# Preface: Nonlinear waves in fluids in honour of Roger Grimshaw on the occasion of his 80th birthday: Part II

# PREFACE: NONLINEAR WAVES IN FLUIDS IN HONOUR OF ROGER GRIMSHAW ON THE OCCASION OF HIS 80TH BIRTHDAY: PART II

KARIMA R. KHUSNUTDINOVA AND DMITRY E. PELINOVSKY

This is the second issue of *Studies in Applied Mathematics* dedicated to Prof. Roger H. J. Grimshaw on the occasion of his eightieth birthday. The first issue focused on modulation theory and other methods of analysis of classical nonlinear wave equations - the area where Roger contributed with many seminal works. The second issue focuses on the theory of nonlinear gravity-capillary and internal waves, effects of variable bottom topography and rotation, and related asymptotic and numerical methods. These topics reflect Roger's life-long interest to the studies of oceanic and atmospheric nonlinear waves.

The first paper of the issue [1] is devoted to the mean flow induced by the reflection of a weakly-nonlinear internal gravity wave beam at a uniform rigid slope. It is shown that the horizontal Eulerian mean flow is controlled by the mean potential vorticity, whereas the corresponding Lagrangian mean flow is generally nonzero, in contrast to two-dimensional flow where the potential vorticity identically vanishes.

The next two papers address the effects of rotation on the propagation of solitary waves generated by the variable topography. The authors of [2] study nonlinear free-surface rotational waves generated through the interaction of a vertically sheared current with topography. Solutions of the reduced model equation, the forced Korteweg–de Vries equation, are compared to solutions of the free surface Euler equations. The author of [3] studies solitary wave dynamics in the Ostrovsky equation with variable coefficients, which models waves in a rotating ocean with a variable bottom topography. It is shown that for solitary waves moving towards the beach, terminal decay caused by the rotation effect can be suppressed by the shoaling effect.

The next two papers focus on the multi-layer interfacial models for internal waves. The authors of [4] obtain a system of four equations in the shallow water limit for three layers of immiscible fluids of different densities bounded above and below by horizontal rigid lids. By using the Boussinesq approximation, they simplify the system of equations to show that the solutions remain on an invariant surface in a symmetric case and leave the invariant surface in non-symmetric cases. The authors of [5] consider a two-layer fluid with variable depth in the special configurations of the bottom topography which allow waves to propagate over large distances. These special configurations represent smooth bottom profiles between two singular points. The two-layer flow at the first singular point transforms into a uniform one, whereas the water depth is unbounded at the second singular point.

The next paper [6] is devoted to the study of resonant triad interactions of gravity-capillary waves. The authors find sufficient conditions for the resonant interactions to not be accompanied by the exchange of energy so that the wave amplitudes remain constant in time. It is shown that the symmetric resonant triad exchanging no energy forms a transversely-modulated traveling wave field, which can be considered to be a two-dimensional generalization of Wilton ripples.

The last two papers of the issue are devoted to initial-boundary-value problems for nonlinear wave equations. The authors of [7] construct a weakly-nonlinear periodic solution of the Cauchy problem for the Boussinesq–Klein–Gordon equation, considering deviation from the oscillating mean value. At leading order, the two counter-propagating waves are described by two uncoupled Ostrovsky equations. By construction, the initial conditions for the Ostrovsky equations have zero mean values, whereas the initial conditions for the Boussinesq–Klein–Gordon equation may have non-zero mean. The authors of [8] study solitary waves and their stability in the focusing thermal optical media, such as lead glasses, in a square cell with mixed boundary conditions. The refractive index of the thermal optical medium depends on temperature so that heating from the optical beam and heat flow across the boundaries can change the refractive index of the medium. It is found that the position of the solitary wave depends on the boundary conditions, with the centre of the beam moving toward the warmer boundaries, as the parameters are varied. The stability of the solitary waves depends on the symmetry of the boundary conditions and the amplitude of the solitary waves.

## References


[1] Kataoka, T.; Akylas, T.R.; "On mean flow generation due to oblique reflection of internal waves at a slope", Studies in Applied Mathematics.

[2] Flamarion, M.V.; Milewski, P.A.; Nachbin, A.; "Rotational waves generated by current-topography interaction", Studies in Applied Mathematics.

[3] Stepanyants, Yu.A.; "The effects of interplay between the rotation and shoaling for a solitary wave on variable topography", Studies in Applied Mathematics.

[4] de Melo Viríssimo, F.; Milewski, P. A.; "Three-layer flows in the shallow water limit", Studies in Applied Mathematics.

[5] Pelinovsky, E.; Talipova, T.; Didenkulova, I.; Shurgalina, E.; "Interfacial long traveling waves in a two-layer fluid with variable depth", Studies in Applied Mathematics.

[6] Chabane, M.; Choi, W.; "On resonant interactions of gravity-capillary waves without energy exchange", Studies in Applied Mathematics.

[7] Khusnutdinova, K.R.; Tranter, M.R.; "D'Alembert-type solution of the Cauchy problem for the Boussinesq-Klein-Gordon equation", Studies in Applied Mathematics.

[8] Louis, S.A.; Marchant, T.R.; Smyth, N.F.; "2-D solitary waves in thermal media with non-symmetric boundary conditions", Studies in Applied Mathematics.



Department of Mathematical Sciences, Loughborough University, Loughborough LE11 3TU, UK

*E-mail address*: K.Khusnutdinova@lboro.ac.uk

Department of Mathematics and Statistics, McMaster University, Hamilton, Ontario, Canada, L8S 4K1

*E-mail address*: dmpeli@math.mcmaster.ca