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A STUDY OF INDIVIDUALISED  
SYSTEMS FOR MATHEMATICS INSTRUCTION  
AT THE POST-SECONDARY LEVELS

by

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A Doctoral Thesis

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## VOLUME II

### SECTION C

#### THE PRESENT STUDY

##### CHAPTER 10

###### BACKGROUND TO THE PRESENT STUDY

10.1	<u>THE PROBLEM AREA UNDER STUDY</u> .....	372
10.2	<u>PILOT STUDIES</u> .....	384
10.3	<u>THE PROPOSED STUDY</u> .....	403

##### CHAPTER 11

###### THE STUDY

11.1	<u>THE RASG-M PROJECT - AIMS AND PROBLEMS</u> .....	410
11.2	<u>THE MATERIALS USED</u> .....	428
11.3	<u>THE SUBJECTS USED</u> .....	431
11.4	<u>THE EXPERIMENTAL METHOD</u> .....	432
11.5	<u>ANALYSIS OF EXPERIMENTAL RESULTS</u> .....	446
11.6	<u>SUBSIDIARY INVESTIGATIONS</u> .....	469

##### CHAPTER 12

###### CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

12.1	<u>CONCLUSIONS FROM THE MAIN EXPERIMENTS</u> .....	477
12.2	<u>CONCLUSIONS CONCERNED WITH THE STUDY AS A WHOLE</u> .....	494
12.3	<u>SUGGESTIONS FOR FURTHER WORK</u> .....	511



## APPENDICES

<u>APPENDIX A</u>	Bibliography and References .....	518
<u>APPENDIX B</u>	Published Papers relevant to the Present Study (the author's) .....	557
<u>APPENDIX C</u>	Strucural Communication and Information Mapping: Descriptions and Examples .....	622
<u>APPENDIX D</u>	The "BASG-M" Materials used in the Author's Experiments .....	722
<u>APPENDIX E</u>	Data from the Experiments .....	797

## SECTION C

### THE PRESENT STUDY

CHAPTER 10 BACKGROUND TO THE PRESENT STUDY

CHAPTER 11 THE STUDY

CHAPTER 12 CONCLUSIONS AND SUGGESTIONS FOR  
FURTHER WORK

## SECTION C

### THE PRESENT STUDY

#### CHAPTER 10

#### BACKGROUND TO THE PRESENT STUDY

##### 10.1 THE PROBLEM AREA UNDER STUDY

10.1.1 The Human Elements in the System - The Teachers.

10.1.2 The Human Elements in the System - Mature Learners.

10.1.3 The Author's Experiences in Teaching Adult  
Non-Specialists.

10.1.4 Identifying the Problem for Study.

##### 10.2 PILOT STUDIES

10.2.1 Remedial Materials at Undergraduate Level.  
The Need for Flexible Materials and Tutor  
Control.

10.2.2 Information Mapping and Structural Communication:  
A Search for Flexible Print-Based Materials  
(the first requirement).

10.2.3 The Council of Europe Study.

10.2.4. Studies of the Keller Plan: A Search for  
Appropriate Systems of Control (the second  
requirement).

##### 10.3 THE PROPOSED STUDY

10.3.1 The Two Requirements together in the  
Context of Mathematics Instruction.

10.3.2 The Hypotheses of the Present Study.

## CHAPTER 10

### BACKGROUND TO THE PRESENT STUDY

#### 10.1 THE PROBLEM AREA UNDER STUDY

##### 10.1.1. The human elements in the system - the teachers.

Research reviewed in Chapter 9 reveals the contrast between the promising initial results (under experimental or small scale implementation) and the disappointing long-term results (once the system has been implemented for some time and become "institutionalised") for most of the systems of individualisation described.

Possible reasons for such trends would include

1. Hawthorne effect. However, in some of the studies quoted, results fall off only after a few years (a long time for a Hawthorne effect to last), and this happens irrespective of the fact that the learners themselves are new to the system every year.
2. A general decline in standards of education. There is some evidence, in the USA of a phenomenon referred to as the "test score decline". However, the drops recorded in some of the quoted studies are too large to be considered solely due to this phenomenon.
3. The way in which materials are utilised has changed. As the materials themselves have not changed, their effectiveness may have decreased due to changes in how they are used. This would be the result of management and control problems.

4. The human instruction component in the system has changed. This could change with time, due again to management/control problems or to replacement of staff by less trained or less motivated instructors. The last two (most probable) causes of decline underline the important role of the human element in any system of instruction.

This phenomenon of decline has been noted by various writers and some have shown that it can be reversed by training and orientation of the staff involved (Romiszowski 1966, Stavert 1969). The author's experiences were concerned with the implementation and long-term operation of an individualised system of apprentice training in the motor car industry. Stavert was concerned with individualised, programmed learning-based, courses in basic electronics for naval cadets. In both cases the systems initially gave very good results, much superior to those achieved previously by traditional instructional methods. However, over a period of some years, the results deteriorated gradually until they were inferior to "traditional instruction". In both cases this was identified as due to changes in teacher attitude, leading to changes in the system of control which they exerted. In both cases, over the two years of study, massive staff changes had taken place (in the author's project, due to the staff originally involved gaining better posts in the local technical college as a direct result of their experience with the system; in the Royal Navy case, due to the built-in mobility of staff in the services). The new staff members had not been involved in the development of

the systems and nobody bothered to get them fully involved (for example, they did not read the materials which the students were studying, consequently had great difficulties in effectively supplementing them in tutorials). In both cases it was shown that careful induction and supervision of new staff entering the system could reverse the downward trend in results and restore the system's effectiveness to its original level.

Both these cases are from the "training", as opposed to "education" field. Both were conducted in environments where it is traditionally much easier (and more acceptable) to specify and control the tasks of the instructor much more closely than is practised in schools or colleges. Most highly developed systems of individualised mathematics instruction do specify to a greater or lesser extent the role that the teacher should play. It may be a highly prescriptive role (as in IPI) or a flexible "guidance" role (as in mathematical laboratories).

It is very probable that many teachers find the "prescriptive" role unsatisfying. If they have been involved in the development of the system and its installation, and are enthusiasts, they will operate it as designed and will transmit their enthusiasm to the students. If, on the other hand (as must be the case in many large-scale "institutionalised" applications of IPI) the teachers have the system given to them as it were "from above", they may often feel dissatisfied with their role and will transmit the dissatisfaction to the students, both by their attitudes and by the way in which they execute their roles.

All is not well in the "guidance" camp either. Here the role of the teacher is much more demanding. He alone is the diagnostician of the student's needs. It is for him to integrate the student's own desires with the objectives of the course, to identify the student's individual weaknesses and learning styles and guide him to appropriate learning activities, and to evaluate the results of these learning activities in order to guide the student to further relevant activities.

However, not all teachers have the necessary insight and abilities to perform this role effectively. Consequently many theoretically well designed "guided discovery" systems, although showing great promise in the hands of the enthusiastic and gifted teachers who developed them, once again do not live up to expectations once they are institutionalised. One need only look at the recent history of 'modern' mathematics teaching to see an excellent case study. The earlier, highly prescriptive, teaching strategies, based on "drill and practice" of specific operations, were replaced by emphasis on the "structure" of mathematics - on the understanding of basic concepts and how they inter-relate. This was accompanied by a stress on guidance and discovery-based instructional strategies, aimed at developing the cognitive structure of the individual. However, the exercising of this role requires a high level of "mathematical perceptual skill" as described by Polya (1962). The teacher must be able to conceptualise the present form of the learner's cognitive structure from observation of his activities and responses, in order to identify where and how

to guide the learner. Unfortunately, few existing teachers have been trained in such skills and it is probable that not all existing teachers of mathematics could be capable of mastering the skills, due in part to gaps in their own mathematical cognitive structure. Hence, the practical application of the modern mathematics curriculum has often drifted either towards a free, "unguided discovery" model in which the children learn despite, rather than because of, the guidance given by the teacher, or else towards a prescriptive model (that the teacher understands because that's the way he learnt) drilling new operations (such as identifying the members of a union of sets). In the first case the learners may learn some concepts and may by chance even form some adequate cognitive structure, but this is more by chance than by planned control. In the second case the learners learn some operations but not operations which are directly relevant to every-day life (such as calculating change, percentages, etc.). In both cases, it is unlikely that many of the learners will achieve the objectives of the curriculum (stated in concept-understanding terms) or the objectives of employers (stated in real-life mathematical operations).

The above comments are not in any way intended to be a criticism of the modern mathematics movement. They do illustrate however the importance (in any innovation project) of involving, motivating and preparing the human elements in the system for the roles that they are expected to play. The earlier case studies, in industry and the Royal Navy, illustrate the need for maintaining the involvement and the skills of the human elements. The general experiences with



large-scale individualisation schemes in the field of mathematics (such as IPI, IMU, the Open Classroom, etc.) illustrate that there are as yet no clear guidelines how one should set about defining the role of the tutor in mathematics, or how one should maintain his performance. This question of the role of the tutor in individualised mathematics systems has been discussed by the author in a previous paper (Romiszowski 1976d- paper attached in Appendix B) in which he suggests that at the operations level (where indeed most teachers are more effective) the instructional strategies are such that they can be easily automated (e.g. by programmed instruction texts), but at the conceptual structure-building level (where generally teachers are less effective) it is currently difficult to automate the process. Whatever the eventual outcome of research such as Pask's (1972), we have at present no substitute for the perceptive, insightful tutor.

We can however give better tools to our tutor, both in the form of better curricular materials supported by suggested systems of application (e.g. Nuffield, School Mathematics Project), or in the form of better information gathering, analysis and diagnosis systems (e.g. Kent Mathematics Project, Computer-managed systems such as PLAN or CAMOL, etc.). Armed with these tools, the "good" tutor should be even better and (more important still) he should be more efficient, being able to handle more individual students. This element of efficiency may be further enhanced if we systematically relieve the good tutor of the need to spend time on routine instruction of concepts or

procedures which are capable of being effectively taught by automated systems.

Hence the general direction of research on the teaching of mathematics should include consideration of how tutors and automated instructional systems should be used to the best advantage. This aspect of individualisation has been chosen as one of the major considerations in the present study.

#### 10.1.2 The human elements in the system - mature learners

The applications of individualisation to mathematics instruction reviewed in Chapters 6 to 9 reveal

- very many, and varied, projects at the primary level
- some projects at the secondary level
- rather few projects at the post-secondary levels.

In recent years, both the Audio-tutorial approach and the Keller plan have grown in popularity in higher education, though few of the applications are directly to the teaching of mathematics. At further education level, or in professional/industrial training still less research has been carried out.

Chapter 4 analyses some of the general research relating to the problems of the mature learner (in particular the learner of mathematics) and relates these to individualised instruction.

It is seen that there is a variety of reasons for supposing that individualised instruction should be at least as effective (and probably more accepted by students) at the post-secondary levels of education. The non-specialist student of mathematics in particular needs careful individual handling due to a combination of different needs (dictated

by the variety of mathematics needs in the other courses he is studying), different entry levels (dictated by the amount that he has remembered from previous learning, the quality of the teaching he received previously, etc.) and different attitudes (formed largely by the previous levels of success, failure, ridicule and frustration of ambitions that he suffered in previous mathematics teaching).

The traditional group-learning situation is particularly badly adapted to his requirements, being fixed in content to the general level of the group, fixed in pace to the "average learning rate" and exactly the system of instruction which he experienced in earlier life and in which he met failure and frustration.

#### 10.1.3 The Author's Experience in Teaching Adult Non-Specialists

This experience has included workshop arithmetic for craft apprentices in industry, basic statistics and quality control for foremen and supervisors in industry, remedial mathematics for social science and business studies degree students and "second chance" continuing education courses for adults who had not had the opportunity to go to school (in Brazil). All these projects involved the production of individualised instructional systems, using a variety of "mixes" of tutor and self-instructional materials. Some of these experiences will be discussed further in the next section of this chapter. The general results of all these experiences were however exceptionally encouraging in terms of their acceptance by the students concerned. The apprentices expressed generally positive attitudes to the use of self-

instructional programmes in the place of traditional classes and asked that more of their course be "packaged" in the same way (Romiszowski 1965). The supervisors, through the use of a statistical laboratory, and simulated games changed their highly negative attitude towards statistical quality control into a highly positive one, and then learned the techniques from self-instructional texts and group exercises. The social science students, whilst initially approaching the computer-managed remedial mathematics course with great reservations concerning the de-humanising aspect of being "treated as a statistic in a computer", generally go out of their way to thank the course tutors at the end, often stating that "it is the only part of the degree course in which I am treated as an individual human being" (Romiszowski 1969). The "continuing education" adults in Brazil participate in a "radio/group meetings/self-study text" instructional system with higher attendance rates over a period of 1½ years and generally doing better in the state examinations, than students studying an equivalent course in a traditional "evening class" system (Romiszowski and Pastor 1977).

In all these systems, the decline in results and attitudes with large-scale institutionalisation has been observed and steps taken to combat the effect, with greater or lesser success.

One research drawback has been that the structure of all these systems has been dictated in the main by practical considerations - by the available resources, by severe restrictions in time for development, by existing practices and by the skills and attitudes of existing staff.

The projects were all of a "consultancy" nature to solve existing problems rather than of a "research" nature to identify general principles for the teaching of mathematics. What research has been carried out has been "grafted on" to the projects.

This research has generally shown the individual systems to give better results than the previous "traditional" systems they replaced. But to what extent these improvements were due to some generalisable "breakthrough" in the strategies used by the systems and to what extent they may have been due to inherent weaknesses in the previously existing systems, is not known. As already mentioned in Chapter 9, and as stressed by such writers as Campeau (1966), and Hartley (1972), these limitations are common to the vast bulk of comparative research on instructional techniques rendering it difficult to generalise the results of any specific study. The plea has been strongly made

for research at a more "micro" level, with closer control of the factors which may affect the efficiency of a given technique. This has been the basis of Piaget's work, using a clinical approach on small numbers of learners over a long period of time. It has also been the basis of Skinner's work in animal learning, based on the cumulative record of observations of individuals. In mathematics teaching, it is the method favoured by such philosophically-opposed researchers as Jerome Bruner and Patrick Suppes. Bruner quotes Piagetian-type case studies of individual discovery of mathematical concepts, (Bruner 1966) whilst Suppes

uses the same technique to log the progress of individual students on a computer-based "drill and practice" arithmetic programme (Suppes and Morningstar 1972 ). Finally, Pask ( 1972) is using these techniques of research in his studies of holist and serialist learners.

There are of course disadvantages to such research strategies. Usually the size of experimental groups is small (sometimes single individuals are used) rendering many of the commonly used statistical techniques (developed for the study of large populations, mainly in the agricultural field) practically useless. In the absence of statistical methods for establishing confidence levels in the results obtained, it is all the more necessary to replicate experiments many times. Whereas this has been done extensively with some basic research, for instance Skinner's conditioning experiments and Piaget's studies of the stages of conceptual development of young children, such replications were performed over several decades before most psychologists were prepared to make generalised statements. The few studies such as Bruner's and Suppes', directly related to mathematics teaching have not been replicated sufficiently often to make generalizations. Furthermore, these studies have in the main been restricted to the teaching of young children in the elementary grades of school. The transferability of their findings to other children is questionable in the absence of sufficient replication studies, even more so to adult learners who differ so much from the young child in their previously developed learning habits, their variety of prior learning experiences and the specificity of their objectives in studying mathematics.

Hence there is a great need for research directed specifically at the problems of the adult, non-specialist, learner of mathematics.

#### 10.1.4 Identifying the problem for Study

Due to the relative dearth of research in this field, it was proposed to study some aspects of individualised instruction applied to the teaching of required mathematics concepts and procedures to mature (i.e. post-secondary age) students who are not mathematics specialists.

Among the problems eliminated by selecting this area are:-

1. The need to consider individualisation at the "course" or "unit" level. In considering a unit of "service mathematics", by definition the unit is required due to the demands of other course units. Thus a student, opting to take the course as a whole, cannot opt out of the required service maths unit. We may therefore concentrate our studies at the "individual objectives" and "learning steps" levels in our taxonomy of individualisation. Furthermore, the objectives of such a unit are generally easier to define precisely.
2. There is much evidence to show that well designed, self-instructional materials are capable of effectively teaching concepts, rules and procedures for their application to problems of a given type (i.e. algorithms for problem solution). There is also evidence that such materials can teach the skills

of problem-solving and the formulation of mathematical problems (model building), but this evidence is very scant and it is by no means convincing that self-instructional materials are the most efficient way of teaching such skills. By restricting the investigation to conceptual and procedural learning one may avoid comparing two, equally inefficient treatments (possibly obtaining statistically significant but practically inconsequential results).

Among the problems posed are those of the special learning problems of the mature learner. The problems to be studied particularly are concerned with

1. the style of instructional materials best suited to the adult learner - in particular whether allowing flexibility of use of the materials is more effective than prescribing a standard way that the learners should study.
2. the style of control applied to the unit - whether it is better to use the instructor to ensure mastery of all objectives, or to leave this to the learner himself.

## 10.2. PILOT STUDIES

### 10.2.1. Remedial Mathematics at Undergraduate level

#### The need for flexible materials and tutor control

The author has since 1969 been concerned with the mounting and operation of a remedial mathematics "clinic" diagnosing and complementing the pre-requisite mathematics skills of first-year undergraduate in Social Science and



Business Studies (Hamer and Romiszowski 1969). (Copy of paper attached in Annex.)

This scheme is based on study units in programmed instruction form, with tutorial support as and when required, on a totally individual basis.

In its original form, the scheme was computer-managed, enabling rapid diagnosis of student weaknesses, together with regular revision of the tests and the programmes used on the basis of analysis of the data collected by the computer.

Over the years, several alternative models were tried. These included

- Computer-Managed Diagnostic Tests and Individual Prescriptions
- Tutor-Marked Diagnostic Tests and Individual Prescriptions
- Tutor-Marked Tests supported by interviews and counselling, as a way to diagnosis and prescription
- Different levels of responsibility for managing the scheme assigned to the tutors - e.g.
  - (a) Team of Part-time Tutors,
  - (b) Full-time tutor-counsellor
  - (c) Secretary/Technician management, with tutors "on call".

The various models are described in detail elsewhere (Romiszowski, Bajpai and Lewis, 1976). (Copy of paper attached in Annex). In general, these studies emphasised the importance of the human intervention in the system. This intervention was not too important in the "tutor" role, as the programmes used

were efficient and rarely needed much tutorial back-up. The "management" role of the tutor seemed to be the important ingredient. Students needed to see that someone cared about how they performed on the programmed materials. Assessments had to be made, and the resulting actions had to be seen to be made. It seemed less important whether this management was performed by a computer, by a tutor, or by a secretary, provided that it was performed systematically. The use of a centralised "learning by appointment" centre rendered the management of the system relatively simple, despite the total individualisation of learning rate, hours for learning, and to some extent, the content. When alternative programmes in printed form were used, and students were able to take them home, the system did not function at all well, mainly due to the difficulties of management and control.

Another aspect of these studies, which was not followed-up systematically, but was supported by much subjective evidence, was the differences among students in the way they used the materials. Printed programmes were not necessarily used in the way they were intended. Some students "browsed" through the material without responding overtly. Others "worked backwards" starting from the post-test and then attempting to refer back to sections which dealt with their particular difficulties. In the case of filmed programmes in teaching machines, where such flexibility of use is impossible, students expressed their different learning styles in their attitudes. The majority enjoyed their work on the machines but expressed the occasional desire to "jump ahead and see if I've got it". A few however came out strongly against the use of the machines specifically because they controlled

their learning style, (Romiszowski 1969). It was felt at the time that a more flexible way of "packaging" the instruction should be adopted. After all, most of the students in this system were revising previously learnt material. The simple diagnostic tests identified deficiencies in performance, but tell us nothing about how much practice the individual learner will require in order to recall and develop previously acquired skills.

The following concept emerged:-

Requirement 1 A set of learning materials to which the learner can have access in a flexible way, for initial study, for rapid revision, possibly for reference. The materials should also accommodate a variety of learning styles.

Requirement 2 A system of management which supplies regular human contact for the purposes of control and can also supply occasional tutorial back-up to the materials as and when required.

10.2.2. Information Mapping and Structural Communication;  
A search for flexible print-based materials.

Requirement 1 would be met by a well-designed computer-assisted-learning system, using both the "tutorial" and the "drill-and-practice" modes of interaction (Suppes et al, 1968). However, such a system was at present beyond the resources available. A paper-programme system called Information Mapping, however seemed to meet most of the criteria, in addition to being potentially cheap and portable. Information Mapping (Horn et al, 1969) is a set of rules and procedures for

the classifying and presenting of information in documents which may be used for the learning, review, or retrieval, of that information. Such documents share some of the features of a Geography Atlas, in that separate pages are used to present the global view, and others present the detail (hence an "atlas" of "information maps"). A user may go from the general to the particular, or from the particular to the general, as he desires. Self-test questions are also grouped on separate pages, so the reader may choose to attempt them first, as diagnostic questions, attempt them later as feedback questions, or skip them altogether if his objective is to refer to the information rather than to learn it.

A full account of the technique of Information Mapping, an example, research on its use, its value and limitations (and a contrast with other possible approaches to the packaging of individualised mathematics materials for adults) is presented in Appendix C to this study.

Another programming technique which appeared in the late 1960's is Structural Communication. This technique, developed by the Institute for Structural Communication (Bennet and Hodgson 1968 ; Hodgson 1974 ) aims to simulate a "good interactive tutorial" in a print-based or computer-based format. The learner, after reading a presentation, is presented with a problem for discussion. He constructs his own response to this problem, using any of the facts, or ideas in the presentation, in any combination. He is assisted in this by a "response matrix" usually of 20 to 36 cells, which contain all the key facts and ideas mentioned in the presentation. By selecting from this

matrix, the learner may, in theory, construct millions of different responses, though in practice many are obviously irrelevant and the author of the learning unit limits himself to a discussion of the more commonly expected responses. However, this programming style may handle a much larger number of alternatives than the multiple choice type of branching programming. A full description of how Structural Communication works, how it is produced, an example, some research on its use, its values and limitations is included in Appendix C to this study.

Both Information Mapping and Structural Communication were tried informally on the social science and business studies remedial mathematics courses at the Middlesex Polytechnic. This work has not been described before as it was performed with small numbers and the data collected is purely anecdotal and based on the verbal reports of the students concerned. However, it will be mentioned here as it gives the background to later work. During 1970 and 1971 certain students were given to study some currently available materials in both "Information Mapping" and "Structural Communication" format. Twelve students were involved, all of them having completed the basic remedial mathematics course and having voluntarily continued to attend the remedial mathematics unit throughout the year in order to study other self-instructional materials or receive tutorials relevant to their course work. These extra voluntary assignments were administered on a learning-by-appointment basis, in a less controlled way (i.e. students were not tested and re-tested, they often tested themselves; students did not necessarily study at the unit, they would often

borrow texts to study at home between tutorials].

The twelve students mentioned above received a unit on Probability in Information Mapping format, (Horn et al, 1969) and a unit on Abstract Mathematics (set theory applied to number systems) in Structural Communication format (Fyfe et al, 1969). They received these materials to take home and study, one unit at a time. They were later asked to rate the units in terms of their acceptability as compared to the linear and branching programmes which they had all been studying for over half a year.

In the case of the Information-Mapped unit, nine out of twelve students said they preferred this format to the linear and branching programmes. All twelve said that they were quicker to use for revision purposes, and seven commented that the style of layout was much clearer and simplified learning.

In the case of the Structural-Communication unit, all twelve said that they preferred a good branching text. Eight said that they were not clear about what they were expected to do. Five said that it was difficult to see if the objectives of the study unit were the same as their own. Two said that their objectives were definitely not the same as those of the study unit; they were interested in understanding a concept and applying it to a problem which was of direct use to them, not in discussing (with live tutor or with the text) the wider implications of the concept. Three said that they liked the general idea of the study units, but did not "relate" to the particular example presented.

It appears from the above that the particular Structural Communication materials used were either irrelevant or ill-adapted to the needs of the students in question. Apparently, the materials had been successfully applied in a number of secondary schools, so it is unlikely that they were basically too difficult for the polytechnic students. Rather, it would seem that the students had very precise needs - to learn concepts and techniques relevant to the problems of social science or business studies which confront them in the main body of their course. They were impatient with a presentation which got them involved in abstract mathematical considerations concerning empty sets, complements, union and intersection, without relating these concepts to their real-life problems. The probability unit, using the Information Mapping format, was more obviously relevant to their needs. There were problems resembling those they face during their own statistics course. Furthermore, they could (due to the organisation of an information-mapped unit) immediately identify which parts of the presentation were relevant to their needs, which parts they already knew, etc.

#### 10.2.3 The Council of Europe Study

An opportunity to systematically evaluate the technique of Information Mapping for the teaching of mathematics to Business Studies undergraduates, presented itself as part of a study for the Council of Europe. Using a linear programme on Matrices, developed experimentally at CAMET, Loughborough University (1972-73 ) an equivalent version was prepared in Information Mapping form. Comparative tests showed that the Information Mapped version was both

significantly more effective (higher mean gains) and more efficient (learning time reduced). (Romiszowski and Ellis, 1973).

A short account of this pilot study is given in Appendix B.

It should be pointed out at this point that the above findings do not necessarily prove anything in general for or against the two programming techniques. It is quite possible that a Structural Communication unit written with the precise needs of the Polytechnic students in mind, would be much better accepted. It is quite possible that for other types of content, the Structural Communication approach would be superior.

One major difference between the two techniques is in the philosophy of control (see Chapters: 3 and 9) on which they are based. Indeed, in the classification chart shown in Chapter 9, we have indicated the positions of the two approaches. Structural Communication is an attempt to simulate the interactive tutorial conversation. Unlike the prescriptive linear programme (and the only-slightly-less prescriptive multiple-choice) a structural communication unit allows the student to construct any response (including any of up to twenty or more factors) to a problem, and then responds to the response (by a process of analysing the factors included or left out of the student's response). It is a technique not far removed from Landa's "Structural-Diagnostic Programmes" which not only do this, but also follow up the "psychological reasons behind wrong responses" by a complicated system of sub-problems and branches (Landa 1976). In the variety of possible interactions it is no less complex than most current



applications of Plato IV, although the computer based system can present the material faster, more elegantly and can handle non-verbal information better. It is certainly capable of inducing a discovery-learning strategy.

Information mapping, on the other hand is a "student-learning/reference system" (Horn 1970). It is based on the philosophy of student-directed learning rather than on planned interactions. It is more akin to a dial-access information system (supported by clear objectives, indexes and diagnostic tests - if you want to use them). Indeed, when presented via a computer terminal, information-mapped programmes are called up map-by-map, block by block at the request of the learner.

The presentation of the material in the blocks is however quite expository. Concepts are defined, then illustrated by examples. Discovery-learning has little place in this system, except at the macro-level of a course unit; a student may "discover" a whole knowledge-structure by using, on his own, a manual in information-mapping format (as he might have done through the use of other alternative media). But during his study of an individual map, the student "receives" rather than "discovers" information.

Thus, referring back to discussions in Chapter 3, on the various philosophies of mathematics instruction and on their relative applicability, one might expect a well-designed structural communication exercise to be superior when a student must be guided towards the discovery of new concepts and new relationships and their assimilation into

his cognitive structure (with consequent possible adaptation of this structure). When on the other hand a student has already a partly-formed cognitive map of a particular subject and has reasonably well-developed study skills that enable him to effectively guide himself towards identification and study of the missing concepts and relationships, then perhaps a student-controlled presentation format, such as information mapping, might be superior.

When dealing on the other hand with simpler learning tasks, such as facts, routine operations and simple concepts, the discovery-learning approach has been generally found less efficient than reception learning (whether based on Ausubel's expository approach or Skinner's operant conditioning approach). Here again, a prescriptive, pre-programmed approach (such as linear or branching P.I.) may be superior when the learners are all starting from the same entry-skills level, have limited learning skills, and there is a limited number of important individual differences between them. When however all these factors vary considerably, as is the case with most mature students of mathematics, then probably, once again, a student-controlled system of presentation (such as information mapping) may be superior.

The above paragraphs are the author's general opinions in relation to the presentation styles of print-based materials which he has studied. These opinions have been formed on the basis of only a small amount of research and a larger amount of hypothesising. Hence the need for further research on this topic.

#### 10.2.4 Studies of the Keller Plan: A search for appropriate systems of control.

In section 10.2.1 of this chapter, two requirements for individualised mathematics courses for non-specialist mature learners were hypothesised on the basis of experiences with remedial mathematics teaching to undergraduate. The first of these, the need for flexible learning materials was further discussed in the last section, indicating the promising nature of the information mapping technique (as far as print-based materials are concerned), particularly for revision courses or when the participants are at various entry levels and with various levels of study skill. In this section we will consider the second requirement hypothesised, namely the need for a management system that supplies regular human contact for the purposes of control and tutorial back-up.

In the work of the Middlesex Polytechnic requirement 2 was being met by the "clinic" atmosphere of the learning-by-appointment scheme. However, we already had evidence that paper-programmes, "to take away", do not work so well in this system, which relies on the student to come to the manager/tutor. In particular the time to complete a given course, and the wastage or dropout rates, increased considerably when students studied at home. However, for various administrative reasons, it was difficult to extend the "clinic" approach to more courses and to more students.

Croxton and Martin (1970) at the University of Aston, showed most conclusively that a paper handout-based system of individualisation can be made to work in undergraduate science and mathematics teaching, within the normal

administrative timetable and staffing constraints, provided that certain feedback loops were built into the system and certain actions taken to make use of the information being fed back. In their case, a computer-management element is introduced into the system in order to "personalise" the attention given to individual students in group tutorials.

At about the same time, a personalised system of individual course control known as the Keller Plan (Keller 1968, Keller & Sherman 1974) was being developed in the USA and in Brazil. Like Croxton and Martin's scheme, the basic learning materials are usually paper reading assignments (sometimes, but by no means always programmed), but the control is exercised by frequent learner-tutor interactions. Typically, less qualified staff are used to do the bulk of the tutoring, the "classic" Keller Plan relying on tutoring (monitoring or proctoring rather, as the tutorial function is very closely defined) by the peer group. The faster students act as tutor/proctors to the slower ones. Alternatively a more advanced group is used as proctors for a less advanced group.

The Keller Plan is very firmly based in the behaviourist camp, and is in many respects an adaptation and refinement of programmed learning to the realities, needs and resources of higher education. Material is no longer "rigidly" programmed, as the relatively sophisticated learners who have "made it" to the university do not need most material to be broken down

so much. It incorporates human contact and discussion, as this is expected by this level of student and often required by the type of learning he is engaged on. Finally, it fits into the administrative constraints of a typical university course.

A full account of the characteristics of the Keller Plan and some of the research on its use, is given in Chapter 6. Reviews of the research and collected papers on typical applications exist (Sherman (editor) 1974; Romiszowski (editor) 1976c). Although most initial applications were to psychology courses, many applications have now been made to the teaching of science (Green, 1971, Elton et al 1973) and also mathematics service courses (McKean, Newman and Purtle 1974; Romiszowski, Bajpai and Lewis 1976). Most studies so far have been comparative and have yielded positive results (though not always supported by sufficient documentary evidence).

The Keller Plan seems to owe much of its success to the learner-tutor interaction. Farmer et al (1972) reported that the more frequent the interaction between student and tutor/proctor the faster the learning rate of the student. Neves and Romiszowski (1976) allowed choice of proctors; although there appeared to be a slight tendency for students to seek out the more lenient proctors, there was a much stronger tendency to stick with one proctor through a learning difficulty irrespective of whether he is lenient or severe. This paper is attached in the Appendix B.

Thus the Keller Plan, or some derivative of it might be expected to satisfy the requirement 2, whilst rendering individualised instruction more adaptable to the "traditional" course timetabling constraints.

Alternatives to the Keller Plan, which are still largely prescriptive in the type of control they exercise and which have been used at the higher or further education level include Computer-Managed-Instructional systems and the Audio-Tutorial Approach (Postlethwait et al 19

The Audio-Tutorial approach has been described fully in Chapter 7 of this thesis. It suffers from limitations as far as mathematics instruction is concerned in that the audio medium is not generally the best suited to the communication of mathematical concepts and techniques. Once the audio medium has been supplemented by some form of visual media (e.g. slide-tape or videotape) the Audio Tutorial system becomes similar to a multi-media programmed package approach. Its difference from, for example, LAP's (also described in Chapter 7) is exactly in its philosophy of control. Like the Keller Plan, the Audio-Tutorial approach adopts the "mastery learning" model of Bloom (1971 ). The use of LAP's is more open-ended and student-directed. Thus as Hess et al (1976) commented, the Audiotutorial system is akin to an audio, or audio-visual version of the Keller Plan. We shall not therefore discuss this approach further.

Computer-managed systems of control are distinctly different from that adopted by Keller. Generally they do not rely so heavily on human tutorial/monitorial

contact. By placing most testing and diagnostic functions "on line" the computer may substitute the monitors in so far as they test, diagnose and re-direct the student to packaged learning materials. This is the function of project PLAN, of the British equivalent project CAMOL, and was also the case in the author's work at the Middlesex Polytechnic (Romiszowski 1969) and Croxton and Martin's (1970 ) at the University of Aston. In both these cases, the tutors were used as learning/teaching resources to back up the printed self-instructional materials used in the first instance. This is not a role that typically the proctor/monitor in "classic" Keller Plan courses is expected to assume.

In practice however, there exist various "hybrid" types of Personalised Instruction Schemes which follow up some of Keller's rules, but break others. At Loughborough University of Technology, for example, the monitors are in fact also tutors. They had both the functions of diagnosing the extent of learning which had taken place from printed study units, and subsequently tutoring those students who had not reached criterion standard. Still later, the diagnostic-control function was dropped; students self-marked their assignments and only contacted tutors when they found themselves in difficulty (Romiszowski Bajpai and Lewis 1976). The existing system of implementation has drifted away considerably from the original Keller Model. Even the tutors are not drawn from the ranks of the students. They are faculty staff. The difference between the Loughborough scheme as now run and Croxton and Martin's 1967 system is very small. Both use self-

instructional linear programmed texts. Both allow the students to mark their own assignments and report their results to faculty. Both run follow-up tutorials on individual or small-group bases, given by faculty staff (teachers and research staff). The small difference is that the tutorial timetable in one case is planned on the basis of student requests, and in the other on the basis of a computer-assisted analysis of the results reported by students.

Many unresolved questions of detail surround the Keller Plan. The use of peer-group monitors as opposed to faculty tutors is one question of difference. For practical reasons, related to course organisation, credit award systems, etc., it is easier to motivate more advanced students to take on the tasks of a tutor/monitor in American universities than appears to be the case in the United Kingdom. In the USA it is common to pay monitors, either in money or in the award of course credits. These methods have not been applied in the U.K. Theoretical viewpoints also differ, from "how can you expect a student who learnt a concept yesterday to identify the learning difficulties of another student today" to "learning of a concept is only complete when one can successfully teach it to others". The practical reply to these extreme viewpoints is that "it is surprising how often students are successful in teaching each other complex subject matter, and, when they are unsuccessful there is always the course teacher or professor in the background to take over".



Another unresolved question, related to the "human-contact" aspect is whether the system of control in the Keller Plan works because the system involves regular contact with other human beings, or simply because the system involves regular control of the mastery of course objectives (and the monitor system just happens to be a practical method of implementing this). Certainly results such as those quoted from Aston, Middlesex and Loughborough suggest that systems without the monitorial element, and therefore with less frequent human contact, may be just as effective as the "clinic" Keller Plan with regular human contact. Early experiments with programmed learning, such as the Royal Navy study (Stavert 1967) suggested that prolonged periods (weeks) of automated instruction without planned human contact were inefficient and profoundly disliked by students. On the other hand, research with computer assisted instructional systems, from the simple drill and practice routines of Suppes to the complex tutorial and dialogue programmes of PLATO IV has shown that effective instruction and high motivation may be achieved over prolonged periods of study without any direct teaching from another human being, (Suppes and Morningstar 1972 ; Bitzer 1976 ) provided of course that normal variety of activities is maintained.

A third unresolved question is "how often should the regular contact take place". We have already mentioned Farmer's (1972) findings of a direct relationship between frequency of student/proctor interactions and student learning rate. This would seem

to imply that the smaller the study unit the better. The extreme case is constant student/proctor interaction - that is, the personal tutorial. But, apart from the economical impracticability of this extreme, other research quoted by Sherman (1974) suggests that Keller Plan courses may be better than tutorials. So where is the limiting point? What is the ideal length for a study assignment? In a small study of the results of three Personalised courses, used for technical teacher training in Brazil, the author found that for these courses, with these students, a length of study unit of between one and two hours seemed to be optimum. But much more research is needed before generalizations could be made.

In this paper the importance of the structure of the assignments in a Keller Plan course was also investigated (Romiszowski 1975). All the courses were written by the same team of technical experts/writers, who also acted as the proctors in the experiments. However the courses differed in (a) the length of the study unit to be read between successive meetings with the proctor, and (b) the degree of internal structuring, or programming of the study units. An analysis of the courses was also performed into the type of objectives or learning tasks which the students had to master. Although many factors were not controlled in this study (the courses dealt with different content; different groups of students were used; an after-the-design analysis was carried out rather than designing materials specifically for the study)

the comparisons showed some significant correlations between the efficiency of a unit and the degree of structure in it, and a negative correlation with the length of the unit. This relationship appeared to be more marked for conceptual learning than for procedural learning. The full paper is attached in Appendix B.

### 10.3 THE PROPOSED STUDY

#### 10.3.1 The two requirements together in the context of mathematics instruction

Much of the experience with the Keller plan is in undergraduate courses, teaching mainstream subject matter to highly selected students. At this level, students have the required pre-requisite experiences and the necessary study skills to cope with "traditional" reading assignments, limited tutor contact, largely independent study, etc.

Mathematics however is in a unique position as compared to other subjects, particularly when we consider the teaching of mature students who are not specialising in mathematics.

First the nature of mathematics concepts is that they are highly structured, highly dependent on adequate mastery of pre-requisite concepts in order to master new concepts. This would suggest the need for more careful structuring of any materials used for mathematics instruction, as compared to materials for other subjects.

Second the nature of the mature non-specialist is that he is generally poorly prepared on pre-requisites, low on basic skills, but very clear on his objectives. He usually sees mathematics as a tool for specific tasks he wishes to accomplish, and the learning of the mathematics as a necessary, but difficult and not very pleasant, chore. As the bulk of the instructional load in a system of the type we are discussing rests on the materials (the tutor only plays a supportive role), we have another reason for the more careful structuring of instructional materials for mathematics.

Third the nature of mathematics is such that there are very clear sub-objectives. The level of mastery of a given concept, relation or operation necessary in order to proceed without difficulty to new material is precisely definable. Therefore a "mastery learning" model for the control of students' progress through a course seems particularly apt to mathematics.

Fourth the nature of the mature learner, as described in Chapter 4, includes a much greater wish to be correct at every stage. Whereas the younger learner is prepared to gloss over his difficulties and be satisfied with lower levels of achievement, the motivated mature learner wishes to be sure he has understood and mastered every step. This was seen to be one source of the apparent difference in learning rate between the young

and the old. The young are prepared to part-learn a topic, for the old it is "all or nothing". Hence the mastery-learning model should be highly acceptable to the mature learner of mathematics.

Fifth the nature of mathematics is such that it takes more than just subject-matter mastery to identify the roots of a learner's difficulty. Bruner and Polya both argue (see Chapter 3) for the necessity of a skilled, perceptive, teacher to guide the student to "see the problem", to go "beyond the information given". Despite Landa's and Pask's assertions that this can and will be automated in computer-assisted instruction (Chapters 3, 8, 9) we have not yet reached this stage in practice. Hence, for the time being at least, it may seem that the inclusion of tutorial contact in any mathematics teaching scheme would be desirable.

Sixth the nature of the mature learner is such that he requires more assurance and "anxiety reduction", as he often has learning difficulties due to the professional importance of the particular course, or to previous frustrating learning experiences with the subject. This might also indicate a high need for understanding, personal, tutorial guidance. However, the mature learner is also more likely to reject a tutor whom he finds unacceptable. Thus

"good" (i.e. acceptable) tutorial support is indicated, but "bad" (i.e. unacceptable) tutors may do more harm at the post-secondary level than at lower levels in the educational system.

Seventh the nature of mathematics and mathematics learning is such that ideas have to be revised, concepts studied from various points of view, with a variety of examples, and a variety of problems have to be attempted to give adequate and generalizable practice in the application of techniques. Experience shows that learners grasp an idea from various starting points, require different examples and different quantities of practice. Not enough is known yet concerning these varied learning styles, therefore a safe "way out" is to delegate a part of the selection of learning strategies (and therefore learning materials) to the student, whilst keeping overall control over progress towards pre-specified objectives.

Eighth the nature of the mature learner is such that he has more highly developed (good or bad) learning styles and study habits. Also he is usually clearer about the objectives he wishes to achieve than the younger student. This is all the more reason to delegate the selection of learning materials to the student, whilst keeping a yet more careful control on

the way he uses them in order to identify and improve any "bad" study habits.

10.3.2. The hypotheses of the present study

The proposed study attempts to investigate some of the factors outlined above, in greater depth.

The main hypotheses to be investigated are that, when teaching procedural and conceptual mathematics content to mature students by means of individualised techniques;

1. Learning is more efficient if the use of the learning resources (both materials and tutor) are student controlled, rather than prescribed by the system.
2. Learning is more efficient if the control of testing for the assessment of mastery is controlled by a tutor, rather than by the student.

Subsidiary hypotheses to be investigated include:

3. That these differences are more marked in the case of adult learners than in the case of younger learners.
4. That highly structured materials (whether allowing student-directed learning or not) are accepted favourably by both adult and younger learners.
5. That different students use the learning resources in different ways.

## CHAPTER 11

### THE STUDY

#### 11.1 THE BASG-M PROJECT - AIMS AND PROBLEMS

- 11.1.1 Introduction - the adult education problem
- 11.1.2 The birth of the BASG-M project
- 11.1.3 The problem of production of the BASG-M materials.
- 11.1.4 Implementation. The College and its problems.
- 11.1.5 The Aims of the experiments.

#### 11.2 THE MATERIALS USED

- 11.2.1 The content of the course
- 11.2.2 Relation of the content to the students
- 11.2.3 Style of the materials

#### 11.3 THE SUBJECTS USED

11.3.1

#### 11.4 THE EXPERIMENTAL METHOD

- 11.4.1 The three sections of the course
- 11.4.2 The 4 main Experiments
- 11.4.3 Subsidiary investigations

#### 11.5 ANALYSIS OF EXPERIMENTAL RESULTS

- 11.5.1 Preliminary testing of groups
- 11.5.2 Experiment 1
- 11.5.3 Experiment 2



11.5.4 Experiment 3

11.5.4.1 Introductory Comment

11.5.4.2 Experiment 3.1

11.5.4.3 Experiment 3.2

11.5.4.4 Experiment 3.3

11.5.5 Experiment 4

11.5.5.1 Introductory Comment

11.5.5.2 Experiment 4.1 (a)

11.5.5.3 Experiment 4.1 (b)

11.6 SUBSIDIARY INVESTIGATIONS

11.6.1 Overall gains during the learning period

11.6.2 The Student Questionnaires

## CHAPTER 11

### THE STUDY

#### 11.1 THE "BASG-M" PROJECT

##### 11.1.1 Introduction -- the adult education problem

This research reported here was carried out during the months of August until December 1976 in Salvador, the capital of the state of Bahia in Brazil. The study involved both adults and upper secondary school children of 15 or 16 years of age studying the same material, namely, a "modern" mathematics approach to arithmetic. The upper secondary school students were participating in the first year of the "segundo grau" which is the 15 to 18 years of age section of the secondary educational system of Brazil. The adults involved in the study were participating in adult continuing education programmes of evening classes designed to bring them up to the standard of the secondary school leaving certificate. As adults are not permitted to take the secondary school certificate on this supplementary continuing education basis until they reach the age of 21, it is normal that they are accepted in these evening classes only after they have passed the age of 19 or even 20. The college at which the present study was based is a very large secondary and upper primary school, state owned, with upwards of 8,000 students and the year group with which this study dealt had 600 students in 16 parallel forms. All these students were in the 15 to 16 years of age bracket, and had entered the segundo grau directly after their completion of the 15+ or "primeiro grau" certificate. The standard for this certificate is not exceptionally high and it is common for a large number of those entering the "segundo grau"

to have large gaps in their primary education and lower secondary education which gives them problems in their upper secondary school career.

The adults were in the age range of from 20 to 35 years of age, were all in full time or part time work and were attending evening classes at the college in order to gain their upper secondary certificate. They would have gained their primary certificate either in a similar manner through evening classes, possibly as recently as the year before, but often several years ago, or alternatively they may have completed their primeiro grau schooling in the normal manner up to the age of 15 and then had left school deciding 5 years later, or more, to return in order to complete their education.

#### 11.1.2 The birth of the BASG-M project

The learning materials used as a basis for this study are a series of self-instructional texts which were developed in a project termed BASG-M, which stands in Portuguese for "Bases for Access to the Secundo Grau/Mathematics". This project is a project of the Federal Ministry of Education designed specifically to deal with the above mentioned problem that many children are entering the upper secondary grades, the "secundo grau", with incomplete preparation due to poor teaching or poor attendance or difficulties in attendance to any school in their earlier years. The particular sub-section of this project ("M" standing for Mathematics,) was a result of some work carried out by the author during 1974 (whilst working in Brazil for the International Labour Organisation, within the Ministry of Education) on Secondary School Curriculum Design. The author's specialism in this

project was the preparation of programmed instruction and other self-instructional materials and audio visual media in general, and among his functions was included the training of teams of programme writers and media producers for the secondary educational school system. One of the teams trained during this period of work was a team for the State Secretariat of Education of the State of Bahia. A course was given during July to October 1974 to 30 members of the staff of the Secretariat, and local school teachers. As a result of this course the BASG-M project was drafted by the Secretariat and later adopted by the Federal Ministry of Education. The general aim of this project was to produce self-instructional materials covering the pre-requisite mathematics that students should know before entering the "secundo grau". These materials were to be used in the "secundo grau" by the teachers to close the gaps in the prior knowledge of the students by setting them self-study assignments over and above the normal classwork expected in the "secundo grau" in mathematics. Materials production commenced in early 1975, continuing to the present time and the testing of some of these materials commenced in August 1976. The present study was attached to the materials testing project in one of the colleges involved. The author had left the United Nations Project in July 1975 but had returned in September 1975 until February 1976 to Brazil when he acted as consultant to the Materials Production Teams. During the period from August 1976 to January 1977 he returned again to take part in the evaluation of these materials and during this period carried out the studies reported in this chapter.

As mentioned above, the author had been responsible, in 1974-5 for the training and supervision of the materials production team. Several problems were encountered in the production of the materials, some of which were due to the historical background to the BASG-M project.

Apart from key management and specialist positions, the State Secretariat of Education is staffed by teachers. Teachers in Brazil are employed on a 4-hour daily "block", (or "shift") basis. Salaries are quoted per "block". Many teachers therefore work only mornings, or only afternoons, on one "block". Some work two blocks, but in different schools. It is not uncommon for a teacher to work half the day in a state school (relatively poorly paid) and the other half in a private school (better paid). Occasionally, some teachers work a 3-block day, though this is not officially encouraged. The effects of this system are often somewhat negative. For example, there is a marked lack of "allegiance" to the school among teachers who work in two or even three different schools. There is a lack of flexibility in timetabling or "covering" during illness, as teachers' "spare" hours are generally committed to other employers. And there is a lack of time for class preparation or markings, as a 4-hour "block" generally means four hours of class contact - thus a full-time teacher would generally teach 8 hours a day (the extreme 3-shiftworker teaches for 12 hours).

The system is however well entrenched and has a foundation in economic reality - few people could make ends meet on the very low salaries paid by the state educational

system. Thus the state relies on a very large proportion of its teachers being part-timers - either full-timers working one shift in the private sector in order to supplement the low income of the other shift, or part-timers working for "pin-money" (about 90% of school teachers in Bahia are women - married or waiting to get married).

The state system however offers "perks" which the private sector does not; almost permanent security of jobs; ability to transfer between jobs; great flexibility in working hours, pregnancy leave, study leave, etc.; promotion opportunities, both academic (through paid scholarships) and political (through the State Secretariat of Education and Culture). These perks, in the author's opinion, contribute greatly to rendering the state system viable at all. The last-mentioned of these is relevant to the present discussion.

Educational projects of all types, whether curriculum development, textbook writing, specialist teacher training, etc., are organised and financed by the State Secretariat. They are staffed by teachers selected from the system, generally on a 1-shift per day basis unless the characteristics of the project demand full time staff. Such projects generally pay better than the schools and are often avenues to promotion or better prospects. Thus there is a fair amount of grass-roots pressure for the development of new projects and, when a project is launched, no shortage of willing workers.

Such was the case with the BASG-M project. The original course, taught by the author in 1974 had been one of the activities of an earlier project whose objectives had been to spread information about (and promote the use of) educational technology among the teachers and administrators of the State educational system in Bahia. Thirty-four teachers and administrators attended the course and subsequent workshops given by the author in 1974.

For the reasons outlined above, most of the personnel involved made subsequent efforts to devise projects of practical application of the systems and techniques they had studied. Three proposed projects have been taken up and are in various stages of progress. One of these is a course "package" for teacher training, specifically to train teachers in "literacy techniques" - methods of teaching reading and writing, diagnosis of student difficulties, etc. - applicable at both the "young child" and the "adult" levels. Another is a "programmed" correspondence course for adults who missed the chance to obtain school qualifications. The third is the BASG-M project, already described in outline, which is the subject of the present study.

This latter project was submitted to the Federal Ministry of Education and Culture, was adopted and receives financial support from the Government. It is intended, eventually, to make the materials available at the national level to any State or any individual school who may wish to adopt them. The materials will eventually comprise a

comprehensive set of self-study modules, covering the whole of the school curriculum of the "primero grau" (the 7 to 15 age groups) in both mathematics and Portuguese language skills. The primary intention was to use the materials for revision and remedial work in the "segundo grau". However, it looks increasingly likely that some of the materials may also find application in the later years of the "primero grau" as "basic texts".

#### 11.1.3 The Production of the BASG-M Materials

The historical background, outlined above, may serve to explain the way that the production team was selected and composed, and also some of the production problems encountered as a result.

The BASG-M project was designed and promoted by personnel who had attended the original courses given by the author. The author was indeed involved, as a consultant, in the design of the project. One question discussed at length was the composition of the production team - whether all the writers should be qualified and experienced teachers of mathematics, or whether the team should be composed of mathematics advisors and "professional writers" who were not necessarily mathematicians.

This is a discussion which has raged for decades among the "programmed instruction" fraternity. Most commercial programme-writing companies swear by the "generalist-programmer" who picks up any subject and, together with a subject matter expert for occasional guidance, develops effective instructional materials. Supposed advantages are that the writer, through having to learn and analyse



the content, prior to organising and presenting it, may have a better and fresher appreciation of the problems of the learner than the expert, who is many years removed from the learning process.

The alternative viewpoint is encountered in the academic world. This holds that the experienced teacher is not removed from the learning process. He observes it daily and has many years of accumulated knowledge not only of the subject but of the typical learning difficulties and pitfalls that learners encounter. It is better therefore that the experienced teachers should be the main producers of the materials. They may learn what they need to know about new techniques of organisation and presentation faster than the expert educational technologist may learn the intricacies of their subject. Thus the team should consist basically of teacher-writers with the consultative aid of an educational technologist.

The author's view, based mainly on experience gained from consulting to industry, is that both approaches can work, but both require rather exceptional personnel in order to work satisfactorily. The "generalist-programmer" to succeed, must have a high level of ability to learn a new topic thoroughly, to organise it in his mind, to identify concepts he does not fully understand and explore them thoroughly by "milking" his subject matter experts. He cannot rely on the experts to present the whole subject, fully worked out and organised for teaching. He must be able to winkle out the "missing links" in his cognitive

schema from the experts or other sources available. He must also have a high level of skill in the organisation of material for learning, in writing, in the use of the visual image and in all the other skills of the educational technologist. These are his stock in trade, and are not as easy to master in practice as the simplicity of the basic principles involved might suggest.

The expert teacher, on the other hand, has not the need to master the subject, but is probably lacking in programming experience. There is a great danger that he may apply certain rules for the production of, say, an instructional programme, or a slide-tape package, in a mechanistic and unimaginative way (as one tends to do with any newly developing skill). His packaged learning materials are often less imaginative, less alive, than his normal classroom performance, not for any limitation in the media themselves but due to his lack of experience in working with the media.

The situation in the BASG-M project was somewhat complicated, in that the personnel most closely concerned with the launching of the project were recent trainees in some (not all) of the techniques of educational technology, and none of them had been specialists in mathematics or mathematics teaching (though they have all taught some mathematics as part of the integrated approach in the elementary schools). They were generally enthusiastic to work in the project, but were all relatively young. They were therefore neither highly

experienced programmers nor highly experienced teachers of mathematics.

The author's original recommendation was that several more experienced mathematics teachers (known to be effective teachers) should be selected and trained in the skills of programme writing. However, the politics of the situation did not make this suggestion easy to accept. There were various vested interests in involving the graduates of the original course in the project. The original structure of the project team thus included: five writers with recent training in the skills of writing (not teachers of mathematics), two subject experts (experienced teachers of mathematics) who were available to the team of writers on a part-time basis and were responsible for defining the content and dividing it into modules and lessons, a resident project manager, who had previous experience of projects of this type and also extensive prior training and experience in educational technology, and the services of visiting consultants (the author and a Brazilian colleague).

This team did not function as effectively as was hoped. Productivity was low initially. The reasons for this were the lack of experience of the writers. They were hesitant to make decisions on their own, without the approval of the subject matter experts or the consultants. They experienced more difficulty than was expected in coming to grips with the learning problems involved in the subject. One or two of the writing team proved to have their own "mental blocks" in respect of learning

mathematics. They attempted to write instructional materials to teach content which they themselves did not understand fully. This of course did not work.

Slowly, the load of responsibility shifted from the writers to the subject matter experts. BY their discussions with the writers over the first few months of the project, the experts learnt a bit of programming and commenced to bring the course modules not only defined in terms of objectives and content, but already organised as regards sequence, pre/post tests, learning exercises, etc.

At the time of writing, the team has a radically different structure. Only one of the original writers is still working in the project, together with two new recruits, who are both experienced teachers of mathematics and have been selected for their proven excellence as teachers. These new recruits received a short period of initial formal training and are now working, together with the one more experienced writer who also performs the duties of the project manager. Other subject matter experts and outside consultants are still used, though on a slightly smaller scale than before. The new team, with less than half the manpower of the first one, is proving to have a higher level of productivity and, in the author's opinion, is producing better quality materials.

It is this second team of three writers, (plus the two original subject matter experts) who have acted as the author's assistants in the execution of the research described in this chapter. They acted the roles of

classroom teachers and individual tutor/monitors as required by the various instructional plans compared in the experiments. They also assisted the author in the preparation of the alternative versions of the materials which are compared in one of these experiments. The work of preparing these alternative versions served as part of the training of this new team in the use of alternative, more flexible and creative approaches to the preparation of self-study packages. The products of this work are the alternative versions of Module VI of the course, which are the subject of Experiment 4 to be described later in this chapter.

The author is also grateful to this team for helping to smooth over various administrative and political problems encountered in performing the study. These included the organisation of discussions and briefing sessions with the staff of the College concerned in the study, the organisation of tutorial space and storage space for the materials at the college (this last was a complicated task, as 1600 separate books were used in the study - 8 volumes x 200 copies - and had to be available to students at any time of the day).

There were also problems of an educational nature in the college, which influenced the form of this study. These will now be described.

#### 11.1.4 The College and its Problems

The Brazilian educational system suffers from a chronic shortage of trained teachers. However, the shortage is not uniformly distributed, being much more acute in the poorer states than in the rich industrialised states. For example, in the State of Sao Paulo, particularly in the urban area of Sao Paulo City, the educational system is about as well provided as in some of the poorer areas of the united Kingdom. There are teacher shortages particularly in science and mathematics. There are building shortages often solved by shift working, morning and evening in schools, and there are equipment shortages which generally result in a somewhat traditional approach to instruction based very much on the spoken word of the teacher rather than on the use of textbooks by students. However, in general this picture is very similar to the more traditional and less well endowed areas of European education.

In the north-east states of Brazil, however, the situation is very different indeed. There were, for example, several states that had no secondary educational system whatever, as recently as 1969, and incidentally two of these have now solved this particular problem by television based educational systems which do not rely on qualified teachers at all. The situation of Bahia, or more specifically of Salvador, its capital, is somewhere in between these two extremes. As Salvador was at one time the capital of Brazil, it has a level of cultural development

which is higher than exists in neighbouring north-eastern states. There is reasonably adequate provision of school places in the urban area and its closer rural districts, so that the problem of plain "non-availability" of education is not as serious in this state as it is in some of the other north-eastern states (or in the states in the Amazon area, for example, in the interior which are very very sparsely populated). However, teacher shortages, specifically qualified teacher shortages, are very much more acute than in the state of Sao Paulo, much more acute than the situation that we know anywhere in the United Kingdom. Equipment is also very limited, as are also textbooks, so that in a situation where teachers are very poorly prepared to teach, or indeed are not available at all, the educational system, although it "exists", becomes hardly functional. In the college in question above it has been known for children to turn up to a day's work and receive only one supervised lesson during the whole day, due to absence or lack of teachers for the other five timetabled lessons. In this sort of situation, it is not surprising that many of the children give up attending school, not due to lack of basic interest or motivation, but on the very reasonable grounds that they are not receiving very much education for the trouble they are putting in in attending. This, therefore, is the general climate into which the BASG-M materials were injected in 1976.

The academic year in Brazil commences in late February or early March, after Carnival time, and continues until just before Christmas, with one month break during July. The author, together with a team of assistants supplied by the State Secretariat, commenced his researches during the second semester of 1976, that is, in the month of August. The assistants included the three writers responsible for the production of the materials being tested, plus two mathematics teachers who had acted as advisors on subject matter. The teachers had not been involved directly in the writing of the material and they were used in these experiments as straight classroom teachers, using the structure and exercises which they had developed for the programmed texts as a basis for normal classroom instruction. The author, together with the three other writers of the material, took the role of tutors, or monitors, in the administration of the materials.

The college supplied eight groups of students for the purpose of the current experiment. Each of the groups had a notional enrolment of between 35 and 40 students, although due to absences or transfer to other cities, the actual number taking part in the experiment was somewhat smaller. Four of the groups were 15 to 16 year old upper secondary students in their first year segundo grau attending during the daytime, four of the groups were adults in the 20 - 35 years age bracket, attending evening classes 5 days a week. In all, the college had 16 forms of 16 - 17 year olds similar to the ones used in the experiment and students were assigned to these forms on a random basis



with no attempts at streaming or any other sub-divisions. This was also true of the adult classes. There were eight adult classes running in parallel at the time of the experiment, and the students who had registered had been assigned in a random manner on a 'first come, first served' basis to any of these eight classes without any attempt at streaming or other sub-divisions. Therefore, although the experimenter was not involved in randomizing the selection, he is relatively confident that the groups used are relatively homogeneous, and similar to other groups in that particular college. One thing was common to all of these groups. They had entered the "segundo grau" in March of 1976 and until the end of that semester, that is until the end of June, they had not received any direct mathematics instruction due to lack of teachers. The time normally allocated to mathematics on the timetable had been allocated to other subjects where teachers were available, or had simply been cancelled. Thus, in the case of the school students, they had passed 6 months without any mathematics instruction before the beginning of the experiment. And in the case of the adults, they had passed at least 6 months without any mathematics instruction but in some cases 5 or 6 or 10 years.

This therefore was the first problem which the college in question faced - an acute lack of mathematics teachers, and indeed it was primarily for this reason that the experimenters were given such a large group of subjects to try out their materials on. The second problem became

apparent at the pre-testing stage of the subjects, to discover which ones were suitable for being used as test subjects for these materials. The objective of the pre-test was to select the weaker students from the groups and to give them extra tuition in the pre-requisite arithmetic knowledge that they needed before entering the secundo grau mathematics curriculum. However, pre-tests showed that levels of prior knowledge were generally very low indeed, and it was quite reasonable not to select the few weak students, but to spend a few months re-teaching the primary school curriculum. This was equally true for the adults who maybe had not studied mathematics for many years, and for the upper secondary school children who had been studying the same curriculum in the previous year. It would seem, therefore, that the success of the elementary school level in teaching the modern mathematics approach to arithmetic had not at all been successful. In the case of the adults, for most of them indeed, the modern mathematics approach would be quite novel as at the time at which they had attended elementary school the system of instruction had been based on the more traditional approach, and had only been generally introduced into the elementary school levels during the last 5 years.

#### 11.1.5 The Aims of the Experiments

It would seem, therefore, that our study started with students who were generally naive as far as modern mathematics was concerned. Their scores on pre-tests were at about the 10% level in all groups, well within the

possibilities of guesswork. The aims of the materials testing exercise was to test the materials firstly to find if they were effective as teaching instruments, and secondly to discover the weak and strong points in the materials for further revision and development. For this reason, firstly, normal teaching of an effective nature was organised in order to have a reasonable comparison between the materials and what one might call "traditional instruction". This was requested by the Secretariat of Education and seemed a sensible approach for the purposes of materials development, irrespective of the reservations which have been mentioned in the previous chapters concerning comparative studies of this nature as far as their generalisability is concerned. Also, in order to gauge the quality of individual sections of the materials, learning times were measured for every module, and all responses to all exercises in all modules were collected and analysed. Every module had its own tests, which were often self-marked by the students, and in addition to these there were several progress tests administered on an examination basis by the experimenters.

The author's interest in researching some of the more detailed aspects of individualisation required the design of specific experiments within the overall materials testing work. Four of these experiments were designed, and they are described further on in this chapter. These experiments required the sub-division and comparison of certain groups on various treatments as regards to the level of control exercised over the student's study methods and over his attainment of course objectives. This also

required the preparation of certain alternative versions of some of the materials in order to test out student-directed and system-directed study methods. These materials are briefly described in the next section of this chapter.

## 11.2 THE MATERIALS USED

### 11.2.1 The Content of the Course

The overall materials planned for the BASG-M project will involve about 20 self-study booklets which in the present context will be termed "modules". At the time of the experiment seven of these had reached the stage of completion, initial small group testing, and the printing of experimental copies. In fact, these are best treated as eight modules, because module 4 was very much larger than the others and was divided subsequently into two separate booklets - 4a, and 4b.

These modules, or booklets, were divided into lessons. A lesson was the study assignment that a teacher was expected to give at one time to a student in order to allow him to study on his own prior to a further meeting with a teacher for discussions. This would normally involve between 1 to 3 hours of student study time. This was the predicted study time prior to materials testing. In practice, some of the lessons proved to be completed rather faster than expected, others took much longer. Module 1 contained 6 lessons, Module 2 three lessons, Module 3 four lessons, Module 4a four lessons, 4b also four lessons, Module 5 three lessons, Module 6 four lessons and finally Module 7 three lessons. This gives a total of 31 lessons.

A full list of the modules and lessons, together with their contents, is given in Appendix D. The content of these first modules deals only with basic arithmetic all the time treating it from a set theory point of view. In summary, the content includes the following topics: sets, properties of sets, operations with sets, relations, functions, the set of natural numbers (N), operations in (N), addition, subtraction, multiplication, powers, division, divisibility, prime numbers, factorisation, HCF, LCM, powers and roots.

#### 11.2.2 Relation of this Content to the Students

As mentioned above, the students typically scored very poorly when pre-tested on the content of these 31 lessons. It was noticeable, however, from their results, that those who scored better than others gained most of their marks on later questions rather than on the earlier ones. In other words, they showed very poor knowledge of set theory and operations with sets and the modern mathematics approach, but could solve some of the more advanced problems such as factorisation and HCF, probably by the application of previously learnt drills rather than by the application of a conceptual modern mathematics-based approach. This result was surprising, as it related to the students of the 15 - 16 age group, because these students had been taught modern mathematics and set theory in previous years of their elementary schooling in the same college. It was not so surprising with respect to the adults in our experiments, as the majority of these would have left school before the modern mathematics approach was adopted as a general rule.

### 11.2.3 Style of the Materials

The printed materials used in this experiment were of course all produced in Portuguese. The style is uniformly a linear programming approach, though not a strictly Skinnerian approach to programmed instruction. The main difference between Skinnerian linear programming based entirely on a conditioning model and the style of programming used in the BASG-M materials is that there is a much more pragmatic approach in these materials to the way that concepts should be presented. Unlike Skinnerian programmes which have a fairly constant frame size, the amount of materials presented to students in the BASG-M materials varies considerably from one frame to another. Unlike classical Skinnerian programmes which demand a response at just about every step, the BASG-M materials often present two or three pages of descriptive presentation without any response required from the student. On the other hand, other pages may require the solution of a whole series of problems before moving on and checking the results of any of them. Finally, unlike classical Skinnerian programmes which generally follow a rule-example structure and are therefore very much of an expositive, reception learning nature the BASG-M materials sometimes use an expositive approach expecting the student to learn and assimilate and at other times use a discovery learning approach where the student has to work out the rules from given examples. An example of a section of these materials has been included in Appendix D. The mathematical content, the sequence of this content, the structure and the general treatment of the mathematics was dictated by the standard

curriculum of the State of Bahia and by the two mathematics teacher-consultants who have been mentioned above. The author's responsibility in this project was limited to the techniques of presentation of the material, not to the mathematical content, nor to the choice of examples used.

As described in Chapters 9, and 10, one of the aims of this study is to compare alternative methods of presentation of material, specifically from the point of view of comparing techniques which are suitable for student-controlled use of the learning materials, as opposed to one which is dictated by the author or by the system of instruction used. Information mapping was identified as a promising way of presenting materials which allows the student to select what he will study and in what sequence. For this reason, several of the modules were re-written by the author in conjunction with one of the Brazilian writers, in an information mapping format, and this was used in some of the experiments described below. An example of the information mapping versions of this material is also included in Appendix D.

### 11.3 THE SUBJECTS USED

As already mentioned above, eight groups of students were supplied for the purpose of these experiments - four groups of adult learners and four groups of schoolchildren. Students had been primarily assigned to these groups at the time of registration to the college. Group size in each case was notionally a minimum of 35 and a maximum of 40. However, in reality, those students who dropped out, changed schools or were very irregular in their attendance were

eliminated from the experiment so that the numbers actively participating were somewhat smaller. For the purposes of this study the groups have been designated A, B, C, D for the adult groups, and P, Q, R, S for the schoolchildren groups. The number of subjects in each of these groups, their average ages, etc. are shown in the table below. Full detailed results are appended in Appendix E.

GROUP	TYPE	MEAN AGE	RANGE OF AGES	NUMBER PARTICIPATING IN STUDY
A	Adults continuing education	28.4	18 - 40	30
B	"	25.8	18 - 43	33
C	"	26.8	18 - 38	33
D	"	24.5	18 - 38	31
P	School-children - secundo grau, first year	15.4	14 - 16	30
Q		15.5	14 - 16	33
R		15.2	14 - 16	33
S		15.1	14 - 16	32

#### 11.4 THE EXPERIMENTAL METHOD

##### 11.4.1 The three sections of the course

The course materials were administered in three major sections, between each of which an examination-type test was taken. Also treatments for different groups were



altered between each section on some occasions. The three sections were

First section - Modules I, II and III

Second Section - Modules IVa, IVb and V.

Third Section - Modules VI and VII.

The groups were assigned to the various sections in a variety of ways. In summary, as follows:

#### FIRST SECTION

Groups A, B and C of the adults worked through this section in three different modes of tutor control. Group C had no tutor available whatever, and Groups A and B had a different frequency of tutorial intervention in the instruction process. Group A received the materials module by module, and reported back to the tutor only after the completion of a module, (i.e. 3, 4 or 5 lessons). This might take them anything up to 10 or even 20 hours of study. Only at the completion of a module did a tutor actively carry out evaluation of what learning had taken place and give guidance as to which section should be studied again before going on to the next module. Throughout the study time tutors were available for consultation, but it was up to the student to ask the tutor for help or for explanation, the tutor did not actively intervene in the learning process unless asked to do so. In between modules the tutor would attempt to send students back to re-study lessons which they had not mastered satisfactorily, and students would then return for re-testing, re-evaluation. Evaluation instruments used were a series of questions covering all the concepts and operations taught in the module. It was expected that students should reach an 80%

level of mastery on these questions before passing on to the next module. However, this was not insisted on if a student failed to make 80% on his second attempt. In this case he automatically proceeded to the next module and the best of his two scores was registered as his pass out mark on the previous module.

Group B received a higher level of tutor intervention. The materials were not given to them module by module, but lesson by lesson, and they then returned to be interviewed by the tutor before receiving the next lesson. This resulted in tutor/student planned interventions at intervals of about 2 - 4 hours of study time. At these interviews the tutor used the lesson post-test which was removed from the lessons, administering this as a check on the student's mastery of the lesson content. In this approach the tutor rigidly adhered to an 80% criterion of mastery, directing students back to study the lesson if they failed to reach 80% on this test, or alternatively giving some supplementary tutorial instructions himself. However, tutors were instructed to limit their contributions to a prompting discovery type of tutorial where they would attempt to guide the student into seeing why his errors were indeed errors, and then guiding him back to appropriate pages in the material where he could study the reasons for his errors at greater depth. One could equate the treatment received by Group B on this first section to roughly the Keller plan model of personalised instruction. The tutors used were not from the peer group, but they acted in the way that a typical tutor in the Keller

plan system is expected to act.

In Group C no tutors were available. Students drew out one module at a time from store and studied this in their own time, assessing their own progress using the same lesson tests as used by the previous groups and having available to them the module tests which for the other groups the tutor applied. Thus all the test instruments used by Groups A, B and C are identical. However, in Group C it is the student's responsibility to apply these evaluation instruments and assess his progress and make decisions about what to do in the way of remedial study. In Group B it is the tutor's responsibility to do this between every lesson and also between modules, and in Group A it is the student's responsibility between lessons and the tutor's between modules.

Group D was instructed by a qualified mathematics teacher who had been involved as a subject expert in the construction of the programmes and was using the same exercises and the same evaluation instruments as were incorporated in the programme. However, the teacher was responsible for delivering a traditional series of group instructional lessons while sticking to the sequence and content of the programme materials used by the other groups.

At the end of Section I all four groups took a cumulative progress test on all the material covered up till then, the results of which are included in Appendix E, together with other results. Also Groups A, B, C were instructed to note their study times on each lesson

and each module. These were kept to the nearest 15 minutes, and were then summed module by module to give a rate of progress through the materials for each individual student. This data is also reported in Appendix E.

The schoolchildren groups received roughly similar treatment to Groups C and D of the adults. Groups P and Q of the schoolchildren received the programmed materials without the aid of tutorial assistance, in exactly the same way as Group C of the adults had received them. Groups R and S received traditional classroom instruction, again based on the content and sequence and evaluation instruments that were incorporated into the programmed instruction texts.

## SECTION 2

This section is composed of Modules IV, part (a) and part (b), and Module V of the materials. The groups were assigned to this section in the following manners:

Group A received the same treatment as group C had received in Section 1 of the course - i.e. independent, free study of the programmed materials, without interventions from a tutor, (except to apply evaluation tests at the end of the whole section of 3 modules). We shall henceforth refer to this treatment as the INDEPENDENT STUDY PLAN.

Group B received the same treatment as Group A had received previously - i.e. tutorial interventions only between modules; tutors role limited to guidance and prompting; student must seek the tutor's help. We shall refer to this treatment as the STUDENT-DIRECTED PLAN.

Group C received the same treatment as Group B had received previously - i.e. tutorial interventions between each lesson; insistence on 80% mastery of each lesson before passing on to further lessons. We could call this the TUTOR-DIRECTED PLAN, but it is more convenient to use the previously discussed term "KELLER PLAN".

Group D continued with classroom instruction as it had done with no change.

The schoolchildren groups P and Q who had been previously working in the INDEPENDENT STUDY mode now received the KELLER PLAN treatment.

Finally, Groups R and S who had been receiving classroom instruction continued to do so in Section 2.

### SECTION 3

Section 3 is made of of Modules VI and VII of the materials. In this section an experiment was carried out on the use of Information Mapping as opposed to the very prescriptive, linear programmed instruction materials.

This experiment was limited to Module VI, which, for this purpose was sub-divided into two smaller modules which we termed VIa and VIb. As already described, Module VI was re-written in the Information Mapping format. The two versions of Module VI were kept as similar as possible in content, sequence, choice of examples, and even number of words. How this was done is explained fully in Appendix D. Every effort was made to ensure that the only differences between the two versions were the layout of the material and the facility, in the information mapping

version, to access material in a random fashion, guided by the indexes.

Groups A, B and C of the adults and groups P and Q of the schoolchildren participated in this experiment. Each of these groups was randomly divided into two sub-groups. One of each sub-groups received the information-mapped version of Module VIa, the other sub-group studying the original linear programmed version. A test was administered before and after study and learning times were recorded. The sub-groups crossed over in Module VIb; those previously studying the linear programmed version now received the information-mapped version and vice versa.

This experiment is really an "experiment within the experiment". It will be described more fully below. However, during Section 3 of the course, the main experiment on frequency of tutorial intervention continued. The three plans of utilization were re-distributed as follows:-  
Group A, which had studied Section 1 in the "student-directed" mode and Section 2 in the "independent study" mode, now studied Section 3 in the INDEPENDENT STUDY mode.  
Group B, already exposed to Keller Plan and Student-Directed study, now studied Section 3 in the INDEPENDENT STUDY mode.  
Group C, already familiar with Independent Study and the Keller Plan, now studied in the STUDENT-DIRECTED mode.  
Group D continued as before under classroom instruction.

The schoolchildren groups were assigned as follows:  
Groups P and Q, which first studied in the independent study mode, and then in the Keller Plan, now studied in the STUDENT-DIRECTED mode.

Groups R and S continued under normal classroom instruction.

This experimental structure is summarised in the Table 11.2 appended here which shows at which points evaluation tests were administered and during which periods learning times were measured. The structure shown here encapsulates four distinct experiments which will be defined in the following section.

#### 11.4.2 The Experiments

11.4.2.1 Experiment 1 - Hypothesis: that there is no difference between the results achieved by groups studying from the self-instructional texts on their own without tutorial assistance and normal classroom group instruction as applied in the experiment. This experiment is repeated thrice, (a) with adults by comparison of Groups C and D during Section 1 of the course, and (b) with the schoolchildren by comparison of Groups P and Q with Groups R and S on section 1 of the course, and finally (c) the adult groups A and D on section

These comparisons can only be made on the basis of scores at the end of the sections as the learning times under group instruction cannot be meaningfully compared with learning time under self-instruction, because we have no measure of how much homework time was given to problems set by the teacher. A record of the number of classroom hours given to the section was kept, but this is probably only a part of the total learning time for most students. However, this will be discussed further later on with the results.

TABLE 11.2

## SUMMARY OF THE EXPERIMENTAL DESIGN

(Note: The \* symbol indicates that learning times were measured for this group)

PRE/POST TEST		TEST (I - III)		TEST (IV - V)		QUESTIONNAIRE		PRE/POST TEST	
GROUP	SECTION I (MODS. I - III)		SECTION 2 (MODS. IV - V)		SECTION 3 (MODS. VI - VII)				
ADULTS	<u>STUDENT-DIRECTED PLAN</u> (Tutor intervention between modules only)		<u>INDEPENDENT STUDY PLAN</u>			Via	Vib	VII	
					A1	Inf.Map	Prog.Inst.	P.I.	<u>Keller Plan</u>
					A2	Prog.Inst.	Inf.Map.		
	<u>KELLER-PLAN</u> (Tutor intervention between each lesson)		<u>STUDENT-DIRECTED PLAN</u>		B1	Inf.Map	P.I.	P.I.	<u>Indep. Study Plan</u>
					B2	P.I.	Inf.Map.		
*C	<u>INDEPENDENT STUDY PLAN</u> (No planned tutorial intervention)		<u>KELLER PLAN</u>		C1	Inf.Map	P.I.	P.I.	<u>Student directed Plan</u>
					C2	P.I.	Inf.Map.		
D					D	Control Group for Tests			
<u>"TRADITIONAL"</u> (classroom group instruction, using the same course content/sequence)									
SCHOOL						Via	Vib	VII	
					P	Inf.Map	P.I.	P.I.	<u>Student directed Plan</u>
	<u>INDEPENDENT STUDY PLAN</u>		<u>KELLER PLAN</u>		Q	P.I.	Inf.Map	P.I.	
*Q									
R					R	Control group			
<u>"TRADITIONAL"</u> (classroom group instruction, using the same course content/sequence),									
S									



11.4.2.2 The Second Experiment - Hypothesis; that no difference exists in the amount of learning taking place, or the learning time, between adults and youths-on the programmed instructional material (with or without tutorial assistance). This may be observed by comparison of the scores and the learning rates of Group C from the adults with Groups P and Q for the schoolchildren, on each section of the course.

11.4.2.3 The Third Experiment - Hypothesis; that the more frequent is the tutorial intervention, the higher the scores achieved by students on end-of-section tests, and the lower the learning time required to complete the section. The experimental design for this experiment is shown in the diagram below. Principally this experiment is comparing Groups A, B + C of the adults under the student directed, independent and Keller-plan modes. The experimental design is a 3 by 3 square comparing groups and sections.

	SECTION 1	SECTION 2	SECTION 3
GROUP A	Student-directed plan	Independent study plan	Keller plan
GROUP B	Keller Plan	Student-directed plan	Independent Study plan
GROUP C	Independent Study Plan	Keller plan	Student-directed plan

11.4.2.4 The Fourth Experiment that Information Mapping format is more effective than the programmed instruction format in both levels of achievement and diminished learning time, and that this benefit to information mapping is greater in the case of adults than in the case of youths. The experimental design here involves four separate experiments one for adults involving Group A-C, and the other one for the schoolchildren involving Group P,Q. In each case a two by two square experimental design has been planned, as shown in the diagram below.

EXPERIMENT 4(a)      (ADULT GROUPS)

		MODULE VIa	MODULE VIb
GROUP "A" (Keller plan)	A1	Information maps	Linear programme
	A2	Linear Programme	Information Maps
GROUP "B" (Independent Study)	B1	Information Maps	Linear Programme
	B2	Linear Programme	Information Maps
GROUP "C" (Student-directed)	C1	Information Maps	Linear Programme
	C2	Linear Programme	Information Maps
GROUP D	Took the tests twice as a control but did not receive any instruction between tests		

EXPERIMENT 4(b)      (SCHOOLCHILDREN GROUPS)

	MODULE VIa	MODULE VIb
GROUP P	Information Maps	Linear Programme
GROUP Q	Linear Programme	Information Maps
GROUP R	Took the tests twice as a control, but did not receive instruction between tests.	

#### 11.4.3 Subsidiary Investigation

Several other aspects of the systems under test were investigated, though not as rigorously as in the four abovementioned experiments.

Some of these arise from the needs of the sponsors - the State Secretariat for Education and Culture of Bahia - to test out the overall effects of the systems on the participating students. Thus:-

- (a) an overall pre-test/post-test was administered, the same test being used at the start and the end of the experimental period (August-December 1976). This test was taken by groups undergoing the experimental treatments and by control groups. This gives a general comparison of achievement by the various groups, which will be presented and discussed briefly later in the chapter.
- (b) Attitude questionnaires were completed by both the adults and the schoolchildren taking part in the experimental systems of instruction. The results of the replies to these questionnaires are also summarised later on.

Other aspects arose out of the theoretical considerations concerning individualised instruction and the adult learner, discussed at length in Chapters 2, 3 and 4 of this study.

One of these aspects concerns the mastery-learning model. The proponents of this model have suggested that

learning to mastery should, to a large extent, eliminate individual differences in attainment between students, by allowing as much time as is necessary for each student to learn. The results of the author's experiments will be considered from this theoretical standpoint.

Another aspect concerns the learning skills of the adult learner. It is commonly asserted that adults are slower learners than schoolchildren, are more set in their ways, require to see the practical relevance of a topic before they can learn it effectively (to a greater extent than young learners). The author has therefore examined the results of his experiments closely from this viewpoint. He had also planned to interview in depth a sample of both adults and children, concerning their learning problems and skills. This however did not prove to be possible.

Yet another aspect concerns the student-directed learning model. The proponents of this model often assert that no-one can know, better than the learner himself, how long to devote to learning a topic, how many exercises to study, how many repetitions, what media to use, etc. The teacher should observe and advise the student, but the "decisions of individualisation" should be as much as possible the student's own decisions. This viewpoint is defended not only on the grounds of "developing initiative in the learner" (a change which would be difficult to measure and prove) but also on the grounds of "more efficient immediate learning" of the topic in question.

This the author attempted to observe during his experiments and to follow up by questionnaires and interviews. Once again it proved impossible to follow this up systematically, but some observations were made, which will be discussed later.

The data on the overall gain scores and the student questionnaires will be presented at the end of this chapter. The other three aspects will be discussed in the next chapter.

## 11.5 ANALYSIS OF EXPERIMENTAL RESULTS

### 11.5.1. Preliminary Testing of Groups

The pre-test scores (on the whole course content) were subjected to analysis on the basis of a Null Hypothesis of "no difference in pre-knowledge of the course content between groups".

Of the adult groups, the largest-divergence groups (groups A and C) gave a "t" score of 0.95 supporting the Null Hypothesis at the 20% level.

Of the schoolchildren groups, the most divergent groups (groups Q and R) gave a "t" score of 2.09 which is significant at the 5% level, suggesting that possibly there was a difference in pre-knowledge between these groups not entirely due to chance. However, this difference, though statistically significant was not very large in absolute terms, being of the order of 6 percentage points.

Furthermore, in the experiments involving schoolchildren, although 4 groups were involved, only two basic treatments were used, the groups P and Q receiving individualised instruction in exactly the same formats, and groups R and S receiving group instruction in an equivalent way. If we analyse the differences between the treatment-groups (i.e. treating groups P and Q as one group and groups R and S as one group), we obtain a "t" score of 1.22 which supports the Null Hypothesis at the 20% level. Therefore, considering these larger groups, we conclude that there were no real differences in prior knowledge of the course content between the groups involved in comparisons in the experiments

discussed below.

Below we give the details of the comparison of pre-test scores.

ADULTS - GROUPS A and C (the most divergent groups).

	GROUP A	GROUP C
MEAN	13.50	16.06
ST.DEV.	10.26	10.95
t	0.95 (NOT SIG. 20%)	
Difference of Means	2.56 $\pm$ 4.50 (90% Confidence Limits)	

SCHOOLCHILDREN - GROUPS Q and R (the most divergent groups)

	GROUP Q	GROUP R
MEAN	18.63	12.54
ST.DEV.	13.76	9.52
t	2.09 (SIG. at 5%)	
Difference of Means	6.09 $\pm$ 4.87 (90% Confidence Limits)	

SCHOOLCHILDREN - AS GROUPED IN THE EXPERIMENTAL TREATMENTS

	P & Q COMBINED	R & S COMBINED
MEAN	19.22	16.80
ST.DEV.	11.22	9.40
t	1.22 (NOT SIG. at 20%)	
Difference of Means	2.42 $\pm$ 3.31 (90% Confidence Limits)	

11.5.2. Experiment 1 - THAT THERE IS NO REAL DIFFERENCE  
BETWEEN THE LEARNING THAT STUDENTS OBTAIN VIA INDEPENDENT  
SELF-INSTRUCTION FROM THE "BASG-M" MATERIALS, AND THAT  
WHICH THEY OBTAIN THROUGH NORMAL CLASSROOM GROUP INSTRUCTION.

The purposes of this experiment were twofold.

- (1) It was requested by the sponsors of the project (the Secretariat of Education of Bahia) to evaluate the materials of BASG-M by comparison to existing systems of instruction. This is a step necessary to collect management decision-making data, irrespective of any broader research intent.
- (2) As a "baseline" for the following experiments (which compare the "Independent Study" mode with the "Student-Directed" and "Keller" modes of course implementation) it was felt necessary to establish a "quality standard", however imperfect, for the materials. The author agrees very strongly with the objections made by Hartley (1968) concerning the limited "generalisability" of the results of comparative studies of this nature, where little is known concerning the characteristics of the teachers, the students, the lesson activities in the group instruction mode, etc. The author therefore did not wish to establish a generalisable comparison between individualised and group instruction, by Experiment 1. Rather he wished to evaluate the effect of the materials in this specific case, as compared to actually existing practice, in order to be able to put



into perspective the results of the later experiments. The intention was to avoid the pitfall apparent in many research reports which identify "statistically significant" differences between certain treatments without identifying that despite these differences, all the treatments are insufficiently better than (or indeed are all much worse than) existing practices to warrant any actions to be taken. We wish therefore either to establish or reject the null hypothesis outlined above.

Two adult groups (groups A and C) were compared with the adult group D which underwent traditional group instruction.

Group C received the SECTION 1 of the course (modules I to III) under the independent study plan.

A comparison of post-test raw scores on Section 1, between groups C and D gave

	GROUP C	GROUP D
MEAN	49.39	47.10
STANDARD DEVIATION	14.51	12.70
t	0.672 (NOT SIG. at 20%)	
Difference of Means	2.29 $\pm$ 5.71 (90% Confidence Limits)	

The t-test showed that the differences between the groups could occur by chance in more than 20% of cases, thus supporting quite strongly the null hypothesis outlined above.

Group A received the SECTION 2 of the course (modules IVa, IVb and V) under the "independent study" conditions.

A comparison of the raw-scores on the post-test for Section 2, between groups A and D, revealed

	GROUP A	GROUP D
MEAN	67.33	64.67
ST. DEV.	8.06	12.51
t	0.98 (NOT SIG. at 20%)	
Difference of Means	2.63 $\pm$ 4.48 (90% Confidence Limits)	

Once again this finding strongly supports the null hypothesis.

For the schoolchildren groups, a similar analysis was carried out. In this case, the groups P and Q both studied the BASG-M materials - SECTION 1, under independent study conditions with no teacher intervention and the groups R and S were both taught by classroom group instruction techniques, by the same teacher. The table below therefore considers the two pairs of groups in combination.

	GROUP P & Q	GROUP R & S
MEAN	49.44	50.09
ST. DEV.	16.61	11.26
t	0.55 (NOT SIG. at 20%)	
Difference of Means	0.65 $\pm$ 1.97 (90% Confidence Limits)	

Thus, among the schoolchildren, we find once again that the BASG-M materials appear to be equivalent to the normal classroom instruction given, in terms of the raw scores achieved on a post-test administered on completion of some 6 weeks of study.

Incidentally, the raw scores have been used in this experiment as it proved impossible to set special pre-tests, section by section, due to the pressure of time during the experiment. The only pre-test set covered the total course content (all 3 sections).

Although it may have been possible to split the scores on this pre-test as between the three course sections, this did not seem appropriate as the pre-test (of say one question on a given module) would in no way be equivalent to the post-test (of perhaps 5 questions on the same module). Also, as the pre-test showed a uniformly low level of prior knowledge on all topics for all groups, (and indeed the pre-test score differences between groups are not significant - see above), it seemed reasonable to use the raw scores on the section tests without any adjustment for prior knowledge.

It should also be noted that the differences in the mean scores between groups, although not statistically significant, do in fact favour the "Independent Study" groups to the extent of between 2 and 3 percentage points in the case of adult learners, but in the case of the schoolchildren, the mean favours the classroom instruction group to the extent of about 1½ percentage points.

11.5.3. Experiment 2 - THAT THERE ARE NO REAL DIFFERENCES BETWEEN THE LEARNING ACHIEVED BY ADULTS AND BY SCHOOLCHILDREN, FROM THE BASG-M MATERIALS, UNDER ANY OF THE THREE IMPLEMENTATION PLANS USED, EITHER IN TERMS OF AMOUNT OR RATE OF LEARNING.

The comment at the end of the previous section on

Experiment 1, suggested that perhaps adults might be gaining more from the BASG-M materials than do schoolchildren. The trend was not significant and, worse still, was based on an upward or downward tendency from "normal classroom instruction", which was probably not equivalent in both cases. The second experiment was an attempt to compare more directly the learning achieved by adults and schoolchildren.

Raw scores on each section test and study times on each section were recorded for these groups. One should note that as many students did not manage to complete the third section of the course, the data used in this experiment is for module VI only (the first module of this section). As groups P and Q received essentially the same treatment, it seems reasonable to consider them as one large group for the purposes of this experiment, as was the case in Experiment 1.

Section 1 of the course was studied in the "independent study" mode, with no interventions by the teacher except to administer section tests. The results were as follows.

	ADULT GROUP C	SCHOOL GROUP P & Q
MEAN	49.39	49.44
ST.DEV.	14.51	16.61
t	0.01 (NOT SIG. at 20%)	
Difference of Means	0.05 $\pm$ 8.35 (90% Confidence Limits)	

As a further test, group C was compared with group P (the more divergent of the two schoolchildren groups). This gave a t-score of 1.00 which though much higher is still not significant. Thus the Null hypothesis seems to be supported for adults and children under the independent study mode, as far as the amount of learning is concerned.

A similar analysis was carried out for Section 2 of the course, which was studied under the "Keller Plan".

	ADULT GROUP C	SCHOOL GROUP P & Q
MEAN	75.90	61.82
ST.DEV.	10.78	14.98
t	4.78 (SIG. at 1%)	
Difference of Means	14.08 $\pm$ 4.89 (90% Confidence Limits)	

Once again a comparison of the group C with the more divergent of the two schoolchildren groups gave an even higher "t" ratio of 5.96.

Thus, the null hypothesis is rejected. There appears to be a real difference in the amount of learning that adults and school children achieved from the materials of Section 2, under the "Keller Plan" conditions. This difference strongly favours the adult group. In absolute terms the mean scores are different by about 14 percentage points - an amount not only significant but "worth having" in practical terms. The same analysis was repeated for Section 3 of the course, studied under the "student-directed" plan. Due to the aforementioned problem that not all

students reached the end of Section 3 by the end of the experimental period, the comparison here was made on module VI of the course only. The Section test (on modules VI and VII) was not used. Instead, the scores on two tests administered during the study of module VI were used. These tests had been prepared especially for module VI as part of Experiment 4, to be described later. They were administered half way through the module (module VIa) and at the end of study of the second part (module VIb). For the purposes of this experiment the results of these two tests were combined and expressed as a percentage score (see table 19 in appendix E. ). It should be noted that partly due to the much shorter study time (caused by the truncation of Section 3 of the course) and partly due to the shorter interval between study and test (caused by the test being administered in two parts), the scores achieved on this test were much higher for all groups than was the case in the first two sections of the course. This cannot however be considered as an effect of the superiority of the "student-directed" plan, due to the abovementioned two factors. However, the groups experienced equivalent treatment during this section of the course. The results were as follows.

	ADULT GROUP C	SCHOOL GROUP P & Q
MEAN	77.12	71.50
ST.DEV.	8.66	11.48
t	2.46 (SIG. at 2%)	
Difference of Means	5.62 $\pm$ 3.79 (90% Conf.Limits)	

Once again, group C was also compared with group P alone. This gave a t-score of 2.45, also significant at the 2% level. Therefore, the null hypothesis is rejected. There appear to be real differences between the adult and schoolchildren groups in the amount of learning achieved under the "student-directed" plan on Section 3 of the course. The absolute differences are not as large as in the case of the Keller plan, amounting probably to between 2 and 9 percentage points in favour of the adults.

Also, the origin of this difference is not clear. Is it a real difference in the suitability of this plan for adults or children, or is it partly a result of the superior learning achieved by the adults on the previous section of the course, "carrying over" to the next section? This question needs further analysis, which the present experiment cannot supply, as there were no schoolchildren groups who underwent the treatments in a different order.

We now come to considering the study times of the adults and the schoolchildren when using the individualised materials of BASG-M.

On Section 1 of the course, the following analysis was obtained.

	ADULT GROUP C	SCHOOL GROUP P & Q
MEAN TIME (hours)	49.66	50.73
ST.DEV.	13.82	12.08
t	0.38 (NOT SIG. at 20%)	
Difference of Means	1.07 $\pm$ 4.67 (90% Conf. Limits)	

Also a direct comparison of the individual group P with C gave the same value for t, also not significant. Therefore

the null hypothesis is retained. It seems that no real differences in study times exist between adults and schoolchildren when studying under the "independent study plan".

On Section 2 of the course, however, the results are somewhat different:

	ADULT GROUP C	SCHOOL GROUP (P & Q)
MEAN TIME (hrs)	18.56	20.62
ST.DEV.	3.13	4.27
t	2.44 (SIG. at 2%)	
Difference of Means	2.06 $\pm$ 1.40 (90% Conf. Limits)	

A comparison of group C directly with group P (the more deviant of the two schoolchildren groups), yielded a t-score of 2.04 which was significant at the 5% level. Thus the null hypothesis is rejected in this case. It appears that there probably are real differences in the study time of the groups when working on Section 2, under the "Keller Plan". These differences favour the adults, who take less time (about 10% on average). The standard deviation is also smaller for the adult group.

On Section 3 of the course, statistically significant results were also obtained.



	ADULT. GROUP C	SCHOOL GROUP P & Q
MEAN TIME (hrs.)	1.42	1.26
ST.DEV.	0.17	0.20
t	3.97 (SIG. at 1%)	
Difference Of Means	0.16 $\pm$ 0.07 (90% Conf. Limits)	

Similar results are obtained if only groups C and P are compared. Once again, therefore, the null hypothesis is rejected. It appears, however, that in this case it is the schoolchildren groups which completed the material significantly faster (about 10 - 15%). This interesting phenomenon will be discussed later. It may well be due to the very short nature of the learning task in Section 3 as compared to the other two sections, rather than to factors linked with the "student-directed" plan of course organisation adopted for Section 3.

11.5. 4. Experiment 3 - THAT THE MORE FREQUENT THE TUTORIAL INTERVENTION; THE MORE EFFECTIVE (HIGHER SCORE) AND MORE EFFICIENT (LESS TIME) THE LEARNING PROCESS.

11.5.4.1 Introductory Comment

The above hypothesis is not necessarily as obvious as it looks. Given self-instructional, individualised, learning materials which are effective in teaching the mathematics content (verified in Experiment 1), it is possible that the more frequent tutorial interventions in the Keller plan will not contribute to the greater

effectiveness of the course. Also some proponents of the learner-centred approach would argue that the learner knows when he has difficulties and needs a tutor, so that the "student-directed" plan is the best one to adopt for maximum learning in minimum time. Alternatively, higher frequencies of tutor intervention may lead to higher scores, but a time-penalty might be involved in that the tutorial time is added to and may considerably increase total learning time (through sending the student to re-learn certain parts).

The second experiment has already shown that, for example, under the Keller Plan, adults seem to learn better and to work faster than youths, whilst under the "student-directed" plan the adults learnt better, but worked slower. Is this connected with the differences in the two course-management plans? This will be discussed further in the last chapter. The third experiment examines in more detail the effects of the three different course management plans on the learning of the adult groups.

The original experimental design, described earlier, was in the form of a 3 x 3 square, all 3 groups experiencing all 3 treatments, on different sections of the course. This design was intended to be treated as a Latin Square, enabling analyses of variance to be computed for the data as one large experiment. However, due to factors beyond the author's control, serious interactions between the stages of the experiment crept in (unequal group sizes due to transfer of some students, unequal length of course sections due to lack of time to complete section 3, hence somewhat different testing techniques between sections). As Lewis (1968) indicates, such interactions would introduce serious errors into a Latin Square design.

It was decided therefore to treat the experiment as a succession of three experiments, each comparing three methods of course management on a particular section of the course.

If each of the three experiments were to lead to similar conclusions, then this would strengthen the case for assuming the measured differences are due to the management methods adopted, as the groups experience different methods in each section.

#### 11.5.4.2 Experiment 3.1 - Section 1 of the course

	SCORE	TIME
GROUP A (SD)	49.66%	36.09h,
GROUP B (K)	63.63	22.69
GROUP C (IND)	49.39	49.66

#### Analysis of variance of the raw scores

SOURCE	SUM.SQUARES	DEG.FREEDOM	MEAN SQUARE
Between Groups	4314	2	2157
Within Groups	17398	93	187
TOTAL	21712	95	

F = 11.53 (Significant at 1% level).

Analysis of variance of the study times.

SOURCE	SUM.SQUARES	DEG.FREEDOM	MEAN SQUARE
Between Groups	11984	2	5992
Within Groups	8517	93	91.58
TOTALS	20501	95	

F = 65.43 (Significant at 1% level)

Once the F ratio has indicated real differences among the group means (as is the case above), "the significance of the differences between any two group means may be tested by the "t - ratio" (Lewis, 1968. p.49).

The t-ratios for the individual group means were calculated and are reproduced below:-

TEST SCORES

Groups	A:B	t = 4.18 (Significant at 1%) Diff = 13.97 $\pm$ 5.58 (90% Conf. Limits)
	B:C	t = 4.25 (Significant at 1%) Diff = 14.24 $\pm$ 5.60 (90% Conf. Limits)
	A:C	t = 0.07 (Not Significant) Diff = 0.27 $\pm$ 6.44 (90% Conf. Limits)

STUDY TIMES

Groups	A:B	t = 8.46 (Significant at 1%) Diff = 13.40 $\pm$ 2.65 (90% Conf.Limits)
	B:C	t = 10.28 (Significant at 1%) Diff = 26.97 $\pm$ 4.38 (90% Conf.Limits)
	A:C	t = 4.89 (Significant at 1%) Diff = 13.57 $\pm$ 4.63 (90% Conf.Limits)

#### 11.5.4.3 Experiment 3.2 - Section 2 of the course

	SCORE	TIME
Group A (IND)	67.33%	25,01 h
Group B (S.D)	71.06	19,03
Group C (K)	75.90	18.56

#### Analysis of variance of the raw scores

SOURCE	SUM. SQUARES	DEG. FREEDOM	MEAN SQUARE
Between Groups	713	2	356.5
Within Groups	9803	93	105.4
TOTALS	10516	95	

$$F = \frac{356.5}{105.4} = 3.38$$

This is significant at the 5% level.

#### Analysis of variance of the study times.

SOURCE	SUM. SQUARES	DEG. FREEDOM	MEAN SQUARE
Between Groups	800	2	400
Within Groups	1885	93	20.27
TOTALS	2685	95	

$$F = \frac{400}{20.27} = 19.7$$

This is significant at above the 1% level.

The t-ratios were therefore calculated for the differences between the individual group means:-

#### TEST SCORES:

Groups	A:B	t = 1.53 (Not Significant) Diff = 3.73 $\pm$ 4.07 (90% Conf.Limits)
	B:C	t = 1.82 (Nearly Significant at 5% level but not quite) Diff = 4.84 $\pm$ 4.44 (90% Conf.Limits)
	A:C	t = 3.54 (Significant at 1% level) Diff = 8.57 $\pm$ 4.04 (90% Conf.Limits)

#### STUDY TIMES

Groups	A:B	t = 5.73 (Significant at 1% level) Diff = 5.98 $\pm$ 1.74 (90% Conf.Limits)
	B:C	t = 0.48 (Not Significant) Diff = 0.47 $\pm$ 1.64 (90% Conf.Limits)
	A:C	t = 5.73 (Significant at 1% level) Diff = 6.45 $\pm$ 1.88 (90% Conf.Limits)

#### 11.5.4.4 Experiment 3.3 - Section 3 of the course

	SCORE	TIME
Group A (K)	85.33 %	1.33hrs.
Group B (IND)	77.12	1.38
Group C (S.D)	77.12	1.42

Analysis of variance of the test scores:

SOURCE	SUM. SQUARES	DEG. FREEDOM	MEAN SQUARE
Between Groups	1391	2	695.5
Within Groups	5250	93	56.5
TOTALS	6641	95	

$$F = \frac{695.5}{56.5} = 12.31$$

56.5

This is significant at the 1% level.

Analysis of variance of the study times:

SOURCE	SUM.SQUARES	DEG.FREEDOM	MEAN SQUARE
Between Groups	0.22	2	0.11
Within Groups	1.94	93	0.02
TOTALS	2.16	95	0.02

$F = 5.5$  (Significant at the 1% level)

The t-ratios for individual group differences were as follows:-

TEST SCORES

Groups	A:B	t = 4.68 (Significant 1%) Diff = $8.21 \pm 2.93$ (90% Conf.Limits)
	B:C	t = 0 (Not Significant) Diff = $0 \pm 0$ (90% Conf. Limits)
	A:C	t = 4.605 (Significant at 1% level) Diff = $8.21 \pm 2.98$ (90% Conf.Limits)

STUDY TIMES

Groups	A:B	t = 2.16 (Significant at 5% level) Diff = $0.07 \pm 0.05$ (90% Conf.Limits)
	B:C	t = 1.31 (Not Significant) Diff = $0.04 \pm 0.05$ (90% Conf.Limits)
	A:C	t = 3.07 (Significant at 1% level) Diff = $0.11 \pm 0.06$ (90% Conf.Limits)

In summary, therefore, one may conclude from Experiment 3 that:-

- In terms of Test Score, the Keller Plan was significantly superior to the other two management plans, on all course sections.

- No significant differences in effectiveness (Test Score) were found between the Independent Study Plan and the Student-Directed Plan.
- In terms of Learning Time, the Keller Plan always took significantly less time than the Independent Study Plan, to complete the course work.
- In general the Independent Study Plan was the slowest of the three, significantly so on Sections 1 and 2 of the course, but not at all different from the Student-Directed plan on Section 3.

The implications of these results will be discussed further in the last chapter.

11.5.5. Experiment 4 - THAT LEARNING MATERIALS PREPARED IN AN INDEXED "STUDENT-DIRECTED" (INFORMATION MAPPING) FORMAT ARE MORE EFFECTIVE (SCORES) and EFFICIENT (LEARNING TIME) THAN EQUIVALENT LEARNING MATERIALS IN A PRE-PLANNED "AUTHOR-DIRECTED" (LINEAR PROGRAMME) FORMAT.

11.5.5.1. Introductory Comment

The structure of this experiment was described earlier in the chapter. The experimental design was based on a "cross-over" 2 x 2 latin square. Despite his reservations concerning the effects of interactions in latin squares with small numbers of rows and columns, Lewis (1968) quotes the 2 x 2 square as a special case, in which the residual effect disappears. Indeed he



recommends the use of this design for educational purposes such as, for example, the determining of a difference in difficulty between two tests.

The author's data, exhibited in tables 9 to 13 of appendix E, is organised as four sub-experiments, involving respectively groups A, B, C and finally P and Q together. Thus giving four  $2 \times 2$  squares of similar design. In each case the group is randomly split into two equal groups who study both types of materials on successive sections of module VI.

Inspection of this data seems to suggest very strongly that the "Information Mapping" version of Module VI is superior to the "Linear Programme" version. In all four sub-experiments, the Information mapped materials result in higher gain-scores (as measured by the same test administered before and after study), and in lower study times.

To test the statistical significance of these observations, the author has performed an analysis of variance, using the model for a  $2 \times 2$  Latin Square given by Lewis (1968) on pages 164-168 of his book. This analysis has been performed on one only of the four experiments, as it yielded highly significant results. There is no reason to suppose that the other experiments, with very similar raw results, would do otherwise.

The results of the analysis of variance performed on Group A, were as follows:-

# 11.5.5.2 Experiment 4.1 (a) - GAIN SCORES

EXPERIMENTAL MODEL	Mod.VIa	Mod.VIb	Person Totals	Totals
GROUP A.1 —	IM	PI		
N = 15	Total Score = 113	Total Score = 102		— 215
GROUP A.2 —	PI	IM		
N = 15	Total Score = 104	Total Score = 114		— 218
TOTALS — — —	217 — — —	216 — — —		433
TOTALS FOR TREATMENTS (DIAGONALS)				
INFORMATION MAPPING — — — — —				227
PROGRAMMED TEXT — — — — —				206

## Analysis of variance

SOURCE	SUM.SQUARES	DEG.FREEDOM	MEAN SQUARE
Between MODULES	0.2	1	0
Between GROUPS	0.1	1	0
Between TREATMENTS	7	1	7
Persons within groups	61.7	28	2.2
Remainder	17	28	0.6
TOTALS	86	59	

$$F \text{ for TREATMENTS} = \frac{7}{0.6} = 11.67$$

(Significant much above 1%)

### 11.5.5.3 Experiment 4.1 (b) - (LEARNING TIMES)

EXPERIMENTAL MODEL	Mod.VIa	Mod.VIb	Person Totals	Totals
GROUP A.1 —	IM Total Times 615	PI Total Times 565		- 1180
GROUP A.2 —	PI Total Times 675	IM Total Times 510		- 1185
<u>TOTALS</u> — — —	1290 — — —	1075 — — —		→ 2365
TOTALS FOR TREATMENTS (DIAGONALS)				
INFORMATION MAPPING — — —				1125
PROGRAMMED INSTRUCTION — — —				1240

### Analysis of Variance

SOURCE	SUM.SQUARES	DEG.FREEDOM	MEAN SQUARE
MODULES	770.8	1	770.8
GROUPS	0	1	0
TREATMENTS	220.8	1	220.8
PERSONS IN GROUPS	1016.7	28	36.3
REMAINDER	1446.6	28	51.7
TOTALS	3455	59	

F for TREATMENTS = 4.27

(Significant at 5% level).

The general conclusions from this analysis are

- Information mapping improved the gain score achieved by students significantly.
- It also reduced the learning time necessary to complete the learning task. This result was not so highly significant.

## 11.6. SUBSIDIARY INVESTIGATIONS

### 11.6.1 Overall Gains during the Learning Period

In order to gauge the overall effectiveness of the materials, a pre/post test was administered to all students concerned in the investigations. This was done chiefly at the request of the sponsors of the BASG-M project - the State Secretariat of Education of the State of Bahia and the Federal Ministry of Education and Culture. The detailed results are presented in tables 1 to 8 of Appendix E. The summary table, shown below, indicates the mean scores achieved by the various groups, together with the mean gain scores and the McGuigan ratios. This is the ratio of the actual gain to the possible gain, or:

$$\text{McGuigan's ratio} = \frac{\text{POST-TEST SCORE} - \text{PRE TEST SCORE}}{100\% - \text{PRE TEST SCORE}}$$

This ratio is an attempt to adjust raw gain scores for the "pre-test effect". A raw gain score will not discriminate between the individual or group who goes from 0% to 50% and the one who improves from 50% to 100%. Obviously the second case is more desirable than the first, and yet the raw gain is in each case 50%. McGuigan's ratio on the other hand would give 0.5 for the first case and 1.0 for the second. It is a measure of "what proportion of the gap has been plugged".

# SUMMARY TABLE

## OVERALL RESULTS AS MEASURED

### BY PRE/POST TEST, FOR ALL GROUPS

PRE-TEST ADMINISTERED EARLY AUGUST 1966

POST-TEST ADMINISTERED LATE DECEMBER 1966

<u>GROUPS</u>		MEAN SCORE PRE-TEST	MEAN SCORE POST-TEST	GAIN SCORE	McGIGANS RATIO
ADULT LEARNERS	A	13.5	53.5	40	.46
	B	18	59.6	41.6	.51
	C	19.6	55.3	35.7	.44
	D (CONTROL)	16.5	40	23.5	.28
SCHOOLCHILDREN	P	17.9	54.8	36.9	.45
	Q	20.5	55	34.5	.43
	R (CONTROL)	14.4	43.7	29.3	.34
	S (CONTROL)	17.8	49.5	31.7	.39

The data presented in this table has not been analysed statistically, for a number of reasons.

Firstly, the test paper used for this comparison was one prepared by the examiners of the State Secretariat, independently of the author and his team of writers. Thus, although they were both working from the same syllabus, there were some discrepancies between the paper set by the examiners and the treatment of the subject matter in the course.

Secondly, there were found to be certain sources of difficulty and confusion in the paper. Some of the questions were misconstrued by some of the students. It is therefore doubtful that performance on this pre/post test was an accurate measure of the learning which had taken place. It was not felt justifiable to analyse these results in detail, particularly as they are not crucial to the main hypotheses of the study.

Thirdly, the overall "blanket" measure of gain over the whole course is of little interest due to the undesirability of making any generalised conclusions from a comparison with so many uncontrolled factors. One hardly needs statistics to see from the table what happened in the particular case under study. And one is not wishing to extract any generalisable conclusions from this case, nor would it be "safe to do so" (Hartley, 1972).

The overall effects of this particular study are seen from inspection of the table. Both adult and school groups undergoing individualised instruction did better than the control groups undergoing group instruction. It does not seem to make much difference in which sequence the various individualised study plans were experienced. There also does not seem to be much difference between adult and school groups undergoing individualised instruction. There does however seem to be a difference between the gains of the adults and schoolchildren undergoing the "traditional" group instruction; the schoolchildren appear to learn better in this system - a not too surprising finding, in line with the generally held

viewpoints concerning adults being slower learners. However, it is interesting that under individualised instruction this difference between adults and schoolchildren tends to disappear.

However, perhaps the most serious comment one should make on these overall results is that the gain scores (and McGuigan's ratios) for all the groups are disappointing. One would have hoped for more complete, more effective long-term learning from a 5-month period devoted to, after all, quite a small section of the mathematics curriculum. To what extent the post-test scores (and hence the gains) were influenced by the abovementioned weaknesses in the State Secretariat's test, is difficult to determine. However, this cannot be the only factor resulting in mean gain scores usually less than half the maximum possible gain (McGuigan's ratio 0.5). Certainly the very high and uniform final test scores, often reported in studies of programmed instruction and mastery-learning schemes (e.g. a 90/90 or similar criterion - 90 percent of the group score at least 90 percent on the post test) was not even remotely approached.

#### 11.6.2 The Student Questionnaires

At the end of the experimental period, a short questionnaire was given to the experimental groups A, B, C, P and Q. This questionnaire included, among other questions designed for collecting data for course revision, four questions concerning students' attitudes to the individualised systems they had experienced. The responses to these four questions are summarised in tables 20 (adults)



and 21 (schoolchildren) of Appendix E.

These results show a generally favourable reaction to the experience. There were however some differences between the adult and schoolchildren groups.

To the first question (Did you like working with BASG-M?) the responses were generally favourable. Of the adults, 88½% said they liked the experience (very much or "quite") and only 4% said they did not. Of the schoolchildren, 92% liked the experience and just under 5% did not. The other students were indifferent.

To the second question (Did you find BASG-M more easy or more difficult than previous experiences of learning mathematics?) the general response suggested overwhelmingly that the BASG-M materials are relatively easy for the students. Of the adults, 71% found the work easier than previous experiences, 22% found it about the same and only 7% felt that BASG-M was more difficult. Of the schoolchildren, 73% found BASG-M easier, 10% found no difference and 17% found BASG-M more difficult.

So far there are no very obvious differences between the adults and the schoolchildren, except that perhaps the children (being more used to traditional school work) were more inclined to find the individualised approach more difficult.

To the third question (concerning preferences among the three study plans utilised) there appear to be some more obvious differences between adults and schoolchildren. 63% of adults placed the Keller Plan as their first preference as compared to only 40% of the schoolchildren. The schoolchildren favoured the student directed plan

- 44% put this in first place. Only 33% of adults put this first. There was general agreement that the independent study plan was the least desirable. Only 4% of adults and 16% of schoolchildren put this plan in their first place.

However, one way to treat these responses is to sum up the rank positions, giving a weighting to each score (first place, weighting of 3; second place, weighting of 2; third place, weighting of 1). The following weighted scores are then obtained for the three study plans.

	ADULTS		SCHOOLCHILDREN	
	Weighted Score	Percent of Total	Weighted Score	Percent of Total
(a) Independent Study Plan	114	20%	96	25½%
(b) Student-directed plan	215	37%	144	38%
(c) Keller Plan	247	43%	138	36½%
TOTALS	576	100%	378	100%

This treatment of the results does not alter the rank order of the three plans. The Keller Plan still remains favourite with the adults and the student directed plan with the schoolchildren. However the differences between the preferences do not appear to be as marked as the "first-place" choices suggested.

To the fourth question (concerning the learners preferences for presentation format) the responses indicated a general tendency to prefer the information mapping format. This tendency was stronger for the adult groups than for the schoolchildren. Of the adults, 72% preferred the information mapped version of module VI, 23% expressed no preference, and only 5% expressed a preference for the linear programme version. Of the schoolchildren 59% preferred the information maps, 24% expressed no preference and 17% preferred the linear programmed texts.

## CHAPTER 12

### CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

#### 12.1 CONCLUSIONS FROM THE MAIN EXPERIMENTS

- 12.1.1 The first experiment
- 12.1.2 The second experiment
- 12.1.3 The third experiment
- 12.1.4 The fourth experiment
- 12.1.5 Subsidiary investigations

#### 12.2 CONCLUSIONS CONCERNED WITH THE STUDY AS A WHOLE

- 12.2.1 Concerning the mastery-learning model of individualisation
- 12.2.2 Concerning the student-directed model of individualisation
- 12.2.3 Concerning the interactive or cybernetic model
- 12.2.4 Concerning the mature learner and his learning skills
- 12.2.5 Concerning the mathematical content of the course.

#### 12.3 SUGGESTIONS FOR FURTHER WORK

- 12.3.1 Connected with the present study
- 12.3.2 Connected with the individualisation of mathematics in general

## CHAPTER 12

### CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

#### 12.1 CONCLUSIONS FROM THE MAIN EXPERIMENTS

##### 12.1.1 The First Experiment

The results of this experiment support fairly strongly the null hypothesis that there are no differences in the amount of learning achieved by students using the BASG-M materials, (totally independently of tutorial interaction) and students receiving equivalent classroom instruction (in a group of 30 to 40). Clearly, this conclusion is subject to various critical comments, particularly on the grounds, discussed earlier, that there are so many undefined and uncontrolled factors in the "classroom instruction" mode, as to render it impossible to generalise the conclusion. The most that can be said with certainty is that in this particular college, with these students and with these particular teachers involved, the learning levels achieved were equivalent.

However, these results are encouraging for two reasons. Firstly, the conditions of comparison were likely to strongly favour the teacher-led group instruction. The independent study plan which we are here comparing is likely to be the least-favourable way of implementing the BASG-M materials (the later experiments confirm that this was so in comparison to two other implementation plans). The two teachers, on the other hand, were probably well above average. They had been originally selected as outstanding teachers suitable to act as subject matter

experts to the programming team. They had been involved in the analysis of the course content, in the structuring of the course modules and in the critical evaluation of the materials in draft form. These two qualified and well-prepared teachers taught the class-instruction groups. The tutoring/monitoring of the Keller-Plan groups, on the other hand, was performed by the programming team. In addition to the author, there was one qualified mathematics teacher and two teachers of other subjects (who incidentally, at the beginning of the project were themselves rather poor at mathematics, even the basic levels with which this experiment is concerned). Considering that the tutor-less utilization proved as efficient as the "good" teachers, and the tutor-led utilization proved better still (see experiment 3) one could expect that in the "real world" situation in Brazilian schools the materials may even prove to be "better than the average".

Secondly, the materials of the BASG-M project were not designed to replace teachers, but to offer a remedial mathematics service in a situation where teachers are currently unavailable, or in short supply. The question is not whether to implement "teacher-less" systems or "teacher-led" systems. Rather it is how to offer an effective service with the very limited teacher resources available. Sometimes these resources are not available at all, so it is encouraging that the BASG-M materials appear to be as effective as they are. If they were significantly inferior to the "average" teacher, but still

reasonably effective, they would still be of use to the Brazilian upper secondary educational system, to cope with a problem of revision for which there are no teacher resources allocated officially.

However, the situation appears a lot better than this - so much that the Ministry of Education has now set up a group to study the feasibility of using some of the BASG-M materials in the lower secondary level, as "basic" learning resources, rather than merely "remedial" texts. One should also mention here that the materials used are the first version of the modules. It is planned to revise and improve the material on the basis of the results of this and other studies. The materials-revision aspect of the BASG-M project is not discussed in detail here. However, data has been systematically collected on student difficulties, error rates on problems, difficulties in tutorials, etc. which will be used to redesign the course.

For the purposes of this study, the first experiment serves mainly to establish a "base-line". Once satisfied that the materials by themselves are reasonably effective (by "traditional" standards) we may proceed to comparisons of alternative plans for their utilization.

#### 12.1.2 Experiment 2

The second experiment yielded results which suggest that under the highly prescriptive Keller Plan adults learn somewhat better than do younger students. They also appear to work faster in this plan. But on the other hand

they appear to learn better but work slower under the Student-Directed plan. However, these latter results are not too convincing. Although statistically significant, the 90% confidence limits show that the real differences in mean study times between groups could be quite small and insignificant from a practical point of view.

Trends in the directions indicated are rather surprising from a theoretical point of view. If, as much experience suggests (Belbin & Belbin, 1972) adults are slower learners, are more conscientious, and have more learning difficulties, one would expect them to do less well (or at least to take more time to reach the same level of proficiency). There is perhaps some reason to believe that, due to the adult's higher level of conscientiousness and self-evaluation, he may eventually do better than the school child, but would probably require a significantly greater learning time. Another theoretical consideration, relevant to this particular study is that the adults concerned included a large number who had already studied mathematics at school, but had failed to learn and had dropped out of school early. One would expect the adult groups to be strongly biased towards the less-able extreme, whilst the schoolchildren, though generally from poor and disadvantaged backgrounds, were much more representative of the normal distribution of abilities in the community. This would again suggest a probable advantage in favour of the schoolchildren groups. One has, however, to offset against these, the



levels of motivation of the two age-groups. The schoolchildren were certainly not highly motivated at the beginning of the experimental period. They had been without mathematics teachers for some months and had been given tasks to perform with inadequate supervision, and with no follow-up. Indeed, the absenteeism rate at the beginning of the experimental period was about 40% to 60% of the group on any given lesson. As the experiment got under way, this rate decreased considerably, eventually steadying out at about the 15% level.

The adults participating in the evening classes were obviously more motivated. They were after all attending quite voluntarily, paying a (small) course fee and in general needed the secundo grau qualification for progress in their job. The absenteeism rates tended to show this. They were generally of the order of 10%, remaining at these levels throughout the experimental period.

A possible explanation for the generally better performance of the adults in our experiments may well be that the effects of the higher level of motivation of the adults more than counterbalanced any greater learning difficulties that they may have had. Certainly the generally shorter learning times recorded by adult groups, could well be due simply to a lower level of procrastination, chatter among students and other timewasting activities. Such activities were often observed in the schoolchildren groups - quite rarely in the adult group.

The reverse result, on section three of the course, when the schoolchildren took less time than the adults to complete module VI, could be due to a variety of factors. It may be that one of Belbin's assertions regarding the adult learner is supported by this result, namely that adults are more conscientious and more perfectionist in their attitude to learning (Belbin & Belbin, 1972). After all, this should lead to higher scores at the cost of greater learning time - the result obtained.

However, one must remember that Module VI was a very much shorter learning task than the previous section, that new-style materials were introduced during this module and that the groups were each split randomly into two sub-groups for the purposes of experiment 4. Therefore one would expect a strong Hawthorne effect to be present during the study of Module VI. Although this effect could well be present in both adult and schoolchildren groups, there may be some reason to expect it to have a greater effect on a basically less motivated group, namely the schoolchildren.

Thus the faster learning time of the schoolchildren on the third section of the course (not a very large difference in any case) could well be due to a compound effect - the "Belbin" and the "Hawthorne" effects).

The advantage to the adult group (in both mean test scores and in learning time) when working under the Keller Plan is perhaps more interesting. It appears to go against one of Belbin's other assertions (see Chapter 4) that adults work better when they have more

complete and direct control over their own learning process. After all the Keller Plan is much more prescriptive, much more "tutor-led" than the student-directed plan. Yet adults seem to take to the Keller Plan better than the younger students and the advantage (in mean score) to adults is much larger under the Keller plan than under the student-directed plan (also we have seen from the questionnaire that adult attitudes were most strongly favourable to the Keller Plan).

This result reminds the author of an interesting finding in a survey of learning and teaching styles at the Middlesex Polytechnic. Students, when completing questionnaires, stressed strongly in one section that they considered "control of the course content and methods by the student body" a desirable (even essential) aspect of undergraduate courses. In another section of the same questionnaire however, when they were rating the performance of particular teachers, and justifying their ratings, the same students indicated a strong preference for those teachers who stated clearly what was to be learnt and who were most systematic (even autocratic) in their teaching methods.

We also noted in Chapter 4 the survey of Adult independent study projects (Tough, 1967) which showed that adults experienced the greatest difficulties with decision-making (the learning goals to adopt, the learning methods to use and how to overcome learning problems they encounter) and that this was where they sought most help from others and "would have used more

help if they could have got it",

Tough's findings, as well as the author's do not support the general assertion that adults are able and willing to exert more control over their learning. Whether this is as true in higher level, university courses, as it seems to be in vocational, technical and remedial courses is not known. Little formal research has been done on adult learning styles.

#### 12.1.3 The Third Experiment

As explained earlier, in Chapter 11, (Section 11.5.4.1) the originally intended 3 x 3 square experimental design was modified to three separate sub-experiments, each one comparing the three study plans on one section of the course. This modification proved to be necessary due to differences in the length of the three course sections, testing methods, group sizes, etc. which crept into the experiment due to factors beyond the author's control. Such factors included the transfer of students to other colleges, the unexpected closure of the college for nearly two weeks during the election period, leading to the shortening of section 3 of the course and modification of the tests on section 3.

Each section of the course was therefore treated as a separate experiment - a straight comparison of the three study plans. Such a direct comparison is valid if we can assume no serious differences between the groups which may lead to strong interactions between groups. As students were assigned to groups in a random manner and as the analysis of the prior knowledge of the

groups with respect to the course content revealed no differences, it is probably reasonable to assume that no serious interactions exist between groups.

Furthermore, as the basic comparison is repeated section by section, and groups take different treatment in each section, we may have yet more confidence in our results if the same trend is shown by all three sub-experiments.

This was the case in some respects. In all three sub-experiments the Keller Plan appeared as the most effective study plan in terms of final test scores. The student-directed and the independent study plans did not turn out to be significantly different in this respect.

The author has already commented when discussing experiment 2, that this finding is somewhat at odds with the often stated assertion that the adult should have more control over his own learning. (A new word has even been coined - andragogy. This contrasts with pedagogy. Whilst pedagogy is characterised as the "teacher teaching", andragogy is more concerned with the "learner learning". A new science of andragogy is currently being mooted in Scandinavia. This new science is specifically concerned with the adult learner and the student-directed learning mode supposedly much better for him.)

In the author's opinion, one needs to distinguish here the different types of adult learner which were mentioned in Chapter 4 - Lewis Elton's "Martha's and

Mary's" of post-secondary education. Whereas the university undergraduate, or the adult participating in continuing education for its own sake, may well be both skilled and interested in taking decisions concerning the course of study he is to follow, the typical vocationally-oriented student (be he interested in obtaining specific job skills or merely a paper qualification) is often neither skilled nor interested in taking over the planning of his course.

This may well be an undesirable phenomenon, indicative of the conditioning process (to follow pre-determined rules and procedures) which occurs in much of elementary and secondary education. Perhaps we should be making every effort to change this situation. Perhaps, if we succeed in the schools, then the future adult learner will be both more skilled and more inclined to take over responsibility for his learning. However, the results of experiment 3 (and the student questionnaires) seem to suggest that this is certainly not yet the case among adults in Bahia, Brazil.

As far as learning time under the various study plans is concerned, the results are less clear-cut. In general however, the independent study plan required the largest amount of learning time. This was almost certainly due to the lack of tutor guidance and control. Procrastination was quite common in groups working under the independent study plan. The author suspects that a proportion of the learning time logged by students studying under this plan was not very productively spent.

The Keller Plan emerges as the fastest of the three plans. It appears therefore that the extra time a student spends with a tutor (time spent on marking, assessment and remedial work) under the Keller Plan, does not carry an overall time penalty. The extra tutorial time appears to be more than compensated by a reduction in learning time.

On the basis of experiment 3, therefore, it would appear that the Keller plan is the most effective (high scores) and most efficient (low learning time) for adults of the type and level encountered in the BASG-M project in Bahia.

The apparently non-systematic differences in learning times encountered in this experiment may have an interesting explanation which we shall consider later in the chapter.

#### 12.1.4 The Fourth Experiment

As explained in Chapter 11 (Section 11.5.5.1) the results of this experiment (which was conceived as four sub-experiments, each of cross-over design - 2 x 2 square) were in all four cases apparently strongly in favour of the information mapping format, both in terms of reduced learning time and increased final test score. One of the four sub-experiments (a typical one) was analysed statistically. This analysis revealed highly significant results, therefore it was not performed for the other three sub-experiments, as there was no reason to suppose that less significant results would be found.

These conclusions regarding information mapping are in accordance with the author's previous findings when working with a course on matrices at the Middlesex Polytechnic, and with the findings of other researchers.

The result, although highly significant, must however be qualified. Firstly, one must note that the comparison involved only Module VI - quite a small module requiring only a few hours of study to complete. Thus the experiment was necessarily short, particularly as any given student only studied a half of module VI in the information mapping format. Furthermore, the information mapping technique was new to the learners, whereas they had been using linear programmed instruction materials almost daily for several months. There must surely be a strong Hawthorne effect favouring the "new" technique (i.e. information mapping). The experiment was so short that it would be unlikely that the Hawthorne effect would have the chance to die-away. It is difficult, in the present experiment, to separate out the effects that may be due to a genuine difference between the techniques of programming, from the Hawthorne effects.

Secondly, the differences noted, though statistically significant are not very large in absolute terms, being of the order of 5 to 10 percentage points on gain scores and of about 5 minutes on study time (approximately a 10% saving in time).



In order to gain more confidence in these results, a much larger experiment is required, involving many hours of study on each type of presentation. This would help to overcome the effects of Hawthorne, by giving students time to familiarise themselves thoroughly with each technique. It would also help to establish any differences between formats more reliably. The author believes that the types of benefit observed here for the information mapping materials, would be even greater in a large course (in which the ease of revision, ease of reference, clarity of presentation etc. become even more critical factors). However, this would require the preparation of suitable materials and the design of a new project to apply them. It is quite possible that this opportunity may arise in 1977.

#### 12.1.5 Subsidiary Investigation

Very little needs to be added here to the comments already made in Chapter 11 (sections 11.6.1 and 11.6.2) concerning the overall performance of students during the course and their attitudes at the end of the course. As already mentioned, the overall pre/post test and the student questionnaires were administered at the request of the project's sponsors (the State Secretariat of Bahia and the Federal Ministry) who required an overall report on the project. The State Secretariat prepared these two documents. The author had little control over the preparation of the pre/post test, and only an advisory function in the preparation of the student questionnaire.

One might, in summary, only comment that despite the poor controls over this part of the study and hence the objections to the making of any generalisable conclusions, it was:-

(a) encouraging to find that the use of individualised systems of instruction as the sole methods of mathematics teaching over a long period (4½ months) did not result in either below-normal performance scores or negative attitudes. On the contrary the performance of the experimental groups (adults and schoolchildren) seemed to be generally slightly better than the performance of control groups undergoing "normal" (if anything, slightly better-than-average) group instruction, and the attitudes of the experimental groups to the individualised course was highly favourable.

(b) discouraging that the overall gains in performance were only of the order of 35 to 40 percentage points, giving McGuigan's ratio values of the order of 0.4 to 0.5. In other words, it appears that the experimental groups learnt about half (or somewhat less) of what they might have learnt in the ideal case. It is some consolation that the McGuigan's ratios for the control groups hover around 0.3 (they learnt about a third of what they might have). However, it is inevitable that over a long period of study (4½ months) some forgetting of earlier work must occur. One could not expect to obtain a McGuigan's ratio close to unity. However, for short study sessions (a few

hours) followed by an immediate post test, it is common among workers in programmed instruction to expect a McGuigan's ratio of about 0.9 (i.e. students learn 90% of the content they did not know at the beginning). In the mastery-learning model, one is even supposed to insist on 100% on each unit, before progressing to new material.

In this context, one may examine the gains obtained on the relatively short learning tasks (between tests) in module VI. An examination of the performance of, say, group A (see Table 9 of Appendix E) reveals the following picture.

SUB GROUP	SECTION OF MODULE	STUDY MATERIALS	MEAN GAIN	McGUIGAN'S RATIO
A <sub>1</sub>	VIa	Info.Map.	7.7	.87
A <sub>2</sub>	VIb	"	7.6	.87
A <sub>1</sub>	VIb	Lin.Prog.	6.8	.80
A <sub>2</sub>	VIa	"	6.2	.80

It appears therefore that over the short-term, module VI (linear programme version) is achieving a McGuigan's ratio of .8 and the information mapping version is achieving a ratio of .87 (nearly the "magic" .9).

These figures can be used as a rough overall guide to the efficiency of the instructional materials.

One should remember that the BASG-M materials, as used in the present study, are the first 'trial' version, as yet to be fully revised in the light of evaluation experience. There appears every possibility therefore that, in so far as Module VI is concerned, the .9 criterion could be reached and indeed surpassed by some improvements to the materials (particularly the information mapping version).

What criterion should one apply to a long-term evaluation of several months of learning? This is not at all clear. The normal "pass mark" on school examinations in Bahia (as in many other places) is taken to be 40%. However, this a norm-referenced measure, as well as often being highly subjective. At best it defines the student's rank position in the group. Objective, criterion-referenced tests give a different picture of the amount of learning which has taken place. The procedures used in this study have all been criterion-referenced. We note that the mean scores on the post-test of the experimental groups only hover around 40 to 50 percent, and the mean of the control groups around 20 to 30 percent. In a norm-referenced procedure, the individual scores would be scaled-up to give a mean of 50 percent for all groups.

The author favours the use of the McGuigan ratio as a comparison (particularly for remedial courses) as it takes into account the pre-test score of the student, (which may vary widely in a remedial course). If the

scores are objective, criterion-referenced tests, the McGuigan ratio can act as a useful figure to judge the quality of the course. One may set a given value of the McGuigan ratio as an objective to be reached.

We have already indicated, that for short periods of study followed by an objective test, it is common to aim for a ratio of 0.9 or better. For long periods of study such as 4 or 5 months, this is obviously unrealistic. Some authors have set their aims as high as .7 or .75 for a one-year course. The author has achieved such a ratio over a one-year craft-apprenticeship course (measured on end-of-year multiple choice tests of the City and Guilds). It must be stressed that this figure is an arbitrary choice for an objective. However, it would seem reasonable to at least set a target that students should retain  $\frac{3}{4}$  of the course content which is new to them. Comparing the performance of the BASG-M materials against this arbitrary target, one sees that there is a need for much course improvement in order to raise the McGuigan ratio from say .45 to .75.

It may well be unrealistic to expect such high levels to be achieved in the current conditions of the Brazilian educational system. It may indeed be unrealistic to expect such high levels anywhere, in a subject such as mathematics. After all we are not dealing with the simple recall of the principles and data of workshop theory (as was the case in the craft

apprenticeship courses) but with the mastery of abstract concepts and their application to unfamiliar problems. A useful area of research would be to establish "targets" for the teaching of mathematics. These would of course be different for different mathematical content - operations, concepts, problem solving, problem formulation. But what exactly would they be? Is it not strange that mathematicians are happy to work as teachers, and yet have no sound, mathematically-based system for measuring the results of their work?

## 12.2 CONCLUSIONS CONCERNING THE STUDY AS A WHOLE

### 12.2.1 Concerning the mastery-learning model

The philosophical position behind the mastery-learning model was outlined in the first section of this study (Chapter 3). Some of the research was reviewed in the second section (particularly Chapter 8).

Block (1971) defines mastery learning by the expression:

$$\begin{array}{l} \text{MASTERY} \\ \text{LEARNING} \end{array} = \begin{array}{l} \text{CRITERION-REFERENCED} \\ \text{LEARNING} \end{array} + \begin{array}{l} \text{INDIVIDUALISED} \\ \text{LEARNING} \end{array}$$

by which he means that progress is measured by reference to pre-determined performance criteria and progress towards these criteria is guided and promoted on an individual-student basis. This usually implies that at least the learning time will vary from student

to student. Often other aspects of instruction (methods, content etc.) will also be individualised.

Both Carroll (1963) and Bloom (1968) suggest that the mastery-learning approach should diminish significantly (if not eliminate totally) the normally encountered individual differences in attainment. We have, in group-paced "traditional" systems, the following relation,

Fixed criteria + Fixed learning time → Variable achievement.

The aim of the mastery learning model, as propounded by Bloom and Carroll is to transform this relation:

Fixed criteria + Variable learning time → Uniform (high) achievement.

It is possible to examine the results of the author's experiments for signs that may support or reject this assertion of the mastery-learning "camp". Decreased variability in performance would imply a smaller standard deviation in test scores. In the author's experiments, the Keller Plan was operated very much in the "mastery learning" mode. Students were constantly assessed, and were expected to reach criterion before passing on to new material. The only departure from Bloom's rules is that a criterion of 80% was accepted as satisfactory on each lesson, rather than the "true" mastery model's 100%. The Student-Directed plan was much less controlled and the Independent Study plan not at all. If any attempt to ensure mastery was made,

it was made by the student himself, and not "built into" the instructional system. Finally, we also have the scores of the control groups, who underwent group-paced instruction.

The relevant results are summarised in the table below.

GROUP	SECTION OF COURSE	STUDY PLAN	MEAN SCORE	STANDARD DEVIATION
B	1	Keller	63,63	12,5
C	2	Keller	75,9	10,6
A	3	Keller	85,33	4,6
A	1	Student-directed	49,66	13,6
B	2	"	71,06	10,7
C	3	"	77,12	8,5
C	1	Independent Study	49,39	14,5
A	2	"	67,33	8,0
B	3	"	77,12	8,4
D	1	"Traditional" Group teaching	47,10	12,70
D	2	"	64,7	12,51
	(no data available for section 3)			

It is not necessary to resort to statistical analysis to observe that there are no stunning differences between the standard deviations on the various study plans. The Keller Plan, although leading



to higher gain scores than the other methods, does not result in a diminution of the variability among individuals. One could object that no difference should be expected from mature students under the different versions of individualised instruction, as in all versions they can take as much time as is required to master the material and they should be able to evaluate their own progress by use of the self-tests. Attractive as this argument is (supporting the position of the proponents of student-directed learning for adults) it was not borne out by the previous experiments. After all, significantly better mean scores were registered by adult groups under the more controlled Keller plan.

But there is no objection that can be raised to account for no difference in the standard deviations between Keller plan and the control group studying in a traditional system. We are left with no option but to say that this experiment does not contribute any evidence to support the position that mastery-learning systems lead to an elimination of individual differences in performance.

The one exception in the data is the case of section 3 of the course, but this can be discounted. Section 3, composed in the final event only of module VI, was studied under the special conditions necessary for the information mapping experiment. Instead of a final post-test, two tests were used at the end of the two sub-sections of module VI. Each sub-section contained only two lessons. Under the Keller

Plan, the student was obliged to repeat a lesson until at least 80% mastery was achieved. The post-tests followed every other lesson. Thus the mechanics of the experiment more or less assured a minimum score near to 80% for all the group taking section 3 in the Keller Plan. This was not so for the other two plans.

It is highly probable that the relatively large differences in standard deviation obtained in the Keller plan (S.D = 4.6) and the other two plans (8.5 and 8.4) on section 3 is due to the short duration of the learning task and the fragmented nature of the testing procedure coupled to the continuous assessment.

It may therefore be hypothesised that if a continuous assessment scheme is used to measure student performance, one may expect less variability under a mastery-learning model than would obtain under a non-mastery model. However, if an end-of-course assessment is made after a relatively long course, there seems little evidence of a permanent reduction in individual differences among students.

This latter finding is in line with the survey of research on the Keller Plan, reported by Kulik et al (1976). They found that in general, the Keller plan courses they analysed did not strongly affect the variability in student performance scores.

However, the author has not encountered any research which sets up the hypothesis in the exact form stated above - i.e. that under a continuous assessment scheme one might expect an effect but under an end-of-course assessment scheme one would not. This aspect of the mastery-learning model certainly would deserve further investigation.

### 12.2.2 Concerning the student-directed model

One key tenet of the philosophy behind the student-directed model (outlined in Chapter 3) is that "the student knows best" what style of learning suits him. The main argument here is that students left to their own devices in a resource-based learning system, will tend to choose learning paths, strategies and materials in general better suited to their needs than can be prescribed for them by a teacher or (worse still) an inanimate system.

The present study has shed little light on this argument. It is true that at course-unit level (within a given module) groups worked occasionally in the student-directed mode. But they were "forced" to work on particular modules in that particular mode by the experimental structure. What if that mode suits some of the group but not others?

The student-directed mode, as applied, did not in general give as good learning results as the Keller plan. But the proponents of student-directed learning might argue that the student should have been involved in the decision-making process at an earlier point. He should have been able to choose whether to study a given module under student-directed, or independent of Keller plans. The proponents might set up the hypothesis that "groups allowed to choose their learning plan, learn better than groups not allowed to do so". In the author's hierarchy of "levels of individualisation" (propounded in Chapter 2), this implies that individualisation of learning plan should be extended to the "course" level.

A similar argument may be applied to the use of alternative presentation styles. In this study a more student-controlled style of presentation (information mapping) was compared to a more system-controlled style (linear programmed texts). The student-controlled style was found to be more efficient in general. But could we not achieve yet greater efficiency by allowing the student to choose the style which suits him best, on a lesson-by-lesson basis? In other words, could we benefit by extending the level of individualisation of the course materials from the "learning step" level in the author's hierarchy, to the "lesson" level?

These are questions which largely remain unanswered. Some research exists, but it is sporadic and uncoordinated. The author feels that his hierarchy of levels of individualisation (Chapter 2) might serve as a tool to coordinate the efforts of various researchers in the field.

#### 12.1.3 Concerning the Interactive or Cybernetic Model

The final philosophical viewpoint discussed in Chapter 3, the cybernetic viewpoint, has not really been touched by the present study.

The various systems of individualisation tested have been rather mechanistic than cybernetic. They have either prescribed work to students by means of a simple, algorithmic procedure based on percentage scores on standard tests (Keller Plan) or they have left the matter open, for the student to decide between a very limited range of alternatives (e.g. Text, friend or tutor in the

student directed model and text or friend only in the independent study model]. There has been no attempt to construct an interactive teaching system, a system that can learn from the learner and automatically adapt its prescriptions or its presentations to the needs of the individual. The theoretical work of Landa and Pask (reviewed in Chapter 3) and the practical realisation of interactive, computer-based learning systems (Chapter 8) and paper-based attempts such as Structural Communication (Chapter 10 and Appendix C), suggest that this avenue of progress is likely to be most fruitful. Unfortunately the resources available to the author (time, experimental groups, access to computers) have been too limited to allow any practical investigations (to date) of interactive teaching/learning systems.

#### 12.2.4 Concerning the Mature Learner and His Learning Skills

The study has produced some interesting findings, concerning the learner, particularly the mature learner studying from individualised systems of instruction.

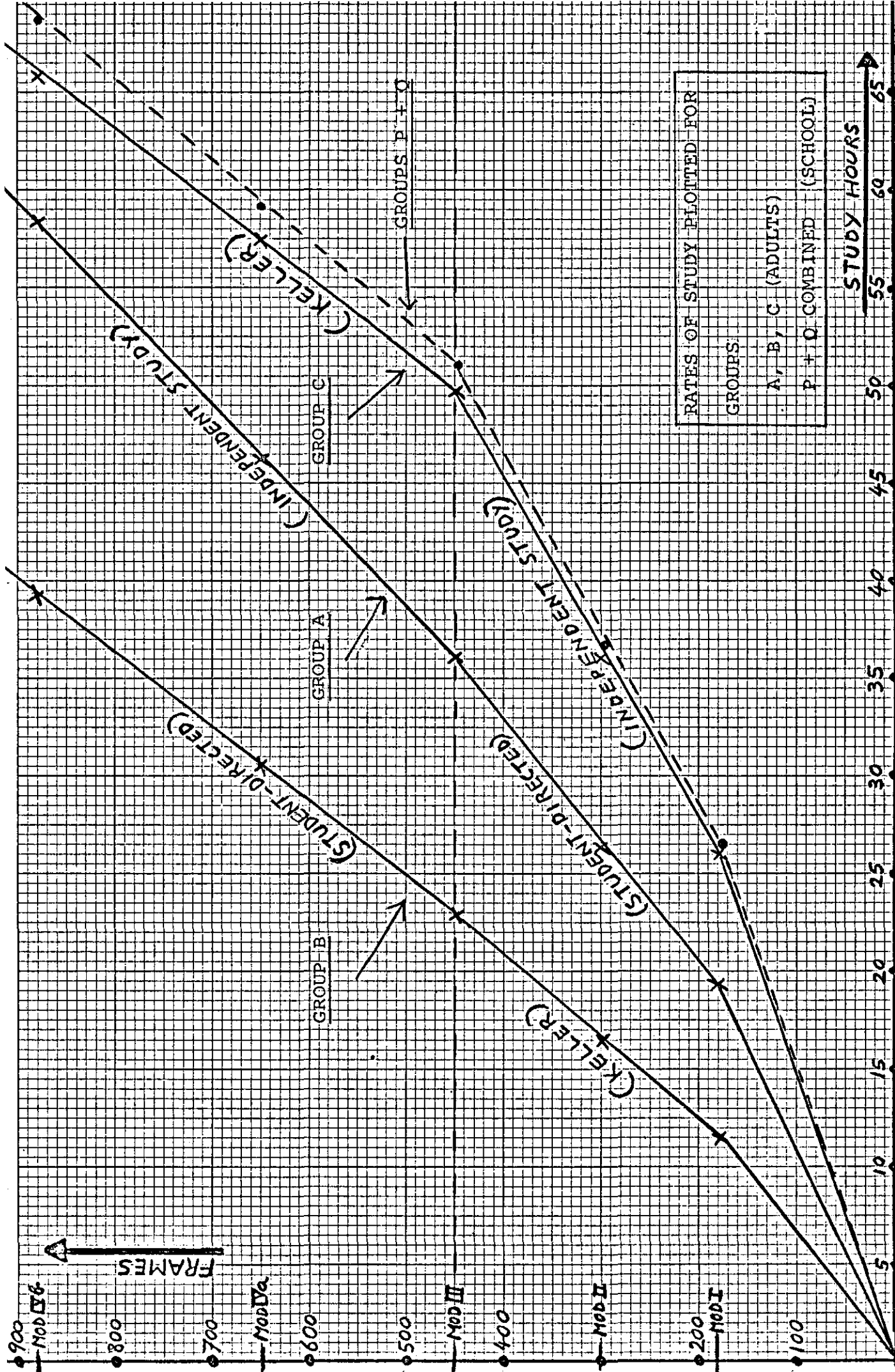
The often-quoted assertion that adults are poorer learners than younger students, seemed to be borne out by the control groups, studying the same course from the same teacher, under a "traditional" group-paced instructional system. However, under some of the individualised systems of instruction, the adults reversed this trend, scoring significantly better than the schoolchildren groups. The greatest difference in favour

of the adult groups was in the Keller Plan, a small difference in the Independent Study plan.

As far as learning rate is concerned, the adult groups were not very different from the schoolchildren, sometimes indeed (e.g. Keller Plan) taking significantly less time to reach a higher level of performance. These findings are quite opposed to the generally held opinions regarding the mature learner, but are in line with some of the observations made by Belbin and Belbin (1972) as regards adult trainees in industry, notably that the re-structuring of the instructional system to take into consideration the special characteristics of the adult learner (see Chapter 4) may eliminate or indeed reverse his apparent handicap as compared to the younger learner.

The author has noted one other aspect of his research on learning times which would benefit from further study. The comparisons of the three study plans, reported in experiments 2 and 3, have taken no account of the possible interactions caused by the sequence in which these plans were experienced. Does experience of, say, the Keller plan, influence the learning under a subsequent plan?

The author believes that such interactions might exist - that there is an element of "learning how to learn" involved in any course. Particularly when dealing with mature students who have not been academic "high-fliers" but are now highly motivated to learn, benefits should be obtained by concentrating on developing the learning skills of the student. In a subject such as mathematics this may be more important than in factual, descriptive subjects as the relevant study skills are less familiar and less



practiced in day-to-day life,

In order to investigate this aspect, the author plotted the learning rates for the first two sections of the course (i.e. modules I to V), calculated in terms of "frames studied per hour" (see the graph attached). This is admittedly a crude measure, as the amount of work required varies widely between frames (ranging from plain reading, to the solution of a set of problems). However, taken module by module, the measure seems to have some value (as the closeness of the points to the "lines of best fit" suggest).

From this graph one may observe several points.

(a) The first module was abnormally slow. This is accounted for by the subjects getting to know what was expected of them, learning to study more independently from printed materials (it was the first time they had experienced anything like this) and by general disruptions in the management of the system, booklet distribution, the tutors learning to do their job efficiently, etc. Thus the rate of work on the first module was atypical.

(b) Ignoring the first module however, and observing the slopes of the lines for the various groups on the rest of section one and then on section two, one may observe that in general the learning rate is somewhat higher on section 2 (modules IV - V) than on section 1 (modules II - III). However, the differences in the Keller Plan are quite small, whereas the differences in the other two plans are larger. The author has estimated the rates as follows:



	MODULES II-III	MODULES IV-V	PERCENTAGE INCREASE IN RATE
KELLER PLAN	24 frames/hr.	27 frames/hr.	12.5%
STUDENT DIRECTED	16 frames/hr.	26 frames/hr.	62.5%
INDEPENDENT STUDY PLAN	11 frames/hr.	19 frames/hr.	72.7%

It would therefore appear that only a part of the difference in rate on the two sections is attributable to the sections themselves. Another (apparently greater) part is attributable either to the sequence in which the plans were studied, or to differences between the groups themselves (e.g. that group B are generally faster learners than group A who are faster than group C). The author feels that whereas this latter variable may play a part in the effect as well, by far the greatest part of the effect could possibly be attributable to a "learning-to-learn" effect. Unfortunately the present study cannot sort out the sources of this effect. An extension of this study could well examine the hypothesis that "experience of a more highly controlled system (e.g. Keller Plan) teaches the learner to exercise "self-control" effectively in a less controlled system" (e.g. Student-directed or Independent study).

If this hypothesis were to be true, it opens up interesting possibilities of maximising the cost/benefit of a scarce resource such as teachers. By starting the

student on a highly controlled, tutor-intensive system, one may be able to "wean" him to operate a similar strategy for himself later on in a more independent (tutor-less) system,

#### 12.2.5 Concerning the Mathematical Content of the Course

In Chapter 3 the author discussed the viewpoints regarding the teaching of mathematics. The "operations" and "expositive teaching" orientation of Ausubel was contrasted to the emphasis on "discovery" to go "beyond the information given" of Bruner. Other viewpoints were related to this as were the practical efforts of such workers as Dienes, Suppes, Scandura and Polya. The value (as conceptual organisers for the various approaches) was pointed out of such hierarchical classifications as Gagne's categories of learning and Bloom's and Krathwhol's taxonomies of educational objectives.

The author stressed his own viewpoint concerning the various viewpoints, namely that the extreme positions adopted by some writers (in an attempt to redress an upset balance) have done more harm than good, in that they have swung the balance to the opposite extreme. A more balanced, "total systems" viewpoint is needed. This viewpoint recognises that the aims of mathematics teaching include the mastery of facts, procedures (operations), concepts, structures, the application of all these to the solution of a given mathematical problem (problem solving), and indeed the formulation of a mathematical problem to fit a real-life situation (model building). It recognises

that all are equally important (in the general sense) but may have different relative importances in a particular case (a particular individual student, studying a particular topic of mathematics, learning it for a particular purpose). Learning and teaching systems should adapt to these particular cases. In order to do this, they must be based on a comprehensive model of mathematics learning and mathematics teaching. A system, in order to learn from reality, must compare the reality with a model of that reality.

We are as yet only at the beginning of formulating a comprehensive model of the mathematics teaching/learning system. Such a model must encompass all the variety of types of mathematics learning.

The present study has concentrated on only two (or three) types of learning - the learning of operations or procedures, the learning of concepts and (to a lesser extent) the learning of simple conceptual structures (i.e. set theory). All these types of learning can be classed as "reproductive" learning, in that the learner, in order to demonstrate mastery need only reproduce what he has been assimilated. No creative or "productive" learning has been attempted in the BASG-M modules tested so far. The student is nowhere asked to solve a problem of a type that he has not yet had demonstrated to him. He is nowhere asked to build a mathematical model. This does not necessarily mean that he has not, incidentally, learnt something which will help him in "productive" learning tasks. But if this occurred, it was incidental, unplanned learning.

The author feels strongly that if we are really interested in our students learning "productive" skills we must plan accordingly.

In relation to the present study, none of the experimental findings will help towards the planning of productive learning experiences.

Indeed, as regards for example problem-solving (true problem solving, in Polya's use of the word) the author believes that the systems he has been testing may well have serious limitations.

One major justification for the development of these systems was to solve a teacher-shortage problem. Now, the model given for the teaching of heuristic problem-solving by Polya, is based firmly on a highly adaptive interaction between the learner and an experienced, insightful teacher. That one can to a certain extent analyse and crystallise this experience is obvious - Polya has done it in 1945 in his book on "How to Solve It". That it is difficult for the average or mediocre teacher to effectively put into practice such an interactive teaching strategy is demonstrated by the level of mathematics teaching in schools and by the products of our teacher training colleges.

How could an individualised system cope with the teaching of problem-solving? Polya's system is individualised - in the sense that it is only really possible in the tutorial one-to-one (or one-to-small group) situation. But as the experienced, insightful, teacher of mathematics is too scarce a resource to use in this way, what are the alternatives?

(a) The prescriptive, teacher-less system is not a viable alternative as there are no opportunities for the two-way learning interaction necessary for the heuristic learning process. Systems such as the Keller Plan, which provide regular interactions between student and tutor, are not likely to solve the problem either. The essence of a Keller Plan course is that the tutor performs a simpler task than was traditionally expected of him. He spends only a small time with each student. His role is more to guide him to other study materials than to teach him directly. The tutor may even be a peer-tutor. This works fine in reproductive learning. But can one really expect the "student who learnt the topic yesterday" to be an experienced and insightful tutor (on Polya's model) for problem-solving? The author thinks not. After all many teachers with years of experience cannot be insightful enough.

(b) The student-directed "open-ended discovery" systems, based chiefly on packaged courses are not alternatives for the same reasons as above, with the additional point that the student is burdened with diagnosing his own needs and problems, which he is ill-equipped to do in a highly structured and abstract subject like mathematics.

(c) The "cybernetics" interactive teaching/learning system might however be an alternative. If, as Polya has done in part, one succeeds in analysing the problem-solving processes in mathematics; if, as Pask has done in part, one succeeds in classifying and describing

learning strategies and develops automated means of identifying them, then one is well on the way to constructing a mathematical model of the process of mathematical problem solving. Once such a model exists (a sufficiently complete and powerful one) modern computer technology is already capable of realising adaptive teaching/learning systems which use the model as a basis for exploring the learner's present state of mathematical competence, his learning problems, etc. The "machine" will then learn about the student and take appropriate decisions to facilitate the student's learning.

If individualised systems, using few teachers (not more than are used in traditional systems) are to help with achieving the "productive learning" objectives of mathematical education, the author feels that they will need to be computer-based, interactive, heuristically programmed systems capable of "learning about the learner" as well as "teaching about the subject".

The present study, and others like it, have provided ample evidence that individualised systems can be most effective in teaching the "reproductive learning" tasks. In so doing, they can "release the teacher to perform the more creative tasks of teaching - the tasks which the machine cannot do" (as any popular article on programmed instruction and teaching machines of the 1960's would point out).

But this has not happened. Partly, the early systems were crude - even for the teaching of "reproductive" skills. But partly, the teachers who were released from

the routine drudgery did not perform the "more creative tasks". They did not perform them because they did not know how to. They still don't. The early crude systems for reproductive learning tasks have improved. The interactive systems capable of some measure of success with "productive learning" tasks are beginning to appear.

Machines will continue to be refined and to improve. Teachers apparently are not improving. How long will we wait to see the revolution?

### 12.3 SUGGESTIONS FOR FURTHER WORK

All the suggestions which follow have indeed been mentioned already in this chapter, as and when they were relevant to the research being discussed. The author will briefly summarise the main suggestions in the paragraphs which follow.

#### 12.3.1 Suggestions Connected with the Present Study

- (a) The author's hierarchy of "levels of individualisation" should be used as a tool to integrate and plan further research work on the characteristics of a mathematics course which benefit from being individualised.
- (b) The study has revealed some interesting trends concerning the efficiency of learning that mature students achieve from individualised courses, as opposed to the younger students. The present study considers only one type of mature learner in one particular type of course. The research could well be extended to other learners and

other courses, to establish whether the greater benefit that adults glean from individualised courses is a general phenomenon or is restricted to only certain types of cases.

- (c) The study has also suggested that the Keller Plan (a prescriptive approach) is more efficient than a more student-directed or independent-study approach. Whilst this was true for the more and the less mature students, it was more marked among the more mature, adult groups. Once again, there is need to replicate the experiment with other types of adult and non-adult groups, and also with other types of mathematical content.
- (d) The study has suggested that information mapping may be a more effective technique of presentation of mathematical content than linear programmed texts. However, there are other possible ways of organising the material. One such way, not investigated in depth in this study (but considered in Chapter 10 and in Appendix C) is Structural Communication. There are also various media based, audio-visual systems and the like. Some work exists in this field, but is inconclusive. More research is required to evaluate other alternatives, (as well as the present alternatives) on different categories of students and on different mathematical content.
- (e) The mastery-learning model should be further investigated, more systematically, to ascertain whether its claims to reduce individual differences in attainment have any basis.



- (f) The student-directed learning model should be further investigated, from the viewpoint of "at which level in the course should the student be the decision-maker?" Is it more important that he should have flexible, easy to access, material (such as information mapping) in order to enable him to plan his own study sequence within a lesson, or is it more important that he can choose between information mapping or some other alternative materials for the whole lesson. The former has been the subject of investigation in this study. The latter has been investigated by other researchers. But no-one has considered the two alternatives together. We are back to "levels of individualisation". Should we plan to give the student control over the materials throughout a lesson, at the beginning of each lesson, at both these points or at neither?
- (g) A study should be mounted to investigate the possible "learning to learn" effect of an individualised system such as the Keller Plan. Such a study should indeed consider other alternatives.. Do students learn to be independent learners better through direct experience of independent learning, through prior experience of more controlled individualised systems or perhaps prior experience of group-based project work?

12.3.2 Suggestions Connected with the Individualisation  
of Mathematics Instruction in General

- (a) Increased effort should be put into the analysis of the mathematics teaching/learning process, and the construction of a comprehensive model of the process.
- (b) This work is a necessary pre-requisite to the development of more general-purpose interactive teaching systems, capable of learning about the learner as well as teaching about the subject.
- (c) The characteristics and structure of existing systems for the individualisation of mathematics should be analysed more thoroughly, in order to ascertain the types of mathematical learning tasks that they are capable of teaching and also the necessary conditions for success.
- (d) The systems for the implementation of individualised instruction should be analysed in order to ascertain the causes for the oft-observed decline in performance standards after some years of functioning, and to evaluate methods for eliminating this decline.
- (e) The objectives of mathematics teaching at various levels and to various groups should be more closely analysed and classified, in order to enable a satisfactory match to be made between the characteristics of the course to be taught, the characteristics of the teaching/learning process required to teach the course and the characteristics of the instructional system necessary to effectively

realise the relevant teaching/learning processes.

In short, research is needed towards a mathematical theory of mathematics learning and mathematics teaching which takes into account all the relevant variables.

## APPENDICES

- APPENDIX A      Bibliography and References
- APPENDIX B      Published Papers relevant to the  
Present Study (the author's).
- APPENDIX C      Structural Communication and  
Information Mapping: Descriptions and  
Examples
- APPENDIX D      The "BASG-M" Materials  
used in the Author's Experiments
- APPENDIX E      Data from the Experiments

APPENDIX A

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## APPENDIX B

### PUBLISHED PAPERS RELEVANT

#### TO THE PRESENT STUDY

1. HAMER, J.W. and ROMISZOWSKI, A.J. (1969)  
"A Computer-managed Individualised Remedial Mathematics Course at Undergraduate Level."  
Aspects of Educational Technology, Pitman, 1969.
2. ROMISZOWSKI, A.J. and ELLIS, P. (1973)  
"Interim Report on Research into Alternative Programming Styles for the presentation of the Correspondence-Course section of the proposed Multi-Media course in Vectors and Matrices."
3. ROMISZOWSKI, A.J. (1975)  
"Factors affecting the programming and control of Individualised Systems of Instruction"  
Aspects of Educational Technology, Kogan Page, 1975.
4. ROMISZOWSKI, A.J. (1976)  
"Individualisation in Higher Education - Introductory Overview". Programmed Learning and Educational Technology, February 1976.
5. NEVES, L.P. and ROMISZOWSKI, A.J. (1976)  
"The Preferences of Students for particular Monitors in a PSI Course". Programmed Learning and Educational Technology, February 1976.
6. ROMISZOWSKI, A.J.; BAJPAI, A.C.; LEWIS, P.E. (1976)  
"The Tutor's Role in the Individualisation of Service Courses in Mathematics". Programmed Learning and Educational Technology, February 1976.
7. ROMISZOWSKI, A.J. (1976)  
"Computer-Generated Examinations and Tests".  
Yearbook of Educational and Instructional Technology, 1976.
8. ROMISZOWSKI, A.J. (1976)  
"Educational Technology in Brazil: A Latin American Case Study". International Yearbook of Educational and Instructional Technology, 1976.
9. ROMISZOWSKI, A.J. (1976)  
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J W Hamer and A J Romiszowski  
*A Computer-managed Individualised Remedial  
Mathematics Course at Undergraduate Level*

(Aspects of Educational Technology III,  
Pitmans, 1969.)

#### THE NEED FOR REMEDIAL MATHEMATICS

Enfield College of Technology has in the last few years undergone a process of expansion, both in numbers of students and in the number and variety of courses offered. The majority of new courses are in business studies and social studies. These courses are at degree level run under the auspices of the Council for National Academic Awards.

As soon as these courses commenced it became apparent that many students were experiencing difficulties with the course material, due to their weakness in basic arithmetic.

It was argued that a mathematics selection test, rigidly adhered to, would result in the turning away of many promising students. It was therefore decided to devise a remedial mathematics course to run in the first few weeks of the 1968-69 Academic Year.

#### COURSE DESIGN

The initial behaviour of the students was likely to vary from quite satisfactory competence in mathematics to near-zero competence. Obviously large group instruction was ruled out. Small group or individual tutorials would impose a teaching load much heavier than the mathematics staff of the College could cope with. It was at this point that the Programmed Instruction Centre was asked to help.

In the time available it was not possible to produce a programmed course to fit our exact requirements, nor was this considered essential, as there is a range of efficient mathematics programmes on the market. The problem was to select appropriate programmes, and integrate them into an efficient system to suit our needs. A four-point 'systems' approach (Romiszowski, 1968) was used to design the course.

#### DEFINE TASK

This was performed by examination of the course syllabus and schemes of work and by discussion with the teaching staff involved. Twenty-two basic mathematic skills were identified as being a pre-requisite for the Social Science Course. The level of proficiency was defined by specifying typical problems. These problems became the basis for a diagnostic test.

#### ANALYSE INPUTS

Subject matter, students, resources

All available programmes which covered the twenty-two mathematic skills were inspected. A final choice was made on the basis of their success and attitudes.

An analysis of the existing first year students suggested that almost all of the following year's entry would require at least part of the remedial course, and about one-third would require to study a large part (over 50%) of the course. On the basis of these predictions, the College resources were analysed.

#### DESIGN OUTPUTS

Content, methods, media

On the basis of analysis and try-out, twenty-two modules of programme were selected as the basic course content. These were reels or parts of reels of Autotutor, Grundy Tutor and Bristol Tutor programmes, together with some texts.

Two diagnostic tests were constructed, each of sixty-six questions — six questions for each module of the course. The acceptable criterion of competence was to be five out of six correct. Students would sit the first of these tests on their first day on the course, and the second whenever they had completed any necessary study of the first eleven modules.

As it was expected that up to 150 students would sit these tests (in fact over 200 did), the marking and assessing problems were likely to be tremendous. Not only must the tests be marked overall, but they must be marked six questions at a time and appropriate modules prescribed for each student. It was mainly for this reason that the computer was first considered. However, once the computer was integrated into the system, it was possible to make use of it for other purposes, e.g. time-tabling, evaluation and progress chasing.

The computer became the main control medium for the course. The teaching media were mainly branching teaching machines (dictated by the choice of programmes). However, the human teacher was to have a very definite role to play in the system. He would monitor the individual course prescriptions which were arrived at by the rather inflexible 'five out of six correct' logic of the computer programme. Secondly, he would be available to give further individual tuition to any student who still could not make the grade after studying the appropriate module.

The resultant system for implementation of the course is shown in Figure 1. This is the final form of the teaching system applied on the course.

#### EVALUATION

Apart from limited try-out of materials on the previous year's students, there was no opportunity for evaluation of the course prior to implementation. The first year's run was to be used for evaluation purposes. This was considered fair as the system allowed for further tutorials if the teaching programme failed.

#### THE ROLE OF THE COMPUTER

The computer has therefore two main roles: (1) Course management — marking, assessing, individual prescriptions, time-tabling, and (2) Course evaluation — test error counts, error analysis, gains, suspect modules.

This second role is simply the standard use of a computer as a data store and calculating machine — the course designer's job aid. The management role is based on the author's experience of similar systems in the United States during a visit in 1967. In particular, the use of computer managed mobile classrooms at Albuquerque, New Mexico (Romiszowski, 1967), and the massive project P.L.A.N. at the A.I.R. Laboratories, Palo Alto, California (Romiszowski, 1967; Hawkrigge, 1967). However, the present study is much more limited, both in size and in the complexity of computer equipment employed. Unlike other systems, we use a simple batch-processing installation with punch-card/tape input and typed output — the Honeywell H.200. However, the system proved adequate for our main problem of handling 200 or more test papers as a batch in minimum time. Two punch-card girls marked 200 tests punched cards and verified them well within two hours. Computer time was about four minutes. The type-out included individual course prescriptions for each student, an error count and analysis for the test items, data on machine and programme requirements for the technician, and a list of student weaknesses for the tutor.

#### THE ROLE OF THE TUTOR

He has several roles: (1) Monitoring the computer's prescriptions. For example, if a student who makes more than one silly mistake in six can, without any tuition, satisfactorily correct his work and explain his errors, then the tutor may allow him to skip certain prescribed modules; (2) Marking and assessing post-tests. He can do this quicker than the present computer system, and can discuss errors and give tuition immediately; (3) Giving extra tuition where required.

In all, eighteen staff members were involved for a total of 190 hours during four weeks. Of these hours 147 were for the Social Science Course,

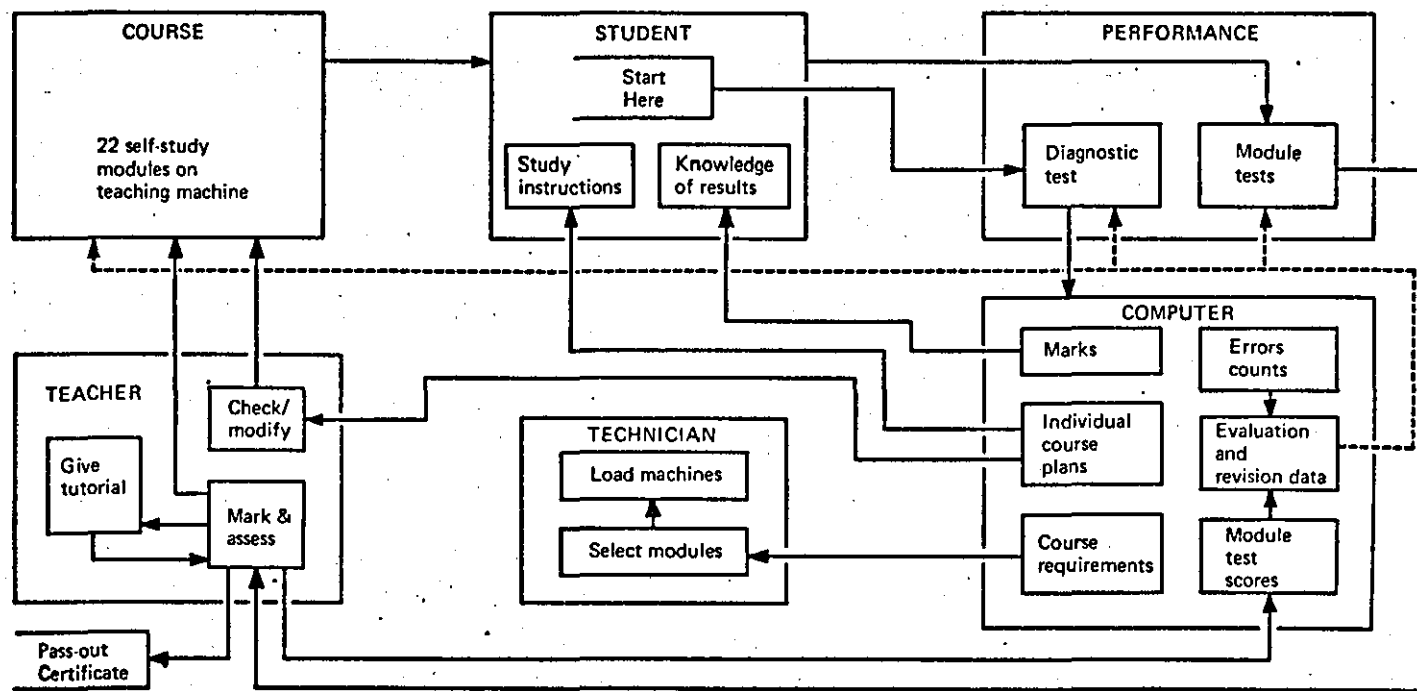


Figure 1. System for remedial mathematics course



and 45 for part-time students (evening). The analysis of the work performed by the Social Science staff gives:

(a) Machine loading, etc.	30 hours
(b) Testing, marking, administration, record-keeping	56 hours
(c) Tutorials, discussions	34 hours
(d) Time unaccounted for (including machine maintenance)	27 hours
	<hr/> 147 hours <hr/>

Categories (a), (d) and the bulk of (b) are routine technician grade of work. By assigning one full time technician to the course, teaching staff time can be cut by 75%. This was done in the later stages of the course and will be a regular part of the system in the future (Figure 1). This will give a substantial saving over traditional staffing costs. On the above figures, staff hours : student hours on the course were approximately 1:6. The planned improvements will raise this ratio to 1:22 at least. Overall staffing on the course is approximately 1:10. Staff-student contact will of course remain entirely on an individual tutorial basis.

### THE PROGRAMMED MATERIALS

Machine utilisation by Social Science students during the four weeks amounted to 859 hours. In addition, about 150 hours were spent unsupervised at home or in the library on programmed texts. After the administration of post-tests, 102 individual tutorials were necessary, totalling about 34 hours. Despite weaknesses in some of the modules, overall failure rate after studying programmes was as low as 13%.

### STUDENTS AND MACHINES

The majority of students booked machine time and studied the prescribed modules very conscientiously. A small minority (about 15%) had to be chased up to complete the course. All other students completed the course well within four weeks. In all, 23 students, that is 12% of the intake, passed all sections of the test and skipped the course altogether. At the other end of the scale, the weakest student studied 20 of the 22 modules, spending a total of 55 hours on teaching machines, and a further  $3\frac{1}{2}$  hours on subsequent tutorials.

It is interesting to note that those modules which were studied from texts were among those requiring most tutorial follow-up. When, coupled with the fact that out of eleven copies of Logarithm texts, eight were 'lost' during the course, it is deduced that the teaching system, if it is to succeed, must impose a modicum of discipline on students, otherwise there is a danger that work is put off indefinitely. In future, if texts are used at all, they will be studied during booked sessions in the Teaching Machines Laboratory.

### COURSE EFFECTIVENESS

In order to assess the long-term effectiveness of the course, a short test consisting of thirty of the original diagnostic test questions were set to all students after a delay of one term. These were the questions which had given the most trouble at the beginning of the course. The overall error rate for these items was just under one-quarter of the original error rate. Individual items varied from one-half to one-ninth of the original error rate. The cost-effectiveness of the first year is not spectacular. Cost of programmes and hire of extra machines was about £400, i.e. £2 per student. However, staffing costs were high, but as outlined above, now that the extent of the problem is known, these can be cut by about 75%.

It would appear that once projected modifications are implemented, the cost of running this course will be no more, and probably less, than traditional group instruction — and of course much greater individual attention is possible.

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COUNCIL OF EUROPE, COMMITTEE FOR CULTURAL CO-OPERATION

INTERIM REPORT ON RESEARCH INTO ALTERNATIVE PROGRAMMING  
STYLES FOR THE PRESENTATION OF THE CORRESPONDENCE-COURSE  
SECTIONS OF THE PROPOSED MULTI-MEDIA COURSE IN VECTORS  
& MATRICES

By A.J. Romiszowski and P. Ellis. March 1973.

The study set out to investigate the relative efficiency of two textual presentation styles in the teaching of introductory matrices material. The styles compared were conventional large step Linear Programming and a relatively new technique developed in the United States, known as Information Mapping. It was suggested at the outset of the investigation that if information mapping seemed to teach better, or even as well as linear programming, then it would be the preferred technique due to the other advantages inherent in the style (ease of use for reference, easy updating etc.)

The matrices material was already available in a Linear version as written at Loughborough. This material was validated and modified as a result of the validation at Enfield College, and a final version produced. The subject matter, as provided by Loughborough, was divided into 3 sections or parts. For the purposes of this study Information Mapping versions of the first two parts were produced at Enfield, and it is these that were used in the comparative study.

It was arranged with the mathematics lecturer of a group of about 90 B.A. Business Studies students, that the materials should be introduced to the students by the authors at the

end of a lecture. The materials were split into two parts and the students were asked to take away the first part, and return it when completed, then take away the second part, this to be returned when completed. The work was on a voluntary basis, although it was pointed out to the students that it covered an area of the course to be covered later in the year. Approximately 50 students took the material away, but only 3 returned the first part. This stage of the investigation was therefore abandoned.

After further negotiations the materials were presented to a second group of Business Studies students. The experimental design was that half the students would initially be given Part I of the materials in Linear Information Mapping form. It was then planned to cross the groups over so that the students who received the Part I in Information Mapping style received Part 2 in Linear Programming style, and the students who received Part I in Linear Programming style received Part 2 in Information Mapping style. A questionnaire for completion afterwards to determine attitudes towards these presentations was also prepared. A lecture was terminated approximately 15 minutes early and a pre-test was given to the students to determine their initial level of knowledge. Part I of the material was given out, and the students were informed that a test would be given on the material after the same lecture next week, and that a second part of the materials would be given out then, to be tested in the same circumstances in the third week. It was not possible to adhere strictly to this plan however, since many people were absent on one or more weeks, and some

who were present on all occasions did not complete the reading in time. A more flexible approach was therefore adopted with students taking away the appropriate texts and tests as they required them.

At the moment we have 3 totally complete returns, and 14 returns complete for Part I. The Pre-test scores, working time required on text, and Post-test scores of the students who have completed Part I are summarised in the attached table. As can be seen with the results available so far the students working on the Information Mapped materials (Unit version) scored slightly better on the Post Test, and completed the work in a much shorter time than the students working on the Linear version. No statistics have, as yet, been carried out on this data as we hope to collect more returns after the Easter Vacation.

The three completed questionnaires from students who have worked through all the materials show that they would like considerable portions of their courses presented in this manner, and that information mapped materials were either liked equally with linear programmed materials, or preferred to them. Obviously, however, little reliability can be attached to an analysis based on 3 returns, and the sample is probably biased since the students who have completed all the materials are probably those who found them most acceptable. Again we hope to enlarge our sample after the Easter Vacation.

However, an interim finding that Information Mapped materials teach at least as well as Linear Programmed materials would suggest that information mapping could profitably be used in the presentation of the matrices materials, especially in view of the easy reference and updating characteristics of the technique.

STUDENT	INFORMATION MAPPING VERSION		Time (mins)	STUDENT	LINEAR PROGRAMME VERSION		Time (mins)
	Pre-Test /5	Post-Test /14			Pre-Test /5	Post-Test /14	
Gibson	1	11	30 mins.	Yin	0	6	40
Bywaters	0	9	30	Kalakoutis	0	12	80
Mitchell	0	8	25	Strudwick	0	11	30
Preston	1	13	?	Synac	0	8	25
Squires	0	11	25	Missing	0	7	35
O'Sullivan	0	8	20	Wallace	0	6	20
				Argyrou	0	11	30
				Mela	0	6	45
MEANS	0.3	10	26	MEANS	0	8.4	38

# Factors affecting the programming and control of individualized systems of instruction

(for the training of technical and science teachers in a developing country)

A J ROMISZOWSKI

(Aspects of Educational Technology IX.

Kogan Page, 1975.)

## ABSTRACT

Individualized or 'personalized' systems of instruction vary widely in structure and in the techniques used for presentation and control. They also vary in what aspects of a course are indeed 'personalized'; variable content, variable learning modes, individual evaluation and feedback, or simply self-pacing. The majority of systems in widespread use are only individualized in the last one or two of these aspects.

One such system which is gaining rapid acceptance at university level, in academic courses mainly, is the Keller Plan. Its popularity stems partly from the ease with which it can be implemented in the traditional structure of a long academic course, partly from the level of sophistication of the typical undergraduate.

Its applicability is less proven in short, intensive courses, with practical rather than academic objectives and with a target population with less experience of studying from the printed word.

After a brief summary of the Keller Plan, this paper discusses five experimental courses for technical and science teachers which are based on a somewhat modified version. Each course consists of about 150 hours of study usually completed in a month. A total of 266 students were involved in the trials. Four courses taught basic subject matter (electrical and electronics procedures); the fifth also taught laboratory practice, lesson planning, materials and visual aids preparation and use, etc.

The philosophy underlying the use of individualized techniques in these courses is discussed and the rationale for modifying certain aspects of the original Keller Plan are examined.

Among the points discussed are:

- The overall effectiveness of the courses.

- The significant early drop-out rate.

- The use of the 'unitmeter' for group motivation.

- The role of the monitor/proctor in the system.

- Student reactions to continuous evaluation and to differences

- in the severity of individual monitors.

- Programming factors - optimal length for study units

- effect of units having varying levels of difficulty

- techniques for rapid revision of the materials.

## INDIVIDUALIZED INSTRUCTION - WHAT IS INDIVIDUALIZED?

'Individualization' is one of those catch-phrases of recent educational jargon which has been used to describe so many patently different systems of instruction that, unqualified by further definitions, it often confuses more than it communicates. At one extreme, individualization is deemed to have taken place if students are working alone at their own pace. At the other extreme the term may imply a system in which any or all of the following factors are adapted to the needs of each individual student - pace, medium of presentation, study style, content, evaluation techniques.

A simple classification is suggested by Edling (1970) which can be summarized in Table I.

Table I.

Media	Objectives	
	School Determined	Learner Selected
System Determined	'Individually diagnosed and prescribed (IPI) (PLAN) (Some CAI)	'Personalized' (many CAI systems)
Learner Selected	'Self-directed' (Learning resource centres) (Some multi-media kits)	'Independent study' (Project QUEST)

This classification, though useful, makes no direct mention of learner groupings or learning pace (often considered key factors in individualization), although all the examples quoted by Edling happen to involve self-instruction or small groups under self-paced learning conditions, as if this was an essential ingredient.

However, it is by no means clear whether a CAI course which allows the learner to select his objectives, is any more or less 'personalized' than the system at Summerhill (Neil, 1960) which also allows the student to select the lessons he attends. If there is a difference then surely it lies in what happens within the lesson. How does the lesson adapt to the learner? Which experience is more individualized, a linear programme in which all students read the same material (albeit at their own pace) or a traditional lesson where all the students have been selected by some diagnostic procedure so that the teacher has an inventory of their learning problems?

We might add a 'third dimension' to Edling's model by constructing a hierarchy of potential individualization among media, based on the degree to which a medium can adapt a presentation automatically to the needs of the individual learner. This is the degree to which a given instructional system is a cybernetic system.

Even in the majority of current CAI adaptive programmes now in existence, one sees only a limited level of adaptability. In highly specific subjects (tracking skills or Suppes' 'drill and practice' mathematics courses, for example) the adaptive machine may have the edge on the human teacher, but

In most academic 'open-ended' disciplines the personal tutor, backed up by efficient diagnostic procedures, is still hard to beat. The more open-ended the subject and the more impulsive and sophisticated the student, the greater the need for adaptive capability. Hence the relative lack of acceptance of 'traditional' programmed instruction at university level, and hence also the origins of the Keller plan.

#### THE KELLER PLAN AND DERIVATIVES

The main features of the Keller plan include (Keller, 1967, 1968):

- (a) Individual study units, usually written matter, which may be (but need not be) specially produced for the course, and may (but certainly need not) be in programmed instruction form.
- (b) Self-tests which the student attempts and then discusses with a proctor or monitor.
- (c) Study guides in the form of detailed objectives, cross-referenced to the reading and practical assignments.
- (d) Individual and/or group practical work and discussion controlled by specially written guide notes.
- (e) The role of the teacher is mainly that of a manager of the system. He has monitors to help him in assessing and tutoring the students. The monitors may be special staff but are often more advanced students who are given the responsibility for the progress of the slower ones. They usually have a monitor's guide book to help them. The teacher evaluates overall progress and revises the course materials, but he does take on monitoring when necessary.
- (f) The teacher also gives a certain amount of face-to-face classes, but these concentrate on enrichment of the course and students have to 'earn' the right to attend these classes by reaching proficiency in certain sets of objectives.
- (g) Proficiency in most Keller courses is taken to mean 100% on each study unit test before moving on to the next one.

The system was first introduced experimentally in the psychology department of the University of Columbia in 1963, and in 1964 was installed at the University of Brasilia (Azzl, 1964, 1965). Since then, use of the plan has spread to other universities and institutions in the USA and also in Brazil, and has been applied to subjects other than psychology, for example, physics MIT (Green, 1971).

Although the Keller plan is the name now in vogue, other attempts were being made to overcome some of the shortcomings of early programmed instruction courses; for example in the UK, Croxton and Martin at the University of Aston were attempting to increase the adaptive nature of programmed courses by roughly similar methods, although also utilizing a computer as a diagnostic aid (Croxton and Martin, 1970). Other examples abound which have some, though not all, of the characteristics of the Keller plan listed above. In the last few years such 'personalized' courses have developed an enthusiastic following in American universities and in other countries including Brazil. At a recent conference/workshop on personalized



Instruction in São Paulo, Sherman (1974) estimated that in the USA "... more than 500 professors have developed their own PSI (personalized systems of instruction) courses and their own materials. At least twice that number have given PSI courses with commercially available materials ..."

#### WHY USE PSI COURSES?

The acceptance of PSI courses at university level is a result of:

- (a) relative ease and speed of materials preparation (not specially programmed and often available from existing sources);
- (b) relative ease of implementation within a traditional university course structure (no major timetable changes, irregular attendance problems minimized, slower students may put in extra time).

The relative success, as compared with other attempts at more rigid programming, is probably explained by:

- (a) familiarity of the style of the learning materials to students;
- (b) the discussion/assessment sessions with the monitors (i) allow for expansion/inquiry/criticism, (ii) ensure full mastery before progress to new materials, (iii) make up for any deficiencies in the quality of the materials, (iv) supply rapid feedback to the teacher, enabling him to revise or add to the course materials on a regular basis throughout the year.

These benefits are particularly marked in long, academic courses and especially in subjects demanding discussion and open-ended responses, involving 'sophisticated' learners.

There is less evidence which supports the use of Keller-type course structures (as opposed to other strategies) in short, intensive courses, or when the subject matter is rigid, or with students not well accustomed to self-study from written materials. These are the characteristics of the courses described below.

#### PSI USED FOR TECHNICAL TEACHER TRAINING

The following discussion is based on the results of a series of PSI courses given in Brazil during 1973/74. The course materials were prepared at the National Foundation for Technical Teacher Training, by a team of specialists trained for the purpose. The courses are intended for trainee technical teachers with generally limited entry experience. A full account of the courses is given elsewhere (Netto, 1974). Table II summarizes the details pertinent to the current discussion. The arguments for using a PSI structure for these courses ran as follows:

- (a) A crash programme of training is required to produce the technical teachers required by current rapid industrialization. Traditional facilities for effecting this training do not exist. Therefore a type of in-service scheme is planned whereby the National Foundation trains and equips a first cadre of teachers, who in turn become trainers of teachers in their locality (the chain-reaction principle).

Table II. Details of three individualized courses for technical teachers

Course Title	Low Tension Electrical Installation (Design)	Basic Electronics	Physics (Experimental)
Objectives (and the main types of Learning Tasks)	Plan, draw, calculate and price an installation for a given building and given needs, following norms laid down and administrative procedures. (Procedural Tasks)	Identify the type, analyse and test basic electronic circuits. Analyse systems into basic circuits and explain function of each. (Conceptual and Discriminatory Tasks - and Laboratory Procedures)	Use laboratory equipment, carry out, write up and criticize experiments. Make and use equipment and teaching aids. Plan a teaching unit based on an experiment. (Conceptual, Procedural and Creative Tasks)
Course Length (Maximum)	150 hours (3 weeks)	156 hours (3 weeks)	150 hours (3 weeks)
Trainees (Number and Background)	48 + 51 + 42 (3 courses) All experienced installers.	41 Low entry level of knowledge.	44 Generally low entry level of physics. Not used to learning through experiments.
Materials	39 obligatory units, 2 optional.	29 obligatory units.	42 obligatory units. 5 optional discussion/ demonstration.
Monitors	Five monitors per course, giving a teaching ratio of about 10:1. All monitors were specially trained staff.		

- (b) Thus effective training systems and materials must be developed to enable relatively inexperienced teachers to instruct others. These 'multipliers', as they are called, must get experience in teaching others during their course. Involving them as monitors will achieve this.
- (c) The multipliers may have to instruct groups or individuals. Individualized materials give the necessary flexibility.
- (d) Finally the concepts of 'mastery learning', behavioural objectives and self-pacing are generally new to the Brazilian educational system. Here is a chance to propagate them 'by example'.

Thus the basic Keller plan seems ideal. However there are several differences between these courses and the classic Keller model:

- (a) The courses as given so far have a fixed maximum time and are very intensive. This was an unavoidable administrative constraint during the experimental testing period. There is no reason why in future the multipliers need to follow the same time-scale.
- (b) So far the students have not acted as monitors, again due to the extra functions the monitors have during the testing period (evaluating and improving the materials). Most of the monitors were also authors of parts of the materials. This again should change with future courses.
- (c) Only very few optional 'motivational' lectures and discussions were included (and then only to present materials which cannot be conveniently presented in print). They were not deemed so necessary in a short course, and probably would not be very motivational as 'extra work' at the end of an eight-hour day of study.
- (d) However, to keep up the pace of work, and also to increase motivation, a pacing device in the form of a graphical display of all students' progress was used. This 'unitmeter' proved very popular in practice.
- (e) One comment made by Sherman (1974) concerning the success of PSI courses, is that the teacher must be there, constantly involved, modifying, improving, discussing, motivating. In a sense the chain-reaction scheme will send fledgling ex-monitors out to repeat their training course without the presence of the teacher/author. A certain contact and feedback from the field is planned, but this will necessarily be sketchy as the scheme expands. How the course efficiency will stand in the second and subsequent phases of the scheme has yet to be seen.
- (f) Finally, as mentioned above, the target populations and the course contents are also very different from the classic university applications of the Keller plan. We shall examine this aspect in detail later.

#### SOME RESULTS IN SUMMARY

##### Completion of course within time

Two of the courses seem to have been relatively well timed. The electronics course, however, fared badly. Analysis showed that the early units were much too difficult and much too long. Unit 5, for example, took a mean study time of 14 hours, and one student spent 27 hours of study on this unit. This was

obviously a great error in programming, which showed up all the more when one noticed that the later units of the course were very easy and quick. The 30% or 40% of the group who managed to struggle through the first 14 units then romped through the remaining 15 units at the rate of 2 to 5 hours per unit with hardly any problems. These trends can be seen on the graphs. So much for the claim that it does not matter about the quality of the course materials. There is obviously a limit to the number of problems caused by poor material which can be left to the monitors to sort out.

Table III

	Electrical Installation	Basic Electronics	Physics
Per cent of students who finished the course within time	84	34	88
Per cent who did not finish or finished later	11	61	5
Per cent of drop-outs	5	5	7

A further interesting point concerned the drop-outs. These almost all dropped out in the first couple of days of the courses, often claiming to have 'prior appointments'. There appears to be a small group in the target population which just does not take to self-instructional techniques. It would have been interesting to analyse this group further, but it vanished so quickly that no one had the chance.

#### Motivation

It is encouraging that the majority, including those who did not finish all their course, worked systematically to the end. Progress was helped by the 'unitmeters'. Students generally took pride in keeping their position on the graph up-to-date. It was found more effective to allow the students rather than the monitors to mark the graph. This competitive 'wish to be graded', much stronger than would be found in similar target populations in Britain, is probably a culture factor. The author has noted the same phenomenon in other developing countries.

#### Role of the Monitors

On the assessment of practical assignments, where the monitor's judgement enters, it proved difficult to standardize assessment procedures. Regular assessment meetings proved necessary. These variations in severity, however, did not seem to be reflected in undue favouritism for the 'lax' monitor on the part of students (Neves, 1974). On the contrary, a student who was failed tended

to stick with the same monitor to be re-tested. He might change monitors for another study unit, however.

A very useful function was played by the monitor/authors, in revising weak points in the materials at evening meetings on a more or less daily basis. Thus, by the end of the three weeks of a course all materials had already been re-written in draft, ready for the next course.

## PROGRAMMING FACTORS AFFECTING EFFICIENCY

### Length of the study unit

The three scatter graphs illustrated below relate the mean study time of the units in a course to the number of students failing to obtain a 100% assessment first time off. There appears to be a certain amount of correlation. Longer units are more likely to give problems (or conversely is it that units which give problems take longer)?

The correlation is not very strong, however. There are several examples of long units with low failure rates (mainly long chunks of descriptive material) but much fewer short units with high failure rates.

The general lesson to learn is to keep the units reasonably short, say, two to four hours of study.

### Effect of the type of learning task

Clearly a unit is not difficult simply because it is long. A further factor is the content of the unit. There are other student-related factors too. To establish some kind of absolute difficulty factor may indeed be an impossible task. If we use the failure rate as a criterion we get the type of chicken-egg argument illustrated in the last section. However, it was felt that the mean failure rate on similar types of units may yield some 'relative difficulty' figures as between different types of learning task.

It so happens that certain groups of study units in certain courses deal with predominantly one type of learning task. An effort was therefore made to analyse the learning tasks within the units and to classify the units accordingly. For this purpose, both Bloom's taxonomy of cognitive objectives and Gagné's hierarchy of learning categories was used. The results of this analysis are shown in Table IV.

Whereas not very much significance can be attached to this data, for the analysis is very general and (as all task analyses) somewhat subjective, a few interesting points emerge.

- (a) The electrical installations course appears to have the lowest levels of learning task, and yet the highest mean failure rate. This illustrates the point that when Gagné talks of learning hierarchies he is referring to prerequisites rather than to absolute difficulty. It is quite easy to learn (and understand) a simple rule of grammar (such as that in Portuguese the adjective agrees with the noun in both number and gender). However, to use the rule one must first know a few nouns and adjectives, and it is at this simple stimulus-response level that all the

sweat of language learning takes place (eg in Portuguese there are hardly any guidelines to help you decide if a particular noun is masculine or feminine - you just 'know').

- (b) The electronics course is difficult to interpret as it was so badly programmed in terms of study unit length. However, the first 14 units (which most people completed) are analysed here. Despite their length the mean error rates are lower than on the electrical installations course. It seems that the programming quality of the electrical installations course, though uniform, is uniformly poor.
- (c) The materials of the physics course would seem to be much better programmed, if mean error rates are a guide. A subjective reading of the various course materials confirms this view. Indeed, the physics programming team had much more extensive experience and training, and had better opportunities for pre-testing their materials. It would seem a sign of good programming too, that errors (and hence the need for monitor assistance) should occur in the conceptual material but should be almost completely eliminated from the simple procedural units.

#### CONCLUSIONS

1. PSI courses can be effectively used for short, intensive periods as well as for the more common applications on long courses.
2. The majority of students, although unaccustomed to this style of learning, showed themselves capable of studying for long periods at a much greater intensity than on courses of a traditional type.
3. For periods of up to several weeks, motivation can be maintained without use of motivational lectures. Simple progress records help to maintain motivation, through competition, even where adult students are concerned.
4. Programming factors are not as unimportant as some proponents of PSI would have us believe. Whereas it is true that weaknesses in the learning materials will 'come out in the wash' and that there is always the possibility of amplification or correction by the monitors, this is always wasteful of monitor time and of student time. In extreme cases (as the basic electronics course, for example) few students may finish the course in reasonable time (or before they lose all motivation), or alternatively monitors may lower standards or start 'expounding' in traditional teacher fashion in an attempt to speed things up.
5. The length of time necessary to complete a study unit should be kept short. Lengthy units should be split or re-written with more self-tests and monitorial interventions.
6. The course producer should be aware of the type of learning task he is setting his student and of its probable difficulty and necessary prerequisites. For this he needs to develop task analysis (preferably behavioural analysis) skills.
7. Thus, whatever the case at university level, Keller-type courses for use at lower levels need careful programming and should therefore be prepared by trained programmers.

## SCATTER GRAPHS FOR THREE PSI COURSES

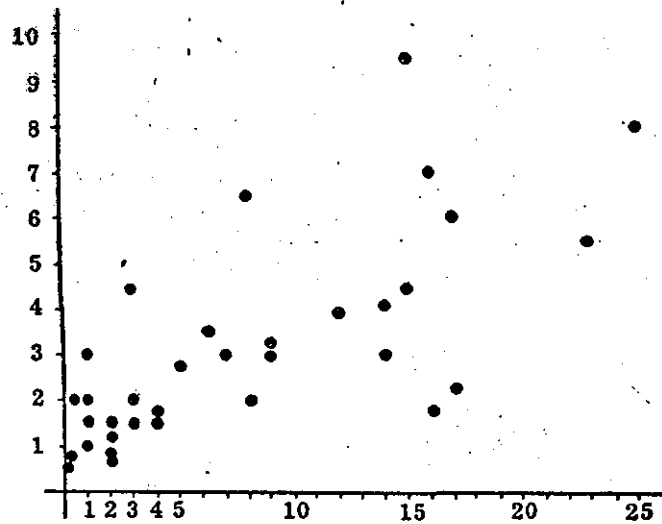
### ELECTRICAL INSTALLATIONS

Results for course involving 42 students.

Each '•' represents a study unit of the course.

Vertical axis: Mean study time to complete first attempt (hours).

Horizontal axis: Number of students failing to obtain a satisfactory assessment (100%) on first attempt

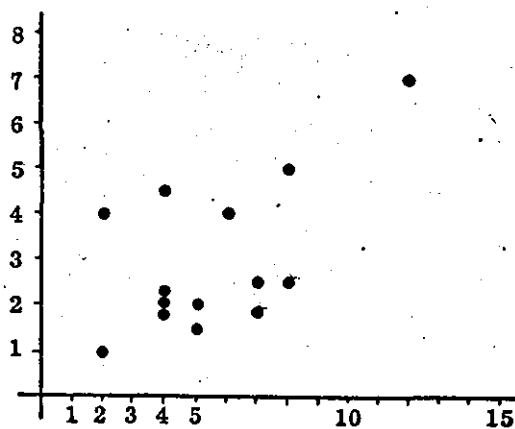


In general, longer study units have higher failure rates.

### PHYSICS

Results of course involving 44 students.

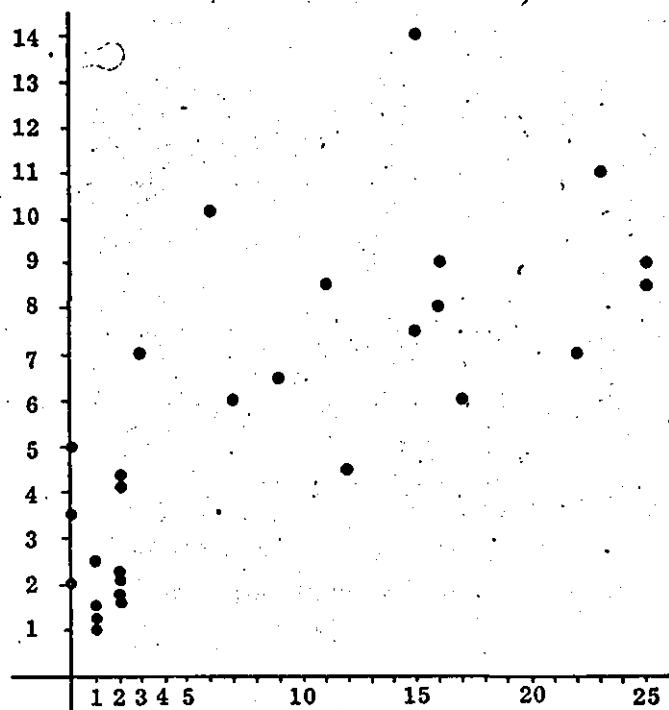
Axes of graph as above.



Note: Only the 14 units teaching theory are plotted here. The 14 laboratory and the 14 materials constructions units, being step-by-step instructions on how to perform self-checking procedures, produced no unsatisfactory assessments whatever the length of the unit involved.

#### ELECTRONICS

Results of course involving 41 students.  
Axes of graph as above.



In general this graph supports the evidence of the earlier two graphs that the length of the study unit is an important factor controlling its difficulty.

However, the results shown here are distorted by the large number of students who did not complete the course.

The cluster of 'low difficulty' is exaggerated. All the shorter units were in the later part of the course, so a diminished number of students attempted the final tests on these units.

All students attempted the final tests on the initial, long study units of the course.



Table IV. A comparison of the relative success of the PSI courses in teaching different learning tasks

Course	Electrical Installations		Basic Electronics		Physics		
Commentary on Content of Study Units Analysed	As the trainees all had experience of installation work, they had to learn few new concepts. Mainly use of reference books, calculations, drawing conventions.		(Analysis of first 14 study units only.) Basically new material involving new ideas and relationships. Having learnt, student uses his information in intellectual tasks of analysing and identifying.		New information, followed by routine experiments to verify it. Also guided instruction on visual aid preparation, lesson planning, etc.		
	7 units theory	18 units practice	7 units theory	7 units practice	14 units basic physics theory	14 of laboratory experiments	14 on visual aids preparation
Main Class of Objectives (Bloom)	Knowledge	Application	Comprehension	Analysis	Comprehension	Application	Application
Main Categories of Learning (Gagné)	Multiple Discriminations	Chains	Concepts + Principles	Discrimination (some problem solving)	Concepts + Principles	Chains + Multiple Discriminations	Chains + Multiple Discriminations
Mean Initial Failure Rate per Study Unit	23.4	23.5	17	14	5.6	0	0

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## Contents

# Programmed Learning & Educational Technology

Journal of APLET

Volume 13 Number 1 February 1976

### Special Issue on 'Individualisation in Higher Education'

Introductory Overview by Alex Romiszowski

3

#### Articles

##### Section 1 — The Personalized System of Instruction (Data and Directions)

The Personalized System of Instruction, or Should University Teaching be Improved? by *Ben A. Green*

9

PSI and the Generic Method of Programmed Instruction by *John H. Hess, Jr.* and *Galen R. Lehman*

13

Research on the Personalized System of Instruction by *James A. Kulik*, *Chen-Lin C. Kulik* and *Beverly B. Smith*

23

Hints to an Effective Personalized Course by *Robert S. Ruskin*

31

PSI: Current Implications — Closing the gap between the theory and the practice of individualized instruction by *J. Gilmour Sherman*

36

##### Section 2 — Applications, Progress and Problems

Two Workshops on the Keller Plan: Content and Evaluation by *B. C. Stace*

45

The Preferences of Students for Particular Monitors in a PSI Course by *Luis Pimenta Neves, Jr.* and *Alexander J. Romiszowski*

54

The Tutor's Role in the Individualisation of Service Courses in Mathematics, or Personalization in an Economic Crisis by *A. J. Romiszowski*, *A. C. Bajpai* and *P. E. Lewis*

59

The Open Access Study Plan, or It Makes Sense to Cooperate by *J. A. Muir*

69

# Individualisation in Higher Education — Introductory Overview

A. J. Romiszowski, Guest Editor

## Individualised instruction — what is individualised?

"Individualisation" is one of those catch-phrases of recent educational jargon which has been used to describe so many patently different systems of instruction that, unqualified by further definitions, it often confuses more than it communicates. At one extreme individualisation is deemed to have taken place if students are working alone at their own pace. At the other extreme the terms may imply a system in which any or all of the following factors are adapted to the needs of each individual student — pace, medium of presentation, study style, content and evaluation techniques.

A simple classification is suggested by Edling (1970) which can be summarised in the following diagram.

TABLE 1.

MEDIA	OBJECTIVES	
	SCHOOL DETERMINED	LEARNER SELECTED
SYSTEM DETERMINED	"Individually diagnosed and prescribed (IPI) (PLAN) (Some CAI)"	"Personalised" (many CAI systems)
LEARNER SELECTED	"Self-directed" (Learning resource centres) (Some multi-media kits)	"Independent study (Project QUEST)"

This classification, though useful, makes no direct mention of learner groupings or learning pace (often considered key factors in individualisation), although all the examples quoted by Edling happen to involve self-instruction or small-groups under self-paced learning conditions, as if this was an essential ingredient.

However, it is by no means clear whether a CAI course which allows the learner to select his objectives, is any more or less "personalised" than the system at Summerhill (1960) which also allows the student to select the lessons he attends. If there is a difference then surely it lies in what happens within the lesson. How does the lesson adapt to the learner? Which experience is more individualised, a linear programme in which all students read the same material (albeit at their own pace) or a traditional lesson where all the students have been selected by some diagnostic procedure so that the teacher has an inventory of their learning problems?

We might add a "third dimension" to Edling's model, by constructing a hierarchy of potential individualisation among media, based on the degree to which a medium can adapt a presentation automatically to the needs of the individual learner. This is the degree to which a given instructional system is a self-regulating system.

In the majority of current CAI programmes now in use, one sees only a limited level of self-regulation or adaptiveness. In highly specific subjects (tracking skills or Suppes' "drill and practice" mathematics courses for example) the adaptive machine may have the edge on the human teacher, but in most academic "open-ended" disciplines the personal tutor, backed up by efficient diagnostic procedures is still hard to beat. The more open-ended the subject and the more inquisitive and sophisticated the student, the greater the need for adaptive capability. Hence the relative lack of acceptance of "traditional" programmed instruction at university level.

A fourth dimension to consider is the level within a course at which individualisation takes place. Is it:

(a) *at the course level* — do students simply exercise their option to take or not to take a given course? Do they do this, incidentally, on the basis of a systematic consideration of the course objec-

#### 4 Individualisation in Higher Education – Introductory Overview

tives or simply by their individual whims?

(b) *at the course unit level – course options* are planned in the light of the overall objectives of the course, the inter-dependencies of the units and the resources available. Neglecting the constraint of resources, *reasonable* course options are few in most courses with tightly defined, specific objectives, and many when objectives are ill-defined.

(c) *at the lesson level* – lessons usually form a sequence, one building on the other. If this is so, is it reasonable at this level to talk of learner-selected objectives? Do we mean simply that learning rate, sequence and perhaps to some extent the methods and media, are individualised? Or do we mean that the learner can select *some* (e.g. enrichment) objectives, over and above the common core of essential objectives?

(d) *at the individual, detailed objective level* – the options would appear much the same as at the lesson level (i.e. learning rate, sequence, methods/media) but on a more “micro” scale, and therefore individual choice is exercised more often.

Trivial as this classification may seem it clarifies some of the confusion which throws such developments as PI, IPI, PSI, PLAN, CAI, into the same bag as Summerhill, QUEST, Resource-based learning, student-directed learning, Dalton Plan, Open Access Plan, Leicestershire Plan, Audio-tutorial system, Siscon Project, Inter-Universities Biology teaching project etc., etc., etc., and then labels the bag “INDIVIDUALISATION”.

##### Individualisation in higher education

Some would claim that the old Oxbridge university system was as individualised as higher education ever has been and is ever likely to be. Individual tutorials, totally optional lectures, some of the best stocked libraries in the world, and every Don an “individual” in every sense. Of course, not much teaching went on. But that was not the point – the university was a “seat of learning”, not teaching. So what’s all this about individualisation?

The plain fact is that universities have become centres of teaching, although many Dons would not wish to recognise this. The pressure for individualisation comes from two directions. The new breed of university teacher tries to adapt to the needs and characteristics of the individual, so that more students learn more of a given prescribed course. Secondly, some universities are seeking new and alternative ways of adapting the content of education to the interests of the individual, so that more students have more choices open to them.

Don’t these two pressures seem to be opposed to each other? Listening to arguments at course plan-

ning sessions, and reading recent literature on the use/misuses of behavioural objectives in higher education, one might be led to this conclusion.

But this is wrong! It is a result of the abovementioned confusion regarding individualisation. The two trends may take place together, in the same university, indeed in the same course, but at different levels. For example, whilst course options are being increased, giving greater choice of objectives at that level, the objectives of a given option may be defined more precisely, objective testing instruments developed, a continuous assessment scheme implanted, all the learning material programmed around behavioural objectives resulting in a “mastery learning” model (completely eliminating the “friendly” normal distribution curve of student achievement). “What about the individual?” I hear your cry. But wait!

At a still more detailed (lesson) level, however, we find that students take varying times to achieve mastery. What to do with the faster students? One answer is to do more work with these students. Let them choose extra learning objectives (enrichment) over and above the course requirements. Here they can exercise their options again. The teacher, in a well planned individualised course, may find that he too has more time than before to devote to these high flyers – more chance to help them follow up their interests.

Another answer is to employ the faster students to help the slower, as in the Keller plan. Here, theoretically, both should gain; the slower student through help, guidance and discussion steps outside the confines of the self-instructional materials; the faster student, by explaining the course content, reinforces his own learning, looking at the material in new ways (if he does not reach new objectives, he reaches new heights of mastery in the basic course).

Is this all a theoretician’s dream? I believe not. A story which illustrates the potential of well-planned individualised learning (and also incidentally the confusion which exists in high places concerning it), tells of a British technological university which some years ago installed an individualised (programmed instruction plus group tutorials) scheme for one subject (structures) of an engineering degree course. When the final examination was taken, the examiners were so shocked by the performance of the students on the structures examination (a greater proportion of distinctions gained than the previously common pass rate) that for a long time they refused to believe that the results were due to improved teaching rather than the per-circulation of the examination paper.

Once they were forced to accept the reality of the

situation, they soon "put things right" for the next year by totally modifying the examination.

### Overview of the papers

In this special issue, I have attempted to collect a selection of papers which illustrate several aspects of the "individualisation" movement. I have concentrated especially however on the systems of "personalised instruction" based on the Keller plan. The reasons for this are:

- It appears that this plan is the first system of individualisation to step out of the "sporadic experiments" stage, and enter the "mass implementation" stage in many universities in the USA and also in other countries. It appears to have features which are particularly well adapted to the perceived needs of higher education.
- It is a plan which, in its purer forms, is the closest practical application to higher education, of the learning principles developed in psychological laboratories, the principles upon which programmed instruction was originally based.

I am most grateful to Dr J. Gilmour Sherman, of the Center for Personalised Instruction, Georgetown University, for his help in assembling the first section of this special issue, which is devoted to the principles, practices, past research and future prospects of PSI.

This section is general in flavour, rather than dealing with specific applications. The opening paper, by Ben Green, defines PSI and justifies its use in higher education. James Kulik, Chen-Sin Kulik and Beverley Smith provide a review of the research into PSI. Robert Rusklin tells us how to do it and Gil Sherman closes this section with an analysis of some of the implications of using PSI.

Also included in this section is the paper by John Hess and Galen Lehman, comparing PSI and other systems of individualised learning, including Individually Prescribed Instruction (IPI), Learner Activity Packages (LAP's), Contingency Management, The Postlewaite Audio Tutorial method, CAI, and of course the generic programmed instruction, from

which most of the other systems are more or less directly descended.

The second section of the journal contains reports of specific applications of individualised learning in higher education. Here I have attempted to include a mixture of papers, illustrating individualisation in various contents and at various levels. The paper by Romiszowski, Bajpai and Lewis discusses ways of individualising to achieve both "mastery learning" and "enrichment" at the same time, illustrating the discussion with two applications to mathematics service courses – one a PSI course, the other a "Learning by Appointment" PI based system. The paper by B. Stace describes some of the PSI workshops at the University of Surrey. Neves and Romiszowski illustrate some Brazilian experiences with PSI, discussing the relationships that exist between students and monitors.

The final two papers deal with individualisation at different levels in the system. Andy Thomas describes the SISCOON project, which is an attempt to spread self-study units throughout a network of colleges in order to implement new courses and assist inexperienced teachers to give these courses. And the final paper by Muir, describes the Open Access System at the Atlantic Institute of Education, Canada. This covers the individualised structuring of postgraduate courses, by the selection of course units, not only from the one institute, nor even from other institutes of academic learning, but from any situation within society which is capable of providing experiences relevant to the overall course objectives.

Thus, in leaps and bounds from one level of individualisation to another, it is hoped the reader will find a useful general overview and also something of specific relevance to his own area of work.

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# The Preferences of Students for Particular Monitors in a PSI Course

Luis Pimenta Neves, Junior and Alexander J. Romiszowski

**Abstract:** PSI courses have been used in Brazil for the training of technical teachers. Whereas they are based on the Keller Plan, they depart from the original in certain aspects. The details of these courses are described briefly by way of introduction.

The study examines whether there is any relationship between the popularity of a given monitor and his leniency. It was found that students tend to choose the more lenient monitor, but if they are failed by a given monitor, they tend to go back to him for re-assessment.

## Introduction

The widespread acceptance of PSI courses at university level is influenced by:

- ... relative ease and speed of materials preparation (not specially programmed and often available from existing sources).
- ... relative ease of implementation within a traditional university course structure (no major timetable changes, irregular attendance problems minimised, slower students may put in extra time).

The relative success, as compared to other attempts at more rigid programming, is probably explained by:

- ... familiarity of the style of the learning materials to students
- ... the discussion/assessment sessions with the monitors (a) allow for expansion/enquiry/criticism (b) ensure full mastery before progress to new material (c) make up for any deficiencies in the quality of the materials (d) supply rapid feedback to the teacher, enabling him to revise or add to the course materials on a regular basis throughout the year.

These benefits are particularly marked in long, academic courses and especially in subjects demanding discussion and open-ended responses, involving "sophisticated" learners.

There is less evidence which supports the use of Keller plan in short intensive courses, or with students not well accustomed to self-study from writ-

ten materials. These are the characteristics of the courses described below.

## PSI Used for Technical Teacher Training

The following discussion is based on research with a series of PSI courses given in Brazil during 1973/74. The course materials were prepared at CENAFOR (the National Foundation for Technical Teacher Training) by a team of specialists trained for the purpose (see BORI, Carolina 1974). The courses are intended for trainee technical teachers with generally limited entry experience. A full account of the courses is given elsewhere (NETTO, W. W. 1974). The following table summarises the details pertinent to the current discussion.

The arguments for using a PSI structure for these courses ran as follows:

- ... A crash programme of training is required to produce the technical teachers required by current rapid industrialisation. Traditional facilities for effecting this training do not exist. Therefore a type of in-service scheme is planned, whereby the National Foundation trains and equips a first cadre of teachers, who in turn become trainers of teachers in their locality (the chain-reaction principle).
- ... Thus effective training systems and materials must be developed to enable the relatively inexperienced teacher to instruct others. Also these "multipliers" as they are called must get experience in teaching others during their course. Involving them as monitors will achieve this.

TABLE I. Details of three individualised courses for technical teachers

Course title	Low tension electrical installation (design)	Basic electronics	Physics (experimental)
Objectives (and the main types of learning tasks)	Plan, draw, calculate and price an installation for a given building and given needs, following norms laid down and administrative procedures. (PROCEDURAL TASKS)	Identify the type, analyse and test basic electronic circuits. Analyse systems into basic circuits and explain function of each. (CONCEPTUAL AND DISCRIMINATORY TASKS - AND LAD. PROCEDURES)	Use lab. equipment, carry out, write up and criticise experiments. Make and use equipment and teaching aids. Plan a teaching unit based on an experiment. (conceptual, procedural and creative tasks)
Course length (maximum)	150 hrs. (3 weeks)	156 hrs. (3 weeks)	150 hrs. (3 weeks)
Trainees (no. and background)	48 + 51 + 42 (3 courses) All experienced installers.	41 Low entry level of knowledge	44 Generally low entry level of physics. Not used to learning through experiment.
Materials	39 oblig. units 2 optional	29 oblig. units	42 oblig. units 5 opt. discussion/demo.
Monitors	Five monitors per course, giving a teaching ratio of about 10:1. All monitors were specially trained staff.		

... The multipliers may have to instruct groups or individuals. Individualised materials give the necessary flexibility.

... Finally the concepts of "mastery learning", behavioural objectives and self-pacing are generally new to the Brazilian education system. Here is a chance to propagate them "by example".

Thus the basic Keller plan seems ideal. However, there are several differences between these courses and the classic Keller model:

... The courses as given so far have a fixed maximum time and are very intensive. This was an unavoidable administrative constraint during the experimental testing period. There is no reason why in future the multipliers need to follow the same time scale.

... So far the students have not acted as monitors, again due to the extra functions the monitors have during the testing period, (evaluating and improving the materials). Most of the monitors were also authors of parts of the materials. This again should change with future courses.

... Only very few optional "motivational" lectures and discussions were included (and then only to present materials which cannot be conveniently presented in print). They were not deemed so necessary in a short course, and probably would not be very motivational

as "extra work" at the end of an eight hour day of study.

... However, to keep up the pace of work, and also to increase motivation a pacing device in the form of a graphical display of all students' progress was used. This "unitmeter" proved very popular in practice.

... One comment made by Sherman (1974) concerning the success of PSI courses, is that the teacher must be there, constantly involved, modifying, improving, discussing, motivating. In a sense the chain reaction scheme will send fledgeling ex-monitors out to repeat their training course without the presence of the teacher/author. A certain contact and feedback from the field is planned, but this will necessarily be sketchy as the scheme expands. How the course efficiency will stand in the second and subsequent phases of the scheme is yet to be seen.

... Finally, as mentioned above, the target populations and the course contents are also very different from the classic, university, applications of the Keller plan.

#### The Study

In an article entitled "The Role of Proctoring in Personalised Instruction" Farmer et al (1972) showed that the greater the interaction between the students and the monitors (proctors) the greater the learning rate of the student. Also students who

had more monitor-content performed better on a final examination.

The present study continues investigating student-monitor interaction, but from the viewpoint of student preferences for a given monitor, in relation to the monitor's relative lenience or severity.

Student behaviour is analysed to identify whether being failed on a unit influences subsequent choice of monitor.

The study has been reported in full (Neves, L. P. 1974) elsewhere. What follows is a summary of the main findings.

### The Course

The course used for this experiment was the PSI Course in Low Tension Electrical Installations, which had been developed in CENAFOR.

The objectives of the course were that students should learn how to design, draw up the plans and other necessary documentation and calculate the costs, of an electrical wiring installation for private houses, blocks of apartment or industrial premises.

The course was in 39 units. It was given in a concentrated period of 150 hours, 8 hours per day, for just over 3 weeks. (These were the hours when monitors were on duty).

### The Subjects

The current experiment involved 42 course participants - one group. 41 were male and 1 female. Ages ranged from 25 to 50 years. The majority were practising teachers of electricity/electronics at technical school level, and all had prior training in physics or engineering.

The course content however was new to all of them, as was the PSI style of course organisation. Most of the subjects were available for the whole period of the course, but this does not mean that they necessarily studied a full 8 hours every day. Indeed, some had job commitments which kept them away from the course at times, but this did not seem to interfere with their rate of progress through the course units. Subjects could of course study at evenings and weekends if they wished, but could be assessed by monitors only in the scheduled course hours.

### The Monitors

Four monitors were used, and the course co-ordinator also stepped in to help with monitoring when required. However, this was not often required, as the ratio of monitors was adequate to avoid queueing problems in general.

Indeed, it was usually possible for a given student to exercise a choice between the available monitors without having to wait too long.

Thus it was logistically possible for some monitors to be "favourites" and thus to work harder, without being overworked.

The four full-time monitors were not from the peer-group but had been specially employed to perform this function.

### Experimental Procedure

During the scheduled course hours, the students worked in one room and when ready for assessment went to another room and chose one of the four (sometimes five) monitors, who would assess the work done awarding either a satisfactory or unsatisfactory grade. The criterion for a satisfactory grade was (theoretically) 100%. However, in the bulk of the unit tests, in addition to some objective "recall"-type questions, there were a large number of "open-ended" problems (alternative layout designs were possible for the same electrical installation). Thus there was an element of subjective assessments which the monitors had to perform.

Students receiving an unsatisfactory grade, in addition to discussions with the monitor, had to repeat the unit, and re-work the unit test.

Thus the tutoring role of the monitors was minimal - they were primarily concerned with marking and assessing the work. Students had to achieve a satisfactory grade on each unit before progressing to the next.

Data was collected on the number of satisfactory and unsatisfactory assessments received by each student, and awarded by each monitor. This was recorded on a session by session basis (a session is a 4 hour period - i.e. a morning or an afternoon), and also accumulated as a running total.

### Hypotheses

The hypotheses investigated were:-

1. Students would tend to choose the more lenient monitors
2. On receiving an unsatisfactory assessment, students would tend to change monitors for the next assessment.

### Results and Conclusions

a) *Were there any preferences for monitors identifiable?*

Preferences were identified in some of the subjects. A few stayed with the same monitor for almost all assessment sessions.



Other subjects exhibited no preference for any monitor, using all four about equally.

An arbitrary criterion of preference was chosen — a subject who chose his "first choice" monitor on at least 5 more occasions than his "second choice" was classified as "having a marked preference". (As there were 39 study units, each subject was assessed at least 39 times, and most had to repeat some units, thus receiving between 40 and 50 assessments approximately. Thus a subject with no preference would choose each of the four monitors about 10 to 12 times.)

Using the above criterion, 23 out of the 42 subjects were found to have a "marked preference".

Analysis of individual performances revealed that the subjects with "preference" tended to have had more unsatisfactory assessments (means 8.3 per cent showing "preference" and 5.5 per cent not showing preferences).

This correlation was quite marked, the "weakest" students showing the "strongest" preferences and the "ablest" students (no unsatisfactory assessments at all) choosing all 4 monitors with almost equal frequency.

*b) Were particular monitors more popular?*

The table II below shows the distribution of the 23 subjects who showed a "marked preference", among their "favourite" monitors.

TABLE II.

Monitor	Number of subjects exhibiting a marked preference
I	15
C	6
E	1
J	1
Total	23

These figures speak for themselves, showing monitors I and C as popular, monitor I being by far the most popular!

*c) Were some monitors more lenient?*

The Figure 1 shows the ratio of unsatisfactory assessments to total assessments made, session by session, for each monitor. Thus each of the four lines shown is a profile (in time) of the leniency or otherwise of the relevant monitor.

Of course the number of unsatisfactory assessments is also a function of the relative difficulties of units. However, this factor should result in undulations in the profiles which are more or less

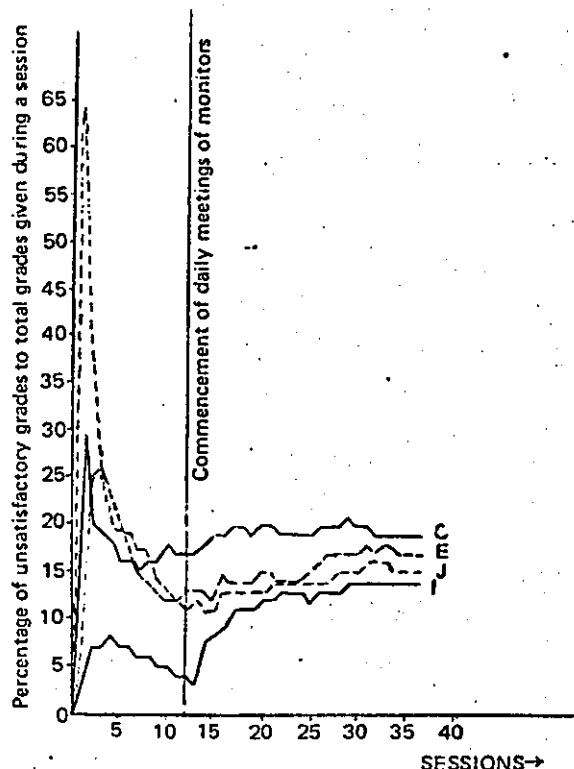


Figure 1. Variation in leniency of the monitors

"in phase" for all four monitors if they are marking with the same level of leniency.

Also, an analysis of the totals of unsatisfactory assessments, unit by unit, shows a "peak" on the first four units and then other peaks on units 15 to 17 and 25 to 30.

Comparison with results from other courses using the same materials but different monitors reveal the existence of the second two peaks in general (thus probably due mainly to the materials) but the general absence of the first peak (thus it was probably due to monitor severity).

(Note that the later two peaks do not show up strongly in Figure 1 because the units are studied at the individual pace of the student so there is no exact relation between unit number and session number.)

It is fairly safe to assume therefore that the differences in "failing rates" shown in Figure 1 are mainly due to differences in leniency/severity between monitors.

It is interesting to note the vertical line shown at session 12 (end of day 6 of course). This marks the implementation of regular (almost daily) meetings of the monitors (together with the course coordinator and a psychologist) in order to discuss

assessment problems and standardise assessment criteria.

The effect of these meetings is obvious; although the most lenient monitor (I) continued to be the most lenient, his performance climbs towards the more uniform performance of the other 3 monitors.

d) *Were the more lenient monitors more popular?*

A comparison of Table II and Figure 1 shows clearly that the most popular monitor (I) was also the most lenient.

However, the second most popular (C) was the monitor who was consistently the most strict, (apart from the exceptional peak during the first few sessions recorded by monitor E). This may probably be explained by the fact that monitor C was the only female monitor, when 41 out of 42 of the students were male.

*Thus to a limited extent, the first hypothesis, that students will tend to choose the more lenient monitor, is supported by the results.*

Another factor which emerged was that those students who did exhibit a preference for a given monitor tended to change monitors in the early units, but would soon settle down to one preferred monitor. Thus the monitor captures his "regular clientele" by his performance early on in the course, later changes in leniency having apparently less effect. This point is linked to the next consideration.

e) *Did subjects change monitors after an unsatisfactory assessment?*

This hypothesis was suggested by the postulate that subjects would tend to seek out positive reinforcement and avoid negative reinforcement (on the assumption that satisfactory and unsatisfactory assessments acted as such reinforcers). This hypothesis was not supported by the results. We have already seen that the "weaker" students tended to favour one monitor, that is they "changed horses" less frequently, whereas the "stronger" students seemed to be picking monitors more or less "at random".

In particular, the frequency of change, immediately after an unsatisfactory assessment (i.e. for re-assessment on the same unit) was analysed. A strong tendency to stay with the same monitor for re-assessment was identified.

*Thus students would in the main tend to seek re-assessment by the same monitor who had failed them on that unit, although they may well then change monitors for assessment on the next unit.*

A sub-group of 5 subjects were identified who exhibited the opposite behaviour – a tendency to always change monitors for re-assessment of repeated units. These five also exhibited a marked tendency for other simplifying (but not necessarily instructionally effective) strategies, such as copying other people's work.

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## Biographical Notes

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# The Tutor's Role in the Individualisation of Service Courses in Mathematics

(OR — Personalization in an Economic Crisis)

A. J. Romiszowski, A. C. Bajpai, P. E. Lewis

**Abstract:** Service courses are more amenable to the specification of precise, "job-oriented" objectives. In a way, the content of such courses acts as "tools" for further courses. Tools must be mastered if they are to be of use — hence the application of the "mastery learning" concept and one argument for individualisation.

But should service courses be solely "mastery" orientated? Should not those students with the desire and ability to extend their studies beyond the "necessary and sufficient" be given the opportunity to do so? This aim can be achieved in individualised service courses, but only if they are individualised in certain specific ways.

The role of the tutor is a key factor to consider.

The paper discusses the roles of the tutor/monitor in personalised courses, from the viewpoint of the "necessary and sufficient" or 'service' objectives, and also the viewpoint of "course enrichment" objectives.

Two ongoing personalised maths courses are described, illustrating how they work, how they have attempted to achieve both types of objectives and how they have adapted to economic constraints which render individualisation of learning somewhat less viable at the present moment.

## 1. Individualisation of "Service Mathematics"

### Service Mathematics

By "service mathematics" we mean the mathematics content of a course which is not primarily a course in mathematics but which uses certain specific mathematical concepts and techniques as "tools" to achieve the course objectives. The mathematical element in the course may be relatively restricted, for example the algebra and statistics concepts required by a sociology student, through the more extensive range of concepts and skills required by a management student, to the major part it plays in, say an engineering course.

The more restricted and marginal the need for maths, the greater is the difficulty in effectively teaching this element of the course.

- (a) problem of motivation due to difficulties in establishing relevance to student objectives;
- (b) problems of the course staff themselves not prepared to teach the content, leading to the frequent farming-off of the mathematics teaching to the mathematics department (b accentuates a)
- (c) a "natural selection" process often occurs, whereby some students on the course have elected to study this course especially be-

cause they were weak in, or averse to, mathematics in earlier schooling.

As McKean, Newman and Purtle (1974) observed when teaching service mathematics sequences for the social and biological sciences;

"To begin with, few of the students taking the sequence really like mathematics: they were taking the sequence only because of a college or departmental requirement. Because the student attitude is relatively negative and mathematical aptitudes largely minimal, most faculty members avoid teaching the sequence like the plague. When "trapped" into teaching such a course the presentation was often sterile.

The result was to be expected — the students actively avoided as much work as possible. Performance levels were marginal. This resulted in grading on a curve where 40 to 50% correct might well mean a passing grade. We understand that the psychometricians believe such a curve to be desirable, but in view of the fact that we had tried to construct our tests to cover the material *essential* to the students future needs, the curve is not appropriate."

In addition to the problems of teaching, we therefore are faced with the problem of testing and grading. If we define service-mathematics as the "tools of the job" then the position adopted by

McKean et al in connection with grading is very appropriate. Tools must be usable, and this implies a high level of mastery of their use.

The "Mastery Learning" model, as described by Bloom (1968) and others, appears appropriate here. However, few university teachers accept this model as of universal relevance to all course content in higher education. Rather they tend to adopt the position that students should be allowed, indeed encouraged, to follow up those subjects which interest them most to the limits of their abilities.

### The Objectives of a Service Course

Now here's the rub. The social science, management or engineering faculty may subscribe to the achievement of a precisely defined set of mathematical objectives. The teacher involved may subscribe to the "mathematical development" of the students. The students themselves may hold objectives which range from a deep interest in all things mathematical, to a fervent desire to get the whole course unit behind them just as fast and as painlessly as possible.

Are these objectives incompatible? We think not.

Firstly, the "faculty" and "teacher" objectives are a question of joint analysis and planning. Clearly the teacher is employed in this "service" context to achieve the objectives of the faculty. If he can do this and still leave time to take students further into the wonders of mathematics, then by all means he should attempt to do so. Indeed, even in service courses, one should support the principle of planning sufficient time and other resources to enable this to happen for those students who desire it.

But what happens in practice? All too often there is a lack of co-ordination between the servicing unit and the faculty being serviced. Planning stops at the stage of identifying major topics to be covered (calculus, differential equations . . .) and the allocation of times and dates. Faculty do not specify the objectives (the operations which the mathematical tools will be used for) very precisely. They cannot therefore, be surprised if the maths teachers put their own interpretations and also their favourite bits of the subject into the course. Courses do not demonstrate clearly the relevance of the "tools" to the job as perceived by the student. Interest and motivation suffer despite all the efforts of the teacher. This is in nobody's interest.

Rather, one should aim at jointly defining the "necessary and sufficient" learning objectives with great precision. Then one should plan resources in such a way as to enable all (or as many as possible) of the students to achieve these, whilst still

allowing the faster, more ably and more interested students to explore more widely.

This aim implies some element of individualisation in the learning process.

Secondly, the varied objectives, both cognitive and affective, generally encountered in groups of students on a service course, coupled to the possible variety of levels of entry behaviour, also imply that an individualised approach should be used.

Furthermore there are at least two levels at which individualisation should be planned.

### Levels of Individualisation

In the introductory paper to this special issue, Romiszowski summarised some of the elements of learning that could be (and sometimes are) individualised.

In particular, Edling's (1970) classification, repeated below was discussed.

Learning rate, methods, media	Objectives	
	Course determined	Learner determined
System determined	A	B
Learner determined	C	D

Where should a mathematics service course (at higher education level) be ideally placed in this matrix?

Well, the answer depends on the *level* within the course, which we are considering.

Looking at the service sequence as a whole, it forms an integral part of other course options. Thus even in a course which has credit options, the students choice is limited by other factors. If for example he chooses the "research models" option in a social science course, he *must* choose the necessary statistics service section. The student may exercise his choice by not selecting the research option, perhaps mainly because he is unsure of coping with statistics. However, this is not really what we mean by individualisation in this context.

Having once chosen his options, however, the content of the main options determine (or should do) the "necessary and sufficient" objectives for the service sequence. It is thus illogical to consider that the student has a choice among the "basic" objectives of the service sequence. He could, however, be given the option of "additional related" objectives and he might indeed suggest some himself.

Finally, as students vary in entry behaviour, rate of learning, etc. the learning methods must at least be individualised in terms of the time factor. It may well be a good idea to offer choices of learning method as well, bearing in mind the diverse experiences in the learning of mathematics that our students inevitably bring from school.

Thus we see two blocks of activities in our "ideal" service course. The activities related to the "necessary and sufficient" conditions as fixed by the exigencies of the main option. We would find these in the A or C boxes of our matrix. Then we have the enrichment element for those who want it and are ready for it. Thus these activities would be, at least in part, in the B or D boxes.

Considering rate, methods and media separately, we might visualise our ideal course as follows:

Learning	Objectives	
	Necessary & sufficient objectives	Enrichment objectives
Rate	C	D
Methods	A/C (choice by learner, and guidance by tutor)	B/D (tutor adapts to learner within limits)
Media	A/C (e.g. resource-based learning)	B (the tutor)

We shall leave this without much comment, merely as an illustration of the possible decisions on individualisation which might be taken when planning a mathematics course. Readers may disagree with certain decisions, due to different positions they may adopt in respect of the philosophy of such courses.

### The Roles of the Tutor

However, the one point which emerges strongly in the suggested scheme above, is the need for both individualised learning materials and able tutors.

We use the word "tutors" intentionally, rather than teachers, monitors or proctors. In the study of McKean et al (1974) mentioned above, a personalised system (PSI) based on the Keller Plan was installed. They used student proctors and report excellent results provided that the proctors are carefully trained and supervised. Such findings are also supported by Elton (1973) and others.

However, it would not be possible to expect student proctors to take students beyond the necessary and sufficient objectives covered by the basic self-instructional materials. If this is desired, a mathematics teacher is required, and, due to the

individualised nature of the proposed course, both in terms of learning rate and methods, his role must be that of an individual tutor.

Of course, it may be possible to arrange in a classic Keller plan course, that student proctors service the students working on the basic course materials, and the course professor, perhaps with assistants takes over the brighter and more advanced students in order to enlarge on certain topics.

As in typical service course the number of students able and willing to proceed beyond the minimum requirements is usually limited, such an arrangement could make the best possible use of available human resources, both students and staff.

### Economic Considerations

One common reason for opting to use student proctors in the economic factor. There are just not enough teachers on the course to staff a P.S.I. scheme effectively. Another economic factor is the argument against using expensive and skilled teaching staff for clerical and supervision tasks – often the case in PSI courses.

In the U.K. recently, the economic climate generally, and particularly the cuts in educational spending, have made us consider more carefully the cost/effectiveness of courses. In efforts to continue to expand the availability of individualised courses, whilst keeping within cost limits, certain reorganisations of existing courses may need to be effected.

One development may be the increased use of student proctors. However, in courses which aim to have an enrichment content as well as a "service" content there are limits to this, as indicated above. An alternative to consider is the more efficient use of the time of existing staff.

### The Two Schemes

The two schemes described below are similar in several respects, but differ in others.

They are similar in that they both are teaching mathematics as a service subject, to university undergraduates. Also they have both had to face up to the current economic climate and attempt to make savings on staff. Thirdly, they both use teacher/tutors rather than student/proctors as a "personalising" element of the total learning system. Fourthly they are both concerned with enriching the good student's mathematical experiences, whilst bringing as many students as possible to full mastery of the basic "service" objectives. Fifthly (and this is perhaps of great importance)

they are both based on carefully designed and validated programme instruction modules, which have been shown to teach reasonably effectively with minimum, or no, tutorial backup. Now the differences:

(a) *The scheme at the University of Loughborough* is a service course in Engineering Mathematics.

- The students involved are second year Electrical Engineering undergraduates.
- The content of the courses given so far are (1) Fourier Series, Partial Differential Equations and (2) Complex Variable Theory.
- The learning materials are sections from the programmed textbook "Mathematics for Engineers and Scientists – Volume 2, by Bajpai, Calus and Fairley, plus special supplementary materials.
- The courses have been given only in the last year and to only a part of the total group – about 40 out of 100.
- They were set up deliberately as personalised courses on the Keller plan, utilising the normal timetable hours.

(b) *The scheme at the Middlesex Polytechnic* is a service, or remedial course in pre-requisite mathematical skills.

- The students are first year social science and business studies undergraduates.
- The course content is in 22 modules, covering basic arithmetic skills, powers, series algebra etc., as required for entry into other service courses on statistics, calculus etc.
- The learning materials are mainly in the form of programmed instruction modules, commercially produced but modified by us. They are administered on teaching machines. In addition there are many programmed and non-programmed booklets, prepared by polytechnic maths staff, supplementing the programmes or giving alternative faster/slower treatments of the same content.
- The course has been given as a standard (obligatory for social science) unit for the last eight years, during the first four weeks (or longer if necessary) of the academic year.
- The course was not constructed to the Keller plan. It was originally a computer-managed "learning by appointment" plan, open to all students at any hour on a come-as-you-please basis. Over the years, however, with the shedding of the computer and other changes, it has

grown to appear more similar to PSI, whilst still maintaining the flexible timetabling.

## 2. The Keller Plan at Loughborough University

### Introduction

One problem with self-study courses of the Keller type is that they can require the presence of a large number of academic staff, postgraduate students, etc. as tutors. Elton suggests that one tutor should be responsible for about ten students and Bridge (1973) mentions the presence of five or six tutors, as well as undergraduate tutors, at "test sessions". Our first attempt at Loughborough at running a self-study course in Mathematics for Engineers involved twenty student volunteers with one member of academic staff and one assistant to act as tutors. Our experience tended to confirm the previous estimates and we felt that this was the largest number of students that could reasonably be coped with without additional tutors. Even then there were some difficulties with students waiting for tests to be marked, and for help with problems. It must be admitted too that the tutors felt that their time could be better spent than in repetitive marking even though the marking sessions did often produce valuable discussion with individual students. (One student, for example, stated that this first self-study course was the first occasion on which he had been treated as an individual, as opposed to one of the herd, in two years at University. He attributed this to the one-to-one tutor-student situation.)

However, at a time of financial stringency we found it difficult to recruit additional tutors, and yet both we and the students were sufficiently enthusiastic about self-study courses to want to extend them to a larger proportion of the total class of over one hundred students. Experiments elsewhere have utilised faster students to act as tutors to slower ones but most of our students – despite encouragement – did not appear very keen to do this.

Accordingly, for our second self-study course we have introduced a scheme whereby the students mark their own tests. Thirty-five students – all volunteers – have studied part of their Mathematics using this approach and the same two tutors were involved.

### Running the Scheme

The students followed a self-study course on the traditional Keller lines, apart from the self-marking innovation. We prepared the standard material – lists of objectives for each Unit, a procedure for them to follow, an introduction and commentary designed to put the mathematics into the context

of their Engineering studies, a graded series of practice examples, and so on. Two – or sometimes three – tests were developed for each Unit and detailed marking schedules prepared for each. The schedules consisted of model solutions, allocation of marks and pass-mark. (We normally use 75% for the latter.)

These solutions were available for students to use after they had attempted tests. If this was during the "official" tutorial period, a tutor was always present to assist in cases of difficulty. If – as often happened – students tried tests at other times during the week, the solutions were available for them but not necessarily the tutorial assistance. In either case the students marked their own tests and had to hand them back, together with the test paper and the marking schedule. The tests were looked at *briefly* later – often several weeks later by the tutors but the onus of passing or failing themselves, as well as the detailed marking rested on the students. The original idea of a tutor check was simply to guard against excessive procrastination.

The views of the students were obtained by discussion and questionnaire, particular interest centering on students who had experience of both the self-study courses – the one tutor-marked and the other self-marked. Student response and our general impressions of the method are discussed below. We concentrate mainly on the self-marking aspect since many discussions on other aspects of self-study have appeared elsewhere.

#### Advantages of Self-Marking

The main advantage – according to the majority of the students – was the far greater smoothness with which the scheme proceeded after self-marking was introduced. There was no queuing in tutorial periods, every minute of which could be utilised efficiently – an important consideration for students with very heavy timetables. Also, the students were able to obtain immediate feedback on their performance – a powerful reinforcement and a useful motivator in a service subject. They were still able to discuss difficulties with tutors – indeed they had better opportunities to do so since the tutors enjoyed much greater freedom with no routine marking. For these students – a substantial minority – who prefer not to attend tutorial periods but to do tests at other times in the week there was the advantage that they could be sure of completing and marking a test regardless of whether or not a tutor happened to be available.

From a staff point of view – apart from our natural antipathy to repetitive marking – we do feel that our time is better spent in helping students with particular difficulties and stretching

good students who have coped with the routine tests to their satisfaction. Above all, we feel we have been able to extend self-study courses to a larger number of students without use of additional staff. We suggest that, with suitable administrative arrangements, two competent tutors should be able to deal with about fifty students. Since the normal University arrangements allow for one tutorial of twenty students for one hour per week it is reasonable to claim that self-study courses, with self-marking as a feature, are at least as efficient in utilisation of staff as lecture/tutorial methods. The former take longer to prepare initially and we must emphasise particularly that the preparation of adequate marking schedules for the students is vital to the success of our approach and that this can be a time-consuming process. Assuming, however, that the material can be used with only minor changes for several years, then this initial effort can be justified both on economic grounds and certainly on the beneficial effects on the students.

#### Disadvantages of Self-Marking

The main disadvantage seen by students – and this was only mentioned by a minority – was a worry that although they might pass themselves in a test an outside marker might not be so lenient. They were particularly concerned that although they "knew what they meant" by what they had written, and could mark themselves accordingly, they might not be expressing their arguments clearly enough.

Another difficulty which arose occasionally was when a student tackled a problem by a quite different method from the "expected solution". This simply emphasises the importance of tutors being available to discuss answers. A further possibility is the inclusion of these alternative methods as part of the marking schedule for later students. It is usually beneficial for all students to realise that there are alternative ways of tackling problems.

#### General Discussion

In some ways the self-marking scheme falls short of the ideal. One of the major advantages claimed for self-study courses is the improved staff-student contact and, inevitably perhaps, a certain degree of impersonality tends to creep in when the student is free, if he wants to, to complete the whole scheme without even speaking to a tutor. In fact, though most of our particular group of students did not seem to be too worried by this. To them the main advantages of self-study were being able to work as and when they chose (within the deadlines that we imposed), perhaps to complete the course very much more quickly than under a lecture system,

and to be tested on their understanding as they proceeded, thus enabling them to identify their difficulties clearly. Other students considered that the incentive value of the tests was their main feature and consequently that the main advantages were not lost by the self-marking.

We have not experienced much difficulty with student procrastination. Perhaps we have been particularly fortunate in having sensible students, but most of them have realised that as the tests do not, under our system, count towards any final assessment then any serious cheating is really only going to harm themselves. This is not to say that some students are not perhaps overgenerous towards themselves. However, we feel that any sacrifice in the rigour of the system has to be weighed against the other considerations we have discussed, and against the general enthusiasm most of the students have shown towards a subject to which engineers are traditionally hostile. As stated earlier, we took the precaution of reading quickly through the student's work at a later stage but we feel that this, though possibly desirable, is by no means essential and if carried out too seriously would involve an intolerable amount of repetitive work which would defeat the whole object. It might be useful for tutors to re-mark a random sample of the students' work as there is evidence that students take more care with writing out answers and with marking them if they know that someone else will see them, particularly if that someone will ultimately have to assess them!

The other obvious method of extending Keller-type courses to larger classes is, as mentioned earlier, to employ faster students to mark the work of slower ones. This has been done elsewhere and has some advantages, e.g. it encourages discussion among students and is particularly beneficial to those who do the marking. However, there are likely to be some personality difficulties and we have found that busy students are not too inclined to want to spend time discussing work they themselves may have completed some time previously. Perhaps this is just a sign of the times!

### Conclusion

We have indicated the possibility of including a self-marking element into self-study courses of the Keller type. Although not an ideal system, this seems a reasonable way of extending such courses to an increased number of students at a time of economic crisis. Most of our students preferred the "smoothness" of the self-marking system to the queuing sometimes involved in the "tutor-marking" system. Most of them preferred self-study courses to lecture-based courses, though most preferred – as we do – a variety of teaching and learning

methods. An important criterion for the success of the self-marking approach seems to be the availability of detailed marking schedules as it must be remembered that students are very inexperienced at assessing their own work critically. The other essential element is the ready availability of tutors and the individual attention accorded to students, particularly those in difficulties. This, surely, is what University teaching should involve, and we believe that the possibilities for this are optimised by our scheme.

### 3. The Remedial Maths Course at Middlesex Polytechnic

#### Introduction

For eight years now, the first-year students on the B.A. Social Science and B.A. Business Studies degree courses at the Middlesex Polytechnic, Enfield site (formerly Enfield College of Technology), have taken a remedial mathematics course. This course forms part of the service mathematics content of these degrees. There are also specialist lecture and seminar courses on statistics, methods and models, business mathematics, and so on. The objective of the remedial mathematics course is to review the mathematics pre-requisites for these various specialist options. As entry to the degree courses is very varied, including many mature students, and since there is no required mathematics entry qualification (at least on the social science degree), the standards at the beginning of the academic year vary wildly. Thus a system of diagnostic tests is used, together with individualised self-study materials (some branching programmes on teaching machines, some linear programmes and some non-programmed booklets) and back-up by individual tutorials if necessary. Students study on a "learning by appointment" system at any convenient time of the day. They are expected to complete all necessary remedial work during the month of October, though some stragglers continue to work on the course during November and early December.

The rationale, structure and initial success of this course has been reported in full elsewhere (Hamer, J. W. and Romiszowski, A. J., 1969). The following notes discuss some aspects of the implementation and running of the scheme, highlighting some of the problems encountered, and describing the changing role of the tutor, in our search for improved efficiency and cost/effectiveness.

#### Running the Scheme

##### (a) *Original Experimental System – The Computer Managed System (1968–69)*

The remedial maths course was first offered in October 1968.



Due to the varied student population, an individualised approach was used. Students only studied those modules which they required. Two diagnostic tests were constructed, each covering eleven modules of the course materials. Students took the first of these tests on their first day on the course, and the second whenever they had completed any necessary study of the first eleven modules.

As it was expected that up to 150 students would sit these tests (in fact over 200 did), the anticipated marking and assessing problems were likely to be considerable. It was mainly for this reason that the computer was first considered. However, once the computer was integrated into the system, it was possible to make use of it for other purposes, e.g. time-tabling, evaluation and progress chasing.

The computer became the main control medium for the course. The teaching media were mainly branching teaching machines. However, the tutor had very definite roles to play in the system. He again had 3 roles. (1) Monitoring the computer's prescriptions. For example, if a student who makes some silly mistakes can, without any tuition, satisfactorily correct his work and explain his errors, then the tutor would allow him to skip certain prescribed modules.

(2) Marking and assessing post-tests. The tutor could do this more flexibly than the present batch-processing computer system, and could discuss errors and give tuition immediately.

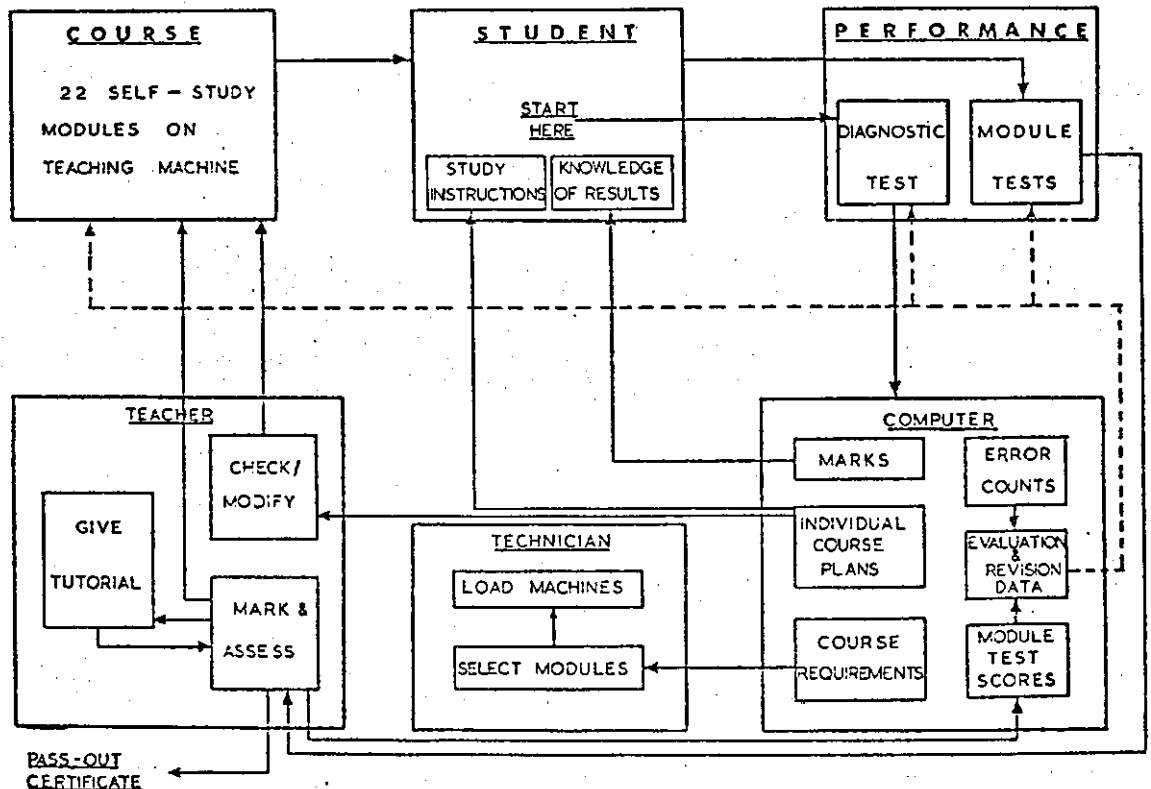
(3) Extra tuition where required.

To allow for any possible difficulties, we time-tabled two tutors to be available whenever machines were in use. This proved to be much too generous. The tutors were trained in operating the system, and were expected to follow a check-list of activities when dealing with a given student.

In all, eighteen staff members were involved for a total of 190 hours during four weeks. Of these, 147 were for the Social Science Course, and 43 for business studies. An analysis of the work performed by the Social Science staff gives

(a)	Machine loading etc	30 hours
(b)	Testing, marking, admin., record keeping	56 hours
(c)	Tutorials, discussions	34 hours
(d)	Time unaccounted for, (including machine maintenance)	27 hours
		<b>147 hours</b>

Fig 1. Original 1968-69 System for Remedial Maths Course



Categories (a), (d) and the bulk of (b) are routine technician grade of work. By assigning one full time technician to the course, teaching staff time could be cut considerably. This was done in the later stages of the course and became a regular part of the system in the second year (see Figure 1).

The course was repeated in 1969, using the same organisational structure, as shown in Figure 1, but of course with improved diagnostic tests and teaching materials, as a result of the data we had collected in 1968. The materials were further revised in 1969 as a result of the second year's experience, and the course was handed over to the mathematics staff.

#### *(b) The Teacher-Managed System (1970-73)*

In the years that followed, certain aspects of the original scheme became unworkable, and the teaching content also evolved. So did the system of management and the role of the tutors.

*Firstly*, the "computer-management" element was discarded. It had been hoped to add flexibility to the course by the use of on-line computer-marked testing. This would have enabled students to take their module tests at any time, and module-by-module rather than the rather large tests they use, covering 11 modules each. The scheme was developed and used experimentally, but unfortunately the new time-sharing computer we expected did not materialise.

In the event, the batch processing of student data, whilst very useful during the first two years for course-revision purposes, was not a great time-saver when used simply for the marking and assessment of up to 200 scripts, particularly as these had to be marked anyway by a punctual girl using a simple template. In any case, most tutors expressed the desire to mark the tests themselves, if possible in the presence of the student, in order to discuss any problems immediately.

*Secondly*, therefore, the role of the tutor changed. The amount of machine-loading and record-keeping was drastically reduced, though not eliminated. Record-keeping was simplified to the minimum. A technician was available at the beginning of each day and at intervals thereafter, and the students were encouraged to load and set their own machines.

The course was staffed on a rota system, 40 hours per week during October and less thereafter.

Tutors adopted a more flexible, less rigidly controlled style of working. This brought the advantages that some tutors became much more involved with the success of the course, and also extended the aid they gave to students beyond the rigidly

defined objectives of the course. Tutorials would often follow-up certain aspects of particular relevance in depth, would suggest further reading or other activities. Unfortunately, the less rigid control over the course administration, coupled with the flexible hours (which meant that tutors were sometimes too busy and sometimes were on duty but no students turned up) led to the disadvantage that some tutors became less involved with the course, irregular in their attendance and superficial in the tutorial aid which they gave to students.

*Thirdly*, therefore, the management of the course changed to eliminate this problem. As far as possible, those tutors who exhibited above-average interest in the course were encouraged to give more hours, and one member of the mathematics staff was assigned full-time to the course. In addition to tutoring he organised the management system, supervised the tutorial system, identified new mathematics needs not covered by existing modules, instigated the production of self-study booklets for these, and generally streamlined and extended the service offered by the remedial mathematics unit from simply "course pre-requisites" to "service assistance during the course".

#### *(c) The Secretary-Managed System (1974-75)*

All good things have to come to an end, and with the advent of economic stringencies, staff and promotion and other changes, it became impossible to maintain the same team of course tutors from year to year, nor to have a full-time staff member involved, and the total number of tutorial hours had, if possible, to be reduced.

This was possible, without serious effect on the quality of the tutorial back-up of the course, by the transfer of all non-teaching duties from the tutors to a secretary.

The day-to-day course running is entirely managed by a secretary, who, although not using all her time on this job, is located permanently in the remedial mathematics room, and is available at any time for the distribution, collection and marking of tests, distribution of programmes, loading of machines, re-testing, and arranging individual tutorials when required. The tutors no longer attend the remedial maths room, but are available by telephone appointment in their offices at appointed hours. The total tutorial hours now timetabled are 20 per week. Tutor-utilisation is now much higher. There are occasional queuing problems, when students have to wait a day or two for an appointment, but these are not severe. On the other hand, the problem of tutors arriving for a session "just in case" a student happens to turn up, has been eliminated.

Some of the "immediacy" of instant tutorials has been lost. On the other hand, the continuity and permanence of the service offered by the secretary avoids earlier problems and frustrations encountered when a tutor just did not turn up for a session.

Tutors still encourage wider study, and the use of optional booklets and modules on advanced topics is on the increase.

The secretary is able to run a similar "remedial English for overseas students" scheme and performs other tasks when the demand from students is slack. Both schemes are eventually to be transferred into the proposed media services section of the library, when they will be open to use by a wider audience.

#### Discussion

The advantages and disadvantages of each system of running the scheme, have been indicated above.

Some general observations will help to put these comments in perspective.

During the eight years and throughout all the changes, the level of 85% mastery has been required from and has been achieved by the students on each module. As the course was refined annually in the light of feedback received, both the study time on most modules, but especially the need for tutorial back-up have been progressively reduced. This has left more time for "enrichment" tutorials. This point, in addition to the gradual elimination of clerical and technician tasks from the role of the academic staff, has improved the staff/student hours ratio considerably.

In 1968 staff/student hours were 1:5. In 1975 the ratio was 1:10 (academic staff only). These figures are the ratio of the total number of academic staff hours given in the first 4 weeks, to the total hours put in by students in the first 4 weeks.

However, looking at the system in terms of full-time-staff-equivalent, in 1975, the equivalent of just over one full-time teacher (plus about half a secretary) for 1 month, serviced the remedial maths needs of 221 students (of whom about half needed some tutoring or counselling) on a totally individual basis.

As a summary of the trends, a few figures taken from the three phases of the course's history are given below. "Weak students" are those who required remedial work on at least two of the 11 modules in either test.

Year	1968	1971	1975
Test 1			
Students completing	200	220	221
Weak students	98	100	94
Test 2			
Students completing	200	145	142
Weak students	102	82	62
Total self-study hours (on programmes) in first 4 weeks	787	686	490
Hours of supplementary tutorials given in first 4 weeks	34	73	52
Hours of Academic Staff theoretically available during the 4 weeks	190	160	80

Note that, although student numbers (on Test 1) have remained reasonably constant,

- The drop-out rate (Test 1 to Test 2) has varied. In 1968, participation was compulsory for both Business Studies and Social Science students. This remained the rule only for Social Science students in following years.
- The time spent on self-study materials has decreased – partly due to improved efficiency of the materials and partly due to increased involvement of tutors in the decisions of what work would be most valuable for a given student.
- As a result, the number of tutorial hours has increased. The peak in 1971 corresponds to the presence of a particularly enthusiastic, full-time tutor responsible for the running of the course. Many tutorial hours are now extension and/or enrichment, over and above the basic skills taught in the self-study modules.
- However, due to the re-organisation of the administration tasks involved in the course, the timetabled hours of mathematics staff have been more than halved and furthermore as tutorials are run by telephone appointment in the tutor's study, any timetabled hours not utilised by students are not lost to the tutor – he may get on with other work.

#### Conclusion

Over the eight years of experience with the course it has been observed that the most critical elements in the system are the human elements.

Individual groups in individual years have partaken

and profited from the course in relation to the importance attached to this by their course heads. Notably, in Social Science, active progress-chasing by the course heads and some tutors, avoided the drop-out rates observed with other groups.

In the day-to-day running of the course, the most positive comments made by students centred around the "personal and flexible" nature of the course - "we get more individual attention than in any other part of the degree course". Also, the strongest criticisms occurred when occasionally students could not get hold of a tutor when required, or a tutor did not turn up.

The tutor's role, as originally planned, was found to be not very motivational for most tutors (particularly the technical and administrative aspects).

The present system, which ensures the smooth running of these aspects as much as is possible, and which ensures regular hours of service, appears more efficient to students and more acceptable to staff. It is hoped that the slight loss of immediate personal contact between tutor and student will not be too big a price to pay for efficiency.

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# Computer-Generated Examinations and Tests

A J Romiszowski (with contributions from G Lippey, IBM Corporation, and F Toggenburger, Los Angeles City Unified School District)

## Introduction

Applications of the computer to instruction vary from the sophisticated computer-based self-instructional systems, such as PLATO, through computer-managed systems such as PLAN, to computer 'back-up' systems for the teacher, usually taking the form of some automatic scoring, ranking, or diagnostic systems. Examples of these latter systems abound. One can mention two UK examples of the 1960s — the computer-marked and validated programmed instruction systems used for remedial mathematics teaching at the then Enfield College of Technology and for a university course in structures at the University of Aston. These latter systems used the computer purely in a data-processing role and could be classified together with other applications of the computer to the replacing of routine clerical functions, such as school record-keeping, processing of psychological tests, even salary payments. The compilation of tests, by computer, from a previously prepared and stored bank of suitable test items, is yet one more such routine application. Yet, it is one which has become common only in recent years, despite the relative simplicity and cheapness of its implementation and its potential usefulness to teachers.

## Computer Applications in Instructional Systems

The diagram below illustrates the continuum of computer applications to instructional systems and suggests how responsibility for the management of the system rests at one extreme largely with the instructor, whilst at the other extreme most of the key functions of the instructor have been delegated to the computer.

An example of a fully computer-based system is PLATO IV. The student interacts with the computer, which is the store of the instructional material, the presentation device, the evaluator of progress and the decision-maker concerning the rate and content of instruction.

Perhaps one of the most developed computer-managed systems is Westinghouse's project PLAN, in use in many areas of the USA and also in some developing countries. In this system, the teacher teaches by more or less traditional methods, with considerable media support, selecting alternative optional lesson plans and materials for the individual student's needs. Students complete post-tests for a unit on mark-sense response cards, which are 'posted' into a reader in the classroom. Thus there is direct student-computer interaction at the test stage. The functions of the computer are to mark, assess, grade and comment on the individual student's tests, and also to compare his progress with that of other typical students in the past. In doing this the computer has access to stored information on

		← A Continuum of Delegation →		
		Computer-based Instructional Systems	Computer-managed Instructional Systems	Computer Support to Instructional Systems
Examples:		'Plato IV'	Project 'Plan'	Aston
			Stanford 'Drill + Practice'	Enfield
Key Instructional Functions	Diagnose Instructional Needs:	Computer	Computer	Instructor with computer support
	Select Content, Methods, Media:	Computer	Instructor with computer support	
	Instruct:	Computer	Instructor and other media	Instructor and other media
	Test:	Computer	Standard tests and computer	Instructor with computer support

previous success with the present material by other students, previous performance on other material (by this student and other students), and a variety of personality and aptitude test data. A diagnosis and prescription for further work is prepared, including a suggestion as to which of the available optional media is likely to be of most value to the student. The teacher can also call up other services, in order to aid in the revision of the

An intermediate example, between PLATO and PLAN, is Patrick Suppes's 'Drill and Practice' arithmetic system. This system gives practice in routine arithmetic procedures, once the basic techniques have been taught (possibly by traditional classroom instruction). The student is presented with an exercise by the computer. His response is used to select the next exercise — either more complex or more simple, depending on his success. The basic instructional materials are not stored in the computer, but a collection of carefully graded test questions is stored, and it is the computer which selects the questions for the student, on an individualized basis.

At the other extreme of our continuum, we have systems such as the one at Aston University, which used the computer simply as a calculating machine. Students take normal, objective paper-and-pencil tests after self-study sessions. These tests are marked by the students (or at certain key points in the course, by the teacher) and the raw results are fed into the computer. The computer produces profiles of progress for individual students and profiles of effectiveness for individual study units. It is entirely up to the teacher, however, to interpret these profiles and to plan remedial actions.

The system in use at Enfield College of Technology (now Middlesex Polytechnic) was intermediary in that it used paper-and-pencil tests, marked by a scoring key, but the raw results were processed by the computer to give not only student profiles, but also individual student prescriptions as in project PLAN. However, the Enfield system used a batch-processing computer to assess a lengthy test and produce a prescription for up to two weeks' work at a time. This lack of sensitivity was one of its weaknesses and led to the abandonment of the computer-generated prescriptions in favour of a tutor-managed system.

The above mentioned projects illustrate quite well the main preoccupations of workers on the computerization of instruction. First and foremost, they have attempted to computerize the decision-making process of the teacher. Secondly, they have attempted to use the computer as the medium of communication with the student, either at the testing stage or for both testing and instruction. It is not surprising that these have been the main areas of research. The first one is obviously of great interest to programmers, systems analysts and students of human decision-making alike. Work has resulted in the development of simple-to-use programming languages for conversational interaction with the computer, and in insights into the problem-solving and decision-making processes in the human brain.

The second has led to developments in man/machine interface hardware, and in some instances to a level of adaptiveness in teaching which was not hitherto possible.

However, progress on the first has necessarily been slow, and implementations have sometimes met with strong opposition from teachers. The second as yet incurs a very high cost, due to the large amount of computer time and large number of terminals necessary to instal a sizeable computer-based instructional system. These reasons have combined to result in the relatively slow growth of CAI systems application on any large scale.

The practical needs of existing educational systems might have been served better, and applications of computers in education might have spread faster, if more emphasis had been placed earlier on developing support systems for our existing teachers — support in the routine functions which teachers must perform, but which are time-consuming and therefore often neglected.

### Computer-Generated Tests

The area of routine teacher activity which has been subjected to computer support for some time, is the marking of tests and the processing of test data. In recent years, attention has been given also to the construction of tests by computer. Much of the pioneer work was performed by Gerald Lippey of the IBM Corporation and Frank Toggenburger, coordinator of the Educational Systems and Programming Branch of the Los Angeles City Unified School District. Their work has led to the development of a 'classroom teacher support system' (CTSS). This system attempts to fulfil the following requirements of today's practising teachers.

Most important, it must be economically feasible. It should not, therefore, depend on communications-oriented equipment; while a conversational or remote job-entry implementation might be desirable, it should be able to

provide adequate service using a messenger or the internal mail. It should accommodate to the existing classroom and school environments; for example, it should not require extensive modification of existing teaching strategies or school logistic patterns. It should, therefore, be possible to introduce the teacher service to, or remove it from, a classroom or school without disruption. It also should be possible to maintain the service for any single school or class at very low usage; there should be no minimum threshold for usage insofar as a single teacher is concerned. Finally, it is essential that the service be optional; at any time, the teacher should be the sole judge of whether or not to take advantage of it.

Lippey and Toggenburger identify questions as one of the teacher's key tools. In addition to their traditional function for grading, questions are frequently used to stimulate student interest, to guide the learner through a sequence, to guide the teacher (behavioural objectives), to diagnose student weakness, to diagnose instructional weakness, to measure achievement, and to practise new skills and concepts. Convenient access to a variety of questions appears to be a need common to all teachers.

Published tests provide much useful information, but do not satisfy the teacher's needs for flexibility and relevancy to immediate classroom goals. He usually desires to use questions frequently and for the many purposes mentioned above. He wishes to set for himself the objectives of the questions, the frequency of their use, and their content. However, he is understandably impatient with the clerical duties that go along with obtaining and scoring his own questions.

A centralized collection of questions could be maintained in machine-readable and machine-retrievable form. Questions could be classified, and selected according to criteria specified by the teacher. This service would:

- (a) Allow many teachers to share a collection of questions. Thus, they all benefit from the advantages of specialized contributions.
- (b) Operate completely under teacher control, supporting the teacher rather than imposing structure.
- (c) Enable the quality of questions to be improved by centralized collection of feedback from teachers and students with subsequent question revision.
- (d) Reduce clerical work associated with preparing exercises and tests.
- (e) Provide scoring statistics that are seldom produced by teachers.
- (f) Operate at low cost.

A preliminary investigation of this concept took place in 1968 at IBM's Advanced Systems Development Division. Near the end of 1968, personnel from the Los Angeles City Unified School District and IBM discussed the possibility of embarking on a joint study to build and test a prototype version. In 1969 a working agreement was reached between the District and IBM's Systems Development Division. System functions were to be specified jointly. IBM would develop the computer programmes, while the District would prepare a collection of questions and take responsibility for all operational aspects.

### **The Classroom Teacher Support System**

Frank Toggenburger describes the CTSS as follows:

CTSS was field-tested in one senior high school early in 1970 and was made available to summer-school teachers the following summer. Since 1971 it has been operating in all of the district's 49 senior high schools and in 30 of the 75 junior high schools, serving about 300 teachers of 'United States History'. This subject was selected in preference to others because it was felt that the subject was not as well-structured or objective as other areas, such as maths or science. Successful use in this discipline would tend to assure success in most other subject areas.

The item collection was produced in the summer of 1969 by 20 experienced teachers and 3 clerks under the direction of a curriculum specialist. This group produced approximately 8,000 items that covered the 8 major units of the revised course of study.

The services which CTSS makes available to teachers are:

- (a) Items are retrieved according to criteria set by the teacher. Items stored in the system are classified among several dimensions. The items selected are listed as an 'exercise'.
- (b) The teacher may specify and receive a modification to an existing exercise. Items can be deleted or new items can be added to produce a new 'generation' of the exercise. This process may be continued until the list of questions meets the teacher's needs.
- (c) An exercise may be printed on reproduction masters for use with a spirit duplicator when desired.
- (d) Upon request, the system will create several copies (or 'versions') of an exercise with the items scrambled.
- (e) Each exercise includes a list of 'references' which contain information associated with the items retrieved. Two references are given for each item.
- (f) Student answer sheets, including those for multiple versions, can be scored. Answers to exercises are remembered by the system, so scoring keys need not be submitted. The teacher may add items of his own to be scored.

Communications between teachers and the data-processing centre go through the existing District internal mail, which services each school once a day.

Items are classified for retrieval in the following ways:

*Course.* This identifies the item collection from which retrieval will take place.

*Category.* This is the basic subject-matter classification system. It is similar to the table of contents of a book. The teacher selects category numbers from a published index when filling out a request block. The category retrieval system is arranged hierarchically so that when a high-level category is specified items in all subordinate categories are eligible also for selection.

*Difficulty Level.* One of the three levels of difficulty is assigned to each item. The teacher may specify one of these.

*Behaviour Level.* Items are classified according to whether they require 'knowledge' only or 'application of knowledge'. A teacher may specify this.

*Keyword.* This dimension can be used like the index of a book. Up to three words or phrases can be associated with each item. The teacher may select a keyword number from an index of keywords and specify it as one of the criteria for selection.

*X-Dimension.* This dimension was built into the prototype in anticipation of future needs.

In CTSS, random selection is used to reduce the number of eligible items when too many are available. That is, if more items than he has requested match the teacher's specifications, items will be selected at random from those eligible. If fewer items are found, some specifications will be relaxed in an attempt to meet the teacher's quantity objectives. Behaviour level, if specified, is disregarded first; then any difficulty level specification is disregarded. No other teacher specifications are relaxed.

## Further Applications

There is a growing interest in the USA in the use of CTSS or similar computer-generated testing systems.

The CTSS prototype is available to other educational institutions; programmes, documentation, and some of the existing item collections may be obtained from the Los Angeles City Unified School District. Several other institutions have installed CTSS and more data banks of questions are becoming available. Also, systems of a similar nature have independently emerged at various other locations, chiefly in institutions of higher learning.

A handbook is now available entitled *Computer-Assisted Test Construction*. This is edited by Gerald Lippey, who also has produced bibliographies on the subject and lists of over 100 existing computer-based test preparation systems.



It would seem reasonable to expect an upsurge of interest in computer-based test preparation in the UK, particularly at this moment in time when economic pressure on education severely limits the funds available for expensive projects such as CAI, but when many institutions, particularly in higher education, are equipped with under-utilized computers.

Furthermore, as Gerald Lippey points out, 'test generation is a natural component of more sophisticated computer-assisted instructional approaches. A few of the existing automated test construction activities are, in fact, parts of larger computer-managed instruction systems. These more extensive systems usually include pedagogical decision-making elements, such as diagnosis of learner difficulties and prescription of assignments. Some of them enable students to proceed through large units of instructional material independently of each other. Those who wish to begin with a small, simple system and grow toward a comprehensive system may find test construction a convenient starting point, since it can stand alone under teacher control as well as fit into an integrated computer-managed instruction system at a later time.'

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# Educational Technology in Brazil: A Latin American Case Study

A J Romiszowski

## Introduction

The sub-title of this chapter does not mean to suggest that as far as educational technology is concerned, Brazil is typical of the rest of Latin America. Being by far the largest country, the only one where Portuguese, as opposed to Spanish, is the official language, and perhaps the most diverse in terms of cultural, ethnic and economic factors, it may well be the exception, rather than the rule. However, there are several factors in common with other Latin American countries, and just about all local educational conditions which exist in other countries in the continent can also be found in certain parts of Brazil. Also, it is probable that Brazil (with the possible exception of Mexico) is the current leader in Latin America, in the practical use of educational technology. Therefore, it will be useful to trace some of the current trends, in order to see where the continent as a whole may be heading in the next few years.

## Factors Affecting Brazil's Use of Educational Technology

Brazil's pre-eminence springs from several factors. Most notable among these is that the last decade, which has seen the growth of a practical set of educational technologies, has been a period of exceptional growth for Brazil — industrial growth, population growth, urban growth, growth of needs for trained manpower, and growth in the educational and material aspirations of the population.

As a result, there has been a considerable growth in education and training services. The private sector has supplied many specialist training needs (eg language, computer staff) but the state educational systems have also expanded.

However, educational provision still lags well behind the needs in most states of the Federation — a gap which shows no sign of being rapidly closed. Indeed, it has been publicly admitted by the Federal Ministry of Education and Culture that the current and future educational needs, both cultural and vocational, cannot possibly be met by traditional methods alone, and that new technologies of teaching must be investigated and used when appropriate.

A second factor which has stimulated projects employing educational technologies, is the great variety both in the educational problems and the extent of educational provision between regions. Most Latin American countries have a great span between the very rich and the very poor — educational progress is geared towards expanding facilities to an increasingly greater proportion of the population. But Brazil has the especial problem of the juxtaposition of relatively rich regions (such as the states of São Paulo or Rio), with a growing middle-class population, highly industrialized, 70 per cent urban dwellers, already equipped with a network of schools not inferior to some European countries, and very poor regions (such as the states of the North East of the country) with no middle class, a largely illiterate, rural, scattered population and hardly any state school system. These glaring regional differences have resulted in some of the more ambitious projects of the last few years, which attempt to create whole new educational systems based on new methods and media.

A third contributory factor is the US influence in Brazil, both in industry and in education (particularly at the university level), which has meant that new ideas emanating from American learning laboratories soon found their way into Brazilian academic circles. Perhaps some of these ideas stayed too long in academic circles before they were applied — we shall discuss this later. However, suffice it to note that the published literature on, for example, programmed instruction, was as prolific in Brazil in 1968 as it was in Britain. Just about all the 'classic' American books on the subject had been translated and published in Portuguese, and were widely read by students of education. It is interesting also to note, however, that although in the late sixties the theories of programmed learning figured much more prominently in teacher-training courses than they did in Britain, there was no great production of home-grown programmed materials at the time.

Brazilian programmes appeared in print. This was probably due in the main to the general scarcity of educational textbook publishing — few schools could afford any books at all, students bought their own books, and programmed texts would be rather expensive items. But a further contributing factor is the organizational set-up of Brazilian schools and of education in general, which render it unlikely for innovations to come 'from below'.

### Some Background on the Brazilian Education System

In theory, education is not over-centralized. Although the federal government establishes broad curricula, academic and administrative norms, it is up to the individual states to implement them as they see fit — a bit like the British system of LEAs. (Indeed, the number of students under a typical state's control is often not very different from a typical LEA — for example, although the state of Bahia is much larger than France, its population is similar to that of Paris.) The state ministries, or 'secretariats' are, however, political organs, elected, so to speak, on the state governor's ticket. They therefore tend to be staffed by bureaucrats rather than educationalists, particularly at the higher levels. This fosters the inevitable tendency to standardize and institutionalize procedures in education. The matter is not helped by the existence in the schools of a much more rigidly defined hierarchy, similar to a management hierarchy. The typical well-endowed school will have a head, who administers; an 'orientator', who plans curricula, tests students and gives guidance; a school psychologist; and several 'supervisors', who see to it that the teachers do their job effectively. Finally, a large proportion of the teachers in certain states have not had any formal teacher training.

All these factors combine to create a tendency to look 'upwards' for decisions. The individual development of new ideas at the 'grassroots' level is rare.

### The Major Trends

Thus, to summarize, educational technology in Brazil is at present applied to large-scale officially sponsored projects, rather than as a general trend throughout education.

Many of these projects are sponsored by the Federal Ministry, or other National bodies, but they are mainly implemented at the state level and usually a proportion of the funds are provided by the relevant state secretariat. Some (see previous paper by Chadwick) also have international backing.

As is the rule in developing countries, the big initial growth area was in the application of educational television. In the richer states this has developed as an adjunct to the existing school system, much as ETV is used in the USA or in Britain, and possibly more for prestige reasons than to fulfil a real educational need. In some of the poorer states, however, TV has become the essential backbone of the educational system.

Recent years have also seen the development of individualized systems of instruction, firstly in higher education but now also applied in secondary education and technical or professional training.

A further recent trend has been a return to the radio as the major factor in educational broadcasting.

On the specialist training front, developments are seen in the use of self-instructional texts, slide-tape presentations, simple simulations and games, group dynamics techniques, etc. More complex and sophisticated (eg computer-based) training systems are rare.

On the hardware front, indeed, there is not much activity. Thankfully, Brazil has been spared (so far) the appearance of special-purpose teaching machines. Such developments are held back by the general poverty of the educational market. It is hoped that as schools get richer, they will also become wiser than the USA or British schools which invested in sophisticated machinery without software backup. Unfortunately, other very useful hardware is also in scarce supply. Although radio and TV are fairly widespread in schools, few schools have slide projectors or film projectors and overhead projectors are almost unused below the further education level. A Brazilian-made OHP has only recently come on the market but so far sales are mainly to universities and specialist training institutions. A recent (1974) survey of language laboratories in use in São Paulo revealed 20 installations in a city the size of London — and almost all of them in privately-owned language schools.

Finally, a very strong trend has been the acceptance of the 'behavioural objectives' creed. It is the rule, rather than the exception, for any documents explaining courses or curricula, to begin with a statement of the objectives in behavioural terms. The Federal Ministry requires this on official submissions; state secretariats therefore require it from schools; supervisors therefore require it from individual teachers. As is the case with many creeds, however, although there are many who go through the motions of writing objectives, those who then 'live by their objectives' are rather few. The use of behavioural objectives has already become 'institutionalized' in Brazil to perhaps a greater extent than it is in the USA or Europe. Many have forgotten why they are stating objectives at all; some never knew.

## **Some Notable Projects**

### **1. Television-based Education in the State of Maranhão**

Whilst the richer states have developed ETV on the US or European model, as an adjunct to normal teaching, the State of Maranhão in 1968 really had no 'normal teaching' to build upon, especially at the secondary level. Only about 4 per cent of the 11 to 19 age groups were in full-time education provided by the state. There was a general lack of school buildings, teachers and the whole infrastructure of a traditional school system.

In 1969 a pilot project was launched, using TV as the backbone of an educational system, administered locally by specially trained organizers who are not qualified teachers. This was sufficiently successful to become the basis of the state's educational system — a system which relies on TV and supporting written materials for all basic teaching, using local churches or halls as meeting places and rapidly trained 'orientators' for motivation, discipline, testing and feedback.

Of course the system has problems, not least due to the unspectacular quality of the rapidly-produced programmes, and the unreliability of the very basic 'household'-type reception equipment. However, these are problems which can be eliminated in time, provided the necessary feedback is collected and resources applied to programme reformulation. There are also some problems with the role of the 'orientators' who in some cases are dissatisfied with their mainly administrative role in the system. Perhaps there could be a planned evolution of the orientator into a qualified teacher, by a programme of in-service training. However, despite these various reservations, the Maranhão project is notable in having in a very short period of time created a system of secondary education where virtually no system existed before.

### **2. Project SACI — TV Education by Satellite**

This project is perhaps one of the few exceptions to the general observation above that Brazil is not engaged in educational hardware development. Project SACI is a 'spin-off' of Brazil's space research programme. As such, it was funded by the Ministry of Technology rather than the Ministry of Education. This was fortunate in that it was funded rather generously, but unfortunate in that its position outside the educational bureaucracy will probably limit its impact.

Like other educational satellite projects around the world, the Brazilian project aims to beam educational broadcasts nationwide, bouncing them from a communication satellite. Advantages of such a system include cheapness (as compared to the short-range microwave transmissions used in normal systems), total coverage (even of outlying, sparsely populated areas), centralized studio and production facilities, etc. The viability of such a system in Brazil should be greater than almost anywhere else in the world. Unlike the Indian satellite project, for example, there are no local language problems. India, with a much smaller land area, has 3 official languages plus over 40 others in local use. Brazil effectively has only one. However, not all authorities are in favour of the project. Arguments against are partly political, partly based on the fact that existing microwave systems already reach the majority of the population in a country which is over 70 per cent urban-dwelling. It is not certain whether the satellite education project will become a permanent element of Brazilian education. However, project SACI has developed the software and specialist reception hardware, which could eliminate the above-mentioned problems associated with the Maranhão experience.

The State of Rio Grande do Norte, also among the poorest of the North-Eastern states and also with a very underdeveloped school system, was chosen as a test-base for project SACL. Special transmission equipment was installed and special battery-operated, tropicalized TV receivers were developed. The software team, trained and aided by USA staff, spent several years developing film, radio and written materials for the primary grade of school, covering virtually the whole curriculum. In addition, teacher-training materials, mainly in programmed instruction form, were developed, so that the teachers could train themselves on an in-service basis. Since 1965, this system has become the educational system of the State of Rio Grande do Norte. Transmissions are by microwave, not via satellite, so they reach only the capital and nearby districts. However, most of the state's population live in this area. The programmes are of a very much higher standard of production than in Maranhão and the equipment more trouble-free. The system, even when using microwave, is eminently transferable to other states. So far, however, there are few signs that it will be transferred.

### 3. Adult Education by Radio

Educational radio, of course, has existed in Brazil for some time. A particularly successful programme, aimed at primary and secondary school levels, is Projecto Minerva. This presents a daily menu of mainly cultural and enrichment programmes which, in addition to use in schools, have captured a sizeable adult audience.

A more recent development, however, has been the appearance of radio-based educational systems. A good example is the State of Bahia which offers adults a second chance (or often a first chance) to complete their basic education. The state's educational radio institute, 'IRDEB', produces, records and broadcasts a complete school course, up to the

15-year age group. Participating adults, who must be over 18, meet in groups together with a trained leader to listen to the broadcasts and to carry out supplementary activities, participate in discussions, etc. The role of the group leaders in this case is somewhere between that of a teacher and the purely administrative role of the group leaders in Maranhão. Over 100 groups exist throughout the state and several thousand adults have qualified through these courses. The state is now experimenting with a correspondence course back-up to some radio programmes and with radiovision. There are no current plans in this state to go for educational TV. It is generally felt that television would give few, if any, benefits over the radio courses, would cost much more and would reach a much smaller proportion of the population in this very large state with many sizeable towns outside the reach of TV signals.

### 4. Correspondence Courses

Whether correspondence courses in general can claim to be among the most important developments in educational technology is doubtful. However, the few mentioned here are notable because they are all using materials and systems developed by educational technologists, and two of them actually teach the principles of educational technology.

The first is a recent project — an attempt to extend the adult education system offered by IRDEB in the State of Bahia to advanced secondary work (the 'segundo grau' as it is called), thus enabling adults who missed school to qualify for the professions or for university entrance. Unlike the 'primario grau' course described above, this is purely a correspondence course — no radio, no group meetings, contact with your tutor mainly by post unless you live close enough to call in at the office. Materials for this course are still in production, although some subjects were first offered in August 1975 on a validation basis. The teaching materials are being prepared in a variety of styles. The science and mathematics courses are in the form of programmed instruction texts and 'newspapers' containing enrichment and reference materials. Some other disciplines are in a more 'traditional' format, although all materials are undergoing a fairly rigorous validation process — the first versions are offered free of charge to students living sufficiently close to enable them to appear for personal tutorial/test sessions with the authors of the modules. Numbers this year are limited to 200 per subject, in order to cope with the work of validation, but several thousand students per annum are expected in the future.

The other two correspondence courses are for teacher training, one in the State of Rio Grande do Sul, in the south of the country, and in the federal capital of Brasilia. Both these regions are richer and more endowed with traditional school systems, but, as in the rest of the country, there is a chronic shortage of trained teachers. In both these projects, 'lay teachers' receive in-service training based mainly on programmed correspondence course material and occasional meetings with tutors. The Rio Grande do Sul project also uses radio broadcasts.

The basic materials in this state are linear programmes and much illustrated, almost cartoon-style booklets. In Brasilia, a style of programming akin to information mapping is used. The content of both these courses places great stress on teaching the trainee teachers to apply a systems approach, to plan courses around clearly defined objectives, and to plan for feedback and continual course revision. It is the intention in both states that all 'lay teachers' should qualify in the near future, through the medium of such courses.

### 5. Personalized Systems of Instruction

The paper by Sherman in this Yearbook describes the Keller Plan, or PSI, and its origins in a psychology course at the University of Brasilia in 1964. Since then the use of the Keller plan has spread to several other Brazilian universities and to some other disciplines. Recently it has also been used in teacher training -- specifically the training of physics and technical teachers. Once again the aim is to develop in-service training, but this time on an individualized basis. The graduate of a course goes to teach in a given school or district. He arrives equipped with personalized course materials. His local colleagues study the course, and he acts as their tutor/monitor. When these teachers graduate, they repeat the course with other colleagues. This 'pyramid selling' approach to teacher training is the basis of a federally-sponsored project to train science and technical teachers, which is being executed by the National Centre for the Development of Technical Teacher Training (CENAFOR).

Some pilot courses have been run with reasonable success, although a number of snags have cropped up. For example, it was observed that although in theory the written learning materials of the courses need not be perfect (as there is a tutor/monitor to help out), in practice the quality of the learning materials was the cardinal factor controlling the efficiency of the course.

These courses are not 'classical' applications of the Keller Plan, in that they have to date been run as short, intensive courses (which necessarily apply a certain amount of forced pacing) and have used the authors of the learning materials and hired helpers as monitors (as opposed to using the peer group). These modifications have been forced by administrative constraints and by the need to evaluate and improve the basic materials before wider dissemination. Once the courses are repeated at the local level, by graduates of the first courses, the self-instructional nature of the materials will enable either an intensive 'off-the-job' or a more spread out, self-paced, 'in-service' system of implementation to be adopted.

Thus the PSI type of course has spread in Brazil from higher education, where it was developed, to teacher training and technician training.

### 6. Application in Technical and Industrial Training

In North American and European experience, the larger industrial courses are often among the pioneers in the use of new training techniques. The Brazilian picture is slightly different, partly because Brazil has a young industrial system, growing rapidly but as yet employing a relatively small proportion of the population, partly because the bulk of Brazilian larger industries are multinational. Thus at the lower levels of operative and semi-skilled personnel, the human resources supply problem is generally seen more as a selection than a training problem, and at the higher levels of management, the typical solutions are the importation of systems and techniques developed overseas. It is in fact the middle region -- middle management and skilled technicians -- where the greatest human resources 'gap' exists, and as industry expands the 'gap' between available and required human resources is growing very rapidly indeed. The problem is accentuated by the 'academic' tradition in education, which renders non-university 'technician' qualifications very much second-class ones.

Traditionally, the technician needs have been supplied by 'SENAI' and 'SENAC' — the National Services for Industrial and Commercial Learning. These organizations operate a network of technical colleges, extremely well-equipped and staffed in general, but no longer adequate to cope with industry's needs. In attempting to extend the numbers of students on courses both SENAI and SENAC have for some time been developing both group- and self-instructional systems. Both organizations have supported teams of writers to produce programmed texts. SENAC had at one time five teams in various parts of the country. SENAI has produced a range of well-structured manuals, some in conjunction with the International Centre for Technical Training (CINTERFOR). These manuals are in use in organizations outside SENAI and in other Latin American countries. Several mass-media and/or correspondence training schemes have been implemented in order to take technical training to a wider population.

A more radical move to meet the skills' shortage was taken by the government a few years ago, when a law was passed which, in effect, obliges all courses in schools to include a vocational component — every school-leaver, even if he is aiming for academic study at university, must also qualify in a trade or profession. Needless to say, the law has not been very fully implemented. Indeed, a growing body in the Ministry of Education is coming round to the idea that even with a massive (unlikely) increase in resources devoted to education, the objectives cannot be fully implemented, and certainly not by simply expanding the traditional systems of education. However, some interesting and potentially useful spin-offs so far have been several large task-analysis projects, in order to define the content of the necessary new courses, and federal funding for several pilot schemes of on-the-job training aided by correspondence, media, or Keller-plan courses.

## 7. Application to Medical Education

At the School of Medicine in the Federal University of Rio de Janeiro, there is the Department of Medical Education, and within this a Nucleus for Educational Technology in Medical Education (NUTES). This Nucleus also acts as the Latin American Centre for Educational Technology in Medicine (CLATES), with support from the Pan-American Health Organization. Consequently, this nucleus is well funded, and has developed many interesting projects, not least of them a programme of staff training in the preparation and use of teaching aids, particularly slide-tape presentations, video-tapes and transparencies, as well as written course units. The emphasis is on the design of the message, rather than the hardware/production problems. A workshop, based on a systems approach to course development, is run regularly. The vast majority of the teaching staff of the School of Medicine have participated in workshops, and many have produced a considerable amount of self-instructional and audio-visual teaching materials. Workshops have now been run for other departments, other universities and indeed other Latin American countries.

This Nucleus is also concerned with the development and use of computer-assisted instruction for the teaching of medical diagnosis, etc.

## Conclusion — A Look to the Future

Brazil is in an interesting position 'ed tech-wise'. Unlike the USA or Europe it does not yet have a fully-developed and entrenched educational system (or systems). There are needs to be filled at all levels.

Unlike many third-world countries it is not in the position of being too poor to do much about its educational needs.

The federal government has publicly stated that the recognized needs cannot be filled by a simple extension of traditional methods and systems.

Both federal and state organizations now exist to foster, guide and finance the development of alternative systems and appropriate technologies. Important among these is PRONTEL (originally standing for the National Programme of Tele-education), now empowered to promote and coordinate any type of educational technology project. Other official organs exist, sometimes overlapping or competing in their areas of activity.

However, despite the typically Latin American bureaucratic jungle which leads to some duplication of effort, some empire building, and a lot of wasted time, the will to improve educational methods is very strong at all levels.

Given these three factors of need, resources and willingness in almost excessive abundance, one can foresee a very busy and interesting future for educational technologists.

## THE TUTOR'S ROLE IN THE INDIVIDUALISATION OF MATHEMATICS

A.J. Romiszowski.

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### INTRODUCTION

Mathematics has been one of the major targets of attempts at individualisation. In the early days of programmed instruction, more programmes were published on mathematics and on mathematics-related subjects than on all other subject areas together (Cavanagh and Jones 1966). This was ascribed to the relative ease with which mathematics could be programmed due to its logical structure and "correct" answers, as well as to fairly marked agreement on instructional objectives and teaching methods as compared to some other disciplines (also large target populations of students as many courses require similar mathematics content, etc.).

This trend continued for several years (Cavanagh and Jones 1970) and not till 1972 did other subject areas come to the fore in instructional programme publishing (Hamer, Howe and Romiszowski 1972), due at least in part to the saturation of the mathematics market.

It is strange therefore that this explosion in systematically produced and (presumably) field-tested instructional materials has not been accompanied by a parallel rise in the efficiency of mathematics instruction. Why is it we still can't do mathematics?!

The truth is that the instances of large-scale and long-term implementation of mathematics instructional systems based on these new materials are few and far between. And not all the projects which have got off the ground have led to improvements in the standards of mathematical attainment.

So it seems that some of the assumptions about the mathematics "market" may not be very sound.

- (a) Perhaps it is not that easy after all to programme mathematics instruction successfully? Perhaps the fact that you can define your objectives with ease and clarity does not necessarily lead automatically to good instructional programming? Perhaps the programmer needs more than a clearly stated objective and an exercise-design model (e.g. demonstrate; prompt; release), in order to ensure success? Perhaps inspired programmers of mathematics are as rare among programmers as inspired teachers of mathematics are among teachers as a whole?
- (b) Or perhaps the differences in teaching approaches among mathematicians are greater and more important than we assumed? This would account in part for the low rate of acceptance of programmed materials. Although it is quite possible to



programme mathematics in the "discovery" method, few published programmes have in fact done so, and programming has grown to be recognised as the antithesis to "discovery". Also the "modern maths" movement, in its re-emphasis on "understanding" rather than mere computation, has tended to view programmed instruction as of very limited application.

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#### INDIVIDUALISATION VIA AUTOMATION OR HUMANISATION ?

However, just about all groups are agreed on the need to "individualise" the teaching of mathematics. Of course, the meaning of "individualisation" varies from simply (a) the need to allow the student to progress at his own pace, through (b) variety in the instructional methods available to students to (c) options in the choice of objectives or content, etc. -- depending on who is using the term. This leads to some confusion and has led writers to attempt to define and classify types of individualisation (Edling, 1970) (Gibbons, 1971).

Programmed Learning in its basic form provides individualised learning rate only, and in attempts to keep the more valuable characteristics of P.I. whilst rendering instruction yet more individualised, innovators have gone in two directions, (1) towards greater automation through the use of computer-based-learning, and (2) back to the well-tried but expensive individual tutorial as part of an integrated man/media system.

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#### EXAMPLES OF MEDIA/TUTOR SYSTEMS

We shall not consider the application of computer-assisted learning to mathematics in this paper, but shall concentrate on those systems which use the human tutor as one of their components. Examples include I.P.I. at the Primary Levels of education (Lindvall and Bolvin, 1967) (Scanlon, 1970) and the team-teaching system based on Autotutor programmes employed by some schools in Surrey (Surrey County Council, 1969) at the secondary level. At the higher education level we have various learning-by-appointment schemes as the one used for remedial mathematics at Middlesex Polytechnic (Hamer and Romiszowski, 1969), audio-tape and tutor schemes such as the Postlethwait Audio Tutorial System (Postlethwait, 1972) and reading assignment and tutor schemes such as the Keller Plan, also called P.S.I. - Personalised System of Instruction, (Keller, 1968) (Sherman (ed.), 1974).

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#### THE KELLER PLAN

The Keller Plan is of particular interest to those engaged in higher education as it has gained rapid acceptance and fairly widespread application in a way that earlier types of programmed instruction did not. As Sherman (Sherman, 1976) puts it:

"There must be disciplines and subject matters as yet untried in a P.S.I. format - but I cannot think of any. With no particularly reliable data I would estimate that more than 1000 professors have developed their own P.S.I. course and their own materials. At least twice that number have given P.S.I. courses using commercially available materials."

Even if the data is not too reliable it is certainly impressive. A little analysis of the characteristics of the Keller Plan (Romiszowski, 1975) suggest that this acceptance may be due to

- (a) the relative ease and speed of materials preparation (not usually specially programmed)
- (b) the relative ease of implementation within a traditional university course structure.

The relative success, as compared to more rigid styles of programming is probably explained by

- (a) familiarity of the style of the learning materials to students
- (b) regular face-to-face meetings with the tutor/monitor.

In short, the Keller Plan invokes very much less disruption of the existing system and is therefore that much easier assimilated into it.

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#### IMPORTANCE OF GOOD MATERIALS

However, there are two factors in the Keller Plan which have sometimes caused controversy. These are the occasional use of non-structured poor-quality materials for the study assignments and the quite common use of members of the peer-group (faster students) as monitors/tutors for the slower students. The former factor has been criticised by the proponents of P.S.I. and is the sign of a poorly designed course (but we should remember that the sort of material which is quite acceptable to an undergraduate may be totally ineffective with other target audiences who are less sophisticated learners - as far as mathematics is concerned, many adults are unsophisticated learners).

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#### THE USE OF PEER-TUTORS

The latter factor is held up as one of the major strengths of P.S.I. and certainly without the use of student-monitors many P.S.I. schemes would be totally unworkable on economic grounds.

However, can we glibly assume that the student who mastered a particular topic yesterday is automatically qualified to assess, correct and perhaps tutor his peers, learning the same topic today? The research supporting the use of peer-tutors in Keller-Plan courses would suggest that the answer is "yes", but there are some disconcerting "no's" as well to cast some doubts.

Particularly in mathematics teaching, if it were that easy to act as an effective tutor, why do we have a mathematics teaching problem. Surely, well-designed instructional materials (which do exist) used as the basis of a P.S.I. course should lead to efficient mathematics learning despite the shortage of teachers. But we don't really believe this do we?

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#### EXPERIENCES WITH PERSONALISED MATHEMATICS SERVICE COURSES

Successful applications of P.S.I. courses in mathematics do exist (e.g. McKeon, Newman and Purtle, 1974). Similarly, two personalised courses analysed by the author in a previous paper (Romiszowski, Bajpai and Lewis, 1976) seemed to be achieving stated objectives effectively. However, experiences with the courses seemed also to suggest that the role of the tutor may be quite critical in such courses. The two schemes were:-

- (a) The scheme at the University of Loughborough is a service course in Engineering Mathematics.
  - The students involved are second year Electrical Engineering undergraduates.
  - The content of the courses given so far are (1) Fourier Series, Partial Differential Equations and (2) Complex Variable Theory.
  - The learning materials are sections from the programmed textbook "Mathematics for Engineers and Scientists - Volume 2, by Bajpai, Calus and Fairley, plus special supplementary materials.
  - The courses have been given only in the last year and to only a part of the total group - about 40 out of 100.
  - They were set up deliberately as personalised courses on the Keller plan, utilising the normal timetable hours.
- (b) The scheme at the Middlesex Polytechnic is a service, or remedial course in pre-requisite mathematical skills.
  - The students are first year social science and business studies undergraduates.

- The course content is in 22 modules, covering basic arithmetic skills, powers, series, algebra etc., as required for entry into other service courses on statistics, calculus etc.
- The learning materials are mainly in the form of programmed instruction modules, commercially produced but modified by us. They are administered on teaching machines. In addition there are many programmed and non-programmed booklets, prepared by polytechnic maths staff, supplementing the programmes or giving alternative faster/slower treatments of the same content.
- The course has been given as a standard (obligatory for social science) unit for the last eight years, during the first four weeks (or longer if necessary) of the academic year.
- The course was not constructed to the Keller plan. It was originally a computer-managed "learning by appointment" plan, open to all students at any hour on a come-as-you-please basis. Over the years, however, with the shedding of the computer and other changes, it has grown to appear more similar to P.S.I., whilst still maintaining the flexible timetabling.

Thus both courses are based on programmed, validated materials. Both are modular. Both use tutorial support. Both have been run successfully (the scheme at Middlesex Polytechnic for a period of 8 years). But neither have used peer-tutors. At Loughborough, the students themselves were unwilling to take on this task. At Middlesex, staff were often unwilling to delegate this task. Marking, yes - but tutoring, definitely not. Thus, to satisfy economic constraints the assessment of student work (the marking) has been delegated - at Loughborough the students mark their own tests and decide if they need tutorial help; at Middlesex this is done by a clerk/secretary. But the tutorial work which may be needed to follow-up the self-study modules is jealously guarded by the mathematics specialists. Tutors have justified this as for example:-

- giving greater freedom to follow up the students particular needs,
- being able to adapt the tutorial in the light of experience, in a way that a peer-tutor would be unable to do,
- being able to test "real understanding" as opposed to the routine procedures tested in the programmes.

Such comments may reflect presumed inadequacies in the objectives and content of the programmed materials, as viewed by the tutors from their own idiosyncratic viewpoints. Or they may reflect some real limitation in written programmed materials, even when supported by relatively inexperienced tutors, which only a highly trained and experienced teacher of mathematics may overcome.

It would seem a good idea to examine this latter possibility a bit more deeply before discarding it and glibly assuming that mathematics teaching is no different from any other teaching.

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#### OME LIMITATIONS F PROGRAMMED INSTRUCTION

An article by Biran (Biran, 1967) gave a check-list of characteristics of a learning problem which would make it more or less suitable for solution through programmed instruction (the term is used here in the restricted sense referring to printed, small-frame, self-instructional materials). Excerpts from this check-list are reproduced in Table I.

On first glance, the factors seem to be balanced strongly in favour of using programmed instruction for the teaching of mathematics. But a more detailed analysis would suggest that some aspects of mathematics are less suitable than others.

Certainly when we consider problem-solving and the learning of problem-solving skills. In this area of mathematics teaching there is no unique answer. The problem may have a unique answer but there may be many ways of arriving at the answer. When teaching problem-solving we are at least as interested in the process of arrival at the solution, as we are in the solution itself. And identifying the process that a particular student is attempting to use certainly qualifies as "difficult to test".

Again, mathematicians are well aware that it is much more difficult to teach a student to formulate a problem in mathematical terms than to then calculate the solution. It is the synthesis of mathematical models that gives major learning problems. Also the judgement of the merits of alternative solutions - the evaluation of steps taken or to be taken as an important factor in mastering efficient problem-solving strategies.

Thus here is one area of mathematics - the learning of problem-formulation and the evaluation of strategies for problem-solving where it seems that the self-instructional mode is perhaps not the ideal.

Biran then goes on to contrast algorithmic and heuristic problem-solving strategies. In this context, an algorithmic strategy is one composed of a set of procedures which, if followed correctly, are guaranteed to lead to a solution. (Such a set of procedures - actions and decisions - may often be presented as a flow-chart, but this is not obligatory). A heuristic strategy, on the other hand is a set of procedures which may, but are not guaranteed to, lead to a solution.

Guessing the answer is a heuristic strategy, albeit generally a most inefficient one. "Guesstimating" however may be a legitimate step in an efficient strategy.

But, "working it out using log. tables" is algorithmic. Algorithmic strategies may be memorised "off the job" or even "looked up" when necessary, whereas heuristic strategies generally develop through experience. Hence the limitation of a printed text as a mode of instruction.

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## THE HEURISTIC TEACHING METHOD

A learner may learn an algorithmic procedure step by step, through demonstration. Or he may discover it through experience. Thus he may learn an algorithm by a heuristic strategy. Once he has learnt the steps of the algorithm, however, he may apply it almost mechanically - the learner's "problem" now is simply to identify the given problem as of the type for a given known algorithm.

Already in the last century a method of mathematics instruction was in vogue - called the "Heuristic Method" and exhibiting all the characteristics of what Gagne refers to as "guided discovery" (Gagne, 1965).

Describing the heuristic method in 1906, Young writes (Young, J.W.A. 1906):-

"the heuristic method is dominated by the thought that the general attitude of the pupil is to be that of a discoverer, not that of a passive recipient of knowledge. The pupil is expected in a sense to rediscover the subject though not without profit from the fact that man had already discovered it. The pupil is a child tottering across a room, not a Stanley penetrating into the heart of Africa .....

..... It is the function of the teacher and of the text so to present the things to be done, so to propose the problems to be solved that they require real discovery from the pupil; that at the same time the steps are within his power and that he attains at the end a good view of his subject."

Young also lists some of the tricks and dangers in the application of the method, giving the following example:

FALSE HEURISTIC	TRUE HEURISTIC
Is ABCD a parallelogram? Is it true that the diagonals of a parallelogram bisect each other?	What sort of figure is ABCD? What do you know about the diagonals of such a figure?
Is therefore AE equal to EC?	What lines in the figure are therefore equal?  How does this knowledge help us in our (main) problem?

## HOW TO SOLVE IT

The heuristic method did not seem to enjoy growing popularity as little is heard of it until it is formulated much more clearly in the excellent book "How to Solve It", (Polya, 1945). (It is interesting to note that this book enjoyed small hardback sales until 1971 when it first appeared in paperback form and soon after became one of the set books of the Mathematics Foundation Course at the Open University).

A summary of the stages in the application of the heuristic method as outlined by Polya, appears in Table II. A study of this table gives some insight into firstly why the method is now creating belated interest and secondly why the application of the method may be difficult by automated self-instruction.

Firstly, any reader versed in current educational technology thinking will immediately recognise a parallel between the four stages outlined by Polya and the four stages -

Define the problem  
Design a solution  
Implement experimentally  
Evaluate

- of the "Systems Approach" as it is called - the systematic application of scientific method to practical problems. As currently the systems approach is "in" among educators and is being applied to course design, for example, its application to the teaching of mathematical problem solving seems "obvious" rather than "revolutionary".

Secondly, the application of the method necessarily involves very careful and detailed observation of the problem-solving processes that the student is attempting to employ. In order to guide the student along a fruitful path, the teacher must choose the questions to ask and the prompts to give in the light of the student's overall behaviour with respect to the problem - not just his last response, but his response-pattern.

Attempts have been made to programme and automate such learning/teaching activity in computer-based learning systems, but in limited areas of application and with limited success. At present, the most readily available (and for a little time yet, the cheapest) instrument for carrying out such conversational, heuristic teaching is still the human tutor. Polya himself says:-

"If the teacher wishes to develop in his students the mental operations which correspond to the questions and suggestions of our list, he puts these questions and suggestions to the students as often as he can do so naturally. Moreover, when the teacher solves a problem before the class, he should dramatize his ideas a little and he should put to himself the same questions which he uses when helping the students. Thanks to such guidance, the student will eventually discover the right use of these questions and suggestions, and doing so he will acquire something that is more important than the knowledge of any particular mathematical fact. "

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#### ROM HEURISTIC O ALGORITHM

We have already mentioned that what was learnt heuristically may be later applied algorithmically. Opinions vary on how algorithmic procedures should be taught. At one extreme, there is the argument that provided the practical constraints allow it, algorithms should not be learned but should be looked up in reference material (Lewis, Horabin and Gane, 1967); another camp maintains that regularly used algorithms should be learnt by demonstration and practice (Gilbert, 1962); yet others maintain that in problem-solving at least, learners should construct their own algorithms (Landa, 1974), formulating them through a heuristic, problem-solving approach. Such a guided discovery approach carries a penalty in terms of learning time, but pays dividends in terms of long-term recall and transfer to other tasks (Gagné, 1967).

But in problem-solving, there always remains the stage of formulating the problem, of identifying which of the learned algorithms are appropriate for the particular case. Duncker (1945) showed that problem solvers use a variety of heuristic strategies, ranging from mechanical (often time-wasting) ones of "putting the problem in a recognised field" and trying all the algorithms one knows in that field until (hopefully) one hits on the right one, to systematic or organic strategies of the type described by Polya, gradually re-stating the problem and successively identifying what is necessary with growing precision, until the appropriate known algorithm is identified (or, if not known, may possibly be discovered anyway). Peel (1960), points out the difficulty in distinguishing between the two extremes by mere inspection of the written work of pupils. Often both extreme strategies will lead to the same result. It is necessary to accompany the problem-solving process stage-by-stage in order to guide students into the use of the organic, more systematic, more powerful strategies. Once again we see the limits of purely self-instructional, written materials.

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## SUMMARY

### SOME POSSIBLE IMPLICATIONS FOR TEACHING OF PROBLEM SOLVING.

From the above considerations it would seem reasonable to hypothesize that

- (a) The teaching of mathematical problem-solving strategies should not be best taught by programmed self-instruction. Programmes have been written with such objectives and have succeeded in some cases, but this does not imply that other, better modes of instruction do not exist.
- (b) Neither should materials/tutorials systems based on peer-tutors be particularly better. We might expect such systems to be very effective for the direct teaching of algorithms (as in the teaching of arithmetic or algebra by traditional methods), and for the teaching of mathematical concepts and concept structures (schemas), but we would expect them to be less effective in teaching heuristic approaches to problems.
- (c) It is also probable that systems employing human teachers who are not trained and skilled in the use of heuristic methods, will not be much better at achieving such objectives.
- (d) Two avenues of progress should therefore be possible:

either (a) the general body of mathematics teachers develop skills in the teaching of heuristic problem-solving,

or (b) Computer-based, conversational teaching programmes will become so sophisticated (and so cheap) as to render the mathematics teacher unnecessary.

Which will happen first?

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TABLE I

Excerpts from the check-list of characteristics in  
favour or against the use of programmed self-instruction.

FACTOR	CHARACTERISTICS IN FAVOUR OF P.I.	CHARACTERISTICS AGAINST USE OF P.I.
STUDENTS	Beginners ... General interests ... Homogeneous initial repertory Lacking in Study ... skills	Advanced Specialised needs Heterogeneous initial repertory Well developed study skills
CONTENT	Symbolic/mathematical Tightly logical .. Difficult to learn .. Retention important Problems with unique answers	Descriptive Loosely connected facts Easy to learn Retention not essential Problems with many answers
OBJECTIVES	Clear and distinct .. Easy to test	Diffuse/interdependent Difficult to test
Types (BLOOM)	Knowledge Comprehension Analysis	Application Synthesis Evaluation
	<u>Algorithmic</u> problem-solving strategies	<u>Heuristic</u> problem-solving strategies

<p>First. You have to understand the problem.</p> <p>Second. Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a plan of the solution.</p> <p>Third. Carry out your plan.</p> <p>Fourth. Examine the solution obtained.</p>	<p>UNDERSTANDING THE PROBLEM</p> <p>What is the unknown? What are the data? What is the condition? Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory? Draw a figure. Introduce suitable notation. Separate the various parts of the condition. Can you write them down?</p> <p>DEVISING A PLAN</p> <p>Have you seen it before? Or have you seen the same problem in a slightly different form? Do you know a related problem? Do you know a theorem that could be useful? Look at the unknown! And try to think of a familiar problem having the same or a similar unknown. Here is a problem related to yours and solved before. Could you use it? Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible? Could you restate the problem? Could you restate it still differently? Go back to definitions.</p> <p>If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or the data, or both if necessary, so that the new unknown and the new data are nearer to each other? Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?</p> <p>CARRYING OUT THE PLAN</p> <p>Carrying out your plan of the solution, check each step. Can you see clearly that the step is correct? Can you prove that it is correct?</p> <p>LOOKING BACK</p> <p>Can you check the result? Can you check the argument? Can you derive the result differently? Can you see it at a glance? Can you use the result, or the method, for some other problem?</p>
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## APPENDIX C

### STRUCTURAL COMMUNICATION AND INFORMATION MAPPING:

#### DESCRIPTIONS AND EXAXPLES.

1. Structural Communication - description.
2. Example of Structural Communication applied to mathematics. Extract from the materials used by the author in a preliminary evaluation.
3. Information Mapping - description.
4. Information Mapping explained by its originator.  
Example of the technique in practice.
5. Example of Information Mapping applied to Mathematics: I.  
Material used by the author in a preliminary evaluation.
6. Example of Information Mapping applied to Mathematics: II.  
Material developed by the author for use in a comparative study of information mapping and linear programmed instruction.

## APPENDIX C

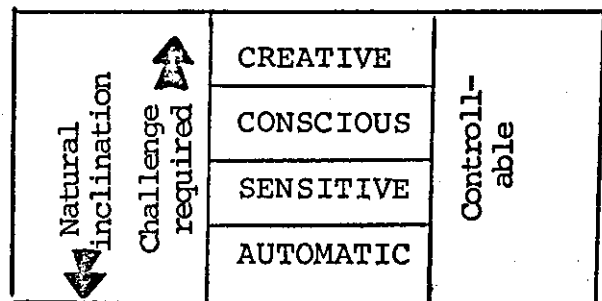
### 1. STRUCTURAL COMMUNICATION

This technique has been developed by the Centre for Structural Communication, Richmond, U.K., notably by A.M. Hodgson and his collaborators. (Zeitlin and Goldberg 1970; Hodgson 1971; 1972; 1973; 1974; Egan 1972).

The roots of this technique are in cognitive and field psychology, with a touch of cybernetics, although Hodgson sees it as basically an extension of Skinner's programmed learning principles to the area of higher order cognitive processes.

To relate these two, generally opposed fields of psychology, Hodgson constructs a model of types of thinking. Egan (1974) describes this model as follows:

"The model distinguishes four more-or-less discrete levels or kinds of intellectual activity. I will try to indicate the distinctions by reference to common experience of reading.



Automatic thinking is the lowest level they distinguish, and it may be recognized in that frustrating experience of reading a paragraph and then realizing one has not "taken in" a word of it. Nevertheless, it is proper to say that one has mechanically or automatically read the words.

Sensitive thinking is that of commonest awake experience; in reading, it is recognizable in the typical condition of reading and understanding the meaning of what is read.

Conscious thinking represents that stage of heightened critical awareness that we recognize in reading and synthesizing what is read with other ideas and areas of knowledge. It is this kind of thinking that is usually referred to rather vaguely as "creative".

Creative thinking is marked by spontaneity, in which one's reading sparks new insights that are not accountable for by any evident lines of association. This rare kind of thinking is perhaps most vividly described as the "Eureka!" experience. "

Hodgson sees behavioural approaches to instruction (e.g. P.I.) as relevant to the automatic and sensitive levels (involuntary habits and reproductive thinking). He sees the need for a cognitive approach at the conscious and creative levels (productive thinking).

"Hitherto cognitive field theorists have been in an awkward position when trying to combat behaviourist claims about the proper method of studying and promoting learning.

Behaviourists have been able to shrug off criticism by pointing to linear and branching programs and experimental results showing their efficacy in improving learning, while field theorists have lacked any technique



drawn from their theories that contributes anything like as directly to pedagogy. They have been restricted to reasserting in a variety of forms that the most effective way of teaching subject matter is "with due regard not only for coverage but also for structure". Without techniques that can implement these recommendations such arguments have little more effect on behaviorists than calling names. J.S. Bruner, who is the most visible proponent of a "structural" approach to learning, has given us *Man; A Course of Study*, but this is a whole curriculum (whose efficacy is enormously difficult to measure reliably) and not a technique that teachers can readily use in order to encourage effective learning of all kinds of subject matter. Structural Communication (S.C.) is, however, just such a technique. "

Structural communication is based on the concept of a "guided dialogue". Hodgson (1974) defines this as follows;

" In depth communication, the 'pre' factors in judgment, conception and disposition are worked on and loosened up by a process of reciprocal action in which aspects of the message are explored, contradicted, negotiated and correlated between the people concerned. This process is known in classical terms as 'dialogue'. When there is present in the communications process

insight and expertise in the discipline of dialogue itself, then we can refer to the process as 'guided dialogue'. The basic philosophy of structural communication as a technique, is that it is the medium of 'guided dialogue' which needs adding to the currently available technologies of communication and learning. This is a richer concept than feedback, since feedback can operate a process that is nothing but conditioning.

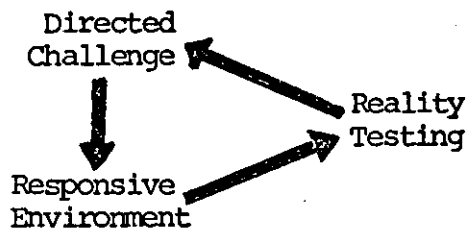
In a guided group dialogue we can discern the following elements:

- (a) Each learner is challenged to think for himself.
- (b) Each has to face the real facts of the case.
- (c) Each is making skilled inferences from the facts.
- (d) Several inter-related perspectives are considered.
- (e) Alternative solutions to each problem are raised.
- (f) The positive imagination of each learner is brought into play.
- (g) The coherence and consistency of the propositions of each learner is analysed.
- (h) Exchanges are adapted to individual differences.
- (i) An optional consensus is reached with commitment to act.
- (j) No learner is left out of, or dominates

over, the group.

We can simplify the process of dialogue to a cycle of directed challenge ('Will you take this up?'), responsive environment ('Tell us how you see it') and reality testing ('Is this really what you intend, mean, feel etc.?'). The cycle is shown in Fig.1.

Fig.1 - Cycle of Dialogue



To be effective, a guided dialogue should set a challenge which is seen to be pertinent, motivates and appeals, and has just the right level of difficulty to 'stretch' the learner without overwhelming his capacity. The responsive environment should be related to the theme of the dialogue, comprehensive towards that theme and rich in the scope for expressions of different reactions to the challenge. The reality testing should be based on analysis of relevance, consistency and as far as possible on an impartial view of the alternatives for viewing the theme. "

This concept of an interactive (to some extent) dialogue brings S.C. close to the cybernetics viewpoints concerning dialogue or conversational programming. It

is perhaps not quite so ambitious as the attempts to automate the instruction process. Certainly Hodgson does not accept the "must be better than a live tutorial" criterion which Pask (1972) sets for CAI. He states that a study unit written in S.C. "approaches the flexibility of a well conducted live tutorial".

The basic element of S.C. is called a Study Unit, and each of these consists of six interdependent parts. The simplest kind of study unit may be contained in a booklet of about 12 - 20 pages. Typically study units are designed for about one hour's work. They may be used individually, or they may be designed for group interactions. Below I will name each section and describe its role in the communication.

INTENTION: This is pretty well self-descriptive. The author uses it to describe what the study unit is about, and may use it to specify certain "behavioral objectives", or provide an "advance organizer", or, if it is one of a series, identify its context and role.

PRESENTATION: Here the student first comes into contact with the subject matter of the study unit. In the simplest units this is a written text, rather like a chapter of a book, though rather more condensed than is usual for a book that lacks the following sections of a study unit.

INVESTIGATION: This section usually comprises between three and five problems about the subject matter of the Presentation. Each of these problems takes a different perspective on the material, and the student attempts to resolve them by composing a response from the next section.

RESPONSE MATRIX: The matrix is a set of between about 12 and 35 "items" that restate in concise and randomized form the significant elements of the study unit theme as outlined in the Presentation. Usually these items will be statements of facts, theories, formulae, parts of strategies, and so on, depending on the subject of the study unit.

The student's task is to compose from the matrix an appropriate response to the challenge posed by each problem. There is nothing on the face of any item that will tell him to which problem or problems it is relevant or in what combination of items it should fit. Each item may be considered a "signifier" of semantic content more fully expressed in the Presentation. Thus each item should carry a fairly rich semantic load. From his reading of the Presentation, which is designed to create conditions of "readiness", the student will be drawn to see complex sets of relationships emerge from the random matrix as he concentrates on it with a particular problem in mind.

DISCUSSION: This is in two parts. Firstly, there is the Guide that analyses the student's responses and directs him to the appropriate discussion comments - which form the second part of the Discussion section. The Guide, in its printed form (there are machines that automate all this - moving the student directly from his response to the appropriate comments) poses diagnostic "tests", as, for example; "If you included in your response items 10 and 18 read comment P" or, "If you omitted from your response items 7 and 11 read comment K". The student will thus be directed to those comments that form the author's response to the student's response.

The possible variety of comments is hardly less than are available to a teacher in a classroom, and because of the author's control over the construction of the problems and response medium, they may discuss his responses with the student in great detail and focus on his difficulties with great precision. One may design a dozen or more comments for each problem, of which any typical student might be directed to only two or three.

VIEWPOINTS: At the conclusion of the discussion the author addresses the student about the material of the study unit. He may use this section to indicate his own biases and suggest further reading that will present another

viewpoint on the material. He may use it also to present a general overview of the theme, which might introduce a more sophisticated investigation of the material in a subsequent study unit. "

An example of a study unit is shown at the end of this appendix. It is taken from a text by Fyfe and Woodrow (1969) entitled "Basic Ideas of Abstract Mathematics". The author has used this text with students at the Middlesex Polytechnic, in informal comparisons of several programming techniques (the results of these comparisons are reported briefly in Chapter 10 of this study). One finding of a somewhat discouraging nature was the apparent difficulty with which these students adapted themselves to working with the S.C. study units. They found the very condensed nature of the presentation difficult and did not find it easy to refer back to the presentation when in doubt over a problem posed in the investigation. Also, they were inclined to omit reading all the discussion comments which were indicated by the discussion guide. Whether this is a characteristic finding with S.C. units presented in text form is not known. Certainly Hodgson (1971) reports remarkable success, both in terms of learning and motivation from groups of managers using S.C. units on management case studies. These were administered by correspondence, with computer-generated discussions to each individual's response pattern. Perhaps the text-format and the individualised self-study mode of usage

at Middlesex Polytechnic had some influence on attitudes. Perhaps also the technique is better suited to open-ended, case-study based, discussion-generating topics such as management, than to more highly structured topics such as mathematics. Finally, perhaps the characteristics of the students at Middlesex (non-specialists in mathematics, attempting to revise some basic pre-requisites for statistics and calculus) were possibly less motivated to participate in a "dialogue", and more attuned to receiving a "prescription".

Be that as it may, the students, participating in a seminar/self-study course on modern mathematics, during which they experienced linear and branching programmed texts, structural communication units and information mapping units, ranked the acceptability of these alternatives as;

1. Information Mapping
2. Branching texts
3. Linear texts
4. Structural Communication units.

This ranking was confirmed by their comments on a questionnaire and also by their choice of alternative units. It should be noted that no great significance is attached to these findings. The units used were all "off-shelf" commercially distributed materials in no way equivalent in content, quality (however that could be measured), style, etc. Also the group size was quite small - twelve students.



A further experience related to S.C. has been in the teaching of teachers, writers and training officers on workshops both in the United Kingdom and overseas. The author has for some years included sessions on S.C. (and on other "new" techniques for programming self-study units, such as Information Mapping or Mathetics) in "Programme Writing Workshops" and a variety of Educational Technology courses. He has noticed that whereas the rationale behind S.C. and the concepts involved are readily understood and generally accepted, few trainees manage to apply them well in practice. In the author's experience, based on 12 years of practical experience of teaching programme writers, it is generally more difficult to teach a novice to produce an S.C. unit than a unit of linear programmed instruction. By contrast, it is generally easier to teach him to produce acceptable units in the Information Mapping format.

One final point concerning the convenience of using S.C. in a text presentation; the author has already mentioned that some students at Middlesex Polytechnic tended to "skip" some of the discussion comments, others occasionally read them all. This can be controlled in a machine-presentation format or a computer-based format. Special machines have been developed by the Centre for Structural Communication, which obviate the necessity of presenting all the discussion comments to the student. An individualised comment is structured "in response to the student's response". On this basis, it is much more convenient to use S.C., much faster, can be studied by individuals

or discussion groups, etc. In this form, S.C. qualifies as a technique for the programming of interactive "dialogue" type CAI materials.

## 2. - EXAMPLE OF STRUCTURAL COMMUNICATION APPLIED TO MATHEMATICS.

EXTRACT FROM THE MATERIALS USED BY THE AUTHOR IN AN  
EVALUATION AT THE MIDDLESEX POLYTECHNIC.

Working through a Study Unit

## WORKING THROUGH A STUDY UNIT

### How is a study unit organized?

Each Study Unit is a communication on a unified theme. The theme is expressed by means of various sections, each section playing a different role in the communication. The purpose of these different sections is to set up a dialogue between you and the author in a way which permits a variety of conversations to take place. These conversations are built around problems which elaborate the theme of the Study Unit. By indicating your own understanding of various problems you enable the author to "talk back" to you about his own view of these problems. These are the sections:

**INTENTION.** This briefly describes the theme which the author intends to discuss. From this you can rapidly judge whether the Study Unit is of interest to you.

**PRESENTATION.** This gives a carefully considered outline of the theme, and any subsidiary themes. From this you can assimilate the primary content of the author's message. If you are already familiar with the material you can use this section simply as an aid to recalling information.

**INVESTIGATION.** This leads to an exploration of the theme in greater depth. Problems are posed which draw your attention to some of the more subtle points in the theme and enable you to develop your understanding. This section is worked on in conjunction with the response indicator section.

**RESPONSE INDICATOR.** This is a set of about twenty meaningful items, all of them relevant to the theme as a whole. They provide the basic vocabulary for you to "talk back" to the author. For any given problem, the relevance of these items changes in such a way that a particular combination of  $N$  items (where  $N$  is a whole number, certainly greater than one and usually less than ten) builds up a statement giving adequate discussion of that problem.

**DISCUSSION.** This is in two parts. Firstly, a DISCUSSION GUIDE which provides the logical information that enables you to find which comment the author considers most suitable to your response. Secondly, a set of DISCUSSION COMMENTS which provides the "rehearsed dialogue" of the Study Unit.

**VIEWPOINTS.** The treatment of any theme by any author always reflects his point of view. This may not be the only admissible point of view. This section therefore provides a more direct statement of the way the author looks at the theme and describes, if required, the more subtle features of the theme. You make up your own mind on the validity of his viewpoint.

### In what order are the sections studied?

The sections are usually arranged in the order in which they are described above. This is not necessarily the order in which it is most useful to study them. If you prefer doing problems to straight reading then you might start at the INVESTIGATION, using the PRESENTATION as an information retrieval aid. If you are concerned first of all with the particular "slant" of the author, you may read the VIEWPOINTS first. At any time you may make cross reference to any other section. All these procedures are correct and desirable uses of the Study Unit. In any case some Study Units may present sections in a slightly different order. The diagram below is provided to assist you to picture the main relationships of the sections.

### How does the DISCUSSION work?

Each problem in the INVESTIGATION can be adequately discussed with a number of possible combinations of items chosen from the RESPONSE INDICATOR. These problems are not intended to be "tests". This means that you may use a variety of ways to arrive at a decision on a given problem. You can start with a small combination and build up a more complete picture. You can make a broad attempt and narrow it down. If completely at sea, you can try anything at random and then see if the comments help you to make sense of it. When a decision on a given attempt at a problem has been reached, the following procedure is recommended:

1. Write down the code numbers of the items chosen for your response.

2. Turn to the DISCUSSION GUIDE and locate the information concerned with the problem you are working on.

3. You will notice that each problem in the DISCUSSION GUIDE has a number of stages. For example:

1. I 3, 7, 9 or 18 A
2. O 4, 8, 10 or 19 B
3. O Any 2 of 2, 6, 11 or 20 C

etc.

These stages are to be interpreted as follows:

Stage 1 above means: If you have *included* any or all of the items 3, 7, 9 or 18, then read comment A.

Stage 2 above means: If you have *omitted* any or all of the items 4, 8, 10, or 19, then read comment B.

Stage 3 above means: If you have *omitted* any 2 or more out of 2, 6, 11 or 20 then read comment C. (This implies that if you have chosen only one of

*Note: The line immediately above should read*  
(This implies that if you have omitted only one of

items 2, 6, 11 or 20, then you may proceed to the next stage, ignoring comment C.)

4. If you are directed to a comment, study it and then make a fresh attempt at the problem, processing your subsequent response by the same procedure.

5. If you are not directed to a comment, then move to the next stage.

6. When you have revised your response in such a manner that you reach the final stage of discussion on a problem, go to the next problem. It is preferable, though not obligatory, to work through the problems in the order they are given.

If you work through a study unit once or twice by the above procedure, you should find that the process becomes automatic and does not interfere with the study of the theme.

## ACKNOWLEDGMENTS

To the cooperation and comments of students and teachers who undertook the various stages of validation, we give credit for any successes we may have in communicating to you, the reader. For all failure, we take the blame. We are indebted to the staff and students of the following education institutions, who gave so much of their time in trying out experimental material:

*Altrincham Grammar School, Cheshire*  
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R. M. Fyfe  
D. Woodrow

## ABSTRACT MATHEMATICS

Abstract mathematics concentrates interest first on awareness of what tools are being used, and then on what necessary or possible results follow. From the constructions formulated, structures appear that are also relevant to other mathematical situations that at first seemed different and unrelated. Then unification becomes possible at still further levels of abstraction.

In fact, all mathematics is abstract. It is a selection of concepts (ideas) and the building up of systems by virtual action. To take a very simple example: the numeral 2 is a symbolic representation equivalent to the sound, or written form, of the word "two." It could also be symbolised as .. or : but the idea, or concept, of two is a mental construct. It does not "exist" in any material sense.

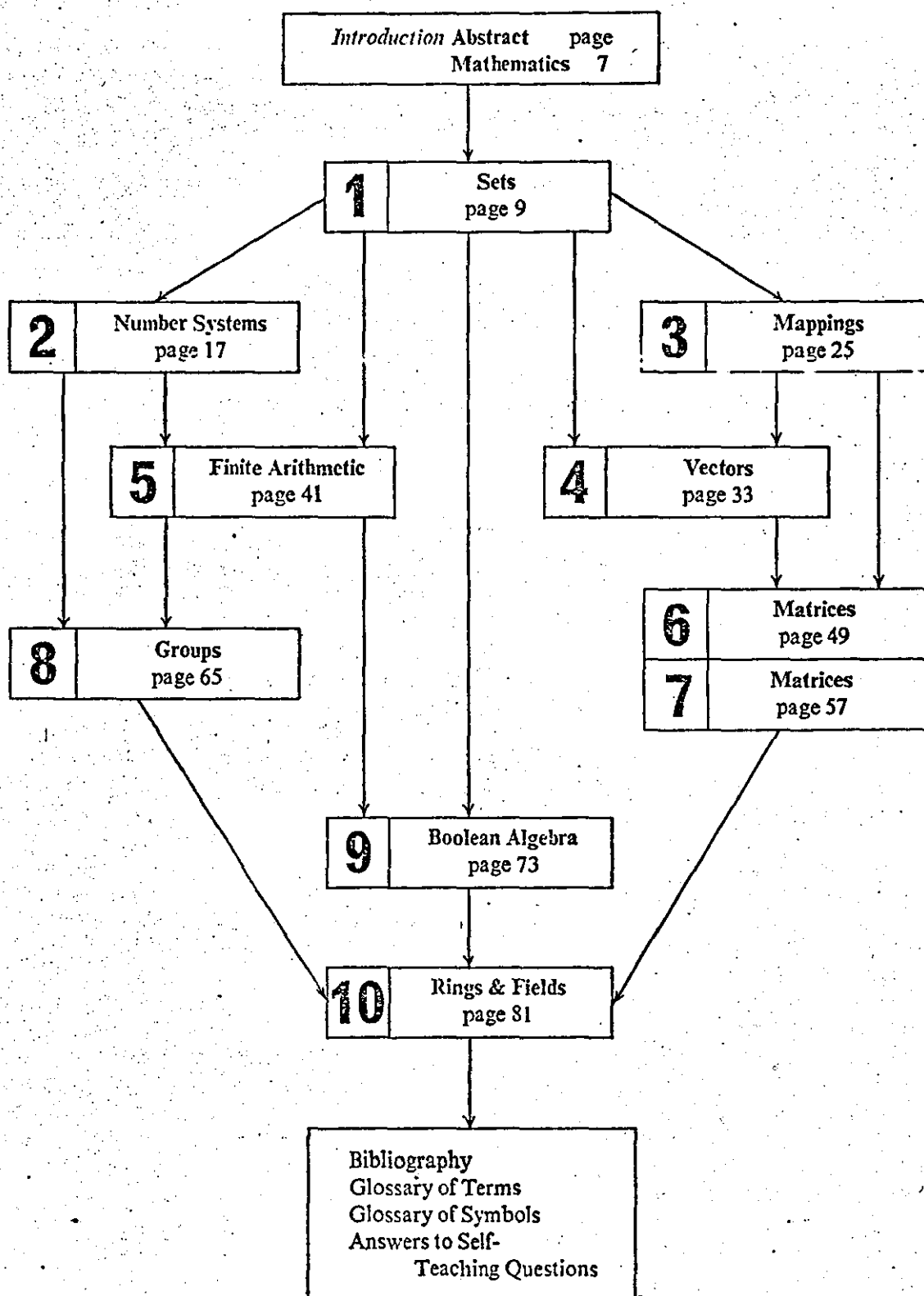
The abstract system may be thought of as a storehouse. Part of it is static—useful for comparison with situations of life and society, either in parallel with them or for giving fresh insights. Another part of the system is dynamic, and can be used actively as a model that simulates some movement or operation that is taking place, or that we want to effect. The abstract mathematical simulation is manipulated by its own rules; conclusions or results may be reached, but they are still abstract. By a return to the actual problem it is possible to verify whether the mathematical solution is valid and useful; at the same time a judgment will be made to decide if the model was well-conceived, adequate or faulty.

The user of the storehouse is one who has a good knowledge of what it contains, why and where, and who knows how to combine its contents. The real mathematician is one who glimpses what it does not contain—yet—and is struggling to fill one of these gaps.

In planning this topic book, we decided that the fundamental idea is that of sets. On the "static" side, the other tools needed are a good grasp of number systems, infinite and finite, and the structure that arises from the operations of either addition or multiplication, together with ideas of identity and inverse. On the Concept Map (page 8) these steps are represented by the chain of arrows connecting Study Units 1, 2, 5 and 8. On the "dynamic" side, from sets we move to relations and mappings, with the idea of function. Vectors and matrices are other tools of action. The relevant Study Units are 1, 3, 4, 6 and 7. Having put something into the storehouse, we are now in a position to make demands. A broader understanding of algebra is offered in the final Study Units 9 and 10.

A topic book such as this can be profitable only to a student who really wants to learn what it has to offer. He has to make an effort, in his own way, to develop strategies of learning that suit his mind. The Presentations of each study unit are limited by space, and rather condensed. A second, and even a third, reading may be required to absorb the message and to become familiar with new notation.

Exercises to be worked are restricted to a few self-teaching questions (described by the letter **EQ**). Their limited number was decided not only by shortage of space. We believe that experimentation with one's own ideas is both more efficient and more fruitful than doing examples conceived by someone else. The student is therefore strongly advised to do all the few key examples that are suggested, particularly those that call for exploration and invention.



CONCEPT MAP OF "ABSTRACT MATHEMATICS"

## SETS

## INTENTION—PRESENTATION

## INTENTION

Set language is basic to mathematical thought; its ideas and notations are described in this first Study Unit.

When sets have been defined, operations are introduced so that the sets can act on one another. An

algebra of sets emerges and certain "laws" are obeyed; uses are found for this algebra. In particular, the understanding gained is needed for Boolean algebra in Study Unit 9.

## PRESENTATION

**Introduction and notation.** A set is any collection of "objects", known as the elements of the set. The set must be well defined, so that there is no doubt which elements are included in the set, or excluded from it. There also has to be a universe of discourse that gives the context within which to consider the set and its complement; the complement being the set of the excluded elements.

A set is equal to another set only if both sets have the same elements. It is not necessary for the elements in the two sets to be arranged in the same order. Also, repeated elements are ignored. Thus, if a set A has elements 4, 2, 3, 1, and a set B has elements 1, 4, 3, 2, 2, 1, we can say that  $A = B$ .

There are different ways of naming a set:

(i) the elements of the set may be enumerated by being shown within braces. The set above could then be written  $\{1, 2, 3, 4\}$ ,

(ii) a set can be described by a sentence, e.g. "all the people in this room whose birthdays are in June",

(iii) a formal statement can be given, e.g.

$S = \{x | x = 2n - 1; n \text{ is a natural number}\}$

is read: "S is the set of  $x$ 's such that  $x = 2n - 1$ , and  $n$  is a natural number".

The set described formally under (iii) could be described by the sentence: "S is the set of positive odd numbers", or be enumerated as

$S = \{1, 3, 5, 7, \dots\}$ .

The symbol  $\in$  is used to say that certain elements are in the set. Thus,  $5 \in S$  is read as: "5 is an element of the set S".

The universe of discourse, called  $\mathcal{U}$ , is the larger set of which other sets are parts. These parts can be called subsets of  $\mathcal{U}$ .

**Operations.** Two (or more) sets can interact if they belong to the same universe  $\mathcal{U}$ .

(i) **Intersection:** the most obvious comparison is to seek the elements which two sets have in common. The set of common elements is called the intersection of the two sets. Intersection has the symbol  $\cap$ . Within  $\mathcal{U} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$  we can have  $S = \{1, 2, 5, 6, 7\}$  and  $T = \{1, 2, 3, 4, 7\}$ . Then  $S \cap T = \{1, 2, 7\}$ .

It may happen that two sets have no common elements. In such a case the sets are said to be *disjoint*, and their intersection is *empty*. For example, within  $\mathcal{U}$  defined above, take  $P = \{1, 7\}$  and  $Q = \{5, 6\}$ ; then  $P \cap Q = \phi$ .  $\phi$  is called "the empty set". Note that if  $X = \{0, 1, 2\}$  and  $Y = \{0, 3, 4\}$  then  $X \cap Y = \{0\}$ . Now, since 0 is an element of  $\mathcal{U}$ ,  $\{0\}$  is not the same as  $\phi$ . However,  $\phi$  is sometimes written  $\{\}$ .

(ii) **Union:** another action on two sets is to ask for the set of elements that are in both sets. This operation is called union, and has the symbol  $\cup$ . In the examples above,  $S \cup T = \{1, 2, 3, 4, 5, 6, 7\}$ ;  $P \cup Q = \{1, 5, 6, 7\}$ ;  $X \cup Y = \{0, 1, 2, 3, 4\}$ .

**Subsets.** Every set is a subset of its universal set  $\mathcal{U}$ . A subset is a set which contains some of the elements of another set. The symbol used is  $\subset$ , which is read either as "is a subset of" or "belongs to". In a visual sense, a subset is contained within the set from which it was derived.  $P = \{1, 7\}$  is a subset of S, or T, of  $S \cap T$ , of  $S \cup T$ , as well as of  $\mathcal{U}$ . We can write  $P \subset S$ ,  $P \subset (S \cap T)$ , etc. It is also true to write  $S \cap P = P$  and  $S \cup P = S$ . Check this.

A set is said to be an *improper* subset of itself. All other subsets are proper subsets, and  $\phi$  is a proper subset of all sets.

There is a "power" set of every set. This is the set of all the subsets of the set. For example, the set  $\{1, 2, 3\}$  has an improper subset  $\{1, 2, 3\}$  and the seven proper subsets,  $\{1, 2\}$ ,  $\{1, 3\}$ ,  $\{2, 3\}$ ,  $\{1\}$ ,  $\{2\}$ ,  $\{3\}$ , and  $\phi$ . The set which has these eight sets as its elements, is called the power set of  $\{1, 2, 3\}$ .

**EQ1.** List the subsets that form the power set of  $\{A, B, C, D\}$ .

**Venn diagrams.** By drawing two or more closed shapes (not necessarily circles) that overlap, it is possible to illustrate definitions and operations on sets. The universal set  $\mathcal{E}$  is shown in fig. 1.2 but not in fig. 1.1.

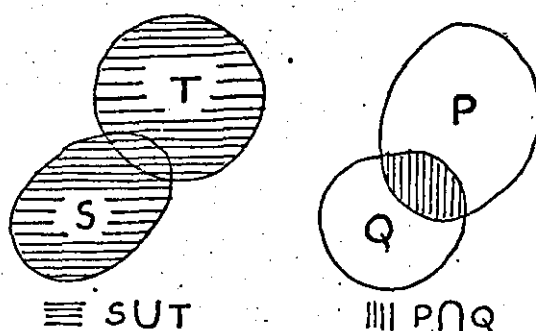


Fig. 1.1. Venn diagrams of union and intersection

**Complements and relative complements.** It has already been mentioned that a complement is a set of elements contained in the universe  $\mathcal{E}$ , but not in a particular subset of it. For  $S = \{1, 2, 5, 6, 7\}$ , its complement not- $S$  (called  $S'$ ) in  $\mathcal{E}$  is  $S' = \{0, 3, 4, 8\}$ ;

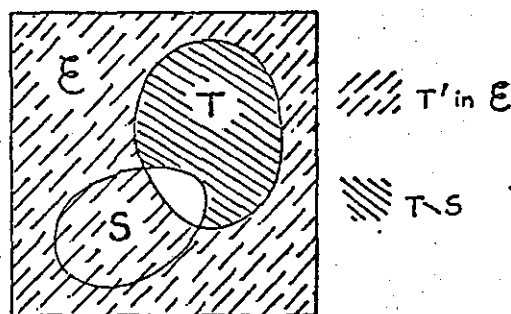


Fig. 1.2. The difference between  $T'$  and  $T - S$

similarly,  $T' = \{0, 5, 6, 8, 9\}$ . Also,  $S' \cap T' = \{0, 8\}$ .  
**EQ2.** Enumerate the sets  $P'$ ,  $X'$ ,  $P' \cup X'$  and  $(P \cup X)'$ . Notice the notation and the results; make Venn diagrams if you have difficulty.

The *relative complement* is a comparison between two sets. Not- $S$  in  $T$  means the set of elements of  $T$  which are not also in  $S$ . This relative complement, symbolized as  $T - S$ , is  $\{3, 4\}$ . Not- $T$  in  $S$  is  $S - T = \{5, 6\}$ . Fig. 1.2 shows the difference in meaning between  $T'$  and  $T - S$ .

**EQ3.** Make a similar diagram to show  $S'$  and  $S - T$ . Also find out whether you agree that  $S - T = S \cap T'$  and  $T - S = T \cap S'$ .

**EQ4.** Draw a Venn diagram of three intersecting sets  $P, Q, R$ . There are eight regions including the part of  $\mathcal{E}$  outside  $P, Q, R$ . Describe these regions in terms of the symbols  $P, Q, R, P', Q', R'$ , and  $\cap$ . e.g. the middle region is  $P \cap Q \cap R$  and the outer part of  $P$  is  $P \cap Q' \cap R'$ .

**EQ5.** Experiment with these ideas and notations. What special results do you get if: (a) the sets  $T$  and  $S$  are disjoint; (b)  $S$  is contained within  $T$ ?

**EQ6.** Satisfy yourself that  $Q' \cap R' = (Q \cup R)'$  and find similar relationships for  $P'$  and  $Q'$ , and for  $P'$  and  $R'$ .

**Laws.** There are certain propositions or laws that apply to numbers and the operations  $+$  and  $\times$ . You have always used them in algebra but may be unaware of their existence. The same laws apply to union and intersection, union taking the place of addition, and intersection taking the place of multiplication. This is shown in table 1.1 on page 11.

Illustrations, by means of Venn diagrams, show whether or not these relationships are reasonable. It must be emphasised that an illustration is not a proof.

The last law is the most surprising, since it is not true for algebra: e.g.

$$6 + (4 \times 7) \neq (6 + 4) \times (6 + 7).$$

Fig. 1.3 reveals that it does make sense with sets.

**EQ7.** Illustrate all the other rules for yourself, and also the following special ones:  $\phi' = \mathcal{E}$ ,  $\mathcal{E}' = \phi$ ,  $S \cup S = S$ ,  $S \cap S = S$ ,  $S \cup S' = \mathcal{E}$ ,  $S \cap S' = \phi$ ,  $S \cup \phi = S$ ,  $S \cap \phi = \phi$ ,  $(S')' = S$ ; De Morgan's Laws:  $(A \cap B)' = A' \cup B'$ ;  $(A \cup B)' = A' \cap B'$ .



Table 1.1

LAW	NUMBERS $a, b, c$	SETS $R, S, T$
commutative	$a + b = b + a$ $a \cdot b = b \cdot a$	$S \cup T = T \cup S$ $S \cap T = T \cap S$
associative	$a + (b + c) = (a + b) + c$ $a \cdot (b \cdot c) = (a \cdot b) \cdot c$	$R \cup (S \cap T) = (R \cup S) \cap T$ $R \cap (S \cup T) = (R \cap S) \cup (R \cap T)$
distributive	$a \cdot (b + c) = a \cdot b + a \cdot c$ ---	$R \cup (S \cap T) = (R \cup S) \cap (R \cup T)$

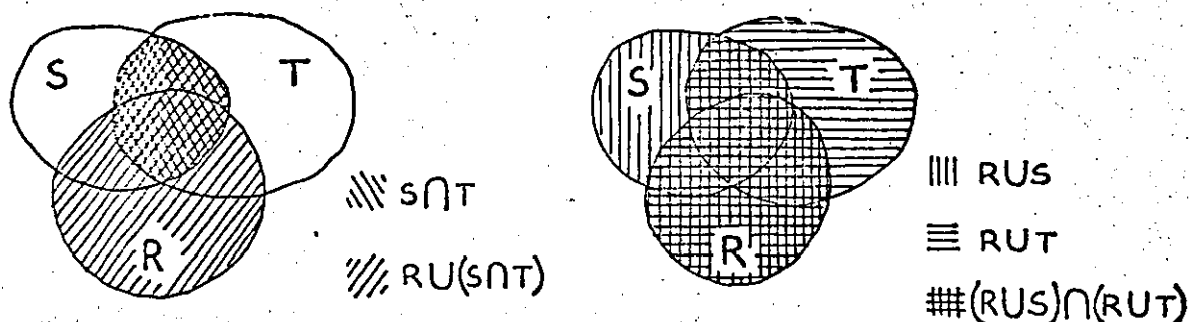


Fig. 1.3. The second distributive law for sets

Number of elements in a set. Example: 100 people were asked if they watched TV and listened to the radio and, if so, which they preferred. 13 said "neither", 72 said "TV" and 59 said "radio". How many of the 100 both watched TV and listened to the radio? A new use of Venn diagrams can help:

$$\xi = 100$$

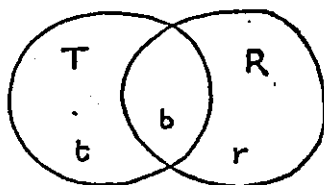


Fig. 1.4.

$$\begin{aligned}
 T \cup R & \quad t + b + r = 87 \\
 T & \quad t + b = 72 \\
 R & \quad b + r = 59 \\
 \text{on solving,} & \quad b = 44
 \end{aligned}$$

Note that  $t, b, r$  stand for the number of elements.

**Theorem:** Call  $n(T)$  the number of elements in  $T$ ,  $n(R)$  the number of elements in  $R$ ,  $n(T \cup R)$  the number in  $(T \cup R)$ ,  $n(T \cap R)$  the number in  $(T \cap R)$ . Then  $n(T \cup R) = n(T) + n(R) - n(T \cap R)$ . For example, since  $87 = 72 + 59 - n(T \cap R)$ , hence  $n(T \cap R) = 44$ .

Q8. Work out the same corresponding theorem for three sets.

Q9. Find or invent some problems of this kind and try out different ways of finding solutions.

Q10. How does the theorem read if  $T$  and  $R$  are disjoint sets?

Q11. A survey of 1200 families disclosed that 900 had a car, 800 had a TV set and 250 had a boat. 100 of the families had all three, and 600 had both a car and TV. 140 had a car and a boat. If the same number of families had none of these things as had only a boat, calculate how many had only TV and how many had both a boat and TV but no car.

## INVESTIGATION

The following problems have been designed to help you to develop your understanding of the new ideas and symbols which have been presented. Each problem can be answered by constructing a response using some of the items on the RESPONSE INDICATOR.

### Problem 1

What remarks can be made about one set that is not the empty set?

### Problem 2

$n(A)$  is the number of elements in a set  $A$ , and  $n(B)$  is the number of elements in a set  $B$ . It is further stated that the sets  $A$  and  $B$  are disjoint. What items on the RESPONSE INDICATOR describe this situation?

### Problem 3

Two sets  $P$  and  $Q$  have some common elements. What operations, laws or problems on  $P$  and  $Q$  can usefully be illustrated or solved with help from Venn diagrams?

### Problem 4

$\mathcal{E} = \{1, 2, 3, 4, 5, 6\}$ ,  $A = \{1, 2, 3\}$ ,  $B = \{4, 5, 6\}$ . What can you say about the sets  $A$  and  $B$ ?

RESPONSE INDICATOR

De Morgan's laws 1	well defined 2	elements are natural numbers 3	commutativity 4
complement in $\mathcal{S}$ 5	there are subsets 6	intersection of two sets 7	within a universal set $\mathcal{S}$ 8
distributivity 9	relative complements 10	equal to all sets with the same elements 11	union of two sets 12
$A \setminus B = A$ and $B \setminus A = B$ 13	sum equal to the union 14	elements or element 15	intersection is the empty set $\phi$ 16
contained within 17	associativity 18	problems and theorems on numbers of elements in sets 19	$\phi$ is a subset 20

## DISCUSSION GUIDE

### Problem 1

1. I  $\{1, 3, 4, 7, 9, 10, 12, 13, 14, 18 \text{ or } 19\}$  A
2. O  $\{2, 5, 6, 8, 11, 15 \text{ or } 20\}$  B
3. I  $\{16 \text{ or } 17\}$  C
4. I  $\{2, 5, 6, 8, 11, 15 \text{ or } 20\}$  D

### Problem 2

1. I  $\{3, 17; 2, 6, 11 \text{ or } 20\}$  E
2. O  $\{13, 14, 16 \text{ or } 19\}$  F
3. I  $\{7, 8, 10 \text{ or } 12\}$  G
4. I  $\{7, 8, 10, 12, 13, 14, 16 \text{ or } 19\}$  H

### Problem 3

1. I  $\{9, 13, 14, 16, 17 \text{ or } 18\}$  J
2. O  $\{1, 7, 10, 12 \text{ or } 19\}$  K
3. I  $\{4, 5 \text{ or } 8\}$  L
4. I  $\{1, 4, 7, 10, 12 \text{ or } 19\}$  M

### Problem 4

1. I  $\{1, 4, 9, 18 \text{ or } 19\}$  N
2. O  $\{3, 5, 7, 8, 12, 13, 16 \text{ or } 17\}$  P
3. I  $\{2, 6, 10, 14 \text{ or } 20\}$  Q
4. I  $\{3, 5, 7, 8, 12, 13, 16 \text{ or } 17\}$  R

## DISCUSSION COMMENTS

### A

We are concerned here with a single set and cannot assume that we know anything about its elements other than that there is at least one. Operations connecting two sets are no use, thus "laws" are ruled out.

### B

A set other than  $\phi$  must have at least one element and be defined within some universal set in which it has a complement. Can such a set have less than two subsets?

### C

Whether you were right to include these items depends on how you were thinking. Since a set A implies its complement A' within  $\mathcal{E}$ , we can say that  $A \subset \mathcal{E}$  and also that  $A \cap A' = \phi$ .

### D

It is important at the outset to be clear about what is implied in mentioning a set. It is equal to all other sets with the same elements. It is well defined, is an improper subset of itself, has the empty set as a

proper subset; if there are two or more elements, there will also be other subsets. It must be conceived as belonging to some universal set within which it has a complement.

### E

It is by no means necessary that the sets A and B should have numbers as their elements. The situation could be, for example, the boys and girls in a classroom. Does "contained within" mean disjoint or its opposite? The other items are not exactly wrong, but they have no relevance to the question. Check them one by one to make sure that you understand why.

### F

To evaluate the relative complements  $A - B$  and  $B - A$  is one way of showing that these sets are disjoint; another way of stating the same fact is by noting that  $A \cap B = \phi$ . Since the number of elements in the sets was mentioned, we can refer to the theorem:  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ . What does this become for disjoint sets? What sentence can you make now, as you read the altered formula?

**G**  
We cannot operate on, or compare, two sets unless they are within the same universal set. To answer this question, we do have to think about union and intersection. One way of stating that the sets are disjoint is to think about the relative complements.

**H**  
One of the interesting things about set language, perhaps confusing at first, is that there are often several ways of stating the same fact. On reflection, you will no doubt agree that this is also true of language in general and is not absent from other branches of mathematics.

For disjoint sets  $A$  and  $B$ ,  $A \cap B = \phi$ ;  $A \setminus B = A$  and  $B \setminus A = B$ ;  $n(A \cup B) = n(A) + n(B)$ .

**J**  
Associativity is a law concerned with three elements and so cannot be demonstrated within the context of two sets. The same remark applies to the distributive law. The other items refer to disjoint sets, or to one set that is a subset of another. Reconsider your answer in view of these comments.

**K**  
These items refer either to illustrations in the PRESENTATION, or to self-teaching questions. Make sure that you can use the Venn diagram technique to illustrate each of them.

**L**  
Commutativity of  $P$  and  $Q$  means  $P \cup Q = Q \cup P$ , and  $P \cap Q = Q \cap P$ . This may seem too easy to require visual representation, but you would be right to mention it.

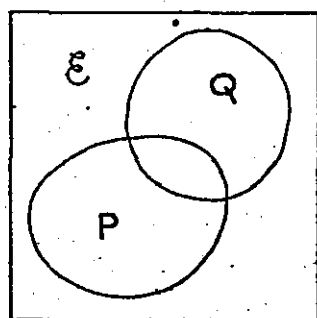


Fig. 1.5.

Fig. 1.5 shows a useful way of representing sets, but it is not an operation. The complements of  $P$  and  $Q$  in  $\mathcal{E}$  could be shaded in the same diagram, but again, this is not an operation.

Read the question again, and reconsider whether you were right or wrong to include 5 and 8.

**M**  
Although they have been considered together in this question, you are advised to distinguish in your mind between "operations" and "laws". It is also useful to be quite sure which of the laws are applicable to algebra, and which are true only for sets.

**N**  
The laws of sets and the theorems about number of elements are always true, but it is difficult to see why you should want to use them, or mention them, in the answer to this question.

**P**  
We first notice that the elements are natural numbers and that the sets  $A$  and  $B$  lie within the universe of  $\mathcal{E}$ . It can also be said that  $A$  and  $B$  are subsets of  $\mathcal{E}$  or contained within  $\mathcal{E}$ . The complement of  $A$  in  $\mathcal{E}$  is  $B$  and the complement of  $B$  in  $\mathcal{E}$  is  $A$ . Also,  $A \cup B = \mathcal{E}$  and  $A \cap B = \phi$ . The relative complements are those of disjoint sets.

**Q**  
You may include these items if you wish, but they do not say anything important about this situation.

**R**  
What is also true here is that  $B = A'$  and  $A = B'$ . How can De Morgan's laws be written if  $B$  is replaced by  $A'$ ? (Hint: find out what  $(A')'$  means.) Do these laws now give  $\mathcal{E} = \mathcal{E}$  for one, and  $\phi = \phi$  for the other? So they are still true? De Morgan's laws are also used in Boolean algebra.

## VIEWPOINTS

The study of modern mathematics forces us to bring into awareness the assumptions we are making, the operations we are using, and the "laws" which limit freedom but give the ground for action.

In a first course, such as this is, there is new symbolism to be memorised. What is more significant is ability to use the new symbols meaningfully. If this can be done, there has been a gain in the understanding of ideas. For example, it is necessary to

distinguish between definitions, such as the meaning of a set, its elements, its complement; the operations of union and intersection; the laws from which simple or complex actions give inevitable results. It may be a surprise to realise that there are different kinds of algebra, and that the algebra of sets is not necessarily concerned with sets whose elements are numbers.

## APPENDIX C

### 3. INFORMATION MAPPING

This technique has been developed, by Information Resources Inc., Lexington, Massachusetts, U.S.A., notably by Robert E. Horn and his collaborators (Horn et al, 1969, Horn 1973, 1974).

The roots of this technique spread into every psychological "camp". It draws its principles from the work of such diverse workers as Gagné, Piaget, Lumsdaine, Skinner, Briggs, Ausubel and Glaser. It knits these principles into a set of rules and procedures for the preparation of written materials which may serve the purposes of instruction, of revision or of reference. It may also serve as the basis of organisation of material for a dial-access, computer-based information system.

If one was to attempt to classify information mapping into one of the major philosophical camps, defined earlier in Chapter 3, it fits best in the "student-directed learning" category. A manual of information maps, somewhat resembles an atlas of geography maps. Each map has a definite purpose, clearly defined. Each map attempts to present the "shape" or "structure" of the information it contains, and for this purpose it uses certain standardised conventions. Finally, just like in a geography atlas, one will find maps which give the "global" view ("overview" maps, summary maps, "structure" maps) and other maps which give more detailed information on certain aspects of the whole topic (individual "concept" maps, procedure maps, compare/contrast tables, etc.)

The whole manual is cross-indexed and cross-referenced to facilitate the student's choice of the material he needs to read. Thus the student can apply his own learning strategy to the material, going from the particular to the general, or vice-versa, from the simple to the complex, or vice-versa, from rules and principles to examples, or vice-versa. The student may follow a holist or a serialist learning procedure. Furthermore, as all exercises (feedback questions) are to be found on separate, labelled pages, he may easily skip them, to use the manual as a reference text, or select those he feels he needs to self-evaluate his learning, or read these pages first, to diagnose which sections he needs to review. The technique is obviously worthy of attention, particularly in the light of the author's observations in Chapters 2, 3 and 4, namely that at the level of the individual learning steps of a course, the methods and materials used should be flexible and adaptive to the learners individual learning style, and that this requirement is even more important for the mature learner than for the younger learner.

The flexibility of use should adapt it well to the varied needs of a typical group of adult learners, coming from different backgrounds, with different levels of prior knowledge.

The emphasis on the presentation of the structure of the topic, and of each concept within the topic, should make it particularly suitable for the presentation of highly structured subjects, such as mathematics. Indeed, the technique was first experimentally applied to the teaching of mathematical concepts (Introduction to Probability, Horn et al, 1971).



The author utilised a section of the abovementioned text on probability, in a small informal comparison of alternative methods of programming. This was already mentioned in Chapter 10 and in the previous section of this appendix. The "information mapped" materials were better liked than the other forms of programmed materials. Also, potential authors learned to produce materials in this style of programming more rapidly.

The technique was applied by the author in some research on the teaching of vectors and matrices, performed as part of a project for the Council of Europe (reported in Chapter 10). This indicated that the use of information mapping, as opposed to linear programmed texts, may improve learning and cut learning time in individualised courses of instruction. This experiment was replicated on a larger scale (but with adult "second chance" students as opposed to undergraduates) in the research reported in the present study (see Chapters, 10, 11).

In order to give a more detailed account of the ideas behind information mapping, and to illustrate its application to mathematics, the author is including in this appendix several examples of the technique.

The first example to follow is a part of Chapter 1 from "Introduction to Information Mapping" by Robert E. Horn (1973). This extract, reprinted here by permission of the author, uses the technique of information mapping to explain the principles on which it is based.

The second example is a part of the instructional materials used in the abovementioned comparison of the acceptability of information mapping and structural communication. It is a part of the course "Introduction to Probability" (Horn et al, 1971).

The third example is a section of the information-mapped version of the course on matrices, produced by the author and his research assistant, Paul Ellis, and used in the comparison of information mapping and linear programmed texts, performed for the Council of Europe (Romiszowski and Ellis, 1973).

## CHAPTER 1 INTRODUCTION TO INFORMATION MAPPING

### OVERVIEW OF THIS REPORT

Introduction	Information mapping is the name given to a method of organizing and displaying information for learning and reference purposes. This report describes the research and development work that has been done with the method in preparing self-instructional books. It also discusses exploratory work with simulated displays for computer-assisted instruction. This report is itself written in a modified information map form.
This Chapter	We describe what information mapping is, how it began, and how it was derived from learning research, educational technology, and other fields of knowledge.
The Next Chapter	The process of writing information map books is explained and illustrated with sample pages. Some content characteristics of a set of information-mapped materials are reported.

#### APPENDIX C

##### 4. - INFORMATION MAPPING EXPLAINED BY ITS ORIGINATOR.

EXAMPLE OF THE TECHNIQUE IN PRACTICE.

EXTRACT FROM THE "INTRODUCTION TO INFORMATION MAPPING"

(HORN ET AL, 1973).

## OBJECTIVES OF INFORMATION MAPPING

**Introduction** In the past twenty years, we have seen a significant increase in research projects concerned with the man-information interface. The reasons for this scarcely need repeating. We have more information to handle in almost every job and discipline. This information is increasingly complex. People switch jobs more often, thus requiring more and speedier retraining. Technology changes; men must learn to use the new. The information-generating capabilities of the computer have surpassed all predictions.

Researchers are following many lines of inquiry in an attempt to augment the ability of human beings to interact with their new information environment. Hardware and software extend in many new and more flexible directions. Retrieval specialists are seeking new ways of indexing, abstracting, sorting, storing, and retrieving information. Computer-driven display units are becoming widely available. Time-sharing is enabling communities of workers to share the same data base. Psychologists and training specialists have given much more attention in recent years to the practical problems of how human beings learn. Enormous efforts are under way to refine programmed instruction and computer-aided instruction in a larger attempt to produce an "instructional technology."

### **Basic Aims**

As one response to the burgeoning educational demands; information mapping has emerged as a system of organizing data bases for self-instructional and reference purposes. Research and development on information mapping have been concerned with these objectives:

- to make learning and reference work easier and quicker
- to make the preparation of learning and reference materials easier and quicker
- to develop economical procedures for designing and maintaining (e.g. updating) training and reference materials

## INFORMATION MAPPING: ITS SCOPE

### Basic Concept

Information mapping is a system of principles for identifying, categorizing, and interrelating the information required for learning-reference purposes.

The system can be applied to production of books for self-instruction or to the specification of data bases for computer-aided instruction. Most of the research and development work described in this report was concerned with information-mapped books.

### Books

Information map books are learning and reference materials in which categories of information are consistently ordered on the page and are clearly identified by marginal labels.

The arrangement of information blocks is dictated not only by logical analysis and classification of subject-matter concepts but also by analysis of the contingencies required for successful learning and reference use. Therefore, in addition to basic content material, information map books also have:

- introductory, overview and summary sequences
- diagrams, charts, trees
- feedback questions and answers in close proximity to material to be learned
- self-tests and review questions
- tables of contents, alphabetic indexes and local indexes with connections to related topics

### Computer Data Base

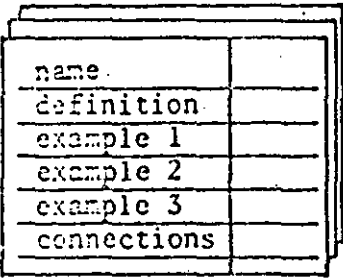
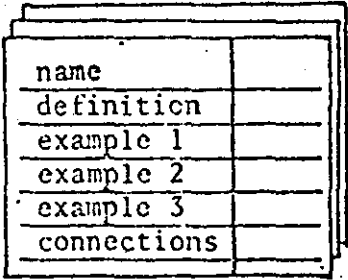
Through our studies with a book version of an information mapped subject, it has become clear that similar techniques could effectively organize a data base for computer-assisted instruction.

The data base would be composed of separable labeled blocks of information together with their interconnections. This would afford a flexibility in using only those parts of the system that are required for a particular purpose.

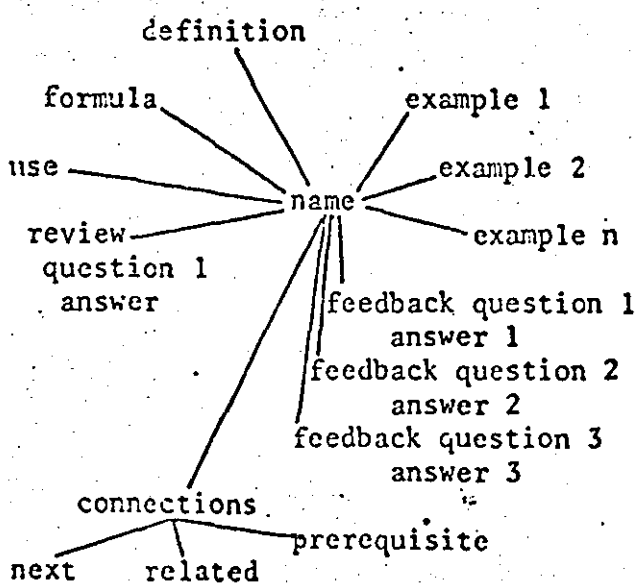
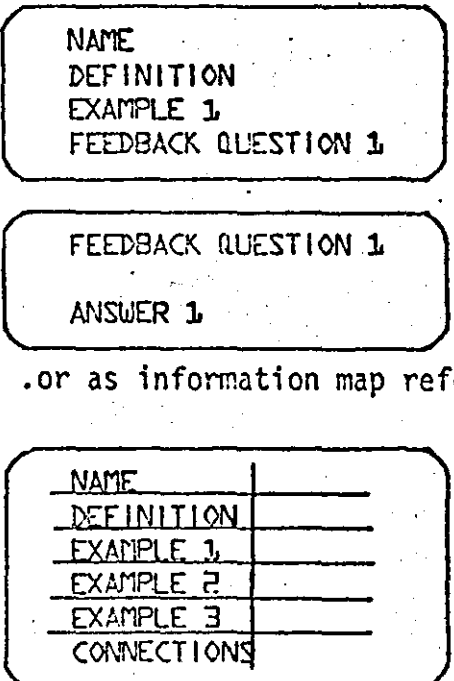
The flexible block-identified data base could be rearranged for:

- initial learning
  - for the naive student
  - for the sophisticated student
- relearning or review
- reference use

IN A BOOK FORM, INFORMATION MAPS ARE. . .

Stored this way. . .	. . .and displayed this way. . .
. . .on printed pages	. . .on the same pages
	

BUT WHEN USED IN A COMPUTER, THE SAME INFORMATION MAPS. . .

. . .are stored this way. . .	. . .and may be displayed in any of the following ways. . .
. . .in interconnected networks of electronically coded information blocks. . .	. . .as programmed instruction-like sequences. . .
	
	. . .or as information map reference.
	. . .and in several other ways dependi upon learner control and dynamic display capabilities.

## VISIBLE AND INVISIBLE FEATURES OF INFORMATION MAP BOOKS

---

**Introduction** Information maps for self-instructional books are conspicuous for their physical features, the format in which they present information.

An equally important aspect of such information maps, however, is that the content itself is selected and organized according to a set of underlying principles.

The method of presentation and the organization of content may be thought of as the visible and invisible features of a mapped page.

---

### Visible Features

The more obvious visible characteristics are these:

- information is presented in blocks
- marginal labels identify the kind of information in each block
- a consistent format is used for each kind of information: procedures follow one format, concept maps follow another distinct format, and so on
- functional and uniform headings and subheadings are used to make scanning easy and to speed up reference work
- each information map begins on a new page and, in programs for initial learning, most maps occupy single pages
- feedback questions and answers are located in close proximity to the relevant information maps
- a local index at the bottom of maps provides page numbers for quick location of prerequisite topics

(The last two features are not used in technical reports.)

---

### Invisible Features

The arrangement and sequencing of materials presented in information map formats are the result of:

- detailed specification of learning and reference objectives in behavioral terms
  - specification of prerequisites for the subject-matter area
- 

continued on next page

## VISIBLE AND INVISIBLE FEATURES OF INFORMATION MAP BOOKS, continued

(continued)  
Invisible  
Features

- 
- classification of the subject matter into component types (concepts, procedures, etc.)
  - definition of the contingencies required for successful learning and reference
-



## ORIGINS OF INFORMATION MAP FEATURES

---

**Introduction** In the effort to design more efficient materials for learning and reference, we drew upon accumulated knowledge in science and technology. Research findings, generalizations, and procedures from many areas were considered with a view to their possible practical value for instruction or reference.

---

**Fields Drawn Upon** Gradually we evolved the set of guidelines and rules for organizing and displaying information that we now refer to as information mapping. These guidelines have their origins in such areas as these:

- logical analyses of subject matters
- learning research findings
- teaching practice
- programmed instruction techniques
- display technology
- human factors research
- communications techniques, including effective writing principles

The implications of the various ideas were translated into practical form and were documented as rules or procedures for preparing information maps.

Example: A common research finding is that learning is enhanced when practice exercises and answers are given in close proximity to new material. This finding becomes the basis for the information map rule that a page of feedback questions and answers should generally be inserted after each map of new information.

---

**Coming Up** In the next few pages, we draw upon the field of education to illustrate how certain information map features were derived. We also outline briefly the process of designing and developing learning materials in book form.

The next chapter traces the actual process of writing maps from the present set of guidelines.

---

## INFORMATION MAP FEATURES DERIVED FROM LEARNING RESEARCH AND TEACHING PRACTICE

**Introduction** Although information map features have their origins in several fields, there is no doubt that their principal foundations lie in education and learning research. On the chart below, we indicate briefly some of the findings that led to the design of certain information map features. This chart (which is not intended to be exhaustive) is one example of the research support behind information mapping.

Naturally, the evidence is not all of equal strength, but we have tried to bring to bear on a practical task some of the most promising factors.

Because the experimental basis for some map features is extensive, we cite wherever possible research review articles to put the reader in touch with the main sources of evidence. In the citations below, such major review articles are marked by asterisks to distinguish them from reports of original research.

These results of educational research lead to . . .



. . . these implications for the design of instructional materials.

Active responding generally aids learning.  
(Lumsdaine and May\*, 1965;  
Briggs\*, 1968; Glaser\*, 1965)

The act of writing responses helps some learners.  
(Edling\*, 1968)

Feedback or knowledge of results (or 'reinforcement') often facilitates learning by:

- confirming or correcting learner's understanding
- providing a motivational effect
- improving scanning behavior

(Lumsdaine and May\*, 1965;  
Smith, 1964; Gagné and Rohwer\*, 1969; Glaser\*, 1965)



Insert feedback questions after introducing new materials.

Locate answers conveniently nearby.

continued on next page

INFORMATION MAP FEATURES DERIVED FROM LEARNING RESEARCH AND  
TEACHING PRACTICE, continued

These results of educational research lead to . . .



. . . these implications for the design of instructional materials.

The insertion of questions, "test-like events," after text segments has a positive effect on learning. Giving knowledge of results further increases the effect.

(Gagné and Rohwer\*, 1969;  
McKeachie\*, 1963)



Use feedback questions after maps with new information and use sets of review questions after natural clusters of maps and at the end of topic treatments. Provide answers as well.

Self-tests, pretests facilitate retention.

(Glaser\*, 1965; Briggs\*, 1968;  
Bloom\*, 1963)

In concept learning, a variety of examples promotes learning.  
(Gagné and Rohwer\*, 1969;  
Lumsdaine\*, 1963)



Use examples and nonexamples to point up differences and similarities among concepts.

Instructions are useful in calling learner's attention to important features.

(Gagné and Rohwer\*, 1969;  
Gagné, 1965)



Use introductory paragraphs or previews to alert learner to importance of upcoming ideas.

Judicious use of underlining often helps to focus attention on key elements.

(Hershberger and Terry, 1963)



Underline important words in definitions.

continued on next page

INFORMATION MAP FEATURES DERIVED FROM LEARNING RESEARCH AND  
TEACHING PRACTICE, continued

These results of educational  
research lead to . . .



. . . these implications for the  
design of instructional materials.

"Cueing" or labelling appears  
to aid by alerting learner to  
nature of upcoming information  
and informing him what his  
learning task is.  
(Glaser\*, 1965)



Use marginal labels and  
informative map titles.

Pictorial materials often  
help learning.  
(Briggs\*, 1968)



Use diagrams and drawings to  
illustrate concepts and procedures

For some kinds of materials,  
charts of the information are  
valuable.  
(Feldman, 1965)



Use tables and verbal matrices  
to display concept relations.

Simple sentence structures in  
the active voice make learning  
easier.  
(Gagné and Rohwer\*, 1969;  
Coleman, 1965)



In general, use active voice  
and simple sentences.

## ORGANIZATION AND INTEGRATION OF INFORMATION IMPORTANT FOR LEARNING

### Introduction

Some important features of information mapping owe their origins to a topic of current theoretical interest among learning psychologists - namely, the logical and psychological structures of knowledge and their impact on learning and retention.

### Theoretical Discussions

Piaget had long ago speculated that "learning . . . is facilitated by presenting materials in a fashion amenable to organization" (Flavell, 1963), but it is only in recent years that psychologists have actively taken up the problems of how cognitive structures develop and of the role of organization in learning and retention.

The 'atomistic' approach of most programmed instruction materials has been criticized (Stafford and Combs, 1967) and a firm case made for the advantages of "meaningful organization and holistic presentation of materials."

In a symposium on "Education and the Structure of Knowledge" (Phi Delta Kappa, 1964), P.H. Phenix remarked: "It is difficult to imagine how any effective learning could take place without regard for the inherent patterns of what is to be learned."

David Ausubel (1960, 1963, 1964, 1968) has developed a logical and psychological case for believing that learning and long-term retention are facilitated by 'organizers' which provide an 'ideational scaffolding.' He has now amassed considerable experimental support for his hypotheses.

The well-known studies of Katona (1940) with college students pointed up the importance of organization for learning and for retention.

The relation of organization of materials to ease of learning also finds support in the area of verbal learning research (Underwood\*, 1966).

### Implications for Information Map Books

Although many issues remain to be settled by research, a strong case can be supported both logically and empirically for the advantages of organizing and integrating features in materials for learning. Both verbal and graphical means can be used to inject a sense of organization and direction into a subject-matter presentation.

In the practical effort to design effective learning materials, we have incorporated a number of features intended to help the learner integrate and organize the ideas for more efficient storage in memory. These are listed on the next page.

## FEATURES TO AID IN ORGANIZATION AND INTEGRATION

### Introduction

The following list of features designed to promote integration of concepts and relationships contains some that we have already adopted on other grounds. For instance, the guidelines called for practice questions and answers throughout the text because learning research suggested their value in several ways; but questions can also be phrased to encourage integration of ideas over sections of learning materials.

Examples of maps showing some of these features are given in Chapter 2.

### List of Features

- reviews and previews: to take stock of the ideas developed up to that point and to prepare the ground for relating them to new concepts about to be encountered
- introductions to each map: to relate new idea to previous concepts or to familiarize with nature and importance of new idea
- recaps or capsules: to summarize succinctly the essential ideas of rules or principles in nutshell form
- tree diagrams: to sketch the ideas and procedures of a topic so as to show the role of each and its links to others
- compare-and-contrast tables: to point up the similarities and differences between two concepts that are sometimes confused
- summary tables: to chart in easy reference form the main concepts of an area
- review tests after short sets of maps and at the end of units: to promote the integration of several concepts and to practice using them in problem solving
- prerequisite charts: to show schematically the paths the learner can take through a subject matter in order to reach the learning objectives

## INFORMATION MAP FEATURES FOR EASE OF REFERENCE

### Introduction

In designing book-type materials for initial learning, we added features to facilitate the return to ideas previously encountered, an activity that is often frustrating with conventional texts where the contents of the paragraphs are unlabelled. Common sense, human factors research, and graphic technology were used in formulating aids for easy access to the learning materials. A list of these aids appears below.

It is clear also that these same features would be important for reference manuals or job aids. If information map materials were designed for those purposes alone, some of the introductions, explanations, and examples needed for initial learning would be omitted.

Again we note that some of the features needed for easy reference purposes have already been mentioned as desirable on other grounds. For example, labels on information blocks aid in quick retrieval of ideas but they also serve to alert the learner to the nature of his learning task and prepare him to take in a specific kind of information.

### List of Features

- Tables of contents for learning books are organized and formatted to speed location of topics and special features. (This report does not use the standard format but follows certain ESD report requirements)
- A predictable format for each type of map (concept, procedure, etc.) facilitates location of needed information.
- Map headings in consistent typography help in scanning for page topic.
- Marginal labels help not only in locating the kinds of information sought but also in skipping those not required.
- Local indexes at foot of each map permit quick location of concepts relevant to the given map.
- Decision tables display the choices appropriate for each possible situation.
- Summary tables assemble main facts and relations for easy review and reference.

continued on next page

## INFORMATION MAP FEATURES FOR EASE OF REFERENCE, continued

(continued)  
List of  
Features

- 
- Capsules provide "kernel" statements of key rules or concepts
  - Flow charts show graphically the sequences of events in a process
  - Indexes aid information retrieval.
-



## OTHER PARTS OF THE INFORMATION MAP SYSTEM

### Introduction

So far we have been concerned with what information maps look like, how they got that way, and how they are written.

But the process of writing cannot begin until fundamental curriculum plans are worked out. Furthermore, the end of the writing task is by no means the end of the production process -- a crucial part of that process is the series of try-and-revise cycles through which the product is refined and the learning outcomes are brought closer to the program objectives.

The information mapping system, then, includes guidelines for curriculum planning and for developmental testing.

### Curriculum Planning

Once the subject-matter area of the project has been agreed upon, a series of interrelated decisions must be settled, including the type of audience for which the program is intended, the conditions under which it is to be used and so forth. When the scope of the program has thus been defined, charts showing the nature of the writing task are evolved through the following steps:

- The nature of the subject matter is explored and the potential topics are listed.
- The learning objectives for the specific program are determined and are stated in behavioral terms.
- The topics that are required to meet the specified learning objectives are organized into a schematic display called the "preliminary prerequisite chart" -- a chart working backward from the objectives to the topics that are required to meet those objectives.
- Analyze the nature of the learning tasks and plan the teaching strategies for achieving them.
- Revise the prerequisite chart to show the assembling of concepts into the networks of associations building toward the final instructional goals.

### The Prerequisite Chart

This chart of the topics and their sequencing plus special learning materials serves as a guide to the writer in his task. The process of writing is illustrated in Chapter 2.

continued on next page

## OTHER PARTS OF THE INFORMATION MAP SYSTEM, continued

### Successive Approxima- tions

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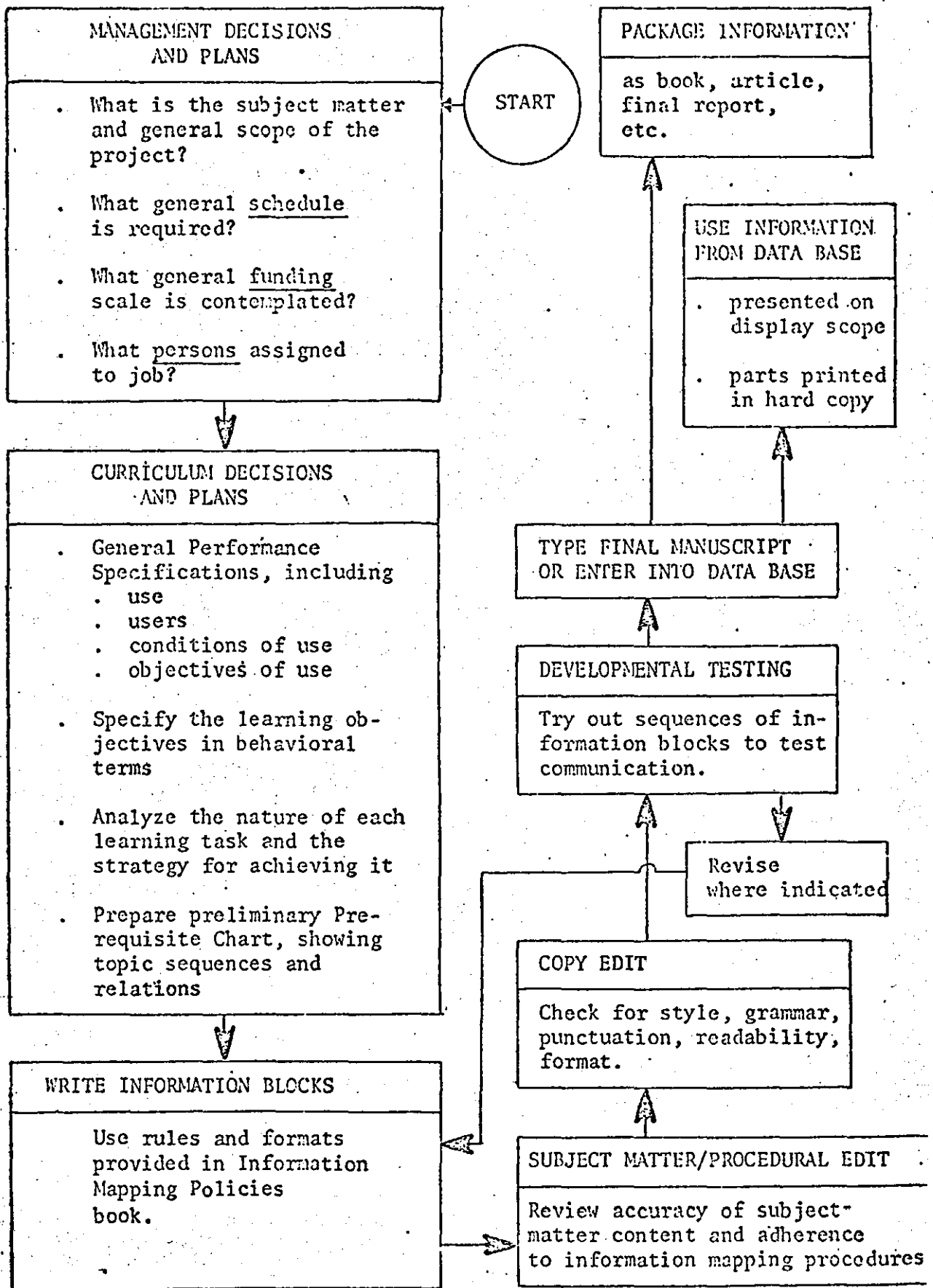
Because teaching and writing are both arts, we do not expect the first draft of a learning program to be totally successful. We rely heavily on the iterative process -- cycles of tryouts with students and revisions of the materials in response to their reactions.

The most important aspect of these tryouts is that the feedback questions and sets of review questions spaced throughout the program give us immediate evidence of the topics that need amendment or expansion.

Developmental tryouts and revisions are key tools in the production of effective information-mapped materials.

---

# THE PROCESS OF INFORMATION MAP PRODUCTION



## THE CONTINUING EVOLUTION OF THE INFORMATION MAP SYSTEM

Introduction	We have mentioned how the guidelines and processes of the information map system first came to be formulated. But the initial statements were only the beginning of a development process that continues into the present.
Developmental Testing	The rules and guidelines were tried out in the preparation of learning materials in several subject-matter areas. As these products took shape, they were subjected to tryout-and-revise cycles with college-age subjects. The students' responses to feedback questions throughout a given map series gave us a basis for continuing improvement of the learning units. But more important in the early stages was the value the responses had for refining the system itself. Rules were amended, format policies were changed, new procedures were introduced. The system continues to evolve gradually as our experience grows and as new situations are encountered.
Subject-Matter Experience	<u>So far we have applied the system mainly to topics in mathematics.</u> The major part of our experience was gained in writing, testing, and reviewing a 150 page <u>introduction to sets and probability</u> . This book of mapped learning materials constitutes a ten hour self-instructional course; it served as the main research vehicle for the studies reported in Chapters 3 to 7.

It was also an important influence on the development of the system itself. Maps from this work are shown in Chapter 2 to illustrate the writing process and to show the nature of the learning materials.

Other subject areas with which the system has been tried are:

SUBJECT MATTER	APPROX. NO. OF INFOMAPS
Computer programming	75
The binary number system	60
Convrs, an experimental computer language	150
Canard, a simulation language	150
Introduction to descriptive statistics	75
Introduction to matrix algebra	35
Permutations, combinations, and the binomial theorem	50

continued on next page

## THE CONTINUING EVOLUTION OF THE INFORMATION MAP SYSTEM, continued

(continued)  
Subject-  
Matter  
Experience.

A wider range of topics was explored by a group of graduate students in a summer school course in 1967. They prepared brief units on:

- basic concepts of operant conditioning
- some topics in American history
- a variety of educational research concepts and procedures
- two dentistry topics: how to extract a tooth, and periodontology
- a topic in chemistry: the structure of the atom
- the Munsel color system in art
- darkroom procedure in photography
- several topics in mathematics

Comment

Experience with other subject areas will undoubtedly raise new classification and display problems for which guidelines will have to be devised. But we expect that the main impetus to the evolution of the system will come from continuing research.

## APPENDIX C

### 5. - EXAMPLE OF INFORMATION MAPPING APPLIED TO MATHEMATICS: I.

Material used by the author in an evaluation of the applicability of the technique to undergraduates at the Middlesex Polytechnic. (Extract from Horn, 1971)

#### TO THE STUDENT

##### About This Book

This Information Map book is an introduction to elementary probability theory for students in the behavioral sciences.

It presents those probability concepts that are the groundwork for a proper understanding of modern statistics.

This book is designed to be adapted by you

- for your own purposes, whether they be initial learning, review, or reference
- for your own level of proficiency, whether you are a novice in mathematics or a competent student of the subject
- for your own choice of mastery level, whether you want only to survey the major ideas of the area or to become proficient in the subject.

The pages have a consistent structure, clearly indicated by marginal labels. Their format makes it easy for you to focus on the kind of information you need.

##### Specific Pre-requisites

The only skills you need to use this book are:

- arithmetic operations, including a moderate facility with decimals and fractions
- a modest acquaintance with algebraic expressions.

##### What Level

This book is appropriate for students at many levels of mathematical competence. In the basic treatment, little formal mathematical training is assumed--yet the clear labelling of information blocks enables the competent mathematics student to scan quickly.

**SELF-TEST:** Use this self-test to see how much of this unit you need to learn. Page numbers below refer to pages in the unit where the topic is introduced.

1. A cube has 4 identical red sides and 2 identical green sides. We throw this cube 3 times in succession.
  - A. List the points of the sample space. [Page 8]
  - B. List the points of the event "at least two red sides appear." [Page 13]
  - C. Define the complement of the event in B. [Page 34]
  - D. Are the events "at least one red side appears" and "at least one green side appears" mutually exclusive? [Page 30]
  - E. Write the event "exactly one green side appears" as the union of elementary events. [Page 20]
2. An urn contains 3 red balls numbered one to three and 2 black balls numbered four and five. We draw a ball at random from the urn.
  - A. Assign probabilities to the elementary events. [Page 39]
  - B. Find the probability of the event "a ball with an even number is drawn." [Page 43]
  - C. Find the probability of the complement of the event "a ball with an even number is drawn." [Page 48]
  - D. What are the odds in favor of drawing a ball with an even number? [Page 51]
  - E. What is the conditional probability that the ball drawn is red given that the number on the ball is even? [Page 62]

**ANSWERS:**

- 1] A) {RRR,RRG,RGR,RGG,GRR,GRG,GGR,GGG}; B) {RRR,RRG,RGR,GRR}; C) "less than two red sides appear" or {RGG,GRG,GGR,GGG}; D) No; E) {RRG}∪{RGR}∪{GRR}.
- 2] A) Since the ball is drawn at random we can assume all outcomes equally lik so  $P(\{R_1\}) = P(\{R_2\}) = P(\{R_3\}) = P(\{R_4\}) = P(\{R_5\}) = 1/5$ ; B)  $P(\text{even}) = P(R_2) + P(B_4) = 2/5$ ; C)  $P(\text{odd}) = 1 - P(\text{even}) = 3/5$ ; D) 2 to 3; E)  $P(R|E) = \frac{P(R \cap E)}{P(E)} = \frac{1/5}{3/5} = \frac{1}{3}$ .

## UNIT 1 SELF TEST (Continued)

3. We have two bags of marbles. The first bag contains 7 white and 3 black marbles, and the second bag contains 4 white and 6 yellow marbles.

A. If we choose a bag at random and then draw a marble at random from the bag selected, what is the probability that a white marble is drawn?

[Page 67]

B. If we choose a marble at random from the first bag and then independently choose a marble at random from the second bag, what is the probability that at least one is white?

[Page 69]

C. If we choose two marbles at random from the first bag, what is the probability that they are both black?

[Page 72]

D. If we choose two marbles at random from the second bag, what is the probability that they are of different colors?

[Page 75]

E. If we choose a marble at random from the first bag and then independently choose a marble at random from the second bag, what is the probability that neither marble is white?

[Page 78]

## ANSWERS:

- 3] A)  $P(W) = 7/20 + 4/20 = 11/20 = .55$ ;  
 B)  $P(W_1 \cup W_2) = P(W_1) + P(W_2) - P(W_1 \cap W_2) = .7 + .4 - .28 = .82$ ;  
 C)  $P(B_1 \cap B_2) = P(B_1)P(B_2|B_1) = (3/10)(2/9) = 6/90$ ;  
 D)  $P(Y_1 \cap W_2) + P(W_1 \cap Y_2) = (4/10)(6/9) + (6/10)(4/9) = 48/90$ ;  
 E)  $P(\bar{W}) = P(B_1 \cap Y_2) = (.3)(.6) = .18$ .



## PREVIEW OF PROBABILITY THEORY

### Importance

Probability theory is the foundation for the statistical tools that are so much a part of the modern scene. Physical and social sciences, engineering and business rely on statistical methods to advance their search for knowledge. To use statistical techniques correctly, the scientist or professional man must know something about the nature of the underlying concepts. The pages that follow will introduce the key ideas of probability theory.

### Intuitive Ideas

Many probability terms are in common daily use. We match coins to see who will buy coffee at coffee-breaks and we expect to win about half the time. We talk about the small probability of getting a perfect hand in bridge and about the large probability that it will rain on the day we plan a picnic.

Our intuitive ideas of probability have much in common with the mathematical theory but they lack precision, cannot cope with complex problems, and sometimes even lead us astray.

One element apparent in our intuitive ideas of probability is the notion of uncertainty -- in each case the event cannot be predicted with confidence. This property of uncertainty is a key attribute of the events with which probability theory is concerned. We often say the events are "non-determined" or "random." Probability theory devises models for describing such random phenomena.

### Relation to Real World

A characteristic of mathematical theory is that it is not directly concerned with real situations but only with conceptual models of them. Just as the theorems of geometry are statements about an idealized model and not about the actual physical triangles we draw, so also do the postulates of probability theory refer to an abstract, hypothetical world. When we apply the ideas of probability to a real situation, we are in fact building a model of the situation, a mathematical representation.

In model building we can define an ideal world where conditions can be simplified so that the central issues of a problem stand out. We assume that dice are perfect, for example, and we assume that they are fairly thrown. Then we compute the probability of getting a certain result, given these assumptions. Whether or not the answer is accurate for describing actual results with a real pair of dice depends on how closely the real conditions parallel the assumptions of the model.

When we apply probability ideas to the data of scientific experiments, we are using a certain probability model to help interpret the results. The accuracy of our interpretations will depend on how closely the assumptions of the model fit the conditions of the experiment.

To apply probability theory correctly to experimental data requires a thorough understanding of the underlying assumptions.

## SIMPLE EXPERIMENT

synonym: "experiment"

## Introduction

Many acts lead to a distinct result each time they are done but the *specific* result cannot be predicted. A tossed coin will land either heads or tails but we cannot predict which with certainty. Such "simple experiments" can be described by telling

- . what is to be done (i.e., the procedure), and
- . what is to be observed and recorded.

## Example One

Procedure: roll a pair of dice.

Observe: what numbers show, or are face up, after the dice stop rolling.

## Definition

A simple experiment is a well-defined, repeatable procedure that leads to a clearly specified result having the following properties:

- . it is capable of being observed and recorded each time the experiment is performed
- . it cannot be predicted with certainty.

## Example Two

Procedure: on a production line, examine products after they have been made.

Observe: the number of defective products.

## Comment

It should be noted that a simple experiment may be either theoretical (imaginary) or actually carried out. In either case we know what the *possible* outcomes could be but not *which one* will actually occur.

As a term used in elementary probability, the simple experiment bears slight resemblance to experiments in the sense used in science where a more elaborate series of trials are carefully carried out under varied, specified conditions.

## OUTCOMES OF A SIMPLE EXPERIMENT

synonym: "sample point"

## Definition

Each possible distinguishable result of a simple experiment is called an outcome.

## Notation

Outcomes are sometimes represented by a lower case "o" and subscript notation. Thus we may write

$$\{o_1, o_2, \dots, o_n\}$$

as the list of all possible outcomes of an experiment.

Sometimes outcomes can be represented by a word or some other symbol enclosed in braces:

{hot, cold}

{all red dogs}

## Comment

Notice that the outcomes are distinctly separate, non-overlapping results. By definition, if one occurs, no other outcome can occur also. This concept is particularly important when we come to assign probabilities.

## Example One

A simple experiment is to toss a tack into the air and to see how many times it falls point up and how many times it falls point down. The only possible outcomes for any one trial are:

{up, down}



## Example Two

An urn contains five numbered grey balls and five numbered white balls, thoroughly mixed.



A simple experiment is to draw out one ball while blindfolded. The outcomes for this can be represented as follows (using "G" for grey and "W" for white and subscripts to tell the balls apart):

$$\{G_1, G_2, G_3, G_4, G_5, W_1, W_2, W_3, W_4, W_5\}$$

(continued)

Related  
Pages

simple  
experiment, 4

# OUTCOMES OF A SIMPLE EXPERIMENT (continued)

## Example Three

A die is thrown three times and we observe whether or not a one turns up. Since we are not interested in any number but one, we can write 1 for the result of a one, and / (or "not one" for any other number on the die (2,3,4,5,6). The outcomes are

{111, 11/, 1/1, /11, 1//, /1/, //1, ///}

where 111 means three ones turned up, 11/ means the first two throws turned up one and the last some other result, etc.

## Example Four

A married couple is asked whether or not they are registered to vote, and the results are recorded. The outcomes of this experiment are:

{RN, NR, RR, NN}

where R stands for registered and N for not registered.

## Example Five

A coin is tossed three times.

Each column represents a possible outcome of the experiment.

1st toss								
2nd toss								
3rd toss								

The outcomes of this experiment can be written

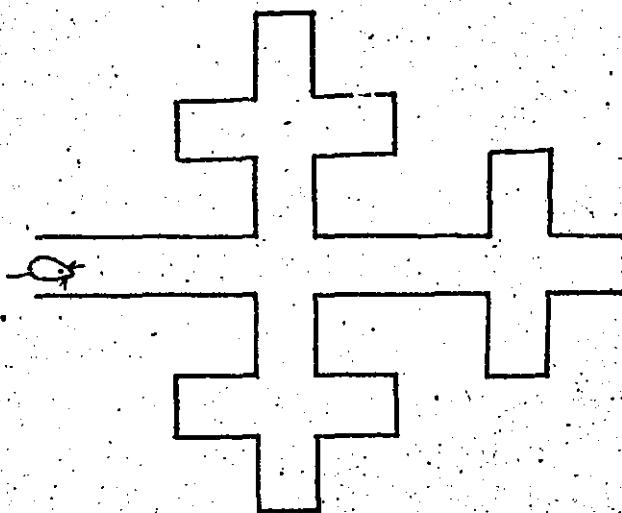
{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

where HHH indicates that the first, second, and third toss all resulted in heads, HHT indicates that the first and second toss resulted in heads and the third toss in tails, etc.

# FEEDBACK QUESTIONS

[Outcomes of a simple experiment]

1. A professor asks a group of three students one question that requires a "yes" or "no" answer. If we consider this process a simple experiment, list all the possible outcomes, using Y for "yes" and N for "no."
2. A maze is constructed so that each path has two forks and at each fork there are three alternatives: straight (S), right (R), left (L). A mouse is placed in the maze. What are the possible paths he can take (i.e., possible outcomes of the experiment?)



## ANSWERS:

- 1] { YYY,YYN,YN Y,YNN,NYY,NYN,NNY,NNN } where YYY indicates that the first, second, and third students all answered yes, etc.
- 2] { SS,SR,SL,RS,RR,RL,LS,LR,LL } where SS indicates that the mouse went straight at the first and second forks he came to.

## SAMPLE SPACE

## SAMPLE POINT

synonym: "sample description  
space"

synonym: "outcome"

## Introduction

In a simple experiment, a number of distinct outcomes are possible. The outcomes that are actually observed must be interpreted in relation to all possible outcomes. A special term is used for this set of all possible outcomes.

## Definition

A sample space is the set of all possible distinct outcomes of an experiment.

A sample point is a single element of a sample space. Every separate possible outcome of an experiment is represented by a single sample point.

Note: "Space" in the term sample space does not refer to any physical dimension. It is merely a term given to a collection of outcomes of an experiment.

## Notation

We frequently let the capital letter "S" stand for a sample space. Sample points are represented by words or symbols contained in braces.

$$S = \{H, T\}$$

is read, "The sample space consists of the sample points H and T."

## Example

The experiment is one toss of a coin. To describe the set of possible outcomes in the sample space, we can use the symbols {H,T}. These are the only two sample points.

## Non-example

The experiment is to select a child from a fourth grade class. We cannot write the sample space as {boy, girl, age 8, age 9} because we would not have distinct outcomes. If we were interested in both the age and sex of the child selected, our sample space would be {8 yr. old boy, 9 yr. old boy, 8 yr. old girl, 9 yr. old girl} (assuming there were no other ages represented in the 4th grade).

## FEEDBACK QUESTIONS

[Sample space ... Sample point]

1. Describe the sample space for the following experiments:

A. A card is drawn from a standard deck of 52 cards.

B. A boy is chosen from the freshman class at a particular all-male college.

C. A mouse is chosen from a group of laboratory mice to run a maze.

2. Check each true statement:

A. \_\_\_\_\_ The elements of a sample space are called sample points.

B. \_\_\_\_\_ A sample point represents one or more possible outcomes of an experiment.

C. \_\_\_\_\_ Words or symbols enclosed in braces are used to represent the sample points in a sample space.

### ANSWERS:

1] A)  $S = \text{All the cards in the deck};$

B)  $S = \text{All freshmen at that college};$

C)  $S = \text{All mice in the laboratory}.$

2] A) True; — B) False (a sample point represents exactly one possible outcome of an experiment); C) True.

## HOW TO USE A TREE DIAGRAM TO FIND THE SAMPLE POINTS IN A SAMPLE SPACE

## Introduction

To insure that all the points in a sample space have been counted, it is best to go about the task of identifying them in a systematic way. One useful method is the tree diagram.

G I V E N	Write out a list of the sample points in the sample space.	<b>EXAMPLE</b> The experiment is tossing a coin 3 times.
STEP	PROCEDURE	
1.	Give a name or a short-hand symbol to each of the kinds of things.	Heads will be labelled H, and tails labelled T.
2.	From a starting point draw a branch for each possible result in the first step	<div style="display: flex; justify-content: space-around; margin-bottom: 10px;"> <span>First Toss</span> <span>Second Toss</span> <span>Third Toss</span> </div>
3.	Next from the end of <u>every branch</u> draw another branch for <u>each</u> outcome that could occur on the second step of the problem.	
4.	Do the same for additional steps and write all the branch labels at the end of each branch.	
5.	To find the sample points, follow each path from start to finish, listing the branch labels.	<div style="display: flex; justify-content: space-around;"> <div>             HHH HHT HTH HTT           </div> <div>             THH THT TTH TTT           </div> </div>

Related  
Pages

sample point, 8

sample space, 8

outcome, 6



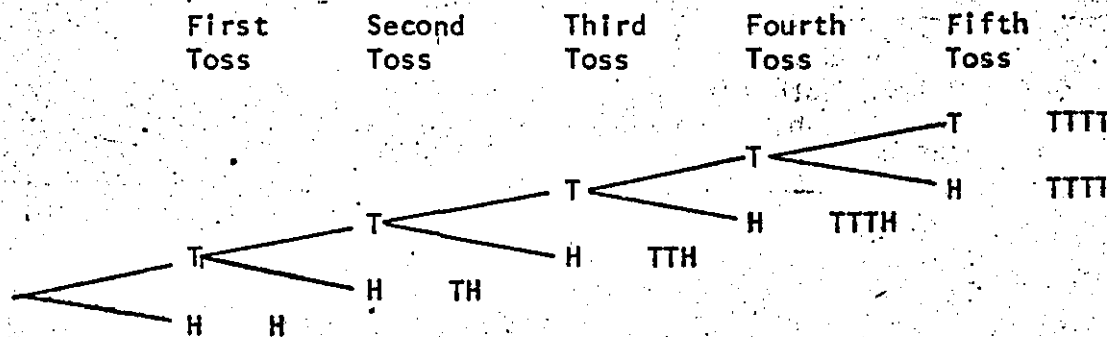
# FEEDBACK QUESTION

[Tree diagrams]

1. A coin is tossed five times or until a head appears, whichever comes first. Use a tree diagram to find the sample points in this experiment.

ANSWER:

1]



The points of sample space are:

{H, TH, TTH, TTTH, TTTTH}

## REVIEW QUESTIONS

[1]

1. An urn contains two red and three green balls. A ball is selected from the urn and its color noted. List the elements of the sample space.
2. Check which of the following is not a property of the result of a simple experiment.
  - A. ☐ It cannot be predicted with certainty.
  - B. ☐ It must support some scientific theory.
  - C. ☐ It is capable of being observed and recorded each time the experiment is performed.
3. A die is rolled and the resulting number is observed.
  - A. List the elements of the sample space  $S$ .
  - B. List the sample points in  $S$  that are even-numbered results.
  - C. List the sample points in  $S$  that are results greater than five.
4. A shipment of 100 precision instruments reached a certain laboratory. Suspecting that some may be defective, a scientist decides to select three instruments in succession. As soon as he comes across one that is defective he will reject the shipment. Otherwise he will accept it. List the elements of the sample space and check those that would result in the return of the shipment.

## ANSWERS:

- 1]  $\{R, G\}$ .
- 2] B.
- 3] A)  $S = \{1, 2, 3, 4, 5, 6\}$ ; B)  $\{2, 4, 6\}$ ; C)  $\{6\}$ .
- 4]  $\{GGG, GGD, GD, D\}$  where GGG means all the instruments are good, GGD means the third instrument inspected is defective, etc.

## APPENDIX C

### 6. - EXAMPLE OF INFORMATION MAPPING APPLIED TO MATHEMATICS: II

Material developed by the author (in conjunction with P. Ellis) for use in a comparative study between information mapping and linear programmed instruction.

#### MATRICES

#### UNIT\_I

NOTE: This material is based on a linear programme developed at the Centre for the Advancement of Mathematical Education in Technology (CAMET), University of Technology, Loughborough. The original programme was developed specifically for an adult education project of the Council of Europe. This version was prepared as an alternative presentation format. The content and examples in the two versions are identical. Only the layout and organisation has been modified.

## INTRODUCTORY NOTES TO THE STUDENT

### This Matrices Unit

This unit has been written to investigate the possibilities of teaching mathematical material by means of a technique known as 'information mapping'.

Since this is designed as an investigation, some people will get the material presented in a different format, but everyone will cover the same information on matrices.

### Content

This unit will relate matrices to charts, and will tell you what matrices are. Some of the technical terms used in referring to matrices will be explained, and the ways that matrices can describe networks will be shown.

The next unit will cover some methods of manipulating matrices.

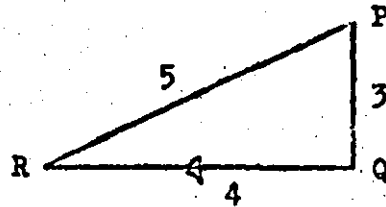
### What Skills you need

You do not need to know any mathematics other than addition, subtraction, multiplication and division.

PRE-TEST

Please complete this test so that we will have some idea how much you already know about matrices. Don't worry if you cannot answer any of the questions, the unit that follows is designed to teach all these points.

1. What is the order of the matrix  $\begin{bmatrix} 3 & 2 & 1 \\ 1 & 4 & 3 \end{bmatrix}$
2. Give the address of the number 4 in the matrix in Q.1
3. Construct a square matrix with 3 columns, with all the elements of the matrix being 1.
4. Write down the distance matrix for this road network. - the arrow represents a one way road



5.  $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \\ 6 & 0 \end{bmatrix}$

What is the transpose of A?

SING CHARTS - A REVIEW

Introduction

When two sets of variables are to be compared, data is often presented in the form of charts. Examples of typical charts are Figures 1, 2 and 3.

Methods of  
Using Charts

METHOD	EXAMPLES
<p>1. A particular entry in a chart may be extracted by referring to the row and column in which it occurs.</p>	<p>In figure 1, by looking for the intersection of the 'Blackpool' row and the 'Hours of sunshine' column we can read off that Blackpool had 75 hours of sun. The arrows illustrate this reading off. Similarly Bristol to Carlisle is 272 miles in the mileage chart (Figure 2)</p>
<p>2. All the data elements for one variable may be considered together. To do this you take either</p> <ul style="list-style-type: none"> <li>a) A whole row</li> <li>b) A whole column</li> </ul>	<p>(a) Fig.1 - Look along the 5th row to find the weather at Lerwick - it is cloudy, wet and cold.</p> <p>(b) Fig.1 - Look down the 1st <del>row</del> <sup>column</sup> to find out the amounts of sun received at the towns. - Weymouth with 11.0 hours received the most.</p>

## THE DISTANCE CHART

### Introduction

This is Figure 2. This is an interesting case, as the labels of the rows and columns are the same. As a result one can find each distance entry twice. (the distance from A to B is the same as the distance from B to A).

### Diagram

Aberystwyth		214	174	244	121	217	100	218	45
Barnstaple	214		180	188	93	239	136	365	202
Birmingham	114	180		160	87	100	102	193	129
Brighton	244	198	160		136	103	176	350	242
Bristol	121	93	87	136		118	43	272	109
Cambridge	211	239	100	105	118		174	256	225
Cardiff	100	136	102	176	43	174		275	65
Carlisle	218	365	193	350	272	256	275		262
Carmarthen	45	202	129	242	109	225	65	262	

Each entry is found in corresponding positions in both the shaded and the unshaded part of the table. The shaded part of the table "mirrors" the unshaded portion. The table thus possesses a certain symmetry about the diagonal line drawn in.

### Alternative Presentation

A different form of presentation is shown in figure 3. This has discarded the duplicated entries resulting in a triangular chart which has been labelled along the diagonal. The arrows show the row and column indicated by Bristol.

To find the distance from Brighton to Cardiff the row and column required are:

Brighton column, Cardiff row  
The entry here is 176

(There is no entry in the Brighton row, and Cardiff column, - that was in the part that was thrown away).

FEEDBACK QUESTIONS

1) Use figure 1 to find:-

- a) The maximum temperature at Weymouth
- b) The rainfall at Lerwick
- c) The highest rainfall, and where this occurred
- d) The highest maximum temperature, and where this occurred.

2) Which of the following best describes the weather,  
at (i) Blackpool (ii) Whitstable (Use Figure 1)

- a) Cold, Wet and Cloudy
- b) Some sunshine, cool, and a little rain
- c) Warm and sunny with a little rain
- d) Sunny, dry, but cool.

3) Use figure 2 to find

- a) The distance from Barnstaple to Cardiff
- b) The distance from Carmarthen to Birmingham

4) Use figure 3 to find the distances

- a) Barnstaple to Cambridge
- b) Carlisle to Brighton.

- ANSWERS:
- 1) (a) 55°F (b) 0.43 ins. (c) 0.43 ins. at Lerwick (d) 59°F at Jersey.
  - 2) (i) Blackpool = (b) Some sunshine cool and a little rain.  
(ii) Whitstable = (c) Warm and sunny with a little rain
  - 3) (a) 136 miles (b) 129 miles.
  - 4) (a) 239 miles (b) 350 miles.



# MATRIX (Plural MATRICES)

## Definition

A rectangular array of numbers.

## Notation

The array is enclosed in square brackets:-  $[ \quad ]$

## Use

Matrices arise from physical situations, that is from data that can be presented in charts.

They have several mathematical properties which can be used in manipulating the data. We will be learning about these properties, and we will apply them to the real physical situations from which the data arose.

## Example 1

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$   $\begin{bmatrix} 3 & 7 & 6 \\ 2 & 9 & 5 \\ 1 & 0 & 3 \end{bmatrix}$   $\begin{bmatrix} 3 \\ 1 \\ 4 \\ 2 \end{bmatrix}$  are all examples of matrices

## Example 2

By omitting the row and column headings in figure 1 (Weather in British towns) the following matrix is obtained

10.9	0	52
7.5	0.04	54
8.1	0.08	54
8.8	0.02	59
0.2	0.43	48
8.6	0	57
11.0	0	55
10.8	0.10	57

## Example 3

Figure 2 (Distance chart) can be converted into a matrix thus:

0	214	114	244	121	211	100	218	45
214	0	180	198	93	239	136	365	202
114	180	0	160	87	100	102	193	129
244	198	160	0	136	105	176	350	242
121	93	87	136	0	118	43	272	109
211	239	100	105	118	0	174	256	225
100	136	102	176	43	174	0	275	65
218	365	193	350	272	256	275	0	262
45	202	129	242	109	225	65	262	0

NOTE: the dashes in the chart have been replaced by zeros (the distance between a town and itself is 0). Zeros are used because '0' is a number '-' is not (see definition of a matrix)

MATRIX (Plural MATRICES) (cont'd.)

Non Example

Figure 3 does not give a matrix since

214								
114	180							
244	198	160						
121	93	87	136					
211	239	100	105	148				
100	139	102	176	43	174			
218	365	193	350	272	256	275		
45	202	129	242	109	225	65	262	

is a triangular array, not rectangular.

# ORDER OF MATRIX/TYPES OF MATRICES

## INTRODUCTION

When discussing charts we talked of rows and columns.  
Matrices are also made up of rows and columns.

## DEFINITION

The ORDER of a matrix is the number of rows and columns it has. The rows are stated first and the columns second.

## EXAMPLE

$$\begin{bmatrix} 1 & 5 & 7 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 4 & 6 \end{bmatrix}$$

is called a 3 x 4 matrix, as it has  
3 rows and 4 columns.

Its ORDER is 3 x 4.

## SPECIAL EXAMPLES

### (a) SQUARE MATRIX

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

is of the order 3 x 3. It has the same number  
of rows and columns. Such a matrix is called  
a SQUARE matrix.

### (b) SINGLE ROW (Row Vector)

$$\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$$

is a single row of numbers.

Order 1 x 4.

### (c) SINGLE COLUMN (Column Vector)

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

is a single column of numbers.

Order 4 x 1.

## COMMENTS (a)

Square matrices have some special properties (see MAPS 10 & 11)

## (b & c)

Single row and single column matrices are also referred to as  
row and column vectors. More information on these is given  
on MAP 7.

FEEDBACK QUESTIONS

- 1) Write down the matrix formed by leaving the row and column headings out of the following table:

	STOCK LIST		
	Floral	Abstract	Plain
Plates	58	94	102
Dishes	64	86	45

- 2) What is the order of these matrices:

(i)  $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$  (ii)  $\begin{bmatrix} 3 & 0 & 6 \\ 2 & 1 & 5 \\ 7 & 2 & 3 \end{bmatrix}$

- 3) Which of the following are square matrices?

(i)  $\begin{bmatrix} 2 & 3 \\ 0 & 4 \end{bmatrix}$  (ii)  $\begin{bmatrix} 1 & 3 & 5 \\ 2 & 7 & 9 \end{bmatrix}$  (iii)  $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

- 4) Which of the following matrices are row vectors, and which column vectors?

(i)  $\begin{bmatrix} 2 & 3 \\ 0 & 4 \end{bmatrix}$  (ii)  $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$  (iii)  $\begin{bmatrix} 3 & 0 & 2 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$  (iv)  $[4 \ 0 \ 3 \ 7]$

- ANSWERS; 1)  $\begin{bmatrix} 58 & 94 & 102 \\ 64 & 86 & 45 \end{bmatrix}$
- 2) (i)  $2 \times 3$ ; (ii)  $3 \times 3$
- 3) (i) and (iii)
- 4) (ii) is a column vector, (iv) is a row vector

## ROW VECTORS AND COLUMN VECTORS

### INTRODUCTION

Row vectors may be combined to make larger matrices and so may column vectors.

It is also possible to split a matrix into row vectors or column vectors.

### EXAMPLE 1

The costs of raw materials, labour, and transport for one manufacturer are:

in England  $\begin{bmatrix} 15 & 45 & 5 \end{bmatrix}$

in Germany  $\begin{bmatrix} 20 & 30 & 10 \end{bmatrix}$

These row vectors can be combined into a single matrix

$$\begin{bmatrix} 15 & 45 & 5 \\ 20 & 30 & 10 \end{bmatrix}$$

### EXAMPLE 2

The matrix  $\begin{bmatrix} 3 & 1 & 5 \\ 2 & 4 & 3 \end{bmatrix}$  can be thought of as.

(a) two row vectors;  $\begin{bmatrix} 3 & 1 & 5 \end{bmatrix}$  and  $\begin{bmatrix} 2 & 4 & 3 \end{bmatrix}$

OR (b) three column vectors  $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$   $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$   $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$

### COMMENT

When you use a chart listing weather conditions and towns, as in Figure 1, to read off the weather at a town - you are effectively splitting it into row vectors. If you look down the columns to find the highest temperature or the shortest sunshine, you are effectively splitting it into column vectors.

## ADDRESS IN A MATRIX

### INTRODUCTION

If we wish to identify a particular position, or data element in a matrix we may name the row and column in which it is found.

### DEFINITION

The row and column numbers in which a particular element is located, make up the ADDRESS of the element.

Row first, column second.

Rows and columns are numbered from the top left corner of the matrix.

### EXAMPLE

$$\begin{bmatrix} 3 & 1 & 4 \\ 2 & \textcircled{7} & \textcircled{5} \\ 3 & 2 & 0 \end{bmatrix}$$

(a) The number 7 is in row 2, column 2.  
Its address is 2,2.

(b) The number 5 is in row 2 column 3.  
Its address is 2,3.

FEEDBACK QUESTIONS

1) Convert the following matrix into

(a) Row vectors

(b) Column vectors

$$\begin{bmatrix} 3 & 4 & 2 & 5 \\ 1 & 2 & 7 & 3 \\ 4 & 1 & 3 & 3 \end{bmatrix}$$

2) Combine these row vectors to make a larger matrix

$$\begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \quad \begin{bmatrix} 4 & 7 & 5 \end{bmatrix}$$

3) Give the address of the number 3 and the number 7 in the following matrix

$$\begin{bmatrix} 2 & 1 & 5 & 7 \\ 6 & 0 & 2 & 5 \\ 8 & 3 & 4 & 2 \end{bmatrix}$$

ANSWERS: 1) (a)  $\begin{bmatrix} 3 & 4 & 2 & 5 \\ 1 & 2 & 7 & 3 \\ 4 & 1 & 3 & 3 \end{bmatrix}$  (b)  $\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 7 \\ 3 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 3 \end{bmatrix}$

2)  $\begin{bmatrix} 2 & 1 & 3 \\ 4 & 7 & 5 \end{bmatrix}$

3) Address of 3 is 3,2; Address of 7 is 1,4.

## LEADING DIAGONAL

### INTRODUCTION

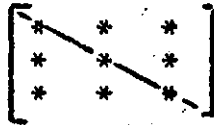
Some technical terms used apply only to square matrices. This is one of those terms.

### DEFINITION

The leading diagonal is made up of the elements in a square matrix which have the same row and column numbers.

### DIAGRAM

In the following representation of a 3 x 3 matrix with the elements represented by stars, the elements forming the leading diagonal have been joined by a line



### EXAMPLE

In this matrix the elements of the leading diagonal all have the value 1.

$$\begin{bmatrix} 1 & 0 & 7 & 0 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 6 & 1 \end{bmatrix}$$

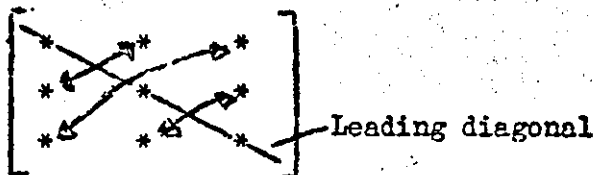


## SYMMETRIC MATRICES

### DESCRIPTION

A symmetric matrix is a square matrix in which, if the row and column number are exchanged, an equal element is found. If the relationship is shown diagrammatically, it is seen that the equal elements are symmetrically placed about the leading diagonal.

### DIAGRAM



In this representation of a matrix the symmetrically placed equal elements have been indicated by double arrows.

### EXAMPLE

$$\begin{bmatrix} 4 & 2 & 1 \\ 2 & 5 & 3 \\ 1 & 3 & 6 \end{bmatrix}$$

is a symmetric matrix. You can confirm this either by visually inspecting the symmetry about the leading diagonal, or by noting down that the values at 1,2 and 2,1 are the same (2), the values at 1,3 and 3,1 are the same (1), and the values at 2,3 and 3,2 are the same (3).

### COMMENT

The mileage chart in Fig.2 forms a symmetric matrix, since the distance from any town to any other is precisely the same if the journey is made in the opposite direction.

FEEDBACK QUESTIONS

- 1) Write down a 3 x 3 matrix with all 1's in the leading diagonal and 0's everywhere else.

- 2) The square matrix below is not symmetric because one pair of elements is not equal - give the addresses of the elements that are not equal.

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & 2 & 1 & 4 \\ 0 & 1 & 3 & 7 \\ 3 & 4 & 4 & 4 \end{bmatrix}$$

- 3) Which of the following square matrices are symmetric?

(i)  $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$  (ii)  $\begin{bmatrix} 0 & 3 & 4 \\ 3 & 0 & 5 \\ 5 & 4 & 0 \end{bmatrix}$  (iii)  $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & 3 & 0 \\ 0 & 3 & 1 & 4 \\ 0 & 0 & 4 & 1 \end{bmatrix}$

- 4) Complete the following matrix to make a symmetric matrix

$$\begin{bmatrix} 1 & \cdot & 5 & \cdot \\ 3 & 2 & \cdot & \cdot \\ 5 & 4 & 3 & \cdot \\ 8 & 7 & 6 & 4 \end{bmatrix}$$

ANSWERS:

1)  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

- 2) The element at 3,4 is not equal to the element at 4,3.

- 3) (i) and (iii).

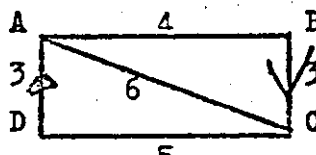
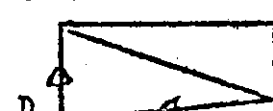
4)  $\begin{bmatrix} 1 & 3 & 5 & 8 \\ 3 & 2 & 4 & 7 \\ 5 & 4 & 3 & 6 \\ 8 & 7 & 6 & 4 \end{bmatrix}$

# REPRESENTING NETWORKS BY MEANS OF MATRICES

## INTRODUCTION

Matrices may be used to describe the relationships within a network, and thus can be used to summarise networks, whether they be about roads, social relationships, electrical connections, etc.

## PROCEDURE TABLE

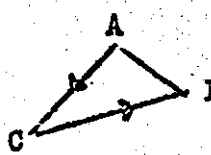
G I V E N	A network of relationships with arrows showing directions of connections which are one way only. Magnitudes may be denoted.	<p>EXAMPLE</p>  <p>Distances = miles.</p>																									
STEP	PROCEDURE	EXAMPLE																									
1	List the starting points down the left hand side and the finishing points across the top	<p>To</p> <p>A B C D</p> <p>From A</p> <p>B</p> <p>C</p> <p>D</p>																									
2	Work out the distances between each starting point and each finishing point.	<p>A → B = 4</p> <p>A → C = 6</p> <p>A → D = 11 (via C)</p> <p>etc.</p>																									
3	Enter the values you obtain in your chart, putting zero's for the distance from any point to itself.	<table><tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr><tr><td>A</td><td>0</td><td>4</td><td>6</td><td>11</td></tr><tr><td>B</td><td>4</td><td>0</td><td>3</td><td>8</td></tr><tr><td>C</td><td>6</td><td>10</td><td>0</td><td>5</td></tr><tr><td>D</td><td>3</td><td>7</td><td>5</td><td>0</td></tr></table>		A	B	C	D	A	0	4	6	11	B	4	0	3	8	C	6	10	0	5	D	3	7	5	0
	A	B	C	D																							
A	0	4	6	11																							
B	4	0	3	8																							
C	6	10	0	5																							
D	3	7	5	0																							
4	Write the chart as a matrix	$\begin{bmatrix} 0 & 4 & 6 & 11 \\ 4 & 0 & 3 & 8 \\ 6 & 10 & 0 & 5 \\ 3 & 7 & 5 & 0 \end{bmatrix}$																									
<p>In some networks a link may either exist, or not exist but no magnitudes are assigned in this case carry out the procedure above, but for your entries place 1 indicating a direct link, 0 indicating no direct link.</p> <p>Thus A B</p>  <p>is represented by</p> $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$																											
Matrices will be symmetrical <u>if</u> there are no one way connections.																											

DRAWING A NETWORK FROM THE MATRIX REPRESENTING IT

INTRODUCTION

Just as matrices may be drawn up to represent a network, so the process may be reversed, so that from a matrix of the type representing open links with a 1 and closed links by a 0, the network being represented can be worked out

PROCEDURE  
TABLE

G I V E N	A matrix representing open and closed connections in a network by 1's and 0's respectively.	EXAMPLE $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$																				
STEP	PROCEDURE	EXAMPLE																				
1	Assign the points labels and put them in alphabetically, top to bottom, and left to right.	<table><tr><td></td><td>A</td><td>B</td><td>C</td></tr><tr><td>A</td><td>0</td><td>1</td><td>1</td></tr><tr><td>B</td><td>1</td><td>0</td><td>0</td></tr><tr><td>C</td><td>0</td><td>1</td><td>0</td></tr></table>		A	B	C	A	0	1	1	B	1	0	0	C	0	1	0				
	A	B	C																			
A	0	1	1																			
B	1	0	0																			
C	0	1	0																			
2	Write 'From' in the left hand margin, 'To' at the top	<table><tr><td></td><td></td><td>To</td><td></td></tr><tr><td></td><td>A</td><td>B</td><td>C</td></tr><tr><td>From A</td><td>0</td><td>1</td><td>1</td></tr><tr><td>B</td><td>1</td><td>0</td><td>0</td></tr><tr><td>C</td><td>0</td><td>1</td><td>0</td></tr></table>			To			A	B	C	From A	0	1	1	B	1	0	0	C	0	1	0
		To																				
	A	B	C																			
From A	0	1	1																			
B	1	0	0																			
C	0	1	0																			
3	Place dots representing the points on a piece of paper, and label them clockwise in alphabetical order	<table><tr><td></td><td>A</td><td></td></tr><tr><td></td><td></td><td>B</td></tr><tr><td>C</td><td></td><td></td></tr></table>		A				B	C													
	A																					
		B																				
C																						
4	Join the points with lines, and indicate one way connections with a single arrow. Connections working both ways may either be left unarrowed, or may be given two arrows - but state which.	 <p>Two way connections = unarrowed</p>																				
	The network may be too involved to be easily laid out if more than 4 or 5 points are involved																					

## TRANSPOSE

### DEFINITION

The transpose of a given matrix is obtained by interchanging the rows and columns.

Thus the element in row 1 column 3, moves to row 3 column 1  
the element in row 2 column 3, moves to row 3 column 2  
and so on.

### NOTATION

The transpose of a matrix is denoted by adding the letter T to it thus:

The transpose of Matrix A, is  $A^T$

Read  $A^T$  as 'the transpose of A'

### EXAMPLE 1

$$\text{If } A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

### EXAMPLE 2

$$\text{If } B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad B^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

### EXAMPLE 3

$$\text{If } C = \begin{bmatrix} 3 & 5 & 7 & 9 \end{bmatrix} \quad C^T = \begin{bmatrix} 3 \\ 5 \\ 7 \\ 9 \end{bmatrix}$$

### COMMENT

If you are dealing with matrices representing networks, taking the transpose of a matrix would be equivalent to reversing the direction of any one way links.

$$D = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{array}{c} A \\ \begin{array}{ccc} & \nearrow & \\ C & \rightarrow & B \end{array} \end{array} \quad D^T = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{array}{c} A \\ \begin{array}{ccc} & \nwarrow & \\ C & \leftarrow & B \end{array} \end{array}$$

REVIEW AND PREVIEW

INTRODUCTION

This information map is designed to help you retrace the ideas covered so far, and tell you what you will be learning next. It is hoped that it will help you to get a clearer picture of how the ideas fit together.

PAST

We have showed the ways in which data can be presented in the form of charts, and how matrices can be derived from charts.

We defined a matrix as a rectangular array of numbers enclosed in square brackets, and mentioned special cases of matrices such as square matrices, row vectors and column vectors.

Row vectors, or column vectors may be combined to form larger matrices, or matrices may be split into row or column vectors.

Order of a matrix - its number of rows and columns

Address of an element - its row number and its column number.

Special Properties of Square Matrices:

- \*Leading diagonal composed of elements with same row and column numbers

- \*Symmetric matrices, with equal elements symmetrically placed about the leading diagonal.

The ability of Matrices to represent networks was shown, and you were shown how to derive a matrix from a network, and a network from a matrix.

Finally the transpose of a matrix was defined, this being the matrix resulting if the row and column numbers of the elements in a matrix are interchanged.

FUTURE

Now you know what matrices are, and some of their properties, we are ready to go on to see some ways in which matrices can be manipulated, and how this can help us to deal with the data represented in the matrices.

## ADDITION OF MATRICES

### DEFINITION

The addition of matrices involves adding together corresponding elements of the initial matrices, to produce the elements of the sum matrix.

### NOTATION

If two matrices, A and B, are added together to form matrix S, we write

$$S = A + B$$

We say that S is the sum of matrices A and B.

### RULES

- (i) The matrices must be of the same order. The sum is a matrix of the same order
- (ii) Add corresponding elements to produce the element in the corresponding position in the sum matrix.

### EXAMPLE

$$A = \begin{bmatrix} 80 & 90 & 50 \\ 120 & 105 & 125 \end{bmatrix}$$

$$B = \begin{bmatrix} 60 & 94 & 63 \\ 130 & 101 & 104 \end{bmatrix}$$

Find the sum S of matrices A and B

$$S = A + B = \begin{bmatrix} 140 & 184 & 113 \\ 250 & 206 & 228 \end{bmatrix}$$

APPLICATION: A manufacturing company produces T.V sets and Hi Fi amplifiers. It has factories in England, West Germany and Italy. Production on two days was

Production, Day 1

	England	W.Germany	Italy
T.V.	80	90	50
Hi Fi	120	105	125

Production, Day 2

	England	W.Germany	Italy
T.V.	60	94	63
Hi Fi	130	101	104

If a manager wishes to find the production over the two days, the charts for Days 1 and 2 can be converted to matrices - the result being matrices A and B. The sum matrix S, then represents production totalled for the two days.

### NON EXAMPLE

$$\begin{bmatrix} 3 & 5 \\ 5 & 10 \end{bmatrix} + \begin{bmatrix} 10 & 4 & 6 \\ 10 & 10 & 20 \end{bmatrix} =$$

Cannot be done when working with matrices pure and simple, because the order of the two matrices is not the same.

### COMMENT

The use of matrices in addition of information found in charts and tables is quicker than rewriting the table complete with headings, although obviously this saving is most pronounced if large tables are used, or there are a large number of additions to be carried out.

SPECIAL CASE OF ADDITION OF MATRICES OF DIFFERENT ORDERS

INTRODUCTION

It is sometimes possible to add together matrices that are not of the same order, if, and only if, it is possible to return to the data which gave rise to the matrix.

RULE

Two matrices which are not of the same order can be added if by returning to the data the matrices were derived from, it becomes possible to complete one of the matrices by enlarging it to the same order as the other matrix.

EXAMPLE

The production matrix describing the number of T.Vs, and Hi Fi's produced in England, West Germany, and Italy for the first two days of a week is:

$$S = \begin{bmatrix} 140 & 184 & 113 \\ 250 & 206 & 228 \end{bmatrix}$$

If the production figures for Italy are not available for the third day, the production matrix for this day might be

$$C^* = \begin{bmatrix} 65 & 83 \\ 103 & 120 \end{bmatrix}$$

By the rules of matrix addition, S and C cannot be added! - They are not of the same order.

But if we know there had been a strike, and there had been no production in Italy on the third day, we could complete the production matrix thus

$$C = \begin{bmatrix} 65 & 85 & 0 \\ 103 & 120 & 0 \end{bmatrix}$$

Now we have matrices of the same order, which can be added.

$$S + C = \begin{bmatrix} 140 & 184 & 113 \\ 250 & 206 & 228 \end{bmatrix} + \begin{bmatrix} 65 & 85 & 0 \\ 103 & 120 & 0 \end{bmatrix} = \begin{bmatrix} 205 & 267 & 113 \\ 353 & 325 & 228 \end{bmatrix}$$



# SUITABILITY OF MATRICES FOR ADDITION

## INTRODUCTION

A point to bear in mind when dealing with matrices is that it may be possible to add matrices, but not be suitable if the sum is meaningless.

## DEFINITION

Two matrices are suitable for addition if the sum is meaningful in terms of the data from which the matrices were drawn.

## ANALOGY

When dealing with numbers we can add 2 and 3, but if we have 2 centimetres and 3 minutes, we cannot make 5 of anything.

## EXAMPLE

Two people counted the numbers of pedestrians walking along a road in two consecutive hours. They got the following results.

Observer A			Observer B		
	Number of Pedestrians			Number of Pedestrians	
	Children	Adults		Children	Adults
Male	5	7	Male	10	13
Female	13	16	Female	8	10

The two matrices  $\begin{bmatrix} 5 & 7 \\ 13 & 16 \end{bmatrix}$  and  $\begin{bmatrix} 10 & 13 \\ 8 & 10 \end{bmatrix}$  derived from this

data could be added to obtain a meaningful result.

## NON EXAMPLE

Two people counted the number of pedestrians walking along a road in two consecutive hours. However, they did not agree on the categories to be recorded:

Observer A			Observer B		
	Number of Pedestrians		DIRECTION OF TRAVEL	Number of Pedestrians	
	Children	Adults		Children	Adults
Male	5	7	East	10	13
Female	13	16	West	8	10

There would be no point in adding the matrices

$\begin{bmatrix} 5 & 7 \\ 13 & 16 \end{bmatrix}$  and  $\begin{bmatrix} 10 & 13 \\ 8 & 10 \end{bmatrix}$  since the categories are different.

Mathematically though it is perfectly possible to add the matrices to obtain the sum matrix

$$\begin{bmatrix} 15 & 20 \\ 21 & 26 \end{bmatrix}$$

FEEDBACK QUESTIONS

- 1) Given that the production figures for a manufacturing company with several factors were as follows for Wednesday of one week:

	England	W.Germany	Italy
T.V	65	83	86
Hi Fi	103	120	93

And that the matrix S, representing the production for Monday and Tuesday of that week, combined, is

$$S = \begin{bmatrix} 140 & 184 & 113 \\ 250 & 206 & 228 \end{bmatrix}$$

Find the matrix representing production for the three days totalled.

2)  $D^* = \begin{bmatrix} 10 & 15 \\ 0 & 0 \\ 16 & 7 \end{bmatrix}$        $E = \begin{bmatrix} 8 & 5 \\ 6 & 3 \\ 12 & 13 \end{bmatrix}$

Find  $D^* + E$ .

- 3) Which of the following matrices can be added? Add those.

i)  $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$  and  $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$  (ii)  $\begin{bmatrix} 1 & 3 & 6 \\ 2 & 5 & 9 \end{bmatrix}$  and  $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

iii)  $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 3 & 4 & 2 \end{bmatrix}$  and  $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$  (iv)  $\begin{bmatrix} 3 & 5 & 7 \\ 1 & 2 & 3 \end{bmatrix}$  and  $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

- 4) In which of the following situations is it suitable to add the matrices derived from the data? Add the matrices where it is suitable.

(1)      8 - 9 a.m.			9 - 10 a.m.		
DIRECTION	Number of Pedestrians		DIRECTION	Number of Pedestrians	
	Male	Female		Male	Female
East	5	7	East	10	13
West	13	16	West	8	10

- (ii) Means used by people travelling to work.

	Car	Bus	Train	Foot	Distance, Travelling	Car	Bus	Train	Foot
Male	10	4	7	3	Less than 3 m.	8	3	0	18
Female	2	8	6	4	3 miles + over	12	16	12	0

# ANSWERS

$$1) \begin{bmatrix} 140 & 184 & 113 \\ 250 & 206 & 228 \end{bmatrix} + \begin{bmatrix} 65 & 83 & 86 \\ 103 & 120 & 93 \end{bmatrix} = \begin{bmatrix} 205 & 267 & 199 \\ 358 & 326 & 321 \end{bmatrix}$$

$$2) D^* + E = \begin{bmatrix} 10 & 15 \\ 0 & 0 \\ 16 & 7 \end{bmatrix} + \begin{bmatrix} 8 & 5 \\ 6 & 3 \\ 12 & 13 \end{bmatrix} = \begin{bmatrix} 18 & 20 \\ 6 & 3 \\ 28 & 20 \end{bmatrix}$$

3) The matrices in (ii) can be added, so can the matrices in (iii)

$$(ii) \begin{bmatrix} 1 & 3 & 6 \\ 2 & 5 & 9 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 7 \\ 2 & 6 & 10 \end{bmatrix}$$

$$(iii) \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 3 & 4 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 \\ 1 & 1 & 0 \\ 3 & 5 & 3 \end{bmatrix}$$

4) (i) Matrices are suitable for addition. (ii) Matrices unsuitable

$$(i) = \begin{bmatrix} 15 & 20 \\ 21 & 26 \end{bmatrix}$$

## EQUAL MATRICES

### DEFINITION

Two matrices are said to be equal if:

- (i) they are of the same order
- (ii) each element of one, is equal to the corresponding element of the other.

### EXAMPLE

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 7 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 7 & 9 \end{bmatrix}$$

### NON EXAMPLE 1

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \text{ is not equal to } \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

since they have different numbers of rows (they are of different order)

### NON EXAMPLE 2

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 7 & 9 \end{bmatrix} \text{ is not equal to } \begin{bmatrix} 1 & 5 & 3 \\ 2 & 7 & 9 \end{bmatrix}$$

because the elements at addresses 1,2 and 1,3 are not equal.

### RELATED MAPS

Order of matrix (7)

Address in a matrix (10)

# EQUALITY OF MATRICES TO THEIR TRANSPOSES

## INTRODUCTION

As has been mentioned earlier, the transpose of a matrix is obtained by interchanging rows and columns.

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

Except in special circumstances matrices are not equal to their transposes.

## DEFINITION

A matrix is equal to its transpose if when its rows and columns are transposed the matrix is unchanged (i.e. an equal matrix results).

## RULES

- (i) Unless the matrix is square its transpose will not even be of the same order, and thus cannot possibly be equal
- (ii) Only in the special case of square matrices which are also symmetric will a matrix equal its transpose.

## EXAMPLE

$$G = \begin{bmatrix} 1 & 4 & 6 \\ 4 & 2 & 5 \\ 6 & 5 & 3 \end{bmatrix} \quad G^T = \begin{bmatrix} 1 & 4 & 6 \\ 4 & 2 & 5 \\ 6 & 5 & 3 \end{bmatrix} \quad G = G^T$$

## NON EXAMPLE 1

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

A is not a square matrix, its transpose is thus not even of the same order.

## NON EXAMPLE 2

$$C = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \quad C^T = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$$

C is not a symmetric matrix - it is thus not equal to its transpose.

## RELATED MAPS

Transpose (17)  
Symmetric Matrices (13)  
Equal Matrices (106)

MATRICES ADDED TO THEIR TRANSPOSES

DESCRIPTION

When a matrix is added to its own transpose, a symmetric matrix always results.

EXAMPLE

$$H = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} \quad H^T = \begin{bmatrix} 3 & 2 \\ 7 & 4 \end{bmatrix}$$

$$H + H^T = \begin{bmatrix} 6 & 9 \\ 9 & 8 \end{bmatrix}$$

COMMENT

This is a general rule true for the addition of any matrix to its own transpose.

RELATED MAPS

Symmetric Matrices (13)

Transpose (17)

FEEDBACK QUESTIONS

1) Which of the following matrices are equal?

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 \\ 2 & 2 \\ 3 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 1 & 2 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 1 & 2 \end{bmatrix} \quad F = \begin{bmatrix} 1 & 3 \\ 2 & 2 \\ 3 & 1 \end{bmatrix}$$

2) Which of the following is equal to its transpose?

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 2 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$$

3) What type of matrix do you obtain if you add a matrix to its transpose?

ANSWERS:

1)  $C = B$ ,  $B = F$ .

2) B is equal to its transpose  $B^T$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \quad C^T = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$$

3) A symmetric matrix.

# SUBTRACTION OF MATRICES

## INTRODUCTION

Subtraction of matrices is subject to rules similar to those controlling addition of matrices.

## DEFINITION

The addition of matrices involves subtracting corresponding elements in the second matrix from their counterparts in the first matrix, to form the elements of the resulting matrix.

## NOTATION

If matrix B is to be subtracted from matrix A to form matrix C, we write

$$C = A - B$$

"Matrix C equals matrix A minus matrix B"

## RULES

- (i) The matrices must be of the same order. The result is a matrix of the same order.
- (ii) Subtract corresponding elements to produce the element in the corresponding position in the resulting matrix.

## EXAMPLE

$$A = \begin{bmatrix} 20 & 11 \\ 35 & 21 \\ 24 & 27 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 5 \\ 7 & 2 \\ 6 & 8 \end{bmatrix}$$

$$C = A - B$$

$$C = \begin{bmatrix} 20 & 11 \\ 35 & 21 \\ 24 & 27 \end{bmatrix} - \begin{bmatrix} 3 & 5 \\ 7 & 2 \\ 6 & 8 \end{bmatrix}$$

$$C = \begin{bmatrix} 17 & 6 \\ 28 & 19 \\ 18 & 19 \end{bmatrix}$$

APPLICATION:- In a wholesale warehouse, there are 3 different sizes of TV sets, both black and white and colour. At the beginning of one day the stock was

	Black & White	Colour
15"	20	11
18"	35	21
21"	24	27

This can be represented by matrix A. During the day the following numbers of sets were despatched to retailers

	Black & White	Colour
15"	3	5
18"	7	2
21"	6	8

This can be represented by matrix B. The stock at the end of the day is then represented by matrix C.

	Black & White	Colour
15"	17	6
18"	28	19
21"	18	19

## NON EXAMPLE

$$\begin{bmatrix} 2 & 6 \\ 10 & 10 \\ 4 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ 3 & 4 \end{bmatrix}$$

is not possible since the matrices are of different order.



NEGATIVE NUMBERS - A REVIEW

INTRODUCTION

This map is designed to remind you about negative numbers. If you do not understand this map, then refer to your supervisor.

APPLICATIONS

When dealing with a quantity that may increase or decrease it is often convenient to refer to the decreases as negative increases

e.g. If I pay £3 into my bank account, there is an increase of £3  
If I withdraw £2, then the balance decreases by £2.

or we may say it increases by -£2.

HOW TO USE  
NEGATIVE  
NUMBERS

An increase of £3 followed by a decrease of £3 leaves the balance unchanged.

$$£3 + (-£3) = 0$$

In general we can say that when using negative numbers, you can reverse the sign in front of the negative number and then use the number as a normal number. Thus

$$3 + (-3) \text{ is the same as } 3 - 3 = 0$$

$$3 - (-3) \text{ is the same as } 3 + 3 = +6$$

Two minus numbers added together give a minus value in the same way that two positive numbers added together give a positive value. Thus

$$(-3) + (-2) = -5$$

If you start off with a negative number then add a number it is as though you started below zero, thus

$$(-3) + 10 = +7$$

since you have started below zero, 3 of the + 10 must be used to cancel out the -3, to get to zero, - the remainder is + 7.

FEEDBACK QUESTIONS

$$1) \quad C = \begin{bmatrix} 17 & 6 \\ 28 & 19 \\ 18 & 19 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 5 & 7 \end{bmatrix}$$

Find matrix E, if  $E = C - D$ .

2) Find where possible:

$$(i) \quad \begin{bmatrix} 6 & 8 \\ 12 & 7 \\ 6 & 9 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 2 & 3 \end{bmatrix} \quad (ii) \quad \begin{bmatrix} 5 & 6 & 2 \\ 1 & 3 & 5 \\ 7 & 2 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

3) Calculate the following

- (i)  $5 + (-3)$
- (ii)  $(-2) + (-4)$
- (iii)  $(-2) + 7$
- (iv)  $2 + (-6)$

ANSWERS:

$$1) \quad E = C - D = \begin{bmatrix} 17 & 6 \\ 28 & 19 \\ 18 & 19 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 15 & 2 \\ 25 & 13 \\ 13 & 12 \end{bmatrix}$$

$$2) \quad (i) \text{ impossible} \quad (ii) \quad \begin{bmatrix} 4 & 6 & 1 \\ 1 & 3 & 4 \\ 6 & 2 & 4 \end{bmatrix}$$

- 3) (i)  $5 + (-3) = +2$
- (ii)  $(-2) + (-4) = -6$
- (iii)  $(-2) + 7 = +5$
- (iv)  $2 + (-6) = -4$

# FEEDBACK QUESTIONS

- 1) When a record of stock for one item is made there are increases and decreases as follows:

Day	1	2	3	4	5
Increase	+2	-3	+5	-3	-2

If initially the stock was 11 items, then:

After 1 day stock was	$11 + 2$	$= 13$
After 2 days stock was	$13 + (-3)$	$= 10$
After 3 days stock was	$10 + 5$	$= 15$
After 4 days stock was		
After 5 days stock was		

Complete the table.

- 2) (a) Complete the following table (Net increase in stock = number received - number despatched)

Day	Number Received	Number Despatched	Net Increase In Stock
Monday	0	7	-7
Tuesday	20	12	+8
Weds.	0	5	
Thurs.	12	17	
Friday	10	0	

- (b) What is the overall increase in stock in the above example?

- 3) In a warehouse the stock of one item was 8, first thing on Monday morning. If the increases for each day of that week were -5, +3, -2, +17, -7, find the stock at end of work on Friday.

## ANSWERS:

- 1)
- |                        |             |        |
|------------------------|-------------|--------|
| After 1 day stock was  | $11 + 2$    | $= 13$ |
| After 2 days stock was | $13 + (-3)$ | $= 10$ |
| After 3 days stock was | $10 + 5$    | $= 15$ |
| After 4 days stock was | $15 + (-3)$ | $= 12$ |
| After 5 days stock was | $12 + (-2)$ | $= 10$ |

- 2) (a)
- | Day       | Number Received | Number Despatched | Net Increase in Stock |
|-----------|-----------------|-------------------|-----------------------|
| Monday    | 0               | 7                 | -7                    |
| Tuesday   | 20              | 12                | +8                    |
| Wednesday | 0               | 5                 | -5                    |
| Thursday  | 12              | 17                | -5                    |
| Friday    | 10              | 0                 | +10                   |

- (b)  $-7 + 8 - 5 - 5 + 10 = +1$  Increase of 1

- 3)  $8 + (-5) + 3 + (-2) + 17 + (-7) = 14$

CALCULATING STOCK CHANGES USING MATRICES

INTRODUCTION

The stock levels of several items represented in matrix form can be manipulated in the same way as stock levels of single items.

PROCEDURE  
TABLE

G I V E N	The matrices representing stock received, stock despatched and initial stock levels - calculate final stock level.	<p>EXAMPLE:</p> <p>articles received: <math>\begin{bmatrix} 6 &amp; 6 \\ 0 &amp; 12 \\ 0 &amp; 0 \end{bmatrix}</math></p> <p>articles despatched: <math>\begin{bmatrix} 3 &amp; 8 \\ 5 &amp; 4 \\ 2 &amp; 7 \end{bmatrix}</math></p> <p>initial stock: <math>\begin{bmatrix} 5 &amp; 8 \\ 9 &amp; 2 \\ 6 &amp; 10 \end{bmatrix}</math></p>
STEP	PROCEDURE	EXAMPLE
1	Calculate the net increase in stock by subtracting the articles despatched matrix from the articles received matrix	$\begin{bmatrix} 6 & 6 \\ 0 & 12 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 8 \\ 5 & 4 \\ 2 & 7 \end{bmatrix}$ $= \begin{bmatrix} 3 & -2 \\ -5 & 8 \\ -2 & -7 \end{bmatrix}$
2	Add the net increase matrix to the initial stock matrix. This gives the matrix for the final stock level.	$\begin{bmatrix} 5 & 8 \\ 9 & 2 \\ 6 & 10 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ -5 & 8 \\ -2 & -7 \end{bmatrix}$ $= \begin{bmatrix} 8 & 6 \\ 4 & 10 \\ 4 & 3 \end{bmatrix}$

# THE SKEW SYMMETRIC MATRIX

## DEFINITION

A skew symmetric matrix is a square matrix in which only the sign of the element changes when row and column numbers are interchanged. The elements in the leading diagonal must be zero.

## EXAMPLE

$$\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix}$$

Note that: element at 1,2 is 2, element at 2,1 is -2  
element at 1,3 is -3, element at 3,1 is 3  
element at 2,3 is 1, element at 3,2 is -1

## NON EXAMPLE

$$\begin{bmatrix} 1 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & -1 \end{bmatrix}$$

is not a skew symmetric matrix since the elements in the leading diagonal are not all 0's.

## PROPERTIES

When a skew matrix is added to its transpose a zero matrix of the same order results.

$$A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & -1 \\ -3 & 1 & 0 \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

a zero matrix of order 3 x 3.

## COMMENT

0's in a skew matrix remain unchanged since  $-0 = 0$ .

## RELATED MAPTS

Leading Diagonal (12)  
Symmetric Matrices (13)  
Transpose (17)

FEEDBACK QUESTIONS

- 1) (i) If the matrix representing items received by a warehouse is

$$\begin{bmatrix} 0 & 0 \\ 12 & 0 \\ 24 & 6 \end{bmatrix} \text{ and the number despatched is } \begin{bmatrix} 5 & 3 \\ 2 & 4 \\ 7 & 5 \end{bmatrix}$$

calculate the net increase in stock.

- (ii) Find the stock at the end of the day if at the beginning it was

$$\begin{bmatrix} 8 & 6 \\ 4 & 10 \\ 4 & 3 \end{bmatrix}$$

- 2) Complete matrix B to make it skew symmetric.

$$B = \begin{bmatrix} 0 & 3 & 4 \\ \cdot & \cdot & \cdot \\ \cdot & 0 & \cdot \end{bmatrix}$$

- 3) What is the result of adding a skew symmetric matrix to its own transpose?

- 4) Which of the following are skew symmetric?

$$(i) \begin{bmatrix} 0 & 4 & 5 \\ -4 & 0 & -2 \\ 2 & -5 & 0 \end{bmatrix} \quad (ii) \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -3 \\ 2 & 3 & 0 \end{bmatrix} \quad (iii) \begin{bmatrix} 1 & 3 \\ -3 & 1 \end{bmatrix}$$

ANSWERS:

$$1) (i) \begin{bmatrix} 0 & 0 \\ 12 & 0 \\ 24 & 6 \end{bmatrix} - \begin{bmatrix} 5 & 3 \\ 2 & 4 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} -5 & -3 \\ 10 & -4 \\ 17 & 1 \end{bmatrix}$$

$$(ii) \begin{bmatrix} 8 & 6 \\ 4 & 10 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} -5 & -3 \\ 10 & -4 \\ 17 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 14 & 6 \\ 21 & 4 \end{bmatrix}$$

$$2) \begin{bmatrix} 0 & 3 & 4 \\ -3 & 0 & 0 \\ -4 & 0 & 0 \end{bmatrix}$$

- 3) a zero matrix of the same order.

- 4) (ii) is skew symmetric.

# MULTIPLYING A MATRIX BY A SCALAR

## INTRODUCTION

Since there is more than one form of matrix multiplication, multiplication of a matrix by one number is called "multiplication of a matrix by a scalar" in order to make clear what form of matrix multiplication we are talking about.

## NOTATION

If the matrix A is to be multiplied by 6, we write

$$6A$$

this means  $6 \times A$ .

More generally, if M is a matrix, and k is a number (or scalar) we write the result of multiplication as kM.

## PROCEDURE TABLE

G I V E N	A matrix, to be multiplied by a scalar	<p>EXAMPLE:</p> $A = \begin{bmatrix} 80 & 90 & 50 \\ 120 & 100 & 120 \end{bmatrix}$ <p>Find 5A</p>
STEP	PROCEDURE	EXAMPLE
1	Multiply each element in the matrix by the number to produce the element in the corresponding position in the resultant matrix.	$5 \times \begin{bmatrix} 80 & 90 & 50 \\ 120 & 100 & 120 \end{bmatrix}$ $= \begin{bmatrix} 5 \times 80 & 5 \times 90 & 5 \times 50 \\ 5 \times 120 & 5 \times 100 & 5 \times 120 \end{bmatrix}$ $= \begin{bmatrix} 400 & 450 & 250 \\ 600 & 500 & 600 \end{bmatrix}$

## USE

Suppose that  $\begin{bmatrix} 80 & 90 & 50 \\ 120 & 100 & 120 \end{bmatrix}$  represented the production figures for various items, on each day for a week of 5 days. Then 5A, or  $\begin{bmatrix} 400 & 450 & 250 \\ 600 & 500 & 600 \end{bmatrix}$  would represent the total production for the week.

TAKING A COMMON ELEMENT FROM A MATRIX

INTRODUCTION

This process is by way of being the reverse of multiplying a matrix by a scalar.

PROCEDURE  
TABLE

G I V E N	A matrix, whose elements can be divided by a common factor	EXAMPLE: $A = \begin{bmatrix} 80 & 90 & 50 \\ 120 & 100 & 120 \end{bmatrix}$
STEP	PROCEDURE	EXAMPLE
1	Divide each element in the initial matrix by the common factor to produce the corresponding element in the resulting matrix	Take the common factor 10 out of the above matrix $\begin{bmatrix} 8 & 9 & 5 \\ 12 & 10 & 12 \end{bmatrix}$
2	Write down the number that the resultant matrix must be multiplied by to get back to the initial matrix. (The number is the common factor)	$A = 10 \times \begin{bmatrix} 8 & 9 & 5 \\ 12 & 10 & 12 \end{bmatrix}$



FEEDBACK QUESTIONS

1) Multiply

(i)  $2 \times \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$

(ii)  $5 \times \begin{bmatrix} 3 & 1 & 0 \\ 2 & 4 & 5 \end{bmatrix}$

(iii)  $10 \times \begin{bmatrix} 4 & 3 & 3 & 2 \\ 5 & 6 & 7 & 2 \\ 7 & 10 & 6 & 8 \end{bmatrix}$

2) Take a common factor from

(i)  $\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$  (ii)  $\begin{bmatrix} 25 & 35 \\ 15 & 75 \\ 40 & 30 \end{bmatrix}$

ANSWERS:

1) (i)  $\begin{bmatrix} 6 & 8 \\ 2 & 4 \end{bmatrix}$  (ii)  $\begin{bmatrix} 15 & 5 & 0 \\ 10 & 20 & 25 \end{bmatrix}$  (iii)  $\begin{bmatrix} 40 & 30 & 30 & 20 \\ 50 & 60 & 70 & 20 \\ 70 & 100 & 60 & 80 \end{bmatrix}$

2) (i)  $2 \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  (ii)  $5 \times \begin{bmatrix} 5 & 7 \\ 3 & 15 \\ 8 & 6 \end{bmatrix}$

## APPENDIX D

### THE "BASG-M" MATERIALS USED IN THE AUTHOR'S EXPERIMENTS

1. Explanatory Notes
2. List of Modules and Lessons used in the Study
3. Lesson 6 of Module I of the BASG-M materials
4. Lesson 1 of Module VI of the BASG-M materials
5. Lesson 1 of Module VI: Manuscript of the alternative, information-mapped, version of the lesson.

## APPENDIX D

### 1. EXPLANATORY NOTES

This appendix lists, and includes examples of, the materials developed by the programming team trained and supervised by the author, in the Secretariat of Education and Culture of the State of Bahia, Brazil, and later used by the author in the research described in this study.

Section 2 of this appendix lists the seven modules (composed of 31 lessons) which the author used in his research. In all, this material ran to nearly 1200 small booklet pages (or "frames"). Most of these "frames" require an active response from the learner. However, unlike the "Skinnerian" style of programme writing, the "frames" of this material seldom require single-word, sentence-completion or blank-filling responses. Most of the responses require mathematical work, often one frame requiring the working of a series of problems, which may take the student some time to complete.

The material was of course produced in the Portuguese language. No translation exists. It was felt that for the purposes of this study, some sample sections of the material, in their original Portuguese version, would serve to illustrate the style of the presentation format.

It should be noted that the BASG-M materials are (or are going to be) much more extensive in their coverage. The modules used in the experiments were those already printed and bound at the time of commencement of the project. At the time of writing, the number of completed modules is about double,

and production of further modules continues.

It should also be noted that the BASG-M materials were initially conceived for use in the first year of the upper-secondary level (the "Segundo Grau") with students of about 15 or 16 years of age. The level of language in the modules, the style of presentation (occasional cartoons) and the rate of progression was geared to the characteristics of Brazilian students in this age-group.

The author has been using the materials with adult students as well. These adults, in the age range from 18 to 40 did not differ appreciably in their language skills or mathematical abilities from the originally intended target population. Indeed, if anything, they were below the cultural level of the typical 14-16 year-olds in schools at the present time. It was felt therefore that the materials would be suitable (though with some reservations as regards the occasional cartoons). In practice, very few adverse comments were received from the adult groups as regards the style of the materials. Indeed, more adverse comments were made by the schoolchildren participating in the research.

However, there were few adverse comments, as compared to the favourable ones from either age group. As the questionnaire results reported in appendix E show, the attitudes of the participating learners were generally favourable to the materials.

In Section 3 of this appendix, is the 6th lesson of the first module of the course. It deals with the "set of the parts of a set", "set of sets", and the "universal set". The content is primarily conceptual. Therefore there are not many problems to work, but rather questions to answer, sets

to form, possible sub-sets of a given set to identify or list, formal definitions to learn and state, etc.

In Section 4 is the first lesson of the 6th Module. This deals with common factors. It is part conceptual and part the application of this and other previously learnt concepts. It therefore includes a higher proportion of problems to be worked.

Incidentally, each lesson has its own post-test which can be student-marked, or extracted and administered by the teacher. These have not been reproduced in the examples.

In the last Section 5 of this appendix, the author wishes to illustrate the process of preparation of the "information mapping" version of Module VI. He has included a copy of the original manuscript and paste-up for the information mapped version of Module VI, lesson 1, in order to enable the reader to compare this with the linear version of the same lesson. The manuscript version has been included to illustrate how closely the information mapped version follows the original. All the printed parts of this exhibit were cut from a copy of the original version. The handwritten words have been added. The author has also added, in English, translations of some of the paragraph, or "block" titles, to aid the reader in understanding the structure of the information maps.

Inspection of the two alternative versions shown in Sections 4 and 5, will reveal that:-

- (a) all the problems and exercises worked by the learner in one version, are also included in the other version,

- (b) all the concepts taught in one version are also taught in the other,
- (c) some repetition of definitions or facts has been eliminated from the information mapped version. The student who needs to read a definition twice, can refer to it again with ease. There is no need to re-print it.
- (d) some extra sentences, mainly instructions, summaries and block titles, have been added to the information mapped version. These do not increase the academic content of the lesson. They are all aids to organisation and clarity of the original content.
- (e) thus, the amount of reading that a student has to perform is about equal in both versions - they have about the same number of words.
- (f) however, this does not mean that all students will read the same amount. The organisation of the information mapped version encourages the student to select only those parts that he needs to read.
- (g) in connection with the latter, one can analyse the structure of the two versions as follows:  
linear programme:- 25 frames, in a linear sequence. All students are expected to read them all, in the sequence in which they are presented. If a student feels he needs to re-cap, it is difficult for him to find the appropriate frame. Finally, the material

is very fragmented. No frame presents the "whole story" of the concept of "common factors". information maps:- 10 pages, which include 3 pages of feedback questions and 3 pages of answers to these questions. Thus all the "academic content" is concentrated on four pages. These are titled

1. The concept of "common factor of two natural numbers"
2. Formal definition of "common factor"
- 3a. Extension of the concept - common factors of 3 or more numbers.
- 3b. How to determine the common factors of 3 or more numbers.

Thus the student requiring only to refer to the chapter, need only read 4 clearly titled pages. The student making reference for revision purposes may not even read all 4. He may also pre-test himself on the feedback questions to "discover" which pages he needs to read and later "evaluate" his learning.

Not only have these four pages clear titles, they also have a clear structure.

1. A "concept" map
2. A "fact" map
- 3a. A combined "fact" and "concept" map.
- 3b. A "procedure" map.

There are rules and hints for authors on how to write any one of these and a number of other "standard" types of information map (see Horn, 1974).

Finally, the author should state that he does not consider the information mapped version which was produced

as a "model" example of a set of information maps. Certain desirable components such as an indexed network of the concepts taught, cross-references between maps, etc., have not been included as it was felt that then the new version would depart substantially from the original version. After all, one could supplement the linear version by indexes and references. The author wished to concentrate on one factor only - the layout of the information being communicated. (linear or structured). Also, in the re-write, it became obvious that the original content could be much improved, by choices of other examples of problems, etc. This was not done for the same reasons of maintaining equivalence between the two versions.



## APPENDIX D

### 2. LIST OF MODULES AND LESSONS USED IN THE STUDY

The material used was originally bound as seven volumes or modules. However, due to its very large size, Module IV was split into two volumes (IVa and IVb) before the beginning of the experiment. These modules are only a part of the total material being prepared by the BASG-M project.

The experimental course was divided into 3 sections as follows:

Section 1: Modules I, II, III.

Section 2: Modules IVa, IVb, V.

Section 3: Modules VI, VII.

The content of the modules is as follows:-

#### SECTION I Module I

Lesson 1 The concept of a "set"

Lesson 2 Notation used to represent sets

Lesson 3 Graphical representation. Venn and Carroll.

Lesson 4 "Belonging to a set"

Lesson 5 The parts of a set: its "sub-sets".

Lesson 6 "Set of sub-sets". "Universal Set"

Module II

- Lesson 1     Union of sets.
- Lesson 2     Intersection of sets
- Lesson 3     Difference of sets

Module III

- Lesson 1     "Ordered pair". "Cartesian product".
- Lesson 2     "Relations"
- Lesson 3     Relations (continued)
- Lesson 4     "Functions"

SECTION 2   Module VIa

- Lesson 1     The set "N".
- Lesson 2     Addition in "N".
- Lesson 3     Subtraction in "N".
- Lesson 4     Multiplication in "N".

Module VIb

- Lesson 5     Powers and Indices
- Lesson 6     "Multiples", "Common Multiples", LCM.
- Lesson 7     "Exact"     Division
- Lesson 8     "Inexact" Division.

Module V

- Lesson 1     "Divisibility"
- Lesson 2     "Prime numbers"
- Lesson 3     "Factorisation"

SECTION 3   Module VI

- Lesson 1      Common factors
- Lesson 2      Highest Common factor (HCF)
- Lesson 3      Practical procedure for determining HCF
- Lesson 4      Practical procedure for determining LCM

Module VII

- Lesson 1      "Square root"
- Lesson 2      Operations with powers and roots
- Lesson 3      "Numerical expressions"

## APPENDIX D

### 3. - LESSON 6 OF THE MODULE I OF THE BASG-M MATERIALS

## 6<sup>a</sup> LIÇÃO

- CONJUNTO DAS PARTES DE UM CONJUNTO
- CONJUNTO UNIVERSO

Na lição “Partes de um Conjunto” você estudou a definição de subconjunto de um conjunto e, a partir da definição, identificou subconjuntos de um conjunto, em vários exercícios, lembra-se? Pois bem, agora vamos determinar os subconjuntos de um conjunto dado. O próprio nome da lição facilita nosso estudo: Conjunto das Partes de um Conjunto.

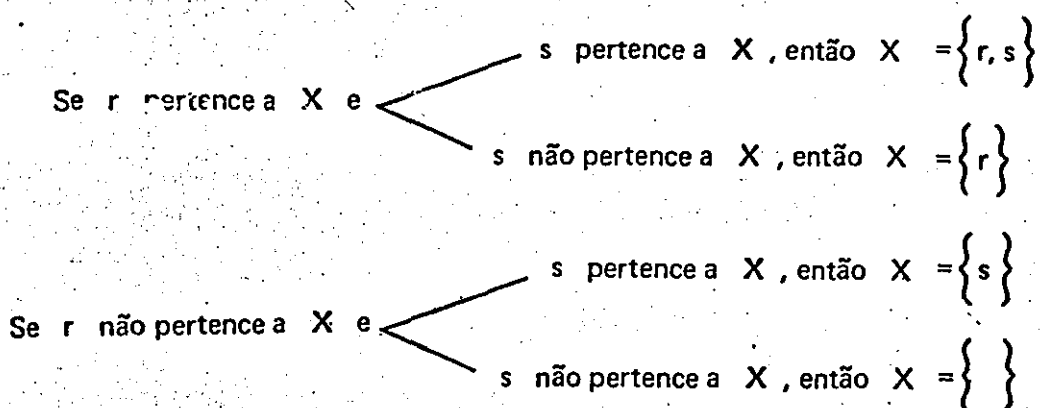
A seguir, estudaremos **CONJUNTO UNIVERSO**, que é um conjunto do qual se parte para a formação de outros conjuntos.

Boa Sorte!

Vamos estudar como determinar **TODAS AS PARTES DE UM CONJUNTO**.

Considere o exemplo  $B = \{r, s\}$

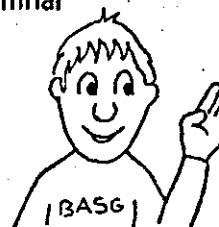
Se indicarmos por  $X$  uma parte qualquer do conjunto  $B$ , pode ocorrer o seguinte



Vá em frente!

Podemos resumir as informações do quadro acima, fazendo um esquema denominado "árvore", e que lhe dará condições para determinar facilmente **TODAS AS PARTES DE UM CONJUNTO**.

Passe ao quadro seguinte e verá.



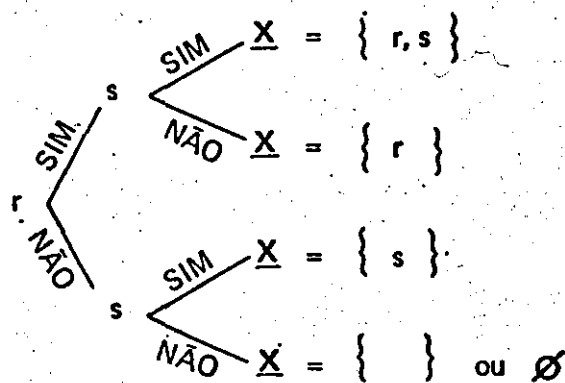
Ainda o mesmo exemplo:  $B = \{r, s\}$

Considera-se as perguntas:

$r$  pertence à parte  $X$ , de  $B$ ?

$s$  pertence à parte  $X$ , de  $B$ ?

e escreve-se SIM ou NÃO conforme o elemento pertença ou não ao subconjunto  $X$ . Assim tem-se:



Logo, você pode concluir que as partes de  $X$ , do conjunto  $B$ , são:

$\{r, s\}$   $\{r\}$   $\{s\}$   $\{ \}$  ou  $\emptyset$

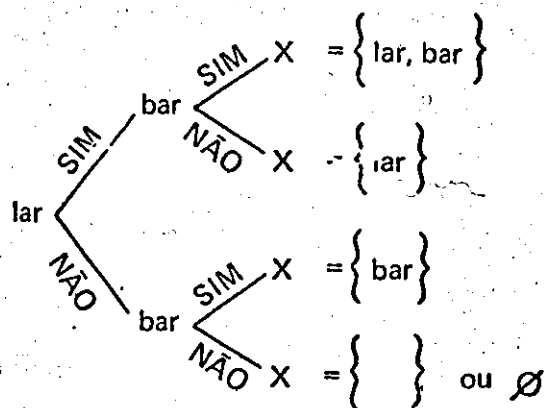
131

Vamos agora determinar o CONJUNTO DE PARTES do seguinte conjunto:

$$M = \{ \text{lar, bar} \}$$

Armamos primeiro o esquema denominado "árvore":

Lembre-se da pergunta: "pertence a X?"



Feito isto, você já pode dizer que as PARTES DO CONJUNTO M, são:

{ \_\_\_\_\_ }, \_\_\_\_\_ , \_\_\_\_\_ , \_\_\_\_\_

132

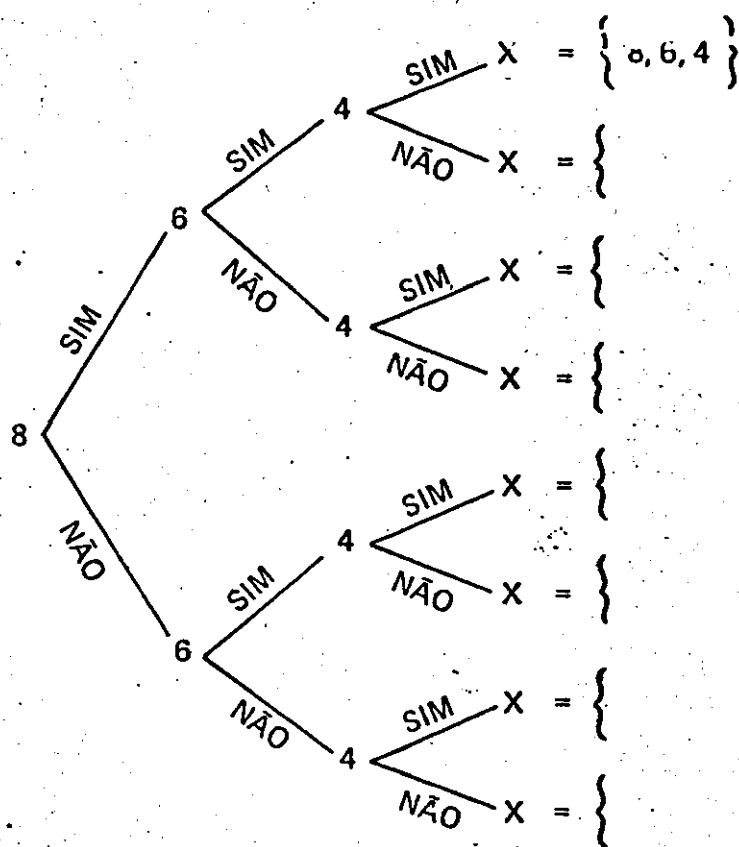
Naturalmente para você completar o quadro acima, olhou para extremidades da direita da "árvore" e viu que as PARTES DO CONJUNTO M, são:  $\{ \text{lar, bar} \}, \{ \text{lar} \}, \{ \text{bar} \}, \{ \} \text{ ou } \emptyset$

Lembrando-se que foi visto nos quadros anteriores, complete o esquema abaixo e determine todos os subconjuntos do seguinte conjunto:

$$S = \{8, 6, 4\}$$

Para facilitar, faça a pergunta: pertence a X?

Lembre-se: X é uma parte qualquer de S.



Portanto, os subconjuntos de S são:

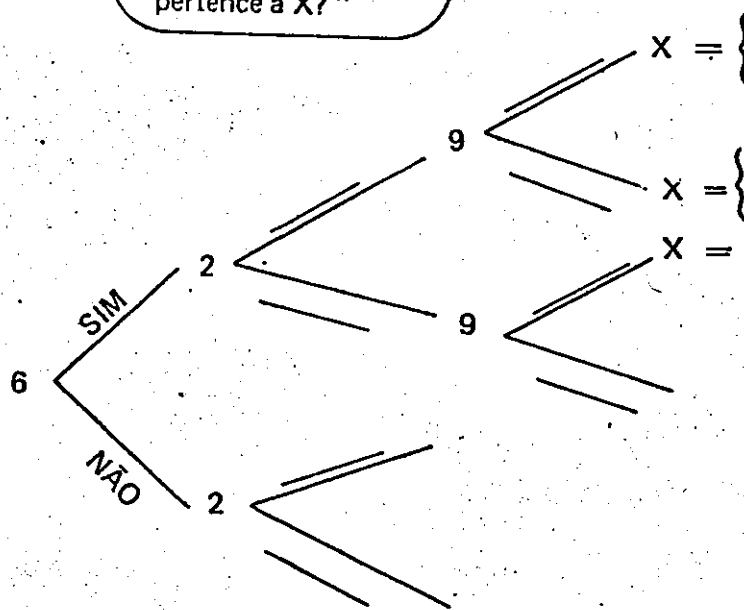
$$\{8, 6, 4\}, \{8, 6\}, \{8, 4\}, \{8\}, \{6, 4\}, \{6\}, \{4\}, \{\}$$



Agora você vai determinar as partes do conjunto  $T = \{6, 2, 9\}$ , a partir de um esquema semelhante ao do quadro anterior.

## OBSERVAÇÃO

Lembre-se da pergunta  
"pertence a X?"

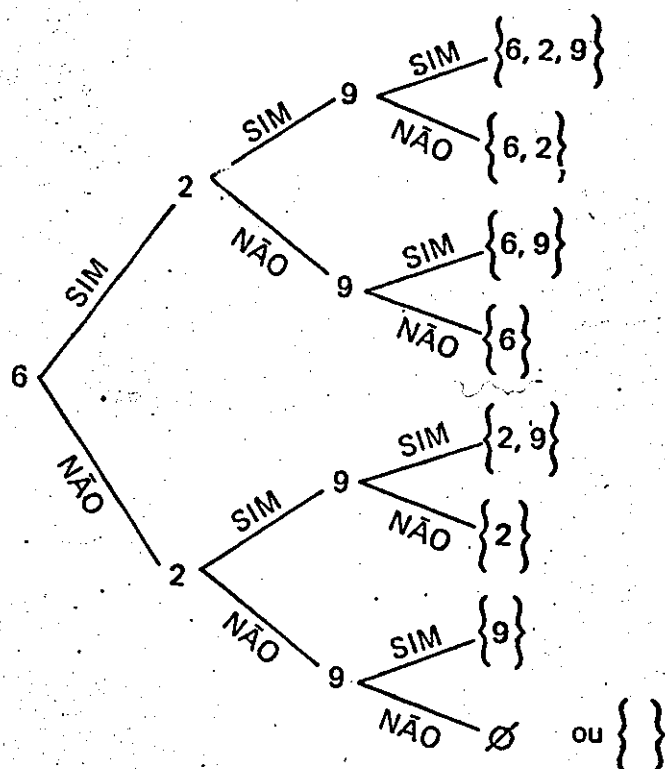


Depois que você completar a árvore, já é capaz de dizer que os subconjuntos de  $T$  são:

$\{6, 2, 9\}, \{ \text{---}, \text{---}, \text{---}, \text{---}, \text{---}, \text{---}, \text{---} \}$

A solução do quadro anterior está logo abaixo.

Mas, só confira depois de ter feito todo o esforço para resolvê-lo.



Os subconjuntos de T são:

$$\{6, 2, 9\}, \{6, 2\}, \{6, 9\}, \{6\}, \{2, 9\}, \\ \{2\}, \{9\}, \{ \} \text{ ou } \emptyset$$

Nos quadros anteriores, você determinou todas as partes dos conjuntos:

$$B = \{r, s\}$$

$$M = \{\text{lar, bar}\}$$

$$S = \{8, 6, 4\}$$

$$T = \{6, 2, 9\}$$

Então poderá responder SIM ou NÃO ao que se pede, colocando S ou N nos quadrinhos abaixo.

B é subconjunto de B? ☐

M é subconjunto de M? ☐

S é subconjunto de S? ☐

T é subconjunto de T? ☐

o conjunto VAZIO é sub-  
conjunto do:

conjunto B? ☐

conjunto M? ☐

conjunto S? ☐

conjunto T? ☐

137

Naturalmente sua resposta foi **SIM** a todas as perguntas. Logo:

$$B \subset B$$

$$M \subset M$$

$$S \subset S$$

$$T \subset T$$

e

$$\emptyset \subset B$$

$$\emptyset \subset M$$

$$\emptyset \subset S$$

$$\emptyset \subset T$$

Isto leva você a concluir que:

Qualquer  
conjunto é  
parte de si mesmo

O conjunto **VAZIO** é  
parte de qualquer  
conjunto

138

Mais uma vez responda:

$$F = \{3, 5\}$$

F é subconjunto de F? \_\_\_\_\_

O conjunto vazio é subconjunto de F? \_\_\_\_\_

139



Resp. Sim, para ambas as perguntas.

Como todo conjunto é \_\_\_\_\_ de si mesmo e tem sempre o conjunto \_\_\_\_\_ como uma de suas partes, então, para qualquer conjunto  $X$ , temos:

$$(X \subset X) \text{ e } (\emptyset \subset X)$$

140

Resp. subconjunto vazio.

Atenção para os seguintes exemplos:

$$A = \{ 2 \}; \text{ partes de } A: \{ 2 \} \text{ e } \{ \}$$

$$B = \{ \}; \text{ partes de } B: \{ \}$$

Sendo todo conjunto parte de si mesmo e o conjunto vazio parte de qualquer conjunto, conclui-se:

A ÚNICA PARTE  
DO CONJUNTO VAZIO É  
O PRÓPRIO CONJUNTO  
VAZIO

UM CONJUNTO  
UNITÁRIO TEM COMO  
PARTES O CONJUNTO  
VAZIO E ELE PRÓPRIO

141

Agora determine as partes do seguinte conjunto **UNITÁRIO**:

$$D = \{ 1 \}; \text{ partes de } D: \underline{\hspace{2cm}}, \underline{\hspace{2cm}}$$

142

★ Resp.  $\{1\}, \{ \}$ 

Neste quadro e no que se segue, daremos mais alguns exemplos de conjuntos, para você determinar suas partes.

CONJUNTOS	TODAS AS PARTES
$F = \{x; x \text{ é capital da Bahia}\}$	$\{\text{Salvador}\}$
$K = \{\text{consoante da palavra "ai"}\}$	$\{ \}$
$P = \{ \star \}$	$\{ \}$

143

Ainda mais alguns exemplos, para você determinar o número de elementos e de subconjuntos existentes em cada um deles.

CONJUNTO	Nº DE ELEM.	Nº DE SUBCONJ.
$K = \{\text{consoante, da palavra "ai"}\}$	—	1
$D = \{1\}$	<u>1</u>	2
$B = \{r, s\}$	—	4
$S = \{8, 6, 4\}$	—	—

144

Aqui estão as respostas dos quadros anteriores.

Todas as partes de F:  $\{\text{Salvador}\}, \{\}$

Todas as partes de K:  $\{\}$

Todas as partes de P:  $\{\star\}, \{\}$

Conjunto	N.º de elementos	N.º de subconjuntos
$K = \{\text{consoante da palavra "ai"}\}$	<u>0</u>	1
$D = \{1\}$	<u>1</u>	2
$B = \{r, s\}$	<u>2</u>	4
$S = \{8, 6, 4\}$	<u>3</u>	<u>8</u>

145

Você já estudou NOTAÇÃO (lição 2) e deve estar lembrado das várias maneiras de se indicar um conjunto.

Considere os conjuntos abaixo:

$$A = \{x; x \text{ é animal}\}$$

$$V = \{x; x \text{ é animal vertebrado}\}$$

$$M = \{x; x \text{ é animal mamífero}\}$$

Observe que M é:

subconjunto de V

e

subconjunto de A



146

Diz-se que  $M$  é subconjunto de  $V$  e que  $M$  é subconjunto de  $A$ .

### ESCREVE-SE

$$M = \{ x \in V; x \text{ é mamífero} \}$$

$$M = \{ x \in A; x \text{ é mamífero} \}$$

### LÊ-SE

$M$  é o conjunto dos  $x$  pertencentes a  $V$ , "tal que"  $x$  é mamífero.

$M$  é o conjunto dos  $x$  pertencentes a  $A$ , "tal que"  $x$  é mamífero.

147

Consideremos, então, o conjunto:

$$A = \{ 1, 2, 3, 4, 5, 6, 7, 8, 9 \}$$

e seus subconjuntos:

$$B = \{ 3, 6, 9 \}$$

$$C = \{ 4, 8 \}$$

$$D = \{ 1, 3, 5, 7, 9 \}$$

Usando uma notação semelhante á do quadro anterior, podemos dizer:

$$B = \{ x \in A; x \text{ é múltiplo de } 3 \}$$

$$C = \{ x \in A; x \text{ é múltiplo de } 4 \}$$

$$D = \{ x \in A; x \text{ é ímpar} \}$$

Baseado no que acabamos de estudar, você já é capaz de fazer sozinho o que se pede:

Considere o conjunto:

$$U = \{x; x \text{ é mês do ano} \}$$

Escreva a listagem dos seguintes subconjuntos de  $U$ :

$$A = \{x \in U; x \text{ tem 30 dias} \}$$

$$B = \{x \in U; x \text{ pode ter 28 dias} \}$$

$$C = \{x \in U; x \text{ tem 31 dias} \}$$

Responda:

$$A = \{ \quad , \quad , \quad , \quad \}$$

$$B =$$

$$C =$$

Resp.:  $A = \{ \text{setembro, novembro, abril, junho} \}$

$B = \{ \text{fevereiro} \}$

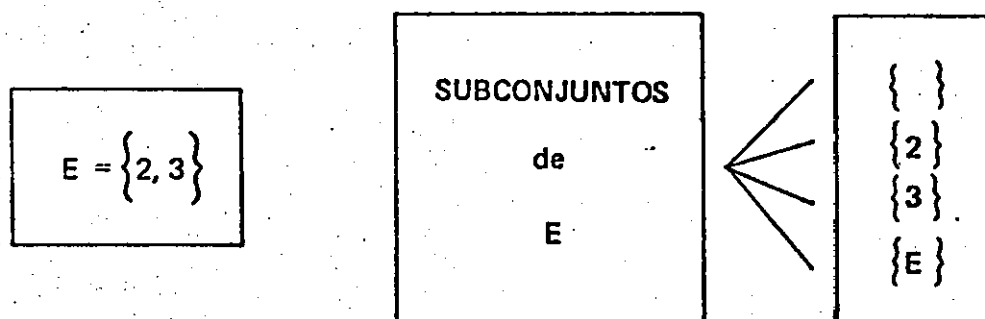
$C = \{ \text{janeiro, março, maio, julho, agosto, outubro, dezembro} \}$

Até agora, nós estudamos conjunto determinando todas as suas partes. Aqui nós veremos que todas as partes de um conjunto formam um novo conjunto, o qual se chama:

### CONJUNTO DAS PARTES

Passe ao quadro seguinte e veja como é simples.

Já vimos que existem conjuntos cujos elementos são também conjuntos.  
Agora observe este exemplo:



Podemos também formar um conjunto cujos elementos são os subconjuntos de  $E$ :

$$\{ \emptyset, \{2\}, \{3\}, E \}$$

A este conjunto chamamos conjunto das PARTES DE  $E$ .

Indicamos este conjunto através do símbolo:

$$\mathcal{P}(E) \text{ que se lê: PARTE DE } E$$

assim:

$$\mathcal{P}(E) = \{ \emptyset, \{2\}, \{3\}, E \}$$

151

Logo, como já se afirmou, existem conjuntos cujos elementos são também conjuntos e, em particular, conjuntos cujos elementos são as partes de um outro conjunto. Esta noção é importante. Vamos repeti-la juntos:

Existem conjuntos cujos elementos são também

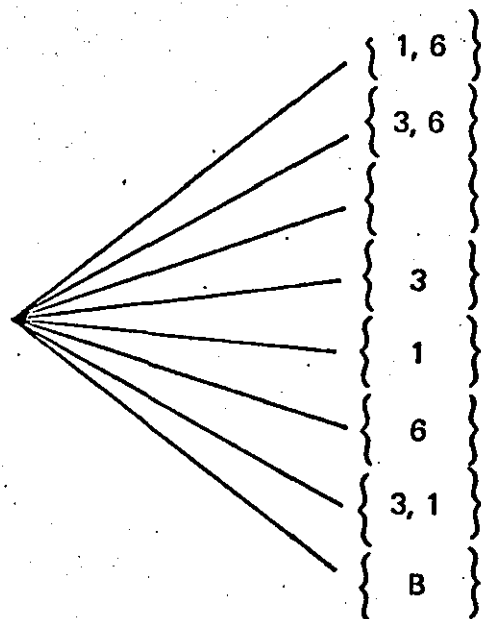
\_\_\_\_\_ e, em particular, conjuntos cujos  
\_\_\_\_\_ são partes de um outro conjunto.

152

Para fixar melhor o que você estudou nos últimos quadros, determine o conjunto das partes do conjunto  $B$ , sendo:

$$B = \{3, 1, 6\}$$

Sabemos que as partes de  $B$  são



Então, você poderá determinar o CONJUNTO DAS PARTES DE  $B$ , assim

$$\mathcal{P}(B) = \left\{ \{1, 6\}, \{ \_, \_, \_, \_, \_, \_, \_ \} \right\}$$

153

★ Resp.:  $\mathcal{P}(B) = \left\{ \{ \}, \{ 3 \}, \{ 1 \}, \{ 6 \}, \{ 3, 1 \}, \{ 3, 6 \}, \{ 1, 6 \}, B \right\}$

Para você completar:

Dado o conjunto  $S = \left\{ \text{consoante da palavra "que"} \right\}$

o CONJUNTO DAS PARTES DE S é:

$$\mathcal{P}(S) = \left\{ \quad S \quad \right\}$$

154

Resp.:  $\mathcal{P}(S) = \left\{ \{ \}, S \right\}$  ou  $\left\{ \{ \}, \{q\} \right\}$ 

Leia com atenção antes de fazer o que se pede.

Considere o conjunto  $F = \{1, 2, 3\}$

Determine o conjunto das partes de  $F$ :

\_\_\_\_\_

Não se esqueça de usar o  
símbolo  $\mathcal{P}(F)$

155

Resp.:

conjunto das partes de F:

$$\mathcal{P}(F) = \left\{ \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, F, \emptyset \right\}$$

156

Exercite um pouco.

a) Determine o conjunto das partes de V, sendo:

$$V = \left\{ \text{número par entre 4 e 6} \right\}$$


---

b) Determine o conjunto das partes de M, sendo:

$$M = \left\{ \text{bola, bela} \right\}$$


---



157

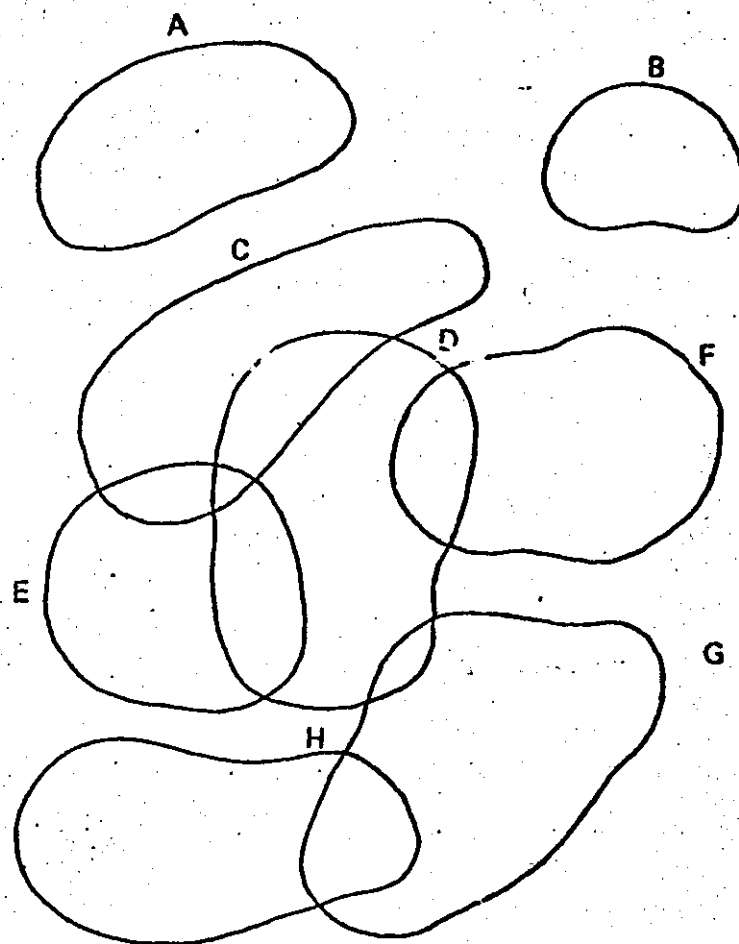
Confira suas respostas:

a) como  $V = \emptyset$

tem-se

$$\mathcal{P}(V) = \emptyset$$

$$b) \mathcal{P}(M) = \left\{ \left\{ \text{bola} \right\} \left\{ \text{bola} \right\}, M, \emptyset \right\}$$



Agora, teremos oportunidade de trabalhar com vários conjuntos ao mesmo tempo. Isso pode lhe parecer complicado, à primeira vista. Mas você mesmo chegará à conclusão de que é simples e interessante, lendo com atenção e seguindo sempre as nossas instruções.

Vamos lá?

Considere os conjuntos:

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$B = \{3, 6, 9, 12, 15\}$$

$$C = \{4, 8, 12, 16\}$$

e a expressão:

"x é número par"

Partindo dos conjuntos A, B e C e considerando a expressão "x é número par", podemos formar outros conjuntos. Por exemplo, o conjunto M :

$$M = \{x \in A; x \text{ é número par}\}$$

Também o conjunto D:

$$D = \{x \in B; x \text{ é número par}\}$$

E ainda o conjunto:

$$P = \{x \in C; x \text{ é número par}\}$$

Lembre-se que havíamos considerado os conjuntos:

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$B = \{3, 6, 9, 12, 15\}$$

$$C = \{4, 8, 12, 16\}$$

**PRESTE ATENÇÃO, RACIOCINE E COMPLETE**

Se  $M = \{x \in A; x \text{ é número par}\}$

$$M = \{ \_, \_, \_, \_ \}$$

Se  $D = \{x \in B; x \text{ é número par}\}$

$$D = \{ \_, \_ \}$$

Se  $P = \{x \in C; x \text{ é número par}\}$

$$P = \{ \_, \_, \_, \_ \}$$



Resp.:  $M = \{2, 4, 6, 8\}$   
 $D = \{6, 12\}$   
 $P = \{4, 8, 12, 16\}$

Como você pode observar, a mesma expressão "x é número par", deu origem a 3 conjuntos distintos: M, D e P.

Para formar cada um desses conjuntos, você partiu, em cada caso, de um outro conjunto mais geral.

A esse conjunto mais geral, dá-se o nome de **CONJUNTO UNIVERSO**, ou simplesmente **UNIVERSO**.

Assim:

O conjunto Universo de M é A

O conjunto Universo de D é B

O conjunto Universo de P é C



165

Sua resposta só poderia ter sido:

$$S = \{ \text{Amazonas, Acre, Minas Gerais, Goiás, Mato Grosso} \}$$

B é o UNIVERSO de S ,  
ou o Conjunto UNIVERSO  
de S

166

Agora, preste atenção para este exemplo, pois ele vai exigir um pouco mais de raciocínio.

Observe os conjuntos:

$$A = \{ \text{animais vertebrados} \}$$

$$B = \{ \text{animais mamíferos} \}$$

Considere a expressão:

"x é ave"

Sejam D o conjunto determinado no Universo A e, C, no Universo B. Tem-se:

$$D = \{ x \in A; x \text{ é } a \_ \}$$

$$C = \{ x \in B; x \_ \_ \}$$

Você deve ter completado:

$$D = \{ X \in A; x \text{ é } \underline{\text{ave}} \}$$

$$C = \{ X \in B; x \text{ é } \underline{\text{ave}} \}$$

Lembre-se dos conjuntos universos considerados:

$$A = \{ \text{animais vertebrados} \}$$

$$B = \{ \text{animais mamíferos} \}$$

### REFLEXÃO QUANTO AO QUE ACABAMOS DE ESTUDAR



A determinação dos conjuntos **D** e **C** por listagem (enunciando seus elementos) não seria conveniente. Veja porque:

Em relação ao conjunto **D**, você teria grande número de elementos, pois toda ave é vertebrado.

Quanto ao conjunto **C**, você sabe que não há nenhuma ave que seja mamífero. Logo, o conjunto **C** é um conjunto vazio.



Leia com atenção e complete:

Sendo  $A = \{x \in U; x \text{ é palavra dissílaba}\}$

Determine as listagens de  $A$ , considerando 3 conjuntos universos diferentes

a)  $U = \{\text{três, dado, balança, bola}\}$

b)  $V = \{\text{João, Ricardo, Pedro, Carlos}\}$

c)  $X = \{\text{palavra que nomeia estação do ano}\}$

a)  $A =$

b)  $A =$

c)  $A =$

170

Resp.:

a)  $A = \{\text{dado, bola}\}$

b)  $A = \{\text{João, Pedro, Carlos}\}$

c)  $A = \{\text{verão}\}$

171

Adiante, você encontrará uma série de exercícios, que lhe ajudarão bastante a fixar melhor todas as informações estudadas nesta lição. Concentre-se e responda cuidadosamente cada questão.

Boa sorte



1 — Represente de 4 maneiras diferentes:

- I) A, conjunto dos algarismos romanos
- II) B, conjunto das cores da bandeira do estado da Bahia
- III) C, conjunto dos nomes dos continentes.

2 — Assinale com X, quais das afirmações abaixo são corretas:

- ( ) a) Os elementos de um conjunto são representados por pontos, escritos sobre a linha do diagrama.
- ( ) b) Os elementos de um conjunto são representados por pontos na parte exterior do diagrama.
- ( ) c) Os elementos de um conjunto são representados por pontos no interior do diagrama.
- ( ) d) Os elementos que não pertencem ao conjunto são representados por pontos, escritos no interior do diagrama.

3 — Determine os seguintes conjuntos, sob a forma de listagem:

$$A = \{ \text{nomes de estados do Brasil cuja sigla tem a letra A} \}$$

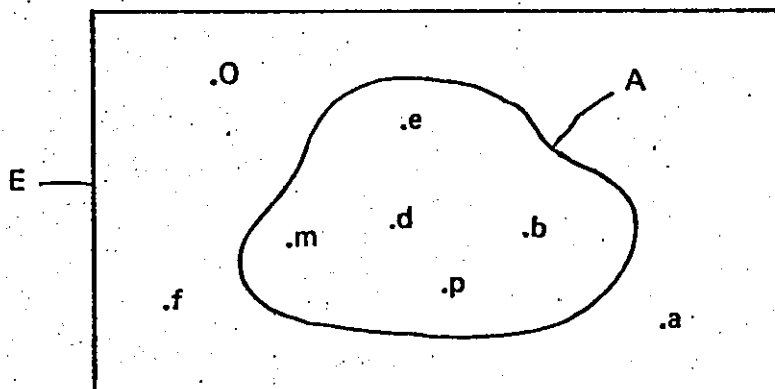
$$B = \{ \text{emissoras de TV de Salvador} \}$$

$$C = \{ P; P \text{ é país da América do Sul de área maior que o Brasil} \}$$

4 — Complete as lacunas abaixo de maneira adequada, usando uma das seguintes expressões: está contido, contém.

- a)  $\{5, 7\}$  ..... A      c)  $\{6, 8\}$  .....  $\{8\}$   
 b) B .....  $\emptyset$       d)  $\{1, 4\}$  .....  $\{1, 2, 3, 4\}$

5 — Considerando o conjunto E (no gráfico abaixo), colocar sobre o traço \_\_\_\_\_ o sinal adequado escolhido dentre:  
 $\in, \notin, \subset, \not\subset$ .



- |                      |                                   |
|----------------------|-----------------------------------|
| I) a _____ A         | VI) $\emptyset$ _____ A           |
| II) a _____ E        | VII) $\{f\}$ _____ $\{m, p\}$     |
| III) $\{a\}$ _____ A | VIII) e _____ A                   |
| IV) $\{a\}$ _____ E  | IX) $\{p, f, o\}$ _____ E         |
| V) $\{b\}$ _____ A   | X) $\{m, e, d\}$ _____ $\{m, d\}$ |

6 — Escreva os conjuntos abaixo por meio de uma propriedade característica de seus elementos.

- a)  $\{ \text{janeiro} \}$
- b)  $\{ \text{Salvador, Ilhéus, Alagoinhas, Feira de Santana} \}$
- c)  $\{ \text{Brasília} \}$

7 — Utilizando o diagrama de Venn, representar os seguintes conjuntos:

- a)  $A = \{ 7, 9, 0 \}$
- b)  $B = \{ 1, 2, 3, 4 \}$

8 — Sendo  $A = \left\{ x \in U; x \text{ é palavra que se inicia com a letra } A \right\}$

determine a listagem de A, sendo o universo:

- a)  $U = \{ \text{estados do Brasil} \}$
- b)  $U = \{ \text{sentidos} \}$
- c)  $U = \{ \text{meses do ano} \}$

9 — Sendo  $A = \left\{ x \in U; s \text{ é consoante da palavra paciência} \right\}$

determine a listagem de A, sendo o universo:

- a)  $U = \{ a, b, c, d, e, f, g, h \}$
- $U = \{ i, j, l, m, n, o, p \}$
- $U = \{ q, r, s, t, u, v, x, z \}$

## Respostas:

- 1 — a) A = { I, V, X, L, C, D, M }  
 A = { x; x é algarismo romano }  
 A = { a, b, c, d, e, f, g }  
 A = { algarismos romanos }  
 b) B = { azul, vermelho, branco }  
 B = { v, a, b }  
 B = { x; x é cor da bandeira do Estado da Bahia }  
 B = { a, b, c }  
 c) C = { Ásia, África, América }  
 C = { a, b, c, d }  
 C = { x; x é nome de continente }  
 C = { nomes dos continentes }

2 — (X) C

$$\begin{aligned}
 3 - A &= \{ \text{Amazonas, Pará, Maranhão, Alagoas,} \\
 &\quad \text{Bahia, Acre} \} \\
 B &= \{ \text{Aratu, Itapoã} \} \\
 C &= \{ \}
 \end{aligned}$$

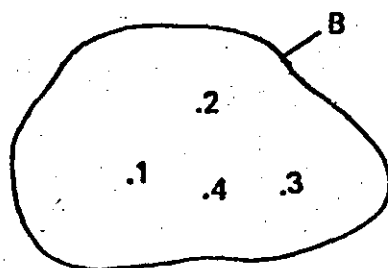
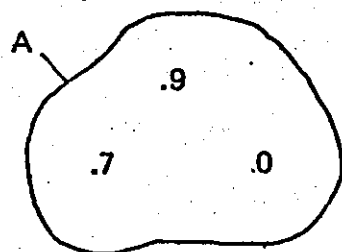
- 4 - a) está contido  
 b) contém  
 c) contém  
 d) está contido.

$$\begin{aligned}
 5 - \text{I)} \quad a &\notin A \\
 \text{II)} \quad a &\in E \\
 \text{III)} \quad \{a\} &\not\subset A \\
 \text{IV)} \quad \{a\} &\subset E \\
 \text{V)} \quad \{b\} &\subset A
 \end{aligned}$$

$$\begin{aligned}
 \text{VI)} \quad \emptyset &\subset A \\
 \text{VII)} \quad \{f\} &\not\subset \{m, p\} \\
 \text{VIII)} \quad e &\in A \\
 \text{IX)} \quad \{p, f, o\} &\subset E \\
 \text{X)} \quad \{n, e, d\} &\supset \{m, d\}
 \end{aligned}$$

- 6 - a)  $\{ x; x \text{ é o nome do } 1.^\circ \text{ mês do ano} \}$   
 b)  $\{ x; x \text{ é nome de cidade do estado da Bahia} \}$   
 c)  $\{ x; x \text{ é capital do Brasil} \}$

7 -



177

8 -  $A_1 = \{ \text{Amazonas, Alagoas, Acre} \}$

$A_2 = \{ \text{audição} \}$

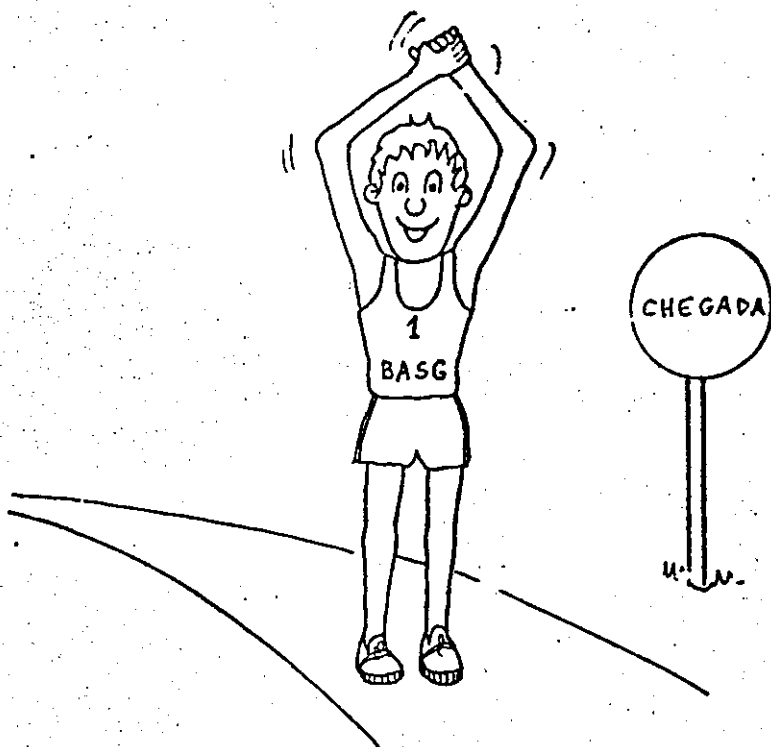
$A_3 = \{ \text{abril, agosto} \}$

9 - a)  $\{ a, c, e \}$

b)  $\{ l, n, p \}$

c)  $\{ \}$





Parabéns, chegamos ao final do 1.º módulo onde estudamos os seguintes assuntos:

- Noção de conjunto
- Notação
- Representação gráfica
- Pertinência
- Partes de um conjunto ou subconjunto
- Conjunto das partes de um conjunto e conjunto universo.

Continue com o mesmo entusiasmo nos módulos que se seguem e você verá como é bom e agradável estudar Matemática.

Adiante por favor!

APPENDIX D

4. - LESSON 1 OF MODULE VI OF THE BASG-M

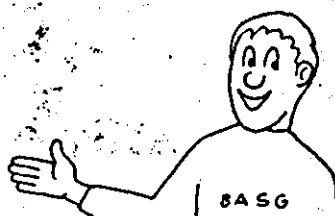
MATERIAL.

THE ORIGINAL LINEAR PROGRAMME VERSION

**1ª LIÇÃO**

**DIVISORES COMUNS ENTRE DOIS OU MAIS NÚMEROS**

Uma nova lição!



O que apresentaremos aqui será bastante simples, para você que já estudou como determinar os divisores de um número natural qualquer. Com facilidade você determinará também o conjunto dos divisores comuns de dois, três ou mais números naturais.

1

Temos os números **30** e **18**. Que faremos para determinar os números naturais que são divisores comuns de ambos?

Primeiro, considere os conjuntos:

(divisores de 30) e (divisores de 18)

$$D(30) = \{ 1, 2, 3, 5, 6, 10, 15, 30 \}$$

$$D(18) = \{ 1, 2, 3, 6, 9, 18 \}$$

Agora responda: quais os elementos comuns aos dois conjuntos?

\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_

2



Resp.: 1, 2, 3, 6

Os Números Naturais:

1, 2, 3, 6

elementos comuns aos  
conjuntos:  $D(30)$  e  $D(18)$

são

**DIVISORES COMUNS**

**DE**

\_\_\_\_\_ e \_\_\_\_\_

3

Resp.: 30 e 18

Um número natural  $x$  será **DIVISOR COMUM** de 30 e 18  
se  $x$  pertence à interseção entre  $D(30)$  e  $D(18)$

INDICA-SE

$$x \in [ D(30) \cap D(18) ]$$

4

Tendo identificado os elementos comuns aos conjuntos  
 $D(30)$  e  $D(18)$  você pode determinar o conjunto dos **DIVISORES**  
**COMUNS** de 30 e 18:

$$D(30) \cap D(18) = \{ \_, \_, \_, \_ \}$$

5

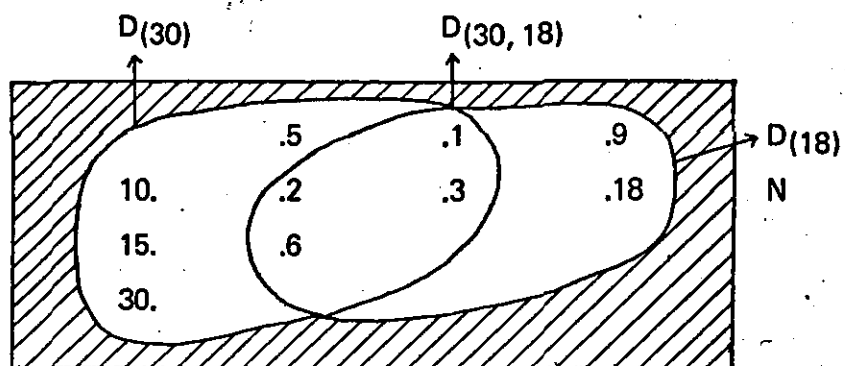
Resp.:  $D(30) \cap D(18) = \underline{1, 2, 3, 6}$ 

Indica-se o conjunto dos **DIVISORES COMUNS** de 30 e 18 por  $D(30, 18)$   
e escreve-se:

$$D(30, 18) = D(30) \cap D(18)$$

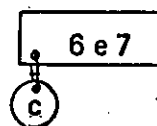
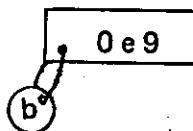
6

Através do diagrama podemos observar melhor:



A parte hachurada no diagrama, representa o conjunto dos naturais que não são **DIVISORES** de 30 nem de 18.

Você agora já é capaz de determinar o conjunto dos **DIVISORES**  
**COMUNS** de:



Complete os espaços:

$$D_{(24)} = \{ 1, \_, \_, \_, \_, \_, \_, \_, \_ \}$$

$$D_{(32)} = \{ \_, \_, \_, \_, \_, \_, \_ \}$$

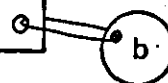
$$D_{(24,32)} = \{ \_, \_, \_, \_ \}$$



$$D_{(0)} = \{ \}$$

$$D_{(9)} = \{ \}$$

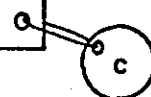
$$D_{(0,9)} = \{ \}$$



$$D_{(7)} = \{ \}$$

$$D_{(6)} = \{ \}$$

$$D_{(6,7)} = \{ \}$$



8

Configura os seus resultados:

a)

$$D(24) = \{ 1, 2, 3, 4, 6, 8, 12, 24 \}$$

$$D(32) = \{ 1, 2, 4, 8, 16, 32 \}$$

b)

$$D(0) = \{ 1, 2, 3, \dots, 20, \dots \}$$

$$D(9) = \{ 1, 3, 9 \}$$

c)

$$D(6) = \{ 1, 2, 3, 6 \}$$

$$D(7) = \{ 1, 7 \}$$

Logo:

$$D(24, 32) = \{ 1, 2, 4, 8 \}$$

$$D(0, 9) = \{ 1, 3, 9 \}$$

$$D(6, 7) = \{ 1 \}$$

9

Se dissermos para você:

"um número natural  $x$  pertence à interseção entre  $D(24)$  e  $D(32)$ "

Como você faz esta indicação?

\_\_\_\_\_ [ ]

10

Resp.:  $x \in [D_{(24)} \cap D_{(23)}]$ 

Através dos exercícios anteriores, você determinou apenas, os divisores comuns de alguns pares de números naturais.

Agora, veremos a definição dos divisores comuns de dois números naturais quaisquer.

Passe ao quadro seguinte.

11

Dados

dois números naturais quaisquer  $a$  e  $b$ , um deles pelo menos, diferente de zero, diz-se

**DIVISOR COMUM de  $a$  e  $b$**

um número

$x \in \mathbb{N}$  tal que

$x \in D_{(a)}$  e  $x \in D_{(b)}$

isto significa que

$x \in [D_{(a)} \cap D_{(b)}]$

Indica-se por:

$D(a, b)$  o conjunto dos **DIVISORES COMUNS** de  $a$  e  $b$

Escreve-se:

$$D(a, b) = D_{(a)} \cap D_{(b)}$$



12

Para fixar a definição do quadro anterior, vamos repeti-la.

Ajude-nos, completando.

Dados dois números naturais quaisquer  $a$  e  $b$ , um deles pelo menos diferente de zero,

diz-se **DIVISOR COMUM** de  $a$  e  $b$ , um número  $x$  ?  $\mathbb{N}$ , tal que;

$x \in D(\text{?})$  e  $x \in D(\text{?})$  isto significa que

$$x \in [D(a) \text{---} \text{?} \text{---} D(b)]$$

13



Resp. ? ;  $D(a)$ ;  $D(b)$ ; ?

Ainda considerando  $D(a)$  e  $D(b)$ , indica-se por  $D(\text{?}, \text{?})$  o conjunto dos **DIVISORES COMUNS** de  $a$  e  $b$  e escreve-se

$$D(\text{?}, \text{?}) = \text{?}$$



Resp.

 $D(a, b);$ 

$$D(a, b) = D(a) \cap D(b)$$

**EXERCÍCIOS:**I – Entre os conjuntos seguintes apenas um é o conjunto  $D(3, 6)$ Marque X ao lado direito da indicação do conjunto correspondente a  $D(3, 6)$  $\{1, 2, 3\}$  ☐ $\{1, 3\}$  ☐ $\{1, 2, 6\}$  ☐II – Complete usando os sinais  $\in$  e  $\notin$ a)  $7 \underline{\hspace{1cm}} D(7, 5)$ b)  $9 \underline{\hspace{1cm}} D(18, 27)$ c)  $10 \underline{\hspace{1cm}} D(20, 150)$ d)  $5 \underline{\hspace{1cm}} D(35, 15)$ 

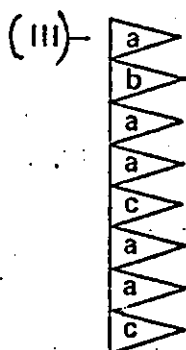
III – Dado conjunto de divisores de dois números, identificar a listagem dos elementos dos conjuntos dados:

$D(3, 7)$	<input type="checkbox"/>
$D(8, 6)$	<input type="checkbox"/>
$D(5, 7)$	<input type="checkbox"/>
$D(3, 4)$	<input type="checkbox"/>
$D(15, 30)$	<input type="checkbox"/>
$D(7, 5)$	<input type="checkbox"/>
$D(4, 3)$	<input type="checkbox"/>
$D(30, 15)$	<input type="checkbox"/>

c  $\{1, 15\}$ a  $\{1\}$ b  $\{1, 2\}$

$$(I) - \boxed{\{1, 3\}} \boxed{X}$$

$$(II) - a) \notin \quad b) \in \quad c) \in \quad d) \in$$



Você deve ter notado, que até agora só determinamos os divisores comuns de dois números naturais. Agora, vamos determinar os divisores comuns de três ou mais números naturais.

Veja como se procede:

Dados os números naturais 3, 6, 12 determinam-se os conjuntos:

$$D(3) = \{1, 3\}$$

$$D(6) = \{1, 2, 3, 6\}$$

$$D(12) = \{1, 2, 3, 4, 6, 12\}$$

Diz-se que um número natural  $x$  é **DIVISOR COMUM** de 3, 6 e 12 se:

$$x \in \begin{cases} D(3) \\ D(6) \\ D(12) \end{cases}$$

isto significa que:

$$x \in [D(3) \cap D(6) \cap D(12)]$$

ou que

$$x \in [D(3, 6) \cap D(12)]$$

Ainda estamos falando sobre o conjunto dos **DIVISORES COMUNS** de 3, 6 e 12

Indica-se por:

$$D(3, 6, 12)$$

o conjunto dos  
divisores comuns  
de 3, 6 e 12

Escreve-se:

$$D(3, 6, 12) = D(3, 6) \cap D(12)$$

$$\text{como } D(3, 6) = \{1, 3\}$$

tem-se

$$D(3, 6, 12) = \{1, 3\} \cap D(12)$$

e finalmente:

$$D(3, 6, 12) = \{1, 3\}$$

Siga o nosso raciocínio e determine os **DIVISORES COMUNS** de 6, 8, 16.

Vá completando convenientemente:

Indicando-se por  $x$  um número natural divisor comum de 6, 8, 16

tem-se  $x \in$   $\begin{cases} D( \quad ) \\ \quad \\ \quad \end{cases}$

Isto significa que:

COMPLETE

$$x \in [D(6) \cap D(\quad) \text{ — } D(\quad)]$$

ou

$$x \in [D(6, 8) \text{ — } D(\quad)]$$

20



Resp.:

$x \in$ 
 $\begin{cases} D(6) \\ D(8) \\ D(16) \end{cases}$

$$x \in [D(6) \cap D(8) \cap D(16)]$$

$$x \in [D(6, 8) \cap D(16)]$$

Determine os conjuntos:

$$D(6) = \{ \underline{\hspace{2cm}} \}$$

$$D(8) = \{ \underline{\hspace{2cm}} \}$$

$$D(16) = \{ \underline{\hspace{2cm}} \}$$

$$D(6, 8) = \{ \underline{\hspace{2cm}} \}$$

21



Resp.:

$$D_{(6)} = \{ \underline{1}, \underline{2}, \underline{3}, \underline{6} \}$$

$$D_{(8)} = \{ \underline{1}, \underline{2}, \underline{4}, \underline{8} \}$$

$$D_{(16)} = \{ \underline{1}, \underline{2}, \underline{4}, \underline{8}, \underline{16} \}$$

$$D_{(6, 8)} = \{ \underline{1}, \underline{2} \}$$

Você conferiu suas respostas? Ótimo!

Uma vez que você já determinou os conjuntos  $D_{(6, 8)}$  e  $D_{(16)}$ , fica mais fácil determinar o **CONJUNTO DE DIVISORES COMUNS** de 6, 8, 16.

Só para lhe ajudar, daremos algumas “dicas” e você deve completar quando se fizer necessário.

$$D_{(6, 8, 16)} = [ D_{(6, 8)} \cap D_{(16)} ]$$

ou

$$D_{(6, 8, 16)} = \{ \underline{\hspace{2cm}} \} \cap D_{(16)}$$

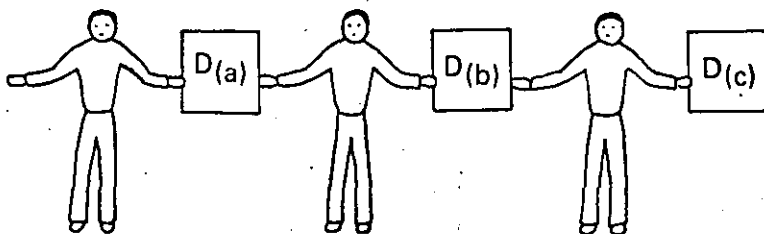
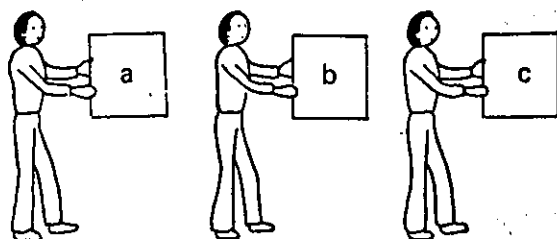
Finalmente:

$$D_{(6, 8, 16)} = \{ \underline{\hspace{4cm}} \}$$

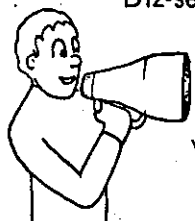


Resp.:  $D(6, 8, 16) = \{ \underline{1}, \underline{2} \} \cap D(16) ; D(6, 8, 16) = \{ \underline{1}, \underline{2} \}$

Revistando o que foi estudado até agora. Dados três números naturais  $a, b, c$  e os conjuntos:



Diz-se que:

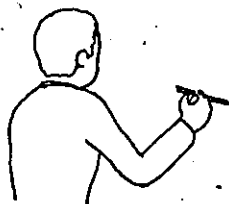


um número natural  $x$

é

divisor comum de  $a, b, c$  se  $x \in D(a, b) \cap D(c)$

ESCREVE-SE



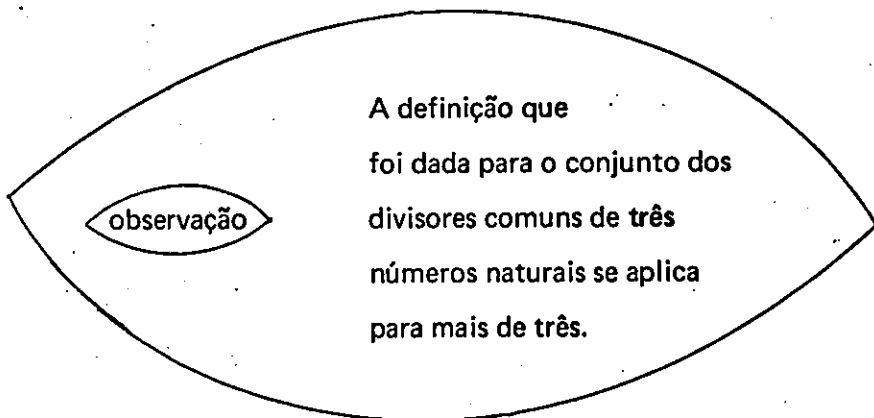
$$D(a, b, c) = D(a, b) \cap D(c)$$

INDICA-SE



$D(a, b, c)$  o conjunto dos divisores comuns de  $a, b, c$

23



24

Considere os conjuntos:

$$D(6) = \{ 1, 2, 3, 6 \}$$

$$D(24) = \{ 1, 2, 3, 4, 6, 8, 12, 24 \}$$

$$D(15) = \{ 1, 3, 5, 15 \}$$

$$D(18) = \{ 1, 2, 3, 6, 9, 18 \}$$

E determine: o conjunto dos **DIVISORES COMUNS** de 6, 24, 15 e 18

$$D(6, 24, 15, 18) = \{$$



Confira sua resposta, acompanhando o nosso raciocínio:

$$D(6, 24, 15, 18) = [D(6, 24) \cap D(15)] \cap D(18)$$

$$\text{como } D(6, 24) = \{1, 2, 3, 6\}$$

tem-se:

$$D(6, 24, 15, 18) = \left[ \{1, 2, 3, 6\} \cap D(15) \right] \cap D(18)$$

ou

$$D(6, 24, 15, 18) = \{1, 3\} \cap D(18)$$

e finalmente:

$$D(6, 24, 15, 18) = \{1, 3\}$$

## APPENDIX D

### 5. LESSON 1 OF MODULE VI OF THE BASG-M MATERIALS.

COPY OF MANUSCRIPT FOR THE ALTERNATIVE, INFORMATION-  
MAPPED VERSION OF THIS LESSON.

The reader may compare this version with the linear programme version shown in the previous section of this appendix, in conjunction with the explanatory notes in Section 1 of the appendix.

# O CONCEITO DE "DIVISOR COMUM" DE DOIS NÚMEROS NATURAIS

1

## DEFINIÇÃO

Um número natural  $x$  será DIVISOR COMUM de 30 e 18 se  $x$  pertence à interseção entre  $D(30)$  e  $D(18)$

INDICA-SE

$$x \in [D(30) \cap D(18)]$$

## EXEMPLO

Temos os números 30 e 18. Que faremos para determinar os números naturais que são divisores comuns de ambos?

Primeiro considere os conjuntos:

(divisores de 30) e (divisores de 18)

$$D(30) = \{1, 2, 3, 5, 6, 10, 15, 30\}$$

$$D(18) = \{1, 2, 3, 6, 9, 18\}$$

Segundo

Identifique a interseção

$$D(30) \cap D(18) = 1, 2, 3, 6$$

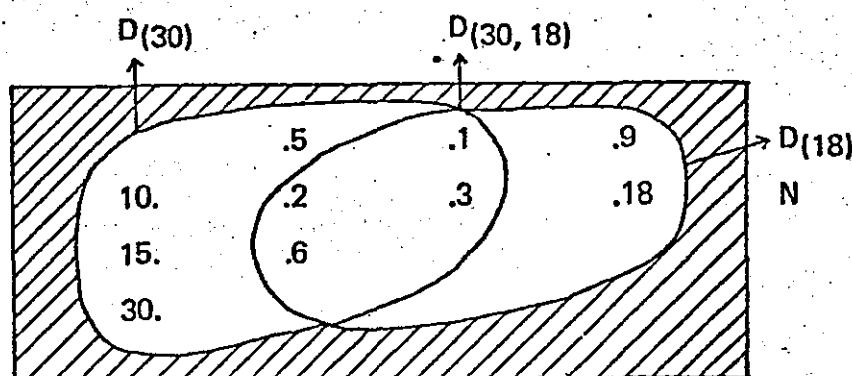
## NOTAÇÃO

Indica-se o conjunto dos DIVISORES COMUNS de 30 e 18 por  $D(30, 18)$  e escreve-se:

$$D(30, 18) = D(30) \cap D(18)$$

## DIAGRAMA

Através do diagrama podemos observar melhor:



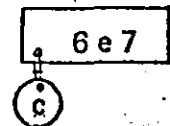
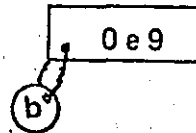
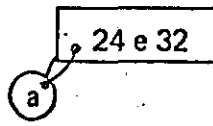
A parte hachurada no diagrama, representa o conjunto dos naturais que não são DIVISORES de 30 nem de 18.

# EXERCÍCIO 1

Ex

1.

Você agora já é capaz de determinar o conjunto dos DIVISORES COMUNS de:



Complete os espaços:

$$D(24) = \{ 1, \_, \_, \_, \_, \_, \_, \_, \_ \}$$

$$D(32) = \{ \_, \_, \_, \_, \_, \_, \_ \}$$

$$D(24, 32) = \{ \_, \_, \_, \_, \_ \}$$

a

$$D(0) = \{ \}$$

$$D(9) = \{ \}$$

$$D(0, 9) = \{ \}$$

b

$$D(7) = \{ \}$$

$$D(6) = \{ \}$$

$$D(6, 7) = \{ \}$$

c

2.

Se dissermos para você:

"um número natural  $x$  pertence à interseção entre  $D(24)$  e  $D(32)$ "

Como você faz esta indicação?

$$\_\_\_\_\_\_ \mid \_\_\_\_\_\_ [ \cdot ]$$

R. 1

# RESPOSTAS 1.

1

a.

$$D(24) = \{ 1, 2, 3, 4, 6, 8, 12, 24 \}$$

$$D(32) = \{ 1, 2, 4, 8, 16, 32 \}$$

Logo:

$$D(24, 32) = \{ 1, 2, 4, 8 \}$$

b.

$$D(0) = \{ 1, 2, 3, \dots, 20, \dots \}$$

$$D(9) = \{ 1, 3, 9 \}$$

$$D(0, 9) = \{ 1, 3, 9 \}$$

$$D(6) = \{ 1, 2, 3, 6 \}$$

$$D(6, 7) = \{ 1 \}$$

c.

$$D(7) = \{ 1, 7 \}$$

2

$$x \in [D(24) \cap D(\underline{23})]$$

$\uparrow$   
 $D(32)$

Observações

(OBSERVATION)

Através dos exercícios anteriores, você determinou apenas, os divisores comuns de alguns pares de números naturais.

Agora, veremos a definição dos divisores comuns de dois números naturais quaisquer.

Definição

(DEFINITION)

Dados

dois números naturais quaisquer  $a$  e  $b$ , um deles pelo menos, diferente de zero, diz-se

**DIVISOR COMUM de  $a$  e  $b$**

um número

$x \in \mathbb{N}$  tal que

$x \in D(a)$  e  $x \in D(b)$

isto significa que

$x \in [D(a) \cap D(b)]$

Notação

(NOTATION)

Indica-se por:

$D(a, b)$  o conjunto dos DIVISORES COMUNS de  $a$  e  $b$

Escreve-se:

$$D(a, b) = D(a) \cap D(b)$$

1

- (I) — Entre os conjuntos seguintes apenas um é o conjunto  $D(3, 6)$ .  
 Marque X ao lado direito da indicação do conjunto correspondente a  $D(3, 6)$

$\{1, 2, 3\}$  ☐

$\{1, 3\}$  ☐

$\{1, 2, 6\}$  ☐

- (II) — Complete usando os sinais  $\in$  e  $\notin$

- a)  $7 \text{ } \underline{\hspace{1cm}} D(7, 5)$   
 b)  $9 \text{ } \underline{\hspace{1cm}} D(18, 27)$   
 c)  $10 \text{ } \underline{\hspace{1cm}} D(20, 150)$   
 d)  $5 \text{ } \underline{\hspace{1cm}} D(35, 15)$

- (III) — Dado conjunto de divisores de dois números, identificar a listagem dos elementos dos conjuntos dados:

$D(3, 7)$		a
$D(8, 6)$		
$D(5, 7)$		
$D(3, 4)$		
$D(15, 30)$		
$D(7, 5)$		
$D(4, 3)$		
$D(30, 15)$		

c  $\{1, 15\}$

a  $\{1\}$

b  $\{1, 2\}$

2.

- (I) Ajude-nos, completando.

Dados dois números naturais quaisquer  $a$  e  $b$ , um deles pelo menos diferente de zero,

diz-se **DIVISOR COMUM** de  $a$  e  $b$ , um número  $x \text{ } \underline{\hspace{1cm}} N$ , tal que:

$x \in D(\text{ })$  e  $x \in D(\text{ })$  isto significa que

$x \in [D(a) \text{ } \underline{\hspace{1cm}} D(b)]$

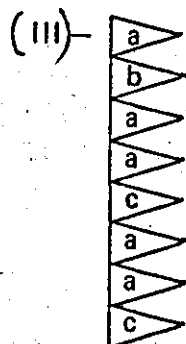
- (II) Ainda considerando  $D(a)$  e  $D(b)$ , indica-se por  $D(\text{ }, \text{ })$  o conjunto dos **DIVISORES COMUNS** de  $a$  e  $b$  e escreve-se

## RESPOSTAS. 2.

1

$$(I) - \boxed{\{1, 3\}} \boxed{X}$$

$$(II) - a) \notin \quad b) \in \quad c) \in \quad d) \in$$



2

$$(I) \quad \left| \quad \underline{\quad} \quad ; D(a); D(b); \underline{\quad} \right|$$

(II)

$$D(\underline{a}, \underline{b}); \quad \boxed{D(\underline{a}, \underline{b}) = \underline{D(a)} \cap \underline{D(b)}}$$



# EXTENÇÃO DA DEFINIÇÃO E CONCEITO

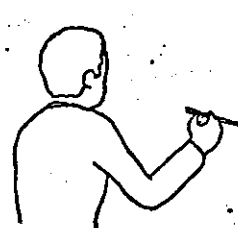
## DIVISORES COMUNS DE 3 OU MAIS NÚMEROS

3

DEFINIÇÃO  
(DEFINITION)

Diz-se que:  
um número natural  $x$   
é  
divisor comum de  $a, b, c$  se  $x \in D(a, b) \cap D(c)$

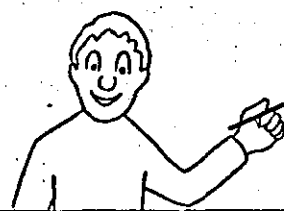
NOTAÇÃO  
(NOTATION)



ESCREVE-SE

$$D(a, b, c) = D(a, b) \cap D(c)$$

INDICA-SE



$D(a, b, c)$  o conjunto  
dos divisores comuns  
de  $a, b, c$

EXEMPLO  
(EXAMPLE)

Diz-se que um número natural  $x$  é DIVISOR COMUM de 3, 6  
e 12 se:

$x \in \begin{cases} D(3) \\ D(6) \\ D(12) \end{cases}$

isto significa que:

$$x \in [D(3) \cap D(6) \cap D(12)]$$

ou que

$$x \in [D(3, 6) \cap D(12)]$$

Indica-se por:

$$D(3, 6, 12)$$

o conjunto dos  
divisores comuns  
de 3, 6 e 12

# Como determinar os divisores comuns de 3 ou mais números - 36

## Introdução (INTRODUCTION)

Você deve ter notado, que até agora só determinamos os divisores comuns de dois números naturais. Agora, vamos determinar os divisores comuns de três ou mais números naturais.

## Procedimento (PROCEDURE)

1

### PASSOS

DADO TRÊS NÚMEROS NATURAIS:  
  
DETERMINAR OS CONJUNTOS DOS DIVISORES

### EXEMPLO

Dados os números naturais 3, 6, determinam-se os conjuntos:

$$D(3) = \{ 1, 3 \}$$

$$D(6) = \{ 1, 2, 3, 6 \}$$

$$D(12) = \{ 1, 2, 3, 4, 6, 12 \}$$

2

DETERMINAR A INTERSEÇÃO DE QUALQUER DOIS DESSES CONJUNTOS

$$D(3, 6, 12) = D(3, 6) \cap D(12)$$

como  $D(3, 6) = \{ 1, 3 \}$

tem-se

$$D(3, 6, 12) = \{ 1, 3 \} \cap D(12)$$

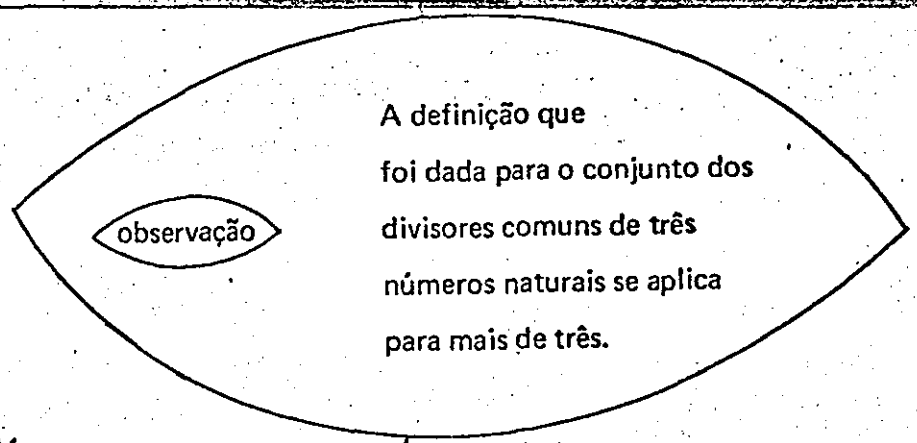
3

DETERMINAR A INTERSEÇÃO ENTRE O CONJUNTO DETERMINADO EM PASSO 2 E O TERCEIRO CONJUNTO DOS DIVISORES

e finalmente:

$$D(3, 6, 12) = \{ 1, 3 \}$$

## Observação (OBSERVATION)



LOGO:

PASSO 3 PODE SER REPETIDO PARA UM QUARTO, QUINTO, ETC, CONJUNTO.

EXEMPLO:

$$D(3, 6, 12, 8) = D(3, 6, 12) \cap D(8)$$

COMO  $D(8) = \{ 1, 2, 4, 8 \}$

$$D(3, 6, 12, 8) = \{ 1 \}$$

1.

Vá completando convenientemente:

Indicando-se por  $x$  um número natural divisor comum de 6, 8, 16

tem-se  $x \in$   $\begin{matrix} D( \quad ) \\ \diagdown \\ \text{---} \\ \diagup \end{matrix}$

Isto significa que:

COMPLETE

$$x \in [D(6) \cap D(\text{---}) - D(\text{---})]$$

ou

$$x \in [D(6, 8) - D(\text{---})]$$

2.

Determine os conjuntos:

$$D(6) = \{ \text{---} \}$$

$$D(8) = \{ \text{---} \}$$

$$D(16) = \{ \text{---} \}$$

$$D(6, 8) = \{ \text{---} \}$$

$$D(6, 8, 16) = \{ \text{---} \}$$

3.

Considere os conjuntos:

$$D(6) = \{ 1, 2, 3, 6 \}$$

$$D(24) = \{ 1, 2, 3, 4, 6, 8, 12, 24 \}$$

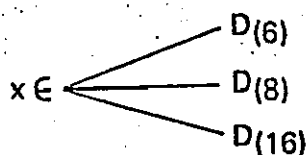
$$D(15) = \{ 1, 3, 5, 15 \}$$

$$D(18) = \{ 1, 2, 3, 6, 9, 18 \}$$

E determine: o conjunto dos DIVISORES COMUNS de 6, 24, 15 e 18

$$D(6, 24, 15, 18) = \{ \text{---} \}$$

1.



$$x \in [D(6) \cap D(8) \cap D(16)]$$

$$x \in [D(6, 8) \cap D(16)]$$

2.

$$D(6) = \{ \underline{1}, \underline{2}, \underline{3}, \underline{6} \}$$

$$D(8) = \{ \underline{1}, \underline{2}, \underline{4}, \underline{8} \}$$

$$D(16) = \{ \underline{1}, \underline{2}, \underline{4}, \underline{8}, \underline{16} \}$$

$$D(6, 8) = \{ \underline{1}, \underline{2} \}$$

$$: D(6, 8, 16) = \{ \underline{1}, \underline{2} \} \cap D(16) : D(6, 8, 16) = \{ \underline{1}, \underline{2} \}$$

3.

Confira sua resposta, acompanhando o nosso raciocínio:

$$D(6, 24, 15, 18) = [D(6, 24) \cap D(15)] \cap D(18)$$

$$\text{como } D(6, 24) = \{ 1, 2, 3, 6 \}$$

tem-se:

$$D(6, 24, 15, 18) = \left[ \{ 1, 2, 3, 6 \} \cap D(15) \right] \cap D(18)$$

ou

$$D(6, 24, 15, 18) = \{ 1, 3 \} \cap D(18)$$

e finalmente:

$$D(6, 24, 15, 18) = \{ 1, 3 \}$$

## APPENDIX E

### DATA FROM EXPERIMENTS

## GROUP "A"

N = 30

Student Ref.No.	RESULTS				TIME (HOURS)					
	Pre- test	Test (I-III)	Test (IV-V)	Post Test	Mod I	Mod II	Mod III	Mod IVa	Mod IVb	Mod V
1	10	65	60	50	24	10	13	12.75	14.5	5
2	10	50	80	85	20.5	7	10.5	8	10.25	2.25
3	20	70	65	50	17.5	5	8.5	5.5	7.25	1.5
4	0	30	70	45	15	4	7	12.5	14.25	3.5
5	5	40	70	40	20.25	8.5	10	10	11.25	2.25
6	10	55	70	45	18.25	6	8.5	12.5	14	3.25
7	20	60	50	45	21	9	11.5	8.25	10.5	2.25
8	15	45	70	55	12	3	5.5	15	14.5	5
9	20	50	65	85	20	7.25	10.25	9.5	12	2
10	15	65	75	50	25	10.5	13	6.25	7.5	1.25
11	10	40	80	30	20.5	7	10.5	10.25	13	3.5
12	15	55	65	45	17.25	6.25	9	7	9	1.5
13	0	40	50	60	20	7.5	10	15	17	4
14	0	40	70	40	16	4.5	5.75	13.5	14.5	3
15	0	50	60	55	21.25	8.5	11.25	9.25	12	2
16	40	80	75	60	22.5	9	11.5	7.25	10	1.5
17	20	60	70	50	19	6	9.75	10	11.25	2
18	10	25	65	40	21	8	11	11.5	15.5	4.25
19	0	45	70	70	17.75	6.5	8	9	11	2.25
20	15	40	75	45	20.5	7	10	10	12.5	3
21	15	65	55	50	13.5	2.5	6	8	10.5	1.5
22	10	45	65	50	21.5	6.5	11	11.25	13.5	3.5
23	40	80	65	90	20.25	7.5	10.25	11.5	15	3.75
24	15	50	60	55	22	12	11.75	9	12.25	2
25	10	35	65	70	19	8	10.5	3.5	11	2
26	20	40	80	40	19.5	7	10	9.25	11.5	2.25
27	15	50	60	85	16.25	5.25	9.5	11.75	15.25	3.25
28	5	30	70	30	20	7	10.5	10.25	12.25	2
29	30	40	80	40	18.25	6.75	9.25	11.75	14	3
30	10	50	65	50	19	8.25	10.25	8	10.5	2.25
MEANS	13.5	49.7	67.3	53.5	19.3	7.0	9.8	10.1	12.2	2.7

Student Ref.No.	RESULTS				TIME (HOURS)					
	Pre- Test	Test (I-III)	Test (IV-V)	Post Test	Mod I	Mod II	Mod III	Mod IVa	Mod IVb	Mod V
31	15	40	65	40	10.5	4	7.5	9	11.5	2
32	20	70	95	90	9	3.5	3.25	7.5	8.25	1.25
33	15	50	80	50	10.25	4.25	5.5	7.75	9	2.5
34	10	60	70	65	13.5	6	8	10.5	12.5	4.25
35	25	85	70	50	9.5	3	5	8	9.5	3.25
36	20	70	65	80	8	2.5	3.25	6.25	7.25	2.25
37	25	45	60	40	12.5	5	6.25	4.5	6.25	1.5
38	5	65	80	35	14	8.5	8.75	9.75	9.75	3
39	25	65	75	80	13	6.25	8.25	6.5	7.5	1.75
40	10	65	75	50	9.25	3.25	4	9	10	4
41	20	85	50	90	17	7.5	9	5	7.25	1.75
42	20	65	65	45	11.25	5.25	6	8.5	9.25	3.25
43	15	45	90	50	9	3.5	4.5	11	12.25	4
44	15	50	65	80	12	4.5	6	9.5	10.75	4.25
45	25	70	60	40	8.5	2.25	4	6.5	7.5	1.5
46	15	60	70	50	15	9.5	10	8.25	8	2.5
47	0	60	75	60	14.25	7	8.25	5	7	1.5
48	30	55	70	60	10	4.5	5.5	10	13	4.75
49	15	65	50	45	9.25	3.75	4	9.5	11.25	4
50	10	75	70	40	10.25	4.25	5	6.75	4.75	2
51	25	50	75	45	14	6.5	6.75	7.25	8.5	3
52	15	60	90	70	10	4	5.25	6	7.25	1.5
53	5	65	70	40	12.25	5	7.25	7.5	8	2.25
54	25	55	70	50	9.5	3.25	4	5.5	5.25	1.5
55	20	60	60	58	13	6.5	6.5	9.25	10.25	3.5
56	20	75	60	65	14.5	8	11	7	8.25	2.5
57	15	60	60	50	10.5	4	5.75	12	10.5	3.75
58	15	45	70	75	9.75	3.5	4.5	6.25	7.5	2
59	15	75	80	75	10.75	4.25	5.5	7.75	8.25	2.5
60	10	75	65	70	14.75	6	9.25	5.75	6.5	1.75
61	20	60	75	70	11.5	5	9	7.5	8.75	2.5
62	0	85	90	95	13.5	7	8	6.5	8	2
63	20	90	80	65	10.5	3	4.5	7	8.25	2.5
MEANS	18	63.6	71.1	59.6	11.5	5.0	6.3	7.7	8.7	2.6

Stud. Ref. No.	RESULTS				TIME (HOURS)					
	Pre- Test	Test (I-III)	Test (IV-V)	Post Test	Mod I	Mod II	Mod III	Mod IV <sub>A</sub>	Mod IV <sub>B</sub>	Mod V
69	15	55	80	40	16	5	5	6.5	8	2.5
70	25	50	80	35	23	6	10	8	9.5	3
71	20	50	70	45	29	11	15	9.5	10	3
72	0	40	60	30	22	8	12	8.5	7.5	2
73	30	65	80	80	25	10	15	8.5	9.25	3.2
74	20	35	85	80	35	15	18	9.25	10.25	3.5
75	0	40	75	40	30	13	16	8.25	8.5	3
76	25	70	90	80	32	14	11	7.25	8.5	2.7
77	15	45	80	40	28	12	16	7	8.25	2.5
78	0	50	85	65	12	3	9	5	7	1.5
79	25	30	60	35	32	14	18	7.75	8.25	2.5
80	15	40	60	35	27	9	13	6.25	7.5	2
81	20	70	80	80	18	5	9	6	7.5	1.5
82	10	30	85	70	30	12	17	9.25	10	3
83	40	70	90	80	20	8	12	7	8.25	2.5
84	10	50	85	85	34	18	22	11.5	10.5	3.2
85	20	65	85	60	24	6	11	9	9.75	3
86	0	40	65	40	29	13	16	8.5	9.25	3.5
87	5	70	85	35	17	4	9	5	7.25	2
88	20	70	75	40	19	8	12	6.5	7.5	1.5
89	10	25	60	50	23	10	15	5.5	6.25	1.5
90	40	70	90	80	28	11	15	7.75	8.25	2.5
91	15	40	80	50	13	2	4	5	7.5	1.2
92	0	45	60	40	31	12	17	9.5	11.25	3.2
93	0	30	60	45	33	19	18	9	10	3.5
94	15	35	70	50	25	9	13	7.5	8.75	2.5
95	15	35	80	60	27	10	16	6.5	8	2
96	25	60	85	80	21	6	10	6.25	7.25	2
97	15	65	90	80	30	12	17	7.5	8	2.2
98	15	45	80	55	26	9	13	7	8.25	2.2
99	25	35	60	60	36	15	19	8.25	8.75	2.5
100	10	45	60	30	28	13	16	7.25	8.25	2.2
101	30	65	75	50	29	11	15	7.75	8.5	2.5
	19.6	49.4	75.9	55.3	26	10	13.8	7.6	8.5	2.5



Stud Ref No.	RESULTS				TIME (HOURS)					
	Pre- Test	Test (I-III)	Test (IV-V)	Post Test	Mod I	Mod II	Mod III	Mod IVa	Mod IVb	Mod V
103	15	25	50	20						
104	15	55	65	35						
105	-	65	70	45						
106	5	45	35	50						
107	20	35	55	20						
108	10	20	50	45						
109	15	40	45	35						
110	20	50	70	40						
111	25	45	80	80						
112	25	35	60	40						
114	5	45	70	20						
115	30	65	80	45						
116	20	50	60	30						
117	15	60	75	50						
118	15	35	60	25						
119	15	60	80	60						
120	-	50	70	30						
121	10	45	60	25						
122	-	50	80	50						
123	30	40	60	60						
124	-	45	60	40						
125	10	60	80	25						
126	25	65	70	40						
128	10	30	65	35						
129	20	75	75	35						
130	15	35	55	35						
131	10	45	75	40						
132	-	40	40	25						
134	25	40	65	35						
135	10	60	60	70						
136	15	50	85	45						
	16.5	47.1	64.7	40						

Stud Ref. No.	RESULTS				TIME (HOURS)					
	Pre- Test	Test (I-III)	Test (IV-V)	Post Test	Mod I	Mod II	Mod III	Mod IVa	Mod IVb	Mod V
137	15	45	50	40	27	9	14	7.5	9	3
138	10	60	60	60	20	8	13	6	7.25	2
139	40	90	90	95	29	11	15	9.5	10.5	3.25
140	20	70	65	65	18	5	11	5.5	7	1.5
141	10	35	70	80	31	12	16	10	12.5	3.75
143	30	45	70	60	26	10	14	8.5	9	3.25
145	10	35	50	45	14	4	11	4.5	6.25	1.5
146	20	35	70	70	30	12	16	9.5	11.25	3.5
147	15	40	60	60	21	6	11	6.25	7.5	2
148	0	50	40	45	29	13	17	9.5	10.75	3.75
149	25	40	40	55	33	14	19	10	11.5	3.5
150	-	30	35	35	31	13	16	9.25	9.75	3
151	15	45	60	60	22	8	13	6.75	5.25	2
152	25	45	45	50	17	5	10	6	7.25	1.5
153	5	30	65	45	24	7	13	8.25	8	2
155	30	35	65	60	34	14	21	10	12.5	4
156	20	50	65	40	28	11	17	9.5	11	3.25
157	-	35	45	50	32	17	18	9.75	10.75	3.5
158	30	35	60	40	30	12	16	9.25	10.5	3.25
159	10	50	60	40	22	6	14	7.25	8.5	3
160	20	50	65	50	15	5	7	5	7.25	1.5
161	30	40	70	60	25	10	17	8.25	9.25	3
162	10	50	70	45	33	14	11	10	12	3.25
163	10	40	60	45	29	13	16	9.5	11.75	3.25
165	30	50	75	65	31	12	17	9	11.5	3.75
166	10	30	55	60	35	10	19	11	12.25	4.25
167	15	35	40	50	20	6	14	7.25	7.75	2.25
168	10	45	45	45	30	13	16	8.75	9.5	3.25
169	25	90	30	65	23	7	14	3.25	8.75	2.5
170	10	40	55	65	29	11	15	9.25	10.5	3.5
MEANS	17.9	45.7	57.7	54.8	26.3	9.9	14.7	8.1	9.6	2.9

Stud Ref. No.	RESULTS				TIME (HOURS)					
	Pre- Test	Test (I-III)	Test (IV-V)	Post Test	Mod I	Mod II	Mod III	Mod IVa	Mod IVb	Mod V
172	5	65	65	40	24	7	14	8.5	8.75	2.5
173	20	50	70	60	32	14	18	9	9.5	3
174	10	25	55	25	25	9	14	8.5	9	2.75
175	6	45	60	50	15	4	6	5.5	7	1.75
176	20	55	60	75	26	10	14	8.5	9.5	2.5
177	25	40	65	50	22	8	12	7	6.5	2
178	35	55	80	60	27	9	14	9	10.5	3
179	6	35	40	60	35	14	21	10	12.5	3.75
180	20	80	90	55	29	11	15	9.25	10.25	3.5
181	45	60	80	55	19	8	12	6	7.5	1.75
182	25	45	70	65	31	12	17	9.5	12.25	3.25
183	20	60	60	55	18	5	7	5.75	7	1.75
184	50	55	60	40	33	16	18	9.75	11.5	3.25
185	15	50	45	50	26	10	14	9	9.75	3.25
186	10	45	70	35	35	14	18	10.75	12.25	4.25
187	20	40	85	75	25	9	14	8.25	9.5	2.75
188	10	35	55	40	32	14	18	9.75	10.5	3.5
189	15	30	35	40	23	7	14	7	6.75	2
190	35	75	80	75	30	11	16	9.25	10.25	3.5
191	25	60	80	70	14	2	5	5.25	6.75	1.25
192	15	25	35	45	27	11	15	9	11	3.25
193	10	55	70	70	24	8	12	8.5	9.25	2.25
194	10	75	65	50	18	4	7	6	6.75	1.75
195	10	20	75	65	29	11	17	9.25	10.5	3.5
196	10	55	85	70	34	14	18	10	11.5	4.25
197	10	40	60	60	26	9	14	7.5	8.5	3.75
198	25	55	70	50	31	11	16	9.5	11.75	3.75
199	45	85	90	95	35	14	20	11.25	12.5	3.75
200	45	40	50	40	20	8	14	7.25	7.25	2.5
201	6	70	55	30	30	12	17	8.25	9.5	3.5
202	10	90	45	40	29	11	15	8.25	10	3.25
203	20	60	70	65	22	7	12	6.5	7.25	2.25
204	0	70	90	60	27	10	14	7.5	8.75	3.25
MEANS	20.5	52.9	65.6	55	26.8	9.8	14.3	8.3	9.5	2.9

Stud. Ref. No.	RESULTS				TIME (HOURS)					
	Pre- Test	Test (I-III)	Test (IV-V)	Post Test	Mod I	Mod II	Mod III	Mod IVa	Mod IVb	Mod V
205	25	40	55	50						
206	10	35	50	30						
207	0	30	40	50						
208	10	35	75	60						
209	-	50	70	45						
210	10	45	60	35						
211	15	25	60	60						
212	10	60	65	55						
213	10	50	50	35						
214	0	40	50	35						
215	10	50	70	30						
216	20	50	50	35						
218	10	55	60	40						
219	10	25	50	30						
220	-	50	55	35						
221	20	55	60	45						
222	15	75	60	40						
223	-	40	60	40						
224	14	40	80	55						
225	20	50	70	80						
226	20	50	60	40						
227	10	50	50	35						
228	15	65	60	45						
229	25	50	70	40						
230	10	50	55	30						
231	-	60	45	20						
232	10	40	50	30						
234	15	50	65	70						
236	20	40	55	35						
237	15	20	40	40						
238	20	40	60	50						
239	45	70	90	85						
241	-	40	45	35						
MEANS	14.4	46.2	58.64	43.7						

Stud Ref. No.	RESULTS				TIME (HOURS)					
	Pre- Test	Test (I-III)	Test (IV-V)	Post Test	Mod I	Mod II	Mod III	Mod IVa	Mod IVb	Mod V
243	15	50	65	65						
244	20	55	70	65						
245	25	55	70	60						
246	10	40	70	50						
247	25	80	85	75						
248	15	35	65	35						
249	30	55	55	50						
250	20	45	65	40						
251	-	65	75	75						
253	30	45	50	50						
254	10	40	50	35						
255	20	40	45	25						
256	10	60	65	30						
257	10	65	65	20						
258	20	65	60	35						
259	-	65	75	75						
261	25	50	75	75						
262	10	60	70	30						
263	10	50	60	45						
265	20	65	80	80						
266	55	85	80	70						
268	30	60	70	50						
269	-	55	50	30						
270	10	65	75	50						
271	-	40	55	30						
272	10	50	55	50						
273	-	90	80	80						
274	15	55	65	30						
275	20	55	75	50						
276	20	65	60	50						
277	15	40	55	40						
278	10	40	50	40						
MEANS	17.8	55.8	65.5	49.5						

(R + S)	16.0	50.9	62.0	46.6
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# DETAILED RESULTS OF MODULE VI (INFORMATION MAPPING EXPERIMENT)

## GROUP "A" (ADULTS)

Stud. Ref. No.	VIA				VIB				Treatments
	Pre- Test (10)	Post Test	Gain	Time (Mins)	Pre- Test (10)	Post Test (10)	Gain	Time (Mins)	
1	2	8	6	40	0	8	8	30	<u>GROUP A1</u>  (VIA) - Inf.Map. (VIB) - Lin.Prog.  N = 15
2	1	9	8	45	3	9	6	35	
4	0	8	8	50	3	8	5	30	
6	1	8	7	40	4	9	5	30	
8	3	9	6	50	3	8	5	40	
12	1	8	7	45	1	8	7	40	
13	0	10	10	40	0	8	8	40	
16	2	9	7	45	3	9	6	45	
18	0	9	9	40	0	8	8	40	
20	1	10	9	40	1	8	7	35	
21	1	9	8	40	1	9	8	40	
22	0	8	8	30	0	8	8	35	
23	2	8	6	40	1	8	7	40	
27	3	8	5	35	2	8	6	40	
28	0	9	9	35	0	8	8	45	
	1.1	8.7	7.7	41	1.5	8.3	6.8	38	
3	2	9	7	40	2	9	7	25	<u>GROUP A2</u>  (VIA) - Lin.Prog. (VIB) - Inf.Map.  N = 15
5	2	8	6	35	2	8	6	45	
7	3	8	5	50	2	9	7	35	
9	0	8	8	45	1	9	8	30	
10	3	8	5	50	2	9	7	35	
11	0	8	8	50	1	8	7	40	
14	0	8	8	45	0	8	8	45	
15	2	8	6	50	1	9	8	20	
17	3	10	7	45	2	10	8	45	
19	2	9	7	40	2	8	6	30	
24	1	8	7	35	2	10	8	45	
25	2	8	6	45	1	9	8	25	
26	0	9	9	45	0	8	8	20	
29	1	8	7	50	0	9	9	45	
30	0	8	8	50	1	10	9	25	
	1.4	8.3	6.9	45	1.3	8.9	7.6	32	

# DETAILED RESULTS OF MODULE VI (INFORMATION MAPPING EXPERIMENT)

## GROUP "B" (ADULTS)

Stud. Ref. No.	Via				Vib				Treatments
	Pre-Test (10)	Post-Test (10)	Gain	Time (Mins)	Pre-Test (10)	Post-Test (10)	Gain	Time (Mins)	
32	2	9	7	45	2	6	4	45	<p><u>GROUP B1</u></p> <p>(VIA) - Inf.Map.</p> <p>(Vib) - Lin.Prog</p> <p>N = 16,</p>
33	0	5	5	40	0	6	6	45	
35	1	7	6	40	2	8	6	40	
36	2	7	5	45	3	8	5	40	
37	0	8	8	40	0	7	7	45	
40	1	10	9	45	1	7	6	40	
43	1	8	7	50	2	6	4	50	
44	2	8	6	50	2	6	4	45	
45	0	8	8	45	0	7	7	45	
48	2	9	7	45	3	7	4	35	
50	3	8	5	40	2	8	6	40	
51	1	7	6	45	0	6	6	40	
53	1	9	8	50	1	8	7	35	
55	0	6	6	40	0	9	9	40	
58	3	8	5	35	1	7	6	30	
59	4	9	5	40	2	10	8	45	
	1.4	7.9	6.5	43	1.2	7.2	6.0	41	<p><u>GROUP B2</u></p> <p>(VIA) - Lin.Pro</p> <p>(Vib) - Inf.Map</p> <p>N = 16</p>
31	1	7	6	50	1	8	7	40	
34	1	8	7	45	0	8	8	45	
38	3	10	7	45	2	9	7	30	
39	2	8	6	50	3	9	6	25	
41	2	8	6	55	4	9	5	20	
42	1	8	7	45	3	10	7	45	
46	2	7	5	50	2	8	6	20	
47	0	7	7	45	0	7	7	35	
49	0	6	6	40	0	8	8	40	
52	1	7	6	40	0	8	7	35	
54	1	8	7	45	1	8	7	45	
56	0	8	8	50	1	9	8	40	
57	1	7	6	35	2	10	8	40	
61	2	6	4	45	2	7	5	35	
62	0	8	8	45	0	7	7	40	
63	1	7	6	45	0	8	8	45	
	1.1	7.5	6.4	46	1.3	8.3	7.0	36	

Stud. Ref. No.	VIa				VIb				Treatments
	Pre- Test (10)	Post Test (10)	Gain	Time (Mins)	Pre- Test (10)	Post Test (10)	Gain	Time (Mins)	
70	0	9	9	40	1	8	7	45	<u>GROUP C1</u>  Inf.Map. - (VIa) P.I. - (VIb)   N = 16
71	0	7	7	45	0	7	7	40	
73	2	8	6	50	1	8	7	30	
74	3	8	5	50	2	9	7	45	
75	0	8	8	45	1	6	5	55	
77	1	8	7	45	3	8	5	40	
78	0	9	9	40	0	7	7	45	
81	1	9	8	45	2	6	4	35	
83	0	8	8	50	0	6	6	50	
85	0	6	6	55	2	5	3	35	
87	1	7	6	55	1	8	7	45	
88	2	7	5	45	1	7	6	45	
89	0	6	6	45	0	7	7	45	
91	3	8	5	50	3	9	6	50	
96	0	5	5	35	2	7	5	40	
98	0	7	7	35	0	6	6	40	
	0.8	7.5	6.7	46	1.2	7.1	5.9	43	
69	0	7	7	40	1	9	8	40	<u>GROUP C2</u>  P.I. - (VIa) Inf.Map - (VIb)   N = 16
72	2	9	7	50	0	7	7	40	
76	1	7	6	55	0	8	8	45	
79	2	7	5	55	2	9	7	30	
80	0	6	6	50	1	8	7	30	
82	2	6	4	45	2	10	8	40	
84	2	8	6	50	2	9	7	25	
86	0	7	7	55	1	9	8	40	
90	1	8	7	50	3	10	7	50	
92	3	8	5	40	1	9	8	30	
93	1	8	7	45	2	9	7	40	
94	0	6	6	50	0	8	8	30	
95	2	8	6	55	2	8	6	25	
97	0	7	7	35	0	9	9	45	
99	2	8	6	40	2	9	7	35	
100	3	10	7	50	1	9	8	25	
	1.3	7.5	6.2	48	1.25	8.75	7.5	36	



TREATMENT - INFORMATION MAPPING - MODULE (Via)  
- LINEAR PROGRAMME - MODULE (Vib)

N = 30.

I.M.

P.I.

TABLE 12

Stud. Ref. No.	MODULE Via				MODULE Vib			
	Pre- Test (10)	Post Test (10)	Gain	Time (Mins)	Pre- Test (10)	Post Test (10)	Gain	Time (Mins)
137	2	7	5	30	0	8	8	30
138	0	9	9	30	0	7	7	20
139	0	5	5	40	1	5	4	45
140	2	8	6	40	3	10	7	30
141	0	6	6	35	2	7	5	45
143	0	7	7	45	1	8	7	40
145	3	8	5	35	2	9	7	20
146	2	4	2	30	0	9	9	30
147	0	7	7	50	1	8	7	40
148	3	8	5	35	1	9	8	50
149	2	6	4	30	2	9	7	35
150	0	5	5	40	3	9	6	35
151	2	6	4	45	1	7	6	40
152	2	8	6	40	3	6	3	30
153	0	7	7	50	0	5	5	20
155	1	8	7	45	1	7	6	45
156	2	5	3	40	2	6	4	20
157	2	9	7	50	3	8	5	30
158	2	9	7	40	0	7	7	40
159	0	8	8	35	2	5	3	40
160	2	9	7	40	0	8	8	30
161	2	6	4	45	4	7	3	45
162	0	8	8	35	2	8	6	40
163	0	8	8	30	2	6	4	30
165	0	6	6	40	1	8	7	40
166	0	8	8	40	2	7	5	50
167	2	9	7	35	0	6	6	30
168	1	6	5	35	2	7	5	45
169	0	6	6	40	2	4	2	45
170	2	6	4	40	2	5	3	20
	1.1	7.1	6.0	39	1.5	7.2	5.7	35

P.I.

I.M.

N = 30

Stud. Ref. No.	MODULE VIA				MODULE VIB			
	Pre- Test (10)	Post Test (10)	Gain	Time (Mins)	Pre- Test (10)	Post Test (10)	Gain	Time (Mins)
172	0	2	2	40	0	7	7	30
173	0	8	8	25	1	6	5	20
174	1	8	7	35	1	10	9	35
175	2	8	6	55	0	6	6	20
176	2	9	7	40	0	7	7	35
177	1	8	7	35	1	4	3	20
178	0	7	7	35	0	2	2	25
179	2	8	6	35	3	9	6	35
180	1	6	5	50	0	8	8	40
181	2	7	5	50	0	5	5	25
182	3	5	2	50	2	6	4	45
183	2	7	5	60	2	9	7	40
184	2	6	4	45	1	8	7	45
185	2	8	6	30	3	8	5	30
186	0	9	9	45	2	8	6	20
187	2	9	7	50	0	8	8	30
188	2	4	2	45	0	9	9	25
189	0	4	4	55	2	7	5	25
190	0	7	7	60	2	9	7	35
191	2	8	6	50	1	8	7	45
192	2	4	2	45	0	8	8	40
193	1	8	7	50	1	7	6	25
194	0	7	7	45	1	9	8	35
195	0	7	7	30	3	7	4	25
196	0	8	8	50	2	9	7	40
197	0	7	7	60	1	6	5	30
198	2	8	6	40	3	10	7	40
199	0	8	8	55	2	10	8	35
200	1	5	4	45	1	8	7	20
201	3	8	5	45	1	7	6	45
	1.1	6.9	5.8	45	1.2	7.5	6.3	33

TABLE 13

TOTAL STUDY TIMES ON SECTIONS

GROUP A

Student Ref. No.	Section 1 (Mod I-III)	Section 2 (Mod IV-V)	Section 3 (Mod VI Only)
1	47 hrs.	32.35 hrs.	1.2 hrs.
2	38	20.5	1.3
3	31	14.25	1.1
4	26	30.25	1.3
5	38.75	23.5	1.3
6	32.75	29.75	1.2
7	41.5	21	1.4
8	20.5	34.5	1.5
9	37.5	23.5	1.25
10	48.5	15	1.4
11	38	26.75	1.5
12	32.5	17.5	1.4
13	37.5	36	1.3
14	26.25	31	1.5
15	41	23.25	1.25
16	43	18.75	1.5
17	34.75	23.25	1.5
18	40	31.25	1.3
19	31.75	22.25	1.2
20	37.5	25.5	1.25
21	22	20	1.3
22	39	28.25	1.1
23	38	30.25	1.3
24	45.75	23.25	1.3
25	37.5	21.5	1.2
26	36.5	23	1.1
27	31	30.25	1.25
28	37.5	24.5	1.3
29	34.25	28.75	1.6
30	37.5	20.75	1.25

TOTAL STUDY TIMES ON SECTIONS

GROUP B

Student Ref. No.	Section 1 (Mod I-III)	Section 2 (Mod IV-V)	Section 3 (Mod VI only)
31	22 hrs.	22.5 hrs.	1.5 hrs.
32	15.75	17	1.5
33	20	19.25	1.4
34	27.5	27.25	1.5
35	17.5	20.75	1.3
36	13.75	15.75	1.4
37	19.25	12.25	1.4
38	31.25	22.5	1.25
39	27.5	15.75	1.25
40	16.5	23	1.4
41	33.5	14	1.25
42	22.5	21	1.5
43	17	27.25	1.7
44	22.5	24.5	1.6
45	14.25	15.5	1.5
46	34.5	18.75	1.2
47	29.5	13.5	1.3
48	20	27.75	1.3
49	17	24.75	1.3
50	19.5	13.5	1.3
51	27.25	18.75	1.4
52	19.25	14.75	1.25
53	24.5	17.75	1.4
54	16.75	12.25	1.5
55	26	23	1.3
56	33.5	17.75	1.5
57	20.25	26.25	1.25
58	17.75	15.75	1.1
59	20.5	18.5	1.4
60	30	14	1.4
61	25.5	18.75	1.3
62	28.5	16.5	1.4
63	18	17.75	1.5

GROUP STUDY TIMES ON SECTIONSGROUP C.

Student Ref. No.	Section 1 (Mod. I-III)	Section 2 (Mod IV-V)	Section 3 (Mod VI only)
69	26 hrs.	17 hrs.	1.3 hrs.
70	39	20.5	1.4
71	55	22.5	1.4
72	42	18	1.5
73	50	21	1.2
74	68	23	1.6
75	59	19.75	1.7
76	57	18.5	1.7
77	56	17.75	1.4
78	24	13.5	1.4
79	64	18.5	1.3
80	49	15.75	1.3
81	32	15	1.3
82	59	22.25	1.4
83	40	17.75	1.7
84	74	25.25	1.25
85	41	21.75	1.5
86	58	21.25	1.6
87	30	14.25	1.7
88	39	15.5	1.5
89	48	13.25	1.5
90	54	18.5	1.7
91	19	13.75	1.7
92	60	24	1.2
93	70	22.5	1.4
94	47	18.75	1.3
95	53	16.5	1.3
96	37	15.5	1.25
97	59	17.75	1.3
98	48	17.5	1.25
99	70	19.5	1.25
100	57	17.75	1.25
101	55	18.75	1.6

TOTAL STUDY TIMES ON SECTIONSGROUP P

Student Ref. No.	Section 1 (Mod. I-III)	Section 2 (Mod IV-V)	Section 3 (Mod VI only)
137	50 hrs.	19.5 hrs.	1.0 hrs.
138	41	15.25	0.8
139	55	23.25	1.4
140	34	14	1.2
141	59	26.25	1.3
143	50	20.75	1.4
145	29	12.25	0.9
146	58	24.25	1.0
147	38	15.75	1.5
148	59	24	1.4
149	66	25	1.1
150	60	22	1.25
151	43	14	1.4
152	32	14.75	1.2
153	44	18.25	1.2
155	69	26.5	1.5
156	56	23.75	1.0
157	67	24	1.3
158	58	23	1.3
159	42	18.25	1.25
160	27	13.75	1.2
161	52	20.5	1.5
162	58	25.25	1.25
163	58	24.5	1.0
165	60	24.25	1.3
166	64	27.5	1.5
167	40	17.25	1.1
168	59	21.5	1.3
169	44	14.5	1.4
170	55	23.25	1.0

TOTAL STUDY TIMES ON SECTIONSGROUP Q

Student Ref. No.	Section 1 (Mod. I-III)	Section 2 (Mod IV-V)	Section 3 (Mod VI only)
172	45 hrs.	19.75 hrs.	1.2 hrs.
173	64	21.5	0.75
174	48	20.25	1.2
175	25	14.25	1.25
176	50	20.5	1.25
177	42	15.5	0.9
178	50	22.5	1.0
179	70	26.25	1.2
180	55	23	1.5
181	39	15.25	1.25
182	60	25	1.6
183	30	14.5	1.7
184	67	24.5	1.5
185	50	22	1.0
186	67	27.25	1.1
187	48	20.5	1.3
188	64	23.75	1.2
189	44	15.75	1.3
190	57	23	1.6
191	21	13.25	1.6
192	53	23.25	1.4
193	44	20	1.25
194	29	14.5	1.3
195	57	23.25	0.9
196	66	25.75	1.5
197	49	19.75	1.5
198	58	25	1.3
199	69	27.5	1.5
200	42	17	1.1
201	59	21.25	1.5
202	55	21.5	1.3
203	41	16	1.2
204	51	19.5	1.3

(TEST (a) + TEST (b) ).

EXPRESSED AS A PERCENTAGE.

GROUP A		GROUP B		GROUP C	
Student Ref. No.	Combined Score	Student Ref. No.	Combined Score	Student Ref. No.	Combined Score
1	80	31	75	69	80
2	90	32	75	70	85
3	90	33	55	71	70
4	80	34	80	72	80
5	80	35	75	73	80
6	85	36	75	74	85
7	85	37	75	75	70
8	85	38	95	76	75
9	85	39	85	77	80
10	85	40	85	78	80
11	80	41	85	79	80
12	80	42	90	80	70
13	90	43	70	81	75
14	80	44	70	82	80
15	85	45	75	83	70
16	90	46	75	84	85
17	100	47	70	85	55
18	85	48	80	86	80
19	85	49	70	87	75
20	90	50	80	88	70
21	90	51	65	89	65
22	80	52	75	90	90
23	80	53	85	91	85
24	90	54	80	92	85
25	85	55	75	93	85
26	85	56	85	94	70
27	80	57	85	95	80
28	85	58	75	96	60
29	85	59	95	97	80
30	90	60	70	98	65
		61	65	99	85
		62	75	100	95
		63	75	101	75



QUESTIONNAIRE

ADMINISTERED IN LATE DECEMBER 1966, ONCE ALL  
SUBJECTS HAD FINISHED STUDYING MODULE VI.  
(N = 96 ADULT LEARNERS IN CONTINUING EDUCATION)

1. How did you like working with the self-study materials of BASG-M during the last 4 months?

LIKED VERY MUCH	31
QUITE LIKED	54
INDIFFERENT	7
DID NOT LIKE	4
HATED IT	0

2. In comparison with your previous experiences of learning mathematics, how easy or difficult did you find the work with BASG-M?

VERY MUCH EASIER	19
SOMEWHAT EASIER	49
NO DIFFERENCE	21
SLIGHTLY MORE DIFFICULT	7
MUCH MORE DIFFICULT	0

3. You have studied the materials in the 3 ways described here. Please place these in your order of preference, by ticking the appropriate columns 1, 2 or 3.

Tick Order

	1	2	3
(a) Independent Study - Without teachers help	4	10	82
(b) Study directed by you - Teachers available when you want them.	32	55	9
(c) Keller-Plan - Your work carefully monitored by teachers.	60	31	5

4. You studied parts of module VI in two alternative styles of presentation; the programmed texts we have been using all along, and the "information maps". Please indicate your preference.

PREFERRED THE PROGRAMMED TEXT	5
NO SPECIAL PREFERENCE	22
PREFERRED THE INFORMATION MAPS	69

QUESTIONNAIRE

ADMINISTERED IN EARLY DECEMBER 1966, ONCE ALL SUBJECTS HAD FINISHED STUDYING MODULE VI AND HAD STARTED MODULE VII. (N = 63 SECONDARY SCHOOLCHILDREN),

1. How did you like working with the self study materials of BASG-M during the last 4 months?

LIKED VERY MUCH	22
QUITE LIKED	36
INDIFFERENT	2
DID NOT LIKE	3
HATED IT	0

2. In comparison with your previous experiences of learning mathematics, how easy or difficult did you find the work with BASG-M?

VERY MUCH EASIER	9
SOMEWHAT EASIER	37
NO DIFFERENCE	6
SLIGHTLY MORE DIFFICULT	8
MUCH MORE DIFFICULT	3

3. You have studied the materials in the 3 ways described here. Please place these in your order of preference, by ticking the appropriate columns 1, 2 or 3.

Tick Order			
	1	2	3
(a) Independent Study - Without teachers help	10	13	40
(b) Study directed by you - Teachers available when you want them.	28	25	10
(c) Keller-Plan - Your work carefully monitored by teachers.	25	25	13

4. You studied parts of module VI in two alternative styles of presentation; the programmed texts we have been using all along, and the "information maps". Please indicate your preference.

PREFERRED THE PROGRAMMED TEXT	11
NO SPECIAL PREFERENCE	15
PREFERRED THE INFORMATION MAPS	37

