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Numerical Quadrature

and its Applications

by

G. A. Evans

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In a period of eighteen years many people influence one's academic development, especially remembering that in the early years one is setting off on a career as a junior lecturer and at the end one as enjoyed senior status for almost a decade.

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It is the personal contacts which keep work progressing on a day to day basis, and result in considerable influence. One man stands out in this respect. Dr. John Hyslop became a friend and colleague and a provider of suberb problems in his area of quantum mechanics in the early stages of this work. He died tragically in February, 1985.

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