Students as Partners in Mathematics Course Design

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In the 2010-2011 academic year, the Department of Mathematical Sciences at Loughborough University secured HE-STEM funding to redesign two of its historically problematic modules, *Vector Spaces and Complex Variables*. The aim of the project was to enhance the second year undergraduate mathematics experience, and increase student engagement and satisfaction with mathematics so that students leave the second year reporting increased satisfaction with their learning experience. The project was a collaboration between staff and second-year students and a unique feature was the active participation of four second-year undergraduate mathematicians in the course design process. The four students were recruited as interns to produce teaching and learning resources that could potentially engage future cohorts of second year students. An ethnographic study was designed to understand the students' role, experiences, working relationship with staff, and the resources that they were able to produce. Data on the students' experiences were collected via diaries kept by the students, self-reflection and evaluation reports produced by the students, participant observation, and fieldnotes. Staff were also interviewed individually in order to collect data to triangulate the students' accounts so as to increase the validity and reliability of the findings. In this paper we report on the data collected from the students. Findings from the study showed that the four students were socialised and drawn from the margins of 'legitimate peripheral participation' in academic practice into full participation of a community of practising mathematicians. The four student interns were able to play an important role as mathematics course designers, and gained a deeper understanding of the mathematics they worked on.

1. Introduction

1.1. Study Background

The Student Experiences of Undergraduate Mathematics [1] study conducted in the UK has shown that beyond the school-university transition year, some students become disengaged and disillusioned with their studies because of poor performance. For some of these students, the difficulties they experience with their studies may be attributed to the on-or-off campus activities that they engage in such as employment, which may leave them with little time to devote to their studies. For others, their lack of success and progression may be attributed to the very nature of undergraduate mathematics which is different from school mathematics, where solutions to mathematics problems are often routine and predictable. Aspects of the design and the delivery of an undergraduate mathematics course could also impact on student engagement with their study of the course and hence on performance. In this paper, the term 'course' is used to describe a course unit that lasts a whole academic year or a module that lasts for one semester. Whatever the attribution of students' underperformance, students will become dissatisfied with their study of mathematics if they persistently underachieve and consequently this may lead to student attrition. Enhancing the student learning experience and increasing student engagement are now hot topics in higher education discourse, as they are believed to improve performance.

In recent years, there have been calls to the higher education community to involve students in the planning and design of courses [2-3]. For example, the 1994 Group of Universities in the UK noted in its policy report entitled

'Enhancing the Student Experience' that member institutions should involve students in the planning and design of courses because students "know how they want to be taught and have ideas about how teaching techniques could be improved."[3 p.12].

1.2. Research Questions.

In the 2010-2011 academic year, in an effort to enhance the second year mathematics experience so that student engagement and achievement can be increased in two historically problematic second year mathematics courses, the Department of Mathematical Sciences, Loughborough University, UK, embarked on a curriculum development project funded by the Higher Education Science, Technology, Engineering and Mathematics (HE-STEM) Programme. The two courses were *Vector Spaces and Complex Variables*. A unique feature of the curriculum development project now called SYMBoL (http:/sym.lboro.ac.uk), is the recruitment of four undergraduate mathematicians as paid summer interns to collaborate with staff to redesign the two courses by producing engaging teaching and learning resources for students. We designed an ethnographic study to understand the experiences of staff and in particular the four undergraduate mathematicians. Among a number of research questions the study aimed to answer were:

1. What role are the four student interns able to play?
2. What resources are the four student interns able to produce?
3. What are the outcomes for the four student interns?

2. Related Research and Theoretical Framework

Between March and August 2011, the first author conducted literature searches on direct student involvement in course design and found some examples where students have been involved in the design of non-mathematical sciences courses. However, to the best of our knowledge, there is a dearth of examples where undergraduate mathematicians have been involved in the design of mathematics courses. In a literature review on student involvement in course design, Bovill, Bulley and Morss [4] found limited examples of direct student participation in the design of Geography, Education and Environmental Justice courses. In further work on evaluating these examples, Bovill, Cook-Sather, and Felten [5] note that staff and students stand to benefit from a collaborative approach to course design. Similarly, Hess [6] provides an account of his own approach to collaborative course design in a graduate law course but his account may be viewed as anecdotal. For the mathematical sciences community, empirical evidence that supports the potential benefits for staff and students in collaborative course design would be informative and increase our knowledge base on tertiary mathematics course design and delivery.

Community of Practice [7] was used as an analytical lens to explore the relationship and interactions between staff and the four student interns. *Community of Practice* is defined as a group of "people who share a concern, a set of problems, or a passion about a topic, and who deepen their knowledge and expertise in this area by interacting on an on-going basis" [7 p. 4]. For a group of people to constitute a *Community of Practice*, the group must have a joint enterprise or domain, mutual engagement or community, and shared repertoire of resources or practice. These defining characteristics, according to Wenger, McDermott and Snyder [7], foster learning and knowledge creation. We also drew on themes from the literature on student participation in course design in non-mathematical sciences disciplines [4, 5] to explore the benefits that accrued to the student interns and their relationship with staff.

3. Methodology

In March 2001, all second year undergraduate mathematicians who had enrolled on and studied *Vector Spaces and Complex Variables* were invited to apply for four positions as student interns. Eight students out of a cohort of about 100 applied for the positions. All eight students were interviewed by the staff who teach *Vector Spaces and Complex Variables* and an additional member of staff. Four students were successful and commenced their internship in March 2011. They worked part-time for two hours per week between March and June 2011, conducting focus groups to collect the views of their peers about the teaching and learning of *Vector Spaces and Complex Variables* in order to inform the course design process. During July and August 2011, the students worked fulltime as student interns for six weeks. They worked closely with staff but with considerable autonomy to design teaching and learning resources. During that period, the first author immersed himself amongst the student interns; sharing an open plan office with them, interacting with them and responding to questions they may have about the use of technology in producing resources. While the student interns worked, Monday to Friday, the first author observed their activities and their interactions with staff and each other and took fieldnotes. The student interns also kept diaries, which they wrote up daily and sent to the first author at the end of each week. At the end of their internship, the student interns also wrote a self-reflection and evaluation report on their six weeks' experience. The qualitative data collected were subjected to thematic analysis [8] using NVivo 8 to generate codes which were later categorised into themes, three of which we describe and discuss in the next section.

4. Findings and Discussion

4.1 Role Played by the Student Interns

In this section we use extracts from transcripts, diaries, self-reflection and evaluation reports, and fieldnotes to provide evidence in support of answers to the research questions. Extracts attributable to the four student interns are identified as P1, P2, P3, and P4. Each of these identifiers is shown on the right of the related extract.

Staff and students formed a community of practising mathematicians. They formed a community of practice because they had a *joint enterprise*; that is, to produce engaging teaching and learning resources to enhance the student learning experience. Throughout their internship, the student interns interacted with staff, discussed the mathematical content of the resources they produced, and built equal but professional relationships with staff. Thus, there was a *mutual engagement* amongst staff and the student interns. During their internship, the four student interns had a one-hour tea break each working day when they met in the office of a member of staff who provided refreshments. Through their *mutual engagement*, staff and students engaged in mathematical discourse in ways that lectures and tutorials do not make possible and developed a *shared repertoire* of resources as the following quotations from the diaries of three of the student interns show:

> *"Meeting up with some of the staff for tea and biscuits was a good opportunity to get to know people a bit more, and made me feel much more involved and valued as a member of the project."* (P2)
>
> *"Its good to be able to comfortably talk to lecturers about interesting points in mathematics, it's also interesting to hear what they do as mathematicians and how they work together or alone."* (P3)
>
> *"I feel Lecturer 1 is more approachable now."* (P4)

During the one-hour afternoon tea break, not only were the students acculturated to academic practice as the above quotations indicate, but they also received feedback on the content of the resources they produced. The mathematical accuracy of the resources the four students produced was of paramount importance since one of the aims of the course redesign process was to make the resources available for use by other institutions. Hence, notwithstanding the autonomy the students had in their role as interns, they felt it was essential that the content of the resources they produced was reviewed by members of staff. Where such feedback was constructive it was often well received and led to revision of the resources as indicated by the following extracts from the diaries of two participants:

> *"Lecturer 1 has reviewed all of the materials that I have produced and provided feedback for each of them, so I now have to amend these."* (P1)

> *"Got feedback which I found helpful and constructive."* (P4)

From the observations and fieldnotes data, we found that the students played two essential roles during their internship; *intermediaries and competent academic apprentices*. These new terms will be discussed in a future publication in *MSOR Connections* and the full research report. However, in this paper, we suggest that the student interns played the role of *intermediaries* between staff and the second year students by soliciting the 'student voice' through focus groups and other informal communication channels. The student voice sought for was more valuable than could be provided by the traditional feedback mechanism, which is perceived to have a different purpose; quality assessment rather than quality enhancement. The richness and depth of the students' views about the teaching and learning of *Vector Spaces and Complex Variables* would not have been obtained with the traditional quantitative survey on course evaluations.

The internship provided the student interns with opportunities to work with the content of *Vector Spaces* and *Complex Variables* as *competent academic apprentices*. Again, although we have not discussed and defined this terminology in this paper, we suggest that the student interns were competent in the content of the mathematics they worked on by the virtue of having taken and passed the examinations. At the start of their internship, three of the student interns, while being competent, showed lack of understanding in some aspects of the content of the courses they were working on. Through the process of resource production and feedback, we observed the students receive informal training and advice akin to the '*apprenticeship model*' in a work place. Hence our introduction and use of the term *competent academic apprentices* to describe the role played by the student interns.

Although the student interns were enthusiastic about their role and sought and received constructive feedback regularly, our observations and fieldnotes data indicated that when feedback was perceived to be overtly critical or unrelated to mathematics, such feedback had an unexpected impact on the way the students sought feedback thereafter. For example, one intern hesitated seeking feedback on a very well produced document with a novel approach to solving a problem on Orthogonal Projections because he did not want to receive what he perceived to be critical feedback. Another participant receiving feedback on the use of good grammatical structures of the English language was not amused. For these students, it was the enjoyment of the mathematics that sustained their interest in their role and anything else seen as not mathematically related was not welcomed. This was particularly evident in week 1 when two student interns, identified as P1 and P3, felt that much of what they were doing was administrative duty and not challenging as can be seen from the following two statements made by the two student interns and recorded in the fieldnotes:

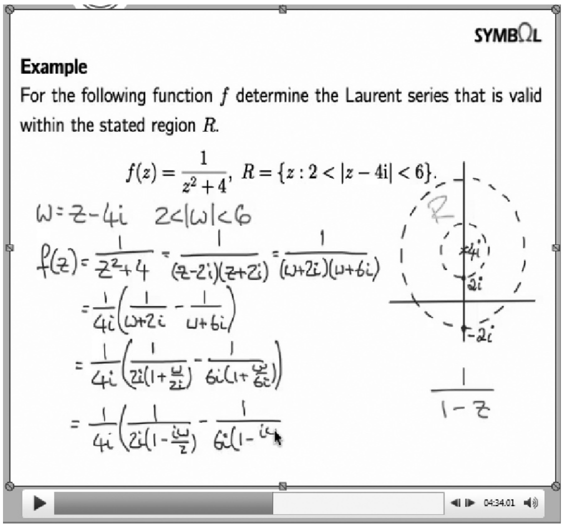
> *"I am getting bored with this [creating LaTex files]"* (P1)

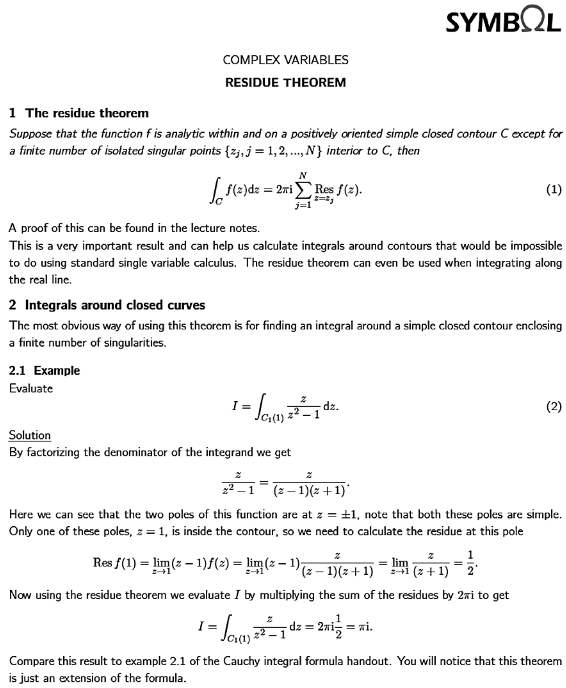
> *"I created LaTex files [all day] which I found boring"* (P3)

4.2 Resources Produced by the Student Interns

While working during the six week internship, the student interns had to liaise with the course leaders, produce teaching and learning resources, and seek feedback on the quality and mathematical accuracy of the content of the resources they produced. Samples of resources that one pair of students produced for the *Complex Variables* course are shown in Figures 1 and 2 below. The screencast videos were produced for use by second-year students independently and out-of-lecture. The videos were produced to supplement lectures but not to replace them.

Also supplementary help sheets in the form of notes or problems and solutions were also produced for either independent use or for use in Peer Assisted Learning (PAL) sessions. These sessions were scheduled for second-year undergraduate mathematicians for the first time at the Department of Mathematical Sciences at Loughborough University in order to enhance the students' learning experiences during the 2011-2012 academic year.





COMPLEX VARIABLES

RESIDUE THEOREM

1 The residue theorem

Suppose that the function f is analytic within and on a positively oriented simple closed contour C except for a finite number of isolated singular points $\{z_j, j = 1, 2, ..., N\}$ *interior to C, then*

$$\int_C f(z)\mathrm{d}z = 2\pi\mathrm{i}\sum_{j=1}^{N} \operatorname*{Res}_{z=z_j} f(z). \qquad (1)$$

A proof of this can be found in the lecture notes.
This is a very important result and can help us calculate integrals around contours that would be impossible to do using standard single variable calculus. The residue theorem can even be used when integrating along the real line.

2 Integrals around closed curves

The most obvious way of using this theorem is for finding an integral around a simple closed contour enclosing a finite number of singularities.

2.1 Example

Evaluate

$$I = \int_{C_1(1)} \frac{z}{z^2-1}\mathrm{d}z. \qquad (2)$$

Solution

By factorizing the denominator of the integrand we get

$$\frac{z}{z^2-1} = \frac{z}{(z-1)(z+1)}.$$

Here we can see that the two poles of this function are at $z = \pm 1$, note that both these poles are simple. Only one of these poles, $z = 1$, is inside the contour, so we need to calculate the residue at this pole

$$\operatorname{Res} f(1) = \lim_{z\to 1}(z-1)f(z) = \lim_{z\to 1}(z-1)\frac{z}{(z-1)(z+1)} = \lim_{z\to 1}\frac{z}{(z+1)} = \frac{1}{2}.$$

Now using the residue theorem we evaluate I by multiplying the sum of the residues by $2\pi\mathrm{i}$ to get

$$I = \int_{C_1(1)} \frac{z}{z^2-1}\mathrm{d}z = 2\pi\mathrm{i}\frac{1}{2} = \pi\mathrm{i}.$$

Compare this result to example 2.1 of the Cauchy integral formula handout. You will notice that this theorem is just an extension of the formula.

Figure 1: Screenshot of screencast video on Laurent series.

Figure 2: Screenshot of a help sheet on the residue theorem.

4.3. Deepening Mathematical Understanding

The internship and the course redesign process provided opportunities for the student interns to gain a much deeper understanding of the course they helped to redesign. Consequently, they gained increased confidence in their abilities as demonstrated through the following quotations:

> *"My knowledge of Vector Spaces is also improving, as I discovered an application for a Theorem that I had not previously realized was possible."* (P1)

> *"I found that as I was creating videos my understanding of the topics is becoming much deeper and I hope these skills will be transferable to other modules I take in the future."* (P2)

> *"I feel [that] my knowledge of the eigenvalue equation has improved a lot. My approach to learning will be very different after this internship."* (P4)

Amongst the four student interns, the student identified as P1 was often positioned by the other three as the most able student. He is believed to be on track for a first class degree in Mathematics. However from the fieldnotes, we note that until the end of the six weeks internship, he did not have secured understanding in all areas of *Vector Spaces*, the module he worked on. He was observed on three occasions using a board and a chalk to devise a solution to a problem on Orthogonal Projection using a geometric approach and then used his solution to produce a supplementary help sheet for student use. He notes in his diary that his solution to the problem on Orthogonal Projection is different from the way the lecturer had previously explained it in lectures and tutorials. The following extract from the diary of P1 is typical of how the student interns believe that the internship experience has impacted on their mathematical understanding:

"I have had to use the blackboard several times to work through a problem, so that I understand it completely and can convey my understanding through the solutions. This has helped me understand the topics within the module better though, which I believe is very helpful." (P1)

4.4. Staff Approach to the Course Design Process

While our focus in this paper is on how the student interns benefited from their internship experience, we mention briefly the impact of the staff-student partnership on the two lecturers who normally teach *Vector Spaces* and *Complex Variables*.

First, we note that the changing relationship between the lecturers and the student interns provided impetus for the two lecturers to become more receptive to students' suggestions for changes to how the two courses had previously been designed. Suggestions for changes to the structure of lecture notes and the provision of additional resources such as mathematics screencasts were acted upon for both *Vector Spaces* and *Complex Variables*. While the lecturer responsible for *Complex Variables* responded to students' suggestions for lecture notes to have gaps ('gappy notes') for additional writing by students during lectures, the lecturer responsible for *Vector Spaces* did not do likewise. Furthermore, the staff member responsible for *Complex Variables* slightly changed the assessment policy in response to students' feedback from focus group discussions. Previously, there was neither a coursework nor a class test component as part of the *Complex Variables* course, but a class test has been introduced for 2011/2012 academic year.

Second, as a consequence of the successful working relationship between the staff and the student interns, the two course leaders provided support to the student interns to produce learning resources for mathematics peer support sessions which have been planned for the 2011/2012 academic year. Also, in addition to their normal office hours, the two lecturers offered to provide additional office hours so that they could provide assistance to peer support facilitators in relation to the mathematical content they will work through during peer support sessions. We intend to report fully on the staff experiences of the summer student internship later, as we collect and analyse more data from staff through interviews.

5. Conclusion

This study showed that students can make a contribution as partners in mathematics course design and that they benefit from the experience in several ways including a deeper understanding of the mathematics on which they work. The limitation of the current study, however, is that the four student interns constituted a convenience sample and hence we do not make generalization from the experiences of these four students. Nonetheless, this study appears to support the call for higher education institutions to involve students in shaping their own learning. The full findings of our study including the discussion of students as *intermediaries* and *competent academic apprentices* will be published in due course.

Acknowledgements

The SYMBOL Project was funded with a grant from the HE STEM Programme and the research element of the project was funded with a PhD studentship from the Mathematics Education Centre, Loughborough University.

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