Forcing awareness

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Dave Hewitt

The phrase ‘forcing awareness’ can create reactions in different ways. Sometimes there is a strong negative reaction due to a sense of what ‘forcing’ might entail. There can be a very understandable reaction of feeling that “forcing is not part of what I do as a teacher”, as if this is a form of abuse. Also a strong sense that a teacher can never force a learner to have an awareness or think anything in particular – a teacher cannot open up a learner’s head and put inside anything. That is not how education works. So what does this term mean? Well, the meaning I have is one in which it is recognised that learners create their own awarenesses and that a teacher cannot impose thoughts on learners; also learners are respected and treated as people who have impressive powers of the mind (Gattegno, 1971; Hewitt, 2015) which can be utilised within the classroom. These powers include: the ability extract relevant information from what might appear at first as a relatively complex scenario; abstract rules from examples (noticing what is the same and what is different); test out those rules (conjectures at this stage) in new situations; create associations between one thing and another; use imagery and, of course use memory. The powers of the mind concern ways in which we all go about our learning and have done so since before we ever entered a school. Careful observation of very young children can see these powers at work.

I consider forcing awareness to involve a carefully constructed activity, usually with a structured line of questioning, where a teacher works with learners so that it is highly probable that the learners will gain a specific awareness, in this case related to mathematics. I am talking about probabilities here and acknowledge that I can never say what all learners will gain from an activity, nor could I even say what any particular learner will gain.

A key aspect of forcing awareness is that there might be something specific that I wish learners to come to know. So I am thinking about ways in which a specific awareness might be gained and offer below examples of activities and ways of working with learners which I feel can result in a high probability that many (I can never say ‘all’) learners with gain that awareness. A principle behind this way of working is that, as a teacher, I try to get learners to use awarenesses they already possess in order to gain this new awareness. Thus, I am working with awarenesses learners reveal. This is one way in which awareness is educated. Rather than thinking in terms of learners gaining something separate from the awarenesses they already possess, education is about educating their existing awarenesses so that they become aware of something new. With this perspective, teaching is seen as working with a learner’s existing awarenesses rather than attempting to inform learners of the teacher’s awarenesses.

The process of forcing awareness is likely to include techniques commonly used by a teacher, such as stressing some things and ignoring others, directing learners' attention to certain aspects, directed questioning. The crucial aspect is being sensitive to the awarenesses shown by learners to judge when and how to act in the moment, so that existing awarenesses can be utilised, shaped, directed, challenged or just appreciated. These decisions are crucial and also lie at the heart of activities being successful or not. The one thing which is not present is explaining, since accounting for what is noticed is part of the process of a learner educating their awareness. Forcing awareness is about the whole package: the sensitive and appropriate use of the teaching skills just mentioned along with the carefully designed activity, and clear direction of travel as the activity unfolds to bring a new awareness to the learners. A key element is establishing a culture where learners will use the powers of their mind and not sit and just expect to be told.

To develop the notion of forcing awareness further, I offer here two contrasting activities. The order of these is chosen so as to offer variation in the extent of teacher involvement within the activity itself. The first example requires the teacher to set up an activity for learners to work in pairs. The second activity is a whole class activity which is far more teacher controlled for some period of time.

Example 1

The key awareness I am thinking of with this activity is that it is possible to make a number smaller by multiplying. Many learners have a sense of a number always getting bigger as a result of multiplication. So a purpose of this activity (amongst others) is for learners to become aware that it is possible to make a number smaller by multiplication and to come to know what sort of multiplier will make another number smaller and by how much. Apart from setting up this activity, it could be seen that a teacher needs to do relatively little. However, it should not be underestimated the importance of the nature of responses, in the moment, a teacher makes to comments from learners as they engage with the activity. These interactions with learners are often crucial to the learners' development of awarenesses yet, due to the unpredictability of the classroom, cannot be determined beforehand. Following the activity a whole class reflection on what individuals noticed when doing that activity can help transfer implicit awarenesses gained during the activity into something more explicit.

> **Activity**
>
> Work in pairs
>
> One calculator for each pair.
>
> One person puts a number between 11 and 19 (inclusive) into the calculator.
>
> They pass the calculator to their partner.

The partner inherits this number and presses the ‘x’ button followed by any decimal number and then presses ‘=’.

Loop back to “They pass the calculator…”

The winner is the person who gets 100 after pressing the ‘=’ sign.

End of activity

This activity also offers many opportunities, especially through seeing the consequences of their actions on the calculator. Future entries into the calculator are often informed through noticing the consequences of previous entries and learning from them. In this way, the factual feedback a calculator provides is sufficient to assist learners in educating their awareness about the effect of multiplicative factors on a number. During this process, there is likely to be a crucial time for many learners where they receive the calculator with a number greater than 100 and feel stuck unless they are allowed to use division or subtraction. At such times a teacher plays a crucial role. This can take different forms. One is to ensure the learner stays with the discipline to keep with multiplication and not cheat. The second is to offer emotional support and encouragement to someone who is feeling frustrated. The third is to ask questions which can assist someone who is feeling stuck. For example, if a learner receives the calculator with a number above 100 then they might feel that it is impossible for them to multiply and make it get lower. I, as the teacher, might ask them what numbers they have tried to multiply by so far. That may be sufficient for them to consider numbers less than one. Or I might ask what they know about multiplying by numbers such as 1.5, 2, 3, 3.2, etc., and when they say it will only make the number bigger then reply that they should try different numbers then; what other sorts of numbers do they know?

Example 2

This example concerns the topic of ratio and the focus is on learners becoming aware of the power of going through the unit in order to address ratio problems, such as dividing up an amount into a given ratio. This activity starts off being highly controlled by the teacher. The activity is such that the desired mathematics is contained within the very structure of the activity itself. So, a learner getting to know the activity means getting to know the underlying mathematics as a consequence. Here, learners are expected to use their powers of the mind, and the awarenesses they already have, to try to make sense of what is happening within the activity. This activity is one in which the teacher takes a prominent role in organising and managing what is done. Having said that, the teacher states almost nothing but uses a sequence of questioning and employs particular visual imagery in terms of hand movements. So, although the teacher is taking on a far greater controlling role, it is still the learners who have to work out the mathematics behind what is going on. I will use “I” when talking about the teacher.

Activity

I choose a learner (call them L1) and ask them to say a times table that they feel they are quite good at. Suppose they say the two times table (and I will label them L2 for the sake of this writing – L for learner and 2 for the two times table).

I say “OK let’s practise that then” and say “three” making a hand movement going from myself to L2 (I will use the notation T→L2).

It is likely that the learner will not really know what is expected. Personally I am content with this as I want there to be a sense of them trying to work out what is going on. That is the very task they have to engage with and so this sets up a sense of curiosity about what is going on and them beginning to work on this. I find it does not take too long before they say “six”. As soon as they do I indicate acceptance of this by moving on and asking my next question (which is stating another number, such as 5, and making the same hand movement). If they happen to say another number or are unable to offer anything, then I usually find that saying “I thought you said you were good at the two times table” usually helps them work out what is expected. A principle I have is that I want to offer the least in order for them to successfully engage in the activity. This is not because I am lazy or that I am trying to be awkward, but that I am expecting learners to be the ones who need to work and the more I do then the less they do. So it is about setting up an expectation that they come into mathematics lessons in order to have to work and think about what is going on and find mathematical connections. That is what part of what being a mathematician is all about.

This stage of the activity continues with several T→L2 ‘movements’ where the teacher says a number and L2 says whatever that number times two is. Each time a hand movement accompanies this. Once this has become fluent, I ask the rest of the class “what is happening here?” whilst making the hand movement T→L2. The expectation is that they say “times two”.

<u>Stage two</u>

I now say “OK now you start”, talking to L2. Again it is likely that L2 might not know what is expected. I find that if I wait long enough then they do say a number. Of course, there are different ways in which the request can be taken and I find that many times L2 thinks that they are going to take my role and that I am going to double whatever number they say. This will not, in fact, be the case. Again, I do not choose to explain this as I want the whole class to stay in the mode of having to work out what is going on. So, supposing L2 says “three”. Then I might reply “Oh, you are making it hard for me, give me an easier number”. They may still be confused but once they do say a number which is in the two times table then I reply by saying half that number, making a hand movement from L2 to myself (L2→T). This is usually accompanied by something like "Oh, I see" from L2. We now practise L2 saying numbers in the two times table and me saying half that number, always accompanying this with the hand movement L2→T.

Once this is fluent, then I ask the class what is happening here, making the hand movement L2→T. Here I look for the language of "dividing by 2" rather than "halving".

Stage 3

I ask for someone else to volunteer and tell me a different times table which they feel confident with. Let me assume that another learner chooses the five times table.

I go through the same process above with this learner (L5), establishing what T →L5 and L5→T hand movements indicate (times by five and divide by five).

Stage 4

The next stage starts off where the last stage finished. This is with L5 starting by saying a number in the five times table. If they say 30, then I make a L5→T hand movement but as I say "six" I suddenly turn to L2 and make a T→L2 hand movement, with the expectation that L2 will now respond. This can be a bit of a shock to them as they were not expecting to be involved. After a while I find they soon realise what is going on and say "twelve". It is possible that they think we are still going to continue with the five times table and end up saying "thirty". If so, I tend to say "but I thought you were good at the *two* times table"; in which case they soon click into saying "twelve". After a few of these the whole class are asked what is happening with the movement from L5→T (making the appropriate hand movement), followed by what is happening from T→L2. This is so that learners articulate the mathematical structure which underpins the movement between the people involved. The 'chant' changes a little over time with the number associated with T being thought rather than said. So, for example, the start number from L5 of 45 would be said by all, then I would make the hand movement L5→T but say "Do not say the number but think it" and then make the hand movement T→L2 with them all saying 18. This is to gain a sense of the T number having the appropriate reduced emphasis as it is just a middle step in the process.

Several passes are made starting with L5, coming back to T and then bouncing that number off to L2. So the movement is L5→T→L2. This is then mixed with movements from L2 to L5 always going through T, so L2→T→L5. Gradually the whole class are involved with asking them to chant what the numbers will be.

These are recorded on the board:

2	**5**
12	30
4	10
18	45
200	500
14	35
22	55
40	100

At some point the ‘:’ sign is introduced between the original 2 and 5 on the board so that is reads ‘2:3’ and the world ratio can be introduced at an appropriate time.

<u>Stage 5</u>

I then choose to ask whether someone will be the seven times table (L7). I choose seven as it is the sum of the two tables already chosen. I am happy to hand out a calculator to be used if it is a times table that may be relatively difficult for some of the learners. I say that I am choosing seven because it is the sum of the two tables so far.

After practising as in stages 1 and 2 with L7, we make movements from L7 to each of the other two, L2 and L5, always going through T. Thus the movements are L7→T→L2 →T→L5 with the numbers at L7, L2 and L5 being noted on the board. Thus a new table of values is formed, something like:

2 : 5	**7**
6 15	21
14 35	49
4 10	14
18 45	63
12 30	42
20 50	70
24 60	84

Movements continue to be indicated by hand movements and are accompanied with the class chanting what numbers appear at L2 and L5. Gradually, over time, the hand movements are gradually withdrawn so that the movements shift from physical to mental imagery.

<u>Stage 6</u>

So far, the activity has been highly structured and controlled by the teacher. In a sense the learners have been taken on a journey even though the teacher has not actually ‘told’ them anything about the mathematics of the situation. This sixth stage is a ‘taking breath’ and reflection period, where learners have an opportunity to ask questions, clarify things about what has just been happening and notice connections between the numbers which appear within the table. There are significant things to notice, such as for every row the numbers under the 2 and 5 add up to the number under the 7 column. Language can be introduced such as ‘ratio’ and the notion of an amount being split up in the ratio of 2:3. Explicit note can be made of the colon notation between the 2 and the 3. There could also be examples placed in contextual situations where the task is to split up a certain number of apples or money in that ratio.

Stage 7

This stage is to help establish generality about the mathematical processes involved rather than be stuck with the particular tables of 2 and 5. So, for example, two, or even three (or four…) other learners can be different times tables (calculators can be provided as appropriate, as the focus is not on the ability to carry out mental arithmetic as this might be a distraction to the fluency of gaining the awarenesses of what is required to work out ratio problems). In fact, after two people have suggest new times tables, such as 3 and 8, when I ask a third person what table they will be – suppose they say 6 – I reply with “six point what?” to create a decimal number times table. This, I feel, can add to a greater sense of generality; that we can do this with any sort of number. So, having got L3, L8 and L6.2, say, the class are asked what other table would we need if we want to split something in these ratios (just as 7 was chosen in the original case). Another learner than becomes L17.2.

Movements are made from L17.2 to L3, L8 and L6.2, always through T. There are other awarenesses which can be obtained, such as realising that once we have a number for L17.2 and, say, L8, we can go L8→T→L3 or L17.2→T→L3 and get the same number for L3. Also, I might suggest at one point that the start number for L17.2 is either a very big number and/or a decimal number. I find that learners are often energised by being faced with large numbers or seemingly ‘difficult’ numbers. In this way, they also come to learn that the mathematics does not actually become any different (provided a calculator is around), no matter what number is chosen. This helps establish the generality and can empower learners into feeling as if they can deal with any situation of this kind. This is an example of gaining ‘mastery’ over this topic, having strong personal sense of being able to deal with any situation of this kind.

Learners could then be asked to create their own situations, choosing whatever times tables they want and create a table of values such as the one above. I tend to encourage many learners to come up with more outrageous situations then we have used so far.

Stage 8

Learners might be presented with more traditional questions relating to ratio. After the sort of situations they have encountered above, these are often seen as being relatively simple and I often frequently hear reactions such as “is that all they want us to do?” A general principle I have is to get learners working on more complex mathematics than they are likely to meet within an examination situation.

Educating awareness

These two examples differ in many ways in terms of the role a teacher might take, but what is common in all of them is that learners are using their existing awarenesses and their powers of the mind to engage in these activities. Each activity, along with the particular line of questioning employed by the teacher, is such that the probability of learners gaining a particular awareness is relatively high. Forcing awareness can only be about percentage success but the design of such activities can help more learners to gain key awarenesses which can give them a sense of empowerment and ‘mastery’ over a particular area of mathematics.

What might be the benefit of forcing awareness be compared with just telling learners? If I take the example of finding percentages, I could just tell learners how to do this by going through the unit. That would take little time compared with the second example above. There are several reasons why such an attractive and seemingly economic alternative is not such a good idea. The first is that just because you tell someone, does not mean they have:

a) listened;
b) managed to transfer the words they heard into meaningful mathematical ideas;
c) educated their own existing awarenesses to become truly aware of the mathematical significance of the statement; or
d) been able to apply this new knowledge in new situations.

There is a real danger that (b) and sometimes (a) does not happen. If (a) does not happen then no-one has got anywhere. If (b) does not happen then also no-one has got anywhere. If (b) does happen, it can still take a lot of work for an individual on their own to achieve (c), educating their own existing awarenesses to become truly aware of the mathematical significance. It is too often the case that, instead of this, a learner tries to memorise this as ‘received wisdom’ from their teacher; trying to memorise someone else’s awareness rather than having educated their own. The problem about trying to memorise something is that this is often accompanied, at a later time, with forgetting. Memory is different to awareness. Memory requires particular effort for that information to be held and maintained over periods of time. This is because it can be unattached to the awarenesses a learner already possesses; it is just a separate piece of information floating in someone’s mind.

Educating awareness skips the first two stages as learners use their own existing awarenesses. What they become aware of, *they* have become aware of and do not have to try to make sense of someone else's awareness.

The process of engaging in the activity involves them in the process of thinking, noticing, conjecturing, testing and this means that what is new is connected with what they knew because it grew out of what was there before. This means the new awareness is not a separate piece of information which has to be remembered but is connected to what was already known.

References

Gattegno, C. (1971). *What we owe children. The subordination of teaching to learning.* London: Routledge and Kegan Paul Ltd.

Hewitt, D. (2015). The economic use of time and effort in the teaching and learning of mathematics. In S. Oesterle and D. Allan (Eds), *Proceedings of the 2014 Annual Meeting of the Canadian Mathematics Education Study Group*, (pp. 3-23). Edmonton, Canada: CMESG/GCEDM.