

INTERNATIONAL APPROACHES TO PROFESSIONAL DEVELOPMENT FOR MATHEMATICS TEACHERS



Edited by
Nadine Bednarz
Dario Fiorentini
Rongjin Huang

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Chapter 1

Situating Mathematics Teacher Education in a Global Context

Barbara Jaworski

Introduction

This chapter stems from an invited presentation at ICME-11 in Topic Study Group 28 focusing on the in-service education, professional life, and development of mathematics teachers. I was to set the scene historically and offer my own perspective, from an international point of view, on where we currently are in this field. I welcomed the invitation as it gave me the chance to reflect on my own years of work in this field and share my reflections with others. What I present here is a personal account.¹ I have tried to be as comprehensive as possible where major events are concerned but cannot guarantee that I have not missed something. I am also strongly aware that my account comes from an English-speaking environment and that language denies me access to events and publications in which the language is not English. However, the rest of this book includes chapters from diverse settings that reflect research activities in a wide range of nations.

Structure of the Chapter

This chapter is composed of three parts: (1) a historical perspective on the (sub)field of mathematics teacher education; (2) some themes in mathematics teaching development; and (3) issues in mathematics teaching development and one way forward.

I consider mathematics teacher education to be a subfield of the discipline of mathematics education. I think that history supports this claim.

I therefore set out what I see as key aspects of the development of this subfield in terms of international events and publications. From there I move into the “content” of the subfield. Here I limit myself mainly to the topic of the study group, which concerns the education and development of *practising* teachers. In the main, this chapter is not about initial or pre-service teacher education, the education of new teachers; that in itself has a considerable literature. I offer what I consider to be main themes in the development of the education of practising teachers. Finally, I recognize a range of issues relating to teacher learning and teaching development in mathematics and offer, tentatively, what I see as being one way forward.

To avoid using the words *mathematics teacher education* repeatedly, I will sometimes use the acronym MTE. Also, MTEs are mathematics teacher educators.

A Historical Perspective on Mathematics Teacher Education

Here I trace the development of our subfield through major events and contributions internationally over the past twenty years. One of the major landmarks for mathematics educators over the years is the annual PME conference (Conference of the International Group for the Psychology of Mathematics Education²), dating back to 1976. I start with the development of MTE as seen through the PME conferences and their proceedings during this time and continue to consider other key events and related publications.

A Reconstruction of the Evolution through PME

I trace events from 1986, the year that I attended my first PME conference. Celia Hoyles (1992) refers to the earlier PME conferences, suggesting that the majority of papers in these early days focused on student understanding of mathematical concepts. Regarding 1979 (PME 3), she writes that “if the teacher was mentioned at all, s/he was purely a facilitator—to dispense facts and information; to identify errors or misunderstandings; to provide materials or strategies to overcome misconceptions; or to promote further mathematical development” (p. 3: 263). It seems to me that the nature of being “purely a facilitator” lies more in how such aspects of role are treated than in the aspects themselves. A suggestion is that perceptions of the role of the teacher at this time were underdeveloped, that the teacher

was almost dismissed in the simplicity of the conception of teaching, and that overcoming misconceptions or promoting development was seen in instrumental ways.

Between 1979 and 1985, Hoyles found a few papers that started to refer to the teacher’s role in the classroom and to consider teachers’ beliefs and their relationship with teaching practice. However, in 1986, there was just one paper explicitly related to teaching: a case study of the socialization of one teacher in his first year of teaching. During the next two decades, a research focus on teachers’ beliefs continued alongside other factors relating to teachers and teaching.

Table 1.1 provides a brief overview of development from 1986.

Table 1.1
Twenty years of PME conferences 1986–2006

| PME # | Year | Papers and their content and other key events |
|-----------|----------|--|
| PME 10 | 1986 | 1 paper relating to teaching |
| PME 11 | 1987 | 19 papers in themes headed as instruction and teacher training |
| PME 14 | 1990 | No thematic categories designated as teaching or teacher education Papers on teaching or learning to teach spread throughout other categories Three working groups in areas of mathematics teacher education |
| PME 15–27 | 12 years | Development of mathematics teaching and teacher education as major research themes Two plenary presentations surveying the subfield Three books arising from working groups at PME |
| PME 28 | 2004 | 50 papers in themes headed teacher classroom practice, teacher education and professional development, and teacher knowledge |
| PME 30 | 2006 | 27 papers in themes on in-service teacher development, pre-service teacher development (elementary), teacher content knowledge, teacher pedagogical knowledge, and teacher thinking A review of 30 years of PME |

PME conference proceedings contain a thematically organized list of the papers presented at a conference. The titles given to these themes are themselves illustrative of thinking at that time. For example, in 1987, we see the thematic titles “Instruction” and “Teacher Training.” More recently, the words *instruction* and *training* have been subject to critique in terms of their perhaps behaviourist connotations, that pupils are to be *instructed* and teachers *trained* according to some given conventions that are the province of experts in the field. The shift in thematic titles to “Teacher Classroom Practice,” “Teacher Education and Professional Development,” and “Teacher Knowledge” in 2004 also reflects a shift in the discourse of teacher education. As more consideration was given to the role of a teacher and the nature of teaching, earlier, more simplistic, notions gave way to wider recognition of context and complexity in teaching. The discourse changed accordingly. In 2006, moreover, the thematic categories emphasized differences between “Pre-Service and In-Service Development” and “Teacher Knowledge and Teacher Thinking.” We see here a maturing within the subfield: finer categorization of papers, more refined discourse, deeper consideration of factors and issues in teaching and its development with relation to classrooms at different levels, and associated education of teachers. Papers classified in these sections number about half of those classified in the subfield in 2004, partly due to new thematic categories in PME, such as “Classroom Culture” and “Social Activity Theory,” which dealt with teaching but whose focus was methodological, institutional, or theoretical. Overall, however, there were fewer papers in 2006, so it remains to be seen whether this is a downward trend.

Alongside developments in the research reported at PME, there was such interest in mathematics teacher education by 1990 that three working groups had been formed. They were known respectively as

1. Psychology of In-Service Education of Mathematics Teachers (a research perspective);
2. Research on Psychology of Mathematics Teacher Development; and
3. Teachers as Researchers in Mathematics Education.

The first focused on in-service teacher education, what we now call education of practising teachers, the substance of this book. The second looked at teacher education and development more broadly, and the third focused specifically on teachers’ engagement in research. These three working groups

continued well into the 1990s with varying membership and leadership. As the work of each group came to an end, it produced a book of papers relating the work of the group over the years. The first volume to be published (Zack, Mousley, & Breen, 1997: Group C) focused largely on teachers' engagement in inquiry and research and corresponding developments in teaching practice; the second volume (Jaworski, Wood, & Dawson, 1999: Group A) concentrated on in-service teacher development; and the third volume (Ellerton, 1999: Group B) focused on both pre-service and in-service development. Looking back on these books, I believe that they offer an important record of the thinking and practice in mathematics teacher education at that time. It was an interesting and in many ways exciting time. Constructivism largely held sway as a basic theoretical perspective (Davis, Maher, & Noddings, 1990), and many of the chapters discussed teacher education initiatives that were rooted in constructivist principles and associated methodology (see also Confrey & Kazak, 2006). We were moving away from visions of mathematics teaching as direct instruction and toward ideas related to problem solving and more creative approaches involving students in mathematics (Grouws & Cooney, 1988; Romberg & Carpenter, 1986). As we thought about student involvement in exploring mathematics, we saw the beginnings of involving teachers in exploring mathematics teaching (ATM, 1987; Krainer, 1998). The roles of teacher educators-researchers in this process started to come into focus (Jaworski, 1994, 1998). The book's chapters offer a panorama of MTE activity related to this landscape.

In the period from PME 15 to 27, two conferences are of particular note (PME 16, 1992; PME 18, 1994), since each had a keynote presentation focusing on mathematics teacher education and offering a survey of research to that time, the first from Celia Hoyles and the second from Joao Pedro da Ponte (Hoyles, 1992; Ponte, 1994). These two papers together chart a progression in ways of seeing teachers and teaching from a one-dimensional, largely instrumental, focus to research in which interpretations seek to account for multiple dimensions and serious complexity in teaching.

In 1992, based on papers at PME up to that time, Hoyles (1992) suggested that up to 1987 the teacher was considered hardly at all other than as an adjunct of students' learning of mathematics, "a passive conveyor of facts and information" (p. 3: 625). According to Hoyles, early papers on teachers and teaching reflected the teacher's focus on curriculum delivery and differentiation of students according to ability. Many papers focused on the beliefs of teachers: "there appears to be an implicit view that teachers have 'beliefs' in some decontextualised sense which need to be accessed and

changed” (p. 3: 266). The focus on beliefs led to suggestions of “true beliefs which may not be enacted in practice” (p. 3: 265). Hoyles had perceived a strong focus on the influence of teacher beliefs on classroom activity and perceptions of a gap between espoused and enacted beliefs. She reported on research that saw inconsistencies between teachers’ beliefs and their practice as obstacles to the success of in-service development and therefore as something to be changed. This suggested a need to investigate the interaction of teacher beliefs and curriculum innovations. Hoyles marked a shift in the late 1980s toward a focus on social interaction and construction in which teachers’ classroom activities were heavily influenced by classroom and school cultures and those of the wider society. Methodologically, the period marked a shift to more qualitative research approaches, particularly case studies, that “illuminate the general through the particular” (p. 3: 282) and go some way toward addressing context and complexity while presenting problems in critical judgment of interpretations.

Two years later, Ponte (1994) examined the place of the teacher in mathematics education research, particularly mentioning negative perspectives—seeing the teacher as an instrument, as a deficient professional, as “a person with deep misconceptions, lack of mathematical knowledge, and inappropriate and inconsistent beliefs, contradictory with current reform efforts” (p. 198). Ponte recognized research that focuses on teachers’ knowledge of mathematical concepts and how to teach them, the assumption being that teachers who do not know their subject cannot do a good job in teaching it. He also pointed to more didactically oriented studies focusing on subject matter and pedagogical knowledge but lacking a clear view of how such knowledge develops and works in practice. Hoyles (1992) had pointed toward teacher research as an emergent focus; Ponte acknowledged research on reflective practice and teachers as researchers, seen as playing an important role in defining the purposes and goals of their work as well as attaining them. Ponte elaborated on the issues raised in his review of research through three short accounts of teachers’ conceptions in relation to mathematical problem solving in the Portuguese curriculum. Although the accounts raised issues relating to teachers’ knowledge and beliefs in much of the research reviewed, Ponte’s analysis took into account social and institutional factors, emotional responses, and self-image. Ponte considered different worlds of experience that give rise to different interpretations, according to changes in context, and different forms of knowing, including knowing in action in contrast to definitive or declarative knowledge. The picture painted in these accounts is one of deep complexity in which

research observations are subject to interpretation relative to theoretical and socio-cultural factors, which need to be made explicit to make sense of what is observed. Overall, there emerges a deep understanding of the complexities that teachers face and the importance of research giving teachers a voice—a stark contrast to the deficiency perspective presented earlier.

In 2006, in a celebration of thirty years of PME, a specially commissioned volume documented research in a number of key areas, one of which was mathematics teacher education. Two review chapters were included: the first from Salvador Llinares and Konrad Krainer and the second from Joao Pedro da Ponte and Olive Chapman (Llinares & Krainer, 2006; Ponte & Chapman, 2006). They bring us approximately to the present in terms of perspectives and issues in the subfield. I will refer to these papers below.

A New Journal in the Field of Mathematics Teacher Education

In parallel with preparation of the books mentioned above, in 1998 a new journal was launched, the *Journal of Mathematics Teacher Education*. It came about from an agreement between the publisher, Kluwer, and Professor Tom Cooney at the University of Georgia, United States, to advertise the publication of a book or books in mathematics teacher education. A call for papers was made, and by the closing date more than 100 papers had been submitted. This quantity suggested the viability of a journal, and a decision was made to launch *JMTE*, with Cooney as the founding editor. To date, the journal has flourished, with increasing numbers of issues per year and new editorial teams.³

The mission statement of *JMTE* is as follows:

*The Journal of Mathematics Teacher Education (JMTE) is devoted to research that seeks to improve the education of mathematics teachers and develop teaching methods that enable mathematics students to learn better. The journal covers all stages of the professional development of mathematics teachers and teacher-educators. It serves as a forum for examining institutional, societal, and cultural influences that impact on teachers' learning and ultimately their students' learning.*⁴

JMTE initially invited papers of two kinds:

1. research papers relating to programs for educating prospective teachers of mathematics, programs for educating practising teachers of mathematics, programs in mathematics teaching development, and theoretical perspectives in mathematics teacher education; and

2. papers relating to mathematics teacher education around the world.

Cooney had the foresight to realize that a journal in English, edited from the United States, might have difficulty attracting papers from a truly international constituency. The section on MTE around the world was designed to provide insight into programs in teacher education worldwide. Nevertheless, throughout ten years of the journal, *JMTE* papers came overwhelmingly from the United States, with a relatively small number coming from a range of other countries. This was despite an international editorial board and requests to members to encourage papers from their own countries. I take up this issue again below.

JMTE has not had a thematic organization of papers: thus, it would be a major task to trace themes through the papers published (though it could be an interesting task). However, a simple count shows that a little more than 10% of papers were published in the category of MTE around the world. During the first ten years of *JMTE*, three special issues were published, one on “Communities in Mathematics Teacher Education” (*JMTE* 6(2), 2003); one on “Inter-Relating Theory and Practice in Mathematics Teacher Education” (*JMTE* 9(2), 2006); and a triple issue on “The Nature and Role of Tasks in Mathematics Teachers’ Education” (*JMTE* 10(4, 5, 6), 2008). In the main, during the first decade of the journal, the editors concentrated on establishing *JMTE* as a major academic journal with a high scientific quality of papers. Doing so had negative associations also, as we shall see below. Currently (in 2008), the journal is anticipating further special issues and moving in new directions. There is a need to find ways to create more opportunities within the international community: for example, national portraits on mathematics teacher education, presentations of research tendencies, results, theoretical perspectives, and issues that could regroup researchers in the subfield internationally.

Mathematics Teacher Education in International Settings

Evidence of activity in mathematics teacher education internationally can be found in a range of other events and publications.

The four-yearly conferences of the International Congress of Mathematics Education (ICME) have always included working groups focusing on different aspects of teacher education, including a group on in-service teachers education and the professional lives of mathematics teachers. Proceedings from these conferences include brief details from the work of these groups. This

book, edited from Canada and Brazil, is the result of such a group meeting at the most recent conference, ICME-11, in Monterrey, Mexico.

ICME conferences are organized by the International Commission on Mathematical Instruction (ICMI), which celebrated 100 years in 2008. Celebrations included a special conference organized in Rome with one group focusing on mathematics teacher education (Grevholm & Ball, 2008).

In Europe in 1997, a European Society for Research in Mathematics Education (ERME) was established. Its congresses, CERME, have been held every two years since 1999. These conferences are based on working groups in key areas of mathematics education chosen by the respective program committee. Mathematics teacher education has been the topic of one group at each of the conferences so far and, in each case, has been one of the largest groups in terms of papers submitted and numbers participating. The group at the first conference produced a special volume of papers relating to research in MTE focusing on issues raised at the conference (Krainer, Goffree, & Berger, 1999). Papers from succeeding conferences can be found in the published proceedings of each conference.

In addition to the regular conferences of established groups, other one-off conferences in mathematics teacher education have been organized internationally, often with invited participants: for example, in Taipei, Taiwan, in 1999, from which a book, *Making Sense of Mathematics Teacher Education* emerged (Lin & Cooney, 2001); in Malmö, Sweden, in 2003, proceedings of which, *Educating for the Future*, were published (Straesser, Brandell, Grevholm, & Helenius, 2004); and in Oberwolfach, Germany, in 2007, with a published report including abstracts of presented papers (Reiss, Schoenfeld, & Törner, 2007). Thus, from these events we see a growing literature base in MTE.

A Special Survey

In preparation for ICME-10 in 2004, a survey was commissioned by ICMI on research in mathematics teacher education. As a result of their literature searches, the researchers identified a number of key issues and made four major claims (Adler, Ball, Krainer, Lin, & Novotna, 2005). The claims are summarized here as follows (adapted from Adler & Jaworski, 2009):

1. Small-scale qualitative research predominates. This most often involves research that focuses on a single teacher or on small groups of teachers ($n < 20$) within individual programs or courses.

2. Most teacher education research is conducted by teacher educators studying the teachers with whom they are working. Most studies are small case studies in which researchers have some direct involvement and thus some interest in the case being studied.
3. Research in countries where English is the national language dominates the literature surveyed. In *JMTE* between 1998 and 2003, this was 80% of published papers, in *Journal for Research in Mathematics Education (JRME)* 71%, and in *PME*, between 1999 and 2003, 43%.
4. Questions that come to constitute the research field are driven by concerns in particular contexts and thus might not reflect the diversity of problems in teacher education that exist globally. Some questions have been studied extensively, though other important questions remain unexamined. Much of the research, particularly in the United States, has been concerned with reform and has involved efforts to show that particular programs of teacher education “work.” As a consequence, for example, we know much less than we should about teachers’ learning from experience: whether teachers learn, what they learn, and what supports learning from experience.

These findings were salutary for the subfield in general and for *JMTE* in particular. Issues are discussed in Adler et al. (2005) and Adler & Jaworski (2009). Briefly here, I see the issues as centring on internationalism in MTE. The language issues noted, the domination of the English-speaking world in published papers, and the domination of the reform movement in the United States on issues in the subfield raise serious concerns for the subfield.

The ICMI Study 15

Since 1990, ICMI has developed a series of studies in specially designated areas of research in mathematics education. In the first study, on mathematics and cognition, Balacheff (1990), in collaboration with a team of researchers from *PME*, wrote as follows: “What we know virtually nothing about is the interactive nature of teachers’ and students’ conceptions: how one influences the others, their origins, the intensity with which they are held, and how permeable they are in the face of classroom dynamics” (p. 141). They wrote further that “what we have developed here confirms the importance of studies taking the teaching situations as a research object:

how they function with the purpose of allowing students' learning, how they affect the meaning of knowledge, and how the various subsystems interact within a given teaching situation" (p. 143).

Research since 1990 has explored not only teaching and teachers' knowledge, with a focus on how they impact students' mathematical learning, but also how teachers come to know what they know and how their practice develops to promote students' learning. The importance of this as a research field was recognized by ICMI in the fifteenth specially commissioned ICMI study, *The Professional Education and Development of Teachers of Mathematics*. The invited international program committee (IPC) met in Prague in 2003, from which meeting a discussion document and call for papers was produced (ICMI, 2004), and worked toward an international conference in Águas de Lindóia in Brazil in 2005. From the papers submitted, 125 were accepted for work at the conference (http://stwww.weizmann.ac.il/G-math/ICMI/log_in.html). In response to recognition that publications in the field are underrepresentative of activities in many countries, a strong effort was made to include all conference participants who wished to be part of writing the study volume (Even & Ball, 2009). The volume includes two major sections reflecting two strands of activity at the conference: Strand 1, "Initial Mathematics Teacher Education," and Strand 2, "Learning in and through Practice." The chapters in the second strand relate particularly to the focus of this book. There are four chapters focusing on (1) factors, benchmarks, and issues in mathematics teacher development; (2) teacher learning as participation in social processes; (3) tools and settings supporting mathematics teachers' learning; and (4) the balance of teacher knowledge in mathematics and pedagogy. In the writing of this strand (in English), editors worked hard to be inclusive of the wide spread of authors and papers. This inclusiveness was not easy to achieve, not least because of the difficulties of communication with a large number of people electronically. Readers will judge the extent to which the goals of inclusivity were achieved.

Handbook of Mathematics Teacher Education

Further recognition of the maturing of our subfield is demonstrated in the first *Handbook of Mathematics Teacher Education*, commissioned by Sense publishers and published in 2008 (Wood, Jaworski, Krainer, Tirosh and Sullivan, 2008). The editors agreed on four areas of research and scholarship to be represented in the handbook and invited contributions from international scholars. The four sets of papers have formed volumes of the handbook as listed below.

- *Knowledge and Beliefs in Mathematics Teaching and Teaching Development* (Sullivan & Wood, 2008)
- *Tools and Processes in Mathematics Teacher Education* (Tirosh & Wood, 2008)
- *Participants in Mathematics Teacher Education: Individuals, Teams, Communities, and Networks* (Krainer & Wood, 2008)
- *The Mathematics Teacher Educator as a Developing Professional* (Jaworski & Wood, 2008)

Themes and issues permeating these volumes are taken up below.

Section Summary

This brings me to the end of an international historical perspective on our field. We see here the growth of mathematics teacher education as a subfield of mathematics education over almost three decades. The main points that I take from this, which influence strongly the themes and issues that follow, are these:

1. During the 1970s and early 1980s, the teacher was seen merely as a facilitator providing for students' learning of mathematics in an instrumental fashion interpreted in essentially simplistic terms often with connotations of deficiency.
2. A major shift toward recognizing context and complexity in teaching and the roles of a mathematics teacher was associated with (a) a shift in theoretical perspectives toward constructivism and social interactionism generally, which are seen to impact how teacher education initiatives are theorized and practically conceived, and (b) a growth in qualitative research methods, particularly case studies, which allow a more in-depth study of teaching and associated treatment of the professional development of teachers.
3. Concerns have arisen about the polarized perspectives in mathematics teacher education arising from the language of publication and conference discourse (English) and the domination of some parts of the world over what is considered worthy of study.

Considerable work is ongoing both in the conferences that continue to happen every year, two years, or four years and in special initiatives internationally. Despite point 3 above, we see more examples of international perspectives permeating the dominant language and discourse. This book

constitutes one of those initiatives. I continue now to address what I see as being important themes in the education of practising teachers of mathematics. Although I strive to keep an eye on the international scene and associated contexts, my main focus is on addressing complexity in MTE and how it plays out in studies of practising mathematics teachers, their learning, and their development.

Themes in Mathematics Teaching Development

Education of Prospective and Practising Teachers

The research literature relevant to this chapter might be divided into papers on teachers and teaching, papers on educating prospective teachers, and papers on educating practising teachers. For a historical dimension, I have drawn extensively on the four review papers mentioned above, which are, chronologically, Hoyles (1992), Ponte (1994), Llinares and Krainer (2006), and Ponte and Chapman (2006). Consistent with the time at which they were published, the first two focused largely on teachers and teaching, whereas the latter two focused more on teacher education. Llinares and Krainer separate into two sections their considerations of the education of prospective and practising teachers, whereas Ponte and Chapman discuss them together, indicating in most cases which type (prospective or practising) is being discussed.

These distinctions raise the question of what we mean by programs for educating teachers. It seems clear, where prospective teachers are concerned, that many countries have a formal program (with a variety of names, including pre-service or initial teacher education programs) through which new teachers are educated and brought into the profession. In the case of practising teachers, terminology includes in-service education and continuing professional development (CPD). The former often refers to formal programs, whereas the latter refers to a range of activities. It is on research in these two areas that the following sections focus.

The Practice of Practising Teachers

As I have indicated in relation to development within PME, the first papers to deal explicitly with our subfield portrayed the teacher as deficient, reporting teacher education activity aimed at changing teaching practice in accordance with curriculum needs. A developmental axis can be seen at this time with a shift from cognitive studies focusing on the individual

learner, on (largely quantitative) studies showing teacher deficiency with a gap between belief and practice, on the teacher as a facilitator or passive conveyor of knowledge, toward more qualitative (case) studies from theoretical perspectives of constructivism or social interactionism emphasizing the importance of context and complexity, a social dimension with situated beliefs, and “sensible” behaviour (Hoyles, 1992, p. 3: 275).

By the time of PME 30 and the associated review of research, there was considerable development but some disappointing conclusions, as reflected in the review papers from the PME 30 compilation (Gutiérrez & Boero, 2006). The two review papers are distinguished by their content, Ponte and Chapman (2006) focusing on teachers’ knowledge and Llinares and Krainer (2006) on teachers (and teacher educators) as learners. Teachers’ knowledge has become a hot topic in recent years. It is clear that teachers employ considerable knowledge in their practice of teaching and that it would be valuable to know more about the nature of this knowledge in order to inform teacher education programs. Ponte and Chapman review the literature to date in this area, focusing on the following.

- *Teachers’ mathematical knowledge*: studies of the teaching of arithmetic, fractions, rational numbers, geometry, functions, word problems, reasoning, and limits. In many cases, such studies drew attention to teachers’ perceived inadequacies in these topics.
- *Teachers’ knowledge of mathematics teaching*: studies focusing on teachers’ awareness of pupils’ misconceptions; their pedagogical content knowledge; and connectedness in the teaching of mathematical concepts.
- *Teachers’ beliefs and conceptions*: studies relating beliefs and knowledge; relating beliefs to particular aspects of teaching and learning, such as problem solving, students’ errors, technology, and gender differences; and relating belief to practice.
- *Teachers’ practices*: studies based on psychological or sociological frameworks; teachers’ biographical studies; curriculum-related studies; and studies related to innovation and reform.

These focuses chart a progression from defining teachers in terms of their facilitative activity in supporting students’ engagement with mathematics, through exploration of teachers’ own conceptions (knowledge and beliefs) in their engagement in practice, to more deeply rooted studies, psychological or sociological, of the practice of teaching in its complexity. Ponte

and Chapman quote Even and Schwartz (2002) in saying that “practice is too complex to be understood by only one perspective” and point to “the significant growth of research in mathematics teachers’ practice” as “the most salient aspect of research concerning the activity of the teacher in recent years” (p. 483). From the wide range of papers reviewed, Ponte and Chapman suggest that “the most common conclusion is that teachers need further learning to carry out ‘better’ practices, *more aligned with the researchers’ espoused perspectives*” (p. 485; emphasis added). They offer one disappointing conclusion, that still “the emergent image of the teacher is that of a professional with deficient knowledge.” They recognize also a research emphasis on the complexity of teachers’ knowledge and its “intimate relation” to practice (p. 486). A challenge for research is to find methodologies to explore this relationship without defining the teacher as deficient according to theoretical perspectives espoused by the researcher. This challenge is addressed further below.

Llinares and Krainer (2006, p. 439-443) focus on studies relating to teachers’ learning, mainly “as a consequence of having participated in some kind of programme or course.” For example, programs focus on

- *raising teachers’ awareness of mathematical process and content*: with a common assumption that “for teachers to become competent in mathematics it was necessary for them to learn mathematics in the same way as they were expected to teach it”;
- *raising teachers’ awareness of children’s mathematical thinking*: with a suggestion that “to provide opportunities for teachers to explore children’s ... activity contributed to teachers’ ongoing professional development”; and
- *emphasizing reflection, collaboration, and community building* in promoting teacher learning and teaching development.

Such programs have “content” goals (see Simon, 2008, discussed below) that arise from how teacher educators perceive the learning needs of teachers. Indeed, research on teaching development is largely conducted by the teacher educators who design such programs. We can see the “content” to include both mathematics per se and ways in which the learning of mathematics can be approached, for example in problem-solving environments or in students’ classroom activity as a basis for learning through analysis of their ways of interacting with mathematics. Focuses on reflection, collaboration, and community building can be seen in how content is structured

and activity designed for teachers' learning in these programs. Llinares and Krainer recognize the centrality of the teacher educator in such programs and ask about the ways in which teacher educators themselves learn, looking at models of learning related to those seen in programs for teachers. However, they also point out important differences in the constraints faced by each group, mainly in terms of working contexts: teachers deciding what to teach face limitations from, for example, curriculum and educational level, whereas teacher educators have more freedom in deciding the content of their work with teachers. This relative freedom extends also to teacher educators defining their own ways in which to grow professionally. I take up these ideas further below. The authors point to a fusion between teacher education and research in intervention programs in which interactions between teachers and teacher educators are a focus of study in considerations of the learning of both groups. They point out that, though such studies provide deep insights into practice through continuous interaction and communication with practice, leading to "interesting research-oriented stories" (p. 451), they are nevertheless local stories and that some means is needed to go beyond such stories to larger-scale perspectives on teachers' learning and practice.

From these reviews of papers presented at PME conferences, we see research that reveals a shift in conceptualizing teachers' thinking and practice in terms of a deeper awareness of complexity and studies that look in detail at teaching practice and teachers' knowledge and learning. Nevertheless, and disappointingly, we still see an emphasis on the teacher as *deficient* according to researchers' (teacher educators') expectations. This raises important issues that relate to the expectations of teacher educators for teaching in schools and why teaching is seen not to fulfill such expectations. The wider socio-systemic factors that contribute to activity in schools have to be recognized and addressed. Relationships between teachers and teacher educators in developing or improving teaching, the sources of knowledge influencing such relationships, and a shift from local stories to more general concepts all need to be addressed.

The Nature of the Education of Practising Teachers

So how are teacher education programs for practising teachers motivated and conceived? How is it decided what is needed and how needs can be addressed and by whom? Writing recently, Ruhama Even (2008, p. 59) refers to "the ill-defined nature of the field of educating practicing teachers." She comments on the diversity of terms for educators who work with

practising teachers: common terms are “professional development (PD) providers,” “professional development teachers,” “professional developers,” “teacher developers,” “facilitators,” “teacher-leaders,” “teachers of teachers,” “teacher educators,” and “in-service teacher educators.” She maintains further, with reference to Wilson and Berne (1999, p. 197), that the system of providing professional development opportunities for practising teachers is a “random, sometimes voluntary, sometimes mandated, always fragmented system” (cited in Even, 2008, p. 60).

My own experience of involvement in what has often been called in-service teacher education in the United Kingdom accords strongly with this evaluation. In 2002, I was part of a team of educators who sought to write a historical account of teacher in-service education in the United Kingdom to date. Although we knew about a variety of initiatives, many of them highly regarded in terms of their developmental outcomes, we found little or no associated published research (so such programs and their outcomes do not feature in the reviews referenced above). It seemed that, historically in the United Kingdom, professional development initiatives had not commonly had associated research programs. Where we did find research, it was mainly in the form of some kind of evaluation of a program (Macnamara, Jaworski, Rowland, Hodgen, & Prestage, 2002). It would be interesting to know the extent to which this pattern mirrors practice elsewhere. The experience certainly seems to fit with the words from Even (2008) above.

Given, then, the “ill-defined nature” of the field in providing professional development opportunities for practising teachers, what can we glean from published research and the experiences of those engaged in such provision? My own experience as a teacher educator, as *JMTE* editor, and as a reader of research studies, such as those reviewed from PME reports, suggests that programs for practising teachers include several models, which I list below. The first three see teachers progressively as *pupils*, *participants*, and *partners* in the educational process (Jaworski, 1999). The third and the latter two emphasize teachers’ engagement in research as a developmental tool.

- Courses or summer institutes for teachers led by teacher educators: teachers as pupils (e.g., Murray, Olivier, & Human, 1999; Schifter, 1998).
- Research and/or developmental programs led by teacher educators: teachers as participants (e.g., Cobb, Wood, & Yackel, 1990; Remillard & Geist, 2002).

- Collaborative research in learning and teaching: teachers as partners in research with teacher educators/academics (e.g., Krainer, 1999; Lin, 2002).
- Research programs in which teachers explore their own practice: teachers as researchers (e.g., Jaworski, 1998; Phillips, 1997).
- Research initiatives by teachers (sometimes as part of a master's or PhD program): teachers as designers of research (e.g., Atkinson, 1994; Lee, 2006).

We might see the first model here as referring to formal education programs for teachers in which teachers are engaged with specific content, mathematical and/or pedagogical, with some attempt to enhance or change teachers' thinking and practice in a decontextualized mode. The second model might also sometimes fit this mode. On the other hand, the second through fifth models can all be seen to draw teachers into active developmental participation with educators and possibly engagement in inquiry or research that is embedded in the context and complexity of practice. We might ask in each case how teachers' learning and development relate to the nature of the program. Those responsible for designing programs need to understand, for any model, the learning taking place in relation to the experiences lived by teachers. Although there are particular examples of well-researched programs,⁵ such learning experiences and their relation to the program model are not generally well documented from the point of view of research.

Programs with “Content” Goals

Martin Simon (2008, p. 18) suggests that teachers' professional development efforts can be sorted into two categories:

1. those with process goals only: the engagement of teachers in inquiry-based, reflective practices in the context of professional support and communication structures; there are no a priori learning goals for teachers involved in these programs (other than learning the processes of inquiry, reflection, etc.); and
2. those that have content and process goals: courses and workshops for teachers in which teacher educators aim to promote particular mathematical and pedagogical concepts, skills, and dispositions.

In terms of my set of models above, perhaps the first two can be seen as having both content and process goals and the last three as having process

goals only. This distinction seems worth exploring further in terms of some of the questions raised above, and I start with Simon's second category: programs with both content and process goals. Llinares and Krainer (2006) have offered a range of examples of such programs. In terms of Simon's wording above, we might ask the following questions.

- What is involved in the “aim to promote”? Who promotes, how do they decide *what* to promote, and how do they *go about* promoting particular mathematical and pedagogical concepts, skills, and dispositions?
- What are the actions and roles of teachers and teacher educators in these programs, and how are they related to the learning of either group?
- What issues, tensions, and contradictions are evident?

The nature of the “content” is clearly an issue—for example, mathematical content and didactical or pedagogical content—as are the kinds of processes used to promote understanding of content. Distinctions between mathematics, didactics, and pedagogy are not clear, as the literature shows. Where mathematical knowledge is concerned,⁶ teachers' knowledge of mathematics is variously referred to as

- mathematical content knowledge (Rowland, Huckstep, & Thwaites, 2005; Shulman, 1987);
- mathematical knowledge for teaching (Ball & Bass, 2003; Schifter, 1998);
- pedagogical content knowledge (Graeber & Tirosh, 2008; Shulman, 1987); and
- didactical knowledge (Bednarz & Larochelle, 1998; Brousseau, 1992; Chevallard, 1992).

Briefly, mathematical content knowledge is knowledge of mathematics *per se*. Mathematical knowledge for teaching goes further in the sense that the mathematical topic is known in a teaching context and in terms of how students can be expected to approach, appreciate, or have difficulty with the topic. Pedagogical content knowledge refers to the overlap between mathematics and pedagogy essential in creating a classroom environment in which students can learn mathematics. And didactical knowledge focuses on the expression of mathematical ideas in terms of

material on which students will work, the tasks, examples, and situations through which mathematics can be accessible to pupils. It also refers to the organization and progression of knowledge developed by the teachers, to the criteria guiding the choice of problems and situations, to the way in which teachers exploit and interpret the solutions developed by students, and to the processes of institutionalization. To design programs in informed ways with respect to teachers' knowledge and learning, we might ask the following questions of such research:

- How do teachers come to know mathematics adequately (?) for the levels at which they teach?
- How do teachers use their mathematical knowledge to create productive (?) opportunities for students to learn mathematics?
- How do teacher educators work with teachers to promote effective (?) learning of mathematics at all levels?

The words *adequate*, *productive*, and *effective* are followed by a question mark to indicate their problematic nature. They are words that we use all the time and assume a common usage—of course we want learning to be effective. However, when we go below the surface and start to analyze just what each one of us means by the term “effective,” we find differing ways of seeing the learning process, seeing how students (or teachers or educators) understand mathematics, which would then make a difference to what is regarded as effective.⁷ For example, the much-quoted work of Skemp (1976) has distinguished between instrumental and relational understandings of mathematics. Effective instrumental understanding seems very different from effective relational understanding. This suggests that, to answer questions such as the three listed above, we have to look critically at what we mean by learning. This takes us into the realm of theory.

Theoretical Perspectives Underpinning the Designs of Programs

A discussion of theoretical perspectives on learning is beyond the scope of this chapter; however, we might distinguish a number of models of learning, each of which is linked to some well-expounded theoretical perspective. The following can be seen as relevant to current situations in classrooms.

- Direct instruction models: teachers showing, telling, demonstrating; metaphors of transmission or conveying of knowledge.

- Constructivist instructional models: the teacher as an individual “cognizer,” making personal sense of what is offered in developmental programs; similarly, the student as an individual “cognizer” of mathematics.
- Reflective practice models: teachers reflecting on and in their teaching, often with an aim for development or change.
- Socio-cultural models: learning as participation; teachers learning in interaction in social settings; importance of culture and context.

Unlike the theories on which they are based, the models are rooted in practice and take on the idiosyncrasies of practice that a purely theoretical approach might question. In any style of teaching, we might recognize elements of more than one of the models with a rationalization that makes sense in context. For example, we might see programs that claim to be socio-culturally rooted, that take participation as a central construct, and that encourage reflection as a means to establish individual perspectives on the classroom. Whereas theorists might argue strongly about the epistemological precedents of such theorizing,⁸ a practical rationale, based on what is done and how, can make realistic sense. However, distinguishing the models allows us to look critically at practical rationales and cut across complexities. This is true for all programs, whether focusing—as suggested by Simon (2008)—on content and process or on process only.

The distinction that Simon (2008) makes between programs that have only process goals compared with those with both content and process goals is more than categorical. It draws attention to the very nature of the teaching-learning process and the roles of those within it. Although there is a spectrum of program modes, and no well-defined format for either mode of program, if there is content to be taught, then some form of teaching or instruction might be assumed, whereas this need not be the case when no content is involved. So the ways in which learning and development are deemed to take place are presumably different in the two categories. This seems to me to be a significant issue in the aim, design, and implementation of programs, and I take it up in the rest of the chapter. A key consideration concerns the roles played by teachers and learners within these programs; indeed, who are the teachers and the learners?

Programs with Process Goals Only

So what kinds of programs fall into Simon’s (2008) first and my last three categories? As examples, Simon mentions programs based on the Japanese

lesson study model and programs focused on teacher inquiry or teacher research: “The basis of these programs is that the engagement of teachers in inquiry-based, reflective practices in the context of professional support and communication structures can support the ongoing professional development of mathematics teachers” (p. 18). What seems to be suggested here are programs with a professional development purpose but without “content” to be taught in any formal instructional sense.

Four papers presented at an international symposium at the American Educational Research Association (AERA) conference in 2008 can be seen to exemplify this category. The symposium focused on partnerships between teachers and academics to promote professional development in teaching, and the four papers, briefly, were as follows.

- A focus on video material recorded in a classroom in Australia. Teachers selected from this material key episodes for discussion in a research group consisting of teachers and academics. The video acted as a shared classroom experience from which participants could discuss issues. This activity provided an opportunity for teachers to modify classroom practice according to new awareness gained from the interactive work. (Gorur & Clarke, 2008)
- A focus on a nationwide bank of PD programs that was made public and financed in Germany. Teachers could elect to take part in advertised programs, choosing what they wanted to learn. Those leading the programs operated in pairs consisting of one academic and one practising teacher. Courses combined theory and practice. They were evaluated by a Centre for Pedagogical Research. (Rösken & Törner, 2008)
- A focus on collaboration between teachers and academics in China to design lessons and “upgrade” teachers’ ideas. One teacher designed a first lesson and taught it, with observation from other teachers and academics. Discussion and reflection between the teacher and the observers led to a redesign of the lesson and learning for all concerned. This was the start of a design cycle involving several iterations, and it was followed by dissemination to other teachers of what had been learned in the iterative process. (Bao & Huang, 2008)
- A focus on lesson study in Japan involving a lesson study model in which teachers planned lessons for observation and analysis. Academics worked with the teachers before and after the lessons,

which were observed by other teachers and academics. Discussion and analysis of the lessons led to new insights, issues, and awarenesses for all those involved, which in turn led to new possibilities for practice. (Shimizu, 2008)

In each case, teachers engaged with opportunities to reflect on their own practice through engagement in a PD program. A distinctive feature of the programs was their creation of partnerships between teachers and academics to promote professional development in teaching. Teachers either led the developmental process, with responses from their academic colleagues, or were partners in creating opportunities for development. Learning and development followed from critical discussion and reflection, building on practical activities in the program. Although professional development was intended for the teachers, the academics were also learners in these programs.

I have provided these examples to emphasize the distinction between programs aiming to teach content (with content and process goals) and those providing opportunities for development (with process goals only). It is clear, from research in the examples, that considerable learning took place for both teachers and academics. However, the programs did not set out to teach particular content. We might ask, what “content” was learned? To what extent can pre-identified content be addressed through such programs? These questions raise obvious issues for teacher educators designing programs, but could these be issues for teachers themselves? Might teachers themselves actually be in a better position to choose what to address in terms of content? The German program mentioned above (Rösken & Törner, 2008) offers such a possibility. Perhaps both are needed?

Teachers and Teacher Educators

As we saw above (Even, 2008), there is a range of terms used in the literature and in practice to describe the professionals who work with teachers to promote development in teaching and learning. It might be that the term used is related to the mode and nature of a particular program. However, the term “teacher educator” is commonly used to describe such professionals internationally. In the AERA symposium, the programs were referred to as partnerships between teachers and *academics*. The academics in each case were professionals from a university setting where the term “academic” would describe the nature of their job. In research projects in which I worked in Norway involving partnerships between academics and teachers, academics called themselves *didacticians*. This

was a deliberate move, first to recognize our role as professionals working in the didactics of mathematics,⁹ and second to avoid the term “educator” in order to avoid (if possible) the perception that our role was to instruct teachers or that teachers were not themselves educators. We wanted to create a partnership in which both partners brought knowledge and experience, and mutual respect for each other’s considerable expertise, and in which both would inquire into developing mathematics teaching and learning (Jaworski, Fuglestad, Bjuland, Breiteig, Goodchild and Grevholm, 2007). Our aim was that both groups would be *co-learners* in the developmental process (Wagner, 1997).

So, in this chapter, when I use the term “teacher educator,” it is (a) as a general term that is largely acknowledged in the field (some term needs to be used) and (b) with a critical view to questioning what it means, its connotations, and the diversity of experience that it encompasses. So we might start by comparing the roles of teachers and teacher educators (see Table 1.2).

By listing the points as in Table 1.2, I draw attention to the parallels in these roles. In each case, the second point emphasizes the instructional aim and the third point the didactic function of creating a learning environment. The fourth point makes clear that both teachers and educators are learners in their respective profession. This means that both generate knowledge related to their particular practice and can learn from their engagement in practice. When they work together, these forms of knowledge must be related in some ways. So, when teachers and educators work together, how is knowledge in the processes of teacher education and teaching development distributed between them? The diagram in Figure 1.1 represents one way of seeing such

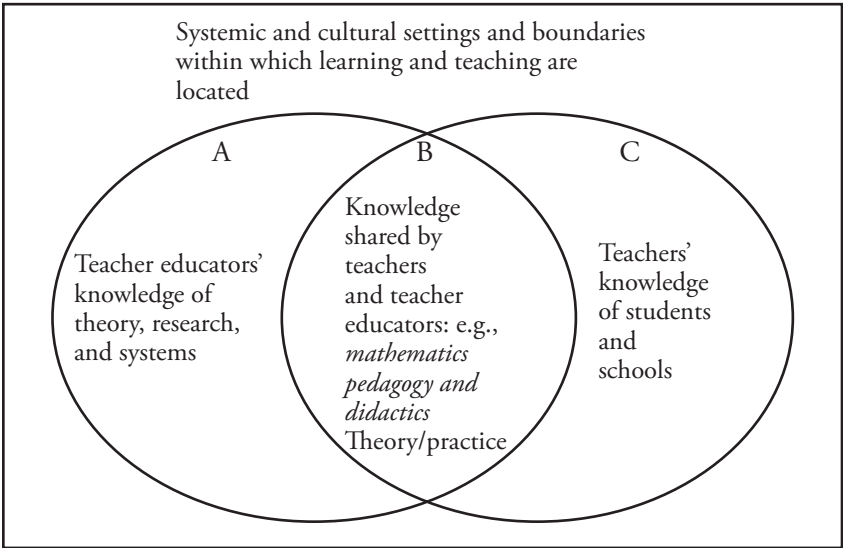
Table 1.2
Comparison of roles of teachers and teacher educators

| Teachers have responsibility to | Teacher educators have responsibility to |
|---|---|
| Work with students | Work with teachers |
| Promote students’ learning of mathematics | Promote teachers’ learning of mathematics teaching |
| Engage with didactics and pedagogy to create learning situations for students | Engage with didactics and pedagogy to create learning situations for teachers |
| Be themselves learners of mathematics teaching | Be themselves learners of mathematics teacher education |

a distribution (Jaworski, 2008, p. 336). In the wider complexity of educational systems and socio-cultural settings in which teaching and learning are located (the large rectangle), the two circles represent knowledge within the two groups. The central section (B) shows that there is considerable common knowledge, albeit knowledge that is highly related to the particular professional practice and context and therefore can seem different in its manifestations. Sections A and C reflect the particular areas of knowledge related to expertise and responsibility in the two groups. It is a simple diagram that avoids many questions about knowledge addressed in the wider literature. Its main purpose is to emphasize the considerable knowledge and expertise in both groups, the professional status associated with such knowledge, and the importance of the co-learning process when the groups work together.

The nature of all these factors will depend of course on the type of program, its design, operationalization, and aim, which brings us back to some of the questions raised above, particularly in regard to the central section (B), where knowledge is shared albeit with different manifestations. It would be valuable to use research to reveal finer details of these manifestations and to explore how working together can enable each group to understand better the aims, motivations, and concerns of the other. Wagner (1997, p. 16), speaking of *co-learning agreements*, noted,

Figure 1.1
Distribution of knowledge in collaboration between teachers and educators



In a co-learning agreement, researchers and practitioners are both participants in processes of education and systems of schooling. Both are engaged in action and reflection. By working together, each might learn something about the world of the other. Of equal importance, however, each may learn something more about his or her own world and its connections to institutions and schooling.

If we replace “researchers and practitioners” with “teachers and educators, both of whom are also researchers,” then this quotation offers a vision of what is possible in such a partnership. Before going further with ideas of collaborative learning, however, it seems worth addressing how educators come to know what they know. How do teacher educators learn?

How Do Mathematics Teacher Educators Learn, and How Similar Is This to How Teachers Learn?

In some cases, new teachers might be accepted into schools with no formal training; however, the principal route into teaching in many countries requires new teachers to have participated and been successful in some initial or pre-service educational program (Llinares & Krainer, 2006). This is their “training” for the job, and the responsibility for this training most often rests with teacher educators, academics in university settings. Given the wide acceptance of such provision for the education of new teachers, should new teacher educators be educated similarly?

When I became a teacher educator in the United Kingdom, I had considerable experience and knowledge but no formal training. I had been a teacher of mathematics myself; I had contributed to courses for teachers at the Open University in the United Kingdom; I had engaged in my own PhD studies and research in mathematics education. In appointing me as a teacher educator, with responsibility for programs in both initial and in-service education, the university accepted these qualifications and presumably assumed that the corresponding knowledge was adequate for the job. It certainly seemed important that I had been a mathematics teacher myself, but this is not true for all academics employed as teacher educators. If educators are to work effectively with teachers who have been through an initial training program and possibly gained considerable experience in school and classroom, then in what ways can educators develop suitable knowledge that allows them to work with teachers in informed ways?

Simon (2008, p. 26) suggests that “being an effective mathematics teacher is necessary, but not sufficient, to be an effective mathematics

teacher educator.” He also questions whether a PhD in education is sufficient training to become a teacher educator: “In the US teacher educators are being prepared in doctoral programs without the conceptual frameworks that they require in order to work with prospective and practicing teachers. Thus the goals of their work and the developmental process that they endeavour to support and promote are under-defined.” So what kinds of definition would be appropriate, and how do educators come to suitable conceptual frameworks for their work with teachers? Zaslavsky (2008, p. 95) elaborates a set of “unifying themes that reflect goals for mathematics teacher education” that “concern qualities and kinds of competence and knowledge that mathematics teacher education seeks to promote in prospective and practicing teachers in a broad sense.” The themes are

- developing adaptability;
- fostering awareness of similarities and differences;
- coping with conflicts, dilemmas, and problem situations;
- learning from the study of practice;
- selecting and using (appropriate) tools and resources for teaching;
- identifying and overcoming barriers to students’ learning; and
- sharing and revealing self-, peer, and student dispositions.

Looking back on my own experience, I can recognize all of these qualities and competencies as being part of my knowledge growth and development. They have come through a variety of activities alongside my professional practice—for example, reading, writing, attending seminars and conferences, undertaking research in classrooms, talking with teachers, supervising research students, and having good, critical colleagues. I suspect that many colleagues around the world have developed as teacher educators similarly. What strikes me particularly as I review this list is that the set of necessary qualities or competencies applies both to teachers and to teacher educators. So how do educators learn or develop these qualities, and is there a parallel to how teachers learn and develop?

To address this question, we might ask, what would or could it look like to design a program to educate teacher educators? Two such programs are the MEd program at the Institute for Educational Development at the Aga Khan University in Karachi, Pakistan (Farah & Jaworski, 2006), and the MANOR program at the Weizmann Institute in Israel (Even 2008). Both programs address Zaslavsky’s (2008) list of competencies, both have formal instructional elements, and both encourage engagement in professional

activities, including reading, writing, attending seminars and conferences, acting as critical friends, undertaking research in classrooms, and working with teachers and pupils. This list of activities is similar to the one that I gave to describe my own learning as a teacher educator, albeit not as part of a professional program. What these two programs recognize is that there are strategic activities that are key to developing as a professional educator that include reading the academic and professional literature, writing scholarly and reflective accounts relating to theory and practice, and engaging in research on mathematics learning and teaching. Although not all educators have taken part in such a professional program, most have engaged in such activity and recognize the importance of learning through such activity. Can this be the same for teachers? What different aspects of context might affect this possibility?

One of the paradoxes of teacher education initiatives is that, despite recognition of how we learn as teacher educators, we subscribe to creating very different learning situations for teachers to which teachers also subscribe. This is captured in the words of Sandy Dawson (1999, p. 148):

This manifestation of in-service culture seems to have the following basic principle: there is something wrong with mathematics teaching worldwide and we, as mathematics educators, must fix it. Many mathematics teachers have bought into this culture. Such teachers seem to be seeking new ways to fix their practice. But this places mathematics teachers in a relationship of co-dependence with mathematics teacher educators. Mathematics teachers need someone to fix them, and mathematics teacher educators need someone to fix. The two groups seem made for each other.

As editor of *JMTE*, I read many submitted papers that seemed to subscribe to this perspective. The programs addressed were largely ones that Simon (2008) would categorize as having both content and process goals and often involved teacher educators as instructors, with teachers as pupils rather than either participants or partners. An assumption was that the expert teacher educators would instruct the teachers in teaching knowledge and the arts of teaching.

A “New Pedagogy” in Teacher Education

Chapman (2008) reviewed a set of research papers related to studies conducted by mathematics teacher educators in North America on their own

programs developed for the education of prospective teachers. Her aim was “to discuss the teacher educators’ learning from researching their own practice” (p. 117). With reference to Lampert, Heaton, and Ball (1994), she writes that

a new pedagogy of teacher education is required if teacher education is to prepare prospective teachers to be responsive to visions of mathematics education advocated by the reform. They [Lampert, Heaton, & Ball] argued that conventional teacher education programs, which present prospective teachers with ideal methods and techniques derived from a synthesis and interpretation of educational theory and research, do not represent the complexity and uncertainty of teaching found in reform-based instruction. (pp. 115-116)

Although Chapman, along with Lampert, Heaton, and Ball, refers here to pre-service programs, the words could apply as well to in-service teacher education. Chapman points out that research conducted by teacher educators on their own practices has the potential to provide professional development for educators and allow them to know something of significance about their own practice (see also Jaworski, 2003). However, her survey of relevant articles shows, for the most part, that this learning was presented in the articles as what other teacher educators could learn about the nature of these approaches to teacher education rather than as the actual learning of those conducting the research. These authors offer what is learned from research as generalizable knowledge for the scientific community or as advice for other professionals rather than as informing their own professional practice.

Thus, what has been found here is that teacher educators’ learning from their own research on their own programs is not usually cast as teacher educators’ learning in practice, though it manifestly is such. Several *JMTE* papers stand out for me as making this link clearer: authors synthesize their findings for a general academic presentation (as encouraged by a quality research journal) and highlight the importance of their findings for their own thinking and practice (see, e.g., Goodell, 2006; Heaton & Mickelson, 2002; McDuffie, 2004). What is clear from a wide range of papers in *JMTE* is that there has been a significant shift from programs in which teacher educators instruct teachers according to teacher educators’ own perceptions of how to know and how to teach toward a more thoughtfully critical conception of teacher education. Simon (2008) marks this as

a contrast between *perception-based* teaching and *conception-based* teaching. In the latter, the teacher seeks to become more aware of the conceptions of the students in order to construct relevant learning experiences rather than pursue mainly the teacher's own perceptions of how things should be (see also Tzur, 2008). It seems clear, and Simon makes this point, that these notions apply similarly to teachers teaching students and educators teaching teachers. Paola Sztajn (2008) takes this further in a discussion of the idea of *reciprocity* in *caring* relationships. When the teacher is seen by the student overtly to enter into students' ways of thinking and to work with students' conceptions, the student is more able to appreciate what the teacher can offer from his or her own knowledge and experience.

From my own experience in professional practice, in international meetings, and as editor of *JMTE*, I detect a shift in perspective in the mathematics education community in addressing teacher education. I wrote about this as follows, referring to current writing:

[There has been] a shift in tone and nuance in the ways educators write about educating teachers. There is less of a surety of models of practice that educators promote with teachers and much more a sense of uncertainty. With this uncertainty comes, almost paradoxically, a strength of purpose, new ways of speaking about mathematics teacher education, and new paradigms of practice. These build on notions of reflection, for both teachers and teacher educators, on teacher-as-researcher and simultaneously educator-as-researcher, and on growing recognitions of epistemology, of complexity and the importance of not trying to oversimplify. (Jaworski, 2008, p. 338)

This quotation comes from the fourth volume of the *Handbook of Mathematics Teacher Education* (Jaworski & Wood, 2008), entitled *The Mathematics Teacher Educator as a Developing Professional*. Perhaps the fact that we have a volume of a handbook focusing on the mathematics teacher educator is itself one source of evidence for what I claim. One important piece of evidence, in my view, is the increasing number of programs reported that seek to build partnerships between educators and teachers and study the processes and outcomes.

Partnerships in Professional Practice

We can see both teachers and teacher educators as learners in practice, reflecting critically on activity, possibly by engaging in research. Both

bring specialist knowledge and expertise to the partnership. Both have specialist roles and responsibilities in the joint practice (e.g., Jaworski, 2001; Krainer, 2008).

The four programs reported above from the AERA symposium are examples of such partnerships, as they claim overtly. Many of Konrad Krainer's writings reflect partnerships through which Krainer (1998) has conceptualized his two-dimensional model of Action, Reflection, Autonomy, and Networking. In one dimension, we see action \leftrightarrow reflection not as polarization but as recognition of their complementary importance for learning and development. Similarly for autonomy \leftrightarrow networking: professionals are both thinking individuals and members of communities of practice in which collaboration supports learning. So one axis concerns engagement, and the other concerns the professional landscape in which engagement is rooted. From my own experience of such partnerships over many years, I suggest the following characteristics of partnerships.

- Partnerships avoid a hierarchy and seek mutual respect and reciprocity—each seeking to fulfill the goals of the other as well as its own goals (e.g., Burton, 1999).
- Partnerships assume a shared power base in which decisions on what is needed and how to fulfill needs are taken jointly through discussion and negotiation (e.g., Farah & Jaworski, 2006).
- Issues and tensions lead to learning and development (e.g., Jaworski & Goodchild, 2006).

At this point, I am ready to take up some of the questions highlighted above and to discuss some of the associated issues for the development of mathematics teaching.

Issues Relating to the Education of Practising Teachers

This book is about the professional development of practising teachers. This involves questions relating to how teachers learn and the kinds of programs that facilitate their learning. It involves considerations of what teachers know, how this knowledge relates to their teaching or is adequate for their teaching, and how this knowledge develops, particularly in relation to practice in the learning and teaching of mathematics. In the above sections, I addressed themes relating to these considerations. Particularly, I have moved toward

considerations of the roles and development of those who teach teachers, the teacher educators, and how their learning and development both parallel and influence that of teachers. In this final section, I recognize issues that arise for research in the teacher education community (including teachers and teacher educators) as it seeks to improve professional development opportunities and concomitantly the experiences widely offered to learners of mathematics.

Theoretical Basis of Research Programs Reporting Teacher Education Outcomes

The move toward a new pedagogy suggests that teachers are not seen as *pupils* in professional programs; they are either *participants* or *partners*. But what do we mean by these terms? How is the nature of participation defined? Well, perhaps a starting point is to avoid seeing teacher knowledge as received wisdom (as in direct instructional models of teacher education) and to conceptualize it either in social constructivist or socio-cultural terms. This theoretical conceptualizing influences the models of practice that emerge. Social constructivist models emphasize social interaction as promoting personal reflection and leading to new insights for individuals in relation to their practices as teachers or educators (e.g., Tzur, 2008). Socio-cultural models emphasize ways of being and thinking in relation to communities of learners and teachers and the institutional settings in which they participate (e.g., Goos, 2008). In my experience, many research papers submitted to a journal, focusing on teacher education programs, are not clear enough about the underlying theories of learning and teaching that influence the programs studied. Better clarity seems to be imperative in understanding the basis of a program and research findings from a study of it. For example, Tzur (2008) makes clear that his research is conducted from a largely cognitive perspective in which conception-based learning is theorized with reference to von Glasersfeld's constructivism and rooted in Piaget and Dewey. Goos (2008) foregrounds the social, embedding her research theoretically in Vygotskian theory and Valsiner's zone theory of child development. With such clarity on theoretical perspectives, it is possible for the reader to judge reported outcomes in relation to the perspectives that have informed interpretation and analysis.

A further consideration lies in the relationships between the theoretical basis of research and its compatibility with the theoretical basis of the program studied; this also requires consideration of how research and development are linked to each other. I address this through consideration of the related roles of those involved in a program.

Collaboration between Teachers and Educators: The Nature and Complexity of Relationships

As indicated in references to Chapman (2008) above, teacher educators as researchers, reporting from their research programs, often seem to objectify their findings and isolate them from the developmental learning experiences to which they relate. My own experience suggests that most research programs on the learning and practice of teachers, in which researchers themselves are teacher educators, are related fundamentally to professional learning both for the participants and for the researchers (Jaworski, 2003). By *participants* here, I refer to both teachers and educators. Thus, the educators are both central participants in the practices studied and researchers in studying them (both insider and outsider researchers; see Goodchild, 2007; Jaworski, 2004). What is reported from the research therefore needs to be acknowledged as being dependent not only on the theoretical stance taken by the researchers but also on the theoretical roots of the developmental program and relationships between the two. The roles of teachers and educators in such programs can be related fundamentally to the theoretical stance, as can the ways in which these roles are interpreted and analyzed. When teachers take on overtly an inquiry or research role in a program, relationships between roles become even more complex—both teachers and educators learn through their roles as insider researchers, and educators take on the additional role of synthesizing more generally from the research.

Thus, when we consider roles of teachers as either *participants* or *partners* in research and development, we are making theoretical statements. We theorize both the positions of teachers within the developmental program and how we observe and analyze these programs. Speaking developmentally, it seems important to juxtapose learning and experience of teachers and educators; in research terms, needed is critical scrutiny of who is making interpretations and judgments and how they relate to the positioning of both groups of participants. I accord with Cochran Smith, and Lytle (1999) and Wells (1999) in suggesting “inquiry” as a theoretical-methodological stance that allows practitioners, teachers, and educators to look critically at their own practice while engaging with it. Thus, the role of insider researcher allows a “critical alignment” with practice that allows practitioners to engage with the norms and expectations within the community of practice of which they are a part and, at the same time, look critically toward ways of developing and improving the practice (Jaworski, 2006).

Proximity to the Classroom: Outcomes for Students' Learning Mathematics

Within the complexity just mentioned is the dimension of educator and teacher learning vis-à-vis student learning of mathematics. We might envisage nested dimensions of

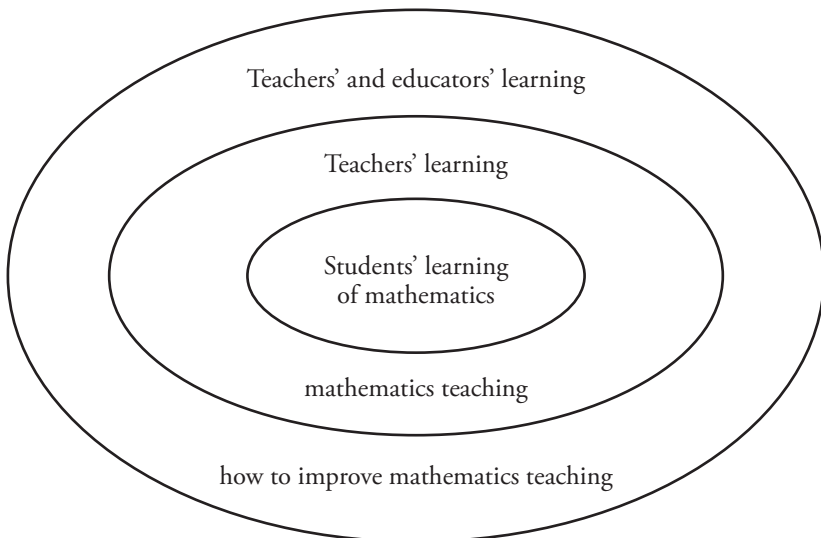
- a mathematics classroom with students learning mathematics;
- a mathematics teacher education program with teachers learning mathematics teaching; and
- research on mathematics teaching development with teachers and educators learning how to improve mathematics teaching and learning.

Zooming in allows us to look closely at students' learning of mathematics and issues in classroom settings. Zooming out takes us into issues of teaching and developing teaching (see Figure 1.2).

The model here might be considered alongside the one presented in Figure 1.1, considering dimensions of teacher and educator knowledge together with the related dimensions of student, teacher, and educator learning in practice.

Figure 1.2

Nested model of educator and teacher learning vis-à-vis student learning



Since all teacher education is premised on the development of opportunities for students to learn mathematics effectively, their mathematical learning needs to be central to all teacher educational concerns.

When we read the literature, we see that students' learning of mathematics is implicitly or explicitly a central factor in most programs, but often both the mathematics and the students are backgrounded in conceptualizations of outcomes. It seems that, when we zoom out to express the learning of teachers and educators, the learning of students, and the mathematics that they learn, become at best implicit in what we report. This has been especially evident in my own research, and I have struggled to make sense of the complexity in a holistic sense. For example, in the Mathematics Teacher Enquiry Project (Jaworski, 1998), my focus as an *outsider* researcher was on teachers' engagement with inquiry processes as they reflected critically on their own teaching, whereas my focus as an *insider* researcher was on how my own practice developed within this program and how my actions and those of a colleague in interactions with teachers influenced the development of the program. In practice, the teacher educators (I and my colleague) were one day in the classroom with teachers and pupils, observing mathematics teaching and learning and discussing the pupils' mathematics with their teachers, on another day leading meetings of all teachers in the project and facilitating their reflections, and on another day reflecting ourselves and struggling with issues within the project. Thus, issues relating to insider and outsider roles were hard to separate in practice. When writing about them, it seemed to make sense to distinguish between the different layers, though this had the effect of fragmenting the overall complexity. Thus, what we might call the "zooming problem" alerts us to ensure (if we can) that we do not lose sight of students' learning of mathematics within the complexity of issues in teacher education.

Formality of Teacher Education Processes

As I wrote the above paragraph, I had in mind a developmental research process in which teachers and educators together explored aspects of learning and teaching and their development. Often such programs are characterized as research and/or development programs without being addressed as mainstream "teacher education." They are programs that have *process* goals rather than explicit *content* to be taught—that is, where there is something to be taught and someone to teach it (Simon, 2008). Research shows, however, that considerable learning happens through such programs, and as discussed above we can point to "content" in what

is learned. For example, in the program mentioned above, I learned about tensions between wanting certain outcomes from practice and not knowing how best to work with the teacher to achieve them.

It might be that a developmental research program highlights learning needs that a more formal program can then address. More often, perhaps, perceived deficiencies in teaching practice (Dawson, 1999) are the motivation for more formal content-based programs. So we recognize differences between formal (content-based) teacher education programs and less formal, more developmental programs. I am not suggesting that content-based programs are not needed—we can find many justifications for them in the literature and in practice, and teachers themselves can identify needs for such provision (Rösken & Törner, 2008). However, it can seem that the outcomes of content-based programs are more distant from developments in practice than are the outcomes of developmental programs. In a more global characterization of mathematics teacher education, how should or can the two kinds of programs sit side by side, complementing each other as they contribute to teaching development? It is important that such questions, which go beyond the nature of specific programs or a specific type of program, be addressed at a socio-systemic or -political level in defining mathematics teacher education.

Degree of Autonomy of the Teacher

Central to such questions, in all of the programs considered, is the position of the teacher and the ultimate mathematics learning of his or her pupils. I have suggested a number of ways of seeing teachers in education programs (pupils, participants, partners, researchers). However, for sustained impact on education more broadly, should it be left to the individual program to decide, implicitly or explicitly, how teachers are treated? It seems fair to say that teacher education programs have moved away from direct instruction models and toward more inclusion of teachers in conceptualization and decision making. I cannot be so confident, however, about the political decision making that commissions and resources programs. It can seem that politicians and those who manage education nationally and internationally work from visions grounded in absolutist epistemological frames (Ernest, 1991) and look for outcomes in terms of measurable evidence and warrants (Shavelson, Phillips, Towne, & Feuer, 2003). One result is that the programs that find favour and support are of the content-based, direct-instructional type in which measuring outcomes seems easier than in developmental models.

Despite these remarks, there is evidence in some parts of the world that national programs move toward more complex models in which critical teacher centrality is acknowledged. This book provides examples from Brazil in which teachers are included centrally in developmental programs (see, e.g., the chapters by Fiorentini et al.; Freitas & Fiorentini; Passos & Lamonato). We can also see this happening in Austria in the IMST program of which Krainer (2008) writes. In the United Kingdom, we have had a long period of political control of education in which measures have predominated through national tests and government curricular control (Alexander & Flutter, 2009). Now we see, encouragingly, more inclusion of teachers in national policies relating to education and in decision making in teacher education programs (<http://www.ncetm.org.uk>). Where mathematics is concerned, there is acknowledgement of mathematics in the curriculum as needing special consideration and new vision—a “radical culture change” in continuous professional development is suggested (Smith 2004, p. 111). Other chapters in this book suggest similar trends in other nations. It therefore seems that the climate of understanding between educators and educational managers is shifting toward more open discussion about how we conceive of teacher education in mathematics globally.

Teachers and Educators as Part of a Culture Change

A climate of understanding in which teachers are included in decision-making processes about educational development brings teachers into a more central position, both of opportunity and of responsibility. As teachers are included centrally in research programs and consulted on developmental issues, they face new demands on their thinking and practice. In socio-cultural terms, we might describe the growth of teachers in new areas of opportunity and responsibility as a *culture change*. Although culture change can seem essential to effective development, it can simply shift the axes without changing practice fundamentally.

What are the implications for mathematics learning and teaching? We might see two fundamentals of mathematics education to involve, for teachers, a deep understanding of the mathematics that they teach and a mature awareness of students’ engagement with mathematics and issues in fostering their understanding. Educators have a responsibility to work with teachers on such fundamentals; their own practice, however, is no more clearly defined than is that of teachers. They too work within the culture and are defined relative to social and political forces. They have to respond to both schools and teachers and to the educational managers

and politicians who resource programs. I refer here to what is expressed in Figure 1.1 as “systemic and cultural settings and boundaries within which learning and teaching are located.” It is the containing area in which teacher and educator knowledge, practice, and interaction are located. Although individual programs can, to some extent, set their own contextual boundaries, the more global context consists of both the sum of its educational parts and its own socio-political frame embedded in global trends and positioning. Krainer (2001, 2008) addresses the institutional demands on mathematics teacher education in relation to the spectrum of fundamentals that concerns not only teachers and educators but also “economics experts, educational policy makers, mathematicians, parents, etc.” (2008, p. 194). This is one approach to a more global frame.

Possibilities: The Macro and the Micro

Lerman (1998) uses the metaphor of the zooming of a lens to discuss alternative perspectives in differing degrees of focus. The complexity of our subfield lies in the need to see at the same time both the macro and the micro—the global picture that I have referred to above in contrast to a focus on the student in the classroom engaging with (struggling with?) mathematics (the central part of Figure 1.2). I know from experience that it is hard to address both perspectives together. It can seem that research focuses finely on classroom interactions and fine-grain details of pupil or teacher learning, ignoring the wider socio-systemic influences, or it can take a more socio-systemic perspective but give little insight into finer details of practice. With my colleague Despina Potari, I have tried recently to bridge the macro/micro gap in a study of teaching using an activity theory perspective (Jaworski & Potari, 2010). With Simon Goodchild, I have also used an activity theory perspective to trace teaching-learning issues across a large developmental project, identifying tensions in practice that illuminate several levels or layers of culture (Jaworski & Goodchild, 2006). Inevitably, such papers leave the authors open to criticisms of insufficiently detailed treatments that allow depth of insight. We need more debate in the research community on how to juxtapose a range of zooms on practice to present more holistic perspectives on teaching and teaching development.

Toward an Agenda for the Future

I offer a number of points as a brief synthesis from themes and issues above to suggest a forward-looking agenda for the education of practising teachers of mathematics.

1. Teachers should be included as partners in teacher education initiatives with their teacher educator colleagues, with mutual respect for the knowledge and expertise of both partners and complementary contributions to the nature and design of programs.
2. Programs with content goals should be linked closely to developmental programs so that content can be related clearly to desired learning outcomes made evident through collaborative process-based activity.
3. The combination of (1) and (2) above should allow for exploration of knowledge in teaching without defining the teacher as deficient.
4. There needs to be recognition that teacher educators are also learners and that developmental programs provide an environment for their learning just as much as they do for teachers.
5. Development needs to be closely linked to research. Research not only studies the progress of development but also provides an inquiry base for the development of practice.
6. Studies of mathematics teaching and its development need to be designed to address complexity so that it is possible to keep in sight students' learning of mathematics while exploring broader aspects of teacher and educator development.
7. The theoretical perspectives underpinning research and development need to be made explicit so that the principles on which programs are founded and on which development is premised are clear.
8. Attention needs to be paid overtly to language issues in reporting research and dominance of the English-speaking world in research agendas. Editors of the major journals and organizers of international events have to be urged to seek new ways of promoting more equitable international communication.

These points are easy to write, and in the writing they seem like common sense. However, each one is challenging and problematic to achieve. We therefore need research in the subfield that addresses how we go about this agenda and what we learn from engaging in studies that seek to achieve these aims.

Acknowledgement

I would like to thank Nadine Bednarz warmly for her very helpful comments on earlier drafts of this chapter.

Notes

- 1 A personal note is perhaps also in order. I came into university education in 1984 after a career in school teaching. In my final five years of school teaching, I was head of a mathematics faculty in a large secondary school teaching mathematics to the pre-university level. I took it as a challenge to look critically at how we taught mathematics in the school and to consider approaches based on investigations and open-ended problems. We held many meetings, seminars, and workshops in the school, inviting colleagues from the academic world to help us think through our possibilities and associated issues. When I was offered the opportunity to move into university education and engage in research, it was with the motivation of investigating mathematics teaching further, of gaining insight into new ways of creating environments for students' mathematical learning. From that time, as a teacher educator and researcher, I had the privilege to work with some outstanding teachers who helped me to work on my visions with a firm base in the reality of schools and classrooms. My book *Investigating Mathematics Teaching: A Constructivist Enquiry* was the outcome of my first years of research in this field (Jaworski, 1994). Since then, I have been concerned with how mathematics teaching can develop, focusing largely on teachers' classroom inquiry and on partnerships between teachers and academics in researching teaching. I was also editor of the *Journal of Mathematics Teacher Education* for six years. I have therefore been personally close to the development of and evolution in mathematics teacher education for more than twenty years.
- 2 In the field of mathematics education, we might see the annual conference of PME, the international group for the Psychology of Mathematics Education, as being the major meeting ground and a principal source of publication. Although PME papers are short, and often cannot do justice to complex areas of research, they are markers in the field. The rigorous review process that has developed over the years and the annual occurrence of the conferences have meant that we have gained a reliable record of research development over more than thirty years.

- 3 For the first three years, three issues a year were published, increasing to four issues in 2001. At the end of 2001, editorship passed to me (I was then at the University of Oxford), and I invited an editorial team to join me (Terry Wood, Purdue University, United States; Konrad Krainer, University of Klagenfurt, Austria; and Peter Sullivan, Monash University, Australia). We continued with four issues a year until 2005. In the meantime, Kluwer had been taken over by Springer, and the Springer editor, along with the *JMTE* team, was keen to see the journal expand. Thus, in 2006, with a new member of the editorial team (Dina Tirosh, University of Tel Aviv, Israel), *JMTE* moved to six issues per year. In 2008, Peter Sullivan took over as editor in chief.
- 4 See <http://www.springer.com/education+%26+language/mathematics+education/journal/10857>.
- 5 We might point particularly to the project in Cognitively Guided Instruction (e.g., Fennema et al., 1996) and the classroom experiments of Cobb, Wood, and Yackel (1990).
- 6 A special edition of *For the Learning of Mathematics* (in production) will focus on this question.
- 7 See Brophy (1986) and Confrey (1986) for a debate on the “teacher effectiveness” research.
- 8 I am thinking of some of the big debates over the years, such as Brophy-Confrey or Lerman-Steffe (Brophy, 1986; Confrey, 1986; Lerman, 1996; Steffe & Thompson, 2000).
- 9 The mathematics education department is called, in Norwegian, Matematikk Didaktikk.

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