	X	X
Money	X	X	X	X			
Time	X	X	X	X	X		
Systems of Measurement		X	X	X	X	X	X
Geometry		X	X	X	X	X	X
Applications		X	X	X	X	X	X

X = Units with objectives.

Each unit is composed of a series of objectives stated in behavioural terms. These objectives are ordered with prerequisite objectives defined in the unit, so that it is not necessary to proceed from the first objective in a unit to the last in an ordinal sequence.

Once the content has been defined and sequences for a curriculum established, the correct placement of the child within the content and sequence is necessary. In I.P.I. this is done by a series of criterion-referenced tests.

The child must have 85 per cent of each test correct in order to be considered to have mastery of the items on the test. To be placed in the curriculum, the student takes a placement test at a level selected by the tester. The test is subdivided into units and selected skills. For each section (or unit) at the level of the placement test in which the student receives 85 per cent or better, he is tested at the next higher level in that area; for each sub-test score of 20 per cent or under, the student is tested at the next lower level in the area; for all scores greater than 20 per cent and less than 85 per cent (20 per cent $< X <$ 85 per cent) the student studies the unit tested.

The materials for each skill are varied. The constant set of materials is the Standard Teaching Sequence Booklet. This booklet is set up in a quasi-program format. Some skills, especially at the lower levels, include lessons that are on taped discs or cassettes, for those students with reading limitations. Lessons that use concrete materials are now being developed for the beginning lessons.

The prescription for each student tells the order in which he will proceed through the objectives in the unit and what work he will do for each objective. If a teacher judges from test information that a student knows most of the information and only needs some quick review to master the objective, the prescription may only consist of two or three pages in the Standard Teaching Sequence Booklet. If, on the other hand, the teacher feels that the student needs a great deal of concept development in an objective, the teacher may assign work with concrete materials, arrange to have a small-group seminar, use taped materials, work on all pages in the Standard Teaching Sequence Booklet, or

work with a combination of any of these materials.

As a child proceeds through a lesson, he is again tested to see if he has reached mastery. These tests are embedded in the teaching materials and are called Curriculum Embedded Tests (CETs). When a student receives mastery on a CET, he proceeds to the next skill to be mastered in the unit. If he receives a score of 85 per cent on the post-test, he proceeds to the next unit. If not, he goes back and continues work on the skills in which he is weak within the unit.

All tests used in the curriculum are parallel tests. The items are designed to be of equal difficulty on each test and to cover the domain of the objective.

6.2.2.3 Research on I.P.I.

Despite its widespread implementation, the research evidence on the efficiency of I.P.I. is not too plentiful. The research carried out by the LRDC and its collaborators has been chiefly formative research, aimed at improving the materials and systems of I.P.I. (Lindvall 1968). Less attention has been devoted to comparative or summative research aimed at evaluating the system as a whole. The various school boards using I.P.I. have published evaluation reports often anecdotal and usually very favourable towards I.P.I. The few documented and controlled studies are not necessarily that favourable. A favourable picture is painted by Johnson, of the system installed in the Minneapolis Public Schools. He gives the results of four years of experience with I.P.I.; concentrating especially on the mathematics learning of disadvantaged groups. He finds that such groups "gained mathematics skills as rapidly as average students throughout the county

and much more rapidly than students in three comparable low-income schools". (Johnson and Ostrum, 1971; Johnson 1973). Gaj were also registered for "average" children, though these were not as significant as for the disadvantaged groups (Johnson, 1972).

Humphrey (1968) reports more favourable attitudes to I.P.I. courses (particularly to mathematics) as compared with non-I.P.I. control groups. In the case of mathematics the positive attitude was relatively independent of the student's self-concept of his own mathematical abilities (not the case for other subjects).

Weinberger (1969) reports good retention over the summer recess of mathematics learned through I.P.I. Booser (1968) reports only a very weak relationship between I.Q. and performance on an I.P.I. course. This would seem to suggest that the theoretical aim of the mastery-learning model (to render achievement independent of ability by varying study time) is at least in part attained.

By contrast however, there are also many not-so-favourable reports. Gallaher (1968) using a well controlled experiment with groups of over 100 students, found that although there was a significantly more positive attitude towards arithmetic among I.P.I. students, there was no significant difference in arithmetic achievement between I.P.I. and non-I.P.I. groups. This was just as true for disadvantaged children as for the "others". Lewey (1969) in an unpublished report (available through the ERIC files) also found no significant differences. The "Second Year Evaluation" of I.P.I. carried out by Research for better schools (1969) found that in general "measured achievement

in I.P.I. is less than that observed in non-I.P.I. classes.

Most disturbing of all, Schoen (1976) has located 36 comparative studies of self-paced mathematics programmes among unpublished doctoral dissertations. Of these systems, 15 were of the I.P.I. system. Figure 6.7 (from Schoen 1976) summarises the results. With the very young age-groups, the results are inconclusive, but with grades 5 to 8 there appears to be a definite advantage to the "traditionally" taught control groups. The I.P.I. system did not fare any better in this comparison than did the other various researcher-developed systems.

Finally Miller (1976) in a more extensive review, going back to 1960 and beyond, located 145 comparative studies on individualized systems of instruction. 88 of these were similar in structure and were therefore compared for achievement as shown in Figure 6.8. The 39 elementary programmes include 31 applications of I.P.I. From the table, we may deduce a somewhat more positive situation with respect to individualisation of mathematics in general (though not an extremely positive situation). However, the elementary level, dominated by I.P.I., gives a less positive picture than the secondary and higher levels - so much so that Miller, among his suggestions for further research includes the following:

"Follow-up investigations on the I.P.I. program should be done to determine if the program has improved."

Schoen (1976) is suggesting that it has not.

FIGURE 6.7 (a) Summary of 36 studies comparing individualised instruction and "traditional" group instruction in arithmetic in the elementary grades of American schools. (Reported by Schoen, 1976a).

		INDIVIDUALISED SYSTEM	
		I.P.I.	RESEARCHER-DEVELOPED
GRADES to 4	INDIV. BETTER	2	3
	CONTROL BETTER	2	3
	NO. SIG. DIFFERENCE	4	4
GRADES 5 to 8	INDIV. BETTER	1	2
	CONTROL BETTER	5	7
	NO SIG. DIFFERENCE	1	2
TOTAL OF STUDIES		15	21

(b) Summary of 12 studies at secondary level mathematics (Schoen 1976b)

INDIVIDUALISED BETTER	CONTROLS BETTER	NO SIG. DIFFERENCE
1	3	8

FIGURE 6.8 Summary of 88 American comparative studies of individualised mathematics instructional systems.
(Reported by Miller 1976)

	ACHIEVEMENT ON TEST	ATTITUDE TO COURSE
INDIVIDUALISED SIG. BETTER	18	3
MIXED RESULTS SOME SIG. FAVOUR INDIVIDUALISED	14	4
NO SIGNIF. RESULTS	42	25
MIXED RESULTS SOME SIG. FAVOUR CONTROLS	10	0
CONTROL GROUPS SIG. BETTER	4	1
TOTAL OF STUDIES REPORTING	88	33

6.2.3 PERSONALISED SYSTEM OF INSTRUCTION (PSI)

6.2.3.1 CHARACTERISTICS

The Personalized System of Instruction gets its name from the fact that each student is served as an individual by another person, face-to-face and one-to-one, in spite of the fact that the class may number 100 students. It is suitable for courses from which the student is expected to acquire a well-defined body of knowledge or skill - the majority of college mathematics courses. The PSI teacher expects almost all of his students to learn his material well and is prepared to award high grades to those who do, regardless of their relative standing in the class. He accepts the responsibility of meeting this goal within the normal limits of manpower, space, and equipment.

This method is associated with the name of Fred S. Keller (1968) who with J.G. Sherman, R. Azzi, and C.M. Bori devised it in 1963 to meet the needs of a new psychology programme in a new university in a new city in the interior of Brazil. It resembles other forms of individualised instruction which date from the late nineteenth century, although most predecessors were of interest only in "lower education". Postlethwaite's Audio-Tutorial method has many elements in common with PSI. There are others with good claim to priority concerning particular features of the method.

The main features of the Keller plan include (Keller, 1967, 1968):

- (a) Individual study units, usually written matter,

which may be (but need not be) specially produced for the course, and may (but certainly need not) be in programmed instruction form.

- (b) Self-tests which the student attempts and then discusses with a proctor or monitor.
- (c) Study guides in the form of detailed objectives, cross-referenced to the reading and practical assignments.
- (d) Individual and/or group practical work and discussion controlled by specially written guide notes.
- (e) The role of the teacher is mainly that of a manager of the system. He has monitors to help him in assessing and tutoring the students. The monitors may be special staff but are often more advanced students who are given the responsibility for the progress of the slower ones. They usually have a monitor's guide book to help them. The teacher evaluates overall progress and revises the course materials, but he does take on monitoring when necessary.
- (f) The teacher also gives a certain amount of face-to-face classes, but these concentrate on enrichment of the course and students have to 'earn' the right to attend these classes by reaching proficiency in certain sets of objectives.

(g) Proficiency in most Keller courses is taken to mean 100% on each study unit test before moving on to the next one.

The system was first introduced experimentally in the psychology department of the University of Columbia in 1963, and in 1964 was installed at the University of Brasilia (Azzi, 1964, 1965). Since then, use of the plan has spread to other universities and institutions in the U.S.A. and also in Brazil, and has been applied to subjects other than psychology, for example, physics at MIT (Green, 1971).

Although the Keller plan is the name now in vogue, other attempts were being made to overcome some of the shortcomings of early programmed instruction courses; for example in the U.K., Croxton and Martin at the University of Aston were attempting to increase the adaptive nature of programmed courses by roughly similar methods, although also utilizing a computer as a diagnostic aid (Croxton and Martin, 1970). Other examples abound which have some, though not all, of the characteristics of the Keller plan listed above.

PSI is now in use in, probably, several thousand college courses. Conferences on it are held once or twice a year now. A newsletter circulates among users. Workshops are being held to help new users learn the details of the method and avoid its pitfalls. The literature about the method numbers several hundred papers. (Sherman, 1974; Ruskin and Hess, 1974).

6.2.3.2 APPLICATIONS

To quote Sherman (1976) on the first of four major characteristics of PSI:

The current status of PSI can be briefly summarized by identifying four major developments. First PSI spread from psychology to physics, then to engineering, and more recently beyond the physical sciences into the social sciences and the humanities. There must be disciplines and subject matters as yet untried in a PSI format - but I cannot think of any. With no particularly reliable data I would estimate that more than 2,000 professors have developed their own PSI course and their own materials. At least twice that number have given PSI courses using commercially available materials. If we estimate the modal teacher has now offered his PSI course twice, we would estimate 12,000 PSI courses have now been given. The actual number could be two or three times that number.

If Sherman is anywhere near the truth in his estimate, then we may safely assume that PSI, after little more than 10 years of application, is the most widespread system of individualisation in higher education in the U.S.A.

The acceptance of PSI courses at university level is probably a result of:

- (a) relative ease and speed of materials preparation
(not specially programmed and often available from existing sources);

- (b) relative ease of implementation within a traditional university course structure (no major timetable changes, irregular attendance problems minimized, slower students may put in extra time).

The relative success, as compared with other attempts at more rigid programming, is probably explained by;

- (a) familiarity of the style of the learning materials to students;
- (b) the discussion/assessment sessions with the monitors (i) allow for expansion/inquiry/criticism, (ii) ensure full mastery before progress to new materials, (iii) make up for any deficiencies in the quality of the materials, (iv) supply rapid feedback to the teacher, enabling him to revise or add to the course materials on a regular basis throughout the year.

These benefits are particularly marked in long, academic courses and especially in subjects demanding discussion and open-ended responses, involving 'sophisticated' learners.

In the U.K. the PSI system has not permeated the higher education scene to the same extent, although much interest is being shown, as witnessed by the response to a special journal issue of "Programmed Learning and Educational Technology" devoted to it and edited by the present author. (Romiszowski - ed., 1976). Some application to science and mathematics teaching are reported at Surrey

University (Elton et al, 1973; Bridge, 1973; Elton, 1975). The system (somewhat modified) is used for mathematics teaching at Loughborough University. This application is fully described in a published paper (Romiszowski, Bajpai and Lewis, 1976) which is included in the appendix to the present study.

The chief differences between the system as used at Loughborough and the "classic" Keller-Plan for PSI are:

- (a) Student-proctors/tutors are not used. The tutoring is performed by members of the teaching staff.
- (b) In one application, evaluation is not performed by the tutors. The students "self-mark" their assignments.

The reasons for and the results of these modifications are described in the attached paper.

American reports of P.S.I. applied to mathematics instruction include applications to introductory statistics (Myers, 1970; Newman et al. 1974) to a service course in mathematics for biologists (McKean et al, 1974), to a course on logic (Cardwell, 1973), to arithmetic at a community college (Dahlke, 1974), to remedial arithmetic (Hecht, 1974), to computer science (Modesitt, 1974), to algebra (Rogers, 1974), to long general mathematics courses (Taylor, 1974), and to calculus (Dahlke and Morash, 1975).

6.2.3.3 RESEARCH

Ruskin and Hess (1974) include over 200 research papers in their review of the literature on the Personalized System of Instruction. Papers on PSI have been appearing

regularly since then, and books and monographs continue to come out describing PSI research. By almost any criterion, "the Personalized System of Instruction has stimulated an unusual amount of research in its short life".

A recent review of the general findings of research on PSI, (Kulik, Kulik and Smith, 1976) was included by the author in a recent special issue of "Programmed Learning and Educational Technology (Romiszowski - ed., 1976).

Kulik's findings are summarised here.

(a) End-of-course performance

Student achievement on a final exam is the most commonly used measure of effectiveness of college teaching methods. When Dubin and Taveggia (1968) reviewed comparative research on college teaching during the years 1924-1965, they found that by this criterion most teaching methods used in higher education seem very similar. They were able to locate, for example, 88 reports comparing lecture and discussion methods of teaching. In 51% of the reports, students learned more from lectures, and in 49% of the reports, students learned more from discussions. Comparisons of other teaching methods ended in the same standoff.

Results with PSI have been strikingly different. We recently searched the literature on college teaching for studies comparing final exam performance of PSI and lecture students, and found 39 studies that met basic requirements of sound experimental design. In 38 of the 39 studies, exam performance was better in the PSI course, and in 34 of these studies, the performance difference between the PSI and comparison groups was great enough to be considered statistically reliable. In one case, lecture performance was slightly better than PSI performance, but the difference did not reach, or even approach, statistical significance.

(b) Retention.

A number of investigators have looked beyond the final exam, and tried to determine whether PSI has a long-term impact on students. In our review, we located 9 studies investigating retention over intervals ranging from 3 weeks to 15 months. In each of the studies, the PSI students performed better on a follow-up examination than students from lecture courses, and in each study the difference between groups reached statistical significance. In most of the studies, differences at the time of follow-up were greater than final exam differences. If anything, final exam

comparisons underestimated the magnitude of PSI's effect.

These 9 studies strongly suggest that PSI promotes something more than rote memorization.

(c) Transfer.

In our review, we located five studies of transfer effects, and in each of these studies, PSI students outperformed other students in a follow-up course - even though the follow-up courses were taught by conventional methods by other teachers.

Transfer studies reinforce the notion that PSI is responsible for a kind of learning that goes much deeper than rote memorization. What is it that transfers?

(d) Attitudes to the course.

Finally, a number of investigators have compared the student ratings from PSI courses to ratings from conventionally-taught courses. In 8 out of 9 cases PSI ratings are higher than those from lecture courses.

Kulik et al. also report studies concerned with why PSI is effective, and for whom. On this latter point the findings do not support the theoretical "mastery-learning model" position that by allowing the learning time to vary we will eliminate individual differences in attainment.

With particular reference to mathematics applications, the pattern of results is not much different from the general picture described above.

Myers (1970) reports,

"There were large individual differences in the time taken to complete the course, in the number of errors made on exams, and in the number of errors made in working through the text."

However, he found no significant correlation between time to complete errors during course, with errors on the final examination. Attitude measures showed a very marked positive attitude to the use of PSI.

Malec (1975) reports similarly favourable results;

"A between-semester comparison of final examinations scores in sociological statistics is presented.

Students in the personalized course performed significantly better and the course was rated second highest out of 22 in the department, with 88% of the students preferring the method to conventional methods. "

In the U.K., Elton et al (1973) also report significantly better results from P.S.I. as compared to traditional lecture-based courses.

Further positive results are reported by Rogers (1974) and Steele (1974). An interesting finding is reported by Zeilek (1974) when using a P.S.I. "crash course" in astronomy. Students completed the year's course in seven weeks, with results equivalent to control groups. The author has also investigated the use of P.S.I. on intensive courses for technical teachers in Brazil. Results of these experiments were encouraging, although no control groups were available to perform a rigorous study. (Romiszowski, 1975). A copy of this paper is included in the appendix.

CHAPTER 7

MULTI-SENSORY INSTRUCTIONAL SYSTEMS

7.1 Concrete Materials and Mathematics Laboratories

- 7.1.1 Introduction: The "Maths Lab" concept.
- 7.1.2 Characteristics of a Maths Lab.
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7.2 Learning Packages, Packets and Activity Packs

- 7.2.1 Introduction
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7.3 The Fife Mathematics Project

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CHAPTER 7

MULTI-SENSORY INSTRUCTIONAL SYSTEMS

7.1 CONCRETE MATERIALS AND MATHEMATICS LABORATORIES

7.1.1 Introduction. The "Maths Lab" Concept

The "Maths Lab" concept grew out of the trend towards concrete operations and "discovery" learning, fostered by the researches of Piaget and the development work of such innovators as Dienes, Cuisenaire and Bruner. Although the approach is most common at the lower primary "beginning mathematics" stage, there are also some attempts to implement laboratory-based mathematics learning systems at secondary and even post-secondary levels. A good example of maths laboratory material applied at the post-secondary level are the experimental kits on basic statistics developed at the Cranfield Institute of Management. These kits have been used with great success by the author to overcome conceptual difficulties experienced by undergraduates on social science degree courses when first confronted with statistics.

The typical situation, with respect to statistics on the Social Science degree at the (then) Enfield College of Technology, in the late 1960's, was that students tended to learn statistical methods as "magical routines" which gave a required result if one followed the steps. The answer to "what is chi-squared" would typically be given as the "formula needed to evaluate it". Even simple concepts such as "median", "standard deviation" or "normal distribution" were not really understood by the general

body of students, but were simply used as pawns in a "game" called "statistics". As a part-solution to this problem the remedial mathematics "clinic" was launched in 1968 (Romiszowski 1969). Another part of the attempted solution was to make available to both staff and students a range of statistical demonstration models and practical kits (some commercially available, some under development at that time at the Cranfield Institute of Management). These included visual demonstrations of the laws of probability, of the formation of normal and skewed distributions, etc., and simulation games which could be played by students to show that such techniques as statistical quality control really work in practice.

The value of these models and games in concept formation, although not evaluated by rigorous experimentation, was deemed to be very great by the lecturers who taught the "Methods and Models" course - a lecture/seminar course intended to develop the students' skills in mathematical model building. After some sessions during which the students were introduced to the "statistical laboratory" and encouraged to experiment with the kits, both the quantity and the quality of student participation in the lecture/seminar work increased.

Games, in particular the quality control simulation, were used with foremen and managers from local industry, in order to successfully change previously negative attitudes towards the implementation of statistical quality control, into strongly positive ones.

Materials for beginning mathematics are numerous and well known - they include Cuisenaire rods, "sets" of objects, shapes for area work, Dienes' Multibase Arithmetic blocks, Geo boards, etc.

The reader may find lists of these in manufacturers' catalogues, or in annotated lists such as that prepared by Davidson (1968). See also Howson (1971) and Turner (1966).

One should include here too the use of electronic or mechanical calculators, slide rules, abacus, etc. when used as instructional devices to teach the concepts behind the operations. (Lewis 1972), and also games (Holt & Dienes, 1973), and "field" activities (Sime and Murray, 1965).

7.1.2 Characteristics of a Maths Lab.

Depending on the resources available to a school and the usage level, a maths lab may be a temporary affair - equipment brought into the normal classroom, or it may be a permanently equipped special purpose room.

In planning schools today, thought should be given to organizing laboratories for mathematics in the same way as they are organized for physics and the other natural sciences. Useful works have been published for classroom designers and suppliers of equipment (Bartnick, 1960; Jones, 1963). These references will give them ideas on classroom sizes, heating, ventilation, lighting and flooring, and advice on the composition and arrangement of blackboards, placards, book cupboards and shelves, display cabinets, storage cupboards, desks, work tables and chairs. The texts are accompanied by plans and photographs of models.

Not all who are concerned with junior education are agreed that a special mathematics room should exist. Many argue that this suggests to the child that mathematics is an exceptional, and perhaps difficult, subject. These, therefore, recommend that mathematics should be taught in an ordinary classroom, thereby enabling the class-teacher to integrate mathematics with the other activities.

Models for the usage of concrete materials vary in terms of the proportions of time spent on laboratory and other classwork. However, in general they follow Bruner's (1966) stages of enactive-iconic-symbolic operations. Thus the laboratory work comes first in the learning of a new concept.

However, in order to maximise teacher utilisation and save on equipment duplication, it is common to break down the class into smaller groups - some groups working with the materials, others performing follow-up activities with the teacher or his assistants.

Kessler (1972) describes the teacher's role as follows:

" The instructor sets the learning atmosphere for his students. In a maths lab, the classroom emphasis shifts from teacher-dominated to child-centered. It is up to the teacher to know when to question, when to be a silent partner and when to withdraw completely.

Instruction in a maths lab should start slowly and easily. The teacher who uses this approach in a self-contained classroom might begin with one small group of students. Some teachers start the whole class with one hour a week; some start with ten minutes a day. Once these students are involved in their projects, he may start a second group, etc. Warning: Don't set up more groups than you can comfortably handle.

All pupils must be allowed to play freely with the materials. The questions the children's undirected explorations generate can be used by the teacher to guide them to further interest and activities.

All the work needn't be done by small groups. For example, materials such as geoboards, calculators, and computers may be introduced to the entire class at one time.

The teacher in this situation - in his pupils' eyes - is a sounding board or resource person. The same concept holds for any aides (paraprofessionals, parents and other children) who may be working in the lab. "

What, then, is a mathematics laboratory? This is the definition offered by the Mathematical Association:

"Let us suppose that we have a space, which is to contain all that we might need for the indoor teaching of mathematics. How might we fill it?

We shall want a classroom, no doubt; that is, a mathematics room. If we want to be able to cut, drill, glue, paint, solder and assemble, then we want a mathematics workshop. Do we want a place where we can use calculating machines without causing disturbance or being disturbed? Then we want a computation room. Probably we want a quiet room for reference and study; so we want a mathematics library. If we add to this complex a store-room, then we have a complete and functional unit.

The mathematics laboratory is the full set, though few, if any of us, have it all. But the name is given deservedly to all those places wherein teachers are striving to make mathematics more intelligible to more and more people, by relating theory and practice. "

(Mathematical Association, 1968)

7.1.3 Applications

As the "Maths Lab." concept is an extremely flexible one, it is difficult to cite "typical" applications - any application has as much right as any other. The Mathematical Association in the abovementioned pamphlet lists some typical organisational structures and operational schemes. In all the schemes, stress is laid on pre-organisation of work plans, record keeping systems and evaluation systems.

In a typical primary school application:

"For lower juniors (7 - 8½ years) tables which are concerned with one particular subject, e.g. shape, can be named; they accommodate a group of up to 6. The names of the tables can vary as new topics are brought in. For each table a list of materials has been prepared and the work is centred around attractive assignment cards covering all types of problems in the different aspects of mathematics. The cards should be constantly amended and developed by observing the reactions and suggestions of the children. The groups can be self-formed or structured and depend upon the nature of the task and the materials available. Children need not, of course, always be in the same group."

In a secondary school in East Ham, London, a specially equipped permanent laboratory houses some 700 books on mathematics, assignment cards, work folders, models, reference books and tables, calculating machines

of various types, audio visual aids and provision to receive radio and television broadcasts. The use made of this room is described as follows:

"The intention is that every form in the school should have at least one lesson a week in the mathematics room and forms taking Additional Mathematics or Applied Mathematics might have more than that. Lessons in the mathematics room could take the form of:

1. Practical lessons with work done individually or in groups.
2. Lessons including demonstrations of apparatus obtained from the near-by mathematics store room.
3. Lessons making use of film strips or other aids.
4. Lessons planning larger mathematical projects, surveys, etc.
5. Periods in which the availability of reference books is necessary.
6. Ordinary lessons which would gain by being held in a mathematical setting.

The use of the room for Out-of-school activities could include:

1. The planning and construction of demonstration or class apparatus.
2. The making of posters to illustrate current work.
3. The making of posters and models of permanent value.
4. The collection and display of statistical information.

5. The carrying out of more detailed surveying and map-work that could be undertaken in class time.
6. The preparation and display of mathematical work for Parents' Evenings and Open Days."

At the post-secondary level, the equipment generally becomes more complex, often including computer facilities, statistical, mechanical or electrical apparatus for experiments. A Mathematics Laboratory at the Brighton Technical College, described in the pamphlet (Mathematical association 1968), contained, in addition to "most of the traditional mechanics apparatus",

"21 Desk calculating machines,
Assorted statistics apparatus,
Polaroid land camera, stroboscope - together with
'dry ice' pucks, linear air-track and adaption of
some of above mechanics apparatus,
Venner stop clocks and photo-cells,
Planimeter,
Three dimensional mathematical models."

The ways in which laboratories can be used also vary exceedingly. Barson (1971) in identifying four types of mathematics laboratories, highlighted the basic principles of usage for each type.

Barson's classification is;

1. Decentralized, or classroom laboratories
 - a) All classrooms containing laboratories
 - b) Some classrooms containing laboratories

2. Centralized laboratory
3. Team-room laboratory
4. Roving, or movable, laboratory.

He describes the ideal usage of each type as follows:

1. A decentralized laboratory provides a situation in which the teacher has a permanent mathematics laboratory in his room. This is ideal because the teacher is responsible for all areas of the curriculum and for a major portion of the time a child is in school. This organization facilitates movement and scheduling (which is a problem with the other types of laboratories). It also allows the children and the teacher to use the material whenever the need arises.
2. Centralized mathematics laboratories are used in many schools where children from all the grades, or from certain grades, share the facilities of the laboratory for part of their mathematics instruction. The ideal program would have a specially trained mathematics teacher in the room full time to instruct the children with the aid of the regular teacher. In this situation the mathematics-laboratory teacher could do the scheduling, take care of the materials, create activities, and use his special talents to help all the children. It also provides an excellent opportunity for the development of the other teachers.

The basic problem is that the teacher can send the class into the laboratory for only a few periods a week and at predetermined times, not necessarily when its service is needed.

3. Team-room laboratories were created as schools became equipped for team teaching. The most distinguishing feature of this type of laboratory is that it is in constant use by the children, and only a small number of them attend the laboratory at a given time. Usually there is no adult in the laboratory because the children are in sight of a teacher at all times. Individual children or small groups work in the laboratory on mathematics concepts of current interest or on any mathematics topic that the team considers useful to the children.
4. The roving, or movable, laboratory is useful when a school cannot afford to buy many materials and doesn't have a room for a central laboratory. The materials are carefully itemized and placed in containers for easy assembling by the teachers. The container for a particular topic has all the necessary materials to allow the child to investigate the related problems freely.

A further discussion of usage problems, as well as some questions concerning the control and evaluation of the work is given by Ewbank (1971).

7.1.4 Research on the Mathematics Laboratory

The school of thought which most strongly supports the use of mathematics laboratories tends also to believe in "individual development" and to be influenced strongly by Piagetian views on "observation of the individual", (T. Lesh, 1974). After all, most of Piaget's work is based on the careful observation of a mere handful of children. This is not to detract from the usefulness and value of Piaget's work. However, there is a difference between the painstaking descriptive analysis of a child's development, and the evaluation of the effects of a particular treatment (e.g. maths lab.) on that development. This latter requires a more formal experimental design, with control groups, and a precise definition of the criteria by which learning will be judged. Perhaps the "process not product" and "individual not group" orientations of the followers of Piaget and Bruner account for the relative scarcity of documented research on the use of mathematics laboratories.

There is ample research on the use of individual manipulative exercises, which goes back a long way. The Eighteenth Yearbook of the National Council of Teachers of Mathematics (1945) was devoted to this topic. Indeed, the Montessori system, with its rulers, squares and cubes illustrating regular divisions goes back to 1914 (Servais

and Varga, 1971). Much research has been based around the "Cuisenaire rods" system and the Dienes multibase arithmetic blocks (Dienes, 1960; Wheeler 1963; Dienes 1964). Logical materials and games such as WFF 'n' PROOF have also been investigated (Dienes, 1964, Allen, 1963). The use of slide-rules and calculating machines as laboratory equipment to "discover" the processes of mathematics has been a recent area of much concentration (French, 1962; Kerr, 1963; Clark, 1964) extending to the use of computers (National Council of Teachers of Mathematics, 1963).

Most of the research is favourable towards the use of the particular item of equipment under test. However, one should distinguish between a small-scale experimental application of one item, and a regular application of many different such items on a regular basis in a mathematics laboratory. Hawthorne effects diminish and the importance of teacher organisation, variety of activities, motivation, control and diagnostic evaluation grow. Thus it is a pity that most reports of the long-term usage of mathematics laboratories are rather general and anecdotal in nature.

One useful analysis is the report prepared by the ERIC information analysis centre for science, mathematics and environmental education (Fitzgerald et. al., 1974). This report is partly historical/descriptive and partly a review of the research. One interesting historical point is that "despite continued interest, the actual numbers of maths labs. declined rapidly after 1970". The review of research concentrates mainly on individual items of equipment (see above). It does however report some long-term studies

(not all favourable to the mathematics laboratory), some unexpected findings (less able children benefit more from the experience, but their attitude towards learning mathematics does not improve) and the ways to employ mathematics laboratories most effectively. Some specific long-term studies illustrate the need for further research, perhaps more rigorous. At the elementary level Wallace (1974) in a Ph.D dissertation compared the effectiveness of mathematics instruction based on the use of manipulative materials in three grades (grades 4, 5, 6) for a period of three weeks. Pre tests, "cognitive" post tests and "manipulative" post tests were administered to all 3 experimental and 3 control groups. All three experimental groups performed better than the controls on both types of post test.

On the other hand, Smith (1973) in another Ph.D dissertation, carried out a very similar study over a longer period of time (one and a half semesters) in the grade levels 6, 7 and 8. The results showed no significant difference between the experimental groups and the controls, neither in achievement nor in attitude.

Mathews (1974) performed a similar but much larger study at the secondary level. Twenty-two experimental classes (taught by 19 teachers) were compared to 20 control classes (taught by 18 of the teachers) over a period of six months. He found a statistically significant difference between the groups in favour of the experimental ones using manipulative materials. However, he points out that with such large numbers of students involved, only very

small "real" differences are statistically significant and in this experiment the differences in terms of mean test points, were very small.

At the post-secondary level, Pigford (1974) reports no significant difference on either post-test or retention test for a group of 28 trainee elementary teachers who used a mathematics laboratory during their course, as compared to a group of 29 who did not. Yet one more experiment on similar lines, but using 3 classes of first year college students again showed no significant differences between groups in attainment, in attitude towards mathematics or in attitude towards the course. (Smith, 1974).

7.1.5 Discussion

The research cited seems to suggest that the use of manipulatives is beneficial for the acquisition of certain specific mathematical concepts. There is a great deal of evidence at the elementary level, less at the secondary and still less (but nevertheless positive) at higher, post-secondary levels. However, the evidence on the extended use of activity methods in a mathematics laboratory as part of a long course, is ambiguous.

There appears to be a need for more research here. In the author's opinion, this research should examine the extent of systematic programming of the course. Are the manipulative activities integrated with the other course work? To what extent is the course as a whole "prescriptive" and to what extent "open"?

Clarke (1969 and 1970) has considered this problem, in an attempt to integrate programmed instruction with the use of concrete materials. He has constructed programmed "activity systems" controlled in part by cards and in part by the teacher. In order to ensure that the systems are implemented as designed, the teachers also receive a programmed course to teach them how to implement, control and evaluate such system. The materials do not resemble programmed texts at all, but nevertheless they do obey the general principles of programmed self-instruction. Vaughan (1972) carried the systematisation of a "manipulatives-based" course yet further. He developed a "PERT" Network of all the planned activities, showing the inter-dependencies between them (sequential and parallel activities) (see figure 7.1. This was used by the teacher and student together to aid the selection of a logical sequence through the material, and also to control the student's progress. The teacher's actions were defined and controlled by a series of flow charts. An example is shown in figure 7.2.

The Fife Mathematics Project, described next is an example of a course which uses manipulatives and other materials in an integrated system, part-prescriptive and part-open.

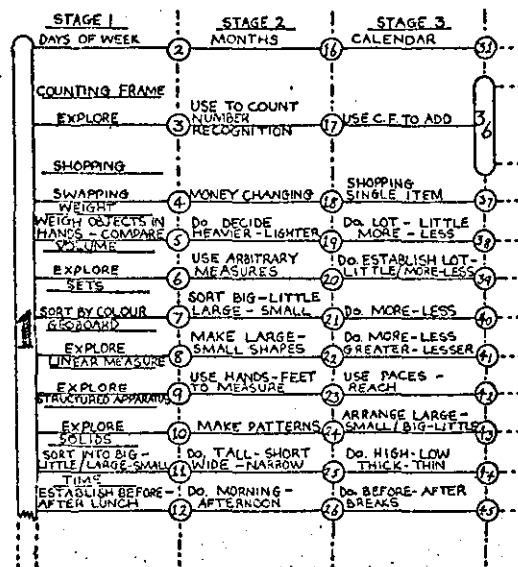


FIGURE 7.1

Section of a network analysis of the activities planned for a primary mathematics course.

(Vaughan 1972)

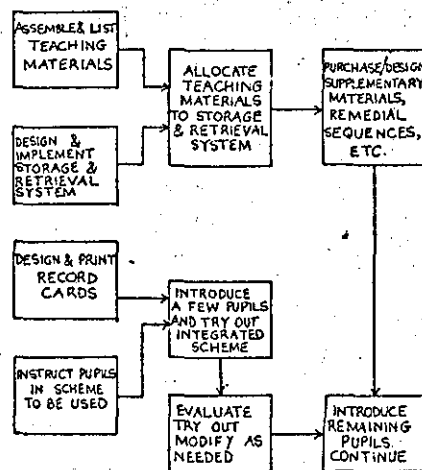
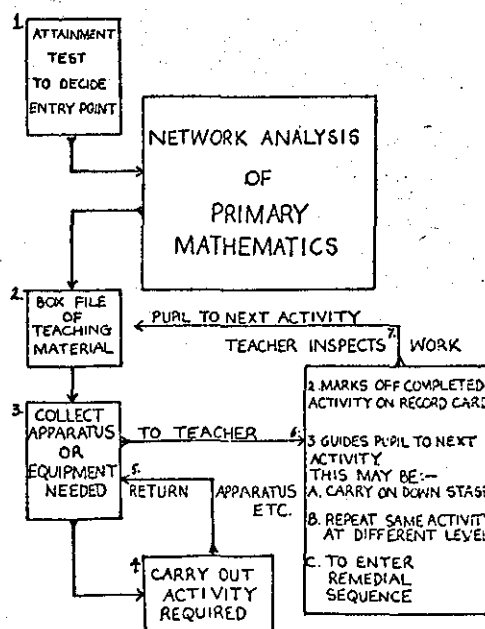


FIGURE 7.2

Flow-charts used to guide the activities of the teacher adopting the integrated scheme of activity-based primary mathematics proposed by Vaughan. The upper chart deals with the overall implementation of the scheme. The lower chart deals with the teacher-student interactions.

(Vaughan 1972)



7.2 LEARNING PACKAGES, PACKETS AND ACTIVITY PACKS

7.2.1 Introduction

Many individualised mathematics projects are based on the use of multi-media packages which may include self-instructional and group-instructional materials. The use of the materials may be teacher-prescribed, as in the I.P.I. system, or student selected, as is often the case in so-called "resource-based learning".

One popular system in the U.S.A. goes by the name of "Learning activity packs" or LAPs". A description of a typical LAP follows (Cardarelli, 1972).

7.2.2 Characteristics of Learning Activity Packs (LAP's)

Basically, the LAP is a booklet on a given topic, containing objectives related to this topic, diverse activities to reach these objectives, and evaluations to determine if the objectives have been met.

The components of the LAP are:

Topic	Pretest
Subtopics	Activities and Self-
Rationale	Evaluations
Behavioural Objectives	Quiz(zes)
	Posttest

Topics and Subtopics.

The student's initial introduction to a LAP is the statement of the topic and subtopics. The extent of coverage of this topic depends on the individual teacher, the type of student utilizing this LAP and how long the child has been using LAPs.

Rationale.

The topic and subtopics are followed ordinarily by the rationale. As the name indicates, the rationale is aimed at providing the student with a reason for studying this topic.

The rationale can take a variety of forms: a stimulating film, a large-group presentation, a challenging experiment, an explorative study or a written rationale which enunciates the relevance of this topic within the framework of the total curriculum, the student's everyday life or his future life.

Behavioural Objectives.

After this brief introduction to the topic, the student reads the objectives for this particular LAP. All objectives are behaviourally stated. The completeness of these objectives should depend on the level of development of the student involved. In the LAP, the function of the objective is to communicate goals to the student, and thus should be written in his language.

Pretest.

For some LAPs the teacher may determine that the pretest is at the option of the student. In such a case, when the objectives unveil a topic or maths concept with which the student is totally unfamiliar, his decision might well be to omit the pretest and immediately begin the LAP. On the other hand, as usually happens in maths, there is built into each unit a review of a number of prerequisite skills, a review which can be a waste of time for many students.

Activities.

The activities of the LAP attempt to provide the student with a multimedia, multi-modal, multi-level road to reach the objectives of the LAP. The multimedia activities, in directing the student to readings, transparencies, tapes, filmstrips, demonstration models, etc., provide for the learning style of the individual child. The multi-modal activities include within the flexible programme large-group, small-group and independent activities. The multi-level activities, provide the pupil with the opportunity to start at the base of his particular weakness. Quizzes.

Frequent quizzes give the student a frequent success record and insure immediate remediation by rerouting the child to activities for which further reinforcement is required. In this way, his learning is a progressive development which will prevent him from getting in over his head.

Posttest.

The posttest is comparable to the pretest in that it evaluates student fulfilment of all objectives of the LAP, at least the terminal objectives if not all subobjectives.

An Aid to Diagnosis and Prescription.

In the LAP model derived by Cardarelli, all number systems are co-ordinated for objectives, activities and testing, in order that evaluations become a diagnostic and prescriptive tool capable of being interpreted by both teacher and student. For example, if the pupil correctly answers all parts of questions 4 and 7 on a

pretest, he has already met LAP objectives 4 and 7 and may skip activities 4 and 7 if he so desires. Incorrect completion of questions 5 and 6 on a pretest would provide an immediate diagnosis of weakness on objectives 5 and 6 and a built-in prescription to complete the corresponding activities 5 and 6.

Cardarelli suggests that this last component should enable the teacher to pass a large part of the diagnosis/prescription function to the student. Her comments emphasise the underlying philosophy of the LAP system;

"The structure of the LAP is only the shell - an empty shell - if implemented without the philosophy behind the LAP programme. What is that philosophy? To my way of thinking, the LAP philosophy could be summarized by these guiding beliefs;

1. Each student is an individual who has a right to receive instruction geared to his needs, his interests and his capabilities.
2. The teacher's role is that of diagnostician, prescriber, motivator and facilitator of learning.
3. The student's role is that of an independent person capable of making decisions, accepting responsibility for his own education, and getting along with others.
4. The atmosphere of the LAP programme classroom or school must reflect an open structure where initiative, creativity, exploration, meaningful interaction and awareness of the needs of others can flourish.

In short, the LAP philosophy would have us take a realistic look at what will be expected of Timmy when he moves into this society of the future, and set up a classroom atmosphere where the qualities he will need as a fully functioning person, not merely as an intellectual, are going to be fostered."

These last comments, place the LAP system closer to the resource-based, student-directed camp, albeit with some limited role for the teacher and the diagnosis aids as "prescribers" of learning activities.

7.2.3 Research

Not much controlled research specifically concerning the use of LAP's has been published. However, the system has such great similarities with the Fife system, the Kent project, even the Nuffield project and innumerable other workcard/workbook/materials-based individualisation schemes. Thus much of the research on these would probably apply.

Two specific studies however do not support the LAP approach unequivocally. Campbell (1972) concluded that although in general attainment from LAP's was as good as from traditional instruction, "relative success with LAP's was related to several student characteristics and LAP's may not be an effective means of individualising instruction for students in the lower ranges of I.Q., reading ability, attitude, etc.". (Schoen (1975) found significantly improved attitudes among users of LAP's but no significant differences in attainment.

7.3 The Fife Mathematics Project

7.3.1 Characteristics

Unlike the I.M.U. project, the Fife mathematics project does not use programmed instruction. The materials are "Workcard Booklets", worksheets and manipulative equipment.

Also the project did not set out to construct a "total" system. It was designed to form a part of the mathematics activities of the first year in the comprehensive school. Normal classroom-teaching activities play a significant (in most schools, the major) part in the mathematics curriculum.

The degree of involvement of the participating schools in Fife vary from 10% to 60% of the total time allotted to mathematics, devoted to the individualized scheme. The mode is 30% - 40%. (Crawford 1975).

The Project was designed as an answer to the problem of teaching mixed ability groups. At least two claims are made for the individualised aspect of the Project. The first is that it helps pupils to understand basic mathematical concepts by using concrete apparatus.

Many textbooks still contain mainly verbally-based material; children must possess a high standard of literacy both to interpret the information given and respond to the questions asked. When pupils are using concrete aids, (or manipulatives,) it is possible for the teacher to assess their understanding of the subject simply by observing how they handle these aids. While it is important that at some stage children learn to express in words the concepts they have grasped, it is

now generally accepted that the first stage of comprehension need not involve words at all.

The second claim made for the Project is that it enables pupils to benefit from the experience of others. The Project incorporates an element of choice, so it seems reasonable to expect that within any class of pupils, some will have experienced in the individualised approach the work which is being discussed in normal class teaching. These pupils can then pass on their experience to others by discussion.

7.3.2 Research results

The results quoted by Crawford are mainly subjective and anecdotal. He defends this position quoting L.C. Taylor's (Taylor, 1971) arguments for "illuminative" evaluation.

"Because human complexity defeats the most skilful attempts at classical 'scientific' evaluation, another less forbidding style of evaluation has begun to develop. It is a style at once more modest and more ambitious: on the one hand it doesn't aim at the authority and the high level of generality that characterizes science; on the other it tries to deal with real, whole situations, not just those parts amenable to scientific treatment. Such evaluation relies less extensively upon the application of validated tests, more upon various evidence derived from observation, questionnaires, interviews, informal conversations, psychological tests, examinations, diaries, documents ...

In such evaluation, as in life, oddities and side-effects may prove particularly illuminating: in scientific evaluation oddities get lost in statistical composites and side-effects excluded because they're not listed in pre-determined objectives. "

With respect to the content, materials and methods, generally favourable teacher reactions are reported. Pupils also "seem satisfied with the content as a whole". The vast majority enjoy working in an individualised system, "choosing their own work and being able to work at their own time". Teachers, on the other hand, observed that prolonged periods of individualised work led to lethargy, remedied by return to classroom-work at intervals.

Discipline problems were rare. The need for efficient organisation was stressed. Assessment created some problems of students "queuing" to see the teacher.

No "hard" results are quoted concerning the amount of learning that has taken place. There is even a doubt expressed about what the students are supposed to be learning - to pass examinations or to work on their own.

7.4. The Audio-Tutorial System

7.4.1 Characteristics

Audio-tutorial instruction was first proposed and developed in 1961 by Samuel N. Postlethwait at Purdue University, primarily to meet the challenge of teaching biology to large numbers of students (Postlethwait, Novak & Murray, 1972). Since that time the method has been adapted to many different subject areas, class sizes, and institutions.

The chief characteristic of the audio-tutorial method in its original form were individualised audio tapes as the main medium of communication, with printed materials in a support role. Over the years, however, the system has taken over most of the critical characteristics of programmed instruction and has tended to rely less exclusively on the audio medium, so much so that Green (1976) in comparing various instructional systems which have roots in programmed instruction, includes the audio tutorial among them, listing the systems benefits as its attention to the presentation of instructional content through a variety of media available in a "self-instructional carrel" equipped with the necessary hardware. The method's strength lies in its attempt to present instructional activities in the sensory mode preferred by the learner and to integrate experiences from various modes into a meaningful whole.

Thus it would seem that there is a danger of the audio-tutorial system losing its independent identity as it becomes less distinguishable from other audio-visual resource-based learning systems.

7.4.2 Applications

Since its development in the early 1960's however, the Audio-tutorial approach has gained a strong following, particularly in higher education in the U.S.A. Fisher and MacWhinney (1976) in a review of research on the system, located 89 documental studies published between 1962-75. However, very few of these are in mathematics, as can be expected for a predominantly auditory medium. Mathematics needs much illustration - it is a predominantly visual subject. Also the linear, fixed pace presentation is not very suitable for many mathematical topics where the ability to go back over the work again is necessary for efficient learning.

Out of 44 comparative studies listed by Fisher and MacWhinney, 37 are at the post-secondary level, but only one of these is in mathematics - an algebra course (Cunningham, 1973). There is also one application to secondary school mathematics (Johnston, 1969),

7.4.3 Research

Both the studies mentioned above gave no significant difference in attainment between the auto-tutorial and lectures. This should be viewed against the background that of the 44 studies cited only one favoured lectures, 13 showed no significant differences and 30 favoured the autotutorial approach.

Similar results were obtained by Apter (1965) in the U.K. when comparing some audio tape versions of linear programmes with the original text versions. In all subject areas investigated, except mathematics the audio programmes

proved to be viable alternatives,

Taylor (1969) however, does describe an experiment comparing "programmed tapes", "non-programmed tapes", and control groups who did not use tapes. The tapes were meant to give further drill practice after initial learning by normal classroom methods. In this case, pupils using programmed tapes made significantly greater achievement gains than those using non-programmed tapes or no tapes.

The general limitations of the audio medium for mathematics teaching do not necessarily hold when there is also a well designed visual component - after all mathematics via TV and radio (with support material) does work more or less. Experiments by Leith at Birmingham University demonstrated the feasibility of applying programmed learning principles to television lessons in mathematics (Leith et al, 1969) and to slide-tape mathematics programmes, which proved as effective, but faster than equivalent linear programmed texts (Leith et al., 1970).

The author, in his literature search, has identified one or two comparative studies favourable to the audio-tutorial approach in mathematics.

At the elementary level, Sganga (1973) compared an audio-tutorial-based system with normal classroom instruction. The A-T method proved significantly better on tests of arithmetic computation, concepts, applications, symbols and vocabulary. The system used however was a

"hybrid" multi-media one, making much use of study-cards at various levels of difficulty, other media and teacher interventions as well as the audio-tapes. At this level also (5 - 6 grade) the audio approach helped to overcome reading difficulties of some students.

At the college level, Chinn (1973) reports favourable results for an A-T course in algebra. 186 students from a total population of 805 used the A-T method. Although significant, the difference between users and non-users was not very large in real terms.

CHAPTER 8

COMPUTER-BASED INSTRUCTIONAL SYSTEMS

8.1 Types of Computer-Based Education

8.1.1 Computer-Managed Instruction (CMI)

8.1.2 Computer-Assisted (or Administered) Instruction (CAI)

8.1.3 Computer-Supported Learning Aids (CSLA)

8.1.4 Some Advantages and Disadvantages of CMI and CAI

8.2 Computer Managed Instruction (CMI)

8.2.1 Characteristics of CMI

8.2.2. Some applications of CMI to Mathematics.

8.3 Computer Administered Instruction (CAI)

8.3.1 Characteristics of CAI

8.3.2 Some typical applications of CAI to Mathematics.

8.4 Research on Computer-based Learning Systems

8.4.1 Comparative studies

8.4.2 Other Research. The computer as a research base

8.4.3 Investigations of the Mastery Learning Model

8.4.4 Investigations of Student Directed Learning

8.4.5 Investigations of Programming and Organisational factors.

CHAPTER 8

COMPUTER-BASED SYSTEMS

At the third International Congress on Mathematics Education, held at Karlsruhe in 1976, R. Heimer stated (Heimer, 1976):

"In 1972, the Carnegie Commission on Higher Education published a report entitled "The Fourth Revolution" in which the role of technology in education is explored. The title of the report was derived from Eric Ashby's observation that four great revolutions have occurred in education. According to Ashby, the first revolution took place when the task of education for the young was shifted in part from parents to teachers and from home to school; the second revolution was the adoption of the written word as a tool of education; the third revolution was the invention of printing and the widespread availability of books; and the fourth revolution was the development of electronics, notably radio, television and the computer, though the computer is distinctly the imperative in the fourth revolution. This is the idea of paramount importance in any analysis of the contemporary technologies; the computer with its vast potential for impact is center stage."

8.1 TYPES OF COMPUTER-BASED EDUCATION

This impact of the computer on education is being felt in several areas, for example, administration, research, computational aids and the instructional process. We shall concentrate our attention on this last area, an area often referred to as computer-assisted instruction. However, even within this area there are several distinct types of systems. We therefore choose to follow Towle's (Towle, 1976) terminology, in referring to "Computer-based-educational systems". These he divides into 3 classes

- Computer-managed instruction (CMI)
- Computer-assisted instruction (CAI)
(we would prefer, for reasons which will be obvious later, if the letters were taken to stand for "computer-administered instruction)
- Computer Supported Learning Aids (CSLA)

Brief descriptions of these three classes follow.

8.1.1 Computer-Managed Instruction (CMI)

CMI encompasses a wide range of computer uses in education that involve the gathering and management of information necessary in developing flexible and individualized learning strategies. CMI utilizes the power of the computer to solve many problems of individualized instruction such as diagnostic testing, test scoring, and instructional prescription management. Management functions of CMI might include test item analysis, scheduling of

instructional resources, and student record keeping. CMI can be implemented in many ways. The computer may process information that students have coded on machine readable answer sheets. Student generated information (answers to test items) may be encoded in punched (Hollerith) cards, or students may interact with a computer terminal to enter information into a CMI system. Regardless of the method used to enter information into the CMI system, CMI uses the capability of the computer to manage the progress of a student through a programme of instruction, testing that progress at several points.

In general the functions of CMI are;

To administer student tests

To score student tests

To store data from student tests

To analyze data from student tests

To provide feedback to students, instructors,
and administrators.

8.1.2 Computer-Assisted (or Administered) Instruction (CAI)

As the word "administered" suggests, this term will be reserved for systems in which the instructional strategies and the bulk of the materials that students study are stored in the computer, or are directly administered and controlled by the computer, on an "on-line" basis during the lesson. There are many varieties, or "modes" of CAI which we shall discuss below.

8.1.3 Computer Supported Learning Aids (CSLA)

CSLA involves the computer as a supportive tool in the learning process but does not use the computer to either perform the functions of a CMI system, or to provide the primary instruction required for the student to master the instructional goals.

The use of the computer by a student in physics to perform numerous calculations required to find the answer to a problem would be a kind of CSLA.

Another example of CSLA is the service of a computer based information retrieval system available in many libraries.

Other uses of the computer that would be classified as CSLA include the capability of storage and retrieval of user generated data; demonstration by an expert of some natural phenomenon that could not otherwise be shown; and communication between users of a computer system using the system itself [such as the CDC-PLATO system] (Bitzer, 1976).

Yet one more use in this category is the growing trend towards computer-generated tests and examinations. A teacher, or indeed a student, may call up a battery of test questions to satisfy stated requirements, taking into consideration the questions used in previous tests, students past performances, etc. (Lippey, 1972; Toggenburger, 1973).

The author has discussed such applications, particularly to test construction, elsewhere (Romiszowski, 1976b). Such applications have some bearing upon the individualisation of instruction, offering sophisticated "tools" to the teacher and thus enabling him to adapt his teaching better to the needs of individual students. However, the CSLA's do not form individualised systems in their own right and will be excluded from the present discussion. We shall concentrate on the characteristics and applications of CMI and CAI.

8.1.4 Some Advantages and Disadvantages of CMI and CAI

Individualised, behaviour based instructional systems have the need for fast and efficient testing, record keeping, analysis, and feedback systems. These needs can be partly met by increasing the available manpower in the instructional system. However, the manpower required to maintain an individualised instructional management system would become too costly and unwieldy to be effective. The computer provides an efficient and effective alternative to human operated management of instruction.

The instructor also benefits from the use of CMI systems. In addition to information on each student's progress through a set of instruction, he/she can also get summary data on a group of students or on all students who have ever used the instruction. This data can guide the instructor on the need for additional learning activities on learning outcomes with which students have problems.

Along with the advantages of using CMI systems, there are also some disadvantages to be considered. Most important is the amount of time and resources needed to create the computer programmes which operate the system.

Another disadvantage to the use of CMI systems is the need for test items to be well constructed and matched to the behaviours required in learning outcomes.

Compared to computer assisted instruction, computer managed instruction can be implemented less expensively. Very often, an institution already has the computer hardware necessary to set up a CMI programme. All that needs to be done is to buy, lease, or produce a computer programme which will do the job. Such programmes have been written for almost every major brand and type of computer.

On the other hand, CAI systems generally require specially designed hardware, specially designed computer languages for course writing and for student-system communication, and sometimes require virtually exclusive use of the computer during operation. All this adds up to very high development and running costs.

A great advantage of CAI systems (at least theoretically) is that the difficulties encountered in programming individualised instruction on the "prescriptive" mode (predicting the difficulties that students will have and prescribing remedial action) can be avoided by "on-line" individualisation. The computer does not follow a programme of pre-set branches, but applies a logic, based on the logic of the knowledge-structure being taught, in order to continually adapt the presentation, in a unique manner, to

the needs of the individual student. This theoretical advantage has been little exploited to date, but some examples (discussed later) exist.

Whether the high cost of CAI will diminish to economical levels for mass use, or whether the educational benefits could justify the expense, are interesting areas of study, but outside the scope of the present work.

8.2. COMPUTER-MANAGED INSTRUCTION

8.2.1 Characteristics of CMI

The chief characteristic of CMI is that the student does not study at the computer, but his progress is monitored, performance is tested, guidance is given, etc. with the assistance of a computer. The extent of the assistance varies from project to project.

The author has been running a remedial mathematics centre for undergraduates in the social sciences, since 1969. This is fully described in papers annexed to this present work (Romiszowski, 1970; Romiszowski, Bajpai and Lewis, 1976). This project included a small CMI element. At the beginning of the year, all students take diagnostic tests which are then marked and transformed into punch-card form. The computer identifies the areas of weakness of each student and, depending upon the pattern of these weak areas, prepares an individual "prescription" for study. In addition, it performs item analysis and error analysis to supply information for the improvement of the diagnostic tests and the instructional materials. As the term progresses the students progress at varying rates through different

course units on a totally individual basis. In our case, using a batch-processing computer with punch-card input, it ceases to be economical to use the computer as a management aid - it turns out cheaper and gives a quicker turn-round time to use human clerical assistance, once the students are at different stages.

The picture will look quite different in a few years' time (economic recession permitting) when a time-sharing computer facility is available. It will be possible for each individual student to take his diagnostic tests and re-tests directly at the computer terminal, to receive immediate assessment, prescriptions and guidance.

Such a "Terminal-Oriented CMI System" (as opposed to "Batch-Oriented CMI") is in use at the Florida State University. Towle (1976) describes its operation as follows:

"When a student is ready to be evaluated over a particular unit, he may sign on to a computer terminal and respond to the randomly selected test items by typing his answers on the terminal keyboard. When the student completes the test, the computer scores his answers and displays a diagnostic learning prescription based on his performance on the test. This prescription will direct the student to proceed on to another unit or to perform specified remedial activities.

After completing the prescribed remedial activities, the student may return to the computer terminal to take another attempt, or a new test composed of randomly selected items over the same unit. This

basic procedure is followed for each unit of the course. "

The main facilities available in the CMI system at the Florida State University are;-

1. Unit Presentation Sequence

a. Linear sequence

All students take the units in a specified sequence.

b. Random selection

A menu of all the available units is displayed and the student may select the units in any order.

c. Stratified random selection

If one set of units is prerequisite to another set, the student can be required to complete the first set before being allowed to select units from the second set. However, selection within a set may be in random order.

2. Pretest

Students may be given a pretest at the beginning of the course. This pretest may be taken on the computer terminal or may be given by conventional pencil and paper means.

3. Selection of Test Items

a. Each student receives the same set of test items.

b. A specified number of test items may be randomly selected from a pool of items for a given unit.

c. Questions on succeeding attempts for the same unit may include or exclude questions from previous attempts for a given student.

4. Feedback

The feedback or diagnostic learning prescriptions given to the student may vary widely in type and amount. Feedback may be given after each question, each set of questions, each unit, or over all completed units to date.

5. Criterion for Proceeding

After completing a test for a given unit, the computer may display to the student diagnostic feedback based on the student's level of performance as compared to a preset criterion. Generally, if the student's performance is at or above the set criterion level, he is instructed to proceed to the next unit. However, if his performance is below the set criterion, he may be instructed to perform remedial activities and return for another attempt to pass the unit test.

6. Record Keeping and Reports

a. The following information may be recorded for any student response:

1. A question identifier
2. A student identifier
3. The current date
4. The current time
5. The response latency
6. An answer choice identifier
7. The complete student response

b. This information is stored on file and may be sorted, merged, and analyzed in a variety of ways to provide information such as:

1. Item analysis information such as the response distribution for specific items
 2. The total or mean response latencies for a given objective or unit
 3. Unit or objective test scores by student
- c. Each student may have a unique record file which has a large number of counters to accumulate and store such information as:
1. The number of questions he answered correctly for a given unit test
 2. The number of attempts made on a given unit or objective
 3. The actual items he received for a given attempt
 4. How many units the student has passed to date etc.
- d. The instructor may be given a periodic report which lists those students who have failed all allowable attempts for a given unit, and/or a distribution which shows the number of students who have passed each unit in the course.

7. Calculation Routine (CSLA)

Before answering a particular question, a student may be permitted to branch to a calculation routine which allows him to utilize the computer as a powerful desk calculator. After using the calculator the student will be automatically returned to the test item from which he was branched.

8. Student Comments

At specific points within the programme, a student may be allowed to enter any comments concerning a particular test item or unit.

9. Bulletin Board

The bulletin board option allows an instructor to send messages to his students through the computer terminals. Similar CMI systems have been developed at the University of Madison, for its own use and also as a service to schools. (Belt, 1975; Spuck and Owen, 1975).

8.2.2 Some Applications of CMI to Mathematics

The system at Florida State University is used to teach a variety of subjects, including mathematics at the undergraduate level. The simpler, batch-oriented system at the Middlesex Polytechnic (Romiszowski, 1970), although used by undergraduates in their first year, teaches remedial units of mathematics which are pre-requisite to the statistics content of the social science degree. Descriptions of this system (which is also programmed-learning based) appear in the papers annexed to this work and also in Chapter 2.

An early and very effective application of CMI, in a simple batch-oriented mode, was attempted by Croxton and Martin at the University of Aston in Birmingham in the late 1960's (Croxton and Martin, 1970).

The course in Structures, which formed a part of the Civil Engineering degree, was individualised by replacing all lectures by written "semi-programmed" handouts. The students attended the timetabled lecture sessions, as before, but these were used for testing the previous week's work.

and distributing the next week's work in the light of individual progress. The students would commence their programmed study assignments during the lecture, then continue them as "homework" devoting as much time as was necessary to complete the assignment. In addition, weekly seminars were held (as they always had been) to discuss points of difficulty and raise questions.

So far there appears to be very little new or revolutionary about the scheme. It fits into the traditional timetable of the university, and employs programmed learning units to promote self-study. The present author was involved as a consultant to this project on the quality of the original programmed materials whilst they were in production and remembers not being unduly impressed by the first drafts.

However, the system at Aston was exceptionally successful and the credit for this may be laid at the door of the computerised management system used.

Each week: (a) Test data on the previous week's work was computer-processed.

(b) Special "student comment cards" were also processed. Together with each assignment the students completed a card showing

- all sections or frames which they found woolly, difficult or misleading.
- all questions or practice problems which they did not solve correctly first time (or at all).

(c) This information was used to:-

- Identify major student problems, and which students had them. Thus it was possible to brief the seminar leaders very precisely, and to notify on an individual basis, those students who particularly should attend the seminar. Seminars were staggered through the week, so that the weaker student could attend more than one weekly seminar, each one devoted to a particular problem that he was experiencing.
- Identify major problems in the learning materials. Thus it was possible to revise and improve the materials, and still test out the new versions on slower students, during the same academic year.
- Identify students who were falling behind unduly and automatically inform them and their teachers that this was the case. Thus the professor could arrange extra meetings (of a motivational or instructional nature as required) for certain students before they fell back too far.

This regular control of the learning process, on an individualised basis resulted in very significantly superior examination results, as compared to the experiences of previous years in the statistics course, as compared to results of other course units in the degree and as compared to other institutions.*

Project P.L.A.N.

Perhaps the largest CMI project in operation is Project P.L.A.N. This project originated in the American Institute for Research, Palo Alto, California, was supported by the Westinghouse Corporation (who now market the system) and now involves hundreds of schools in all parts of the U.S.A. (Flanagan, 1968).

* Croxton and Martin quoted the following anecdote during a paper delivered at the APLET Conference of 1969 -

When the first group of students to experience the individualised system right through to the end of their course sat their final examinations, the University was still working to the London University External Degree. The difference in results between this group and groups from other universities (and also the same group on other examinations papers) was so startling that initially the examiners were convinced that the structures examination paper had been circulated beforehand and attempted to demand a re-sit.

It started in 1967 on an experimental basis, with about 20 schools (some in California and some in New York State) all linked to a computer at the Westinghouse Corporation.

The author spent some weeks in the summer of 1967 sitting in on the summer school at Palo Alto, where teachers, under the direction of Robert Mager, were jointly developing behavioural objectives and tests for the primary school curriculum. One result of this work was a book of several thousand precise objectives in elementary mathematics, published as a guide to teachers and curriculum designers (Mager, 1969). These objectives were then used by the participating schools to develop lesson plans and select or prepare teaching materials. The teaching methods and materials are not standard among the schools, but the objectives and tests are.

Data from these tests are collected centrally at the Westinghouse Corporation's computer, which also stores a vast amount of related data on the students' aptitudes etc., and control-data which had been collected by project T.A.L.E.N.T. (Flanagan, 1968).

Project T.A.L.E.N.T., has been proceeding for some years. It involves the establishment of a "data bank" of information about American students. Data on achievements, aptitudes, learning strategies, etc., have been stored on tape for thousands of students. Many interesting findings have already come to light. Individual differences among students are much greater than was suspected. Their performance

on some materials is very different from their performance on other basically similar materials. Many of the findings from T.A.L.E.N.T. have been used in deciding the form of project P.L.A.N. This project (Program for Learning in Accordance with Needs) has developed an individualised course of study in a range of subjects for students from grades 1 to 12. The course is managed by a central computer located at Palo Alto. A number of schools located as far apart as California and New York are in contact with the computer via normal telephone lines. This project differs from C.A.I. projects in that there is no direct student-computer exchange. The student works through written or filmed materials which have been selected to prepare him to achieve specified behavioural objectives. He then takes a test, completes a computer card, feeds the card into the one terminal in the school. The card is processed by the central computer, which marks and records his results and recommends the next unit of work to be tackled. The student may also feed in information regarding his interests and aspirations, and this is taken into account when planning his course. The computer may offer a selection of units of material (named Teaching-Learning-Units or T.L.U's,) all designed to reach the same objective, and the student may choose the one he finds most attractive. The computer is therefore used to plan, to record and to manage the individual student's course of work.

It is not, however, used to generate or to present the learning materials.

Much comment and research has been published concerning Project PLAN (Dunn, 1972; Lipe et.al, 1970; Patterson, F, 1973; St. Louis Board of Education, 1973)

Most of the reports emanating from the personnel involved with the project (both designers and users) has been positive. Some criticisms have also appeared, on the grounds that the project is not really all that cost/beneficial or that it leads to too high a degree of standardisation of objectives and methods among schools.

In connection with this last objection, please note the comments of Suppes which follow in the next section on CAI.

Other important CMI applications include the system used as part of the Individually Prescribed System of Instruction (IPI) developed by the University of Pittsburgh, (Ferguson, 1969; Cooley and Glaser, 1969; Carlson, 1974); the Title III project of the Pennsylvania State University (Igo et.al., 1969) and, nearer home, the Hertfordshire Computer-Managed Mathematics Project (Mathematics Association, 1976.)

Hatfield School experimented with the computer management of all-ability first-year classes and were sufficiently enthusiastic over the results to commit the whole first year to a computer managed course. The course contained two distinct components - E.S.Y.M.R.K., which managed a 'conventional' worksheet-based course and produced diagnostic information for the teacher and S.A.M. (Set and Mark) which produced, and subsequently marked,

a 'random' arithmetic test for each pupil.

In 1973, the National Development Programme in Computer Assisted Learning indicated a willingness to make a major investment in the Project, which by now was being used in two Hertfordshire schools.

Funds were made available for the appointment of a 7-man development team, for the Project to produce a series of video-taped television programmes to support the course, and for the controlled expansion of the Project to cover the first two years of secondary mathematics in 12 Hertfordshire schools.

The educational materials are produced by existing L.E.A. resources and computer processing is currently effected by the Hatfield Polytechnic computer which provides a service to all schools and colleges in the authority's area.

Pupils code their answers on mark-sensed documents. Schools submit documents via a courier van and receive results by the following morning.

The E.S.Y.M.R.K. course is divided into 3-week (approximately) modules, each with up to 15 worksheets, studyshee teachers' notes, answers, ancillary activity sheets, wall-charts, etc. There are 13 modules in the basic course in the first year and 13 further modules in the second. These are supplemented by additional and alternative modules and a number of 'revision interludes'.

Materials in the S.A.M. course are rather more ephemeral, being produced to need. There is scope for up to 1,000 skeleton specifications of question-paper types, each of which may be used in the production of an arithmetic question-paper. Two papers produced to the same specification will in general be different, effectively providing an inexhaustible supply of papers of a given standard.

Other British applications of CMI to mathematics teaching are at Glasgow University (Dunn & McGregor, 1969) and in the schools of the London Borough of Havering.

The London Borough of Havering has been supporting an experiment in the use of computer managed learning in various subject areas, including biology, and this is now being extended to other schools and other local authorities. The basic system of operation is as follows:

- (a) The student receives his work in the form of one or more CALTs (Computer Aided Learning Tasks). As far as the student is concerned each CALT contains a teaching section and a testing section. The completed test answer sheets are sent to the computer for processing at the end of the lesson.
- (b) The computer aided learning system is devised to operate without on-line terminals in the classroom and all the students responses to questions are written on answer sheets which are later punched onto paper tape and entered into the computer by

computer operators. The computer builds up a history of the student's learning activities and records his attainments. By comparing this history with the pre-requisites of certain teaching lessons, the computer endeavours to allocate to a student the most suitable teaching material available in its CALT library.

- (c) The allocation of CALTs by the computer is based upon its establishing a match between the CALT prerequisite capabilities and the capabilities present in the student's learning profile.
- (d) The computer tries to fill all of a student's lesson time with CALTs. As students work at different rates each CALT has three completion times specified, one fast, one slow, the other average. After a student completes a CALT he is asked to record the time.
- (e) The computer is able to determine the student's work rate and uses it in its next allocation of CALTs.
- (f) The CALTs may utilize any instructional technique available. They may be film loops, video or television programmes, experiments, demonstrations or teacher given lectures and tutorials.
- (g) The CALTs are of three types: Main line CALTs, Remedial CALTs, Enrichment CALTs.
- (h) Main line CALTs. These contain the tests and teaching material to cover the subject matter objectives of the course as described in the planning specification. They vary in length between about 20 and 60 minutes each.

Most of the lesson is made up of a selection of these CALTs which contain teaching material at one of the three levels of complexity and difficulty.

- (i) Remedial CALTs. Capability can be specified as absolutely essential in the course and should the student fail to achieve a pass mark in the questions testing the capability, then he is automatically assigned a remedial CALT which attempts to teach it.
- (j) Enrichment CALTs. After the computer has filled a student's lesson time with as many main line CALTs as possible, some unfilled time may remain. To fill this time the computer makes a selection from its list of enrichment CALTs. These are short work modules of about 10 minutes duration which cover objectives not specified in the original planning objectives, but which are interesting and relevant if the student has time to spare.

8.3 COMPUTER ADMINISTERED INSTRUCTION (CAI)

8.3.1 Characteristics of CAI

Instruction may be administered by a computer in several different ways, or modes.

Jerman (1969) described three approaches to CAI which represent different levels of interaction between student and instructional system, first suggested by Suppes (1966)

Drill-and-practice system

The instruction provided by this approach is supplementary to the regular curriculum taught by the classroom teacher. Each new topic or concept is introduced by the classroom teacher. Students work at the instructional terminals on exercises or review material previously introduced in class. Skills are maintained at a high level and difficult exercises are mastered through daily interaction with an eternally patient and tolerant instructional system.

Tutorial system

A tutorial system is intended to provide as much of the actual instruction as possible. A tutorial system assumes the major burden of instruction rather than play a supplementary role like a drill-and-practice system.

Dialogue systems

In this mode, a free exchange of questions and answers takes place between student and machine. Programmes are now being developed which will recognize certain spoken words and numbers, while other programmes being developed will generate spoken messages to students. Although the problems to be solved are substantial, much effort is being directed toward their solution. The PLATO IV system described later, approaches the dialogue mode in some of its applications.

Towle (1976) gives a further breakdown:-

- drill and practice mode
- tutorial mode
- simulation
- + instructional games
- modelling

The last three modes listed can all be thought of as types of simulation, and simulation as a type (but, in the author's opinion, not the only possible type) of dialogue between student and computer.

Towle defines the modes in his classification as follows:-

Drill and Practice

In the drill and practice mode of CAI, the aim is to take over the main responsibility for developing a learner's skill in the use of a given concept. Drill and practice lessons typically lead learners through a series of examples where they can practice material already learned or they can have the examples worked out for them step by step by the computer.

Drill and practice lessons are usually developed based on the assumption that the learner has already learned that which he is practising. The instructional purpose, then, is for the learner to gain familiarity and develop dexterity and speed with the skill or idea being practised.

Drill and practice lessons may be very simple, acting as a teacher with "flash cards" or may be very sophisticated in diagnosing learner weaknesses and immediately providing instruction to counteract those weaknesses.

Tutorial

In tutorial CAI applications the computer functions like a very sophisticated programmed textbook but allows a tremendously more complex network of instructional pathways through the material than a book could provide. The purpose of the computer in this mode is to present instruction that is simple, straight-forward, and individualised for each learner. This CAI mode can relieve the instructor of the burden of individualising instruction simultaneously for many different learners.

Tutorial CAI lessons are quite difficult to design and develop if there is to be more than just a few pathways through the instruction. The ability to branch to an appropriate set of materials depending upon the student's current response is an ability that is uniquely provided by the computer and probably cannot be provided in any way other than one instructor for each learner. But, the success of tutorial lessons depends heavily upon the creativity and careful planning of the instructional developer.

Simulation

Simulation techniques use the computer to simulate (or model) the behaviour of real systems. The learner may investigate the potential impact of alternatives in systems that would be too costly, too dangerous, too time consuming, or, for some other reason, impractical for him or her to experience in the real situation.

Instructional Games

We give this mode of CAI a separate category because of the competition present in instructional games that is not present in most CAI simulations. Competition can be between individuals, between teams of learners, between the computer and an individual, or between performance of one individual at different times.

A simple computer-presented instructional game might involve the presentation to a student of 10 simple addition problems one at a time. The computer records the amount of time the student required to give correct answers to these problems and displays this time to the student. The student then is asked to solve 10 more problems similar to the first 10 and try to "beat" his former time. Games of this kind are useful in providing motivation for students to practice a repetitious activity.

Modelling

Modelling is much like simulation in that the computer simulates a system of some kind.

However, the modelling mode of CAI allows the learner to have control over the model of the system being simulated.

Applications of modelling techniques might be involved in the development of a new aeroplane wing design where the designers could model the effect that various changes in the design would have on speed, range, etc. of the aeroplane.

8.3.2 Some Typical Applications of CAI to Mathematics

Perhaps the most well-documented applications of CAI to mathematics are those of Patrick Suppes and his collaborators at the Institute for Mathematical Studies in the Social Sciences. (Suppes, P. et.al., 1968; Stanford University Institute of Mathematics in Social Science, 1968 and 1969; Suppes, P., 1971).

Most of Suppes' work has been confined to the "Drill-and-Practice" and "Tutorial" modes, and has dealt with the teaching of elementary mathematics. Some recent work (Goldberg and Suppes, 1976) however, concerns the teaching of elementary logic at the University level.

Work started at Stanford with drill-and-practice arithmetic programmes in the early 1960's (Suppes, Jermyn and Brian, 1968) and has continued in an every-expanding form till now. The system now used was perfected and tested in 1966-68 (Suppes and Morningstar, 1972). The subject matter (e.g. arithmetic skills) is divided into concept blocks. The structure of the course for each concept block is similar and is shown schemetically in figure 8.1. A concept block is planned to occupy seven working days, a few minutes (15 - 30) per day. Students have already received prior instruction in the content of the concept block before commencing to work on the computer terminal.

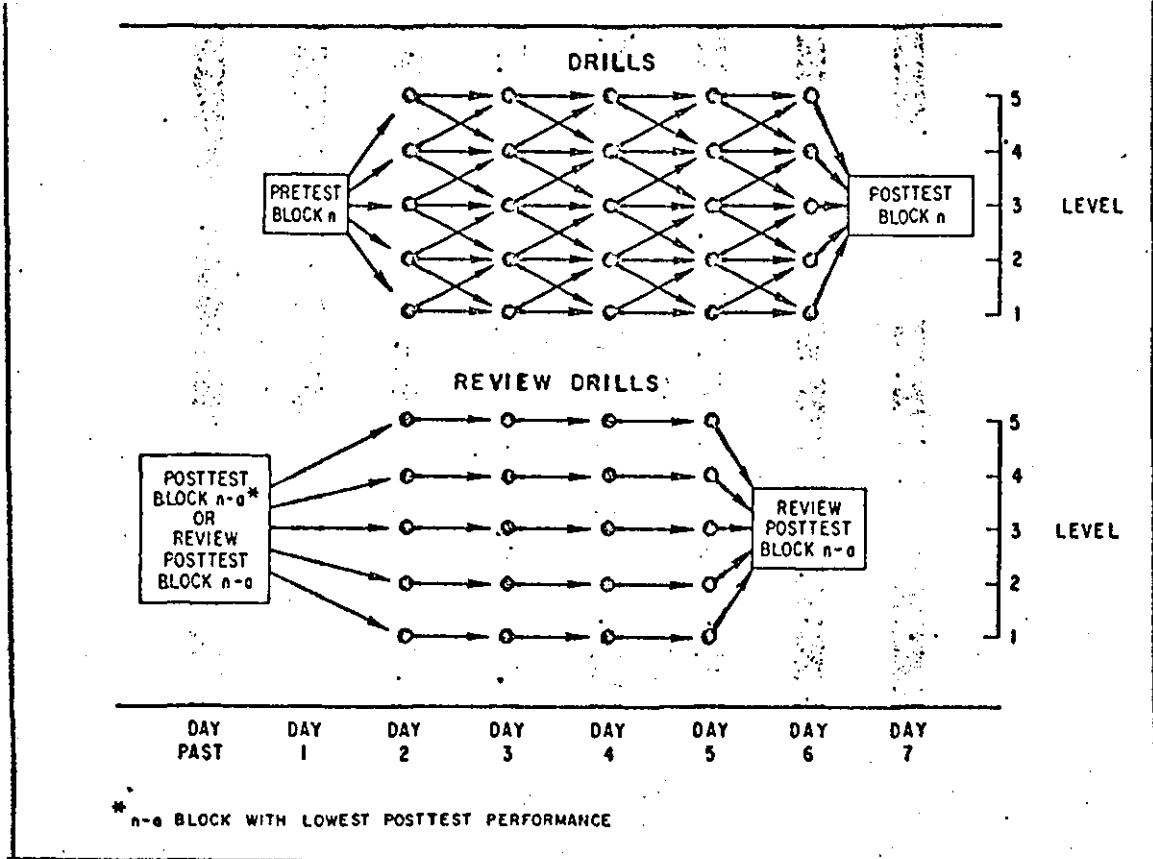


FIGURE 8.1 Branching structure of a seven-day concept block.

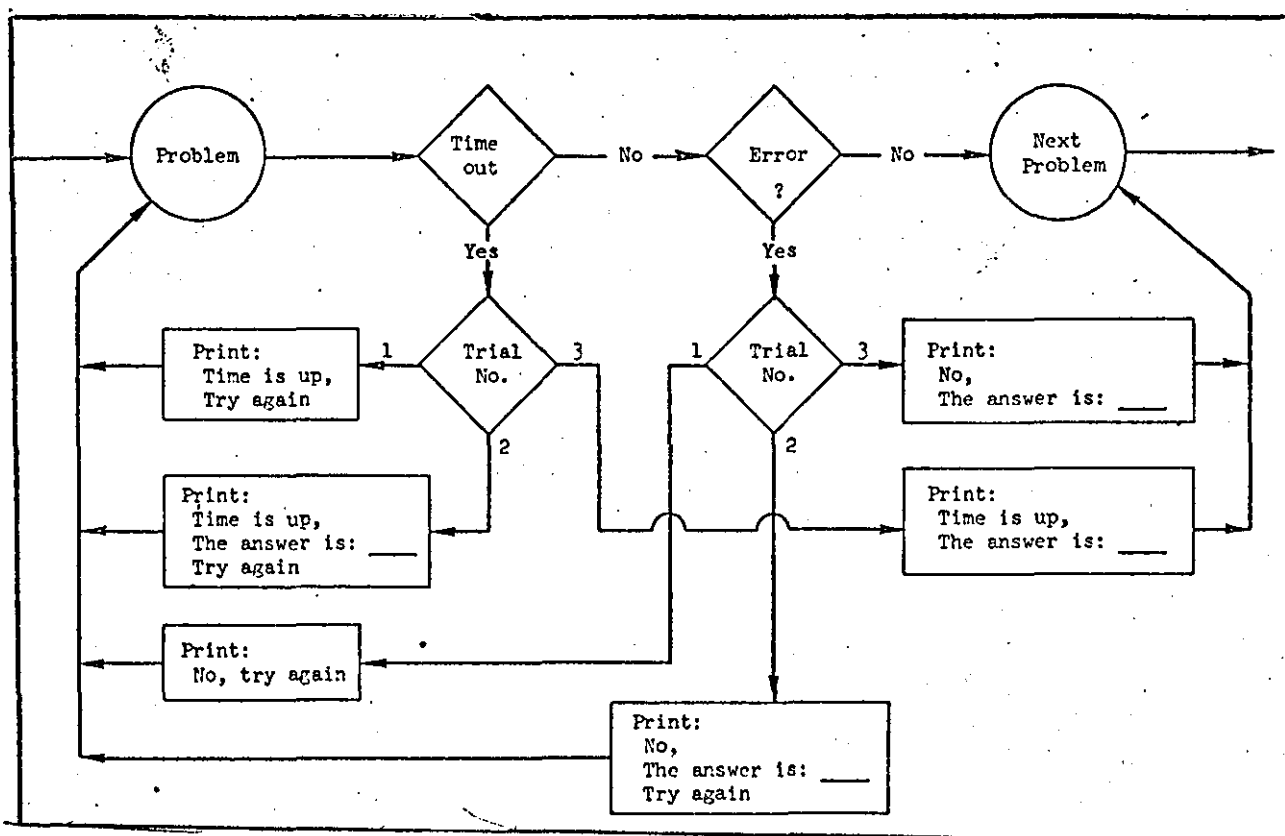


FIGURE 8.2 Flow-chart of programme logic for teletype drill programme.

ADAPTED FROM: "Computer Assisted Instruction at Stamford 1966-68" (Suppes & Morningstar 1972)

The student may work on practice drills of the concept block most recently taught, or on revision, or review, drills on previous concept blocks not yet fully mastered. The practice drills are arranged in 5 difficulty levels. The computer branches the student between the drills (within the same difficulty level, or to a higher or lower level) depending on the student's performance pattern. The procedure runs as follows: (Suppes & Morningstar, 1972):

Each student is given his problems on a computer-based remote control terminal in the school. After the student signed on to the programme by typing his student number and his first name, the teletype printed his last name and the set of problems appropriate for the grade, block, and difficulty level for that student.

The materials presented to the student for the seven days required for each concept block were:

- Day 1 pretest;
- Days 2-5 drill and review drill;
- Day 6 drill and review posttest;
- Day 7 posttest.

In presenting the drill, the teletype printed out a problem, positioned itself to accept the answer in the appropriate place, and waited for the student to type the answer. If the student answered correctly, he proceeded to the next problem; if he input the wrong answer, the teletype printed NO, TRY AGAIN and presented the problem again. If he made a second error, the teletype printed NO, THE ANSWER IS ... and presented the problem once more; if the student input the wrong answer for the third time, he was

given the correct answer and the teletype automatically proceeded to the next problem. The student was allowed from 10 to 40 seconds to respond, depending upon the type of problem presented. If a student took more than the allotted time to answer, the teletype printed TIME IS UP, TRY AGAIN instead of NO, TRY AGAIN. A flow chart of the programme logic is given in Figure 8.2.

Upon completion of a set of problems, the student was given a summary of his work that indicated the number of problems completed, the percentage correct, and the amount of time, in seconds, taken to complete the set.

A daily evaluation was sent over the teletype to each teacher giving individual progress reports and a class summary.

During 1966/67 nearly 900 students used the system and a further 1800 or so in 1967/68. The number of students and schools involved has been steadily growing ever since.

Results indicate consistently that groups of students undergoing drill-and-practice exercises after basic instruction, score better on tests (both immediate and recall) than students receiving the same instruction but not the drills (or receiving non-computer-based drills). These gains are more significant in less favourable socio-economic environments, where the general standard of mathematics teaching and learning is poorer.

Whilst most of the work has been restricted to arithmetic and algebraic operations (i.e. totally symbolic), some work has been done also with problem-solving of "word-problems" (Suppes, Loftus and Jerman, 1969). Some difficulties were encountered here in achieving effective communication between student and computer. The wording of questions was found to bear significantly on student success and to interfere with the "transformation of the problem into a mathematical problem" - the first stage of Descartes' model for problem-solving. Here we observe one of the difficulties which is slightly hampering the use of CAI in the tutorial mode, and posing very difficult problems (of 2-way verbal communication between the student and the computer) for researchers attempting to employ the "dialogue" mode of CAI.

The PLATO Systems

However, some progress has been made towards true "dialogue" CAI in the last few years, particularly in research on the PLATO IV system at the University of Illinois.

The University of Illinois has been involved in using the computer for direct education since 1960. Beginning with a single terminal system, PLATO I, the University of Illinois has progressed through a series of four systems. Each of these systems was designed to gather data and to improve upon the use of computers for direct education.

The first four years of the PLATO project were devoted to determining what subject matter could be taught, in what areas, to whom, and with what kind of effectiveness; as well as what computer processing power, memory, and communication rates would be necessary.

The PLATO I system consisted of a single student terminal connected to the Illiac I computer at the University of Illinois. (Bitzer et.al., 1961). The Illiac I, by today's standards, was a very small computer. It consisted of 1024 40 bit words of 16 microsecond memory.

Soon after the testing of PLATO I in a few areas, the design of the PLATO II began (Lichtenberger et.al., 1962). The purpose of the PLATO II system was to develop software that would permit more than one terminal to operate independently in the same lesson. Although the software for PLATO II was written to handle several terminals, memory limitations of the Illiac I computer permitted only two terminals to be connected.

Evidence gained by student interactions at even these two terminals indicated that in fact computers could be used successfully for direct education in a variety of areas. Consequently, a PLATO III system was designed. This system consisted of twenty terminals, connected to a Control Data Corporation 1604 computer. The PLATO III terminals, although using a more expensive television monitor, also utilized the storage tube for generating graphics

information from the computer and an electronic slide selector for selecting pictures. This new electronic slide selector could select or display any of 122 slide images in less than a microsecond. Eventually, four classrooms were connected to the system providing teaching experiments in a community college, a university, an elementary school, and a school of nursing.

With the PLATO III system, there were for the first time classroom experiments of sufficient size to provide evidence of the effectiveness of teaching with a computer. (Bitzer et al, 1969). In addition, analysis of the operational data provided new information on communication rates, computational power, and memory space needed to run large systems of several thousand terminals.

The present PLATO IV system consists of a large central computer to which approximately 1000 graphic terminals are connected. These terminals are located at over 160 different sites around the world in classrooms which may have as few as one terminal or as many as eighty. Over 3000 authors currently produce lesson material on the system using the system's TUTOR language.

Many thousands of hours of material already exist and new material is produced at the rate of 50 to 100 hours of instruction per week, (Lyman, 1975). The wide variety of lesson material ranges from the fields of science and mathematics to areas such as music and social studies and also includes special areas of education such as medical education and teaching of the handicapped.

The student terminal of PLATO IV is very sophisticated, allowing the computer to communicate with the student by print, speech and high quality graphics (both still and moving). The student may communicate with the computer by typing, or pointing to specific areas of the display panel.

Each terminal also has an additional input-output connector for attaching any number of devices. Tape recorders, film projectors, music boxes, or various types of instruments may be attached. Data are transmitted to and from these devices through the terminal. This capability not only allows the attachment and control of experimental laboratory apparatus, but it also permits users to explore their creativity in developing new hardware devices.

We shall not here go into details concerning the hardware systems of PLATO IV, nor the author-languages, but simply illustrate some of the dialogue-like facilities which are available to authors when constructing courses.

Authors frequently store concepts in the computer to interact with the student instead of storing all possible answers to a problem. An example of this, shown in Figure 8.3 is taken from a geometry course (Dennis, 1968). Geometry is an area where precise concepts can be defined and stored in the computer. In this lesson, the student can construct polygons of different size, shape, orientation, and location. The computer then identifies the figure drawn by responding in this case with "That is a bow tie, not too neatly tied". The computer did not identify the geometric figure by searching a long list of figures stored

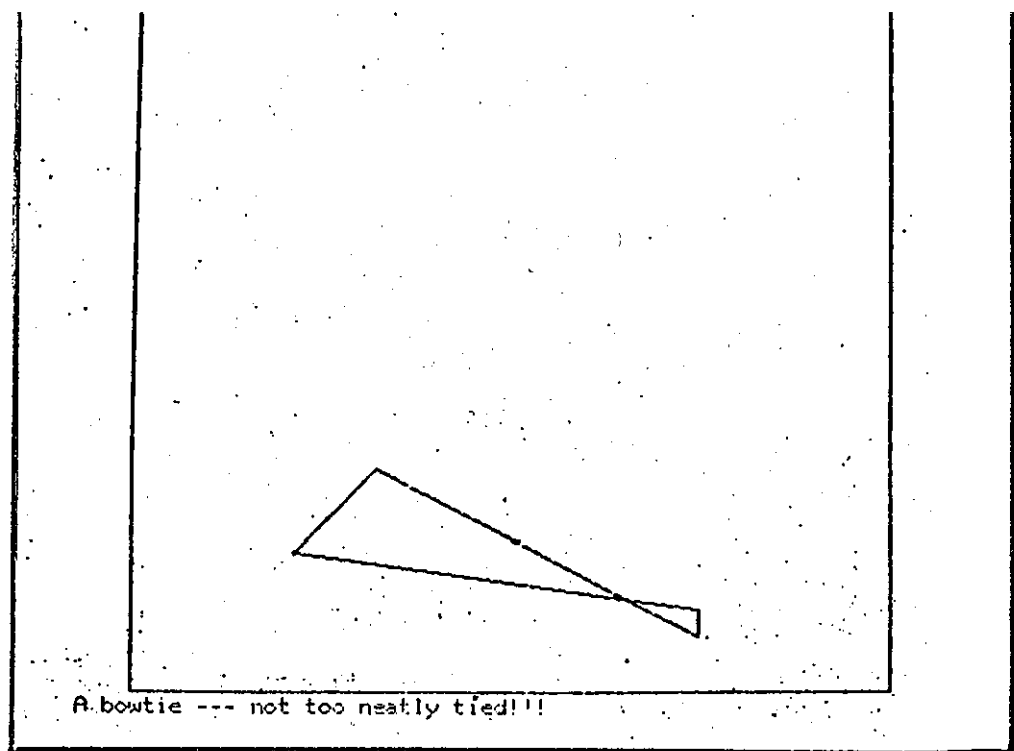


FIGURE 8.3 The computer's response to the figure drawn by the student (in response to the computer's request "Draw a figure - I'll guess what it is."). The computer is able to analyse any polygon constructed by the student and tell him what he has drawn.

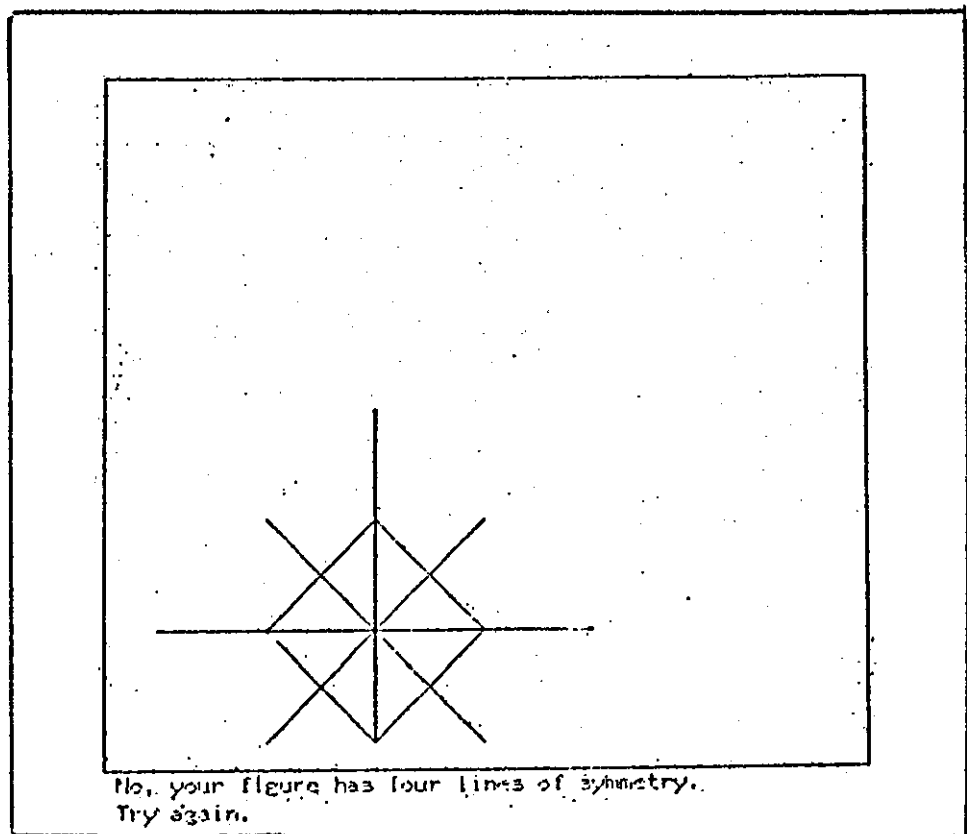


FIGURE 8.4 Response to student's attempt to draw a figure with 2 lines of symmetry. The computer not only tells the student that he is wrong but also attempts to tell him what he did wrong.

in the computer and comparing them to the student's. Instead, it computed the figure drawn and determined its characteristics. Briefly, here is how this feature is used in the class. In Figure 8.4 the student was asked to construct a quadrilateral with only two lines of symmetry. After drawing what he thought was a quadrilateral with only two lines of symmetry, he asked the computer to look at it. The computer responded in this case with, "No, your figure has four lines of symmetry", and in addition, also drew the symmetry lines into the student's figure in case he could not find them. Thus, the computer was able to correctly respond to this unique figure. It was able to do this because the geometric concepts have been programmed in it rather than a list of geometric figures.

For subject matter where algebraic responses are required of the student, the system has the ability to determine if the student's response is the same as that given by the author independent of its form, (Lederman, 1975).

An example which illustrates both the use of stored concepts and the use of the system as a simulated laboratory is one taken from a genetic biology course. This lesson is designed to replace the traditional fruit fly laboratory. The computer is programmed with the genetics rule for fruit flies. Each student begins with two stocks of fruit flies and can specify which of the flies he wishes to cross. His task is to perform a sufficient number of appropriate experiments to enable him to write a report describing the hidden and visible traits of fruit flies and to demonstrate that he understands the genetic rules

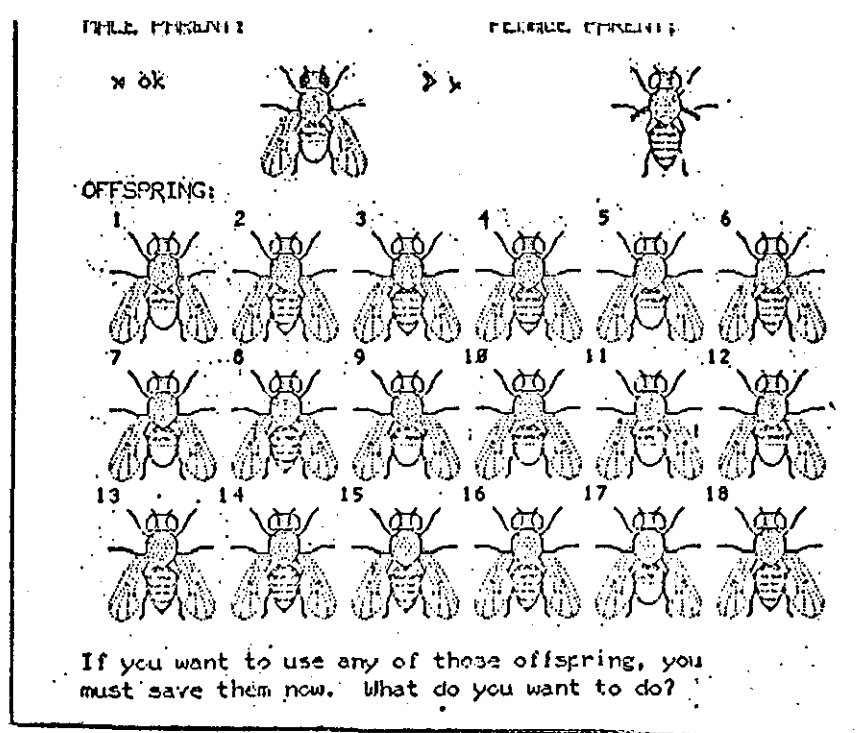


FIGURE 8.5 Mating of Fruit Flies
First Generation

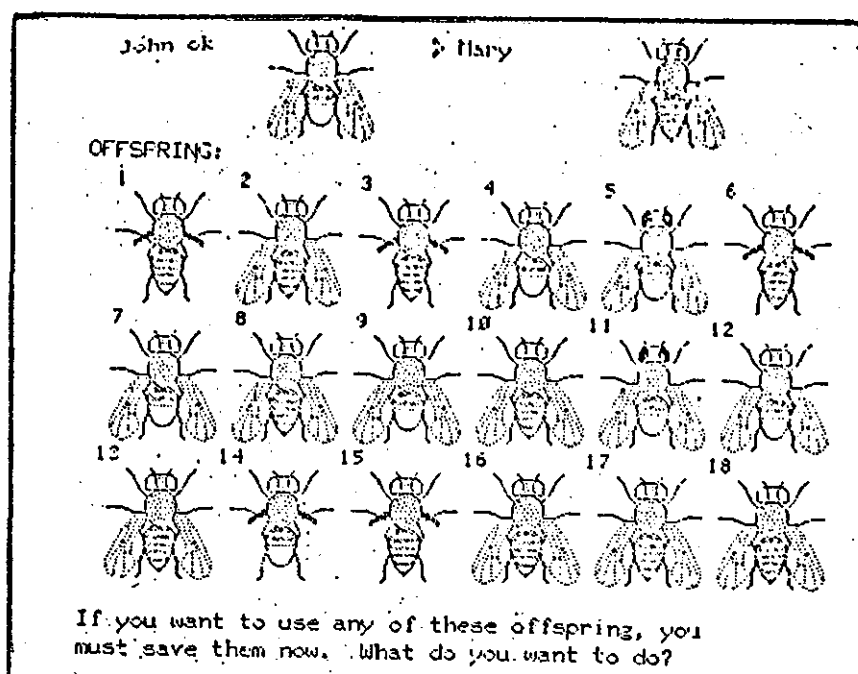


FIGURE 8.6 Mating of Fruit Flies
Second Generation

Adapted from "PLATO: An adventure in learning with
computer-based education" (Bitzer 1976).

for fruit flies. Figure 8.5 shows the display of a student who has begun by mating a male from stock X with a female from stock Y. The computer used the genetic rules to produce the genes of the offspring, and then drew pictures of the offspring flies on the display screen. To carry the experiment through one more generation, the student has asked the computer to save some of the flies - fly 1 which is given the identifying name of John and fly 2, the name of Mary. John and Mary are mated to obtain the next generation of offspring. In this generation of offspring, shown in Figure 8.6, one can see some of the characteristics that were present in the grandparents but not the parents. Computer simulated laboratory situations such as this one are used in a variety of areas.

Another example of the data processing capability of the system can be found in some example material shown in Figure 8. This programme, developed by Paul Handler at the University of Illinois (Klauff and Handler, 1976) is designed to present information and projections to aid planners in decision making. The complete programme deals with food, economies, energy, and population. The material pictured here deals with the population aspect of the programme.

Here the planner has asked the computer to plot the projected population of Mexico and the United States from 1970 to 2070. This graph indicates that if the present trends continue, Mexico's population will surpass that of the United States in 50 years' time. A planner trying to solve the problem of a rapidly expanding population

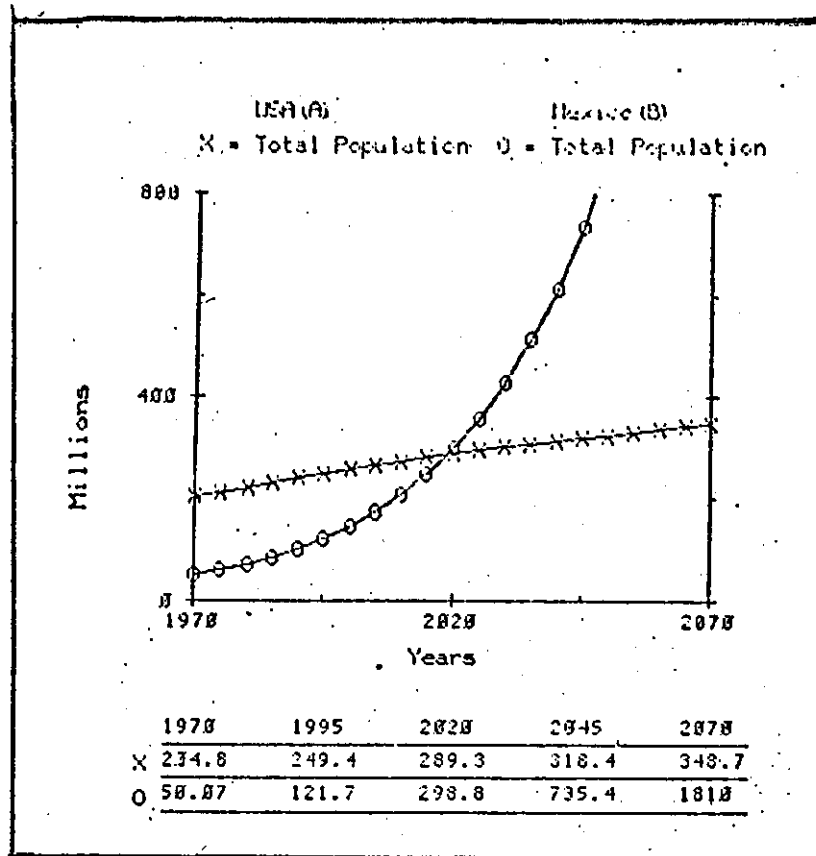


FIGURE 8.7 Strategists can use the computer to make projections based on their individual assumptions and plans.

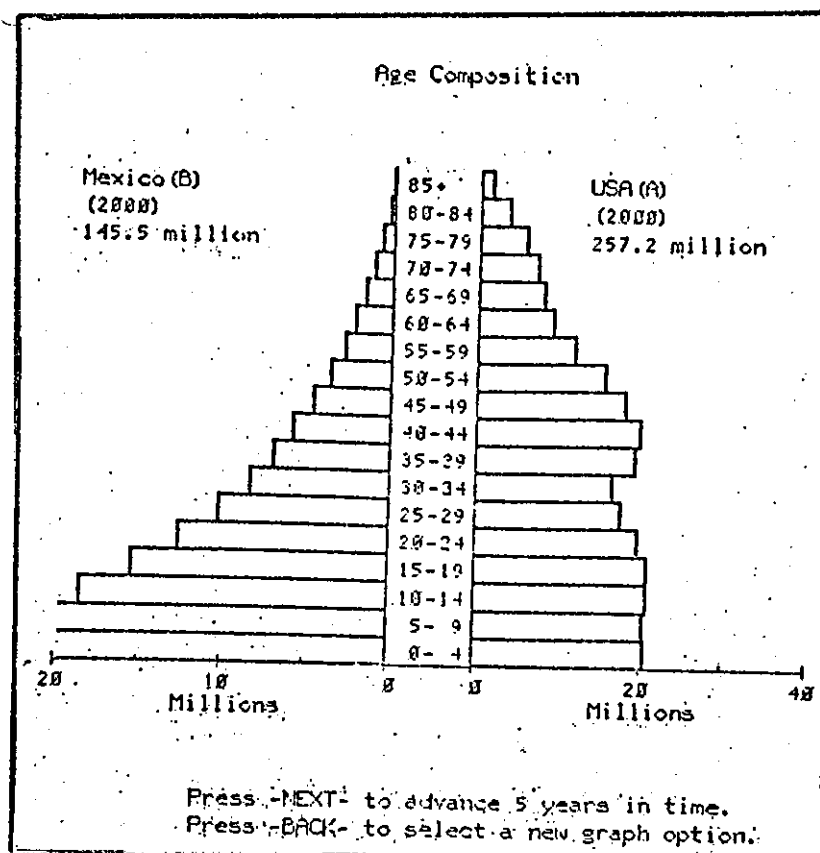


FIGURE 8.8 The computer is used here to project age composition plots to aid planners.

would, in using this programme, change some variable such as the fertility rate to modify the growth dynamics. He would then have the computer plot the new results and would request the new graph be plotted alongside of the old one so that the two could be compared. The planner can quickly change the variables until he reaches his objective in terms of optimal population, food, land, etc.

The computational power of a large computer is also made available to the users. The TUTOR language facilitates storage of student-generated algebraic and alpha-numeric strings which can be compiled into machine code and executed.

In addition to the computational and instructional mode of the system, intercommunication from terminal to terminal is also possible. This intercommunication permits electronic mail services as well as the transmission of computer graphics. Applications for this mode of communication include interactive games and permits the teachers to help students who may be geographically distant from them by viewing the student's display.

Presently, plans are being made for a PLATO V system to be used in the early 1980's (Bitzer, 1976). A considerable amount of new design and development is now underway for this system. Some of the engineering work which is being done will provide new types of service, but for the most part, the new developments will improve upon the present services and will provide them at a lower cost.

It appears from projections that there are likely to be over a million graphic terminals by the early 1980's. Projections made by extending the past must, of course, be considered with caution. However, since most of the technology mentioned here is already in existence in the research laboratory, there is a good possibility that these projections will be reached in the next five to ten years.

The PLATO V system is being designed to meet the anticipated needs of users in 1980. This terminal will contain a microprocessor with memory and thus will be intelligent. This capability will provide faster and more dynamic graphics at the terminal than could otherwise be provided when a telephone line is used as the connection to the main computer.

The PLATO V system will consist of several hundred interconnected centres. Each of these centres will be capable of operating concurrently over 4000 graphic terminals. The central computer will provide each terminal with more than twice as much memory and processing power as presently provided in the PLATO IV system. Although terminals may be connected to different centres, they will be able to communicate with terminals at other centres and share lessons. Although many new services will be available, the cost of using the PLATO V system is expected to be reduced by a factor of 5 to 10 from the present day cost.

Other systems

Other CAI systems currently being used to teach mathematics, include the system at the Pennsylvania State University, based on an IBM 1500 computer and used to teach elementary school mathematics in schools. (Heimer, 1973) and to teachers in training (Hall et al., 1970), and the system used by the Montgomery County Public School, U.S.A. since 1968 (Dunn et al., 1974). This latter system operates in both elementary and junior high schools, offering a modular course. The project was federally funded till 1971, but is since supported by the school board - an indication that the costs of CAI are indeed reducing to practically acceptable levels. At the post-secondary level, the STAT-CONCEPT project offers a coordinated collection of interactive computer programmes and printed materials (part CAI/part CMI) designed to enrich graduate instruction in educational statistics. By doing the computation and providing a responsive "dialogue-type" environment and computer allows the student to explore statistical concepts in a "what happens if .." laboratory-type atmosphere (Rubner et al., 1974).

The Purdue Instructional and Computational Learning System (PICLS) offers similar facilities, and has been applied in the teaching of numerical methods on undergraduate courses (Oldehoeft, 1970). First there is a CAI "tutorial" presentation of the mathematics behind a particular algorithm. The student then solves several problems to demonstrate and develop his mastery of the algorithm. Finally, the student passes to an exploratory stage, formulating his own problems (the computer performs all necessary calculations).

In Britain the British National Development Programme for CAI is supporting the Computer Assisted Learning (CAL) projects at the University of Glasgow (Hunter, 1976).

The main objective of the project is to provide a CAL service in basic mathematics for first year students at the University of Glasgow, in which the aim is to help students by an interactive dialogue to gain confidence in their knowledge of mathematical topics and in problem solving. The basic structure of a topic with illustrative examples will still be presented in lectures, and tutorials, problems hours, weekly exercises and progress-assessing examinations will still be held. In spite of this array of help, too many students at present make little or no real progress; it is hoped that the possible advantages of CAL will be helpful, in that, compared with other media, CAL can be more personalised, self-paced and interactive, more respondent to student's progress by frequent and relevant feedback, more demanding of a student in that full attention is required (especially during the important first 15 to 20 minutes), and has excellent dynamic graphics capabilities.

The first units produced have been in calculus (integration) and geometry (3-dimensional).

Further CAL units are being prepared on various topics such as sets, functions, number systems, differentiation, further integration, differential equations etc. Units in Physics have been prepared using the author language, MALT, developed for the Mathematics Project. Since September 1975 the project has investigated the possibility of

producing self-help materials in another medium but designed in relation with the CAL materials. This other medium will cope with areas not suitable for CAL presentation (e.g. requiring tabular work, heavy technical algorithms or manipulation, difficult notation or being unsuitable for interactive presentation), but important for student progress. It has been decided to use an audio-cassette presentation with prepared texts and slides.

However, the main concern is to explore the self-learning mode, keeping in mind such questions as: For a given concept, how many problems of a certain type are needed (on the average) to ensure that the concept has been grasped?; Is one solution strategy for a problem more acceptable to students than another?; When (if ever) is a student in a strong enough position to communicate the essentials of a topic to others?; When is a student ready to construct his own problems?; For a difficult topic, what proportion of students can be helped to eventually solve problems away from a prompt situation?

Another centre of activity is Leeds University. Experiments commenced in 1967 with "Suppes-type" drill-and-practice programmes in elementary arithmetic (Woods et al. 1969). This was extended to other applications (Sleeman and Hartley 1969). Work is continuing, with the support of the National Development Programme in Computer Assisted Learning (Hooper, 1974).

8.4 RESEARCH ON COMPUTER-BASED LEARNING SYSTEMS

Whilst there is a vast bibliography of articles concerning computers in education, there are fewer directly related to the teaching of mathematics. Of these, fewer still report formal research. For example, the proceedings of a National Conference on "Computer-Assisted Instruction and the Teaching of Mathematics" (National Council of Teachers of Mathematics, 1969) contain fourteen papers but do not include a single research study. Due no doubt to the complexity and the cost of computers, the subject has to date created more talk than action.

8.4.1. Comparative Studies

There are however, some studies reported. The extensive reports on the work at Stanford (Suppes and Morningstar, 1972) investigate characteristics of the system and factors within the system which influence learner performance. No direct comparisons with "traditional" teaching have been attempted, partly because no exact equivalent to "drill and practice" exists in traditional courses. Comparisons have however been made by others, both of the Drill and Practice mode of CAI, the Tutorial mode, and of Computer Managed Systems. In all, 16 comparative studies were located in the literature search. Of these 12 were significantly favourable towards the computer based system as opposed to the control. The other four gave no significant differences.

Concerning the Drill-and-Practice mode of CAI, Davies (1972) in a Ph.D dissertation compared the performances and attitudes of 240 low-ability elementary school students, using a drill-and-practice routine developed to accompany an existing set of programmed curriculum materials. He found that students using the computer performed significantly better in computational skills than students not using the computer. No differences in attitudes to the course were noted.

Palmer (1973) reports a series of three large comparative studies, involving 14 School districts in the Los Angeles County. Results indicated that

1) the mean post-test scores for the experimental groups exceeded those of the control groups; 2) a higher percentage of experimental than of control students exceeded their expected growth rates for the period; and 3) the students receiving CAI experienced growth rates substantially beyond normal expectations. Control group students performed better on tests of reasoning ability, perhaps because the CAI did not stress this skill. Stovall (1969) reported gains in computational skills averaging $1\frac{1}{2}$ years after some months of use of the Stanford drill and practice programme.

Woods et al. (1969) at Leeds University compared computer-based and non-computer-based drill and practice. The authors observed a significant improvement in performance on problems when the problems were worked "on line" at a computer terminal (with immediate feedback of results) as opposed to being worked in a paper-and-pencil format with feedback only at the end of each session.

Finally, Castleberry (1970) reports favourable results for a "mixed" (part drill-and-practice, part simulation) mathematics programme, designed as a remedial, or service, course for college chemistry students.

No unfavourable reports concerning the use of drill-and-practice programmes have been encountered. This is not the case for experiments which set out to be the primary instructional agent - tutorial programmes.

Hall et al. (1969) report favourable results for a CAI course in modern mathematics and mathematics teaching methods for elementary school teachers.

The course, called "elmath", consisted of 80% mathematical content and 20% teaching methods, with the methods units interspersed throughout so that each would be studied immediately following the presentation of related content. Results of various analyses of data gathered during the course from the 130 participants showed that the programme was effective in providing inservice education for teachers of elementary school mathematics, that the programme increased favourable attitudes towards mathematics, that the content of the course was probably learned faster in the CAI format than in a conventional classroom, that the course needed several revisions, and that both high and low achievers expressed favourable opinions toward CAI.

A more complex example, involving both the tutorial mode and a form of CSLA (Computer-supported learning aids) is reported by Bitter and Slaickert (1970). They developed a CAI package on Calculus.

Experimental and control classes were compared at each of three participating colleges. The experimental classes solved calculus homework assignments using timesharing remote computer terminals. These classes used programmed materials during the first week, outside of class, to learn BASIC. The experimental and control classes at each school were taught by the same instructor and covered the same calculus content. Many of the usual homework assignments were replaced in the experimental classes with assignments designed for computer application. The subjects had to write their own programmes to solve these assignments. Statistical analysis showed that the subjects which were provided with computer assisted instruction achieved significantly higher in the college introductory differential calculus course than those who did not have use of a computer. Further, female students achieved significantly higher than male students.

Finally, Attala and Howard (1974) report on a system using a minicomputer linked to audiovisual media, for the teaching of computer programming at the college level. The "Learning Resource Aided Instruction (LRAI) System centred around a Data General NOVA minicomputer augmented with slide projector-audio cassette media was designed and developed at the University of California, Santa Barbara.

A similar system was integrated into the educational programme at California Polytechnic State University and experimentally evaluated for two courses with large enrolments. These courses were an introduction to COBOL and an introduction to BASIC respectively. The evaluation procedure consisted of comparing the curricula of the two courses taught in a traditionally-aided instruction (TAI) mode with that taught in a LRAI mode using appropriately structured questionnaires, common examinations, and statistical analyses. The results of the evaluation strongly demonstrated that the performance of students using the LRAI approach was better than those in the TAI mode of instruction.

The four studies located which fail to show significant gains for CAI are all examples of the tutorial mode of programming. A direct comparison of CAI and printed programmed instruction is reported by Phillips (1971).

The purpose of this study was to compare experimentally the relative effectiveness of two instructional media - a computer-based display unit and a programmed-text booklet - for presenting selected instructional units of a common programme for teaching FORTRAN IV. In conducting the study a common programme for teaching FORTRAN was prepared and presented by the media being tested to 49 University students.

It was found that students taught by programmed-text booklets made significantly fewer errors when working with the media than did students taught by computer-based display units. However, it was concluded that neither of the two media is superior to the other when they are used for presenting programmed instruction to "Computer Science

103"students in regard to knowledge of language rules, ability to solve programming problems, and attitude.

This study simply lends support to the obvious contention that it is not effective (and certainly not cost-effective) to use a computer to present instruction which is structured sufficiently simply to be presented by direct printed or audiovisual means.

A study by Dasenbrock (1970) supports this view. Dasenbrock prepared a CAI programme based on existing (not programmed instruction) materials.

This investigation was conducted to determine the validity of the use of Computer Assisted Instruction (CAI) as a tool in formative curriculum evaluation. The instructional materials were selected from 1968 grade seven materials produced by the Intermediate Science Curriculum Study (ISCS). The comparison of student performance within these instructional materials was made between CAI (20) and non-CAI students. The non-CAI students sample consisted of 40 students from several schools using ISCS materials.

The questions within the ISCS instructional sequence were classified into eight process categories. The number of correct responses to the questions within each of the categories was considered to be a measure of student performance. Student performance in each of the categories was correlated with logical reasoning, reading and general ability measures. The change in correlations between the first half and second half of the instructional sequence was not significant in 22 of the 24 cases. The results of the study indicate that CAI and non-CAI student performance was similar with the ISCS materials.

These findings suggest that either the "programming" that went into the materials to transform them into the CAI version resulted in no improvement in materials quality, or that the original ISCS printed materials were already so efficient that there was no room for improvement in test scores. Once again one observes that CAI contributes nothing to the presentation of simple textual material. The general intention behind this study, however, of formatively evaluating curriculum materials on the computer, is very sound indeed. One may programme the computer to collect and analyse data on learner/ performance "on line". One can modify faulty sequences by "overtyping" without any delays for printing, thus developing and refining the materials much more rapidly. Once the materials function satisfactorily, one produces permanent paper versions for use "off-line" (the computer might even be employed to print these).

A study with a slight variation is reported by Durrall (1972). This is really a mixed CMI/CAI system. Initial study is from programmed instruction, followed by computer-managed diagnosis. The study compared two forms of remedial action.

Approximately 70 seventh grade mathematics students worked individually in self-instructional booklets for a period of 15 weeks. Upon completion of each booklet, the student was evaluated by direct contact with a computer through teletype terminals. If criterion was not attained, half the students received first remediation through the computer and half from the teacher. Further remediation,

if necessary, was from the teacher for both groups. The two methods of remediation were equivalent overall, but there was some indication that low ability students found teacher remediation more supportive.

Whether this result was due to inherent weaknesses in the CAI remedial materials, whether the teacher indeed adopted a richer, more adaptive remedial strategy or whether the "human-contact" factor was partly responsible is not clear from this study.

Another study of a CMI/CAI system which gave no significant differences is reported by Rockhill (1971). This study is an attempt to develop and evaluate an individualised instructional programme in pre-calculus college mathematics. Four computer based resource units were developed in the areas of set theory, relations and functions, algebra, trigonometry, and analytic geometry. Objectives were determined by experienced calculus teachers, and multiple-choice questions were written for each objective. Programmed instructional materials were selected for use by the students. Computer programmes were written for each unit which diagnosed student difficulties and provided printed outputs of instructional materials for each objective not satisfied by the student. One of two college pre-calculus classes used the resource units while the other class acted as a control group. No significant differences in achievement were found.

Research on the value of CMI is generally ambiguous. For example, a large year-long study of the effectiveness of Project PLAN was performed in the St. Louis public schools system (Patterson 1973).

The study was conducted during the 1972-73 school year in both control and PLAN classrooms for grades 1-7. The PLAN classrooms did significantly better in grades 1 and 3 and significantly worse in grades 4 and 6. Grades 4 - 7 were handicapped by a midyear start up and a contamination of pretest data. Measures of self-esteem and anxiety were also taken and on these three grade levels scored significantly better with PLAN and one significantly worse. Questionnaires of student and teacher opinion were also used, and a cost analysis revealed that the cost of PLAN (10 times greater than the control) could be expected to decrease to normal levels once normal operation begins.

It is not clear from the report whether the change in results between grades could be ascribed to the age-group effects, to a Hawthorne effect (this was not the first year of Project PLAN in the school system so older children may have already studied in the system, whilst younger children perhaps had not), to the different content of the mathematics taught in the various grades, to the commitment and skills of the teachers, or to several other possible factors.

A more rigorous study, by Ferguson (1969), investigated the implementation of a CMI system of control in conjunction with the IPI system of mathematics instruction.

A model for computer-assisted branched testing was developed, implemented, and evaluated in the context of an elementary school using the system of Individually Prescribed Instruction. A computer was used to generate and present items and then score the student's constructed response. Using Wald's sequential probability ratio test, the computer determined whether the examinee was or was not proficient in the skill being tested. If such a decision could be made, he was branched to another objective according to specified criteria based upon the hierarchy. Otherwise, another item was generated and the cycle repeated. Results showed that the computer test was highly successful in providing reliable information in substantially less time than that which was required by the conventional paper and pencil test.

8.4.2 Other Research. The computer as a research base,

As mentioned above, one benefit of a CAI (or a CMI) course is the ease with which evaluation data may be collected and processed. The "computer-based textbook development" function may well be one of the greatest short-term contributions of computers to the instructional process. In the remedial mathematics course which the author installed at the Middlesex Polytechnic (Romiszowski, 1970 - annexed to this study) this is exactly what happened.

A CMI system was operated for the first 3 years, during which data was continually collected and analyzed, concerning all aspects of the course. The diagnostic tests, learning materials and tutorial activities were validated and revised "on line", until they were operating at a high level of reliability. Then the CMI element of the scheme was abandoned (or rather, put into cold-storage whilst waiting for a time-sharing computer system to be installed).

Another result of the data-gathering and processing capabilities of computer-based systems has been to encourage researchers to perform studies on programming variables, learner variables, control variables, etc., using computer-based courses. Another factor to consider is the ease with which one can ensure that independent variables (for example teacher-personalities) do not contaminate the research results. Some of this research is reported here, as, apart from being illustrative of yet more applications of computer-based systems, it will be of relevance to some theoretical considerations to be developed in later chapters of this study. Only recent studies directly concerned with mathematics education and highlighting certain factors of later importance are included.

8.4.3 Investigations of the Mastery Learning Model

The mastery learning model suggested by Bloom (1968), Carrol (1963), and others is described fully elsewhere in this study (Chapter 3). One key principle of this model is that learners should not pass to new materials before achieving mastery of all necessary prerequisites. Mastery in this context is usually defined as 100% correct performance (occasionally 90% or 80% is accepted) on a given test or exercise. In classroom instruction, resource-based learning and even programmed instruction this is difficult to ensure. The computer is one way of effectively controlling the learner's progress, thus the use of CAI as a test-bed for the mastery model. The review of the literature revealed two studies investigating the mastery model applied to mathematics instruction. Magidson (1974) developed a CAI package for use on the PLATO system.

The objective was to apply mastery learning principles in the development of a computer-based instruction lesson on "Divisibility Rules", which was designed for students preparing for the General Education Development (GED) examination, to demonstrate that computer-based instruction which follows mastery learning principles facilitates student learning and fosters positive student attitudes toward learning. The lesson on "Divisibility Rules" follows a systematic approach to instruction that offers the student a rationale, objectives, pretest, alternative learning activities, and posttest with provision for revision.

No control groups were used. The programme results were compared with a "criterion-referenced" standard of 80/100 (at least 80% of the learners should achieve 100% correct performance on the final evaluation).

The achievement results of the target group failed to measure up to the goal that 80 per cent would achieve mastery. Technical difficulties hampered the results. The attitudinal results, however, were unanimously positive; this demonstrates that mastery learning strategies can provide students with enjoyable learning experiences.

The failure to reach the desired criterion suggests that either the programme of instruction or the system of administration was imperfect (and no doubt capable of improvement in the light of data gathered).

Another study is reported by Taylor (1975).

Three learning models (adaptive mastery, typical mastery, and traditional non-mastery learning models) which employed different criteria for terminating computer-based practice in order to determine mastery or non-mastery of arithmetic skills were compared.

All treatments involved the teaching of basic arithmetic skills to seventh-grade students. The adaptive mastery learning model produced the same high level of performance on both the post-test and a delayed retention test as the other two models, but required less time, fewer practice items, and minimized overpractice.

This result suggests a possible superiority of adaptive control over both the "mastery" and the "student-directed" learning models.

8.4.4 Investigations of student-directed learning

In the last-quoted study, there was no significant difference observed between the "mastery" and the student-directed learning models. As these two models spring from radically different philosophical positions, the result is particularly interesting and relevant to our later discussions.

Several other studies have compared student-directed and system-directed (though not necessarily "mastery") models. Fisher et al. (1974) performed a study involving choice or no choice of level of difficulty. The Stanford drill and practice arithmetic programmes were used. The experimental group used a modified version allowing the student to choose his level of difficulty. The control group used the standard course which directs the student automatically to an "appropriate" level of difficulty.

Among the major finds were: (1) that the "choice" group was significantly higher in engagement, (2) for both groups, engagement decreased significantly over a 15 day period, and (3) distinctive choice patterns did occur. The findings also showed that children will choose problems that result in poor academic performance; if performance in choice situations is to be improved, training methods that use information about children's unique patterns of choices should be designed.

Jacobson and Thompson (1975) compared two versions of a CAI programme in elementary mathematics for grades 4 and 5. One version gave control over the sequence of presentation to the student, the other followed a pre-determined decision algorithm in reaction to student performance.

However, before being given the CAI course, the experimental group were given training in self-management and in making curriculum-choice decisions.

Students in fourth and fifth grade were able to effectively manage their learning in elementary mathematics and apparently learned faster and retained material better than a comparative group of students.

At the post-secondary level Judd et al. (1970) investigated student-directed learning in a course labelled "MATHS".

MATHS is a computer-assisted instruction course in remedial mathematics for college level students. The course offers tutorial help in the areas of exponentiation, logarithms, and dimensional analysis, and drill in a number of basic mathematical skills. An attempt was made to allow the student to control his own course of study when using the programme. To validate this approach, an experiment was conducted which separated students into five groups, each with different degrees of control over the course flow. An evaluation of course effectiveness and student attitudes was undertaken. It was concluded that the programme is an effective instructional experience for the students who participated and that student attitudes

concerning the programme are generally favourable. In the area of student control, the results suggest that if students are to be given the option of deciding whether or not to enter a particular instructional segment, they should also be given control options within the instructional segment. If this is not the case, there is a tendency for students who need the instruction to avoid entering the instructional segment. Increased student control failed to improve student attitudes concerning the programme.

These, somewhat ambiguous results suggest that the question of student-directed/system directed learning is not one that has a simple answer. Many factors - learner variables, content variables, type of objective - may have an influence. We shall return to discuss this important point later on.

8.4.5 Investigations of programming and organisational factors

This last group of studies includes only a few examples of research into the structures and methods of CAI applied to mathematics instruction.

Hill (1971) compared a tutorial programming strategy with a drill-and-practice strategy for the teaching of orthographic projection. The tutorial strategy was shown to be significantly better.

Schoen (1972) prepared several versions of a CAI unit designed to teach the concept of function. The versions differed in the degree of individualisation (the frequency of adaptation to the incorrect responses of the individual). No significant differences were observed.

In a remedial mathematics course for electronic technicians, McCann (1975) compared various mixtures of the drill-and-practice, tutorial and dialogue/game modes using the facilities of the PLATO IV system. The six groups did not differ significantly in attainment or in learning time. However, there was a strong preference for the game mode among those who had experienced it.

Finally, Okey and Majer (1975) investigated the use of a CAI terminal by groups of learners.

Students worked together at the PLATO IV CAI terminal and then completed criterion-based tests individually on the material covered and an attitude questionnaire. No significant differences in achievement were found, but very significant differences in the time to complete the module were observed. Pairs of students required the most time, and groups of 3 or 4 required the least time.

These few investigations cannot be taken as conclusive. Rather they illustrate that more research is needed on a variety of factors influencing mathematics learning in general and learning through computer-based systems in particular. All the factors investigated in the above mentioned studies are relevant to the planning and execution of mathematics instruction in general. The CAI laboratory is an excellent, controllable test-base to investigate them.

CHAPTER 9

SOME PROBLEMS IDENTIFIED

9.1 Summary of the Systems Reviewed

The prescriptive, student-directed and cybernetic approaches to course control. Comparison of the systems reviewed - media used and control philosophy adopted. Different trends in USA and U.K. Trends in post-secondary education. The growing importance of the computer.

9.2 Summary of the Research Reviewed

The comparative surveys of Miller and Schoen. Negative results. Discrepancies. The limitations of comparative studies. The problems of institutionalisation of innovation.

9.3 Directions for Further Research

Research into methods of control.
Research into student choice of objectives, methods, etc.
Research into learning styles and strategies.

9.4 Conclusion

Factors to consider for future research.
Style of learning materials, frequency of decision-making regarding course alternatives.

9.1 SUMMARY OF THE SYSTEMS REVIEWED

In Chapter 3 when discussing the theoretical viewpoints regarding the individualisation of mathematics, we identified three characteristic positions regarding the control of the course:

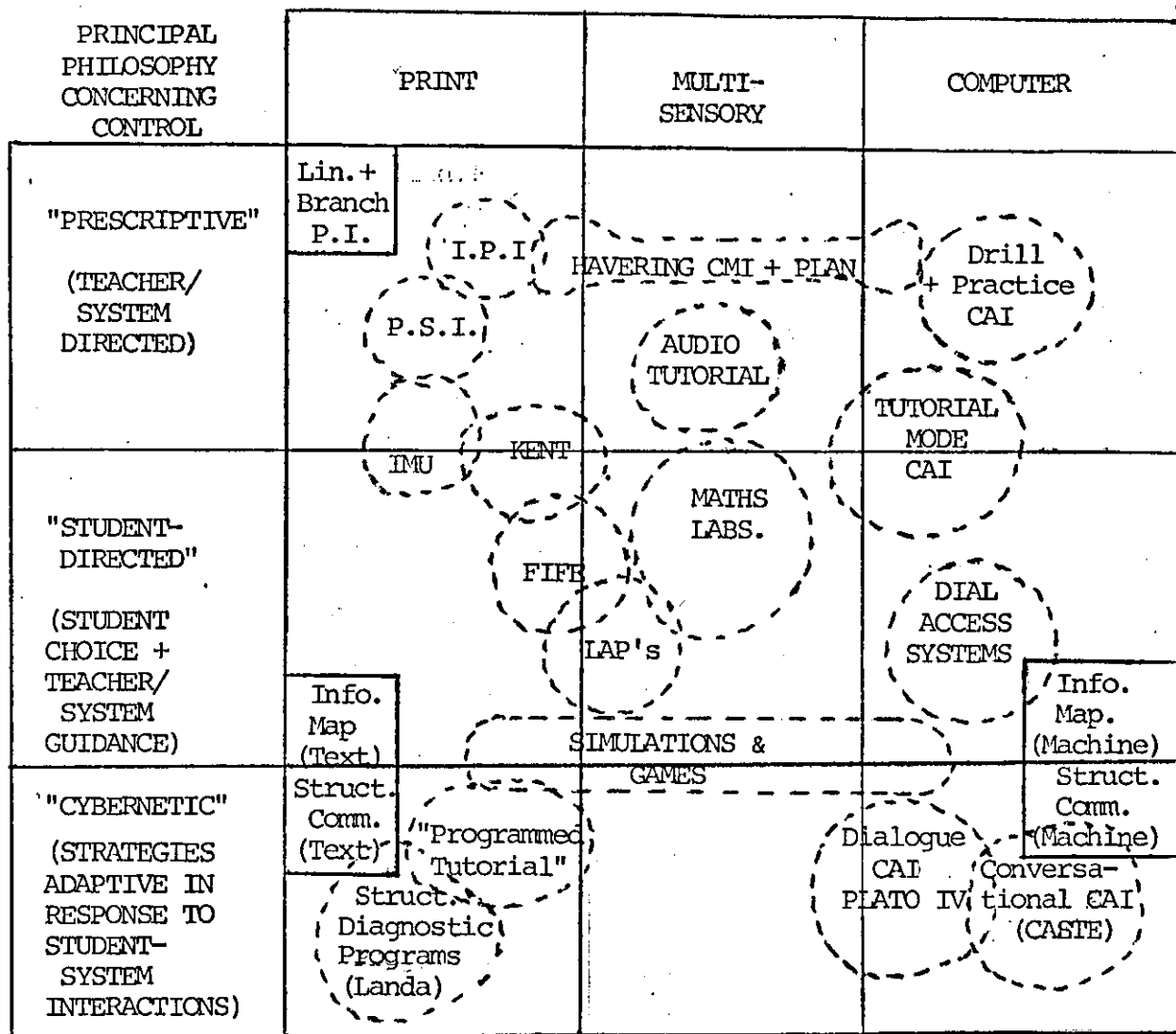
- (a) the prescriptive approach, which attempts to measure students individual differences and match instructional strategies to them according to a pre-determined algorithm,
- (b) the student directed, "open", or "free-learning" approach, which attempts to let the student have maximum control over the choice of learning strategies, media (even sometimes content and objectives).
- (c) the cybernetic approach, which attempts to set up an interactive system, adaptive to the student's needs in an on-line manner, based on what the system has learned concerning the student's needs, learning styles, difficulties, etc.

The chart appended here is an attempt to compare in summary all the main systems of individualised mathematics instruction discussed in Chapters 6 to 8. The classification used in these chapters, according to the primary media used, is combined with the abovementioned classification according to the chief philosophy of control adopted.

This comparison is necessarily somewhat crude, as many of the systems are capable of being prescriptive or student-directed to greater or lesser degrees (often depending on the

FIGURE 9.1 CHART COMPARING THE SYSTEMS DISCUSSED

PRINCIPAL INSTRUCTIONAL MEDIA



- NOTES
1. The circles and ovals are not drawn to any scale, they simply indicate approximately where into the suggested classification, the systems described fit.
 2. The items in the rectangles refer to techniques of materials preparation (programming) which will be discussed more fully in Chapter 10.

personalities of the teachers and learners involved). No quantitative implications should be read into the chart.

However, it does help to illustrate certain general trends.

(a) The systems reviewed cover the whole field of media/control combinations fairly comprehensively. Two types of systems which were not reviewed have been added, in order to "complete the picture". These are "simulation and gaming" and the "programmed tutorial".

Simulation techniques may be computer-based (and in this format there are several applications to mathematics, e.g. geometrical problem-solving (or probability problems - statistical maths lab.) or they may be group games or even individualised paper-and-pencil games (mathematical puzzles, Wff-n-Proof,). The computer-based type of simulations were briefly mentioned in Chapter 8. The non-computer-based games are rare and/or poorly documented. Bloomer (1974) reviews the simulations and games related to mathematics listed in a recently published "Handbook of games and simulation exercises" (Gibbs, 1974). She quotes Allen (1971) that in a formal sense mathematics cannot be simulated "The player of EQUATIONS is not setting and solving problems in a system which models elementary arithmetic. He is setting and solving problems in elementary arithmetic itself." Thus the CAI simulations which were discussed earlier, are in fact highly mathematical simulations of science problems. This does not necessarily detract from their value as methods of teaching mathematics. Not many true simulations with high mathematical content are listed in the handbook. There are some examples in probability.

The case is however different for the non-simulation game, which simply gives practice in mathematical computation or problem solving through the medium of playing a game. The handbook lists over 250 such games. Unfortunately, few of them have been subjected to rigorous evaluation.

The "programmed tutorial" is a technique which has been variously employed to make use of less trained, "para-educational" staff in face-to-face teaching. The "tutor" follows a set of very rigid lesson plans. It can also be used as a research device, whereby a human tutor adopts a strategy as laid down by a computer programme or an algorithm, in order to test out its efficiency without the expense of using actual computers. This was used effectively by Dodd (1969) to develop and validate drill-and-practice routines (similar to Suppes 1968) in fractions. These fractions programmes, though destined for computer-assisted instruction, have also been successfully used in worksheet form. Similar tactics have been used by Pask in his experiments on conversational programming.

(b) There appears to be a general trend, at least so far as print-based systems are concerned, for a concentration of "prescriptive" projects in the U.S.A., and "student-directed" projects in Europe. The big "growth" projects in the U.S.A. have been IPI at the elementary level and the Keller Plan (PSI) in higher education. Both of these systems are directly descended from the programmed instruction movement of the late 50's and early 60's, have abandoned many of the early P.I. characteristics, but have not abandoned the highly prescriptive nature of P.I. The computer-managed system,

project PLAN, is somewhat less prescriptive, in that the teacher or the students are free to choose from a limited range of suggestions generated by the computer. In Europe, on the other hand, programmed-instruction based systems such as IMU in Sweden or the Kent Mathematics project in the U.K, have from the beginning attempted to build in a strong element of student-directed learning. The Kent project, from its beginnings, which go as far back as IPI (Banks 1969), has organised its learning materials on a matrix structure, specifically to facilitate student choice of pathways. The Fife project is even more student-directed, as well as using a greater quantity of manipulatives and multi-sensory learning aids.

In the multi-sensory area, once again, the Audio-Tutorial System (Postlethwait, 1972), which has become very popular in many North American universities, is highly prescriptive. The more student-directed LAP's and mathematics laboratories, although much publicised and practiced, are reported to be on the wane in some areas of the U.S.A. (Fitzgerald 1974). In the U.K., however, to judge from the literature of mathematical education, from the topics of discussion at the 3rd International Congress on Mathematical Education and from the curriculum materials being produced by such projects as SMP, Nuffield, Mathematics Applicable, etc., the "mathematics laboratory" is a "growth-area".

(c) The systems which have been developed specifically for use at the university level have generally been prescriptive (e.g. P.S.I. and Audio-Tutorial). This is rather surprising considering the lip-service paid to the need for greater

student responsibility for the learning process.*

It is true that these systems have not yet "caught on" to the same extent in Great Britain as they have in the United States. Also one should remember that higher education has a long tradition of individualised teaching based on the tutor, on projects and essays, on small group seminars - methods which can be much less prescriptive and much more student-directed, if handled in the right way.

It seems to the author that there is need for both types of model in higher education. There are bodies of knowledge to be mastered, techniques to be practised as well as opinions to be formed and intellectual curiosity to be satisfied. In mathematics and in the sciences in particular, a large proportion of any course is inevitably "required", otherwise there could be no coherent structure to the course of study. One may offer alternative methods of reaching certain goals, but the goals are "prescribed". One can see the value of such systems as P.S.I. in enabling certain basic "prescribed" goals to be reached rapidly and with economy of teacher-contact hours, thus releasing more teacher time to devote to those types of goals that cannot be prescribed in advance. Every subject has such goals - in structural engineering, one talks of "elegant" designs for a bridge, one observes "creative" solutions, one can hold a debate on the "best" approach to estimating the stresses and strains in the structure (there are five

* The author was once involved in preparing a survey of undergraduate opinion about learning/teaching problems at the (then) Enfield College of Technology. Many students were very outspoken in favour of more say in course curricula, methods etc. However, these same students expressed critical opinion of those staff who passed such responsibilities to them and were most happy with staff who "told them clearly what to do".

possible approaches), but one cannot benefit fully from any of these open ended activities if one has not previously mastered the basic principles of design, studied other people's solutions to problems and learnt the five ways of calculating stresses in beams. In all these areas of learning, it is probably quicker, and harmless (in terms of inhibiting creativity); to adopt a prescriptive rather than a student-directed "discovery" strategy. Of course, one may offer alternative learning resources in order to reach the goals, provided that it is feasible economically to do so and there is some evidence to suggest that the alternatives do match up in some way to individual differences in the learning group. Very often this is not so -- the research on alternative media has been generally inconclusive (Caplan 1966, 1972). Sometimes one type of presentation has proved better for most students, other times it has proved worse. Differences have usually been traced more to the intrinsic quality of the particular product (e.g. a well written book, a well made film), rather than to general characteristics of the medium, which can in some way be matched to learner characteristics. It would seem therefore that one cannot, at our present state of knowledge, be very effectively "prescriptive" with regard to the media and methods of learning.

But with regard to the objectives of learning, one can, and indeed should, be prescriptive whenever there is a strong logical structure to the subject being taught. This is largely so in the case of mathematics.

(d) The area of greatest promise for the future, particularly with regard to mathematics teaching would appear to be the use of the computer and the adoption of adaptive "cybernetic" strategies of programming. In the conversational or dialogue mode, the computer is programmed to learn from the learner as well as to teach him - the computer learns to interpret the individual, perhaps even idiosyncratic, learning strategies of each separate learner, his preferences for teaching style, media and so on. Thus the need for a "theory of individual differences" is eliminated. One no longer needs to develop general rules for matching learner characteristics to methods and media. These rules are developed "on line" for each individual learner. They do not have to be generalised to groups of learners. Johnny learns to solve equations better if they are explained step by step in an expositive manner, and needs many examples, whilst Mary learns better if she is prompted to discover the rules for herself. But Mary needs geometry theorems explained and drilled whilst Johnny discovers such rules almost without prompting. The computer learns these individual learning styles and adapts the presentation accordingly. There is no need to classify the differences between Johnny and Mary according to some theory of learning, of cognition of personality or of aptitudes.

In particular, there is a need for adaptive student/teacher response in that most crucial and most difficult area of mathematics teaching - problem-formulation and problem-solving. The prescriptive, programmed-instruction-based, systems have largely avoided this issue. There were some

fairly successful attempts to teach productive thinking by linear programmes (Covington and Crutchfield 1965) but not in mathematics. The sort of approach to the teaching of problem-solving suggested by Polya in "How to Solve It". (Polya 1945, 1971) depends on a skilled tutor/mathematician observing the learner's attempts and carefully prompting his "discovery" process by suggesting appropriate heuristics. Even Polya does not claim that his approach is a universally teachable system - "experience shows that the questions and suggestions of our list, appropriately used, very frequently help the student". He suggests that the teacher, by using the approach himself and "dramatising" his problem-solving might have success in transferring this skill to his students (some of them). This is the traditional "teaching by setting an example - learning by osmosis" approach, the basis of the case-study method. The approach has not been packaged (in a prescriptive manner) because the rules are insufficiently understood and the learning process insufficiently analysed. But Pask (1972b) in his work with CAI lays claim to being able to programme computers to behave just like Polya's skilled tutor/mathematician. Landa (1976) claims to have produced texts which can also diagnose the reader's learning style and adapt the presentation accordingly. The Structural Communication approach to programming (Bennet & Hodgson 1968) can do this in a limited way, although what the text learns about the learner is not stored in a cumulative profile, as can be done in the case of a computer-based system. Certainly one might hope for breakthroughs in the teaching of problem-solving to emanate from the CAI laboratories during the next few years.

(e) In conclusion, we may observe the two extremes of programming at the top left and bottom right corners of our classification diagram. At the top left the rigidly programmed, prescriptive approaches based on the assumption that there is a "best" content to be learnt and a "best" way to learn it. At the bottom right we have the conversational simulation of the ideal tutor, still knowing what is "best" taught but willing to learn from the learner how best to teach it. In the middle lies the unprogrammed, student-directed approach which makes fewer assumptions about what must be taught and very few indeed about how.

9.2 SUMMARY OF THE RESEARCH REVIEWED

The research reported in Chapters 6, 7 and 8 has been primarily to illustrate how the various systems of instruction are applied in practice and to demonstrate the benefits, or potential benefits of the various systems. By far the largest class of experiments has been the comparative study between the "new" system and what is often termed "traditional" instruction (often not too clearly defined in the experiments). The results reported have been mixed. Some studies located by the author have been very strongly positive in favour of the "new" system, almost as many have given no difference or mixed results and a few have been unfavourable to individualised instruction. The author has not attempted to make a thorough review of the literature on comparative studies on the individualisation of mathematics instruction. This has been done by other authors recently (Miller 1976; Schoen 1976a; 1976b) but limited to the research in the U.S.A. on highly

prescriptive systems, such as P.I., I.P.I., or PLAN. These studies are interesting as they paint a far-from-totally-optimistic picture and also because they illustrate the limitations of this type of research.

Miller (1976) located 145 studies meeting the two criteria "dealing with individualised instruction in mathematics". The studied samples ranged from 13 students to 1766 students at all age groups. He quotes no other criteria for the studies. He selects 88 of these which gave comparisons between the individualised system and "traditionally taught" control groups. The tables shown below summarise his findings. Table 1 illustrates an "average" for achievement and attitude across all the studies reported. This was calculated by multiplying the number of studies in a category by the score (0 to 4), then summing these weightings. Hence "208" for achievement - divide by the 88 studies to get an "average". The scores "0 to 4" were assigned as follows:

- 0 - Significant results in favor of control group
- 1 - Mixed results, some significant, some not significant, in favor of control group
- 2 - No significant results for either individualized instruction or control group
- 3 - Mixed results, some significant, some not significant, in favor of individualized instruction
- 4 - Significant results in favor of experimental (individualized instruction) group.

Table 1 Research Results on Achievement and Attitude

	Score					
	0	1	2	3	4	Total
Achievement	4	10	42	14	18	88
Percent	4.5%	11.4%	47.7%	15.9%	20.5%	100%
Attitude	1	0	25	4	3	33
Percent	3%	0%	75.8%	12.1%	9.1%	100%
Average for achievement = $\frac{208}{88} = 2.36$						
Average for attitude = $\frac{74}{33} = 2.24$						

Miller concludes that:

"research does lend some support to the use of individualized instruction in mathematics. However, possible difficulties such as student motivation, lack of student interaction, time limitations, expense, difficulties in obtaining appropriate materials, and so on, may need to be overcome in order to have a successfully operating program. Nevertheless, the possible benefits, such as increased achievement in some instances, may provide the incentive to overcome these potential problems. "

He also lists suggestions for further research, including the need for a similar survey of CAI (not included in the above table) and also a closer look at the I.P.I. system which came out rather badly giving an "average" for achievement of 1.85.

Schoen (1976) performs a similar study concentrated solely on elementary arithmetic and on prescriptive systems such as I.P.I. or PLAN. He mentions 8 studies favouring the individual system, 17 favouring the controls and 11 showing no difference. Most of the studies were unpublished dissertations, using research-prepared materials (21 splitting, 6 positive, 10 negative, 6 neutral). Thus the commercial systems (15 - all using I.P.I.) gave 3 positive, 7 negative and 5 neutral. On the basis of these findings, together with other measures of attitude etc., Schoen comes out strongly against the use of individualisation of mathematics, and reinforcing his arguments with very sketchy data on applications at the secondary and post-secondary levels (Schoen 1976b) says:-

"For the moment, a mathematics teacher or principal who adopts in its entirety a program such as those described in this paper can be sure that (1) it will be more expensive and (2) it will be more work for the teacher. In addition, (3) mathematics achievement is not likely to increase and may very well decrease, and (4) student attitude is not likely to improve except perhaps in the primary grades."

It is interesting to contrast the findings of Miller and Schoen with a summary of the well-documented studies which the author has mentioned in this thesis so far (poorly-documented, poorly controlled, second-hand and anecdotal studies, even if mentioned elsewhere have not been included in the table below). Miller's "0 to 4" scoring has been used to facilitate comparison.

		Score				
		0	1	2	3	4
Type of system	P.I. - based	-	1	2	3	-
	I.P.I.-based	1	1	1	2	1
	P.S.I.	-	-	-	-	5
	Audio-Tut.	1	-	2	2	3
	Maths.Lab.	-	-	3	2	1
	LAP's	-	-	1	1	-
	CAI/CMI	-	-	4	-	12

The P.I/I.P.I. groups together give a breakdown very similar to Miller's with about twice as many instances to the positive of neutral as to the negative. This is nothing like Schoen's distribution. The author's sample is very much smaller, but includes all the well-documented studies that the author has located (mostly American as very few English studies are in fact well documented). One might speculate on the differences between the pictures represented. The author has not had access to unpublished doctoral dissertations in the U.S.A. - the source of the majority of Schoen's studies. Perhaps there is a tendency for these studies to give less favourable results? Perhaps, as many of them are based on researcher-produced materials, they are not very good examples of their breed? Perhaps only the positive results get published and the unfavourable studies remain as unpublished dissertations? Perhaps some other selection factor is in operation.

We have already discussed the limitations of comparative studies. As George (1976) says, we should "avoid typical global comparisons (i.e. "traditional" instruction versus "individualised" instruction) and,

instead, begin developing "a framework for the empirical evaluation of the separate variables involved in methods of individualised instruction."

The surveys of Miller and Schoen should therefore be viewed in this statistical context; as inconclusive even if apparently convincing, due to lack of any control of a multitude of relevant variables. From a non-statistical viewpoint however, one can say that the investment (in the U.S.A.) in (prescriptive) individualisation of mathematics has not yet really paid off. In particular, Miller's finding that in large institutional applications (in a total school district, for example) results deteriorate with time, poses serious questions both for administrators and for researchers. As prescriptive systems do not change much from year to year, one would expect constant results, assuming the school intake does not vary wildly in abilities or previous learning. The students coming into the system are always new. Not only do students in their second or third year of an individualised system generally do worse than they used to earlier on, but also the new entrants do not do as well as entrants of previous years. It appears that the most likely variable to account for this is the teacher (his attitudes, work, organisational pattern). Little research has been done to identify the roots of such a decline in performance standards. The author noted the phenomenon in a programmed course of 1 year in workshop technology for engineering craft apprentices (Romiszowski 1966). It has also been reported by the Royal Navy in programmed electronic courses, (Stavert, 1967). In both cases the

source was traced as loss of instructor interest or competence (due to staff changes not accompanied by orientation and training in the operation of the system). Perhaps some systems of individualisation require such extra effort or extra skills from the teacher that a high level of commitment to "make it work" is required. The problems of the "institutionalisation" of an educational innovation are not very much understood at present.

9.3 DIRECTIONS FOR FURTHER RESEARCH

One aspect which no doubt affects the success of an innovation is the attitude of the teachers involved to the underlying philosophy. For this reason, in Chapter 3 we have examined the various psychological positions regarding learning and we have identified the philosophies with regard to instruction which spring from them. We have identified the "Mastery" model, chiefly attractive to the behaviourists and usually (not always) implemented by a prescriptive and expositive course strategy. In contrast we identified the "Cyclic" model, springing from cognitive and developmental psychology, usually implemented by a student-directed "free discovery" strategy. These two positions may be considered as two ends of a continuum. Most individualisation schemes lie somewhere in between, being a mixture of prescriptive and free-learning strategies.

We shall close this review of the research by considering this factor of who primarily controls the learning process. In Chapter 2 the author developed a classification form

on which one may indicate (for various course factors, at various levels of frequency during the course) who decides to adapt the course - teacher, student or inanimate system or any combination of the three. This question is one major aspect of the author's research reported in the later chapters. We shall therefore briefly summarise the research on this question.

The research on the Mastery Learning model, reported in Chapter 3 has been generally very encouraging. One type of individualisation - P.S.I. - which embraces this model totally was incidentally not reported in the studies of Miller and Schoen, yet the author has only come across very positive research results where the technique has been applied to mathematics teaching.

The mastery model generally (not always) implies a standard course for all students. It certainly implies a set of objectives which are to be reached by all students, time of study being a variable. Thus much of the control in a P.S.I. course is with tutor or system. True, the student could select the course objectives, but once these are defined, the system attempts to ensure that they are achieved by regular control of each learner's progress.

The "free'discovery", student-directed approach on the other hand does not offer so much hard research data as, by implication, if students are totally free to choose objectives, media, etc. it becomes very difficult to define how one measures the success of the system as a whole. However, when objectives are fixed one can compare their achievement under student-control or system-control

conditions. It should be noted that comparative research of this type is not quite the same as that criticised earlier. It is not comparing two media in a "blanket" fashion. It is evaluating the effect of varying one factor in otherwise basically the same system, and is thus a move towards the type of research called for by Hartley, George and Campeau.

A review of some research on this question has been published by George (1976). He summarises the findings of 14 studies under four headings.

- (1) Programmed Instruction in which the student is controlled as to methods and sequence of study or alternatively is free to select his own methods. He reports two studies in which there were no significant differences in achievement (but marked preference for the self-planning mode), one study in which a "trained" student-control group did significantly better than an "untrained" group and a programme-control group and one study in which a group having "total" control over the learning did significantly worse than groups controlled by an instructor and partly instructor/students. All in all, no clear indications emerge, but the number and scale of the research is very small. Also most of this research dates from the early and mid-1960's when programmed instruction texts were still hidebound by Skinnerian and Crowderian, highly prescriptive programming rules. No attempt was made to modify the materials in order to make it easier for students

to choose their sequence. Techniques such as Information Mapping have since appeared - an ideal technique for presenting materials destined for student control. The author has made use of Information Mapping in his research for this thesis.

- (2) Student choice of instructional objectives. The two studies reported claim improved results in both achievement and attitudes when students have been allowed to choose the objectives of their course. Unfortunately George does not indicate at which level of detail this choice takes place. As already discussed, in a highly structured subject like mathematics, decisions at one level (e.g. course unit) largely determine the necessary objectives at other levels (e.g. topic or lesson). There may be some reasonable choices left to the student, but which are "reasonable" is dictated very much by the subject matter itself. More research is needed on this topic.

(3) Students choice of Learning Activities

Here again George records only two studies, in this case both inconclusive. That only two studies should be reported, one rather old (Newman 1957) the other recent and by the author of the review (George 1973) is surprising, considering that "allowing students to choose learning strategies (activities) is better than controlling them" is one of the cardinal beliefs of the "open-ended", "free-learning" school of thought. The author may recall here the significant recent research by Pask (1972) reported in Chapter 3 concerning the

Holists and Serialists in problem solving tasks. It is this type of research which is now beginning to give interesting results where a century of "matching of teaching methods to aptitudes, personality traits or other learner characteristics" has failed. Much more research of this type is needed,

(4) Student choice of performance standards

George reports 6 experiments, 4 of them showing significant improvements either in learning or in concentration and general behaviour when learners are allowed to set their own performance standards. The other two studies were inconclusive. The technique being investigated here implies the concept of "contracting" - the learner and the teacher agree on a performance objective to be achieved to a mutually agreed standard by a mutually agreed date - (an application of the "management-by-objectives" concept to the management of the learning process). In the studies reported by George, the onus for decision lies mainly on the student. In the more general case of the "learning contract" a joint teacher-student decision is negotiated. Of course, in practice, it is often very much the teacher who decides and then persuades the learner that it is a desirable and realistic objective for him to accept.

The literature on learning contracts is much more extensive than indicated by George, both in schools (elementary to higher education), and also in industrial training, where it has come to be called "training by objectives".

It forms a part of the philosophy of many individualisation schemes, even the more highly prescriptive ones (particularly P.S.I.), being a means of negotiating reasonable deadlines and targets between tutor and learner on an individual basis. The ultimate objective is fixed perhaps, but the time taken to get there and the number of intermediate learning steps (and tutorial meetings) are variable by mutual contract. When the concept is well applied, it is not an imposition of the tutor's will, but an expression of the learner's perceived readiness and previously mastered capability with respect to the course objectives. As such, contracting has a place in both the "mastery" and the "free-discovery" philosophies.

It forms part of the system used by the author in his research described in later chapters.

9.4 CONCLUSION

This chapter has summarised the philosophical positions and practical "delivery systems" of the various models for individualised mathematics which were described earlier.

The research on the use of these models has also been summarised, the futility of much past research indicated, new directions mapped out in general and, in particular, certain key factors of interest in any individualised scheme have been identified for further study.

These factors are

- (1) the style of learning materials which may facilitate learner choice of strategies,
- (2) the levels at which learner choice is exercised (course, unit, lesson or task),

- (3) the degree to which it is logically dictated by the subject matter whether certain factors should or should not be subject to learner choice. (This particularly applies to objectives in mathematics "service courses"),
- (4) the need for much more research on learning strategies and how to optimise them. This is particularly necessary in mathematics as the "breakthrough" in CAI will depend on a theory of learning strategies (or styles).
- (5) the "learning contract" as a means of sharing the decision making process between tutor and student (or indeed, in CAI, between machine and student). As a means of controlling (in the cybernetic sense) the learning contract process, the "learning contract + mastery learning" approach is analogous to project-management-by-objectives in industry.

The abovementioned factors have been linked to the analysis of individualisation (Chapter 2), the analysis of teaching and learning of mathematics (Chapter 3) and the problems and needs of the adult non-specialist student of mathematics (Chapter 4) in the design of the research for this study, outlined in the following chapter.

